JOURNAL OF APPLIED PHYSICS 122, 064902 (2017)

The constructal size of a heat exchanger

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(Received 19 June 2017; accepted 23 July 2017; published online 9 August 2017)

The architecture of heat exchangers is a classical subject that has been studied extensively in the past. In this paper, we address the fundamental question of what the size of the heat exchanger should be, in addition to what architectural features it should have. The answer to the size question follows from the tradeoff between (1), the useful power lost because of heat transfer and fluid flow and (2), the power destroyed during transportation, manufacturing, and maintenance. Changes in heat exchanger size induce changes in the opposite sign in the power requirements (1), and (2). This fundamental tradeoff regarding size is illustrated by considering one side of a heat exchanger (one flow passage) in laminar flow and in fully rough turbulent flow, with several duct cross sectional shapes and arrays of channels in parallel. The size tradeoff is present in heat exchanger applications across the board, from vehicles to stationary power plants. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.4991014]

NOMENCLATURE

a	optimal size of duct cross section, A_f, Eq. (18) (m^2)
A	surface area (m^2)
A_f	area of duct cross-section (m^2)
b	slenderness of flow passage, Eq. (19)
c_{1-5}	constant factors
c_P	specific heat (J kg^{-1} K^{-1})
D_h	hydraulic diameter (m)
D_{ref}	reference diameter (m)
$\tilde{D}$	dimensionless diameter
f	friction factor, Eq. (2)
g	gravitational acceleration (m s^{-2})
h	heat transfer coefficient (W m^{-2} K^{-1})
L	flow length (m)
L_{ref}	reference flow length (m)
$\tilde{L}$	dimensionless flow length (m)
M	mass (kg)
$\dot{m}$	mass flow rate (kg s^{-1})
$\dot{m}_{ref}$	reference mass flow rate (kg s^{-1})
$\tilde{m}$	dimensionless mass flow rate
N	number of channels
p	perimeter, m
Po	Poiseuille constant
$\dot{Q}$	heat transfer rate (W)
$\dot{Q}_{ref}$	reference heat transfer rate (W)
$\tilde{Q}$	dimensionless heat transfer rate
r	dimensionless factor, Eq. (10)
Re	Reynolds number (UD_h/ν)
St	Stanton number
$\dot{S}_{gen}$	entropy generation rate (W K^{-1})
t	wall thickness (m)
T	temperature (K)
U	mean fluid velocity (m s^{-1})
V	vehicle speed (m s^{-1})
Vol	volume (m^3)
W	width (m)
$\dot{W}$	power (W)

Greek symbols

ΔP	pressure drop (Pa)
ΔT	temperature difference (K)
μ	viscosity (kg s^{-1} m^{-1})
ν	kinematic viscosity (m^2 s^{-1})
ρ	fluid density (kg m^{-3})
ρ_s	solid density (kg m^{-3})

I. INTRODUCTION

Heat exchangers are an established technology for power generation, refrigeration, and heat transfer engineering in general. They also serve as one of the pillars of thermal science. Books and articles on heat exchangers appear regularly,[1–10] and they document the permanent interest in this subject.

The fundamental question in heat exchanger design is how the flow configuration affects the performance of the flow component. The performance has a thermal aspect (the degree of thermal contact, e.g., the effectiveness) and a fluid-mechanical aspect (the pressure drop or the pumping power). The two aspects of performance are pursued separately or together, as in the search for the reduced entropy generation rate or exergy destruction rate.[11,12] Inside a specified size (volume, weight, or cost), the designer's search is for heat and fluid flow configuration—the aspect ratio and the shape of the profile of each flow passage—that offers improved thermal and mechanical performance. As we will show in

Sec. VI, this traditional activity concerns the designs that occupy the "shape" plane in Fig. 6.

In this paper, we propose a fundamental step beyond the traditional course and question the size itself. What size should the heat exchanger have in order to serve best the larger installation (a vehicle, for example) that must use the heat exchanger as a component? Along with this question, we will consider the configuration (the aspect ratios) and the thermo-fluid performance as well; however, the new aspect is the size. It is the problem of determining the size, and the effect that the size has on performance. We show how and why larger heat exchangers perform better, and as a consequence, we unveil the physics basis for the phenomenon of economies of scale[13] in heat exchangers.

A fundamental step is most clear when presented in the simplest setting. As their name indicates, heat exchangers are devices that facilitate the transfer of heat between two or more fluid streams, which are usually separated by thin solid walls (heat transfer surfaces). The wall acquires a temperature distribution that results from the thermal interaction between the mating streams. The simple setting chosen for the following analysis consists of only one of the streams and its thermal contact and friction with the wall, which has an assumed temperature. This simple model applies to one side of a two-stream heat exchanger and also to one-stream cooling systems, as in packages of electronics. The objective of the following analysis is not a specific application; rather, it is to unveil the size and the relationship between the size and global performance.

II. SIZE

The size of a flow component in a complex flow system can be determined from the tradeoff between the losses associated with the flow component and the losses associated with moving the complex flow system on the landscape.[12,14] This tradeoff is illustrated in Fig. 1. We demonstrated the existence of the tradeoff by determining the diameter of a single tube with a specified fluid mass flow rate and the surface of a heat transfer area with a specified heat transfer rate. In this article, we consider the more complex configuration where both the fluid stream and the heat current are present. As in all problems of constructal design, the configuration (shape and size) is the unknown that must be determined.

One stream in a heat exchanger has the specified mass flow rate $\dot{m}$ (Fig. 2). The stream receives the specified heat transfer rate $\dot{Q}$ from the solid surface that it bathes. The flow path length is L, and the flow cross sectional area is A_f. The wetted perimeter of A_f is p, and therefore, the heat transfer area is $A = pL$. The hydraulic diameter associated with A_f and p is $D_h = 4A_f/p$.

There are three sources of thermodynamic losses (irreversibility, entropy generation, and exergy destruction) in the operation of the heat exchanger. One is the pumping power

$$\dot{W}_1 = \frac{\dot{m}\Delta P}{\rho}, \quad (1)$$

where ΔP is the pressure drop along the flow path length L

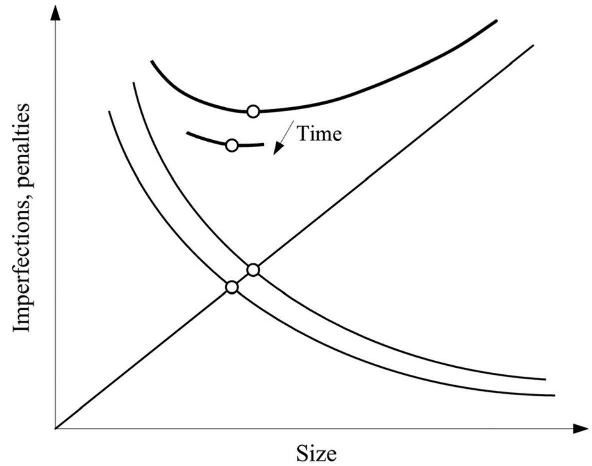

FIG. 1. Every flow component has a characteristic size, which emerges from two conflicting trends. The useful energy dissipated because of the imperfection of the component decreases as the component size increases. The useful energy spent by the efficient system (vehicle and animal) increases with the component size. The sum of the two penalties is minimum when the component size is finite, at the intersection between the two penalties. In time, the component evolves toward smaller sizes because it improves and its penalty (the descending curve) slides downward.

$$\Delta P = f\,\frac{4L}{D_h}\,\frac{1}{2}\rho U^2, \quad (2)$$

and the mean flow velocity is $U = \dot{m}/(\rho A_f)$. The definition of friction factor f used in this paper is given by Eq. (2). The f value generally decreases with the Reynolds number (Re) based on the duct hydraulic diameter D_h. If the flow is fully developed laminar, the friction factor is equal to Po/Re, where Po is the Poiseuille constant (for example, Po = 16 for a round flow cross section). If the flow is fully developed turbulent, and if the duct wall is smooth, f decreases considerably more slowly (as $Re^{-1/4}$ or $Re^{-1/5}$) as Re increases. Many f(Re) correlations exist, for example, Refs. 7 and 15. For the sake of analytical simplicity, in the following analysis, we assume that the duct is smooth, the flow regime is fully developed turbulent, and the Re interval is sufficiently narrow, such that for this interval, the f value for Eq. (2) is approximated by a constant equal to the friction factor averaged over that interval. For example, according to the classical Moody chart (Ref. 15, p. 373), in the Re interval $10^4 - 10^5$, the f value decreases from 0.0075 to 0.0045, which recommends approximately $f \cong 0.006$ for the assumed constant f. In summary, the pumping power is

$$\dot{W}_1 = c_1 \frac{L}{D_h A_f^2}, \quad (3)$$

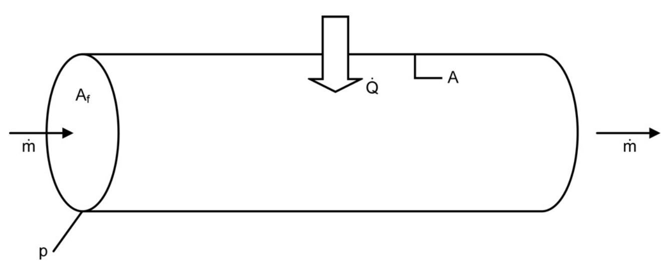

FIG. 2. General notation for a duct with a specified mass flow rate and a heat transfer rate.

where c_1 is a constant parameter:

$$c_1 = 2f\,\frac{\dot{m}^3}{\rho^2}. \tag{4}$$

Another source of imperfection is the heat transfer $\dot{Q}$ through the surface A

$$\dot{W}_2 = T\,\dot{S}_{gen,\dot{Q}}, \tag{5}$$

where T is the absolute temperature of the environment (and of the same order as the temperature of the heat exchanger surface), and $\dot{S}_{gen,\dot{Q}}$ is the entropy generation rate due to the flow of heat across the temperature gap ΔT, which is the temperature difference between the average surface temperature and the bulk temperature of the fluid,

$$\dot{S}_{gen,\dot{Q}} = \frac{\dot{Q}}{T-\Delta T} - \frac{\dot{Q}}{T} \cong \frac{\dot{Q}\Delta T}{T^2}, \tag{6}$$

where in addition we assumed that $\Delta T \ll T$. The temperature difference that drives the heat current $\dot{Q}$ is

$$\Delta T = \frac{\dot{Q}}{hpL}, \tag{7}$$

where $h = \rho c_P U\,St$, which is valid for single phase flow, and here, St is the Stanton number. Condensers and boilers require a different formulation. The Stanton number St is treated as a constant based on the same argument as the one made for f under Eq. (2). Eliminating ΔT and $\dot{S}_{gen,\Delta T}$ from Eqs. (5)–(7), we conclude that

$$\dot{W}_2 = c_2\frac{D_h}{L}, \tag{8}$$

where c_2 is a second constant parameter:

$$c_2 = \frac{\dot{Q}^2}{4\,\dot{m}\,c_P\,T\,St}. \tag{9}$$

Finally, there is a power loss associated with carrying the heat exchanger on the horizontal landscape with the specified speed V,

$$\dot{W}_3 = r\,Mg\,V, \tag{10}$$

where Mg is the weight of the heat exchanger, and r is a dimensionless factor that accounts for the medium in which the vehicle moves (water $r \sim 1$, air $r \sim 10^{-1}$, and land $10^{-1} < r < 1$)[14] and for the current state of the technology of transportation. The mass of the heat exchanger is due to the fluid of density ρ and the solid walls of density ρ_s and thickness t

$$M = \rho A_f L + \rho_s pLt. \tag{11}$$

When the fluid is a gas, $\rho_s \gg \rho$ and M is dominated by the second term, and $\dot{W}_3$ becomes

$$\dot{W}_3 \cong c_3\,A_f\,\frac{L}{D_h}, \tag{12}$$

where

$$c_3 = 4r\,g\,V\,\rho_s\,t. \tag{13}$$

Taken together, Eqs. (3), (8), and (12) yield the total expenditure of power $\dot{W} = \dot{W}_1 + \dot{W}_2 + \dot{W}_3$, which can be expressed as

$$\dot{W} = \frac{L}{D_h}\left(\frac{c_1}{A_f^2} + c_3 A_f\right) + c_2\,\frac{D_h}{L}. \tag{14}$$

There are two degrees of freedom, A_f and D_h/L, and $\dot{W}$ can be minimized with respect to both, by solving $\partial\dot{W}/\partial A_f = 0$ and $\partial\dot{W}/\partial(D_h/L) = 0$. The minimum-$\dot{W}$ design is represented by the constants a and b reported below

$$(A_f)_{opt} = \left(2\,\frac{c_1}{c_3}\right)^{1/3} = a, \tag{15}$$

$$\left(\frac{D_h}{L}\right)_{opt} = c_1^{1/6}\,c_2^{-1/2}\,c_3^{1/3}\,\left(2^{-2/3} + 2^{1/3}\right)^{1/2} = b. \tag{16}$$

The minimum destruction of useful power is

$$\dot{W}_{min} = \frac{1}{b}\left(\frac{c_1}{a^2} + c_3 a\right) + c_2 b. \tag{17}$$

The constants are shorthand for the following groups of physical parameters:

$$a = \left(\frac{f\,\dot{m}^3}{r\,\rho^2\rho_s g\,V\,t}\right)^{1/3} = (A_f)_{opt}, \tag{18}$$

$$b = 2^{3/2}\,3^{1/2}\,f^{1/6}\,\frac{\dot{m}}{\dot{Q}}\,(c_P T\,St)^{1/2}\left(\frac{\rho_s}{\rho}\,rgVt\right)^{1/3} = \left(\frac{D_h}{L}\right)_{opt}. \tag{19}$$

These groups indicate that when other parameters are fixed, $(A_f)_{opt}$ varies proportionally with $\dot{m}$ and $(D_h/L)_{opt}$ is proportional to $\dot{m}/\dot{Q}$. In particular, the proportionality between $(A_f)_{opt}$ and $\dot{m}$ means that the mean fluid speed ($U_{opt} = \dot{m}/\rho A_{f,opt}$) does not depend on the fluid flow and heat transfer duties of the system ($\dot{m},\ \dot{Q}$)

$$U_{opt} = \left(\frac{r}{f}\cdot\frac{\rho_s}{\rho}\,g\,Vt\right)^{1/3}. \tag{20}$$

We discover in this way that the internal flow should be faster in faster vehicles (larger V), as in the case of blood flow in the whale versus the minnow. The internal flow should also be faster in media that are more resistive to vehicle movement (larger r), such as movement in water versus movement in air. Furthermore, if we combine Eqs. (17)–(19) and then use the definitions of c_1, c_2, and c_3, we find that the minimal power required by heat exchanger operation is

$$\dot{W}_{min} = 2^{2/3}\,3^{1/2}\left(\frac{f}{2}\right)^{1/6}\,\dot{Q}\,(c_P\,T\,St)^{-1/2}\left(rgVt\,\frac{\rho_s}{\rho}\right)^{1/3}. \tag{21}$$

In conclusion, the overall power requirement scales in proportion with the heat transfer rate duty $\dot{Q}$.

When the fluid is a liquid and ρ is comparable with ρ_s, M is dominated by the first term in Eq. (11), and $\dot{W}_3$ becomes

$$\dot{W}_3 = c_4 A_f L, \tag{22}$$

where $c_4 = rg\ V\rho$. The results (14), (18), and (19) are replaced by

$$\dot{W} = \left(\frac{c_1}{A_f^2} + c_4 A_f D_h\right)\frac{L}{D_h} + c_2\frac{D_h}{L}, \tag{23}$$

$$(A_f)_{opt} = \left(\frac{f\ \dot{m}^3}{r\rho^3 g\ V D_h}\right)^{1/3}, \tag{24}$$

$$\left(\frac{D_h}{L}\right)_{opt} = 2^{3/2}\ 3^{1/2}\ f^{1/6}\ \frac{\dot{m}}{\dot{Q}}\ (c_P T\ St)^{1/2}\left(\frac{\rho_s}{\rho}\ rg\ VD_h\right)^{1/3}. \tag{25}$$

The only difference between Eqs. (18) and (19) and Eqs. (24) and (25) is that the wall thickness t is replaced by D_h, which now plays the role of an additional degree of freedom. Unlike in Eq. (19), now the optimal duct length L is proportional to $D_h^{2/3}$.

Figure 3 shows the D_h/L value for minimum $\dot{W}$ as a function of $\tilde{m}$ for different values of $\tilde{Q}$ for circular tubes, where $\tilde{m} = \dot{m}/\dot{m}_{ref}$, $\tilde{Q} = \dot{Q}/\dot{Q}_{ref}$, $\dot{m}_{ref} = 1$ kg/s, and $\dot{Q}_{ref} = 1$ kW. Figure 4 shows the diameter $\tilde{D}$ and the heat exchanger length $\tilde{L}$ for minimum $\dot{W}$ as a function of $\tilde{m}$ for different values of $\tilde{Q}$, where $\tilde{D} = D/D_{ref}$, $\tilde{L} = L/L_{ref}$, $D_{ref} = 1$ m, and $L_{ref} = 1$ m. The working fluid was air at 300 K. The wall material was copper with the thickness t = 3 mm. The speed V was set at 1 m/s.

III. ROUND OR POLYGONAL CROSS SECTIONS

The configuration of the heat exchanger is now complete. The configuration means aspect ratios and sizes. For example, if the shape of the duct cross-section is round or a regular polygon, D_h is the diameter of the inscribed circle[15] [Fig. 5(a)]:

$$A_f \cong D_h^2, \tag{26}$$

and Eqs. (15) and (16) yield

$$D_h \cong a^{1/2}, \qquad L \cong \frac{a^{1/2}}{b}. \tag{27}$$

Equations (26) and (27) are exact when the duct cross section is square. If the cross section is flat (elongated), for example, as the channel between parallel plates of width W, Fig. 5(b), then

$$A_f \cong \frac{1}{2} D_h W, \tag{28}$$

and Eqs. (15) and (16) yield

$$D_h = \frac{2a}{W}, \qquad L = \frac{2a}{bW}. \tag{29}$$

The duct volume that corresponds to designs (27) and (29) is, respectively,

$$Vol = \frac{a^{3/2}}{b}, \qquad Vol = \frac{2a^2}{bW}. \tag{30}$$

IV. CHANNELS IN PARALLEL

If the wetted surface is more complicated, for example, with channels in parallel [Fig. 5(c)] or with one channel perpendicular to other tubes or fins, the hydraulic diameter is smaller than in Eq. (26). We write

$$A_f \cong N D_h^2, \tag{31}$$

where N is the number of parallel channels or the apparent "pores" visible in the cross section A_f.

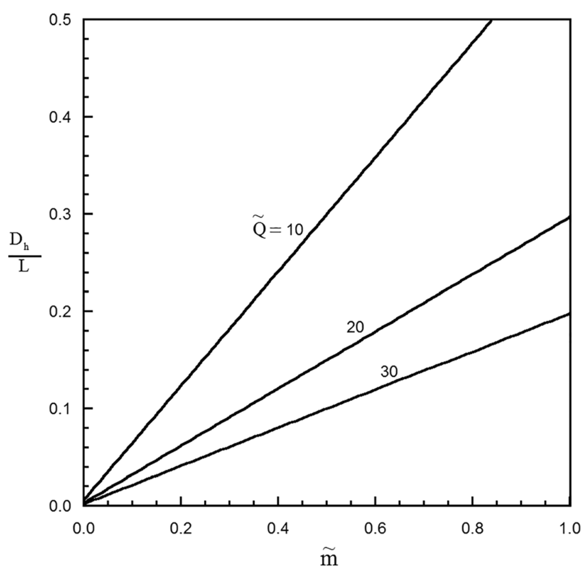

FIG. 3. The optimal D_h/L ratio for different heat transfer rates for air flowing in a pipe at T = 300 K and St = 0.0015.

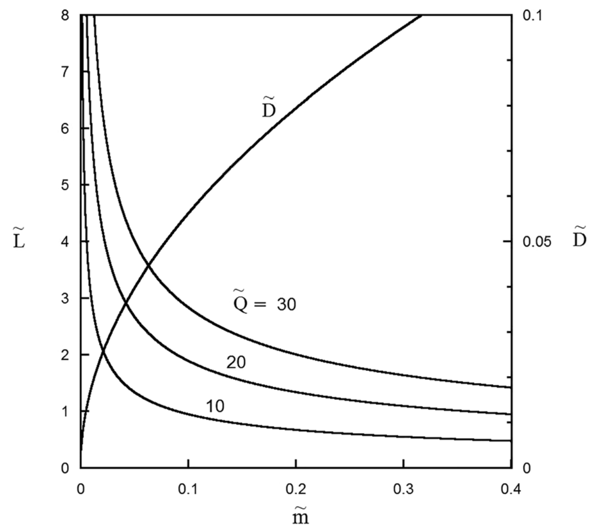

FIG. 4. The optimal $\tilde{D}$ and $\tilde{L}$ values for different heat transfer rates, for air flowing in a pipe at T = 300 K and St = 0.0015.

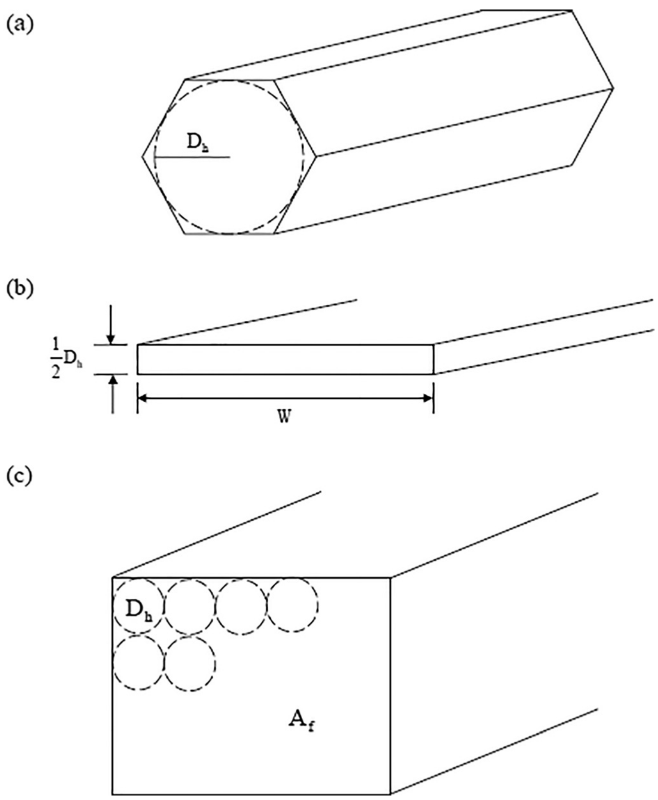

FIG. 5. (a) Duct with a cross section shaped as a regular polygon. (b) Duct with a flat rectangular cross section. (c) Cross section consisting of several channels in parallel.

In such designs, Eqs. (27) are replaced by

$$D_h \cong \left(\frac{a}{N}\right)^{1/2}, \qquad L \cong \frac{a^{1/2}}{bN^{1/2}}. \tag{32}$$

The flow volume

$$\mathrm{Vol} = A_f L \cong \frac{a^{3/2}}{bN^{1/2}} \tag{33}$$

decreases as N increases, i.e., as the details of the internal structure become finer. In the same direction of miniaturization, the minimal power required by the heat exchanger remains constant (independent of N), cf. Eq. (17).

V. LAMINAR FLOW

The formulation developed so far can be extended to additional classes of applications. For example, as heat exchangers evolve toward miniaturization, from compact heat exchangers to microscales (cf. Ref. 14, Fig. 4.2), the flow regime prevails as laminar throughout the flow architecture. For such applications, the analysis of Sec. II can be repeated by starting with Eq. (2), in which the friction factor for laminar flow is

$$f = \frac{Po}{Re} \tag{34}$$

where Po is the Poiseuille constant (Po = 16 for a round cross section and Po = 24 for a flat cross section). The new feature in the analysis is that Eq. (3) is replaced by

$$\dot{W}_1 = c_5 \frac{L}{D_h^2 A_f}, \tag{35}$$

where

$$c_5 = \frac{4\nu\,Po\,\dot{m}^2}{\rho}. \tag{36}$$

Because the dependence of $\dot{W}_1$ on L, D_h, and A_f in Eq. (35) is qualitatively the same as in Eq. (3), the ensuing analysis that replaces Eqs. (15)–(21) reveals the existence of the same kind of tradeoffs, optimal A_f and D_h/L, and minimal overall power requirement, $\dot{W} = \dot{W}_1 + \dot{W}_2 + \dot{W}_3$.

VI. STATIONARY APPLICATIONS

The design domain occupied by heat exchangers is considerably wider than the applications analyzed in Secs. II–V. The common features of these applications are that the heat exchangers belong to vehicles that move on the landscape with the speed V. At least as prevalent are the stationary heat exchangers, such as in plants for power, refrigeration, and air conditioning. For these, the power required for transporting the heat exchanger, Eq. (10), does not hold. There is however an additional power requirement ($\dot{W}_3$) that is of economic origin, and just like in Eq. (10), it is greater when the heat exchanger is larger.

This contribution to $\dot{W}_3$ is a function of factors that vary from one installation to the next, stationary or not. Examples are the power spent on manufacturing and assembling the heat exchanger, the power spent on bringing the heat exchanger components to the site, and the power spent on maintaining

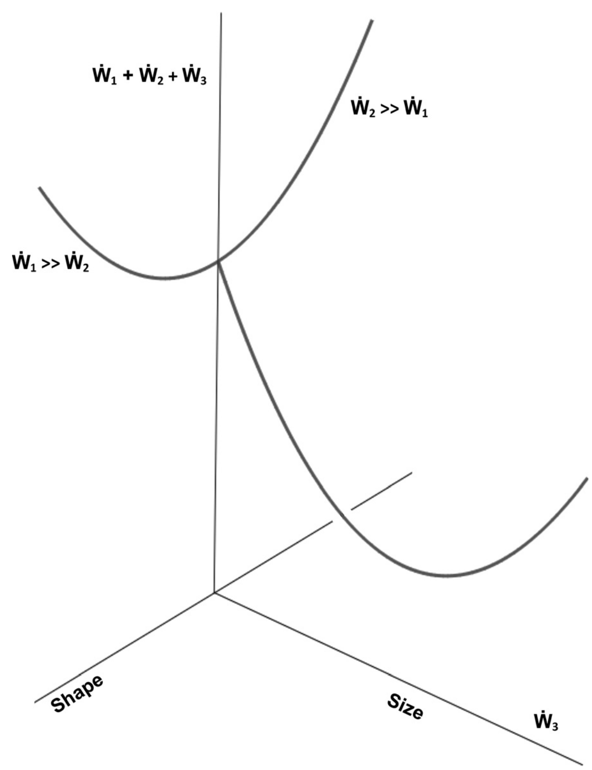

FIG. 6. A flow system has shape and size. The shape is the tradeoff between heat transfer and fluid flow irreversibilities.[11,12] The size is the tradeoff between the sum of the first two irreversibilities and the irreversibility associated with carrying and constructing the flow system.

and cleaning periodically the heat exchanger. All these features of stationary heat exchanger design and operation contribute to the new $\dot{W}_3$ requirement such that each contribution increases monotonically with the size of the heat exchanger.

In conclusion, the new $\dot{W}_3$ increases monotonically with the size M, and this means that the qualitative impact of the new $\dot{W}_3(M)$ function has the same character as in the analysis developed for moving heat exchangers. Said another way, the tradeoffs unveiled in Secs. II–V are also present in stationary applications, albeit with a different (empirical) expression in place of Eq. (10).

Figure 6 is a qualitative summary of the fundamental step made in this paper. Any flow configuration has two basic characteristics, shape and size. The overall irreversibility of the heat exchanger $(\dot{W}_1 + \dot{W}_2 + \dot{W}_3)$ can be minimized in two ways: in the "shape" plane at fixed size, (i) by balancing the heat transfer and fluid flow irreversibilities, and (ii) by exploring the "size" dimension of the design space, as was anticipated in Fig. 1. The present results indicate the best combination of the shape and size that endow the flow configuration with minimum thermodynamic losses.

ACKNOWLEDGMENTS

The work by Professor Bejan's was supported by the U.S. National Science Foundation.

[1] *Heat Exchangers: Thermal-Hydraulic Fundamentals and Design*, edited by S. Kakac, A. E. Bergles, and F. Mayinger (Hemisphere, New York, 1981).
[2] W. M. Kays and A. L. London, *Compact Heat Exchangers*, 3rd ed. (McGraw-Hill, New York, 1984).
[3] A. P. Fraas, *Heat Exchanger Design*, 2nd ed. (Wiley, New York, 1989).
[4] S. Kakaç, *Boilers, Evaporators and Condensers* (Wiley, New York, 1991).
[5] D. Kröger, *Air Cooled Heat Exchangers and Cooling Towers* (PenWell Books, Tulsa, OK, 2004).
[6] A. D. Kraus, "Heat exchangers," *Heat Transfer Handbook*, edited by A. Bejan and A. D. Kraus (Wiley, Hoboken, 2003), Chap. 11.
[7] S. Kakaç, R. K. Shah, and W. Aung, *Handbook of Single-Phase Convective Heat Transfer* (Wiley, New York, 1987).
[8] B. S. Baclic, "1-(2N-1) Shell-and-tube exchanger effectiveness: Explicit equations," Heat Mass Transfer **33**, 163–165 (1997).
[9] D. P. Sekulic, R. K. Shah, and A. Pignotti, "A review of solution methods for determining effectiveness-N_{tu} relationships of heat exchangers with complex flow arrangements," Appl. Mech. Rev. **52**(3), 97 (1999).
[10] A. D. Kraus and J. R. Welty, *Extended Surface Heat Transfer* (Wiley, New York, 2001).
[11] A. Bejan, *Entropy Generation through Heat and Fluid Flow* (Wiley, New York, 1982).
[12] A. Bejan, *Advanced Engineering Thermodynamics*, 4th ed. (Wiley, Hoboken, 2016).
[13] A. Bejan, A. Almerbati, and S. Lorente, "Economies of scale: The physics basis," J. Appl. Phys. **121**, 044907 (2017).
[14] A. Bejan, *The Physics of Life: The Evolution of Everything* (St. Martin's Press, New York, 2016).
[15] A. Bejan, *Convection Heat Transfer*, 4th ed. (Wiley, Hoboken, 2013).