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NUMERICAL STUDY OF FLUID FLOW AND HEAT TRANSFER OVER A ROTATING DISK AT ARBITARY ANGLE OF INCLANATION IN LAMINAR FLOW

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ABSTRACT

This article presents an experimental and numerical study of heat transfer from a rotating disk in still air for a large range of angle of inclination and laminar boundary layer over the rotating disk. The Nusselt number over the rotating disk was measured and compared with the numerical results data obtained from numerical study. The goal of the present research is to develop a semi empirical correlation in the familiar classical form for a circular rotational disk for any arbitrary angle of inclination, velocity of rotation and various size over a wide range of the Reynolds numbers. Our results show that the local Nusselt number has not a strong relation with angle of inclination.

INTRODUCTION

The flow field and convective heat transfer in rotating systems are both experimentally and theoretically relevant. In a broad spectrum of practical applications we can find an example such as electronic components, flywheels, disks turbine blades are attached to, disk brakes and even modern high speed CD-ROMs.
The first work in this geometry was done by von Karman in 1921. The classical model of von Karman (1921) dealt with an infinite disk, which rotates at a rotation rate, and the fluid far away from the disk is assumed to be at rest. He gave a formulation of the problem and then introduced his famous transformations which reduced the governing partial differential equations to ordinary differential equations. Since the pioneering work [1] of von Karman many authors have studied flow and heat transfer problems due to rotating disks such as Zandbergen and Dijkstra [2], Kreith [3], Owen and Rogers [4] or Dorfmann [5] and so on [6-10].
In the majority of prior studies, the problem of heat transfer from a rotating disk maintained at a constant temperature for a variety of Prandtl numbers in the steady state, Millsaps and Pohlhausen [11], Sparrow and Gregg [12]. Later, many authors have studied the heat transfer near a rotating disk considering different thermal conditions [13-19]. Recently for the case of an isothermal circular disk, several papers have addressed some new experimental data and empirical correlation [20-23]. From the standpoint of practical engineering applications, the problems of constant flux can be found in a variety of industrials example. At this time no empirical data exists in the available literature for natural or forced convection from stationary (rotation) circular disks with an imposed heat flux, which is a more realistic condition in many practical applications. Such is the objective of the current research.
The present paper intends to provide descriptions of flow field and characteristics heat transfer in of a rotating disk with this type of boundary condition.

NOMENCLATURE

C empirically determined coefficient
n empirically determined exponent
c_p isobar specific heat capacity . . . . KJ/KgK
h_m average heat transfer coefficient . . . . KW/m^2K
k_f heat conductivity (fluid) . . . . KW/mK
g acceleration due to gravity. . . . . m/s^2
d diameter of circular disk heat transfer model. . . . . m
Nu Nusselt number, hd/k
Nu_m average Nusselt number, hd/k
p pressure . . . . Pa
$\Pr$ Prandtl number, $\mu c_p/k$
$\dot{Q}$ Heat flux . . . . KW/m^2
r Radial co-ordinate . . . . m

R Radious of circular disk heat transfer model. . . . . m
t thickness of circular disk heat transfer model. . . . . m
Re Reynolds number, $\rho v d / \mu$
T Temperature. . . . . K
v_r, v_θ, v_z velocity component. m/s
V volume. . . . . m^3
X, Y, Z Cartesian co-ordinates . . . . m

Greek symbols
ν kinematic viscosity . . . . m^2/s
ρ density . . . . Kg/m^3
ω angular velocity . . . . rad/s

Experimental Apparatus

The experimental setup is shown in Figure 1. It consist of a thin circular disk with radius equal to R=12.5cm. The rotating speed of the disk was fixed on ω (rad/s).

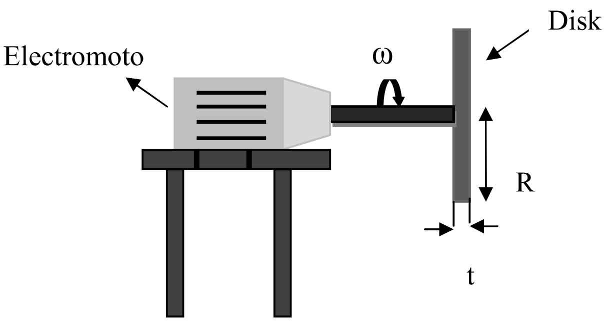

Figure 1. Schematic of experimental apparatus

The Rotating Reynolds number is defined by:

$$\mathrm{Re}_\omega = \frac{VD}{\nu} = \frac{\mathrm{R}\,\omega R}{\nu} = \frac{R^2\omega}{\nu} \tag{1}$$

A difference of the temperature between the rotating disk and Air was varied as a parameter. Five situations were tested for a wide range of rotating Reynolds number for every different angle of inclination.
The circular disks that were used as heat transfer models for the experimental data presented in this paper were commercially available. Disks are made by pressing its material under high pressure in a round die to produce flat coin like pieces. These pieces are then coated with silver on the two flat surfaces. Because of this silver coating, the electrical flux, and thus heat generation by Joule heating, will be uniform in the heat transfer model.
Using an electrical circuit can be provided accurately and simultaneously the convective heat transfer rate. It should be noted that this assumes the temperature to be uniform within and over the surface of the disk. Since the disk is thin, with a uniform voltage difference between its flat surfaces, the current flux will be uniform, resulting in a uniform Joule heating within the disk.
The average convective heat transfer coefficient, h_d can be expressed in terms of the heat transfer rate, $\dot{Q}$ the heat transfer area, A, and the temperature difference, $(T - T_f)$ between the surface and the fluid from its definition; thus:

$$h_m = \frac{\dot{Q}}{(T - T_f)} \tag{2}$$

Where, as mentioned earlier, the convective heat transfer rate, $\dot{Q}$ can be measured by analyzing the electrical circuit represented.
The main objective of the present study is to calculate the mean Nusselt number at steady state defined by means of:

$$Nu_m = \frac{h_m R}{k_f} = \frac{\dot{Q}\,R}{k_f\,(T - T_\infty)} \tag{3}$$

Where the mean heat flux $\dot{Q}$ is the surface-averaged quantity (at steady state):

$$\dot{Q}_m = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \dot{Q}\, r dr d\varphi \tag{4}$$

A dimensional analysis shows that the mean Nusselt number (at steady state) can present as:

$$Nu_m = Nu_m(\mathrm{Pr}, \mathrm{Re}_\omega) \tag{5}$$

Depends on the Prandtl number Pr and the rotational Reynolds number Re_ω given by:

$$\mathrm{Re}_\omega = \frac{R^2\omega}{\nu} \tag{6}$$

In the present study, only the case $Pr = 0.7$ is considered (air at normal conditions). The limit case of a rotary disk in an air $(Pr = 0.7)$ is typically correlated by an expression:

$$Nu_m = c.\mathrm{Re}_\omega^n \tag{7}$$

Problem formula

A thin circular disk with outer radius, R rotates in the X,Y-plane that have an angle of θ with horizontal plane with angular velocity ω create a complex flow; far away from the disk the velocity field is given by the rather simple potential flow. The disk surface is kept at constant flux. Furthermore, the material properties (density and transport coefficients) are assumed to be constant, too. In the flow domain, the governing equations call as:

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0 \tag{8}$$

$$v_r\frac{\partial v_r}{\partial r}-\frac{v_\theta^{\ 2}}{r}+v_z\frac{\partial v_r}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$+\nu\left[\frac{\partial^2 v_r}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)+\frac{\partial^2 v_r}{\partial z^2}\right] \tag{9}$$

$$v_r\frac{\partial v_\theta}{\partial r}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}=\upsilon\left[\frac{\partial^2 v_\theta}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\upsilon\left[\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$\rho C_P\left(v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}\right)=k\frac{\partial^2 T}{\partial z^2}$$

$$v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2} \tag{10}$$

Eq. (8) is the continuity equation for an incompressible fluid, Eq. (9) represents the Navier–Stokes-equation for a Newtonian fluid, and the energy equation (10). The flow over a stationary disk is described efficiently by means of the boundary layer theory [24, 25]. In case of an additional rotational motion of the disk the situation changes dramatically.

Numerical approach

To solve the above system of equations a numerical approach based on the finite-volume-method is chosen. The discretisation of the partial differential equations is performed on a cylindrical mesh with staggered grid arrangement for the velocities. The chosen semi-implicit formulation for the pressure forces results in coupled sets of equations that must be solved by an iterative technique. At the x,y-planes and x,z-planes the slip-condition is prescribed. The disk with radius R and thickness $d/R = 0.1$ is located at the origin in the x,y-plane . With regard to the global co-ordinate system the rotation is with angular velocity ω. At the disk surface the velocity is prescribed by the non-slip condition leading to non-trivial velocity boundary conditions at the obstacle's boundary:

$$z=0: \qquad v_r=0, \quad v_\theta=r\omega, \quad v_z=0 \tag{11}$$

The computational region extends from $x=3.5R$ to $x=-3.5R$, from $y=3.5R$ to $y=-3.5R$, and from $z=0$ to $z=1.5R$, respectively. These dimensions have to be found sufficient after some preliminary studies. The boundary conditions far from the disk were:

$$z=\infty: \qquad v_r=0, \qquad v_\theta=0 \tag{12}$$

Close to the disk, the mesh size has been refined, whereas far away from the disk a coarser mesh size has been chosen. The material and flow domain parameters used for calculation are summarized in Table1.

Table 1
Simulation parameters

Density of air ρ	$1.20Kg/m^3$
Kinematic viscosity of air ν	$15\times10^{-6}\,m^2/s$
Specific heat of air c_p	$1006KJ/KgK$
Thermal conductivity of air k_f	$26KW/mK$
Disk radius R	$25cm$
Computational domain in *x*-direction	$x=3.5R$ to $x=-3.5R$
Computational domain in *y*-direction	$y=3.5R$ to $y=-3.5R$
Computational domain in *z*-direction from	$z=0$ to $z=1.5R$

Results and Discussion

Considerable experimental data and numerical results were obtained in the present research. As was mentioned earlier, our investigations are limited to a single disk model of diameter, d=25cm, included vertical, horizontal and three other angle of orientation.

The experimental data for laminar convection are depicted in dimensionless form in Fig. 1, where Num is plotted as a function of the Reynolds number, Re. The data presented in this figure consist of multiple heat transfer models at various angles of inclination. Fig. 1 illustrates the cumulative experimental data of all the heat transfer models. The range of Reynolds numbers given in Fig. 1 represented the limits of the existing experimental apparatus and heat transfer models, not necessarily the limit of the existing correlation. The data indicate that heat transfer coefficients are generally higher for disk in a vertical orientation than for disks in a horizontal orientation, at least over this range of the Reynolds number except for θ=90. It is clear from this figure that the influence of

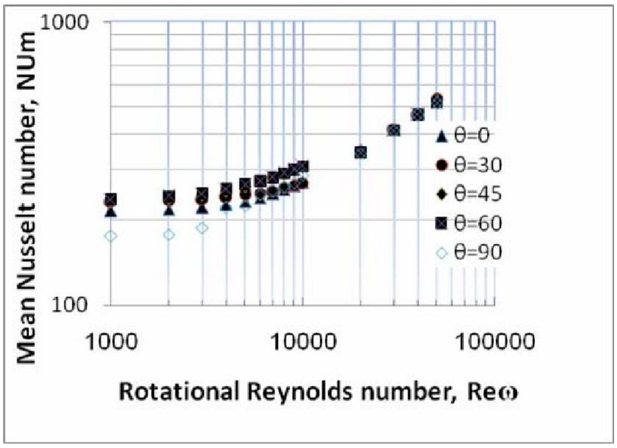

Figure 1. Experimental data of convection heat transfer for rotationary disk at various angles of inclination from 0 to 90^

inclination angle appears to be continuous, and is greatest at the lowest Reynolds number, and at least at the highest. The influence of inclination angle that is clear in Fig. 1 is diminished for four highest value of the Reynolds number. This figure which illustrates the experimental data for the lowest range of the Reynolds number, shows very little difference between the vertical, horizontal, or inclined disk orientation (within experimental uncertainty).

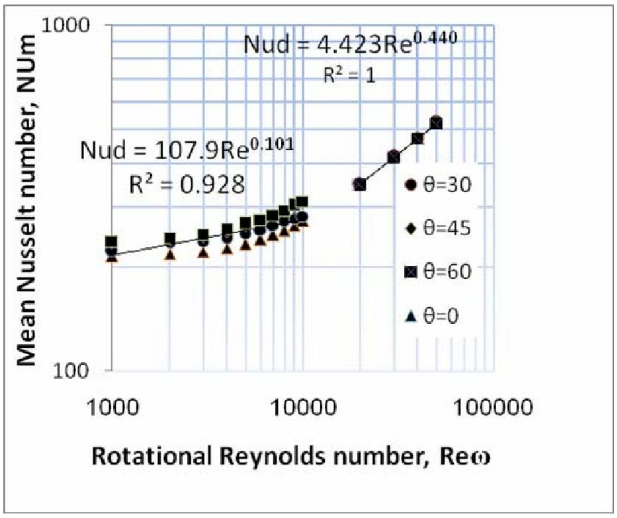

Figure 2. Experimental data of convection heat transfer for rotationary disk at various angles of inclination from 0 to 60^ with correlation

An empirical correlation which fits all of the data is given in Fig. 2. As can be seen, there does not appear to be an appreciable difference in the heat transfer characteristics for the various inclination angles over this range of Reynolds numbers. Although the experimental data covers three orders of magnitude in the modified Reynolds number, it would be interesting to see if there are inclination angle effects at lower Reynolds numbers. The above correlation has the advantage of being valid over the full range of inclination angles between the vertical and horizontal limits.

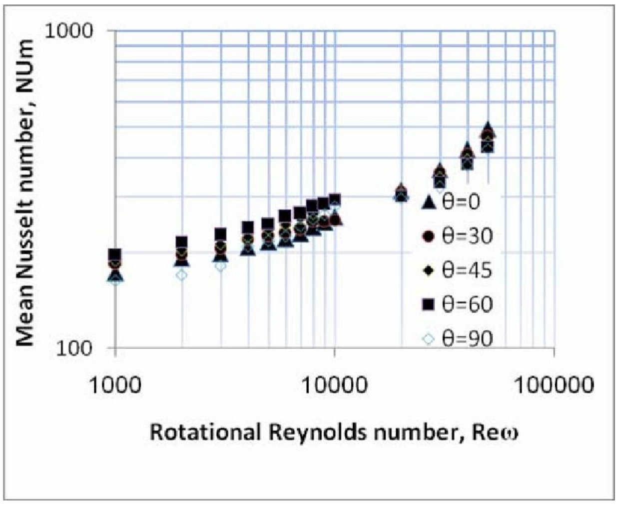

Figure 3. Numerical results of convection heat transfer for rotationary disk at various angles of inclination from 0 to 90^

The numerical results for laminar convection heat transfer are depicted in dimensionless form in Fig. 3, where Nud is plotted as a function of the Reynolds number, Re. The results presented in this figure consist of multiple heat transfer models at various angles of inclination. Fig. 3 illustrates the cumulative numerical result of all the heat transfer models. The numerical results indicate that heat transfer coefficients are generally higher for disk in a vertical orientation than for disks in a horizontal orientation, at least over this range of the Reynolds number except for θ=90. Excellent agreement exists between the experimental data and the numerical results for 1000< Reω < 50000, Fig. 5, 6, 7. Fig. 3 is the same type of data as Fig. 1.

An empirical correlation which fits all of the numerical results is given in Fig. 4. As can be seen, there does not appear to be an appreciable difference in the heat transfer characteristics for the various inclination angles over this range of Reynolds numbers.

Table 2

Dimensionless correlation: $Nu_m = Re_\omega^n$ Comparison of experimental and numerical convection correlation results

Re range	C Exp.	n Exp.	C Num.	n Num.	C Diff %	n Diff %
10^3-10^4	108	0.101	68	0.141	37	40
10^4-10^5	4.423	0.440	4.035	0.436	9	1

Fig. 3 and 4 display both the experimental data and numerical results correlation for inclination angles from 0 to 90 from vertical, respectively (except for θ=90). The correlation in Fig.

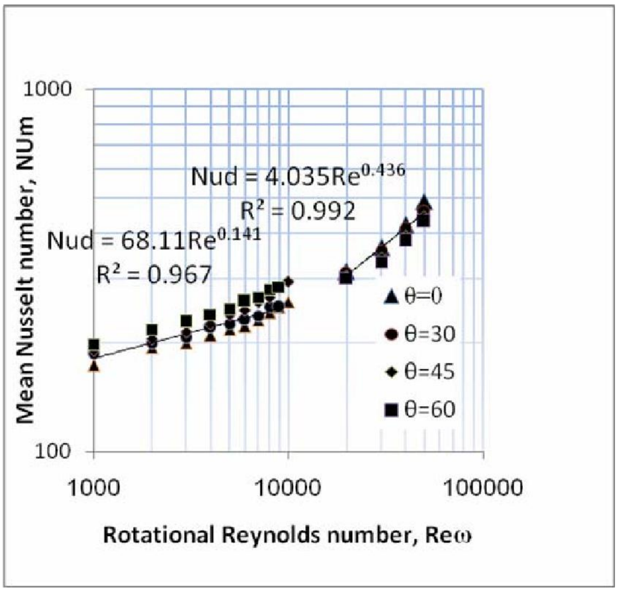

Figure 4. Numerical results of convection heat transfer for rotationary disk at various angles of inclination from 0 to 60^ With correlation

3 and 4 are valid for 1000<Re<50000. Where the coefficients C and exponent n are given in table 2. The average correlation coefficient for the coefficient, C is 4.035, with a maximum deviation of less than 9% and for the exponent, n is 0.436, with a maximum deviation of less than 1%. Therefore although for inclined circular disks at Reynolds number less than 10000, we have not good agreement between the experimental data and numerical results, the Nusselt-Reynolds correlations expressed by Eq. 7 yield very good estimation between the mean Nusselt number and rotational Reynolds number.

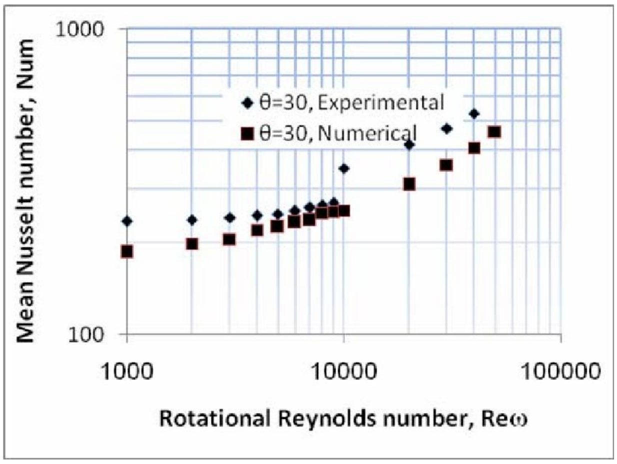

Figure 5. Comparison of convection heat transfer for experimental data and Numerical results for rotationary disk at 30^ angles of inclination

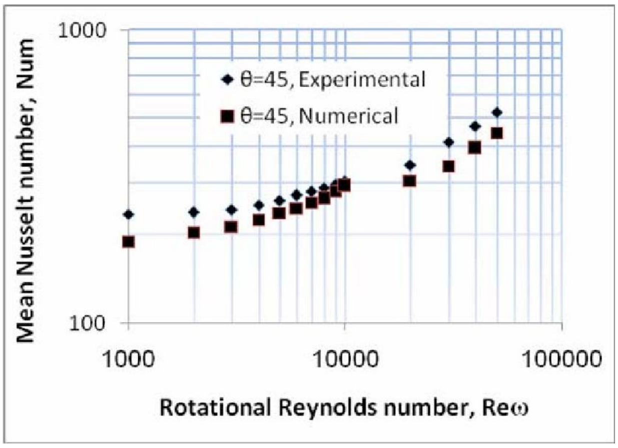

Figure 6. Comparison of convection heat transfer for experimental data and Numerical results for rotationary disk at 45^ angles of inclination

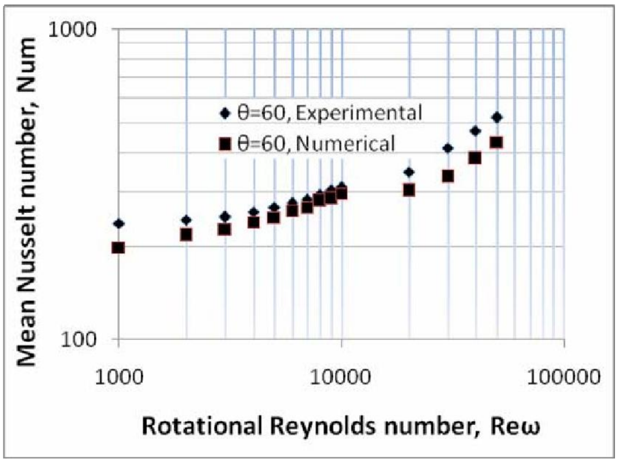

Figure 7. Comparison of convection heat transfer for experimental data and Numerical results for rotationary disk at 60^ angles of inclination

In all of the tests, only thin circular disks were tested, the thickness-to-diameter aspect ratio, t/d, was equal to 0.1. For this range, if any influence of the aspect ratio exists, it must be less than that of the experimental uncertainty in the measurements, since no discernible pattern of influence was observed for any data presented, regardless of the mode of heat transfer. Therefore, a range restriction on the aspect ratio cannot yet be made. However, our investigation was limited to a single disk model of diameter, d = 250mm, and a thickness-to-diameter aspect ratio, t/d = 0.1

The data point for the mean Nusselt number of a rotating disk in an air with angle inclination of 30 to 60^ calculated (from numerical results) and compared with the available experimental data in this research. Excellent agreement exists between the experimental data and the numerical results for 1000< Reω < 50000, Fig. 5, 6, 7. In case of dominant rotational heat transfer the agreement is good, but a significant shift to higher values has been noticed for the experimental mean Nusselt numbers. This conclusion is clearly show table 2.

Conclusion

Experimental heat transfer data and numerical results have been presented and a dimensionless correlation proposed for convection heat transfer from rotationally circular disks (at constant heat flux) over a wide range of the Reynolds number and various angles of inclination. The experimental heat transfer data appears to be lower than the numerical results in this range. For $Re_\omega > 10000$, the experimental heat transfer data are virtually indistinguishable from the numerical results. The significant volume of experimental data presented agreed well with the proposal empirical correlation over a wide range of Reynolds numbers and angles of orientation. All of data showed a good fit to the two correlations. Also of interest is the similarity in the coefficients for the experimental and numerical empirical correlations. Referring to these correlations depicted in Table 2, it appears that only the coefficient, C for $Re_\omega <$ 10000, is different. This interesting find lead encouragement for the future research with additional data must be obtained at arbitrary inclination angles. This Conclusion was obtained in one case (for $\theta=0$) and agreed with work of Stefan aus der Wiesche [26].

Since only air was tested, the current correlation is recommended for Prandtl numbers near unity, which includes most common gases. The correlation may be valid for Prandtl numbers outside this range, however, this is not known at this time since no experimental data is available. Also the maximum aspect ratio recommended should not be much more than the maximum of the heat transfer models tested, thus $(t/d)<0.1$. The influence of larger aspect ratios should be also be the subject of ongoing research. According to the experimental results in the current research, disk orientation cannot greatly affect forced convection heat transfer for $1000 < Re_\omega < 50000$. This effect is completely diminished for $Re_\omega > 10000$.

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