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CONTEXT IN GEOMETRY IN SECONDARY SCHOOL MATHEMATICS TEXTBOOKS

Halalisani MTHETHWA, Ugorji OGBONNAYA, Sonja VAN PUTTEN

Abstract. Mathematics textbooks are commonly considered to be tools for efficient teaching and implementing change in mathematics classrooms worldwide, helping teachers to teach and learners to learn. When the content as described in the curriculum is not faithfully and relevantly represented in textbooks, learners' opportunity is limited. Against this background, opportunity to learn Euclidean geometry was investigated by scrutinising four Grade 11 mathematics textbooks used in South African schools, using deductive content analysis. The Kurz Opportunity to Learn (OTL) Model was the theoretical foundation focusing on content coverage and the quality of tasks. In this paper, the embedding of context within geometry questions is investigated as one of the criteria in judging the quality of tasks. The study revealed that although the four textbooks covered almost all the Euclidean geometry topics in depth, contextualisation was absent, despite its ability to improve learners' problem recognition, interpretation and understanding abilities. All the questions were intra-mathematical. With the use of contextualised tasks, possibly the real world can enter the mathematics classroom, so that the age-old learner question, "Why am I doing this?" may be answered with a demonstration of relevance.

Keywords: Context; Euclidean geometry; mathematics textbooks; opportunity to learn.

1. Introduction

Many learners in South Africa find the study of Euclidean geometry challenging (Author, 2010; Ngirishi & Bansilal, 2019). In fact, Ngirishi (2015) found that these learners had little understanding of many geometry concepts and terms, and could not recognise or describe interrelationships between properties and shapes, since they possessed neither conceptual nor procedural knowledge in this regard. However, low performance in geometry is not isolated to South Africa – for example, a study in India (Ali et al, 2014), stated that of the 120 learners in their study, 92 failed the Euclidean geometry test. In South Africa, Euclidean geometry had been optional until 2014, when it again became an obligatory topic. In the final exams of 2014, the percentage of learners who passed Grade 12 mathematics examinations dropped by 5,6% (DBE, 2015).

Given that mathematics textbooks have a great influence on teaching and learning mathematics (Fan et al., 2013; Reys et al., 2004; Tarr, 2008), with a strong correlation between the textbook used and students' mathematics performance (Törnroos, 2005; Xin, 2007), and against the background of consistent challenges in the teaching of mathematics in South Africa, investigating how geometry is offered in South African textbooks was deemed both current and essential. In this descriptive analysis of opportunities to learn through contextualised problems (also called modelling or using real life problems) in the geometry sections of Grade 11 textbooks, the focus is on a qualitative description of the incorporation of real life problems into geometry exercises as per the Curriculum and Assessment Policy Statement (CAPS) of 2011.

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2. The curriculum, textbooks and the opportunity to learn

Improving mathematics teaching practice is one of the primary goals of mathematics education research (Rezat et al., 2021). Mathematics textbooks and, more broadly, curricular resources are commonly considered to be tools for implementing change in mathematics classrooms in educational systems all over the globe (Hwang & Ham, 2021; Remillard, 2018; Stols, 2013). Textbooks have been defined as printed mathematics material used to support teachers' teaching and learners' learning (Gueudet & Pepin, 2018), but, of course, a textbook can also be digital.

Generally, a mathematics curriculum focuses on the sequencing of mathematics content, the cognitive levels to be used in assessments in a particular grade, and content coverage in order to address all parts of the curriculum specification (Gueudet & Pepin, 2018). Textbooks should reflect the requirements of the curriculum and are utilised as tools to improve students' knowledge and transform their ideas of and attitudes to mathematics (Cai et al., 2016). A number of researchers argue that inconsistencies between curriculum documents and instructional materials (textbooks), such as the sequencing of subject content, content coverage, cognitive levels of questions and types of tasks in textbooks, limit learners' opportunity to learn mathematics (Hadar, 2017; Hwang & Ham, 2021), and could thus be responsible, in part, for some learners' poor performance in mathematics. Therefore, textbooks should theoretically be purveyors of opportunities to learn (OTL). They also play an important role in explaining and deciphering the enacted curriculum for the information of teachers (Adler & Ronda, 2015). Hence, textbooks "act as a bridge between the intended curriculum and the implemented curriculum" (Lui & Leung, 2012, p. 56). In this regard, Polikoff (2015) goes so far as to say that any study of textbooks is in fact based on OTL, and that poor alignment between the curriculum and the textbooks leads to a deficient implementation of the content and intent of the curriculum.

Many mathematics assignments, the ways in which learners are taught, the types of assessments, and much more, are textbook based (Lepik et al., 2015). As devices for teaching and learning, textbooks have an impact on instructors' instructional decisions, particularly in structuring lessons (Knight, 2015; Remillard & Heck, 2014). The result is that most students rely on textbooks as study material (Lepik et al., 2015; Van den Ham & Heinze, 2018), which means that they shape what students learn, how they learn, and the cognitive levels at which they learn (Hiebert et al., 2019; Nachlieli & Tabach, 2019). Therefore, the nature of textbook tasks can have an impact on and shape the way students think, and can narrow or broaden their understanding of mathematics. Herein lies the opportunity to learn.

3. South African context

Recent studies indicate that learners' underperformance in mathematics is a matter of serious concern in South Africa (Alex & Roberts, 2019; Bansilal & Ubah, 2019; Tachie, 2020). South African learners have one of the worst track records among the countries participating in international comparative studies like the Trends in International Mathematics and Science Study (TIMSS) and the Southern and Eastern Africa Consortium for Monitoring Education Quality (SACMEQ) (Bansilal & Ubah, 2019; Sandefur, 2018; Venkat & Spaull, 2015). This has been the status quo despite curricular changes that have been implemented since 1994.

After the elections of 1994, the Department of Basic Education initially introduced Curriculum 2005 (C2005), which formalised the tenets of outcomes-based education (OBE) (Jojo, 2019); subsequently this curriculum was replaced by National Curriculum Statement for Grades 10 to 12 and the Revised National Curriculum Statement (RCNS) for Grade R through to Grade 9, which were put into effect in 2002. These did not work well either, so the Curriculum and Assessment Policy Statement (CAPS), which was implemented in stages, was introduced, with Subject Statements, Learning Program Guidelines, and Subject Assessment Guidelines in Grades R-12 being replaced by a single comprehensive CAPS as of January, 2012.

The quick succession of one curriculum after the other resulted in many changes in the structure of mathematics education, with topics either being added or removed, and new textbooks being published. Alex and Mammen (2018), Machisi (2021) and Tachie (2020) argue that these curriculum transformations revealed gaps in teachers' content knowledge because the majority of teachers had themselves never done some of the newly-introduced topics at the high school or tertiary level, and now they were expected to teach those very topics. In this regard, according to a report from the National Education Evaluation and Development Unit (NEEDU, 2013), which is at the head of the Department of Basic Education, low student performance in many schools was found to be caused mostly by teachers' inadequate understanding of the subjects they taught, particularly mathematics. Teachers lacking content knowledge, inter alia, base their classroom practice entirely on textbooks (Jones & Tarr, 2007; McIntyre, 2017; Remillard & Heck, 2014; Van den Ham & Heinze, 2018).

Textbooks are, however, the products of the authors' understanding of the curriculum documents of a specific learning area (Lui & Leung, 2012). Therefore, logically, if the author has certain misconceptions about the content, there is a strong possibility that those misconceptions will be adopted by teachers, as well as their learners. Not only do textbooks directly affect what is taught, but also how that content is taught. Wijaya et al. (2015) found that pupils taught from different textbook series varied significantly in their mathematics achievements. The implication is that the pedagogy reflected in the textbook will, in all likelihood, be practised in the classroom.

4. Theoretical framing of the study

4.1 Contextualisation

According to Larochelle and Bednarz (1998) in their treatise on constructivism in education, there needs to be a "narrowing [of] the gap separating what the student knows from the subject matter to be taught" (p. 3) (emphasis added). This notion relates to what was taught by such constructivists as Piaget (1954), Dewey (1929) and Vygotsky (1978): that students come into any learning situation with prior knowledge that informs their uptake of new information (Hyslop-Margison & Strobel, 2007). It makes sense then, to extrapolate this thinking to the teaching and learning of geometry: learners may do better if the knowledge to be acquired and the work to be done were contextualised within their frame of reference. In fact, the CAPS document for Grades 10 -12 (DBE, 2011) is very clear about the importance of modelling/contextualisation/real life problems into all mathematical topics, not excluding geometry: "Mathematical modeling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible" (p. 8).

4.1.1 Context in textbooks

The capacity of learners to apply mathematics to daily life is seen as a primary aim of mathematics education. Context refers to the degree and manner in which real-world experiences have been incorporated into textbook activities (Gracin, 2018). Wijaya et al. (2015) found that the textbooks in their study only contained approximately 10% context-based tasks. In addition, in 85% of these exercises, the information needed to complete them had already been provided, with students not being given the freedom to choose the pertinent material on their own. Purnomo et al. (2022) found that only 30% of the geometry tasks in their textbook research involved high level mathematical thinking, while more than 80% of the geometry problems were related to intra-mathematical contexts without any real-life, practical application.

Zhu and Fan (2006) suggest that there are three contextual features in mathematics tasks: intra-mathematical context i.e. with no connections to the real world (Schukajlow et al. 2012); realistic context - where a mathematical topic is "camouflaged in a real-life setting" (Botha & van Putten, 2018, p. 95), and authentic context which requires the learner to "transfer processes between reality and mathematics" (Schukajlow et al., 2012, p. 220).

4.1.2 Context of questions

Mathematics researchers have several explanations for the definition of contexts. Sullivan et al. (2003) define context as a circumstance in which a problem is rooted to provide information that may allow students to solve the problem, and it may even be fun doing so (Laurens et al, 2017). Gravemeijer et al (2017) contend that mathematics education should prepare students "to apply mathematics in all sorts of work- and everyday-life situations" (p. 108). This type of context may be referred to as the task context, whereas the pedagogical context denotes the context of the classroom (Mbekwa & Julie, 2009). Gracin (2018) defines context used in mathematics textbooks as the extent to which textbooks integrate real-life experiences into mathematical tasks. Zhu and Fan (2006) refer to such tasks as application tasks, using realistic or authentic contexts. They describe non-application tasks as intra-mathematical, providing suitable mathematical data, but without context; they are entirely based on mathematical symbols, whereas a realistic context refers to those mathematical tasks situated in an artificial or imaginary reality.

Promoters of contextual mathematics propose that the usage of contexts in mathematics enables conceptual understanding, advances mathematical ideas and procedures, and contextually driven reasoning (Mbekwa & Julie, 2009). So, the it would seem that according to the literature, the best tasks provide appropriate settings and levels of complexity; promote the growth of cognitive networks, thinking, creativity, and reflection; and explicitly address crucial mathematical concepts (Chotimah et al. 2018; Lutfianto et al., 2013). Such tasks would address what Gravemeijer et al (2017) refer to as the necessity to understand what the demands of mathematics in reality are. The context of tasks as a factor in OTLs in this study was analysed using Zhu and Fan's (2006) classification.

4.2 Opportunities to learn

A reasonable question when specific learning outcomes are not attained by learners is whether they were empowered to grasp the skills articulated in these outcomes. OTL is an approach that looks at whether learners had access to all the components that ensure quality learning, or, in this case, the measure of adequate opportunities afforded to learners to do mathematics (Ogbonnaya, 2021; Wijaya et al., 2015). Stols (2013) states that OTL is used in various studies to determine or evaluate the conditions inside a school or classroom that encourage or hinder learning. Factors influencing OTL include human and non-human resources, such as the availability of qualified teachers, quality of textbooks, teachers' content knowledge, quality of instruction, and access to a conducive learning environment.

The OTL in a textbook determines what the textbook provides for teachers and learners for them to acquire the anticipated information and skills in the planned curriculum (Hong et al., 2020; Otten et al., 2014). This includes the textbook content, cognitive levels of tasks, and types of tasks (Gracin, 2018). To investigate OTL in a textbook, Charalambous et al. (2010) defined three types of analyses: horizontal, vertical, and contextual. The horizontal analysis looks at the general characteristics of textbooks, such as physical properties and content organization. The vertical analysis deals with mathematical content, and the contextual analysis, the third category, is focused on how textbooks are employed in instructional activities (Wijaya et al., 2015). Wijaya et al. (2015) found a relationship between a lack of OTL in contextual tasks and student performance in such tasks, finding that textbooks have a greater direct impact on what is taught in class than the curriculum itself. In fact, Leshota (2020) found that the teachers in her study based their lessons on the textbook, not the curriculum document. However, herein lies core of the problem: Van Zanten and van den Heuvel-Panhuizen (2018) found that textbooks provided very limited OTLs for problem solving in that, "Our study clearly shows how complex the concept of opportunity to learn is from the perspective of the textbook. Just exposure of the content does not tell the whole story" (p. 837). Mostly tasks are regarded as devices for initiating learning. Therefore, the nature of textbook tasks can possibly affect and shape the way learners think, which is likely to limit, or broaden, their knowledge of the subject matter (Gracin, 2018). The latter, according to the NCTM (1995) are situated in real-life, meaningful tasks.

4.2.1 Kurz's Opportunity to Learn model

Opportunity to learn is a key indicator of the opportunities afforded to learners about a specific concept. Such opportunities include content structure, resources available, cognitive demand, qualified teachers, and quality of tasks (Ayieko, 2018; Hadar & Ruby, 2019). The opportunity to learn from a textbook provides teachers and learners access to needed information and skills in the planned curriculum (Barnard-Brak et al., 2018; Camburn et al., 2017; Hadar, 2017). Specific to classroom teaching time, OTL involves such factors as content, and quality of teaching (Camburn et al.; Irvin et al., 2017). In this study, Kurz's (2011) conceptual model of opportunity to learn, which uses these factors as main variables i.e. content, quality, and time of teaching in the classroom, was implemented. The embedding of contextual features is part of the quality aspect.

4.3 Research purpose and question

This study investigated the Euclidean geometry learning opportunities found in four randomly selected Grade 11 mathematics textbooks in South Africa by applying the tenets of Kurz's Opportunity to Learn model. Four aspects of geometry presentation in these textbooks were scrutinised: (1) the extent to which Euclidean geometry content aligned with the prescribed curriculum standards; (2) the distribution of cognitive levels of questions; (3) the nature of tasks and cognitive levels of questions; and (4) the use of contextual features. In this article, bearing in mind the curriculum statement which says, "Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible" (DBE, 2011, p. 8), we pose the question, how are contextual features embedded in the presentation of Euclidean geometry in the textbooks? This question relates directly to the fourth of Kurz's OTL model.

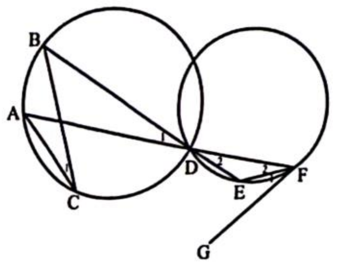
5. Descriptive analysis of the textbooks

5.1. Textbook A

Using Zhu and Fan's (2006) context analysis framework, these tasks were identified as involving only pure mathematical facts without any relation to a real-world context. This analysis approach was applied consistently not only to Textbook A, but to all four textbooks that were analysed. In Textbook A, there were 324 mathematical questions, of which 51 were worked examples and 273 exercise tasks. In the example tasks, all 51 were intra-mathematical tasks, as were all 273 exercise tasks. An example is provided in Figure 1.

EXAMPLE 17

In the following sketch, FG is a tangent to the smaller circle at F. ADF and BDE are straight lines.



Prove that $\hat{C}_1 = \hat{F}_1$.

Solution

$\hat{C}_1 = \hat{D}_1$ (∠s in same segment)

$\hat{D}_1 = \hat{D}_2$ (vert opp ∠s)

$\therefore \hat{C}_1 = \hat{D}_2$

$\hat{D}_2 = \hat{F}_1$ (tan chord thm)

$\therefore \hat{C}_1 = \hat{F}_1$

Figure 1. *Example from Textbook A*

5.2. Textbook B

Textbook B has a total of 293 mathematical tasks, of which 30 are worked examples and 253 exercise tasks. All the example and exercise tasks are intra-mathematical. There are no tasks that involve any imaginary or real-life context to which learners can relate. Figure 2 presents an example.

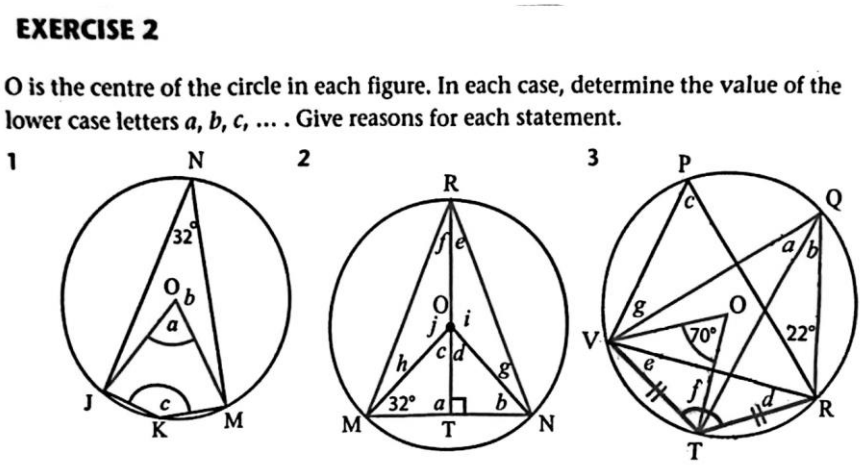


Figure 2. *An Example from Textbook B*

5.3. Textbook C

In this textbook, a total of 286 mathematical tasks was analysed, of which 28 were example tasks and 258 were exercise tasks, all of which were categorised as intra-mathematical, as in the example Figure 3.

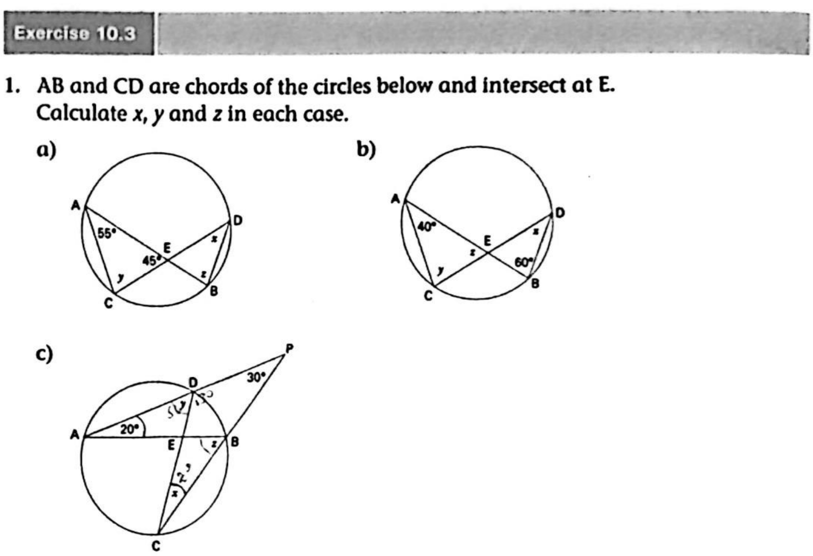
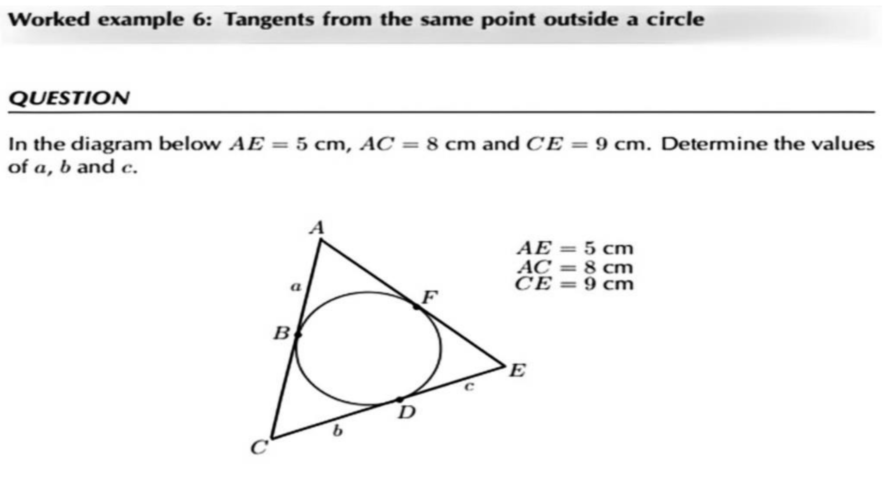


Figure 3. *An example from Textbook C*

5.4. Textbook D

A total of 118 mathematical tasks were examined in Textbook D. Of the 100 tasks analysed, there were 18 example tasks and 100 exercise tasks, all intra-mathematical. See Figure 4.



Worked example 6: Tangents from the same point outside a circle

QUESTION

In the diagram below $AE = 5$ cm, $AC = 8$ cm and $CE = 9$ cm. Determine the values of a, b and c.

Figure 4. *A worked example from Textbook D*

6. Demonstration of contextualistation

Although none of the four textbooks that were analysed for this study presented any geometry that was contextualized or that related to the learners' lived experiences in any way, such contextualisation is in fact possible, although not for every type of geometry sum. The worked example from textbook D is a case in point. This example can not only be contextualised as a sum per se, but the contextualisation presented below can in fact also be used to teach the theorem upon which it is predicated.

Below is an example of how this sum could be presented.

The principal of a secondary school gives the Grade 11 class a piece of the school grounds that is triangular in shape and fenced off. The fence is seen in the diagram as AC, CE and EA. AC is 8m long, CE is 9m and EA is 5m. Their job is to look after this piece of land and to prepare the ground so that eventually they will be able to plant cabbages for the school kitchen's use. They begin by watering the piece of land to soften the soil. The school gives them a water sprinkler that sprays the water in a circle. They set it so that the spray of water reaches all the way to the fence. Very soon, lush grass begins to grow inside the circle created by the sprinkled water. The neighbour's goats, seeing the grass, break parts of the fence labelled a, b and c. The principal wants to replace those broken pieces. How much fencing will he need to buy? Show how you worked this out.

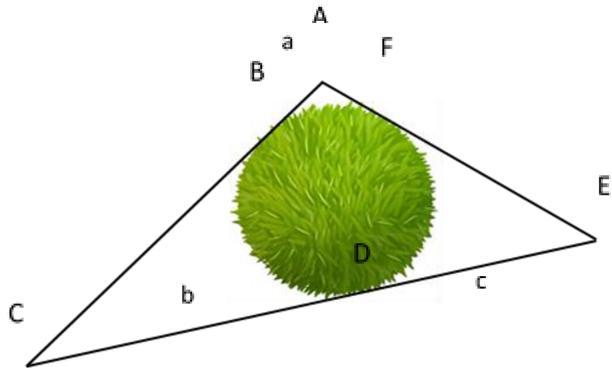


Figure 5. *Contextualising the worked example from Textbook D*

This particular contextualisation lends itself to taking the class outside. Learners placed at point A, C and E can be given a long piece of string to hold, and perhaps 10 learners can be asked to stand in a circle inside the triangle formed by the string, with their backs touching the string at the tangential points. Now, remaining learners can be asked to measure the required lengths of string by pacing the out the distances.

7. Conclusion

The four textbooks that were examined in this study did not integrate any real-life context into the Euclidean geometry tasks; all tasks were purely intra-mathematical. The literature indicates that textbooks typically present exercises with little context that do not require students to engage in mathematisation or modelling activities. This means that the mathematics procedure to be used is more or less predetermined, and learners are not required to identify an appropriate mathematics procedure to solve the tasks. As a result, they are not given the opportunity to develop their ability to transform a context-based task into a mathematical problem. These findings are consistent, for example, with the findings of three recent studies in other parts of the world: Ayyildiz and Aktaş (2022), who examined the context of assignments in various Grade 5 and Grade 6 mathematics textbooks approved by the Turkish Ministry of National Education for the 2021/2022 academic year; Gracin (2018) also observed that the majority of exercises in the most commonly used Croatian mathematics textbooks in Grades 6-8 were entirely intra-mathematical; in a study investigating the opportunities to develop concept images of polygons in middle school (Grades 6-8) mathematics textbooks, Cannon (2021) discovered that all three middle school mathematics textbook series examined used intra-mathematical tasks and almost none related to the real world.

Our contextualised example shows that it is possible that some theorems or riders can be transformed into a real-life situation. However, contextualising takes time and creative thinking. This may be why contextualisation is not often used in geometry tasks: it just is too time-consuming.

It is possible that this lack of real-life context in Euclidean geometry tasks could be why most learners view geometry as being irrelevant to their lives (Patkin & Levenberg, 2012). This lack remains prevalent despite the fact that contextualised exercises have the potential to draw students' attention and to demonstrate how they can make sense of the world of mathematics (Sullivan et al., 2012). Just as real-life situations play a significant part in the development of various abilities in students, so authentic, real-life contexts could and should be included in textbooks. We posit that textbooks with contextualised tasks provide better OTLs. In the words of Hwang et al (2019), "Although geometry is important and strongly related to our surroundings, students still have few chances to learn it through practicing in authentic contexts" (p. 269).

Implications and recommendations

Reys and Chavez (2004, p. 61) suggest that "the choice of textbooks often determines what teachers will teach, how they will teach it, and how their students will learn". Therefore, a teaching strategy that relates to reality assists learners to develop skills in applying mathematics to a situation so that it makes sense to them. Students using any one of the four textbooks investigated in this study would have a very limited opportunity to figure out how to "transfer processes between reality and mathematics" (Schukajlow et al., 2012, p. 220) using geometry principles. Thus this problem-solving skill is not developed. The omission of contextual interpretation tasks can effectively narrow students' conceptual understanding of Euclidean geometry and its application in the real world. The result of this would be the perpetuation of the age-old student question: "Why am I learning this? I am never going to use this in my life!"

The authors report there are no competing interests to declare.

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Authors

Halalisani MTHETHWA, Science, Mathematics and Technology Education, University of Pretoria, Pretoria (South Africa). Email address: u16302011@tuks.co.za

Ugorji OGBONNAYA, Science, Mathematics and Technology Education, University of Pretoria, Pretoria (South Africa). Email Address: ugorji.ogbonnaya@up.ac.za

Sonja VAN PUTTEN, Science, Mathematics and Technology Education, University of Pretoria, Pretoria (South Africa). Email Address: Sonja.vanputten@up.ac.za – *corresponding author*

All three authors have contributed equally to this paper.