

A note on applying the Markowitz portfolio selection model as a passive investment strategy on the JSE

ABSTRACT

Harry Markowitz is generally acknowledged as the father of modern portfolio theory after publishing his seminal paper in 1952, for which he (jointly) received a Nobel Prize in 1990. Markowitz (1952) and Tobin (1958) showed that it was possible to identify the composition of an optimal portfolio of risky securities, given forecasts of future returns and an appropriate covariance matrix of share returns. This research endeavours to apply the theory of Markowitz to the Johannesburg Securities Exchange (JSE) to establish whether an optimal portfolio can be identified and used as an effective trading rule.

Weekly data over 11 years on the top 40 JSE listed companies was analysed to construct Markowitz mean-variance optimised portfolios using *ex-ante* data. The optimal portfolio was then selected and re-balanced periodically, and the returns compared against the FTSE/JSE ALSI40 index. The study found that the trading strategy significantly outperformed the market in the period under review.

1. INTRODUCTION

The JSE has been found to be 'operationally efficient' (see for example: Thompson and Ward, 1995) and it has been shown that relatively few active fund managers in South Africa are able to consistently outperform the market (Oldham and Kroeger, 2005). In terms of the categories of Fama's (1965) Efficient Market Hypothesis (EMH), trading rules belong at the 'weak form' efficiency level, and are therefore not expected to form the basis of a mechanism to outperform a benchmark index.

The objective of this study is to test whether a trading rule based on Markowitz's (1952) mean-variance optimal portfolio construction can outperform an appropriate index.

2. LITERATURE REVIEW

Varian (1993) succinctly reviews the history of modern portfolio theory as follows:

Markowitz's groundbreaking research on portfolio optimisation was published in March 1952 in an article titled 'Portfolio Selection' in the *Journal of Finance*. Thirty-eight years passed before he was jointly awarded the Nobel Prize for Economics with Merton Miller and William Sharpe (Varian 1993:159f). Markowitz solved the problem of minimising a portfolio's variance, given an expected return and covariance matrix of shares in a portfolio, and demonstrated the importance of this to investors. In 1958 Tobin included the risk-free asset, and showed that the set of efficient risk-return combinations was in fact a straight line, consisting of an optimal portfolio of risky assets and the riskless asset.

Sharpe (1963) simplified the computational burden of Markowitz's model with his 'single factor model':

$$R_{at} = \alpha + bR_{mt} + \varepsilon_{at}, \quad \dots (1)$$

where

R_{mt} is the return on the market,

ε_{at} is an error term with expected value of zero,

α is the expected return of the share if the market is expected to have a zero return and

b measures the sensitivity of the share to 'market conditions' (Sharpe 1963:281).

The single factor model assumes that the return on each security is linearly related, through b , to the market index. Sharpe's single factor model dramatically reduced the computational burden of Markowitz's model by assuming that Tobin and Markowitz's optimal portfolio of risky assets was in fact the market itself. This led to the development of Sharpe's Capital Asset Pricing Model (CAPM) (Varian 1993:164).

The most familiar expression of the CAPM is the expected return-beta relationship:

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f] \quad \dots (2)$$

where

$E(r_i)$ is the expected return of asset i ,

r_f is the risk free rate of return,

$E(r_m)$ is the expected return of the market and

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β_i is the beta coefficient of asset i .

Sharpe's (1963) simplified model for portfolio analysis, which he called the 'critical line method', showed that any set of efficient portfolios can be described in terms of a set of 'corner portfolios'. Adjacent corner portfolios are related in the following way: the one will contain either all the assets of the other, except for one, or all the assets of the other plus one additional one. This implies that moving along the minimum variance frontier from one corner portfolio to another will have the effect that one share is either added to the portfolio or removed from the portfolio. Individual securities can be examined to see whether they will either fall out of the portfolio or enter the portfolio. This radically reduced the computational effort of determining the portfolio with the maximum risk-reward ratio.

In a comparative study of the Markowitz model and the Sharpe model (using the critical line method described above), Affleck-Graves and Money (1976) noted interesting links between the two. Their study used expected index portfolio returns and standard deviations and they observed that the results obtained with Sharpe's model became progressively better with every index that was added. It was further noted that if more portfolios are added, to the point that each share was its own portfolio, the model simulates the Markowitz model. Furthermore it was found that if very low upper boundaries (in terms of the percentage holding of any one share) were enforced on Markowitz's model, the one-index model was a close approximation of the optimal portfolio. Their study also found that Markowitz's model naturally limits the maximum weight invested in any one share to about 40 percent (if no upper boundaries are enforced) and has in the region of six shares in the efficient portfolio, which they felt gave it a natural diversification.

However, one of the biggest criticisms of Markowitz's model, is that it does not produce portfolios that are adequately diversified. McLeod (1998) noted that portfolio managers believe that the Markowitz model gives unrealistic portfolios, which are not properly diversified. When the model was applied to a South African dataset he found only four out of seven indices were ever included, with one of them never having more than 3% of funds allocated.

Bowen (1984) noted that the Markowitz model required large volumes of data and found that it was difficult to estimate covariances. He doubted whether the predictions from the model would be reliable and concluded that 'semantic and statistical barriers exist that prevent the average businessman from coming to grips with the approach' (Bowen 1984:21).

In commenting on why the Markowitz optimisation is not used more in practice, despite its theoretical

success, Michaud (1989:31) gave the following possible reasons:

- the conceptually demanding nature of the theory;
- the fact that most investment companies are not structured to use a mean-variance optimisation approach; and
- anecdotal evidence that portfolio managers find the composition of optimised portfolios counter-intuitive.

Portfolio mathematics and the theory of Markowitz minimum-variance frontier is well documented in the literature (see for example: Bodie, Kane and Marcus, 2005). In its simplest form Markowitz's theory states that a portfolio that will give a minimum variance for a target expected return can be unambiguously selected from a collection of assets. In other words, for every possible target portfolio return there is a unique portfolio of assets that will give the required return at a minimum variance. These define Markowitz's 'efficient frontier'.

Tobin's (1958) addition of the risk-free asset to the set of risky assets re-defined the efficient frontier as a straight line. According to Tobin, all investors would select the optimal risky portfolio – the point where the straight line from the risk free rate is tangent to the efficient frontier. Individual investors will add more or less of the risk-free asset to their complete portfolios, according to their risk averseness. This implies that the only difference in approach amongst investors would be where they would position their portfolio along the straight line between the risk-free asset and the optimal risky portfolio. The straight line is therefore the capital allocation line (CAL).

3. HYPOTHESIS

The main hypothesis of this study, expressed in the null form, is that a passive investment strategy, following a trading rule based on Markowitz's optimal portfolio theory, cannot outperform the South African stock market in the medium-to-long term. Two sub-hypothesis examine constraints; the first that no short-selling be permitted and the second that a maximum of 10% be invested in any one share in the portfolio.

4. METHODOLOGY

Data covering an eleven year period, commencing on 3 January 1997 and ending on 31 December 2007 was obtained from McGregor BFA. Two datasets were provided – one containing the ALSI40 weekly closing prices and the other the weekly closing prices and market capitalisations of all shares listed on the JSE for the period. The second dataset was required to re-construct the ALSI40 constituents, as these altered

periodically, and over the 11 year review period, a total of 109 companies were at one time or other in the weekly 'Top 40' list. Dividends were not included. Survivor bias was minimised by including newly listed shares into the analysis, and by maintaining de-listed shares with cash returns until they could be appropriately excluded.

Information on share splits was not available, and the return data was inspected to locate and correct this problem in 18 instances. Further anomalies were

observed in the data as a consequence of 'bundling' or 'unbundling' of companies, and these were identified and corrected in nine instances.

A model was developed in Microsoft Excel, the data being extracted from Microsoft Access, and Solver™ was used for the mean-variance optimisations required to identify the Markowitz efficient frontier. The model was automated with code written in Visual Basic for Applications (VBA) to allow for parameterised inputs; as shown in Table 1.

Table 1: Model parameters

SharesInIndex	40	0-100	Number of shares to include in "index"
StartDate	04/07/1997	>= 03/01/1997	The date when analysis starts
EndDate	31/12/2007	<= 31/12/2007	The date when analysis ends
Optimise	TRUE	True/False	Does iterative solve for efficient frontier or creates explanatory sheet.
WeeksPerRebalancingPeriod	26		The rebalancing period in weeks
NumberOfIterations	20	1-20	The number of iterations for searching max Sharpe (Minimum 4 suggested)
FineSearch	TRUE	True/False	Does a second round of searching for more accurate maximum Sharpe ratio
NumberOfFinIterations	4	1-20	The number of iterations for fine search (equal number suggested)
EvaluatePerformance	TRUE	True/False	False: Efficient Frontiers will be determined; True: Portfolio performance also evaluated.
MinimumWeightPerShare	-0.1	x to 1	
MaximumWeightPerShare	0.1	0 to x	
MaxNumberOfIterations	24		This stops solver if the Sharpe Ratio does not max out
InitialInvestment	100		
EvaluateMomentum	TRUE	True/False	
SharesUsedInMomentum	4		The number of shares invested in if momentum is evaluated
RiskFreeRate	10		
NameExtension	40 26 20 4 10 ss		This will be added to the names of all sheets and to the name of the directory

The selection of the constituents of FTSE/JSE Top40 index is mainly based on their market capitalisations, but market capitalisation is not the only criteria. Other criteria include rules of eligibility, free float and liquidity. The selection of the FTSE/JSE Top40 constituents falls beyond the scope of this research, but it is important to note that the proxy 'Top40' used in this study was selected purely on market capitalisation and therefore differs slightly from the FTSE/JSE Top40.

Ex ante returns were used to predict both the expected future returns and the covariance matrix. The *ex ante* returns were replicated from the period directly preceding the re-balancing period, with a duration equal to that of the re-balancing period. It should be noted that this methodology implies 'momentum' in portfolio returns; i.e. that shares which out-performed in a prior period will continue to out-perform in the next (see for example Fraser and Page (2000) who found evidence of a momentum effect on the JSE.)

The model was designed to accept any length of re-balancing period. Results indicated that shorter re-

balancing periods yielded better results, but to keep transaction costs down a re-balancing period of 26 weeks was arbitrarily decided upon.

A two step search process was used to identify the optimal portfolio. A 'coarse search' followed by a 'fine search', dramatically improved the accuracy of the results without adding too much of a computational burden. Sharpe's measure for the risk-return ratio of a portfolio was used to maximise the slope of the CAL and thereby determine the optimal market portfolio (Equation 3):

$$S_p = \frac{(E(r_p) - r_f)}{\sigma_p} \quad \dots (3)$$

where

S_p is the Sharpe-ratio of portfolio p ,

$E(r)_p$ is the expected return of portfolio p ,

r_f is the return of the risk-free asset and

σ_p is the standard deviation of the returns of portfolio p .

Minimum and maximum weight per share were parameters used to dictate how investment funds are allocated to individual shares. Short-selling was accomplished by specifying a value less than zero for 'minimum weight per share'. Furthermore, unit trusts have certain rules imposed on them in terms of the maximum percentage of their funds being invested in a single share. The 'maximum weight per share' parameter imposes this maximum on the model.

Changes to the risk-free rate of return, within its normal band of movement, only causes marginal differences to the composition of the optimal portfolio. A risk-free rate of 10% pa throughout was therefore selected to simplify this study.

The optimisation process was executed for each 26 week re-balancing period within the 11 year review period under analysis. The optimal portfolio, once identified, formed the basis of the selection of shares and their weightings over the next review period. Various statistics were collated and the results compared with a buy-and-hold the FTSE/JSE ALSI40 index strategy.

5. RESULTS

The following four optimisation procedures were performed:

Portfolio 1: Short selling allowed, and no other restrictions.

Portfolio 2: Short selling allowed with a minimum of -10% and a maximum of +10% invested in any one share.

Portfolio 3: No short selling and no other restrictions.

Portfolio 4: No short selling allowed with a maximum of 10% invested in any one share.

The comparative graphs (see Figures 1 – 4) illustrate the performance of a R100 investment in the four portfolios against that of the FTSE/JSE ALSI40 index. The initial investment was made on 27 June 1997 and liquidated on 31 December 2007 – a period of ten-and-a-half years.

A more detailed statistical summary of the results is shown in Table 2:

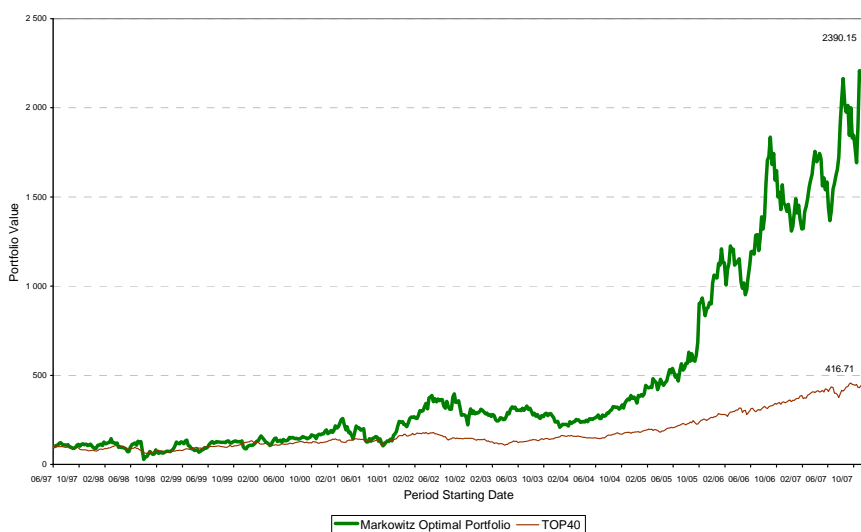


Figure 1: Portfolio 1 (Short-selling allowed, and no other restrictions)

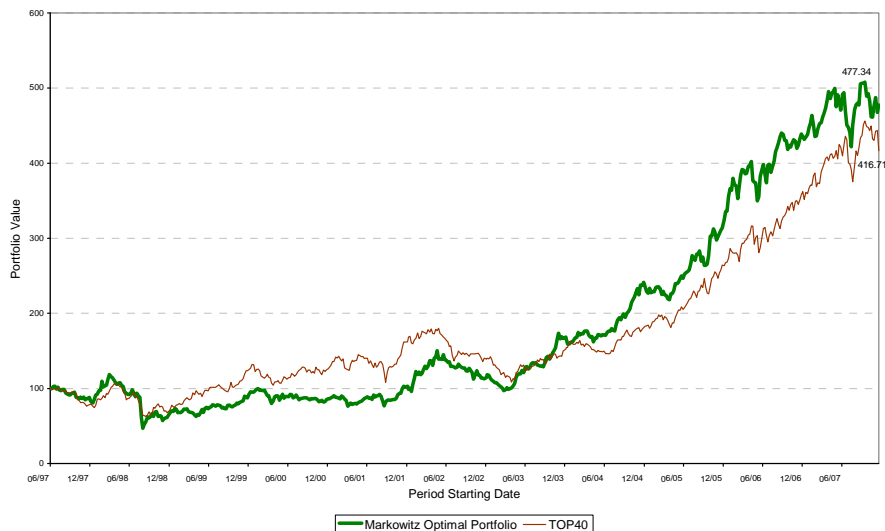


Figure 2: Portfolio 2 (Short-selling allowed with min of -10% and max of 10%)

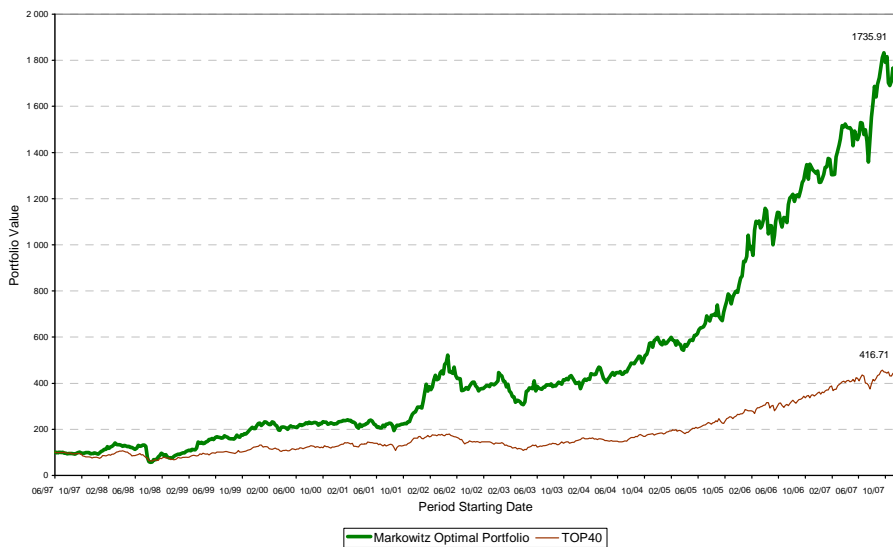


Figure 3: Portfolio 3 (No short-selling allowed, and no other restrictions)

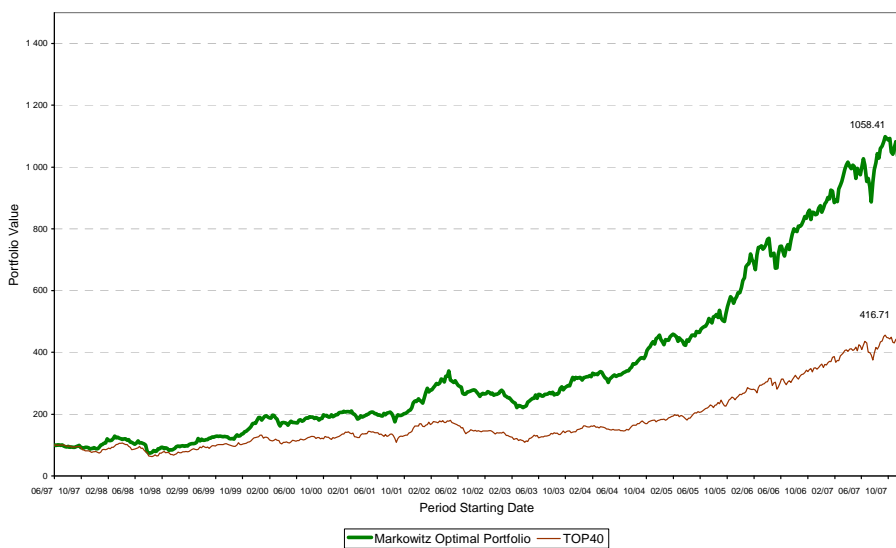


Figure 4: Portfolio 4 (No short-selling allowed with a max of 10%)

Table 2: Summary of results

Description	Portfolio 1 Short selling Unrestricted	Portfolio 2 Short selling Restricted	Portfolio 3 No short selling Unrestricted	Portfolio 4 No short selling Restricted	Top40
i. Portfolio start value (R)	100	100	100	100	100
ii. Portfolio end value (R)	2 390	477	1 736	1 058	417
iii. Avg number of shares in portfolio	38,3	38,4	6,5	12,3	40,0
iv. Avg min of absolute invested in any share (%)	1,7	1,5	3,3	2,7	-
v. Avg max of absolute invested in any share (%)	80,5	10,0	36,7	10,0	-
vi. Start-to-end real annualised return (%)	35,3	16,1	31,2	25,2	14,6
vii. Avg weekly year-on-year return (%)	46,6	19,2	34,5	26,3	15,9
viii. Six-monthly average return (%)	19,5	9,1	16,8	13,0	8,2
ix. Six-monthly variance (%)	746,5	292,6	593,1	244,3	236,7
x. Six-monthly standard deviation (%)	27,3	17,1	24,4	15,6	15,4
xi. Six-monthly Sharpe-ratio	0,536	0,249	0,491	0,518	0,215

As can be seen from Table 2, all the Markowitz-based portfolios outperform the Top40 index both in terms of pure return and in terms of risk-adjusted returns as measured by their respective Sharpe-ratio.

A comparison of the four Markowitz portfolios shows that Portfolio 1, the unrestricted portfolio, is optimal across all the aspects measured. However, closer inspection reveals that the relationship between variance and return of the portfolios is not commensurate with their Sharpe-ratios. The reason is that the Sharpe-ratio uses standard deviation as the denominator and not variance. Variance can be interpreted as a 'measure of expected surprises' (Bodie *et al.*, 2005:1010). The question of what a portfolio's excess returns are, measured in terms of the magnitude of its expected surprises, arises. This can be answered if the denominator in the Sharpe-ratio, portfolio standard deviation, is replaced by portfolio variance. The Sharpe-ratio function then

becomes the excess-returns-to-expected-surprise-ratio:

$$RES_p = \frac{(E(r_p) - r_f)}{\sigma_p^2} \dots (4)$$

where

RES_p is the excess-returns-to-expected-surprise-ratio of portfolio p ,

$E(r)_p$ is the expected return of portfolio p ,

r_f is the return of the risk-free asset and

σ_p^2 is the variance of the returns of portfolio p .

If the excess-returns-to-expected-surprise-ratio of the portfolios is calculated and multiplied by a thousand for readability, the results in Table 3 are obtained.

Table 3: Excess-returns-to-expected-surprise-ratios

Description	Portfolio 1 Short selling Unrestricted	Portfolio 2 Short selling Restricted	Portfolio 3 No short selling Unrestricted	Portfolio 4 No short selling Restricted	Top40
Excess-returns-to-expected-surprise-ratio	19,608	14,544	20,175	33,162	13,998

Table 3 illustrates that the Markowitz portfolios once again outperform the Top40 portfolio in terms of the excess-returns-to-expected-surprise-ratio. The relative performance of the Markowitz portfolios changes, though. Portfolio 4, the portfolio without short selling and limited to a maximum of 10% invested in any share, is optimal. It is important to note that this portfolio has a variance similar to that of the Top40 portfolio, but its return is significantly higher per year. Portfolio 4 has the added advantage that it is less

concentrated, the maximum percentage of the portfolio invested in one share being limited to 10%.

6. CONCLUSION

The main hypothesis of the study, and the sub-hypotheses, are rejected in this study.

The simulations and optimisations done using 11 years of data from the JSE show that a trading rule strategy, based on Markowitz's optimal portfolio theory,

outperformed the market, as represented by the FTSE/JSE Top40 index. The results of the four different portfolio constructions show that even under the constrained conditions of no short-selling and/or no more than 10% in any single security, the strategy works.

Further research using a longer time period and more shares would enhance the quality of this study. Furthermore, the inclusion of more onerous constraints as well as alternative approaches to estimating future returns (such as analyst consensus earnings forecasts) might improve the methodology. Nevertheless, these findings present strong evidence that the Markowitz optimal portfolio does provide the basis of a useful trading rule strategy.

Since the covariance matrix and the expected returns used in the analysis were based on the *ex ante* period returns, there is reason to believe that these results support findings of momentum in share returns.

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