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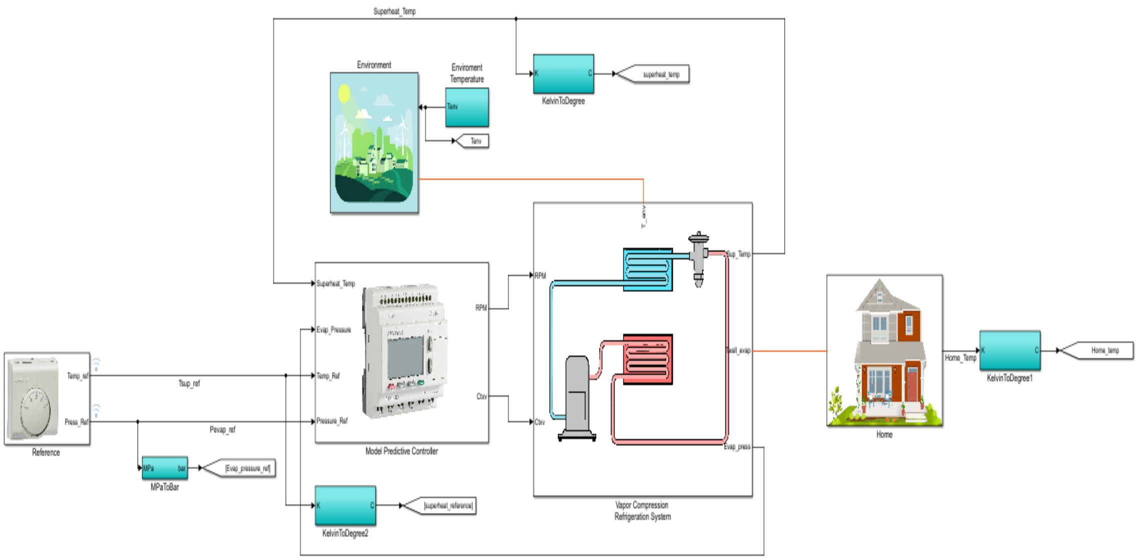
ADDENDUM

A.1. SIMULINK MODEL OF A VC SYSTEM



A.2. HEAT TRANSFER CORRELATIONS

Heat exchanger	Parameter	Formula	Description
Dry zone	Chilton - Colburn factor	$J_{hx,dry} = 0.023Re_{hx,dry}^{-0.2}$	○ $Re_{hx,dry}$: Reynolds number at the dry zone of the heat exchanger
	Prandtl number	$Pr_{hx,dry} = \frac{C_{air}\mu_{hx,dry}}{k_{hx}}$	○ $\mu_{hx,dry}$: dynamic viscosity of air at the dry zone of the heat exchanger in $(kg/m.s)$ ○ k_{hx} : thermal conductivity of the heat exchanger in $(kw/m\,°C)$

	Reynolds number	$Re_{hx,dry} = \frac{\rho_{air} u_{hx,dry}}{\mu_{hx,dry}} l_{hx,dry}$	○ $l_{hx,dry}$: length of the dry zone of the heat exchanger in (m)
Wet zone	Chilton - Colburn factor	$J_{hx,wet} = 0.023 Re_{hx,wet}^{-0.2}$	○ $Re_{hx,wet}$: Reynolds number at the wet zone of the heat exchanger
	Prandtl number	$Pr_{hx,wet} = \frac{C_{air}\mu_{hx,wet}}{k_{hx}}$	○ $\mu_{hx,wet}$: dynamic viscosity of air at the wet zone of the heat exchanger in $(kg/m.s)$ ○ k_{hx} : thermal conductivity of the heat exchanger in $(kw/m\,°C)$
	Reynolds number	$Re_{hx,wet} = \frac{\rho_{air} u_{hx,wet}}{\mu_{hx,wet}} l_{hx,wet}$	○ $l_{hx,wet}$: length of the wet zone of the heat exchanger in (m)

A.3. DERIVATION OF DISCRETIZED EQUATIONS

Conservation of mass and energy in the condenser:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{A.1}$$

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial(\rho u h)}{\partial x} + \delta\dot{Q}_{ref} = 0 \tag{A.2}$$

$\delta\dot{Q}_{ref}$ is defined as follows:

$$\delta\dot{Q}_{ref} = \frac{\alpha}{N_{cells}} \frac{\left(T_{ref,cond} - T_{wall}\right)}{\delta x} \tag{A.3}$$

α is the heat transfer coefficient (kw/m^2°C). $T_{ref,cond}$ is the refrigerant temperature in the condenser (°C). T_{wall} is the wall temperature in the condenser (°C). δx is the distance travelled by the refrigerant stream (m).

Eq. (A.1) and (A.2) could be discretized across the i^{th}-control volume as follows:

$$\int_{CV}\left(\frac{\partial\rho}{\partial t}\right)dV_i+\int_{CV}\left(\frac{\partial(\rho u)}{\partial x}\right)dV_i=0 \tag{A.4}$$

$$\int_{CV}\left(\frac{\partial(\rho h)}{\partial t}\right)dV_i+\int_{CV}\left(\frac{\partial(\rho uh)}{\partial x}\right)dV_i+\int_{CV}\left(\delta\dot{Q}_{ref}\right)dV_i=0 \tag{A.5}$$

The volume integrals with spatial terms in (A.4) and (A.5) could be re-written as surface integrals by adopting the divergence theorem of Gauss. For a vector λ this theorem states:

$$\int_{CV}\frac{\partial\lambda}{\partial x}dV=\int_{CV}div(\lambda)dV=\int_{A}n.\lambda dA \tag{A.6}$$

$n.\lambda$ represents the component λ in the direction of vector n normal to the bounding surface element dA. Applying Gauss's theorem, (A.4) and (A.5) can be written as follows:

$$\frac{\partial}{\partial t}\int_{CV}(\rho)dV_i+\int_{A_i}n.(\rho u)dA_i=0 \tag{A.7}$$

$$\frac{\partial}{\partial t}\int_{CV}(\rho h)dV_i+\int_{A_i}n.(\rho uh)dA_i+\left(\delta\dot{Q}_{ref}V\right)_i=0 \tag{A.8}$$

Re-arranging (A.7) and (A.8) yields to:

$$\frac{\partial(\rho V)_i}{\partial t}+[\rho uA_i]_{in}^{out}=0 \tag{A.9}$$

$$\frac{\partial(\rho hV)_i}{\partial t} + [\rho u A_i h]_{in}^{out} + \delta \dot{Q}_{ref,i} A_i \delta x, i = 0 \tag{A.10}$$

By definition,

$$\rho u A_i = \dot{m} \tag{A.11}$$

$$\delta \dot{Q}_{ref,i} = \frac{\alpha_i}{N_{cells}} \frac{\left(T_i - T_{wall,i}\right)}{\delta x, i} \tag{A.12}$$

Therefore,

$$[\rho u A_i]_{in}^{out} = -\left(\dot{m}_{cond,ref}\right)_{in} + \left(\dot{m}_{cond,ref}\right)_{out} \tag{A.13}$$

$$[\rho u A_i h]_{in}^{out} = -\left(\dot{m}_{cond,ref}\right)_{in} h_{cond,in} + \left(\dot{m}_{cond,ref}\right)_{out} h_{cond,out} \tag{A.14}$$

$$\delta \dot{Q}_{ref,i} A_i \delta x, i = \alpha_i \frac{A_i}{N_{cells}} \left(T_i - T_{wall,i}\right) = \dot{Q}_{ref,i} \tag{A15}$$

Eq. (A.9) and (A.10) could therefore be re-written as follows:

$$\frac{\partial(\rho V)_i}{\partial t} - \left(\dot{m}_{cond,ref}\right)_{in} + \left(\dot{m}_{cond,ref}\right)_{out} = 0 \tag{A.16}$$

$$\frac{\partial(\rho hV)_i}{\partial t} - \left(\dot{m}_{cond,ref}\right)_{in} h_{cond,in} + \left(\dot{m}_{cond,ref}\right)_{out} h_{cond,out} + \dot{Q}_{ref,i} = 0 \tag{A.17}$$

Applying the chain rules with $\rho = f(P, h)$, (A.16) and (A.17) could be arranged as follows:

$$V_i\left[\frac{\partial \rho_i}{\partial P}\bigg|_{h,i}\frac{dP_{cond}}{dt}+\frac{\partial \rho_i}{\partial h_i}\bigg|_P\frac{dh_i}{dt}\right]=\left(\dot{m}_{cond,ref}\right)_{in}-\left(\dot{m}_{cond,ref}\right)_{out} \tag{A.18}$$

$$V_i\left[\left(h_i\frac{\partial \rho_i}{\partial P}\bigg|_{h,i}-1\right)\frac{dP_{cond}}{dt}+\left(h_i\frac{\partial \rho_i}{\partial h_i}\bigg|_P+\rho_i\right)\frac{dh_i}{dt}\right] = \left(\dot{m}_{cond,ref}\right)_{in}h_{cond,in}-\left(\dot{m}_{cond,ref}\right)_{out}h_{cond,out}-\dot{Q}_{ref,i} \tag{A.19}$$

Inserting (A.18) into (A.19) yields to:

$$\left(\dot{m}_{cond,ref}\right)_{in}h_i-\left(\dot{m}_{cond,ref}\right)_{out}h_i-V_i\frac{dP_{cond}}{dt}+V_i\rho_i\frac{dh_i}{dt} = \left(\dot{m}_{cond,ref}\right)_{in}h_{cond,in}-\left(\dot{m}_{cond,ref}\right)_{out}h_{cond,out}-\dot{Q}_{ref,i} \tag{A.20}$$

Eq. (A.20) is for one control volume and it is equivalent to (3.6) and could be re-arranged as follows:

$$\left(\dot{m}_{cond,ref}\right)_{in}h_i-\left(\dot{m}_{cond,ref}\right)_{out}h_i-V_i\frac{dP_{cond}}{dt}+V_i\rho_i\frac{dh_i}{dt} + \left(\dot{m}_{cond,ref}\right)_{out}h_{cond,out} = \left(\dot{m}_{cond,ref}\right)_{in}h_{cond,in}-\alpha_i\frac{A_i}{N_{cells}}\left(T_i-T_{wall,i}\right) \tag{A.21}$$

- Within $(i-1)^{th}$-control volume:

Inlet conditions : $\left(\dot{m}_{cond,ref}\right)_{in}$ and $h_{cond,in}$

Centre or nodal point: m_{i-1} and h_{i-1}

Outlet conditions: $m_{i-1,i}$ and $h_{i-1,i}$

Applying Eq. (A.21) within the $(i-1)^{th}$-control volume yields to:

$$\left(m_{cond,ref}\right)_{in} h_{i-1} - m_{i-1,i} h_{i-1} - V_{i-1}\frac{dP_{cond}}{dt} + V_{i-1}\rho_{i-1}\frac{dh_{i-1}}{dt} + m_{i-1,i} h_{i-1,i} = \left(m_{cond,ref}\right)_{in} h_{cond,in} - \alpha_{i-1}\frac{A_{i-1}}{N_{cells}}\left(T_{i-1} - T_{wall,i-1}\right) \tag{A.22}$$

○ Within i^{th}-control volume:

Inlet conditions: $m_{i-1,i}$ and $h_{i-1,i}$

Centre or nodal point: m_i and h_i

Outlet conditions: $m_{i,i+1}$ and $h_{i,i+1}$

Applying Eq. (A.21) within the i^{th}-control volume yields to:

$$m_{i-1,i} h_i - m_{i,i+1} h_i - V_i\frac{dP_{cond}}{dt} + V_i\rho_i\frac{dh_i}{dt} + m_{i,i+1} h_{i,i+1} = m_{i-1,i} h_{i-1,i} - \alpha_i\frac{A_i}{N_{cells}}\left(T_i - T_{wall,i}\right) \tag{A.23}$$

○ Within $(i+1)^{th}$-control volume:

Inlet conditions: $m_{i,i+1}$ and $h_{i,i+1}$

Centre or nodal point: m_{i+1} and h_{i+1}

Outlet conditions: $\left(\dot{m}_{cond,ref}\right)_{out}$ and $h_{cond,out}$

Applying (A.21) within the $(i+1)^{th}$-control volume yields to:

$$m_{i,i+1}h_{i+1} - \left(m_{cond,ref}\right)_{out}h_{i+1} - V_{i+1}\frac{dP_{cond}}{dt} + V_{i+1}\rho_{i+1}\frac{dh_{i+1}}{dt} + \left(m_{cond,ref}\right)_{out}h_{cond,out} = m_{i,i+1}h_{i,i+1} - \alpha_{i+1}\frac{A_{i+1}}{N_{cells}}\left(T_{i+1} - T_{wall,i+1}\right) \tag{A.24}$$

Regrouping (A.22) to (A.24) into a system and re-arranging each equation yields to:

$$\begin{cases} \left(m_{cond,ref}\right)_{in}h_{i-1} - m_{i-1,i}h_{i-1} - V_{i-1}\frac{dP_{cond}}{dt} + V_{i-1}\rho_{i-1}\frac{dh_{i-1}}{dt} + m_{i-1,i}h_{i-1,i} \\ \quad = \left(m_{cond,ref}\right)_{in}h_{cond,in} - \alpha_{i-1}\frac{A_{i-1}}{N_{cells}}\left(T_{i-1} - T_{wall,i-1}\right) \\ m_{i-1,i}h_{i} - m_{i,i+1}h_{i} - V_{i}\frac{dP_{cond}}{dt} + V_{i}\rho_{i}\frac{dh_{i}}{dt} - m_{i-1,i}h_{i-1,i} + m_{i,i+1}h_{i,i+1} \\ \quad = -\alpha_{i}\frac{A_{i}}{N_{cells}}\left(T_{i} - T_{wall,i}\right) \\ m_{i,i+1}h_{i+1} - \left(m_{cond,ref}\right)_{out}h_{i+1} - V_{i+1}\frac{dP_{cond}}{dt} + V_{i+1}\rho_{i+1}\frac{dh_{i+1}}{dt} - m_{i,i+1}h_{i,i+1} \\ \quad = -\left(m_{cond,ref}\right)_{out}h_{cond,out} - \alpha_{i+1}\frac{A_{i+1}}{N_{cells}}\left(T_{i+1} - T_{wall,i+1}\right) \end{cases} \tag{A.25}$$

A.4. MODEL TRANSFORMATION FROM PREDICTIVE TO CONTROL-ORIENTED

To convert the transient model into a control-oriented model more suitable for controller implementation the Navier-Stokes equations adopted for the transient model in the previous section would need to be considered with additional parameters to capture with more accuracy the phase changes undergone by the refrigerant.

Therefore, starting from the conservation equations for mass and energy we previously had as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{A.1}$$

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial(\rho u h)}{\partial x} + \delta \dot{Q}_{ref} = 0 \tag{A.2}$$

We need to derive the governing equations of the refrigerant dynamics along within the evaporator and the condenser considering the phase changes. The transient pressure p needs to be considered to effectively capture the refrigerant dynamics therefore (A.2) could be written into generic energy equation as:

$$\frac{\partial(\rho h - p)}{\partial t} + \frac{\partial(\rho u h)}{\partial x} + \delta \dot{Q}_{ref} = 0 \tag{A.26}$$

A.4.1. Evaporator modelling

The refrigerant flow through the evaporator undergoes phase change due to energy gains, therefore it is divided in two zones namely, the two-phase and superheated zones. The refrigerant temperature is at saturation and is spatially invariant along the two-phase zone whilst it increases along the single-phase zone through the evaporator outlet.

Adopting time-invariant principle the following Leibniz equation could be adopted:

$$\int_{x_1(t)}^{x_2(t)} \left(\frac{\partial g(x,t)}{\partial t}\right) dx = \frac{d}{dt}\int_{x_1(t)}^{x_2(t)} g(x,t)\, dx - g(x_2(t),t)\frac{dx_2(t)}{dt} + g(x_1(t),t)\frac{dx_1(t)}{dt} \tag{A.27}$$

Mass conservation equation along the two-phase zone could be integrated from $x_1 = 0$ to $x_2 = l_{evap,2ph}$ and expressed as:

$$A_{evap}\int_0^{l_{evap,2ph}}\left(\frac{\partial\rho_{evap,2ph}}{\partial t}\right)dx = \left(m_{evap,ref}\right)_{in} - \left(m_{evap,ref}\right)_{int} \tag{A.28}$$

with,

$$\rho_{evap,2ph} = \rho_{evap,liq}\left(1-\bar{\gamma}_{evap}\right) + \rho_{evap,gas}\bar{\gamma}_{evap} \tag{A.29}$$

we have,

$$\begin{aligned} A_{evap}\int_0^{l_{evap,2ph}}\frac{\partial\rho_{evap,2ph}}{\partial t}dx \\ &= \frac{d}{dt}\int_0^{l_{evap,2ph}}\left(\rho_{evap,liq}\left(1-\bar{\gamma}_{evap}\right) + \rho_{evap,gas}\bar{\gamma}_{evap}\right)dx \\ &- \rho_{evap,gas}\frac{dl_{evap,2ph}}{dt} \\ &= \frac{d}{dt}\left(l_{evap,2ph}\frac{1}{l_{evap,2ph}}\int_0^{l_{evap,2ph}}\left(\rho_{evap,liq}\left(1-\bar{\gamma}_{evap}\right)\right.\right. \\ &\left.\left.+ \rho_{evap,gas}\bar{\gamma}_{evap}\right)dx\right) - \rho_{evap,gas}\frac{dl_{evap,2ph}}{dt} \\ &= l_{evap,2ph}\frac{d\rho_{evap,2ph}}{dp}\frac{dp}{dt} + \left(\rho_{evap,liq} - \rho_{evap,gas}\right)\frac{dl_{evap,2ph}}{dt} \end{aligned} \tag{A.30}$$

Therefore, the conservation of mass along the two-phase zone is expressed as:

$$A_{evap} l_{evap,2ph} \frac{d\rho_{evap,2ph}}{dp} \frac{dp}{dt} + A_{evap} \left(\rho_{evap,liq} - \rho_{evap,gas}\right) \frac{dl_{evap,2ph}}{dt} = \left(m_{evap,ref}\right)_{in} - \left(m_{evap,ref}\right)_{int} \tag{A.31}$$

Energy conservation equation along the two-phase zone could be integrated from $x_1 = 0$ to $x_2 = l_{evap,2ph}$ and expressed as:

$$A_{evap} \int_0^{l_{evap,2ph}} \frac{\partial \rho_{evap,2ph} h_{evap,2ph}}{\partial t} dx - A_{evap} l_{evap,2ph} \frac{dp}{dt} = \left(m_{evap,ref}\right)_{in} h_{evap,in} - \left(m_{evap,ref}\right)_{int} h_{evap,int} + Q_{evap,2ph} \tag{A.32}$$

where,

$$\int_0^{l_{evap,2ph}} \frac{\partial \rho_{evap,2ph} h_{evap,2ph}}{\partial t} dx$$

$$= \frac{d}{dt}\int_0^{l_{evap,2ph}} \rho_{evap,liq} h_{evap,liq}\, dx - \rho_{evap,gas} h_{evap,gas} \frac{dl_{evap,2ph}}{dt}$$

$$= \frac{d}{dt}\int_0^{l_{evap,2ph}} \Big(\rho_{evap,liq} h_{evap,liq} \left(1 - \bar{\gamma}_{evap}\right)$$

$$+ \rho_{evap,gas} h_{evap,gas} \bar{\gamma}_{evap}\Big)\, dx - \rho_{evap,gas} h_{evap,gas} \frac{dl_{evap,2ph}}{dt}$$

$$= \frac{d}{dt}\Bigg(\rho_{evap,liq} h_{evap,liq} \int_0^{l_{evap,2ph}} \left(1 - \bar{\gamma}_{evap}\right) dx$$

$$+ \rho_{evap,gas} h_{evap,gas} \int_0^{l_{evap,2ph}} \bar{\gamma}_{evap}\, dx\Bigg)$$

$$- \rho_{evap,gas} h_{evap,gas} \frac{dl_{evap,2ph}}{dt} \qquad \text{(A.33)}$$

$$= \frac{d}{dt}\Big(l_{evap,2ph} \rho_{evap,liq} h_{evap,liq} \left(1 - \bar{\gamma}_{evap}\right)$$

$$+ l_{evap,2ph} \rho_{evap,gas} h_{evap,gas} \bar{\gamma}_{evap}\Big) - \rho_{evap,gas} h_{evap,gas} \frac{dl_{evap,2ph}}{dt}$$

$$= l_{evap,2ph} \left(\frac{d\left(\rho_{evap,liq} h_{evap,liq}\right)}{dt}\left(1 - \bar{\gamma}_{evap}\right)\right.$$

$$\left. + \frac{d\left(\rho_{evap,gas} h_{evap,gas}\right)}{dt} \bar{\gamma}_{evap}\right)$$

$$+ \left(1 - \bar{\gamma}_{evap}\right)\left(\rho_{evap,liq} h_{evap,liq} - \rho_{evap,gas} h_{evap,gas}\right) \frac{dl_{evap,2ph}}{dt}$$

Therefore, (A.32) could be rewritten as:

$$A_{evap} l_{evap,2ph} \left(\frac{d\left(\rho_{evap,liq} h_{evap,liq}\right)}{dt} \left(1 - \bar{\gamma}_{evap}\right) + \frac{d\left(\rho_{evap,gas} h_{evap,gas}\right)}{dt} \bar{\gamma}_{evap} - \frac{dP_{evap}}{dt} \right) + A_{evap}\left(1 - \bar{\gamma}_{evap}\right)\left(\rho_{evap,liq} h_{evap,liq} - \rho_{evap,gas} h_{evap,gas}\right) \frac{dl_{evap,2ph}}{dt} = \left(m_{evap,ref}\right)_{in} h_{evap,in} - \left(m_{evap,ref}\right)_{int} h_{evap,int} + Q_{evap,2ph} \quad (A.34)$$

A.4.2. Condenser modelling

The condenser equations to be adopted for control-oriented modelling could be derived similarly to the evaporator from Navier-Stokes equations using Leibniz equations however, three zones instead of two should be considered namely, superheat, two-phase and subcool zones.

A.4.3. Derivation of steady state equations

Considering (A.20) as follows:

$$\left(\dot{m}_{cond,ref}\right)_{in} h_i - \left(\dot{m}_{cond,ref}\right)_{out} h_i - V_i \frac{dP_{cond}}{dt} + V_i \rho_i \frac{dh_i}{dt} = \left(\dot{m}_{cond,ref}\right)_{in} h_{cond,in} - \left(\dot{m}_{cond,ref}\right)_{out} h_{cond,out} - \dot{Q}_{ref,i} \quad (A.20)$$

At steady state operating conditions, all time derivatives must be equal to zero therefore:

$$\left(\dot{m}_{cond,ref}\right)_{in} h_i - \left(\dot{m}_{cond,ref}\right)_{out} h_i = \left(\dot{m}_{cond,ref}\right)_{in} h_{cond,in} - \left(\dot{m}_{cond,ref}\right)_{out} h_{cond,out} - \dot{Q}_{ref,i} \quad (A.35)$$

At steady state condition the refrigerant mass flow rate at the condenser inlet and outlet could be assumed identical and equal to the refrigerant mass flow rate at the compressor or thermostatic expansion valve and therefore:

$$\dot{m}_{comp}\left(h_{cond,out} - h_{cond,in}\right) = \dot{Q}_{ref,i} = Q_{cond} \tag{A.36}$$

Eq. (A.36) is equivalent to (3.4) and similar approach could be adopted for the evaporator equation in Table 3.3. Darcy-Weisback correlation is adopted for pressure drop estimation within the condenser and evaporator to determine (3.5) assuming constant HTC, friction factor and neglected water-side pressure drop. Mass flow rate through the expansion valve was correlated following ASHRAE guidelines considering isenthalpic process and constant valve flow coefficient. The refrigerant mass flow rate through the compressor was correlated following **[145]**.