Appendix A. Supplementary data

1 The likelihood function in general form

We model the green and brown energy stock index returns using four GARCH-type models, standard GARCH, GRJGARCH, EGARCH, and APARCH. For the exogenous shocks from climate risk factors, we use MIDAS regression to include alternatively the Wall Street Journal climate change news (CCN) index and Crimson Hexagon negative climate change news index (NCC), and the Climate Policy Uncertainty index (CPU). The specification in general form is,

$$r_t - \mu_t = \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_{t,\tau} z_t \tag{2}$$

$$\sigma_{t,\tau}^2 = m_\tau \times g_{t,\tau} \tag{3}$$

$$\ln m_\tau = m + \theta \sum_{k=1}^{K} \varphi_k(w,1) X_{\tau-k} \tag{4}$$

The mean equation is $\mu_t = E_{t-1}(r_t) = \mu - \rho r_{t-1}$; the volatility equation $g_{t,\tau}$ can be GARCH, GRJGARCH, EGARCH, and APARCH alternatively, and $X_{\tau-k}$ in the MIDAS regression is alternatively CCN, NCC and CPU; $\varphi_k(w,1)$ is beta function.

We can use MLE to estimate all parameters in a united log-likelihood function as follows,

$$LLF(\boldsymbol{\theta}) = \sum_{t=1}^{T} lnf\left(\frac{r_t-\mu_t}{\sigma_{t,\tau}}, \boldsymbol{\theta}\right) - ln\sigma_{t,\tau} \tag{5}$$

where $\sigma_{t,\tau}$ is given by the combination of Eq. (3) and (4); the last in RHS is Jacobi term; $\boldsymbol{\theta}$ is coefficients vector including all parameters in Eq. (1), (4) and those appears in GARCH, GRJGARCH, EGARCH, and APARCH alternatively; $f(\cdot)$ is density function of the conditional distribution whose shape parameters are included in $\boldsymbol{\theta}$, such as degree of freedom η from standard *t*, and λ and η from *skew-t* distribution. We implement MLE based on Eq. (5) with the help of the open source project, 'rumidas' that is available from CRAN.r-project.

In the similar spirit of Eq. (5), we can write the log-likelihood function of (A)DCC-MIDAS as

$$LLF(\boldsymbol{\theta}) = \sum_{t=1}^{T} lnbiskewt\big((z_{1,t}, z_{2,t}), \boldsymbol{\theta}\big) - \frac{1}{2} ln(|\boldsymbol{H_t}|, \boldsymbol{\theta}) \quad (6)$$

where $\boldsymbol{\theta}$ is the parameters vector including that of *biskewt* distribution, $(z_{1,t}, z_{2,t})$ is the standardized innovation vector of green and brown stock returns after estimated GARCH-MIDAS from Eq.(2); $\boldsymbol{H_t}$ is the quasi variance-covariance matrix given by,

$$\boldsymbol{H}_t = diag(\boldsymbol{Q}_t)^{-1/2} \boldsymbol{Q}_t diag(\boldsymbol{Q}_t)^{-1/2} \quad (7)$$

the element of $\boldsymbol{Q}_t$ for ADCC-MIDAS is,

$$q_{i,j,t} = \bar{\rho}_{i,j,\tau}(1 - a - b) - g\bar{n}_{i,j,\tau} + a\varepsilon_{i,t-1}\varepsilon_{j,t-1} + gn_{i,t-1}n_{j,t-1} + bq_{i,j,t-1} \quad (8)$$

$$\bar{\rho}_{i,j,\tau} = \Lambda(2x_\tau), \;\; x_\tau = m_c + \theta_c \sum_{l=1}^{L} \varphi_l(w_c, 1) X_{\tau-l} \quad (9)$$

Obviously, we also borrow the spirit of two-step estimation in to do MLE for ADCC-MIDAS as Amendola et al. (2019). That is, $(z_{1,t}, z_{2,t})$ comes respectively from MLE based on Eq. (5). We implement MLE based on Eq. (6) with the help of the open source project, 'dccmidas' that is available from CRAN.r-project.

2 The omitted results for Table 4 and 5

We present the omitted results in Table 4 and 5 as following Table 4' and 5'.

Table 4' The estimated parameters of various GARCH for the brown stock index volatilities

Coef.	GARCH-MIDAS		GJRGARCH-MIDAS		EGARCH-MIDAS		APARCH-MIDAS	
	Std-t	Skew-t	Std-t	Skew-t	Std-t	Skew-t	Std-t	Skew-t
Panel A: MIDAS with NCCN								
ω	0.015**	0.029**	0.038**	0.024***	0.024**	0.018***	0.046**	0.018**
	(0.006)	(0.011)	(0.016)	(0.009)	(0.010)	(0.007)	(0.019)	(0.007)
α	0.013**	0.032**	0.059**	0.018**	0.022***	0.039**	0.027***	0.036**
	(0.005)	(0.013)	(0.024)	(0.007)	(0.008)	(0.015)	(0.010)	(0.015)
β	0.877**	0.858***	0.831**	0.872**	0.868**	0.851***	0.863***	0.854***
	(0.350)	(0.311)	(0.337)	(0.356)	(0.351)	(0.304)	(0.332)	(0.328)
γ			0.040***	0.051**	-0.030**	-0.026***	0.412**	0.319**
			(0.015)	(0.020)	(0.012)	(0.010)	(0.162)	(0.124)
δ							1.209**	1.515**
							(0.552)	(0.764)
Panel B: MIDAS with CPU								
ω	0.037***	0.020***	0.050**	0.044***	0.056***	0.036***	0.023**	0.051**
	(0.014)	(0.007)	(0.020)	(0.017)	(0.022)	(0.014)	(0.009)	(0.020)
α	0.018***	0.013**	0.035**	0.025**	0.015***	0.056**	0.022***	0.016**
	(0.006)	(0.005)	(0.014)	(0.010)	(0.005)	(0.022)	(0.008)	(0.006)
β	0.872**	0.877**	0.855***	0.865***	0.875***	0.834***	0.868***	0.874**
	(0.357)	(0.345)	(0.313)	(0.322)	(0.331)	(0.298)	(0.337)	(0.345)
γ			0.056**	0.058**	-0.042**	-0.031***	0.468***	0.558**
			(0.022)	(0.023)	(0.017)	(0.012)	(0.172)	(0.221)
δ							1.370**	1.515**
							(0.594)	(0.769)

Table 5' The estimated parameters of various GARCH for the green stock index volatilities

Coef.	GARCH-MIDAS		GJRGARCH-MIDAS		EGARCH-MIDAS		APARCH-MIDAS	
	Std-t	Skew-t	Std-t	Skew-t	Std-t	Skew-t	Std-t	Skew-t
Panel A: MIDAS with NCCN								
ω	0.010***	0.019***	0.051***	0.055***	0.059***	0.045**	0.024**	0.058**
	(0.004)	(0.007)	(0.019)	(0.021)	(0.022)	(0.018)	(0.010)	(0.023)
α	0.041**	0.029***	0.038***	0.037***	0.024***	0.055**	0.024**	0.027***
	(0.017)	(0.011)	(0.014)	(0.014)	(0.009)	(0.022)	(0.010)	(0.011)
β	0.849***	0.861**	0.852***	0.853**	0.866**	0.835**	0.866***	0.863**
	(0.313)	(0.344)	(0.308)	(0.332)	(0.338)	(0.329)	(0.333)	(0.341)
γ			0.046**	0.042***	-0.037***	-0.045***	0.372**	0.427***
			(0.019)	(0.016)	(0.014)	(0.017)	(0.148)	(0.158)
δ							1.295**	1.312***
							(0.508)	(0.469)
Panel B: MIDAS with CPU								
ω	0.015**	0.041**	0.051***	0.026***	0.056**	0.012***	0.053***	0.054***
	(0.006)	(0.016)	(0.019)	(0.010)	(0.022)	(0.004)	(0.020)	(0.021)
α	0.023**	0.059***	0.046**	0.016**	0.048***	0.030***	0.025**	0.017***
	(0.009)	(0.022)	(0.019)	(0.006)	(0.018)	(0.011)	(0.010)	(0.006)
β	0.867**	0.831***	0.844***	0.874**	0.842***	0.860***	0.865***	0.873***
	(0.337)	(0.315)	(0.313)	(0.346)	(0.321)	(0.310)	(0.309)	(0.319)
γ			0.059**	0.058**	-0.036**	-0.031**	0.464***	0.315***
			(0.024)	(0.023)	(0.015)	(0.012)	(0.177)	(0.116)
δ							1.234**	1.323***
							(0.481)	(0.498)