Endogenous Long-Term Productivity Performance in Advanced Countries: A Novel Two-Dimensional Fuzzy-Monte Carlo Approach[1]

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Abstract

Better performance at a country level will provide benefits to the whole population. This issue has been studied from various perspectives using empirical methods. However, little effort has as yet been made to address the issue of endogeneity in the interrelationships between productive performance and its determinants. We address this issue by proposing a Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach. By applying the proposed method to a sample of 23 countries for 1890-2018, our results show that the best and worst-performing countries were Norway and Portugal, respectively. We further found that the intensity of human capital and the age of equipment (capital stock) have different impacts on productive performance – it has been established that capital intensity and total factor productivity are influenced by productivity performance, which, in turn, has a negative impact on labor productivity and GDP per capita. Our analysis provides insights to enable government policies to coordinate productive performance and other macroeconomic indicators.

Keywords: Productivity and competitiveness; endogeneity; type-2 fuzzy sets; 2DFMC; stochastic performance

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1. Introduction

The quest to understand and explain the productivity performance of economies has grown since the seminal work of Solow (1957), which accorded Total Factor Productivity (TFP) a crucial role in generating and predicting economic growth. TFP is an important tool for policy analysis, as it measures the performance of economies. Specifically, it is the unexplained or residual portion of output and gives an idea of how efficiently and intensely inputs are used in the production process (Şeker and Saliola, 2018).

The debate about the causes of widespread productivity slowdown within and across countries is ongoing (Tang and Wang, 2020). There are some arguments in favor of both supply and demand factors. The supply factors identified in the literature include a slowdown in important innovation, a strong decline of entrepreneurship, and the waning of the ICT-related (Information and Communications Technology) productivity boom (Decker et al., 2016). On the demand side, weak aggregate demand, great uncertainty and financial market disruption could be critical factors explaining productivity slowdowns. However, Blanchard et al. (2017) noted that an expectation of lower future productivity growth might cause weak demand, that is, reverse causality (Blanchard et al., 2017).

Empirical evidence shows that differences in the growth patterns and income levels of countries are associated with differences in their productivity levels, and often TFP accounts for a greater portion of these cross-country differences than physical and intangible capital (Hsieh and Klenow, 2010; Jones and Romer, 2010; Prescott, 1998). Nonetheless, the process of economic growth is also based on technology and human capital (Teixeira and Queiros, 2016; Bustos and Yildirim, 2020). Thus, the intensity of productivity differentials depends on the interaction of forces acting in opposite directions, some increasing and others limiting productivity differentials (Tamberi, 2020).

The need to monitor productivity performance cannot be overstressed, as this ensures that appropriate and timely decisions, plans and policies are put in place. Therefore, the use of suitable performance measures can assist a country to adopt a long-term perspective and allocate its resources more efficiently. The paths of country development have been studied from different theoretical perspectives. One of the rare consensuses is that gains in productivity are key to promoting economic progress and social welfare. This research digs into historically related productivity data to explore the underlying endogenous relationships that may exist among different but related metrics for wealth, and physical and human capital accumulation over the course of time. More specifically, the current study has the following objectives. First, unlike the previous literature studies, we aim to provide an accurate measure of performance at the country level by proposing a novel Multi-attribute Decision-Making (MADM) model based on Type-2 Fuzzy Sets (T2FS). This objective was motivated by the need – considering the importance of performance at a country level – to give priority to providing an accurate performance measurement before making relevant policies, without which the policy implications related to improving performance will be biased and inaccurate. Second, we aimed to investigate not only the inter-relationships between performance and its potential determinants, but also the potential interrelationships among the potential determinants. This objective was motivated by the focus in the majority of the literature studies on a one-way relationship, i.e. by only investigating the determinants of performance – ignoring the potential two-way relationships – neglects the importance of understanding potential determinants, which albeit to a limited extent, would be helpful in improving performance. In addition, the relative lack of consideration of the potential interrelationships among the determinants makes the policy implications less accurate; our consideration of this perspective was aimed at developing relevant policies that could promote developing performance as well as its potential determinants.

Given the intrinsic epistemic uncertainty of dealing with such a long time series – compiled in a decentralized fashion across several countries with distinct cultural and technological backgrounds – this research was aimed at employing fuzzy reasoning techniques to investigate the impacts of such underlying vagueness on long-term productivity performance. Vague or imprecise pieces of information increase as much as time series go back into the past. Hence, this paper addresses the literature gap by developing a novel 2DFMC analysis based on the Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach, first proposed by Kentel and Aral (2005). This approach combines the probability and possibility theories, bridging gaps between both imprecise and probabilistic pieces of information, the remote origins of which may be questionable. We are the first study to apply this method in the area of productivity performance analysis at a country level.

In the first dimension of the 2DFMC approach proposed here, a novel MADM based on T2FS was developed for computing and ranking the long-term productivity performance of advanced countries. This model (UP-IS) innovates by using a power tower function of order two as the cornerstone for building type-2 hesitant fuzzy sets, representing the membership degrees of productivity attributes and their distances to ideal solutions. Also, unlike previous studies, this novel model enables the computation of positive and negative decision-making biases that eventually arise from the inherent trade-offs of representing vagueness using T2FS. In the second dimension, an additional contribution of this study is related to the development of a novel Stochastic Structural Relationship Programming (SSRP) Model based on neural networks to evaluate the endogenous feedbacks among long-term productivity performance and its different attributes related to wealth accumulation and physical and human capital intensities, by means of maximal entropy functions. This model allows the cause-effect direction mapping between distinct productivity attributes and respective long-term productivity performance levels in advanced countries, computed at different threshold quantiles. The results have indicated that UP-IS performance scores presented better discriminatory power than the TOPSIS base case. Furthermore, long-term productivity performance in advanced countries is positively impacted by Human K (Human capital intensity) but negatively by Age K (Average age of equipment capital stock). On the other hand, KI (capital intensity) and TFP (total factor productivity) are positively impacted by productivity performance, while LP (Labor productivity) and GDPpc (GDP per capita) are negatively impacted by the same.

In summary, to our knowledge, there are numerous research articles investigating performance at a country level. Although various methods have been applied, few studies have attempted to address the issue of uncertainty in the modeling framework through the proposal and application of interval fuzzy set multi-criteria decision-making. An optimal method for computing positive and negative decision-making bias has not been found yet. We contribute significantly to the literature by providing a solution in this regard. Of the empirical studies evaluating performance at a country level, some have focused on investigating the factors influencing performance; however, they suffer from the limitation that there is no clear attempt made to investigate the two-way relationships between performance and other attributes; also no internal mechanism has been proposed related to the interrelationships among the attributes. We fill in this gap with the current paper.

The remainder of this paper is organized as follows. Section 2 presents the literature review and indicates the gap found. Section 3 describes the dataset and the novel methodology developed. The results are analyzed and discussed in Section 4. Section 5 concludes the discussion and shows the limitations of the research while giving suggestions for future studies.

2. Literature Review

The economic and productivity performance of countries have been evaluated within both microeconomic and macroeconomic frameworks using different quantitative techniques, including Multi-Criteria Decision-Making (MCDM) methods. Moreover, the drivers of productivity differences, including the conventional factors of production and non-conventional demand and supply variables, have been analyzed in the literature. This section provides an overview of these studies. For instance, Färe et al. (1994) used the Malmquist productivity index and DEA to analyze productivity growth in 17 OECD countries from 1979 to 1988. Decomposing productivity growth into changes in technical efficiency (catching up) and shifts in technology (innovation), and using GDP as the measure of aggregate output, and capital stock and employment as the aggregate inputs, they found that TFP growth was driven predominantly by innovation, while in contrast, technical efficiency has deteriorated slightly over time.

Gouyette and Perelman (1997) estimated the productivity performance and convergence in service and manufacturing industries of 13 OECD countries over the period 1970-1987 using alternative frontier analysis and Divisia index approaches. They found that productivity levels converged in the services sector despite low growth rates, unlike in the manufacturing sector. Furthermore, new investments in capital had a negative effect on total factor productivity growth in service activities, while the effect was positive in manufacturing industries. There have been additional studies investigating the issue of total factor productivity, but in different contexts, by focusing on a specific industry or considering the interrelationships between total factor productivity and another issue concerned, as well as methodological contributions in estimating productivity (Gonzalez and Gascon, 2004; Bolli and Somogyi, 2011; Kancs and Siliverstoves, 2016; Pieri et al., 2018; Fonseca et al., 2018; Hu et al., 2021).

Naastepad and Kleinknecht (2004) analyzed two aspects of the performance of the Dutch economy from 1982 to 2001, namely a rapid growth in employment and a significant slowdown in labor productivity growth. Based on their growth accounting analysis, a large part of the slowdown of growth of Dutch labor productivity was due to the slowdown in wage growth. Broadberry and Ghosal (2005) compared the productivity performance of the United States and Britain while illustrating the importance of the development of services. They concluded, among others, that adaptation in the technology-using sectors may be made difficult by technological change if it is not suited to the society's capabilities; also, a reversal of technological trends could lead to the reversal of comparative productivity performance.

Ball et al. (2005) constructed an alternative productivity growth measure, the Malmquist cost productivity (MCP) index, which integrates the externality/social output into a generalized productivity measure reflecting social responsibility. Applying this to the US agriculture sector, they show that conventional measures of productivity are biased upward (download) when the production of negative externalities (or bad) outputs increases (decreases). Amendola et al. (2005) argue that:

> … productivity is the outcome of an out-of-equilibrium process triggered by a technological shock and that the potential gains of a superior technology may only be appropriated if agents succeed in reshaping the productive capacity, and in recovering the intertemporal coordination disrupted by the introduction of the new technique. Physical, human, and financial capital are complementary in this process of reshaping and may constrain each other. The outcome of the disequilibrium process depends then on the interaction of accumulation choices, learning, and money supply rules.

They further argue that the differences in performance between the US and Europe in the last two decades may be explained along these lines.

Palazuelos and Fernández (2009), in their analysis of the causes of the slowdown in growth in labor productivity in the European economies, found that weak domestic demand and the features of the labor markets in European countries were the main determinants. Maroto-Sánchez and Cuadrado-Roura (2009) investigated the role of the service sector, specifically tertiary activities, in productivity growth, using data on 37 OECD countries from 1980 to 2005. Their results show that the contribution of the service sector to productivity growth has increased in contrast with historical trends.

Azomahou et al. (2013) analyzed productivity growth for a panel of developed and developing countries from 1998 to 2008 using three measurements of the following frontier: the economy with the highest level of productivity growth, world productivity growth and the productivity growth of the USA. The results based on a semi-parametric generalized additive model show a high degree of nonlinearity between productivity growth and its determinants (human capital, R&D expenditure, international trade), including a U-shape, inverted U-shape and W-shape, depending on the determinant and the frontier in question.

Wu et al. (2014) evaluated the performance efficiency of 21 OECD countries and assessed whether the undesirable outputs were over-produced relative to desirable outputs using four data envelopment analysis (DEA) models. Their results support the above arguments and also show that knowledge capital, proxied by R&D, improves countries' efficiency scores, thereby supporting the endogenous growth theory in respect of OECD countries.

Karabiyik and Kutlu (2016) evaluated the international trade performance of OECD countries from 1999 to 2014 using TOPSIS and AHP methods. Their analysis, based on three foreign trade performance indicators, namely, Volume of Exports Per Capita, Normalized Trade Balance and Terms of Trade, ranked Norway, Ireland and Germany among the top three countries, while Turkey, the USA and Greece were the bottom three. Pan and Ngo (2016) analyzed the regional performance of 64 Vietnamese provinces using a panel dataset. Their results show that FDI, openness, and capital investment had positive impacts on GDP growth, although no support for the endogenous growth model was found in cases where regional per capita income tended to converge across different regions. They also found that for provinces that had established special economic zones through liberal state regulation, internationalization activities had a positive effect on regional performance.

Millemaci and Ofria (2016) report a positive effect for labor costs, R&D and railway infrastructure with regard to Italian productivity growth, based on results from pooled cross-section OLS and time series LIML estimators, using data from 1964–2009. However, Padilla-Pérez and Villarreal (2017) show that highly qualified production factors (both labor and capital) have not shown a significant contribution to value-added growth in Mexico. The long-run relationship between GDP growth and the job "required" by such growth was examined by Compagnucci et al. (2018) using data from 1970 to 2015. They found that the break in the relationship between GDP per capita growth and employment was due to the decoupling of productivity, labor compensation and utilization from each other. They also found that the technological and knowledge intensities of different economic sectors played a significant role in the productivity change. Fernándeza and Palazuelos (2018) could find neither empirical support for the thesis that the contribution of manufacturing to aggregate productivity in European countries was high but lower than that of the service sector, nor that the growth pattern of technology-intensive branches of the manufacturing sector were more oriented towards productivity.

Using the subsystems approach and Chinese data from 1995 to 2009, Brondino (2019) found that the major source of productivity growth across subsystems was direct labor savings and that the best-performing subsystems were targeted by industrial policy for promotion. Based on the tradable-nontradable framework, Friesenbichler and Glocker (2019) found that increases in overall productivity among EU Member States was predominantly due to the tradable, and not nontradable, sectors of production. They found that productivity growth differentials could be explained by differences in the legal systems and the quality of public institutions, among others.

A decomposition analysis by Moussir and Chatri (2020) shows that the intersectoral component accounted for much of the labor productivity growth in Morocco. Their results show an increase in the income, education and human capital determinants of labor productivity growth and, hence, structural transformation, while labor market flexibility, inflation and the financial system had an adverse effect on the competitiveness of the economy. Tang and Wang (2020), using microdata, decomposed productivity in Canada into technological frontier and technical efficiency. They found that Canada's productivity was mainly due to the retreat of the aggregate technological frontier, driven by large and high-productivity firms, rather than by factors such as R&D, ICTs and other intangibles. Rouyendegh et al. (2020) used an integrated intuitionistic fuzzy Technique for Order of Preference by Similarity to Ideal Solution (IF-TOPSIS) and Data Envelopment Analysis (DEA) to evaluate the performance of the retail industry. They show that a combination of the two methods is suitable for any number of DMUs. Chen et al. (2013) developed the non-radial fuzzy slack-based model to measure technical efficiency in the Taiwanese banking industry. Their findings suggest that the efficiency scores estimated by the Fuzzy SBM model were subordinate to functional form. Not used only in the context of performance evaluation, the fuzzy environment is also considered to price the options through proposing a double-exponential jump-diffusion model (Zhang et al., 2012).

Yapa et al. (2020) used the Best Worst Method (BWM), an MCDM method and several macroeconomic indicators (GDP per capita, unemployment, inflation, real interest rates, and growth rates) to compare the performance of EU countries and

Turkey. They found that Luxembourg ranked first, Denmark second, and Sweden third, while Portugal, Croatia, and Greece were ranked last, respectively. Turkey ranked 24th. Azenui and Rada (2021), studying 30 sub-Saharan African LDCs over the 1991–2018 period, foumd that manufacturing and FDI contributed positively towards labor productivity growth – with the exception of mineral exporting countries – while FDI had a negative effect, whereas the effect of global integration was weak.

With this study, we contribute to the literature on country-level productivity performance by developing a novel 2DFMC analysis based on the Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach, the merits of which have been stated previously.

3. Methodology

The methodology developed in this study is shown in the flowchart below. Every step in this diagram is explained in detail in the following subsections.

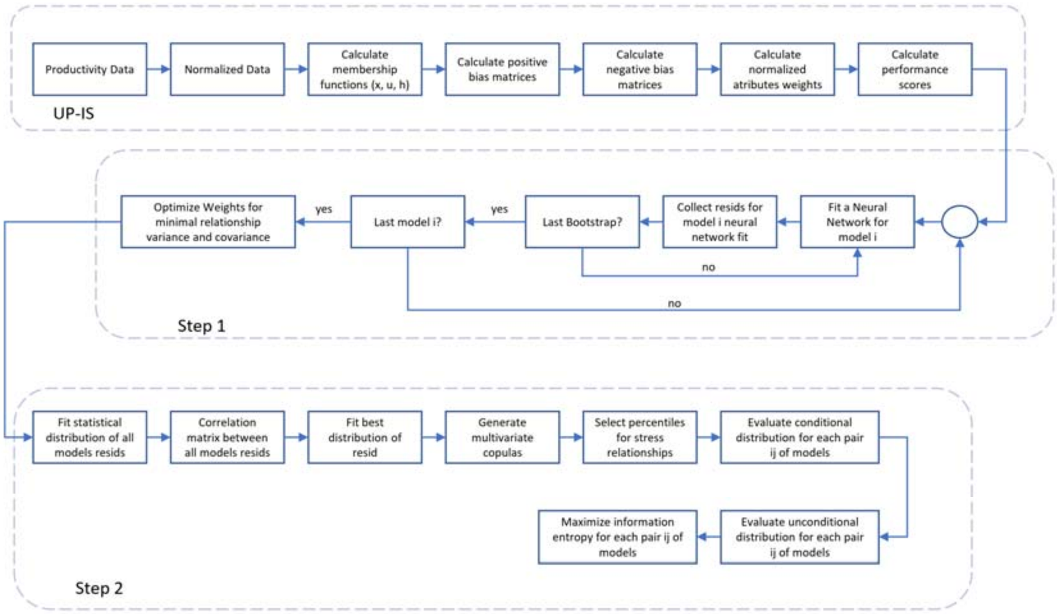

Fig. 1. Flowchart of methodology

3.1. The Data

We used annual time series data from 1890 to 2018 for 23 countries as well as the Euro Area (Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States). The data were obtained from the long-term productivity project (http://www.longtermproductivity.com/), a database developed and regularly updated by Bergeaud et al. (2016). Table 1 presents the summary statistics for the data. Total factor productivity (TFP) is the Solow residual from a constant return to scale Cobb-Douglas production function, with capital stock and hours worked as input. Labor productivity is the ratio of GDP in respect of total hours worked. Capital intensity (KI) is the ratio of total capital stock in respect of total hours worked. Age K is the average age of equipment capital stock in years. Human capital intensity (Human K) is proxied by education attainment (Teixeira and Tavares-Lehmann, 2014). GDP per capita is the Gross Domestic Product per capita. Variables were calculated using GDP, and capital stock series were converted into US dollars of 2010 ppp. All the variables have a positive mean with a moderate variation. All the variables are positively skewed, with the exception of human capital intensity and the average age of equipment capital stock, which are negatively skewed.

Table 1. Descriptive statistics for long-term productivity of advanced countries.

Variable	**Type**	**Min**	**Max**	**Mean**	**SD**	**CV**	**Skewness**	**Kurtosis**
GDP per capita	Positive Criteria	1496.44	74972.89	16016.38	13788.82	0.86	1.27	1.26
Human K per capita	Positive Criteria	582.42	2726.59	1483.22	362.54	0.24	-0.07	-0.28
Labor Productivity	Positive Criteria	0.93	103.95	20.36	19.14	0.94	1.25	1.17
TFP	Positive Criteria	0.96	18.63	5.47	3.56	0.65	0.82	-0.15
Age K	Negative Criteria	-10.72	11.91	6.46	1.14	0.18	-0.69	17.12
KI	Positive Criteria	0.24	380.37	63.94	65.73	1.03	1.33	1.25

3.2. Performance Measurement and Ideal Solutions (IS)

Performance measurement refers to a broader benchmarking concept that can be structured by using either scalar or ratio variables, or even a mix of them. It is usually employed when there are difficulties in comparing with peers – they may not be homogeneous in quantifying monetary or physical values for inputs and outputs, namely, the negative and positive attributes for decision-making (Mihaiu et al., 2010). Performance scores are often assessed by MADM matrix-based methods like TOPSIS, VIKOR, or COPRAS, for instance, where specific functions are assumed (e.g. ideal solutions, compromise solutions, utility solutions, among others) (Behzadian et al., 2012). More specifically, with respect to ideal solutions, TOPSIS is a well-known MADM model that develops cardinal or scale metrics within the range delimited by positive and negative ideal solutions through a linear combination of attributes. The performance distance in TOPSIS is cardinal, consisting of a Euclidean distance (Olson, 2004). Otherwise put, TOPSIS computes cardinal distances (scores) from ideal positive solutions, while simultaneously presenting an ordinal ranking of them (Behzadian et al., 2012). The positive ideal solution has the best level for all attributes considered, while the negative ideal is the one with the worst values (Wanke et al., 2016a). While determining the weights of the relative importance of each attribute is exogenously defined in TOPSIS, it might also be considered a drawback. TOPSIS is computationally simpler because there are almost no constraints with respect to the number of attributes that can be assessed (Wanke et al., 2016a).

3.3. Type-2 Fuzzy Sets (T2FS)

Treating normalized productivity attributes and their distances to ideal solutions by means of T2FS yields more freedom for capturing inherent vagueness related to data collection and measurement when compared with Type 1 fuzzy sets (Mendel and John, 2002). This happens because T2FS can reflect the uncertainty of inaccurate data by means of primary and secondary membership functions related, respectively, to normalized attributes and their distances to ideal solutions (Turk et al., 2014). As regards its application in advanced countries, while distances to ideal solutions are usually computed at each normalized attribute level, very often identifying an unbiased global ideal solution is difficult due to multicollinearity issues among attributes. This being the case, extensive pairwise comparisons may assist in developing unbiased evaluation alternatives (Zavadskas et al., 2014).

T2FS are defined by primary and secondary membership functions in order to represent uncertain information more effectively (Hu et al., 2015). In this section, the basic concepts and arithmetic operations of T2FS are summarized from Lee and Chen (2008), Chen and Lee (2010a, 2010b), Mendel et al. (2006), Hu et al. (2015), and are given as follows.

Definition 1. A T2FS $\tilde{\tilde{A}}$ in the universe of discourse can be represented by a type-2 membership function, as shown below (Mendel et al., 2006):

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{A}}}(x,u) \right) | \forall_{x} \in X, \forall_{u} \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1 \right\} \qquad (1)$$

Where J_x denotes an interval in [0, 1] which represents support for the secondary membership function. Moreover, the T2FS $\tilde{\tilde{A}}$ can also be represented as follows (Mendel et al., 2006):

$$\tilde{\tilde{A}} = \iint_{x \in X \; u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u) / (x,u), \qquad (2)$$

where $J_x \subseteq [0,1]$ and the double integral denotes union over all admissible x and u.

Definition 2. The concept of a hesitant T2FS (HT2FS) was introduced to represent a particular subset of the original T2FS and can be used to explore alternative membership possibilities with respect to x and u (Hu et al., 2015).

Definition 3. Let $\tilde{\tilde{A}}$ be a fixed set. An HT2FS on $\tilde{\tilde{A}}$ is expressed in terms of a function that returns a type-2 fuzzy subset when applied to each (x,u) pair in $\tilde{\tilde{A}}$ (Xia and Xu, 2011). To make it more easily understood, this HT2FS is represented by $E = \{< (x,u), \tilde{h}_E(x,u) > | x \in X, u \in J_x\}$, where $\tilde{h}_E(x,u)$ denotes the possible membership degrees for each pair (x, u) with respect to the subset E.

3.4. Unbiased-Power functions for Ideal Solutions (UP-IS) based on T2FS

Let us consider a set of d advanced countries' sections, each one of them formed by I positive attributes in terms of productivity performance -$pos_{d,i}$- and o negative attributes in terms of productivity performance -$neg_{d,o}$-, where $d = \{1..n\}$, $i = \{1..m\}$, $o = \{1..s\}$, **pos** and **neg**, are positive and negative attribute matrices, respectively, with dimensions

nxm and nxs. The negative ideal solution for all d countries for each negative attribute o is given by max(neg_o), while the positive ideal solution for each positive attribute i is given by max(pos_i), for all DMU (Decision-Making Units), i.e., country, d. These ideal solutions are the cornerstones for computing the normalized values for each productivity performance positive attribute i and for each productivity performance negative attribute o at each country level, with respect to their ideal solutions, such as:

$x_{d,i}$ = ($pos_{d,i}$- min(pos_i))/(max(pos_i)-min(pos_i)), $x_{d,I}$ ranges between 0 and 1 for all i and d (3)

$x_{d,o}$ = (max(neg_o) - $neg_{d,o}$)/(max(neg_o)-min(neg_o)), $x_{d,o}$ ranges between 0 and 1 for all o and d(4)

where $x_{d,i}$ is the normalized positive attribute i for each country d, while $x_{d,o}$ is the normalized negative attribute o for each country section d. One can see that the maximal values of positive attributes would correspond to a normalized value of 1. Conversely, maximal values for negative attributes would correspond to a normalized value of zero, thus allowing us the treatment of both sets of attributes simultaneously in a matrix **x** with dimensions nx(m+s). Lastly, **u** represents the nx(m+s) distance matrix to each ideal solution. For positive attributes, it follows that $u_{d,i}$= 1 - $x_{d,I}$ and, for negative attributes, $u_{d,o}$= $x_{d,o}$. Analogously to normalized the attributes, the distances to ideal solutions range between zero and one. The UP-IS computational steps are detailed below.

Suppose $\mu_{\tilde{A}}(x)$ denotes the partial membership function for the elements of the normalized attribute matrix **x**, expressed as a power tower function of order (or height).

2. Precisely, power tower functions denote interactive exponentiation (tetration).

$$\mu_{\tilde{A}}(x) = \frac{(1-x^{x})}{k_1}, \qquad (5)$$

where all x range between 0 and 1, k_1 is a constant given by $\left(1-e^{(-e^{-1})}\right) \approx 0.3078$, $\mu_{\tilde{A}}(0) = \mu_{\tilde{A}}(1) = 0$, and $\mu_{\tilde{A}}(x) = 1$ for $x = e^{-1} \approx 0.3679$.

The partial membership function for any given country's productivity attribute distance to the ideal solution, $\mu_{\tilde{A}}(u)$, can be defined by solving $\mu_{\tilde{A}}(x) = u$ in terms of x, thus expressing x as a function of u. This way, J_x, a wavy slice representation of $\mu_{\tilde{A}}(u)$ is obtained as an embedded set within the type 1 fuzzy set *X*. This wavy slice is also known as the Mendel-John representation theorem. This theorem enables the creation of a 3D membership function, $\mu_{\tilde{\tilde{A}}}(x,u)$, based on a bidimensional space (*x*, *u*) defined by the union of two membership functions of type 1, $\mu_{\tilde{\tilde{A}}}(x)\text{and}\mu_{\tilde{\tilde{A}}}(u)$, both ranging over the interval between 0 and 1.

$$\mu_{\tilde{\tilde{A}}}(u) = \frac{\beta}{eW(\beta/e)}, \tag{6}$$

where β is a function of *u* given by $\beta = [\ln(u(1 - e^{-e}) + e^{-e})e - 1]$, W is the Lambert function, all *u* range between 0 and 1, and $\mu_{\tilde{\tilde{A}}}(u) = 1$ for $u = 0$. W is the transcendent function that satisfies $W(z)e^{W(z)} = z$ and which can be numerically evaluated using MAPLE functions or R libraries (codes are available to readers upon request). W is also related to the integrated exponentiation problem, or power tower functions, namely $f(z)$, such that $f(z) = -W(-\ln(z))/\ln(z)$, converging for real values of z in the range from $e^{-e} \approx 0.066$ to $e^{1/e} \approx 1.44$ (Lynch, 2017).

Fig. 1 depicts the type 1 representations of partial membership functions $\mu_{\tilde{\tilde{A}}}(x)\text{and}\mu_{\tilde{\tilde{A}}}(u)$ and the type 2 representation of $\mu_{\tilde{\tilde{A}}}(x,u)$ – recall that $\mu_{\tilde{\tilde{A}}}(x,u) = \mu_{\tilde{\tilde{A}}}(x) \cap \mu_{\tilde{\tilde{A}}}(u)$. Fig. 2 depicts the *x* and *u* axis-slices representing the hesitant membership degrees for $\tilde{h}_{\tilde{E}}(x,u)$. One can note from Fig. 1 that higher membership grades are assigned to smaller distances towards the ideal solutions $(\mu_{\tilde{\tilde{A}}}(u))$, although lower membership grades are assigned to the extreme measurement of normalized attributes $(\mu_{\tilde{\tilde{A}}}(x))$. This being the case, this T2FS – for assessing each country's productivity performance – counterbalances the vagueness inherent in the extreme measurements of normalized attributes with their distances to ideal solutions. These characteristics can also be inferred from the hesitant memberships depicted in Fig. 2. Shorter distances to ideal solutions (lower values of *u*) imply higher membership grades for normalized attributes (especially for those mid-valued on the *x*-axis).

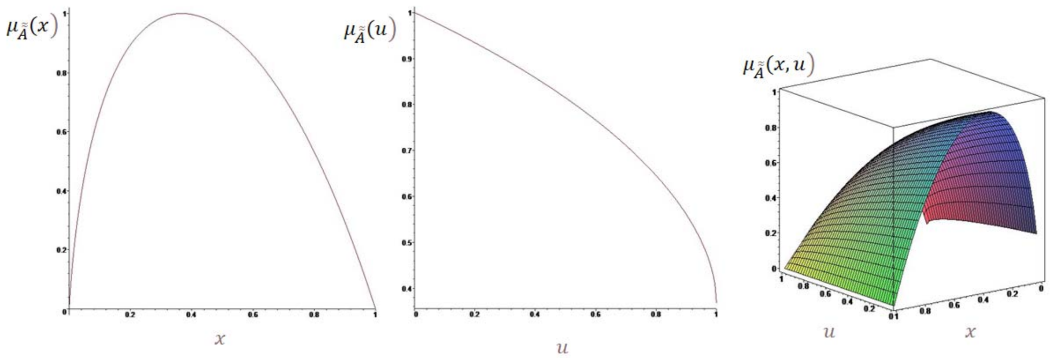

Fig. 2. Membership functions.

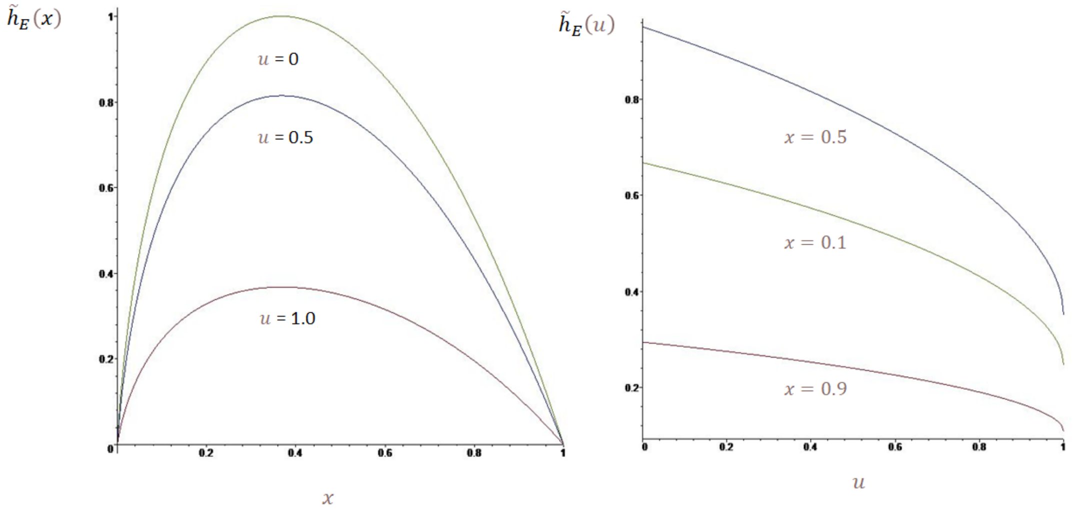

Fig. 3. Hesitant fuzzy sets.

This intrinsic counterbalance or trade-off between the partial membership grades assigned to each (x, u) pair yields a measurement bias that should be accounted for when computing productivity performance for country sectors based on their normalized productivity attributes, their distances to ideal solutions and their consequent weights.

<u>Positive bias</u>

Consider $\mathbf{x}^{+}$,$\mathbf{u}^{+}$, and $\mathbf{h}^{+}$, three nx(m+s) positive bias matrices for normalized attributes, distances to ideal solutions, and type-2 membership functions. The positive bias integration departs from **Definition 3**. As regards normalized attributes and distances to ideal solutions, these are computed as the average positive deviation weighted by the respective membership degrees for each element *x* and *u* with respect to the subset *E*, as defined by the integral limits. As regards type-2 membership functions, it is the average difference in membership degree for each pair (*x*, *u*) with respect to the subset *E*.

$$\mathbf{x}^{+} = \int_{x=\mathbf{x}}^{1} \frac{\mu_{\tilde{\tilde{A}}}(x)(x-\mathbf{x})}{(1-\mathbf{x})} dx \tag{7}$$

$$\mathbf{u}^{+} = \int_{u=\mathbf{u}}^{1} \frac{\mu_{\tilde{\tilde{A}}}(u)(u-\mathbf{u})}{(1-\mathbf{u})} du \tag{8}$$

$$\mathbf{h}^{+} = \iint_{x=\mathbf{x};u=\mathbf{u}}^{1;1} \frac{\mu_{\tilde{\tilde{A}}}(x,u)}{(1-\mathbf{x})(1-\mathbf{u})} du dx - \mu_{\tilde{\tilde{A}}}(\mathbf{x},\mathbf{u}) \tag{9}$$

Negative bias

The negative bias computation is defined analogously to positive bias.

$$\mathbf{x}^{-} = \int_{0}^{x=\mathbf{x}} \frac{\mu_{\tilde{\tilde{A}}}(x)(\mathbf{x}-x)}{(\mathbf{x})} dx \tag{10}$$

$$\mathbf{u}^{-} = \int_{0}^{u=\mathbf{u}} \frac{\mu_{\tilde{\tilde{A}}}(u)(\mathbf{u}-u)}{(\mathbf{u})} du \tag{11}$$

$$\mathbf{h}^{-} = \iint_{0;0}^{x=\mathbf{x};u=\mathbf{u}} \frac{\mu_{\tilde{\tilde{A}}}(x,u)}{(\mathbf{x})(\mathbf{u})} du dx - \mu_{\tilde{\tilde{A}}}(\mathbf{x},\mathbf{u}) \tag{12}$$

Unbiased or bias-corrected T2FS

While the computation of unbiased normalized attributes, distance to ideal solutions, and membership functions are straightforward, readers should note that two sets of resulting matrices are produced in accordance with the bias removal: positive (denoted by extension_**pos**) and negative (denoted by extension_**neg**).

$$\mathbf{x\_pos} = \mathbf{x} + \mathbf{x}^{+} \tag{13}$$

$$\mathbf{x\_neg} = \mathbf{x} - \mathbf{x}^{-} \tag{14}$$

$$\mathbf{u\_pos} = \mathbf{u} + \mathbf{u}^{+} \tag{15}$$

$$\mathbf{u\_neg} = \mathbf{u} - \mathbf{u}^{-} \tag{16}$$

$$\mathbf{h\_pos} = \mu_{\tilde{\tilde{A}}}(\mathbf{x},\mathbf{u}) + \mathbf{h}^{+} \tag{17}$$

$$\mathbf{h\_neg} = \mu_{\tilde{\tilde{A}}}(\mathbf{x},\mathbf{u}) + \mathbf{h}^{-} \tag{18}$$

Normalized attribute weights

Positive (w_k^+) and negative (w_k^-) weights for each normalized attribute$k$ $(k = i \cup o)$ can be defined based on the following equations:

$$w_k^+ = \frac{\sum_{d=1}^{n} u\_pos_{d,k} * h\_pos_{d,k}}{\sum_{k=1}^{m+s} \sum_{d=1}^{n} u\_pos_{d,k} * h\_pos_{d,k}} \text{for all } k \tag{19}$$

$$w_k^- = \frac{\sum_{d=1}^{n} u\_neg_{d,k} * h\_neg_{d,k}}{\sum_{k=1}^{m+s} \sum_{d=1}^{n} u\_neg_{d,k} * h\_heg_{d,k}} \text{for all } k \tag{20}$$

In what follows, normalized attribute weights are given by the pounded unbiased distances to ideal solutions in terms of the unbiased respective membership degrees.

Performance scores for each country *d*

Positive (s_d^+) and negative (s_d^-) productivity performance scores for each country d were computed based on the following equations. Readers should note that positive and negative cross-performance scores for each country *d* may also be computed (cs_d^+ and cs_d^-, respectively).

$$s_d^+ = \sum_{k=1}^{m+s} x\_pos_{d,k} * w_k^+ \text{for all } d \tag{21}$$

$$s_d^- = \sum_{k=1}^{m+s} x\_neg_{d,k} * w_k^- \text{for all } d \tag{22}$$

$$cs_d^+ = \sum_{k=1}^{m+s} x\_pos_{d,k} * w_k^- \text{for all } d \tag{23}$$

$$cs_d^- = \sum_{k=1}^{m+s} x\_neg_{d,k} * w_k^+ \text{for all } d \tag{24}$$

3.5. Stochastic Structural Relationship Programming (SSRP) Model

This paper proposes a novel SSRP model based on neural network architecture to unveil the existing endogenous relationships among productivity performance relative to ideal solutions and countries´ normalized productivity attributes, while identifying the relevant structural cause-effect relationships that may exist. Specifically, neural networks are employed to unveil endogeneity among a given country's productivity performance and its normalized productivity attributes, whether positive or negative, in terms of the residuals produced by the following models.

Model 1: Performance ~ f(GDP; Human K; Labor Prod.; TFP; Age K; KI)
Model 2: GDP ~ f(Performance; Human K; Labor Prod.; TFP; Age K; KI)
Model 3: Human K ~ f(GDP; Performance; Labor Prod.; TFP; Age K; KI)
Model 4: Labor Prod ~ f(GDP; Human K; Performance.; TFP; Age K; KI)

Model 5: TFP ~ f(GDP; Human K; Labor Prod.; Performance; Age K; KI)
Model 6: Age K ~ f(GDP; Human K; Labor Prod.; TFP; Performance; KI)
Model 7: KI ~ f(GDP; Human K; Labor Prod.; TFP; Age K; Performance)

These residuals are used subsequently to generate a full set of conditional residual distributions between the respective dependent variable pairs identified in each model. These conditional residual distributions allow exploring the directional relationships that may exist among variables. The novel SSRP model is structured in two consecutive steps that allow unveiling endogeneity while identifying relevant cause-effect relationships among the reminder variables. These steps are described next.

Step 1: Minimal Endogenous Relationship Variance

The relative importance of models (1)-(7) in explaining the feedback process between normalized attributes and long-term productivity performance levels in advanced countries, besides the endogenous nature of these variables, were explored by the variances for each model and the covariances between models, respectively. Variances and covariances of the residuals (R_i) of these seven models were simultaneously minimized by a non-linear stochastic optimization problem, as presented in Model (25), where w_i stands for the weights – which range from 0 to 1 – assigned, respectively, to the residual vectors of each one of the seven models described previously. The values of *w* were optimized so that the variance (*Var*) and covariance (*Covar*) of the pooled residuals were minimal. Model (25) was solved by means of differential evolution (DE). DE is a research stream of genetic algorithms, emulating also natural selection and evolution.

$$\min\left[Var\left(\sum_{i=1}^{7} w_i * R_i\right)+\left(2*\sum_{i,j=1}^{7} Covar\,(w_i * w_j * R_i * R_j), i \neq j, j < i\right)\right]$$

$$subject\ to$$

$$\textstyle\sum_{i=1}^{7} w_i = 1 \qquad (25)$$

$$0 \leq w_i \leq 1 \quad \forall i$$

The residuals of the MLP models were bootstrapped 100 times, allowing the collection of a distributional profile of *w* for the most accurate prediction of productivity performance scores and contextual variables.

Step 2: Maximal Information Entropy for Directional Weighted Residuals

The principle of maximum entropy states that the probability distribution which best represents the current state of knowledge is the one with the largest entropy where there isprecisely testable information. Subsequently, in the second step, a full combinatorial set of conditional distributions of residuals ($\mathrm{CR_k}$) was computed. The previous 100 bootstrapped replications of each model's unconditional individual residuals ($\mathrm{R_i}$) served as cornerstones for this computation, where $\mathrm{CR_k} \sim \mathrm{f(R_i/R_j)}$ for all i and j, $\mathrm{i \neq j}$, and $\mathrm{K = i * j - i = 7 * 7 - 7 = 42}$. Similarly, DE was employed to solve the following non-linear integer-programing model in order to diagnose whether conditional distributions for each residual pair presented significant differences in terms of directions. For instance, the weights assigned to $\mathrm{f(R_i/R_j)}$ could yield higher entropy than those assigned to $\mathrm{f(R_j/R_i)}$ levels, compared with the unconditional residuals analyzed in Step 1. This non-linear integer programming problem is depicted in model (26).

$$\max\left[\left(\sum_i \sum_j H\left(f\left(\frac{R_i}{R_j}\right) * w_i * w_j\right)\right) OR \left(\sum_i \sum_j H\left(g\left(R_i, R_j\right) * w_i * w_j\right)\right), i \neq j\right]$$

$subject\ to$

$$\sum_{i=1}^{7} w_i = 1 \ (26)$$

$$0 \leq w_i \leq 1, \qquad \forall i$$

Where:

H(.) denotes the information entropy function,

$g\left(R_i, R_j\right)$ denotes the unconditional marginals of the residuals from models (1) – (7)

, $\forall i, j,\ i \neq j$,

$f(R_i/R_j)$ denotes the conditional distribution of the residuals from models (1) - (7),

$\forall i, j,\ i \neq j$.

This non-linear integer programming model returned the structural relationship among dependent variables defined in models (1)-(7) for which information entropy was maximal. This assured the uniqueness and consistency of the probabilistic weight profile computed in Step 1, for which the overall residual variance was also minimal. Hence, the weights computed in Step 1 were used as starting values for the Step 2 optimization. Again, differential evolution was employed to find optimal solutions in terms of maximal entropy for each *ij* pair. This model returned whether for a given pair, *ij*, the relationship was endogenous or whether *i* caused *j* (or the other way around). Table 2 presents the pseudo code used for computing the f(.) and *g*(.) estimates used in the Step 2 optimization model.

Table 2. Pseudo code used for computing *f*(.) and *g*(.) estimates used in Step 2

```
1 Fit the statistical distributions for each model's residuals
2 Calculate the correlation matrix for all models' residuals
3 Fit the best distribution of residuals for each model (cf. Table 3)
4 Generate multivariate Copulas, preserving the correlation and distribution structures for all models' residuals
5 Select percentile thresholds to stress directional relationships under extreme Copula distributions
a for p in percentiles do
b   for i in 1 to models do
c     for j in i to models do
d       Evaluate the conditional distribution of Copula ij
e       Evaluate the unconditional marginal distributions of copula ij
f       Solve model (26) using Differential Evolution Approach
```

Table 3. Distributional fit for residuals obtained in models (1)-(7).

Model	Distribution	Parameters	
		meanlog	sdlog
SD+	lnorm	-2.03	0.04
		shape1	shape2
GDP per capita	beta	21.62	15.82
		Mean	sd
Human K per capita	norm	-0.36	0.19
		shape1	shape2
Labor Productivity	beta	19.68	10.50
		shape1	shape2
TFP	beta	15.46	12.98
		shape1	shape2
KI	beta	24.93	25.25
		shape1	shape2
Age K	beta	17.27	6.49

4. Analysis and Discussion of Results

4.1 Characteristics of performance attributes and comparison with TOPSIS

Correlograms for the normalized attributes (x), their respective distances to ideal solutions (u), and their resulting hesitant membership function values (h) are depicted in Fig. 3 (top-left, top-right, and bottom). With the exception of Age K, all other productivity attributes tend to be strongly correlated with one another. However, as regards the distances to ideal solutions, Age K presents negative, even if weak, correlations with the remainder of the productivity attributes. The age of the capital stock being the only negative attribute, the implication is that the higher it is, the further from their ideal levels are the other productivity attributes. Yet, an analogous interpretation can be considered for Human K as regards membership functions. Lower productivity attributes such as GDPpc, LP, and TFP can be compensated by higher levels of human capital pertaining to a higher membership grade of long-term productivity performance in advanced countries. Hence, Age K and Human K appear to be the two cornerstones for long-term productivity performance in advanced countries. Age K is a necessary condition since a newer stock of capital raises all other productivity attributes towards their ideal solutions. Human K is a sufficient condition since it is capable of compensating deficiencies in other productivity attributes. These issues are elaborated in the reminder of the text. Although considering the inputs and outputs to derive total factor productivity as the performance indicator, which is different from the method adopted in the current study, Bergeaud et al. (2017) found similar results which indicate that a significant contribution is made by Age K in terms of productivity performance. Similarly, total factor productivity was also measured by Mannasoo et al. (2018) in assessing the performance of 31 European countries over the period 2000-2013. Their results indicate that human capital does not have a consistent impact on productivity growth. The difference between their study and ours can be explained by the different samples used; also, we used a more advanced technique.

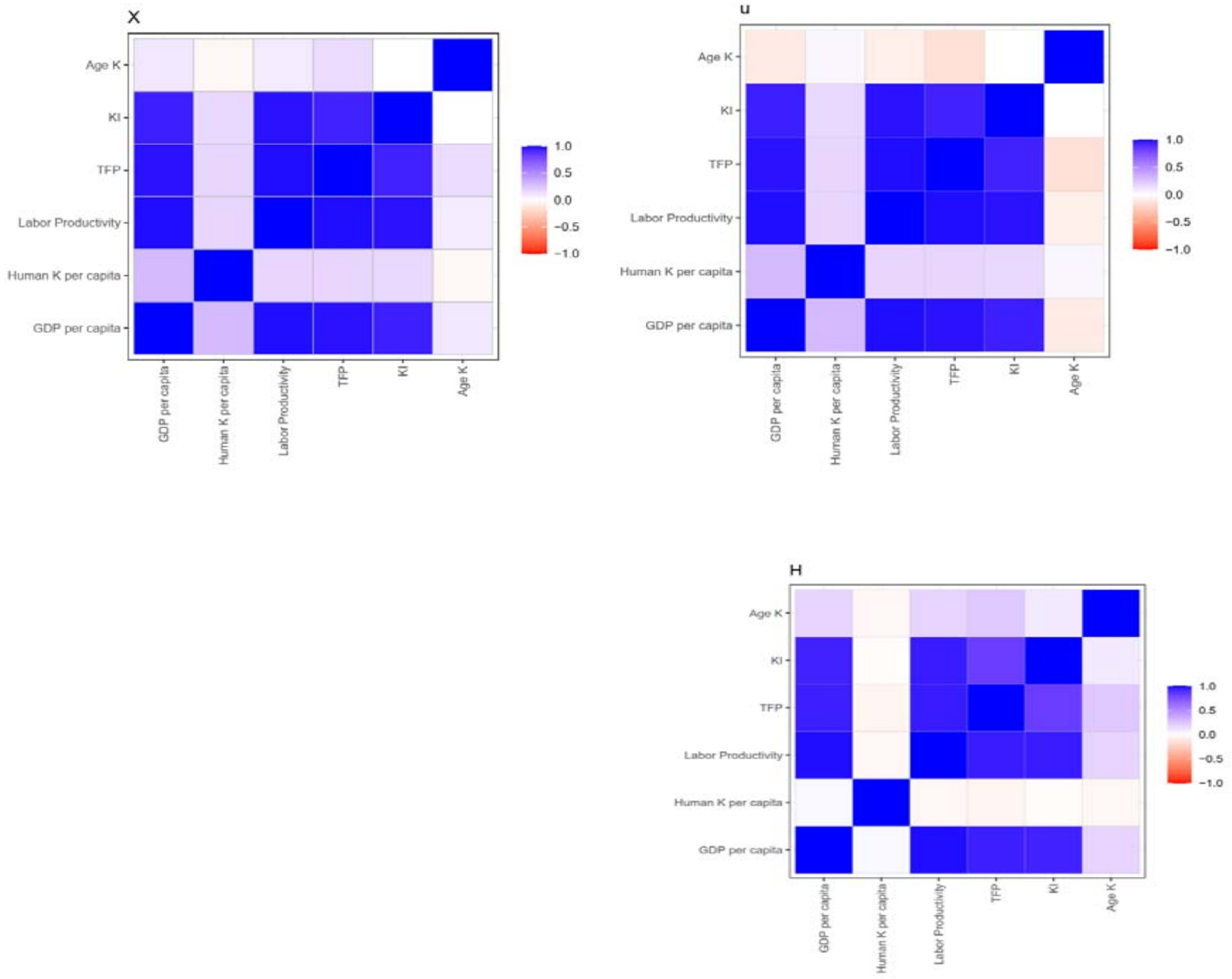

Fig. 4. Correlograms for the performance attributes. (Top left) X represents the partial membership function for the normalized attributes. (Top right) shows the partial membership function of distance to ideal solution. (Bottom right) H represents the hesitant fuzzy set considering both normalized attribute and distance to ideal solution.

Fig. 4 reports the normalized attribute weights obtained after removing the positive and negative biases inherent in the measurements comprising **x**, **u**, and **h**. Readers should recall that these weights were derived as per the pounded averages of the unbiased distances to ideal solutions in terms of the respective unbiased membership degrees. Readers should also note that the normalized attribute weights obtained after removing positive bias are more balanced than those obtained after removing negative bias. This suggests that the key to long-term productivity performance in advanced countries is a balance among capital intensity, labor productivity, total factor productivity, and GDP per capita – productivity attributes that presented largely the same positive weights. Less important for better long-term productivity performance are Human K and Age K, when all other attributes are balanced. On the other hand, lower levels of longer-term productivity performance indicate a conjunction of attributes that reinforce lower levels of human capital, higher capital stock age, and lower TFP. This lower productivity performance seems to have been created by an unfortunate combination of the two cornerstone productivity attributes identified in the correlation analysis: Human K and Age K.

Figure 4 shows the result of our analysis of better long-term and worse long-term productivity performance. This not only from the perspective of potential determinants – as derived with our advanced technique, in analyzing the different scenarios related to productivity performance. We were also able to identify the coordinating role played by various potential determinants, not only filling in the gap in the literature, but also enabling more accurate and concrete policies for governments regarding performance regulation.

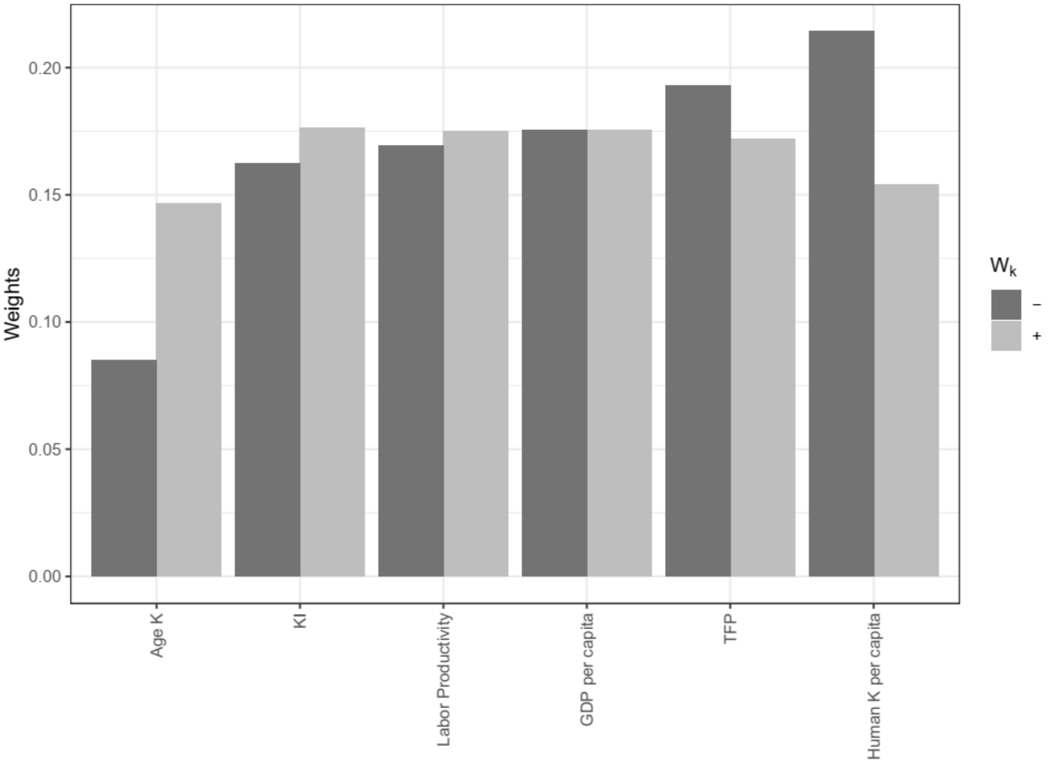

Fig. 5. Positive and negative weights for each normalized attribute.

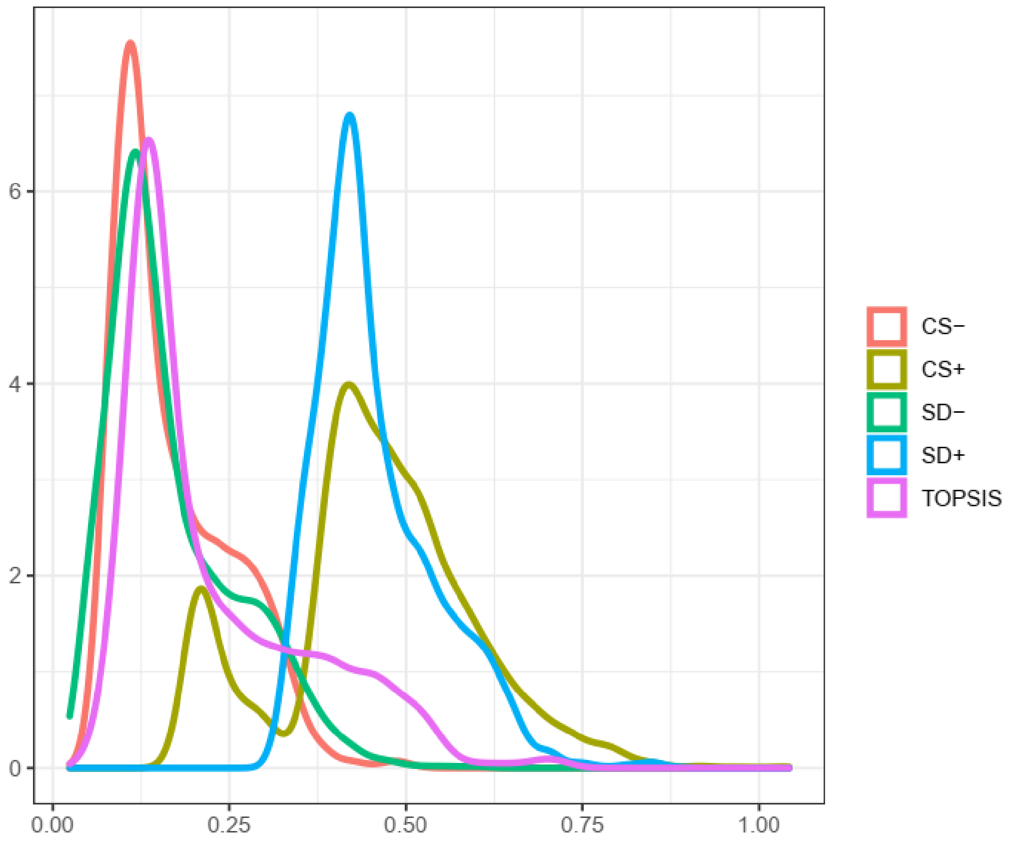

Fig. 6. Density plot comparison for UP-IS performance under alternative weighting schemes in light of basic TOPSIS results.

The density plots for the four alternative UP-IS performance scores obtained using weights from eqs. (21)-(24) are given in Fig. 5, together with the TOPSIS base-case comparison for equal weights. As can be seen, the UP-IS performance scores presented better discriminatory power – a widespread shape, although still in-between the 0 and 1 boundaries – in comparison with the TOPSIS base case. As expected, the performance and cross-performance scores computed with the weight set obtained after having removed the positive bias presented median values higher than the TOPSIS base case. The contrary was verified for the performance and cross-performance UP-IS scores computed with the weight set that was determined after having removed the negative bias.

4.2 Performance distributions, performance scores

Table 4 presents the descriptive statistics, together with the information entropy computation for each of the distributions depicted in Fig. 5. According to the maximal entropy principle, whenever deciding in favor of a given distribution to the detriment of the other without any prior information, maximal entropy distributions should be preferable for the sake of robustness, as long as they are not impacted, in subsequent analysis, by any modelling bias that has escaped the decision makers.

Table 4. Descriptive statistics for alternative performance distributions.

Variables	**Mean**	**SD**	**CV**	**Kurtosis**	**Skewness**	**IE**
TOPSIS	0.228	0.130	0.569	0.555	1.160	0.493
SD+	0.458	0.086	0.187	1.315	1.073	0.494
SD-	0.170	0.092	0.542	0.588	1.002	0.497
CS+	0.457	0.136	0.298	0.260	0.045	0.496
CS-	0.170	0.082	0.480	0.196	0.916	0.502

The best and worst rankings of the countries with respect to their performance scores are presented in Table 5. The results show that most countries had their best productivity performance in 2018, while their worst performance for most countries occurred from around the late 1800s to the mid-1900s. Norway was the best-performing country, with a score of 0.854, while Portugal was the least, with a score of 0.347. We argue that our paper contributes not only from a methodological perspective, but from others too: a special feature is that it covers the longest period of investigation for the 23 countries and the Euro Area; it also used a big sample in terms of the number of countries included in the study, which enabled an opportunity to provide a historical review of productivity performance. Most literature studies have focused on a smaller sample in terms of the number of countries studied; in particular, they focused on a significantly shorter period (see Levenko et al., 2019, as one of the recent studies on this topic).

Table 5. Positive productivity performance scores by country

S/N	Country	Best Rank Order (Year)	Worst Rank Order (Year)	Score for Best Rank	Score for Worst Rank
1	Norway	2018	1905	0.854	0.397
2	Ireland	2018	1921	0.751	0.371
3	Switzerland	2018	1891	0.722	0.424
4	United States	2016	1894	0.676	0.377
5	Australia	2018	1897	0.652	0.389
6	Netherlands	2018	1945	0.644	0.379
7	Denmark	2017	1891	0.641	0.394
8	Belgium	2016	1918	0.634	0.366
9	Germany	2016	1947	0.633	0.385
10	Sweden	2018	1921	0.616	0.381
11	Canada	2017	1890	0.612	0.391
12	France	2018	1944	0.611	0.377
13	Austria	2017	1919	0.609	0.366
14	Finland	2018	1918	0.601	0.361
15	Euro Area	2018	1946	0.596	0.383
16	New Zealand	2018	1890	0.590	0.397
17	Japan	2017	1949	0.585	0.386
18	United Kingdom	2018	1893	0.575	0.388
19	Italy	2007	1945	0.575	0.372
20	Spain	2017	1896	0.557	0.364
21	Chile	2018	1944	0.510	0.366
22	Portugal	2018	1919	0.499	0.347
23	Greece	2018	1945	0.494	0.348
24	Mexico	2016	1932	0.463	0.355

Endogeneity performance results using the novel SSRP model are henceforth discussed with reference to the UP-IS performance set, computed using a positive weight set.

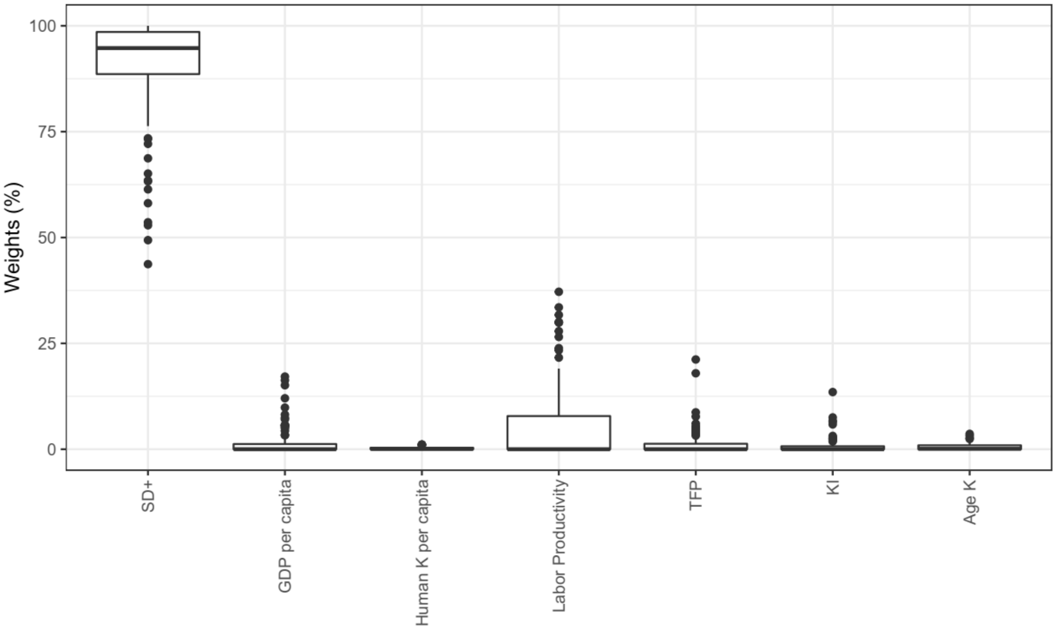

Fig. 7. Relative importance of models (1)-(7).

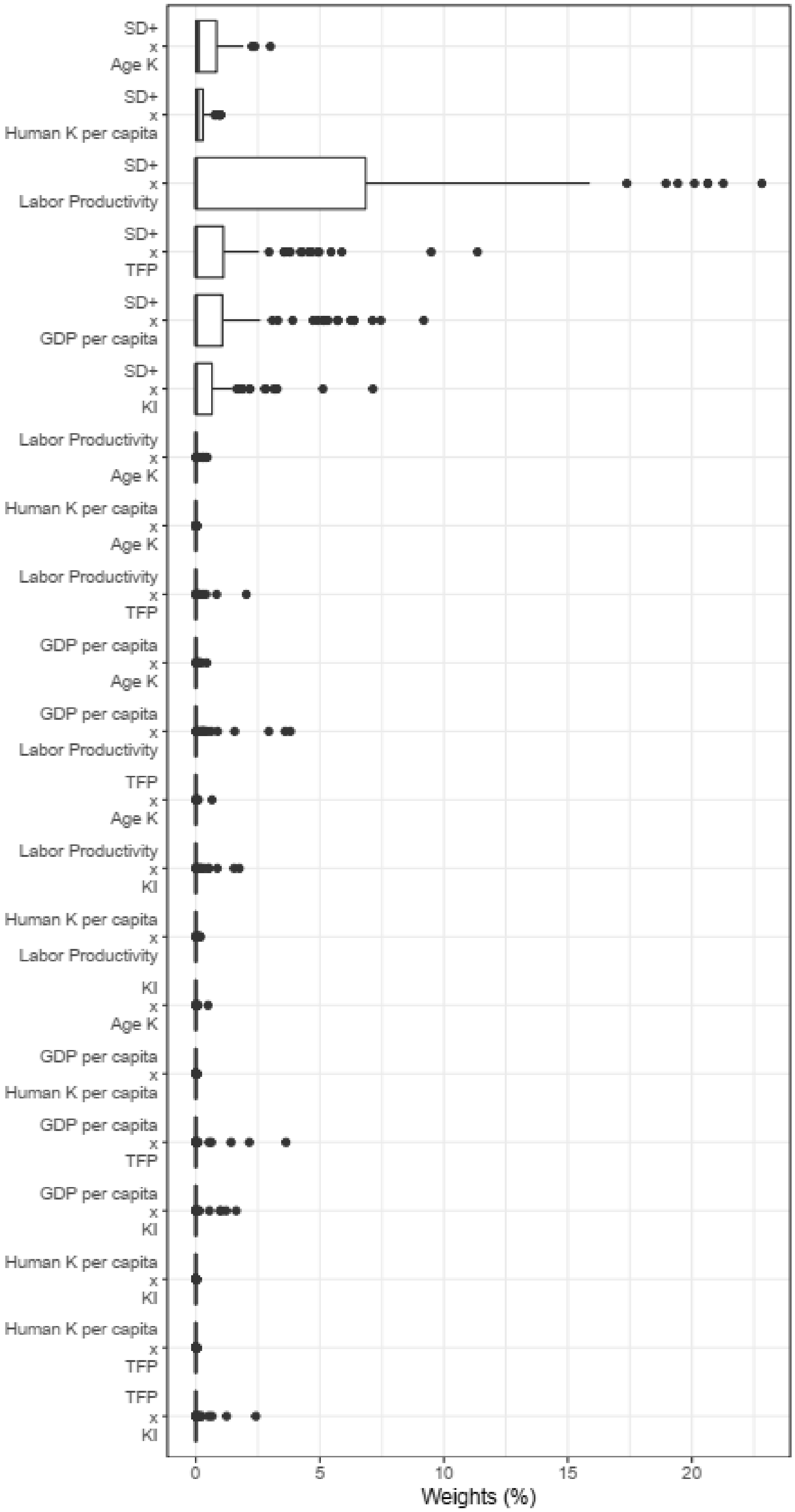

Fig 8. Endogeneity weights for pairs of models (1)-(7): major combinations.

4.3 **Association among performance and key attributes, and endogeneity**

The relative importance of models (1)-(7) for achieving minimal residual variance is heavily concentrated in terms of resulting productivity performance (cf. Fig. 6), accounting for almost the total median weights. Yet, long-term productivity performance appears to be associated with feedback or cause-effect processes that may occur with the other productivity attributes, particularly with labor productivity. the results for the relative importance of the major interaction pairs in explaining overall residual variance are presented in Fig. 7. These results also confirm the aforementioned issues further, where productivity performance also appeared to be associated with the six productivity attributes detailed in Table 1. As reviewed in section 2, there are quite a few studies investigating the issue of labor productivity; however, it seems that little effort has yet been made to look at the association of productivity performance at a country level and labor productivity. We fill in the gap from this perspective; also, our results will be helpful in generating policies for improving country-level productivity performance by aiding managing labor in the productivity process.

The joint feedback effect of all model pairs on overall residual variance is around 20% (cf. Fig.7), pretty low compared with the maximal possible endogeneity effect, achievable when all seven models would account individually for 14.29% of the total residual variation. Under these equal weight cases, the maximal joint effect totals 85.76% = 2*42.88% = 2*14.29%*14.29%*21, where 21 is the number of combinations taken two by two as obtained from the joint variations of models (1)-(7). This first screening of the weights computed in Step 1 suggests that endogeneity appears not to be as relevant as directional cause-effect for apprehending the relationships among productivity attributes and productivity performance. The comparison between endogeneity and the cause-effect relationship is also made in the literature, but in a different context. An attempt has been made by Antunes et al. (2021) at investigating the endogeneity and cause-effect relationships in the Chinese banking performance context. Similar results were obtained relative to the low endogeneity level.

Table 6. Information entropy of conditional distribution of [ROW] given that [COLUMN] for 0.975 percentile.

Models	SD+	GDP per capita	Human K per capita	Labor Productivity	TFP	KI	Age K
SD+		0.669876	0.668941	0.669588	0.669308	0.669085	0.667061
GDP per capita	0.652034		0.655528	0.657469	0.653186	0.652839	0.653042
Human K per capita	0.670181	0.671262		0.670389	0.670138	0.669353	0.668754
Labor Productivity	0.646793	0.655154	0.650398		0.65079	0.65472	0.648201
TFP	0.660281	0.660774	0.660747	0.661244		0.659537	0.659065
KI	0.640757	0.636911	0.638873	0.639065	0.640965		0.637307
Age K	0.671979	0.672483	0.672182	0.671076	0.672242	0.671298	

Table 7. Information entropy of unconditional distributions for 0.975 percentile. (*)

Models	SD+	GDP per capita	Human K per capita	Labor Productivity	TFP	KI	Age K
SD+		0.510284	0.496786	0.534606	0.498858	0.504953	0.550925
GDP per capita	0.510284		0.547034	0.575171	0.532416	0.539707	0.558466
Human K per capita	0.496786	0.547034		0.550645	0.573801	0.539375	0.55573
Labor Productivity	0.534606	0.575171	0.550645		0.59235	0.573361	0.559225
TFP	0.498858	0.532416	0.573801	0.59235		0.573366	0.582221
KI	0.504953	0.539707	0.539375	0.573361	0.573366		0.492459
Age K	0.550925	0.558466	0.55573	0.559225	0.582221	0.492459	

(*) Unlike the conditional distribution matrix, the unconditional distribution is a symmetric one.

4.4. Cause-Effect Relationships

Tables 6 and 7 report on the conditional and unconditional distribution results used in model (26), obtained for the 0.975 percentile after bootstrapping the fitted individual residual distributions as displayed in Table 3. Figs. 8 and 9 present the directional relationships among variables, and their respective signs, based on the optimal weights computed in Step 2 for the 0.975 percentile threshold. Readers should observe that the median expected weight for each conditional distribution would be 0.0238 (1/42), considering a balanced bi-directional relationship among variables.

Considering the percentile threshold of 0.975, where relationships are stressed to a 2.5% false positive rate, long-term productivity performance in advanced countries is impacted by both Human K (positively) and Age K (negatively). It is interesting to note that Age K and Human K are endogenously related, in the sense that newer capital stock and a highly skilled labor force present a feedback process relevant for achieving higher productivity performance. Age K and Human K also exert cause-effect relationships with other productivity attributes, acting either as countervailing forces with respect to labor productivity[2], or in the same direction with respect to KI and GDPpc (negatively) and TFP (positively). The positive impact of Human K on labor productivity can be explained from the perspective that either more highly educated workers or companies providing in-service training to improve the level of human capital will have a positive impact on labor productivity. This is evidenced in the study of Rukumnuaykit and Pholphirul (2016) and Kaiser and Kuhn (2016). As far as we are concerned, the literature has not yet looked at the effect of the age of equipment capital stock on labor productivity, although an attempt has been made to investigate the relationship between workers' age and labor productivity (Iparraguirre, 2019). Readers should note that Human K can lead to KI destruction due to the movement towards economies based on services and less dependent on heavy industries. This being case, however, GDPpc is also put into jeopardy, as long as heavy industries are more value-added-intensive and scalable, even compared with high-tech sophisticated services such as IT. On the other hand, the interpretation for Age K is more straightforward, given that older capital stocks may even be physically depreciated or obsolete, and unable to catch up with newer technological advances in the production of goods and services.

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[2] As expected, higher levels of Human K impact labor productivity positively, while higher levels of Age K impact it negatively, thus suggesting capital obsolescence may limit productivity gains due to better schooling, for instance.

However, productivity performance itself impacts the other productivity attributes: KI and TFP (positively) and LP and GDPpc (negatively). To interpret the signs of these effects, productivity performance scores should be apprehended as the joint effect between Human K and Age K. The positive impacts on KI and TFP suggest that increased productivity performance simultaneously opens up room for capital renovation and the adoption of newer production technologies (higher KI). Although there is no evidence in the literature showing this relationship, at a firm level, the evidence shows that more productive firms have the competitive advantage of introducing new and improved products (Wadho and Chaudhry, 2018). This indirectly indicates that a higher level of productivity enhances the level of research and innovation; this can be reflected and achieved by renovating capital and adopting new technologies, but it also contributes to a better balance between capital and labor production factors (higher TFP). This productivity performance spillover is achieved when Human K is high, and Age K is low. On the contrary, a negative influence of higher productivity performance on LP and GDPpc may result from third-party contractors and, more recently, from the "uberization" phenomenon in different economic sectors. Wealth generation per worker under these circumstances has dropped dramatically in the last decades, also contributing to a loss of impetus in GDP pc increases. These results suggest a dichotomous development path that should be followed by advanced countries: high-tech versus "uberized" economies. On a more general level, we could explain the negative influence of higher productivity on LP in the context of firms. A higher level of productivity gives a firm a competitive advantage, through which higher market power can be accumulated at the expense of less productive firms. The resulting increase in the level of monopoly power will reduce the managerial working effort, one aspect of which entails reducing the incentive to monitor employees; this leads to a further reduction in LP. Compared with a scenario of higher LP, lower LP will have a resulting negative impact on GDPpc. The negative influence of monopoly power on working effort is documented in the banking industry (Tan and Anchor, 2017). We argue that this is also applicable to the country level: the countries with higher levels of productivity enjoy a competitive advantage compared with the countries with lower productivity; this induces management from different industries to reduce the managerial and monitoring effort. As a result, LP as well as GDP per capita will suffer.

It is interesting to note that KI is the ultimate consequence of the countervailing forces that co-exist in these dichotomous development paths, which commence with Human K and Age K. All remaining variables – productivity performance, TFP, LP, and GDPpc – either act as causes or consequences. It is also worth noting that productivity performance scores computed by UP-IS and TFP, LP, and GDPpc attributes consist of different expressions or metrics for the amount of wealth generated, given the number of resources (either capital or labor) used. Taking a closer look among them, another endogenous relationship is found between labor productivity and GDPpc, and its interpretation is straightforward: per-capita welfare is built upon higher levels of labor productivity, and per-capita welfare engenders the means of becoming more productive. The literature studies have not yet investigated the interrelationships between GDPpc and labor productivity; however, an effort has been made to use GDP per worker as the standard measure of labor productivity (Filippetti and Peyrache, 2015). From this evidence, we can, to some extent, confirm our findings regarding the endogenous relationship between LP and GDPpc. As regards their ordinal rank, and as expected, UP-IS productivity performance is the most important, impacting LP, GDPpc, and TFP. TFP is the second in the ranking, impacting GDPpc and LP, while being impacted by productivity performance. LP and GDPpc are third in the ranking, being impacted by productivity performance and TFP. This analysis embeds a rationale for attribute hierarchy:

- **Human K and Age K** are the descriptors of labor and capital quality.
- **UP-IS productivity performance** is the synthetic measure derived by both resource quality descriptors, reflecting the countervailing forces of a dichotomous development path based on high-tech industries versus third-party contractors.
- **TFP** is the measure that captures the resource allocation between capital and labor, which should to some extent reflect this dichotomous path.
- **GDPpc and LP** are the measures of wealth generation at the individual level, which are "supposed" to have opportunities on both sides of this dichotomous path.
- **KI** is the ultimate result of the developing process, the importance of which has dramatically changed due to transformations in society with respect to IT, labor relations, environmental pressures, the emergence of China as a novel superpower, and the like (Schweinberger, 2014).

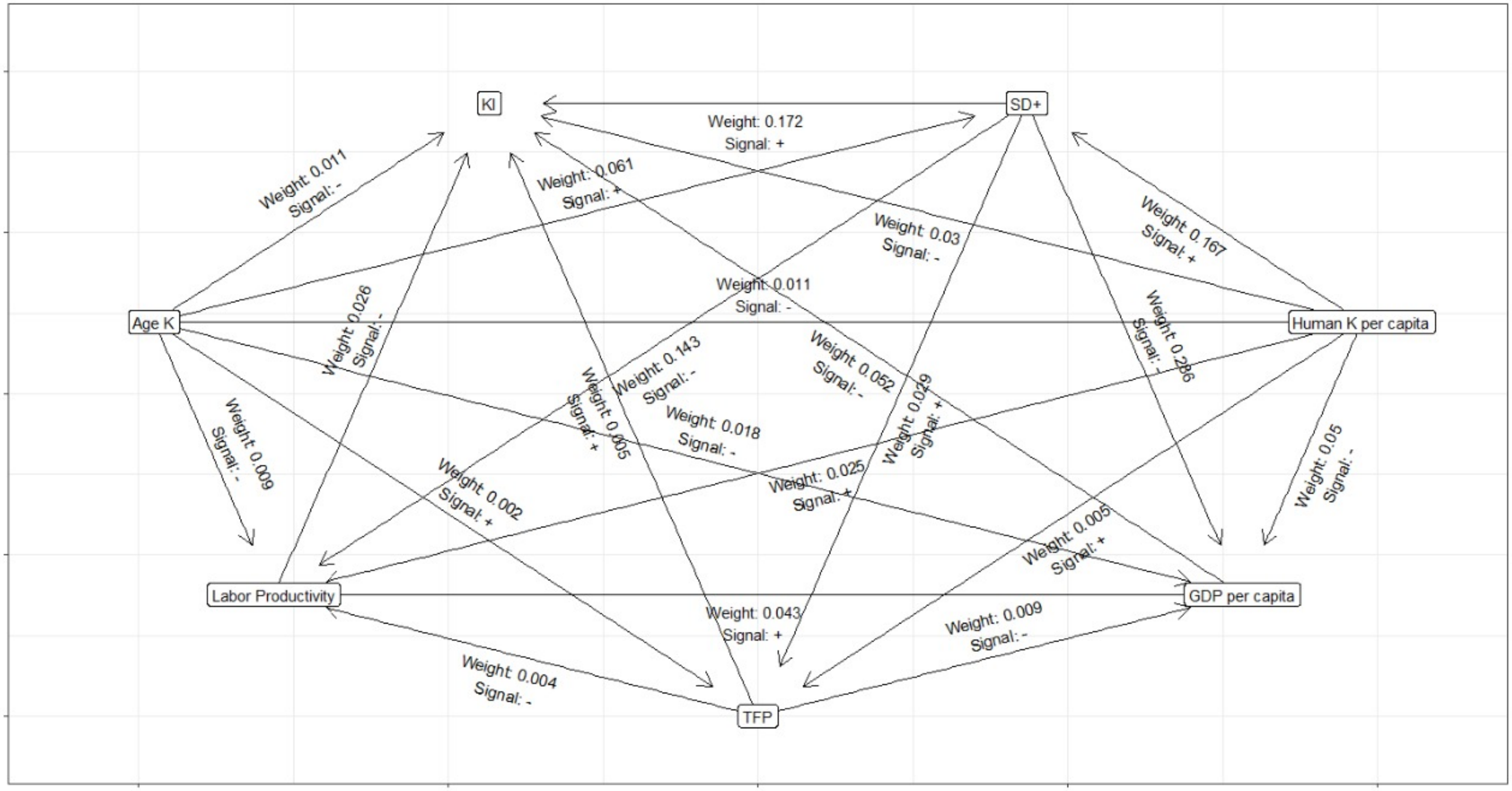

Fig. 9 Cause and effect framework for selected attributes at 0.975 percentile (weights are derived from Step 2; signs are derived from Olden´s (2002) sensitivity analyses).

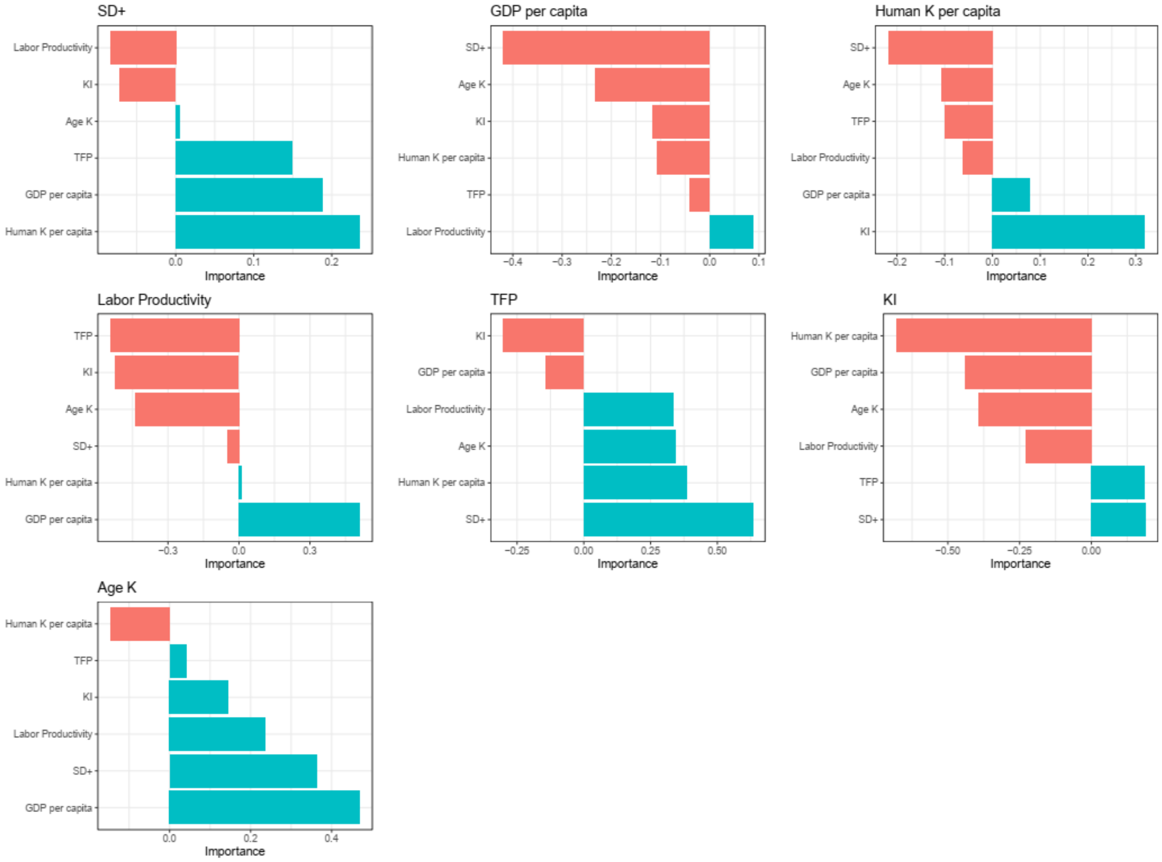

Fig. 10. Results for Olden´s sensitivity analysis in terms of signs and relative importance for models (1)-(7).

5. Conclusions

In this paper, the endogenous sources of long-term productivity performance in different advanced countries are decomposed using a Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach. We found that the UP-IS presented higher cross-performance scores than the TOPSIS base-case. Further, Norway was the best-performing country with a positive performance score of 0.854 in 2018, while Portugal was the worst, with a score of 0.347 in 1919. In terms of the ordinal ranking of long-term productivity performance, UP-IS ranked first, followed by TFP and next, LP and GDPpc. More so, the average age of equipment capital stock and human capital intensity have positive and negative impacts respectively on long-term productivity performance in advanced countries. Conversely, productivity performance has a positive impact on KI and TFP but a negative impact on LP and GDPpc. These results suggest a dichotomous development path that should be followed by advanced countries: high-tech versus "uberized" economies. As far as we are concerned, there have not yet been any studies attempting to use the two-dimensional fuzzy-monte Carlo analysis approach to analyze country-level productivity performance. However, we notice that certain literature studies have already used this method to evaluate health risk (Kentel and Aral, 2005). One study attempted to use a two-dimensional Monte Carlo analysis to study dose reconstruction (Simon et al., 2015). Our study is the pioneer for the use of the proposed method in the economics context and would have a certain level of significance in terms of policy-making, not only to improve productivity performance at a country level, but also simultaneously to consider the endogeneity between performance and potential determinants, as well as the potential endogeneity among the determinants.

The policy implications for both types of economies should focus on, as a starting point, improving physical and human capital levels, which involve massive investments in logistics and energy infrastructure, in parallel with integral elementary and fundamental schooling. However, "uberized" countries will still need increased levels of capital intensity and renewal, which requires WTO tariff incentives and waivers for (re)industrialization to some extent, aligned with the needs of industry 4.0 and their embeddedness with technological start-ups. However, based on our findings regarding the negative impact of productivity performance on labor productivity and GDP per capita, we recommend that relevant government policies should be established and implemented during the period of higher productivity, in the form of providing a signaling mechanism to the management levels of different economic sectors to enhance the level of control and monitoring of labor in their operational processes, the results of which will alleviate reducing labor productivity, and it will further promote GDP per capita.

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