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Weighted averages and variance

If the values of a property of a set of samples are normally distributed, the average is easily calculated. The variability around the average (often expressed as a standard deviation or as confidence limits around the average) can then also be calculated using well-known methods. In the present study, the samples did not all represent similar production or usage volumes. For example, the samples from power generation are representative of a greater share of the country's coal usage than those from smaller users. Thus, to calculate the average emission factor for the country, different weightings must be applied to the values. The variability or standard deviation must then also take the weightings into account. This was done using the method of Bevington (quoted by Kirchner[1]) with the assumption that the variability of the individual samples is similar, but weightings assigned to samples differ.

The equations are:

$$Var(x)_{wtd} = (s_x^2)_{wtd} = \frac{\sum_{i=1}^{n} w_i(x_i - \bar{x}_{wtd})^2}{\sum_{i=1}^{n} w_i} \frac{n_{eff}}{n_{eff}-1} = \left[\frac{\sum_{i=1}^{n} w_i x_i^2}{\sum_{i=1}^{n} w_i} - (\overline{x_{wtd}})^2\right] \frac{\left(\sum_{i=1}^{n} w_i\right)^2}{\left(\sum_{i=1}^{n} w_i\right)^2 - \sum_{i=1}^{n}\left(w_i\right)} \quad \text{Equation 1}$$

and

$$(s_{\bar{x}})_{wtd} = \frac{(s_x)_{wtd}}{\sqrt{n_{eff}}} = \sqrt{\frac{Var(x)_{wtd}}{n_{eff}}} = \sqrt{\left[\frac{\sum_{i=1}^{n} w_i x_i^2}{\sum_{i=1}^{n} w_i} - (\bar{x}_{wtd})^2\right] \frac{\sum_{i=1}^{n}(w_i^2)}{\left(\sum_{i=1}^{n} w_i\right)^2 - \sum_{i=1}^{n}(w_i^2)}} \quad \text{Equation 2}$$

where n_{eff} is now the effective number of values, given by

$$n_{eff} = \frac{\left(\sum_{i=1}^{n} w_i\right)^2}{\sum_{i=1}^{n}(w_i^2)} \quad \text{Equation 3}$$

In these equations, the overbar over symbols indicates the weighted average values of *x* and
$Var\,(x)_{wtd}$ is the variance of the weighted average,
s_x is the standard deviation,
w_i is the weight assigned to individual values,
n is the number of values,
x is the values, with $\bar{x}$ the weighted average of the values, and
wtd indicates the weighted values.

An often-used measure of the variation of values around an average is the standard deviation. For a normal distribution, 68% of all values will lie within one standard deviation on either side of the average. The relative standard deviation (often expressed as a percentage or %RSD) is the percentage of the standard deviation divided by the average value. As an example, for a RSD of 5%, 68% of the values will lie within 5% on either side of the average.

Results of the analysis of reference materials

The certified reference materials were as follows:

1. ERM-EF411 Hard Coal (classified as bituminous coal) from Belgium supplied by Merck. Certified for gross calorific value (GCV, 29.0 ± 0.4 MJ/kg), net calorific value (NCV, 28.0 ± 0.4 MJ/kg), carbon (71.4 ± 1.0 g/100g), volatile matter (38.1 ± 1.0 g/100g), ash (8.3 ± 0.7 g/100g), H, N, S, Cl and Se. Three samples from the same lot (#244, 247 and 248) of 50 g each were combined and submitted as one sample for laboratory analysis.
2. ERM-EF412 Brown Coal (classified as bituminous coal) from Belgium supplied by Merck. Certified for gross calorific value (GCV, 26.02 ± 0.22 MJ/kg), net calorific value (NCV, 24.98 ± 0.25 MJ/kg), carbon (66.2 ± 0.7 g/100g), volatile matter (50.1 ± 0.7 g/100g), ash (4.11 ± 0.23 g/100g), H, N, S, Ca, Hg, K, Mn, Na, Se and V. Three samples from the same lot (#158, 159 and 160) of 50 g each were combined and submitted as one sample for laboratory analysis.
3. 502-683 Metallurgical Coke (classified as bituminous coals) all lot number 22256 from the USA supplied by Leco Corporation. Certified for carbon (88.4 ± 1.4%), volatile matter (1.1 ± 0.6%), ash (9.52 ± 0.13%), as well as H, N and S. Two samples of 50 g each were combined and submitted as one sample for laboratory analysis. The third sample was submitted as received (i.e. as one sample of 50 g).
4. 502-680 Prox-Plus Coal (classified as bituminous coals), all lot number 23132 from the USA supplied by Leco Corporation. Certified for carbon (81.8 ± 1.0%), volatile matter (19.3 ± 1.6%), ash (8.32 ± 0.11%), as well as H, N and S. Two samples of 50 g each were combined and submitted as one sample for laboratory analysis. The third sample was submitted as received (i.e. as one sample of 50 g).

Supplementary table 1: Results from analysed reference materials

Certified reference material	Total %C (certified value)	Total %C (reported value)	% error %C (reported vs certified average)	Calorific value (MJ/kg) (certified value)	Calorific value (MJ/kg) (reported value)	% error calorific value (reported vs certified average)
Leco 502-680 Prox-Plus Coal (i)	81.8±1.0	83.43	2.0	(33.04)	33.03	-0.03
Leco 502-683 Metallurgical Coke (i)	88.4±1.4	89.43	1.2	(28.80)	26.74	-7.2
Leco 502-680 Prox-Plus Coal (ii)	81.8±1.0	87.31	6.7	(33.04)	32.92	-0.4
Leco 502-683 Metallurgical Coke (ii)	88.4±1.4	92.08	4.2	(28.80)	26.99	-6.3
ERM-EF411	71.4±1.0	73.05	2.3	29.0±0.4	28.22	-2.7
ERM-EF412	66.2±0.7	71.75	8.4	26.02±0.22	25.67	-1.3

Note: Certified values in brackets indicate that these values are reported on the certificate of analysis for the material but they are not certified and no uncertainty measurement is included.

Reference

1. Kirchner J. University of California at Berkeley Seismology lab. Toolkit no 12 [webpage on the Internet]. c2006 [cited 2024 Apr 05]. Available from: https://seismo.berkeley.edu/~kirchner/toolkits.html