
Supplementary Material: MOTA: a Many Objective Tuning Algorithm Specialized for Tuning under Multiple Objective Function Evaluation Budgets

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1 IGD tables

Table 1: The Nadir and Utopia points of the respective NSGA-II tuning problems, which were used by tPSO to compare performances. IGD values are multiplied by 10^3 for readability.

	ZDT ₁	ZDT ₂	ZDT ₃	ZDT ₄	ZDT ₆
max.	78.830	121.652	85.624	3197.750	279.499
min.	0.387	0.422	0.473	1.387	0.661

	DTLZ ₁	DTLZ ₂	DTLZ ₃	DTLZ ₄	DTLZ ₅	DTLZ ₆	DTLZ ₇
max.	18222.560	47.663	52108.309	13.593	46.092	677.292	182.925
min.	17.100	0.243	460.708	0.108	0.234	7.668	0.336

Table 2: The Nadir and Utopia points used to compare the NSGA-II based tuning problems. IGD values are multiplied by 10^3 for readability.

	ZDT ₁	ZDT ₂	ZDT ₃	ZDT ₄	ZDT ₆
max.	78.746	121.304	85.624	3140.462	275.842
min.	0.364	0.402	0.471	1.204	0.620

	DTLZ ₁	DTLZ ₂	DTLZ ₃	DTLZ ₄	DTLZ ₅	DTLZ ₆	DTLZ ₇
max.	17557.518	45.638	51911.085	13.593	45.951	673.232	174.187
min.	12.777	0.232	363.496	0.086	0.231	6.189	0.335

	WFG ₁	WFG ₂	WFG ₃	WFG ₄	WFG ₅	WFG ₆	WFG ₇	WFG ₈	WFG ₉
max.	12.397	4.845	1.655	3.474	5.766	6.889	4.640	3.620	6.516
min.	3.483	0.171	0.714	0.092	0.605	0.679	0.081	0.745	0.180

Table 3: The Nadir and Utopia points used to compare the MOEA/D based tuning problems. IGD values are multiplied by 10^3 for readability.

	ZDT ₁	ZDT ₂	ZDT ₃	ZDT ₄	ZDT ₆
max.	81.068	123.746	88.729	3236.527	280.509
min.	0.129	0.112	0.402	0.797	0.056

	DTLZ ₁	DTLZ ₂	DTLZ ₃	DTLZ ₄	DTLZ ₅	DTLZ ₆	DTLZ ₇
max.	18895.532	47.975	54699.749	13.922	47.198	680.422	182.987
min.	2.101	0.174	228.864	0.063	0.177	0.120	0.194

	WFG ₁	WFG ₂	WFG ₃	WFG ₄	WFG ₅	WFG ₆	WFG ₇	WFG ₈	WFG ₉
max.	12.434	5.066	1.699	3.740	6.059	7.189	4.864	3.743	6.863
min.	1.820	0.285	0.707	0.172	0.557	0.251	0.118	0.714	0.209

2 Specialist tuning problem τ results

Table 4: Performances on the NSGA-II specialist tuning problems.

suite	problem	$\tau(\times 10^3)$		
		tMOPSO	MOTA	RAND ^M
ZDT	1	946.743 [†]	946.265	944.384
	2	<i>934.845</i> [‡]	935.562	931.651
	3	932.770	931.857	929.537
	4	<i>960.120</i>	960.496	955.470
	6	938.548	939.447	935.492
DTLZ	1	966.481	<i>966.385</i>	963.242
	2	969.295	969.101	967.557
	3	950.034	950.824	946.185
	4	936.064	<i>934.882</i>	<i>935.222</i>
	5	<i>969.339</i>	969.345	967.734
	6	<i>808.841</i>	810.097	801.350
	7	957.173	956.581	953.861
WFG	1	795.679	805.681	793.968
	2	<i>912.616</i>	913.655	908.411
	3	969.683	969.162	967.491
	4	932.710	<i>931.937</i>	928.284
	5	951.742	<i>951.456</i>	950.191
	6	<i>928.772</i>	930.064	921.503
	7	934.793	<i>934.787</i>	932.267
	8	927.275	<i>927.262</i>	924.992
	9	939.018	941.294	938.045

Friedman test: χ^2 24.100, p-value 5.8×10^{-6}

† **bold entries** indicate the best value in each row.

‡ *Italic entries* indicate samples whose difference in mean relative to the sample with the best mean is not statistically significant according to Mann-Whitney U-test with a 95% confidence.

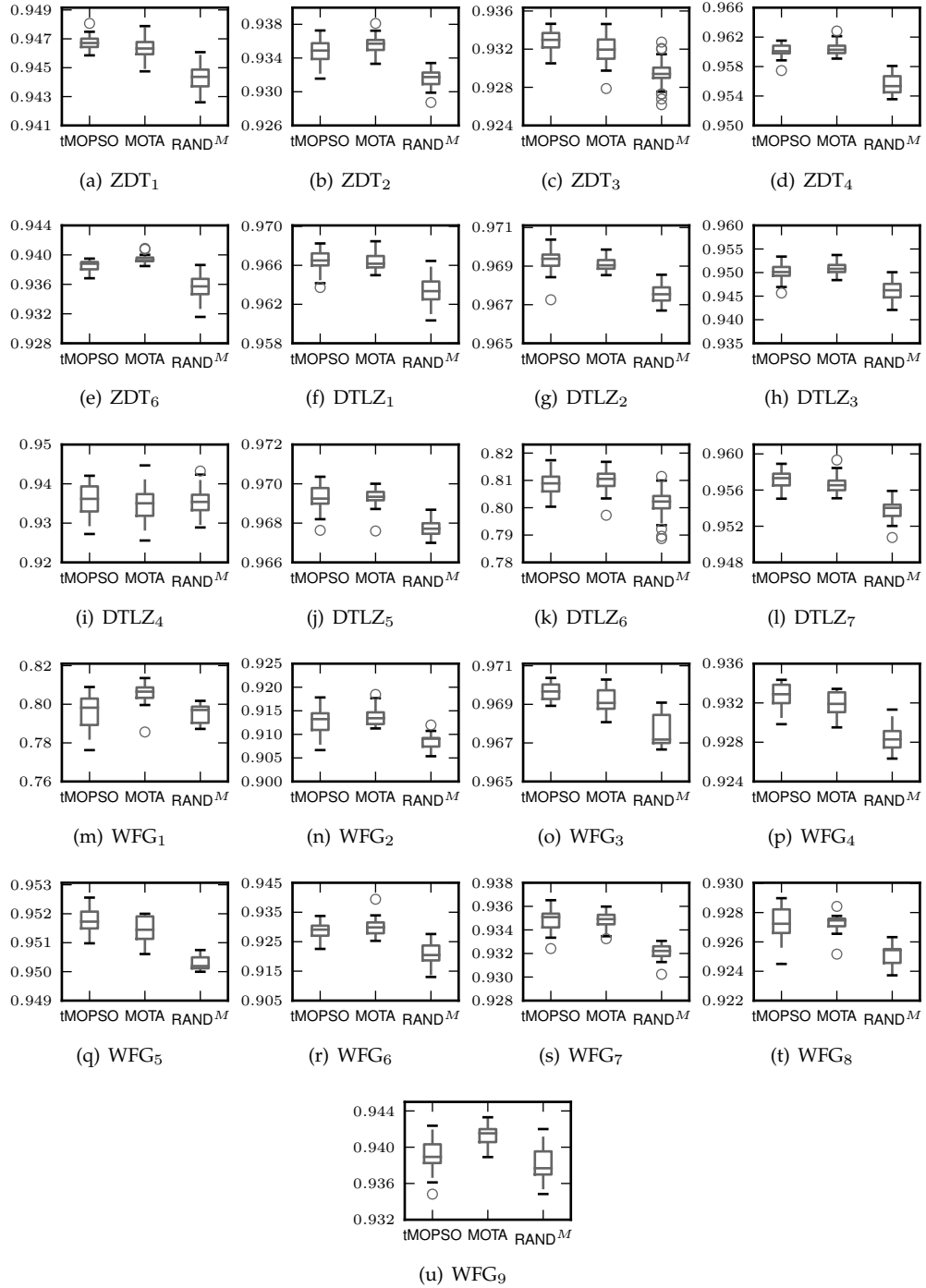


Figure 1: τ distributions for the NSGA-II specialist problems

Table 5: Performances on the MOEA/D specialist tuning problems.

suite	problem	$\tau(\times 10^3)$		
		tMOPSO	MOTA	RAND ^M
ZDT	1	978.496 [†]	<i>978.324</i> [‡]	975.777
	2	<i>975.150</i>	975.154	972.183
	3	<i>963.370</i>	963.768	960.094
	4	<i>969.652</i>	969.875	965.638
	5	987.639	<i>987.392</i>	985.198
	6	967.943	969.541	965.065
DTLZ	2	978.276	977.818	976.321
	3	947.300	955.056	944.539
	4	975.730	974.372	971.954
	5	977.851	977.553	976.276
	6	994.090	993.891	993.032
	7	982.132	981.473	979.793
	7	795.654	796.439	773.524
WFG	2	897.245	<i>897.224</i>	892.643
	3	976.787	975.889	974.403
	4	912.752	910.685	909.565
	5	988.313	987.905	987.498
	6	<i>900.856</i>	902.272	898.140
	7	932.921	<i>932.576</i>	930.612
	8	920.745	<i>920.254</i>	917.489
	9	930.300	<i>928.280</i>	<i>927.276</i>

Friedman test: χ^2 25.200, p-value 3.4×10^{-6}

† **bold entries** indicate the best value in each row.

‡ *Italic entries* indicate samples whose difference in mean relative to the sample with the best mean is not statistically significant according to Mann-Whitney U-test with a 95% confidence.

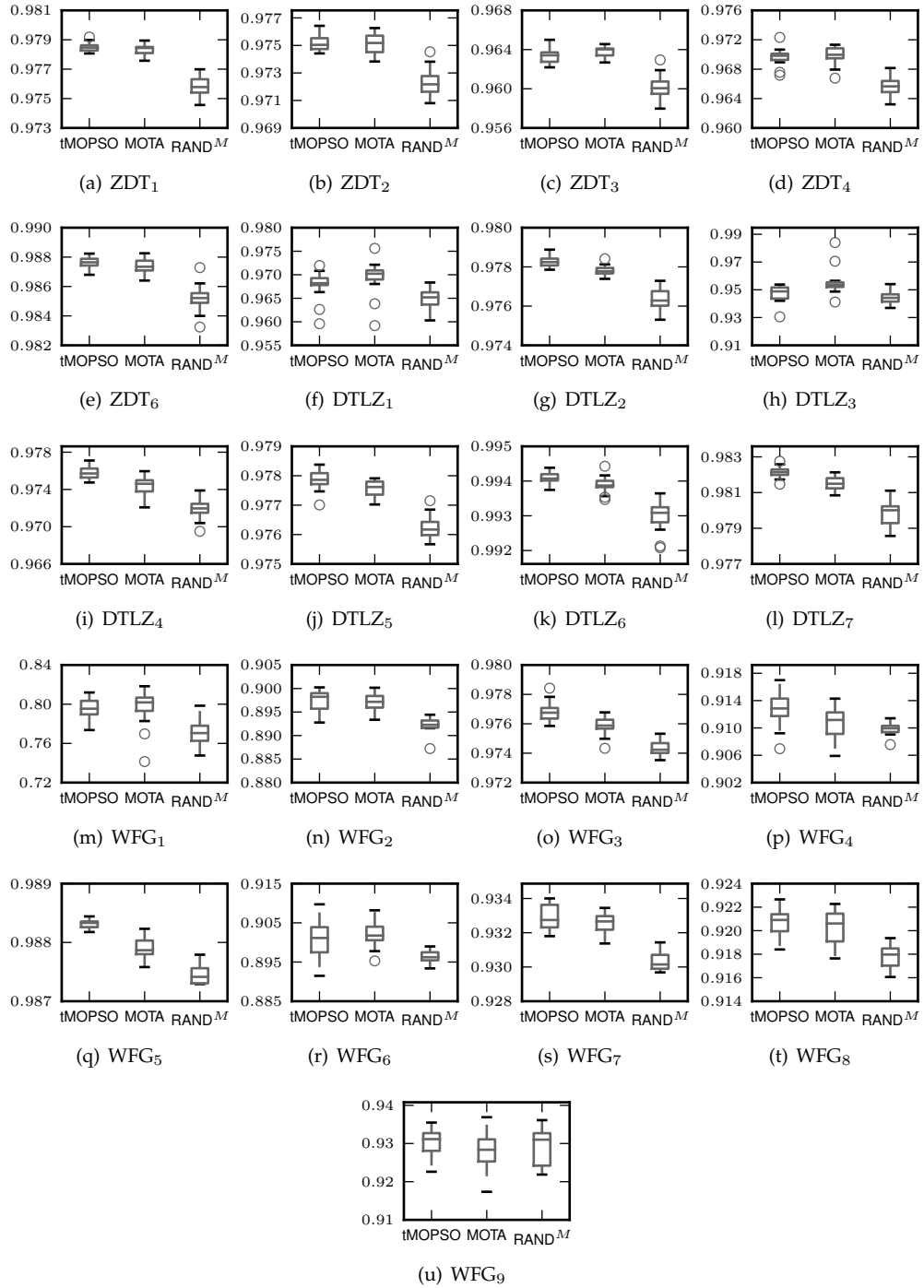


Figure 2: τ distributions for the MOEA/D specialist problems

3 Specialist tuning problems IGD versus OFE

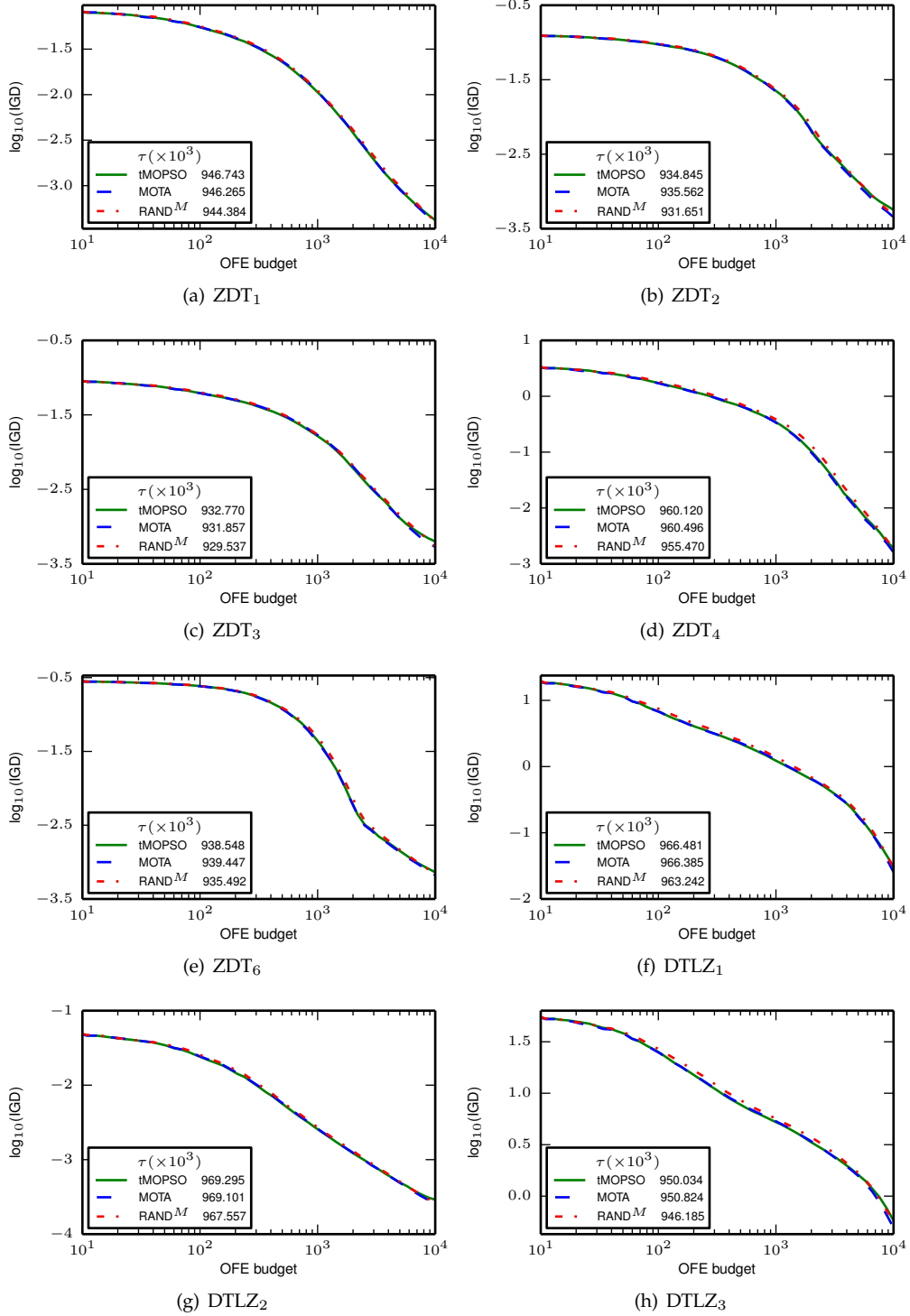


Figure 3: NSGA-II specialist results ($\gamma = 10^7$), IGD versus OFE budget (table 1/3)

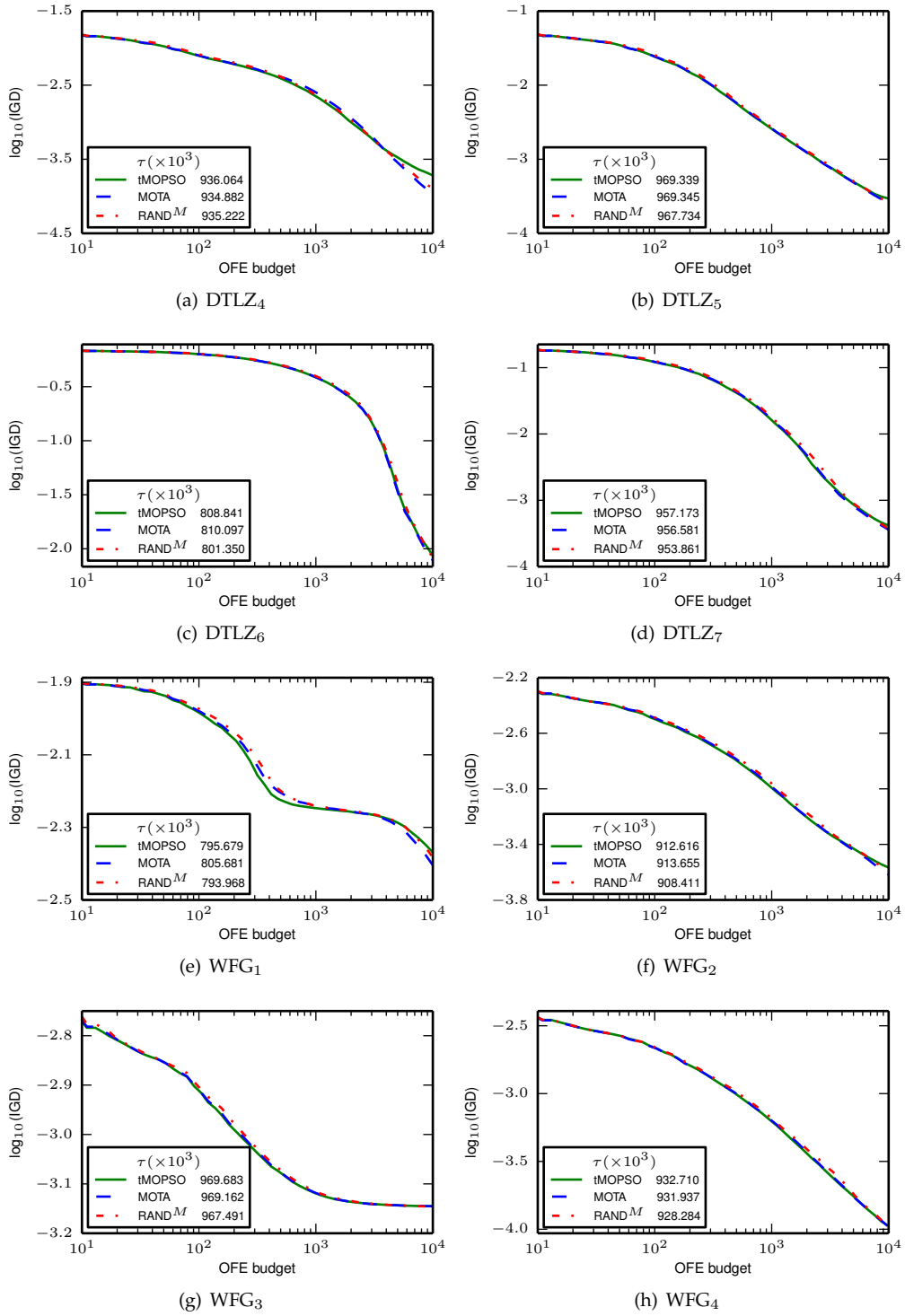


Figure 4: NSGA-II specialist results ($\gamma = 10^7$), IGD versus OFE budget (table 2/3)

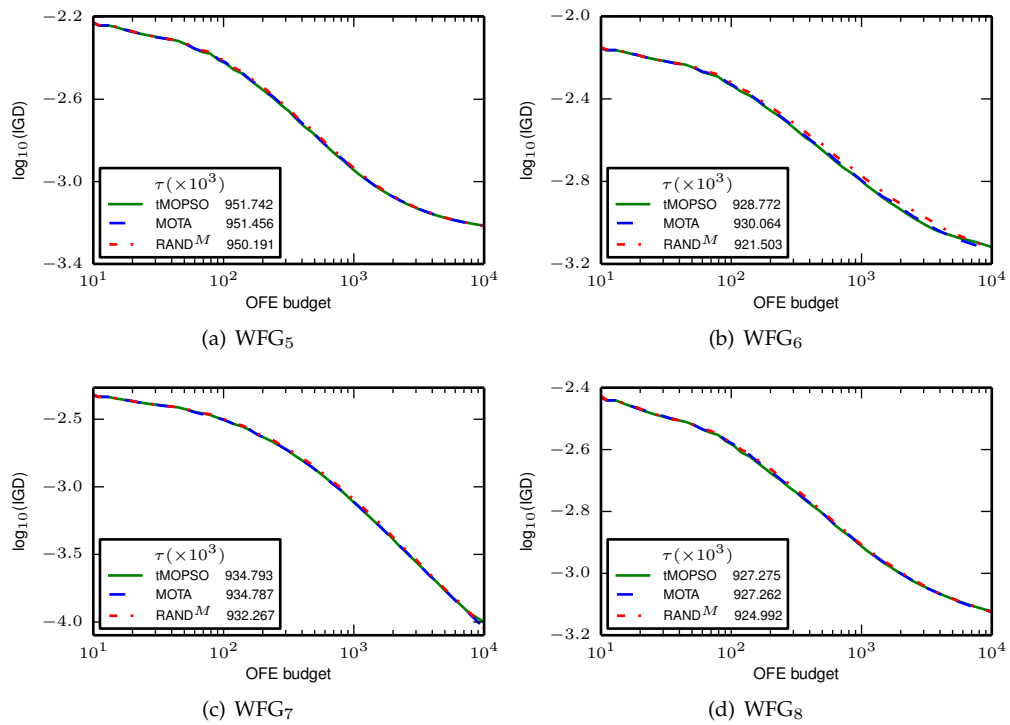


Figure 5: NSGA-II specialist results ($\gamma = 10^7$), IGD versus OFE budget (table 3/3)

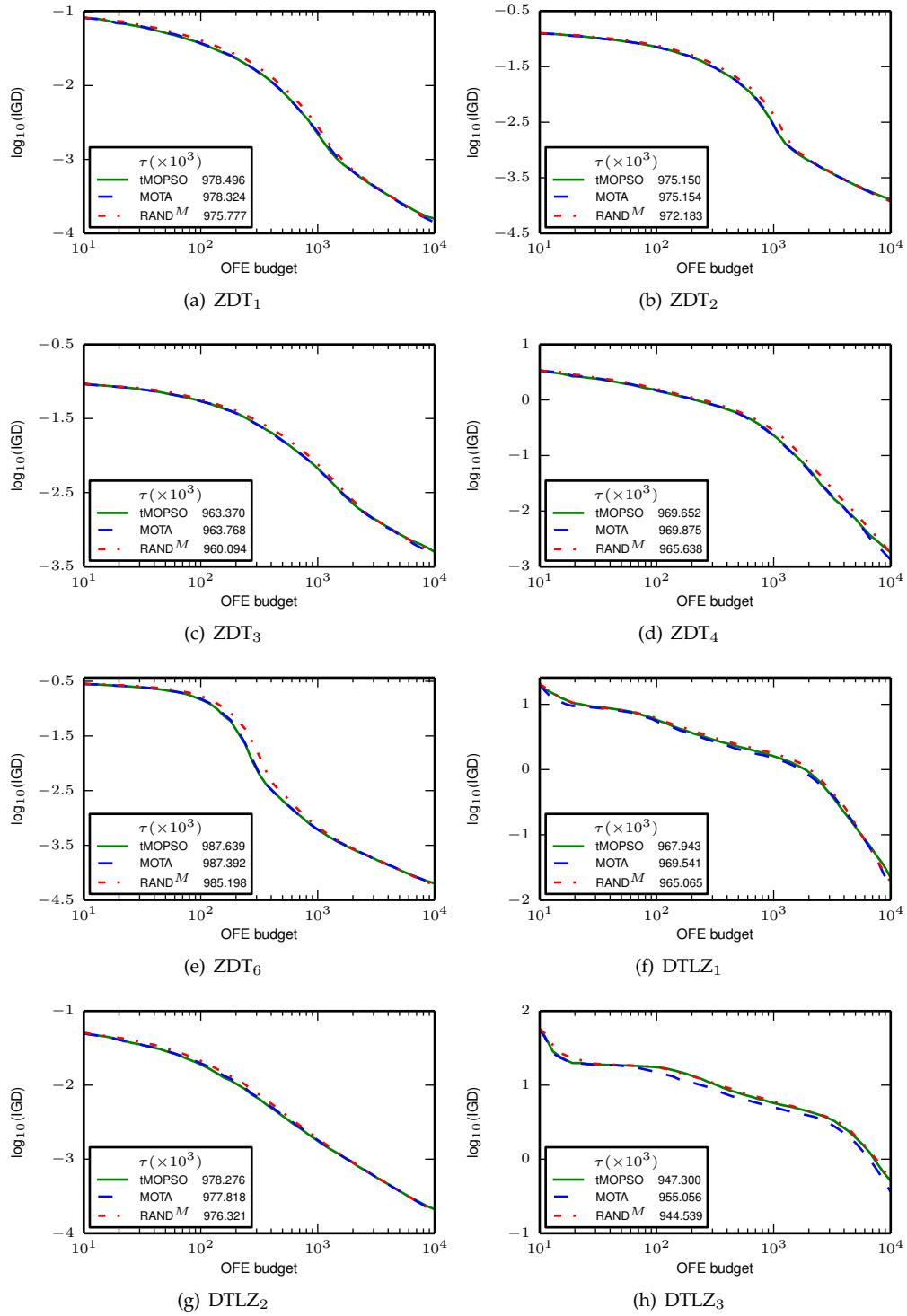


Figure 6: MOEA/D specialist results ($\gamma = 10^7$), IGD versus OFE budget (table 1/3)

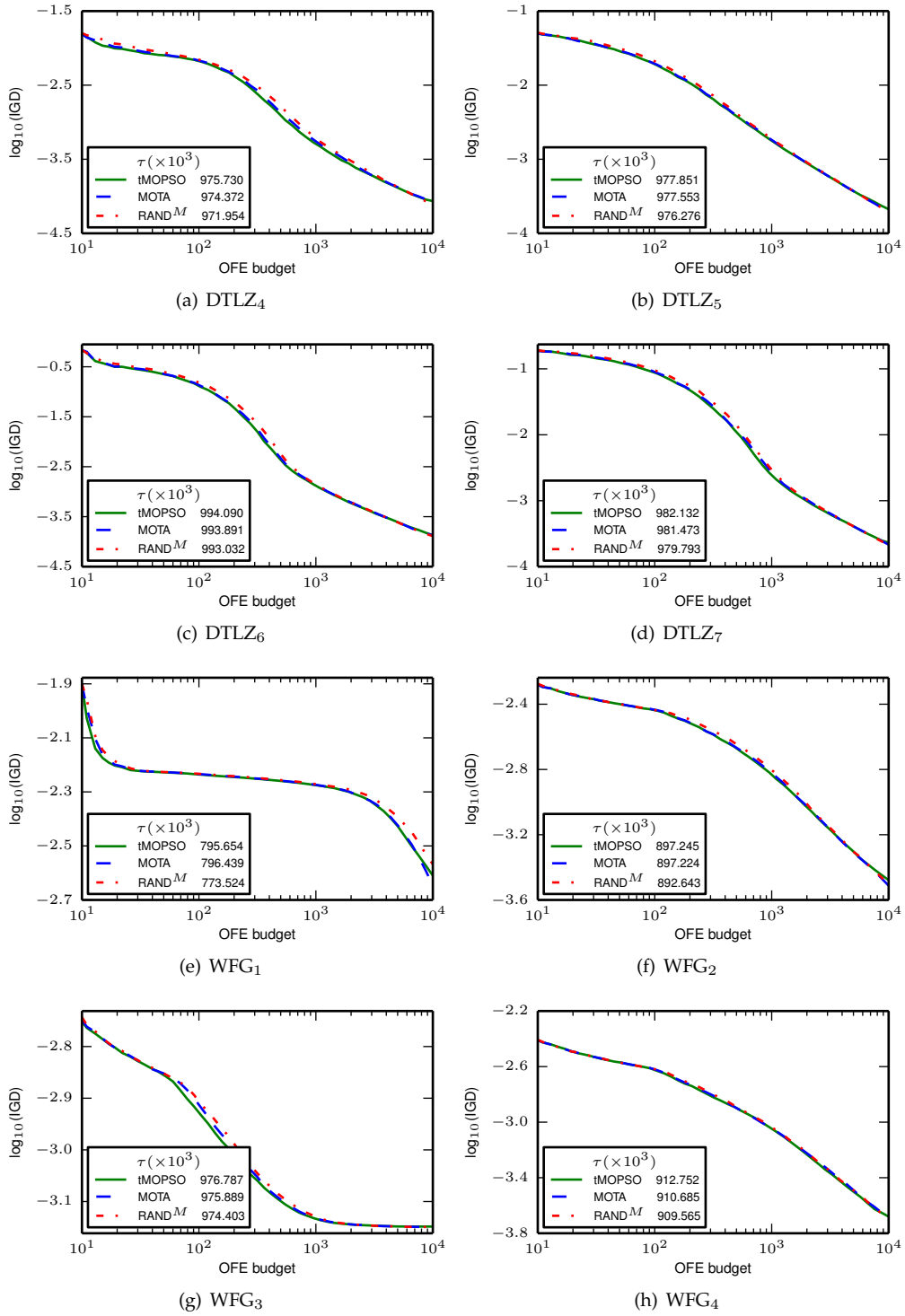


Figure 7: MOEA/D specialist results ($\gamma = 10^7$), IGD versus OFE budget (table 2/3)

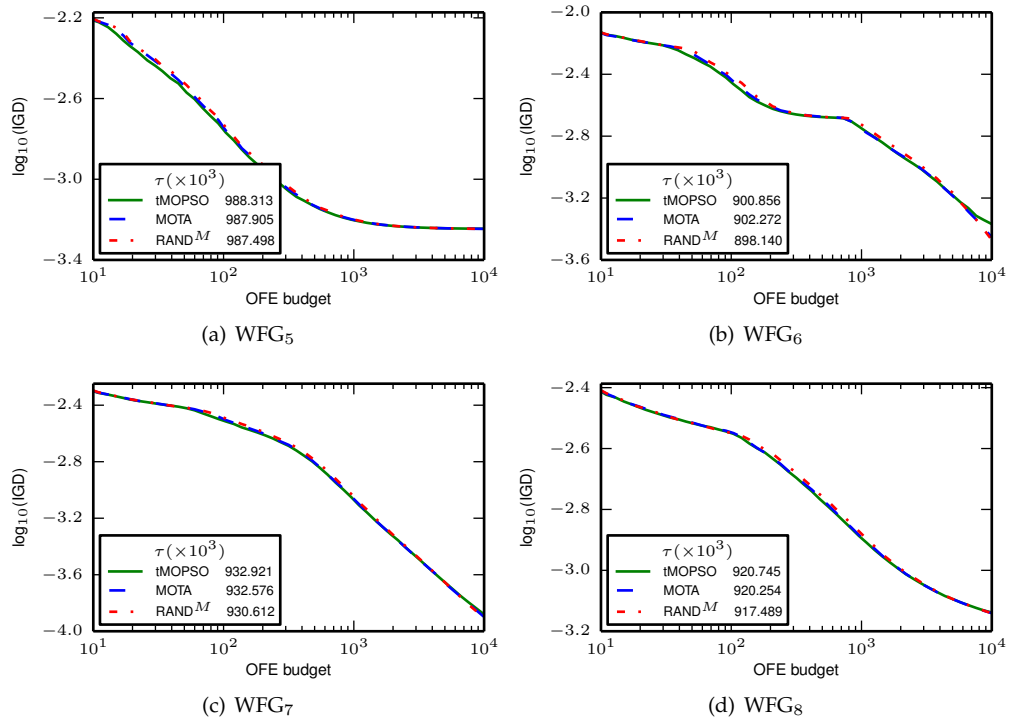


Figure 8: MOEA/D specialist results ($\gamma = 10^7$), IGD versus OFE budget (table 3/3)

4 tMOPSO results for the specialist problems

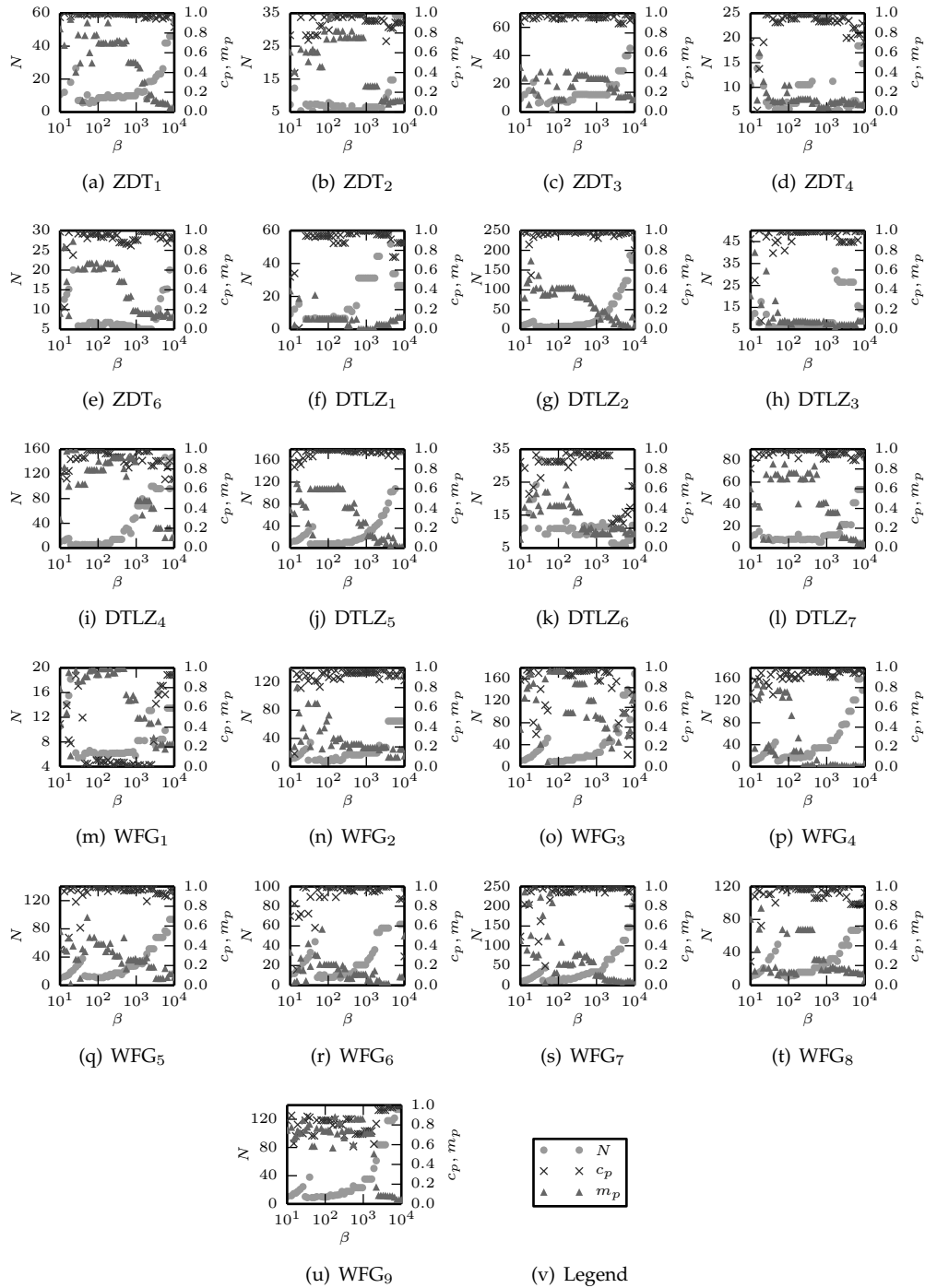


Figure 9: The best tMOPSO results for the NSGA-II specialist tuning problems. The recommended CPVs are shown for differing OFE budgets, β .

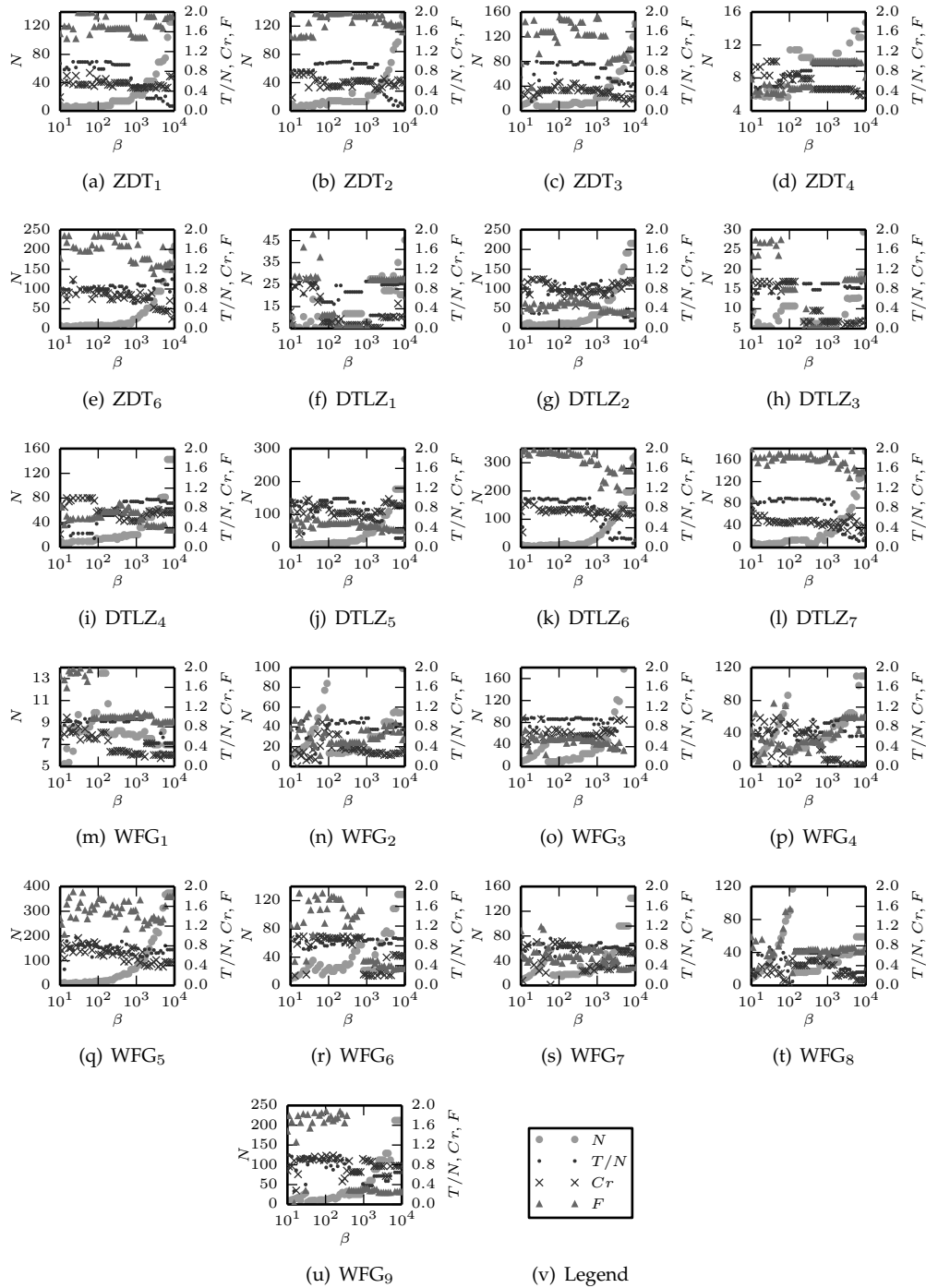


Figure 10: The best tMOPSO results for the MOEA/D specialist tuning problems. The recommended CPVs are shown for differing OFE budgets, β .

5 Generalist tuning problems τ results

Table 6: Performances on the NSGA-II generalist tuning problems. The $\mathbf{1}^{i \neq j}$ notation indicates a vector whose elements are all equal to 1, with exception to the j 'th element which is equal to 0.

suite	w	$\tau \times 10^3$		
		tMOPSO	MOTA	RAND ^M
ZDT	1	882.125	932.601 [†]	929.575
	$\mathbf{1}^{i \neq 1}$	902.718	929.089	923.368
	$\mathbf{1}^{i \neq 2}$	913.863	932.617	928.565
	$\mathbf{1}^{i \neq 3}$	920.730	935.314	929.664
	$\mathbf{1}^{i \neq 4}$	907.071	928.597	925.018
	$\mathbf{1}^{i \neq 5}$	908.475	931.625	927.993
DTLZ	1	834.185	884.049	880.356
	$\mathbf{1}^{i \neq 1}$	826.849	864.007	857.315
	$\mathbf{1}^{i \neq 2}$	823.796	865.996	857.257
	$\mathbf{1}^{i \neq 3}$	844.050	869.802	<i>869.114</i> [‡]
	$\mathbf{1}^{i \neq 4}$	858.144	904.283	891.732
	$\mathbf{1}^{i \neq 5}$	832.369	864.767	857.945
	$\mathbf{1}^{i \neq 6}$	894.699	921.736	<i>920.040</i>
	$\mathbf{1}^{i \neq 7}$	847.747	868.590	859.925
WFG	1	851.983	879.122	876.701
	$\mathbf{1}^{i \neq 1}$	890.162	916.305	<i>915.295</i>
	$\mathbf{1}^{i \neq 2}$	852.251	870.980	<i>870.457</i>
	$\mathbf{1}^{i \neq 3}$	840.971	861.910	<i>860.170</i>
	$\mathbf{1}^{i \neq 4}$	856.030	874.207	872.268
	$\mathbf{1}^{i \neq 5}$	839.367	866.008	862.065
	$\mathbf{1}^{i \neq 6}$	848.161	871.661	868.776
	$\mathbf{1}^{i \neq 7}$	850.533	869.538	867.419
	$\mathbf{1}^{i \neq 8}$	848.610	869.767	865.051
	$\mathbf{1}^{i \neq 9}$	847.614	869.408	<i>866.949</i>

Friedman test: χ^2 39.000, p-value 3.4×10^{-9}

[†] **bold entries** indicate the best value in each row.

[‡] *Italic entries* indicate samples whose difference in mean relative to the sample with the best mean is not statistically significant according to Mann-Whitney U-test with a 95% confidence.

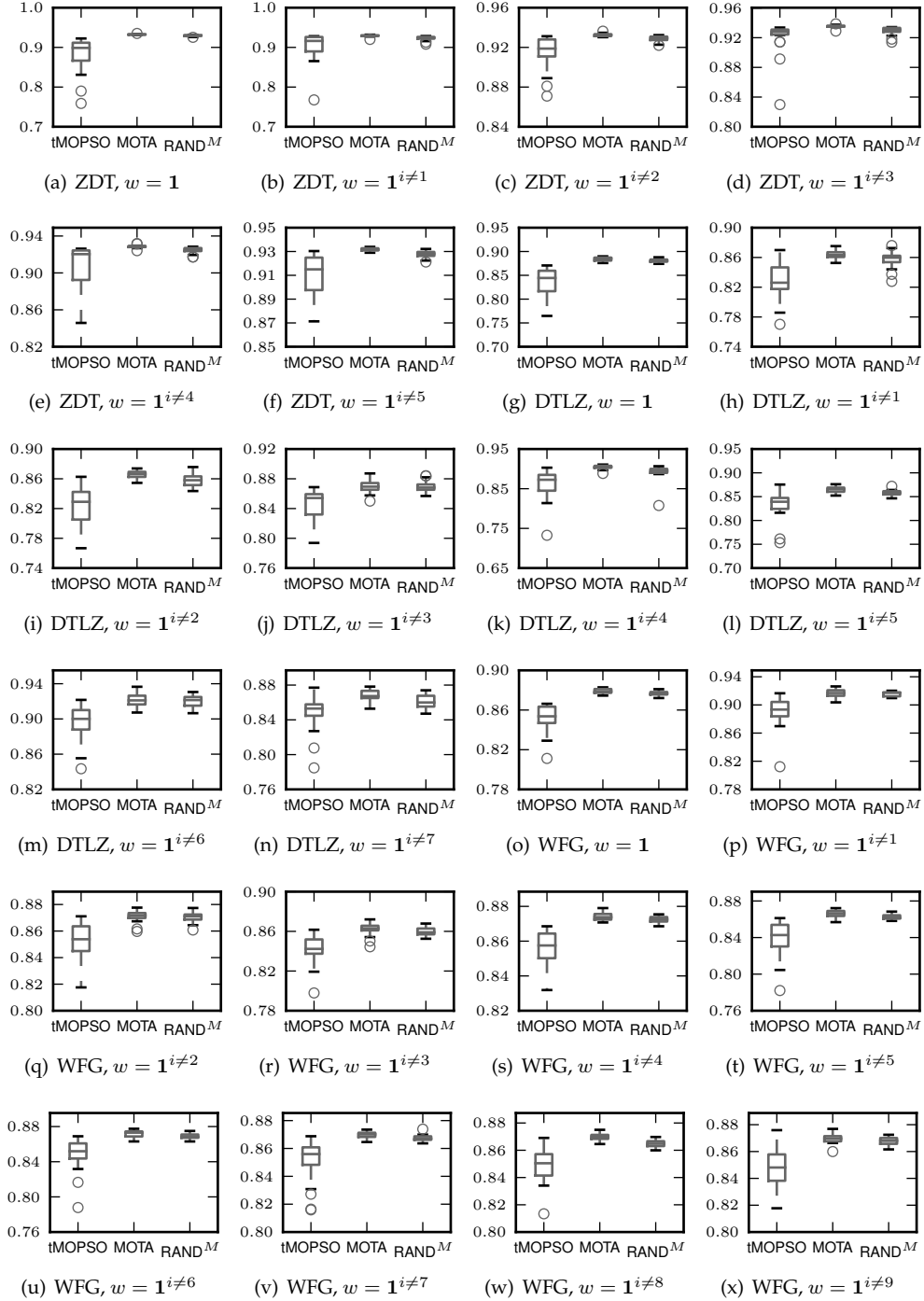


Figure 11: τ distributions for the NSGA-II generalist problems

Table 7: Performances on the MOEA/D generalist tuning problems. The $\mathbf{1}^{i \neq j}$ notation indicates a vector whose elements are all equal to 1, with exception to the j 'th element which is equal to 0.

suite	w	$\tau \times 10^3$		
		tMOPSO	MOTA	RAND ^M
ZDT	1	954.444	964.325 [†]	963.128
	$\mathbf{1}^{i \neq 1}$	955.393	961.119	958.943
	$\mathbf{1}^{i \neq 2}$	957.256	963.529	961.398
	$\mathbf{1}^{i \neq 3}$	962.893	965.680	963.138
	$\mathbf{1}^{i \neq 4}$	<i>968.530</i> [‡]	969.579	969.613
	$\mathbf{1}^{i \neq 5}$	951.967	957.607	955.255
DTLZ	1	930.843	958.360	953.098
	$\mathbf{1}^{i \neq 1}$	934.621	955.802	948.353
	$\mathbf{1}^{i \neq 2}$	929.119	952.000	944.280
	$\mathbf{1}^{i \neq 3}$	939.228	962.001	956.258
	$\mathbf{1}^{i \neq 4}$	930.771	953.190	944.010
	$\mathbf{1}^{i \neq 5}$	927.883	954.376	944.924
	$\mathbf{1}^{i \neq 6}$	923.124	950.179	941.941
	$\mathbf{1}^{i \neq 7}$	929.391	954.263	944.859
WFG	1	847.868	882.793	880.510
	$\mathbf{1}^{i \neq 1}$	881.356	904.044	898.567
	$\mathbf{1}^{i \neq 2}$	850.824	880.573	877.219
	$\mathbf{1}^{i \neq 3}$	831.903	866.881	864.155
	$\mathbf{1}^{i \neq 4}$	836.309	878.954	876.121
	$\mathbf{1}^{i \neq 5}$	836.376	866.981	863.358
	$\mathbf{1}^{i \neq 6}$	854.657	879.328	876.132
	$\mathbf{1}^{i \neq 7}$	845.944	872.224	868.259
	$\mathbf{1}^{i \neq 8}$	850.214	874.633	870.659
	$\mathbf{1}^{i \neq 9}$	849.236	881.582	878.529

Friedman test: χ^2 40.333, p-value 1.7×10^{-9}

[†] **bold entries** indicate the best value in each row.

[‡] *Italic entries* indicate samples whose difference in mean relative to the sample with the best mean is not statistically significant according to Mann-Whitney U-test with a 95% confidence.

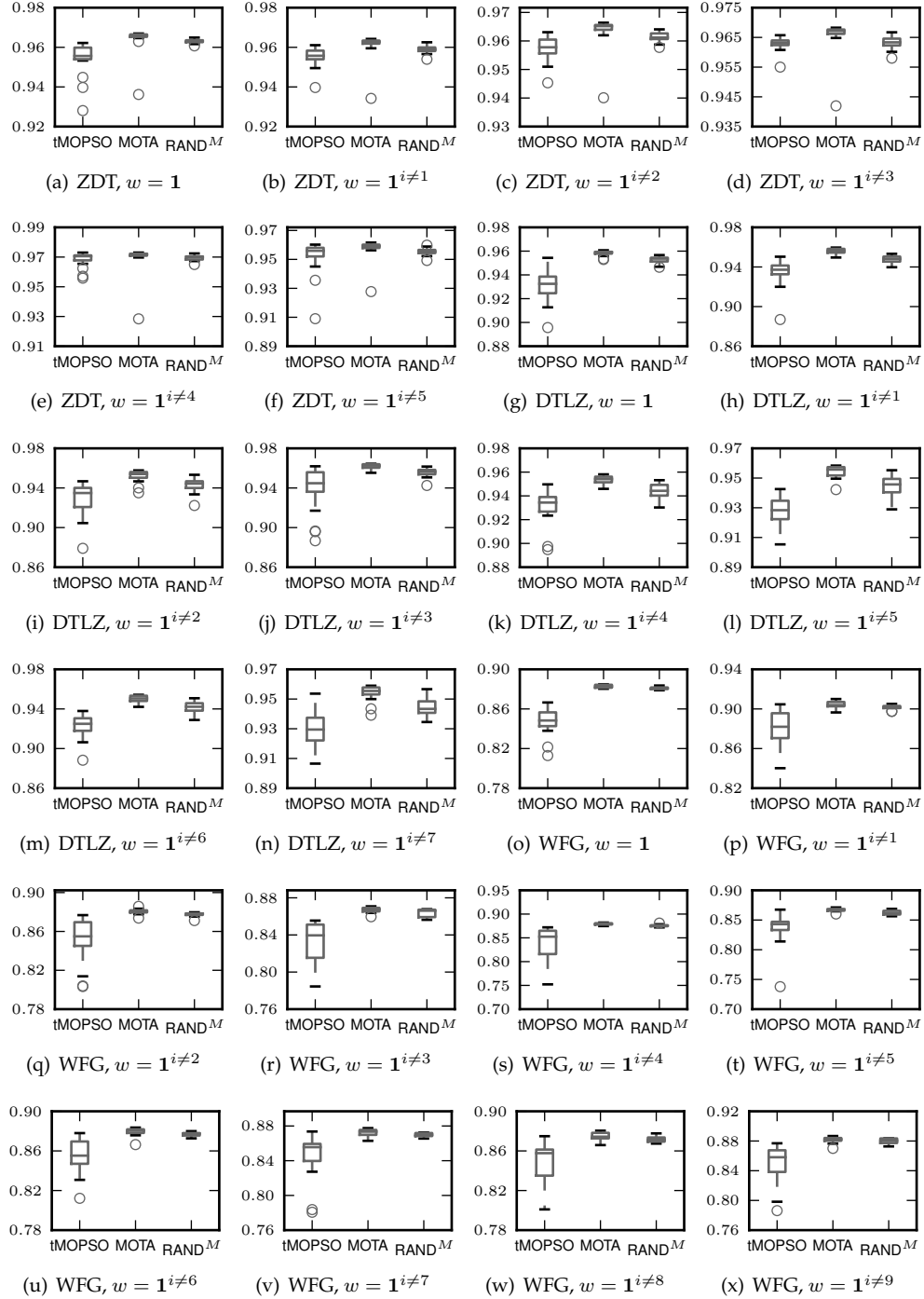


Figure 12: τ distributions for the MOEAD generalist problems

6 Generalist tuning problems IGD versus OFE

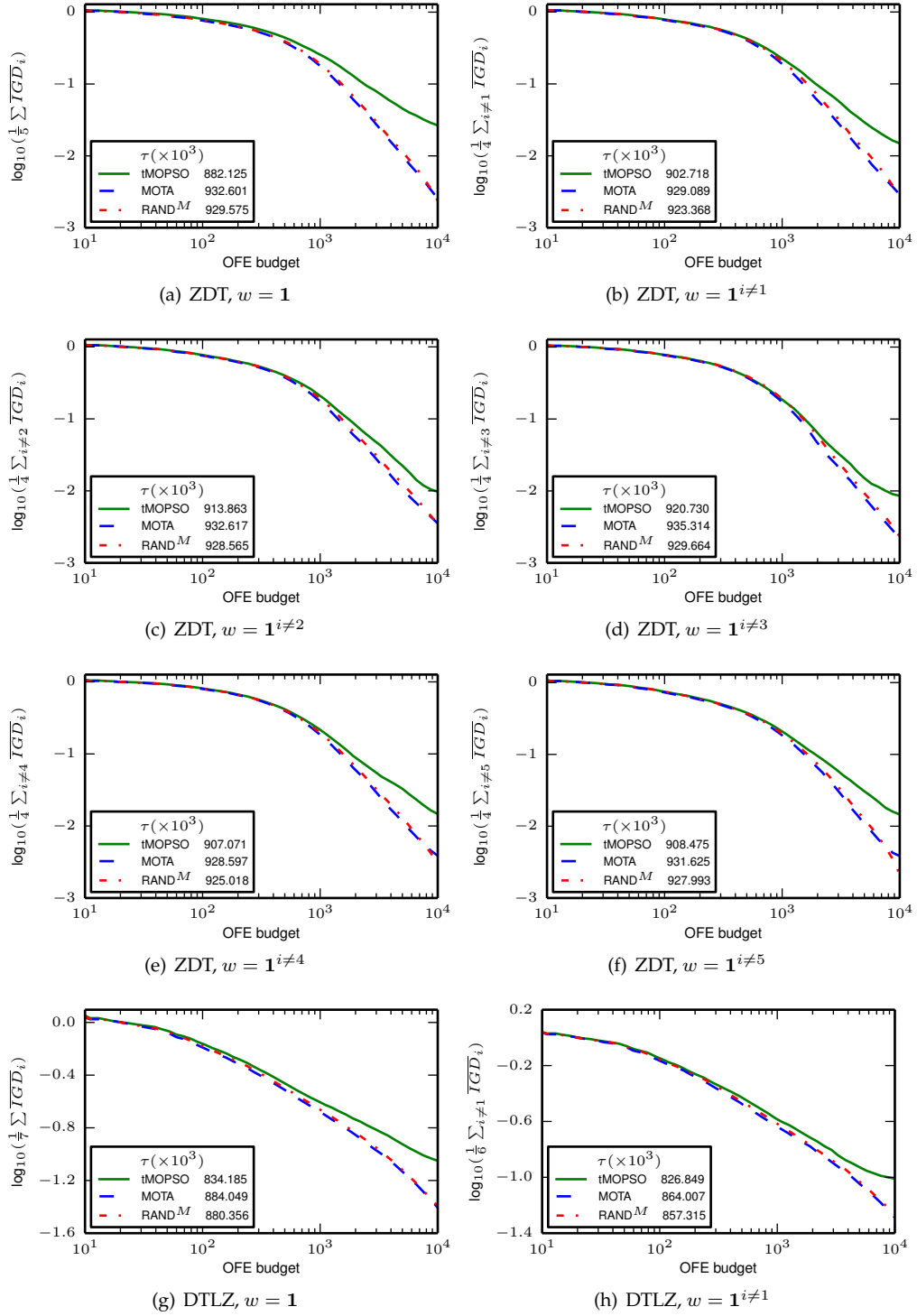


Figure 13: NSGA-II generalist results ($\gamma = 10^7$), Weighted sum of normalized IGD values (\overline{IGD}) versus OFE budget, with \overline{IGD} normalized between the Nadir and Utopia point. (table 1/3)

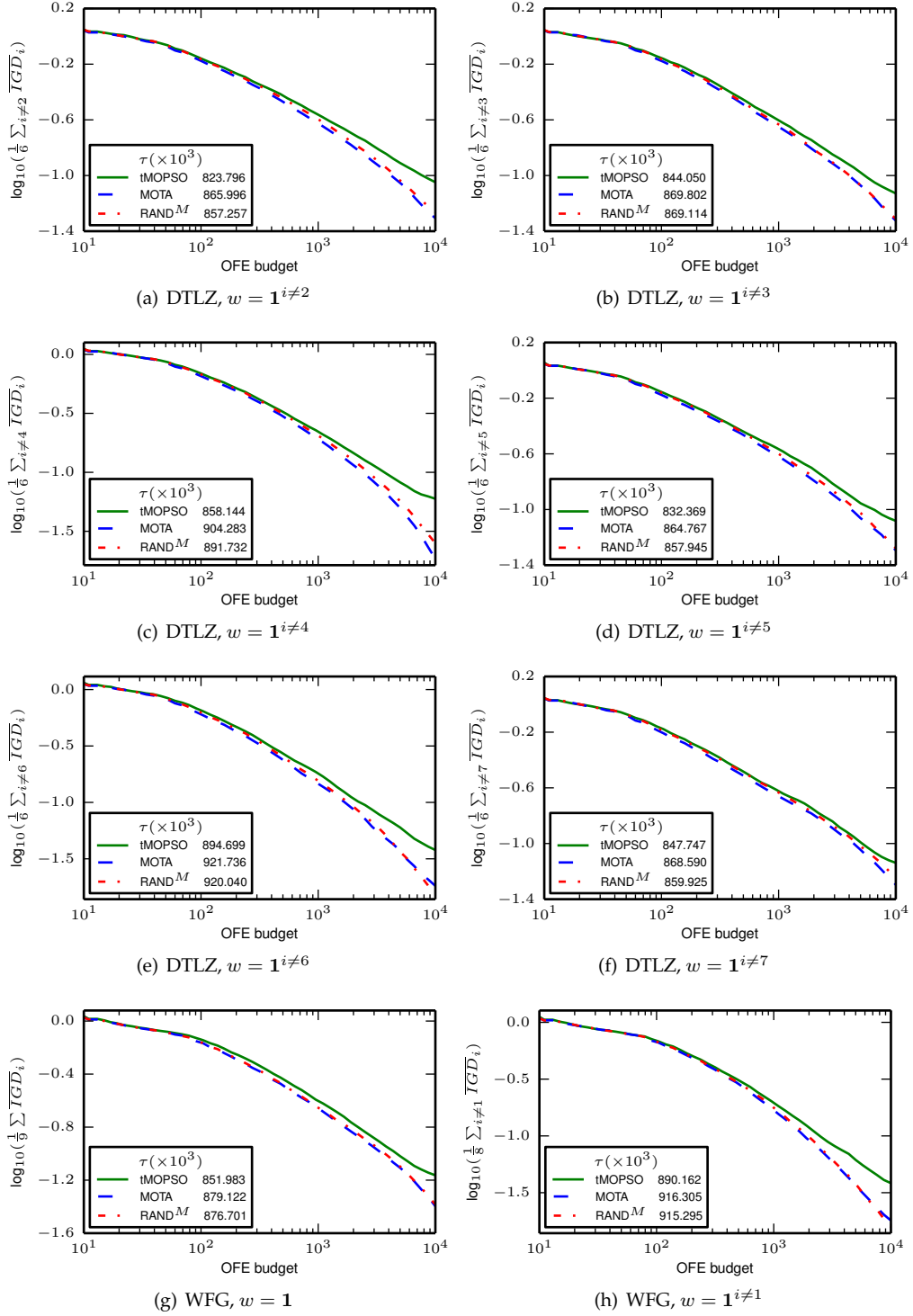


Figure 14: NSGA-II generalist results ($\gamma = 10^7$), Weighted sum of normalized IGD values (\overline{IGD}) versus OFE budget, with \overline{IGD} normalized between the Nadir and Utopia point. (table 2/3)

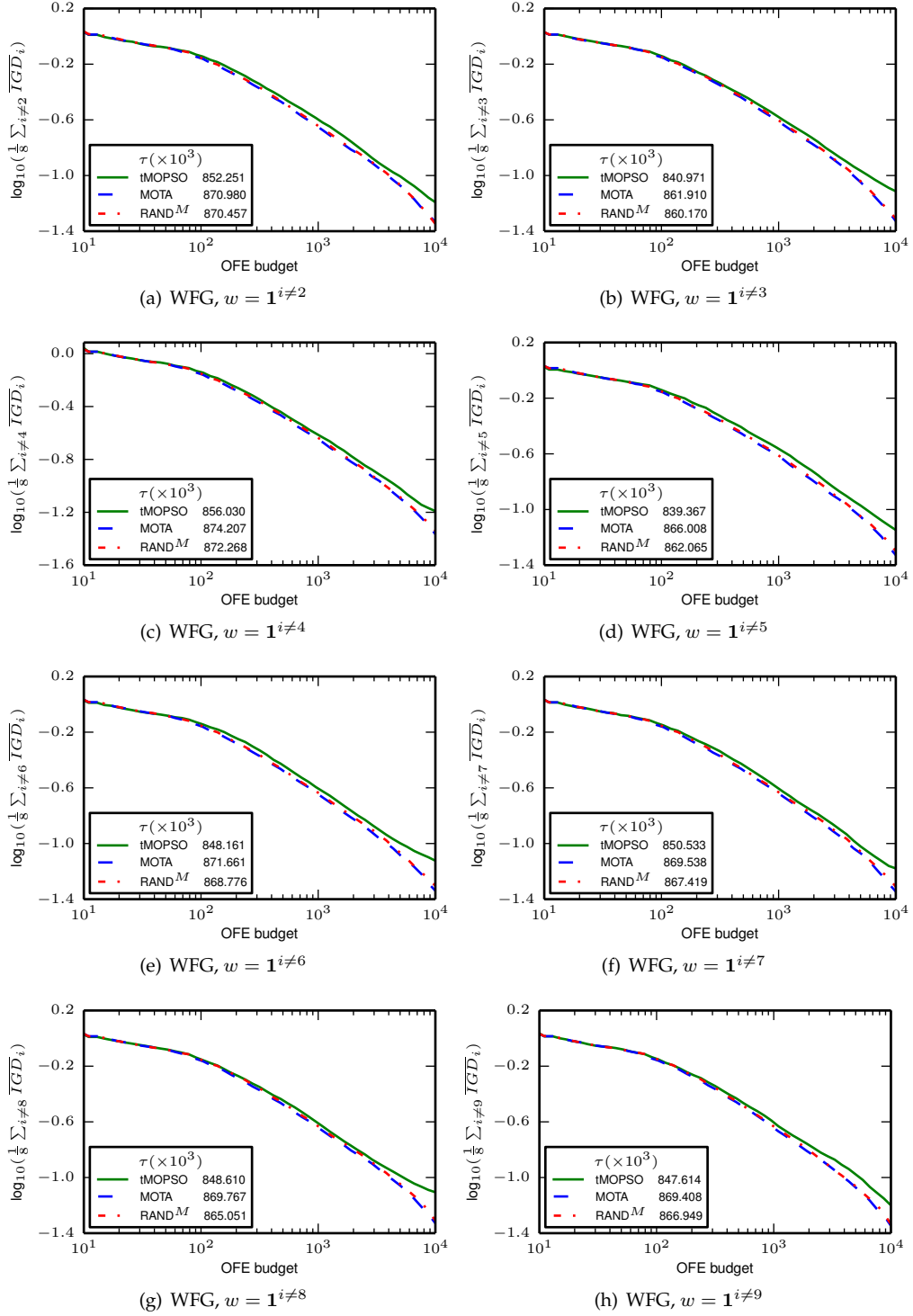


Figure 15: NSGA-II generalist results ($\gamma = 10^7$), Weighted sum of normalized IGD values (\overline{IGD}) versus OFE budget, with \overline{IGD} normalized between the Nadir and Utopia point. (table 3/3)

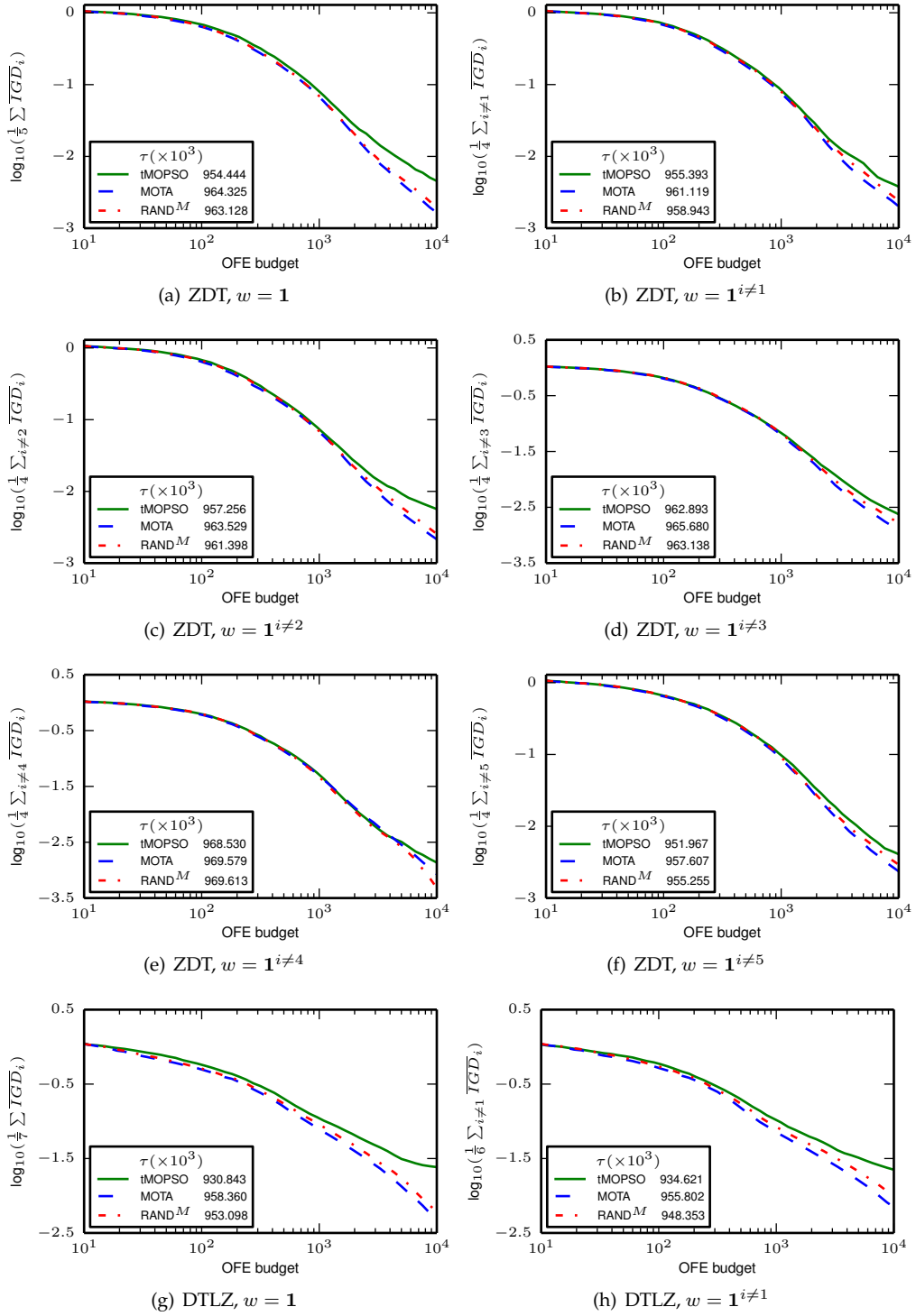


Figure 16: MOEA/D generalist results ($\gamma = 10^7$), Weighted sum of normalized IGD values (\overline{IGD}) versus OFE budget, with \overline{IGD} normalized between the Nadir and Utopia point. (table 1/3)

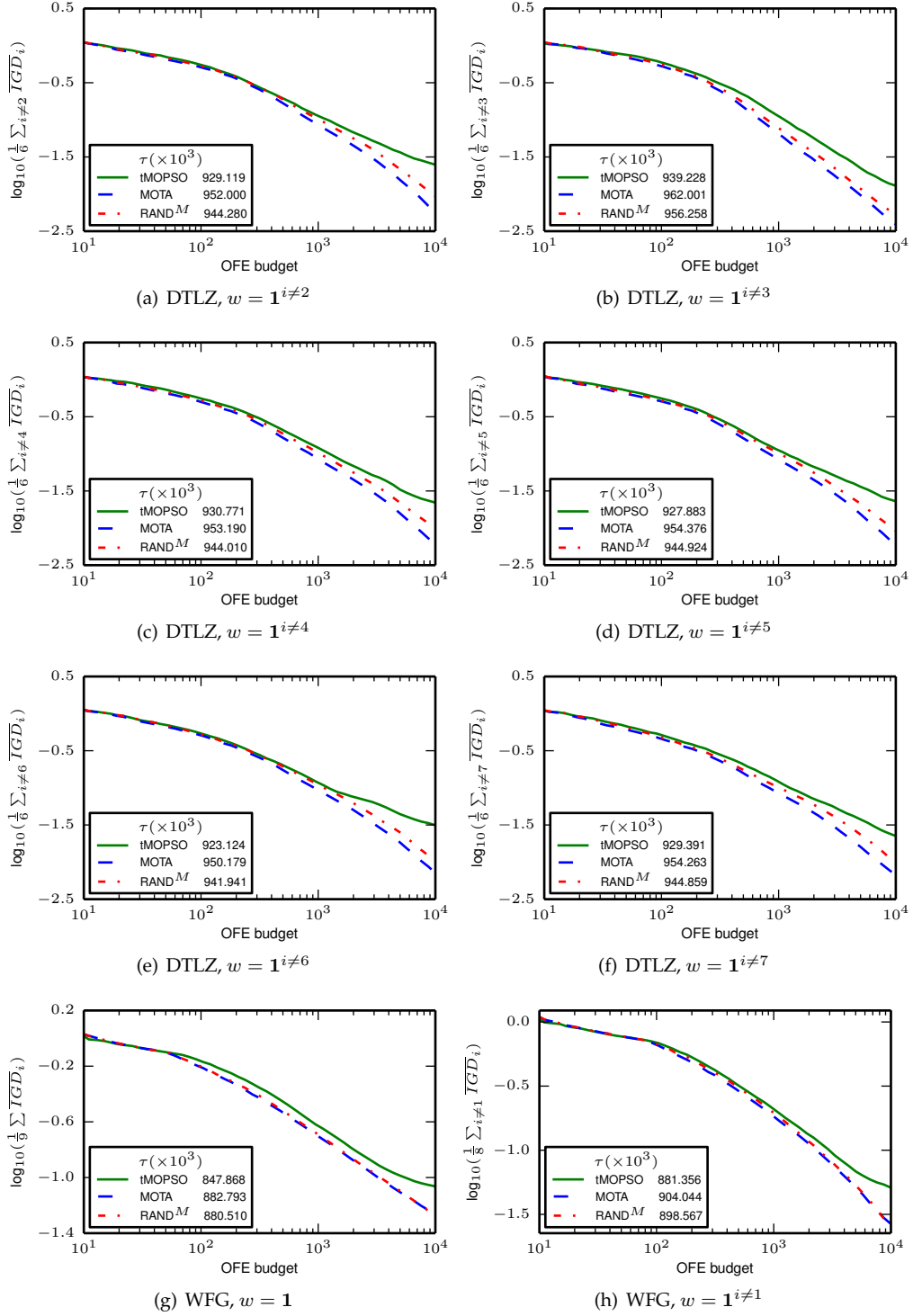


Figure 17: MOEA/D generalist results ($\gamma = 10^7$), Weighted sum of normalized IGD values (\overline{IGD}) versus OFE budget, with \overline{IGD} normalized between the Nadir and Utopia point. (table 2/3)

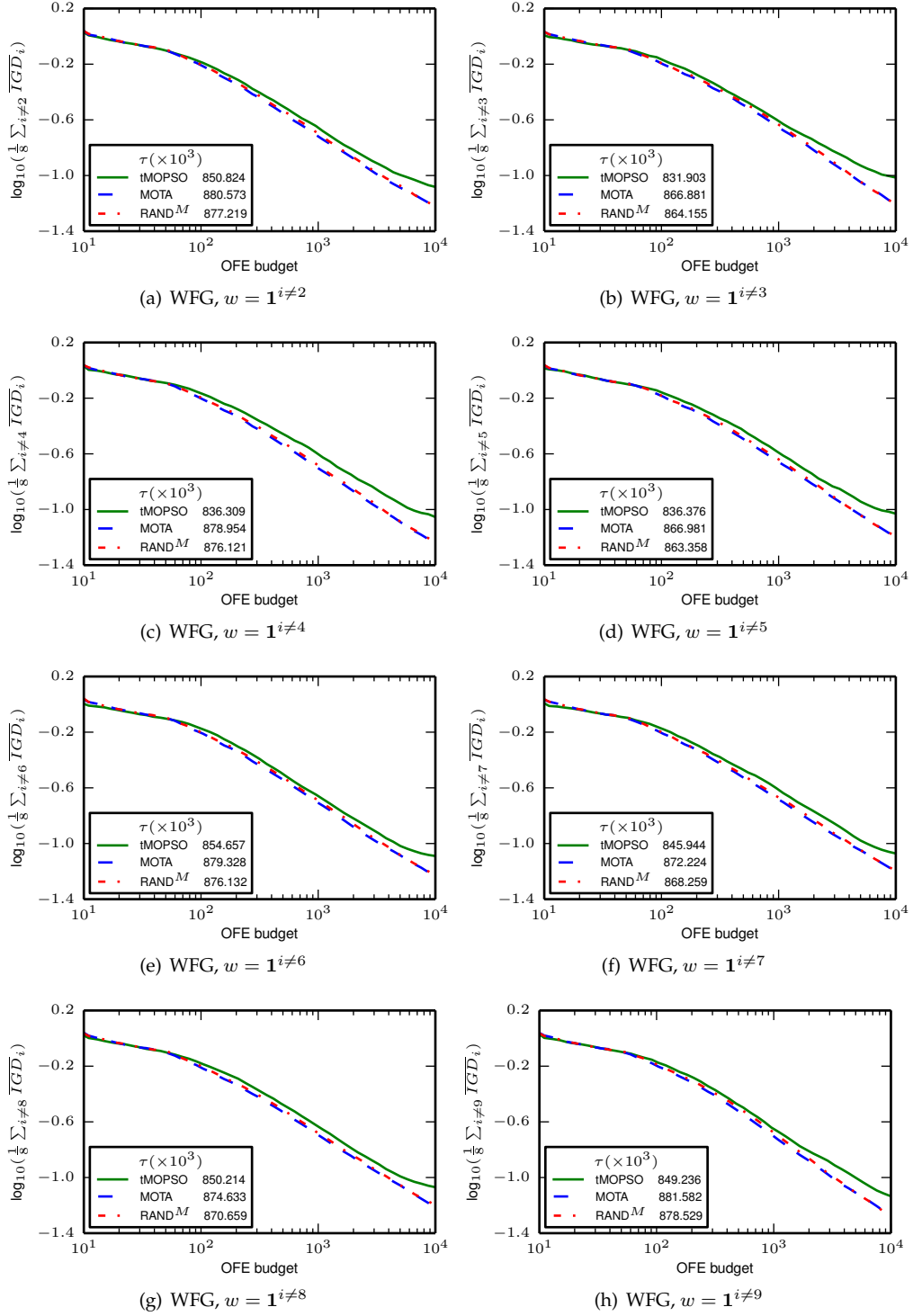


Figure 18: MOEA/D generalist results ($\gamma = 10^7$), Weighted sum of normalized IGD values (\overline{IGD}) versus OFE budget, with \overline{IGD} normalized between the Nadir and Utopia point. (table 3/3)

7 MOTA's results for the generalist problems

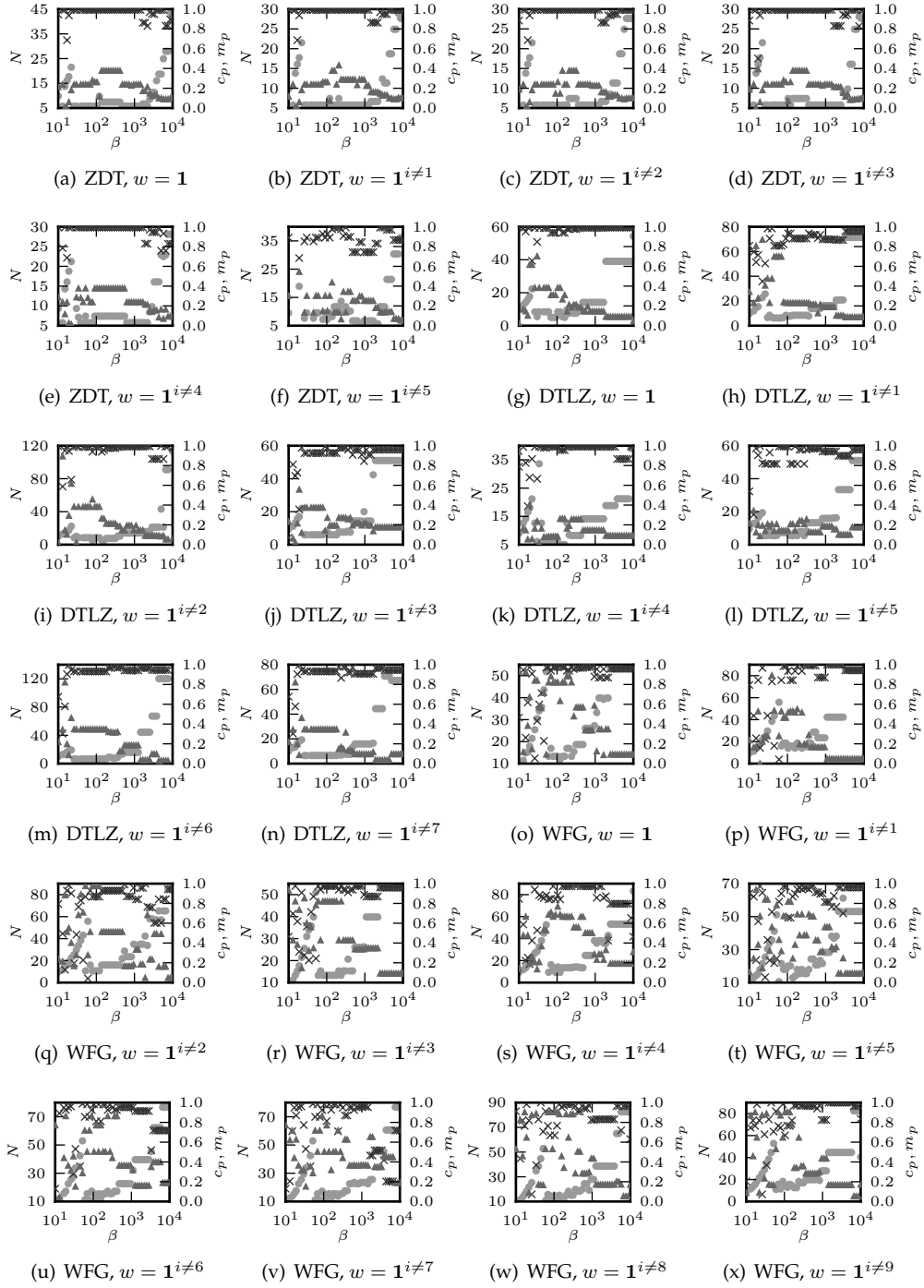


Figure 19: The best MOTA results for the NSGA-II generalist tuning problems. The recommended CPVs are shown for differing OFE budgets, β . The legend for the above subfigures is the same as in Figure 9

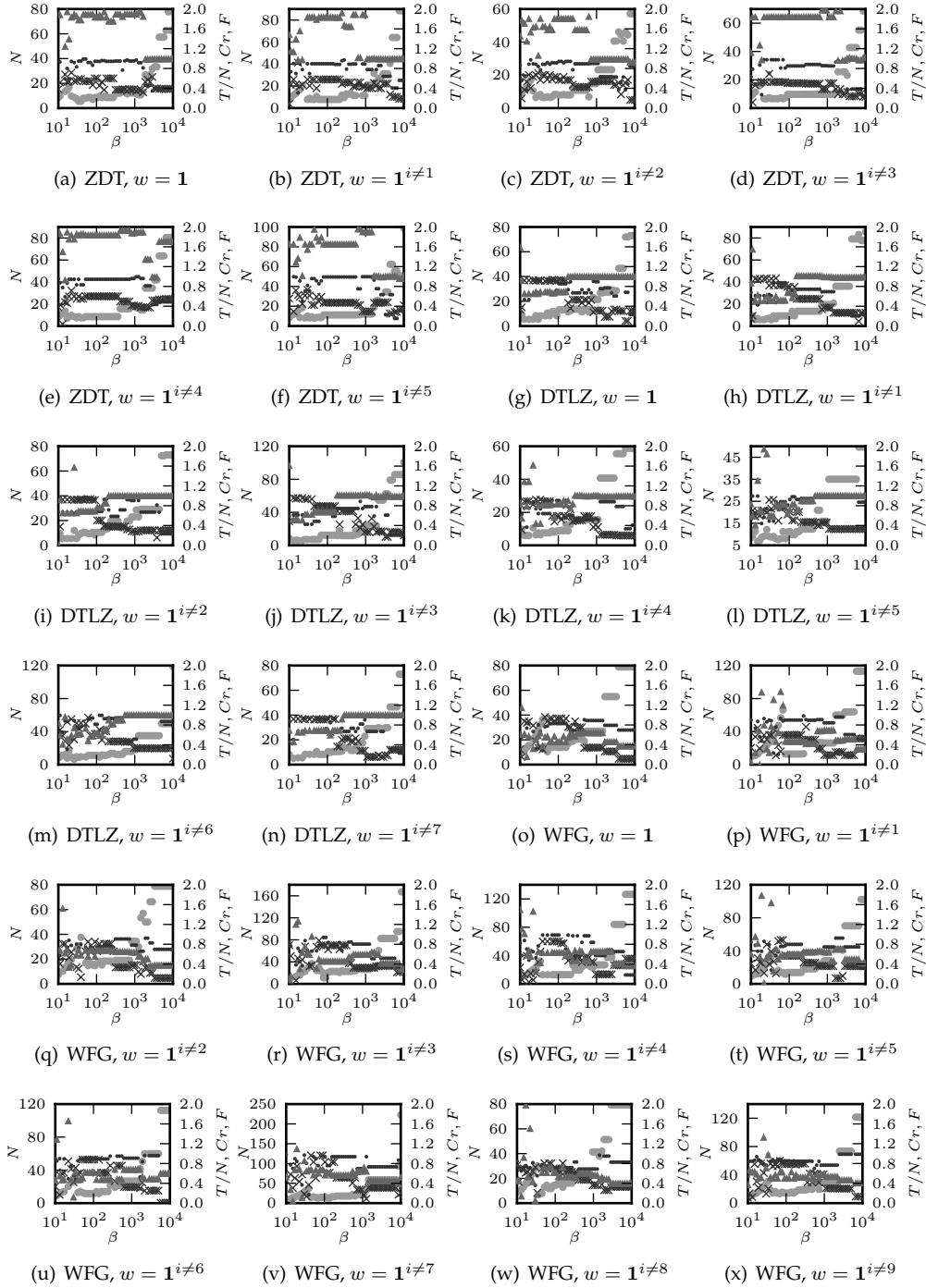


Figure 20: The best MOTA results for the MOEA/D generalist tuning problems. The recommended CPVs are shown for differing OFE budgets, β . The legend for the above subfigures is the same as in Figure 10