

Table S1. Estimated marginal means (emmeans) model output comparing means for each sex (F: female, M: male) within each treatment ambient temperature (T_a) for body temperature (T_b), whole animal resting metabolic rate (waRMR), whole animal evaporative water loss (waEWL), and the ratio of evaporative heat loss/metabolic heat production (EHL/MHP) for *E. fuscus*. Standard error (SE), degrees of freedom (df), lower critical limit (lower.CL), and upper critical limit (upper.CL), were reported, with significant differences being determined when critical limits do not overlap between the sex categories. Significant sex differences within each T_a are in bold.

T_b						
T_a	Sex	emmean	SE	df	lower.CL	upper.CL
32	F	34.9	0.222	138	34.5	35.4
	M	34.8	0.191	138	34.4	35.2
36	F	37.2	0.173	138	36.9	37.6
	M	37.1	0.153	138	36.8	37.4
40	F	39.5	0.148	138	39.2	39.8
	M	39.4	0.148	138	39.1	39.7
42	F	40.7	0.149	138	40.4	41
	M	40.5	0.16	138	40.2	40.8

	Sex	emmean	SE	df	lower.CL	upper.CL
44	F	41.8	0.16	138	41.5	42.1
	M	41.7	0.18	138	41.3	42
	Sex	emmean	SE	df	lower.CL	upper.CL
46	F	43.0	0.178	138	42.6	43.3
	M	42.8	0.204	138	42.4	43.2
	Sex	emmean	SE	df	lower.CL	upper.CL
48	F	44.1	0.202	138	43.7	44.5
	M	44.0	0.233	138	43.5	44.4
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waRMR						
T_a	Sex	emmean	SE	df	lower.CL	upper.CL
36	F	0.131	0.007	128	0.118	0.145
	M	0.121	0.005	128	0.111	0.131
	Sex	emmean	SE	df	lower.CL	upper.CL
40	F	0.168	0.005	128	0.158	0.179
	M	0.158	0.005	128	0.149	0.167
	Sex	emmean	SE	df	lower.CL	upper.CL
42	F	0.187	0.005	128	0.177	0.197
	M	0.177	0.005	128	0.166	0.187
	Sex	emmean	SE	df	lower.CL	upper.CL
44	F	0.205	0.005	128	0.195	0.216
	M	0.195	0.006	128	0.183	0.207
	Sex	emmean	SE	df	lower.CL	upper.CL
46	F	0.224	0.006	128	0.213	0.235
	M	0.214	0.007	128	0.200	0.227

	Sex	emmean	SE	df	lower.CL	upper.CL
48	F	0.242	0.007	128	0.23-	0.255
	M	0.232	0.008	128	0.216	0.248
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waEWL						
T_a	Sex	emmean	SE	df	lower.CL	upper.CL
36	F	0.003	0.034	98	-0.065	0.070
	M	0.082	0.027	98	0.028	0.135
40	F	0.232	0.022	98	0.189	0.276
	M	0.312	0.018	98	0.276	0.346
42	F	0.346	0.019	98	0.310	0.383
	M	0.425	0.017	98	0.392	0.459
44	F	0.461	0.018	98	0.426	0.496
	M	0.550	0.020	98	0.501	0.579
46	F	0.575	0.020	98	0.536	0.615
	M	0.654	0.025	98	0.606	0.703
48	F	0.690	0.025	98	0.640	0.739
	M	0.769	0.031	98	0.708	0.830
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EHL/MHP						
T_a	Sex	emmean	SE	df	lower.CL	upper.CL
36	F	0.460	0.079	128	0.304	0.617
	M	0.922	0.059	128	0.805	1.039

	Sex	emmean	SE	df	lower.CL	upper.CL
40	F	0.930	0.063	128	0.804	1.055
	M	1.391	0.057	128	1.279	1.504
	Sex	emmean	SE	df	lower.CL	upper.CL
42	F	1.164	0.061	128	1.044	1.285
	M	1.626	0.063	128	1.502	1.750
	Sex	emmean	SE	df	lower.CL	upper.CL
44	F	1.399	0.063	128	1.275	1.523
	M	1.861	0.072	128	1.719	2.002
	Sex	emmean	SE	df	lower.CL	upper.CL
46	F	1.634	0.068	128	1.498	1.769
	M	2.096	0.083	128	1.932	2.260
	Sex	emmean	SE	df	lower.CL	upper.CL
48	F	1.868	0.077	128	1.715	2.022
	M	2.330	0.096	128	2.141	2.520

Table S2. Linear mixed effect model output for body temperature (T_b ; °C), whole animal resting metabolic rate (waRMR; W), whole animal evaporative water loss (waEWL; g/h), and the ratio of evaporative heat loss/metabolic heat production (EHL/MHP) of *E. fuscus* with respect to experimentally increasing ambient temperature (T_a ; °C), sex, and body mass (mass; g). The reference category for sex was male. Bold indicates significance at $\alpha = 0.05$.

Dependent Variable	Fixed Effect	Estimate	Degrees of Freedom	t	P
T_b	Intercept	16.54	134.07	23.27	<0.0001
	T_a	0.57	125.95	32.66	<0.0001
waRMR	Intercept	-0.33	114.53	-7.02	<0.0001
	T_a	0.01	109.22	11.15	<0.0001
	Sex (Male)	0.16	117.45	2.91	0.004
	T_a *Sex (Male)	-0.004	109.69	-3.16	0.002
waEWL	Intercept	-2.63	81.92	-12.56	<0.0001
	T_a	0.07	79.11	14.76	<0.0001
	Sex (Male)	0.87	82.85	3.39	0.001
	T_a *Sex (Male)	-0.02	79.16	-3.11	0.003
EHL/MHP	Intercept	-5.03	116.13	-9.35	<0.0001
	T_a	0.15	108.98	11.77	<0.0001
	Sex (Male)	1.81	119.58	2.92	0.004
	T_a *Sex (Male)	-0.03	109.32	-2.25	0.030

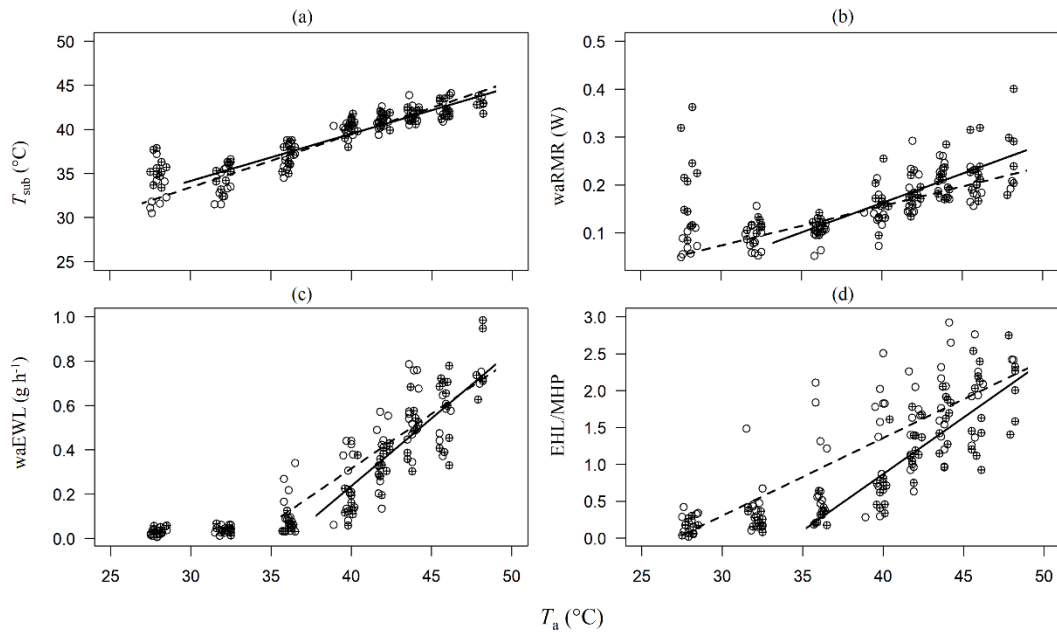


Fig. S1. (a) Subcutaneous body temperature (T_{sub}), (b) whole animal resting metabolic rate (waRMR), (c) whole animal evaporative water loss (waEWL) and (d) evaporative heat loss (EHL)/metabolic heat production (MHP) at high air temperatures (T_a (°C)) of male ($n = 11$) and female ($n = 11$) big brown bats (*Eptesicus fuscus*) from Lillooet B.C., Canada. Dashed and solid lines represent regressions for males and females respectively above inflection points (Table 1). Open circles indicate males and filled circles indicate females. Note absence of an inflection point in T_{sub} , waRMR and EHL/MHP in males.

Supplementary Materials and Methods

Endotherm model

We set wind speed (VEL) and relative humidity (RH) to match the conditions of the respirometry chamber. The core temperature (TC), minimum core temperature (TC_MIN) and maximum core temperature (TC_MAX) was set using the T_b values measured during the respirometry trials. The SHAPE, SHAPE_B and SHAPE_B_MAX values were set to fit *E. fuscus*. The base skin wetness (PCTWET) was set assuming only the nose region of the bat was wet initially. Maximum skin wetness (PCTWET_MAX) was set for the bat licking its arms while intervals by which skin wetness is increased (%) (PCTWET_INC) was set to match our respirometry data. Increment by which panting is increased (PANT_INC), maximum panting rate (PANT_MAX) and multiplier on basal metabolic rate at maximum panting level (PANT_MULT) was adjusted to fit our respirometry data. Thermoregulation mode (TREGMODE) was adjusted to reflect the thermoregulatory patterns of bats and TORPOR was set to 1 as most individuals entered or was beginning to enter torpor at 28 °C. All other values were set to default. We compared the endotherm model output for each individual bat for its experimental T_{as} and compared with the respirometry data using R2, Pearson correlation and Root mean square deviation tests (Table S4).

Table S3. Variables used in the endotherm model to calculate evaporative water loss for *Eptesicus fuscus* (females and males).

Variable	<i>Eptesicus fuscus</i> (female)	<i>Eptesicus fuscus</i> (male)
VEL: Wind speed (m/s)	0.5	0.5
RH: Relative humidity (%)	3	3
TC: Core temperature (°C)	36	36
TC_MAX: Maximum core temp (°C)	43	43

TC_INC: Increment by which TC is elevated (°C)	0.1	0.1
AMASS: Body mass (g)	20.7	19.6
SHAPE_B	1.1	1.1
SHAPE_B_MAX: maximum ratio of length to width/depth	5	5
UNCURL	0.1	0.1
SHAPE	4	4
SAMODE	2	2
DHAIRD: Hair diameter, dorsal (m)	1.2E-05	1.2E-05
DHAIRV: Hair diameter, ventral (m)	8E-06	8E-06
LHAIRD: Hair length, dorsal (m)	7.7E-03	8.3E-03
LHAIRDV: Hair length, ventral (m)	7.5E-03	7.3E-03
RHOD: Hair density, dorsal (1/m ²)	1.83E+09	1.5E+09
RHOV: Hair density, ventral (1/m ²)	1.52E+09	1.81E+09
: Fur reflectivity, dorsal	0.248	0.248
: Fur reflectivity, ventral	0.351	0.351
ZFURD: Fur depth, dorsal (m)	4E-03	4E-03
ZFURV: Fur depth, ventral (m)	4E-03	4E-03
PCTWET: base skin wetness (%)	0.5	0.5
PCTWET_MAX: Maximum skin wetness (%)	10	10
PCTWET_INC: intervals by which skin wetness is increased (%)	0.05	0.05
PCTBAREVAP: surface area for evaporation that is skin	10	10
Q10	2.5	2.5
QBASAL: Basal heat generation (W)	$(70 * AMASS ^ 0.75) * (4.185 / (24 * 3.6)) / 2$	$(70 * AMASS ^ 0.75) * (4.185 / (24 * 3.6)) / 2$
DELTAR: offset between air temperature and breath (°C)	5	5
EXTREF: O ₂ extraction efficiency	25	25
PANT_INC: increment by which panting is increased	0.05	0.05
PANT_MAX: Maximum panting rate	7	7
PANT_MULT: multiplier on basal metabolic rate at maximum panting level	1	1
AK1: initial thermal conductivity of flesh	0.412	0.412
AK1_INC: increment by which AK1 is increased per iteration	0.1	0.1
TREGMODE	2	2
TORPOR	1	1
AK1_MAX: maximum flesh conductivity (W/mK)	2.8	2.8
TC_MIN: Minimum core temperature (°C)	8	8

Table S4. Results of the correlation tests between respirometry results and endotherm model predictions for female and male big brown bats (*Eptesicus fuscus*). Tests included R^2 , Pearson correlation and Root mean square deviation (RMSD).

Sex	Variable	R^2	Pearson's Correlation					RMSD
			r	t	df	p	CI	
Female	T_b (°C)	0.845	0.919	20.643	78	< 2.2e-16	0.877 0.948	1.958
Female	waRMR (W)	0.636	0.797	11.594	77	< 2.2e-16	0.699 0.865	0.042
Female	waEWL (H ₂ O g/h)	0.931	0.965	32.274	77	< 2.2e-16	0.945 0.977	0.079
Male	T_b (°C)	0.887	0.942	23.386	70	< 2.2e-16	0.908 0.963	1.693
Male	waRMR (W)	0.574	0.757	9.6367	69	2.097e-14	0.637 0.842	0.044
Male	waEWL (H ₂ O g/h)	0.754	0.868	14.651	70	< 2.2e-16	0.797 0.916	0.145