UNIVERSITEIT VAN PRETORIA
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Proposing the variable finance capacity model for fundamental moments in turnarounds

Submitted in fulfilment of the requirements for the degree in

PhD Business Management

In the Faculty of Economic and Management Sciences

At the University of Pretoria

Under the supervision of Professor Marius Pretorius

By

Karl Johannes Gribnitz

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-PhD dissertation-

Proposing the variable finance capacity model for fundamental moments in turnarounds

by

Karl Johannes Gribnitz

Institution	University of Pretoria
Home department	Business Management
Supervisor	Prof M. Pretorius
Strategy of inquiry	Theory and model description

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January 2022

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The best opportunities always arrive at the most inopportune time.

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Acknowledgements

The origins of this study date back to 2013, but it was only in 2017 that I sat down to develop the set of equations contained in this study. Then life happened, and I became preoccupied. I had every intention to take the research further but could not find the time to complete the study. As we all know, 2020 brought about the Covid-19 pandemic, which saw an increase in the restructuring of businesses as firms faced financial distress and required business rescue or turnarounds. June 2020 brought a personal crisis that led to various unexpected trials and tribulations and resulted in my personal life being turned upside down. In August 2020, I decided to complete this study and dedicate my free time to its completion. During this time, a handful of people supported me, assisted me in my issues, and supported me in various ways to complete this study.

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The best opportunities always arrive at the most inopportune time, and maybe I can take up golf again.

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Preface

Undertaking this study has surprised me in many ways. I am surprised at how enormous the chasm between practice and research is in dealing with turnarounds. Researchers dealing with the process of turnarounds have created theories that are elegant but fail to offer measurements thereto. Ironically, these researchers quote Bibeault (1982), who interviewed turnaround practitioners, and describe in colourful language how managing cash flow and obtaining stakeholder support is critical to the firm's survival. I was surprised to discover that the research I have studied seeks to find elegant theories to describe the process during a turnaround and then create statistical methods to substantiate these theories. At the same time, the measurement of a turnaround is calibrated in a manner that cannot contradict the outcomes they seek to confirm.

In my studies, I found that researchers do not seek to understand that strategy selection is a much more integrated process that aims to optimise the best outcomes given the limited available resources and not provide the best out of given unlimited resources. Undertaking a turnaround is a stressful affair. Time becomes compressed. It is like a spring uncoiled – as a leader arrives to take charge, everybody wants their issues addressed immediately. Making sense of the chaos and mountains of information presented and determining a clear and concise implementation strategy signify a successful turnaround practitioner.

I was surprised by the results of this study, where every unique set of inputs led to a unique set of outputs. Measuring turnarounds in a calibrated manner allows for comparison, whether it is between different strategies of different industries, countries, or periods. It offers the tools to take a fresh look at the research related to the process of turnarounds. It offers turnaround practitioners the ability to engage with stakeholders in a meaningful manner as the measurement tools are calibrated. An established nomenclature to facilitate discussion may increase trust amongst the parties, leading to enhanced decision-making.

This thesis was completed on 28 February 2022.

Pecanwood Golf Estate, Broederstroom, Northwest Province, South Africa.

Karl Johannes Gribnitz

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Table of contents

Declaration regarding plagiarism i
Acknowledgements iii
Preface v
Table of contents vi
List of tables xii
List of figures xiii
List of abbreviations and acronyms xv
Glossary xvi
Chapter 1: Background and introduction to fundamental moments during a turnaround 23
Abstract 23
1.1 Introduction and background to this study 24
1.2 Problem statement of this study 26
1.3 Purpose statement in this study 27
1.4 Research questions considered in this study 27
1.5 Unit of investigation of this study 28
1.6 The significance of this study and its contribution to science 28
1.7 Utilisation in the field of turnaround around process research 29
1.8 Application in the turnaround industry 31
1.9 Summary of the chapter 32
Chapter 2: Exploring the salience of fundamental moments in turnaround research 33
Abstract 33
2.1 Introduction and background 34
2.2 The phenomenon of investigation of this study 35
2.2.1 The purpose of the investigation 36
2.3 An overview of the literature dealing with the process of turnarounds 36
2.4 The methodology used to perform the study 42
2.4.1 Using the foundation of strategy process research 43
2.4.2 Research design and methodology used in the study 44
2.5 Findings: In search of fundamental moments 50
2.5.1 Analysis performed on the studies dealing with the turnaround process 50
2.5.2 Discussion about fundamental moments in turnaround research 56
2.6 Conclusion 61

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References 62

Chapter 3: Using traditional cash flow "TCF" calculations to determine fundamental moments during turnarounds 67

Abstract 67

3.1 Introduction to the role of the turnaround plan and the TCF calculations in the management of turnarounds 68

3.2 Stakeholder theory as a framework for this research 69

3.2.1 Overview of the stakeholder theory for going concerns 70

3.2.2 Stakeholder theory or relationships in ZoI 72

3.2.3 Stakeholder theory as a framework for this study 73

3.3 The purpose and construction of TCF calculations 74

3.3.1 What is the purpose of TCF calculations? 74

3.3.2 The rules and construction of the TCF calculation 76

3.3.3 Turnaround plans – the reams of paper very few stakeholders read 98

3.4 Summary when dealing with the TCF calculations and a turnaround plan 107

Chapter 4: Research methodology used to develop the proposed VFC model to inform fundamental moments 110

Abstract 110

4.1 Introduction and background to the methodology applied and purpose of the . various chapters in this study 111

4.2 The research methodology and modelling process used in this study 112

4.2.1 The modelling cycle used in the development of this study 113

4.2.2 Establishing the problem statement and research questions in the real world (Stages 1 and 2) 114

4.2.3 Mathematising and development of the VFC model (Stages 3, 4 and 5) 117

4.2.4 Establishing derivatives, validating the values for fundamental moments (Stages 6, 7, and 8) 118

4.2.5 The utilisation of the VFC model and establishing a standardised nomenclature (Stages 9 and 10) 119

4.2.6 Summary of the research methodology implemented in this study 120

4.3 Mapping the research problem, purpose, and questions to the chapters of this ... study that deal with the development of the VFC model and outcomes thereof 120

4.3.1 A brief overview of the chapters included in this study 123

4.4 Summary 124

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Chapter 5: Proposing the VFC model that informs fundamental moments during a turnaround 125
Abstract 125
5.1 Introduction and background 126
5.2 Background to proposing the VFC model 126
5.2.1 The purpose statement of this chapter 127
5.3 Introducing the VFC model 129
5.4 The methodology used to derive the model 130
5.4.1 The inputs that are used to create a VFC model 131
5.4.2 The equations used to constitute the VFC model 133
5.4.3 The appropriateness, consistency, and accuracy of the VFC model as a fit to that of the TCF calculations 135
5.5 Introducing fundamental moments as derived from the VFC model 144
5.5.1 The duration of the turnaround 145
5.5.2 Breakeven of a turnaround 147
5.5.3 The period when the nadir will occur 148
5.5.4 Cash flow required to fund the nadir 149
5.5.5 Presentation of fundamental moments 150
5.6 Findings and recommendations 152
5.6.1 Using the VFC model to create boundary values for the same strategy 152
5.6.2 Using the VFC model to evaluate the different strategies 153
5.7 Summary and conclusion 155
Chapter 6: Discussion and recommendations on the implementation of the VFC model that informs fundamental moments during a turnaround 156
Abstract 156
6.1 Introduction and background 157
6.2 Discussion of the outcomes of this study 158
6.2.1 Can the VFC model and fundamental moments be defined as a theory? 158
6.2.2 Some observations and nomenclature derived from the VFC model and fundamental moments 167
6.3 The potential benefits of using the VFC model and fundamental moments in the turnaround industry 169
6.3.1 Future specific areas of research that may be developed using the VFC model and fundamental moments 170
6.4 Limitations of this study 171
6.5 Summary and conclusion 174

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List of references 176

Annexure 1: Developing the mathematical **VFC** model and fundamental moments 186

Abstract 186

A1.1 Introduction 187

A1.2 Summary of assumptions used to develop the **VFC** model 188

A1.3 The methodology used to create the **VFC** model 190

A1.4 The assumptions used to create a **VFC** model 193

A1.5 Development of the mathematical model to determine **VFC** 194

A1.6 Creating the **VFC** model for any period of **tx**, subject only to condition **α** (no AC (**μ**) incurred and PC (**ψ**) implemented from the period **t1**) (A1) 194

A1.6.1 Calculating the duration (the period funders will recover the funding used to fund the turnaround), assuming condition α is present (no AC (μ) and the PC (ψ) implemented from the period t1) (B1) 199

A1.6.2 In which period will the business reach breakeven assuming condition α is present? (no AC (μ) and the PC (ψ) implemented from the period t1) (C1) 200

A1.6.3 In which period will the cash flow nadir be reached, assuming condition α is present? (no AC (μ) and the PC (ψ) implemented from the period t1) (D1) 201

A1.6.4 What is the size of the funding required or the value of the cash flow nadir (Acc λmin(ψ)) assuming condition α is present? (no AC (μ) and the PC (ψ) implemented from the period t1) (D1) 202

A1.7 Creating the **VFC** model for any period of **tx**, subject to conditions **α and β** (where AC (**μ**) incurred and the PC (**ψ**) implemented from the period **t1**) (A2) 202

A1.7.1 Calculating the duration (when the funders will recover the funding used to fund the turnaround), assuming conditions α and β are present (with AC (μ) being incurred from the period t0 and the PC (ψ) implemented from the period t1) (B2) 206

A1.7.2 In which period will the business reach breakeven, assuming conditions α and β are present? (no AC (μ) and the PC (ψ) implemented from the period t1) (C2) ... 207

A1.7.3 In which period will the cash flow nadir be reached, assuming conditions α and β are present? (with AC (μ) incurred from the period t0 and the PC (ψ) .implemented from the period t1) (D2) 208

A1.7.4 What is the size of the funding required for the value of the cash flow nadir (Acc λmin(n)), assuming conditions α and β are present? (with AC (μ) incurred from the period t0 and the PC (ψ) implemented from the period t1) (E2) 208

A1.8 Creating a **VFC** model for any period of **tx**, subject to conditions **α, β and γ** (where AC (**μ**) incurred and the PC (**ψ**) not implemented from the period **t1** as a delay incurred of **k** periods) (A3) 210

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A1.8.1 Calculating the duration (when the funder will recover the funding used to fund the turnaround), assuming conditions $\boldsymbol{\alpha, \beta}$ **and** $\boldsymbol{\gamma}$ are present (with AC ($\boldsymbol{\mu}$) incurred from period $\mathbf{t0}$ and a delay of $\mathbf{k}$ periods before the implementation of PC ($\boldsymbol{\psi}$) from period $\mathbf{tk+1}$) (B3) 213

A1.8.2 In which period will the business reach breakeven, assuming conditions α, β and γ are present? (no AC (μ) and the PC (ψ) implemented from the period $t1$) (C3) 217

A1.8.3 In which period will the cash flow nadir be reached, assuming conditions α, β and γ are present? (with AC (μ) incurred from the period $t0$ and a delay of k periods before the PC (ψ) occurs from the period $tk+1$) (C3) 217

A1.8.4 What is the size of the funding required for the cashflow nadir ($\mathrm{Acc}\ \lambda\min(k)$), assuming conditions α, β and γ are present? (with AC (μ) incurred from the period $t0$ and a delay of k periods before the implementation of the PC (ψ) from the period $tk+1$) (E3) 218

A1.9 Conclusion 219

Annexure 2: Testing the fit for appropriateness, consistency, and accuracy between the **VFC** model against that of the **TCF** calculations 221

Abstract 221

A2.1 Introduction and background 222

A2.2 Setting the parameters to evaluate the fit of the **VFC** model to that of the **TCF** calculations 223

A2.2.1 The computer and software used to perform the test sets and the accuracy level required for tests 223

A2.2.2 Setting the parameters against which the test sets are measured to determine if the *VFC* model is an appropriate, consistent, and accurate fit to that of the TCF calculations 224

A2.2.3 The design of the master test set that will be used to generate the simulations to verify the fit of the VFC model to that of the TCF calculations 229

A2.2.4 Design of the master test set that will be used to conduct the various tests for different input values 230

A2.2.5 Utilising the master test set to verify the different values for the various input values 231

A2.3 A detailed description of the test sets to determine the accuracy of the fit between the **VFC** model and the **TCF** calculations 232

A2.3.1 Design and creation of the input table and the calculation of the VFC values for periods 0 to 32 232

A2.3.2 The design and construction of the TCF calculations used in the master test set 233

A2.3.3 Creating outputs using the TCF calculations and the VFC model to verify the fit between the two 237

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A2.3.4 Identification and labelling of the tests 238
A2.4 Performing the tests for the different values of $-\lambda, \psi, -\mu$ and k 240
A2.4.1 Creation of the summary of the various tests using SCT 241
A2.5 Findings of the tests performed 243
A2.5.1 Testing for appropriateness 243
A2.5.2 Testing for consistency 244
A2.5.3 Testing the various models for accuracy 246

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List of tables

Table 2.1: Analysis performed in search of fundamental moments 51

Table 2.2: A summary of studies reviewed in search of fundamental moments as set out in Table 2.1 54

Table 5.1: Summary of the outcomes for the evaluation tests performed to determine appropriateness, consistency, and accuracy of fit of the VFC model to that of the TCF calculations 138

Table A1.1: Summary of the time and financial consequences when evaluating a turnaround 189

Table A1.2: A representation of the losses that keep recurring for every period if no action is taken 196

Table A1.3: The cumulative effect of the PC (ψ) implemented for every period 197

Table A1.4: How exceptional or AC (μ) arise over time 203

Table A1.5: Overview of the various elements required to create a VFC subject to conditions α and β 204

Table A1.6: Overview of the various elements of the VFC model what happens when the PC required are delayed (conditions α, β and γ) 210

Table A2.1: Summary of the outcomes for the evaluation tests performed to determine appropriateness, consistency, and accuracy of fit of the VFC model to that of TCF calculations 227

Table A2.2: A cluster error map of tests 1 to 100, where the difference was multiplied by 1E12 for $[a; b; 0; -10]$, showing that the errors only arise after period 25 and only for tests 21, 31, 41, 51, 61, 71, 81, and 91, representing 54 errors of 3 200 values 248

Table A2.3: A cluster error map of tests 101 to 200, where the difference was multiplied by 1E12 for $[a; b; 1; -10]$, showing that the errors only arise after period 26 and only for tests 11, 21, 31, 41, 51, 61, 71, 81, and 91, representing 52 errors of 3 200 values 249

Table A2.4: A cluster error map of tests 201 to 300, where the difference was multiplied by 1E12 for $[a; b; 2; -10]$, showing that the errors only arise after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 91, and 92, representing 51 errors of 3 200 values 250

Table A2.5: A cluster error map of tests 301 to 400, where the difference was multiplied by 1E12 for $[a; b; 3; -10]$, showing that the errors only arise after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 82, 91, and 92, representing 48 errors of 3 200 values 251

Table A2.6: A cluster error map of tests 401 to 500, where the difference was multiplied by 1E12 for $[a; b; 4; -10]$, showing that the errors only arise after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 82, 91, and 92, representing 44 errors of 3 200 values 252

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List of figures

Figure 1.1: A graphical representation of the relationship between the VFC model and fundamental moments of a turnaround, where these values are currently derived using TCF calculations 26

Figure 2.1: The four fundamental moments during a turnaround 36

Figure 3.1: A deconstruction of TCF calculations to determine the VFC required for each period, and that informs the fundamental moments of the turnaround 77

Figure 3.2: Example of how the accumulated cash flow and cash per period is created by using the TCF calculations to inform fundamental moments 81

Figure 3.3: The impact of stretching accounts payable and accelerating accounts receivable as the firm "creates" loans by using suppliers' funds to fund other activities of the firm 84

Figure 3.4: The level of detail created in TCF calculations to be fit for purpose versus the ability of the audience capability to understand the content thereof 94

Figure 3.5: Observations made about the interaction of projected and actual outcomes and the impact thereof on the confidence levels of stakeholders (as an approximation)..... 97

Figure 3.6: The process followed, and the content included in a turnaround plan 100

Figure 4.1: The process used to understand, create, and verify the VFC model that informs the fundamental moments 113

Figure 4.2: Positioning the development of the VFC model as being a dynamic deterministic model using discrete time intervals..... 118

Figure 4.3: Mapping the research problem, purpose, and questions to the chapters that deal with the development of the VFC model and outcomes thereof 122

Figure 4.4: Summary of the chapters contained in this study and how these interlink 123

Figure 5.1: The relationship between the inputs, the VFC model, and fundamental moments based on different assumptions given certain inputs..... 128

Figure 5.2: An example of an assumption sheet to determine the input values for the VFC model based on a specific turnaround strategy 142

Figure 5.3: Defining the fundamental moments of a turnaround 145

Figure 5.4: An analysis of what happens to duration (ρ) if there is a delay (k) in an adopted strategy, thereby delivering a boundary value for the same adopted strategy..... 147

Figure 5.5: Presentation of the boundary outcomes for the same strategy (A) using the VFC model assuming no AC (B) or anticipating different delays $(k = 1, 2, 3, 4)$ 153

Figure 5.6: A presentation of the different outcomes based on different strategies resulting in different inputs using the VFC model and fundamental moment sets 154

Figure 6.1: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (side view) 162

Figure 6.2: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (bottom view)..... 163

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Figure 6.3: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (front view) 164

Figure 6.4: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (front-side view) 165

Figure A2.1: The design and construct of the master VFC parameter table 234

Figure A2.2: Legend to the input parameter table as set out in Figure A2.1 235

Figure A2.3: Legend to the calculation of the auxiliary model as set out in Figure A2.1 237

Figure A2.4: The design of the creation of TCF calculations using an Excel® spreadsheet to determine fundamental moments 239

Figure A2.5: Legend to the TCF calculations to determine output values as shown in Figure A2.4 240

Figure A2.6: Comparing the values of the VFC model against the TCF model values 241

Figure A2.7: Comparing the VFC model against the Auxiliary model values 242

Figure A2.8: A graphical representation of the errors and anomalies of tests 1 to 100, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal 253

Figure 2.9: A graphical representation of the errors and anomalies of tests 101 to 200, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal 254

Figure A2.10: A graphical representation of the errors and anomalies of tests 201 to 300, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal 255

Figure A2.11: A graphical representation of the errors and anomalies of tests 301 to 400, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal 256

Figure A2.12: A: graphical representation of the errors and anomalies of tests 401 to 500, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal 257

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List of abbreviations and acronyms

AC	abnormal cost
AEA	autoethnographic approach
CEO	chief executive officer
DHET	Department of Higher Education and Training
HRD	human research development
IFRS	international financial reporting standards
PC	planned changes
PDF	portable document format
SCT	summary comparison tabs
SOCI	statement of comprehensive income
SOFP	statement of financial position
TCF	traditional cash flow
VFC	variable finance capacity
ZOI	zone of insolvency

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Glossary

The following equations, formulas, and definitions are used in this study to develop the algebraic model to calculate the VFC model and fundamental moments.

Definition	Description
a	is the average planned changes (PC) per period expressed as a proportion of loss $(-\lambda)$, as $\psi = \lambda/a$, where $\psi, a \in \mathbb{R}$ and $a > 0$, so that a is expressed as a denominator for the duration of the turnaround.
α	No abnormal costs (AC) exist, and the implementation of PC happens as planned.
Abnormal costs (AC) $(-\mu)$	are the adjusted average AC $(-\mu)$ per period t, calculated as the total estimated AC divided by the estimated turnaround duration where any such cost is deemed not to be an operational cost in the ordinary course of business and can include the costs of the professional team, such as the legal, financial structuring, and accounting team, plus any other costs associated with the rescue or turnaround that would not occur during the ordinary course of business. AC is represented by μ in this study and where the average AC per period expressed as a proportion of loss $(-\lambda)$, as $\mu = \lambda/b$, where $\mu, b \in R$ and $b > 0$, so that b is expressed as a denominator for the duration of the turnaround.
$Acc(\lambda_{\psi,\check{t}})$	returns the accumulated cash flow at the end of each period $\check{t}$ so that $Acc(\lambda_{\psi,x-1}) + f(t_x) = Acc(\lambda_{\psi,x+1})$. It does not include any AC for any period. It is calculated from period $f(t_0)$, being the first period of commencement, and for each period of $f(t_x)$, where $x = 1, 2, 3$ up and until the end of period $f(t_{\check{n}})$ as delays occurred in the PC. It represents a boundary value of the selected strategy, indicated by $\check{t}$, but with no AC (μ).

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$Acc(\lambda_{\psi,t})$ returns the accumulated cash flow at the end of each period t, so that $Acc(\lambda_{\psi,x-1}) + f(t_x) = Acc(\lambda_{\psi,x+1})$. It does not include any AC for any period. It is calculated from period $f(t_0)$, being the first period of commencement, and for each period $f(t_x)$, where $x = 1, 2, 3$ up and until the end of period $f(t_n)$, as no delays in implementation of the PC have occurred. It represents a selected strategy, indicated by t, but with no AC (μ).

$Acc(\lambda_t)$ returns the accumulated cash flow at the end of each period t, so that $Acc(\lambda_{x-1}) + f(t_x) = Acc(\lambda_{x+1})$. It includes any AC for any period and is calculated from period $f(t_0)$, being the first period of commencement, and for each period $f(t_x)$, where $x = 1, 2, 3$ thereafter, up and until the end of period $f(t_n)$, as no delays in implementation of the PC have occurred. It represents a selected strategy, indicated by t, that includes AC $(-\mu)$.

$Acc(\lambda_{\check{t}})$ returns the accumulated cash flow at the end of each period $\check{t}$ so that $Acc(\lambda_{\psi,x-1}) + f(t_x) = Acc(\lambda_{\psi,x+1})$. It includes any AC for any period. It is calculated from period $f(t_0)$, being the first period of commencement, and for each period of $f(t_x)$, where $x = 1, 2, 3$ up and until the end of period $f(t_{\check{n}})$, where $\check{n} = n + k$ as k represents the delays that occurred in implementing changes. It represents a boundary value of the selected strategy, indicated by $\check{t}$ and that includes AC $(-\mu)$.

$Acc\ \lambda_{min(\psi)}$ represents the nadir value of the cash flow requirements, where the maximum funding will be required to implement the turnaround for a selected strategy with no AC $(-\mu)$ associated in addition to that, and no delays (k) have occurred or are expected. This value can also serve as a boundary value for a selected strategy with AC. It cannot be implemented in any shorter period than represented in this equation.

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$Acc\ \lambda_{min(n)}$	represents the nadir value of the cash flow requirements, where the maximum funding will be required to implement the turnaround for a selected strategy, including AC $(-\mu)$ associated with the turnaround, and no delays (k) have occurred or are expected to occur.
$Acc\ \lambda_{min(k)}$	represents the nadir value of the cash flow requirements, where the maximum funding will be required to implement the turnaround for a selected strategy, including AC $(-\mu)$ associated with the turnaround, and delays (k) have occurred or are expected to occur.
b	is the average AC $(-\mu)$ per period expressed as a proportion of loss $(-\lambda)$, as $\mu = \lambda/b$, where $\mu, b \in \mathbb{R}$ and $b > 0$, so that b is expressed as a denominator for the duration of the turnaround.
β	AC exists, and the implementation of PC happens as planned.
Columns	refer to a vertical column of cells shown in the test set, consisting of two tabs as contained in the spreadsheet.
$E_{(\psi)}$	is defined as the period where the business will reach breakeven, which is where the loss $(-\lambda_0)$ is equal to the PC $(\sum_{x=1}^{y} \psi_x)$, given there are no delays (k), and there is no AC $(-\mu)$, and it is a boundary value to the selected strategy.
$E_{(n)}$	is defined as the period $f(t_b)$, where the business will reach breakeven, which is where the loss $(-\lambda_0)$ is equal to the PC $(\sum_{x=1}^{y} \psi_x)$, given there are no delays (k) but incurring AC (μ), and it is a part of the selected strategy.
$E_{(k)}$	is defined as the period $f(t_b)$, where the business will reach breakeven, which is where the loss $(-\lambda_0)$ is equal to the PC $(\sum_{x=1}^{y} \psi_x)$, where actual or expected delays (k) may occur while incurring AC (μ), and it is a boundary value to the selected strategy.
Equations	refer to the equations developed as set out in Annexure 1.

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Excel formulas	refer to the formulas that contain calculations and conditions, which may use absolute or relative references as input values in various cells forming a part of the test set that forms part of the spreadsheet.
γ	AC exists, and the implementation of PC is delayed.
k	is defined as the number of periods by which implementation of the plan is delayed compared to the original or previous estimate and where $k \in \mathbb{R}$.
Inputs or input parameters	refer to the inputs calculated from various estimates so that: the value of the loss $(-\lambda)$ in the period immediately preceding implementing/actioning the turnaround, the improvements to be achieved and defined as planned changes (ψ), AC (-μ) relating to costs associated to implementing/actioning the turnaround, and if there have been any delays (k) in the implementation of the planned changes. These inputs are combined, and different combinations thereof are applied to the test sets.
Planned changes (PC) (ψ)	are the adjusted average PC (ψ) per period (t), calculated as the total estimated PC divided by the estimated turnaround duration. A planned change can consist of a saving to be implemented (retrenchment), improving the efficiency of operations, or strategically reorientating the firm's operations during a turnaround that would not occur during the ordinary course of business. Planned changes are represented as (ψ) in this study. The adjusted average PC per period is expressed as a proportion of loss $(-\lambda)$, as $\psi = \lambda/a$, where ψ, $a \in R$ and $a > 0$, so that a is expressed as a denominator of the duration of the turnaround.
$-\lambda_0$	is defined as the loss that the business made before the commencement of the turnaround at period (t_0) and where $-\lambda \in \mathbb{R}$ per period t. It is assumed that the business will continue making this loss for every period of t after that if no further intervention is undertaken. The loss can be measured in accounting terms or can be deemed to be a net cash loss. It can be argued that a loss usually grows because of the interest or other compounding

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factors; however, such additional losses are defined as additional costs to the loss ($-\lambda_0$) and are added to the AC.

$-\mu$ is the AC per period (t) and can be calculated as the total estimated AC divided by the estimated turnaround duration. Any such cost not deemed to be an operational cost in the ordinary course of business and can include the costs of the professional team, such as the legal and accounting team, plus any other costs associated with the rescue or turnaround, which would not occur during the ordinary course of business. Suppose there is no AC, then $\mu = 0$.

ψ_t is the planned changes per period (t) and can be calculated as the total savings to be implemented divided by the duration of the individual saving that will take the most prolonged period to implement. Alternatively, the weighted average period of all the PC can be used. These PC can be achieved by cost-cutting, efficiency improvements, growth in sales, or any other savings proposals to be included in the calculation.

$\breve{n}_{min(\psi)}$ represents the period where the business will reach nadir ($Acc\ \lambda_{min(\psi)}$) as no AC ($-\psi$) are incurred, and no delays (k) in PC (ψ_t) have occurred. The value $\breve{n}$ indicates that it is a boundary value of a selected strategy.

n_{min} represents the period where the business will reach nadir ($Acc\ \lambda_{min(n)}$), where AC ($-\mu$) are incurred, and no delays (k) in PC (ψ_t) have occurred. The value n indicates that this forms a part of the selected strategy.

$\breve{n}_{min(k)}$ represents the period where the business will reach nadir ($Acc\ \lambda_{min(k)}$), where AC ($-\mu$) are incurred, and delays (k) in PC (ψ_t) have occurred. The value $\breve{n}$ indicates that it is a boundary value of a selected strategy.

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$t_{\breve{n}} or\ \breve{n}$	returns the duration of the turnaround or being the point where the capital to fund the turnaround is recovered so that $Acc(\lambda_{\psi}) = 0\ or Acc(\lambda_k) = 0$. It is the period where the accumulated cash flow is equal to zero as the accumulated losses $(-\lambda(\breve{n} + k + \rho))$ plus any accumulated AC $(-\mu(\breve{n} + k + \rho))$ are equal to the accumulated PC $(\sum \psi_t)$. Alternatively stated, it is the period where the accumulated cash inflows are equal to the accumulated cash outflows, and there have been delays of k periods in the PC (ψ_t) that implied that $t_{\breve{n}} > t_n$, and $t_{\breve{n}} \neq t_n$ and represent a part of the boundary values as indicated by $\breve{n}$.
$t_n or\ n$	returns the duration of the turnaround or being the point where the capital to fund the turnaround is recovered so that $Acc(\lambda_n) = 0$. It is the period where the accumulated cash flow is equal to zero as the accumulated losses $(-\lambda(n + 1))$ plus any accumulated AC $(-\mu(n + 1))$ are equal to the accumulated PC $(\sum \psi_t)$. It also indicates that there are no delays of (k) periods in the PC (ψ_t) and that this is part of the selected strategy.
ρ	is defined as the additional number of periods it would take to recover the additional cash flow shortfall due to the delay by k periods to implement the plans compared to the original or previous estimate and where $\rho \in \mathbb{R}$.
Rows	refer to a horizontal row of cells shown in the test set, consisting of two tabs as contained in the spreadsheet.
Spreadsheet	refers to Microsoft® Excel® 2019 spreadsheets containing tabs (pages) consisting of rows and columns used to perform the various calculations to verify if the models and calculations perform the various individual tests.
SCT	refers to summary comparison tabs where the outcome values for a specific set of tests performed are consolidated into a single tab per spreadsheet. Every spreadsheet contains an SCT, where the comparison test between the VFC model and TCF calculations occur, numerically verifying the fit between the VFC model and the TCF calculations.

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Tab	refers to the individual pages in a standard spreadsheet that can be copied, retaining all the absolute and relative formulas used to create a test set. New input values can be inserted to derive new output values for the VFC model.
Test set	refers to two tabs containing various calculations consisting of formulas used to determine and compare the output values of the VFC model and the traditional cash flow (TCF) calculations.
TCF calculations	refer to TCF calculations consisting of various income items and various expenses grouped and summed. The total expenses are deducted from the income to provide a cash flow for the period (t). Cash flow at the end of the previous period (t_{x-1}) is also referred to as cash flow at the beginning of the existing period. These cash flows are added together and referred to as cash flow at the end of the period (t_x).
Turnaround	the process where an acute or severe management intervention occurs after a decline in business operations due to experiencing losses for consecutive periods, thereby leading to the actual or threat of capital loss by funders and suppliers.
x	Represents the index used when calculating the VFC model, which is in equal periods, where $x \in \mathbb{R}$. A business rescue practitioner may want to measure in weeks, months, or quarters, while a turnaround practitioner may think about months or even years.
Variable Finance Capacity (VFC)	is represented by accumulated cash flow $Acc(\lambda_{,n})$ at the end of each period and is critical in determining if the company will have the necessary working capital requirements to fund the operations. VFC is a registered trademark.

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Chapter 1: Background and introduction to fundamental moments during a turnaround

Abstract

There is currently no standardised manner to measure, evaluate, and define the various fundamental moments or determine the status of a turnaround. The fundamental moments of a turnaround can be defined by asking (1) how long is the turnaround going to last; (2) when is the firm going to breakeven; (3) when is the cash flow nadir going to occur; and (4) how much funding is required to cover the cost of the turnaround? Currently, to answer these questions, a string of numerical solutions in elaborate bespoke spreadsheets of varying complexity are created. Then, fundamental moments are determined by inspecting the string of the numerical values.

As a result of the lack of standardisation in the process of turnarounds, intra- or inter-comparisons of performance are cumbersome and unlikely to be measured. The inability to evaluate the performance of a turnaround results in a lack of support from potential funders, as they do not have the tools to assess if the indicated fundamental moments of the turnaround are calculated correctly. This study provides a standardised method to measure the progress and turnaround process, allowing the stakeholders to make enhanced decisions when dealing with turnarounds.

This chapter sets out the introduction and background of this study, which incorporates the problem statement, purpose of the study, and research questions provided.

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1.1 Introduction and background to this study

Research dealing with the turnaround process offers no single standardised single definition for a turnaround. However, various authors have presented similar themes that state that company performance has declined and posted losses for several periods and require drastic management intervention to change the performance (Barker & Duhaine, 1997; Chowdhury, 2002). Grinyer, Mayes and McKiernan (1990:116) argue that "… a sharp, sustained improvement in performance from the base of relative decline is termed a 'sharpbend' and companies that successfully negotiated this change are known as 'sharpbenders'." Bibeault (1982:1) states that a turnaround requires management interventions distinctly different from those used in managing a firm in stable or growth periods. He continues by stating that "… growth considerations no longer exercise their appeal; even the bottom line takes a back seat to cashflow objectives". Having a calibrated measurement tool to measure and evaluate these objectives is the essence of this statement and encapsulates the importance of this study.

This study introduces the variable finance capacity (VFC) model that is the result of the mathematisation of the traditional cash flow (TCF) calculations undertaken using spreadsheets. The VFC model is utilised to determine the fundamental moments during a turnaround. A turnaround practitioner must convince stakeholders to support the turnaround process. Making an informed decision to implement a specific turnaround strategy is predicated on having enough funds to affect such an implementation. The practitioner can obtain the funds to affect the turnaround from the existing internal resource slack availability (Audia & Greve, 2006; Barker & Duhaime, 1997) or stakeholder support (Arogyaswamy, Barker & Yasai-Ardekani, 1995; Gopinath, 1991). A turnaround is a dynamic process (Chowdhury, 2002), where a sequence of actions has to occur (inputs) in a specific time frame that will lead to a successful implementation of a turnaround or not. A plan must be prepared to ensure enough resources or "resource slack" (Greve, 2011:104) to implement the turnaround strategy. Stakeholders want to measure the progress made with the implementation of the turnaround, which may affect stakeholder support and lead to a change in the implementation strategy for the turnaround.

Traditionally, fundamental moments are determined by bespoke spreadsheets that are cumbersome to analyse, difficult to verify, complicated to understand, and open to manipulation. This results in a reluctance by third parties to engage with these spreadsheets,

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meaning very little value is attached by stakeholders to the fundamental moments calculated in this manner. The more complex the spreadsheet, the more difficult it may be to verify the fundamental moments or the turnaround status. By creating standardised predictions and measurement tools with the resulting standardised nomenclature, stakeholders are provided with the ability to engage in the turnaround in an informed manner, enabling enhanced decision-making.

This study proposes the VFC model that maps the financial and time consequences of a turnaround, summarised in a set of fundamental moments. Figure 1.1 (see overleaf) shows graphically how this study attempts to define and calculate these fundamental moments so that:

1. the duration of the turnaround;
2. breakeven period;
3. the period when nadir is going to occur; and
4. what funding is required to fund the nadir can be determined in a standardised manner.

These four values form a defined set of fundamental moments based on the unique input values used and result in different fundamental moment sets throughout the turnaround duration. The ability to map these fundamental moment sets allows the practitioner to select an appropriate strategy based on the resources available and stakeholder support for such a selected strategy.

The contribution of this study is to provide a standardised prediction and measurement tool to determine the fundamental moments during a turnaround. By considering inputs, the VFC model enables enhanced decision-making when committing to a specific turnaround strategy based on the outcomes of various predicted fundamental moments. It enables researchers, banks, and other stakeholders to perform intra- and inter-comparisons of the progress and proposed strategies of turnarounds in real-time and make decisions in an informed manner. As the old adage goes, “you cannot manage if you cannot measure”.

Figure 1.1 (see overleaf) provides an overview of the relationship between the research questions, problem statement, purpose statement, and process to deliver the model.

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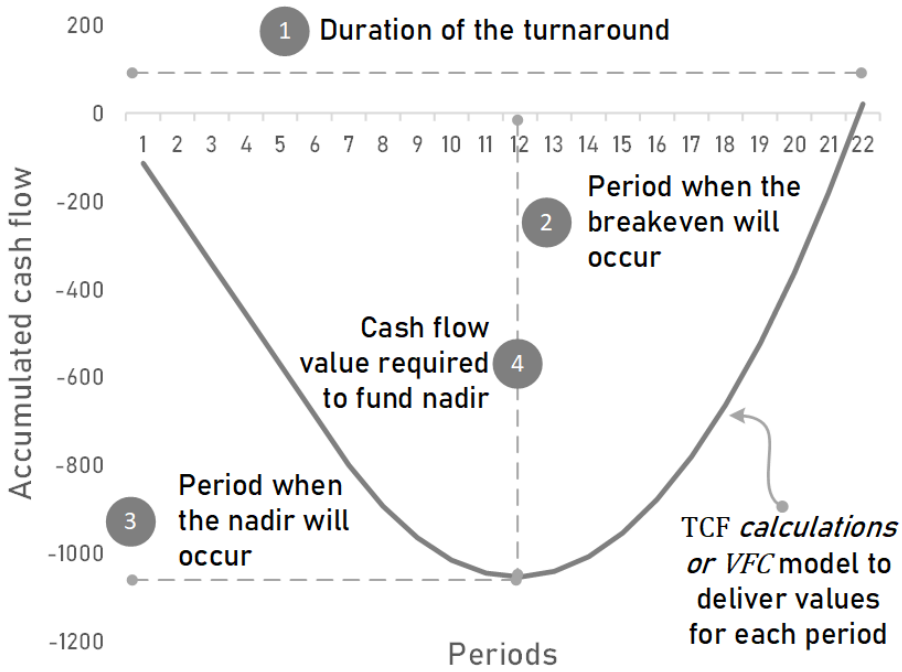

Figure 1.1: A graphical representation of the relationship between the VFC model and fundamental moments of a turnaround, where these values are currently derived using TCF calculations

Source: Author (2021)

This chapter sets out the problem statement, the purpose of the study, the research questions considered, and the unit of investigation and concludes by explaining what contribution this study can make to science.

1.2 Problem statement of this study

There are currently no quantitative expressions to measure fundamental moments, thus limiting the ability to conduct research and make enhanced decisions to finance turnarounds. The research undertaken in Chapter 2 of this study shows that in the past 42 years, research dealing with the process during turnarounds has not defined fundamental moments, postulating different views based on semantics without providing accurate and verifiable descriptions of fundamental moments. Research undertaken in this study shows that only 16 per cent of the research undertaken refers to the funding of the nadir and its timing, and then only by way of a vague implied reference. This study introduces the VFC model that uses quantitative expressions to replace the need to create bespoke spreadsheets to predict future cash flows based on implementing a selected strategy. It can be argued that fundamental

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moments are the loadstar whereby turnaround practitioners should manage existing turnarounds. Research dealing with the turnaround process seemed to have ignored the use of fundamental moments, how it is determined, and how it may influence the outcome of a turnaround.

Fundamental moments calculated using spreadsheets result in a lack of universal acceptance by stakeholders. The answers derived via a bespoke spreadsheet by a practitioner can be easily manipulated and, therefore, requires a careful analysis to accept these answers. Providing a mathematical model that calculates the fundamental moments can result in universal acceptance by stakeholders and offer a basis for further research opportunities, such as creating a nomenclature, when dealing with the status of a turnaround. Research on the turnaround process does imply but does not formally discuss or define that the practitioner's decision horizons are bound by the realities such as resource availability and stakeholder support. While a practitioner has infinite choices to implement any strategy, these choices are bounded by the reality of the time and the unique financial effects of each turnaround.

1.3 Purpose statement in this study

The purpose of this investigation is to create, verify, and introduce the VFC model:

1. that replaces the TCF calculations to determine the accumulated cash flow values for each period;
2. to determine the fundamental moments for a turnaround based on different input values; and
3. to establish a standardised nomenclature for fundamental moments, defining what each element means and how these are to be presented in fundamental moments sets.

1.4 Research questions considered in this study

The main research questions considered in this study are:

1. Can the VFC model be algebraically derived to replace the TCF calculations to determine the accumulated cash flow for each period during the turnaround?
2. Can the VFC model provide the foundation for a set of algebraic equations to determine fundamental moments?

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Further research sub-questions are:

1. How appropriate, consistent, and accurate is the fit between the VFC model and the TCF calculations?
2. Can a standardised nomenclature be developed for individual fundamental moments, leading to enhanced decision-making?
3. What role may fundamental moments play in future research?

1.5 Unit of investigation of this study

The unit of investigation is a mathematically calibrated measurement tool to measure the fundamental moments that may lead to enhanced decision-making when dealing with turnarounds. A firm has experienced a decline in performance (Weitzel & Jonsson, 1989) and undergoes a drastic management intervention to return the firm to levels of performance as differently defined by various authors. This study falls in the research domain on the process of turnarounds, which forms a part of change management and is, in turn, a part of management strategy. The VFC model would enable practitioners and researchers to use the model on an *apriori* basis when deciding on the best strategy to implement a turnaround. In contrast, researchers can use the model on a *posteriori* basis to evaluate the turnaround.

1.6 The significance of this study and its contribution to science

This study provides a measurement tool by proposing the VFC model that can be used to measure the projected (*apriori*) and implemented (*posteriori*) strategies of a turnaround. This study shows that the VFC model is a perfect fit for TCF calculations that may improve the trust between the parties that may use the VFC model. The calculation and presentation of fundamental moments can inform enhanced decision-making when dealing with turnarounds. Standardising the nomenclature of each element of the fundamental moments allows for universal acceptance and utilisation by researchers, turnaround practitioners, and stakeholders. The VFC model and fundamental moments have far-reaching implications for research on turnarounds. It offers a metric that can further the development of turnaround theory and serves as a tool to undertake new empirical research. It enables a conversation on the subject of turnarounds, allowing for enhanced decisions while providing a new platform that can propel the stagnated field of research in the process of turnarounds.

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1.7 Utilisation in the field of turnaround around process research

Turnaround management forms a part of the business or strategy domain. It is a relatively new field of research, with fewer than 300 studies dealing with content, context, and processes published on the subject matter in the past 45 years. According to Schweizer and Nienhaus (2017), it seems that research into turnarounds has stagnated for approximately 20 odd years, and research undertaken in Chapter 5 of this study shows that 42 of 50 studies reviewed were published in or before 2010, with only three studies being published after 2015. The majority of empirical studies dealing with the process of turnarounds have adopted a positivist approach, where the researchers seek to confirm their prior theoretical concepts and models adopted from broader literature. Solnet, Paulsen and Cooper (2010:154) suggest that further research is required to "… adopt interpretive approaches to provide 'thick descriptions' of phenomena" and assist in building theories pertaining to turnarounds in the future.

Schweizer and Nienhaus (2017) performed an extensive literature study and identified nine gaps or lenses in the agenda for future research. Of the nine gaps identified, six require a standardised measurement system providing for:

1. the theoretical grounding;
2. avoiding sample biases;
3. comprehensively linking content (input) to process and context;
4. serving as a basis for refining longitudinal turnaround research design;
5. measuring the performance in the recovery phase; and
6. understanding the relationship between distress and success measurement.

Poole, van de Ven, Dooley and Homes (2000) state that better interpretive tools are required to understand the difference between turnarounds and decline, thereby indicating a lack of foundational theory on this subject matter. Chapter 2 of this study shows that criticism by one researcher of another researcher's findings seems disingenuous, as a lack of a uniform measurement system precludes any meaningful comparison of turnarounds.

The ability to measure performance during a turnaround is the single most crucial aspect as it:

1. determines the impact of the operational or strategic levers used during the turnaround;

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2. is able to determine the interaction of financial and operational restructuring during a turnaround;
3. allows for determining what role timing plays when implementing retrenchment or strategic actions;
4. allows for the measurement of the quantum of resources required and when those resources must be available (within time frames);
5. develops an indicator regarding what constitutes a successful turnaround or not;
6. determines what level of exploitation versus exploration is required to implement a successful turnaround plan;
7. provides a measurement tool to measure the impact of the cash flow requirements during a turnaround;
8. delivers a quantified timeline that measures expected outcomes forming an integral part of future research;
9. calculates how the various interdependencies impact the outcome of the turnaround; and
10. determines the impact a turnaround has on financial performance and how it converts into market expectations.

The introduction of the VFC model proposes an algebraic measurement system that emulates the existing numeric solutions used by turnaround practitioners to manage turnarounds. The measurement of fundamental moments enables the evaluation of the various strategies available to a practitioner during the turnaround. It offers the ability to compare the performance of the turnaround to that of the initial strategy selected. Researchers can now define the fundamental moments, creating a standardised nomenclature algebraically, thereby eliminating the confusion on defining duration, breakeven, or when and what quantum of resources will be required to implement the turnaround. For instance, if the duration of the turnaround is determined mathematically, it will allow the researcher to compare the duration of various turnarounds in different industries or compare the actual duration versus the planned duration of the turnaround. These equations can provide the basis for further development of the foundational theory of the process of a turnaround. The VFC model is a simulation of the planning tools that practitioners use, with the implication that new insights into the process of turnarounds that supersedes the various multi/two-stage models as proposed (Bibeault, 1982; Chowdhury, 2002; Hofer, 1980; Robbins & Pearce, 1993) can be

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achieved. This study proposes a toolset to enable future research in turnarounds and possibly the field of start-ups.

1.8 Application in the turnaround industry

Business cycles of growth and decline are part of the business life cycle. While the growth of the business is systematic and stable over a long period, a decline in business is represented by a combination of events that bring about aggressive declines occurring over a concise period. The decline results in a real or threat of flight of capital from the business, which, in turn, requires a focussed intervention to restore trust, thereby stemming the flight of capital and attracting new capital to continue on the path of growth. A turnaround forms a critical part of the business cycle. As the modern economy develops, businesses will find themselves moving from business-as-normal to turnaround and back again more frequently. Adopting a standardised measurement system allows stakeholders and new funders to make enhanced decisions, whether to adopt the proposed plan, stop the threat of flight of capital, or support the business with new capital.

When dealing with a turnaround, the ability to make enhanced decisions leaves little room for error or interpretation of how the turnaround practitioner derived and provided a proposal for implementation. Trust in the measurement system allows the stakeholders to deal with the issues at hand: how much is required, for how long, when will it be over, and what happens if there is a delay.

Providing the measurement tools to evaluate the proposed and actual performance of a turnaround may allow for more turnarounds or business rescues to be completed. Undertaking a turnaround and garnering stakeholders' support of stakeholders will result in retaining more jobs and reducing the destruction of capital due to liquidating a firm. More successful turnarounds will provide researchers with more case studies and the tools to advance turnaround management as a science.

This study proposes a presentation of fundamental moments in a manner that is easy to understand and compare, enabling a constructive conversation between the practitioner and stakeholders.

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1.9 Summary of the chapter

This chapter showed in the problem statement that there is no standardised measurement tool to measure fundamental moments during a turnaround. Fundamental moments measure the time and the financial consequences of a turnaround. This study aims to establish a model that mathematises the current method practitioners use to determine the outcomes based on the strategy selected to implement the turnaround.

The research questions expand the purpose of this study by determining:

1. How appropriate, consistent, and accurate the fit of the VFC model is to that of the TCF model;
2. Can fundamental moments be mathematically derived from the VFC model; and
3. Can a nomenclature and presentation method for fundamental moments sets be established?

The unit of investigation forms a part of the domain on the process of turnarounds, which forms a part of change management, which, in turn, is a part of management strategy.

The chapter concludes by explaining what contribution this study can make to science. This study provides a measurement tool that:

1. researchers can use to further the theory related to turnarounds and develop new empirical research designs; and
2. practitioners can make informed decisions during strategy selection based on the available resource slack and stakeholder support.

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Chapter 2: Exploring the salience of fundamental moments in turnaround research

This chapter represents and article that is envisaged for submission to an accredited Department of Higher Education and Training (DHET) journal. Accordingly, the referencing style is adapted to suit that of the journal. The reader may find certain concepts repetitive, but it is required to set out the arguments made herein.

Abstract

When selecting a turnaround strategy, a turnaround practitioner aims to understand: (1) how long the turnaround is going to last; (2) when the firm is going to breakeven; (3) when the cash flow nadir is going to occur; and (4) how much funding is required to cover the cost of the turnaround. These are defined as fundamental moments of a turnaround, which can become the loadstar for selecting and implementing an appropriate strategy. This study determined how research focusing on the process of turnarounds deals with fundamental moments. Results show that researchers do not consider fundamental moments as a key measurement tool to plan and implement a turnaround, and consequently, no standardised nomenclature dealing with the process of turnarounds exists.

Keywords: fundamental moments, turnaround, turnaround process, restructuring

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2.1 Introduction and background

How long is it going to last, and what is it going to cost? These are the two most basic questions asked during a turnaround from a practitioner by stakeholders. Bergh, Johnson and Dewitt (2008) found that managers adopt strategies that mitigate information asymmetries and maximise financial value. These questions form the composite for understanding: (1) the duration of the turnaround; (2) when the turnaround is going to break even; (3) when the cash flow nadir is going to occur; and (4) the size of the cash flow nadir. These four points describe the fundamental moments of a turnaround. This research study analysed 50 studies published over the past 42 years to determine how they dealt with turnarounds and considered these fundamental moments in their empirical research design or when propagating theories about the process of turnarounds.

Turnaround practitioners create spreadsheets to determine the cash flow impact of the possible strategies they can select and what resources are required to implement the turnaround and engage with stakeholders for support. These answers may *inter alia* influence an employee to leave or stay for the duration of the turnaround or whether an equity provider may want to invest in the turnaround. All firms operate under various constraints, including market conditions, technological limitations, and financial or human capital constraints. However, resource constraints become more pronounced during a turnaround and directly impact the selected strategy. Obtaining stakeholder support for the selected strategy is critical, and mapping progress is essential throughout the turnaround (Arogyaswamy, Baker & Yasai-Ardekani, 1995; Hambrick, 1985; Slatter, 1984).

Pandit (2000) defined the turnaround process, which was further expanded by Schweizer and Nienhaus (2017), describing what occurs in terms of actions and timeframes during a turnaround. Ultimately, capturing the effects of actions within timeframes should directly impact the fundamental moments of a turnaround, making it the loadstar influencing decisions in the turnaround process, both in research and practice. This study focused on how researchers have encapsulated fundamental moments into their research and investigated whether it refers wholly or partly and how it is defined or calculated. Some researchers estimate the duration, and others estimate the duration by determining the nadir of earnings

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on a *post priori* basis and then arbitrarily, including several periods before and after the nadir event.

The lack of calibrated measurement tools results in some empirical research to analyse those firms that had undergone a successful turnaround, implying a survival bias in the conclusions reached. Careful data selection may skew the focus and results of research, impacting future research questions (Barker & Mone, 1994).

2.2 The phenomenon of investigation of this study

Figure 2.1 shows how fundamental moments are defined during a turnaround: (1) *duration* – referring to the length of time it would take to complete the turnaround (what constitutes a start and end of a turnaround); (2) *breakeven* – the point where the cash flow inflows are equal to the cash flow outflows (measuring when exactly the fortunes of the firm changes); (3) *period when the cash flow nadir will occur* – in which period will the accumulated cash flow reach a nadir (measuring when the maximum cash commitment will be required to fund the turnaround); and (4) *how much cash flow will be required to fund the nadir* – where during the turnaround is the point where the maximum cash flow commitment will be required to fund the turnaround (how much is needed to fund the turnaround). This research aimed to determine the occurrence, status, acknowledgement, and calculation of fundamental moments in the salient studies on the turnaround process and if it is collectively or individually referred to as (1) implied, (2) estimated, or (3) mathematically derived. Therefore, this study aimed to understand how research relating to turnarounds deals with fundamental moments as it substantially impacts research on turnarounds.

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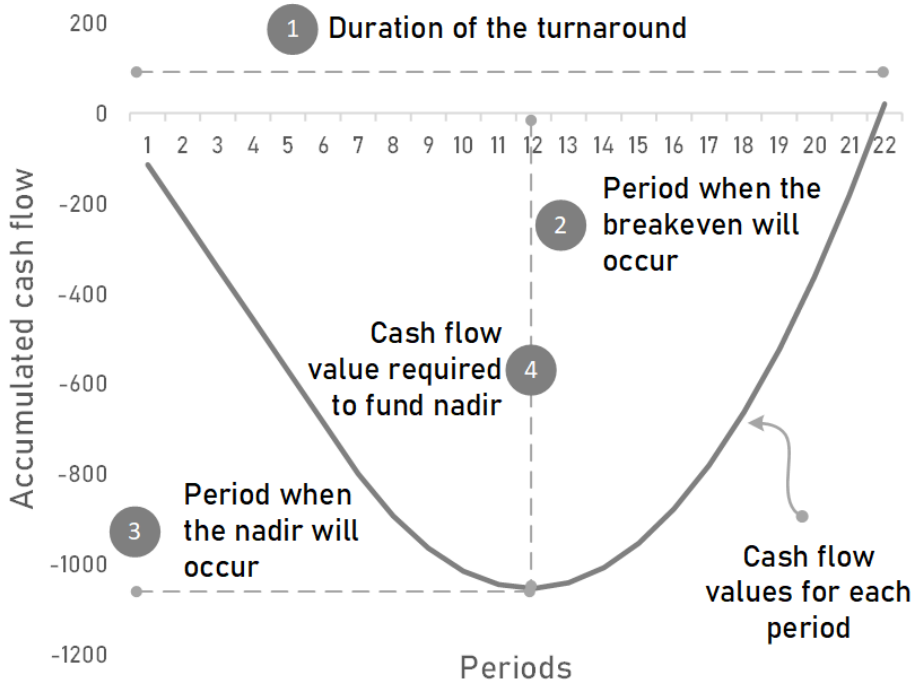

Figure 2.1: The four fundamental moments during a turnaround

Source: Author (2021)

2.2.1 The purpose of the investigation

A practitioner determines fundamental moments by creating a "cash flow spreadsheet" containing the projected periodic inflows and outflows to determine the cumulative impact. This cash flow spreadsheet determines the firm's breakeven (Plane, 1994; Ragsdale, 2017). These cash flow spreadsheets can vary in content, complexity, and style; however, regardless of how they are constructed, the purpose appears to be always the same – how much and when funds are required to fund the turnaround, how long it will last, and when will it be completed.

2.3 An overview of the literature dealing with the process of turnarounds

This section provides an overview of literature dealing with the process of turnarounds (Pandit, 2000; Schweizer & Nienhaus, 2017). Some researchers criticise the lack of measurement, while others propose that the cash flow forecast is critical at the commencement of the turnaround. Various authors refer to the nadir during a turnaround, using arbitrary references such as performance, and others seek to define it using financial ratios (Barker & Mone, 1994; Pearce,

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2007; Tangpong, Abebe & Li, 2015). These arguments stand in strong contrast to the original works by Hofer (1980) and Bibeault (1982), which indicated that cash flow models are critical during a turnaround as it informs the selected strategy. This section, therefore, focuses on themes that have a direct relationship with fundamental moments, namely: (1) the lack of measurement tools to measure the outcome of a turnaround; (2) the importance of breakeven during a turnaround; (3) the use of duration estimation when performing empirical studies; (4) the use of financial forecasting and cash flow models during a turnaround; and (5) references to resource slack.

Lack of measurement tools to measure the outcome of a turnaround – There is an adage in management circles that "if you cannot measure, you cannot manage". The single most significant pitfall of research dealing with the process of turnarounds to date is that there is no universal measurement system to determine whether a turnaround will be successful or not (Bhattacharyya & Malik, 2019; Filatotchev & Toms, 2006; Hoffman, 1989; Pandit, 2000; Schweizer & Nienhaus, 2017). Various researchers acknowledge that their research is subject to limitations as they cannot objectively measure how close the firm is to insolvency (Hambrick & Schecter, 1983; Hofer, 1980).

Various researchers use diagrams with horizontal and vertical axes that show the performance of the turnaround but provide no definition or metric of what such performance should be and do not offer any measure of performance or specificity of time (Battacharyya & Malik, 2019; Chowdhury, 2002; Filatotchev & Toms, 2006; Grinyer, Mayes & McKiernan, 1990; Pearce, 2007). Weitzel and Jonsson (1989) offer a diagram that shows the decline of a firm but does not offer any description for the vertical or horizontal axis, which can only be presumed to be "performance" and "time period". This lack of measurement hinders the advancement in research on the process of turnarounds and the theory of turnaround in general. Some researchers use financial ratios, or discriminants thereof (Pearce, 2007), while researchers pre-select data from listed entities that reported losses for various periods followed by a period of profits and then use five to eight years to determine the turnaround duration (Hambrick & Schecter, 1983; Morrow, Johnson & Busenitz, 2004; Ndofor, Vanevenhoven & Barker, 2013; Schendel & Patton, 1976; Schendel, Patton & Riggs, 1976). Barker and Mone (1994, p. 403) state that they "… lack the knowledge base to predict accurately which particular retrenchment strategies and practices promote recovery."

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Barker and Duhaime (1997) argue that large-sample studies seek to emphasise: (1) the measure of management response to the decline of the firm through the use of financial ratios, but the Statement of Financial Affairs obscures the actions the managers took in dealing with the decline; and (2) those firms selected for studies have been heterogeneous or had a low-level need for strategic change to recover.

Chowdhury (2002, p. 250) states, "A theory of turnaround is lacking because of a wide separation between empirical findings (based on either large samples or case descriptions) and work done toward systematically uncovering the causal structure of events from the onset of the firm's decline to its ultimate recovery or death." The inability to measure the impact of the financial and time consequences implies that empirical research on change management is fumbling in the dark trying to confirm hypotheses. It seems that the research into turnarounds has stagnated for approximately 20 odd years and that "… they are a long way up the empirical creek without a theoretical paddle" (Meyer, 1988, p. 413). Confirming or rejecting a hypothesis as to one strategy being better than another without measuring the initial and prevailing fundamental moments during the turnaround can only lead to biased answers. Comparing the outcomes of the turnarounds without taking into consideration *inter alia,* the severity of the decline, lack of stakeholder support (D'Aveni, 1989; Arogyaswamy *et al.*, 1995), and extraneous events further skew the potential bias of the arguments advanced by researchers in the field of the turnaround process.

What is the importance of a breakeven during a turnaround? – A turnaround refers to when a firm experiences a survival-threatening declining performance over months or years and can reverse the declining performance through an intervention that will restore the firm to profitability on a sustainable basis (Barker & Duhaine, 1997; Bibeault, 1982; Pearce, 2007). A turnaround is a process that enables a firm to move from a poor to a sustainable performance position, implying that this is fundamentally a two-stage process (Arogyaswamy *et al.*, 1995; Brandes & Brege, 1993; Chowdhury, 2002; Dewitt, 1993; Hambrick & Schecter, 1983). Robbins and Pierce (1993) refer to a turnaround as a reversal of poor performance. The firm faces an existence-threatening situation and can recover from a loss-making position through the combination of strategies, skills, and capabilities and return it to profitability. Khandwalla's (2001) definition changes from a loss-making position to a profit-making position. Grinyer *et al.* (1990, p. 116) state, "This sharp, sustained improvement in performance from the base of

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relative decline is termed a 'sharp-bend' and firms that successfully negotiated this change are known as 'sharp-benders.'"

The importance of breakeven – Barker and Mone (1994, p. 403) state, "We think breakeven (sic) performance is an appropriate floor that the firm's performance must fall below for it to be considered to be a declining firm." Breakeven is precisely the point where the change of direction of the performance or the 'sharp-bend' occurs, and this is the point where the outflows are equal to the firm's inflows. It is the crucial point of a turnaround and implies that this is the nadir of the accumulated cash flow requirement, a maximum immediately before the breakeven occurs. Therefore, determining when breakeven occurs should be the focal point of researchers to understand when the fortunes of the firm change.

The estimation of duration in empirical studies dealing with the process of turnarounds – It seems duration is estimated based on specific observations made by researchers of individual turnarounds. Researchers seek to determine which strategy may result in a turnaround success using general organisational theories, referred to as content (Bibeault, 1982; Chowdhury, 2002; Hambrick & Schecter, 1983; Hofer, 1981; Khandwalla, 2001; Lohrke, Bedeian & Palmer, 2004; Pandit, 2000; Pretorius, 2009; Schweizer & Nienhaus, 2017; Trahms, Ndofor & Sirmon, 2013).

Some researchers seem to attempt to identify a statistical significance to determine how many turnarounds have been successful. However, this research appears to be based on choosing an arbitrary measurement, such as reported share prices or earnings, and does not add much value to a predictive theory for turnarounds (Hambrick & Schecter, 1983; Morrow *et al.*, 2004).

The studies reviewed seem to refer to a turnaround in broad descriptive terms. Most studies dealing with turnarounds adopt a positivist approach, where the researchers seek to confirm their prior theoretical concepts and models adopted from broader literature. Solnet, Paulsen and Cooper (2010, p. 154) suggest that further research is required to "… adopt interpretive approaches to provide 'thick descriptions' of phenomena" and assist in building theories pertaining to turnarounds in the future.

The use of cash flow projections during a turnaround – Experienced practitioners know that doing things in a usual manner is simply not going to work, as crises require immediate, directed, and measured actions when undertaking a turnaround (Bibeault, 1982). These

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actions are informed by a selected strategy that forms the foundation formulated in the first month and sometimes in the first week of the turnaround, indicating urgency for what needs to be done. This sense of urgency is conveyed via various interviews with actual turnaround practitioners (Bibeault, 1982). Arogyaswamy *et al.* (1995) refer to the firm as being maladapted to the environment when faced with a turnaround. Slatter (1984) argues that a turnaround is a short-term strategy devised under special conditions to bring about organisational change. Stace and Dunphy (2001) state that in the interest of survival, a radical approach is necessary as time is of the essence. A turnaround should be a radical frame-breaking form of organisational change as required. A turnaround can be defined as the process whereby turnaround practitioners actively seek and successfully implement strategies and management methodologies to turn the distressed firm around, saving the firm from failure (Fredenberger & Bonnici, 1994).

Gribnitz and Appelbaum (2015) argue that a restructure involves changes in equity, assets, and liabilities as set out in the Statement of Financial Affairs (IFRS Foundation, 2020). A turnaround is operational and focuses on the activities reported in the statement of comprehensive income (SOCI). The litmus test for any proposed strategy, action, event, or outcome comes down to the financial requirements and the timing of when it has to occur, as represented by the fundamental moments during a turnaround. Once the implementation commences, the practitioner must revisit the fundamental moments at regular intervals throughout the turnaround.

Answering these fundamental questions on an ongoing basis informs the ability to implement a proposed strategy or amend the proposed strategy during the turnaround. Suppose a proposed strategy cannot be financially sustained. It should be abandoned and replaced with an alternative strategy that fits within the financial or logistical constraints of the firm's financial capability. During a turnaround, the availability of resources informs what strategy can be implemented that is funded through a combination of resource slack and stakeholder support. Arogyaswamy *et al.* (1995) state that the resource requirement is determined as a projection and assumes that it has to be delivered promptly for the turnaround to succeed. If these resources are not delivered at all or timeously, the proposed strategy cannot be implemented. These estimations and projections become an exercise in combining various beliefs and optimism into a planning document. Grinyer and Spender (1979, p. 126) state,

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"Even financial forecasts are located within patterns of operational beliefs, and without understanding these, the group management can scarcely hope to grasp the real operational meaning of forecasts it receives."

According to McRobert and Hoffman (1997), a firm collapses due to human behaviour. Therefore, research dealing with turnarounds also focuses on management strategies of human behaviour. Ormerod (2005) states that "… intent is not the same as outcome". The turnaround practitioner will envisage a plan, but the implementation of such a plan may and usually will never realistically be the same as the original plan. While all individual agents are connected in specific networks, which evolve over time, the implication is that they are constrained by their development of a plan, or in this case, the implementation of the original plan. Even with the best analysis availed, there remains uncertainty about the plan's outcome. When dealing with a turnaround, the leader and his turnaround commando must perform an analysis and compile a management strategy, which has to be presented in a financial firm plan (Nueno, 1993). Blayney (2005) posits that people are the reason for a firm's collapse, and it is also people that want to perform a successful turnaround on the firm. A practitioner states their intent, given the information available at a predetermined time, to what is believed to be achievable by implementing the proposed turnaround strategy. Any stakeholder requires assurance of whether the presented plan is a reasonable reflection of what can be achieved during the turnaround, even if it is not necessarily an over-optimistic view of what can be done.

In summary, proposing a theory with no precise performance measurements means that empirical research can be manipulated to verify selected biased positivist views by selecting a sample set that fits the positivist outcome pursued. Every turnaround should be informed by the fundamental moments, as this is the key to deciding whether to proceed with a particular strategy or not.

Resource slack – a firm can only implement a turnaround if enough resource slack arises from its own resources and creditor support (Arogyaswamy *et al.*, 1995). Resource slack is required to create a financial buffer for the delays in implementation or variation in performance viability (Bourgeois, 1981; Cyert & March, 1963). As long as the creditors continue supporting the turnaround, implementation can succeed, but it will fail once they withdraw support. Perception by the creditors and maintaining their trust is key to a turnaround. Barker and Duhaime (1997, p. 20) state that "The level of firm resources at the time of the turnaround

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attempt may affect the declining firm's capacity to implement strategic change. Maintaining adequate resources while responding to the decline process destroys firm resources over time." Singh (1986, p. 567) refers to absorbed and unabsorbed slack, where the latter has not been consumed by the company and refers to "… uncommitted, liquid resources in organizations". According to Castrogiovanni and Bruton (2000, p. 26), "… absorbed slack refers to excess organizational costs".

Chattopadhyay, Glick and Huber (2001) argue that the availability of resources influences how management reacts to threats and opportunities. At the same time, the type of slack may influence the strategy that management may follow internally (requiring less slack according to Cook, Shortell, Conrad and Morrisey (1983)) or externally directed actions.

Audia and Greve (2006) argue that the firm's resources affect the decision makers' risk tolerance during a turnaround where the availability of a buffer or resources will lead to riskier decisions by management if faced with declining performance. On the other hand, the management of a firm with limited resources will not undertake risky decisions when close to failure as performance declines.

While research refers to the resource slack as a general concept, typically, they do not show how it should be calculated or when it should be available. The availability thereof is essential for determining fundamental moments, as a strategy in turnarounds is always dictated by the resources available and, therefore, limits the available choices to the practitioner. They are bound to implement the best strategy subject to the constraints of what is available to affect the turnaround and may not be able to select the most optimal strategy for the firm.

2.4 The methodology used to perform the study

This section sets out (1) why research dealing with the process of turnarounds was selected as a unit of investigation; (2) what design methodology was followed to determine how the research dealt with fundamental moments; and (3) how the research was divided into sub-categories so that the results could be interpreted with more accuracy.

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2.4.1 Using the foundation of strategy process research

The field of strategy process research describes how organisational strategies are formulated and implemented to bring about change (Chakravarthy & Doz, 1992; van de Ven, 1992). Van de Ven (1992, p. 169) argues that the meaning of process is used in different ways when dealing with strategic management literature and indicates three different uses: "… (1) a logic that explains a causal relationship between independent and dependent variables, (2) a category of concepts or variables that refer to actions of individuals or organisations, and (3) a sequence of events that describes how things change over time."

The search for fundamental moments seeks to understand if it was used as a measurement tool when dealing with the research regarding the process of turnarounds. Chowdhury (2002) questions the use of "variance theory" as proposed by Mohr (1982) when using statistical analysis to determine the relationship between dependent and independent variables. He argues that it represents static descriptions of the turnaround strategies to explain outcomes or what effect other external or contextual factors may have on the outcome of the turnaround. Variance theory also fails to explain the direction of the causality and incorporate the complex interrelationships among various variables (Langley & Traux, 1994).

The unit of investigation for this study dealt with turnaround research which refers to the turnaround process. Schweizer and Nienhaus (2017) based their research on organisational change theory which was also referred to by Pandit (2000). They classified 260 studies into three categories, namely content (what was done), context (why it was done), and process (how the organisational changes occur). Research on content and context would either: (1) present a statistical analysis of the performance of multiple firms to describe a phenomenon that is linked to an existing body of research (content); (2) deal with a particular aspect of the turnaround by describing a very case-specific experience or event that occurred; or (3) may be dealing with bankruptcy research; (4) while describing the process of a turnaround is focussed on what happened, and what stages occur during the turnaround. Turnarounds are dynamic and fluid, bringing about change in an organisation that must happen within a certain period. At the same time, a multitude of external and internal forces and factors can impact the outcome of the turnaround. Combining these dynamic events indicates that a turnaround should be viewed as a process (Chowdhury, 2002).

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Schweizer and Nienhaus (2017) classified 42 studies as research on the process of turnarounds, content, and context research, which were used as the foundation for this study. Therefore, it can be expected that research dealing with a turnaround process should supposedly describe some dynamic process: what and when incidents, events, and concepts occurred as contemplated in stage theory (Bhave, 2002; Langley, 1999; van de Ven, 1992). Fundamental moments are key in any discussion of process research and are vital in understanding the chosen strategy's impact on the outcomes.

The 42 studies were expanded by searching for publications dealing with turnarounds published after 2016, regardless of whether they were classified as a process, context, or content, and included research on bankruptcy prediction. Keywords searched for included: turnaround, corporate distress, restructuring, bankruptcy, crisis, and reorganisation. A total of 627 studies were found that dealt with the various aspects of business rescue, turnarounds, bankruptcy prediction, and restructuring. A final collection of 50 studies were selected, including the most recent studies of Bhattacharyya and Malik (2019), Rico, Pandit and Puig (2021), and Tangpong *et al.* (2015). Additionally, a further five publications published before 2016 (Bibeault, 1982; Pearce, 2007; Pearce & Robbins, 2008; Robbins & Pearce, 1992; Weitzel & Jonsson, 1989) were added to the collection, as these have been seminal to later research on the process of turnarounds. Weitzel and Jonsson (1989) deal with the stages of decline, while the remaining publications *inter alia* deal with the stages of a turnaround. Therefore, the final publication collection included in this study ranged from 1973 to 2021, providing an extensive overview of research dealing with the turnaround process.

2.4.2 Research design and methodology used in the study

The presence of fundamental moments was divided into four categories: (1) *no reference* – where no reference was made to the specific fundamental moment whatsoever, nor was it implied and no possible reference can be made that implies that such a moment was even considered; (2) *implied reference* – where authors indirectly implied or possibly referred to a fundamental moment, even if such reference was not made intentionally, it would be deemed to be implied, for instance, where ratios, factor analysis or discriminant analysis was used to imply but not calculate the breakeven point, it would imply that it would refer to the nadir of the turnaround and, therefore, would be deemed to be an implied reference (Barker &

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Duhaime, 1997); (3) *estimated reference* – while there is no basis in theory or process to substantiate such a reference, authors have estimated a specific fundamental moment in some arbitrary manner; it is particularly prevalent when dealing with the duration of a turnaround, where it was arbitrarily estimated by various authors when undertaking an empirical study to substantiate their views; and (4) *calculated reference* – where the authors set out how they calculated the specific fundamental moment, where the reader can verify such calculation.

Each study review that forms part of the collection of publications was reviewed, and a table was constructed (see Table 2.1), showing what reference was made to each individual fundamental moment as the: (1) *duration* – referring to the duration of the turnaround (measuring the length of time); (2) *breakeven* – the point where the cash inflows are equal to the cash outflows (measuring when exactly the fortunes of the firm changes); (3) *period when the nadir will occur* – the period where the accumulated cash flow reaches a nadir (measuring when the maximum cash commitment will be required to fund the turnaround); and (4) *the value of the nadir* – where during the turnaround there is the point where the maximum cash flow commitment will be required to fund the turnaround (see Figure 2.1).

Publications were classified into three sub-categories (see Table 2.1) that describe the different types of processes during a turnaround, namely: (1) process change management; (2) theory of turnaround process; and (3) process affirmation by empirical study, as some of these publications dealing with the turnaround process merely discuss the stages of the turnaround process.

Process change management – these studies utilised various theories dealing with human interaction as a part of the turnaround process that may form part of the organisational theory. Chattopadhyay *et al.* (2001) discuss the integration of the threat rigidity theory (Staw, Sandelands & Dutton, 1981) and explore the behavioural or prospect theory (Kahneman & Tversky, 1979) when dealing with organisational actions in response to threats and opportunities. Agarwal, Sarkar and Echambadi (2002) integrate research on technology management, organisational ecology, and evolutionary economics as a platform and utilise a life cycle approach to determine these firms' survival. Bruton, Ahlstrom and Wan's (2003) study deals with cultural issues and investigates how the overseas Chinese community in East Asia deals with turnarounds and how this differs from turnarounds in Europe and the USA.

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Balogun and Johnson (2004, pp. 523, 546) believe that researchers need to understand how middle managers interpret change and how this impacts their perceptions, enabling them to change during the process. They further argue that change management brings about further change. A study undertaken by Amburgey, Kelly and Barnett (1993) on the Finnish newspaper industry over 192 years shows that organisational change can be adaptive and disruptive. While these papers deal with change management on broader issues, they are cast against the backdrop of decline or turnaround. Agarwal and Gort (2002) investigate the determinants of firm survival: it is linked to the product and firm lifecycle. It is interlinked over the firm's life, operating at a particular hazard rate function that varies over various phases of the firm's life. The reference to fundamental moments was minimal. Most of these studies contained a narrative dealing with case studies and where the financial and time effects were ignored. However, the proximity to failure and uncertainty has a substantial impact on the behaviour of various stakeholders.

Theory of turnaround process – is classified as those studies that seek to describe the turnaround process and is defined as the theory of turnaround process. These studies have limited or no empirical evidence to substantiate their contribution to the theory of the turnaround process (Arogyaswamy *et al.*, 1995). They form the bedrock of the theory of the turnaround process and create the framework used by studies that seek to affirm or disprove certain aspects of the theory of the turnaround process via empirical studies (Barker & Mone, 1994; Pearce, 2007). The studies usually have broad descriptions of the stages that occur during the turnaround but do not provide a method of calculating the time and financial consequences and, therefore, cannot be measured. Some of these studies also rely on the work of previous studies to expand their theories, only to build thereon with broad descriptions (Arogyaswamy *et al.*, 1995; Chowdhury, 2002). Where an empirical study was undertaken, data were carefully selected on a *post priori* basis to ensure that the results confirmed the theory proffered. It would be expected that these studies would define fundamental moments and provide a method of calculating these moments during a turnaround.

Process affirmation by empirical study – represents those studies where statistical analysis was performed to substantiate their hypotheses on the affirmation of one aspect or another of the theory of the turnaround process. These studies seek to criticise a part of the theory of the

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turnaround process by creating a hypothesis based on some arbitrary hypothesis, using historical financial results from listed companies over several years from a specific industry or financial results from various industries. They use financial ratios to accept or reject the hypothesis. It would be expected to refer to fundamental moments to ensure enough capital to fund the turnaround and how long it lasted. Arogyaswamy *et al.* (1995, p. 517) describe how authors follow a specific research design to identify firms with declining performance without regard to the reasons or causes of such decline.

This combination creates heterogeneity by combining firm-based and industry-contraction-based decline in the same sample. Using these conflated samples, researchers attempt to express views on the strategy selection and suitability of the outcomes.

2.4.2.1 Age by sub-classification of the type of studies

Analysis of the various studies published tells an interesting story of what happened to research related to the turnaround process and the lack of research progress made over the past 42 years within this field of study, which is in line with the research of Schweitzer and Nienhaus (2017, p. 7). They provide a good analysis of when research on turnarounds was undertaken before 2016. The 50 studies included in this research study provided a comprehensive overview of the turnaround process and were classified into several sub-categories. The studies were divided into three groups: those published prior to 2000, those published from 2001 to 2010, and, lastly, publications from 2011 to 2021. These studies were further divided into different classifications of the sub-categories. An interesting overall pattern appears since of the 50 studies reviewed, 86 per cent were published before 2010. A total of 18 papers dealt with the theory of the turnaround process, and 17 of these studies were published prior to 2010. In fact, of these 18 studies, 14 were published before 2000, and only three studies in the following ten years. The majority of the studies dealing with process affirmation deal with the issues of the stage models as proposed by Bibeault (1982) and Robbins and Pearce (1993) and are reflected in a relatively even spread over the study period. Interest in the theory of the turnaround process, as well as what is defined as process change management that forms a part of organisational change management, has waned since 2010, save for the study published by Greve (2011), which is an empirical study dealing with the effects of change management.

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Stage theory of turnarounds – the current standard when dealing with the process of turnarounds – Chowdhury (2002, p. 252) proposes a four-stage process model (decline, response initiation, transition, and outcome), which is an adaption of earlier research by various authors (Arogyaswamy *et al.*, 1995; Bibeault, 1982; Finkin, 1985; Gopinath, 1991; Grinyer & Spender, 1979; Grinyer *et al.*, 1990; Hambrick & Schecter, 1983; Hofer, 1980; Krueger & Willard, 1991; Moulton & Thomas, 1993; Rosenblatt, Rogers & Nord, 1993; Weitzel & Jonsson, 1989). Pearce and Robbins (2008) published a study that summarises the work by previous authors and adds to the previous work stating that strategic transformation is key to turnarounds.

Process affirmation by an empirical study of stage theory – of the 20 studies classified as process affirmation, 75 per cent (15 studies) seek to confirm or reject certain aspects of the stage theory, where the hypotheses are focused on retrenchment and recovery strategies (Barker & Duhaime, 1997; Barker & Mone, 1994; Burton, Ahlstrom & Wan, 2003; Castrogiovanni & Bruton, 2000; Chowdhury & Lang, 1996; Filatotchev & Toms, 2006; Lim, Celly, Morse & Rowe, 2013; Morrow *et al.*, 2004; Ndofor *et al.*, 2013; Pearce, 2007; Rico *et al.*, 2021; Robbins & Pearce, 1992; Schmitt & Raisch, 2013; Sudarsanam & Lai, 2001; Tangpong *et al.*, 2015). Of the remaining five studies, four deal with other aspects of turnarounds that do not specifically focus on stage theory. Bergh *et al.* (2008) believe that when a firm owns a diverse array of operations or investments, there is an asymmetry in the information between how the managers and the market may value such a business. They propose that selling- or spinning-off may lead to the better unlocking of value for such a firm. Lamont, Williams and Hoffman (1994) investigate how the change in organisational structure to an M-form can benefit a firm with deteriorating performance, especially when speedily implemented. Sheppard (1994) seeks to understand the linkages between cooperative inter-organisational and financial strategies and what these may have on the outcomes of firms that are close to bankruptcy or not. Based on their longitudinal data research, Zajac and Kraatz (1993) propose a diametric forces model highlighting the different conflicting pressures that higher education faces if the need to restructure arises.

Studies published from 2011 to 2021 – of the seven studies published from 2011 to 2021, only one study by Battacharyya and Malik (2019) can be classified as a contribution to the theory of the turnaround process. They deal with the process of turnarounds and propose an

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integrated turnaround canvass. The authors attempt to explain what levers are available to a turnaround practitioner and what the impact may be on future cash flows and outcomes. Unfortunately, they do not measure these outcomes but believe it will impact future discussion and research.

The remaining six studies are all empirical studies. Greve (2011) does not seek to confirm any part of the stage model proposed by different authors dealing with the theory of the turnaround process but rather shows that risk-taking is linked to aspirational levels. Low performance leads to restricted information processing, especially if there is a threat to the proximity of bankruptcy.

The remaining five studies are empirical studies that further the debate about the two-stage model proposed by Robbins and Pearce (1993) and continue discussing the impact of retrenchment on turnaround outcomes. Rico *et al.* (2021) argue that it is not the retrenchment of assets and operational expenses that are important but rather the retrenchment of debt. Tangpong *et al.* (2015) state that their empirical research shows that when the retrenchment occurs earlier in the cycle of decline, as defined by Weitzel and Jonsson (1989), it leads to better outcomes than when retrenchment takes place in the latter part of the cycle of decline. Ndofor *et al.* (2013) conducted an empirical study of packaged software firms in the 1990s. They found that retrenchment actions such as layoffs, asset reductions, and product withdrawals were negatively associated with performance recovery. These findings were in line with a study published in the same year by Lim *et al.* (2013) and posit the concept of Schumpeterian rent creation associated with explorative capabilities against that of Ricardian rent creation that is of an exploitative nature. They propose that retrenchment negatively impacts Ricardian-type firms versus Schumpeterian-type firms. In the same year, Schmitt and Raisch (2013) also performed a study in Germany, Austria, and Switzerland, interviewing turnaround practitioners, and proposed a duality between retrenchment and recovery that is interrelated and seems both contradictory and complementary.

It is clear that the research dealing with the turnaround process has ground to a halt and that a new impetus is required to progress the research in this field. The first step is to understand how authors deal with fundamental moments and any other or indirect references to such moments.

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2.5 Findings: In search of fundamental moments

This section describes how the 50 selected studies dealt with the fundamental moments collectively and individually. The analysis in Table 2.1 shows a list of the authors studied with columns adjacent thereto that indicate the four classifications for each fundamental moment. The last column shows how the study was classified.

2.5.1 Analysis performed on the studies dealing with the turnaround process

None of the 50 selected studies attempted to indicate how to calculate any of the fundamental moments. None of the studies attempted to estimate the fundamental moments save for the turnaround duration. The estimation method used for the duration is very rough, relying on historical annual financial statements. On a *post priori* basis, these studies selected companies that have experienced financial distress and have had a change in financial fortunes. They use the year where the nadir occurred as the middle point and add information for three to four years before the nadir in losses occurring and two to five periods after the nadir of losses occurring.

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Table 2.1: Analysis performed in search of fundamental moments
Source: Author (2021)

No	Author	Duration				Breakeven				Value of the nadir				Period of the nadir				Sub-classification of the type of studies as part of the turnaround process
		Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	
1	Agarwal and Gort (2002)				✓				✓				✓				✓	Process change management
2	Agarwal, Sarkar and Echambadi (2002)				✓				✓				✓				✓	Process change management
3	Amburgey, Kelly and Barnett (1993)				✓				✓				✓				✓	Process change management
4	Arogyaswamy, Barker III and Yasai-Ardekani (1995)			✓					✓				✓				✓	Theory of turnaround process
5	Audia and Greve (2006)				✓				✓				✓			✓		Process change management
6	Balogun and Johnson (2004)				✓				✓				✓				✓	Process change management
7	Barker III and Duhaime (1997)		✓						✓				✓				✓	Process affirmation by empirical study
8	Barker III and Mone (1994)		✓					✓				✓				✓		Process affirmation by empirical study
9	Bibeault (1982)			✓				✓				✓				✓		Theory of turnaround process
10	Bhattacharyya and Malik (2019)			✓				✓				✓				✓		Theory of turnaround process
11	Bergh, Johnson and Dewitt (2007)				✓				✓				✓				✓	Process affirmation by empirical study
12	Bruton, Ahlstrom and Wan (2003)		✓						✓				✓				✓	Process affirmation by empirical study
13	Castrogiovanni and Bruton (2000)			✓					✓				✓			✓		Process affirmation by empirical study
14	Chattopadhyay, Gick and Huber (2001)		✓						✓				✓			✓		Process change management
15	Chowhurry (2002)			✓				✓				✓					✓	Theory of turnaround process
16	Chowhurry and Lang (2002)		✓						✓				✓				✓	Process affirmation by empirical study
17	Denis and Rogers (2007)		✓					✓					✓				✓	Process change management

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		Duration				Breakeven				Value of the nadir				Period of the nadir				
No	**Author**	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	**Sub-classification of the type of studies as part of the turnaround process**
18	Domadenik, Prasnikar and Svejnar (2008)				✓				✓				✓				✓	Process change management
19	Finkin (1995)				✓				✓				✓				✓	Theory of turnaround process
20	Filatotchev and Toms (2006)			✓				✓				✓					✓	Process affirmation by empirical study
21	Gopinath (1991)				✓				✓				✓				✓	Theory of turnaround process
22	Greve (2011)				✓				✓				✓				✓	Process affirmation by empirical study
23	Grinyer and Spender (1979)				✓				✓				✓				✓	Theory of turnaround process
24	Grinyer, Mayes and McKieran (1990)			✓				✓				✓					✓	Theory of turnaround process
25	Hambrick and Schecter (1983)		✓						✓				✓				✓	Theory of turnaround process
26	Hofer (1980)			✓					✓				✓			✓		Theory of turnaround process
27	Jansen (2004)				✓				✓				✓				✓	Process change management
28	Ketchen and Palmer (1999)				✓				✓				✓				✓	Process change management
29	Krueger and Willard (1991)				✓				✓				✓				✓	Theory of turnaround process
30	Lamont, Williams and Hoffman (1994)		✓						✓				✓				✓	Process affirmation by empirical study
31	Lim, Celly and Morse (2013)		✓						✓				✓				✓	Process affirmation by empirical study
32	Maitlis and Sonenshein (2010)				✓				✓				✓				✓	Process change management
33	Miller and Chen (2004)				✓				✓				✓				✓	Process change management
34	Morrow, Johnson and Busenitz (2004)		✓					✓					✓				✓	Process affirmation by empirical study

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No	Author	Duration				Breakeven				Value of the nadir				Period of the nadir				Sub-classification of the type of studies as part of the turnaround process
		Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	
35	Moulton and Thomas (1993)			✓					✓				✓				✓	Theory of turnaround process
36	Ndofor, Vanevenhoven and Barker (2013)		✓						✓				✓				✓	Process affirmation by empirical study
37	Pearce (2007)		✓					✓				✓					✓	Process affirmation by empirical study
38	Pearce and Robbins (2008)		✓						✓				✓				✓	Theory of turnaround process
39	Pearce and Robbins (1993)		✓						✓				✓				✓	Theory of turnaround process
40	Rico, Pandit and Puig (in press)				✓				✓				✓				✓	Process affirmation by empirical study
41	Robbins and Pearce (1992)		✓						✓				✓				✓	Process affirmation by empirical study
42	Rosenblatt, Rogers and Nord (1993)			✓				✓					✓				✓	Theory of turnaround process
43	Schmitt and Raisch (2013)		✓					✓					✓				✓	Process affirmation by empirical study
44	Sheppard (1994)			✓					✓				✓				✓	Process affirmation by empirical study
45	Suradsanam and Lai (2001)		✓					✓					✓				✓	Process affirmation by empirical study
46	Tangpong, Abebe and Li (2015)		✓						✓			✓				✓		Process affirmation by empirical study
47	Vaara, Tienari and Laurila (2006)				✓				✓				✓				✓	Process change management
48	Weitzel and Jonsson (1989)				✓				✓			✓					✓	Theory of turnaround process
49	Wicks (2001)				✓				✓				✓				✓	Process change management
50	Zajac and Kraatz (1993)				✓				✓				✓				✓	Process affirmation by empirical study

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Table 2.2: A summary of studies reviewed in search of fundamental moments as set out in Table 2.1.

Sub-classification of the type of studies as part of the turnaround process	**Duration**				**Breakeven**				**Value of the nadir**				**Period of the nadir**				Total studies
	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	Calculated	Estimated	Implied	No reference	
Relative percentage per class	Duration				Breakeven				Cashflow nadir				Max cash required				Total studies
Process change management	0%	17%	0%	86%	0%	0%	7%	93%	0%	0%	0%	100%	0%	0%	14%	86%	28%
Theory of turnaround process	0%	19%	50%	31%	0%	0%	31%	69%	0%	0%	31%	69%	0%	0%	19%	81%	32%
Process affirmation by empirical study	0%	65%	15%	20%	0%	0%	30%	70%	0%	0%	20%	80%	0%	0%	15%	85%	40%
Total	0%	36%	22%	42%	0%	0%	24%	76%	0%	0%	18%	82%	0%	0%	16%	84%	100%

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Table 2.1 shows that of the 50 studies reviewed, 42 per cent did not refer to duration, 22 per cent made an implied reference, and 36 per cent estimated the turnaround duration. Only 24 per cent of the studies referred to an implied breakeven, while the remaining studies did not refer to breakeven. When dealing with when the cash flow nadir will occur, only 18 per cent of the studies made any reference thereto, while the remaining studies remained silent thereon. Again, only 16 per cent of the studies implied some cash requirement, while the remaining studies did not address the issue.

Of the 14 publications classified as *process change management*, 86 per cent of these studies did not refer thereto, zero per cent implied in some manner, and 17 per cent estimated (be it extremely vaguely) the turnaround duration. Only seven per cent of the publications implied breakeven, while 91 per cent did not refer to when the breakeven occurs in any manner whatsoever. None of these publications made any reference to when the cash flow nadir would occur, while only 20 per cent of the publications made an implied reference thereto. When dealing with how much capital/cash will be required to fund the turnaround, 86 per cent did not refer to that. In contrast, only 14 per cent of these publications created an implied reference to the capital required. None of these publications indicated how to calculate the duration, when the breakeven will occur, when the cash flow nadir occurs, and how much funding is required to cover the turnaround cost.

Of the 16 publications dealing with the *theory of turnaround process research*, 31 per cent of these studies did not refer thereto, 50 per cent implied a duration in some manner, and 19 per cent performed some form of estimation for the turnaround duration. Only 31 per cent of publications made an implied reference to breakeven, while 69 per cent made no reference to when the breakeven occurs in any manner whatsoever. Of these publications, 69 per cent had no reference to the cash flow nadir, while only 31 per cent had made an implied reference thereto. Regarding how much capital/cash will be required to fund the nadir, 81 per cent did not refer to the capital/cash requirement. In comparison, only 19 per cent of the publications created an implied reference to the capital/cash required. None of these publications indicated how to calculate the duration, when the breakeven will occur, when the cash flow nadir occurs, and how much funding is required to fund the turnaround. This study includes 20 publications classified as a *process affirmation by empirical studies*, where 20 per cent of these studies did not refer thereto, 15 per cent implied in some manner, and 65 per cent

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estimated (be it inaccurately) the turnaround duration. Only 30 per cent of publications made an implied reference to breakeven, while 70 per cent made no reference to when the breakeven occurs in any manner whatsoever. Of these publications, 80 per cent did not refer to when the cash flow nadir would occur, while only 20 per cent of the publications made an implied reference thereto. When dealing with how much capital/cash will be required to fund the turnaround, 85 per cent have made no reference thereto. In comparison, only 15 per cent of these publications created an implied reference to the capital required. None of these publications indicated how to calculate the duration, when the breakeven will occur, when the cash flow nadir will occur, and how much funding is required to cover the turnaround cost.

Of the 50 studies reviewed, 17 did not refer to fundamental moments. In contrast, only three studies referred to all the fundamental moments, even though this is done by implication, as no direct reference was made.

2.5.2 Discussion about fundamental moments in turnaround research

The findings in this study indicate that researchers in the turnaround process field do not recognise fundamental moments individually or collectively. No researcher has identified or defined fundamental moments as a critical management tool. At best, there are vague or implied references to certain individual elements of the fundamental moments. A turnaround practitioner will constantly update his evaluation tool to determine the fundamental moments to inform him how the strategy must or may be adapted. Fundamental moments act as the loadstar in directing the selected strategy, influencing stakeholder engagement, and improving the chances of a successful outcome. Tracking the financial and time consequences of the cash flow on an ongoing basis allows the practitioner to tweak the strategy as required and communicate the same to stakeholders, thereby establishing trust in the process. Therefore, fundamental moments should also become a keystone for future research dealing with turnarounds.

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2.5.2.1 Engagement with stakeholders dealing with the proposed strategy

Communicating the financial and time effects of a turnaround succinctly is essential in stakeholder management, which is in- and outward-focused and intertwined, as each part plays a critical role in delivering a successful outcome for a turnaround.

Inward-focused stakeholder management seeks to improve cost optimisation, implying that the same output must be achieved with fewer human and physical resources. It may also require improved efficiencies, which implies that more must be produced within a given time frame, suggesting that the firm's output per time unit must improve. The strategy may require an overall increase in output or product. The selected strategy may combine these three components but ultimately implies that the firm must work harder and smarter. Implementing the selected strategy occurs against the backdrop of constrained financial resources and retrenchment of staff and possibly assets. Enabling the employees to understand the proposed turnaround strategy and what is expected will inform what is required. Understanding what progress has to be made and has been made allows employees to make sense of what is going on and embrace and influence the proposed and actual change. Suppose the employees do not buy into the proposed strategy or purposefully delay (very plausible) the implementation aspects of the turnaround. In that case, it will result in costly delays, which will result in a new set of fundamental moments. In other words, if a delay occurs, there is an increasing chance that the turnaround will not be successfully implemented, as the resources necessary to complete such a turnaround are being depleted until the business cannot continue operating. The funders and customers, especially external stakeholders, lose patience with the process and withdraw support.

The turnaround practitioner must keep the management and staff motivated, which is achieved by explicit direction and direct communication. Experiencing a turnaround as an employee means living during a time of upheaval and change. Making sense of the events requires understanding the purpose and progress to keep the team motivated. If the staff understands the objective and progress, it may lead to fewer staff leaving the firm for greener pastures elsewhere. Keeping the institutional memory intact while bringing about change in habits forms a part of the process of turnarounds. Future research on the process of change management during a turnaround should incorporate research into how and what information

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the practitioner communicates when dealing with employees. Assuming the use of fundamental moments would lead to better implementation of a proposed strategy, thereby assisting the employees in making sense of what is occurring during the turnaround.

The external focus of a turnaround practitioner requires obtaining support from funders and customers, defined as external parties. Obtaining the support of these external parties requires different messages from the practitioner, but both are equally important during a turnaround. Obtaining support from funders includes acquiring support from suppliers, banks, and equity providers. For instance, a practitioner may request an extension of credit from existing suppliers while negotiating a moratorium on the company's existing debt.

There may be reasons why the practitioner may not want to specifically share the details of the fundamental moments with a supplier. However, it is essential to understand what is happening in the cash flow movements and how, where, and when the fundamental moments occur during a turnaround. If the turnaround is still declining, as it has not reached the nadir, it implies that the firm is absorbing financial resources. Once the firm has reached breakeven, it can start repaying the absorbed resources. Providing undertakings to effect repayments without being sure that such payments can be affected may be deemed reckless. It may be subject to various sanctions depending on the jurisdiction in which the firm operates. Undertakings given to clients are also informed by fundamental moments, as the deliveries may be influenced by available resources to fund the working capital requirements to fulfil such orders. Suppose the suppliers are willing to extend credit terms or the debt or equity funders are willing to provide the funding to deliver such orders. In that case, firm commitments can be provided to the customers. The alternative is to make promises based on a "wing and prayer", only to find that deliveries will be delayed, eroding trust with the client, who will seek alternative suppliers if the firm does not deliver on time.

When dealing with equity or debt providers, it becomes essential for the practitioner to communicate what specific fundamental moments may apply to a specific proposed strategy, whether private or public debt. Obtaining finance for the firm is about establishing a sense of trust with the funder or supplier that the capital will be returned with an appropriate reward attached for making such capital available. When a firm is not undergoing a turnaround, the funder may pay less attention to the risk of the non-repayment of capital provided. However, this becomes the central focus of any cash flow discussion during a turnaround. Accordingly,

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informing funders about a selected strategy inevitably leads to a discussion about the time and financial consequences of the specific strategy being implemented, which returns to the fundamental moments attached to that specific strategy. The practitioner will continuously change or adapt the proposed strategy, depending on the support obtained from the funders. Maybe future research on selected strategies and outcomes should focus on the initial conditions, how these were communicated, and how they changed for the duration of the turnaround to determine if the selected strategy was the correct one for the business.

2.5.2.2 Time and turnarounds

Implementing the selected strategy in a timeous manner is as important as the proposed actions that are to take place. Only three of the publications under review made implied references to all the fundamental moments. However, not a single publication discusses the fundamental moments as an essential measurement tool and does not provide a model or method to measure these moments (Barker & Mone, 1994; Bhattacharyya & Malik, 2019; Bibeault, 1982). Interviews with turnaround practitioners by Bibeault (1982) provide urgency and sincerity with which a turnaround has to be undertaken. Survival pressure becomes an interplay of resources and actions that must be extra carefully coordinated in time. In other words, measuring time during a turnaround is key to devising and implementing a turnaround strategy. Agarwal *et al.* (2002, p. 989) provide an excellent argument for why time is a key element in dynamic processes, which is precisely what a turnaround is – "… time changes the rules of the game. Time impacts existing precepts and makes a subtle mockery of static tenets. Time is endemic, and its effects are ubiquitous. Therefore, it is to incorporate the systematic and fundamental effects of time in evolutionary studies". A turnaround is a dynamic process where everything is predicated on happening on time and in time.

Any practitioner that has undertaken a turnaround will know that the dimension of time becomes critical. As the firm experiences a shortage of cash resources or slack, every day can make a difference. Suppose the debtor does not pay on time. In that case, the cash available to pay for the next shipment is delayed, causing a stoppage in the operations, and exacerbating the firm's losses even further. The success of a turnaround is predicated on measuring the sequential actions that have to occur in a particular time sequence. It seems odd that the organisational change theorists offer no accurate time perspective on their theories.

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Chowdhury (2002, p. 251) states, "Any dynamic phenomenon is a combination of sequential events over some time and can therefore be viewed as a process rather than a state of affairs."

Fundamentally, no measurement model in the turnaround process and theory can be considered unless such a model incorporates the effect of the delay or accelerated implementation on the success of the turnaround. Measuring for a given time or at a given time goes to the core of any management undertaking. Regardless of equity or debt, resource providers will always measure the return on the capital provided within a timeframe. Accordingly, the time it takes to achieve the outcome relates to the availability of financial support, which may lead to the success of the turnaround. During a turnaround, a selected strategy will contain a chain of events that have to occur in perfect sequence to deliver a successful outcome. However, life is never straightforward, especially during a turnaround. As things go wrong, which, in turn, affects other things that also go wrong, this cascade of events determines a cumulative result that informs the overall outcome. Therefore, when things go wrong at a certain point, it results in cumulative adverse outcomes. In that case, the proposed strategy may not be achieved on time or at all unless further resources and more time are provided or allowed to achieve the necessary results. Any future measurement model must provide for the effects of time. Future research should focus on how the effects of time pressure influence decisions in an organisation undergoing a turnaround. When researchers deal with the theory of the turnarounds process, time is viewed as being amorphous. They offer theories, discussions, and figures about turnaround stages or phases without measuring time. Time is depicted as a horizontal line in a figure with the word "time" written next to it, and there is no way to determine if these time frames for implementation are achievable or realistic. The lack of measurement in the theory of the process of turnarounds results in researchers who undertake empirical research creating a research design based on the availability of financial information. The only information these researchers use for the financial figures appears in the published annual financial statements. Using this information, they rely on financial ratios and statistical analysis to identify the specific implementation strategy that led to the specific outcome achieved during a turnaround.

Due to the inability to measure the role of financial and time consequences, empirical researchers have resorted to using duration periods artificially manipulated to fit their purpose, where the selected duration is exceptionally long. Schendel *et al.* (1976) were the

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first to study and compare the results of various listed firms that had undergone a turnaround. Based on that study, various subsequent empirical studies followed the same line of argument. This selection implied that the seven-year financial information would be considered the turnaround's duration and is, therefore, suitable for evaluation (Pearce & Robbins, 1993, p. 629). This selection period became the accepted manner of performing an evaluation and the subsequent substantiation of a hypothesis for this research method. Selecting data in this manner implies that the fundamental moments have occurred during the period selected, and their moments do not have to be determined. However, their data selection has a survivor bias that does not consider those firms that did not succeed in implementing the turnaround. The measurement tools used are so imprecise that they lose the entire detail of what happened and cannot be juxtaposed to those firms that have failed in undertaking turnaround.

2.6 Conclusion

This study shows that there may be limited awareness of fundamental moments. No effort has been made to define or discuss these moments, nor has any attempt to link fundamental moments impact the decision-making in the formulation of the strategies in a turnaround. A turnaround requires a necessary quantum of resources at particular moments to ensure success. This study shows that only 16 per cent of all the reviewed publications implied that funding is required to cover the turnaround cost. The remaining research did not refer to the required capital to fund the turnaround, nor when it was made available wholly and timeously. Bibeault (1982) refers to creating a cash flow model to simplify the in-and outflows of cash without becoming trapped in various accounting issues that are echoed in his interview conducted with Frank Grissini, a turnaround practitioner. The latter avers that understanding what is happening in the cash flow is vital before deciding what should be paid or not. Future research should develop a model to measure the fundamental moments mathematically, enabling universal acceptance and adoption. The inputs, such as the rate of losses incurred and the rate at which the planned changes occur with its related cost, will return a unique set of fundamental moments every time. Understanding how fundamental moments are affected by the inputs may lead to a revival in the research into the process of turnarounds.

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Chapter 3: Using traditional cash flow "TCF" calculations to determine fundamental moments during turnarounds

Abstract

With the commencement of a turnaround, a turnaround practitioner must decide on an applicable strategy to undertake and implement such a turnaround. The selection of such an implementation strategy is informed by the time and financial consequences of each strategy, where the time and financial consequences are defined by the fundamental moments of a turnaround, and where the fundamental moments can be determined by asking: (1) how long is the turnaround going to last; (2) when is the firm going to breakeven; (3) when is the cash flow nadir going to occur; and (4) how much funding is required to cover the cost of the turnaround? This chapter discusses how turnaround practitioners determine these fundamental moments by creating a string of numerical solutions in elaborate bespoke spreadsheets as TCF calculations that may vary in complexity and then obtaining the values of the fundamental moments through inspection and selection.

The TCF calculations form the pivot around which a turnaround plan is created. These TCF calculations drive the plan's narrative and inform the projected annual financial statements that usually accompany the plan.

The practitioner will prepare a turnaround plan, an extensive document, that they will use to engage stakeholders to obtain support for the selected implementation strategy. The design, usefulness, and efficacy of a turnaround plan are predicated on the stakeholders' trust in the contents thereof. Trust is informed by believing the future delivery values (a.k.a. promises) and understanding the plan's contents.

This chapter discusses TCF calculations, their limitations, their purpose, and how it determines fundamental moments. It also discusses how practitioners use the TCF calculations as a pivot when creating a turnaround plan and how stakeholders engage with these documents.

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3.1 Introduction to the role of the turnaround plan and the TCF calculations in the management of turnarounds

This chapter is written in the narrative format based on the participant-observer view. It tells the story of the typical turnaround and TCF interaction.

Cash flow forecasting, -budgeting, or -projection is the loadstar during any turnaround (Bibeault, 1982). According to Bibeault (1982:112), "The key thing to realize in a turnaround situation, more than at any other time in the corporate life span, is that 'cash is king.'" Hofer (1980:21) states that "... to determine an indicator of an impending disaster, create pro forma cash flow projections and cash flow breakeven charts. It also requires how the inventories and accounts receivable can be manipulated to release cash into the system". The turnaround practitioner measures the cash flow impact of the strategy on an ongoing basis from the commencement to the completion of the turnaround. Measuring the cash flow impact determines what amount of resources will be required to fund a turnaround so that the practitioner can obtain the support of the stakeholders.

This chapter describes how TCF calculations inform specific fundamental moments and the construction of the turnaround plan containing the narrative and projected financial statements used to seek support from stakeholders. Based on the author's autoethnographic experience and observations as a turnaround practitioner for the past 35 years, it became clear that preparing an extensive document may be good for formulating your own decisions about the implementation of a chosen strategy. However, using this extensive document to convince stakeholders is generally met with suspicion. Little to no value may be attached to such a document, even if a financial institution may require the same for their records. However, ultimately, the decision to gain the support of stakeholders turns on two simple questions:

1. How long is it going to last?
2. How much is needed?

Stakeholders ask these questions for a firm requiring a turnaround, as they want to minimise their losses and maximise their chances of recovery from distress. In other words, they want to be sure of the outcomes proposed before providing support for a selected strategy. As the entire engagement between practitioner and stakeholders is a discussion about future outcomes, it requires the stakeholder's trust to evaluate the documents presented to motivate

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the selected strategy. However, no matter how many pages are contained in the document, the same questions posed above form the fundamental moments of a chosen strategy.

Therefore, Chapter 3 discusses the role of TCF calculations and fundamental moments in preparing the documentation by practitioners to promote stakeholder support. It further discusses the issues that arise when engaging with stakeholders that may lead to a successful outcome. Fundamental moments describe a proposed strategy's salient financial and time effects, and the TCF calculations determine the quantum of and the period when resources are required. Furthermore, an overview is provided of the various parts usually contained in a turnaround plan but does not attempt to explain or discuss the content thereof. It also expands on the issues and shortcomings in dealing with TCF calculations, leading to the requirement for an alternative to informing fundamental moments.

Chapter 3 concludes with observations and conclusions on the efficacy of a turnaround plan when seeking support from stakeholders based on the author's experience in turnarounds and firm rescues.

3.2 Stakeholder theory as a framework for this research

Freeman (1984) established that individuals and groups could, as stakeholders, affect the ability to achieve the firm's goals. Stakeholder theory is a body of research focused on the relationship of networks between stakeholders and the firm being a going concern. Mitchell, Agle and Wood (1997) proposed a model that formed the cornerstone of stakeholder theory, while Weitzner and Deutsch (2015) added to the stakeholder prioritisation.

However, stakeholder theory focuses on the interactions between management and the various stakeholders when and only when the firm is a going concern (Decker, 2018; Parmar, Freeman, Harrison, Wicks, Purnell & de Colle, 2010). The implementation of the principles of stakeholder theory is a function of the moral, relational, or instrumental motives adopted by management that should align with the firm's goals (Jahavar & McLaughlin, 2001).

This chapter discusses the observations by the author as a turnaround and business practitioner based on his autoethnographic experience. It specifically argues that a successful turnaround or business rescue requires "buy-in" from various stakeholders to achieve a successful outcome. It proposes that once there is a breakdown in trust between the stakeholders, it becomes very difficult to convince stakeholders to support the turnaround or

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restructure efforts. In addition, the practitioner faces the complex task of presenting a complex turnaround plan in a manner that is simple to understand and engenders trust and belief to support such a plan. At the same time, it can be argued that the quality and presentation of the information set out in this study may form a subset of the stakeholder theory; it is premised on the firm's going concern in a profit-making position.

When a firm ceases to be a going concern, it enters a state of distress or is in the zone of insolvency (ZoI). Pretorius (2018) concludes that the network of relationships as defined in stakeholder theory changes as the various stakeholder positions are amplified or diminished depending on their position.

3.2.1 Overview of the stakeholder theory for going concerns

Freeman, Phillips and Sisodia (2020) argue that the value creation stakeholder theory provides a framework for responsible management of the firm, but it is not an all-encompassing theory. It is, for instance, not the same as corporate social responsibility. Graham (2020) states that stakeholder theory is "corporate responsibility", where the firm is responsible for all the stakeholders involved through its decisions and actions. According to Barney and Harrison (2020:209), "A corporation that provides an excellent and safe product at a fair price, pays its suppliers fairly and on time, treats its employees well, is respectful to the local communities in which it operates, pays its financiers as promised, provides a reasonable return to its shareholders, obeys applicable laws, and exhibits trustworthy behavior in its interactions with all its stakeholders is *responsible*."

Freeman et al. (2020:5) summarise the stakeholder theory's essence: "The real issue is not shareholder versus stakeholder, but a narrow/reductionist versus broad/holistic perspective on business. It is the difference between a value chain (linear and singularly focused on financial value) and a value network (which includes the importance of shared purpose and values). A value chain has one endpoint and one desired outcome for one stakeholder, the shareholder; all other players in the system are a means to that ultimate end. In an interconnected and interdependent system, each stakeholder must be a means and an end."

Barney and Harrison (2020) argue that there has been no consensus about the general applicability of stakeholder theory regardless of its existence for half a century. They further posit that this theory has been used, is being used, and should be used in business decision-

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making. However, those who propose this theory agree that there is no consensus on what stakeholder theory means, which led to a meeting in Zion, USA, to agree on what questions should be addressed to standardise the theory. It led to the special issue of Business & Society in 2020, dealing with various questions raised, and this, in turn, was edited by Barney and Harrison (2020). This study does not seek to repeat the 12 questions raised but rather focuses on informing three (their sequencing) specific questions:

2. "Is the primary role of stakeholder management one of satisfying stakeholders, or is it seeing them as joint partners in co-production?"
4. "From a firm value creation perspective, is it really optimal to treat all essential stakeholders unusually well (within reasonable limits)? If, on the other hand, a firm focuses on a narrower group of stakeholders, to what extent are the benefits of generalized exchange lost?"
6. "Are decision rights primarily a function of cash flow considerations, or are they implicit to stakeholders that do not have explicit rights but are still critical to the creation of value?"

These three questions need specific attention once the firm enters the ZoI, as defined by Pretorius (2018). Once the going concern relationship ends, the stakeholder relationship and expectation change become unbalanced. It becomes impossible to satisfy all the stakeholders as there are simply insufficient resources. As a result, the unequal ranking of creditors, combined with preferential treatment of employees under certain jurisdictions, changes the power dynamic on what the outcomes will be during the ZoI. Cashflow considerations (as per the TCF) drive all actions when a firm is in a ZoI as there are not enough reserves to go around.

Freeman *et al.* (2020) identify further distinctive elements of the stakeholder theory, where:

(a) Human actors cooperate in value creation;
(b) their values, norms, and ethics are aligned to produce outcomes for all the organisations involved in creating value; and
(c) these actors have a worldview of business with a higher state of consciousness about its short and long-term role in society.

Bridoux and Vishwanathan (2020) discuss the issues that arise when a powerful stakeholder, usually the banks (as a secured creditor), promotes their demands and thereby constrains the firm's choices to progress. They tend to use their power disproportionately by appropriating as much value as possible. Alternatively, they may withhold such actions if they perceive the present value of the long-term gains exceeds the value of short-term gains that they may

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extract. These powerful stakeholders have a profound impact once the firm exits the going concern status.

Crane (2020) explains that trust is a key element in the stakeholder perspective, where stakeholders act with each other through various interconnected networks of relationships, which are influenced by the direct and indirect actions of the firm. He further argues that trust is a fundamental need of all stakeholders enabling mutually beneficial and productive relationships resulting in value creation.

Therefore, value creation stakeholder theory ceases when a firm transitions from a going concern to operating in the ZoI, as discussed in the next section.

3.2.2 Stakeholder theory or relationships in ZoI

Pretorius (2018) proposes that influence and prioritisation by stakeholders change when a firm transitions from a going concern to being in distress, requiring a turnaround or restructure. He proceeds to investigate the ensuing effects on stakeholders, resulting in him restating the theories of stakeholder, firm, and resource dependence.

He further investigates how new stakeholders (usually providing new funding) interact with the turnaround, how the stakeholders prioritise their decision-making during distress, and what consequences and effects this has on the industry. Decision-making is framed by the context and situation that, in turn, influence the levels of uncertainty and position while being modified by levels of power that the various stakeholders may have (Becker-Ritterspach & Dörrenbächer, 2011).

According to Pretorius (2018), the ZoI is an imprecise construct (Barondes, Fairfax, Hamermesh & Lawless, 2007). The various decision-makers may have different biases that compete with the finite and limited resources available. The outcome is measured as a zero-sum game, where one stakeholder is benefitted in favour of another. When a firm is in distress, the conditions become ambiguous, unclear, and uncertain, resulting in a negative and combative interaction between stakeholders and management. (Laplume, Sonpar & Litz. 2008). Parmar *et al*. (2010) proposed that the stakeholder theory is primarily concerned with the distribution of financial resources. When a firm is in distress, focusing on the division or retention of the financial resources becomes more pronounced. The practitioner may seek new funding while ensuring the existing capital does not leave the firm. Jahavar and McLaughlin (2001) defined

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this process as resource dependence, implying that certain decision-makers may be prohibited from extracting their committed resources from the firm, at least while the firm is undergoing a turnaround. Stakeholders become more focused on the potential outcomes of a turnaround and seek to evaluate their risk position on an ongoing basis, thereby limiting their downside of losing more than what they have committed (Francis & Desai, 2005; Schmitt, Barker III, Raisch & Whetten, 2015). The prioritisation of competing stakeholder claims is guided by the power, legitimacy, and urgency that the stakeholder may impose on management (Agle, Mitchell & Sonnenfeld, 1999). Parent and Deephouse (2007) argue that the more attributes the stakeholder possess, the greater the influence it will have on the management and, therefore, the outcome of the turnaround. This chapter differentiates between stakeholders who have an existing exposure versus attracting new stakeholders to the firm to provide new capital to fund the turnaround. The relationship of power and influence of existing stakeholders influences the choices and selected strategies available to the firm's management. It may, in turn, influence decision-makers' bias and, therefore, the choices and responses they make to the proposed outcomes of a turnaround plan (Davila & Molina, 2015). Every stakeholder has a different reference framework, viewpoint, and expectation of the turnaround outcome should provide (Zavyalova, Pfarrer, Reger & Hubbard, 2016).

3.2.3 Stakeholder theory as a framework for this study

Stakeholder theory is focused on the social networks that exist while the firm is a going concern. It is also a theory being developed, as highlighted by Barney and Harrison 2020. When a firm is in distress, the dynamics in dealing with stakeholders change completely, as shown by Pretorius (2018). Ultimately, stakeholders want to be treated fairly, and there are not enough resources for a firm in distress, exacerbating the suspicion of stakeholders. Providing information on how stakeholders will be paid is contained in a turnaround plan. The turnaround practitioner devises the plan and then has to try and convince stakeholders to support the plan by either maintaining or providing new capital during the turnaround. This chapter discusses the autoethnographic journey the author undertook over a period of 30 years of dealing with turnarounds and restructures. The amount of time a stakeholder will invest in engaging with the practitioner is limited, whether a stakeholder asked to provide new capital or those already committed who are looking to extract their capital as soon as possible. This chapter focuses on preparing information and communication and how this may have

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influenced stakeholder support during a turnaround. This study seeks to create a model that can replace the use of TCF calculations. Such a model may be able to allow stakeholders to make better-informed decisions and allow them to be able to trust the outcomes predicted by the model. In other words, this study is about creating a model that can be used in engaging with stakeholders and not about the stakeholder theory. It may also be useful in dealing with the certain questions highlighted above, a subset of the 12 questions discussed by Barney and Harrison (2020). However, stakeholder management during a turnaround usually is a combative process and not necessarily one of cooperation (Pretorius, 2018). Providing information that all stakeholders can trust may reduce the combativeness and lead to increased cooperation.

Therefore, the core of this research leans on stakeholder theory and specifically relevant and accurate information for stakeholders to allow for decision-making. It proceeds now to the TCF calculations (see Section 3.3.), while the VFC framework is discussed in Chapter 5.

3.3 The purpose and construction of TCF calculations

This section explains the purpose of the TCF calculations, as it is the pivot around which a turnaround plan is created. It shows how cash flow availability is determined during the progress of the turnaround by capturing the changes in inputs that have occurred or in the event of new information becoming available. Wruck (1990:421) argues that "… in an insolvent firm with a negative net worth: the present value of its cashflows is less than its total obligations, and a firm in financial distress is insolvent on a cash flow basis if it is unable to meet current cash obligations". The primary purpose of TCF calculations is for management to use it as an internal navigation tool for the cash resource supply and utilisation. This section sets out the rules and general principles of how the TCF calculations are constructed and used to calculate the fundamental moments.

3.3.1 What is the purpose of TCF calculations?

TCF calculations capture the proposed and intended outcomes based on the selected strategy in a time-delineated format, linking the inflows and outflows of changes to be implemented and creating an overall accumulated view of the cash flow at the end of every period.

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TCF calculations aim to inform the:

1. narrative of the turnaround plan;
2. projected financial statements; and
3. fundamental moments based on a selected turnaround strategy.

Fundamental moments are essential to determine the quantum of resources required and when they must be available to deliver on the promised outcome, which relates to when breakeven will occur and how long the turnaround will last (duration). The accumulated values at the end of each period in the TCF calculations can be classified as the engine that generates the outcome as reflected in the projected financial statements. Changing the strategy or inputs and considering non- or partial delivery of resources results in new estimates and assumptions that will change the projected outcome. TCF calculations are a cumulative set of numerical calculations that inform the outcomes discussed in a turnaround plan while informing the fundamental moments. Therefore, even one change in the assumptions has a cascading effect on the entire document creation process.

TCF calculations are created using bespoke spreadsheets that generate these values that are neither calibrated nor standardised. Every practitioner creates their own version thereof; even though the same rules or construction may have been used, the level of detail and complexity varies from one practitioner to another.

The utilisation of TCF calculations is not only confined to turnarounds and is often used as a tool to assist the financial management team in making decisions when making commitments to make a payment or not; in other words, micro cash flow management. Depending on the bank and supplier credit terms available to the firm, the cash receipts become vital to operating the firm, significantly if the firm is constrained for liquidity. If the firm can acquire materials only on a cash basis, the firm operations become beholden to the whims of the debtors paying the firm on time. Usually, if a firm is undergoing a turnaround, it is cash-constrained, as there has been a threat or actual flight of capital.

The ultimate purpose of the TCF calculations is to determine what resources will be required to ensure that the promised outcomes will materialise. Failing to obtain enough resources at the right time implies a delay in implementing the proposed turnaround strategy. This delay leads to more resources being committed and extends the period to achieve the same result.

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Therefore, the purpose of the TCF calculations is to determine the VFC requirement to fund the turnaround given a specifically selected strategy.

3.3.2 The rules and construction of the TCF calculation

From the outset, the turnaround practitioner usually prepares a TCF calculation as a matter of urgency, based on a set of rules that incorporate the income and expense items divided into periods for the estimated implementation term (Bibeault, 1982).

Publications dealing with spreadsheet modelling and decision-making do not always show how to construct a TCF calculation (Brigham & Houston, 2009; Plane, 1994; Ragsdale, 2017). However, there are a few very basic rules on how to construct a TCF calculation applicable to any firm, regardless of whether it is a turnaround or not. Effectively, there are three parts to a TCF calculation:

1. income;
2. expenses; and
3. net difference for the period.

It is then added to the accumulated difference of the previous period to provide the accumulated difference of income and expense for the period.

Constructing total operational income: Figure 3.1 shows estimated sales based on an estimate by the person who prepares the TCF calculation. The estimate would be based on the income for every sub-period, which can vary from monthly, quarterly, or annually and lasts for the duration of the turnaround. Other income and interest earned (if any) are usually based on historical assumptions and are added to the gross income to provide the total income for a specific period. The cost of sales is derived using historical or researched assumptions that are, in turn, based on the historical information of what the cost of sales in the firm has been to date. The cost of sales impacts the assumed variable expenses, as this may vary based on what is required to produce the projected sales, thereby informing the cost of sales. When preparing the TCF calculation, the practitioner must decide how income recognition will be treated.

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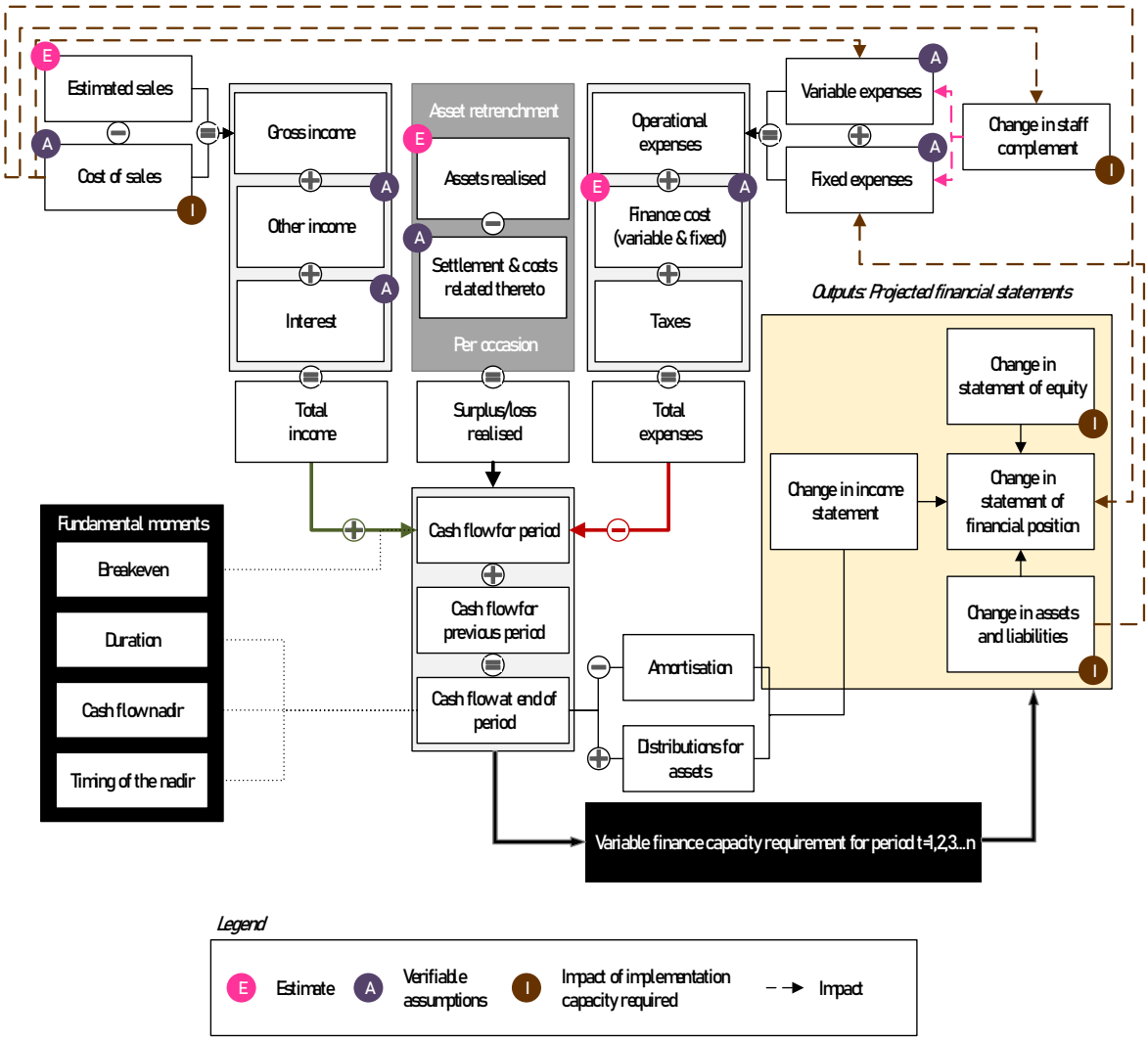

Figure 3.1: A deconstruction of TCF calculations to determine the VFC required for each period, and that informs the fundamental moments of the turnaround

Source: Author (2021)

In most cases, when creating the TCF calculation, income recognition occurs when the actual cash is received and not when sales are realised. The purpose of a TCF calculation is to understand the cash flow impact for that period and how the accumulated cash flow behaves during the turnaround. However, this merely remains a projection based on estimates and assumptions, where one estimate or assumption impacts various other inputs, which may result in further assumptions and estimates that ultimately produce a different outcome.

Constructing total expenses: As stated, the estimate of the future sales results in an assumption of the concomitant cost of sales, which indicates what resources must be acquired from suppliers to sustain the projected sales level. Estimated sales, therefore, require a specific

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implementation capacity consisting of resources, assets, and staff. The change in capacity and staff complement may impact the variable expenses, such as electricity costs. It may even impact the fixed expenses, where the premises may have to increase or be downsized. Adding these expenses together can be defined as operational expenses. Such expenses are usually well known and based on reasonably accurate estimates that can be included in the TCF calculation. The fixed finance costs relate to long-term loans that may be asset-backed or not, where the interest and capital repayment expenses are well known and based on historical information. The variable finance loan cost relates to the ongoing working capital requirements funding and can be an estimate at best. Usually, this type of funding is provided in the form of a bank overdraft or some short-term funding, such as a debtor discount facility arrangement, which can be called up with a 30-day notice or sooner. All taxes are treated as an expense when constructing a TCF calculation. In contrast, the payment of payroll and value-added taxes will feature prominently in each period as these are preferent monthly payments. The sum of the operational expenses, fixed and variable finance costs, and taxes are the total operational expenses.

Cash flows for the period: The total expenses are deducted from the total income, resulting in a surplus or loss – being the cash flow for that specific period. However, the cash flow sans/without the surplus or loss on the asset retrenchment may be zero, positive, or negative. If the cash out-and-in-flow is zero or becomes positive, it indicates that the firm has reached breakeven and is producing more cash than it consumes and can, therefore, repay some of the debts it has incurred. If the cash flow is negative, it should be determined how many periods away the firm is from breakeven (where the cash flow is equal to zero). Breakeven is one of the elements contained in the fundamental moments, and one of the elements used to determine the progress of the turnaround. As discussed below, breakeven is informed by the cash flow being equal to zero for a specific period and not by the overall accumulated cash flow.

The cash flow for the period is then added to the accumulated total of the previous period, which is also referred to as the cash flow for the previous period, which is equal to the cash flow at the end of the period. The accumulated cash flow at the end of the current period will become the cash flow of the previous period in the period to follow. Figure 3.2 shows that this series of accumulated cash flows at the end of each period becomes the VFC requirement

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that differs for each period. During a turnaround, the cash flow at the end of each period is usually negative as there is not enough capital to fund the consequential losses being suffered, leading to an increased accumulated shortfall in cash flows. Figure 3.2 graphically depicts that the sequential negative cash flows show an increase in the negative balances up to a certain point in time. The negative cash flows reduce sequentially until such time that the accumulated cash flow becomes positive.

Estimated asset retrenchment: The proposed turnaround strategy may include the retrenchment of certain assets. Devising the strategy, the practitioner will estimate the value of the assets to be realised in that specific period that may provide resource slack to affect the turnaround. Usually, expenses are associated with the realisation of assets, such as commission, decommission and transport costs. These costs are typically added together and deducted from the value realised from the sale of the assets. Asset retrenchment may have two outcomes when performing TCF calculations. At the end of each period, the accumulated cash flow may lift the cash flow nadir (in the event of a surplus occurring). Alternatively, the cash flow nadir may decline further in the event of further realisation of losses. Any ongoing costs associated with these assets will not form a part of the future expenses and would lead to a faster recovery in the turnaround strategy.

Integrating TCF calculations in the projected financial statements: Since the TCF calculations are cash-focused, they would not include amortisation, depreciation, or any other non-cash flow items. TCF calculations can and usually are used as a basis to estimate the projected statement of comprehensive income (SOCI) at some future date, where the non-cash flow items are deducted, and capital distributions (repayment of debt capital) for assets must be added back to the accumulated cash flow at the end of the period at a future period of the TCF calculations. However, if the funding is not provided or not provided timeously at any stage, projected financial statements become an exercise in futility.

In addition to adding the distributions for assets back to the SOCI, the same amounts can be deducted against the liabilities or added to the cash position shown on the statement of financial position (SOFP). The value of the assets (physical or non-physical) is reduced based on any disposals. Assets may also be reduced with depreciation or amortisation in the projected SOFP at the future selected date based. Lastly, the projected SOFP may also be

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adapted to incorporate changes in equity and debt that may result from the implementation of the turnaround strategy.

The resources and assets of the firm are funded by equity and liabilities, and this funding underpins the turnaround strategy. Suppose the firm does not have enough equity to part-fund the assets, resources, and staff. It implies that the selected strategy cannot be implemented, and an alternative should be considered. Suppose the strategy is to be funded by debt. In that case, the fixed cost component (interest cost) is shown in the SOCI. At the same time, the repayment of the capital is shown in the SOFP; however, in the TCF calculations, the interest and capital are added together and deducted as expenses.

TCF calculations do not attempt to separate capital items and expenses as set out in annual financial statements prepared on international financial reporting standards (IFRS) as published from time to time. It seeks to understand what overall cash inflows and outflows will be made from the firm's bank accounts. TCF calculations may, therefore, also include cash disbursements for the acquisition of certain assets and are contained in the projected SOFP.

3.3.2.1 How to determine fundamental moments using TCF calculations

A practitioner will prepare TCF calculations to determine the fundamental moments for a specific turnaround. Figure 3.2 shows a summary of a spreadsheet that would be prepared, reflecting a sum of the income, being the cash inflows for each specific period, kept the same for the duration of 12 periods in this example.

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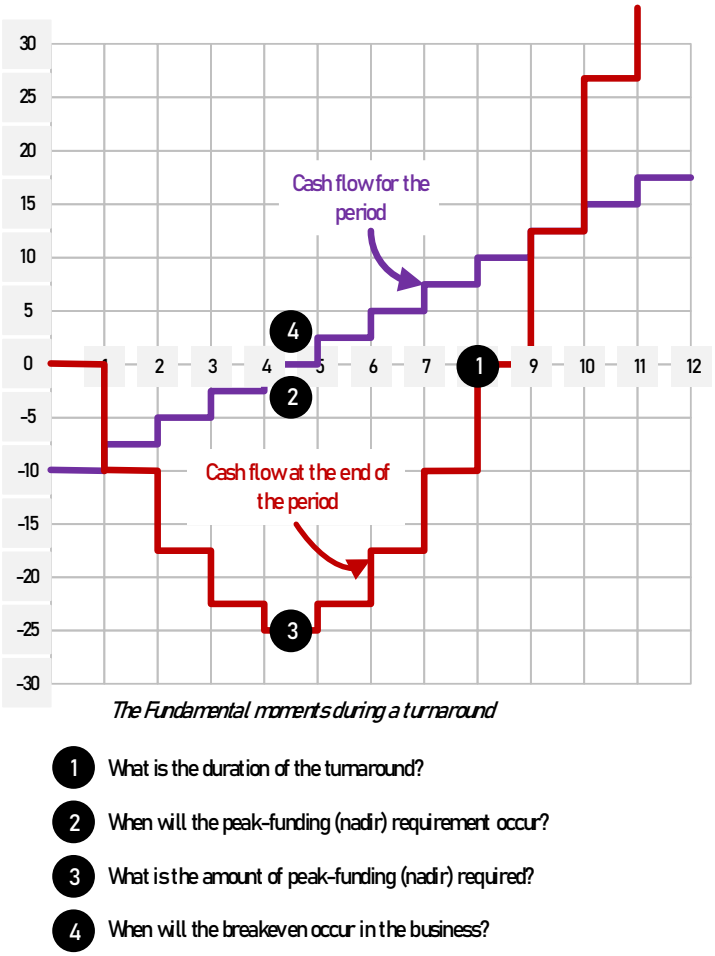

Description of items	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
Sum of income	40,0	40,0	40,0	40,0	40,0	40,0	40,0	40,0	40,0	40,0	40,0	40,0
Sum of expenses	48,0	45,5	43,0	40,5	38,0	35,5	33,0	30,5	28,0	25,5	23,0	20,5
Abnormal expenses	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0
Cash flow for the period	- 10,0	- 7,5	- 5,0	- 2,5	-	2,5	5,0	7,5	10,0	12,5	15,0	17,5
Cash flow for the previous period	0	- 10,0	- 17,5	- 22,5	- 25,0	- 25,0	- 22,5	- 17,5	- 10,0	-	12,5	27,5
Cash flow at the end of the period	- 10,0	- 17,5	- 22,5	- 25,0	- 25,0	- 22,5	- 17,5	- 10,0	-	12,5	27,5	45,0

Figure 3.2: Example of how the accumulated cash flow and cash per period are created by using the TCF calculations to inform fundamental moments

Source: Author (2021)

TCF calculations are an exercise to predict future cash flow outcomes based on estimates (a.k.a. best guesses) and assumptions based on verifiable historical or research information while creating a construct that links certain relationships. Figure 3.1 shows how, while various estimates and verifiable assumptions can be present, only a few implementation options may

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be available to the practitioner. Many possible strategies will be constrained by the lack of resources, as reflected in the SOFP.

TCF calculations do not remain static and change as new information becomes available. At the same time, projections for longer than three months are usually an expression of best intent, and should they realise it, it would be pure coincidence. However, TCF calculations serve as the loadstar of the practitioner, as it provides the basis to determine the appropriate fundamental moments for a specifically selected strategy and a specific given moment in time. Should any assumptions or estimates change or time elapses without a specific element of the strategy being implemented, a new set of fundamental moments will arise. Therefore, while TCF calculations or any other model can never predict the future, if properly constructed, it provides a fair indication of what is required to implement the selected strategy. It must be updated with new information to map what has happened and determine what resources are required to continue the turnaround.

All TCF calculations are created using basic rules and seek the same outcome to measure the cash flow movements on a period-by-period basis. Regardless of the levels of sophistication that may be added to TCF calculations, ranging from semi-automated assumptions using stochastically derived estimates or automating the impact of the implementation capacity required, it is created to determine the VFC. Adding complexity to the TCF calculations does not seem to improve the accuracy of the estimates but only increases the accuracy of the impact of the implementation capacity. Regardless of how sophisticated or straightforward the spreadsheets may be, the purpose of TCF calculations is to determine the amount of cash the firm may have or not and what should be done to manage it through the cash flow shortfall. TCF calculations are created to be a prediction tool of cash flow fluctuations, usually measured periodically, focusing on the sequential impact of each period. However, management may use a TCF calculation as a payment allocation tool by ranking the payment's importance in delivery and production activities. The management then micromanages or "cash flows" (manipulates) the creditors by stretching payments beyond the agreed payment terms to manage the firm's overall cash flow position.

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A selected strategy may assume that a retrenchment of assets and/or human resources will occur over 12 months. The sum of expenses refers to all cash-based expenses incurred in the normal day-to-day operations of the firm. In addition to the sum of expenses are the abnormal expenses, including legal costs, practitioner fees, and any other ancillary costs associated with the turnaround that would not form a part of the normal day-to-day operational costs. The expenses (outflows) are deducted from the income (inflows), providing the difference for the period, referred to as the "cash flow for the period." It is then added to the "cash flow for the previous period," referred to as "net cash flow as at the end of the period." It is repeated every period until the accumulated cash flow becomes positive (period 10). Note that the example in Figure 3.2 reflects 12 periods. The four fundamental moments can now be derived from the example shown in Figure 3.2.

Using the TCF calculations, the following four points are derived and can be defined as fundamental moments can be determined:

1. the duration of the turnaround, which will occur when the accumulated cash flow, being the cash flow at the end of the period, is equal to zero (see period 9);
2. the cash flow nadir or peak funding requirement, which is -25;
3. when the peak-funding requirement or nadir occurs (see period 5).
4. Using the "cash flow for the period," the following fundamental moments can be determined: the breakeven is informed by the period where the cash flow for the period is zero.

Therefore, the fundamental moments can be determined by "looking up" or "reading off" the values from the prepared spreadsheet. Once the spreadsheet has been created, the same numerical calculation set is used, and by only updating the estimates or assumptions that may have changed, a new set of fundamental moments is delivered.

Assuming the practitioner proposes a strategy for product rationalisation, this may result in lower inflows, resulting in lower outflows that may lessen the level of the cash flow nadir that will impact the turnaround duration. Ultimately, TCF calculations are a very labour-intensive method to determine the fundamental moments. It is clear that several resources must be committed to complete a successful turnaround. Failing to commit such resources in a timeous manner will lead to failure of the implementation thereof. No matter how many projected

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annual financial statements are prepared, if the funding does not arrive wholly and timeously, whatever is projected thereafter simply does not matter.

3.3.2.2 The difference between profit and cash flow for a period

TCF calculations determine the cash flow requirements for each period, informing fundamental moments of the turnaround. Suppose the firm makes a profit in a specific period. In that case, it will result in a positive cash flow contribution in either that period or several ensuing periods.

There may be an impact on the firm's cash flow depending on the terms provided to and by the firm for accounts payable and receivable. If the firm operates under normal profitable conditions, it is reasonable to assume that the debtors' and creditors' payment terms should coincide. In other words, provide the accounts receivable with payment terms of 30 days and obtain credit for payment terms of 30 days.

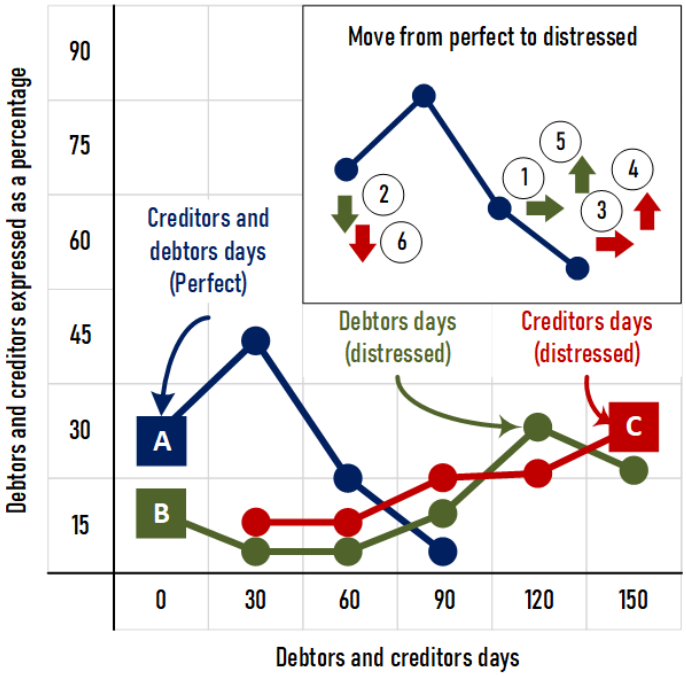

Figure 3.3: The impact of stretching accounts payable and accelerating accounts receivable as the firm "creates" loans by using suppliers' funds to fund other activities of the firm

Source: Author (2021)

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However, assuming that a firm is about to commence with a turnaround and the average accounts receivable is 30 days and the accounts payable is 45 days, it indicates that the firm has "created" a loan from its suppliers. Suppose such a differential is present at the beginning of the turnaround. In that case, it is also reasonable to assume that it will be maintained throughout the turnaround unless the accounts payable change the payment terms required by stakeholders. Stretching the accounts payable payment terms means that the cash received from accounts receivable was not applied to settle the accounts payable. Stretching accounts payable for payment ends up becoming a permanent mismatch between the accounts payable and accounts receivable for the duration of a turnaround, as there is not enough equity or capital to catch up with the delayed payments in the following periods. The consequences of not paying suppliers on the agreed credit terms mean that the firm may lose settlement discounts, thereby affecting the firm's margins. Suppliers may not have credit cover for the firm's exposure and may have to place the firm on stop-supply until the account is normalised. It results in the firm not obtaining material for its production purposes on time, resulting in further losses.

Utilising the TCF calculations, management may consider ways of extracting working capital or creating money (also referred to as "cash-flowing suppliers") by stretching the creditor payments beyond agreed payment terms. It creates "cash" that does not exist in the firm, but this "cash" is merely retained for the period it exceeds the repayment terms. This process of stretching accounts payable while accelerating the collection of those collectable receivables is analogous to "phase-shifting." Assume a firm is in a perfect state of making profits and the terms of accounts payable and receivable are received as agreed. Line A (the blue line), as shown in Figure 3.2, shows the distribution of accounts payable and receivable ranging from current to 150 days.

As the firm is purchasing the items precisely on the same terms as it is selling them, the spread or distribution of creditors and debtors are deemed the same. As the accounts receivable are collected, the accounts payable payment will occur. Line A represents the percentage over each period; however, it is assumed that there is a difference in the overall value between accounts payable and receivable, being the markup.

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Assume that the firm finds itself in a distress spiral:

1. due to non-payment by debtors, implying that the debtors are ageing; the firm is not generating enough cash to pay for its operations;
2. the firm attempts to collect debtors that are not due for collection by offering a cash discount for early payments or delivering goods against cash payments but at a discount;
3. alternatively or concurrently, the management may choose not to pay certain suppliers and instead use the cash payable to these suppliers to pay other suppliers to keep the production or sales cycle turning;
4. as the firm environment declines, these suppliers are stretched for more than one consecutive period and end up being owing in a 90-day-plus position, where more and more accounts payable are owed 60 to 90-days and longer than agreed in terms of the original supply agreement;
5. other accounts receivable may also stop paying and, regardless of the reason for doing so, may add further pressure to extract working capital to keep the firm turning, and so the accounts receivable age and accumulates debtors in the 60 to 90-day plus period;
6. while some suppliers may then place the firm on stop supply and insist on delivery against cash only, resulting in the firm having fewer credit terms with which to conduct its business, payment to other suppliers is delayed so that the accounts payable keeps on increasing until one of its creditors brings an application for the liquidation of the firm;
7. and the beginning of the "end" ensues – the company faces financial distress.

As the firm faces increasing financial distress, management will squeeze every part of their accounts receivables and payable to unlock cash ("create" money) to continue trading. Management lives in the belief or hopes that they will make enough profit to settle their debts, even to the extent of obfuscating costs and manipulating earnings (Bergh, Johnson & Dewitt, 2007:134). Over a period of time, the "inverted-v-shape" of accounts receivable becomes a "flattened-out-u-shape curve" as shown by line B (green line), and the accounts payable becomes a "rising-incline" as shown by line C (red line) over the distribution of days receivable or payable.

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Once phase-shifting has occurred, it implies that the firm has incurred an unfunded loss or funds for non-working capital-related activities. In other words, the firm is borrowing short-term funding from suppliers to fund activities that should be funded by equity or long-term debt. The nature of phase-shifting becomes semi-permanent unless a serious intervention occurs from equity or long-term debt injection. Based on the author's experience, it is nearly impossible to raise debt funding without an equity injection to repay some existing debt. By implication, by the time phase-shifting occurs, management has exhausted all other avenues of funding and the necessary collateral to fund its activities.

Once the TCF calculations are made, a specific differential in the distribution between accounts receivable and payable will be present, maintaining this position throughout the turnaround. Undertaking phase-shifting during a turnaround distorts the required VFC and will increase the risk of the turnaround not being implemented as planned.

3.3.2.3 Determining the variable finance capacity for each period

Figure 3.1 shows that the TCF calculations inform what VFC is required for each period during the turnaround. It follows that the practitioner will enter into a discussion with stakeholders (equity or other funders) of the firm and request capital to fund the proposed turnaround strategy based on the fundamental moments of the proposed strategy to be implemented. A practitioner will always present the best scenario when proposing a turnaround strategy. It is because the funders are already nervous or hesitant about turnarounds. If they are informed that there may be delays or things may not work out precisely as set out in the plan, they would be even more reluctant to invest or support the turnaround. The only certainty about turnarounds is that there will be a delay in obtaining the necessary resources timeously, resulting in a delay in implementation. However, the lack of delivery of adequate resources when required has a knock-on effect, resulting in further or more resources required for a more extended period, which, in turn, creates uncertainty and mistrust, with the possibility of stakeholder support for a successful outcome declining substantially. They believe that the practitioner is incompetent and should have foreseen these delays. A practitioner must provide the best scenario with caution, as things can go wrong (Murphy, 1949). It is preferable to make the funders aware of possible delays than to try "hooking in" the funders to the turnaround process, only to realise that more resources and time is required to complete the

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turnaround. Stakeholders should be informed that the proposed outcomes as presented in the plan show the various fundamental moments if the turnaround is delayed.

Consider a firm that does not require a turnaround and is making a profit in every sequential period; it has the luxury of utilising its retained equity (cash flow) to fund the firm's future growth. The retained equity can be utilised to reduce the cost of finance by repaying debt, acquiring more assets to facilitate growth, or distributing them to the shareholders. A profitable firm implies a third party requires no immediate intervention if a profit is not made at the end of the period, as it has enough reserves to draw on. Since the firm has reserves to draw on, the directors can choose the timing to expand the firm's operations. They may use the retained income for additional periods until a surplus develops, as this is cheaper than incurring the cost of outside funding.

Unfortunately, during a turnaround, when the TCF calculations indicate a loss in a specific period or for a sequential number of periods, the management cannot rely on internally generated funds to fund the firm and has to rely on the support of stakeholders. Once the company shows a loss at the end of the period(s), funding must match the indicated accumulated loss, as the company now has negative cash flow. This issue is further compounded if the net cash flow at the end of the period shows sequential losses, implying that the funding must be maintained over some time (as is the case in a turnaround).

This compounded shortfall implies a requirement for a VFC that must be obtained from a debt funding facility or equity funding. This facility must remain in place for some time and should be sufficient to fund the duration of the periods showing negative cash flow or losses. Therefore, calculating the quantum required and the different periods in terms of the VFC is the primary function of creating a TCF calculation. A firm in turnaround requires very intensive management with tight control over the cash flow performance. There are not enough unencumbered liquid and near liquid assets available in the firm to allow for any variations in the VFC. A firm constrained in its access to working capital means that the slightest variation or change in the working capital movements disproportionately affects that firm's trading. For instance, assume that a debtor representing two per cent of the total accounts payable does not pay on time. The management planned to use the cash received to pay cash for raw materials to complete an order. The firm operations may include a standstill until the payment occurs. The cost of such a late payment may be various multiples in losses for not having the

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capital to absorb a payment delay by a single debtor. During a turnaround, a firm does not have enough cash flow resources to absorb shocks in non-payment or reduce credit terms by suppliers. Therefore, the practitioner must ensure adequate funding as indicated by the VFC requirements for every period.

3.3.2.4 Issues with the use of spreadsheets in mission-critical decisions, such as used in turnarounds

Powell, Barker and Lawson (2009) argue that any person can prepare a spreadsheet. It does not require the services of a professional programmer, allows for more freedom to customise the spreadsheet, and can be created with speed. Madahar, Cleary and Ball (2007) classified three types of spreadsheets with the following dimensions: dependency, magnitude, and time/urgency.

Dependency – how much does the firm rely on the spreadsheet for its operational, tactical, or strategic decisions? A TCF calculation is crucial in a turnaround as it makes strategic decisions to obtain stakeholder support. During the turnaround, it may also be used as a part of tactical or operational decisions, such as delaying payments to suppliers to "manage cash flow."

Impact – what is the impact on the severity of consequences in potential errors contained in the spreadsheet? A mistake in the TCF can lead to the stakeholders being misled, allowing them to believe that the turnaround duration will be shorter or will require fewer resources, resulting in a breakdown in trust when projected outcomes are not met. This breakdown in trust means that the stakeholders will not want to continue with their support of the turnaround, which can be fatal.

Time/urgency – what deadlines must be met due to using the spreadsheet? Making decisions and timeous implementation thereof is the key to progressing with the turnaround.

The problem is that a high percentage of these operational spreadsheets contain errors. Panko (2005) showed that the spreadsheet error rates in operational spreadsheets could be as high as 94 per cent (Powell *et al.*, 2009). Croll (2005) and Grossman, Mehorta and Özluk (2005) indicated that spreadsheets are increasingly used to make mission-critical decisions, and this is precisely what TCF calculations represent.

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Research on spreadsheet errors provides an insight into the fallibility of humans when preparing spreadsheets. Powell, Lawson and Barker (2007) argue that poor design contributes to the number of errors encountered in a spreadsheet. The reasons for poor design may be:

1. time pressure;
2. organic design;
3. changing specifications;
4. lack of testing; and
5. lack of knowledge and skills to construct a spreadsheet.

When commencing with a turnaround, certain aspects are always present, such as: being under time pressure, creating an organic design specific to the firm; and having the specifications changed that may vary based on the inputs, assumptions, or inputs. A lack of testing is usually a function of the time constraints to make an informed decision and not necessarily an unwillingness to perform such testing. It is generally expected that a practitioner will have the skills and knowledge to construct a TCF calculation and know what it should contain.

Powell *et al.* (2007:62) state: "Because a spreadsheet platform is so unstructured, and because end-users generally follow unique designs, errors and poor practices can arise in thousands of different guises." They refer to research indicating that spreadsheets:

1. are chaotic in design;
2. have embedded numbers;
3. are special cases;
4. non-repeating structures; and
5. contain complex formulas that are difficult to audit.

These factors are usually an indicator that they may contain more errors than a spreadsheet that is well laid out and presented. Brown and Gould (1987) and Panko and Halverson (1996) found that end-user spreadsheet developers begin building without any adequate design or consideration of presentation or layout. An experienced practitioner has built various spreadsheets over the years and has templates that may be used to construct a TCF calculation. However, adopting a TCF calculation prepared for another firm may result in

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artefacts of calculations and errors remaining in the spreadsheet and being carried over into the new spreadsheet.

Researchers (Panko, 2005; Powell *et al.*, 2007) have identified various errors such as:

1. logic errors, where the wrong formula is used;
2. reference errors, where it points or refers to the wrong cell;
3. hardcoding of numbers, which are included in a formula, indicating poor practices;
4. copy and paste errors as a result of incorrect use;
5. data input errors; and
6. the most difficult error to determine – omission errors.

Panko and Halverson (1996) divide logic errors into pure logic and domain errors, which stem from the end user's misunderstanding of the knowledge domain in dealing with a spreadsheet. Pure logic errors arise from using the wrong mathematics or logic in general. Panko (2008) refines the measurement of errors between slip and lapse errors where "A slip is an error made during a sensory-motor action, such as typing the wrong number or pointing at the wrong cell," while "… a lapse is a failure in memory that is caused by overloading the limited human memory capacity". Rasmussen and Jensen (1974) state that mistakes or errors occur when undertaking rule-based thinking or knowledge-based thinking, resulting in different types of errors. Panko and Halverson (1996) divide the errors that can be made when dealing with spreadsheets into quantitative and qualitative errors. A quantitative error affects the bottom line or final value.

In contrast, a qualitative error may distort the entire outcome of the spreadsheet and specifically will do so when dealing with "what-if" scenarios. Hayes and Flower (1980) discuss the taxonomy of concerns when writing and argue that the author must consider all levels of planning the context simultaneously at the document-, chapter-, paragraph-, and word level. Panko (2008) argues that this overload on memory and planning capacity holds true when creating a spreadsheet. Turnaround practitioners have the added challenges that they have to understand the issues of why the firm is in distress, deal with the emotional insecurities of the staff and draft a plan that will convince the stakeholders to support the turnaround. Their cognitive process is overloaded, as they must either correctly build the TCF calculation themselves or oversee the team that does it, which increases the overload on the memory and planning capacities, making it prone to errors.

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The creators of spreadsheets do make errors and mistakes, even if they do not believe they have made any mistakes. Researchers have shown that end-user developers may be overconfident about their spreadsheets being error-free (Brown & Gould, 1987; Floyd & Pyun, 1987). Panko and Halverson (2001) performed a study where they asked what the estimated error rate may be of spreadsheets prepared by the developers and found that an actual error rate was between two to four times more than the estimated error rate by the developers. Panko (2005:20) states that "Overconfidence is corrosive because it tends to blind people to the need for taking steps to reduce risks."

Suppose TCF calculations are the loadstar used to plan and execute the turnaround of a firm. It is the practitioner's responsibility to ensure that it is always error-free by installing the proper auditing controls and ensuring that the necessary protections are included in the spreadsheet.

3.3.2.5 Shortcomings and issues when utilising TCF calculations in determining the fundamental moments of the turnaround

While TCF calculations can determine the fundamental moments of turnaround, various issues arise from utilising it for its actual intended purpose and how it is implemented. This section expands on the shortcomings and issues that may arise when utilising TCF calculations as a part of a turnaround plan and how to obtain the resources required to deliver the outcomes as promised in the turnaround plan.

Using the correct dimension when creating the TCF calculations

Management may believe that TCF calculations can become a ubiquitous tool. It can predict and manage cash flow by delaying paying certain accounts payable and mapping historical cash flow performance against the predicted values. They may even integrate the spreadsheet with the firm's enterprise management systems and create various scenarios based on current performance.

Figure 3.4 shows a relationship between the purpose and dimensions of TCF calculations, the target audience, and how information is transferred. The purpose of the TCF calculation morphs from being:

1. a prediction tool to determine the fundamental moments;
2. to being a spreadsheet that is manipulated continuously to determine who gets paid and what happens if a particular debtor does not pay (cash flow); and

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3. to a verification tool to measure the predicted cash flow to actual cash flow realised.

The spreadsheet morphs to be much more than intended, so complexity is added to it. Management seeks to create smart input analysis tables and may use various features available to create what they believe will be more accurate projections. However, adding additional features contributes to the complexity of understanding the spreadsheet and the process. The audience that engages with the spreadsheet declines until only the person who created the spreadsheet understands how it works and where to tweak inputs in the spreadsheet. At the same time, the complexity increases to such a level that it becomes difficult to verify the TCF calculations. The stakeholder may still be able to understand what estimates or assumptions are being used; however, the calculations to achieve the answer become difficult to follow and require exponentially more time to understand and verify. In order to determine the cash flow outcome for every period requires a calculation of value in every cell corresponding with an expense and income line. Meaning that if there is a change in estimates or assumptions pertaining to one or more items, every cell in a specific row or even multiple rows has to change to provide a new outcome as the calculations are repeated for every period, and verification requires reviewing every cell in the spreadsheet. It implies that the time cost involved in verifying these calculations becomes prohibitively expensive. Therefore, these third parties choose not to engage with these spreadsheets and simply look at the static information provided, using a judgment call of how much security is available to lend against or arbitrarily decide what funding they are willing to make available to the firm. It results in stakeholders deciding to support the turnaround based on collateralised lending or subjective criteria rather than an objective measurement.

Complexity may also lead to the probability of the projections being manipulated to improve the outcomes. These calculations purposefully or accidentally do not multiply, divide, add or subtract specific values to indicate the cash flow performance. The stakeholder has to verify all the estimates and assumptions and then ensure that the calculation engine of the TCF calculations and inputs are correctly reflected. In the author's experience, it is clear that stakeholders, especially potential equity and debt providers, do not trust TCF calculations, as each is bespoke to the person who designed it and is specific to the proposed implementation strategy for that specific firm.

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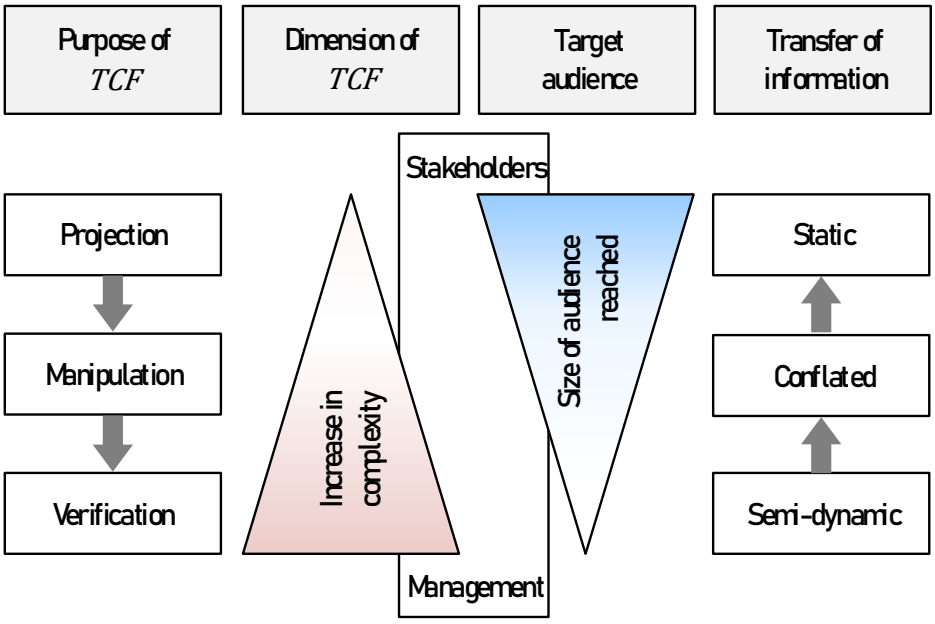

Figure 3.4: The level of detail created in TCF calculations to be fit for purpose versus the ability of the audience capability to understand the content thereof

Source: Author (2021)

TCF calculations are usually created in electronic format by one or a few individuals in management for internal use to understand what and when resources will be required and what should be done during the turnaround. While some TCF calculations may be semi-dynamic and others may be very sophisticated, communicating all this information in a static format becomes a compounded dilemma. Management may choose to create a "what-if" scenario planning that requires creating multiple estimate and assumption tables that read to different sheets in the spreadsheet to calculate the outcome of the cash flow and accumulated cash flow for every scenario.

As a result, the number of calculations increases exponentially, leading to more confusion for those third parties who did not create the spreadsheet. This complexity is further exacerbated when management may want to update the projections, only to find it challenging to track the updated projections against the actual performance.

As shown in Figure 3.1, once the spreadsheet is created, it is delivered in a static format that spans numerous A4 pages with innumerable rows and columns of income and expenses repeated for each period together with the VFC for each period. The complexity increases as the spreadsheet become an expanded mixture of projected and actual cash flows. It becomes exponentially more difficult to evaluate the projected initial outcomes versus the outcomes

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realised to date versus the newly adapted projected performance based on new information provided.

A turnaround plan is presented to the stakeholders in printable format, either paper or portable document format (PDF), thereby immediately rendering the presentation static. Pinning the outcomes onto a specific date means that all the sophisticated semi-dynamic calculations are reduced to a single static view of specific inputs producing certain outputs at a given point in time. A turnaround is a dynamic process, and the TCF calculations inform the specific narrative in the turnaround plan as captured at that moment in time.

The requirements for presenting the plan to the stakeholders may mean that after all the work has been done to generate possible outcomes, a single version is created, which may be subject to change for various reasons that will invalidate these projections as soon as these are published. Creating various scenarios requires the delivery of multiple printed pages, which one would refer to as low, middle, and high road scenarios. The narrative needs to convey these scenarios, which may confuse the reader even further, as it seems that they are not sure what the outcome will be. It creates a paradox: the more the practitioner explains, the more paperwork is created, and the more likely the practitioner will lose their audience in the process. The more complex the formulas included in the spreadsheet, the more it requires blind trust by the stakeholders to believe the practitioner that the projections are correct.

Even if the practitioner should provide a stakeholder with a copy of the TCF calculations in electronic format, they have no interest in investigating or changing the figures or looking at the scenarios created. The final summarised version (in printed or PDF format) is presented to their credit committee for approval. The stakeholder has little time capacity to evaluate the content of the assumptions and estimates used in the TCF calculations and simply has to accept that it is correct. Typically, the final decision-making body, being that faceless credit committee, never engages with the dynamic outcomes of the TCF calculation and simply bases their decision on a single view submitted by the relationship manager.

Spreadsheets act as a form of comfort to the banks that you have done some planning, though they rarely read and interrogate these spreadsheets, based on the author's experience. It is simply easier to decline a facility than invest the time and effort to understand what is contained in the spreadsheet unless a specific bank stands to lose a substantial amount of money.

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In addition to the TCF calculation, the banks will call for a projected SOCI and SOFP but rarely call for a cash flow statement, as contemplated in IFRS. These documents are required for the analysis systems used by the banks to evaluate a firm to provide a facility. All banks have some form of a credit rating system that requires specific standardised numbers for their analytical software to deliver a credit score rating, which then serves as the basis for their decision-making. Their systems are designed to deal with a profitable firm and have enough retained income to fund its growth. When dealing with a turnaround, the projected financial statement at some future date has very little value as a turnaround fails on the ability to fund and implement the turnaround timeously. The key lies in the fundamental moments predicated in the TCF calculations being bespoke to every practitioner and unique to the strategy selected for that firm. Predicted performance cannot be independently verified or compared with historical data, extending the trust gap between stakeholders and practitioners.

The interaction between projections, actual occurrence, and the confidence level of stakeholders

Once a practitioner has developed a turnaround plan, it implies that a promise is made to deliver an outcome at a future date. Figure 3.5 show that the outcome requires the stakeholder's support and is often based on confidence levels that may barely exceed fifty per cent, based on the author's estimation. Stakeholders may view a turnaround in the same category as liquidations, where stakeholders believe there is a strong chance of being compromised, even if the compromise implies delayed payment. The practitioner is operating from a level of optimism and belief that everything will work according to plan. However, as stated before, nothing in a turnaround ever works precisely to plan. Based on the author's experience, the practitioner always relies on resources arriving as and when promised. However, these are, more often than not, delayed for various reasons that will cause a delay in the implementation of the plan. The moment there are delays in the implementation of the plan, there will be an immediate and drastic decline in the confidence levels of the stakeholders. Their immediate reaction would be to believe that the practitioner has misled them and that they cannot trust the process. The promised outcomes become obsolete and are no longer applicable, as an increased and prolonged resource commitment is required to achieve a new outcome.

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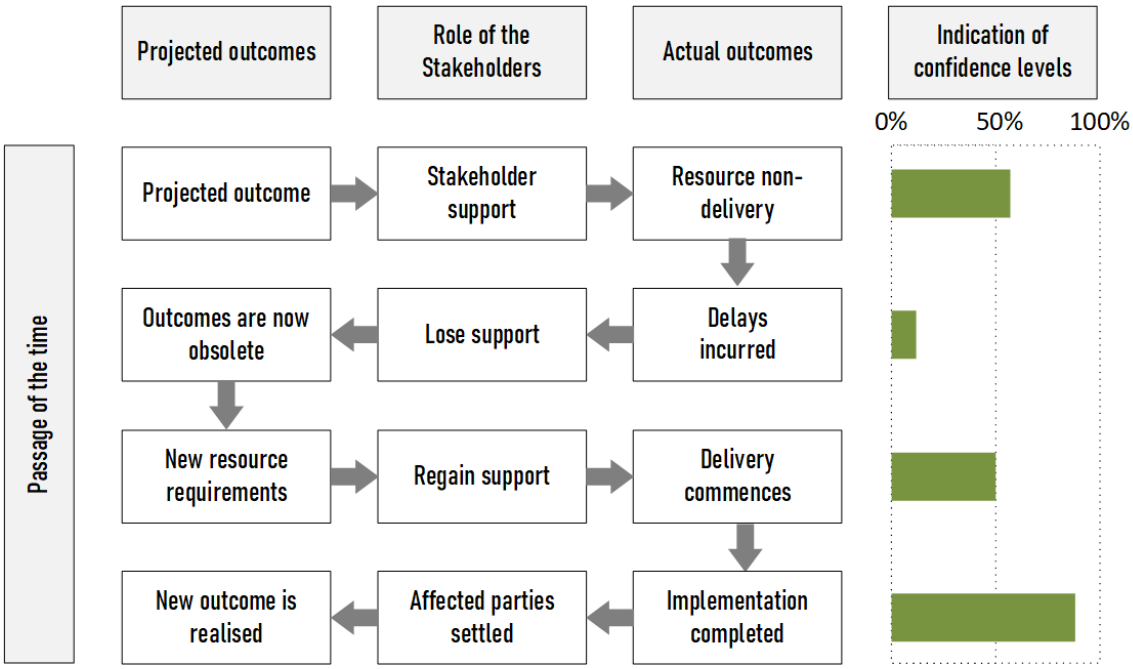

Figure 3.5: Observations made about the interaction of projected and actual outcomes and the impact thereof on the confidence levels of stakeholders (as an approximation)

Source: Author (2021)

The practitioner must convince the stakeholders to commit more resources to implement the plan and wait for a certain period before being paid. Even in the event of such a commitment being achieved, the confidence level of the stakeholders will not return to the same level that existed at the commencement of the turnaround. Assuming that the turnaround starts delivering in terms of the revised projections, the confidence levels of the stakeholders will increase gradually over time until these stakeholders are settled or the firm reaches the pre-determined outcome, as promised in the revised projections.

In the author's experience, the success or failure of a turnaround lies in maintaining the confidence of the stakeholders at all times, which can be achieved by:

1. communicating in addition to the original outcomes how these outcomes and resource requirements will change if there is non-delivery of the committed resources and what the effect of delays will have on the projected outcomes;
2. making sure that if there are any non-deliveries, new outcomes and resource requirements are reported to the stakeholders as soon as possible; and
3. reporting the progress as often as possible to the stakeholders to ensure they support the process.

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In other words, communication is key to the success of the turnaround, regardless of what strategy is being implemented or what methodology is followed to restructure the firm. The problem remains what amount of information should be provided to the stakeholders and how such information should be provided.

3.3.3 Turnaround plans – the reams of paper very few stakeholders read

This section discusses how the TCF calculations form the pivot of the turnaround plan. A turnaround plan is based on a conflation of what a financial institution believes it needs and requires in making an informed decision and some template that the practitioner may have developed or begotten from the Internet. A search on Google for "firm plan template" reveals 8,3 billion results; in other words, there is no standardised form for a firm plan. However, these templates follow a similar approach to what should be contained in a firm plan, where the purpose thereof is to:

1. formulate a strategy to be implemented, and
2. obtain support for implementing the plan from stakeholders.

The form and format of the plan do not matter as long as the practitioner knows what needs to be done and that the stakeholders continue supporting the turnaround.

A plan template used under normal firm conditions does not cater to potential variables that may become an issue during a turnaround. For instance, a turnaround plan may incorporate debt moratoriums, participation rights on specific revenue streams, or how much the creditors will be writing off. This document may present a duality, asking stakeholders to accept a loss while at the same time asking for support, making the audience suspicious of anything proposed in the turnaround plan. Generally, stakeholders are not keen to participate, as their sense of loss, a pain felt now, influences their ability to make informed decisions about the outcomes that may bring superior returns in assisting the turnaround firm.

A turnaround plan is a sales tool seeking to convince the stakeholders to support the proposed turnaround. Therefore, the contents thereof should address the concerns of various stakeholders involved in the turnaround and convince them that it is in their best interest to support such a plan. Based on the author's experience, a turnaround plan consists broadly of various discrete parts:

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1. a narrative;
2. historical and projected financial statements; and
3. the TCF calculations.

Figure 3.6 shows the process of creating the content usually included in a turnaround plan and the narrative describing what inputs are required, supported by a set of projected financial statements.

The narrative contains the proposed implementation strategy, incorporating the following:

1. estimates;
2. assumptions; and
3. implementation capacity required to implement the plan, together with a summary of the projected outcomes.

Projected outcomes refer to the cash flow impact the strategy will have and for how long the firm will be consuming cash before it can return cash to the funders. During the process of drafting a turnaround plan, it is not only necessary to create the projected outcomes (based on best estimates) of what is envisioned, but also to make sure that the necessary inputs (the resources required based on best estimates) are available to complete the proposed turnaround. Resources can be:

1. the services of specialist advisors such as lawyers, engineers, or human resource specialists;
2. requiring capital to expand the firm or reorganise the affairs thereof; and
3. retaining the capital in the firm under the threat of being extracted by its funders.

These resources can be translated into the actual costs incurred during the turnaround and must be reflected in the turnaround plan. In other words, the turnaround plan is a marketing document to deliver some future outcome. It is based on a proposed strategy, specific resources being available, and inputs occurring at specific moments in time. While a turnaround plan may incorporate narrative and supporting documents such as historical and projected financial statements, the key to a successful turnaround plan pivots on the ability to communicate the impact of the fundamental moments of specific strategy implementation.

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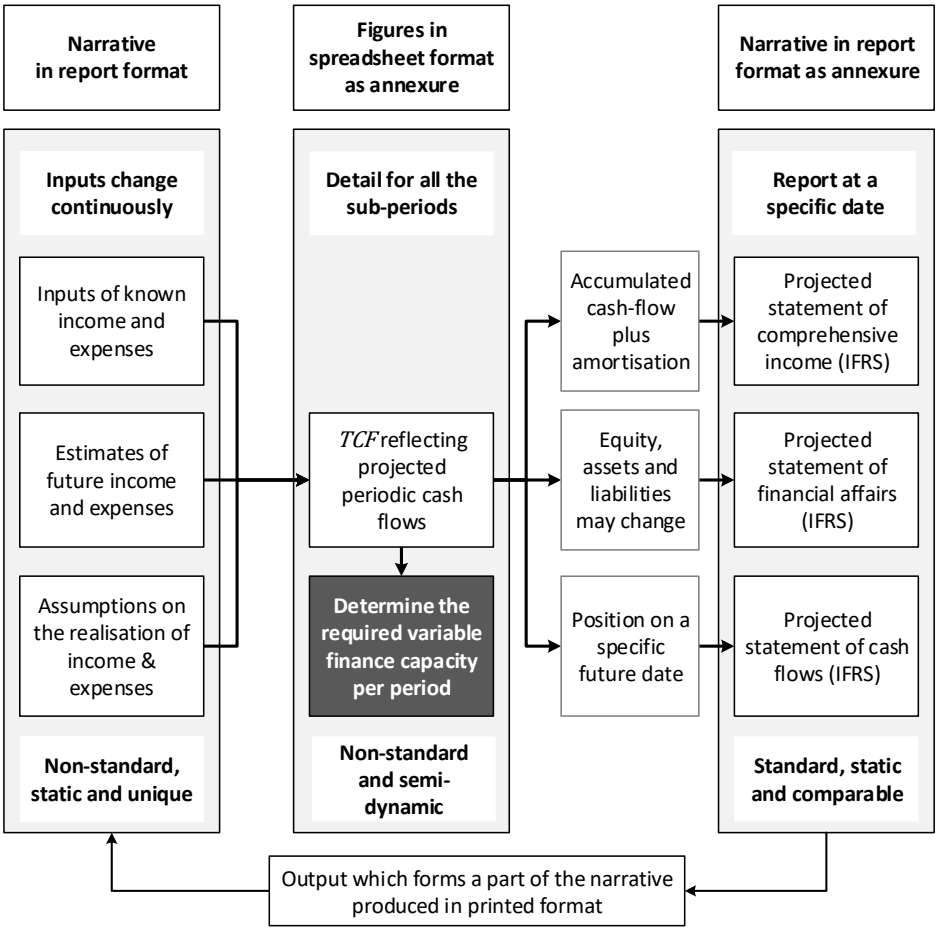

Figure 3.6: The process followed, and the content included in a turnaround plan

Source: Author (2021)

The narrative also deals with the support required from stakeholders, which may, *inter alia*, include asking:

1. employees to work overtime and receive delayed payment thereon;
2. suppliers to provide material on credit while their current account is not being settled;
3. the bank or other debt providers to increase the overdraft or provide further debt to the firm; or
4. shareholders to commit more equity to the firm.

Therefore, the information it contains and its presentation are vital in obtaining support from stakeholders.

Projected and historical financial statements: Financial statements are compiled in line with the requirements on financial statements, being the SOCI (IFRS: IAS 7 – Statement of Cash Flows (SOCI) (1976) 2017) or the SOFP (IFRS: IAS 1 – Presentation of Financial Statements

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(SOFP) (1974) 2020) for some future date, which is randomly chosen to coincide with a year-end or simply for 12 months in the future. Usually, the projected cash flow statement (IFRS: IAS 9) is optional and may be presented for completeness' sake. Turnaround plans are usually prepared for the projected duration of the turnaround. Therefore, the future date is the anticipated date of when the turnaround will be completed. The expected duration of the turnaround forms part of the fundamental moments, the focus of this study. Suppose the turnaround duration lasts longer than 12 months and extends past a year-end. In that case, the stakeholder may require projected annual financial statements and an interim set of the projected financial statements to show the effects of the turnaround. These projected figures may be shown side by side with the existing historical financial information to allow the audience whose support will be required during the turnaround to gain an informed opinion.

TCF calculations form the "pivot" around which a turnaround plan is created, as shown in Figure 3.6. It holds the entire turnaround plan together, as it determines what necessary inputs (resources) are available or should be obtained at specific dates. The plan's narrative informs the TCF calculations by providing the assumptions and inputs based on the selected strategy. However, it also informs the projected financial statements that inform the narrative of the plan of the expected outcomes. It also informs the narrative of the fundamental moments, indicating the financial and time effects of the turnaround. Since the TCF calculations are a set of sequential numerical calculations contingent on various other calculations occurring, it is clear that the slightest change in an assumption or input will change the projections and requirements in terms of fundamental moments. Failure to provide the resources as informed by fundamental moments timeously and sufficiently implies that the plan cannot be actioned. The irony is that the essence of a turnaround plan is predicated on TCF calculations that are non-standard and bespoke to every practitioner who prepares the same.

The reams of paper produced are designed to convince stakeholders to support the plan, which always starts with the questions, how much is needed, and how long will it last? Producing a hundred- or two-hundred-page plan may (or not) provide some extra comfort to stakeholders. Nonetheless, they seek to know the fundamental moments of the turnaround and that they can trust the information provided.

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3.3.3.1 Why should a turnaround practitioner prepare a firm plan?

Once management or new management appointed by shareholders decide to commence with the implementation of a turnaround, it is the job of the turnaround practitioner to devise a strategy that maps the possible outcomes thereof. Bibeault (1982) interviewed various practitioners who indicated that the first step is to prepare a cash flow forecast (TCF calculation). Various authors insist that a change in management is a pre-condition to a successful turnaround. They show that the critical difference between success and failure when in distress is directly linked to the firm's top leadership change (Barker & Mone, 1994; Hofer, 1980; Lohrke, Bedeian & Palmer, 2004; Nystrom & Starbuck, 1984; Schendel, Patton & Riggs, 1976; Slatter, 1984; Warner, Watts & Wruck, 1988). Further research by Murphy and Zimmerman (1993) and Warner *et al.* (1988) indicates that changing the senior management in a firm may have the inverse effect to that of changing the leadership or chief executive officer (CEO) of the firm. According to Bibeault (1982), Chowdhury (2002), and Harker and Sharma (2000), leadership changes may temporarily assist in dealing with a lack of confidence during the turnaround, as stakeholders' trust is the key currency to bringing about and sustaining change during a turnaround. Arogyaswamy *et al.* (1995) and Barker and Duhaime (1997) argue that as the trust in leadership dwindles, there is a decline in the firm's output, implying that the turnaround is stalling or failing. The decisiveness of adopting a specific strategy and meticulously implementing the selected strategy leads to a successful turnaround. It implies that time is critical, and the strategy must be implemented timeously and sequentially as planned. Failure to do so leads to further delays, costs, and depletion of resources that may collapse the trust of stakeholders.

Therefore, a turnaround plan is time-delineated, attempting to predict future outcomes based on the existing information and current assumptions while assuming enough resources are available to undertake the actions required. The availability of resources often informs the selection of a strategy based on resources and not necessarily the most optimal return for shareholders. Once a strategy is selected, management must determine:

1. an action list of the implementation activities to be undertaken;
2. what resources are needed; and
3. what outcomes must occur before other actions or outcomes can occur?

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In other words, a turnaround plan is a narrative of:

1. what is going to be done;
2. when is it going to be done;
3. how it is going to be done;
4. who is going to do it;
5. what resources are required;
6. what the expected outcomes will be; and
7. when it will be implemented.

This information is contained in a narrative that may include tables, graphs, projected financial statements, and possibly a copy of the TCF calculations used to determine the required resources and the achievable outcomes if these necessary resources are delivered timeously.

The turnaround strategy drives the turnaround plan

The strategy component is contained in a narrative of the plan explaining what actions will be undertaken to bring about the desired turnaround outcomes. A turnaround strategy usually focuses on activities measured in the SOCI as per IFRS IAS1, including cost-cutting and retrenchment activities. It may also focus on improving efficiencies of firm activities or any other activities and a combination thereof.

Barker, Mone, Mueller and Freeman (1998) maintain that a reduction in the cost base or assets forms a part of and should be included in the retrenchment strategy within the turnaround strategy. The cost and asset reductions should be the first strategic step implemented by the turnaround practitioner (Hofer, 1980; Pearce & Robbins, 1993). They believe that in an organisational turnaround, the turnaround practitioner should sharply reduce operational expenses by implementing aggressive retrenchment, regardless of the reasons for the decline. These sentiments are further echoed by Pearce and Robbins (1994), Robbins and Pearce (1992), and Slatter (1984), where research dealing with turnarounds has been focusing on retrenchment as a strategy to ensure that negative cash flow is converted to positive, thereby avoiding insolvency.

Arogyaswamy *et al.* (1995), Barker and Mone (1994), and Barker *et al.* (1998) hold the view that aggressive downsizing will affect customer service, product quality, and/or shipping and delivery delays. Morrow, Johnson and Busenitz (2004) found that asset reduction and/or

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retrenchment reflected a positive correlation. However, retrenchment showed little, if any, benefits in growth industries. They also found that cost reduction and/or retrenchment in declining industries had a positive correlation. In contrast, asset reduction and/or retrenchment had a negative correlation with the performance of the company. Arogyaswamy *et al.* (1995) and Stahl, Chua and Pablo (2003) argue that an ill-executed retrenchment strategy may have a negative impact on the morale and internal climate of the organisation. Various schools of thought brought about by research lead to alternative views pertaining to the success or failure of retrenchments in a turnaround strategy. According to Hambrick and Schecter (1983), there is no clear evidence that the retrenchment strategy is necessarily more effective than improving efficiency in a firm.

Some research argues that retrenchment is not the key but that change in how efficiently the firm operates seems to be the secret to a successful turnaround. Grinyer *et al.* (1990) argue that there is a difference between improvement programmes and rescue attempts. Hambrick and Schecter's (1983) and Robbins and Pearce's (1993) research indicates that improvement in efficiency strategies was usually the key strategy in a successful turnaround.

Gribnitz and Appelbaum (2015) provide a table indicating the difference between operational turnaround, focusing inwardly on activities focusing on the SOCI, versus restructuring, focusing outwardly by reorganising the SOFP. Efficiency improvements usually require a change in operational effectiveness given a particular set of assets. When dealing with a firm in distress, the selection of a strategy requires changes to the SOFP and SOCI.

Altering the firm's strategy is defined as strategic reorientation (Barker & Duhaime, 1997). Pearce and Robbins (2008) argue that a dramatic transformation of the strategy must occur after the firm has been stabilised. Arogyaswamy *et al.* (1995) believe that a weak strategic position leads to decline, and if the firm is to undergo a turnaround, it requires a change in strategy, while failure to do so may lead to the firm's demise. Hofer (1980), O'Neill (1986), and Starbuck, Greve and Hedberg (1978) state that firms in decline need strategic change or a reorientation to recover from the decline.

As proposed in IFRS (IAS 7 – Statement of Cash Flows (SOCF) (1976) 2017), the cash flow statement is merely an indicator of the source and application of cash and how cash was

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applied during the previous year. It does not add any specific value to a turnaround plan other than determining the cash balance at the end of the period as reflected in the SOFP.

Based on the author's experience, the selected strategy is unique to every firm, and it is determined by:

1. what capital resources are available currently, and what can be obtained to continue with operations;
2. if the firm model or products are still relevant in the current market conditions;
3. if the firm can be realigned or requires an entire refocus;
4. if the quality of the human capital available in the firm is sufficient to support the proposed outcome; and
5. whether the stakeholders will continue supporting the firm during and after the turnaround.

This study does not focus on "what actions are to be undertaken" but on how the information is prepared and presented. It seeks to understand "what are the financial and time effects (outcomes) going to be" subject to what resources are required to achieve the outcome."

A mixture of standardised and non-standardised inputs and outputs

As described in Figure 3.6, a turnaround plan becomes a mixture of bespoke and standardised inputs and outputs. The historical and projected financial statements are delivered in line with the standards set by IFRS, ensuring that the presentation and delivery of these financial statements are always the same, thereby enabling inter- and intra-comparison. While the source information for the annual historical information is a consequence of a rigid process of applying the accounting standards, the projected annual financial statements become an exercise of budgeting based on assumptions. A construct is created as a result of the TCF calculations.

However, the TCF calculations are a non-standardised process to periodically measure the cumulative cash flow, ensuring that the firm has enough cash and can continue operating on a period-by-period basis for the turnaround duration. The TCF calculations use a spreadsheet, entering inputs, assumptions, and estimates, to map the cumulative effects of the periodic and cumulative cash flows for each sequential period for the turnaround duration. Such a spreadsheet can be complex to create, read, and analyse and becomes unwieldy to manage.

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Since the TCF calculations are unique to the person who created them, their uniqueness is exacerbated by it being only applicable to the selected strategy applied to a specific firm at a particular time. The cumulative results of the TCF calculations inform the projected financial statements. In other words, sans/without the annual historical information prepared in terms of the IFRS rules, the rest of the turnaround plan is entirely non-standardised.

Constructing the TCF calculations is based on specific input assumptions of a strategy that may be implemented, subsequently, only to find the strategy PC continuously as new information pertaining to events, resource availability, and where sensemaking crystallises as time unfolds. This new information may lead to adopting new strategies or adapting the existing strategy, but all this information must be converted into numerical outcomes as contained in the TCF calculations. Since a turnaround is dynamic, sometimes volatile, and new information may come to light daily, the inputs may change quickly, rendering the outputs obsolete as the plan is published.

A turnaround plan is forward-looking, based on the newest information available on the date that the plan was published and the best estimates and assumptions of the practitioner, implying that the planned and actual outcome will seldom be the same. To combat this divergence of planned versus actual results, the practitioner will continuously revise the implementation based on the new information but not necessarily issue a new turnaround plan every time. It results in stakeholders being unaware of a change in the turnaround plan, leading to a breakdown of trust between the parties. A plan may contain various scenarios based on different input assumptions, complicating the plan further as stakeholders are unsure which strategy will be followed, making it even more challenging to read and comprehend.

While the plan must convey a message of the potential variance of outcomes, creating a meaningful turnaround plan that is simple to understand and yet dynamic is near impossible since the slightest change in the extensive spreadsheet will affect the projected financial statements of the turnaround plan. A dynamic turnaround plan must anticipate events that may be delayed or events that will not occur and show the potential outcomes. Creating a simple turnaround plan that is simple enough to understand but can address the dynamic elements of the possible outcomes is essential. Therefore, it creates trust and support from the stakeholders in the proposed plan.

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The narrative is always non-standard, static, and unique, as every turnaround may require a different strategy or combination of strategies that may apply to that firm. The TCF calculations are non-standard and may be semi-dynamic while presented in an electronic format. The problem is that when a turnaround plan is presented to the stakeholders, it is done in paper format or in an unalterable electronic format such as PDF, which cannot render the dynamic effects of a turnaround.

The lack of standardisation of the contents and presentation compounds the suspicion of stakeholders who generally do not believe that the firm can be returned to profitability, especially if it has been posting losses for a while. They will not want to invest time to read reams of paper to tell them something they believe cannot work. Figure 3.6 shows that the heart of the turnaround plan is to determine what funding will be required per period. From these calculations, the fundamental moments can be determined. The basic information that stakeholders would like to see is the fundamental moments for a given strategy that will be implemented. This information develops a sense of trust in what can be achieved. This study does not seek to explain what should be contained in a turnaround plan but merely aims to investigate the role of the TCF calculations in determining the fundamental moments of a turnaround.

3.4 Summary when dealing with the TCF calculations and a turnaround plan

Conventional wisdom dictates that a turnaround requires a plan containing outcomes based on estimates and assumptions and requires an implementation capacity set out in the TCF calculations. However, what is the plan's purpose, and who is the audience for which the plan is prepared? Turnaround plans seem to have become a catch-all document that contains information dealing with various aspects of the turnaround that may not apply to every stakeholder. If the purpose is to obtain stakeholder support, ensuring that the implementation capacity can be sufficiently and timeously resourced provides a substantial increase in the probability of success. Delays in implementing a turnaround are usually a result of non- or under-delivery of resources as requested by the practitioner. A delay in the implementation of the turnaround plan may be the result of approval having been obtained, but the stakeholders have not delivered the promised resources. Communication with the stakeholders and the impact of the delays in implementation due to the non-delivery of resources are critical in

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providing the promised outcome. Communication should further extend to inform the stakeholders of the progress of the turnaround implementation at regular intervals.

Presentation forms the cornerstone of communication, and this presentation must inform in a manner that allows the stakeholders to make correct and necessary decisions. The decisions they have to make rest on what is the right decision for them and not the right decision for the company. If the presentation does not portray the benefits for the stakeholders so that they can see this benefit, they will not support the plan. Promising an outcome is based on a commitment of specific resources, where a delay may result in the total or partial non-payment of staff and existing suppliers. Obtaining new financial support from an outside funder for the firm to deliver resources may be a very hazardous strategy if the practitioner does not inform the stakeholders about the risks.

Whether stakeholders consider supporting the turnaround plan or not, they should be informed what the fundamental moments for the selected strategy are and the chances of it being realised as promised. The narrative and supporting outcome documents, such as the projected SOFP and SOCI, serve as material to demonstrate to the stakeholders that the practitioner is competent and has devised an implementable plan.

Due to content being added to the turnaround plan to convince the stakeholders of the possible success of the outcome, vast additional information is added that does not sway the stakeholders to commit new capital. The answer lies in re-imagining the presentation by delivering the core and necessary information to the stakeholders visually and graphically, allowing them to visualise the different time-delineated trajectories of the VFC of the firm based on the projections and realised actuals to make an informed decision.

This presentation should also be accompanied by material to educate the stakeholders on necessary or optional information during the turnaround. Providing a set of projected annual financial statements may enable the bank to perform an evaluation, but this is predicated on the allocated resources being made available timeously. It is necessary to ensure that support is provided as promised while mapping the potential promises against delivery dates that, in turn, provide outcomes for the turnaround.

Preparing the TCF calculations is a manual process, the output of which is complicated to read and difficult to manipulate. It combines a large array of cells displayed in a spreadsheet. It is also difficult to present the TCF calculations in a manner that is easy to understand. Suppose

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the practitioner provides the VFC for every period as a single line. In that case, it leads the stakeholders not to trust the information provided, as they may want to verify the calculations themselves.

Trust in the answers can only be brought about where these vast calculations can be expressed mathematically, allowing for easy evaluation and creating a simple input mechanism that incorporates the benefits derived from the proposed strategy implementation. The current tools used to evaluate a firm making profit (annual financial statements) are not sufficiently refined in terms of time scales to be adaptable to measure the risks in a firm in distress. They do not provide a dynamic view of the periodic sequential financial effects of the turnaround, nor do they provide any comfort about what will happen if the allocated resources materialise. Even using evaluation tools such as financial ratios and z-scores on an annual or bi-annual period on the calculations is not sufficient, as these measurements are not accurate enough to determine the impact of non- or partial delivery of the resources. Resource shortfalls compound the need for further resources and imply that the outcome is at risk. It can be like using a speedometer to measure kilometres, but the actual measurement needs to be done in millimetres. Developing a mathematical model that can deliver various trajectories of the possible outcomes per period establishes the trust required in the calculation, allowing for a much more accurate manner for the stakeholders to evaluate their risks when supporting a turnaround. This process simplifies how outcomes are presented, leading to enhanced decision-making when considering a turnaround.

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Chapter 4: Research methodology used to develop the proposed $\boldsymbol{VFC}$ model to inform fundamental moments

Abstract

This chapter discusses the research methodology used to develop and introduce the VFC model and fundamental moments. An autoethnographic approach was adopted, resulting in an iterative process to understand the research problem. Once the research problem was formulated, the process of mathematising the knowledge gained in the real-world experiences of what worked or not. Practitioners use TCF calculations to determine fundamental moments that lead to process developing and testing various refinements resulting in the VFC model. Fundamental moments are determined using derivatives and maxima and minima settings in the VFC model. The values of the fundamental moments were validated by performing a test to determine the accuracy of the fit of the VFC model to that of the TCF calculations. The study then describes how to utilise these fundamental moments to make informed decisions and establish a standardised nomenclature for these moments.

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4.1 Introduction and background to the methodology applied and purpose of the various chapters in this study

This chapter deals with the research methodology applied to arrive at the problem statement and research questions, which form the pillars of this study. It discusses the research path, utilising an autoethnographic approach (AEA) to identify the problem statement and research questions that arose from my experiences as a turnaround practitioner. Building cash flow utilisation calculations (TCF calculations) is essential to a turnaround, but obtaining support from stakeholders for a turnaround is often met with limited results, as stakeholders ask "what if?" questions. These questions become challenging to answer using the numerical solutions created in spreadsheets. "What if?" questions imply that changes to the variables result in different outcomes, which have to be captured in a manner that allows stakeholders to understand the differences in potential outcomes. While creating spreadsheets containing various values for every period under consideration, it became clear that the crux of these discussions with all involved stakeholders is the turnaround's fundamental moments.

The study shows how these fundamental moments can be used to evaluate the different strategies. Practitioners are forced to select a specific strategy that will be accepted by the stakeholders even though there may be many other strategies available during the turnaround. Practitioners are limited by the resource availability or the lack of stakeholder support for the various proposed strategies, leading to the implementation of strategies that may not be optimal but are the only ones available.

The research journey ventured from an AEA to define the research problem to developing the VFC model, mathematically derived from first principles to provide an algebraic model that replaces the need to create a numerical solution using a spreadsheet. Using the VFC model, values could be derived to determine the values of the fundamental moments. A test had to be designed to evaluate how accurately the VFC model compares to the TCF calculations. Accordingly, 16 000 iterations were created between the VFC model and TCF calculations. These values were deduced to ensure that the VFC model and the TCF calculations are an appropriate, consistent, and accurate match for each other. Once the fit was confirmed for the VFC model to that of the TCF calculations, the VFC model could be used to inform the fundamental moments. The mathematical basis of these equations leads to a standardised

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nomenclature of the fundamental moments that can be used in any discussion on turnarounds.

4.2 The research methodology and modelling process used in this study

This study introduces the mathematically derived VFC model that informs fundamental moments of a turnaround. These can be used for research and enhanced decision-making when dealing with turnarounds. Therefore, the VFC model replaces the numerical solutions as a part of the TCF calculations. The purpose of this study is to propose a nomenclature of fundamental moments that can be universally accepted and used in research and by the industry that can only be achieved using mathematical modelling as the basis for research modelling to establish such nomenclature.

Perrenet and Zwaneveld (2012) argue that mathematical modelling follows a cycle that starts and ends with a problem in a real-life situation that forms part of the non-mathematical world. By identifying the problem and finding a mathematical solution that can be simulated, the application of the solution in the real world can be evaluated. This modelling process was extensively described in many studies, showing how the mathematical model is developed through an iterative process of mathematising, simulating, and measuring (Berry & Davies, 1996; Blomhøj & Hoff Kjeldsen, 2006; Blum, 1996; Blum & Leiß, 2006; Borromeo Ferri, 2006; Kaiser, 1995; Kaiser & Schwartz, 2006; Langley, 2002; Perrenet & Zwaneveld, 2012) state that mathematical modelling requires a:

1. "Problem translation into a mathematical model;
2. the solution to mathematical problems and
3. interpretation of the solution in terms of the context of the original model."

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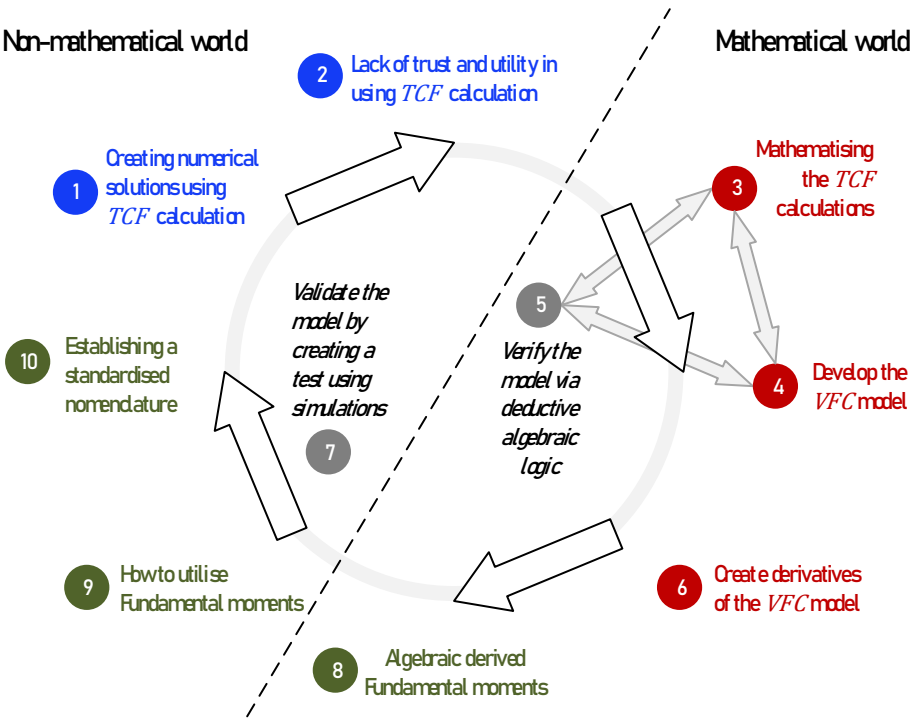

Figure 4.1: The process used to understand, create, and verify the VFC model that informs the fundamental moments

Source: Author (2021)

This study seeks to convert the calculation of the financial and time consequences that exist in the real world into a mathematical model that can deliver the fundamental moments of a turnaround. The outcome of this study enables these fundamental moments to:

1. make the calculation thereof much easier and quicker;
2. become a universally accepted method to calculate these fundamental moments;
3. create intra- and inter-comparisons of turnarounds; and
4. deliver a standardised nomenclature when dealing with these moments.

4.2.1 The modelling cycle used in the development of this study

Figure 4.1 shows the process or cycle used to develop the VFC model that informed the fundamental moments that:

1. commence with the understanding that there are currently no standards when creating a numerical solution that determines these fundamental moments;
2. there is a lack of trust by stakeholders that affects the utility in the veracity of TCF calculations when stakeholders have to make decisions in dealing with these moments;

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3. the TCF calculations had to be mathematised on a first-principle basis by developing a basic model and then expanding it with the impact of Abnormal Costs (AC) and delays in the implementation thereof;
4. the VFC model was derived by incorporating AC and delays;
5. the model was verified using deductive algebraic logic;
6. upon the verification of the VFC model, derivatives were used to determine the fundamental moments;
7. 500 test sets consisting of 1 559 calculations and 1 708 cells with data for every test set were created —varying one input value at a time, resulting in 16 000 iterations with 779 500 calculations to determine if the fundamental moments are accurate and a good fit against those values generated using the TCF calculations;
8. a comparison of the VFC model and TCF calculations verify that the fundamental moments can be algebraically derived;
9. a method was devised on how to utilise and implement these fundamental moments showing the impact during a turnaround; and
10. establishes a standardised nomenclature that deals with issues raised relating to trust and the utility of financial projections during a turnaround.

This process led to different research methodologies being used to conclude this study. It commenced through the author's autoethnographic experiences in turnarounds and restructuring to understand why the current method of determining fundamental moments has shortcomings. The research methodology then moved to mathematical modelling that has been verified and can inform fundamental moments more efficiently.

4.2.2 Establishing the problem statement and research questions in the real world (Stages 1 and 2)

Stages 1 and 2 of the modelling cycle, as shown in Figure 4.1, were derived via an AEA to define the research problem and understand the research questions that this study seeks to address. A turnaround is a real-world phenomenon subject to constraints that are dealt with using best practice methods currently available to determine fundamental moments. Based on the author's experience in the field of turnarounds and business rescues led to understanding:

1. how a turnaround plan was to be used;
2. what details should be included or not;

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3. what shortcomings and issues exist with how fundamental moments have been determined, and ultimately;
4. what factors led to successfully obtaining approval from the stakeholders.

As stated in Chapter 3, a practitioner will prepare a cash flow projection via a spreadsheet using numeric values. There is currently no algebraic manner to determine a turnaround's fundamental moments. The lack of quantitative expressions implies a lack of standardisation, resulting in mistrust by the stakeholder over the accuracy thereof.

The author's journey started when a banker argued that the AC incurred in a business rescue was excessive and should be substantially less. It led to an argument that getting the rescue completed as quickly as possible was more important than worrying about the absolute value of the AC incurred. At the same time, trying to prove the argument led to the understanding of the author that TCF calculations are not suitable or adequate as a measurement and evaluation tool. Therefore, the problem statement for this study was formulated/postulated on a *posteriori* basis, based on the author's experiences. It provided the knowledge and understanding of how turnarounds are managed and measured in commerce and the issues and shortcomings that exist when delivering a turnaround plan and relying on the TCF calculations that are the actual basis of measurement for the practitioner.

The author spent over three decades in turnarounds and business rescues in South Africa, completed 73 restructures as a principal, and was an advisor on 47 business rescues as contemplated in the Companies Act, Act No. 71 of 2008 (South Africa, 2008). Some of these restructurings involved the closure of the business, while others were traded to profitability and then on-sold. These experiences led the analytical AEA to create a turnaround plan and cash flow projections to obtain support for a turnaround. It can be defined as convincing those who have capital (equity or debt) committed to the business to leave their capital in the business and convincing others to support the turnaround process by making more capital available for the potential growth of the business. This ability to make informed or enhanced decisions during a turnaround led to the formulation of this problem statement.

Over 30 years, the author prepared bespoke numerical solutions in spreadsheets to determine the fundamental moments in the firm undergoing a turnaround. Over this time, it became abundantly clear that, regardless of what plans were laid to turn around the business, a lack of cash flow immediately scuppers any business rescue or turnaround.

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During this period, the author dealt with financially knowledgeable parties, like bankers and those who did not know about financial affairs, while seeking support from those already involved in the turnaround (committed and busy dealing with loss minimisation).

As a part of obtaining support, the author tried to convince stakeholders to provide new capital to fund the turnaround (while providing an excellent return) but was always met with suspicion by the stakeholders and had very little success obtaining new capital. The plan was always adopted by those already committed (as creditors) when the turnaround commenced, not because they understood the plan but because they understood that the outcome being offered was better than the alternative. However, when dealing with potential private or public investors in debt or equity, the author realised that very few of these parties read the plan's narrative, let alone analyse the supporting figures. The author also noticed that no person ever asked for an electronic copy of the extensive spreadsheet to verify if the assumptions, predictions, inputs or even spreadsheet were indeed correct. Investors are negatively predisposed to turnarounds and assume that they will fail more often than not. Their perceived risk of failure is because they do not have an accurate way to measure risk and, therefore, cannot manage their risk when investing in a turnaround. Using a turnaround plan that also incorporates a numerical bespoke TCF calculation in the hope of convincing an investor to invest is like using a hammer to loosen a bolt; it is entirely inefficient, and it does not work.

Some of the insights developed came at great expense, and as the saying goes – "… there is no substitute for experience by cheque book". The author questioned the purpose and use of spreadsheets when engaging with third parties, resulting in the extreme property space of an unorthodox academic (Lazarsfeld, 1937; Siggelkow, 2007). The author examined turnarounds across different economic cycles, industries, and businesses, identifying extreme examples in the turnaround space (McClelland, 1998). It, in turn, allowed for an expansion of the theoretical understanding of what does or does not work (Yin, 2003) "… because the dynamics being examined tend to be more visible than they might be in other contexts" (Pratt, Rockmann & Kaufmann, 2006:238). Gold (1958) described four categories of an observer: complete participant, participant-as-observer, observer-as-participant or complete observer.

The cases where the author acted as a turnaround principal made him a complete participant. The author's beliefs, theories, techniques, and applications were put to the test, leaving no room for error. Trading in distressed companies was not only challenging conventional thinking

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by reviewing assumptions, accepted principles, and rules (Lunenburg, 2012) but also challenged the outcome by which the research was conducted with a simple question – "did they support the plan or not?" Accordingly, some conclusions to develop the research question are based on subjectivity, relying on an analytic autoethnographic qualitative research approach, characterised by analytic reflexivity where the complete participant is interested in what other parties think and will do based on their own value systems. "Autoethnographic interrogation of self and others may transform the researcher's own beliefs, actions and sense of self" (Anderson, 2006:383). As an objective realist, the author gathered knowledge from every experiment (transaction) and performed these experiments in the real world, where a mistake could be costly. The author could draw conclusions from these experiences by observing participants' responses. The resulting analysis led to question the efficacy of using traditional turnaround numerical solutions generally relied on, as evaluated through the author's experiences, which led to the problem statement and research questions contained in this study.

4.2.3 Mathematising and development of the $\boldsymbol{VFC}$ model (Stages 3, 4 and 5)

Stages 3, 4, and 5 of the modelling cycle, as shown in Figure 4.1, describes how the VFC model was developed, using first principles to establish a set of quantitative expressions that form the foundation of the fundamental moments of a turnaround. Creating a deterministic mathematical model, developed from first principles, describes the sequential consequences that follow when formulating the calculations that simulate the overall cash flow over the various turnaround periods. The model describes the specific relationships between time and financial consequences when undertaking a turnaround. Stage 3 was the conceptualising stage in converting the TCF calculations into a model using algebraic concepts. The mathematising led to the creation of the VFC model, as shown in Stage 4. This model had to be algebraically verified using deductive algebraic logic to ensure that the model was correct. Stages 3, 4, and 5 became an iterative process, where the calculations' algebraic interpretation were numerically calculated and compared against the TCF calculation currently in use to determine fundamental moments. This process was to establish a model that is an accurate and appropriate fit to the TCF model.

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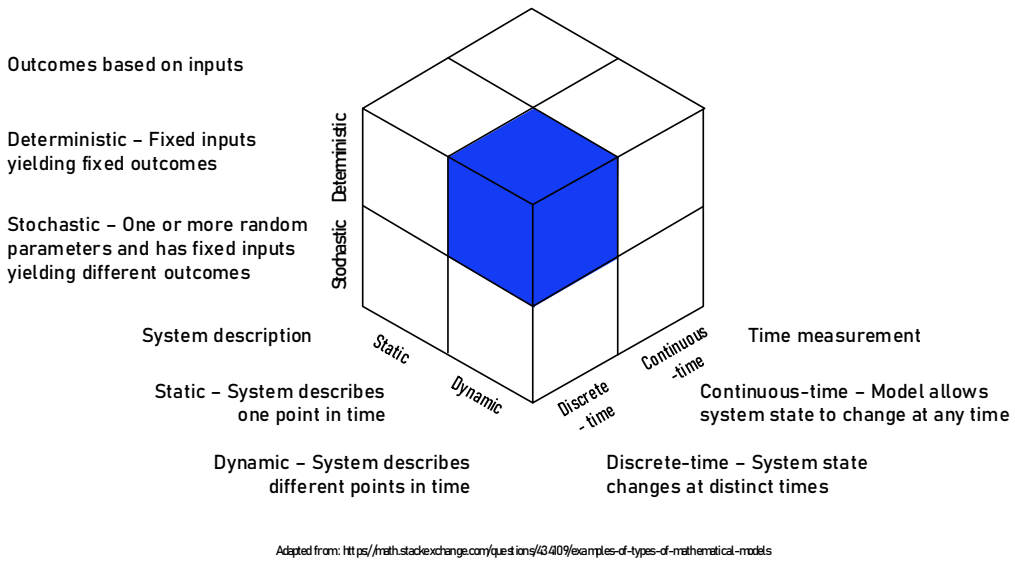

Figure 4.2: Positioning the development of the VFC model as being a dynamic deterministic model using discrete time intervals

Source: Partially adapted from Stack Exchange (2014)

Figure 4.2 summarises how the VFC model is positioned in time, system, inputs, and outputs. The VFC model can be classified as a dynamic deterministic model that works on discrete time intervals. It is dynamic because it describes different points in time, thereby offering a dynamic view of the changing outputs. The outputs of fundamental moments are interlinked, leading to a unique outcome for every set of inputs, making the VFC model deterministic. The inputs result in a specific and unique set of fundamental moments and do not lead to random outcomes. The model is deemed a discrete-time model as it measures progress in sequential periods. These periods can be equated to months, quarters, years, or even any other length of time as long as these intervals are equidistant. Ultimately, the VFC model is an approximation of the future cash flow projections of the firm. The model is driven by the assumptions used as inputs to determine future estimated outcomes.

4.2.4 Establishing derivatives, validating the values for fundamental moments (Stages 6, 7, and 8)

Stages 6, 7, and 8 of the modelling cycle, as shown in Figure 4.1, result from the algebraic nature of the VFC model, where the derivatives are used to establish the fundamental moments as contemplated in Stage 6. It was achieved by determining the maxima and minima for the VFC model to determine the fundamental moments of a turnaround and using parameters as constraints.

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Stage 7 required a test to be developed to determine how accurately the VFC model fits the TCF calculations. The primary test was constructed by simulating values for the VFC model and TCF calculations using the same input parameters. The values for each period were deducted from each other to determine if there was a difference and, if so, what level of accuracy the fit could be judged to be a: no fit, good fit, or perfect fit. The level of accuracy is vital as it determines the order of magnitude a practitioner or researcher should expect errors to occur when using the model. A simulation of 16 000 iterations was generated to ensure that the level of fit for this model was accurate and meaningful.

Upon completion of the simulation and tests, Stage 8 could follow. The fundamental moments are algebraically derived and deemed a fit with a measured level of accuracy, allowing for a stated margin of error between the TCF calculation and VFC model.

4.2.5 The utilisation of the VFC model and establishing a standardised nomenclature (Stages 9 and 10)

Stages 9 and 10 of the modelling cycle, as shown in Figure 4.1, take the mathematically derived VFC model back into the real-world domain. Stage 9 shows how to apply and implement the model to determine the fundamental moments. The VFC model and the determination of fundamental moments are derived on a *priori* basis where it was derived from first principles through deductive reasoning, which reports the cause and effect but remains independently valid from preconceived observations. Therefore, the outcome and finding of this study is a mathematically derived model that fits the problem statement in the real world, as identified by experience and observation.

This study is neither empirical nor stochastic in nature. It does not predict how the system behaves but rather shows how the two components, income, and expenses (operational and abnormal), work together, showing the turnaround's effects. Its purpose is to provide an appropriate, consistent, and accurate fit to the TCF calculations so that stakeholders can use fundamental moments and make enhanced decisions. These standardised quantitative expressions create the ability to measure and test the effects of a change in input parameters.

Stage 10 of the mathematical model also enables a method of description that allows a standardised nomenclature on turnarounds, as the definitions provided can now be mathematically determined and leave no room for error in the determination thereof

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regardless of context, content, or process when dealing with a turnaround. It forms the foundation for the nomenclature of fundamental moments and allows for easy evaluation of the impact of selecting a specific strategy. Understanding the required fundamental moments will allow the ability to determine when and what finance will be required and the turnaround duration. It also allows precise and concise language to formulate underlying assumptions while utilising well-defined rules to perform mathematical manipulations. These manipulations aid in decision-making that may include tactical or strategic decisions by management and researchers.

A discrete model enables "time" to be input as an integer value, representing a homogeneous and discrete-time continuum that can be adapted according to the measurement requirements. This feature allows for intercomparisons between historical and actual performance.

4.2.6 Summary of the research methodology implemented in this study

Practitioners currently use TCF calculations as a tool to determine the fundamental moments when dealing with turnarounds and business rescues. This study seeks to establish a formal model to replace the TCF calculations and requires a clear and precise process to achieve this outcome. This model must be mathematically developed and interpreted, verified by simulation, and presented in a manner that would easily be adopted in industry and research and form the basis of the status theory of turnarounds.

4.3 Mapping the research problem, purpose, and questions to the chapters of this study that deal with the development of the $\boldsymbol{VFC}$ model and outcomes thereof

Chapter 1 sets out the research problem, purpose statement, and questions. Figure 4.3 provides an overview of how the chapters in this study were constructed to achieve the objectives set out in Chapter 1. In essence, this study proposes that turnaround practitioners seek to answer specific vital questions when considering various strategies to undertake a turnaround. The problem statement of this study revolves around the fact that there are no quantitive equations to determine fundamental moments. The study investigates how fundamental moments are addressed by practitioners in Chapter 3, while it seeks to understand how research on the turnaround process deals with fundamental moments in

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Chapter 2. The purpose statement is set out in Chapter 1. The methodology used to achieve the purpose is set out in Chapter 4, which explains how TCF calculations are used in practice and will result in the development of the VFC model. The main- and research sub-questions are set out in Chapter 1 to determine if it is possible to derive/design/create an algebraic model to replace the TCF calculations and, if so, how appropriate, consistent, and accurate the model will fit that of the TCF calculations, and whether a standardised nomenclature can be established to define fundamental moments.

Annexure 1 deals with the algebraic development of the VFC model, which informs the tests performed in Annexure 2 to determine the appropriateness, consistency, and accuracy of the VFC model to that of the TCF model. Chapter 5 provides the research findings as set out in Annexures 1 and 2. These annexures and Chapter 5 further propose a method of presenting fundamental moments that makes it easy to understand the outcome of a selected strategy coupled with a set of input assumptions.

Chapter 6 discusses the recommendations and potential benefits of the findings as set out in Chapter 5. It explains the benefits of using fundamental moments in practice and the research dealing with the turnaround process. The chapter concludes by discussing the research lenses or foci that have been identified by various researchers and how fundamental moments may be used to develop the research of turnarounds in the future.

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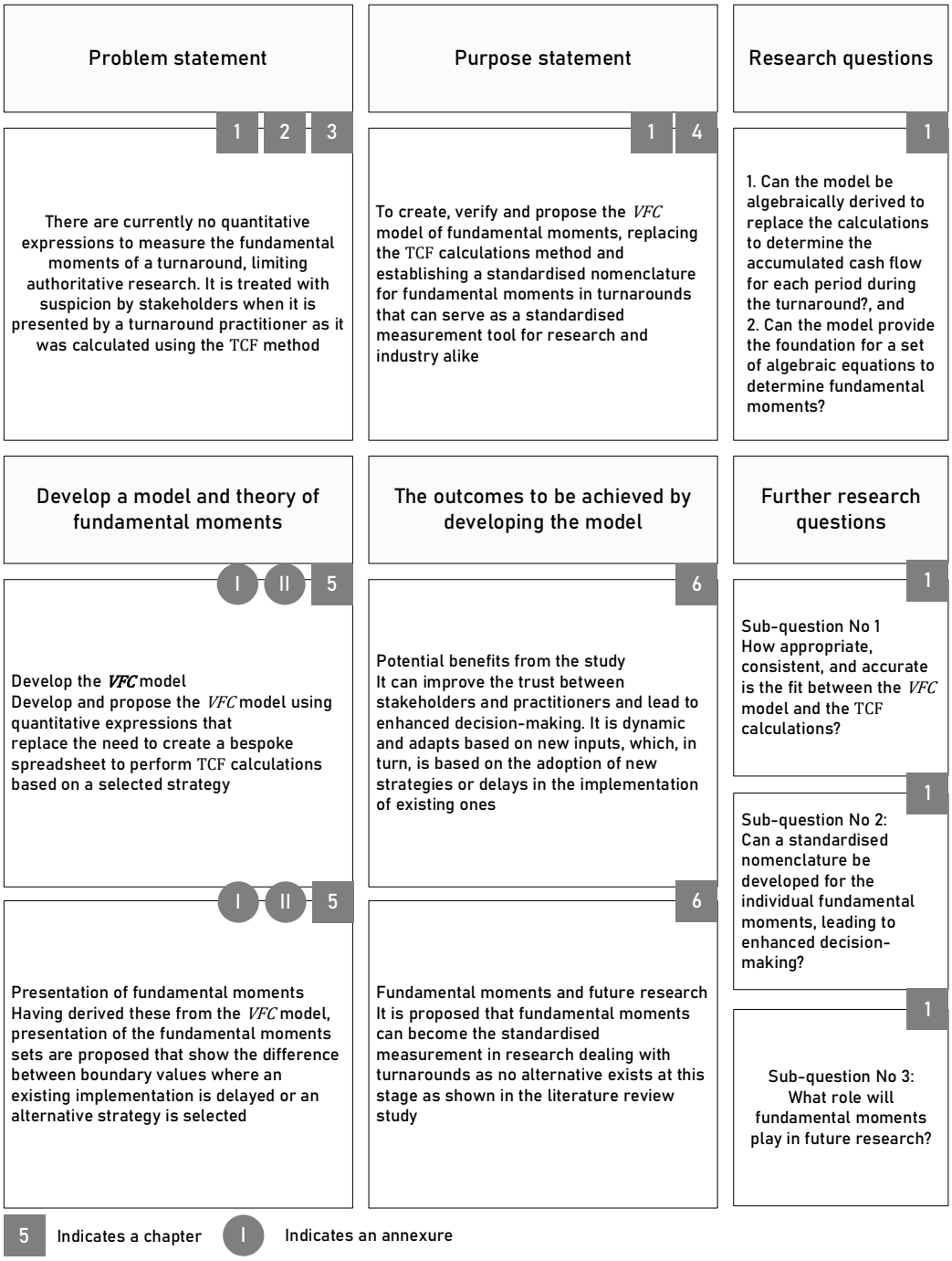

Figure 4.3: Mapping the research problem, purpose, and questions to the chapters that deal with the development of the VFC model and outcomes thereof

Source: Author (2021)

This section shows that the research problem, purpose, and questions in Chapter 1 are mapped to specific annexures and chapters of this study. Figure 4.3 shows how the process described in Figure 4.4 is presented in this study.

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4.3.1 A brief overview of the chapters included in this study

The previous section mapped the objectives of Chapter 1 to the rest of the chapters and annexures of this study. This section discusses the abstract of each of the chapters and annexures contained in this study.

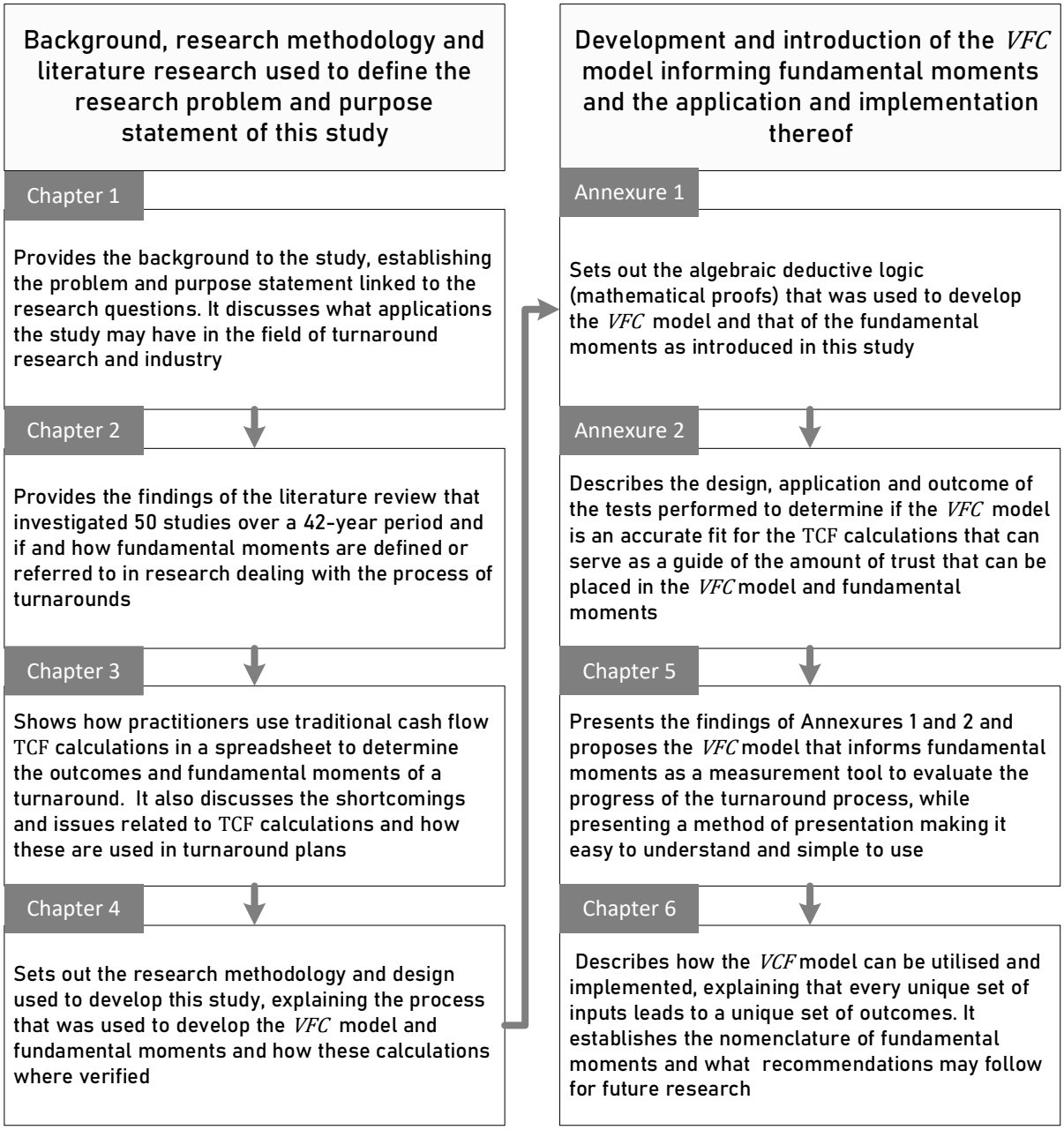

Figure 4.4: Summary of the chapters contained in this study and how these interlink

Source: Author (2021)

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Figure 4.4 provides a brief overview of the framework of the six chapters and two annexures contained in this study. The first four chapters deal with the introduction, background, overview, description of how fundamental moments are dealt with in practice, literature review of research dealing with the turnaround process, and research methodology. Annexures 1 and 2 provide the research used to inform the findings as presented in Chapter 5, which informs the discussions and recommendations set out in Chapter 6.

4.4 Summary

This chapter discussed how the type of research methodology that commenced with the author's experiences as a practitioner dealing with restructures and turnarounds resulted in an AEA informing the problem statement of this study. The research methodology then morphed into constructing the VFC model that informed fundamental moments. It further discussed the ten-step process employed to develop, verify, and present the VFC model informing the fundamental moments.

The chapter also showed how the research problem, purpose, and questions, as set out in Chapter 1, are mapped to chapters and annexures of this study that guides how the answers sought were derived. Finally, this chapter concluded with a brief overview and summary of the chapters included in this study.

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Chapter 5: Proposing the VFC model that informs fundamental moments during a turnaround

Abstract

This chapter introduces the VFC model and explains how to use an assumption sheet to record the turnaround strategy, thereby determining the input values to be used in the VFC model based on the chosen strategy. It shows that using the different inputs derived from an assumption sheet, the VFC model delivers various unique outputs. The VFC model delivers the accumulated cash flow at each period and shows the impact on the turnaround when there is a delay in the planned changes (PC) as determined by the strategy.

This chapter also shows how the VFC model was verified as a perfect fit for the TCF calculations. Using the VFC model, different equations were algebraically derived from calculating the various individual fundamental moments, as determined by the various assumptions and input values. It concludes by establishing a precedent for presenting these fundamental moments as a succinct array of values.

This presentation method underpins the finding that every unique set of inputs has a unique set of outputs, thereby creating a unique set of fundamental moments.

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5.1 Introduction and background

When turnaround practitioners build a spreadsheet to predict or budget future cash flows, they create a spreadsheet that is unique to their skills and specific to that business. While the spreadsheet may be bespoke, it ultimately requires a particular set of outputs to be the same in all budget or cash flow projections. The income items (cash inflows) and expenses (cash outflows) are summed, deducted from each other, and added to the difference from the previous period to provide the cash flow at the end of the period, also referred to as the accumulated cash flow for the period. Given the requirements to calculate these outputs, an algebraic model was created to map the VFC for each period, representing the accumulated cash flow requirement that the firm will have at the end of each period. The creation of the VFC model enables the creation of derivatives, showing how long the turnaround will last when the breakeven occurs during the turnaround, what the required peak finance amount will be, and what happens if the implementation is delayed. While the VFC model is predictive, the calculations should also evaluate and compare the historical to the previously predicted or actual performance as the turnaround develops.

5.2 Background to proposing the VFC model

Chapter 3 showed how fundamental moments are currently determined using the TCF calculations. It also discussed various shortcomings in using the TCF calculations and concluded that stakeholders lack universal trust when dealing with spreadsheets. Chapter 2 provided a literature study that discusses how current research deals with fundamental moments as a part of the turnaround process. It was found that no proper nomenclature existed in research when dealing with fundamental moments. Both these chapters established that the fundamental moments of a turnaround should be defined algebraically, as it would: lead to fewer errors when calculating these values; better analysis and presentation to obtain stakeholder support; offer a method to create a standardised nomenclature for fundamental moments and allow for inter-and intra-comparisons between turnarounds. It argues for the creation of a universally accepted measurement system to improve the research on the turnaround process and how practitioners engage with stakeholders. This chapter introduces the VFC model that has been algebraically derived and verified utilising a fit-test to ensure appropriateness, consistency, and accuracy with the TCF calculations currently in use. It also

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sets out how the fundamental moments are calculated and can be presented in an array for ease of use.

5.2.1 The purpose statement of this chapter

Figure 5.1 provides an overview of the taxonomy of this chapter and:

1. presents an explains the inputs required for the VFC model;
2. introduces the VFC model to calculate the output values given certain assumptions relating to AC (ubiquitous) or if delays are anticipated; and
3. delivers four fundamental moments of a turnaround depending on the selected assumptions.

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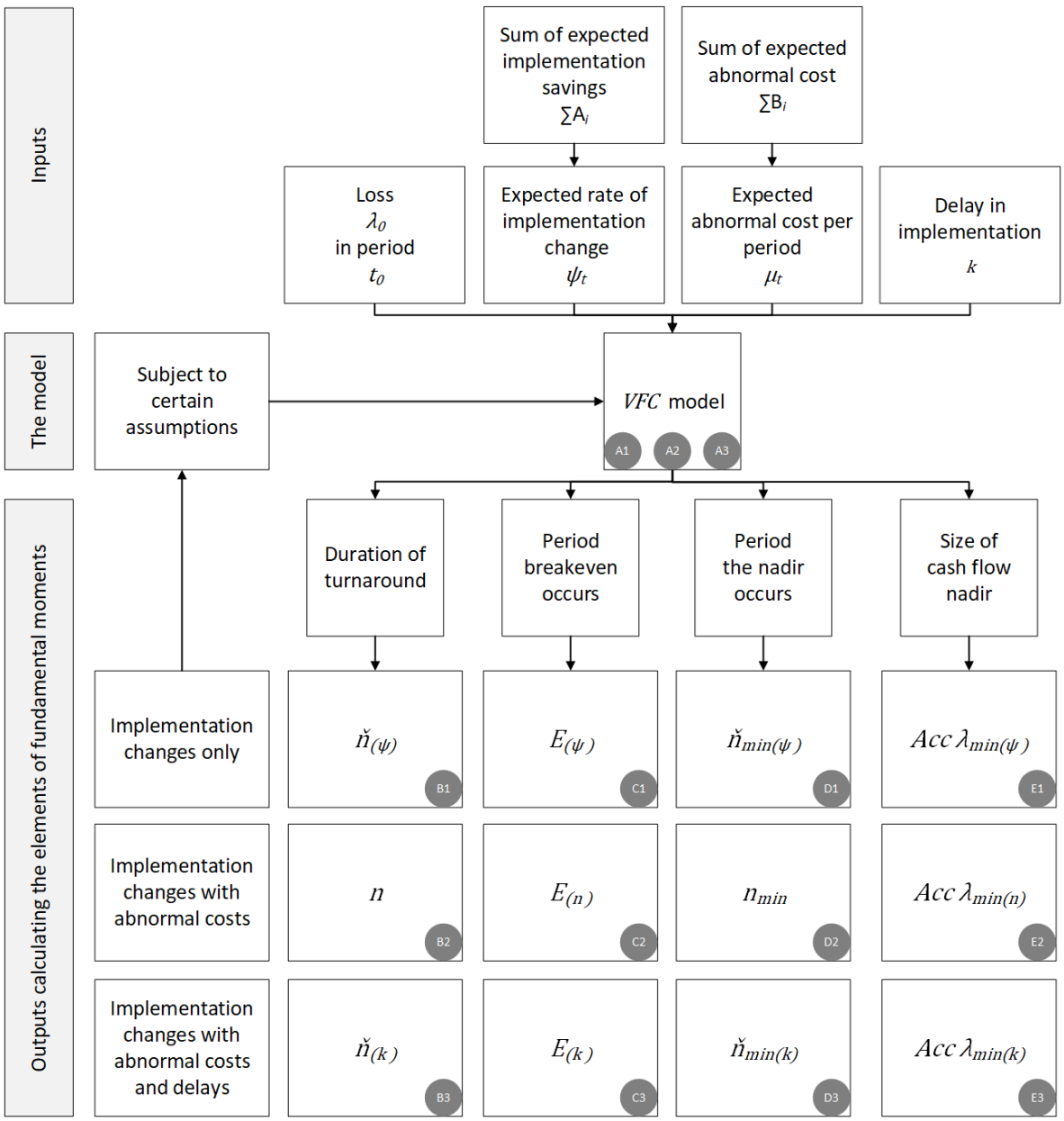

Figure 5.1: The relationship between the inputs, the VFC model, and fundamental moments based on different assumptions given certain inputs

Source: Author (2021)

Accordingly, this chapter endeavours to set out the various calculations for each case: where AC or delays are not present (the absolute best scenario but least likely); where no delays are present, but AC are present (the preferred scenario); and where there are AC and delays (the most likely scenario).

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5.3 Introducing the VFC model

Figure 5.1 summarises the relationship between inputs, the model, and how fundamental moments are derived based on certain assumptions being present. This section discusses how these inputs are determined and introduce the equations used in the VFC model. This section also shows how the different input values produce different outcomes for each period of t.

The VFC model provides the mathematical foundation from which the fundamental moments are derived and for the establishment of the nomenclature of the fundamental moments. It further attempts to illustrate how the slightest change in input or an assumption leads to a different set of fundamental moments. The ability to calculate the various outcomes can lead to enhanced decision-making when dealing with turnarounds.

The VFC model: This model shows the results of the mathematical modelling used to progress from:

1. understanding the simple dynamic between the proposed implementation changes (ψ_t) and the rate at which the loss (λ_o) is incurred for each period t;
2. the VFC model is further expanded to take into consideration the effects of AC (μ_t), and how it impacts the outcomes of the fundamental moments; and
3. as delays can occur and usually do for various reasons, the model is expanded to consider what effect the delay (k) will have on the fundamental moments.

The VFC model enables the calculation of the cash flow for each period (t_x) at the end of the period, referred to as the accumulated cash flow, so that:

1. $Acc\ \lambda_{(k)}$ assumes AC and a delay in the implementation of changes (PC) has occurred;
2. $Acc\ \lambda_{(n)}$ assumes there are no delays, but AC are present; while
3. $Acc\ \lambda_{(\psi)}$ assumes there are no delays and no AC present.

The cornerstone for introducing the VFC model rests on the appropriateness, consistency, and accuracy with which it will return the same value as generated using the TCF calculations. Confirmation of the fit between the VFC model and the TCF calculations was determined using 16 000 iterations. The VFC model was required to return the correct outputs regardless of the frequency/period of the measure, planned changes proposed, and AC to be incurred to

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affect the turnaround. Return values were also required if there were delays, regardless of the inputs or assumptions used.

The fundamental moments: The mathematically derived fundamental moments derived using the VFC model are the basis of measuring cash flows during a turnaround. The VFC model thus offers a simplified numerical array to determine the outcomes without creating spreadsheets or graphs to calculate the critical values required to address the time and financial consequences of a turnaround. It also allows for establishing a standardised nomenclature for the financial and time consequences of the turnaround.

5.4 The methodology used to derive the model

This study shows that a dual research methodology approach was used, where:

1. in Chapter 2, current research dealing with the process of turnarounds has not identified or defined fundamental moments as the loadstar used by practitioners in managing turnarounds;
2. in Chapter 3, an AEA led to the understanding of the shortcomings and issues that exist with TCF calculations and why there is a need to find an alternative; and
3. in Chapter 4, the research methodology was proposed to mathematise an alternative to the TCF calculations that sought to deliver the same values and resulted in the VFC model.

Therefore, it implies understanding a real-world problem in the non-mathematical world, creating a very basic model, and refining deductive algebraic logic to establish the VFC model. The VFC model was subjected to testing to determine if it delivers the same values for every period as delivered by the TCF calculations. Once suitability, consistency, and accuracy were established, the fundamental moments of a turnaround could be algebraically established using the VFC model. Using mathematical manipulation and differencing, the process was achieved to establish a verifiable calculation method for turnaround outcomes. The use of fundamental moments allows a measurement system to discuss any turnaround strategy or research thereon, as it establishes a standardised nomenclature for turnarounds.

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5.4.1 The inputs that are used to create a VFC model

This section discusses the various inputs used in the VFC model and how assumptions around these inputs were used to devise the VFC model. The inputs consist of:

1. what is the loss (-λ_0) in the period prior to commencing the turnaround;
2. what are the additional (abnormal) costs (-μ) associated with the turnaround;
3. what are the proposed planned changes (ψ) that will be implemented as a result of the turnaround; and
4. what happens if there is a delay (k) in the implementation of the proposed turnaround plan.

Loss λ_0 as at period t_0: A loss or a profit at the end of a period, as shown in any financial report, assumes that it is a differential between the sum of all the income generated and the sum of all the expenses generated, to date, for a given period. This differential is a good indicator of what may happen if no intervention occurs, assuming no additional new expenses are incurred. Assuming a change in management occurred to affect a turnaround, the newly appointed turnaround practitioner will be presented with the firm's financial statements showing that the firm has been incurring losses for some time, deemed to be accumulated losses. The loss made for the period prior to the implementation of the turnaround is an indicator of the operational state of the business. Assuming that no action is undertaken and no new or extra expenses are to be incurred, it would imply that the firm will continue to make the same loss for every period thereafter so that:

$$\lambda_0 = \lambda_1 = \lambda_2 = \cdots = \lambda_n \,; where\ t = 1, 2, 3 \ldots n \qquad (1)$$

Planned changes (ψ_t): are the improvements that were derived from the planned changes (A_i) that the turnaround practitioner expects to implement during the turnaround. Typically, improvements occur from period t_1 as the practitioner will require the first period (t_0) to analyse the firm's affairs and devise a strategy to implement the turnaround. Accordingly, the model assumes that the planned changes are only implemented from the end of t_0, thereby implying that the results of the planned changes will only be measured at the end of t_1. Planned changes refer to any action that may be implemented during the turnaround. It may include:

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1. savings in overhead costs;
2. any improvements brought about by enhancing efficiencies that would imply an improved cost of sales or margins; and
3. the anticipated increase or decrease in the sales to be achieved during the turnaround. The practitioner and their team will estimate how long it would take to implement each individual change (action) during the turnaround.

Every implementation change will have a different outcome in terms of duration, implying that the values for each period will be different for the duration of the turnaround. It can be achieved by calculating the average of all the individual periods (t_{ave}) or merely taking the individual task that will take the most prolonged period to implement as the expected period of duration (t_{max}). As an alternative, an expected implementation period can also be the weighted average for each implementation change. However, underestimating the expected duration is usually a common mistake that the turnaround practitioner makes, as the actual duration inevitably exceeds the original estimated duration. Accordingly, it is better to use (t_{max}) as the expected period of implementation. The implementation change for each period can, therefore, be calculated as:

$$\frac{\sum \mathrm{A}_i}{t_{ave}\ or\ t_{max}} = \psi_t$$

$$where\ \mathrm{A}_i\ is\ the\ sum\ of\ all\ the\ expected\ implementation\ changes\ and \quad (2)$$

$$t_{max}\ or\ t_{ave}\ is\ the\ expected\ period\ of\ the\ implementation$$

AC $(-\boldsymbol{\mu_t})$**:** are the additional costs explicitly associated with the actions undertaken during the turnaround and do not occur in the normal cause of the business. These may include *inter alia*:

1. fees and costs for outside advisors or even the practitioner fee in the event of being a contractor;
2. additional labour costs associated with the termination of the services;
3. costs associated with closure and disposal of assets;
4. interest cost during the duration of the turnaround;
5. any other costs associated with implementing a turnaround; or
6. any combination of the above;

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7. Abnormal or exceptional costs (μ) in certain jurisdictions are deemed a non-recurring expenditure in terms of IFRS 2013: IAS 1 and exceptional items in terms of FASB, 2002: No 145.

The VFC model, therefore, differentiates between AC and operational costs incurred in the normal course of business, as the AC (μ) are not contained in the loss (λ_0) where the loss will be repeated for every period thereafter. The estimated total sum of the estimated AC (B_i) are divided by the estimated or expected implementation period (where the expected implementation period has to be the same one used for determining the implementation change (ψ_t) to provide what the average AC per period will be so that:

$$\frac{\sum \mathrm{B}_i}{t_{ave}\ or\ t_{max}} = \mu_t$$

$$where\ \boldsymbol{B}_i\ is\ the\ sum\ of\ all\ the\ expected\ AC\ and \tag{3}$$

$$t_{max}\ or\ t_{ave}\ is\ the\ expected\ period\ of\ the\ implementation$$

Estimating delays ($\boldsymbol{k}$): The VFC model incorporates measurement for delays (k) in the planned changes during a turnaround. By establishing and incorporating anticipated delay in the turnaround, the VFC model delivers a new set of fundamental moments, providing the stakeholders with the ability to form an opinion on the potential successful implementation of a turnaround. It allows for the determination of outcomes based on the delays given a set of inputs, thereby enabling enhanced decision-making. Due to the lack of resources, failure to complete a turnaround represents the worst outcome for any stakeholder. Understanding the impact a delay may have on the resource requirements can lead to improved scenario planning. It allows for buffers to be held in reserve by a stakeholder to ensure that the turnaround can be completed.

5.4.2 The equations used to constitute the $\boldsymbol{VFC}$ model

Based on the assumptions and inputs that determine the output values for the different periods of t_x, Equations (1), (2), and (3) are expanded to form the VFC model. The model consists of three equations, where the equation used depends on whether there is:

1. no provision for AC (μ) and no delay (k);
2. the inclusion of AC (μ) and no delay (k); and

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3. AC (μ) and delays (k) are present.

In Equation (4) A1 of the VFC model, only planned changes (ψ) are considered (no AC and no delay); Equation (5) A2 incorporates the planned changes (ψ) and AC (B); while Equation (6) A3 expands Equation (5) A2 to incorporate the effect of expected delays (k) to the planned changes.

Equation (4) A1 returns accumulated cash flow $(Acc\ \lambda_\psi)$, also known as the cash flow at the end of each period for the VFC model, assuming there are no AC and delays. The implementation change (ψ) is calculated as discussed above and shown in Equation (4) A1 below. If we take a to represent the planned changes (ψ) expressed as a proportion of the operational loss (λ_0), and then equate the value of ψ to the operational loss (λ_0) divided by a so that $\psi = \frac{\lambda}{a}, with\ a > 0\ where\ [\lambda, \psi, a \in \mathbb{R}]$. Since there are no delays, the expected duration $(\breve{n}_{(\psi)})$ of the turnaround forms a part of the fundamental moments. Equation (4) A1 returns the value for any period of t during the duration of the turnaround, so that:

$$Acc\ \lambda_\psi = -\lambda\left((t_x + 1) - \left(\frac{t_x}{2a}(t_x + 1)\right)\Big|_{x=1}^{x=\breve{n}_{(\psi)}}\right) \qquad \text{(4) A1}$$

$$where\ x = 0, 1, 2, \ldots\ \breve{n}$$

Equation (5) A2 returns accumulated cash flow $(Acc\ \lambda_n)$ at the end of each period for the VFC model, assuming AC are incurred, but there are no delays in the planned changes. The implementation change (ψ) is calculated as in Equation (2) and as shown in Equation (5) A2 below. Again, taking the value of (ψ) as equal to the loss (λ_0) divided by a so that $\psi = \frac{\lambda}{a}, with\ a > 0\ where\ [\lambda, \psi, a \in \mathbb{R}]$, where a represents the planned changes (ψ) expressed as a proportion of the loss (λ_0). AC (μ) is calculated as shown in Equation (3). The value of -μ is also equal to a proportion of the loss (λ_0) so that $\mu = \frac{\lambda}{b}, with\ b > 0\ where\ [\lambda, \mu, a \in \mathbb{R}]$, were b represents the AC $(-\mu)$ as a proportion of the loss (λ_0). Since there are no delays, the expected duration (n) of the turnaround forms a part of the fundamental moments. Equation (5) A2 returns the value of the accumulated cash flow $(Acc\ \lambda_n)$ for any chosen period during the duration of the turnaround, so that:

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$$Acc\ \lambda_n = -\lambda_0\left((t_x+1)\left(1+\frac{1}{b}\right)-\left(\frac{t_x}{2a}(t_x+1)\right)\Big|_{x=0}^{x=n}\right) \qquad \text{(5) A2}$$

$$b \neq 0, t_x = 1, 2, 3, \dots, n$$

Equation (5) A2 returns the accumulated cash flow ($Acc\ \lambda_k$), taking into consideration any delay $k, where\ k \in \mathbb{R}$ that may occur or is expected to occur, where the planned changes are not implemented from the end of the period t_0, but will be implemented in period k, so that:

$$Acc\ \lambda_k = -\lambda\left(\left(1+\frac{1}{b}\right)(t_x+1)\Big|_{x=0}^{x=\breve{n}_{(k)}}-\left(\frac{t_y}{2a}(t_y+1)\right)\Big|_{y=1}^{y=\breve{n}_{(k)}-k}\right)$$

$$a \neq 0; b \neq 0; k < 0, k \in \mathbb{R}; and \qquad \text{(6) A3}$$

$$x = 0,1,2, \dots \breve{n}_k;\ and\ y = 1,2,3, \dots \breve{n}_k - k\ and$$

$$where\ y = t_x - k, t_x > k$$

When the duration is denoted by $(\breve{n}_k)$, it implies that there has been a delay (k) in the planned changes (ψ), while if it is indicated with $(\breve{n}_{(\psi)})$, it means there are no delays (k) and no AC (μ). When duration is indicated with (n), AC (μ) has been incurred, but no delays (k) have occurred. When a strategy is devised, it is forward-looking, and the best intent is assumed. However, delays may occur, and the stakeholders may want to know the impact of the AC (μ) on the turnaround. Accordingly, all values with (n) as a subscript indicate the planned strategy, which includes AC (μ) but with no delays (k). In contrast, the psi (ψ) shows the boundary value, assuming that no AC (μ) is present, while the subscript (k) indicates a boundary value due to delays.

5.4.3 The appropriateness, consistency, and accuracy of the VFC model as a fit to that of the TCF calculations

To verify if the VFC model is an appropriate, consistent, and accurate fit to that of the TCF calculations, a rigorous test had to be designed to evaluate the differences between the VFC model and TCF calculations. The test requires the simulation of the output of the VFC model and TCF calculations using the same input values. After calculating the output values for both, they are deducted to compile the differences or error rates. Based on the error size, the difference between these values informs whether the VFC model is a fit that is consistent,

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appropriate, and accurate. A sufficient number of simulations had to be generated to create a representative sample for the various input values. A series of truncated tests were performed to accept or reject the model's fit. Table 5.1 summarises the designs of the tests performed, where: if test No. 1 fails, no further testing occurs; if test No. 2 fails, no further testing occurs; and if test No. 3 shows that there is not a sufficient level of accuracy with the overall fit; the test also fails. Assuming that tests No. 1 and 2 had passed, test No. 3 will use the level of accuracy as an indication of suitability. Therefore, the suitability indicator determines if the model could be used in a research or business application, or both.

Evaluation test No. 1 – Is the fit between the models appropriate? - This evaluation test checks the appropriateness of the VFC model. If any test of the differences returns an integer value larger than zero, the VFC model cannot be accepted as appropriate and must be rejected.

Evaluation test No. 2 – Is the fit between the models consistent? - The second evaluation test checks for consistency or the lack thereof. This test investigates the spread or distribution of errors and anomalies. If errors and anomalies occur randomly for different places (bands) after the decimal, it implies that the model cannot be considered consistent, and the VFC model should be rejected. In other words, if an error or anomaly occurs in the second place and the next value occurs in the tenth place after the decimal, the model cannot be deemed consistent. However, if the first error or anomaly occurs in the eleventh place and the value adjacent thereto returns an error in the tenth place after the decimal, this indicates there may be a cumulative process present. Due to the cumulative nature of the model and calculation, where the value of the previous period is added to that of the current period, it implies that there can be a cumulative increase in error values in the calculations. If the errors appear randomly, it means that the model is not consistent. So, suppose adjacent differences show small increases or increments, as the adjacent values follow each other. In that case, the VFC model can be accepted as consistent, and the next set of truncated tests can be performed.

Evaluation test No. 3 – Is the fit accurate, and if so, to what level? - To determine the level of accuracy of the model, all the anomalies or errors must occur in a specific band or place after the decimal. In other words, if the errors occur in the twelfth place after the decimal and possibly, in the thirteenth, fourteenth, and fifteenth place thereafter, it can be accepted that the model is accurate to the twelfth decimal. Once the band or place after the decimal where the error has occurred has been determined, it becomes possible to determine the level of

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accuracy in adopting the model. The level of accuracy is determined in Evaluation tests 3.1 to 3.5.

Evaluation test No. 3.1 – The model is deemed not to be an accurate fit - If any single test returns an anomaly or error that occurs between zero and three places (bands) after the decimal, the model will be deemed to be not an appropriate and accurate fit. Table 5.1 shows that if the model is accurate to three places after the decimal, the model can only be accurate to one thousand Rand (or $ or whatever currency is used). The VFC model would be deemed unsuitable for research purposes and cannot be utilised as an alternative in evaluating cash flows and determining the variable finance requirements.

Evaluation test No. 3.2 – The model is deemed a rough fit - If an error or anomaly occurs between four and six places (bands) after the decimal, the model could be considered a rough fit. Table 5.1 shows that six places after the decimal, the model can only be accurate to one million Rand. The VFC model may be used in research but cannot be relied on as an alternative to determine the variable finance requirements and the moments during a turnaround.

Evaluation test No. 3.3 – The model is deemed a good fit - If an error or anomaly occurs between seven and nine places after the decimal, the model will be considered a good fit. Table 5.1 shows that the model can only be accurate to one billion Rand at nine places after the decimal. The VFC model may be used in research and may be used as an alternative to determining the variable finance requirements, as long as the calculation is less than one billion Rand.

Evaluation test No. 3.4 – The model is deemed a perfect fit - If an error or anomaly occurs between ten and twelve places after the decimal, the model will be considered a perfect fit. Table 5.1 shows that the model is accurate to one trillion Rand at twelve places after the decimal. It allows the VFC model to be used for research and can be utilised as an alternative in evaluating the variable finance requirements.

A further reason for declaring the VFC model as a perfect fit up to 12 places after the decimal is informed by the capability of the Intel® processor. It is limited to returning error-free values above 12 places after the decimal. Complex calculations in a spreadsheet are impacted by how the Intel® processor calculates the floating-point. As set out above, the evaluation tests are

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summarised in Table 5.1 below to clarify what measurement will be used to determine if the *VFC* model is appropriate and consistent and to what level it is accurate.

Table 5.1: Summary of the outcomes for the evaluation tests performed to determine appropriateness, consistency, and accuracy of fit of the *VFC* model to that of the TCF calculations

Source: Author (2021)

Summary of the evaluation tests to be performed for appropriateness, consistency and accuracy		
Evaluation test 1 (*Testing for appropriateness*)		
Check if any difference has an integer value	If a single integer greater than zero occurs in single difference regardless of the type of test performed, excluding when a singularity occurs, the entire model is rejected and deemed as not being appropriate.	Reject the ***VFC*** model and do not perform further tests
Evaluation test 2 (Testing for consistency)		
Check if the values vary across various places after the decimal	Check the spread of the difference errors and anomalies are sequential and clustered together, value errors exceeding ***1E-12*** is ignored for the purpose of the analysis, but that it excludes singularities which will cause a jump in value.	Reject the ***VFC*** model and do not perform further tests

Evaluation test 3.1 to 3.4 (Testing for accuracy)

Places after the decimal	Expected level of error or anomaly of the difference as an indication of the accuracy per the indicated value	Fit indicator used to express level of confidence	
1 places	Accurate to - 0,9 to 0,9 per 10	No fit	
2 places	Accurate to - 0,9 to 0,9 per 100	No fit	
3 places	Accurate to - 0,9 to 0,9 per 1 000	No fit	
4 places	Accurate to - 0,9 to 0,9 per 10 000	Rough fit	
5 places	Accurate to - 0,9 to 0,9 per 100 000	Rough fit	
6 places	Accurate to - 0,9 to 0,9 per 1 000 000	Rough fit	
7 places	Accurate to - 0,9 to 0,9 per 10 000 000	Good fit	
8 places	Accurate to - 0,9 to 0,9 per 100 000 000	Good fit	
9 places	Accurate to - 0,9 to 0,9 per 1 000 000 000	Good fit	
10 places	Accurate to - 0,9 to 0,9 per 10 000 000 000	Prefect fit	
11 places	Accurate to - 0,9 to 0,9 per 100 000 000 000	Prefect fit	
12 places	Accurate to - 0,9 to 0,9 per 1 000 000 000 000	Perfect fit	

Suitability indicator to utilise the model

Places after the decimal	1	2	3	4	5	6	7	8	9	10	11	12
Fit indicator	No fit			Rough fit			Good fit			Perfect fit		
Suitability for proposed use	Not suitable for research or business application				Can be used in research but not in business application				Can be used in research and in business application			

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5.4.3.1 The methodology used to establish and perform the tests to measure the accuracy between the VFC model and the TCF calculations

Findings of the truncated tests performed

The following findings were established from the truncated tests performed for appropriateness, consistency, and accuracy of the VFC model to that of the TCF calculations:

Test for appropriateness: No error was found in Evaluation test No. 1, as no integer greater than zero occurred in the 16 000 individual tests. Therefore, the VFC model is deemed an appropriate fit against the values generated by the TCF calculations. The testing can, therefore, progress to testing for consistency and accuracy.

Test for consistency: The truncated tests imply that the most significant error occurs 12 places after the decimal. A map was created by sorting the tests according to the output values in a manner where b is displayed first so that $[b, a, -\lambda_0, k]$. Since Evaluation test No. 2 is the test for consistency, it requires that the errors be clustered together and not distributed in different places (bands) after the decimal. The error map shows that the first errors or anomalies occur in the fourteenth and thirteenth place only to progress to the twelfth place after the decimal. The values increase in small increments in those tests with input values of $\psi = 1 \; or \; a = 10$, and where $\mu = 1,2, \ldots 10$. The change in decimal place only occurs in period 25 for tests 21, 31, 41, 51, 61, 71, 81, and 91 and increases in small increments up to period 32. The movement in the differences in sequential values after the decimal can be linked to limitations of the Intel® processor with a limited floating-point of 12 places after the decimal. Thomas (personal communication, 9 October 2021) confirmed that the cumulative nature of the models, where the value of the previous period is added to that of the current period, implies that there can be a cumulative increase in error values. Based on the 16 000 tests performed, the VFC model is, therefore, deemed to be a consistent fit with that of the TCF calculations.

Test for accuracy: Of all the 16 000 iterations performed, less than two per cent of the errors occurred in twelfth place after the decimal, with the rest of the errors ranging from the thirteenth place after the decimal to the fourteenth place to zero errors.

Errors showed in tests 1 to 500 suggest that the maximum error value is shifted to later periods as there is a delay in the implementation of the turnaround. For example, the errors shown in

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tests [1, 11, 21, 31, 41, 51, 61, 71, 81, 91], which have a very low-level PC ($\psi = \frac{1}{10}$), shown at 10% and a very high AC ($\mu = 1$). It means the implementation will be very slow and exceeds the 32-period limit, even if $k = 0$. Accordingly, if there is any delay in the PC (ψ), it will exacerbate the periods for which the turnaround must be undertaken. The researcher considered performing a hypothesis test to pronounce the accuracy of the VFC model to that of the TCF calculations. It was initially considered to use all the 16 000 iterations that were performed; however, upon reflection, it was decided to evaluate only the tests that occurred during the duration, thereby reducing the number of observations to 6 672. A standard deviation and average for these values were calculated only to realise that there is no reason to calculate these values. Of the 6 672 observations, 249 observations occur between the twelfth and eleventh place after the decimal. However, the sequential values have a cumulative error. Once a value has exceeded the twelfth place after the decimal, the next value in the sequence will continue with errors, increasing with minor incremental values due to the errors arising from the math coprocessor's inability to calculate 13 places after the decimal.

Accordingly, any statistical analysis simply analyses the noise created by the equipment used. Based on the number of tests performed, all errors occurred in the fifteenth, fourteenth, thirteenth, and twelfth place after the decimal, resulting in the *VFC* model being declared a perfect fit for the TCF calculations.

Based on the tests performed, the VFC model is deemed a perfect fit in terms of accuracy with that of the TCF calculations. Therefore, the VFC model is an appropriate, consistent, and perfect fit when measured against the TCF calculations.

It further follows that all the derivatives that have been derived to determine the fundamental moments will be accurate, as these values have been determined algebraically from the VFC model. Various tests were performed to ensure that the fundamental moments return the correct values and that these values match against the VFC model. These calculations were also found to be accurate to 14 and 13 places after the decimal.

5.4.3.2 Implementation and presentation of the VFC model

Having established that the VFC model can be used in research and business, determining the input values and how changing the primary inputs affects the outcomes calculated by the VFC

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model are essential. This section concludes by offering a map that reflects the various outcomes based on an input set with 100 different inputs.

How to determine the input values for the VFC model

This section shows how the VFC model simplifies the calculation of the value of accumulated cash flow for any period and can be adapted by taking into consideration the following variables:

1. Abnormal costs (AC); and
2. delays in planned changes (PC).

The key to using the VFC model is establishing the inputs and assumptions that apply to the specific turnaround strategy.

Figure 5.2 shows an example of how an assumption sheet can be constructed to establish the input values based on a specified turnaround strategy. The first column describes all the activities generally associated with a COSI as defined by IFRS (IAS 1 Presentation of Financial Statements, 2020). However, the user may expand this assumption sheet with the level of detail that they may require to map the envisioned turnaround strategy.

Historical information: In Figure 5.2, column No. 1 contains the detail of the historical performance for the immediate past period prior to undertaking the turnaround. The duration of the selected period must be the same length of time as the duration of the periods in the turnaround. In other words, if the turnaround is to be measured in months, the historical figures must be in months, or if measured quarterly, then it must be continued to be measured quarterly, and so forth.

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An example of how to determine the input values for the *VFC* model

Input values to determine the value of ψ and a		Historical performace (1)	Planned changes (2)	Expected period (3)	Relative change (4)	
Change in revenue	A		-12%	3		
Margin (on cost of sales)	B	28%	32%	5		
Revenue	C	1,000.0	880.0		- 120.0	A_1
Cost of sales	C	720.0	598.4		121.6	A_2
Gross margin		280.0	281.6		1.6	
Expenses		345.0	220.0		125.0	
Salaries	D	120.0	80.0	3	40.0	A_3
Wages	D	100.0	50.0	3	50.0	
Travel	D	25.0	10.0	1	15.0	⋮
Production costs	D	100.0	80.0	6	20.0	A_n
(Loss)/ Profit		- 65.0	61.6	3.5	- 126.6	

Input values to determine the value of μ and b	Total estimated cost (5)	
Abnormal costs per period	295.0	
Turnaround team costs	100.0	B_1
Legal costs	80.0	B_2
Retrenchment costs	40.0	⋮
Other costs	75.0	B_n

$$t_{max} = \textit{max period}\,[6,0] \textit{ or } t_a = \textit{average period}\,[3,5]$$

$$\psi = \frac{(A_1 + A_2 + \cdots + A_n)}{t_{max} \textit{ or } t_a} = 21,1 \text{ or } 36,2$$

t_{max} t_a

Equation 2

$$\mu = \frac{(B_1 + B_2 + \cdots + B_n)}{t_{max} \textit{ or } t_a} = 49,2 \text{ or } 84,3$$

t_{max} t_a

Equation 3

Where: λ_0 *is loss at the beginning*
ψ *is the incremental changes, and*
a *is the proportion of* ψ *in terms of* λ_o
μ *is the abnormal costs, and*
b *is the proportion of* μ *in terms of* λ_o

Figure 5.2: An example of an assumption sheet to determine the input values for the VFC model based on a specific turnaround strategy

Source: Author (2021)

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Planned changes: Column No. 2 contains the planned changes considered under the proposed turnaround plan. The assumption sheet makes provision for any implementation strategy that may be proposed and may consist of:

1. [A] a strategic reorientation or realignment, implying that there will be a change in revenue;
2. [B] a change in the product mix to be sold that brings about a change in the margin realised;
3. [C] any efficiency improvements that may be brought about that may result in a change of the cost of sales; and
4. [D] any retrenchments that may be considered in the turnaround strategy. The assumption sheet captures the multiple actions that may be considered part of the turnaround strategy. The changes shown in this column represent the planned outcome of the intended turnaround plan once the total planned changes have been fully implemented.

Expected period: Column No. 3 indicates the expected time it would take to implement each of these changes. The periods must be calculated using the same time interval as that of the historical performance and the selected measurement frequency. The practitioner and their team will provide their best estimates of how long it will take to implement the PC.

Relative change: Column No. 4 indicates the relative change that will be achieved once the plan is fully implemented, being the difference between the historical performance and the PC. These differences are summed together as contemplated in Equation (2).

AC: Column No. 5 sets out the proposed list of the AC associated with a specific turnaround strategy. The amount can constitute expenses that may not form a part of normal operating expenses. It may include any other costs that may be incurred but were not provided for in the historical costs. The AC is calculated as contemplated in Equation (3).

Once the planned changes and AC have been calculated, these calculations must be converted into ratios that are proportional to the loss ($-\lambda_0$) where a is the proportional implementation change to be implemented and b is the proportional AC to be implemented relative to the loss ($-\lambda_0$) that was incurred before the commencement of the turnaround. The assumption sheet enables the turnaround practitioner to clearly understand the impact of the proposed strategy

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on the outcome of the turnaround. As time elapses, the practitioner can update the assumption sheet with new information as it becomes available and may update certain assumptions about the implementation periods associated with the various planned changes.

5.5 Introducing fundamental moments as derived from the VFC model

In the previous section, the VFC model has derived algebraically and was verified to be a perfect fit for the TCF calculations. This section introduces fundamental moments derived from the VFC model. Fundamental moments can be determined by asking:

1. how long is the turnaround going to last;
2. when is the firm going to breakeven;
3. when is the cash flow nadir going to occur; and
4. how much funding is required to cover the cost of the turnaround?

These questions are graphically depicted in Figure 5.3. The questions are interrelated, as the duration is intersected by the nadir, which is connected to when the breakeven occurs. The ability to calculate these fundamental moments allows for a unique set of values to determine the impact of the turnaround. Presenting the stakeholders with a set of fundamental moments offers key indicators that are easy to understand with an indicative presentation of the entire curve of the values. It can be deemed akin to an identity vector (eigenvalue) used when dealing with matrices.

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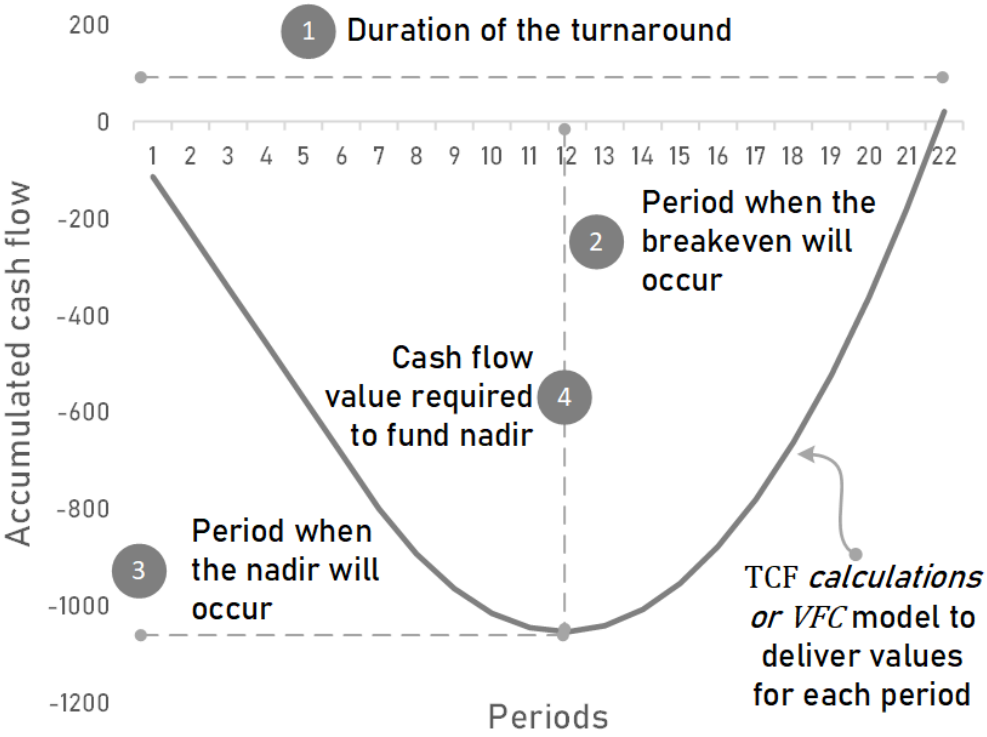

Figure 5.3: Defining the fundamental moments of a turnaround

Source: Author (2021)

5.5.1 The duration of the turnaround

"How long is it going last?" is the most common question often asked when dealing with turnarounds. As there is no standardised nomenclature, the answer becomes a moving target. The answer will usually start with "Well, it depends …". By mathematically manipulating the input values using the VFC model, it becomes possible to answer this question definitively.

Given that the duration is the point where the accumulated cash flow is equal to zero and that the turnaround will result in a nadir, it follows that the accumulated cash flow will decline and recover thereafter. As the accumulated cash flow reaches zero, the turnaround has been completed, as the firm is making profits and no longer incurring losses. Equations (4) A1, (5) A2, and (6) A3 are employed and equated to zero and mathematically manipulated to determine the duration of the turnaround. Equation (4) A1 is used to obtain Equation (7) B1, implying no AC and no delays present. Equation (7) B1 is equated to zero so that $Acc\ \lambda_{\psi} = 0$ and where $a > 0, a \in \mathbb{R}$, so that:

$$\breve{n}_{(\psi)} = 2a \qquad \text{(7) B1}$$

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Equation (5) A2 is used to obtain Equation (8) B2, implying AC but no delays. Equation (5) A2 is equated to zero so that $Acc\ \lambda_n = 0$ and where $a > 0, b > 0, a, b \in \mathbb{R}$, so that:

$$n = 2a\left(1 + \frac{1}{b}\right) \qquad \text{(8) B2}$$

Assume there are delays of k periods in undertaking the planned changes during the turnaround, implying that there should be a delay in the duration of the turnaround. It means that the duration will last $\breve{n}$ periods and let $\breve{n} > n$ and consists of k periods, which represents the delay in the planned changes, which results in $Acc\ \lambda_k = 0$ and where $a > 0, b > 0, \rho > 0, k > 0, a, b, k, \rho \in \mathbb{R}$, so that.

$$\breve{n}_k = \rho + n + k$$

$$where\ n = 2a\left(1 + \frac{1}{b}\right)\ and$$

$$\rho = a\left(-\left(\frac{1}{2a} + \varpi\right) \pm \sqrt{\left(\frac{1}{2a} + \varpi\right)^2 + \frac{2k\varpi}{a}}\right)\ where\ \varpi = 1 + \frac{1}{b} \qquad \text{(9) B3}$$

Figure 5.4 illustrates what happens to the duration (ρ) if there is a delay in implementing the changes contemplated in a given strategy. It shows that there is the initial period (1) that is the same as if there is no delay, followed by the delay (k) and planned changes $(n - 1)$. At the end of this period, the accumulated cash flow is equal in value as to when the implementation started. It follows that if the initial investigation period (1) is added to this period, it will deliver (n) as contemplated in Equation (8) B2. However, the turnaround duration is not complete at this point $(Acc\ \lambda_k \neq 0)$. The tunaround has to last (ρ) for $Acc\ \lambda_k = 0$, as is reflected in Equation (9) B3.

Therefore, once a specific strategy has been selected and a delay in the planned changes occurs, it results in a new outcome. This new set of outcomes is defined as boundary values. In other words, a boundary value can only occur if there is a delay in implementing a specific strategy.

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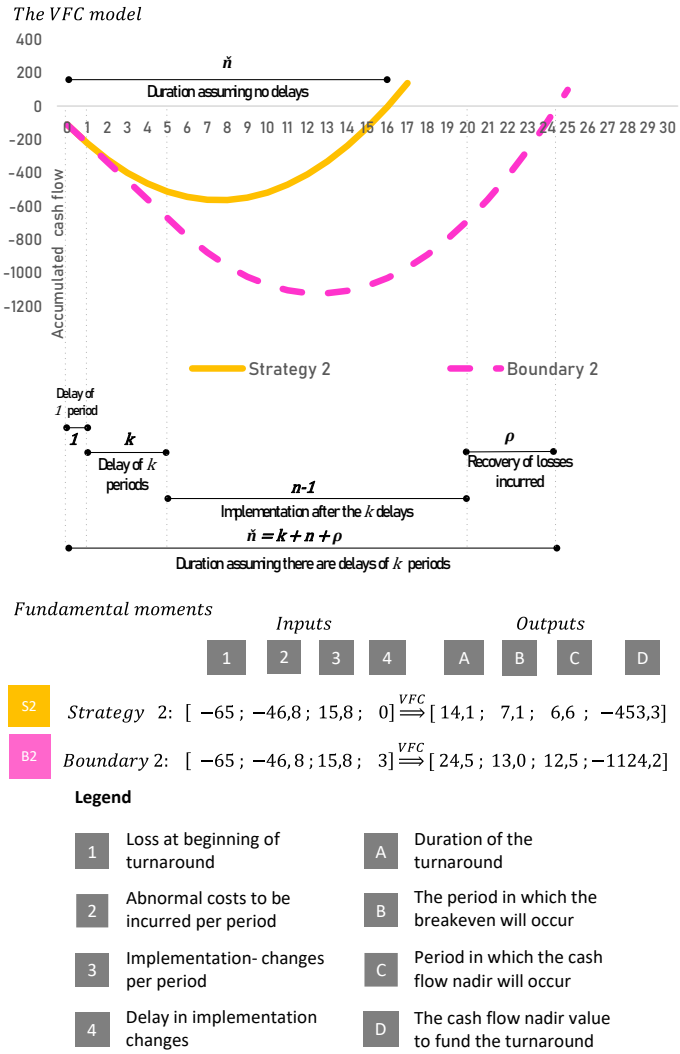

Figure 5.4: An analysis of what happens to duration (ρ) if there is a delay (k) in an adopted strategy, thereby delivering a boundary value for the same adopted strategy

Source: Author (2021)

5.5.2 Breakeven of a turnaround

The breakeven (E) of a turnaround occurs in a period where the operational inflows are equal to the outflows. It is the period preceded by periods of losses or cash flow shortfalls in ever-decreasing size, followed by periods of profits or cash flow surpluses made in each period thereafter. (E) is not determined by the accumulated cash flow at the end of the period but rather by the cash flow or profit/loss generated for that specific period. Assuming there are no AC or delays during turnaround, then the breakeven $(E_{(\psi)})$ for a turnaround is indicated as the loss for the period (λ_0) being equal to the sum of the planned changes (ψ) for that period and $\lambda_0, \psi_x \in \mathbb{R}$, so that:

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$$E_{(\psi)} = a \qquad \text{(10) C1}$$

Equation (11) C2 includes AC (μ) during turnaround. The breakeven ($E_{(n)}$) for the turnaround is indicated as the loss for the period (λ_0) being equal to the sum of the planned changes (ψ) for that period and $\lambda_0, \mu, \psi_x \in \mathbb{R}$, so that:

$$E_{(n)} = a(1 + \frac{1}{b}) \qquad \text{(11) C2}$$

If the breakeven ($E_{(k)}$) is delayed by k periods, then the planned changes will come into effect after period k. Since Equation (11) C2 calculates the breakeven point during the turnaround, it simply shifts the breakeven period by k periods, so that:

$$E_{(k)} = a\left(1 + \frac{1}{b}\right) + k \qquad \text{(12) C3}$$

Whether delays are present or not, the definition for breakeven for a firm remains the same; it is when the operational performance reaches zero and does not seek to calculate the accumulated cash flow impact.

5.5.3 The period when the nadir will occur

Determining when the cash flow nadir will occur is critical to the success of the turnaround, as discussed in Chapters 3 and 4 of this study. Even if stakeholder support has been obtained, should such support be slow in arriving, it delays the implementation of the turnaround. Delay in support leads to further support required to fund the more severe nadir that follows soon thereafter, as shown in Figure 5.4. Equations (4) A1, (5) A2, and (6) A3 are employed and equated to zero, and by differencing these equations, the period in which the minima or nadir will occur can be determined. By applying this treatment to Equation (4) A1, Equation (13) D1 is obtained, implying there is no AC ($-\mu$) and no delays present and where $a > 0, a \in \mathbb{R}$, so that:

$$\breve{n}_{min(\psi)} = a - \frac{1}{2} \qquad \text{(13) D1}$$

Equation (5) A2 is differenced and equated to zero to obtain Equation (14) D2, implying there are AC but no delays and where $a > 0, b > 0, a, b \in \mathbb{R}$, so that:

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$$n_{min} = a(1 + \frac{1}{b}) - \frac{1}{2} \qquad \text{(14) D2}$$

Equation (6) A3 is differenced and equated to zero to obtain Equation (15) D3, implying there are AC and delays present and where $a > 0, b > 0, k > 0;\ a, b \in \mathbb{R}$, so that:

$$\breve{n}_{min(k)} = a\left(1 + \frac{1}{b}\right) - \frac{1}{2} + k \qquad \text{(15) D3}$$

This section shows that the nadir period will occur a half period before the breakeven, as indicated above, which is in line with the saying that the "coldest time of the night is just before the sun rises". The equation also shows that the nadir period is the direct result of the delays by (k) but proportionally $(1 + 1/b)$. The notation is deliberately different between Equations (14) D2 and (15) D3 as the value of (n_{min}) indicates the planned strategy while the value of the $\breve{n}_{min(k)}$ indicates a boundary value (the result of a delay of a chosen strategy).

5.5.4 Cash flow required to fund the nadir

As discussed in Chapter 3 of this study, establishing what funding is required to implement a turnaround is key. When a practitioner engages with a stakeholder to provide financing, the practitioner will request the funding that occurs at the nadir. Accordingly, determining the size of the nadir is critical to the success of the turnaround. To determine the extent of nadir, Equation (4) A1 is adapted by inserting Equation (13) D1 therein so that $\breve{n}$ is substituted with $\breve{n}_{min(\psi)}$ and $a > 0, and\ where\ a \in \mathbb{R}$ that results in:

$$Acc\ \lambda_{min(\psi)} = -\lambda\left(\frac{1}{2}a + \frac{1}{2} + \frac{1}{8a}\right) \qquad \text{(16) E1}$$

When AC is present, and there are no expected delays, the size of the nadir is determined by adapting Equation(5) A2 and inserting Equation (14) D2 therein so that n is substituted with n_{min} and $a > 0, b > 0\ and\ where\ a, b \in \mathbb{R}$ that results in:

$$Acc\ \lambda_{min(n)} = -\lambda\left(\frac{1}{2}a\left(1 + \frac{1}{b}\right)^2 + \frac{1}{2}\left(1 + \frac{1}{b}\right) + \frac{1}{8a}\right) \qquad \text{(17) E2}$$

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Assuming AC is present and there are expected delays, the size of the nadir is determined by adapting Equation(6) A3 and inserting Equation (15) D3 therein so that $\breve{n}_k$ is substituted with $\breve{n}_{min(k)}$ and $a > 0, b > 0, k > 0 \; and \; where \; a, b, k \in \mathbb{R}$ that results in:

$$Acc\ \lambda_{min(k)} = -\lambda\left(k\left(1+\frac{1}{b}\right)+\frac{1}{2}a\left(1+\frac{1}{b}\right)^2+\frac{1}{2}\left(1+\frac{1}{b}\right)+\frac{1}{8a}\right) \qquad \text{(18) E3}$$

Equations (16) E1 to (18) E3 show the size of the nadir is directly influenced by the periods of delay that have occurred and the AC incurred. The notation is deliberately different between Equations (17) E2 and (18) E3 as the value of $(Acc\ \lambda_{\min(n)})$ indicates the planned strategy while the value of the $(Acc\ \lambda_{min(\psi)})$ indicates a boundary value of a planned strategy, but that has been delayed.

5.5.5 Presentation of fundamental moments

This chapter showed how to calculate the individual fundamental moments as shown in Equations (10) C1 to (18) E3 and offers a simple method of presenting these figures in a combined manner. This chapter has shown that input values return unique output values. Accordingly, to show the cause and effect of input and output values requires a method in which fundamental moments should be presented. It proposed that a set of fundamental moments are always shown in the same manner so that:

$$[-\lambda_0; -\mu;\ 0;\ 0] \underset{VFC}{\Longrightarrow} [\ \breve{n}_{(\psi)};\ E_{(\psi)};\ \breve{n}_{min(\psi)}; Acc\lambda_{min\ (\psi)}\ ], where\ \mu, k = 0$$

$$or$$

$$[-\lambda_0; -\mu;\ \psi;\ 0] \underset{VFC}{\Longrightarrow} [\ n;\ E_n;\ n_{min}; Acc\lambda_{min(n)}\ ], where\ k = 0 \qquad \text{(19) F1}$$

$$or$$

$$[-\lambda_0; -\mu;\ \psi;\ k] \underset{VFC}{\Longrightarrow} [\breve{n}_{(k)}; E_{(k)}; \breve{n}_{min(k)}; Acc\lambda_{min(k)}], where\ k > 0$$

Equation (19) F1 describes the presentation covenants that describe fundamental moments. For example, a set of fundamental moments can be shown as:

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$$[-10\,;-5;\ 5;\ 0] \underset{VFC}{\Longrightarrow} [\,6;\ 3;\ 2,5;\ 30{,}6]\,,$$

The values shown in the first square bracket set are the inputs, and the values in the second square bracket set are the outputs. The input values are always shown in the same sequence so that:

1. the first position shows the loss (-10);
2. the second position shows the AC (-5);
3. the third position shows the planned changes (5); and
4. the fourth position shows the delay (0).

The outputs are also shown in the same sequence:

1. the first position returns the duration (6) of the turnaround;
2. the second position returns the period in which the firm will break even (3);
3. the third position returns the period $(2{,}5)$ in which the nadir will occur;
4. while the fourth position returns the nadir value $(30{,}6)$.

Extending this example assumes there is a delay of the two periods to implement the turnaround, which results in a new set of fundamental moments so that:

$$[-10\,;\ 5;\ 5;\ 2] \underset{VFC}{\Longrightarrow} [\,9{,}4;\ 6;\ 5{,}5;\ 60{,}6]$$

Comparing the two sets shows that only the value of the delay in the fourth position of the inputs has changed, but this delay results in a completely new set of outputs, where all the values have changed. It represents a boundary value for the previous set of fundamental moments. The turnaround duration will be one and half times longer, while the breakeven period of the nadir and the value of the nadir has doubled.

This section showed the different equations required to establish a set of fundamental moments and where these equations are derived from the VFC model. Additionally, calculating the inputs used for the VFC model and fundamental moments was further expanded. It also discussed what tests were conducted to ensure that the VFC model is a perfect fit for a TCF calculation and how an assumption sheet could be used to plan the turnaround and determine the input values for the VFC model. It showed that the assumption sheet incorporates strategic orientation, changes in margins, efficiency improvements, and

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retrenchments. Therefore, it is agnostic as to what the preferred strategy of the practitioner may be.

5.6 Findings and recommendations

Chapter 3 discussed how the information about the turnaround plan is presented to the stakeholder and identified this as a significant stumbling block. Using the tools created in this chapter enables a presentation of the turnaround information that is easy to understand. A practitioner can prepare graphs using the VFC model to show the impact of different scenarios, simply provide the fundamental moment sets, or even provide a hybrid of these to a stakeholder for discussion.

5.6.1 Using the VFC model to create boundary values for the same strategy

Once a strategy has been selected, the input values can be determined using an assumption sheet. Suppose there is a delay in the chosen strategy. In that case, it becomes relatively simple to calculate a set of fundamental moments representing a single value or various boundary values based on the expected delays that may occur. Various outcomes (boundary values) can be graphed on the same page, allowing easy comparison and discussion with any stakeholder.

Figure 5.5 shows the different expected outcomes, assuming strategy (A) is delayed by $k = 1, 2, 3, 4$ periods, while all other inputs remain the same. Line B in Figure 5.5 represents the same strategy but assumes that no AC is present and no delays occur. This line is a boundary value of a given strategy. Line A represents the selected strategy, while the other lines in Figure 5.5 represent potential boundary values caused by delays, assuming there will be no change in the strategy or AC associated in addition to that. All the lines indicating the delays include AC. Figure 5.5 also shows that the downward trajectory or slope remains the same for all the boundary values, indicating a delay. The decline continues, and the nadirs for the different levels of delay follow in rapid succession. It is also clear that the delay increases the duration of the turnaround. Effectively, a delay has a scaling impact on the time and financial consequences of the turnaround.

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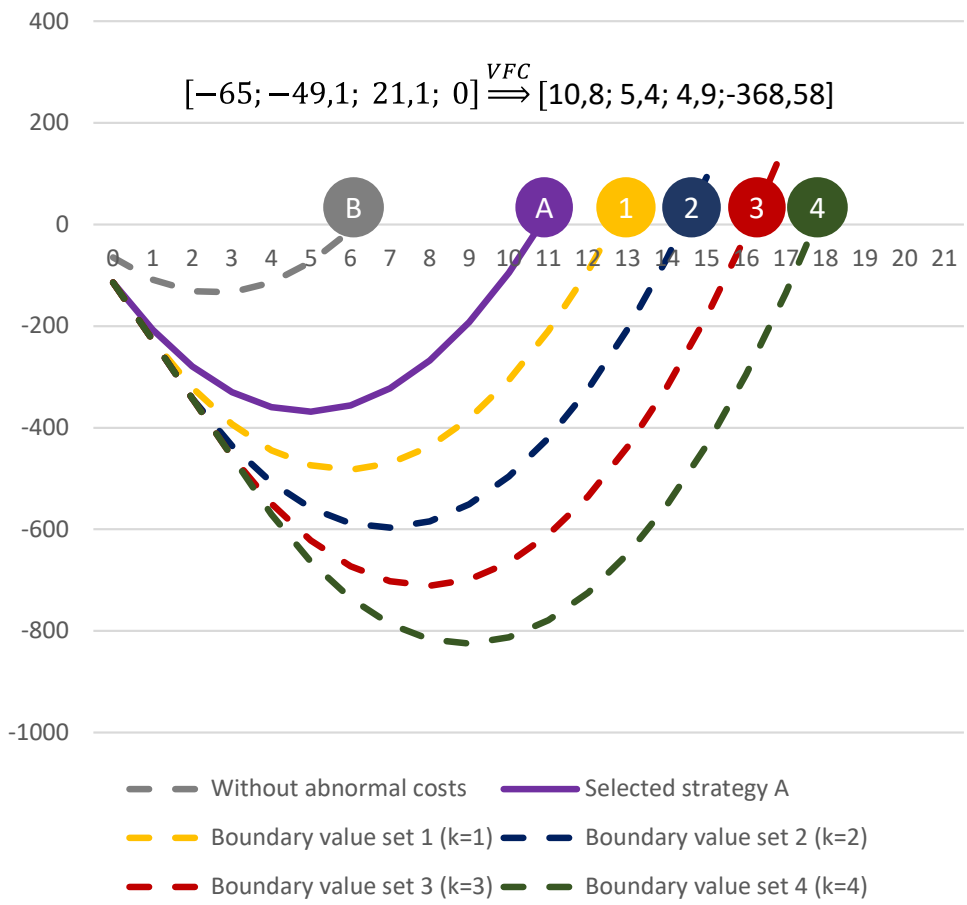

Figure 5.5: Presentation of the boundary outcomes for the same strategy (A) using the VFC model assuming no AC (B) or anticipating different delays $(k\ =\ 1, 2, 3, 4)$

Source: Author (2021)

Figure 5.5 shows the fundamental moments for strategy A, based on the input values used in the assumption sheet, as shown in Figure 5.2. It shows what the values of the intended strategy are and what variation in boundary values can be expected. The VFC model also offers the practitioner the ability to map different boundary values of the same strategy to determine what the impact of costs and delays will have thereon. The ability to create these boundary values is critical when dealing with stakeholders to obtain their support.

5.6.2 Using the VFC model to evaluate the different strategies

The VFC model can also be used to evaluate different proposed strategies. Figure 5.6 displays two strategies with varying values of the boundary of those strategies. Strategy 1 (S1) requires the least variable finance but is very likely to slip to the boundary of Strategy 1, as shown in line B1. In contrast, Strategy 2 (S2) may require more funding to implement the strategy

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initially but has a substantially lesser chance of falling into the boundary of Strategy 2, as shown in line B2.

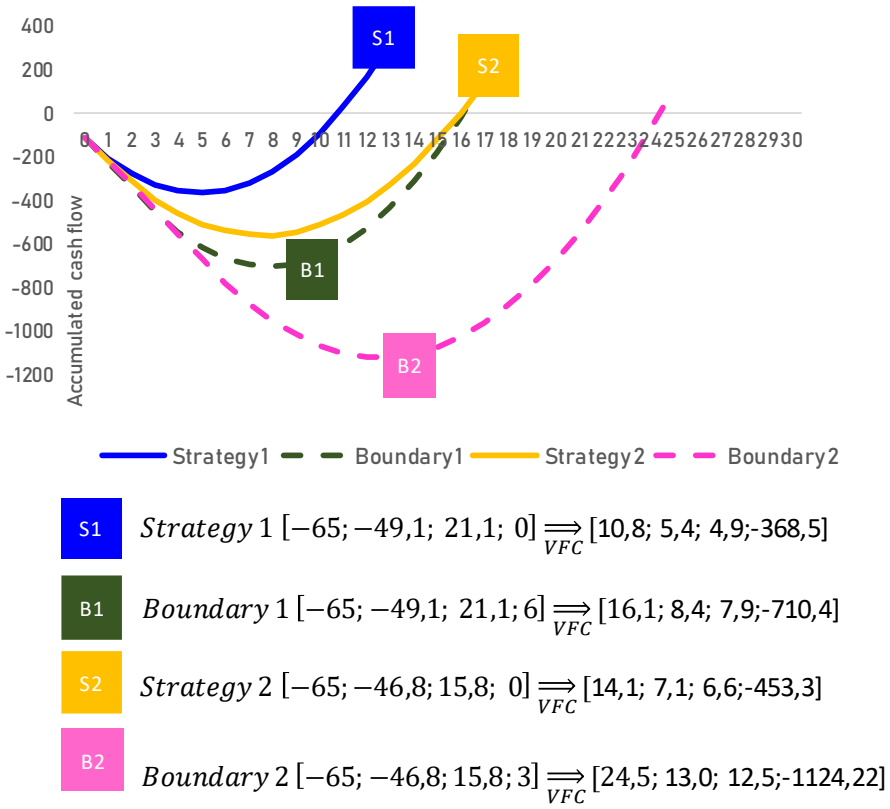

Figure 5.6: A presentation of the different outcomes based on different strategies resulting in different inputs using the VFC model and fundamental moment sets

Source: Author (2021)

The VFC model and fundamental moments enable the practitioner to provide the outcomes of various strategies on a single graph. A single overview of inputs and outcomes can be presented in this format, allowing stakeholders to comprehend what is required and what will be delivered for each strategy. The fundamental moments indicate inputs and outputs that can be presented to stakeholders. In other words, it provides a simplified manner to present the very complex and dynamic data as it captures the process of a turnaround.

Different strategies lead to different inputs that deliver different outcomes. The different strategies may range from retrenchment with a few efficiency improvements to re-equipping the plant and changing the firm's product mix. The latter may require more capital, which means the turnaround will last longer but may provide better returns in the long term. Future

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research may be able to gainfully use the VFC model and fundamental moments to evaluate the level of support that stakeholders may have in supporting one strategy above another.

5.7 Summary and conclusion

This chapter introduced the VFC model and showed how to use an assumption sheet to determine the turnaround strategy and the inputs used for the VFC model. It also revealed various graphs derived from the VFC model. It showed the impact of a delay in the planned changes to the turnaround plan. Equation (19) F1 showed a unique set of outputs for every set of inputs.

This chapter also discussed the methodology used to verify if the VFC model is a perfect fit for the TCF calculations. This verification is intended to establish trust in the VFC model, whereby the TCF calculations method can be tested and the outputs verified. Different what-if scenarios can be examined and evaluated.

This chapter also presented equations representing the fundamental moments based on certain assumptions and specific input values, concluding with establishing a precedent for presenting fundamental moment sets or displaying VFC values. This presentation method underpins the finding that every unique set of inputs has a unique set of outputs, thereby creating a unique set of fundamental moments.

The VFC model and the presentation of fundamental moments presented in this chapter are designed to create a measurement system that allows for a ubiquitous and generally accepted tool that can lead to enhanced decision-making.

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Chapter 6: Discussion and recommendations on the implementation of the $\boldsymbol{VFC}$ model that informs fundamental moments during a turnaround

Abstract

This chapter discusses and provides recommendations on the findings of the previous chapter. It investigates if the VFC model and fundamental moments can be classified as a theory based on the elements of theory-building research as proposed (Dubin, 1978, 1983; van de Ven, 2003). This chapter also investigates if the model could be analogous to the strategy process proposed by van de Ven (1992) and provides the findings derived from the research undertaken. It shows how the different inputs lead to unique outcomes using the VFC model, implying that every fundamental moment set is unique. It further discusses the benefits that may exist for practitioners and stakeholders in that fundamental moments can enhance decision-making when dealing with turnarounds. The chapter concludes with a section indicating how this measurement tool can be further developed to enhance research dealing with turnarounds.

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6.1 Introduction and background

Chapter 2 showed that research about the process of turnarounds provides a fleeting acquaintance with fundamental moments by having indirect references to fundamental moments. It showed a lack of calculating or defining fundamental moments. At best, duration is estimated on a *post priori* basis. It is obtained by determining the cash flow nadir and then arbitrarily adding periods before and after the nadir to determine the duration. To ensure that all the fundamental moments are captured, researchers review the historical performance, identify the nadir and its value, and when it occurs, then add a certain number of periods before and after that date for their study. Therefore, the data selection appears to be pre-selected to confirm a specific hypothesis and is not predictive.

Chapter 3 of this study showed that the fundamental moments could be derived from TCF calculations; however, it was revealed that there are potentially severe shortcomings in using this method. TCF calculations derived from a spreadsheet may result in errors, and because these spreadsheets are complex and bespoke to the person who prepared them, stakeholders may lack trust. The chapter also showed that there is no simple way to present various outcomes of different strategies when using numerical solutions. The nature of TCF calculations makes it challenging to create intra- and inter-comparisons of the potential outcomes of multiple strategies, limiting the possible applications should such comparisons be available. There is no uniform method to communicate the outcome of a strategy to stakeholders using the TCF calculations.

Chapter 4 outlined the methodology and modelling process used to develop the VFC model and fundamental moments. It also sets out how the problem statement and research questions are mapped to the chapters of this study.

Chapter 5 proposed the VFC model, and Annexure 1 shows how the VFC model was derived and verified to be deemed a perfect fit for the TCF calculations as set out in Annexure 2. The VFC model was then mathematically manipulated based on changing or manipulated assumptions associated with the chosen strategy to demonstrate the various elements of the fundamental moments. The chapter showed the optimal method of presenting fundamental moments to represent any turnaround concisely.

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This chapter discusses the findings set out in Chapter 5 of this study. It provides recommendations on how practitioners may use this VFC model and fundamental moments in practice. This chapter also informs how future research may utilise fundamental moments to progress empirical research on turnarounds.

6.2 Discussion of the outcomes of this study

This section discusses the findings as set out in Chapter 5, the research methodology as set out in Chapter 4 and compares these to the views expressed in the research of van de Ven (1992, 2003) and Dubin (1978, 1983). Van der Ven (1992) provides views on how to approach research on the strategy process, and in the latter, he provides his views on theory-building. As presented in Chapter 5, the findings are evaluated against the standards set out by Dubin (1978, 1983) and the requirements of creating a theory as proposed by van de Ven (1992, 2003) and Storberg-Walker (2003). The latter compares various authors' different theory-building research proffered (Dubin, 1983; Lynham, 2002; van de Ven, 2003). This section uses the requirements of these authors for the VFC model and fundamental moments to be deemed a theory.

6.2.1 Can the VFC model and fundamental moments be defined as a theory?

Van de Ven (1992) made suggestions for studying the strategy process, which can be expanded to evaluate the outcomes of this study. To understand the strategy process, van de Ven (1992) argues that the *meaning must be defined*, it must clarify the theory, and a *research design must be provided* to observe the same. He continues that the meaning of process must be clearly defined to *explain the causal relationship* between independent and dependent variables that follows from the actions and describe the process that happens over time.

The strategy process meaning must be defined: Figure 5.2, as set out in Chapter 5, shows that any strategy effect can be estimated using the assumption sheet to determine the VFC model or create a fundamental moment set. The strategy may include practices such as cost savings, efficiency improvements, or strategic reorientation of the firm. Based on the researcher's experience, a turnaround strategy will seldom consist of only one set of practices to bring about change. Utilising the assumption sheet allows for the practitioner's level of detail when estimating the PC and the AC value. In addition to defining the inputs clearly, the consequences

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of the outcomes of the strategy process are also clearly defined. The VFC model shows how a selected strategy will result in a set of inputs resulting in a dynamic pattern involving a decline and recovery of the accumulated cash flow resources that produce a set of fundamental moments to describe a specifically selected strategy. Delays in the implementation of the planned changes (PCs) will lead to boundary values of the specifically selected strategy, as shown in Chapter 5, Figure 5.6.

The research design must be provided: Chapter 4 of this study sets out the research methodology that showed various shortcomings in using the TCF calculations to determine the fundamental moments as set out in Chapter 3. It further showed the process used to understand, create, and verify the VFC model. Van de Ven (2003:4) argued, "A central mission of scholars in a professional school is to conduct research that contributes knowledge to a scientific discipline, on the one hand, and to apply knowledge to advance the practice … on the other." Storberg-Walker (2003) believes that to advance human research development (HRD), research must advance the knowledge for the HRD and guide practitioners to solve real-world problems. This study approaches the problems that practitioners face (Chapter 3) and the issues with the research on the process of turnarounds (Chapter 2). Van de Ven (2003) proposed a professional research diamond model comprising: (1) conceptualisation; (2) theory-building; (3) research design; and (4) problem-solving. Van de Ven (2003) proposed this model from which elements are used to create a comparison to determine if the VFC model is deemed to be a theory so that:

1. *Problem formulation:* The researcher questions the existing body of knowledge to determine the research questions that lead to the problem formulation. The author started questioning the effectiveness of the TCF calculations as set out in Chapter 3 hereof and how the results thereof were presented, as described in Chapter 4 of this study.
2. *Conceptualisation:* The VFC model was developed mathematically using the first principles to emulate the TCF calculations set out in Annexure 1. The model was then verified by performing the 16 000 tests in Annexure 2. The findings represented in Annexures 1 and 2 are also presented in Chapter 5, where the theory that the VFC model can replace the current TCF calculations was proven and found to be a perfect fit.

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3. *Theory building:* The derivates of the VFC model led to presenting fundamental moments as the key elements to describe the: (1) duration of the turnaround; (2) when breakeven will occur; (3) when the cash flow nadir will occur; and (4) the size of the nadir. These equations were derived from the VFC model and verified as a perfect fit for the TCF calculations.
4. *Problem-solving:* The study also presented nomenclature for the various fundamental moments that can facilitate discussions between practitioners and stakeholders. The various parties can now understand what is being referred to, thereby engendering trust between the parties.

Ultimately, van de Ven (2003) argues that any theory must have the following criteria or relevance: validity, truth, and impact. This study has provided a rigorously implemented methodology to pursue these criteria. Future research will be able to use fundamental moments to further research dealing with turnarounds. In contrast, practitioners can use these tools daily in managing turnarounds.

Explaining the causal relationship: The VFC model and fundamental moments proposed in this study deliver an input-process-output model, where fundamental moments, as presented in this study, link inputs to specific outputs, establishing causality. Figure 6.1 to Figure 6.4 show the different views of 100 VFCs using different input sets where only one value of either (μ) or (ψ) were varied by one. The results of each set was calculated for 32 periods, and the outcomes are displayed in simulated 3D from four different angles. These sets assume no delays and PC (ψ) in incremental steps from one to ten. Figure 6.1 shows when the PC (ψ) is rapidly implemented (as shown in sets [10, 20, ... 100], and where there is an implementation proportional to 100 per cent of the loss value), AC does not have a pronounced effect on the turnaround. Figure 6.1 also shows the difference in the incline between sets [10, 20, ..., 100], and sets [1, 11, 21, ..., 91]. For instance, there is a steep incline in the one end (right) represented by sets [91, 81, 71, 61, 51, 41, 31, 21, 11, 1], where there is a sharp drop in the incline of the nadir value, as opposed to the mild incline of sets [100, 90, 80, 70, 60, 50, 40, 30, 20, 10]. When a turnaround is executed with great alacrity, the turnaround can absorb higher AC associated in addition to that, and the AC do not materially affect the depth of the nadir. Figure 6.1 shows that every set of inputs results in a different and unique set of outputs. The flat surface (shown in green) indicates zero values. Figure 6.1 to Figure 6.4 further shows how

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the values of the nadir change with the change of the input values. It also indicates a relationship between the duration (length) and the depth of the nadir as the losses are incurred for an extra four periods before the PC is implemented. The results indicate a relationship between the speed and PC's size to be implemented and the depth of the nadir. The PC size is determined by the quantum of proposed changes and how quickly these can be implemented.

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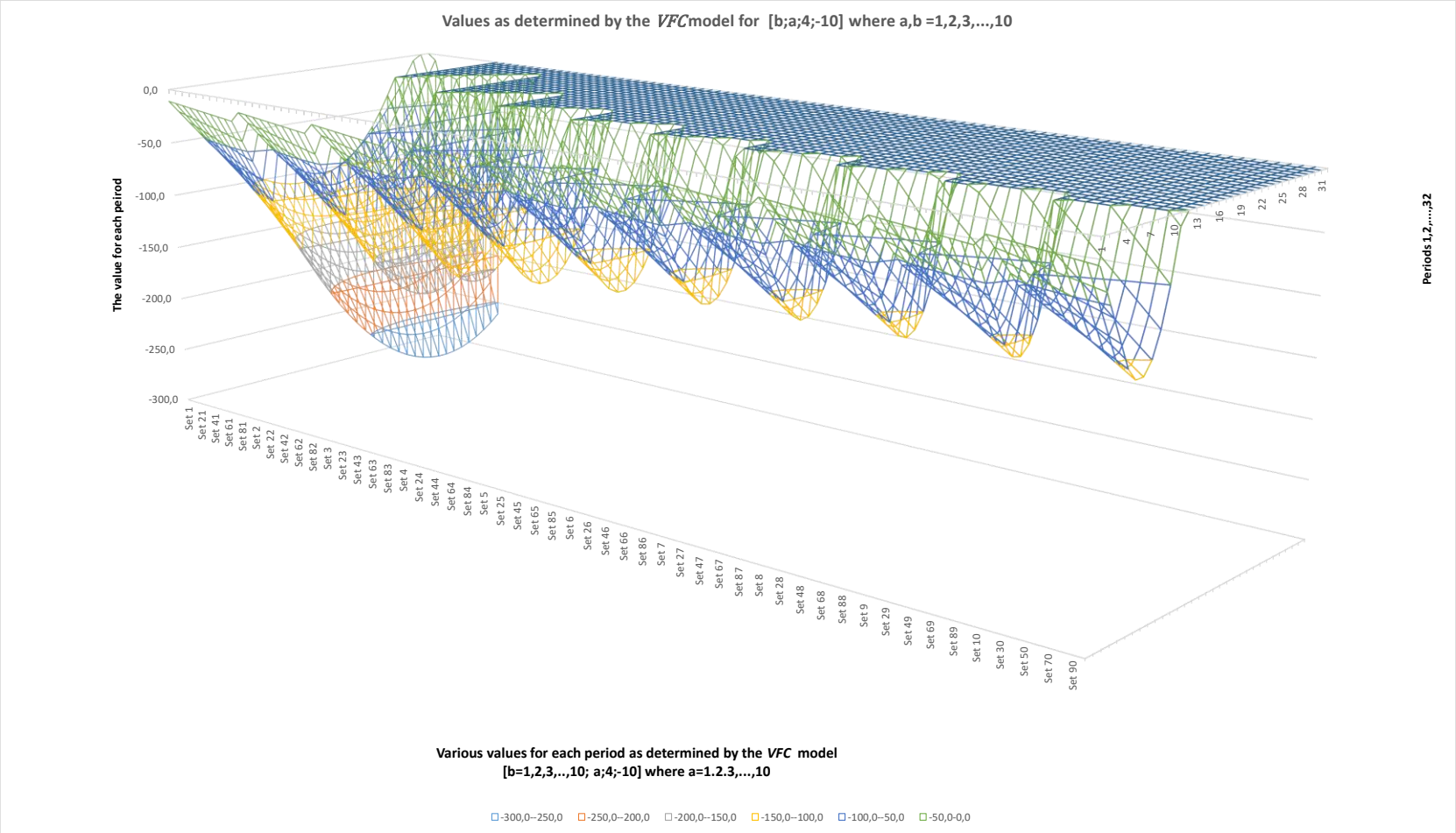

Figure 6.1: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (side view)

Source: Author (2021)

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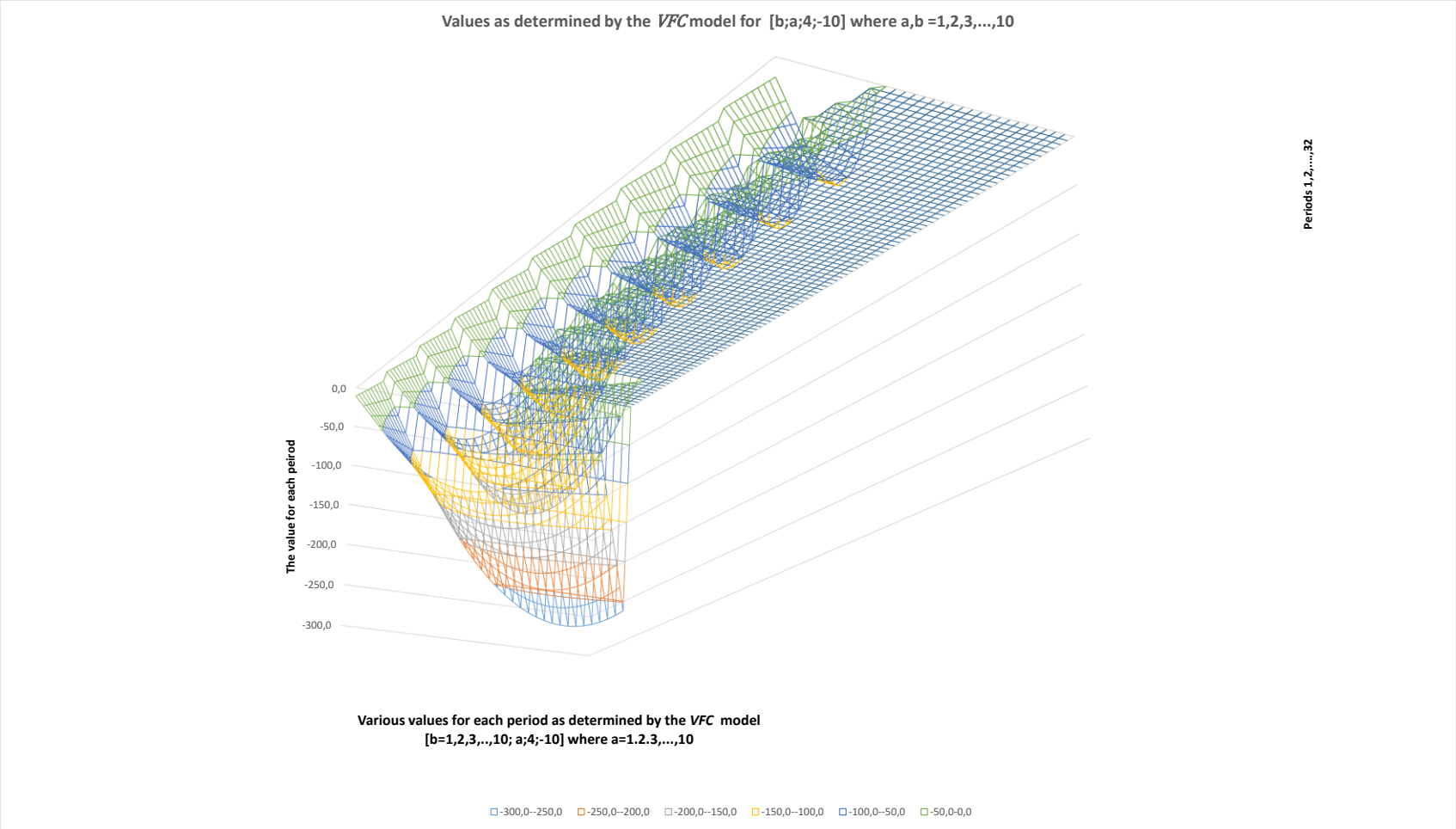

Figure 6.2: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (bottom view)

Source: Author (2021)

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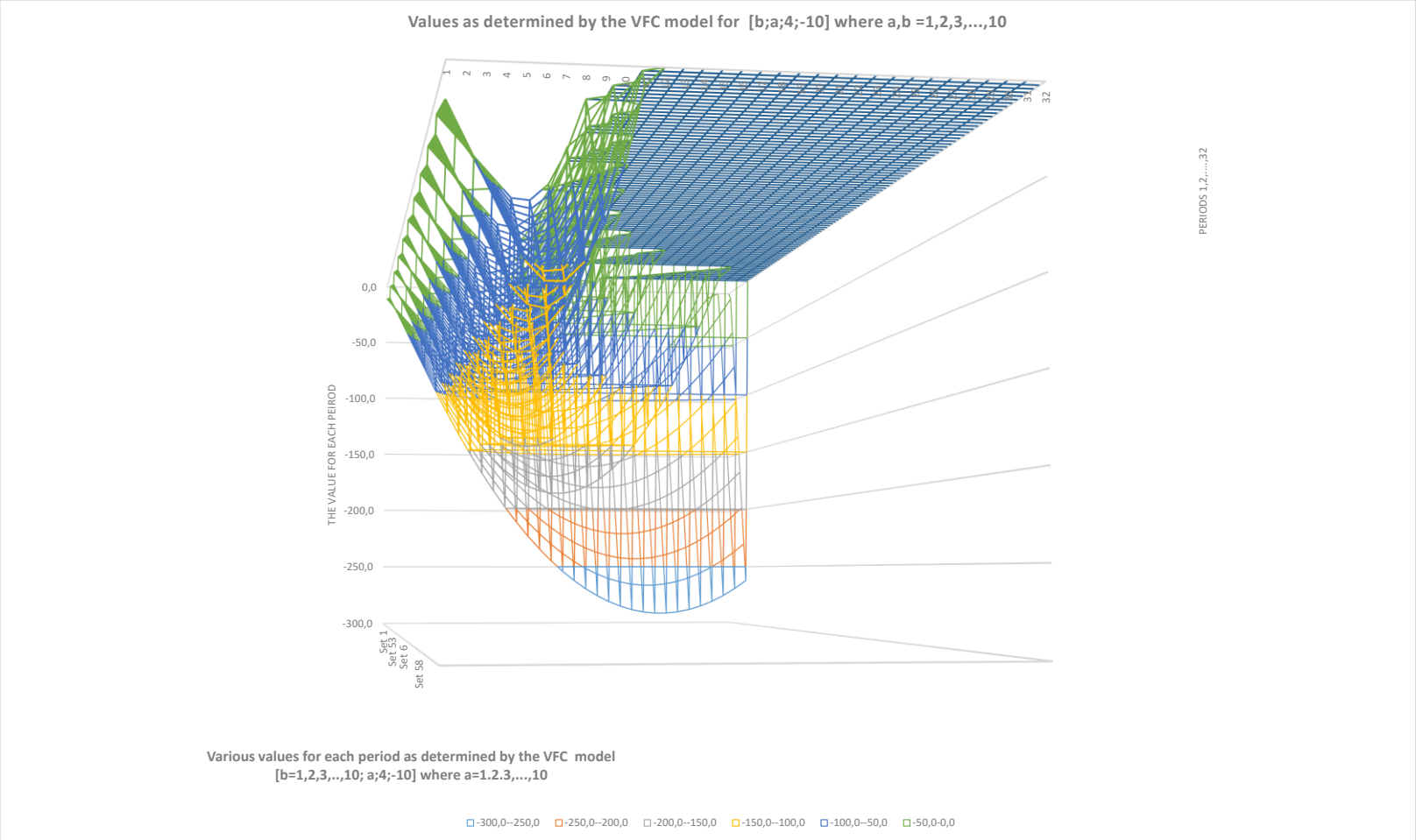

Figure 6.3: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (front view)

Source: Author (2021)

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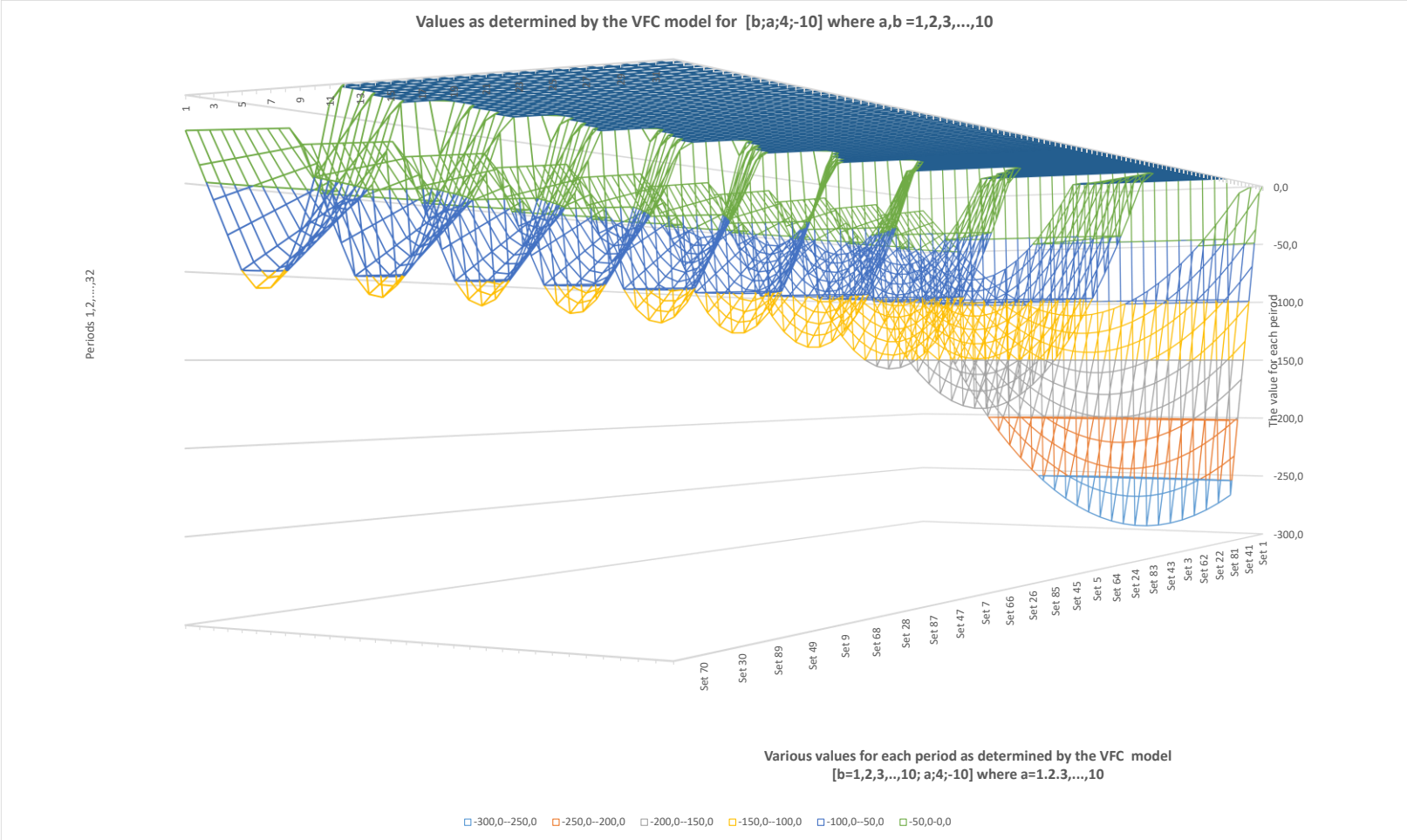

Figure 6.4: Graphing the VFC models using 100 different input sets, where the results for every input set provide a unique set of output results (front-side view)

Source: Author (2021)

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AC influences the output values but has a limited effect on the duration of the turnaround in the event of the PC being implemented speedily.

Storberg-Walker (2003) summarises the work by Dubin (1978, 1983) as a "… theory-building research method, comprising of the following:

(a) The theory has units that interact with one another,
(b) the theory posits laws of interaction among units,
(c) the theory describes boundaries within which the relationships are supposed to hold,
(d) the theory describes different system states that exist in the real world,
(e) the theory should provide propositions,
(f) the propositions need to have empirical indicators,
(g) the theory needs to provide a testable hypothesis, and
(h) the theory should continuously be applied (e.g., continual research).

Dubin argued that a positive working relationship between theorist and practitioner is required to develop and exercise the theory in daily application".

Utilising the elements of theory-building research, the VFC model and fundamental moments can now be analysed to determine if it complies with the rules to create a theory as set out by Dubin. The objective of the exercise is to confirm if the VFC model and fundamental moments could serve as a theory for the measurement of the process of turnarounds.

(a) The VFC model and fundamental moments clearly show the input values and output values as derived in Annexure 1. A fundamental moment set proposed in Equation (19) F1 in Chapter 5 shows a single array's input and output values.
(b) The equations provide concise and unambiguous laws or relationships determining the interaction between input and output values.
(c) The VFC model is subject to limitations (boundaries) regarding the inputs and application and how these relationships operate.
(d) The VFC model was rigorously tested as a fit to the TCF calculations that are used by practitioners to inform which strategy should be selected. The VFC model provides a dynamic map of the impact on the accumulated cash flow for any period during the turnaround.

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(e) The VFC model provides propositions on calculating the input values using the assumption sheet, thereby establishing the PC and AC associated with the proposed selected strategy.

(f) The proposed fundamental sets offer a clear empirical indicator of system states showing the input and output values to return concise information.

(g) The VFC model is a measurement system that may be used to perform intra- and inter-comparisons of selected strategies and the efficacy of the implementation thereof.

(h) As a measurement system, the VFC model and fundamental moments can be used on an ongoing basis to perform research on the process of the turnarounds and may be incorporated into empirical research design.

Practitioners and stakeholders may adopt fundamental moments to become the preferred method to discuss a turnaround and what a selected strategy may bring about.

6.2.2 Some observations and nomenclature derived from the **VFC** model and fundamental moments

This chapter discusses the verifiable nomenclature derived from the VFC model and fundamental moments. Each fundamental moment can be precisely defined:

Duration indicates the recovery of capital used to fund the turnaround: Duration is determined by the point where the accumulated cash flow returns to zero. It indicates how long the turnaround will last, implying when the turnaround practitioner's work and associated costs will terminate. The duration also indicates the period to recover the capital available to fund the turnaround. By definition, once the decision to commence with the turnaround is made, things will worsen before it gets better from an accumulated cash flow perspective. Ensuring the success of a turnaround requires funding, as indicated by the VFC model. The model shows how much and when this funding will be needed. It does not address what capital may be required to acquire assets unless the payments for such assets are included explicitly in the VFC model.

The cash flow nadir will always occur half a period before breakeven occurs: The maximum funding is required just prior to the firm reaching breakeven. This insight means that the stakeholders committed to funding the turnaround need to know when the breakeven is expected to indicate the cash flow nadir. Understanding that once support is provided, the

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stakeholders should continue doing so to ensure the successful implementation of a turnaround. Suppose the stakeholders can measure the progress of the turnaround and can determine when the breakeven and nadir occur – it enables the stakeholders to decide not to cut support for the turnaround too early.

Managing turnarounds is an iterative process: The VFC model and fundamental moments form a measurement tool that enables management to manage a turnaround as an iterative process. Management selects a strategy based on experience, facts, and information available to them at that moment in time. They provide their best estimates of the AC incurred and the PC that can be achieved. These outcomes of the estimates are returned either via the VFC model or expressed using fundamental moment sets. As time elapses, new information becomes available that may lead to the management changing their estimates relating to planned changes, AC, or expecting further delays in the implementation thereof. The ability to present the information in a simple format allows for creating various outcomes based on various possible strategies. Access to these measurements throughout the duration allows for continuous evaluation of the selected strategy. As new information becomes available, the chosen strategy may be refined. However, changing a strategy can become costly during a turnaround. Therefore, carefully selecting the strategy in the first instance is of primary importance.

Delays can occur at any period during the turnaround: The VFC model shows the delays occurring at the beginning, prior to the planned changes occurring. However, the delays may occur at any time during the duration period. Delays can also be used as a measurement indicator for the growth in working capital requirements. In other words, assuming that accounts receivable will grow as a part of the turnaround, the practitioner may elect to use a delay of one (assuming one period is equal to a month) to provide a rough calculation of the effect on the nadir. However, the VFC model calculates the funding required to fund the turnaround. Any further capital requirements, such as acquiring new assets or funding growth in working capital, should be calculated separately and added to the VFC requirements.

The impact of non-implementation on the nadir: The nadir depth will increase sequentially for each period if there is a delay in the PC. The only way to change the downward trajectory of

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the VFC model is to implement the PC and sustain such implementation until the nadir is reached.

Standardised nomenclature is interlinked to the measurement model, allowing for verification and discussion when dealing with turnarounds, regardless of context, content, or process. Future research on the process of turnaround should move away from operational and strategic implementation and view these as tactics to achieve an outcome to implement a turnaround that the standardised nomenclature will underpin. Using fundamental moments as a measurement model enables the measurement of the performance during a turnaround and the effects of time, and the sequential effects of the turnaround. Fundamental moments can measure a turnaround's planned and actual outcomes, thereby enabling inter-comparison of performance. Future empirical research designs can incorporate fundamental moments as an essential measurement, as this is how practitioners measure turnarounds, even if these are derived from varying TCF calculations.

To summarise: a measurement model will inform the nomenclature and allow for the theory of turnarounds to be developed while considering the pressures and constraints present in an organisation while undertaking such a turnaround. The model also informs future empirical research design, where it is proposed that future research may focus on the outcomes achieved versus the adopted strategy. The VFC model and fundamental moments provide the ability to create a dynamic picture that demonstrates the different outcomes based on the different strategy selections proposed.

6.3 The potential benefits of using the VFC model and fundamental moments in the turnaround industry

Various benefits in utilising the VFC model and fundamental moments can be identified, such as:

1. it can improve trust between stakeholders and the practitioner;
2. it is a dynamic model that describes the process of the turnaround and how different scenarios influence the outcomes; and
3. it was designed to provide specific answers to the questions that stakeholders may want to answer before deciding to support the turnaround.

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The VFC model and fundamental moments can improve trust between stakeholders and the practitioner: Fundamental moments do not require calculating values for every period to determine the characteristics of a turnaround. As explained in this study, the presentation of fundamental moments enables the user to understand cause and effect immediately, given a particular set of assumptions. Therefore, it allows for comparing various strategies or understanding the boundary values for a given strategy. The ability to calculate these values mathematically means that any party can recreate the calculations and can verify the presented fundamental moments at any time. The stakeholder can request that the practitioner provide the assumption sheet used to determine the values in the fundamental moments. An assumption sheet provides a structured presentation method to determine PC (ψ) and AC (μ). Verification of fundamental moments by multiple parties establishes trust between the users. Therefore, stakeholders and the practitioner can provide support for the turnaround, as there is no reason not to trust the valuations presented.

The VFC model and fundamental moments provide a dynamic model: The model describes the process of a turnaround based on the assumptions derived from the selected strategy to be implemented. The ability to map and present various strategies or boundary values promotes an understanding of the outcomes, as they can be verified and can ultimately lead to enhanced decision-making.

The VFC model and fundamental moments are designed to be fit for purpose: These tools can be used to measure the selected strategy's outcomes and serve as part of the communication strategy when engaging with stakeholders. Its role is not to project or manage cash flow on a daily or weekly basis but to provide a holistic overview of the funding requirements and obtain support from the stakeholders.

6.3.1 Future specific areas of research that may be developed using the VFC model and fundamental moments

The previous section described how the VFC model and fundamental moments could be used in the broader turnaround research agenda. This section deals with specific future research areas that may be considered.

Introducing stochastics in establishing inputs: Future research may introduce a stochastic treatment of the PC (ψ) and AC (-μ). Such a study could conceivably use, for instance, a Monte

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Carlo simulation using the initial assumptions prepared by the practitioner. It could estimate a variation in estimating the period required to implement a specific planned change. The practitioner may also apply a stochastic treatment to estimate a particular planned change. Research may also result in understanding the relationship between the estimation of the time it takes to complete a task and the possible result.

Expand the nomenclature on the turnaround process: The VFC model may also be adapted to establish a standardised nomenclature about the process of a turnaround. It could deliver a mathematical classification of what constitutes decline and recovery. Such research can define the nomenclature pertaining to the phases or stages of the turnaround. By manipulating the equations of the VFC model, it would create the definitions of various stages describing a turnaround. Such research may lead to a new approach to dealing with the process of turnarounds by establishing or re-purposing existing definitions to fit start-ups or entrepreneurial research and address how the entrepreneur may engage with stakeholders' support for a specific start-up.

Creating a "what-if" analysis on turnarounds: The VFC model can be mathematically adapted to create a "what-if" analysis that measures different strategy outcomes in terms of boundary values or across various strategies. Such research would answer the questions: what relative change in PC is required to achieve a different duration outcome or what relative change in percentage of PC would be equal to the relative change in AC to achieve the same outcome? Such research would assist in enhanced decision-making by stakeholders and practitioners when dealing with turnarounds.

Apply fundamental moments to start-up research: Research on start-ups may consider adapting fundamental moments, and where the VFC model may also be purposed to track the process of a start-up. In such a case, the AC may indicate the setup and once-off expenses required to establish the business.

6.4 Limitations of this study

This study is subject to the following limitations:

(1) it is a deterministic and not a probabilistic measurement of outcomes;
(2) the VFC model uses average inputs;

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(3) the VFC model uses real numbers as input values;

(4) the VFC model does not propose an optimal strategy;

(5) the VFC model and fundamental moments do not measure total capital requirements for a selected strategy; and

(6) TCF calculations remain a management tool to be used internally.

These limitations are further expanded upon below.

It is a deterministic and not a probabilistic measurement of outcomes: The VFC model and fundamental moments, as introduced in this study, do not predict the turnaround's failure or success, as that falls outside the scope of this study. Predicting the failure or success of an outcome requires a probabilistic treatment that falls outside the current deterministic model. Fundamental moments may indicate various outcomes of various strategy options available to the practitioner. It can also provide the boundary values of a selected strategy, assuming delays have occurred. It does not consider what constraints are coupled to each strategy. It, therefore, does not inform how the selection may be skewed by constraints and possibly not be the best strategy for the business. This study provides a ubiquitous calibrated measurement tool for outcomes of a turnaround.

The model uses average inputs: The VFC model uses "average" input parameters, where the sum of AC and the sum of PCs are divided by period and estimated period (t). As shown, this period may be the most prolonged estimated period (t_{max}) or the weighted average period (t_{ave}) to determine the periodic average value to be utilised. The VFC model ignores the possibility of a specific planned change occurring in a shorter timeframe than was initially estimated. Since the entire process is an exercise in predicting the future, delays matter, and the model provides that. Optimising for every eventuality only adds complexity but no extra benefit. The VFC model and fundamental moments can be updated at the end of each period, providing an improved estimate based on the new information that has become available. The VFC model measures the intervals in months. If a turnaround requires measurement weekly, then management is busy with the micromanagement of cash flow, and the TCF calculations will serve this purpose best. On the other hand, if the intervals used to measure are too far apart, for instance, on an annual basis, the detail of when the fundamental moments occur may be lost, as the overall scaling used does not provide the effect of the turnaround.

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The VFC model and fundamental moments use real numbers as input values: While the simulations were performed using integers as input values, there is no limitation to using any fraction or non-integer as an input value. The VFC model and fundamental moments do not cater to irrational or complex numbers.

The study does not describe or explain how to draft a turnaround plan: It does not propose an optimal strategy; nor does it seek to explain what narrative to use, how to present the plan, or what level of detail to include in a turnaround plan. It does not explain how to prepare projected annual financial statements or other supporting documents required to prepare a turnaround plan. Every turnaround practitioner may have different views on what documents they would like to combine within their turnaround plans. The study seeks to establish a measurement tool where all parties can rely on a calibrated measurement system and use the standardised nomenclature when dealing with turnarounds.

The VFC model and fundamental moments do not measure for total capital requirements for a selected strategy: The VFC model measures the operational VFC requirement to fund the selected turnaround strategy. Any capital requirements such as funding the accounts receivable or inventory; or acquiring assets to implement the selected strategy must be added to the nadir requirements calculated. The purpose of the VFC model is to understand the time and financial effects that a selected strategy will have on the outcomes of a turnaround. The VFC measures the impact of income and expenses as they are accrued and not as they are received or paid. Delays (k) in the projected income allow a mechanism to compensate for the timing differences between receipts and accruals.

TCF calculations remain a tool for management to manage their business internally: The study does not propose that the VFC model replace the TCF calculations as an internal management tool to manage the allocation of payments to suppliers and the like. Instead, it is proposed that the VFC model and fundamental moments are calibrated measurement tools that can efficiently facilitate the practitioner and stakeholders' engagement, ultimately leading to enhanced decision-making.

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6.5 Summary and conclusion

This study sought to answer the questions dealing with the time and financial effects of turnarounds. It investigated how practitioners determined the time and financial effects using TCF calculations. It was determined that the duration period, the period when breakeven occurs, the size of the nadir, and when the nadir occurs are the fundamental moments of a turnaround.

A mathematical model was created to determine these fundamental moments, and where this model was tested and verified to be a perfect fit for the TCF calculations. The method was proposed using the assumption sheet to show and determine the inputs required for the VFC model or when calculating fundamental moments.

The VFC model proves a mathematical relationship between time and the financial consequences for any selected strategy. It leads to a unique set of outcomes presented as fundamental moments. These fundamental moments offer a simple manner to provide all the relevant information for stakeholders to make an informed decision. The model also differentiates between the boundary value of a selected strategy and the presentation of the different strategies. It enables the stakeholder to determine if the practitioner has changed strategy or if there has been a delay in implementing the selected strategy.

The study shows an interconnectedness between the different elements in each unique fundamental moment set. The implementation rate is the key to a turnaround. The more rapidly the PC is implemented, the better AC can be absorbed without significantly impacting the outcomes of the fundamental moments.

The study also investigated how research dealing with turnarounds deals with fundamental moments. The investigation showed that research does not describe or define fundamental moments. It was found that when empirical research is undertaken, it is based on selecting an extended period on a *post priori* basis to ensure that all the fundamental moments are covered.

This study concluded by showing the benefits of using the VFC model and fundamental moments and how this could possibly lead to enhanced decision-making. It also investigated what research questions or lenses have been identified by various authors who have

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undertaken literature studies on the research field of turnaround. It showed that the VFC model and fundamental moments could become the central pivot of future research dealing with turnarounds. As stated at the beginning of this study – you cannot manage if you cannot measure. The VFC model and the fundamental moments measure time and financial consequences and might be a helpful tool for future research.

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Annexure 1: Developing the mathematical $\boldsymbol{VFC}$ model and fundamental moments

The content of this annexure was developed prior to the commencement of this study.

Abstract

This annexure provides the mathematical proofs used to develop the VFC model, from which the fundamental moments were derived.

The VFC model aims to enable the user to calculate the VFC algebraically, allowing for the evaluation of any turnaround plan with or without delays in the plan's implementation. The VFC model may also be applied to the evaluation of start-ups.

The VFC model was algebraically manipulated to calculate the various elements of the fundamental moments set, including how long the turnaround is going to last, in which period breakeven will occur, what the peak level of required funding will be, and the value of the accumulated cash flow for each period for the duration of the turnaround.

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A1.1 Introduction

It is safe to assume that every business owner, financial manager, or director has probably built a spreadsheet to predict their business's future cash flow or profit or loss. These spreadsheets are referred to as TCF calculations. The process of building a spreadsheet follows a specific format. Income items and expenses are summed as separate groups, deducted from each other, and added to the difference from the previous period to provide the cash flow at the end of the period.

This annexure takes these basic practices of the TCF calculations and describes a basic algebraic model to map the outcomes for each period, referred to as the VFC model, where the VFC represents the cash flow requirement at the end of each period for the turnaround duration. Once created, the VFC model enables the creation of derivatives to determine mathematically:

1. how long the turnaround will last;
2. when the breakeven will occur during the turnaround or a start-up;
3. what the required peak finance amount will be; and
4. what happens if the implementation is delayed.

Since this is a predictive model, the model serves as a measurement tool to evaluate and compare the historical to the previously predicted or actual performance as the turnaround develops.

This annexure provides mathematical proofs for equations used to propose the VFC model and the fundamental moments derived therefrom. It provides a rigorous algebraic framework for the equations presented. It sets out the assumptions used to develop the equations and follows the methodology discussed in Chapter 4.

This annexure deals with the mathematisation of the TCF calculations by commencing with an elementary model that is expanded by adding further appropriate and necessary variables. After each expansion, the equations are developed afresh to verify these equations with mathematical proofs. The development of these equations is subject to the following assumptions:

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1. that the VFC model returns the exact value as would be generated in a spreadsheet regardless of the period in which it occurs;
2. that the savings implemented will be averaged over the most prolonged period of the turnaround;
3. that the AC is averaged using the prolonged period over the estimated duration of the turnaround; and
4. the loss that arose in the period immediately prior to the commencement of the turnaround will be repeated for every period thereafter.

These calculations can also be used to perform intra-comparisons between different periods to create visibility on how the turnaround is performing, which is critical for developing trust with the affected parties.

A1.2 Summary of assumptions used to develop the VFC model

The VFC model, as proposed, replaces the numerical solution derived from the TCF calculations. Therefore, this model must accurately emulate the numerical model and must be able to determine any specific value when required to enable decision-making when evaluating the turnaround. The VFC model must be tested to verify if it is a perfect fit based on the mathematisation that has occurred. It is achieved by performing numerical simulations usually created in a spreadsheet to determine the VFC.

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Table A1.1: Summary of the time and financial consequences when evaluating a turnaround

Source: Author (2021)

Deriving the equations for the $\boldsymbol{VFC}$ model (Equations A)
Derive equations for the VFC model that return the accumulated cash flow for each period, as set out in Equations A1 (subject to condition α); A2 (subject to conditions α and β); and A3 (subject to conditions α, β and γ)
Establish the duration of the turnaround (Equations B)
Use the VFC function to derive an equation that determines the duration of the turnaround, indicating when the capital utilised in the turnaround will be recovered, as set out in Equations B1 (subject to condition α); B2 (subject to conditions α and β); and B3 (subject to conditions α, β and γ)
The period in which breakeven will occur (Equations C)
Use the VFC function to derive an equation that determines when the firm breaks even; where the loss $(-\lambda)$ plus the AC $(-\mu)$ and the sum of the planned changes $(\sum \psi_t)$ is equal in that period, as set out in Equations C1 (subject to condition α); C2 (subject to conditions α and β); and C3 (subject to conditions α, β and γ)
The period in which cash flow nadir will occur (Equations D)
Use the VFC function to derive an equation that determines in which period the cash flow nadir takes place as set out in Equations D1 (subject to condition α); D2 (subject to conditions α and β); and D3 (subject to conditions α, β and γ)
The value of the maximum funding requirement (Equations E)
Use the VFC function to derive an equation that determines the size of the cash flow nadir required to fund the business, as set out in Equations E1 (subject to condition α); E2 (subject to conditions α and β); and E3 (subject to conditions α, β and γ)

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The VFC model determines the four elements that form the fundamental moments set:

(1) develop the VFC model to determine the accumulated cash flow (Equations A1, A2 and A3)

(2) how long is the turnaround going to last (Equations B1, B2, and B3);

(2) when is the firm going to break even (Equations C1, C2, and C3);

(3) when is the cash flow nadir going to occur (Equations D1, D2, and D3); and

(4) how much funding is required to cover the cost of the turnaround (Equations E1, E2, and E3)?

Once the implementation of the strategy commences, the answers to these four questions are revisited throughout the duration of the turnaround.

Table A1.1 above maps the equations derived subject to variations of conditions α, β *and* γ to ensure that the mathematical model addresses the questions as set out above.

A1.3 The methodology used to create the VFC model

This approach couples the VFC model to the fundamental moments, ensuring that the time and financial consequences are interrelated. The model seeks to understand how the rate at which the planned changes coupled with delays impact the turnaround, given the AC and losses associated with a turnaround.

The methodology used to create the algebraic model consists of using the simplest assumptions to build the basic model. Further conditions and assumptions are added to determine their impact on the model. To develop the equations for the VFC model, the following conditions apply:

1. condition $\boldsymbol{\alpha}$ – no AC (μ) exists, and the implementation of PC (ψ) happens as planned;
2. condition $\boldsymbol{\beta}$ – AC (μ) exists, and the implementation of PC (ψ) happens as planned;
3. condition $\boldsymbol{\gamma}$ – AC (μ) exists, and the implementation of PC (ψ) is delayed.

The VFC model is set out in Equations A1, A2, and A3 and repeated in summary below:

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condition $\boldsymbol{\alpha}$

$$Acc\ \lambda_{\psi} = -\lambda\left((t_x + 1) - \left(\frac{t_x}{2a}(t_x + 1)\right)\Big|_{x=1}^{x=\check{n}_{(\psi)}}\right), \quad \text{(20) A1}$$

$$where\ x = 0, 1, 2, \dots, \check{n}$$

condition $\boldsymbol{\beta}$

$$Acc\ \lambda_n = -\lambda_0\left((t_x + 1)\left(1 + \frac{1}{b}\right) - \left(\frac{t_x}{2a}(t_x + 1)\right)\Big|_{x=0}^{x=n}\right) \quad \text{(21) A2}$$

$$b \neq 0, t_x = 1, 2, 3, \dots, n$$

condition $\boldsymbol{\gamma}$

$$Acc\ \lambda_k = -\lambda\left(\left(1 + \frac{1}{b}\right)(t_x + 1)\Big|_{x=0}^{x=\check{n}_{(k)}} - \left(\frac{t_y}{2a}(t_y + 1)\right)\Big|_{y=1}^{y=\check{n}_{(k)}-k}\right) \quad \text{(22) A3}$$

$$a \neq 0; b \neq 0; k < 0, k \in \mathbb{R}; and$$

$$x = 0,1,2, \dots, \check{n}_k;\ and\ y = 1,2,3, \dots, \check{n}_k - k\ and$$

$$where\ y = t_x - k, t_x > k$$

Once the fundamental moments have been derived from the VFC model equations, the four elements of a fundamental set should also be calculated subject to conditions $\alpha, \beta\ and\ \gamma$ being present. These values are set out in Equations B1, B2, and B3 to E1, E2, and E3 and repeated in the summary below:

Duration of turnaround:

condition $\boldsymbol{\alpha}$

$$\check{n}_{(\psi)} = 2a \quad \text{(23) B1}$$

condition $\boldsymbol{\beta}$

$$n = 2a\left(1 + \frac{1}{b}\right) \quad \text{(24) B2}$$

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$$\breve{n}_k = \rho + n + k$$

$$where\ n = 2a\left(1 + \frac{1}{b}\right)\ and$$

condition

γ

$$\rho = a\left(-\left(\frac{1}{2a} + \varpi\right) \pm \sqrt{\left(\frac{1}{2a} + \varpi\right)^2 + \frac{2k\varpi}{a}}\right) \quad (25)\ B3$$

$$where\ \varpi = 1 + \frac{1}{b}$$

Breakeven of a turnaround:

condition

α

$$E_{(\psi)} = a \quad (26)\ C1$$

condition

β

$$E_{(n)} = a(1 + \frac{1}{b}) \quad (27)\ C2$$

condition

γ

$$E_{(k)} = a\left(1 + \frac{1}{b}\right) + k \quad (28)\ C3$$

The period when the nadir will occur:

condition

α

$$\breve{n}_{min(\psi)} = a - \frac{1}{2} \quad (29)\ D1$$

condition

β

$$n_{min} = a(1 + \frac{1}{b}) - \frac{1}{2} \quad (30)\ D2$$

condition

γ

$$\breve{n}_{min(k)} = a\left(1 + \frac{1}{b}\right) - \frac{1}{2} + k \quad (31)\ D3$$

Cash flow required to fund the nadir:

condition

α

$$Acc\ \lambda_{min(\psi)} = -\lambda\left(\frac{1}{2}a + \frac{1}{2} + \frac{1}{8a}\right) \quad (32)\ E1$$

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condition $\boldsymbol{\beta}$

$$Acc\ \lambda_{min(n)} = -\lambda\left(\frac{1}{2}a\left(1+\frac{1}{b}\right)^2 + \frac{1}{2}\left(1+\frac{1}{b}\right) + \frac{1}{8a}\right) \quad \text{(33) E2}$$

condition $\boldsymbol{\gamma}$

$$Acc\ \lambda_{min(k)} = -\lambda\left(k\left(1+\frac{1}{b}\right) + \frac{1}{2}a\left(1+\frac{1}{b}\right)^2 + \frac{1}{2}\left(1+\frac{1}{b}\right) + \frac{1}{8a}\right) \quad \text{(34) E3}$$

The algebraic framework for the mathematical proofs of the equations used to propose the VFC model and the derived fundamental moments together with the assumptions used to develop the equations that follow.

A1.4 The assumptions used to create a $\boldsymbol{VFC}$ model

During the process of mathematising the VFC model, the following assumptions (input values) were used:

Assumption No. 1

The cornerstone assumption used in the VFC model returns the exact value derived from the TCF calculations for every period, using the same assumptions for the model and calculations. The TCF calculations group the income items and expense items together, which are then deducted to deliver the net income generated for that period. The "result for the period" is added to the "result at the end of the previous period", sometimes referred to as the "result of the beginning of the period". The sum of these two items is the "result at the end of the period". Therefore, the result for the end of the period is the VFC required for the turnaround. Depending on the conditions used, the value at the end of each period is expressed as the accumulated cashflow $Acc(\lambda_{\psi,\ or\ n,\ or\ k})$ for that period.

Assumption No. 2

The total PC (ψ) achieved during the turnaround is aggregated and divided by the most prolonged period of any individual implementation change. In other words, if implementation change number one is estimated to take six months and number two is estimated to take twelve months, the values of these two PCs (ψ) can be added together and

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divided by twelve. The aggregate PC (ψ) is divided by the most prolonged period of a single activity to be undertaken. All PC (ψ) are deemed to be permanent.

Assumption No. 3

All costs associated with the turnaround that do not form a part of normal operating costs that may have existed prior to the turnaround are deemed AC (μ). These costs are aggregated and divided by the same number of periods as determined for the treatment of the PC (ψ).

Assumption No. 4

The loss that occurred in the period t_0 will be repeated in every subsequent period as the model assumes that the PC (ψ) reflects the improvements. Should additional costs increase on a period-by-period basis, such an increase, for example, interest, will be added to the AC (μ).

A1.5 Development of the mathematical model to determine VFC

The departure point for the development of the VFC model was to create a ubiquitous algebraic function that maps the numerical calculations contained in a TCF calculation for any period, subject only to the α condition (no AC (μ) exists and the implementation of PC (ψ) happens as planned). This function is then compared to the numerical values generated in the spreadsheet, which should accurately fit these values for any period and is subject to Assumptions No. 1, No. 2, and No. 4 above. The resulting function is then manipulated to determine the fundamental moments upon determining such a function.

Once the primary model was mathematised, the process was then repeated subject to conditions α and β being present. It is based on Assumptions No. 1 to No. 4, upon which the process is then repeated but now incorporates conditions α, β and γ.

A1.6 Creating the VFC model for any period of t_x, subject only to condition α (no AC (μ) incurred and PC (ψ) implemented from the period t_1) (A1)

Assume a function, $f(t_x)$, exists for every period of t, equally spaced in time, to express the loss ($-\lambda$) or profit for each period, which results in the accumulated net cashflow ($Acc\ \lambda_{\psi,\check{t}}$) with no AC ($\mu$) and a cumulative implementation change (ψ) occurring for every period of t

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starting in the period t_1. It is assumed that period t_0 is used for analysis and preparation of the strategy, as no PC (ψ) occurred during this period. Therefore let:

$$Acc\ \lambda_{\psi,\breve{n}} = f(t_0) + f(t_x) + \cdots + f(t_{\breve{n}}), where\ x = 1, 2, 3, \ldots, \breve{n} \tag{35}$$

Assume that $-\lambda_0$ represents a loss in the period prior to the turnaround team taking control of the business. The team takes no further actions to evaluate their strategy to turn around the business.

$$f(t_0) = -\lambda_0 \tag{36}$$

Based on Assumption No. 4, the loss ($-\lambda_0$) will be repeated for every period so that:

$$= -\lambda_0 = -\lambda_1 \cdots = -\lambda_{\breve{n}} \text{ and that:}$$
$$= -\lambda_0 + -\lambda_1 \cdots + -\lambda_{\breve{n}} = -\lambda_0(\breve{n} + 1) \tag{37}$$

An example of how the loss arises in each period is shown in Table A1.2 below, where the accumulated losses $-(n+1)\lambda_0$ are shown in the last vertical column, and loss per period on the bottom horizontal row.

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Table A1.2: A representation of the losses that keep recurring for every period if no action is taken

Source: Author (2021)

Periods of t	Loss incurred in each period						Total per period t
	0	1	2	$\cdots$	$\cdots$	n	
0	$-\lambda_0$						$-\lambda_0$
1		$-\lambda_1$					$-\lambda_0 - \lambda_1$
2			$-\lambda_2$				$-\lambda_0 - \lambda_1 - \lambda_2$
$\cdots$				$\cdots$			$\cdots$
$n-2$				$-\lambda_{n-2}$			$-(n-1)\lambda_0$
$n-1$					λ_{n-1}		$-n\lambda_0$
n					$\cdots$	$-\lambda_n$	$-(n+1)\lambda_0$
	$-\lambda_0$	$-\lambda_1$	$-\lambda_2$	$-\lambda_3$	$-\lambda_4$	$-\lambda_n$	

Table A1.2 above assumes that loss $(-\lambda_0)$ shown in period t_0 is, therefore, the accumulated cash flow at the end of that period, so that $Acc(\lambda_{-2}) + f(t_{-1}) = Acc(\lambda_0)$. It implies that all the information related to the income and expenses is included in the loss $(-\lambda_0)$, which existed in the period t_0.

Further assume, subject to Assumption No. 2, that PC (ψ) is to be implemented during the period t_1 as a result of the actions taken by the management team, so that:

$$f(t_1) = -\lambda_0 + \psi_1 \tag{38}$$

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The total PC that was implemented before t_x is expressed as $\sum_1^{x-1} \psi_t$ plus PC to be undertaken in the period t_x and applies anew in the next period t_{x+1}. Therefore, the loss remains the same as at the commencement of the turnaround. The PC (ψ) are measured separately as the cumulative measurement of the PC (ψ) leads to the turnaround. This measurement shows what actions were proposed and used to reduce the losses as implemented and planned.

Table A1.3: The cumulative effect of the PC (ψ) implemented for every period

Source: Author (2021)

Periods of t	Savings per period						Total per period t
	0	1	2	$\cdots$	$\cdots$	n	
0							0
1		ψ_1	ψ_1	ψ_1	$\cdots$	ψ_1	$(n)\psi_1$
2			ψ_2	ψ_2	$\cdots$	ψ_2	$(n-1)\psi_2$
$\cdots$				$\cdots$	$\cdots$	$\cdots$	$\cdots$
$n-2$				ψ_{n-2}	ψ_{n-2}	ψ_{n-2}	$3\psi_{n-2}$
$n-1$					ψ_{n-1}	ψ_{n-1}	$2\psi_{n-1}$
n					$\cdots$	ψ_n	ψ_n
$n+1$	0	1ψ	2ψ	3ψ	$\cdots$	$n\psi_n$	$½n(n-1)\psi_n$

As shown in Table A1.3 above, the cumulative effect of the PC (ψ) over the preceding periods, taken in the period t_1, will be repeated in the period t_2 for every period thereafter. In period t_y, the sum of the PC (ψ) from the previous periods plus the PC (ψ) of the current period is added together.

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This process will continue until the PC (ψ) stops due to the completion of the turnaround. Returning to Equation (38), the loss incurred and the PC (ψ) undertaken in period $t = y$, can be expressed as:

$$f(t_y) = -\lambda_0 + \sum_{x=1}^{y} \psi_x \tag{39}$$

Equation (39) is inserted into Equation (35) to show the cumulative effect for the period $f(t_{\breve{n}})$ representing the duration of the turnaround. Equation (40) can further be mathematically adapted to yield Equation (4) A1 so that:

$$f(t_{\breve{n}}) = (-\lambda_0) + \left(-\lambda_0 + \sum_{t=1}^{\breve{n}} \psi_t\right) + \left(-\lambda_0 + \sum_{t=1}^{\breve{n}-1} \psi_t\right) \cdots + (-\lambda_0 + \psi_{\breve{n}}) \tag{40}$$

$$= (-\lambda_0 + \breve{n}\psi_1) + (-\lambda_0 + (\breve{n} - 1)\psi_2) + \cdots + (-\lambda_0 + 2\psi_{\breve{n}-1}) + (-\lambda_0 + \psi_{\breve{n}}) \tag{41}$$

Further, assume that the PC (ψ) will be constant throughout all periods of t. Based on Assumption No. 2, it follows that $\psi = \psi_1 = \psi_2 = \cdots = \psi_{\breve{n}}$ and based on Assumption No. 4, it implies that $\lambda = -\lambda_0$ results in:

$$Acc\ \lambda_{\psi,\breve{n}} = -(\breve{n} + 1)\lambda + \sum_{t=1}^{\breve{n}} \psi_t + \sum_{t=1}^{\breve{n}-1} \psi_t + \cdots + \psi_1 \tag{42}$$

$$Acc\ \lambda_{\psi,\breve{n}} = (\breve{n} + 1)(-\lambda) + \breve{n}\psi + (\breve{n} - 1)\psi + \cdots + 2\psi + \psi \tag{43}$$

$$= -\lambda(\breve{n} + 1) + \psi(\breve{n} + (\breve{n} - 1) + \cdots + 2 + 1) \tag{44}$$

$$= -\lambda(\breve{n} + 1) + \frac{\psi}{2}(\breve{n}(\breve{n} + 1)) \tag{45}$$

$$Acc\ \lambda_{\psi,\breve{n}} = -(\breve{n} + 1)\lambda + \psi\,(1 + 2 + \cdots + (\breve{n} - 1) + \breve{n}) = -(\breve{n} + 1)\lambda + \frac{\psi}{2}(\breve{n}(\breve{n} + 1)) \tag{46}$$

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$$Acc\ \lambda_{\psi,\breve{n}} = -\lambda(\breve{n}+1) - \left(\frac{\breve{n}\psi}{2}(\breve{n}+1)\right) \tag{47}$$

$Let\ \psi = \frac{\lambda}{a}, with\ a > 0\ so\ that:$

$$Acc\ \lambda_{\psi,\breve{n}} = -\lambda(\breve{n}+1) + \frac{\frac{\lambda}{a}}{2}\left(\breve{n}(\breve{n}+1)\right) \tag{48}$$

$$Acc\ \lambda_{\psi,\breve{n}} = -\lambda(\breve{n}+1) + \frac{\lambda}{2a}\left(\breve{n}(\breve{n}+1)\right) \tag{49}$$

$$Acc\ \lambda_{\psi,\breve{n}} = -\lambda\left((\breve{n}+1) - \left(\frac{\breve{n}}{2a}(\breve{n}+1)\right)\right) \tag{50}$$

A general function for the VFC model, subject to the condition $\boldsymbol{\alpha}$, can therefore be defined as:

$$Acc\ \lambda_{\psi,t} = -\lambda\left((t_x+1) - \left(\frac{t_x}{2a}(t_x+1)\right)\Big|_{x=1}^{x=\breve{n}}\right) \tag{51} \text{ A1}$$

$$where\ a > 0, a, -\lambda \in \mathbb{R}, and\ x = 1, 2, 3 \ldots, \breve{n}$$

Equation (4) A1, therefore, calculates the accumulated cash flow at the end of each period t and by inserting any values of $-\lambda, a\ or\ t_x$ into this equation will return the accumulated cash flow $(Acc(\lambda_{\psi,t}))$ for that period.

A1.6.1 Calculating the duration (the period funders will recover the funding used to fund the turnaround), assuming condition $\boldsymbol{\alpha}$ is present (no AC ($\boldsymbol{\mu}$) and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (B1)

A fundamental question for any turnaround is: how long will the turnaround be? Using the VFC model, as shown in Equation (4) A1, such an answer can be mathematically derived. Measuring the duration is critical for funders, employees, and suppliers (stakeholders), whose support is critical for saving the business. It is the point where the business will return to normal trading activities and does not require the services of the turnaround practitioner and any other third parties. The duration of the turnaround lasts until period $(\breve{n})$ and this

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is also the period where the funders of the business will have recovered the capital used during the turnaround. Using Equation (4) A1 to calculate the period $(\breve{n})$ implies that it will occur where $Acc\ \lambda_{\psi,t} = 0$.

$$0 = -\lambda\left(\breve{n} + 1 - \frac{1}{2a}\breve{n}(\breve{n} + 1)\right) \tag{52}$$

$$0 = \breve{n} + 1 - \frac{1}{2a}\breve{n}(\breve{n} + 1) \tag{53}$$

$$\breve{n} + 1 = \frac{1}{2a}\breve{n}(\breve{n} + 1) \tag{54}$$

$$1 = \frac{1}{2a}\breve{n} \tag{55}$$

$$\breve{n}_{(\psi)} = 2a \qquad \text{(56) B1}$$

Equation (56) B1 implies that the turnaround duration will last twice as long as the average PC $\boldsymbol{a}$ expressed as a proportion of the initial loss. Therefore, regardless of the size of the loss in the period t_0 or the rate of the PC $\boldsymbol{a}$, beginning at period t_1, it will take two times the proportional fraction (a) in periods to recover the funding that was made available to fund the turnaround.

A1.6.2 In which period will the business reach breakeven assuming condition $\boldsymbol{\alpha}$ is present? (no AC $(\boldsymbol{\mu})$ and the PC $(\boldsymbol{\psi})$ implemented from the period $\boldsymbol{t_1}$) (C1)

Assume there is a period $f(t_E)\ or\ E_{(\psi)}$, where the business will reach breakeven, where the loss $(-\lambda_0)$ is equal to the PC $(\sum_{x=1}^{E}\psi_x)$ for that period. It is achieved by equating Equation (39) to zero so that:

$$0 = -\lambda_0 + \sum_{x=1}^{y}\psi_x \tag{57}$$

$$0 = -\lambda_0(1 - \frac{y}{a}) \tag{58}$$

$$E_{(\psi)} = a \qquad \text{(59) C1}$$

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$E_{(\psi)}$ represents the period when the firm stops absorbing and returns cash flow to repay stakeholders who have funded the turnaround, regardless of the size of the loss at period t_0 or the rate at which the PC is undertaken, beginning at period t_1.

A1.6.3 In which period will the cash flow nadir be reached, assuming condition $\boldsymbol{\alpha}$ is present? (no AC ($\boldsymbol{\mu}$) and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (D1)

Assume there is a period where the business will reach a cashflow nadir ($Acc\ \lambda_{min(\psi)}$), shown in Equation (69) E1 below. By differencing Equation (4) A1 provides a minimum so that:

$$\frac{\Delta}{\Delta \breve{n}} Acc\ \lambda_{\psi,\mu} = \frac{\Delta}{\Delta \breve{n}}\left(-\lambda\left((\breve{n}+1) - \frac{1}{2a}\breve{n}(\breve{n}+1)\right)\right) \tag{60}$$

$$\frac{\Delta}{\Delta \breve{n}} Acc\ \lambda_{\psi,\mu} = \frac{\Delta}{\Delta \breve{n}}\left(-\lambda\left((\breve{n}+1) - \frac{1}{2a}\breve{n}^2 - \frac{1}{2a}\breve{n}\right)\right) \tag{61}$$

After differencing and equating Equation (4) A1 to zero, it returns a result of $\breve{n}_{min(\psi)}$, which is the period in which the cash flow nadir exists, so that:

$$0 = -\lambda\left(-\frac{1}{a}\breve{n} - \frac{1}{2a} + 1\right) \tag{62}$$

$$\frac{1}{a}\breve{n} = -\frac{1}{2a} + 1 \tag{63}$$

$$\breve{n}_{min(\psi)} = a - \frac{1}{2} \qquad \text{(64) D1}$$

$\breve{n}_{min(\psi)}$ represents the period where the firm requires the maximum to fund the turnaround, regardless of the size of the loss at period t_0 or the rate at which the PC is undertaken, beginning at period t_1.

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A1.6.4 What is the size of the funding required or the value of the cash flow nadir ($\boldsymbol{Acc\ \lambda_{min(\psi)}}$) assuming condition $\boldsymbol{\alpha}$ is present? (no AC ($\boldsymbol{\mu}$) and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (D1)

The cash flow nadir is the point where the negative cash flow reaches the absolute minimum, implying that it is the point where maximum funding is required. By substituting $\breve{n}$ with $\breve{n}_{min(\psi)}$ in Equation(4) A1 enables the calculation of $Acc\ \lambda_{min(\psi)}$ so that:

$$Acc\ \lambda_{min(\psi)} = -\lambda\left(\breve{n}_{min(\psi)} + 1 - \left(\frac{1}{2a}\breve{n}_{min(\psi)}\left(\breve{n}_{min(\psi)} + 1\right)\right)\right) \tag{65}$$

$$Susbstitue\ \breve{n}_{min(\psi)}\ with\ a - \frac{1}{2}, so\ that$$

$$= -\lambda\left(a - \frac{1}{2} + 1 - \left(\frac{1}{2a}\left(a - \frac{1}{2}\right)\left(a - \frac{1}{2} + 1\right)\right)\right) \tag{66}$$

$$= -\lambda\left(a + \frac{1}{2} - (\frac{1}{2} - \frac{1}{4a})\left(\frac{1}{2} + a\right)\right) \tag{67}$$

$$= -\lambda\left(a + \frac{1}{2} - (\frac{1}{4} - \frac{1}{8a} + \frac{a}{2} - \frac{1}{4})\right) \tag{68}$$

$$Acc\ \lambda_{min(\psi)} = -\lambda\left(\frac{1}{2}a + \frac{1}{2} + \frac{1}{8a}\right) \tag{69 E1}$$

$Acc\ \lambda_{min(\psi)}$ represents the cash flow nadir, implying this is the maximum funding required during the turnaround based on condition α being present.

A1.7 Creating the $\boldsymbol{VFC}$ model for any period of $\boldsymbol{t_x}$, subject to conditions $\boldsymbol{\alpha\ and\ \beta}$ (where AC ($\boldsymbol{\mu}$) incurred and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (A2)

Assume a function, $f(t_x)$, exists for every period of t, equally spaced in time, to express the loss ($-\lambda$) or profit for each period, which results in the accumulated net cashflow ($Acc\ \lambda_t$). Further, assume that for every period of t, the AC ($-\mu$) is incurred, while the cumulative PC (ψ) occurs from the period $t = 1$. It is assumed that period t_0 is used for analysis and

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preparation of the strategy, as no PC occurred during this period. The AC ($-\mu$) occurs in each period as estimated with the commencement of the turnaround.

Table A1.4: How exceptional or AC (μ) arise over time

Source: Author (2021)

Periods of t	*Abnormal costs in each period*						*Total per period t*
	0	1	2	...	...	n	
0	$-\mu_0$						$-\mu_0$
1		$-\mu_1$					$-\mu_0 - \mu_1$
2			$-\mu_2$				$-\mu_0 - \mu_1 - \mu_2$
...				...			...
$n-2$				$-\mu_{n-2}$	...		$-(n-1)\mu_0$
$n-1$					$-\mu_{n-1}$		$-n\mu_0$
n					...	$-\mu_n$	$-(n+1)\mu_0$
	$-\mu_0$	$-\mu_1$	$-\mu_2$	$-\mu_3$	...	$-\mu_n$	

Table A1.4 above shows that the AC is incurred for each period of t starting in the period t_0. The accumulated AC ($-(n+1)\ \mu_0$) are shown in the last vertical column in the second last row. The AC for each period is shown on the bottom horizontal row.

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Table A1.5: Overview of the various elements required to create a VFC subject to conditions α and β

Source: Author (2021)

Description	*Periods*							
	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
Loss per period	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
Abnormal cost	μ_0	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7
Incremental Savings		ψ_1	ψ_1	ψ_1	ψ_1	ψ_1	ψ_1	ψ_1
			ψ_2	ψ_2	ψ_2	ψ_2	ψ_2	ψ_2
				ψ_3	ψ_3	ψ_3	ψ_3	ψ_3
					ψ_4	ψ_4	ψ_4	ψ_4
						ψ_5	ψ_5	ψ_5
							ψ_6	ψ_6
Term (no delay)	1	n						

Combining Tables A1.2, A1.3, and A1.4, provides an overview of the elements that should be present in the function, assuming conditions α *and* β are present. Table A1.5 above shows how the PC ψ_x, *where* $x = 1, 2, 3, \ldots, n$ accumulate in each subsequent period, and the loss (-λ) and AC ($-\mu$) are also included. Period t_0 is the time used to analyse and prepare the strategy; therefore, no PC can be implemented during this period. As shown in Table A1.5, a VFC model can be constructed that is subject to conditions α *and* β, so that:

$$f(t_0) = -\lambda_0 - \mu \tag{70}$$

$$f(t_y) = -\lambda_0 - \mu + \sum_{x=1}^{y} \psi_x \tag{71}$$

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Replace the $\check{n}$ *with* n and using Equations (35) so that:

$$Acc\ \lambda_t = f(t_0) + f(t_x) + \cdots + f(t_n)\ where\ x = 1, 2, 3, \ldots. \tag{72}$$

Equation (75) is expanded by inserting elements of Equation (74) therein so that:

$$= (-\lambda_0 - \mu) + (-\lambda_0 - \mu + \psi_1) + (-\lambda_0 - \mu + \sum_{x=1}^{2} \psi_x) + \cdots + (-\lambda_0 - \mu + \sum_{x=1}^{n} \psi_x) \tag{73}$$

$$= (-\lambda_0 - \mu) + (-\lambda_0 - \mu + \psi_1) + (-\lambda_0 - \mu + \sum_{x=1}^{2} \psi_x) + \cdots + (-\lambda_0 - \mu + \sum_{x=1}^{n} \psi_x) \tag{74}$$

$$= (-\lambda_0 - \mu)(1 + n) + (n - 1)\psi_2 + \cdots + 2\psi_{n-1} + 1\psi_{\mathrm{n}} \tag{75}$$

Given Assumption No. 2 that $\psi = \psi_1 = \psi_2 = \cdots = \psi_n$

$$= (-\lambda_0 - \mu)(1 + n) + n\psi_1 + \psi(n + (n - 1) + \cdots + 2 + 1) \tag{76}$$

$$= (-\lambda_0 - \mu)(1 + n) + \frac{\psi}{2}(n(n + 1)) \tag{77}$$

Let $\psi = \frac{\lambda}{a}$, *with* $a > 0$ *and let* $\mu = \frac{\lambda}{b}$, *with* $b > 0$:

$$= (-\lambda_0 - \frac{\lambda_0}{b})(1 + n) + \frac{\lambda_0}{2a}(n(n + 1)) \tag{78}$$

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The general function to determine the VFC model, where α *and* β conditions are present, is, therefore:

$$Acc\ \lambda_t = -\lambda_0 \left((t_x + 1)\left(1 + \frac{1}{b}\right) - \left(\frac{t_x}{2a}(t_x + 1)\right)\Big|_{x=0}^{x=n} \right) \qquad (79)\ \text{A2}$$

$$where\ a > 0, b > 0;\ a, b, -\lambda \in \mathbb{R}, and\ x = 1,2,3, \dots, n$$

A1.7.1 Calculating the duration (when the funders will recover the funding used to fund the turnaround), assuming conditions $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ are present (with AC ($\boldsymbol{\mu}$) being incurred from the period $\boldsymbol{t_0}$ and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (B2)

Equate Equation (79) A2 to zero ($Acc(\lambda_t) = 0$) to calculate what the duration of the turnaround will be, which is subject to the conditions α *and* β. The duration for the turnaround lasts until the period n and this is also the period where the funders of the business have recovered the capital used during the turnaround.

$$0 = -\lambda \left((n + 1)\left(1 + \frac{1}{b}\right) - \frac{1}{2a} n(n + 1) \right) \qquad (80)$$

$$0 = (n + 1)(1 + \frac{1}{b}) - \frac{1}{2a} n(n + 1) \qquad (81)$$

$$(n + 1)\left(1 + \frac{1}{b}\right) = \frac{1}{2a} n(n + 1) \qquad (82)$$

$$\left(1 + \frac{1}{b}\right) = \frac{1}{2a} n \qquad (83)$$

$$n = 2a\left(1 + \frac{1}{b}\right) \qquad (84)\ \text{B2}$$

$$where\ a > 0\ and\ b > 0; a, b \in \mathbb{R}$$

Equation (84) B2 implies that the turnaround duration will take two times the proportional fraction (a) representing the PC (ψ), times one plus the inverse of b that represent the AC($-\mu$) as a fraction of the loss, in periods to recover the funding what was made available to fund the turnaround. It is regardless of the size of the loss in the period t_0 or the rate of the (a), at the beginning of period t_0.

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A test can be performed if the duration period is correctly derived by inserting Equation (84) B2 into Equation (79) A2 to confirm that the answer returned is zero.

$$Acc\ \lambda_n = -\lambda\left(\left(2a\left(1+\frac{1}{b}\right)+1\right)\left(1+\frac{1}{b}\right) - \left(\frac{1}{2a}(2a\left(1+\frac{1}{b}\right)\left(2a\left(1+\frac{1}{b}\right)+1\right))\right)\right) \tag{85}$$

$$Acc\ \lambda_n = -\lambda\left(\left(2a\left(1+\frac{1}{b}\right)^2+\left(1+\frac{1}{b}\right)\right) - \left(2a\left(1+\frac{1}{b}\right)^2+\left(1+\frac{1}{b}\right)\right)\right) \tag{86}$$

$$Acc\ \lambda_n = 0 \tag{87}$$

A1.7.2 In which period will the business reach breakeven, assuming conditions $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ are present? (no AC ($\boldsymbol{\mu}$) and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (C2)

Assume there is a period $f(t_E)$ *or* $E_{(n)}$ where the business will reach breakeven, where the loss ($-\lambda_0$) plus the AC ($-\mu$) is equal to the PC ($\sum_{x=1}^{E}\psi_x$) for that period. Using Equation (71) and equating this to zero, so that:

$$0 = -\lambda_0 - \mu + \sum_{x=1}^{y}\psi_x \tag{88}$$

$$0 = -\lambda_0(1+\frac{1}{b}-\frac{y}{a}) \tag{89}$$

$$E_{(n)} = a(1+\frac{1}{b}) \tag{90 C2}$$

$E_{(n)}$ represents the period when the firm stops absorbing and returns cash flow to repay stakeholders who have funded the turnaround. It is regardless of the size of the loss at period t_0 or the rate at which the PC *(a)* are undertaken, beginning at period t_1.

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A1.7.3 In which period will the cash flow nadir be reached, assuming conditions $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ are present? (with AC ($\boldsymbol{\mu}$) incurred from the period $\boldsymbol{t_0}$ and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (D2)

Assume there is a period where the business will reach a cashflow nadir ($Acc\ \lambda_{min(n)}$), shown in Equation (103) E2 below. By differencing Equation (79) A2 provides a minimum so that:

$$\frac{\Delta}{\Delta n}Acc(\lambda_n) = \frac{\Delta}{\Delta n}\left(-\lambda\left((n+1)\left(1+\frac{1}{b}\right)-\frac{1}{2a}n(n+1)\right)\right) \tag{91}$$

$$\frac{\Delta}{\Delta n}Acc(\lambda_n) = \frac{\Delta}{\Delta n}\left(-\lambda\left(-\frac{1}{2a}n^2-\left(\frac{1}{2a}-\frac{1}{b}-1\right)n+\left(1+\frac{1}{b}\right)\right)\right) \tag{92}$$

$$0 = -\lambda\left(-\frac{1}{a}n-\left(\frac{1}{2a}-\frac{1}{b}-1\right)\right) \tag{93}$$

$$\frac{1}{a}n = -\frac{1}{2a}+1+\frac{1}{b} \tag{94}$$

$$\breve{n}_{min} = a(1+\frac{1}{b})-\frac{1}{2} \qquad \text{(95) D2}$$

n_{min} is the period where the company requires the maximum funding regardless of the size of the loss at period t_0 or the rate of implementation of the PC (ψ) beginning at period t_1, and where the company is incurring AC ($-\mu$) associated with the turnaround for period t_0. This period is a function of the PC and the AC and is determined regardless of the size of the loss ($-\lambda$).

A1.7.4 What is the size of the funding required for the value of the cash flow nadir ($\boldsymbol{Acc\ \lambda_{min(n)}}$), assuming conditions $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ are present? (with AC ($\boldsymbol{\mu}$) incurred from the period $\boldsymbol{t_0}$ and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (E2)

The cash flow nadir is defined as the point where the negative cash flow reaches the absolute minimum, implying that it is the point where maximum funding is required. By substituting n with n_{min} in Equation(79) A2 enables the calculation of $Acc\ \lambda_{min(n)}$ so that:

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$$Acc\ \lambda_{min(n)} = -\lambda\left((n_{min}+1)\left(1+\frac{1}{b}\right) - \left(\frac{1}{2a}n_{min}(n_{min}+1)\right)\right) \quad (96)$$

$$= -\lambda\left(\left(a\left(1+\frac{1}{b}\right)-\frac{1}{2}+1\right)\left(1+\frac{1}{b}\right) - \left(\frac{1}{2a}(a\left(1+\frac{1}{b}\right)-\frac{1}{2})\left(a\left(1+\frac{1}{b}\right)-\frac{1}{2}+1\right)\right)\right) \quad (97)$$

$$= -\lambda\left(\left(a\left(1+\frac{1}{b}\right)+\frac{1}{2}\right)\left(1+\frac{1}{b}\right) - \left(\frac{1}{2a}\left(a\left(1+\frac{1}{b}\right)-\frac{1}{2}\right)\left(a\left(1+\frac{1}{b}\right)+\frac{1}{2}\right)\right)\right) \quad (98)$$

$$let\ \left(1+\frac{1}{b}\right) = \varpi$$

$$= -\lambda\left(\left(a\varpi+\frac{1}{2}\right)\varpi - \left(\frac{1}{2a}(a\varpi-\frac{1}{2})\left(a\varpi+\frac{1}{2}\right)\right)\right) \quad (99)$$

$$= -\lambda\left(\left(a\varpi^2+\frac{1}{2}\varpi\right) - \left(\frac{1}{2a}\left(a^2\varpi^2-\frac{1}{4}\right)\right)\right) \quad (100)$$

$$= -\lambda\left(a\varpi^2+\frac{1}{2}\varpi-\frac{1}{2}a\varpi^2+\frac{1}{8a}\right) \quad (101)$$

$$= -\lambda\left(\frac{1}{2}a\varpi^2+\frac{1}{2}\varpi+\frac{1}{8a}\right) \quad (102)$$

$$Acc\ \lambda_{min(n)} = -\lambda\left(\frac{1}{2}a\left(1+\frac{1}{b}\right)^2+\frac{1}{2}\left(1+\frac{1}{b}\right)+\frac{1}{8a}\right) \quad (103)\ E2$$

$Acc\ \lambda_{min(n)}$ represents the cash flow nadir implying this is the maximum funding required during the turnaround based on conditions $\alpha\ and\ \beta$ being present. Without funding the nadir and ensuring that it is fully funded and funded on time, a turnaround cannot be implemented and may only lead to further delays.

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A1.8 Creating a $\boldsymbol{VFC}$ model for any period of $\boldsymbol{t_x}$, subject to conditions $\boldsymbol{\alpha, \beta\ and\ \gamma}$ (where AC ($\boldsymbol{\mu}$) incurred and the PC ($\boldsymbol{\psi}$) not implemented from the period $\boldsymbol{t_1}$ as a delay incurred of $\boldsymbol{k}$ periods) (A3)

Assume a function, $f(t_x)$, exists for every period of t, equally spaced in time, to express the loss ($-\lambda$) or profit for each period, which results in the accumulated net cashflow ($Acc\ \lambda_k$). Further, assume that for every period of t, the AC ($-\mu$) is incurred, while the cumulative PC (ψ) is only from the period $t = k$. It is assumed that period t_0 is used for analysis and preparation of the strategy, and no PC occurred during this period. The AC ($-\mu$) occurs in each period as estimated with the commencement of the turnaround. It implies that the firm is incurring the losses ($-\lambda$) and the AC ($-\mu$) for $1 + k$ periods.

Table A1.6: Overview of the various elements of the VFC model what happens when the PC required are delayed (conditions $\alpha, \beta\ and\ \gamma$)

Source: Author (2021)

Description	$Periods$							
	t_0	t_k	t_{k+1}	t_{k+2}	t_{k+3}	t_{k+4}	t_{k+5}	t_{k+6}
Loss per period	$-\lambda_0$	$-\lambda_k$	$-\lambda_{k+1}$	$-\lambda_{k+2}$	$-\lambda_{k+3}$	$-\lambda_{k+4}$	$-\lambda_{k+5}$	$-\lambda_{k+6}$
Abnormal cost	$-\mu_0$	$-\mu_k$	$-\mu_{k+1}$	$-\mu_{k+2}$	$-\mu_{k+3}$	$-\mu_{k+4}$	$-\mu_{k+5}$	$-\mu_{k+6}$
Incremental savings			ψ_{k+1}	ψ_{k+1}	ψ_{k+1}	ψ_{k+1}	ψ_{k+1}	ψ_{k+1}
				ψ_{k+2}	ψ_{k+2}	ψ_{k+2}	ψ_{k+2}	ψ_{k+2}
					ψ_{k+3}	ψ_{k+3}	ψ_{k+3}	ψ_{k+3}
						ψ_{k+4}	ψ_{k+4}	ψ_{k+4}
							ψ_{k+5}	ψ_{k+5}
								ψ_{k+6}
Term (no delay)	1	k	$\breve{n}$			ρ		
Term (delay)	n							

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Let k be the periods by which the plan's implementation is delayed, implying additional losses and AC ($-\mu$) are incurred for k period. Since a delay was incurred with the PC (ψ), Equation (79) A2 can be expanded by n, being a date after period k but before period $\breve{n}$, so $k < n < \breve{n}$.

Assume that a function exists where loss and AC ($-\mu$) are added for each period commencing in the period t_0, while there is a delay in the PC (ψ), which only occurs from the period t_{k+1} to establish an accumulated cash flow for every period, where $Acc(\lambda_k) = 0$, so that:

$$\begin{gathered} Acc\ \lambda_k = f(t_0) + f(t_1) + \cdots + f(t_k) + \cdots + f(t_n) + \cdots + f(x_{\breve{n}}) \\ where\ x = 1,2, \ldots, k, n, \ldots, \breve{n};\ and\ where\ n + \rho = \breve{n} \end{gathered} \tag{104}$$

Since there is a delay of the PC by k periods, it is assumed that the VFC model, as set out in Equation (79) A2, is undertaken from period $k+1$ up to and until period n. However, the capital recovery of the losses incurred for k periods have to be recovered from period $n+1$ up to and until the period $\breve{n}$. The duration of the periods between $n+1$ and $\breve{n}$ is measured in ρ periods so that:

$$\begin{aligned} = (-\lambda_0 - \mu_0) + (-\lambda_1 - \mu_1) + \cdots + (-\lambda_k - \mu_k) \\ + (-\lambda_{k+1} - \mu_{k+1} + \psi_{k+1}) + (-\lambda_{k+2} - \mu_{k+2} \\ + \sum_{x=k+1}^{k+2} \psi_x) + \cdots + (-\lambda_{k+n} - \mu_{k+n} + \sum_{x=k+1}^{n} \psi_x) \\ + \cdots + (-\lambda_{n+1} - \mu_{n+1} + \psi_n) + \cdots + (-\lambda_{n+\rho} \\ - \mu_{n+\rho} + \sum_{x=n+1}^{n+\rho} \psi_x) \end{aligned} \tag{105}$$

where ρ represents some arbitrary length of time, which is unknown, and k represents the periods of delay, which resulted in the delay of the plan. Further assume that:

$$\begin{gathered} -\lambda_0 = -\lambda_1 = -\lambda_k = -\lambda_\rho \\ -\mu = -\mu_1 = -\mu_k = -\mu_\rho \end{gathered} \tag{106}$$

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$$= (-\lambda_0 - \mu)(1 + k) + (-\lambda_{k+1} - \mu_{k+1} + \psi_{k+1}) + (-\lambda_{k+2} - \mu_{k+2} + \sum_{x=k+1}^{k+2} \psi_x) + \cdots + (-\lambda_{k+n} - \mu_{k+n} + \sum_{x=k+1}^{n} \psi_x) + \cdots + (-\lambda_{n+1} - \mu_{n+1} + \psi_n) + \cdots + (-\lambda_{n+\rho} - \mu_{n+\rho} + \sum_{x=n+1}^{n+\rho} \psi_x) \quad (107)$$

Equate Equation 107 to zero so that:

$$(\lambda_0 + \mu)(k) - (-\lambda_{n+1} - \mu_{n+1} + \psi_n) - \cdots - (-\lambda_{n+\rho} - \mu_{n+\rho} + \sum_{x=n+1}^{n+\rho} \psi_x) = (-\lambda_0 - \mu) + (-\lambda_{k+1} - \mu_{k+1} + \psi_{k+1}) + (-\lambda_{k+2} - \mu_{k+2} + \sum_{x=k+1}^{k+2} \psi_x) + \cdots + (-\lambda_{k+n} - \mu_{k+n} + \sum_{x=k+1}^{n} \psi_x) \quad (108)$$

The right side of Equation 108 is equivalent to Equation (79) A2 so that if the PC (ψ) are implemented from the period t_{k+1} *until period* t_n. From Equation (84) B2 it is also clear that this equation implies that this is a measure for the turnaround duration, which implies that the left side is also equal to zero. If the left side of the equation is equal to zero, it implies that the losses and AC ($-\mu$) incurred for the period t_0 *to* t_k is equal to additional savings, which occur in periods t_{n+1} *until period* $t_{n+\rho}$ and are required for the remaining duration of the turnaround to recover the funding provided for the turnaround. Therefore, the left side of the equation can be written as follows:

$$(\lambda_0 + \mu)(k) = (-\lambda_{n+1} - \mu_{n+1} + \psi_n) + \cdots + (-\lambda_{n+\rho} - \mu_{n+\rho} + \sum_{x=n+1}^{n+\rho} \psi_x) \quad (109)$$

$$(\lambda_0 + \mu)(1 + k + n + \rho) = n\rho + n\psi_{k+1} + (n-1)\psi_{k+2} + \cdots + 2\psi_{n-1} + 1\psi_n + \rho\psi_{n+1} + (\rho - 1)\psi_{n+2} + \cdots + 2\psi_{\rho-1} + 1\psi_\rho \quad (110)$$

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Based on Assumption No. 2, it is clear that $\psi_1 = \psi_1 = \psi$

$$= (-\lambda_0 - \mu)(1 + k + n + \rho) + n\rho + \psi(n + (n-1) + \cdots + 2 + 1) + \psi(\rho + (\rho - 1) + \cdots + 2 + 1) \quad (111)$$

$$= (-\lambda_0 - \mu)(1 + k + n + \rho) + \psi\left(\frac{1}{2}\big(n(n+1)\big) + n\rho + \frac{1}{2}\big(\rho(\rho+1)\big)\right) \quad (112)$$

Let $\breve{n} = n + \rho$, and $n \neq \breve{n}$, and where the planned changes (ψ) commence in period $k + 1$ and endure until period n

$$= (-\lambda_0 - \mu)(k + \breve{n} + 1) + \frac{\psi}{2}\big(\breve{n}(\breve{n} + 1)\big) \quad (113)$$

Based on Equation 73 $let\ \psi = \frac{\lambda}{a}, with\ a > 0\ and\ let\ \mu = \frac{\lambda}{b}, with\ b > 0$

$$= (-\lambda_0 - \frac{\lambda}{b})(1 + k + n + \rho) + \frac{\lambda}{a}\left(\frac{1}{2}\big(n(n+1)\big) + n\rho + \frac{1}{2}\big(\rho(\rho+1)\big)\right) \quad (114)$$

$(k + \breve{n} + \rho)$ equates to the entire period and is denoted by t_x, while the sum of $n\ and\ \rho$ would be the period without delay and is thus denoted by t_y. Therefore, the general function when there is a delay is:

$$Acc\ \lambda_{\psi,\mu,n} = -\lambda\left(\left(1 + \frac{1}{b}\right)(t_x + 1)|_{x=0}^{x=n} - \left(\frac{t_y}{2a}(t_y + 1)\right)|_{y=1}^{y=n-k}\right),$$
$$a \neq 0; b \neq 0; k < 0, k \in \mathbb{R}; and \quad (115)\ A3$$
$$x = 0,1,2, \ldots\ n;\ and\ y = 1, 2, 3, \ldots, n - k\ and\ where\ y = t_x - k, t_x > k$$

A1.8.1 Calculating the duration (when the funder will recover the funding used to fund the turnaround), assuming conditions $\boldsymbol{\alpha, \beta\ and\ \gamma}$ are present (with AC ($\boldsymbol{\mu}$) incurred from period $\boldsymbol{t_0}$ and a delay of $\boldsymbol{k}$ periods before the implementation of PC ($\boldsymbol{\psi}$) from period $\boldsymbol{t_{k+1}}$) (B3)

Equate Equation (115) A3 to zero ($Acc(\lambda_k) = 0$) to calculate what the duration of the turnaround will be, which is subject to the conditions $\alpha, \beta\ and\ \gamma$. The duration for the turnaround lasts until the period $\breve{n}$ and is the period where the funders of the business have recovered the capital used during the turnaround.

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$$0 = -\lambda(1+k+n+\rho)\left(1+\frac{1}{b}\right) + \lambda\frac{n(n+1)}{2a} + \lambda\left(\frac{n\rho}{a} + \frac{\rho(\rho+1)}{2a}\right) \tag{116}$$

$$0 = -\lambda\left(\left((1+n)\left(1+\frac{1}{b}\right) - \frac{n(n+1)}{2a}\right) + \left((k+\rho)\left(1+\frac{1}{b}\right) - \frac{n\rho}{a} - \frac{\rho(\rho+1)}{2a}\right)\right) \tag{117}$$

$$0 = (1+k+n)\left(1+\frac{1}{b}\right) - \frac{n(n+1)}{2a} - \frac{n\rho}{a} - \left(\frac{\rho(\rho+1)}{2a}\right) + \rho\left(1+\frac{1}{b}\right) \tag{118}$$

$$0 = (1+k+n)\left(1+\frac{1}{b}\right) - \frac{n(n+1)}{2a} - \frac{1}{2a}\rho^2 - \frac{1}{2a}\rho + \rho\left(1+\frac{1}{b}-\frac{n}{a}\right) \tag{119}$$

$$0 = -\frac{1}{2a}\rho^2 + \left(-\frac{1}{2a} - \frac{n}{a} + 1 + \frac{1}{b}\right)\rho + (n+k+1)\left(1+\frac{1}{b}\right) - \frac{n(n+1)}{2a} \tag{120}$$

$$As\ n = 2a\left(1+\frac{1}{b}\right)$$

$$0 = -\frac{1}{2a}\rho^2 + \left(-\frac{1}{2a} - \frac{2a\left(1+\frac{1}{b}\right)}{a} + \left(1+\frac{1}{b}\right)\right)\rho + \left(2a\left(1+\frac{1}{b}\right)+k+1\right)\left(1+\frac{1}{b}\right) + \frac{2a\left(1+\frac{1}{b}\right)\left(2a\left(1+\frac{1}{b}\right)+1\right)}{2a} \tag{121}$$

$$and\ let\ \left(1+\frac{1}{b}\right) = \varpi \tag{122}$$

$$0 = -\frac{1}{2a}\rho^2 + \left(-\frac{1}{2a} - \frac{2a\varpi}{a} + \varpi\right)\rho + (2a\varpi+k+1)\varpi - \frac{2a\varpi(2a\varpi+1)}{2a} \tag{123}$$

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$$0 = -\frac{1}{2a}\rho^2 + \left(-\frac{1}{2a} - \varpi\right)\rho + (2a\varpi + k + 1)\varpi - \varpi(2a\varpi + 1) \tag{124}$$

$$0 = -\frac{1}{2a}\rho^2 - \left(\frac{1}{2a} + \varpi\right)\rho + k\varpi \tag{125}$$

$$\rho = a\left(-\left(\frac{1}{2a} + \varpi\right) \pm \sqrt{\left(\frac{1}{2a} + \varpi\right)^2 + \frac{2k\varpi}{a}}\right) \tag{126}$$

$$Where\ \varpi = 1 + \frac{1}{b}$$

Assume that the duration of the turnaround will last $\breve{n}$ periods and let $n < \breve{n}$ consisting of k periods, which represents the delay in PC (a), and once the implementation occurs, there will be a duration of n periods, which results in $Acc(\lambda_n) = k\left(1 + \frac{1}{b}\right)$. It implies that the duration of the turnaround will continue for a further ρ periods until $Acc(\lambda_{\breve{n}}) = 0$.

$$n = \rho + n + k$$

$$\rho = a\left(-\left(\frac{1}{2a} + \varpi\right) \pm \sqrt{\left(\frac{1}{2a} + \varpi\right)^2 + \frac{2k\varpi}{a}}\right)\ where\ \varpi = 1 + \frac{1}{b} \tag{127) B3}$$

Equation (127) B3 implies that the duration of the turnaround will last twice as long as the average PC (a) expressed as a proportion of the initial loss plus the delay caused by $1 + k$ plus the recovery period (ρ), to recover the funding that was made available to fund the turnaround.

A test can be performed to determine if this duration calculation is correct by inserting $\rho = 0\ and\ k = 0$ into Equations (127) B3 and (84) B2 to confirm that it would return Equation(79) A2, so that:

$$let\ k = 0\ and\ let\ \rho = 0$$

$$0 = -(0 + n + 0 + 1)\lambda\left(1 + \frac{1}{b}\right) + \lambda\frac{n(n+1)}{2a} + \lambda\left(\frac{n0}{a} + \frac{0(0+1)}{2a}\right) \tag{128}$$

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$$0 = -(n+1)\lambda\left(1+\frac{1}{b}\right)+\lambda\frac{n(n+1)}{2a} \tag{129}$$

$$0 = -\lambda\left((n+1)\left(1+\frac{1}{b}\right)-\frac{n}{2a}(n+1)\right) \tag{130}$$

$$which\ is\ equal\ to\ (79)\ \mathrm{A2}$$

Test if the duration being $n = \rho + n + k$ inserted into Equation (115) A3 returns Equation (127) B3

$$Acc\ \lambda_{\psi,\mu,n} = -\lambda\left(\left(1+\frac{1}{b}\right)(t_x+1)|_{x=0}^{x=n}-\left(\frac{t_y}{2a}(t_y+1)\right)|_{y=1}^{y=n-k}\right) \tag{131}$$

$$a \neq 0; b \neq 0;\ x = 0,1,2,\ldots\ and\ y = 1,2,3,\ldots$$

$$0 = -\lambda\left(\left(1+\frac{1}{b}\right)(n-0+1)-\left(\frac{n-(k+1)}{2a}(n-(k+1)+1)\right)\right) \tag{132}$$

$$0 = -\lambda\left(\left(1+\frac{1}{b}\right)(n+1)-\left(\frac{n-k-1}{2a}(n-k)\right)\right) \tag{133}$$

$$0 = -\lambda\left(\left(1+\frac{1}{b}\right)(\rho+n+k+1) - \left(\frac{\rho+n+k-k-1}{2a}(\rho+n+k-k)\right)\right) \tag{134}$$

$$0 = -\lambda\left(\left(1+\frac{1}{b}\right)(\rho+n+k+1)-\left(\frac{\rho+n}{2a}(\rho+n+1)\right)\right) \tag{135}$$

$$0 = -\lambda\left(\left(1+\frac{1}{b}\right)(\rho+n+k+1)-\left(\frac{\rho+n}{2a}(\rho+n+1)\right)\right) \tag{136}$$

$$0 = -\lambda\left(\left(1+\frac{1}{b}\right)(\rho+n+k+1)-\frac{\rho^2+2n\rho+n^2+\rho+n}{2a}\right) \tag{137}$$

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$$0 = -\lambda\left(\left(1+\frac{1}{b}\right)(\rho+n+k+1) - \frac{n\rho}{a} - \frac{\rho^2+\rho}{2a} - \frac{n^2+n}{2a}\right) \tag{138}$$

$$0 = -(\rho+n+k+1)\lambda\left(1+\frac{1}{b}\right) + \lambda\frac{n(n+1)}{2a} + \lambda\left(\frac{n\rho}{a} + \frac{\rho(\rho+1)}{2a}\right) \tag{139}$$

A1.8.2 In which period will the business reach breakeven, assuming conditions $\boldsymbol{\alpha, \beta\ and\ \gamma}$ are present? (no AC ($\boldsymbol{\mu}$) and the PC ($\boldsymbol{\psi}$) implemented from the period $\boldsymbol{t_1}$) (C3)

Assume there is a period $f(t_E)\ or\ E_{(k)}$ where the business will reach breakeven, where the loss ($-\lambda_0$) plus the AC ($-\mu$) is equal to the PC ($\sum_{x=1}^{E+k}\psi_x$) for that period using Equation (71) and equating this zero to denote the breakeven. However, this breakeven period would have shifted by k periods as the PC occur from period $k+1$ so that:

$$f(t_y) = (-\lambda_0 - \mu) + \sum_{x=1}^{y}\psi_x \tag{140}$$

However, the breakeven has to shift by k periods and added to the breakeven calculation so that:

$$E_{(k)} = a\left(1+\frac{1}{b}\right) + k \tag{141 C3}$$

$E_{(k)}$ represents the period when the firm stops absorbing and returns cash flow to repay stakeholders who have funded the turnaround, further delayed by k periods. It is regardless of the size of the loss at period t_0 or the rate at which the PC (a) are undertaken, beginning at period t_1.

A1.8.3 In which period will the cash flow nadir be reached, assuming conditions $\boldsymbol{\alpha, \beta\ and\ \gamma}$ are present? (with AC ($\boldsymbol{\mu}$) incurred from the period $\boldsymbol{t_0}$ and a delay of $\boldsymbol{k}$ periods before the PC ($\boldsymbol{\psi}$) occurs from the period $\boldsymbol{t_{k+1}}$) (C3)

Evaluating Table A1.6, it becomes clear that losses ($-\lambda$) plus AC ($-\mu$) continue to be incurred from the period $t_0\ until\ t_k$, until the PC (ψ) are implemented in the period t_{k+1}. Based on

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this observation, it is clear that the period between t_0 $until$ $\check{n}_{min(\psi,\mu)}$, as set out in Equation (95) D2, can be used to determine the value of $n_{min(k)}$, where :

$$n_{min(k)} = \check{n}_{min(n)} + k \tag{142}$$

$$n_{min(k)} = a\left(1+\frac{1}{b}\right) - \frac{1}{2} + k \tag{143}$$

D3

A1.8.4 What is the size of the funding required for the cashflow nadir ($\boldsymbol{Acc\ \lambda_{min(k)}}$), assuming conditions $\boldsymbol{\alpha, \beta\ and\ \gamma}$ are present? (with AC ($\boldsymbol{\mu}$) incurred from the period $\boldsymbol{t_0}$ and a delay of $\boldsymbol{k}$ periods before the implementation of the PC ($\boldsymbol{\psi}$) from the period $\boldsymbol{t_{k+1}}$) (E3)

The cash flow nadir is defined as the point where the negative cash flow reaches the absolute minimum, implying that it is the point where maximum funding is required. By substituting n with n_{min} in Equation (115) A3 enables the calculation of $Acc\ \lambda_{min(k)}$, so that:

$$Acc\ \lambda_{(k)} \ = \ Acc\ \lambda_{(x)}|_0^k + Acc\ \lambda_{(x)}|_{k+1}^{\check{n}} \tag{144}$$

$$Acc\ \lambda_{(k)} = -\lambda\left(\left(\check{n}_{min(k)} + k + 1\right)\left(1+\frac{1}{b}\right) - \left(\frac{1}{2a}\check{n}_{min(k)}\left(\check{n}_{min(k)} + 1\right)\right)\right) \tag{145}$$

$$\begin{aligned} = -\lambda\Bigg(&\left(a\left(1+\frac{1}{b}\right) - \frac{1}{2} + k + 1\right)\left(1+\frac{1}{b}\right) \\ &- \left(\frac{1}{2a}\left(a\left(1+\frac{1}{b}\right) - \frac{1}{2}\right)\left(a\left(1+\frac{1}{b}\right) - \frac{1}{2} + 1\right)\right)\Bigg) \end{aligned} \tag{146}$$

$$\begin{aligned} = -\lambda\Bigg(&\left(a\left(1+\frac{1}{b}\right) + \frac{1}{2} + k\right)\left(1+\frac{1}{b}\right) \\ &- \left(\frac{1}{2a}\left(a\left(1+\frac{1}{b}\right) - \frac{1}{2}\right)\left(a\left(1+\frac{1}{b}\right) + \frac{1}{2}\right)\right)\Bigg) \end{aligned} \tag{147}$$

$$let\ \left(1+\frac{1}{b}\right) = \varpi$$

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$$= -\lambda\left(\left(a\varpi + \frac{1}{2} + k\right)\varpi - \left(\frac{1}{2a}(a\varpi - \frac{1}{2})\left(a\varpi + \frac{1}{2}\right)\right)\right) \tag{148}$$

$$= -\lambda\left(\left(a\varpi^2 + \frac{1}{2}\varpi + k\varpi\right) - \left(\frac{1}{2a}\left(a^2\varpi^2 - \frac{1}{4}\right)\right)\right) \tag{149}$$

$$= -\lambda\left(a\varpi^2 + \frac{1}{2}\varpi + k\varpi - \frac{1}{2}a\varpi^2 + \frac{1}{8a}\right) \tag{150}$$

$$= -\lambda\left(\frac{1}{2}a\varpi^2 + k\varpi + \frac{1}{2}\varpi + \frac{1}{8a}\right) \tag{151}$$

$$= -\lambda\left(\frac{1}{2}a(1 + \frac{1}{b})^2 + k\left(1 + \frac{1}{b}\right) + \frac{1}{2}\left(1 + \frac{1}{b}\right) + \frac{1}{8a}\right) \tag{152}$$

$$Acc\ \lambda_{min(\psi,\mu,n)} = -\lambda\left(\left(\frac{1}{2}a\left(1 + \frac{1}{b}\right)^2 + k\left(1 + \frac{1}{b}\right) + \frac{1}{2}\left(1 + \frac{1}{b}\right) + \frac{1}{8a}\right)\right) \tag{153}$$

E3

$Acc\ \lambda_{min(k)}$ represents the cash flow nadir, implying this is the maximum funding required during the turnaround based on conditions $\alpha, \beta\ and\ \gamma$ being present. Without funding the nadir and ensuring that it is fully funded and funded on time, a turnaround cannot be implemented and may only lead to further delays.

A1.9 Conclusion

This annexure set out to explain the VFC model, a proposed measurement tool to replace the role of the TCF calculations. The VFC model was created using certain constraints and assumptions as a part of mathematising the solutions. Following systematic and logical processes using rigorous mathematical proofs, the model was expanded to include more complexities usually associated with turnarounds.

A set of algebraic equations being (4) A1, (79) A2, and (115) A3, were derived, thereby establishing the VFC model based on the various assumptions as set out in the beginning of

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this annexure. From the VFC model, Equations (56) B1, (84) B2, and (127) B3 were created to determine the turnaround duration; Equations (59) C1, (90) C2, and (141) C3 determine when the firm will breakeven; (64) D1, (95) D2, and (143) D3 determine in which period the nadir will occur; while (69) E1, (103) E2, and (153) E3 determine the size of the nadir value during the turnaround. These elements can be combined into a set of fundamental moments that can describe the turnaround, showing the input and outcome in a single line.

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Annexure 2: Testing the fit for appropriateness, consistency, and accuracy between the VFC model against that of the TCF calculations

Abstract

To verify the accuracy of the VFC model, simulations using the same input values for the VFC model and TCF calculations were generated and deducted from each other for each specific period.

These tests were performed to verify the appropriateness, consistency, and accuracy of the VFC model to that of the TCF calculations.

A master test set has been created to evaluate the differences between the results of the two simulated values. This annexure shows the considerations taken into account in the test set design. It shows how the formulas are created in Microsoft Excel® to evaluate the differences between the VFC model and TCF calculations. Various tests were performed using different input values of $(a, b, k, and - \lambda)$ and show how the results of these tests were summarised into a summary sheet. The summary sheet was further analysed for the size, distribution, and quantum errors between the VFC model and TCF calculations.

It concludes with the results and findings of the tests and a recommendation regarding the level of assurance with which the VFC model can be used to replace the TCF calculations.

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A2.1 Introduction and background

The equations for the VFC model were algebraically derived as set out in Annexure 1. However, the output values of the VFC model must be compared to the output values generated by the TCF calculations. This annexure investigates if the VFC model is an appropriate, consistent, and accurate fit to that of the values generated from the TCF calculations.

The output values generated are deducted to establish a difference between them, and then various tests must be performed to determine the level of the fit. A sufficient number of simulations were generated for various input values to create a sample set that allows for accepting or rejecting the model's fit.

This annexure sets out the criteria of an acceptable and sufficient test to ensure that the various models map each other accurately and correctly, using the same input values. The appropriateness, consistency, and accuracy level must be pre-defined to determine what error rate is acceptable and to measure the differences generated from the test sets. It also describes how a master test set was designed to compare the output values for the various periods for the models. Test sets were then duplicated, and different inputs were used to ensure an acceptable sample population existed.

It describes how the tests were performed, and the results were summarised to compare the values generated between the TCF calculations and VFC model. These results were evaluated using the established parameters to show any variations or anomalies between the models. The prevalence of these variations or anomalies influences the assurance level with which the model can be considered a good fit or not for the TCF calculations. It concludes with assurance statements on the accuracy of the fit of the VCF model that is based on a series of hypothesis tests, which were performed on the test results that were generated.

The purpose of this annexure is to prove that the VCF model is an appropriate, consistent, and accurate alternative to the TCF calculations to determine the fundamental moments during a turnaround. Upon reaching such a conclusion, the universal utility of the VFC model can be used to establish the nomenclature of the process of a turnaround.

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A2.2 Setting the parameters to evaluate the fit of the VFC model to that of the $\mathbf{TCF}$ calculations

The only way to verify the appropriateness, consistency, and accuracy of the VFC model is to:

1. establish a set of parameters against which the fit of the VFC model to that of the TCF calculations will be measured;
2. create a master test set that translates the equations of the VFC model derived into Microsoft Excel® formulas;
3. generate various simulations using the master test set by using different combinations of input values; and
4. perform the analysis on the differences between output values of the VFC model and TCF calculations.

Authoritative simulations should be performed to ensure that the conclusions are sufficient, utilising various input values and determining if the compared models return the same values. The level of consistency, appropriateness, and accuracy can be utilised in research or business applications.

A2.2.1 The computer and software used to perform the test sets and the accuracy level required for tests

All tests in this annexure were performed during 2021. These tests were performed on a laptop computer with the following specifications:

Device specifications

Machine	2021 Surface Book 3
Processor	Intel® Core™ i7 – 1065G7 CPU@ 1.30 GHz 1.50 GHz
Installed RAM	32,0 GB (31,6 usable)
System type	64-Bit operating system, x64-based processor

Window specifications

Edition	Windows® 10 Pro
Version	1909

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Excel specifications

Excel version	Microsoft® Excel® 2019
Version	(16.013127.2066) 32-bit

Date tests were created

Period	1 January 2021 to 6 November 2021

A2.2.2 Setting the parameters against which the test sets are measured to determine if the VFC model is an appropriate, consistent, and accurate fit to that of the TCF calculations

This section sets out the parameters against which the VFC model is measured to determine if it is an appropriate, consistent, and accurate fit to that of the TCF calculations. It sets out the rationale for the parameters used and how these output values are compared by applying truncated evaluation tests that inform the decision to determine the level of the fit between the VFC model and TCF calculations.

A2.2.2.1 Background to the requirements to measure the fit of the VFC model to that of the TCF calculations

The master test set is developed to determine if the VFC model and TCF calculations are an appropriate, consistent, and accurate fit. It is based on creating a set of formulas in Microsoft Excel® consisting of 32 periods and then evaluating their difference. These test results enable the analysis of the size and distribution of the differences thereof. The truncated evaluation parameters inform the fit of the VFC model to that of the TCF calculations.

A2.2.2.2 Measurement for the size and distribution of errors between the two models

To verify if the VFC model is an appropriate, consistent, and accurate fit to that of the TCF calculations, a rigorous test had to be designed to evaluate the differences between the VFC model and TCF calculations. The test requires the simulation of the output of the VFC model and TCF calculations using the same input values. After calculating the output values for both, they are deducted from one another to compile the differences or error rates. Based on the error size, the difference between these values informs whether the VFC model is a fit that is consistent, appropriate, and accurate. A sufficient number of simulations had to be generated

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to create a representative sample for the various input values. A series of truncated tests were performed to accept or reject the model's fit. Table A2.1 summarises the designs of the tests performed, where:

1. if test No. 1 fails, no further testing occurs;
2. if test No. 2 fails, no further testing occurs;
3. and if test No. 3 shows that there is not a sufficient level of accuracy with the overall fit; the test also fails.

Assuming that tests No. 1 and 2 had passed, test No. 3 will use the level of accuracy as an indication of suitability. Therefore, the suitability indicator determines if the model could be used in a research or business application, or both.

Evaluation test No. 1 – Is the fit between the models appropriate? - This evaluation test checks the appropriateness of the VFC model. If any test of the differences returns an integer value larger than zero, the VFC model cannot be accepted as appropriate and must be rejected.

Evaluation test No. 2 – Is the fit between the models consistent? - The second evaluation test checks for consistency or the lack thereof. This test investigates the spread or distribution of errors and anomalies. If errors and anomalies occur randomly for different places (bands) after the decimal, it implies that the model cannot be considered consistent, and the VFC model should be rejected. In other words, if an error or anomaly occurs in the second place and the next value occurs in the tenth place after the decimal, the model cannot be deemed to be consistent. However, if the first error or anomaly occurs in the eleventh place and the value adjacent thereto returns an error in the tenth place after the decimal, this indicates there may be a cumulative process present. Due to the cumulative nature of the VFC model and the TCF calculation, where the value of the previous period is added to that of the current period, it implies that there can be a cumulative increase in error values in the calculations. If the errors appear randomly, the model is not consistent. So, suppose adjacent differences show small increases or increments as the adjacent values follow each other; in that case, the VFC model can be accepted as consistent, and the next set of truncated tests can be performed.

Evaluation test No. 3 – Is the fit accurate, and if so, to what level? – To determine the level of accuracy of the model, all the anomalies or errors must occur in a certain band or place after the decimal. In other words, if the errors occur in the twelfth place after the decimal and

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possibly in the thirteenth, fourteenth, and fifteenth place thereafter, it can be accepted that the model is accurate to the twelfth decimal. Once the band or place after the decimal where the error has occurred has been determined, it becomes possible to determine the level of accuracy in adopting the model. The level of accuracy is determined in Evaluations 3.1 to 3.4.

Evaluation test No. 3.1 – The model is deemed not to be an accurate fit – If any single test returns an anomaly or error that occurs between zero and three places (bands) after the decimal, the model will be deemed to be not an appropriate and accurate fit. Table A2.1 shows that if the model is accurate to three places after the decimal, the model can only be accurate to one thousand Rand. The VFC model would be deemed unsuitable for research purposes and cannot be utilised as an alternative in evaluating cash flows and determining the variable finance requirements.

Evaluation No. 3.2 – The model is deemed a rough fit – If an error or anomaly occurs between four and six places (bands) after the decimal, the model could be considered a rough fit. Table A2.1 shows that six places after the decimal, the model can only be accurate to one million Rand. The VFC model may be used in research but cannot be relied on as an alternative to determine the variable finance requirements and the moments during a turnaround.

Evaluation No. 3.3 – The model is deemed a good fit – If an error or anomaly occurs between seven and nine places after the decimal, the model will be considered a good fit. Table A2.1 shows that the model can only be accurate to one billion Rand at nine places after the decimal. The VFC model may be used in research and may be used as an alternative to determining the variable finance requirements, as long as the calculation is less than one billion Rand.

Evaluation No. 3.4 – The model is deemed a perfect fit – If an error or anomaly occurs between ten and twelve places after the decimal, the model will be considered a perfect fit. Table A2.1 shows that the model is accurate to one trillion Rand at twelve places after the decimal. It follows that the VFC model may be used for research and can be utilised as an alternative in evaluating the variable finance requirements.

A further reason for declaring the VFC model as a perfect fit at 12 places after the decimal is informed by the limitation in the capability of the Intel® processor to return error-free values above 12 places after the decimal. Complex calculations in a spreadsheet are impacted by how the Intel® processor calculates the floating-point.

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Table A2.1: Summary of the outcomes for the evaluation tests performed to determine appropriateness, consistency, and accuracy of fit of the VFC model to that of TCF calculations

Source: Author (2021)

Summary of the evaluation tests to be performed for appropriateness, consistency and accuracy

Evaluation test 1 (*Testing for appropriateness*)

Check if any difference has an integer value	If a single integer greater than zero occurs in single difference regardless of the type of test performed, excluding when a singularity occurs, the entire model is rejected and deemed as not being appropriate.	Reject the ***VFC*** model and do not perform further tests

Evaluation test 2 (Testing for consistency)

Check if the values vary across various places after the decimal	Check that the spread of the difference errors and anomalies are sequential and clustered together, value errors exceeding ***1E-12*** is ignored for the purpose of the analysis, but that it excludes singularities which will cause a a jump in value.	Reject the ***VFC*** model and do not perform further tests

Evaluation tests 3.1 to 3.4 (Testing for accuracy)

Places after the decimal	Expected level of error or anomaly of the difference as an indication of the accuracy per the indicated value	Fit indicator used to express level of confidence	
1 places	Accurate to - 0,9 to 0,9 per 10	No fit	
2 places	Accurate to - 0,9 to 0,9 per 100	No fit	
3 places	Accurate to - 0,9 to 0,9 per 1 000	No fit	
4 places	Accurate to - 0,9 to 0,9 per 10 000	Rough fit	
5 places	Accurate to - 0,9 to 0,9 per 100 000	Rough fit	
6 places	Accurate to - 0,9 to 0,9 per 1 000 000	Rough fit	
7 places	Accurate to - 0,9 to 0,9 per 10 000 000	Good fit	
8 places	Accurate to - 0,9 to 0,9 per 100 000 000	Good fit	
9 places	Accurate to - 0,9 to 0,9 per 1 000 000 000	Good fit	
10 places	Accurate to - 0,9 to 0,9 per 10 000 000 000	Prefect fit	
11 places	Accurate to - 0,9 to 0,9 per 100 000 000 000	Prefect fit	
12 places	Accurate to - 0,9 to 0,9 per 1 000 000 000 000	Perfect fit	

Suitability indicator to utilise the model

Places after the decimal	1	2	3	4	5	6	7	8	9	10	11	12
Fit indicator	No fit			Rough fit			Good fit			Perfect fit		
Suitability for proposed use	Not suitable for research or business application				Can be used in research but not in business application				Can be used in research and in business application			

A2.2.2.3 The methodology used to establish and perform the tests

The master test set was designed using an Excel® spreadsheet to calculate values for each consecutive period of the accumulated cash flow, using the TCF calculations and the VFC model. These values were generated utilising the same input values of $[-\lambda, \mu, \psi, k]$ so that a comparison of these values for every consecutive period could be evaluated. The evaluation was achieved by deducting the value generated for the VFC model from those generated using

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the TCF calculations. A difference between the two values is deemed a deviation or an indication that the VFC model may not fit the TCF calculations. Upon detecting a variation of the values between the two models, a determination must be made on how accurately the VFC model is an algebraic and numeric fit to that of the TCF calculations.

The master test set allows for the variation of inputs $[-\lambda, \mu, \psi, k]$ that can be varied singularly or collectively. Allowing for the duplication, variation, and verification thereof for different values, 500 test sets consisting of 1 559 calculations and 1 708 cells per data for every test set were created —varying one input value at a time, resulting in 16 000 individual tests with 779 500 calculations. Each test set evaluated the TCF calculations and the VFC model for 32 periods. These values were deducted from each other and recorded in a separate tab in the spreadsheet classified as a summary comparison tab (SCT). This tab collates all the differences from the tests performed into a single tab.

A truncated test was performed to determine if any differences did not return a non-zero value as contemplated in Evaluation No. 1. The values were evaluated for the most significant error to determine if any level of accuracy exists across the 500 tests performed.

A2.2.2.4 Findings of the truncated tests performed

The following findings were established from the truncated tests performed for appropriateness, consistency, and accuracy of the VFC model to that of the TCF calculations.

Test for appropriateness: No error was found in Evaluation test No. 1, as no integer greater than zero occurred in the 16 000 iterations. Therefore, the VFC model is deemed an appropriate fit against the values generated by the TCF calculations. The testing can, therefore, progress to testing for consistency and accuracy.

Test for consistency: The truncated tests imply that the most significant error occurs 12 places after the decimal. A map was created by sorting the tests according to the output values in a manner where b is displayed first so that $[b, a, -\lambda_0, k]$. Since Evaluation test No. 2 is the test for consistency, it requires that the errors be clustered together and not distributed in different places (bands) after the decimal. The error map shows that the first errors or anomalies occur in the fourteenth and thirteenth place only to progress to the twelfth place after the decimal. The values increase in small increments in those tests with input values of $\psi = 1 \; or \; a = 10$, and where $\mu = 1, 2, \ldots, 10$. The change in decimal place only occurs in

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period 25 for tests 21, 31, 41, 51, 61, 71, 81, and 91 and increases in small increments up to period 32. The changeover in the places (bands) after the decimal is linked to the calculations performed on a computer with an Intel® processor with a limited floating-point of 12 places after the decimal. Thomas (personal communication, 9 October 2021) confirmed that the cumulative nature of the models, where the value of the previous period is added to that of the current period, implies that there can be a cumulative increase in error values. Based on the 16 000 tests performed, the VFC model is deemed to be a consistent fit with the TCF calculations.

Test for accuracy: Of all the 16 000 individual tests performed, less than two per cent of the errors occurred in twelfth place after the decimal, with the rest of the errors ranging from the thirteenth place after the decimal, to fourteenth place to zero errors. Based on the 16 000 tests performed, the VFC model is, therefore, deemed a perfect fit for accuracy with the TCF calculations. The VFC model is, therefore, deemed to be appropriate, consistent, and a perfect fit when measured against the TCF calculations.

After performing the different tests using the different input combinations, the differences were analysed, as set out in the SCT. The output values for each period are captured, creating a representative population sample size that will return anomalies or errors between the VFC model and the TCF calculations. All the calculated differences will be measured to 12 places after the decimal point, thereby indicating the size and distribution of the anomalies or errors. A set of truncated tests has been designed to verify the appropriateness, consistency, and accuracy of the VFC model and the TCF calculations. The results of the five spreadsheets are combined to determine the level of fit between the VFC model and TCF calculations.

A2.2.3 The design of the master test set that will be used to generate the simulations to verify the fit of the VFC model to that of the TCF calculations

A master test set was designed to deliver values derived from the TCF calculation and the output values of the VFC model for consecutive periods. The test sets use the same input values to determine the output values for every period. The values for each period are deducted from one another to detect any deviation in the values generated between the two

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values. Upon detecting a deviation between the two models, a determination must be made on how accurately the VFC model is an algebraic and numeric fit to the TCF calculations.

The master test set must perform all the calculations based on the inputs that can be varied in specific cells in the tab and are referred to as an input table. Creating a master test set allows for duplication, and only the input values are changed, thereby ensuring that all the subsequent calculations are the same. The input values that can be changed are the initial loss $(-\lambda)$, PC (ψ) or AC (-μ), and the delay (k) in PC.

The test set must, therefore, calculate the value of the VFC model for every period compared to the TCF calculations for the same periods to determine the variation or anomalies.

A2.2.4 Design of the master test set that will be used to conduct the various tests for different input values

The master test set consists of two tabs created in a spreadsheet, and that is designed for various parts. The test set consists of the input parameters, the calculation of the VFC values, the traditional cash-flow projection, an auxiliary test to verify the model, and a Momentum calculation. The first page of the first tab provides the input parameters from which all other values are calculated. It shows the value of the loss $(-\lambda)$, incremental savings (ψ), AC (-μ), and if there have been any delays (k) in the implementation of the incremental savings.

The remaining part of the first page on the first tab will show the auxiliary test to verify the VFC model as described in Equation (81) A3, as set out in $I11\ to\ 145$ for every period ranging from periods 0 to 32. The second tab consists of a few pages, deemed a traditional cash flow spreadsheet. It follows the discussion set out in Chapter 3 and calculates the cash flow periodically to compare each period against the values of the VFC model. The headings at the top of each tab indicate the value of the loss $(-\lambda)$, AC (μ), incremental savings (ψ), and if there are any expected delays in the implementation (k) for every period. The name of the first tab is $VFC - 10, 1, -1, 0,$ indicating that it deals with input parameters with a loss of -10; planned changes of 1; AC of -1 and the delay in PC of 0 for the calculation of the VFC, and auxiliary model and TCF calculations for each period $t = 0\ to\ t = 32$ for the indicated values of $t_x; a\ and\ b$. The name of the second tab is $TCF\ 10, 1, -1, 0,$ indicating it deals with input values: a loss of -10; PC of 1; AC of -1; and the delay in PC of 0 for the TCF calculations for each period $t = 0\ to\ t = 32$. These values are compared with the values of the VFC model

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generated in the first tab. A test set is limited to 32 periods as 95 per cent of the tests performed fall in these periods, using integer values as inputs being $[\psi, -\mu]$ between $1\ and\ 10$ that assume that no delays are required to complete the turnaround within the 32 periods. The master test set contains 1 559 calculations to calculate the tests for a single set of inputs, which then test and verify a single test set.

A2.2.5 Utilising the master test set to verify the different values for the various input values

The master test set is duplicated, with the input values of $-\lambda, a, b\ and\ k$ indicated in the tab for this specific combination of input values. This process is repeated for the value of $a = (1, 2, \dots, 10)\ for\ a\ given\ value\ of\ b\ and\ k$ and then the entire process is repeated by changing the value of $b = (1, 2, \dots, 10)\ for\ a\ given\ value\ of\ a\ and\ k$. The first spreadsheet is then duplicated with variations in the input values of the $[a, b]$ as described above, but the values of k are now changed to $k = 1$. The process is repeated for values of $a, b\ and\ k = 1, 2, \dots, 5$.

Therefore, the master test set will be duplicated 99 times in a single spreadsheet, resulting in 100 test sets in 200 tabs, 100 test sets consisting of 1 559 calculations, and 1 708 cells with data contained in a single spreadsheet. The spreadsheet is copied another four times to measure the impact of the delays in implementation k of the incremental savings.

A further copy of the spreadsheet deals with input values that will arbitrarily vary the losses $(-\lambda)$ and delay in savings (k) to verify that the models address all values. These calculations serve as an adequate sample size to check for any variances between the VFC model and the TCF calculations. The auxiliary method and additional process ensure that the values are accurate.

The 500 test sets in the five spreadsheets are summarised, showing the various values generated for each given set of inputs. It is assumed that any other outcome values for a given set of input values of $a,\ b\ and\ k$ will fit the TCF calculations at the same level of confidence as determined in simulations performed by employing mathematical induction.

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A2.3 A detailed description of the test sets to determine the accuracy of the fit between the VFC model and the TCF calculations

The first part of this section describes the detailed design of the first tab of the test set that deals with the input table that captures the input values used to calculate the VFC model, auxiliary model, and TCF calculations. The section shows the formulas utilised in Excel® to calculate the VFC model's equations and the fundamental moments, as shown in Annexure 1. Figure A2.1 shows a facsimile of the various tests created in the first tab of the master test set. The facsimile layout is identical to the spreadsheet, indicating what Excel® formulas are used to calculate the fundamental moments and the VFC model. Figure A2.2 shows the legend of the VFC inputs and the fundamental moments. It compares the equations developed in Annexure 1 to the Excel® formulas.

The second part of this section describes the second tab of the master test set that sets out the TCF calculations, followed by a legend showing the Excel® formulas used, including the conditional tests that read off the input table as contained in the first tab of the test set. The values for each period are displayed to make for easy comparison. These values of the VFC model and the TCF calculations are incorporated into the SCT, discussed later in this annexure.

A2.3.1 Design and creation of the input table and the calculation of the VFC values for periods $0\ to\ 32$

Figure A2.1 shows the layout design and constructs of the input table as set out in $C4\ to\ H7$,and from which the output values are derived for the VFC model, the auxiliary projection, and the TCF calculations.

The VFC inputs show the different inputs used in the VFC model, auxiliary test, and the TCF calculations. The output values for the VFC model for each period are displayed in $H13\ to\ H45$, using the index displayed in $I13\ to\ I45$, by where the index changes as the value of (k) change. The VFC model is specifically displayed next to the values generated from the auxiliary tests for ease of comparison.

It shows the columns and rows applicable to the tab, enabling the reader to verify if the Excel® formulas are positionally and relatively correct. The headings to all the tabs in the master test set contain the relative input values: the loss ($-\lambda_o$=-10); incremental savings ($\psi = 2$), AC ($\mu = -5$), and delay in implementation ($k = 0$). These values are concatenated into the heading,

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which reads from $\$C\$5\ (-\lambda_0);\ \$D\$5(\psi);\ \$E\$5(\mu)\ and\ \$C\$7(k)$, enabling the identification of these input values.

The black number discs and squares correspond with the legend as set out in Figure A2.2 and A2.3; compare the Excel® formulas to the equations from Annexure 1. It also contains a description of the construct of the Excel® formulas used in the input table, enabling verification that the Excel formula is a true reflection of the appropriate equations in Annexure 1.

A2.3.2 The design and construction of the $\mathbf{TCF}$ calculations used in the master test set

Figure A2.12 shows the design used to create simulated TCF calculations. It shows the different income items and various expense items, which are separately totalled and deducted from one another to provide the cash flow for the period t_x. In the examples, the amounts shown for income were averaged, and so were the amounts averaged for the expenses. It is not essential if the PC is achieved by increasing the revenue without increasing the expenses or using a single item or savings of many small items. A turnaround is about making the differential between the income and the expenses as significantly and quickly as possible.

An equation was created to handle any changes in lambda

Income: $9 * \lambda/-10$

Expenses: $5 * \lambda/-10$

These equations have used the ratio of the income and expenses of the TCF calculations according to the new lambda value with a baseline of $\lambda = -10$. The second tab dealing with the TCF calculations shows that the PC can be achieved with savings in 20 expense items.

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Tests for the VFC equations where $-\lambda = -10, -\mu = -5, \psi = 2, k = 0$

Excel columns

A	B	C	D	E	F	G	H	I
3	**The VFC inputs**		=D7*(1+1/E7)+C7 (8)		=$D$7*(1+1/$E$7)-(1/2)+$C$7 (1)			**The fundamental moments**
4		$-\lambda_0$	ψ	$-\mu$	n_{min}	n	$\breve{n}$ (2)	=2*$D$7*(1+1/$E$7)
5	A	-10	B 2	C -5	7,0	15	0	
6	D	k	a	b	E_k	ρ	$Acc\lambda_{min(k)}$ (3)	=IF($F$7=0;0;$G$5+$C$7+$G$7)
7		0	5	2	7,5	0	62,5	
8			=-C5/D5 (4)	=C5/E5 (5)	=$D$7*(-(1/(2*$D$7)+(1+1/$E$7))+((1/(2*$D$7)+(1+(1/$E$7)))^2+((2*$F$7*(1+1/$E$7))/$D$7))^(1/2)) (6)		=-C5*(1/2*D7*(1+1/E7)^2+(1/2)*(1+1/D7)+1/(8*D7)+F7*(1+1/D7)) (7)	
9	**The VFC verification**							
10		**Auxillary test to verify the model**					**The VFC model**	
11		t_x	$-\lambda_0$	$-\mu$	ψ	$f(t_x)$	$Acc(\lambda_k)$	$Acc(\lambda_k)$	t_k
12		Excel rows						
13		0	-10,0	-5,0				-15,0	0
14		1	-10,0	-5,0				-28,0	1
15		2	-10,0	-5,0				-39,0	2
16		3	-10,0	-5,0	6,0	-9,0	-48,0		3
17		4	-10,0	-5,0	8,0	-7,0	-55,0		
18		=1+B19	=$C$5	=$E$5	10,0	=C20+E20+D20	-60,0		
19		6	-10,0	-5,0	12,0	-3,0	-63,0		
20		7	-10,0	-5,0	14,0	-1,0	-64,0	-64,0	7
21		8	-10,0	-5,0	16,0	1,0	-63,0	-63,0	8
22		9	-10,0	-5,0		3,0		-60,0	9
23		10	-10,0	-5,0		5,0	-55,0	-55,0	10
24		11	-10,0	-5,0		7,0	-48,0	-48,0	11
25		12	-10,0	-5,0	24,0	9,0	-39,0	-39,0	12
26		13	-10,0	-5,0	26,0	11,0	-28,0	-28,0	13
27		14	-10,0	-5,0	28,0	13,0	-15,0	-15,0	14
28		15	-10,0	-5,0	30,0	15,0	0,0	0,0	15
29		16	-10,0	-5,0	32,0	17,0	17,0	17,0	16
30		17	-10,0	-5,0	34,0	19,0	36,0	36,0	17
31		18	-10,0	-5,0	36,0	21,0	57,0	57,0	18
32		19	-10,0	-5,0	38,0	23,0	80,0	80,0	19
33		20	-10,0	-5,0	40,0	25,0	105,0	105,0	20
34		21	-10,0	-5,0	42,0	27,0	132,0	132,0	21
35		22	-10,0	-5,0	44,0	29,0	161,0	161,0	22
36		23	-10,0	-5,0	46,0	31,0	192,0	192,0	23
37		24	-10,0	-5,0	48,0	33,0	225,0	225,0	24
38		25	-10,0	-5,0	50,0	35,0	260,0	260,0	25
39		26	-10,0	-5,0	52,0	37,0	297,0	297,0	26
40		27	-10,0	-5,0	54,0	39,0	336,0	336,0	27
41		28	-10,0	-5,0	56,0	41,0	377,0	377,0	28
42		29	-10,0	-5,0	58,0	43,0	420,0	420,0	29
43		30	-10,0	-5,0	60,0	45,0	465,0	465,0	30
44		31	-10,0	-5,0	62,0	47,0	512,0	512,0	31
45		32	-10,0	-5,0	64,0	49,0	561,0	561,0	32

(9) =$C$5*(1+1/$E$7)*(B14+1)-$C$5*(I14/(2*$D$7))*(I14+1)

(10) =IF($C$7<B14;I13+1;0)

(13) =IF(B20>=$F$7;IF($F$7>=B20;0;$D$5)+E19;0)

(15) =F20+G19

Figure A2.1: The design and construct of the master VFC parameter table

Source: Author (2021)

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No	Input paramaters	Description
A	$-\boldsymbol{\lambda_0}$	This value is to be provided from the loss as at the end of the period in period t_0, with the definition in Chapter 5.
B	$\boldsymbol{\psi}$	This value represents the planned changes (PC) per period associated with the turnaround or rescue as defined on and calculated in Chapter 5.
C	$-\boldsymbol{\mu}$	This value represents the estimated average abnormal costs (AC) per period associated with turnaround or rescue, as fully defined and calculated in Chapter 5.
D	$\boldsymbol{k}$	This value represents the estimated or actual delay in the implementation of the incremental savings, measured in periods, which may occur during the turnaround or rescue, as fully defined and calculated in Chapter 5.
No	**Excel equation used vs derived equation**	**Description**
1	=$D$7*(1+1/$E$7)-(1/2)+$C$7 $\breve{n}_{min(k)}$ $\breve{n}_{min(k)} = a\left(1+\frac{1}{b}\right)-\frac{1}{2}+k$	Based on equations (30)D1, (61)D2 and (109)D3 shown in Annexure 1 that determine in which period the cash flow nadir occurs $(\breve{n}_{min(k)})$, assuming a delay of (k) as derived from the Excel formula No. 1, where $a = \$D\$7; b = \$E\$7; k = \$F\$7$; and $\rho = \$G\$7$, *and where* $b > 0$
2	=2*$D$7*(1+1/$E$7) $n = 2a\left(1+\frac{1}{b}\right)$	Use equation (50)B2 to determine when the turnaround will be completed (n), assuming there is no delay in the implementation of the planned changes (PC) as derived from the Excel formula No. 2, *where* a = $D$7 and b = $E$7 *and where b>0*
3	=IF($F$7=0;0;$G$5+$C$7+$G$7) $n = \rho + n + k$	Use equation (93)B3 to determine when the turnaround will be completed (n) periods, assuming there is a delay or multiple delays in the implementation of the planned changes (PC) as derived from the Excel formula No. 3, *where* $a = \$D\$7$ and b = $E$7 *and where b>0*
4	=-C5/D5 $a = -\frac{-\lambda}{\psi}$	Where the loss $(-\lambda_0)$ at the end of the period immediately prior to the commencement of the turnaround is divided by the planned changes (ψ) to be implemented per period as set out in page 9 of Chapter 5 and page 14 of Annexure 1, as calculated in Excel formula
5	=C5/E5 $b = \frac{-\lambda}{\mu}$	Where the loss $(-\lambda_0)$ at the end of the period immediately prior to the commencement of the turnaround is divided by the abnormal costs (ψ) to be incurred per period as set out in page 9 of Chapter 5 and page 20 of Annexure 1, as calculated in Excel forumla No.

Figure A2.2: Legend to the input parameter table as set out in Figure A2.1

Source: Author (2021)

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No	Excel equation used	Description
11	=1+B19	Excel Formula No. 11 represents the periods for which the auxillary model is to be calculated being an index increasing by one from the previous value.
12	=$C$5	Excel formula No. 12 returns the loss $-\lambda_0$ at the end of the peirod that arose immediately prior to the commencement of the turnaround and where $\$C\$5$ reads from the input values displayed for every row in column C.
13	=$E$5	Excel formula No. 13 represents the abnormal costs ($-\mu$) to be incurred per period that reads from $\$E\$5$.
14	=IF(B20>=$C$7; IF($F$7>=B20;0;$D$5)+E19;0)	Excel formula No. 14 caculates planned changes (ψ) generated in this and the previous period. It checks the period and indicate if in $B20$ exceeds the value of k shown in $\$C\$7$ and if true, returns the sum of the $E19$ representing the cash flow at the end of that period plus the additioinal planned changes as set out in $\$D\$5$ for the current peirod, which is subject to checking if k exceeds $B20$ and if true, returns a zero.
15	=C20+E20+D20	Excel Equation 15 represents the sum of the loss, the abnormal costs and the accumulated planned changes for the period and is equal to the value of $f(t_x)$.
16	=F20+G19	Excel Equation 16 adds the cash flow for the current peirod to that of the accumulated cash flow of the previous period to give the cash flow at the end of the current period

Figure A2.2: Legend to the input parameter table as out in Figure A2.1 (continued)

Source: Author (2021)

The model can also be adapted to show some increase in income or concentrate the savings on a single item, but nothing turns on what strategy is followed to achieve the result. All that matters is how the differential is achieved per period. The cash flow projection is created in a spreadsheet and contains 1 120 cells containing calculations. It reads the input information used to create the cash flow projection from the parameter table and reads from $\$C\$5\ (-\lambda_0);\ \$D\$5(\psi);\ \$E\$5(\mu)\ and\ \$C\$7(k)$ as set out in Figure A2.1.

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No	Excel equation used	Description
1	=1+B19	Excel Formula No. 11 represents the periods for which the auxillary model is to be calculated being an index increasing by one from the previous value.
12	=$C$5	Excel formula No. 12 returns the loss $-\lambda_0$ at the end of the peirod that arose immediately prior to the commencement of the turnaround and where $\$C\$5$ reads from the input values displayed for every row in column C.
13	=$E$5	Excel formula No. 13 represents the abnormal costs ($-\mu$) to be incurred per period that reads from $\$E\$5$.
14	=IF(B20>=$C$7; IF($F$7>=B20;0;$D$5)+E19;0)	Excel formula No. 14 calculates planned changes (ψ) generated in this and the previous period. It checks the period and indicate if $B20$ exceeds the value of k shown in $\$C\$7$ and if true, returns the sum of the $E19$ representing the cash flow at the end of that period plus the additioinal planned changes as set out in $\$D\$5$ for the current period, which is subject to checking if k exceeds $B20$ and if true, returns a zero.
15	=C20+E20+D20	Excel Equation 15 represents the sum of the loss, the abnormal costs and the accumulated planned changes for the period and is equal to the value of $f(t_x)$.
16	=F20+G19	Excel Equation 16 adds the cash flow for the current period to that of the accumulated cash flow of the previous period to give the cash flow at the end of the current period

Figure A2.3: Legend to the calculation of the auxiliary model as set out in Figure A2.1

Source: Author (2021)

The design includes the limits in the calculation. If the cash flow at the end of the previous period is equal to or greater than zero, the calculations will cease and return zero values for the remaining periods. The design also uses Excel formulas to emulate any delays in the PC.

A2.3.3 Creating outputs using the $\mathbf{TCF}$ calculations and the $\boldsymbol{VFC}$ model to verify the fit between the two

Annexure 1 dealt extensively with the design and creation of the VFC model to determine fundamental moments. However, Chapter 3 discussed how the TCF calculations create output values for each period using Excel® spreadsheets to determine the fundamental moments. It

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also shows that the purpose of the TCF calculations is to determine the fundamental moments that are also the touchpoint with the VFC model.

A2.3.4 Identification and labelling of the tests

Each test set consisting of two tabs in a spreadsheet is labelled and grouped. Each test set represents a unique combination of inputs and requires the ability to find a specific test and confirm the output values of each such test. Accordingly, the tab labels or names in the spreadsheet are named to reflect the value of the loss ($-\lambda$), the incremental savings (ψ), AC ($-\mu$), and delay (k), so that tab label "$VFC\ -10, 2, 5, 0$" reflects the values that were tested, enabling the duplication of tests. A test set consisting of two tabs is colour-coded, enabling easy verification of which two tabs form a test set. Naming the tab labels in this manner becomes very useful when performing an audit to ensure that correct values emanating from the correct tabs in the summary comparison sheets have occurred.

The first spreadsheet was labelled "$Testing\ for\ -10, a, b, 0$", where such a spreadsheet contains one hundred test sets or two hundred and one tabs. The first test set represents the value of $-10, 1, 1, 0$, followed by $-10, 2, 1, 0$, where the value of a is varied by one until the test set reaches $-10, 10, 1, 0$. The entire process is repeated, but where the value of $b = -2$, and again, each subsequent test set for a is varied by one. In this manner, every combination of (a, b) for each integer between $1\ to\ 10$ is evaluated.

Once this spreadsheet had been completed, the spreadsheet was copied and saved as "$Testing\ for\ -10, a, b, 1$", where implementation is delayed for one period. This same process was then repeated for "$Testing\ for\ -10, a, b, 2$", "$Testing\ for\ -10, a, b, 3$" and "$Testing\ for\ -10, a, b, 4$."

A test varying the values of losses was also created by creating a copy of the original test spreadsheet named "$Testing\ for\ -loss, a, b, k$" wherein this specific spreadsheet the value of the loss was varied to display different values $-\lambda\ and\ k$, of in the specific spreadsheet to ensure that the necessary verification of the loss has occurred.

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Utilising the TCF calculations to determine output values for every period

A	B	C	D	E	F	G	H	
								Excel columns
3	Excel rows							
4	**Period**		**-1**		**0**		**1**	Period
5								
6	Income No	1	9		9		9	=IF(F$43>=0;0;F6)
7	Income No	2	9		9		9	=IF(F$43>=0;0;F7)
8	Income No	3	9		9		9	:
9	Income No	4	9		9		9	17
10	Income No	5	9		9		9	:
11	Income No	6	9		9		9	:
12	Income No	7	9		9		9	:
13	Income No	8	9		9		9	:
14	Income No	9	9		9		9	=IF(F$43>=0;0;F15)
15	Income No	10	9		9		9	=SUM(H6:H15)
16	Total income		90		90		90	
17								
18	Expense No	1	5,0		5,00		4,9	=IF(F$43>=0;0;IF(H$4> 'VFC a=1,b=1,k=0'!$F$7;F18- ('VFC a=1,b=1,k=0'!$D$5/$C$37);$F18))
19	Expense No	2	5,0		5,00		4,9	
20	Expense No	3	5,0		5,00		4,9	
21	Expense No	4	5,0		5,00		4,9	=IF(F$43>=0;0;IF(H$4> 'VFC a=1,b=1,k=0'!$F$7;F19- ('VFC a=1,b=1,k=0'!$D$5/$C$37);$F19))
22	Expense No	5	5,0		5,00		4,9	
23	Expense No	6	5,0		5,00		4,9	
24	Expense No	7	5,0		5,00		4,9	:
25	Expense No	8	5,0		5,00		4,9	18
26	Expense No	9	5,0		5,00		4,9	:
27	Expense No	10	5,0		5,00		4,9	:
28	Expense No	11	5,0		5,00		4,9	:
29	Expense No	12	5,0		5,00		4,9	:
30	Expense No	13	5,0		5,00		4,9	:
31	Expense No	14	5,0		5,00		4,9	:
32	Expense No	15	5,0		5,00		4,9	:
33	Expense No	16	5,0		5,00		4,9	=IF(F$43>=0;0;IF(H$4> 'VFC a=1,b=1,k=0'!$F$7;F37- ('VFC a=1,b=1,k=0'!$D$5/$C$37);$F37))
34	Expense No	17	5,0		5,00		4,9	
35	Expense No	18	5,0		5,00		4,9	
36	Expense No	19	5,0		5,00		4,9	
37	Expense No	20	5,0		5,00		4,9	19
38	Abnormal expenses		0		1,0		1,0	=IF(F43>=0;0;-'VFC a=1,b=1,k=0'!$E$5)
39	Total expenses		100		101		99	=SUM(H18:H38) 20
40								
41	Net cash flow for the period		-10		- 11		- 9	21 =H16-H39 22
42	Cash flow at beginning of period		0		-		- 11	=IF(F43>=0;0;F43) 23
43	Cash flow at end of period		-10		- 11		- 20	=H41+H42

Figure A2.4: The design of the creation of TCF calculations using an Excel® spreadsheet to determine fundamental moments

Source: Author (2021)

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No	Excel equation used	Description
17	=IF(F$43>=0;0;F6) and repeat for every income cell there after up to IF(F$43>=0;0;F15) and sum these values so that =SUM(H6:H15)	Perform a test if the cash flow at the end of the previous period is equal to or exceeds zero. If true, return zero. Alternatively, repeat the same income as set out in the previous period. Then sum all the income items together to create a total income.
18	=IF(F$43>=0;0;IF(H$4> 'VFC a=1,b=1,k=0'!$F$7;F18-('VFC a=1,b=1,k=0'!$D$5/$C$37);$F18))	Perform a test if the cash flow at the end of the previous period is equal to or exceeds zero. If true, return zero. Alternatively,check if the period exceeds the value of the expected delay in implementation as shown in Table A1.6, where if true, return the value of expense in the previous period less the saving value implemented as set out in Table A1.3, which is divided equally amongst all the of expenses in this example. Should the value of the period not exceed the delay, return the value of the expense in the previous period.
19	=IF(F43>=0;0;-'VFC a=1,b=1,k=0'!$E$5)	Perform a test if the cash flow at the end of the previous period is equal to or exceeds zero. If true, return zero. Include the abnormal expense as set out in Table A1.4 for that period.
20	=SUM(H18:H38)	Sum all the expense items together to create the total expenses.
21	=H16-H39	Deduct the total expenses from the total income, representing the net cash flow for the period. These figures indicate when breakeven occurs and what the momentum of the business will be.
22	=IF(F43>=0;0;F43)	Perform a test if the cash flow at the end of the previous period is equal to or exceeds zero. If true, return a zero. Alternatively copy the cash flow at the end of the previous period.
23	=H41+H42	Add the cash flow for the period to the cash flow at the end of the previous period to represent the cash flow as at the end of the period. In the model it is referred to as accumulated cash flow.

Figure A2.5: Legend to the TCF calculations to determine output values as shown in Figure A2.12

Source: Author (2021)

A2.4 Performing the tests for the different values of $-\lambda, \psi, -\mu$ and k

The previous section explained how the master test set was designed to be duplicated, and the input values could be changed to verify the different output values of the models. This

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section describes how the test set was implemented, and input values varied, comparing the VFC model and the TCF calculations.

A2.4.1 Creation of the summary of the various tests using SCT

Each spreadsheet contains an SCT that contains a heading describing each specific test, and it contains all values for the various test sets contained in the spreadsheet. The SCT verifies the differences between the VFC and TCF calculations for the values of $-\lambda, \psi, -\mu \; and \; k$.

It also checks the values of the VFC model against the auxiliary model of $-\lambda, \psi, -\mu \; and \; k$.

The SCT shows the input and output values of the VFC models and the TCF calculations, comparing these for each period. Each test was allocated a number ranging from 1 to 500, enabling verification that no duplication of the tests has occurred. The results of each test set are captured in the SCT, enabling further analysis of the tests performed.

1	B	C	D	E	F	G	H	I	J	K	L	M	N	O
2	**Summary of the tests 1 to 100 performed to check each of the the values of the VFC model against the TCFM for the values of *ψ, μ, and k=0***													
3														
4					1	2	3	4	5	6	7	8	9	10
5	**Test 1**	$-\lambda_0$	ψ											
6		-10	1	TCFM	-11	-22	-33	-44	-55	-66	-77	-88	-99	-110
7		$-\mu$	k	VCF	-11	-21	-30	-38	-45	-51	-56	-60	-63	-65
8		-1	0	Test	0,00E+00	-1,00E+00	-3,00E+00	-6,00E+00	-1,00E+01	-1,50E+01	-2,10E+01	-2,80E+01	-3,60E+01	-4,50E+01
9														
10	**Test 2**	$-\lambda_0$	ψ											
11		-10	2	TCFM	-11	-22	-33	-44	-55	-66	-75	-82	-87	-90
12		$-\mu$	k	VCF	-11	-20	-27	-32	-35	-36	-35	-32	-27	-20
13		-1	0	Test	0,00E+00	-2,00E+00	-6,00E+00	-1,20E+01	-2,00E+01	-3,00E+01	-4,00E+01	-5,00E+01	-6,00E+01	-7,00E+01
14														
15	**Test 3**	$-\lambda_0$	ψ											
16		-10	3	TCFM	-11	-22	-33	-44	-52	-57	-59	-58	-54	-47
17		$-\mu$	k	VCF	-11	-19	-24	-26	-25	-21	-14	-4	9	0
18		-1	0	Test	0,00E+00	-3,00E+00	-9,00E+00	-1,80E+01	-2,70E+01	-3,60E+01	-4,50E+01	-5,40E+01	-6,30E+01	-4,70E+01
19														
20	**Test 4**	$-\lambda_0$	ψ											
21		-10	4	TCFM	-11	-22	-33	-40	-43	-42	-37	-28	-15	2
22		$-\mu$	k	VCF	-11	-18	-21	-20	-15	-6	7	0	0	0
23		-1	0	Test	0,00E+00	-4,00E+00	-1,20E+01	-2,00E+01	-2,80E+01	-3,60E+01	-4,40E+01	-2,80E+01	-1,50E+01	2,00E+00
24														
25	**Test 5**	$-\lambda_0$	ψ											
26		-10	5	TCFM	-11	-22	-33	-39	-40	-36	-27	-13	6	0
27		$-\mu$	k	VCF	-11	-17	-18	-14	-5	9	0	0	0	0
28		-1	0	Test	0,00E+00	-5,00E+00	-1,50E+01	-2,50E+01	-3,50E+01	-4,50E+01	-2,70E+01	-1,30E+01	6,00E+00	0,00E+00
29														
30	**Test 6**	$-\lambda_0$	ψ											
31		-10	6	TCFM	-11	-22	-27	-26	-19	-6	13	0	0	0
32		$-\mu$	k	VCF	-11	-16	-15	-8	5	0	0	0	0	0
33		-1	0	Test	0,00E+00	-6,00E+00	-1,20E+01	-1,80E+01	-2,40E+01	-6,00E+00	1,30E+01	0,00E+00	0,00E+00	0,00E+00

Figure A2.6: Comparing the values of the VFC model against the TCF model values

Source: Author (2021)

The summary displayed in SCT in Figure A2.6 is an example of a test performed to determine if the values generated by the VFC model are identical to that of the TCF calculations, as explained above. The example shows the test number that ranges from tests 1 to 100 for

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$-10, a, b, 0$; 101 to 200 for $-10, a, b, 1$; 201 to 300 for $-10, a, b, 2$; 301 to 400 for $-10, a, b, 3$; 401 to 500 for $-10, a, b, 4$, 501 to 535 for $-\lambda, a, b, 0$ and 536 to 570 for $-\lambda, a, b, 2$. Tests 501 to 570 use random values for the $-\lambda$ and k.

Row 4 shows the periods for each test and ranges up to period 32, which is deemed to be sufficient periods over which the model must be tested, as 95 per cent of the tests in test batch 1 to 100 incorporate the recovery of the capital provided in this period so that $Acc(\lambda_{\psi,\check{n}})$ = 0. The values, in summary, are read directly from each tab used to create the tests. The values in row 6 relate to the values generated by using the TCF model, and values in row 7 are values generated by the VFC model. The values in rows 6 and 7 are deducted for each period, ensuring that these values match for each period.

1	B	C	D	E	G	H	I	K	L	M	O	P	Q	S	T	U
2	**Summary of the tests 101 to 200 performed to check each of the the values of the VFC model against the Auxilary model for the values of *ψ, μ, and k=0***															
3																
4																
5																
6		**Test 101**	$-\lambda_0$	ψ	**Test 102**	$-\lambda_0$	ψ	**Test 103**	$-\lambda_0$	ψ	**Test 104**	$-\lambda_0$	ψ	**Test 105**	$-\lambda_0$	ψ
7			-10	1		-10	2		-10	3		-10	4		-10	5
8			$-\mu$	k		$-\mu$	k		$-\mu$	k		$-\mu$	k		$-\mu$	k
9			-1	0		-1	0		-1	0		-1	0		-1	0
10		Auxilary	VFC	Test	Auxilary	VFC	Test	Auxilary	VFC	Test	Auxilary	VFC	Test	Auxilary	VFC	Test
11		test	model		test	model		test	model		test	model		test	model	
12	1	-11,0	-11,0	0,00E+00	-11,0	-11,0	0,00E+00	-11,0	-11,0	0,00E+00	-11,0	-11,0	0,00E+00	-11,0	-11,0	0,00E+00
13	2	-21,0	-21,0	0,00E+00	-20,0	-20,0	0,00E+00	-19,0	-19,0	0,00E+00	-18,0	-18,0	0,00E+00	-17,0	-17,0	0,00E+00
14	3	-30,0	-30,0	0,00E+00	-27,0	-27,0	0,00E+00	-24,0	-24,0	0,00E+00	-21,0	-21,0	0,00E+00	-18,0	-18,0	0,00E+00
15	4	-38,0	-38,0	0,00E+00	-32,0	-32,0	0,00E+00	-26,0	-26,0	0,00E+00	-20,0	-20,0	0,00E+00	-14,0	-14,0	0,00E+00
16	5	-45,0	-45,0	0,00E+00	-35,0	-35,0	0,00E+00	-25,0	-25,0	0,00E+00	-15,0	-15,0	0,00E+00	-5,0	-5,0	0,00E+00
17	6	-51,0	-51,0	0,00E+00	-36,0	-36,0	0,00E+00	-21,0	-21,0	0,00E+00	-6,0	-6,0	0,00E+00	9,0	9,0	0,00E+00
18	7	-56,0	-56,0	0,00E+00	-35,0	-35,0	0,00E+00	-14,0	-14,0	0,00E+00	7,0	7,0	0,00E+00	28,0	28,0	0,00E+00
19	8	-60,0	-60,0	0,00E+00	-32,0	-32,0	0,00E+00	-4,0	-4,0	0,00E+00	24,0	24,0	0,00E+00	52,0	52,0	0,00E+00
20	9	-63,0	-63,0	0,00E+00	-27,0	-27,0	0,00E+00	9,0	9,0	0,00E+00	45,0	45,0	0,00E+00	81,0	81,0	0,00E+00
21	10	-65,0	-65,0	0,00E+00	-20,0	-20,0	0,00E+00	25,0	25,0	2,84E-14	70,0	70,0	0,00E+00	115,0	115,0	0,00E+00
22	11	-66,0	-66,0	0,00E+00	-11,0	-11,0	0,00E+00	44,0	44,0	0,00E+00	99,0	99,0	0,00E+00	154,0	154,0	0,00E+00
23	12	-66,0	-66,0	0,00E+00	0,0	0,0	0,00E+00	66,0	66,0	0,00E+00	132,0	132,0	0,00E+00	198,0	198,0	0,00E+00
24	13	-65,0	-65,0	0,00E+00	13,0	13,0	0,00E+00	91,0	91,0	0,00E+00	169,0	169,0	0,00E+00	247,0	247,0	0,00E+00
25	14	-63,0	-63,0	0,00E+00	28,0	28,0	0,00E+00	119,0	119,0	0,00E+00	210,0	210,0	0,00E+00	301,0	301,0	0,00E+00
26	15	-60,0	-60,0	0,00E+00	45,0	45,0	0,00E+00	150,0	150,0	0,00E+00	255,0	255,0	0,00E+00	360,0	360,0	0,00E+00
27	16	-56,0	-56,0	0,00E+00	64,0	64,0	0,00E+00	184,0	184,0	0,00E+00	304,0	304,0	0,00E+00	424,0	424,0	0,00E+00
28	17	-51,0	-51,0	0,00E+00	85,0	85,0	0,00E+00	221,0	221,0	0,00E+00	357,0	357,0	0,00E+00	493,0	493,0	0,00E+00
29	18	-45,0	-45,0	0,00E+00	108,0	108,0	0,00E+00	261,0	261,0	0,00E+00	414,0	414,0	0,00E+00	567,0	567,0	0,00E+00
30	19	-38,0	-38,0	0,00E+00	133,0	133,0	0,00E+00	304,0	304,0	0,00E+00	475,0	475,0	0,00E+00	646,0	646,0	0,00E+00
31	20	-30,0	-30,0	0,00E+00	160,0	160,0	0,00E+00	350,0	350,0	0,00E+00	540,0	540,0	0,00E+00	730,0	730,0	0,00E+00
32	21	-21,0	-21,0	0,00E+00	189,0	189,0	0,00E+00	399,0	399,0	0,00E+00	609,0	609,0	0,00E+00	819,0	819,0	0,00E+00
33	22	-11,0	-11,0	0,00E+00	220,0	220,0	0,00E+00	451,0	451,0	0,00E+00	682,0	682,0	0,00E+00	913,0	913,0	0,00E+00
34	23	0,0	0,0	0,00E+00	253,0	253,0	0,00E+00	506,0	506,0	0,00E+00	759,0	759,0	0,00E+00	1012,0	1012,0	0,00E+00
35	24	12,0	12,0	0,00E+00	288,0	288,0	0,00E+00	564,0	564,0	0,00E+00	840,0	840,0	0,00E+00	1116,0	1116,0	0,00E+00
36	25	25,0	25,0	0,00E+00	325,0	325,0	0,00E+00	625,0	625,0	0,00E+00	925,0	925,0	0,00E+00	1225,0	1225,0	0,00E+00
37	26	39,0	39,0	0,00E+00	364,0	364,0	0,00E+00	689,0	689,0	0,00E+00	1014,0	1014,0	0,00E+00	1339,0	1339,0	0,00E+00
38	27	54,0	54,0	0,00E+00	405,0	405,0	0,00E+00	756,0	756,0	0,00E+00	1107,0	1107,0	0,00E+00	1458,0	1458,0	0,00E+00
39	28	70,0	70,0	0,00E+00	448,0	448,0	0,00E+00	826,0	826,0	0,00E+00	1204,0	1204,0	0,00E+00	1582,0	1582,0	0,00E+00
40	29	87,0	87,0	0,00E+00	493,0	493,0	0,00E+00	899,0	899,0	0,00E+00	1305,0	1305,0	0,00E+00	1711,0	1711,0	0,00E+00
41	30	105,0	105,0	0,00E+00	540,0	540,0	0,00E+00	975,0	975,0	0,00E+00	1410,0	1410,0	0,00E+00	1845,0	1845,0	0,00E+00
42	31	124,0	124,0	0,00E+00	589,0	589,0	0,00E+00	1054,0	1054,0	0,00E+00	1519,0	1519,0	0,00E+00	1984,0	1984,0	0,00E+00
43	32	144,0	144,0	0,00E+00	640,0	640,0	0,00E+00	1136,0	1136,0	0,00E+00	1632,0	1632,0	0,00E+00	2128,0	2128,0	0,00E+00

Figure A2.7: Comparing the VFC model against the Auxiliary model values

The summary displayed in Figure A2.7 is an example of a test performed to determine if the values generated by the VFC model are identical to that of the $auxillary$ model as explained above. The example shows the test number that ranges from tests 1 to 100 for $-10, a, b, 0$;

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101 to 200 for $-10, a, b, 1$; 201 to 300 for $-10, a, b, 2$; 301 to 400 for $-10, a, b, 3$; and 401 to 500 for $-10, a, b, 4$.

Column B shows the periods for each test and ranges up to period 32, which is deemed to be sufficient periods over which the model must be tested, as 95 per cent of the tests in test batch 1 to 100 incorporate the recovery of the capital provided in this period so that $Acc(\lambda_{\psi,\breve{n}}) = 0$. The values, in summary, are read directly from each tab set used to create the tests.

The information contained in rows 4 to 7 deals with the input values used in each test, and these input values read directly for the inputs used in the VFC parameters, as discussed in the section above. Rows 8 and 9 show the headings of the values in Column C and D below them. The values of $C10\ to\ C41$ reflect the values generated by using the $auxillary$ method of estimating the cash flow. The values shown in $D10\ to\ D41$ contain the values generated by the VFC model, while the values shown in $E10\ to\ E41$ contain the test value by deducting the two values in column C from column D and ensure that these values match for each of these periods.

A2.5 Findings of the tests performed

Once all the test sets have been populated with the correct input values and the output values have been captured in the relevant SCT, it requires a further test where all the differences are added together. This process highlights where anomalies and errors occur.

A2.5.1 Testing for appropriateness

Description of test sets Test No $[\boldsymbol{\psi}; \boldsymbol{\mu}; \boldsymbol{k}; -\boldsymbol{\lambda}]$	Evaluation test No. 1 for appropriateness	Findings based on the test
1-100 $[a; b; 0; -10]$	No integers are present, and no singularities exist	Appropriate
101-200 $[a; b; 1; -10]$	No integers are present, and no singularities exist	Appropriate
201-300 $[a; b; 2; -10]$	No integers are present, and no singularities exist	Appropriate

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301-400 $[a; b; 3; -10]$	No integers are present, and no singularities exist	Appropriate
401-500 $[a; b; 4; -10]$	No integers are present, and no singularities exist	Appropriate

A2.5.2 Testing for consistency

Description of test sets Test No $[\boldsymbol{\psi}; \boldsymbol{\mu}; \boldsymbol{k}; -\boldsymbol{\lambda}]$	Evaluation test No. 2 for consistency	Findings based on the test
1-100 $[a; b; 0; -10]$	Table A2.2 shows the errors that occur 12 places after the decimal, with errors only arising after period 25 and only for tests 21, 31, 41, 51, 61, 71, 81, and 91, representing 54 errors in the 12 places after the decimal of 1 012 values.	The test is consistent as the errors grow consistently and sequentially
101-200 $[a; b; 1; -10]$	Table A2.3 shows the errors that occur 12 places after the decimal, with errors only arising after period 26 and only for tests 11, 21, 31, 41, 51, 61, 71, 81, and 91, representing 52 errors in the 12 places after the of 1 195 values.	The test is consistent as the errors grow consistently and sequentially
201-300 $[a; b; 2; -10]$	Table A2.4 shows the errors that occur 12 places after the decimal, with errors only arising after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 91, and 92, representing 51 errors of 1 342 values.	The test is consistent as the errors grow consistently and sequentially

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301-400 $[a; b; 3; -10]$	Table A2.5 shows the errors that occur 12 places after the decimal, with the errors only arising after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 82, 91, and 92, representing 48 errors of 1 492 values.	The test is consistent as the errors grow consistently and sequentially
401-500 $[a; b; 4; -10]$	Table A2.6 shows the errors that occur 12 places after the decimal, with the errors only arising after period 25 and only for tests 1,11,21, 31, 41, 51, 61, 71, 81,82, 91, and 92, representing 44 errors of 1 631 values.	The test is consistent as the errors grow consistently and sequentially

A2.5.2.1 Discussion of the test for consistency of the $\boldsymbol{VFC}$ model to that of the $\boldsymbol{TCF}$ calculations

The consistency test indicates that the errors only occur after period 25, only in those tests where the PC (ψ=1), implying implementation savings of 10 per cent. The AC ($\mu = 1, 2, 3, \ldots, 10$) is increased by 10 per cent for each test from test 1, 20 per cent for test 11, and so on; test 91 has an AC of 100 per cent. A closer inspection of these tests shows that the errors are cumulative. Errors occur in the fifteenth, fourteenth, and thirteenth place after the decimal from period 11 onwards. These minor errors accumulate into errors in period 25 and grow for every period after that. Mr Clive Thomas, an expert programmer, informed the author that these errors are directly related to the floating-point limitations of the math coprocessor used by Intel® and AMD® when calculations occur 13 places after the decimal.

Given the accumulative nature of how the errors occur from the fifteenth, fourteenth, and thirteenth place, it can be declared that the VFC model fits consistently with that of the TCF calculations.

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A2.5.3 Testing the various models for accuracy

Description of test sets Test No. $[\boldsymbol{\psi}; \boldsymbol{\mu}; \boldsymbol{k}; -\boldsymbol{\lambda}]$	Evaluation test No. 3 for accuracy	Findings based on the test
1-100 $[a; b; 0; -10]$	All errors and anomalies occur at the 12th, 13th, 14th, and 15th place after the decimal. Values that fall in the error range 12 places after the decimal range from $-1.02E-12\ to\ -1{,}80\ E-12$, indicating 54 errors of 3 200 values.	Accurate to 12 places after the decimal and is, therefore, a perfect fit
101-200 $[a; b; 1; -10]$	All errors and anomalies occur at the 12th, 13th, 14th, and 15th place after the decimal. Values that fall in the error range 12 places after the decimal range from $-1.02E-12\ to\ -1{,}73\ E-12$, indicating 52 errors of 3 200 values.	Accurate to 12 places after the decimal and is, therefore, a perfect fit
201-300 $[a; b; 2; -10]$	All errors and anomalies occur at the 12th, 13th, 14th, and 15th place after the decimal. Values that fall in the error range 12 places after the decimal range from $-1.02E-12\ to\ -1{,}61\ E-12$, indicating 51 errors of 3 200 values.	Accurate to 12 places after the decimal and is, therefore, a perfect fit
301-400 $[a; b; 3; -10]$	All errors and anomalies occur at the 12th, 13th, 14th, and 15th place after the decimal. Values that fall in the error range 12 places after the decimal range from -$1.02E-12\ to\ -1{,}58\ E-12$, indicating 48 errors of 3 200 values.	Accurate to 12 places after the decimal and is, therefore, a perfect fit
401-500 $[a; b; 4; -10]$	All errors and anomalies occur at the 12th, 13th, 14th, and 15th place after the decimal. Values that fall in the error range 12 places after the decimal range from $-1.01E-12\ to\ -1{,}44\ E-12$, indicating 44 errors of 3 200 values.	Accurate to 12 places after the decimal and is, therefore, a perfect fit

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A2.5.3.1 Discussion on the accuracy of the fit of the VFC model to that of the TCF calculations

Errors showed in tests 1 to 500 suggest that the maximum error value is shifted to later periods as there is a delay in the implementation of the turnaround. Since the tests were designed for 32 periods tests [1, 11, 21, 31, 41, 51, 61, 71, 81, 91] exceed the 32 period limit, even if $k = 0$. Accordingly, if there is any delay in the PC (ψ), it will exacerbate the periods for which the turnaround must be undertaken. The author considered performing a hypothesis test to pronounce the accuracy of the VFC model to that of the TCF calculations. It was initially considered to use all the 16 000 tests that were performed; however, upon reflection, it was decided to evaluate only the tests that occurred during the duration, thereby reducing the number of observations to 6 672. A standard deviation and average for these values were calculated only to realise that there is no reason to calculate these values. Of the 6 672 observations, 249 observations occur between the twelfth and eleventh place after the decimal. However, the sequential values have a cumulative error. Once a value has exceeded the twelfth place after the decimal, the next value in the sequence will continue with error increasing with minor incremental values due to the errors arising from the math coprocessor's inability to calculate 13 places after the decimal.

Accordingly, any statistical analysis simply analyses the noise created by the equipment used. Based on the number of tests performed, all errors occurred in the fifteenth, fourteenth, thirteenth, and twelfth place after the decimal, resulting in the *VFC* model being declared a perfect fit for the TCF calculations.

Tests were also performed on ranging values of $(-\lambda)$ by multiplying the value by a factor of 10. The error behaved as expected by scaling up the error in decimal places to account for the higher value of lambda, i.e., if the value of lambda increased by a factor of 10^5, the error increased from the twelfth decimal place to the eighth decimal place accordingly.

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Table A2.2: A cluster error map of tests 1 to 100, where the difference was multiplied by 1E12 for $[\boldsymbol{a}; \boldsymbol{b}; \mathbf{0}; -\mathbf{10}]$, showing that the errors only arise after period 25 and only for tests 21, 31, 41, 51, 61, 71, 81, and 91, representing 54 errors of 3 200 values

Source: Author (2021)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Test 1	0,00	-0,03	-0,06	0,00	0,00	0,00	-0,06	-0,10	-0,10	-0,13	-0,14	-0,14	-0,23	-0,30	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 11	0,00	-0,03	-0,06	0,00	0,00	0,00	-0,06	0,00	0,00	-0,13	-0,14	-0,18	-0,18	-0,26	-0,30	-0,33	-0,38	-0,47	-0,54	-0,58	-0,61	-0,68	-0,78	-0,87	-0,98	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 21	0,00	-0,03	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,27	-0,27	-0,31	-0,34	-0,40	-0,48	-0,55	-0,60	-0,63	-0,70	-0,80	-0,88	-0,99	-1,09	-1,17	0,00	0,00	0,00	0,00	0,00
Test 31	0,00	-0,03	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,27	-0,34	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	-0,91	-1,02	-1,12	-1,19	-1,29	-1,44	0,00	0,00	0,00
Test 41	0,00	-0,03	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,27	-0,34	-0,38	-0,38	-0,44	-0,53	-0,60	-0,64	-0,67	-0,74	-0,84	-0,92	-1,04	-1,14	-1,21	-1,31	-1,45	-1,55	-1,68	0,00
Test 51	0,00	-0,03	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,28	-0,34	-0,40	-0,43	-0,43	-0,51	-0,57	-0,63	-0,65	-0,72	-0,82	-0,91	-1,02	-1,12	-1,19	-1,29	-1,44	-1,53	-1,66	-1,75
Test 61	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	0,00	0,00	-0,26	-0,34	-0,40	-0,43	-0,48	-0,48	-0,57	-0,63	-0,65	-0,74	-0,85	-0,94	-1,05	-1,15	-1,22	-1,32	-1,46	-1,56	-1,69	-1,78
Test 71	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,26	-0,34	-0,40	-0,43	-0,48	-0,57	-0,57	-0,63	-0,65	-0,74	-0,85	-0,94	-1,05	-1,14	-1,19	-1,29	-1,44	-1,53	-1,66	-1,75
Test 81	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,68	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,59	-1,72	-1,80
Test 91	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,68	-0,74	-0,74	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,59	-1,71	-1,79
Test 2	0,00	-0,03	0,00	0,00	0,00	-0,06	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 12	0,00	-0,03	0,00	0,00	0,00	-0,06	-0,06	-0,10	-0,16	-0,23	-0,26	-0,34	-0,40	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 22	0,00	-0,03	0,00	0,00	0,00	-0,06	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 32	0,00	-0,03	0,00	0,00	0,00	-0,06	-0,14	-0,14	-0,20	-0,27	-0,30	-0,38	-0,44	-0,51	-0,57	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 42	0,00	-0,03	0,00	0,00	0,00	-0,06	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 52	0,00	-0,03	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,18	-0,26	-0,28	-0,37	-0,43	-0,50	-0,55	-0,58	-0,67	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 62	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 72	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,24	-0,27	-0,36	-0,41	-0,48	-0,54	-0,57	-0,65	-0,72	-0,78	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 82	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	-0,85	-0,88	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 92	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,31	-0,40	-0,45	-0,53	-0,58	-0,61	-0,70	-0,77	-0,82	-0,85	-0,88	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 3	0,00	0,03	0,00	0,04	0,04	0,06	0,14	0,18	0,23	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 13	0,00	0,03	0,00	0,04	0,04	0,06	0,14	0,18	0,23	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 23	0,00	0,03	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 33	0,00	0,03	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 43	0,00	0,03	0,00	0,00	0,00	0,00	0,13	0,17	0,21	0,30	0,28	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 53	0,00	0,03	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 63	0,00	0,03	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 73	0,00	0,00	0,00	0,00	0,00	0,06	0,06	0,10	0,14	0,23	0,21	0,21	0,27	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 83	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 93	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 4	0,00	0,03	0,06	0,04	0,01	0,03	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 14	0,00	0,03	0,06	0,06	0,03	0,04	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 24	0,00	0,03	0,06	0,04	0,00	0,03	0,07	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 34	0,00	0,03	0,06	0,04	0,00	0,03	0,07	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 44	0,00	0,03	0,06	0,00	0,00	0,03	0,07	0,09	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 54	0,00	0,03	0,06	0,00	0,00	0,06	0,10	0,11	0,14	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 64	0,00	0,03	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 74	0,00	0,03	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 84	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 94	0,00	0,00	0,06	0,00	0,00	0,00	0,06	0,07	0,10	0,16	0,17	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 5	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 15	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 25	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 35	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 45	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 55	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 65	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 75	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 85	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 95	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 6	0,00	-0,03	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 16	0,00	-0,03	-0,03	-0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 26	0,00	-0,03	-0,04	-0,03	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 36	0,00	-0,03	-0,04	-0,03	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 46	0,00	-0,03	-0,04	-0,03	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 56	0,00	-0,03	-0,04	-0,03	0,00	-0,01	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 66	0,00	-0,03	0,00	0,00	0,00	-0,01	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 76	0,00	-0,03	0,00	0,00	0,00	-0,03	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 86	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 96	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 7	0,00	-0,03	0,00	-0,03	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 17	0,00	-0,03	0,00	-0,03	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 27	0,00	-0,03	0,00	-0,03	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 37	0,00	-0,03	-0,03	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 47	0,00	-0,03	0,00	-0,03	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 57	0,00	-0,03	0,00	-0,03	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 67	0,00	-0,03	0,00	-0,03	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 77	0,00	-0,03	0,00	-0,03	-0,03	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 87	0,00	-0,03	0,00	0,00	-0,03	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 97	0,00	0,00	0,00	0,00	-0,03	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 8	0,00	0,03	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 18	0,00	0,03	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 28	0,00	0,03	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 38	0,00	0,03	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 48	0,00	0,03	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 58	0,00	0,03	0,03	0,06	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 68	0,00	0,03	0,00	0,03	0,04	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 78	0,00	0,03	0,00	0,03	0,04	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 88	0,00	0,03	0,00	0,03	0,04	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 98	0,00	0,03	0,00	0,03	0,04	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 9	0,00	0,03	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 19	0,00	0,03	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 29	0,00	0,03	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 39	0,00	0,03	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 49	0,00	0,03	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 59	0,00	0,03	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 69	0,00	0,03	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 79	0,00	0,03	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 89	0,00	0,03	0,04	0,05	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 99	0,00	0,03	0,00	0,05	0,03	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 20	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 40	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 50	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 60	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 70	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 80	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 90	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 100	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

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Table A2.3: A cluster error map of tests 101 to 200, where the difference was multiplied by 1E12 for $[\boldsymbol{a}; \boldsymbol{b}; \mathbf{1}; -\mathbf{10}]$, showing that the errors only arise after period 26 and only for tests 11, 21, 31, 41, 51, 61, 71, 81, and 91, representing 52 errors of 3 200 values

Source: Author (2021)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Test 101	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,14	-0,23	-0,30	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	-0,91	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 111	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,18	-0,26	-0,30	-0,33	-0,38	-0,47	-0,54	-0,58	-0,61	-0,68	-0,78	-0,87	-0,98	-1,08	0,00	0,00	0,00	0,00	0,00
Test 121	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,27	-0,27	-0,31	-0,34	-0,40	-0,48	-0,55	-0,60	-0,63	-0,70	-0,80	-0,88	-0,99	-1,09	-1,17	-1,26	0,00	0,00	0,00
Test 131	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,27	-0,34	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	-0,91	-1,02	-1,12	-1,19	-1,29	-1,44	-1,53	0,00
Test 141	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	0,00	-0,26	-0,34	-0,40	-0,40	-0,45	-0,54	-0,63	-0,67	-0,70	-0,77	-0,87	-0,95	-1,07	-1,17	-1,24	-1,34	-1,48	-1,58	-1,71
Test 151	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,26	-0,34	-0,40	-0,43	-0,43	-0,51	-0,57	-0,63	-0,65	-0,74	-0,85	-0,94	-1,05	-1,15	-1,22	-1,32	-1,46	-1,56	-1,69
Test 161	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,54	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,26	-1,36	-1,51	-1,61	-1,73
Test 171	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,61	-1,73
Test 181	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,68	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,59	-1,71
Test 191	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,68	-0,74	-0,74	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,59	-1,71
Test 102	0,00	0,00	-0,03	0,00	0,00	0,00	-0,06	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 112	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,10	-0,16	-0,23	-0,26	-0,34	-0,40	-0,47	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 122	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 132	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,14	-0,20	-0,27	-0,30	-0,38	-0,44	-0,51	-0,57	-0,60	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 142	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 152	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,18	-0,26	-0,28	-0,37	-0,43	-0,50	-0,55	-0,58	-0,67	-0,74	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 162	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	-0,85	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 172	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,24	-0,27	-0,36	-0,41	-0,48	-0,54	-0,57	-0,65	-0,72	-0,78	-0,81	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 182	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	-0,85	-0,88	-0,91	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 192	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,23	-0,28	-0,28	-0,37	-0,43	-0,50	-0,55	-0,58	-0,67	-0,74	-0,80	-0,82	-0,85	-0,94	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 103	0,00	0,00	0,03	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 113	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 123	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 133	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 143	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,13	0,17	0,21	0,30	0,28	0,28	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 153	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 163	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 173	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,23	0,21	0,21	0,27	0,33	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 183	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 193	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 104	0,00	0,00	0,03	0,06	0,04	0,00	0,03	0,07	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 114	0,00	0,00	0,03	0,06	0,06	0,03	0,04	0,09	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 124	0,00	0,00	0,00	0,06	0,00	0,00	0,03	0,07	0,09	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 134	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 144	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 154	0,00	0,00	0,00	0,06	0,00	0,00	0,06	0,10	0,11	0,14	0,20	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 164	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 174	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 184	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,09	0,11	0,17	0,18	0,24	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 194	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,10	0,16	0,17	0,23	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 105	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 115	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 125	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 135	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 145	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 155	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 165	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 175	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 185	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 195	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 106	0,00	0,00	-0,03	-0,04	-0,03	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 116	0,00	0,00	-0,03	-0,03	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 126	0,00	0,00	0,00	0,00	-0,03	0,00	-0,01	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 136	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 146	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 156	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 166	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 176	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,07	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 186	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 196	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 107	0,00	0,00	-0,03	0,00	-0,03	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 117	0,00	0,00	-0,03	0,00	-0,03	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 127	0,00	0,00	0,00	0,00	-0,03	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 137	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 147	0,00	0,00	0,00	0,00	0,00	-0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 157	0,00	0,00	0,00	0,00	0,00	-0,03	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 167	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 177	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 187	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 197	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 108	0,00	0,00	0,03	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 118	0,00	0,00	0,03	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 128	0,00	0,00	0,03	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 138	0,00	0,00	0,00	0,00	0,03	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 148	0,00	0,00	0,00	0,00	0,03	0,04	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 158	0,00	0,00	0,00	0,00	0,06	0,07	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 168	0,00	0,00	0,00	0,00	0,00	0,04	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 178	0,00	0,00	0,00	0,00	0,00	0,04	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 188	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 198	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 109	0,00	0,00	0,03	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 119	0,00	0,00	0,03	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 129	0,00	0,00	0,03	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 139	0,00	0,00	0,00	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 149	0,00	0,00	0,00	0,00	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 159	0,00	0,00	0,00	0,00	0,05	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 169	0,00	0,00	0,00	0,00	0,05	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 179	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 189	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 199	0,00	0,00	0,00	0,00	0,00	0,03	0,03	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 110	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 120	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 130	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 140	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 150	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 160	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 170	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 180	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 190	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 200	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

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Table A2.4: A cluster error map of tests 201 to 300, where the difference was multiplied by 1E12 for $[\boldsymbol{a}; \boldsymbol{b}; \mathbf{2}; -\mathbf{10}]$, showing that the errors only arise after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 91, and 92, representing 51 errors of 3 200 values

Source: Author (2021)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Test 201	0,00	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,14	-0,23	-0,30	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	-0,91	-1,02	0,00	0,00	0,00	0,00	0,00
Test 211	0,00	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,18	-0,26	-0,30	-0,33	-0,38	-0,47	-0,54	-0,58	-0,61	-0,68	-0,78	-0,87	-0,98	-1,08	-1,15	0,00	0,00	0,00
Test 221	0,00	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,27	-0,27	-0,31	-0,34	-0,40	-0,48	-0,55	-0,60	-0,63	-0,70	-0,80	-0,88	-0,99	-1,09	-1,17	-1,26	-1,41	0,00
Test 231	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	0,00	-0,26	-0,34	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	-0,91	-1,02	-1,12	-1,19	-1,29	-1,44	-1,53
Test 241	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,45	-0,51	-0,60	-0,68	-0,74	-0,77	-0,85	-0,95	-1,04	-1,15	-1,25	-1,32	-1,42	-1,56	-1,66
Test 251	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,48	-0,57	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,21	-1,28	-1,38	-1,52	-1,62
Test 261	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,54	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,61
Test 271	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,59
Test 281	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,68	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,59
Test 291	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,68	-0,74	-0,74	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51	-1,59
Test 202	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 212	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,16	-0,23	-0,26	-0,34	-0,40	-0,47	-0,53	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 222	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 232	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,14	-0,20	-0,27	-0,30	-0,38	-0,44	-0,51	-0,57	-0,60	-0,68	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 242	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 252	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,18	-0,26	-0,28	-0,37	-0,43	-0,50	-0,55	-0,58	-0,67	-0,74	-0,80	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 262	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	-0,85	-0,88	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 272	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,24	-0,27	-0,36	-0,41	-0,48	-0,54	-0,57	-0,65	-0,72	-0,78	-0,81	-0,84	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 282	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,31	-0,40	-0,45	-0,53	-0,58	-0,61	-0,70	-0,77	-0,82	-0,85	-0,88	-0,97	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 292	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,28	-0,37	-0,43	-0,51	-0,57	-0,60	-0,68	-0,75	-0,81	-0,84	-0,87	-0,95	-1,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 203	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 213	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 223	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 233	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 243	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,13	0,17	0,21	0,30	0,28	0,28	0,34	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 253	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 263	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 273	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,23	0,21	0,21	0,27	0,33	0,36	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 283	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,45	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 293	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 204	0,00	0,00	0,00	0,00	0,06	0,00	0,00	0,03	0,07	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 214	0,00	0,00	0,00	0,00	0,06	0,06	0,00	0,00	0,09	0,10	0,13	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 224	0,00	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 234	0,00	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 244	0,00	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 254	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,10	0,11	0,14	0,20	0,21	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 264	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,09	0,11	0,17	0,18	0,23	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 274	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,23	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 284	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,24	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 294	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,16	0,17	0,23	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 205	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 215	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 225	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 235	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 245	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 255	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 265	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 275	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 285	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 295	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 206	0,00	0,00	0,00	0,00	0,00	-0,03	0,00	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 216	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 226	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 236	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 246	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 256	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 266	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 276	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,07	-0,07	-0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 286	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 296	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 207	0,00	0,00	0,00	0,00	0,00	-0,03	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 217	0,00	0,00	0,00	0,00	0,00	-0,03	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 227	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 237	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,07	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 247	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 257	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 267	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 277	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 287	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 297	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 208	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 218	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 228	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 238	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 248	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 258	0,00	0,00	0,00	0,00	0,00	0,06	0,07	0,07	0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 268	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 278	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 288	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 298	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 2209	0,00	0,00	0,00	0,00	0,04	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 219	0,00	0,00	0,00	0,00	0,00	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 229	0,00	0,00	0,00	0,00	0,00	0,05	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 239	0,00	0,00	0,00	0,00	0,00	0,05	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 249	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 259	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 269	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 279	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 289	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 299	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 210	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 220	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 230	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 240	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 250	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 260	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 270	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 280	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 290	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 300	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

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Table A2.5: A cluster error map of tests 301 to 400, where the difference was multiplied by 1E12 for $[\boldsymbol{a}; \boldsymbol{b}; \mathbf{3}; \mathbf{-10}]$, showing that the errors only arise after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 82, 91, and 92, representing 48 errors of 3 200 values

Source: Author (2021)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Test 301	0,00	0,00	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,14	-0,23	-0,30	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	-0,91	-1,02	-1,12	0,00	0,00	0,00
Test 311	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,18	-0,26	-0,30	-0,33	-0,38	-0,47	-0,54	-0,58	-0,61	-0,68	-0,78	-0,87	-0,98	-1,08	-1,15	-1,25	0,00
Test 321	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	0,00	-0,26	-0,26	-0,28	-0,31	-0,37	-0,45	-0,53	-0,57	-0,60	-0,67	-0,77	-0,85	-0,97	-1,07	-1,14	-1,24	-1,38
Test 331	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,40	-0,43	-0,48	-0,57	-0,63	-0,68	-0,71	-0,78	-0,88	-0,97	-1,08	-1,18	-1,25	-1,35	-1,49
Test 341	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,45	-0,51	-0,60	-0,68	-0,74	-0,77	-0,85	-0,97	-1,05	-1,17	-1,26	-1,34	-1,44	-1,58
Test 351	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,48	-0,57	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,35	-1,49
Test 361	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,54	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51
Test 371	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51
Test 381	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,68	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36	-1,51
Test 391	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,45	-0,51	-0,57	-0,63	-0,68	-0,68	-0,74	-0,85	-0,91	-1,02	-1,14	-1,19	-1,31	-1,45
Test 302	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 312	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,16	-0,23	-0,26	-0,34	-0,40	-0,47	-0,53	-0,55	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 322	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 332	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,14	-0,20	-0,27	-0,30	-0,38	-0,44	-0,51	-0,57	-0,60	-0,68	-0,75	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 342	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	-0,85	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 352	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,18	-0,26	-0,28	-0,37	-0,43	-0,50	-0,55	-0,58	-0,67	-0,74	-0,80	-0,82	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 362	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,31	-0,40	-0,45	-0,53	-0,58	-0,61	-0,70	-0,77	-0,82	-0,85	-0,88	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 372	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,23	-0,26	-0,34	-0,40	-0,47	-0,53	-0,55	-0,64	-0,71	-0,77	-0,80	-0,82	-0,91	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 382	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,31	-0,40	-0,45	-0,51	-0,57	-0,60	-0,68	-0,75	-0,81	-0,84	-0,87	-0,95	-1,02	0,00	0,00	0,00	0,00	0,00	0,00
Test 392	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,28	-0,37	-0,43	-0,51	-0,57	-0,60	-0,68	-0,75	-0,81	-0,84	-0,87	-0,95	-1,02	-1,08	0,00	0,00	0,00	0,00	0,00
Test 303	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 313	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,14	0,18	0,23	0,31	0,30	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 323	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 333	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 343	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,13	0,17	0,21	0,30	0,28	0,28	0,34	0,40	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 353	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 363	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 373	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,23	0,21	0,21	0,27	0,33	0,36	0,36	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 383	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,45	0,46	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 393	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,44	0,45	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 304	0,00	0,00	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,07	0,09	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 314	0,00	0,00	0,00	0,00	0,00	0,06	0,06	0,00	0,00	0,09	0,10	0,13	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 324	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 334	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 344	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,09	0,11	0,17	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 354	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,14	0,20	0,21	0,26	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 364	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,23	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 374	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,23	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 384	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,24	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 394	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,16	0,17	0,23	0,30	0,33	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 305	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 315	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 325	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 335	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 345	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 355	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 365	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 375	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 385	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 395	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 306	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 316	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 326	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 336	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 346	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 356	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 366	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 376	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,07	-0,07	-0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 386	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,09	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 396	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,09	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 307	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	-0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 317	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 327	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 337	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,07	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 347	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 357	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 367	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 377	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 387	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 397	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 308	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 318	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 328	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 338	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,05	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 348	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 358	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,07	0,07	0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 368	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 378	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 388	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 398	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 309	0,00	0,00	0,00	0,00	0,00	0,00	0,05	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 319	0,00	0,00	0,00	0,00	0,00	0,00	0,05	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 329	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 339	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 349	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 359	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 369	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 379	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 389	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 399	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,10	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 310	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 320	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 330	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 340	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 350	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 360	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 370	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 380	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 390	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 400	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

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Table A2.6: A cluster error map of tests 401 to 500, where the difference was multiplied by 1E12 for $[\boldsymbol{a}; \boldsymbol{b}; \mathbf{4}; -\mathbf{10}]$, showing that the errors only arise after period 25 and only for tests 1, 11, 21, 31, 41, 51, 61, 71, 81, 82, 91, and 92, representing 44 errors of 3 200 values

Source: Author (2021)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Test 401	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,14	-0,23	-0,30	-0,34	-0,37	-0,43	-0,51	-0,58	-0,63	-0,65	-0,72	-0,82	-0,91	-1,02	-1,12	-1,19	0,00
Test 411	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,13	-0,14	-0,18	-0,18	-0,26	-0,30	-0,33	-0,38	-0,47	-0,54	-0,58	-0,61	-0,68	-0,78	-0,87	-0,98	-1,08	-1,15	-1,25
Test 421	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,31	-0,34	-0,37	-0,43	-0,51	-0,57	-0,61	-0,64	-0,71	-0,81	-0,90	-1,01	-1,11	-1,18	-1,28
Test 431	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,40	-0,43	-0,48	-0,57	-0,63	-0,68	-0,71	-0,80	-0,90	-0,98	-1,09	-1,19	-1,26	-1,36
Test 441	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,45	-0,51	-0,60	-0,68	-0,74	-0,77	-0,85	-0,97	-1,05	-1,17	-1,26	-1,34	-1,44
Test 451	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,48	-0,57	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36
Test 461	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,54	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36
Test 471	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,48	-0,54	-0,63	-0,63	-0,68	-0,71	-0,80	-0,91	-0,99	-1,11	-1,19	-1,25	-1,36
Test 481	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,31	-0,40	-0,45	-0,45	-0,51	-0,57	-0,63	-0,63	-0,68	-0,74	-0,85	-0,91	-1,02	-1,14	-1,19	-1,31
Test 491	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	0,00	0,00	-0,45	-0,45	-0,51	-0,57	-0,63	-0,68	-0,68	-0,74	-0,85	-0,91	-1,02	-1,14	-1,19	-1,31
Test 402	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 412	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,16	-0,23	-0,26	-0,34	-0,40	-0,47	-0,53	-0,55	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 422	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 432	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,14	-0,20	-0,27	-0,30	-0,38	-0,44	-0,51	-0,57	-0,60	-0,68	-0,75	-0,81	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 442	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,14	-0,18	-0,24	-0,31	-0,34	-0,43	-0,48	-0,55	-0,61	-0,64	-0,72	-0,80	-0,85	-0,88	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 452	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,26	-0,34	-0,40	-0,47	-0,53	-0,55	-0,64	-0,71	-0,77	-0,80	-0,82	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 462	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,31	-0,40	-0,45	-0,53	-0,58	-0,61	-0,70	-0,77	-0,82	-0,85	-0,88	-0,97	0,00	0,00	0,00	0,00	0,00	0,00
Test 472	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,23	-0,26	-0,34	-0,40	-0,45	-0,51	-0,54	-0,63	-0,70	-0,75	-0,78	-0,81	-0,90	-0,97	0,00	0,00	0,00	0,00	0,00
Test 482	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,31	-0,40	-0,45	-0,51	-0,57	-0,60	-0,68	-0,75	-0,81	-0,84	-0,87	-0,95	-1,02	-1,07	0,00	0,00	0,00	0,00
Test 492	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,23	-0,28	-0,28	-0,37	-0,43	-0,51	-0,57	-0,60	-0,68	-0,74	-0,80	-0,82	-0,85	-0,94	-1,01	-1,07	-1,10	0,00	0,00	0,00
Test 403	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 413	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 423	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 433	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 443	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,13	0,17	0,21	0,30	0,28	0,28	0,34	0,40	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 453	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 463	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,18	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,45	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 473	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,16	0,24	0,23	0,23	0,28	0,34	0,37	0,38	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 483	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,23	0,31	0,30	0,30	0,36	0,41	0,44	0,45	0,46	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 493	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,23	0,31	0,28	0,28	0,34	0,40	0,43	0,43	0,44	0,48	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 404	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,07	0,09	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 414	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,09	0,10	0,13	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 424	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,07	0,09	0,11	0,17	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 434	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,09	0,11	0,17	0,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 444	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,23	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 454	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,14	0,20	0,21	0,26	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 464	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,23	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 474	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,17	0,18	0,23	0,31	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 484	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,14	0,16	0,21	0,28	0,32	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 494	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,16	0,17	0,23	0,30	0,33	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 405	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 415	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 425	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 435	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 445	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 455	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 465	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 475	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 485	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 495	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 406	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 416	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 426	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 436	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 446	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 456	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 466	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,06	-0,09	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 476	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,07	-0,10	-0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 486	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,09	-0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 496	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,09	-0,07	-0,14	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 407	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 417	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 427	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,04	-0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 437	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,06	-0,07	-0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 447	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 457	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,03	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 467	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 477	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 487	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 497	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 408	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 418	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 428	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,05	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 438	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,05	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 448	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 458	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,07	0,07	0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 468	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 478	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 488	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 498	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 409	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 419	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 429	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 439	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 449	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 459	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 469	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 479	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,09	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 489	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 499	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,10	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 410	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 420	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 430	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 440	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 450	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 460	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 470	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 480	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 490	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Test 500	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

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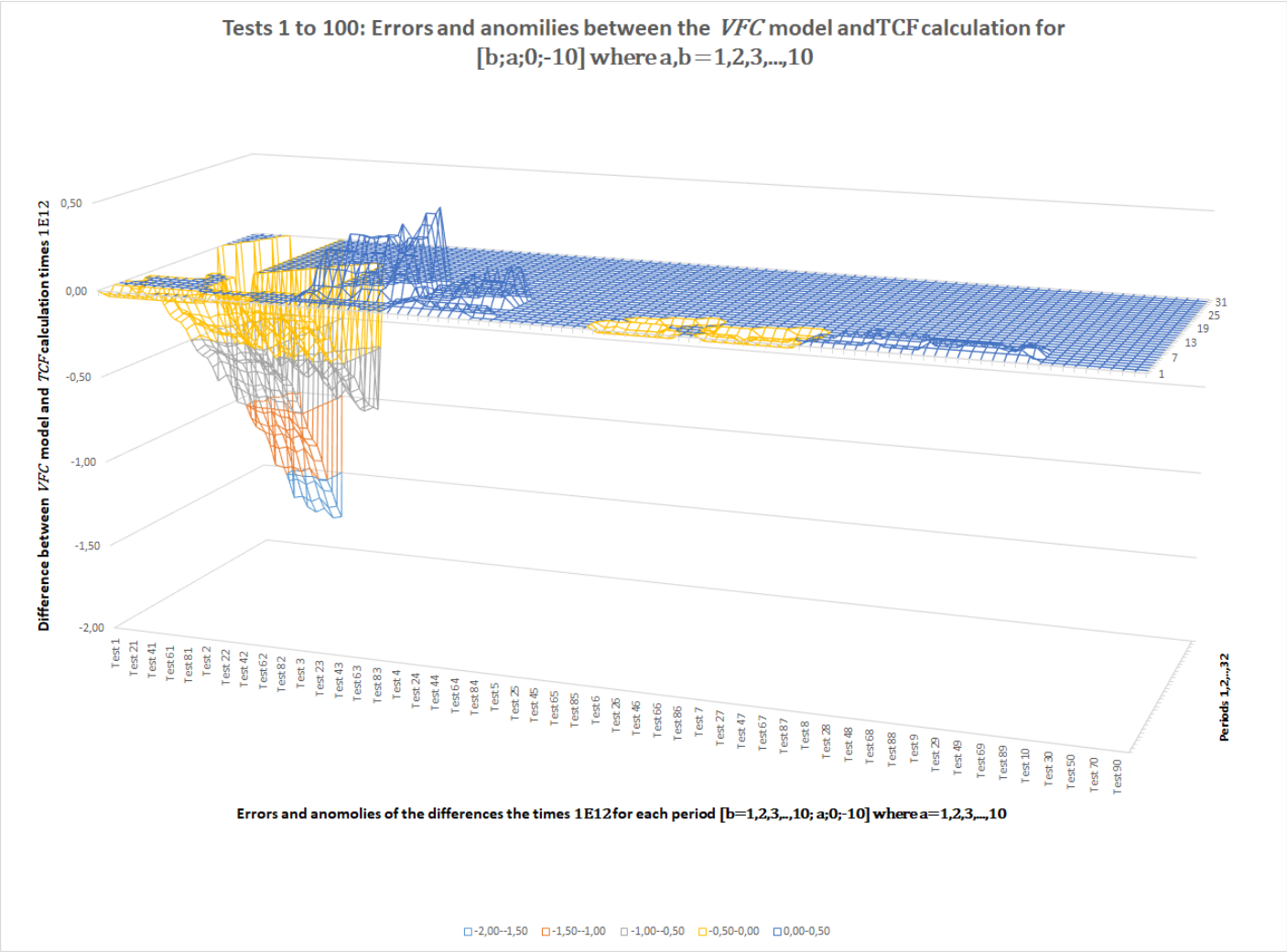

Figure A2.8: A graphical representation of the errors and anomalies of tests 1 to 100, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal

Source: Author (2021)

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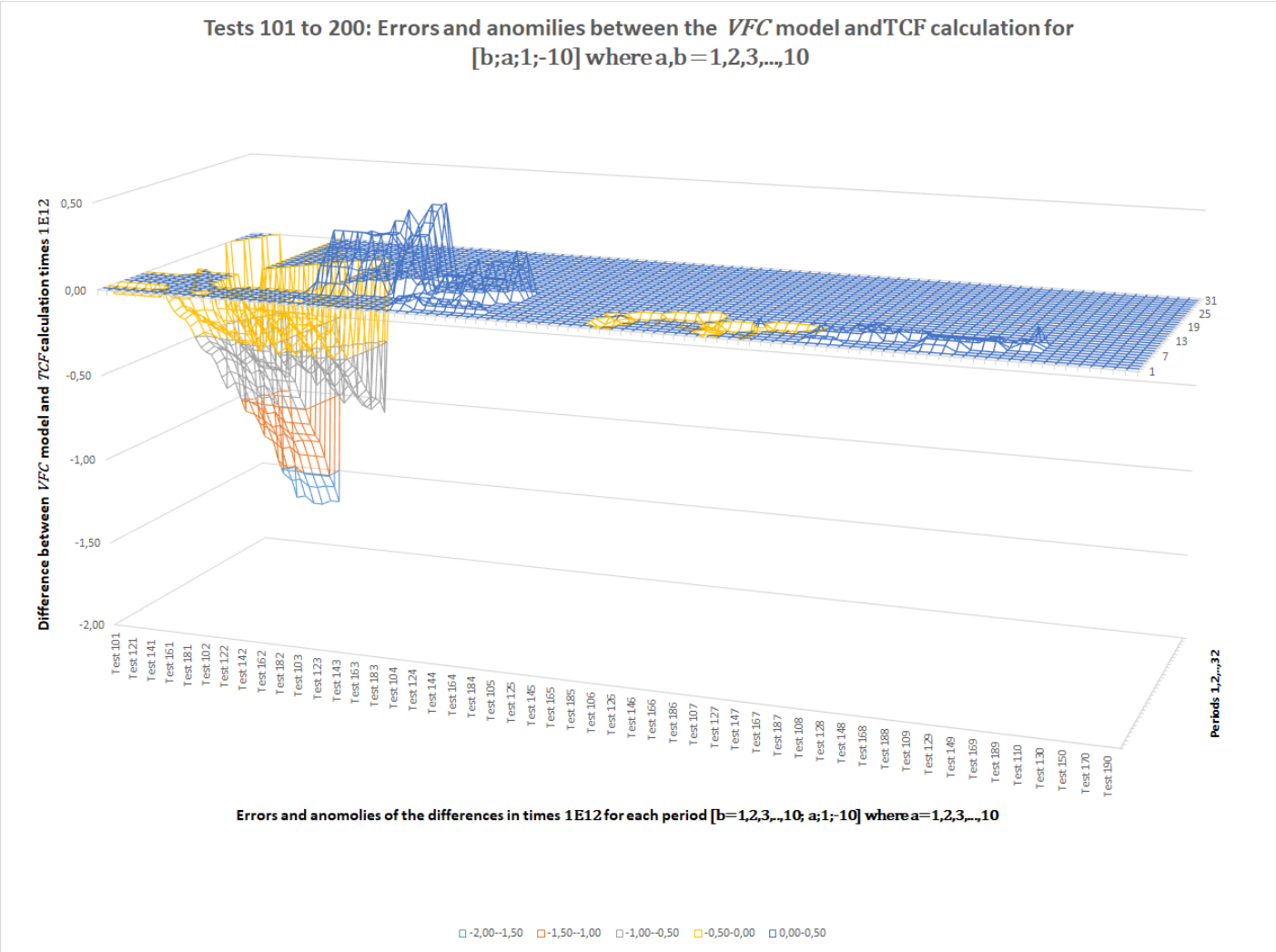

Figure 2.9: A graphical representation of the errors and anomalies of tests 101 to 200, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal

Source: Author (2021)

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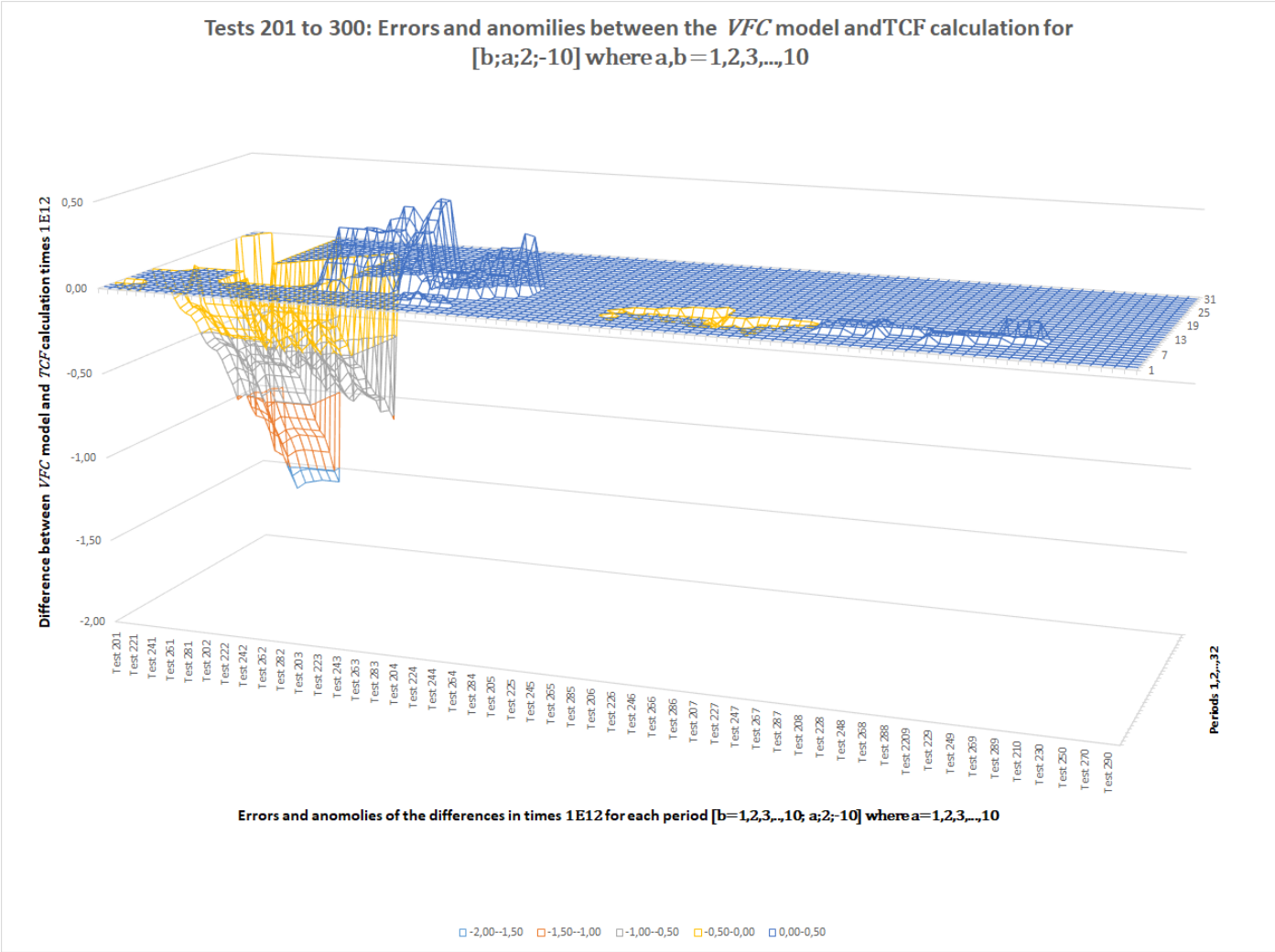

Figure A2.10: A graphical representation of the errors and anomalies of tests 201 to 300, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal

Source: Author (2021)

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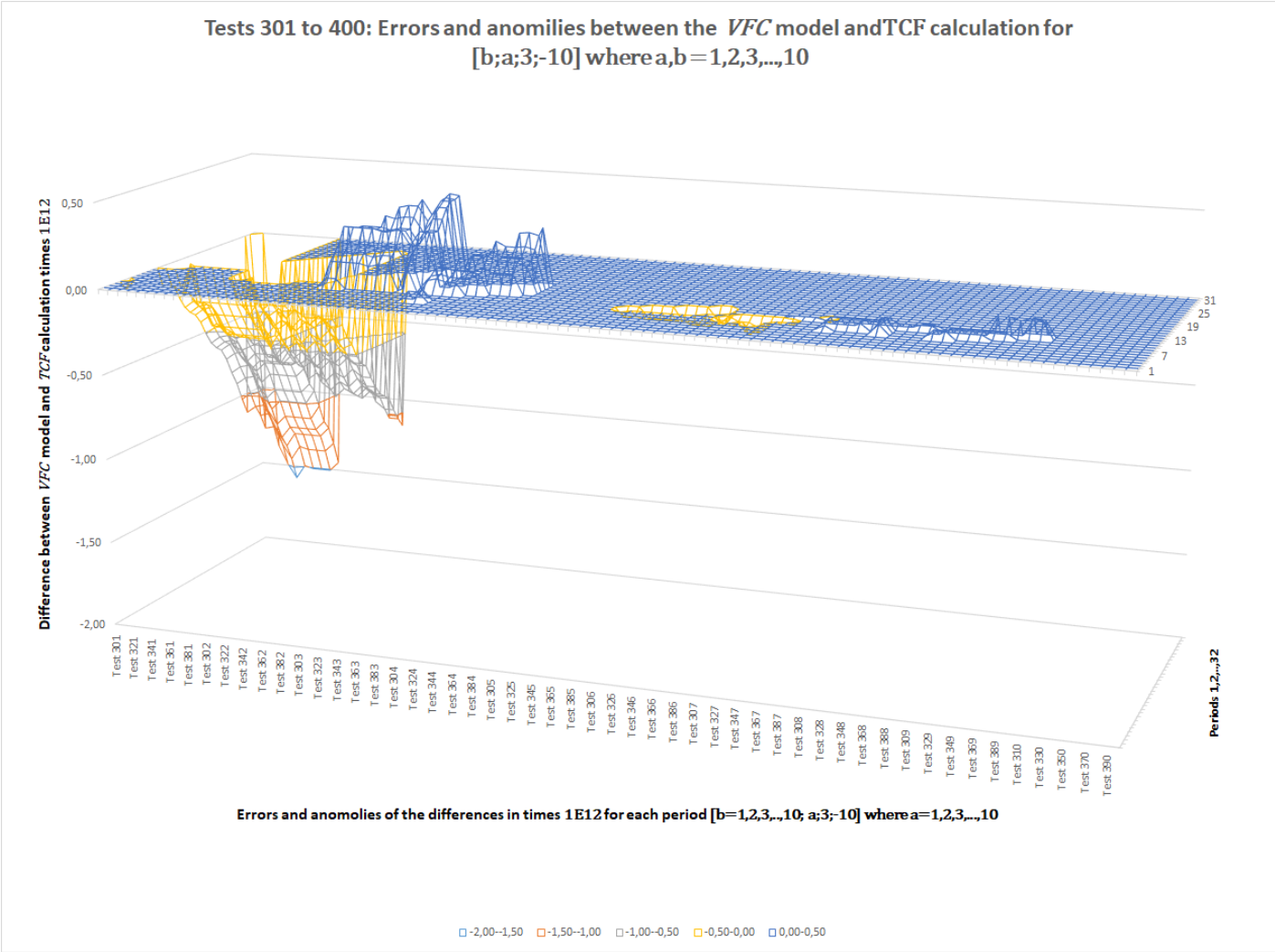

Figure A2.11: A graphical representation of the errors and anomalies of tests 301 to 400, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal

Source: Author (2021)

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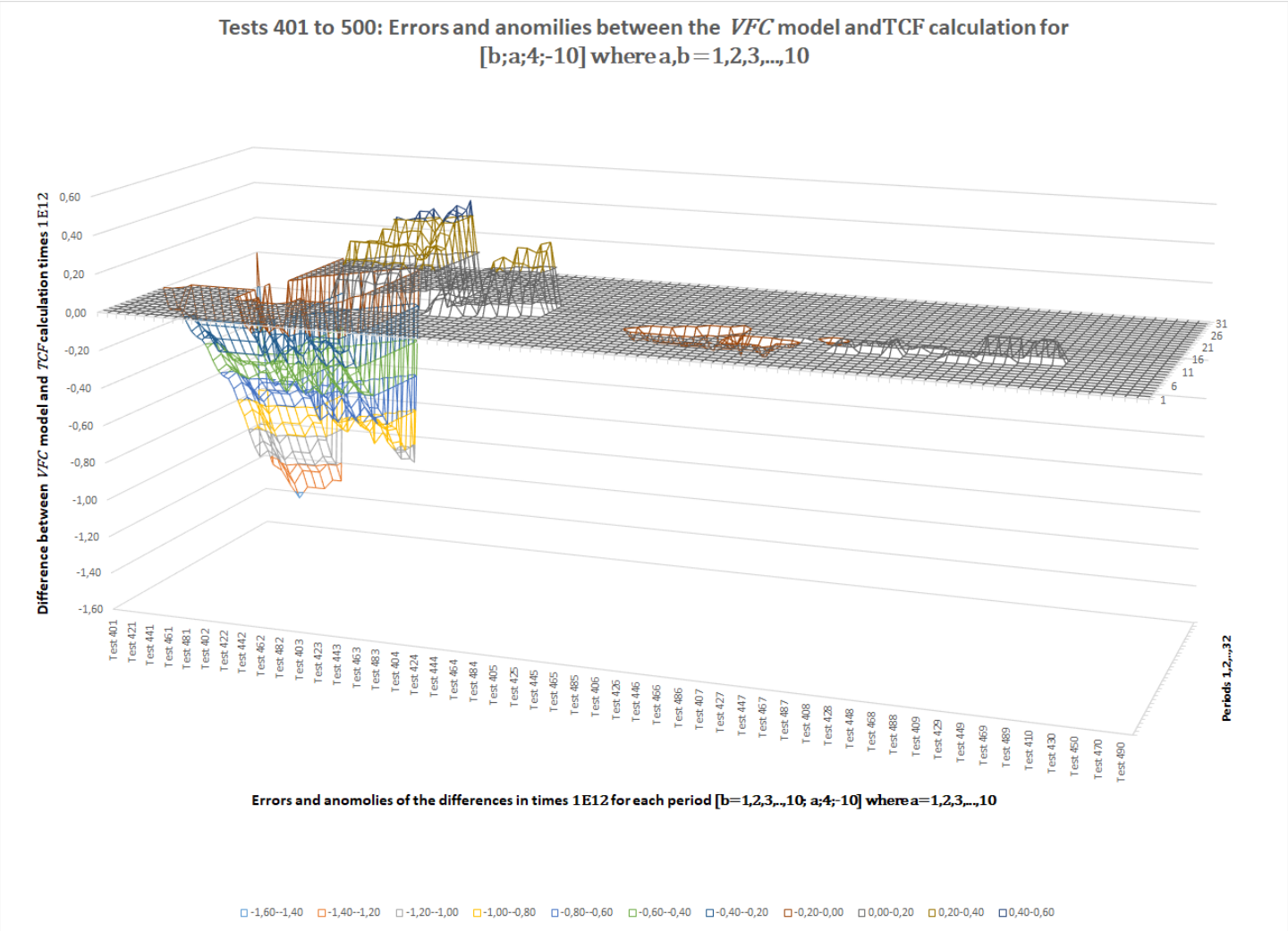

Figure A2.12: A: graphical representation of the errors and anomalies of tests 401 to 500, where each test was multiplied by 1E12, where only errors greater or lesser than one indicates an error 12 places after the decimal

Source: Author (2021)

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