



# Investment adjustment costs and growth dynamics<sup>☆</sup>

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## ARTICLE INFO

### JEL classification:

E22  
E58  
O42

### Keywords:

Investment adjustment costs  
Endogenous growth  
Inflation-targeting  
Growth dynamics

## ABSTRACT

We develop a monetary endogenous growth overlapping generations model characterized by investment adjustment costs as a negative function of productive government expenditures, and an inflation-targeting central bank. We show that growth dynamics arise, otherwise not possible in a standard monetary endogenous growth model with a money growth-rule and an exogenous adjustment cost parameter. Furthermore, hinging crucially on the strength of the response of the adjustment cost to productive public spending, single or multiple equilibria emerge, with the high-growth (low-growth) equilibrium in the latter case being stable (unstable), but locally indeterminate (locally determinate).

## 1. Introduction

In this paper, we develop a monetary endogenous growth overlapping generations (OLG) model characterized by an inflation-targeting central bank and capital adjustment costs, to analyze growth dynamics. The growth process is endogenized by allowing for a Romer (1986)-type production function with externalities, while money is introduced via a cash-in-advance constraint outlined in Lucas and Stokey (1983). Importantly, following Turnovsky (1996), in our model, productive government expenditures (for example, in infrastructure, research and development) are assumed to reduce the costs associated with investment, and facilitate the accumulation of new capital. In such a set-up, we show that interesting growth dynamics, otherwise not possible in a standard monetary endogenous growth model with a money growth-rule and an exogenous adjustment cost parameter, arise, hinging crucially on the strength of the response of the adjustment cost to productive public spending.

In the process, our paper adds to the existing literature of endogenous growth OLG models that have analyzed growth dynamics (see, for example, Gupta and Vermeulen (2010), Gupta (2011), Kudoh (2013), Gupta and Stander (2018), Gupta and Makena (2020), Bittencourt et al. (2022), Gupta et al. (2024)) through an alternative channel, namely, by incorporating the role of productive government expenditures to affect the investment adjustment costs, for the first time. In this regard, note that to create growth dynamics in OLG models, Gupta and Vermeulen (2010), Gupta (2011) and Gupta and Stander (2018) had to respectively introduced probability of survival as a function of private and public

investment, and lagged inputs respectively, while Kudoh (2013) had to rely on lump-sum, rather than income taxation, with Gupta and Makena (2020), Bittencourt et al. (2022) and Gupta et al. (2024) having to incorporate the role of inflation targeting, socio-political instability, and costs of climate change respectively. It must be pointed out that, these studies produce various types of growth dynamics ranging from convergent and divergent growth paths with and without oscillations, multiple equilibria with indeterminacy, and chaos.

The rest of the paper is organized as follows: Section 2 defines the economic setting of the theoretical model, with the solution detailing the process of the growth dynamics outlined in Section 3, and then Section 4 concludes the paper.

## 2. The model

### 2.1. The environment

We consider a growing economy consisting of an infinite sequence of two period-lived overlapping generations of individuals, the initial old generation, and an infinitely lived government. Let  $t = 1, 2, \dots$  index time. In each period, a new generation of unit measure is born. Each agent is endowed with one unit of labor when young and is retired when old. The initial old agents are endowed with  $k_1 > 0$  units of capital, and receive  $M_0$  units of fiat money from the central bank.

The production technology employed is motivated by Romer (1986), whereby a single final good is produced using the production function:

<sup>☆</sup> We would like to thank an anonymous referee for helpful comments. The usual disclaimer applies.

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$y_t = Ak_t^\alpha (n_t \bar{k}_t)^{1-\alpha}$  where  $A > 0$  is a technology parameter,  $0 < \alpha(1-\alpha) < 1$  represents the elasticity of output with respect to capital,  $k_t$ , labor,  $n_t$ , or aggregate capital,  $\bar{k}_t$ , respectively. The aggregate capital stock enters the production function because of the production externality, in the sense that labor productivity rises as the overall economy increases its stock of capital. In particular,  $k_t = \bar{k}_t$  in equilibrium.

In line with Turnovsky (1996), the capital evolution equation with a quadratic adjustment cost function is outlined as:  $k_{t+1} = (1 - \delta_k)k_t + i_{kt} - \frac{1}{2}c \frac{i_{kt}^2}{k_t}$ , where  $c = c(\frac{g_t}{k_t})$ , where  $i_{kt}$  is the capital investment,  $g_t$  is real government spending, with  $c' < 0$ , and  $c'' > 0$ , and  $\delta_k$  is the depreciation rate of capital. More specifically, the ratio of real government expenditures to capital lowers capital adjustment costs, though these reductions are assumed to occur at a declining rate.

Factor markets are perfectly competitive, and hence, the factors of production receive their respective marginal products. When maximizing profits, firms take the aggregate stock of capital,  $\bar{k}_t$ , as given, and we preclude labor-leisure choices by assuming that young agents supply their labor endowment inelastically in the labor market, i.e.,  $n_t = 1$ , we have:  $r_t = \alpha Ak_t^{\alpha-1} (n_t \bar{k}_t)^{1-\alpha} = \alpha A$ , and  $w_t = (1 - \alpha)Ak_t^\alpha n_t^{1-\alpha} \bar{k}_t^{1-\alpha} = (1 - \alpha)Ak_t^\alpha$ , wherein,  $r_t$  is the real gross return on capital, and  $w_t$  is the real wage at time  $t$ .

### 2.2. Households

Let  $c_{1t}$  and  $c_{2t}$  denote consumption when young and when old, respectively, for an agent corresponding to generation- $t$ . We assume that agents care about consumption only when old, i.e.,  $c_{1t} = 0$ , so that all income is saved, and there is no endogenous saving decision to make, thus allowing us to focus on portfolio choices of the individuals,

In this economy, there are two assets: money and capital, i.e., there are two means of saving from the perspective of portfolio choice. It is well-known that fiat money cannot coexist with an interest-bearing asset like capital in our case, unless money plays a role other than store-of-value. Given this, we follow Lucas and Stokey (1983), and utilize the cash-in-advance constraint to formalize the structure of the model economy with multiple financial assets. Specifically, consumption goods are divided into ‘‘cash goods’’ and ‘‘credit goods’’. Cash goods must be purchased with cash, and hence, agents intending to consume cash goods need cash in advance. At the same time, agents do not need cash to purchase credit goods. Let  $c_{mt}$  denote the amount of cash goods, and  $c_{nt}$  the same involving credit goods that are consumed when old. Then  $c_{2t} = c_{mt} + c_{nt}$  must hold, suggesting that the marginal rate of transformation between the two goods is unity, which, in turn, implies that the price  $p_t$  is the same for the two goods. The cash-in-advance constraint is given by:

$$p_{t+1}c_{mt} \leq M_t \tag{1}$$

where  $M_t$  is the nominal money balance. As per (1), a young agent must set aside cash in advance for purchasing cash goods when old.

Besides money, agents may hold capital. Given the income tax rate of  $\tau_t$  and gross nominal rate of return on capital investment being  $I_{t+1}$ , the budget constraints for young and old agents of generation  $t$  are:  $M_t + p_t i_{kt} = p_t(1 - \tau_t)w_t$ , and  $p_{t+1}c_{2t} = M_t + I_{t+1}p_t i_{kt}$ , respectively. Then, in real terms, these constraints are written as:

$$\frac{M_t}{p_t} + i_{kt} = (1 - \tau_t)w_t \tag{2}$$

$$c_{2t} = \frac{M_t}{p_{t+1}} + r_{t+1}i_{kt} \tag{3}$$

where  $r_{t+1} = \frac{I_{t+1}p_t}{p_{t+1}} = \frac{I_{t+1}}{\Pi_{t+1}}$ , with  $\Pi_{t+1}$  being the gross inflation rate at  $t + 1$ . Note that, the cash-in-advance constraint binds if and only if money is (weakly) dominated by capital in rates of return, i.e., if and only if  $I_{t+1} \geq 1$ .

Preferences are such that the agent cares only about old-age consumption and derives utility from both cash and credit goods. Given this, we specify the utility function as:

$$U(c_{mt}, c_{nt}) = [\phi(c_{mt})^{1-\rho} + (1 - \phi)(c_{nt})^{1-\rho}]^{\frac{1}{1-\rho}} \tag{4}$$

where  $0 < \phi < 1$ , and  $\rho > 0$ . The elasticity of substitution between  $c_{mt}$  and  $c_{nt}$  is  $1/\rho$ . As part of the utility-maximization problem, each young agent chooses  $c_{mt}$  and  $c_{nt}$  to maximize (4) subject to (1), (2), and (3). The first-order necessary conditions for the maximization problem requires  $U_1/U_2 = I_{t+1}$ , which gives the real money demand function:  $M_t/p_t = \mu(I_{t+1})(1 - \tau_t)w_t$ , and investment (savings) function:  $i_{kt} = [1 - \mu(I_{t+1})](1 - \tau_t)w_t$ , where  $\mu(I) = [1 + (\phi/(1 - \phi))I^{\frac{1-\rho}{\rho}}]^{-1}$ .<sup>1</sup>

### 2.3. Government

An infinitely-lived consolidated government, with two wings involving the Central Bank and the Treasury, purchases  $g_t$  units of consumption goods, and government expenditure (as percentage of the capital stock, to ensure that the ratio of all real variables are constant along a balanced growth path) is assumed to play a productive role in reducing investment adjustment costs, as discussed earlier. The government finances its productive consumption expenditure through the collection of income taxes and seigniorage income, and ensures a balanced-budget at each  $t$ :

$$g_t = \tau_t w_t + \frac{M_t - M_{t-1}}{p_t} \tag{5}$$

Also, to capture the recent practice of monetary policy making, the central bank follows a Taylor (1993)-type interest-rate rule rather than controlling the monetary base (i.e.,  $M_t = \theta_t M_{t-1}$ ), such that:

$$I_t = I^* \left( \frac{\Pi_t}{\Pi^*} \right)^\beta \tag{6}$$

where  $I^*$  and  $\Pi^*$  are the implicit targets for  $I_t$  and  $\Pi_t$  respectively, and  $\beta$  is a non-negative parameter capturing the degree of aggressiveness of monetary policy, such that  $\beta > (<) 1$  corresponds to active-(passive)-type monetary policy.

### 3. Growth dynamics

A monetary equilibrium under perfect foresight is defined as a set of sequences for real allocations  $m_t, k_t$ , relative prices  $r_t, \Pi_t$ , the initial price level  $p_1$ , and initial conditions  $k_1 > 0$ , and  $M_0 \geq 0$ , such that each household maximizes utility, asset and factor markets both clear, the budget constraint of the consolidated government is satisfied, with fiscal policy specifying  $g$  and  $\tau$ , and monetary policy setting  $I$ .

The equilibrium results in a growth path at time  $t + 1$  for the gross growth rate based on the capital evolution equation with adjustments costs, whereby  $\Omega_{t+1} = f(\Omega_t)$ . Formally, we have:

$$\Omega_{t+1} = (1 - \delta_k) + B_t \left[ 1 - \frac{1}{2} B_t c \left( \frac{g_t}{k_t} \right) \right] \tag{7}$$

where  $\frac{g_t}{k_t} = A(1 - \alpha)[\tau_t + (1 - \frac{1}{\Omega_t \Pi_t})(1 - \tau_t)\gamma(I_{t+1})]$ ;  $B_t = (1 - \tau_t)A(1 - \alpha)(1 - \gamma(I_{t+1}))$ ;  $\gamma(I_{t+1}) = \left[ 1 + \left( \frac{1 - \phi}{\phi} \right)^\frac{1}{\rho} I_{t+1}^{\frac{1 - \rho}{\rho}} \right]^{-1}$ ;  $I_t = (\alpha A \Pi^*)^{\frac{\beta}{\beta - 1}} I^{*\frac{1}{1 - \beta}}$ ; and  $\Pi_t = \frac{\alpha A}{(\alpha A \Pi^*)^{\frac{\beta}{\beta - 1}} I^{*\frac{1}{1 - \beta}}}$ .<sup>2</sup>

<sup>1</sup> It is easy to show that  $\mu(I)$  satisfies  $0 < \mu(I) < 1$ ,  $\mu'(I) < 0$  under  $\rho \in (0, 1)$ , and  $I\mu'(I)/\mu(I) = -[1 - \mu(I)](1 - \rho)/\rho$ . Thus, as the nominal interest rate increases, the household substitutes away from money to reduce its demand. But, at the same time, an increase in the nominal rate raises the earnings from the interest-bearing capital investment, which, in turn, raises money demand through the income effect. The former dominates the latter if  $0 < \rho < 1$ , in which case the elasticity of substitution between  $c_{mt}$  and  $c_{nt}$  is  $1/\rho > 1$ . When  $\rho > 1$ , the elasticity is low, and the income effect ends up being dominant.

<sup>2</sup> Note that,  $r_t = \alpha A$ , so the Fisher-equation would imply that  $I_t = \alpha A \Pi_t = \alpha A \Pi^*(I_t/I^*)^{1/\beta}$ . Hence,  $I_t = (\alpha A \Pi^*)^{\beta/(\beta - 1)} I^{*1/(1 - \beta)}$ . Therefore, an interest-rate rule is equivalent to a strict-targeting of the nominal interest rate, which, in turn, is also equivalent to a strict-targeting of the rate of inflation, with this occurring because the real interest rate is constant in this one-sector growth model.

Without loss of generality, let the capital depreciate completely across the periods:  $\delta_k = 1$ ; substitution and income effects cancels out:  $\rho = 1$ ; the weight of cash and credit goods are equal in the utility function:  $\phi = \frac{1}{2}$ ; monetary policy follows a fixed interest rate-rule:  $\beta = 0$ ; and government spending is purely financed through seigniorage:  $\tau_t = 0$ , then Equation (7) can be simplified as:

$$\Omega_{t+1} = f(\Omega_t) = \frac{A(1-\alpha)}{2} \left[ 1 - \frac{A(1-\alpha)}{4} c \left( \frac{g_t}{k_t} \right) \right], t = 0, 1, 2, \dots \quad (8)$$

where  $g_t/k_t = A(1-\alpha)[1 - (\alpha A)/(\Omega_t I^*)]/2 (= h(\Omega_t))$ . It is easy to see that an increase in  $\Omega_t$  would lead to a rise in  $g_t/k_t$ , due to collection of more seigniorage, which, in turn, will reduce the adjustment cost parameter:  $c(g_t/k_t)$ , given that  $c'(g_t/k_t) < 0$ , and, hence, produce an increase in  $\Omega_{t+1}$  due to higher capital accumulation, implying that  $f'(\Omega_t) > 0$ . Moreover, since  $c''(g_t/k_t) > 0$ , it translates into  $f''(\Omega_t) < 0$ . So  $f(\Omega_t)$  is strictly increasing and strictly concave.

At the same time, it must be realized that, if instead of inflation-targeting, the central bank pursued a money growth rate rule, then  $(g_t/k_t) = A(1-\alpha)[(\theta_t-1)/\theta_t]/2$ . In other words, while having government expenditures impacting adjustment costs in our model, i.e.,  $c(g_t/k_t)$ , is a necessary condition for producing growth dynamics, inflation-targeting serves as the sufficient requirement in this regard.

To characterize the growth dynamics in detail, let us define  $\Omega_0$  as the initial gross growth rate, and then we distinguish between three cases:

Case (I):  $\Omega_1 > \Omega_0$ , there is a unique equilibrium. To see this, note, on the one hand, that  $\Omega_1 > \Omega_0$  means  $f(\Omega_0) > \Omega_0$ . On the other hand,  $\lim_{\Omega_t \rightarrow \infty} f(\Omega_t) = \frac{A(1-\alpha)}{2} \left[ 1 - \frac{A(1-\alpha)}{4} c \left( \frac{A(1-\alpha)}{2} \right) \right]$ , and therefore  $f(\Omega_t) < \Omega_t$  when  $\Omega_t$  is sufficiently large. By the intermediate value theorem, there exists an  $\Omega$  such that  $f(\Omega) = \Omega$ , and because of the strict concavity of  $f$ , such  $\Omega$  must be unique.

Case (II):  $\Omega_1 < \Omega_0$ . In this case, the number of equilibria depends on the derivative of the function  $c(g/k)$ . Specifically, direct calculation shows:  $f'(\Omega_t) = \frac{1}{\lambda_t} c' \left( \frac{g_t}{k_t} \right)$ , where, for notational convenience, we define:  $\frac{1}{\lambda_t} = -\frac{[A(1-\alpha)]^3 \alpha A}{16 I^* \Omega_t^2}$ . If  $c'(g/k)_0 \geq \lambda_0$  and hence  $f'(\Omega_0) \leq 1$ , it follows from the strict concavity of  $f$  that  $f(\Omega_t) < \Omega_t$  for all  $t$ , and hence there is no equilibrium. Now suppose  $c'(g/k)_0 < \lambda_0$ , so that  $f'(\Omega_0) > 1$ . As  $\lim_{\Omega_t \rightarrow \infty} f'(\Omega_t) = 0$ , there exists a unique  $\Omega^*$  such that  $f'(\Omega^*) = 1$  or, equivalently,  $c' \left( \frac{g}{k} \right)^* = \lambda^*$ , where  $\left( \frac{g}{k} \right)^* = \frac{A(1-\alpha)}{2} \left( 1 - \frac{\alpha A}{\Omega^* I^*} \right)$ , and  $\frac{1}{\lambda^*} = -\frac{[A(1-\alpha)]^3 \alpha A}{16 I^* (\Omega^*)^2}$ .

The number of equilibria depends on the function value of  $f$  at  $\Omega^*$ . If  $f(\Omega^*) = \Omega^*$  or, equivalently,  $c \left( \frac{g}{k} \right)^* = \eta^*$ , where  $\eta^* = \frac{4[A(1-\alpha)-2\Omega^*]}{[A(1-\alpha)]^2}$ , there is a unique equilibrium, otherwise if  $c \left( \frac{g}{k} \right)^* > \eta^*$ , there is no equilibrium, and if  $c \left( \frac{g}{k} \right)^* < \eta^*$ , there are two equilibria.

Case (III):  $\Omega_1 = \Omega_0$ . If  $c'(g/k)_0 \geq \lambda_0$ , there is a unique equilibrium, which is  $\Omega_0$  itself. If  $c'(g/k)_0 < \lambda_0$ , there are two equilibria.

When we have two equilibria under the conditions defined in Cases (II) and (III) above, given  $c'(g_t/k_t) < 0$  and  $c''(g_t/k_t) > 0$ , translating into  $f'(\Omega_t) > 0$  and  $f''(\Omega_t) < 0$ , under perfect foresight, the low-growth equilibrium is unstable (as the locus  $f(\Omega_t)$  cuts the 45° line from below), while the high-growth equilibrium is stable (as the locus  $f(\Omega_t)$  cuts the 45° line from above). In addition to the situation where two balanced-growth equilibria exists, there are infinitely many rational expectations paths leading to the stable balanced-growth equilibrium from any given initial condition  $k_1$ , making high-growth (low-growth) equilibrium locally indeterminate (locally determinate). The main reason behind this indeterminacy is the inability of the model to supply the initial conditions. To see this clearly, let us rewrite the dynamics  $\Omega_{t+1} = f(\Omega_t)$ , and as  $k_{t+1}/k_t = f(k_t/k_{t-1})$ , which is a second-order difference equation of  $k_{t+1}$ , and, hence, to start the economy, we require two initial conditions:  $k_1$  and  $k_2$ , but only  $k_1$  can be provided exogenously.

Moreover note, given that,  $I^* = \alpha A \Pi^*$  from the Fisher-equation,  $g_t/k_t$  can be re-written as:  $A(1-\alpha)[1 - 1/(\Omega_t \Pi^*)]/2$ . Understandably,

an increase in the inflation target of  $\Pi^*$  will increase the revenue collection of the government through seigniorage, i.e.,  $g_t/k_t$  will increase, and, in turn, reduce investment adjustment costs (as  $c(g_t/k_t)$  will fall) to shift the  $f(\Omega_t)$  locus upwards. Hence, from a policy perspective, unless we have the case of a unique equilibrium, an increase in the inflation target does not necessarily guarantee the attainment of a higher steady-state growth rate, as under the two-equilibria case, the high-growth steady-state, though stable, is indeterminate.

#### 4. Conclusion

In this paper, we develop a monetary endogenous growth OLG model characterized by an inflation-targeting central bank, and investment adjustment costs as a negative function of productive government expenditures. In such a set-up, We show that interesting growth dynamics arise, otherwise impossible in a monetary endogenous growth OLG model with a money growth-rule and an exogenous adjustment cost parameter. Furthermore, depending on the strength of the response of the adjustment cost to productive public spending, single or multiple equilibria emerge, with the high-growth (low-growth) equilibrium in the latter situation being stable (unstable), but locally indeterminate (locally determinate). A higher inflation-target increases the collection of government revenue via seigniorage, and hence, is unambiguously growth-enhancing under the single steady-state case, but this is not guaranteed when we have multiple equilibria.

Empirically, the importance of adjustment costs in the process of investment (Groth and Khan, 2010), and the role of productive public expenditures in driving economic growth (Irmen and Kuehnel, 2009), have been well-recognized. However, the fact that public expenditures can boost investment via reduction of adjustment costs have not been investigated in the data. And, crucially, as we show theoretically, if this is indeed the case, then interesting growth dynamics contingent on the elasticity of the abovementioned link can lead to possible multiple equilibria and unambiguous growth-enhancing impact of productive public expenditures.

#### Data availability

No data was used for the research described in the article.

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