Aspects of perspective and ambiguous space in the work of M.C. Escher

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Cartographers, mathematicians and artists discovered many of the rules for linear perspective before the Renaissance, but the mathematical basis to represent objects three-dimensionally was developed only in the 15th century. Brunelleschi was the first to demonstrate geometric perspective with his famous experiments in 1425. The humanist Alberti recorded the geometrical principles for creating three-dimensional space on a two-dimensional surface during 1435. This was, in other words, the tools to translate mental vision onto paper. The fact that, in human vision, the parallel lines from the edges of objects converge at an infinite point (the vanishing point) on the horizon, is an illusion. Escher wanted to make concrete representations of infinity and he used traditional perspective to create impossible worlds.
Key words: linear perspective, vanishing point, horizon line, zenith, nadir

Die ontwikkeling van perspektief en die gebruik daarvan in die werk van M.C. Escher
Kartograwe, wiskundiges en kunstenaars het sommige van die reëls vir lineêre perspektief intuïtief ontdek, maar die wiskundige gronde vir drie-dimensionele voorstelling is eers tydens die 15de eeu ontdek. Brunelleschi het tydens 1425 meetkundige perspektief gedemonstreer met behulp van sy bekende eksperimente van 1425. Die humanis Alberti het tydens 1435 die meetkundige beginsels vir die voorstelling van drie-dimensionele ruimte op 'n twee-dimensionele oppervlakte gedokumenteer. Hierdie beginsels was die metode waarvolgens die kognitiewe visie omgesit word in 'n voorstelling op papier. Die feit (volgens menslike visie) dat die ewewydige lyne vanaf die rand van voorwerpe by 'n oneindige punt (die verdwyningspunt) op die horison konvergeer, is 'n illusie. Escher wou konkrete voorstellings van oneindigheid maak en hy het tradisionele perspektieftekening gebruik om 'n onmoontlike wêreld te skep.
Trefwoorde: lineêre perspektief, verdwyningspunt, horisonlyn, toppunt, voetpunt

Space and time in Western culture

Heidema and Schmidt[1] distinguish between three approaches regarding space and time in Western culture. These approaches occurred in history as follows:

1. First approach (heterogeneous treatment of space) – prehistoric times to the 5th century BC
2. First and second approach coexist – mainly in Greco-Roman culture between 500 BC and 300 AD
3. First approach is dominant – from 300 AD up to about 1450 AD
4. Second approach rediscovered in 15th century (homogeneous treatment of space) – between 1450 AD and 1880
4. Third approach (multivalent treatment of space) – from about 1880 onwards

The first approach is revealed in painting by means '... of abstraction, a collective iconography and the heterogeneous treatment of space.'[2] In science this approach is revealed 'through a conception of space as the hierarchical ordering of juxtaposed places.' Historically the heterogeneity of space in art corresponds with the ordering of space in science.[3] Heterogeneous space in painting was determined by iconography and during this historical period it was based on religion. There was no attempt to create homogeneous scenes according to which figures were enlarged or foreshortened in perspective. Heterogeneous ordering was done according to a hierarchy which was determined by the iconography (religion). Artists depicted invisible aspects of religion in a symbolic way. Their aim was not to present visible reality. Reality was of secondary importance in relation to abstraction. Religion, presented symbolically in a painting, transcended time.

According to the second approach artists regarded space as homogeneous; other important aspects were realism and central perspective. In paintings physical space was presented from a single viewpoint and a three-dimensional picture could be created. Central perspective corresponds with the geometrical concept of optics in the Ancient World. The basic principles of central perspective were known in Greek and Roman civilisations. An example of such an ancient source is the Geographica (an atlas) of Claudius Ptolemaeus (born in c. AD 90), known as Ptolemy. He was a Roman citizen of Greek or Egyptian ancestry. In his atlas he described three methods of representing a sphere (the earth) on to a flat surface with the least distortion. It is important to note that this approach 'was synonymous with the adoption of Euclidian space as an independent framework'[4] in science and mathematics. According to this approach time is continuous.

With the multivalent treatment of space (third approach) the painter has a choice from alternative paradigms. He can use the heterogeneous treatment of space as well as abstraction (Expressionism) to create a painting; use both homogeneous space and realism (Contextualism); or use geometrical forms to reduce space to structural frameworks (Structuralism).[5] The Surrealists Salvador Dalí (1904-1989) and René Magritte (1898-1967) were concerned with the world of dreams and the subconscious and used traditional techniques to portray this. They created ambiguous space with the aid of the basic rules of Renaissance perspective. Escher also used traditional perspective to create impossible worlds.

Perspective before the Renaissance

From the viewpoint of a scientist perspective is closely related to optics. According to O'Connor and Robertson[6] al-Haytham gave the first correct explantation of vision (about 1000 A.D) by showing that light is reflected from an object into the eye. Roger Bacon (c. 1220-c.1292), a Franciscan monk, wrote a long treatise *Opus majus* and included a section on optics[7]. Bacon intended to show that the geometrical laws of optics reflected the way in which God distributed His grace through the entire universe. Spirituality played an important role in all the arts of the early Medieval period, but at the end of the Medieval period this was gradually replaced by secularism, capitalism and worldly pleasure.[8] The Ptolematic system (the theoretical system of planetery movements according to which the sun, moon, and planets revolve around a stationary earth) gave a geometric structure to the celestial world while linear perspective gave geometric structure to the terrestrial world[9]. The medieval artist did not try to depict objects in perspective, or for that matter, as if there was a spatial connection between objects. 'The Ancient Egyptians, who had a rich artistic tradition, showed little interest in creating realistic illusions of space and depth. Their art used a rigid language of religious and social symbols.'[10] Hellenistic painters were able to create the illusion of depth in their work, but due to the lack of evidence it is not possible to say that they used mathematical laws to do it.

Reverse perspective is 'The phenomenon whereby the sides of rectilinear, three-dimensional objects seem to diverge rather than converge toward a common vanishing point.'[11] Reverse perspective was an attempt to create the illusion of three dimensions. Giotto (13th century) used reverse perspective and the rules he followed for this, were the following:

> He inclined lines above eye-level downwards as they moved away from the observer, lines below eye-level were inclined upwards as they moved away from the observer, and similarly lines to the left or right would be inclined towards the centre. [12]

The altarpiece of Duccio below suggests a spiritual space depicted by the gold background. The individual holy figures are important, but realistic space and linear perspective do not play a role here.

Figure 1
Duccio di Buoninsegna, *The Virgin and Child with Saints*
(c.1315, centre panel of altarpiece: 61,5x39cm
(source: http:www.museumsyndicate.com/images/3/23907.jpg).

In artworks of the Medieval period and early Renaissance, the perspective is not accurate. The following is an illustration from the Kaufmann Haggadah (Spain, late 14th century). The perspective lines converge, but not towards a vanishing point and not on the horizon.[13] The manuscript was produced in 14th century Catalonia and contains prayers, poems and narrative texts to be recited on the eve of the festival of the Jewish Easter etc. Children are seen with the bearded Moses in a pointed red hat and he is leading the Jews. On the left is an Egyptian city and the inhabitants watch the Jews passing.

Figure 2
Ms A422 from Kaufmann Haggadah. Spain, late 14th century
(The Jews: A Treasury of Art and Literature. NY: Levin Assoc. 1992)
(source: http://kaufmann.mtak.hu/img/illustration-haggadah-exodus.jpg).

The *Grandes Chroniquest de* France is a royal compilation of the history of France which was compiled between the 13th and 15th century. In *Banquet given by Charles V in honour of his uncle Emperor Charles IV in 1378* (Figure 3) reality is presented intuitively, before the mathematical basis for perspective had been developed, but it is still an acceptable picture of reality.[14]

Figure 3
Jean Fouquet, *Grandes Chroniques de France* (Ms Fr 6465)
(ca 1455-1460, illuminated manuspcript, 46 x 61 cm, *banquet given by Charles V in honour of his uncle Emperor Charles IV in 1378*)
(source: http://www.picassomio.es/jean-fouquet-poster-artist.html).

Linear perspective

Linear perspective is a mathematical method used to translate the mental vision of the artist onto paper. In other words, it is a method to translate space (three dimensions) to paper (two dimensions). More technically, it is a geometrical way of creating the illusion of the convergence of parallel lines into the distance (vanishing point or distance point). The vanishing point (also the point of infinity) lies on the horizon line. In this sense the horizon line is, according to the vision of the observer, the end of the earth. It is important to note that mathematically the horizon line can be identified with the infinite limit of the theoretical plane of the earth.[15]

The Italians Leon Battista Alberti (1404-1472) and Filippo Brunelleschi (1377-1466) both made fundamental contributions towards the development of linear perspective. Brunelleschi is famous for his peepshow experiments (c.1413) and he made a panel with a painted picture of the Baptistery in Florence. He made a hole in this panel at a point which was later called the vanishing point. A person viewing the Baptistery could then hold the unpainted side of the panel against one eye while holding a mirror in the other hand. The mirror would reflect the painting of the panel. The idea was that the viewer could compare the real Baptistery in front of him with the painted illusion reflected in the mirror. While Brunelleschi did the experiments, it seems as if Alberti was responsible for the fundamental geometry.[16] Fact is that they defined a very important aspect of linear perspective, namely the vanishing point. Brunelleschi made a correct formulation of linear perspective because he understood that there should be a single infinite point on the horizon towards which all parallel lines in a plane (excluding the plane of the painting itself) should converge.

As far as we know Masaccio (1401-1428) was the first painter and Donatello the first sculptor who used linear perspective rigorously. Masaccio constructed the vanishing point in the '*The Holy Trinity*' (figure 4) at eye level, the height of the average person standing in front of the painting. In this painting the orthogonals (a line representing an edge of an object receding into distance, the vanising point) recede from the barrel-vaulted ceiling to the vanishing point which is below the feet of Christ (at floor level of the two kneeling figures outside the chapel. A skeleton lies on a tomb and the inscription is: I was what you are, and what I am you shall be. Above the inscription the donor of the painting kneels with his wife. According to Honour[17] the donor would have been much smaller in a medieval painting, but Masaccio used the perspective system and painted the figures to scale in a 'single unified space'. The Virgin and St John, standing either side of the crucifix, are foreshortened as they stand inside the chapel and not outside like the donors. God, Christ and the Holy Ghost (Trinity) are depicted without foreshortening and therefore the importance of the spiritual is emphasized in this fresco.

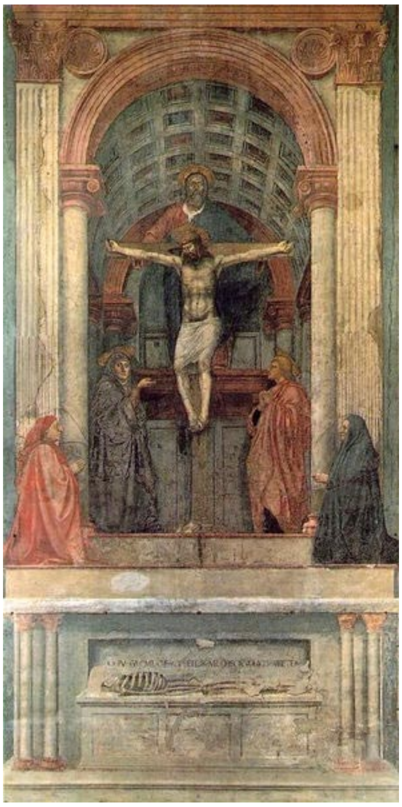

Figure 4
Masaccio, *The Holy Trinity*
(c.1427, fresco, 6,7x3,2 m, S Maria Novella, Florence).

In the next two examples the orthogonals recede from a chequered floor. The floor 'tiles' decrease in size (diminution) according to the increasing distance from the lower edge of the artwork. A chequerboard floor often provides a linear framework for the linear perspective in many early Renaissance paintings.[18] The Renaissance artist often depicted the world as a view from outside, as if the painting was a window between the artist and the object. The painting (figure 5) of Antonello da Messina illustrates the picture in a window frame, including a window ledge. The horizontal lines of the floor 'tiles' are parallel to the lower frame (window sill) of the painting and they remain parallel. The spaces between these parallel lines appear to be smaller the further away they are (foreshortening). The vertical lines which run at 90° to the window ledge do not stay parallel, but converge towards the vanishing point. The light rays (orthogonals) converge at the saint's bust and hands. Objects which are further away from the viewer are smaller (diminution). A Mediterranean landscape is visible through the windows of the study.

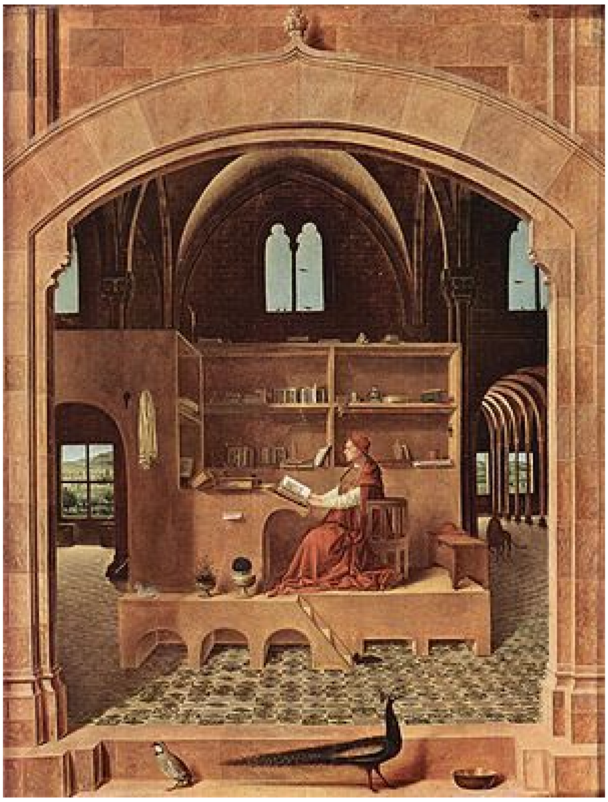

Figure 5
Antonello da Messina, *St. Jerome in his Study*
c.1475, oil on lime wood, 46x36cm,
(http://en.wikipedia.org/wiki/Antonello_da_Messina).

The Annunciation, with Saint Emidius (1486), figure 6 on the following page, is a good illustration of the mathematical basis of art used by the Renaissance painter. This painting shows '...this skill through beautifully constructed perspectives and clever foreshortenings.'[19] Cole continues and says that the annunciation (when Gabriel appeared to Mary) was often used for perspective display and therefore perspective becomes almost symbolic: 'The perspective therefore helps to focus the eye on the "moment" of Christ's Incarnation as the Son of God.' The orthogonals recede from a chequered floor and they converge at the vanishing point (window grid in the distance).

Peepshow and camera obscura

Hoogstraten's peepshow 'is created through a masterly manipulation of simple central perspective and its distorted – or "anamorphic" – form.'[20] Cole defines anamorphosis as 'a deliberately distorted image which, when viewed head on, is almost unrecognizable.'[21] Samuel van Hoogstraten (1627-1678) was a pupil of Rembrandt and he is famous for his perspective peepboxes. A peepbox is a rectangular box of which 5 of six sides are painted on the inside. The sixth side is open and originally light entered the box from this side by placing the box close to a window. In *Peepshow Box* (Hoogstraten, 1650) a three-dimensional illusion (scene) of the interior of a house is created when the viewer looks through a small peephole. The *Peepshow Box* of Hoogstraten is a fine example of very few surviving examples of three-dimensional scenes created by peepboxes.

Figure 6
Carlo Crivelli, *The Annunciation, with Saint Elmidius,*
1486, egg and oil on canvas, 207 x 146,7 cm
(source: http://upload.wikimedia.org/wikipe7dia/commons/4/4d/carlo_crivelli_Annunciation_with_St_ Emidius_1486_London.jpg).

The Dutch painter Vermeer (1632-75) used a camera obscura (meaning 'a dark chamber') and his painting *The Music Lesson* (figure 7) suggests this. A camera obscura is a dark room or cubicle with a small hole (sometimes with a magnifying lens) through which rays from an illuminated scene pass. The scene is projected through the dark room and small hole and appears as a reduced, inverted image on a white wall or screen. Canaletto also used a camera obscura and his painting *The Grand Canal with San Simeone Piccolo* (figure 8) shows a Venetian view with buildings receding in perspective. The panoramic view suggests the use of a camera obscura.

Figure 7
Jan Vermeer, *The Music Lesson,*
1660s, oil on canvas, 73 x 64,5cm.

Figure 8
Canaletto, *The Grand Canal with San Simeone Piccolo,*
c.1738, oil on canvas, 1,25x2,05m.

Artists used a wide variety of devices to create more accurate perspectives, but are not discussed in this article.

The impossible worlds of M.C. Escher (1898-1972)

Escher himself explained what he understood about classical perspective.[22] When sitting in front of a window in a room, you can draw the scenery outside with a piece of soap on the window. The drawing on the window represents the scenery in perspective. There are numerous straight and parallel lines in the world we live in. Buildings (walls, windows, doors), books and tables consist mainly of rectangles. When we draw railway tracks, they disappear on the horizon line at the vanishing point. On the drawing the railway tracks do not seem to be parallel while telephone poles (next to the tracks) stay parallel. Escher believes that these basic rules of classical perspective are not always true. Things can become unsettled, depending on whether you look upwards, downwards or straight ahead of you. If you stand in a corner room on the 20th floor of a building and look down, the vertical lines of the building intersect at the nadir vanishing point while the street and sidewalks continue to run parallel to each other. If you are still looking from the 20th floor upwards towards the 40th floor of a building accross the street, the vertical lines will converge towards the zenith vanishing point. Upon receiving the Culture Prize of the City of Hilversum on 5 March 1965 Escher[23] said that it is ‘a pleasure knowingly to mix up two- and three dimensionalities, flat and spatial, and to make fun of gravity.’ He was a master of the infinite and he had several tools for this. He used regular division of a plane to present infinity in two dimensions (e.g. *Symmetry Work 25,* 1939) and regular division of three dimensional objects (e.g. *Carved Beechwood Ball with Fish,* 1940). He also used hyperbolic tesselation (e.g. *Circle Limit,* 1959).

The Surrealists Salvador Dalí (1904-1989) and René Magritte (1898-1967) used conventional perspective, but created perspective illusions ('impossible' images). These ambiguities were also used by Escher (1898-1972) and his work (Cole, 1992:58) 'show that perspective illusion relies for its effect on the viewer's expectations – and these are easily confused!'[24]

The Penrose triangle (or Penrose tribar) is an impossible object and it was first created by the Swedish artist Oscar Reutersvärd in 1934. The mathematician Roger Penrose invented and popularised it during and after 1950. The tribar appears to be a solid object which is constructed of three straight beams which meet at right angles at the vertices of the triangle. The viewer interprets it as a three-dimensional triangle made up of three right angles, but this is not possible in ordinary Euclidean space. Escher was inspired by these impossible objects to create his own impossible perspectives. In the lithograph *Waterfall* (1961) Escher linked together three Penrose tribars to create an impossible world.

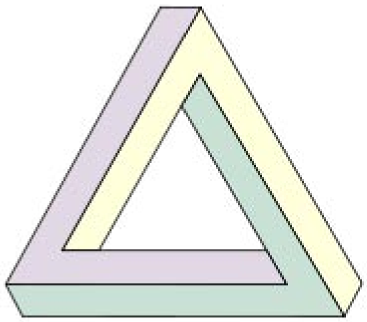

Figure 9
Penrose triangle
(source: http://en.wikipedia.org/wiki/File:Penrose_triangle.svg).

Escher makes use of the nadir and zenith in his work. According to the New Penguin English Dictionary the nadir is 'in astronomy, the point of the celestial sphere that is directly opposite the zenith and vertically downward from the observer' and the zenith is 'the point of the celestial sphere that is directly opposite the nadir and vertically above the observer.' If a person is standing between railway tracks, the tracks seem to converge at a single point (zenith) on the horizon. In a woodcut *Tower of Babel* (1928) Escher made the drawing according to what a viewer at an angle above the tower would see and here he used the nadir.

Escher deals, inter alia, with infinity, relativity and curvature of space in his work. He also admitted that mathematicians (especially the crystallographers) had a considerable influence on his work. He spent hours on a gallery of the dome to do the sketch of the wood engraving *St. Peter's, Rome* (1935) and while looking down he realized that all the vertical lines culminated in the nadir. In Escher's own words[25] : 'So this print may be the primary cause of the series of perspective fantasies I developed many years later.' Escher worked strictly according to the rules of classical perspective and this is the reason why 'they are so suggestive of space.'[26] In the litograph 'Cubic Space Division' (figure 10) Escher aimed at depicting the infinite extent of space by using linear perspective. When the vertical bars are projected they seem to meet at a vanishing point, the nadir (lowest point). There are also two other vanishing points which can be obtained by projecting the bars that point upward to the right and the bars that point upward to the left. The three vanishing points lie outside the litograph. The litograph further consists of three systems of parallel beams intersecting at right angles and the beams are divided into parts of equal length. Here Escher filled space with an infinite number of cubes which all have the same volume. For Escher perspective drawing was a very time consuming task and he used very large sheets of paper to create this litograph.

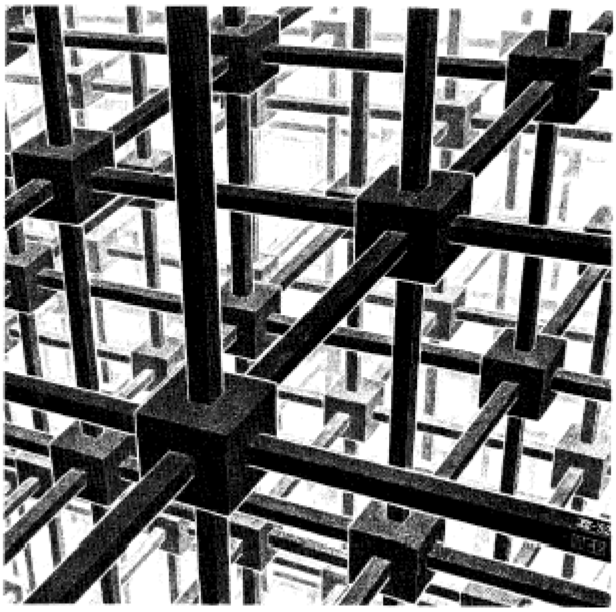

Figure 10
M.C. Escher, *Cubic Space Division*
(1952, litograph).

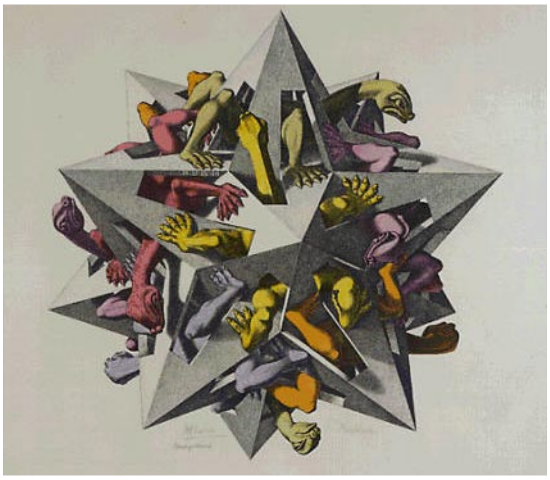

Figure 11
M.C. Escher, *Gravity*
(1952, litograph).

The litograph *Gravity* (figure 11) is a stellated dodecahedron which is one example of regular solids discovered by Kepler. The inside consists of a regular twelve-faced polyhedron (dodecahedron) and on each of those twelve faces a regular pentagon is constructed. On the faces of the pentagon a regular five-sided pyramid is superimposed. Escher shared his interest in crystallography with his brother, the geologist Professor B.G.Escher. *Gravity* was the last of the series of regular polyhedrons and it is important because it serves as a transition to a new period in the work of Escher which concerns relativity.[27] In *Gravity* relativity plays a roll: sides function as floors and walls at the same time. For instance, the plane that is a floor for the green animal is at the same time an upstanding wall of the pyramid that covers the body of the yellow animal. In *Other World I* (1946) *Other World II* (1947) Escher deals with the relativity of the vanishing points.

When several lines converge at a single point, it can represent the zenith, nadir or point of distance. It all depends on the the viewpoint. Ernst[28] argues that Escher demonstrates this aspect in the prints *Other World I* (figure 12) and *Other World II* (figure 13). In the mezzotint of 1946 a long tunnel seems to vanish somewhere in a single dark point far away. If the viewer looks at the right or left tunnel wall, he sees a horizontal lunar landscape. The vanishing point is then a distance point. If the viewer looks at the top tunnel wall, he looks down on a lunar landscape. In the latter case the same vanishing point has the function of a nadir. In *Other World II* the illusionistic architecture, lunar landscape and other objects can be seen from three contradictory viewpoints. Although the three viewpoints lead to the same vanishing point, the concepts floor, wall and ceiling have lost their relevance.[29]

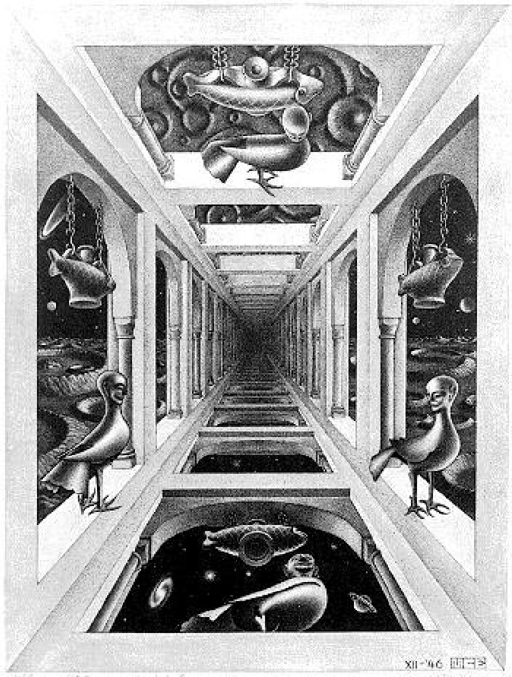

Figure 12
M.C. Escher, *Gallery*
(1946-1949, mezzotint, 21,3 x 15,9 cm).

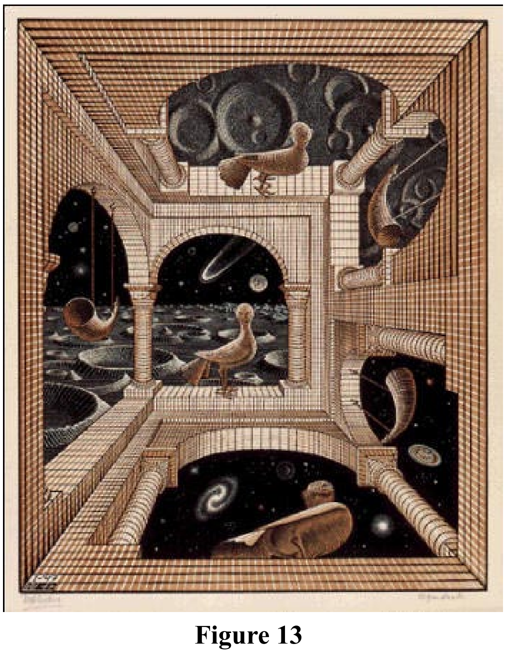

Figure 13
M.C. Escher, *Other World*
(1947, wood engraving, 31,8 x 26,1 cm).

The *Other World* prints both have only one vanishing point, but in *Relativity* (Figure 14) Escher uses three vanishing points and '...three gravitational forces operate perpendiculartly to one another.'[30] Vero says that '...artists can be challenged, impressed, and influenced by scientific and mathematical concepts and, wishing to illustrate such concepts, arrive at new ideas substantially structured on new perspective combinations'.[31] The three vanishing points in *Relativity* lie beyond the area of the drawing and are situated at the three vertices of an equilateral triangle. This triangle can be seen on the preparatory sketches Escher made for the work. The concept of up, down, left and right depend on which person the viewer is looking at. For instance, the person carrying a bag in the middle of the litograph puts his right foot on the floor (for him) but for the seated person it is a wall. This is the way in which Escher illustrates relativity.

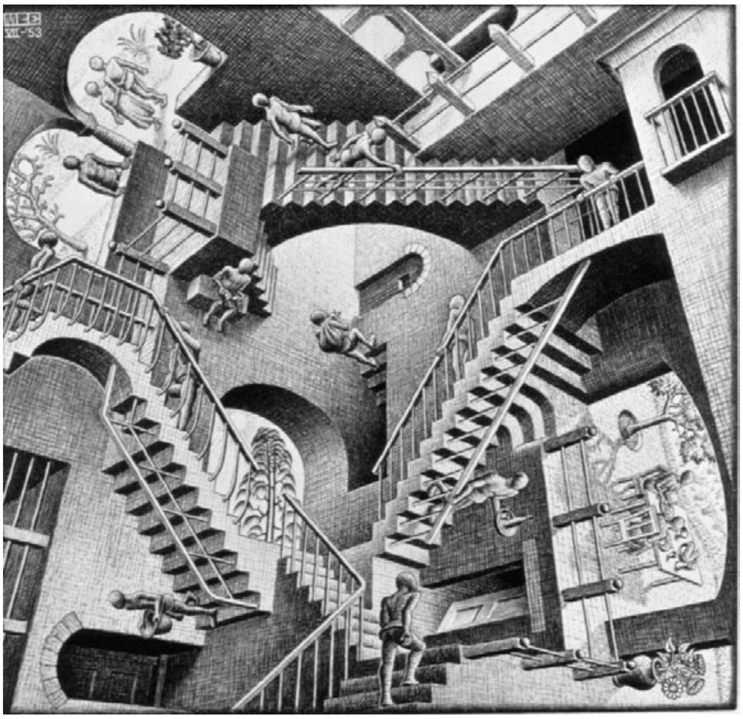

Figure 14
M.C. Escher, *Relativity*
(1953, litograph, 27,7 x 29,2 cm).

Escher is a very creative innovator and he takes the matter of perspective even further with aspects of curvature of space. In the litograph *High and Low* (figure 15) all the vertical lines converge towards a single point (the vanishing point) at the center of the lower edge of the drawing, the nadir. It is interesting that the lines are curved and not according to classical perspective where straight lines are always used.

In Figure 16 the 15th century French painter uses curved lines for the buildings and tiles. Vero says: 'Both perspective drawing and photography suffer from undesirable distortions, which stem from our technical inability to render in two dimensions the complex perceptions of our eyes.'[32] Spherical correction is present in ancient Greek architecture. Vero also illustrates how curvature of the Greek columns corrected the fact that a conical column looks thinner in the middle. Curvature is only used for the upper 66% of the column. Greek architects made empirically based corrections which were in accordance with spherical perspective. During the 15th century the French painter Jean Fouquet used curvature in *Arrival of Emperor Charles IV at the Basilica St. Denis in 1378* (figure 16) according to the perception of the human eye.

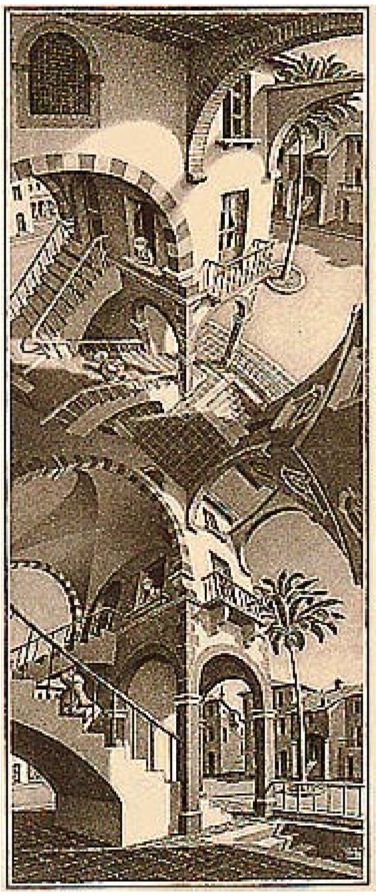

Figure 15
M.C. Escher, *High and Low,*
1947, lithograph, 19,75 x 8,125 inches.

Figure 16
Jean Fouquet, *Grandes Chroniquest de France,* Ms Fr 6465,
ca 1455-1460, illuminated maniscript, *Arrival of Emperor Charles IV at the Basilica St. Denis in 1378*
(source: http://upload.wikimedia.org/wikipedia/commons/e/e9/Entr%C3%A9e-de-/-empereur-Charles/IV-à-Saint-Denis).

It seems as if the human eye perceives space rather with curved lines than straight lines and Escher sometimes drew buildings with curved lines because that is the way he saw them.[33] The initial sketches for this work reveal that curves in *High and Low* correspond to the the mathematical sine curve, but Escher was not aware of this. He created the curves intuitively.[34] The vanishing point has two functions: for the upper part of the drawing it is the nadir and for the bottom part it is the zenith. The top half of this litograph is not a mirror image of the bottom half.

Escher often used a cycle to represent infinity. Examples of the latter is found in *Cycle* (1938), *Reptiles* (1943), *Magic Mirror* (1946), *Swans* (1955) etc. In *Ascending and Descending* (figure 17) the monks move upwards, but if one looks very carefully they just move in a cycle, without getting higher. Escher learnt about quasi-endless ascent or descent in an article by the mathematician Professor L.S. Penrose. Lionel and Roger Penrose made the impossible staircase (figure 17) famous, but it was independently discovered previously by the Swedish artist Oscar Reutersvärd. Lionel and Roger Penrose (and Escher) were not aware of Reutersvard's designs. The staircase is a two-dimensional depiction of a staircase in which the stairs make four 90° turns while ascending or descending, but also forms a continous loop. This is impossible in three dimensions, but the two-dimensional figure is possible by the paradox of distorting perspective. The human eye tries to interpret it as a three-dimensional object in Euclidean space while it is a two-dimensional object drawn on a flat surface which is possible.

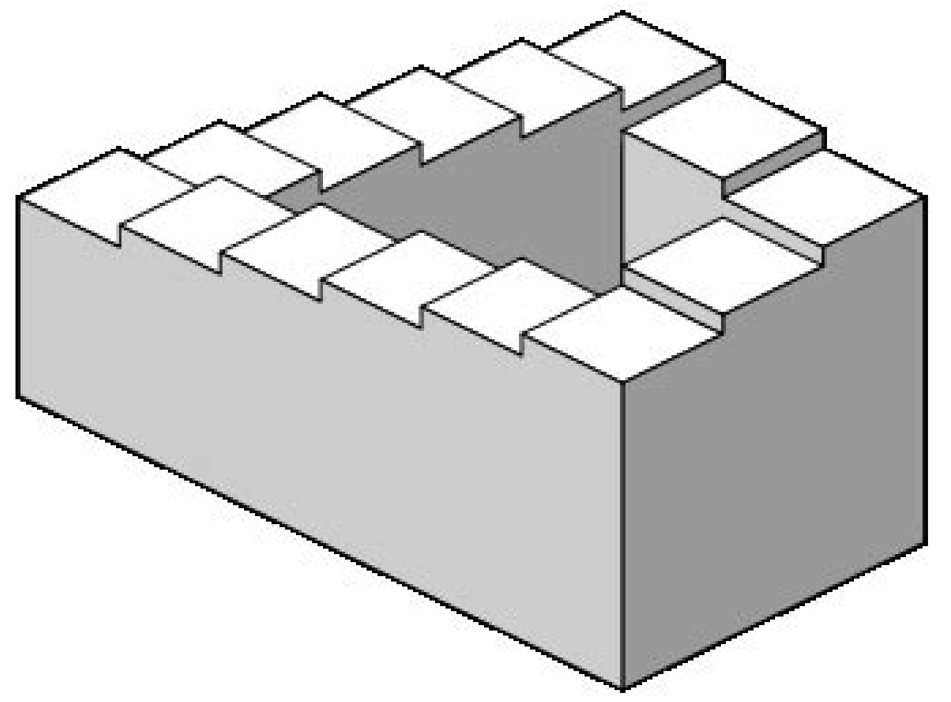

Figure 17
(Source: http://en.wikipedia.org/wiki/File:Impossible_staircase.svg).

The theme of *Ascending and Descending* is the continuous staircase of Penrose. In the place of a roof, there is a closed circuit of steps. The monks (members of an unknown sect) walk around on the top of their home and it is seen as a spiritual exercise.[35] If the building is cut into slices, the sections are not horizontal planes. The sections are constructed in the form of a spiral and only the staircase of the monks is a horizontal plane.

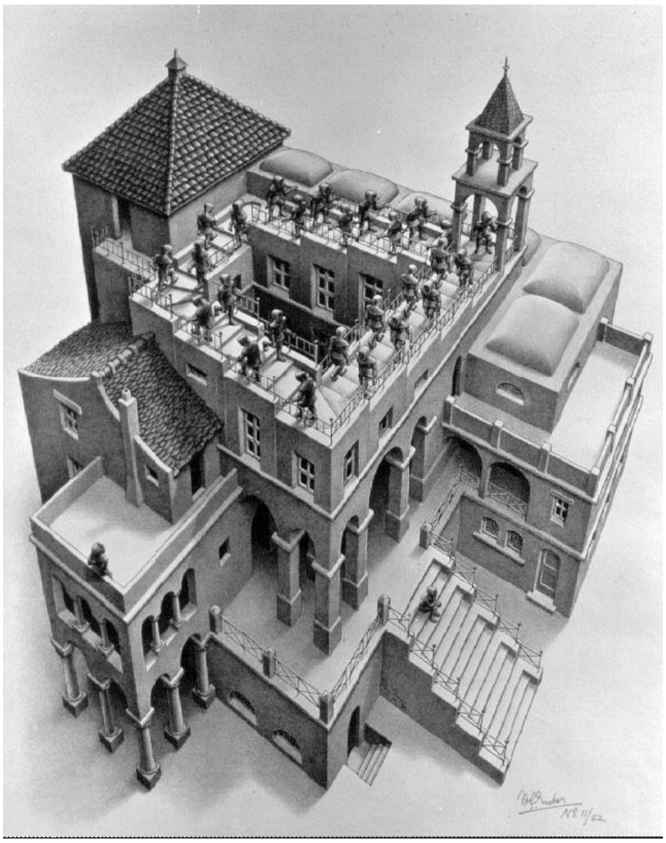

Figure 17
M.C. Escher, ***Ascending and Descending,***
(1960, litograph, 35,5 x 28,5 cm).

When making a map of the world, one is faced with the problem of creating a two-dimensional (flat) representation of a sphere. It is not possible to make a flat representation of a sphere without distortion. On the other hand it is possible to flatten out a cylinder. Escher uses curved lines in *House of Stairs* (figure 18), like in *High and Low* (figure 15).

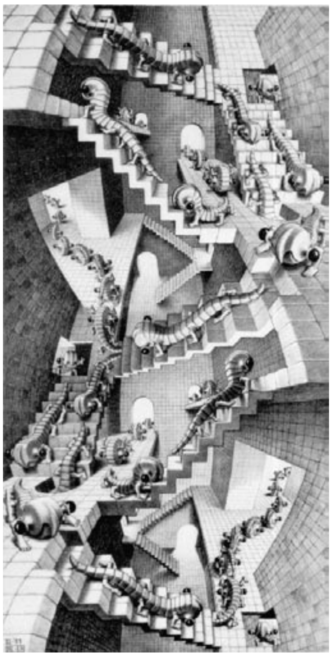

Figure 18
M.C. Escher, ***House of Stairs,***
1951, litograph, 47,2 x 23,8 cm.

Figure 19 shows a preparatory sketch for the litograph *House of Stairs* (figure 18), but the actual litograph deviates from it.

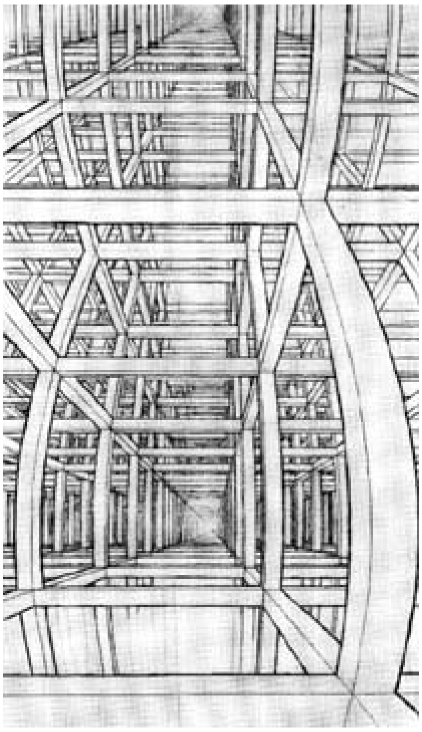

Figure 18
M.C. Escher, preliminary study for *House of Stairs,*
1951, India ink and pencil.
(source: www.clowder.net/hop/persp/persp.html).

Escher used curved lines in *House of Stairs* (1951) and he '...develops cylindrical perspective empirically....'[36] Vero carries on and says that Escher realizes that the human eye records long, straight lines as curves and, '..situating himself on the axis of a cylinder, finds that the virtual line joining his eye to each point of the straight line intersects the cylinder in an ellipse.' Ernst[37] calls *House of Stairs* an 'infinitely complicated print' and a print of infinite length could be constructed by alternating sections with their mirror images. Escher wants to depict an infinite extent of space in *House of Stairs* as was the case with *Cubic Space Division*, but here he uses cylinder perspective (not spherical perspective). Mechanical animals move up and down flights of stairs. The top half is a mirror image of the bottom half. Relativity plays a role here as in other litographs and Escher also uses glide reflection.

Coda

The aim of this article was not to give a detailed account of the historical development of perspective in art. Some aspects of the historical development of perspective were discussed. A brief overview of the impossible worlds of Maurits Cornelis Escher, the master of infinity, followed. There are many other aspects of perspective which could be pursued: for instance, the approaches to spatial composition according to Futurists, Cubists and Surrealists.

Notes

1. Heidema & Schmidt (1980: 3).
2. Heidema & Schmidt (1980: 3).
3. Heidema & Schmidt (1980: 8).
4. Heidema & Schmidt (1980: 3).

5. Heidema & Schmidt (1980: 4).
6. O'Connor (2003: 1).
7. Edgerton (1975: 16).
8. Politoske (1984: 82).
9. Calter, (1998: 1).
10. Cole, (1992: 8)..
11. Edgerton, (1975: 199).
12. O'Conner (2003: 1).
13. Calter, (1998: 2).
14. Ernst, (1985: 42-44).
15. Vero, (1980: 34).
16. Cole, (1992: 12).
17. Honour (1984: 372).
18. Cole, (1992: 7).
19. Cole, (1992: 22).
20. Cole, (1992: 36).
21. Cole, (1992: 32).
22. Escher (1986: 128).
23. Escher (1986: 19).
24. Cole (1992: 58
25. Escher (1986: 54).
26. Ernst, (1985: 42).
27. Escher (1986: 73).
28. Ernst (1985: 46).
29. Cole (1992: 59).
30. Escher (1986: 76).
31. Vero (1980: 113).
32. Vero (1980: 116).
33. Ernst (1985: 50).
34. Ernst, (1985: 51).
35. Escher (1986: 78).
36. Vero (1980: 129).
37. Ernst (1985: 57).

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