Bargaining over loan contracts with signaling*

Alexander Zimper† Mpoifeng Molefinyane‡

September 27, 2020

Abstract

This paper combines a sequential bargaining game between an enterprise and a fixed number of banks with a signaling game through which the enterprise reveals her project quality as well as her market speed on the lending market. We characterize subgame-perfect Nash equilibrium loan contracts that are supported by separating perfect Bayesian equilibria in the signaling game. In contrast to existing models of lending markets, low quality investment projects might be rewarded with more favorable equilibrium loan contracts than high quality projects. Also in contrast to existing models, an increase in the competitive pressure between banks reduces the aggregate welfare in our model. The reason is that more favorable loan conditions come with a greater incentive for the 'strong' entrepreneur to distinguish herself from her 'weak' counterpart through socially wasteful signaling costs.

Keywords: Sequential bargaining; Signaling; Relationship lending
JEL Classification Numbers: C78; G21

*Parts of this paper had been written when the first author was a guest at the Center for Interdisciplinary Research (ZiF), University of Bielefeld, as well as at the SAFE Research Center, University of Frankfurt. We are grateful to comments and suggestions from Roman Inderst, Alex Ludwig, Frank Riedel, and Myrna Wooders. We are also grateful to the helpful comments from an associate editor and two anonymous referees.

†Corresponding author. Department of Economics, University of Pretoria, Private Bag X20, Hatfield 0028, South Africa. E-mail: alexander.zimper@up.ac.za

‡Department of Economics, University of Pretoria, Private Bag X20, Hatfield 0028, South Africa. E-mail: mpoifeng.molefinyane@up.ac.za

1 Introduction

The welfare analysis of loan markets typically focuses on situations in which investment projects with a positive net-present value (=NPV) remain unfunded. This literature distinguishes between welfare losses that arise (i) from a monopolistic or oligopolistic lending market and (ii) from credit-rationing where borrowers have a higher demand for loans at given interest rates than actually supplied by lenders. In monopoly models of banking (cf., e.g., Klein, 1971; Monti, 1972; Dermine, 1986) the bank's profit-maximizing interest rate leaves projects with positive NPVs unfunded that cannot repay the artificially high interest rate of the lending monopoly. In standard industrial organization models of oligopolistic Cournot- or Betrand competition between lenders an increasing number of banks would drive down welfare losses so that all projects with positive NPVs become funded in the competitive limit.[1] Starting with the seminal paper by Stiglitz and Weiss (1981), the vast literature on credit-rationing argues that investments projects with positive NPVs might–regardless of the competitive pressure on banks–remain unfunded due to asymmetric information between lender and borrower. The informational asymmetries considered in these credit-rationing models either concern the borrower's subsequent actions (moral hazard) or the quality of her project (adverse selection).[2]

This paper complements the existing literature by analyzing welfare losses which exclusively arise from signaling costs in an economy in which all investment projects with positive NPVs become funded. The equilibrium loan contracts of our economy are negotiated in a sequential bargaining game between a small or medium enterprise (=SME) and a fixed number of banks. The SME is either of a 'strong' or a 'weak' type. Although the SME's type is private information–which is not directly observable by the banks–the SME can signal her true type to the banks through the quality of her loan application. The 'strong' SME is more cost-efficient than her 'weak' counterpart in the twofold sense that (i) she can move with a greater market speed, i.e., at less time-delay costs, to a new bargaining partner on the lending market and (ii) that she can achieve the same quality level of her loan application at less costs. Our preferred interpretation of the SME's type is that the 'strong' SME is more market-savvy than the 'weak' SME when it comes to the

[1] For models of Cournot output competition between banks over the number of loans see Freixas and Rochet (2008, Chapter 3.2.2) and references therein. For models of Bertrand price competition between banks with and without product differentiation see, e.g., Stahl (1988), Yanelle (1988), and Parlour and Rajan (2001).

[2] According to moral hazard models credit-rationing happens because high interest rates might induce rogue behavior in borrowers. In adverse selection models of credit-rationing, high interest rates combined with limited liability would attract investment projects with low success probabilities as interest rates are only payable in the event of success. For an overview on both classes of asymmetric information models–moral hazard and adverse selection–see Tirole (2006).

complex interactions on the lending market including loan applications and bargaining processes. The strong SME type would, e.g., naturally correspond to an SME with an experienced financial management team whereas the weak type corresponds to an SME with an inexperienced financial management team.[3]

Investing into the quality of a loan application can take on very different forms in real life. The SME might, e.g., develop a reputational track record of good standing, design a flashy ad campaign, work on self-marketing skills, purchase the appropriate business attire, buy pralines for the bank employees etc. Whereas some of these activities might actually improve the quality of the SME's investment project, we follow Spence's (1973) classic contribution and restrict attention to signaling costs that are a complete waste of resources which would not happen in a world with perfect information. More precisely, welfare losses will arise in our economy exclusively through the signaling costs of the 'strong' SME type who wants to overcome asymmetric information in order to obtain a more favorable loan contract than her 'weak' counterpart. While we adopt Spence's (1973) notion of socially wasteful signaling costs, we go in one important modeling choice beyond his approach. Recall that in Spence's (1973) original model signals about a worker's productivity type are sent to a perfectly competitive labour market that is not explicitly modeled. In contrast, we explicitly model the bargaining process between the SME and the banks to the effect that the resulting equilibrium loan contracts–as well as the corresponding signaling costs–are characterized as functions in the exogenously given economic fundamentals of our model. Our subsequent welfare analysis can therefore investigate the impact of changes in the values of relevant model parameters on the socially wasteful signaling costs that will be chosen in an equilibrium.

Starting with Shaked and Sutton (1984), there exists a substantial literature on bilateral bargaining for which the prospect of an alternative bargaining partner constitutes a valuable outside option. These models impose different rules for bilateral bargaining such as, e.g., Rubinstein's (1982) infinite horizon model versus the take-it-or-leave-it approach (for a discussion of both approaches see Muthoo, 1993). The literature also makes different assumptions about whether an outside option arises either through random matching (e.g., Rubinstein and Wolinsky, 1985; Gale, 1987) or by active choice (e.g., Wolinsky, 1987; Bester, 1988, 1989, 1993). In our own bargaining model the SME can actively choose to move on to a new bargaining partner whereby we combine (i) Bester's (1988, 1989) concept of spatial bargaining under 'time-delay' costs with (ii) ordered take-it-or-leave-it bargaining (cf. Raskovich (2007) and references therein). Whereas the value of an outside option arises in Bester's model from a random first-mover advantage in Rubinstein's (1982) alternating offers model, the banks in our model have the power to make

[3]We owe this straightforward and intuitive interpretation of SME types to an anonymous referee.

take-it-or-leave-it offers to the SME, which rules out any first-mover advantage of the SME. If the SME and a given bank cannot agree on a loan contract, the SME moves on to the next bank, until she runs out of banks. The strong type of the SME thereby moves between banks with greater market speed than her weak counterpart, which translates into less time-delay costs for the strong SME.

Given that there is no random first-mover advantage in our bargaining model, the question arises why it might be a valuable outside option for the SME to move on to the next bank. In line with the literature on relationship versus transactional banking (see, e.g., Boot and Thakor, 2000; Inderst and Müller, 2007), the banks in our economy either offer loan contracts under a relationship or a transactional lending technology. Besides increased customer loyalty, a bank benefits in the long run from relationship lending through her privileged access to soft information about the SME (Sharpe, 1990; Boot 2000; Stein, 2002; Liberti and Petersen, 2018). However, relationship lending is costly or/and faces resource constraints as it depends on the employment of highly qualified loan officers. As a consequence, the banks also engage in transaction lending whereby we assume that transaction loan contracts aim to maximize the bank's profit from the investment project at hand without any consideration about future benefits that might arise from relationship banking. For the sake of analytical convenience, we make the simplifying assumption that the use of either lending technology is not a strategic choice of the banks but determined by a chance move of nature.[4] That is, either lending technology is offered with a fixed probability which is i.i.d. across banks. The SME will benefit in the equilibrium of our model from relationship lending through a loan contract that comes with lower interest rates than a transactional loan contract through which the bank would maximally exploit the SME in the short run. To move on to the next bank might thus be a valuable outside option in our sequential bargaining model, because the SME expects to be offered with some positive probability an attractive relationship banking contract rather than an exploitative transaction banking contract.[5]

We comprehensively solve the sequential bargaining game for subgame perfect equilibrium (SPE) loan contracts under the assumption that the banks know the SME's true

[4]It would be an interesting task for future research to endogenize the possibility of different lending technologies in terms of the bank's strategic choice. For example, Konrad and Thum (2020) argue in the context of ultimatum bargaining games that an employer (e.g., the bank) may have strategic incentives to delegate bargaining to an 'opaque' employee whose decision making process (e.g., about lending technologies) appears to be random.

[5]There exists a large literature which argues that SMEs benefit from relationship lending through a combination of easily available future loans and favorable interest rates (cf., e.g., Petersen and Rajan, 1994, 1995; Cole, 1998; Elsas and Krahnen, 1998; Harhoff and Körting, 1998; Berger and Udell, 2002, 2006). In this paper we focus on favorable interest rates only.

type. The comparative statics of the resulting SPE loan contracts are in line with standard economic intuition:

The SME receives a more favorable loan contract if

(i) *the probability of relationship lending increases;*

(ii) *the number of banks in the economy increases;*

(iii) *the SME's market speed increases.*

To justify our maintained assumption that the banks know the SME's true type, we close off the model by combining the second stage sequential bargaining game with a first stage signaling game. We speak of *equilibrium loan contracts* whenever a given pair of type-dependent SPE loan contracts provides both SME types with incentives to reveal their true types in a separating perfect Bayesian Nash equilibrium (PBE) of the signaling game. Our welfare analysis investigates how changes in the exogenous parameters of the economy impact on the signaling costs in cost-efficient separating PBEs. In such cost-efficient PBEs, the strong SME chooses the minimal signaling costs that can distinguish herself from her weak counterpart who chooses a zero cost level.

Our comparative statics welfare analysis establishes the following main findings:

The aggregate welfare of our lending economy increases if

(i) *the number of banks in the economy decreases;*

(ii) *the probability of relationship lending decreases.*

Both welfare findings are counterintuitive as they are seemingly at odds with traditional industrial organization models that focus on the negative welfare impact of too little competition in monopolistic or oligopolistic banking economies. To see the economic intuition behind our welfare findings, note that the incentive compatibility condition for the weak type is binding at the cost-efficient separating PBE. The strong SME type can therefore save on signaling costs if and only if the weak type has less incentives to mimic the strong type. Both SME types receive a better equilibrium loan contract if the number of banks increases and/or if the probability of relationship lending increases. In our economy, however, the strong type would benefit to a greater extend from a more favorable equilibrium loan contract than her weak counterpart. As a consequence, the weak type's incentives for mimicking the strong type increase through more favorable loan conditions. This drives up the strong SME type's wasteful signaling costs in the overall equilibrium for our two-stage model.

The remainder of our analysis proceeds as follows. Section 2 provides an overview on our model and introduces basic concepts. Section 3 constructs and solves the signaling game. Section 4 constructs and solves the sequential bargaining game under the assumption that banks have perfect knowledge about the SME's true type. In Section 5 we bring the signaling and the sequential bargaining game together in order to characterize equilibrium loan contracts for our two-stage model. Section 6 analyzes how the socially wasteful signaling costs could be driven down. Section 7 concludes. Formal proofs are relegated to the Appendix.

2 Set-up

2.1 Preliminaries

A risk-neutral SME requires the loan amount I from a risk-neutral bank in order to fund a risky investment project. If the project is successful, it yields return R. The SME is either of a 'strong' or a 'weak' type, i.e., $\theta \in \{S, W\}$. The informal interpretation of the SME's type is that the 'strong' SME operates more cost-efficiently on the lending market than her 'weak' counterpart. At the end of this section, we will provide a formal characterization of both SME types through Assumption 1.

If performed by type $\theta \in \{S, W\}$, the project succeeds with probability $p_\theta \in (0, 1)$. The NPVs of the project are strictly positive for both types, i.e.,

$$p_\theta R - I > 0 \tag{1}$$

so that the projects of both SME types are creditworthy. A loan contract between the bank and the SME, denoted $\alpha(\theta)$ for $\theta \in \{S, W\}$, specifies the SME's share $\alpha(\theta) \in [0, 1]$ versus the bank's share $1 - \alpha(\theta)$ of the successful project's return R. Due to limited liability, the SME will end up with a zero profit if the project fails whereas the bank will loose its investment I. For a fixed loan contract $\alpha(\theta)$ the payoffs of the SME of type θ and of the bank are respectively given as follows

Entrepreneur	Bank	event
$R\alpha(\theta)$	$R(1 - \alpha(\theta)) - I$	success (with prob. p_θ)
0	$-I$	failure (with prob. $1 - p_\theta$)

The expected payoff of the SME of type $\theta \in \{S, W\}$ from the loan contract $\alpha(\theta)$ is

$$p_\theta R\alpha(\theta)$$

whereas the expected payoff of the bank is

$$p_\theta R (1 - \alpha(\theta)) - I.$$

2.2 The two-stage structure of the model

The SME's type is private information and cannot be directly observed by the banks. However, we model the first stage of the lending market as a signaling game where the SME can make a costly investment into the quality level of her loan application, denoted q, which can be observed by the banks. We will exclusively focus on separating (perfect Bayesian) equilibria in this signaling game for which the true type of the SME becomes revealed to the banks. The signaling costs through which the strong type distinguishes herself from her weak counterpart correspond in our model to wasteful social costs. More specifically, we assume that the corresponding signaling costs are given as $c_\theta q$ where $c_\theta > 0$ denotes the type-dependent constant marginal costs. For a fixed loan contract $\alpha(\theta)$, the utility of an SME of type $\theta \in \{S, W\}$ from choosing quality level $q \geq 0$ is given as

Entrepreneur	state of the world
$R\alpha(\theta) - c_\theta q$	successful project
$-c_\theta q$	failure

The second stage of the lending market is modeled as a sequential bargaining game in which the SME–with her true type being revealed–negotiates with the bank(s) the conditions of her loan contract. Whenever a bank is approached by an SME, it offers to each type of the SME either a relationship loan contract or a transaction loan contract. To keep the model simple, we assume that the relationship loan contract, denoted $\alpha^*(\theta)$, will be offered by any given bank with the exogenously given probability $\lambda \in (0, 1)$ whereas the transaction loan contract, denoted $\alpha^{tr}(\theta)$, will be offered with probability $1 - \lambda$.

Our preferred interpretation of the transaction loan contract $\alpha^{tr}(\theta)$ is that the bank will try to maximally exploit the type θ SME through this contract in order to maximize its short-run profit. The transaction loan contract $\alpha^{tr}(\theta)$ will be derived endogenously when we solve the sequential bargaining game between the SME and the banks. Although every bank will have the power to make a profit-maximizing take-it-or-leave-it offer to the SME, the exogenously given parameters of our economy will determine–through their impact on the value of the SME's outside option–the transaction loan contract that will be offered by the bank in the equilibrium of the bargaining game. In contrast, the relationship loan contract $\alpha^*(\theta)$ is not subject to any short-run profit maximization considerations by the bank and we treat is as exogenously given to the model. Our preferred interpretation of the relationship loan contract is that it comes with highly attractive conditions for the SME because the bank uses this 'good-will' loan contract to initiate a long-term relationship with the SME.

Technically speaking, the possibility of being offered a relationship loan contract at the next bank establishes a valuable outside option for the SME during its negotiations with the current bank. However, to take up this outside option by moving to the next bank comes with type-dependent time-delay (i.e., traveling) costs. The SME of type $\theta \in \{S, W\}$ requires time τ_θ to reach the next bank whereby we refer to τ_θ^{-1} as the *market-speed* of type θ SME. In line with our interpretation that the strong SME operates more cost-efficiently than her weak counterpart, we assume that the strong SME can move on with a greater market-speed to a new bargaining partner whenever she is not happy with the loan contract offered to her by her current bargaining partner.

Whether there exist separating PBEs in the signaling game of the first stage of the model or not, will depend on the type-dependent SPE loan contracts that obtain in the second stage of the model. As a consequence, we have to close-off our model through a dynamic consistency condition which amounts to a simple backward-induction argument for the two stages of our model. More precisely, a pair of SPE loan contracts will constitute equilibrium loan contracts of the two-stage model whenever these SPE loan contracts provide both SME types with incentives to choose a separating PBE in the signaling game. The minimal signaling costs of the strong SME type which are required to support type-dependent equilibrium loan contracts will then become the subject of our welfare analysis.

2.3 The two SME types

The type $\theta \in \{S, W\}$ of the SME determines (i) the quality of her project through the success probability p_θ, (ii) her signaling costs through the marginal investment costs c_θ into the quality of her loan application, and (iii) her time-delay costs on the lending market through her market-speed τ_θ^{-1}. We conclude this section by a formal characterization of our assumption that the strong SME type operates more cost-efficiently on the lending market than her weak counterpart.

Assumption 1. *Throughout this paper we assume that the strong type is more cost-efficient than her weak counterpart in the specific sense that*

$$c_S < c_W \text{ and } \tau_S^{-1} > \tau_W^{-1}.$$

Remark. Note that Assumption 1 does not impose any restrictions on the success probabilities, i.e., the project quality, of both types. We will later introduce such restrictions–depending on the marginal signaling cost parameters–through Assumption 2 in order to ensure the existence of separating PBEs of the signaling game. Importantly, the analysis in this paper will also apply to situations in which the strong type's success probability is (slightly) smaller than the low type's success probability (cf. Proposition 6 below).

3 Quality of the loan application: A signaling game

Before the SME of type $\theta \in \{S, W\}$ approaches a bank for a loan, she does not know whether she will be offered a relationship loan contract $\alpha^* (\theta) \in [0, 1]$ or a transaction loan contract $\alpha^{tr} (\theta) \in [0, 1]$. From her ex ante perspective, she faces therefore a *random loan contract*, denoted $\mathcal{L} (\theta)$, which we define as the following lottery over the relationship and transaction loan contracts

$$\mathcal{L} (\theta) = \left(\alpha^* (\theta), \lambda; \alpha^{tr} (\theta), (1 - \lambda)\right) \text{ for } \theta \in \{S, W\} \tag{2}$$

for a fixed relationship lending probability $\lambda \in (0, 1)$. The expected payoff of the SME of type $\theta \in \{S, W\}$ from the random loan contract $\mathcal{L} (\theta)$ is given as

$$p_\theta R \mathbb{E} [\mathcal{L} (\theta)]$$

such that

$$\mathbb{E} [\mathcal{L} (\theta)] = \lambda \alpha^* (\theta) + (1 - \lambda) \alpha^{tr} (\theta)$$

denotes type θ's expected share of the project's return under the random loan contract $\mathcal{L} (\theta)$. If the SME type θ chooses quality level q for her loan application, her expected utility from the random loan contract $\mathcal{L} (\theta)$ and quality level q becomes

$$U_\theta (q; \mathcal{L} (\theta)) = p_\theta R \mathbb{E} [\mathcal{L} (\theta)] - c_\theta q.$$

To formally describe the signaling game, denote by $P (\theta) \in (0, 1)$ the bank's prior belief that the SME is of type θ. Upon observing quality level q of the loan application the bank updates this prior belief to the posterior $P (\theta \mid q)$. Denote by $\sigma_\theta (q)$ the probability with which type θ chooses quality level q. For convenience, we simply refer to a probability distribution σ_θ as a *strategy* of type θ. In a *perfect Bayesian equilibrium* (=PBE), each type of the SME chooses an expected payoff maximizing strategy over quality levels subject to the consistency condition that the bank has correct beliefs about which type chooses which equilibrium strategy.

Definition: Loan contracts supported by a Perfect Bayesian Nash Equilibrium. *The pair of random loan contracts* $\mathcal{L} = (\mathcal{L}(S), \mathcal{L}(W))$ *is supported by the PBE* $(\sigma_S^*, \sigma_W^*, P(\theta \mid \cdot))$ *if and only if the following two conditions are satisfied.*

(i) *The equilibrium strategy of the SME of type* $\theta \in \{S, W\}$ *satisfies: If* $\sigma_\theta^*(q^*) > 0$*, then*

$$q^* \in \arg\max_{q \in \mathbb{R}_+} \sum_{\theta \in \{S,W\}} U_\theta(q; \mathcal{L}(\theta)) P(\theta \mid q).$$

(ii) *The bank's posterior beliefs* $P(\theta \mid q)$ *are formed in accordance with Bayes' rule*

$$P(\theta \mid q) = \frac{\sigma_\theta^*(q) P(\theta)}{\sigma_S^*(q) P(\theta_S) + \sigma_W^*(q) P(\theta_W)} \text{ for } \theta \in \{S, W\}$$

whenever at least one type chooses q *with strictly positive probability (i.e., whenever* $\sigma_S^*(q) > 0$ *or* $\sigma_W^*(q) > 0$*). If there is no type who chooses quality level* q *with strictly positive probability in an equilibrium, then the out-of-equilibrium belief* $P(\theta \mid q)$ *can be formed arbitrarily.*

The remainder of our analysis restricts attention to *separating* PBEs in *pure strategies.* A PBE in pure strategies, denoted $(q_S, q_W, P(\theta \mid \cdot))$, is characterized by probability-one equilibrium strategies

$$\sigma_\theta^*(q_\theta) = 1 \text{ for } \theta \in \{S, W\}.$$

A PBE in pure strategies is *separating* if and only if both types choose different quality levels, i.e., $q_S \neq q_W$.[6] Such PBEs must satisfy the following incentive constraints for the strong type

$$U_S(q_S; \mathcal{L}(S)) \geq U_S(q_W; \mathcal{L}(W))$$

$$\Leftrightarrow$$

$$p_S R \mathbb{E}[\mathcal{L}(S)] - c_S q_S \geq p_S R \mathbb{E}[\mathcal{L}(W)] - c_S q_W$$

and for the weak type, respectively

$$U_W(q_W; \mathcal{L}(W)) \geq U_W(q_S; \mathcal{L}(S))$$

$$\Leftrightarrow$$

[6] In a *pooling* PBE both types would choose the same quality levels to the effect that the banks cannot distinguish between both types. Pooling PBEs are not very interesting from a welfare analysis perspective as they can always be supported by zero signaling costs. To gain interesting welfare insights from a comparative statics analysis, this paper therefore focuses on separating PBEs only.

$$p_W R\mathbb{E}\left[\mathcal{L}\left(W\right)\right] - c_W q_W \geq p_W R\mathbb{E}\left[\mathcal{L}\left(S\right)\right] - c_W q_S.$$

Note that the weak type will always choose the minimal quality level, $q_W = 0$, in a separating PBE. As a consequence, the weak type's participation constraint

$$\begin{aligned} U_W\left(q;\mathcal{L}\left(W\right)\right) &\geq 0 \\ &\Leftrightarrow \\ p_W R\mathbb{E}\left[\mathcal{L}\left(W\right)\right] &\geq 0 \end{aligned} \tag{3}$$

is always satisfied in a separating PBE as the left hand side of inequality (3) cannot be negative because of $\mathbb{E}\left[\mathcal{L}\left(W\right)\right] \geq 0$.

Rearranging the above incentive compatibility constraints gives us, together with the participation constraint of the strong type, the following characterization of quality levels that might be chosen in a separating PBE for a given random loan contract.

Proposition 1. *The pair of random loan contracts $\mathcal{L} = \left(\mathcal{L}\left(S\right), \mathcal{L}\left(W\right)\right)$ can be supported by a separating PBE $\left(q_S, q_W, P\left(\theta \mid \cdot\right)\right)$ in pure strategies if and only if*

$$\begin{aligned} q_S &\in \left[\frac{p_W}{c_W} R\left(\mathbb{E}\left[\mathcal{L}\left(S\right)\right] - \mathbb{E}\left[\mathcal{L}\left(W\right)\right]\right), \frac{p_S}{c_S} R\left(\mathbb{E}\left[\mathcal{L}\left(S\right)\right] - \mathbb{E}\left[\mathcal{L}\left(W\right)\right]\right)\right], \\ q_W &= 0 \end{aligned}$$

whereby q_S has to satisfy the strong type's participation constraint

$$\begin{aligned} U_S\left(q;\mathcal{L}\left(S\right)\right) &\geq 0 \\ &\Leftrightarrow \\ \frac{p_S}{c_S} R\mathbb{E}\left[\mathcal{L}\left(S\right)\right] &\geq q_S. \end{aligned} \tag{4}$$

Supporting equilibrium beliefs $P\left(\theta \mid \cdot\right)$ for the PBE quality levels q_S, q_W are, for example, given as[7]

$$\begin{aligned} P\left(W \mid q\right) &= 1 \text{ for all } q < q_S, \\ P\left(S \mid q\right) &= 1 \text{ for all } q \geq q_S. \end{aligned}$$

[7] In words: Whenever the SME chooses a quality level q below q_S, the bank assumes that the SME is of the weak type. In contrast, the bank assumes that the SME is of the strong type, if she chooses at least quality level q_S.

In order for a separating PBE to exist, the interval

$$\left[\frac{p_W}{c_W}R\left(\mathbb{E}\left[\mathcal{L}\left(S\right)\right]-\mathbb{E}\left[\mathcal{L}\left(W\right)\right]\right),\frac{p_S}{c_S}R\left(\mathbb{E}\left[\mathcal{L}\left(S\right)\right]-\mathbb{E}\left[\mathcal{L}\left(W\right)\right]\right)\right] \tag{5}$$

must be a non-empty subset of the positive real numbers. Together with the strong type's participation constraint (4), we thus obtain the following necessary and sufficient conditions for the existence of a separating PBE in the signaling game.

Proposition 2. *The pair of random loan contracts $\mathcal{L}=\left(\mathcal{L}\left(S\right),\mathcal{L}\left(W\right)\right)$ can be supported by a separating PBE if and only if the following three conditions hold:*

$$\frac{p_W}{c_W} \leq \frac{p_S}{c_S}, \tag{6}$$

$$\mathbb{E}\left[\mathcal{L}\left(S\right)\right] > \mathbb{E}\left[\mathcal{L}\left(W\right)\right], \tag{7}$$

$$\mathbb{E}\left[\mathcal{L}\left(S\right)\right] \geq \frac{c_S}{p_S}\frac{q_S}{R}. \tag{8}$$

By condition $c_S < c_W$ of Assumption 1, the parameter condition (6) would always be satisfied if the strong type's project has a greater success probability than the project of her weak counterpart, i.e., if $p_S \geq p_W$. However, condition (6) also allows for the possibility that the weak type's success probability is strictly greater as long as the strong type has a sufficiently great cost advantage. In order to ensure the existence of separating PBEs we will henceforth restrict attention to situations that satisfy parameter condition (6).

Assumption 2. *Throughout this paper we assume that the success probabilities of both SME types satisfy, for fixed marginal signaling costs c_S and c_W, the parameter condition*

$$\frac{p_W}{c_W} \leq \frac{p_S}{c_S}.$$

Our subsequent welfare analysis will exclusively focus on the minimal quality level in the interval (5). Given that the choice of positive quality levels by the strong SME is socially wasteful, this minimal quality level stands for the least wasteful quality level through which the strong SME can distinguish herself from her weak counterpart in a separating PBE.

Definition: Cost-efficient quality level. *For a given pair of random loan contracts* $\mathcal{L} = (\mathcal{L}(S), \mathcal{L}(W))$*, the cost-efficient quality level is given as follows:*

$$q_S^*[\mathcal{L}] = \frac{p_W}{c_W} R\left(\mathbb{E}[\mathcal{L}(S)] - \mathbb{E}[\mathcal{L}(W)]\right). \tag{9}$$

For the cost-efficient quality level the participation constraint (8) of the strong SME type becomes

$$\begin{aligned} \mathbb{E}[\mathcal{L}(S)] &\geq \frac{c_S}{p_S}\frac{q_S^*[\mathcal{L}]}{R} \\ &\Leftrightarrow \\ \left(\frac{c_W p_S - c_S p_W}{c_W p_S}\right)\mathbb{E}[\mathcal{L}(S)] &\geq \frac{c_S p_W}{c_W p_S}\left(-\mathbb{E}[\mathcal{L}(W)]\right) \end{aligned} \tag{10}$$

By Assumption 2, we have that $c_W p_S - c_S p_W \geq 0$. Moreover, since

$$\mathbb{E}[\mathcal{L}(S)], \mathbb{E}[\mathcal{L}(W)] \geq 0,$$

the left hand side of inequality (10) will always be non-negative whereas the right-hand side will always be non-positive. Consequently, the participation constraint (8) for the strong SME type is always satisfied for the cost-efficient quality level $q_S^*[\mathcal{L}]$ in any separating PBE.

Corollary 1. *By Assumption 2, the pair of random loan contracts* $\mathcal{L} = (\mathcal{L}(S), \mathcal{L}(W))$ *can be supported by a separating PBE in which the strong SME chooses the cost-efficient quality level* $q_S^*[\mathcal{L}]$ *given by (9) if and only if*

$$\begin{aligned} \mathbb{E}[\mathcal{L}(S)] &> \mathbb{E}[\mathcal{L}(W)] \\ &\Leftrightarrow \\ \lambda\alpha^*(S) + (1-\lambda)\alpha^{tr}(S) &> \lambda\alpha^*(W) + (1-\lambda)\alpha^{tr}(W). \end{aligned} \tag{11}$$

Remark. Corollary 1 will be crucial for the dynamic consistency of our two-stage model. Condition (11) will justify our maintained modeling assumption that the true type of the SME is common knowledge to the banks after they have observed the quality of the SME's loan application (cf. Proposition 5 below). Namely, there will exist a separating PBE in the first stage signaling game if and only if the loan contracts $\mathcal{L}(S)$

and $\mathcal{L}(W)$ that obtain in the subgame-perfect equilibrium of the second-stage sequential bargaining game satisfy condition (11). In words: In the overall equilibrium of our two-stage-model, the strong type will distinguish herself from her weak counterpart if and only if she is going to receive a strictly greater expected share of the project's return than her weak counterpart.

4 Sequential bargaining over loan contracts

4.1 Set-up

We construct a sequential bargaining game with $n \geq 1$ banks under the assumption that the true type of the SME is common knowledge to the banks. The game is finite and consists of n different stages such that the SME approaches at stage $i \in \{1, ..., n\}$ bank i for a loan in case she did not already reach an agreement with some bank $j < i$.

The SME starts the bargaining process by making an initial offer to bank i about a loan contract. The bank can either accept this offer to the effect that the game stops. Or the bank makes in turn a take-it-or-leave-it offer about a loan contract to the SME. If the SME accepts the bank's offer, the game stops. If she rejects this offer, she moves on to bank $i+1 \leq n$ facing the same situation as with bank i, i.e., she makes an initial offer and so forth. At the final stage n either (i) bank n accepts the SME's initial offer, or (ii) the SME accepts bank n's take-it-or-leave-it-offer, or (iii) she rejects the bank's offer to the effect that the investment project will not be financed.

To keep the game-theoretic exposition as simple as possible, we assume that bargaining happens under perfect information so that the banks observe the previous bargaining history. As a consequence of this perfect information assumption, we can formally identify any history leading to stage i as the origin of a bargaining subgame that starts at stage i. Moreover, the specific history that leads to some bargaining subgame at stage i is irrelevant to the players' payoffs at this subgame. Since all bargaining subgames starting at stage i are thus strategically equivalent regardless of their histories, any subgame-perfect Nash equilibrium (SPE) of a bargaining subgame starting at stage i will also be a SPE for all other bargaining subgames starting at stage i.

Our sequential bargaining game will be driven by the existence of a valuable outside option at type-dependent costs. Every bank offers to the SME type θ with probability $\lambda \geq 0$ the relationship loan contract $\alpha^*(\theta)$. As a consequence, the SME's outside option of moving on to the next bank becomes valuable whenever this relationship loan contract is superior to the transaction loan contract. However, moving on to the next bank is also costly. We follow Bester (1988, 1989) and formally model these costs as 'time-delay' costs. More precisely, we assume that ability type θ moves with market-speed $\tau^{-1}(\theta)$ from bank

i to the next bank $i+1$ for all $i \in \{1,...,n-1\}$. Moving on the next bank is costly for the SME because she is time-impatient whereby she discounts the amount of time $\tau(\theta)$ by $\delta^{\tau(\theta)}$ where $\delta \in (0,1)$ denotes her time-discount factor. By Assumption 1, the strong type moves with greater market-speed than her weak counterpart, implying

$$\begin{array}{ccc} \tau(S) & < & \tau(W) \\ & \Leftrightarrow & \\ \delta^{\tau(S)} & > & \delta^{\tau(W)}. \end{array}$$

In words: The strong type discounts the value of her outside option strictly less than her weak counterpart. Not surprisingly, this difference in the evaluation of outside options will translate into a greater bargaining power for the strong than for the weak type.

4.2 Solving the sequential bargaining game

We solve the sequential bargaining game through backward induction. Fix some bargaining subgame at stage $i \in \{1,...,n\}$. At the initial node of each bargaining subgame Nature chooses, unobserved by the SME, with probability λ a relationship and with probability $1-\lambda$ a transaction loan contract offer by bank i.

1. Let us start at the end of stage i where the SME has to decide between either accepting a take-it-or-leave-it (t-o-l) counter offer $a_i(\theta)$ from bank i or moving on to bank $i+1$. The SME of type $\theta \in \{S,W\}$ would accept any t-o-l counter-offer such that the amount $a_i(\theta)R$ is at least as good as the time-discounted expected payoff she would receive in a SPE of the bargaining subgame starting at stage $i+1$. Denote the *expected SPE continuation value at stage* $i+1$ of type θ (before time-discounting) as $V(i+1;\theta)$. Note that $V(1;\theta)$ stands for type θ's expected SPE continuation value at the first stage, i.e., her expected value of the whole game.

2. Move up to the transaction loan contract situation where the bank i makes some t-o-l counter offer to the SME of type $\theta \in \{S,W\}$ because she had rejected the SME's initial offer. By correctly understanding the situation under Step 1, the bank offers in any SPE $a_i^{tr}(\theta) = \delta^{\tau(\theta)}\frac{V(i+1;\theta)}{R}$, which will be just accepted by type θ thereby ensuring the maximally possible expected profit for bank i while avoiding that the SME moves on to bank $i+1$.

3. Now consider the relationship loan contract situation where the bank i makes the t-o-l counter offer $a^*(\theta)$ to the SME of type $\theta \in \{S,W\}$ because she has rejected

the SME's initial offer. The t-o-l counter offer $\alpha^*(\theta)$ will be accepted by the SME whenever

$$\delta^{\tau(\theta)} \frac{V(i+1;\theta)}{R} \leq \alpha^*(\theta).$$

4. Move now up to the situation where type $\theta \in \{S, W\}$ of the SME makes her initial offer $\alpha^{SME}(\theta)$ under uncertainty about the lending contract type. Under transaction lending, any initial offer $\alpha^{SME}(\theta) > a_i^{tr}(\theta)$ would be rejected whereas any initial offer $\alpha^{SME}(\theta) \leq a_i^{tr}(\theta)$ would be accepted. Note that any $\alpha^{SME}(\theta) \geq a_i^{tr}(\theta)$ would result in the same SPE outcome

$$a_i^{tr}(\theta) = \delta^{\tau(\theta)} \frac{V(i+1;\theta)}{R}$$

which is the optimum that can be achieved under transaction lending. Under relationship lending any initial offer $\alpha^{SME}(\theta) > \alpha^*(\theta)$ would be rejected whereas any initial offer $\alpha^{SME}(\theta) \leq \alpha^*(\theta)$ would be accepted. In the SPE outcome, any $\alpha^{SME}(\theta) \geq \alpha^*(\theta)$ results in the optimum $\alpha^*(\theta)$ achievable under relationship lending.

As the above reasoning applies to every bargaining subgame starting at any stage $i \in \{1, ..., n\}$, we obtain the following characterization of the SPE in terms of expected SPE continuation values.

Proposition 3. *Suppose that the expected SPE continuation value $V(i+1;\theta)$, $\theta \in \{S, W\}$, is given for all $i \in \{1, ..., n\}$ such that*

$$\delta^{\tau(\theta)} \frac{V(i+1;\theta)}{R} \leq \alpha^*(\theta) \tag{12}$$

Then the following actions constitute a SPE.

(i) *The stage $i \in \{1, ..., n\}$ SPE actions of type $\theta \in \{S, W\}$ of the SME are:*

$$\text{make initial offer } \alpha^{SME}(\theta) = \alpha^*(\theta);$$

$$\text{accept t-o-l counter offer } a_i \text{ if } a_i \geq \delta^{\tau(\theta)} \frac{V(i+1;\theta)}{R}$$

$$\text{else: reject t-o-l counter offer}$$

(ii) *The SPE actions under transaction lending by bank $i \in \{1, ..., n\}$ are:*

$$\text{accept initial offer } \alpha^{SME}(\theta) \text{ if } \alpha^{SME}(\theta) \leq \delta^{\tau(\theta)} \frac{V(i+1;\theta)}{R}$$

$$\text{else: make t-o-l counter offer } a_i^{tr}(\theta) = \delta^{\tau(\theta)} \frac{V(i+1;\theta)}{R}$$

(iii) *The SPE actions under relationship lending by bank $i \in \{1, ..., n\}$ are:*

$$\text{accept initial offer } \alpha^{SME}(\theta) \text{ if } \alpha^{SME}(\theta) \leq \alpha^*(\theta)$$

$$\text{else: make t-o-l counter offer } \alpha^*(\theta)$$

For the SPE of Proposition 3 the game ends at the first stage such that the SME of type $\theta \in \{S, W\}$ receives from bank 1 with probability λ the relationship loan contract $a^*(\theta)$ and with probability $1-\lambda$ the transaction loan contract $a_1^{tr}(\theta)$. The following proposition characterizes the loan contract, denoted $\mathcal{L}^*(\theta)$, that obtains as outcome in this SPE; (the formal proof is relegated to the Appendix).

Proposition 4.

(i) *For the SPE loan contract*

$$\mathcal{L}^*(\theta) = \left(a^*(\theta), \lambda; a_1^{tr}(\theta), (1-\lambda)\right) \text{ for } \theta \in \{S, W\}$$

we have that

$$a_1^{tr}(\theta) = \delta^{\tau(\theta)} \lambda \frac{1 - \left((1-\lambda)\,\delta^{\tau(\theta)}\right)^{n-1}}{1 - \left((1-\lambda)\,\delta^{\tau(\theta)}\right)} \alpha^*(\theta).$$

(ii) *Type θ's expected share from the SPE loan contract $\mathcal{L}^*(\theta)$ is*

$$\begin{aligned} \mathbb{E}[\mathcal{L}^*(\theta)] &= \lambda a^*(\theta) + (1-\lambda)\, a_1^{tr}(\theta) \\ &= \lambda \frac{1 - \left((1-\lambda)\,\delta^{\tau(\theta)}\right)^{n}}{1 - \left((1-\lambda)\,\delta^{\tau(\theta)}\right)} \alpha^*(\theta). \end{aligned} \tag{13}$$

(iii) *Type θ's expected share (13) increases if*

- *the probability λ of relationship lending increases;*
- *the market speed $\tau^{-1}(\theta)$ increases;*
- *the number of banks n increases.*

If the number of banks approaches infinity, we obtain for the expected share (13) of type θ that

$$\lim_{n\to\infty} \mathbb{E}\left[\mathcal{L}^*(\theta)\right] = \frac{\lambda}{1-\left((1-\lambda)\,\delta^{\tau(\theta)}\right)}\alpha^*(\theta)\,. \tag{14}$$

For a finite market-speed we have $\delta^{\tau(\theta)} < 1$ so that the expected share (14) is strictly smaller than the share $\alpha^*(\theta)$ under relationship lending. If type θ's market speed approaches infinity for a fixed number of banks, (13) becomes in the limit

$$\lim_{\tau(\theta)\to 0} \mathbb{E}\left[\mathcal{L}^*(\theta)\right] = \left(1-(1-\lambda)^n\right)\alpha^*(\theta)\,,$$

which is again strictly less than $\alpha^*(\theta)$ for any fixed number n of banks. Consequently, for a fixed $\lambda < 1$ the expected share of the SPE loan contract $\mathcal{L}^*$ for type θ converges to this type's share from the relationship loan contract α^* if and only if both, the number of banks and type θ's market speed, approach infinity.[8]

Remark. The determination of SPE through backward induction is exclusively driven by the fact that each banks knows in our model the (finite) number of remaining banks that might still be approached by the SME. Instead of assuming perfect information about the SME's bargaining history, we could have alternatively assumed that the SME bargains with the banks in a fixed order, say $(1,...,n)$, such that this order is common knowledge. Our model of sequential bargaining could thus alternatively be interpreted as an ordered bargaining model (cf. Raskovich, 2007). A spatial version of this ordered bargaining process corresponds, e.g., to the one-way version of Salop's (1979) circular city such that SMEs have to move in a fixed direction from bank to bank whereby the banks know the location from where the SME has started.

[8]Note that this limit result is different from the according limit result in Bester's model of spatial bargaining (1989, Theorem 8) where the SPE outcome converges to the competitive (here $\alpha^*(\theta)$) outcome when the number of banks becomes arbitrarily large. The reason is that we keep the distance between banks fixed whereas this distance becomes zero in Bester when the number of firms gets large. That is, in Bester's model an increase in the number of firms (here banks) is mathematically equivalent to an increase in the SME's market speed, which is not the case in our model where we keep the market-speed constant.

5 Equilibrium loan contracts

5.1 Closing-off the two-stage model

The type-dependent SPE loan contracts of Proposition 4 have been derived under the maintained assumption that the true type of the SME is common knowledge to the banks. It remains to close off our two-stage model by ensuring that these type-dependent SPE loan contracts provide both SME types indeed with the necessary incentives to reveal their true types.

Definition: Equilibrium loan contracts. *We call a pair of random loan contracts $\mathcal{L} = (\mathcal{L}(S), \mathcal{L}(W))$ equilibrium loan contracts (of the two-stage game) if and only if both $\mathcal{L}(\theta)$, $\theta \in \{S, W\}$, are SPE loan contracts of the second stage sequential bargaining game that induce both SME types to reveal their true types in a separating PBE in the first stage signaling game.*

For the formal characterization of any pair of equilibrium loan contracts, we combine the existence condition (11) for separating PBEs from Corollary 1 with the characterization of SPE loan contracts (13) from Proposition 4.

Proposition 5. *Any SPE loan contracts $\mathcal{L}^*(\theta)$, $\theta \in \{S, W\}$, are equilibrium loan contracts of the two-stage game if and only if*

$$\begin{aligned} \mathbb{E}[\mathcal{L}^*(S)] &> \mathbb{E}[\mathcal{L}^*(W)] \\ &\Leftrightarrow \\ \frac{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^n}{1-(1-\lambda)\,\delta^{\tau(S)}}\alpha^*(S) &> \frac{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^n}{1-(1-\lambda)\,\delta^{\tau(W)}}\alpha^*(W)\,. \end{aligned} \tag{15}$$

In other words, any equilibrium loan contracts of our two-stage model are pairs of type-dependent SPE loan contracts such that the expected value of the SPE loan contract for the strong SME type must be strictly greater than the expected value of the SPE loan contract for the weak SME type. For this reason the notion of equilibrium loan contracts can only apply to a given pair of random loan contracts for both types and not to any type-dependent loan contract in isolation. That is, to call, e.g., $\mathcal{L}^*(S)$ an equilibrium loan contract without having also specified $\mathcal{L}^*(W)$ would be meaningless in our model.

For fixed model parameters δ, $\tau(S)$, $\tau(W)$, λ, and n the existence of equilibrium loan contracts depends on the design of relationship loan contracts $\alpha^*(\theta)$ for both SME types $\theta \in \{S, W\}$. The next result establishes a necessary and sufficient condition for the existence of equilibrium loan contracts in terms of the relationship loan contracts for both types. While we keep the parameters δ, $\tau(S)$, $\tau(W)$, and λ fixed, we consider the possibility that there is a different number n of banks in the economy. Formally, the following corollary to Proposition 5 pins down the n-dependent number $\varepsilon(n) \geq 0$ which solves the equation

$$\frac{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^n}{1-(1-\lambda)\,\delta^{\tau(S)}}\,(1-\varepsilon(n))\,\alpha^*(W) = \frac{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^n}{1-(1-\lambda)\,\delta^{\tau(W)}}\,\alpha^*(W).$$

Corollary 2. *Fix any relationship loan contract $\alpha^*(W) \in (0,1)$ for the weak SME type.*

(i) *Then inequality (15) holds for all relationship loan contracts $\alpha^*(S)$ for the strong SME type that satisfy*

$$\alpha^*(S) > (1-\varepsilon(n))\,\alpha^*(W)$$

such that

$$\varepsilon(n) = 1 - \left(\frac{1-(1-\lambda)\,\delta^{\tau(S)}}{1-(1-\lambda)\,\delta^{\tau(W)}}\right)\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^n}{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^n}\right). \tag{16}$$

(ii) *Because of $\tau_S^{-1} > \tau_W^{-1}$ (cf. Assumption 1), the function (16) is strictly increasing in $n \geq 1$ whereby*

$$\varepsilon(1) = 0 \text{ and } \lim_{n\to\infty} \varepsilon(n) = 1 - \left(\frac{1-(1-\lambda)\,\delta^{\tau(S)}}{1-(1-\lambda)\,\delta^{\tau(W)}}\right) > 0.$$

By Corollary 2, there always exist equilibrium loan contracts $\mathcal{L}^*(\theta)$, $\theta \in \{S, W\}$, in our model if the strong type receives a more favorable relationship loan contract than her weak counterpart, i.e.,

$$\alpha^*(S) > \alpha^*(W). \tag{17}$$

In the case of a banking monopoly, i.e., $n = 1$, this inequality (17) is also necessary for the existence of equilibrium loan contracts. For an economy with $n > 1$ banks, however, equilibrium loan contracts do exist for which the strong type's relationship loan contract is (slightly) less favorable than the weak type's relationship loan contract in the specific sense that

$$(1-\varepsilon(n))\,\alpha^*(W) < \alpha^*(S) < \alpha^*(W)$$

whereby $\varepsilon(n)$, given by (16), is strictly increasing in the number n of banks. That is, even if the strong SME's value of her outside option in the form of the relationship loan contract is (slightly) less worth than the outside option for the weak SME, she might nevertheless end up with a more favorable equilibrium contract $\mathcal{L}^*(S)$ due to her superior market speed. This role of the strong type's superior market speed is the more relevant the more banks exist in the economy.

5.2 Break-even relationship lending

So far, we have treated type-dependent relationship loan contracts as exogenously given to the model. Our preferred interpretation was thereby that relationship loan contracts stand for 'favorable' loan conditions through which the bank wants to get the SME hooked on a long-term relationship which lies outside of our short-term model. As one plausible candidate for such 'favorable' loan conditions, we consider in this subsection a relationship loan contract which ensures that the bank just breaks even with regards to the SME's short-term investment project.

Definition: Break-even relationship lending. *We call $\alpha^*(\theta)$ a break-even relationship loan contract for SME type $\theta \in \{S, W\}$ if and only if*

$$\begin{aligned} p_\theta R\,(1-\alpha^*(\theta)) - I &= 0 \\ &\Leftrightarrow \\ \alpha^*(\theta) &= \frac{p_\theta R - I}{p_\theta R}. \end{aligned} \tag{18}$$

For break-even relationship loan contracts the inequality (17) becomes

$$\begin{aligned} \alpha^*(S) &> \alpha^*(W) \\ &\Leftrightarrow \\ p_S &> p_W. \end{aligned} \tag{19}$$

That is, the break-even relationship loan contract (18) is more favorable for the strong SME than for the weak SME type if and only if the strong type's success probability is higher than the weak type's success probability. If the number of banks is greater than one, we can, by Corollary 2, relax the inequality (19) to the effect that there exist equilibrium loan contracts for situations in which the success probability of the strong type is (slightly) smaller than the success probability of the weak type.

To make this argument precise, we, firstly, rearrange the inequality

$$\begin{aligned} (1-\varepsilon(n))\,\alpha^*(W) &< \alpha^*(S) \\ &\Leftrightarrow \\ (1-\varepsilon(n))\,\frac{p_W R - I}{p_W R} &< \frac{p_S R - I}{p_S R} \end{aligned}$$

for p_S. Secondly, we use our positive NPV assumption (1) which implies

$$\frac{p_W R}{I} > 1 \Rightarrow (1-\varepsilon(n)) + \varepsilon(n)\,\frac{p_W R}{I} > 1$$

for $\varepsilon(n) > 0$. This gives us the following result.

Proposition 6. *Suppose that $\alpha^*(\theta)$ is a break-even relationship loan contract for $\theta \in \{S, W\}$. If the number of banks in the economy satisfies $n > 1$, then there exist equilibrium loan contracts for situations in which the strong SME type's project comes with a (slightly) less success probability than the weak type's project in the specific sense that*

$$\frac{p_W}{(1-\varepsilon(n)) + \varepsilon(n)\,\frac{p_W R}{I}} < p_S < p_W$$

such that $\varepsilon(n) > 0$ is given by (16).

Let us formulate the main qualitative insight of Proposition 6 as follows. The strong SME type might receive a more favorable loan contract than the weak SME type–in the sense that $\mathbb{E}[\mathcal{L}^*(S)] > \mathbb{E}[\mathcal{L}^*(W)]$ holds–not necessarily because her investment project is of superior quality but because she can operate with greater market speed on the lending market. Simply speaking, in such situations not the better investment project but the more cost-efficient negotiator is awarded with the better deal.

6 Welfare analysis: Bringing down the signaling costs

As in the original signaling game of Spence (1973) signaling costs are socially wasteful in our model. Since all creditworthy projects will be financed, any aggregate welfare losses are exclusively given as the strong type's positive signaling costs. This section investigates in how far changes in the values of relevant model parameters might reduce the signaling costs of the strong type.

Let us apply the definition (9) of the cost-efficient quality level to a pair of equilibrium loan contracts $\mathcal{L}^*(\theta)$, $\theta \in \{S, W\}$ to obtain, by (13),

$$\begin{aligned} q_S^*[\mathcal{L}^*] &= \frac{p_W}{c_W} R\left(\mathbb{E}[\mathcal{L}^*(S)] - \mathbb{E}[\mathcal{L}^*(W)]\right) \qquad (20) \\ &= \frac{p_W}{c_W} R\lambda \left(\frac{1-\left((1-\lambda)\delta^{\tau(S)}\right)^n}{1-(1-\lambda)\delta^{\tau(S)}}\alpha^*(S) - \frac{1-\left((1-\lambda)\delta^{\tau(W)}\right)^n}{1-(1-\lambda)\delta^{\tau(W)}}\alpha^*(W)\right). \end{aligned}$$

Through expression (20) the socially wasteful signaling costs $c_S q_S^*[\mathcal{L}^*]$ of the strong SME type are explicitly described as a function in the exogenous parameters of our model, including the relationship loan contracts for both types. With respect to the type-dependent relationship loan contracts, we impose the following assumption which ensures the existence of equilibrium loan contracts irrespective for all admissible parameter values (cf. Corollary 2).[9]

Assumption 3. *For the welfare analysis of this section we assume that the strong SME type always receives a more favorable relationship loan contract than her weak counterpart, i.e.,*

$$\alpha^*(S) > \alpha^*(W).$$

Our comparative statics analysis treats the model parameters p_W, p_S, c_W, c_S, R, I, $\tau(S)$, $\tau(W)$ and δ as economic fundamentals whose values cannot be influenced by any welfare-maximizing policy maker. As relevant candidates for policy interventions–which would impact on the minimal quality level (20)–we investigate changes in (i) the number of banks n, and (ii) the probability λ of relationship lending. To this purpose, rewrite the minimal quality level (20) as a function $q_S^*[\mathcal{L}^*](x)$ in the selected parameter $x \in \{n, \lambda\}$. The economic interpretation is as follows: If $q_S^*[\mathcal{L}^*](x)$ is a decreasing function in x, then the socially wasteful signaling costs of the strong type would be brought down whenever the value of the parameter x decreases through some policy intervention. The following proposition, formally proved in the Appendix, constitutes our main welfare result.

[9] Note that Assumption 3 is satisfied for break-even relationship loan contracts if and only if the success probability of the strong type is greater than the success probability of the weak type.

Proposition 7. *Suppose that Assumption 3 holds. Then the strong type's signaling costs would strictly decrease if*

(i) *the number of banks n strictly decreases;*

(ii) *the probability λ of relationship lending strictly decreases.*

At a first glance, Proposition 7 is somewhat surprising. On the one hand, an increase in the number or banks or/and an increase in the probability of relationship lending improves in our model the loan conditions for the SME of either type (compare Proposition 4(iii)). Consequently, more banks or/and more relationship lending is desirable from the perspective of both SME types. On the other hand, however, either increase diminishes in our economy the overall welfare as both–an increase in the number or banks and an increase in the probability of relationship lending–would be driving up the strong SME type's socially wasteful signaling costs! The following two examples illustrate this point.

Example: Banking monopoly versus competitive limit. By Proposition 7(i), a banking monopoly would maximize the aggregate welfare in our economy whereas the competitive limit with infinitely many banks would minimize the aggregate welfare. More precisely, for the banking monopoly we obtain the following socially wasteful signaling costs

$$c_S q_S^* \left[\mathcal{L}^*\right](n=1) = c_S \frac{p_W}{c_W} R\lambda \left(\alpha^*(S) - \alpha^*(W)\right). \tag{21}$$

If the number of banks goes to infinity, these signaling costs converge to

$$\begin{aligned} & c_S q_S^* \left[\mathcal{L}^*\right](n \to \infty) \\ = \; & c_S \frac{p_W}{c_W} R\lambda \left(\frac{1}{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)}\alpha^*(S) - \frac{1}{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)}\alpha^*(W) \right). \end{aligned} \tag{22}$$

Since our market speed assumption $\tau_S^{-1} > \tau_W^{-1}$ (cf. Assumption 1) implies

$$\begin{array}{ccc} \delta^{\tau(S)} & > & \delta^{\tau(W)} \\ & \Rightarrow & \\ \dfrac{1}{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)} & > & \dfrac{1}{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)}, \end{array}$$

the socially wasteful signaling costs (22) associated with the competitive limit are–for our parameter ranges $\lambda, \delta \in (0,1)$–always strictly greater than the corresponding costs (21) for the banking monopoly.□

Example: Taking the relationship lending probability to its limits. Assume, at first, that the relationship lending probability $\lambda \in (0,1)$ becomes arbitrarily small. Then the corresponding signaling costs will converge to zero, i.e.,

$$c_S q_S^* [\mathcal{L}^*] (\lambda \to 0) = 0. \tag{23}$$

Next, consider the opposite limit case where the relationship lending probability $\lambda \in (0,1)$ converges to one. As the limit for the corresponding signaling costs, we obtain, for any number n of banks, that

$$c_S q_S^* [\mathcal{L}^*] (\lambda \to 1) = c_S \frac{p_W}{c_W} R (\alpha^* (S) - \alpha^* (W)), \tag{24}$$

which is, by Assumption 3, strictly greater zero. By a continuity argument, small relationship lending probabilities will thus come with socially wasteful costs close to zero whereas large probabilities drive up theses costs towards the upper cost limit (24).□

To see the rationale behind these seemingly counterintuitive findings of Proposition 7, recall the economic intuition for separating PBEs in the signaling game. The strong type has to invest the more socially wasteful signaling costs in a separating PBE the more attractive it is for the weak type to mimic her. But this is exactly what happens in our model if the number or banks or/and an increase in the probability of relationship lending increases: The equilibrium loan contract for the strong SME type becomes comparatively more attractive than the equilibrium loan contract for her weak counterpart.

Let us summarize the main insight from our welfare analysis as follows. Better economic conditions on the lending market–from the perspective of the SME of either type–come with stronger incentives for the strong SME type to distinguish herself from her weak counterpart. This drives up the socially wasteful signaling costs of the strong SME type.

7 Concluding remarks

This paper combines a sequential bargaining game between an SME and a fixed number of banks with a signaling game through which the SME can reveal her true type to the banks. We characterize the parameter conditions for which subgame-perfect equilibrium loan contracts are supported by separating perfect Bayesian equilibria in the signaling game. The analysis of the resulting equilibrium loan contracts comes with two novel theoretical insights.

Firstly, low quality investment projects might be rewarded with more favorable loan conditions than high quality projects. The reason is that the bargaining power of the SME does not only depend on the project's quality but also on the SME's market speed. As a consequence, there exists a parameter range such that a sufficiently high market speed compensates for an inferior quality of the strong SME's project.

Secondly, socially wasteful signaling costs increase in the number of banks as well as in the probability of relationship lending. That is, although more competitive pressure between banks improves the loan conditions for both SME types, it also results in greater aggregate welfare losses. The reason for this effect is that the strong SME type benefits comparatively more than her weak counterpart from competitive pressure between banks due to her greater market speed. In a separating PBE the strong type must thus incur greater signaling costs in order to dissuade the weak type to mimic her. This latter finding adds a new insight to the existing welfare analysis of lending markets which only focuses on welfare losses due to unfunded projects with positive NPVs. According to this literature, more competitive pressure between banks would always increase the aggregate social welfare. In contrast, we show that more competitive pressure between banks may drive up the incentives to invest into socially wasteful signaling.

Appendix: Formal proofs

Proposition 4

To prove Proposition 4, we use the following Lemma (which is basically a corollary to Proposition 3).

Lemma 1. *Expressed in terms of the continuation value* $V(2;\theta)$*, the loan contract* W* *that obtains in the favorable SPE is given as*

$$\mathcal{L}^*(\theta) = \left(a^*(\theta), \lambda; a_1^{tr}(\theta), (1-\lambda)\right) \text{ for } \theta \in \{S, W\}$$

such that

$$a_1^{tr}(\theta) = \delta^{\tau(\theta)} \frac{V(2;\theta)}{R}.$$

Next we derive explicit expressions for the expected SPE continuation values $V(i+1;\theta)$, $i \in \{1, ..., n\}$. As point of departure, note that the following recursive relationship holds for all i

$$V(i;\theta) = \lambda\alpha^*(\theta) R + (1-\lambda)\delta^{\tau(\theta)} V(i+1;\theta).$$

To solve for the expected SPE continuation values, we proceed by backward induction.

1. Observe that
$$V(n+1;\theta) = 0$$
for all $\theta \in \{S, W\}$ since the SME cannot move from the last bank n to any other bank.

2. Consequently, the SPE actions under transaction lending at stage $i = n$ are given as
$$a_n^{tr}(\theta) = \delta^{\tau(\theta)} \frac{V(n+1;\theta)}{R} = 0$$
resulting in the expected SPE continuation value
$$\begin{aligned} V(n;\theta) &= \lambda a^*(\theta) R + (1-\lambda) a_n^{tr}(\theta) R \\ &= \lambda\alpha^*(\theta) R. \end{aligned}$$

3. Now move up to stage $n-1$ and observe that
$$a_{n-1}^{tr}(\theta) = \delta^{\tau(\theta)} \frac{V(n;\theta)}{R} = \delta^{\tau(\theta)} \lambda\alpha^*(\theta),$$

implying

$$\begin{aligned} V(n-1;\theta) &= \lambda a^{*}(\theta) R + (1-\lambda) a^{tr}_{n-1}(\theta) R \\ &= \lambda \alpha^{*}(\theta) R + (1-\lambda) \delta^{\tau(\theta)} \lambda \alpha^{*}(\theta) R \\ &= \lambda \left(1 + (1-\lambda) \delta^{\tau(\theta)}\right) \alpha^{*}(\theta) R \\ &= \lambda \sum_{j=0}^{n-(n-1)} \left((1-\lambda) \delta^{\tau(\theta)}\right)^{j} \alpha^{*}(\theta) R. \end{aligned}$$

4. Repeating the above argument gives us for any stage $n-k$, $k \in \{0, ..., n-1\}$,

$$a^{tr}_{n-k}(\theta) = \delta^{\tau(\theta)} \lambda \sum_{j=0}^{k-1} \left((1-\lambda) \delta^{\tau(\theta)}\right)^{j} \alpha^{*}(\theta),$$

which results in the expected SPE continuation value

$$\begin{aligned} V(n-k;\theta) &= \lambda \alpha^{*}(\theta) R + (1-\lambda) \delta^{\tau(\theta)} \lambda \sum_{j=0}^{k-1} \left((1-\lambda) \delta^{\tau(\theta)}\right)^{j} \alpha^{*}(\theta) R \qquad (25) \\ &= \lambda \sum_{j=0}^{k} \left((1-\lambda) \delta^{\tau(\theta)}\right)^{j} \alpha^{*}(\theta) R \\ &= \lambda \frac{1 - \left((1-\lambda) \delta^{\tau(\theta)}\right)^{k+1}}{1 - \left((1-\lambda) \delta^{\tau(\theta)}\right)} \alpha^{*}(\theta) R. \end{aligned}$$

5. Note that condition (12) of Proposition 3 is satisfied with strict inequality because, by construction, for all $k \in \{0, ..., n-1\}$,

$$\frac{V(n-k;\theta)}{R} < \alpha^{*}(\theta).$$

6. Substitute $n-k=2$ in Formula (25) to obtain

$$V(2;\theta) = \lambda \frac{1 - \left((1-\lambda) \delta^{\tau(\theta)}\right)^{n-1}}{1 - \left((1-\lambda) \delta^{\tau(\theta)}\right)} \alpha^{*}(\theta) R. \qquad (26)$$

Substituting (26) in Lemma 1 proves Proposition 4.□□

Proposition 7

Proof of Statement (i). It is easy to see that

$$q_S^* [\mathcal{L}^*] (n) = \frac{p_W}{c_W} R\lambda \left(\frac{1 - \left((1-\lambda) \delta^{\tau(S)} \right)^n}{1 - (1-\lambda) \delta^{\tau(S)}} \alpha^* (S) - \frac{1 - \left((1-\lambda) \delta^{\tau(W)} \right)^n}{1 - (1-\lambda) \delta^{\tau(W)}} \alpha^* (W) \right)$$

must be a monotonic function in $n \geq 1$. To prove our claim we have to show that $q_S^* [\mathcal{L}^*] (n)$ strictly increases in $n \geq 1$ (to some upper limit).

Observe, at first, that

$$q_S^* [\mathcal{L}^*] (n = 1) = \frac{p_W}{c_W} R\lambda \left(\alpha^* (S) - \alpha^* (W) \right).$$

Next, take the limit $n \to \infty$ to obtain

$$q_S^* [\mathcal{L}^*] (n \to \infty) = \frac{p_W}{c_W} R\lambda \left(\frac{1}{1 - (1-\lambda) \delta^{\tau(S)}} \alpha^* (S) - \frac{1}{1 - (1-\lambda) \delta^{\tau(W)}} \alpha^* (W) \right).$$

Finally, comparing $q_S^* [\mathcal{L}^*] (n = 1)$ with $q_S^* [\mathcal{L}^*] (n \to \infty)$ gives

$$\begin{aligned} q_S^* [\mathcal{L}^*] (n \to \infty) &> q_S^* [\mathcal{L}^*] (n = 1) \\ &\Leftrightarrow \\ \frac{1}{1 - (1-\lambda) \delta^{\tau(S)}} \alpha^* (S) - \frac{1}{1 - (1-\lambda) \delta^{\tau(W)}} \alpha^* (W) &> \alpha^* (S) - \alpha^* (W) \\ &\Leftrightarrow \end{aligned}$$

$$\begin{aligned} & \left[\left(1 - (1-\lambda) \delta^{\tau(W)} \right) - \left(1 - (1-\lambda) \delta^{\tau(W)} \right) \left(1 - (1-\lambda) \delta^{\tau(S)} \right) \right] \alpha^* (S) \\ \geq & \left[\left(1 - (1-\lambda) \delta^{\tau(S)} \right) - \left(1 - (1-\lambda) \delta^{\tau(W)} \right) \left(1 - (1-\lambda) \delta^{\tau(S)} \right) \right] \alpha^* (W). \end{aligned}$$

But, by Assumption 3, the last inequality is always satisfied if

$$\begin{aligned} & \left(1 - (1-\lambda) \delta^{\tau(W)} \right) - \left(1 - (1-\lambda) \delta^{\tau(W)} \right) \left(1 - (1-\lambda) \delta^{\tau(S)} \right) \\ \geq & \left(1 - (1-\lambda) \delta^{\tau(S)} \right) - \left(1 - (1-\lambda) \delta^{\tau(W)} \right) \left(1 - (1-\lambda) \delta^{\tau(S)} \right) \end{aligned}$$

$$\Leftrightarrow$$

$$\delta^{\tau(S)} > \delta^{\tau(W)},$$

which holds by Assumption 1.□

Proof of Statement (ii). To prove the statement, we are going to show that

$$\frac{d}{d\lambda}\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(S)}}\alpha^{*}(S)-\frac{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(W)}}\alpha^{*}(W)\right)>0. \tag{27}$$

By Assumption 3, (27) is satisfied, if

$$\frac{d}{d\lambda}\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(S)}}\right)\geq\frac{d}{d\lambda}\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(W)}}\right) \tag{28}$$

holds. Substitute

$$\begin{aligned}&\frac{d}{d\lambda}\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(S)}}\right)\\ =\;&\frac{n\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n-1}\delta^{\tau(S)}\left(1-\left((1-\lambda)\,\delta^{\tau(S)}\right)\right)+\delta^{\tau(S)}\left(1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}\right)}{\left(1-(1-\lambda)\,\delta^{\tau(S)}\right)^{2}}\end{aligned}$$

and

$$\begin{aligned}&\frac{d}{d\lambda}\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(W)}}\right)\\ =\;&\frac{n\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n-1}\delta^{\tau(W)}\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)+\delta^{\tau(W)}\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}\right)}{\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)^{2}}\end{aligned}$$

in (28) to obtain

$$\frac{d}{d\lambda}\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(S)}}\right)\quad\geq\quad\frac{d}{d\lambda}\left(\frac{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}}{1-(1-\lambda)\,\delta^{\tau(W)}}\right)$$

$$\Leftrightarrow$$

$$\begin{aligned}&\frac{n\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n-1}\delta^{\tau(S)}}{1-(1-\lambda)\,\delta^{\tau(S)}}+\frac{\delta^{\tau(S)}\left(1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}\right)}{\left(1-(1-\lambda)\,\delta^{\tau(S)}\right)^{2}}\\ \geq\;&\frac{n\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n-1}\delta^{\tau(W)}}{1-\left((1-\lambda)\,\delta^{\tau(W)}\right)}+\frac{\delta^{\tau(W)}\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}\right)}{\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)^{2}}.\end{aligned} \tag{29}$$

To see that inequality (29) holds for any $n \geq 1$, observe, at first, that

$$\frac{n\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n-1}\delta^{\tau(S)}}{1-(1-\lambda)\,\delta^{\tau(S)}} > \frac{n\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n-1}\delta^{\tau(W)}}{1-(1-\lambda)\,\delta^{\tau(W)}} \tag{30}$$

because of $\delta^{\tau(S)} > \delta^{\tau(W)}$. Next, we argue that we also have

$$\frac{\delta^{\tau(S)}\left(1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}\right)}{\left(1-(1-\lambda)\,\delta^{\tau(S)}\right)^{2}} > \frac{\delta^{\tau(W)}\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}\right)}{\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)^{2}}, \tag{31}$$

which, together with (30), would prove (29).

To this purpose, define the function

$$f[\lambda](n) = \frac{\delta^{\tau(S)}\left(1-\left((1-\lambda)\,\delta^{\tau(S)}\right)^{n}\right)}{\left(1-(1-\lambda)\,\delta^{\tau(S)}\right)^{2}} - \frac{\delta^{\tau(W)}\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)^{n}\right)}{\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)^{2}}.$$

Note that $f[\lambda](n)$ is monotonic in $n \geq 1$ such that

$$\begin{aligned} f[\lambda](n \to \infty) &= \frac{\delta^{\tau(S)}}{\left(1-(1-\lambda)\,\delta^{\tau(S)}\right)^{2}} - \frac{\delta^{\tau(W)}}{\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)^{2}}, \\ f[\lambda](n = 1) &= \frac{\delta^{\tau(S)}}{\left(1-(1-\lambda)\,\delta^{\tau(S)}\right)} - \frac{\delta^{\tau(W)}}{\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)}. \end{aligned}$$

By a similar argument as in the proof of Statement (i), we have that

$$f[\lambda](n \to \infty) > f[\lambda](n = 1)$$

so that $f[\lambda](n)$ is monotonically increasing in $n \geq 1$. Evaluating $f[\lambda](n)$ at its minimum $n = 1$ and observing that

$$f[\lambda](n=1) = \frac{\delta^{\tau(S)}}{\left(1-(1-\lambda)\,\delta^{\tau(S)}\right)} - \frac{\delta^{\tau(W)}}{\left(1-\left((1-\lambda)\,\delta^{\tau(W)}\right)\right)} > 0,$$

therefore shows that, for all $n \geq 1$, $f[\lambda](n) > 0$. This gives us the desired inequality (31) for any $n \geq 1$.□□

References

Berger, A.N., Udell, G.F., 2002. Small business credit availability and relationship lending: The importance of bank organisational structure. Economic Journal 112, F32-F53.

Berger, A.N., Udell, G.F., 2006. A more complete conceptual framework for SME finance. Journal of Banking and Finance 30, 2945-2966.

Bester, H., 1988. Bargaining, search costs and equilibrium price distributions. Review of Economic Studies 55, 201–214.

Bester, H., 1989. Bargaining and spatial competition. Econometrica 57, 97-113.

Bester, H., 1993. Bargaining versus price competition in markets with quality uncertainty. American Economic Review 83, 278–288.

Boot, A.W.A., 2000. Relationship banking: What do we know? Journal of Financial Intermediation 9, 7-25.

Boot, A.W.A., Thakor, A.V., 2000. Can relationship banking survive competition? Journal of Finance 55, 679-713.

Cole, R., 1998. The importance of relationships to the availability of credit. Journal of Banking and Finance 22, 959-977.

Dermine, J., 1986. Deposit rates, credit rates and bank capital: The Monti-Klein model revisited. Journal of Banking and Finance 10, 99-114.

Elsas, R., Krahnen, J., 1998. Is relationship lending special? Evidence from credit-file data in Germany. Journal of Banking and Finance 22, 1283-1316.

Freixas, X., Rochet, J.C., 2008. Microeconomics of banking. The MIT Press.

Gale, D., 1987. Limit theorems for markets with sequential bargaining. Journal of Economic Theory 43, 20-54.

Harhoff, D., Körting, T., 1998. Lending relationships in Germany: Empirical results from survey data. Journal of Banking and Finance 22, 1317-54.

Inderst, R., Müller, H.M., 2007. A lender-based theory of collateral. Journal of Financial Economics 84, 826-859.

Klein, M., 1971. A theory of the banking firm. Journal of Money, Credit and Banking 3, 205-218.

Konrad, K.A., Thum, M., 2020. Equilibrium opacity in ultimatum-offer bargaining. Journal of Public Economic Theory. Published online. https://doi.org/10.1111/jpet.12471

Liberti, J.M., Petersen, M.A., 2018. Information: Hard and soft. Working paper.

Monti, M., 1972. Deposit, credit, and interest rate determination under alternative bank objectives. In Szego, G.P., Shell, K., (Eds.) Mathematical Methods in Investment and Finance. Amsterdam: North-Holland.

Muthoo, A., 1993. Sequential bargaining and competition. Economic Theory 3, 353-363.

Parlour, C.A., Rajan, U., 2001. Competition in loan contracts. American Economic Review 91, 1311-1328.

Petersen, M., Rajan, R., 1994. The benefits of firm-creditor relationships: Evidence from small business data. Journal of Finance 49, 3-37.

Petersen, M., Rajan, R., 1995. The effect of credit market competition on lending relationships. Quarterly Journal of Economics 110, 407-443.

Raskovich, A., 2007. Ordered bargaining. International Journal of Industrial Organization 25, 1126-1143.

Rubinstein, A., 1982. Perfect equilibrium in a bargaining model. Econometrica 50, 97-109.

Rubinstein, A., Wolinsky, A., 1985. Equilibrium in a market with sequential bargaining. Econometrica 53, 1133-1150.

Salop, S., 1979. Monopolistic competition with outside goods. Bell Journal of Economics 10, 141-156.

Shaked, A., Sutton, J., 1984. Involuntary unemployment as a perfect equilibrium in a bargaining model. Econometrica 52, 1351-1364.

Sharpe, S.A., 1990. Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships. Journal of Finance 45, 1069-1087.

Spence, A.M., 1973. Job market signaling. Quarterly Journal of Economics 87, 355-74.

Stahl, D.O., 1988. Betrand competition for inputs and Walrasian outcomes. American Economic Review 78, 189-201.

Stein, J.C., 2002. Information production and capital allocation: Decentralized versus hierarchical firm. Journal of Finance 57, 1891-1921.

Stiglitz, J.E., Weiss, A., 1981. Credit rationing in markets with imperfect information. American Economic Review 71, 393-410.

Tirole, J., 2006. The theory of corporate finance. Princeton University Press.

Wolinsky, A., 1987. Matching, search, and bargaining. Journal of Economic Theory 42, 311-333.

Yanelle, M.O., 1988. The strategic analysis of intermediation. European Economic Review 33, 294-301.