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Evaluation of the seismic response of a reinforced concrete footing to
increasing peak ground acceleration using pseudo-dynamic
experimentation

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Evaluation of the seismic response of a reinforced concrete footing to increasing peak ground acceleration using pseudo-dynamic experimentation

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THESIS SUMMARY

Evaluation of the seismic response of a reinforced concrete footing to increasing peak ground acceleration using pseudo-dynamic experimentation

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Southern Africa is characterised as a region of moderate seismicity with several instances of both natural seismicity and mining-related seismicity occurring within the last century. Evaluating the performance of a structure due to increasing seismic intensity is traditionally calculated post-earthquake using statistical means. However, the limited network of accelerometers in South Africa has prevented a detailed statistical analysis to be undertaken to determine the resultant structural damage to South African designed structures with increasing earthquake intensity. Therefore, this research investigates a method to relate damage to a structure with earthquake intensity by performing numerical analysis in combination with physical experimentation.

The pseudo-dynamic experimentation technique was utilised to evaluate the damage occurring in a reinforced concrete footing due to the overall response of a linear elastic two-storey, two-bay moment resisting steel frame structure that is subjected to earthquake excitation. The implicit Newmark's method with static condensation was utilized in the present study to solve the governing equation of motion of the multi-degree of freedom system. Five pseudo-dynamic

experiments were performed by scaling the El Centro ground motion record, which occurred in California on May 18, 1940, to produce peak ground accelerations that ranged between 0.34 g and 2 g. To supplement the pseudo-dynamic tests, two cyclic load tests were also undertaken. All the laboratory experiments were undertaken under a constant axial load for the duration of the applied earthquake excitation and utilised Rayleigh damping to model the energy loss with the overall linear elastic frame structure. Utilising the results produced during the experiments, an analytical hysteretic model and a damage index was formulated for the analysed reinforced concrete footing with the aim of interpolating damage at peak ground accelerations and overall structural fundamental period of vibration that were not evaluated during the laboratory test. The Park and Ang damage index was used in combination with the results to formulate damage curves and fragility curves for the reinforced concrete footing.

The pseudo-dynamic method provides a reliable method to relate damage suffered by the footing due to the overall structure's response to the applied earthquake excitation. The method enables the structural capacity and failure mechanisms of the reinforced concrete footing to be observed in relation to the seismic demand. The hysteretic response of the footings and energy dissipation characteristics were determined and was shown that the yield strength of the longitudinal reinforcement within the footing has a significant impact on the maximum shear capacity and damage incurred by the footing. The reinforced concrete footing could only sustain a maximum PGA before failure, which is related to the structure's natural frequency and overall energy loss within the structure. Five damage states can be determined for the reinforced concrete and are related to the design of the footing and material properties that comprise the footing. The damage is more pronounced with an increase in the number of cycles of vibration, particularly at displacements that exceed the yield strength of the reinforcement. An increase in the hysteretic energy dissipated by the reinforced concrete footing results in a concomitant increase in the observed damage to the footing in the form of concrete cracking, reinforcement yielding and spalling of the concrete. The investigation shows that the resultant damage to an individual structural component is complex and is dependent on several characteristics that define the structure.



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LIST OF SYMBOLS

ω	Natural frequency of the structure in rads/s
μ	Ductility ratio
ζ	Structural damping ratio
γ	Newmark's parameter equalling $\frac{1}{2}$ for the linear acceleration method
β	Either indicates Newmark's parameter equalling $\frac{1}{4}$ for the linear acceleration method or Park and Ang parameter indicating the extent of damage due to hysteresis
u	Displacement
du	Change in displacement
PGA	Peak ground acceleration
DI	Damage index
ρ_{st}	Steel density
ρ_m	Masonry density
ρ_c	Concrete density
Δt	Change in time
g	Gravity acceleration (9.81 m/s ²)
ρ_w	Confinement ratio
β_i	Logarithmic standard deviation
u_s	Non-linear spring displacement
p_t	Longitudinal steel ratio as a percentage
n_0	Normalised axial stress
k_s	Non-linear spring stiffness
a_0 & a_1	Rayleigh damping coefficients having units s ⁻¹ and s respectively

W_{SDL}	Distributed superimposed dead load
W_{LL}	Distributed live load
R_i	Measured restoring force from tests specimen
P_m	Point load due to masonry infill panels
P_{DL}	Point dead load
$[M]$	Mass matrix
$[K]$	Overall structural stiffness matrix
$\{I\}$:	Influence vector that accounts for the direction of the earthquake loading
F_s	Non-linear spring force
E_s	Strain energy
E_m	Kinetic energy
E_l	Total energy
E_c	Viscous damping energy
E_H	Hysteretic energy
$[C]$	Viscous damping matrix
h_t	Overall height of the structure used in building period formulas
$\{u_i\}$	Displacements at time step i
$\{\dot{u}_i\}$	Velocity vector at time step i
$\{\ddot{u}_i\}$	Acceleration vector at time step i
$\{\ddot{u}_{gi}\}$:	Ground acceleration at time step i

1 INTRODUCTION

1.1 BACKGROUND

The performance of structures during an earthquake of given intensity dictates the extent of damage and loss of life that becomes associated with the earthquake event. Quantifying the level of damage within a structure that has occurred during an earthquake is traditionally undertaken post-earthquake using statistical methods. However, this method is not suitable in areas with moderate seismicity as insufficient data are available to calibrate structural damage to an earthquake intensity parameter.

Southern Africa is characterised as a region of moderate seismicity, and due to the limited network of accelerometers within South Africa, a detailed statistical analysis of the level of damage that could occur within structures during future earthquakes has been prevented (Brandt, 2011). The previous century has been characterised by several earthquakes in South Africa, with the most widely documented being the Ceres-Tulbagh earthquake in 1969 of Richter magnitude 6.7. The Ceres-Tulbagh earthquake had an insured loss of US\$ 7.4 million and a total uninsured loss amounting to approximately 3.5 times that of the insured loss. Extensive mining in South Africa, particularly in the gold mining districts of the Witwatersrand Basin, has resulted in at least four events in recent years that have caused significant structural damage (Kijko and Davies, 2003). Therefore, as noted in Kijko and Davies (2003), seismic risk faced by South African structures cannot be met with complacency and structures need to be evaluated against a range of earthquake intensities to quantify their seismic capacity and performance during an earthquake.

The typical engineering design process does not adequately quantify the level of performance that a structure can sustain during an earthquake. The procedure entails the analysis and design of structural components to satisfy the requirements of the South African structural design codes of practice, which tend to be prescriptive (Kijko and Davies, 2003). Therefore, the resultant performance of the structure and the components making up the structure are not thoroughly investigated at various earthquake intensities.

Quantifying the level of damage incurred by a structure due to increasing earthquake intensity is a complex task. Typically, structural components are evaluated using quasi-static methods to determine the response due to increasing load. However, the slow rate of the load applied onto the structure results in the inertia of the structure not being considered resulting in the response

of the structure being independent of the applied earthquake loading. To relate earthquake intensity to damage, shake table testing or pseudo-dynamic testing provide a more accurate damage correlation with earthquake intensity. Shake table tests provide the most realistic means to evaluate damage at various intensities as it accounts for the inertial effects, and the time and frequency content of the ground motion. However, shake tables are very expensive, and it is difficult to evaluate large scale multi-story structures.

During pseudo-dynamic experimentation, part of the structure under investigation is physically tested in the laboratory in parallel with the dynamic time-stepping structural analysis of the overall structure that is mathematically modelled on the computer. The mathematical model of the overall structure incorporates both the mass and damping properties of the structure during the analysis with the physical model only accounting for the static force-displacement response of the test specimen. Pseudo-dynamic experimentation uses well established time integration numerical methods to determine the resultant displacement at the degree of freedom that couples the numerical model with the physical model. The displacement determined at the coupled degree of freedom is physically applied to the test specimen using a servo-controlled linear actuator. The resultant force on the structure is measured using a load cell and is subsequently fed back into the computational model and used in successive iterations and computations to calculate the new displacements.

The disadvantages associated with pseudo-dynamic tests is that they exclude time-dependent effects and can result in cumulative errors in the computational process. However, the advantages associated with the method include using the same equipment that is used to perform quasi-static tests and because of the time-independent nature of the experiment, the structural damage can be observed at each time step due to the slow application of the load. Advancements in computer software, the increase in the resolution of the control systems and data acquisition systems and the ability of pseudo-dynamic tests to incorporate the dynamic characteristics of a structure has made the method a feasible alternative to shake table tests to evaluate the performance of a structure at various earthquake intensities.

1.2 OBJECTIVE OF THE STUDY

The primary objective of the study was to use the pseudo-dynamic experimental method to relate structural damage incurred by a single axially loaded reinforced concrete footing, which forms part of a two-bay two-story moment resisting frame structure, to increasing earthquake intensity. Therefore, the purpose of the study was separated into the following objectives:

- To evaluate the feasibility and viability of using the pseudo-dynamic method with implicit time-stepping numerical methods to accurately relate the level of damage encountered by a reinforced concrete footing structural subcomponent, which forms part of an overall structural system subjected to an earthquake excitation, to increasing earthquake intensity;
- To utilise the pseudo-dynamic method to determine the structural capacity and failure mechanisms of the reinforced concrete footing structural component to be observed in relation to the seismic demand at various earthquake intensities. To determine the relationship between earthquake intensity and structural damage for the reinforced concrete footing by analysing the hysteretic cyclic response and energy dissipation characteristics at various earthquake intensities. To use the results obtained from experiments to formulate a damage index that can be used to quantify the expected level of damage at various earthquake intensities;
- To evaluate the response of the reinforced concrete structure at high peak ground accelerations (PGA) and to determine the maximum peak ground acceleration that the concrete section can endure for the analysed structural design and configuration; and
- To investigate the feasibility of using the pseudo-dynamic testing method to formulate damage curves and fragility curves for a structural subcomponent that forms part of an overall structural system. To determine the influence of the overall structure's fundamental period of vibration has on the level of damage experienced by the footing for the analysed structural design and configuration.

1.3 SCOPE OF THE STUDY

The susceptibility of damage to a reinforced concrete footing during an earthquake is dependent on several factors that relate to the type of structure placed on the footing and the type of earthquake ground motion that is imparted to the structure. The study focused on evaluating the pseudo-dynamic testing method using only the Newmark's implicit numerical time integration method to correlate damage incurred by the reinforced concrete footing to different earthquake intensities. Only a single ground motion record was used in the analysis with changes in the peak ground acceleration being the only variable. The El Centro earthquake record was selected as the input ground motion for the pseudo-dynamic analysis and the change in the peak ground acceleration was achieved by amplifying the record to obtain the required peak ground acceleration (PGA).

The reinforced concrete footing was subjected to a constant axial load for the duration of the earthquake record, and therefore the response of the footing due to a varying axial load was not investigated. Only a reinforced concrete footing of a single design that satisfied the minimum reinforcement requirements contained in SANS 10100-1:2000 was considered in this research. Therefore, the reinforced concrete footing dimensions, reinforcement and concrete mix design were all kept constant for each of the tests. All the footings were constructed using the same batch of concrete and reinforcement. Additional quasi-static cyclic load tests were conducted on the footings to determine the cyclic response to increasing deformation and to determine the maximum stiffness of the footing.

The frame structure, which was placed on the footing, remained linear elastic for the duration of the earthquake record. Only Rayleigh damping was incorporated into the analysis to account for damage and energy loss in the overall frame structure. The capacity of structural members and their connections within the overall frame structure is not considered during the analysis, and therefore, the formation of plastic hinges and the resulted loss of stiffness within the overall structure is not accounted for in the response of the reinforced concrete footing. Future studies can incorporate plastic hinges and determine the influence it has on the damage and fragility of the footings and the overall structure.

An analytical material model was developed by only using the results produced from the pseudo-dynamic experiments and the cyclic load tests to interpolate damage at peak ground accelerations and fundamental periods of vibration that were not undertaken during the laboratory experiments. The typical range of the fundamental periods of vibration were determined using building period formulas obtained from SANS 10160-4:2017. The fundamental period of vibration of the overall frame structure was varied by multiplying the stiffness of the each of the members by the same constant value, therefore ensuring that there is the same proportional variation in stiffness between each of the structural members. The fundamental period of vibration was not varied by altering the structural configuration or type of structure placed on the footing. A damage index was formulated using on the results produced in this research.

Damage curves and fragility curves were only produced for the designed reinforced concrete footing and moment resisting frame structure that was subjected to a single ground motion record. The fragility curves only consider a uniform distribution of fundamental periods of vibration of frame structures that could reasonably be placed on the reinforced concrete footing as the actual distribution of structures in an area is not known. The procedure following in this

research could be used in future research to analyse the response of structures and structural components with varying configurations and designs to develop damage and fragility curves.

1.4 METHODOLOGY

The influence of earthquake intensity on the level of damage to the reinforced concrete footing was evaluated by using seven test specimens that were constructed from the same batch of reinforcing steel and 30 MPa concrete. Two cyclic load tests and five pseudo-dynamic tests were undertaken at PGAs of 0.34 g, 0.68 g, 0.78 g, 1 g and 2 g, which were obtained by amplifying the El Centro ground motion record. The reinforcement and concrete were obtained from commercial suppliers, which ensured that consistency was maintained between the materials used to construct the footings and the materials typically used in industry.

The pseudo-dynamic experimental method was used to establish a correlation between the level of damage encountered by the reinforced concrete footing, which forms part of a two-bay, two-story moment resisting steel frame structure, with earthquake intensity. Two stages were required to perform the pseudo-dynamic experiments with the first stage requiring the establishment of a physical test setup that would allow for a constant axial load to be applied to the test specimen for the duration of the earthquake record while allowing for a varying horizontal load, which was servo-controlled. Both the horizontal load and axial load needed to be applied simultaneously without any disruptions during the applied earthquake excitation.

The second stage required to undertake pseudo-dynamic experiments required the development and programming of an algorithm that would be used to control the actuators. The computer algorithm had to model the overall frame structure's mass, stiffness and applied static loading and was formulated using the equation of motion and Newmark's implicit time integration numerical method. The algorithm required two steps, with the first step requiring the formulation of the frame structure and the calculation of the initial state of the structure under static loading conditions. The second step entailed subjecting the structure to the earthquake excitation, and due to the implicit nature of the numerical method, the analysis required iteration until convergence at each time step. The horizontal displacement of the footing that was calculated from the numerical model was applied directly to the test specimen in the laboratory at each time increment, and the restoring force was measured using a load cell that was subsequently fed back into the algorithm to be used in further calculations.

Strain gauges were attached to the reinforcing bars to determine the strain at each time step, and four external displacement transducers were distributed along the height of the column to

obtain additional displacement information at each time increment. Due to the implicit nature of the time-stepping algorithm used to perform the pseudo-dynamic tests, cyclic load tests were undertaken to determine the maximum stiffness of the footing, which was then used as an input into the pseudo-dynamic tests.

Due to the limited number of pseudo-dynamic tests undertaken, an analytical hysteretic model was developed from the results obtained from the cyclic load tests and pseudo-dynamic tests to complement and enable the interpolation of damage at peak ground accelerations and overall structural fundamental periods of vibration that were not undertaken during the pseudo-dynamic experiments. The result obtained from the cyclic load tests and pseudo-dynamic tests were subsequently used to formulate a damage index for the reinforced concrete footing. Ten-thousand numerical analysis were undertaken using the formulated analytical hysteretic model and damage index to determine the level of damage incurred by the footing over a range of peak ground accelerations and fundamental periods of vibration. The numerical analysis undertaken using the analytical hysteretic model utilised the same algorithm used to perform the pseudo-dynamic experiments, which entailed substituting the physical test setup with the analytical hysteretic model.

The results obtained by using the analytical hysteretic model were used to develop damage curves and contour damage plots, in terms of the overall structural fundamental period of vibration, for the reinforced concrete footing. The contour damage plots and curves were subsequently used to develop fragility curves assuming a uniform distribution and representative range of fundamental periods of vibration that could typically be obtained for structures placed on the footing. The results produced from the damage curves and fragility curves are related to predicted earthquake intensities and historical earthquake intensities in South Africa to infer predicted damage that a structure may sustain.

1.5 ORGANISATION OF THE REPORT

This report consists of the following chapters:

- Chapter 1 serves as an introduction to the report;
- Chapter 2 provides a literature study of the different concepts covered during the study;
- Chapter 3 provides the experimental test setup including the analysis and design of the overall frame structure and the reinforced concrete footing. The pseudo-dynamic experimental method is presented showing the physical test setup and the formulation of the algorithm used to perform the experiments;

- Chapter 4 discusses the analysis and detailed description of the results obtained from the cyclic load tests and pseudo-dynamic experiments;
- Chapter 5 provides the formulation of the analytical hysteretic model and damage results. The formulated damage index, the resultant damage and fragility curves are also presented and discussed;
- Chapter 6 contains the final conclusions and recommendations of the study; and
- Chapter 7 provides a list of references.

2 LITERATURE STUDY

This chapter explores the current understanding of pseudo-dynamic experimentation techniques used to analyse engineering structures and includes information on the non-linear time-stepping numerical models for solving the equation of motion. In the literature, numerous studies have been conducted on the pseudo-dynamic method; however, only the important contributions relevant to the current study are discussed. This literature review commences with a discussion on seismic risk in South Africa, followed by a discussion on the pseudo-dynamic experimentation concerning previous studies and the cyclic response of reinforced concrete. The final section of the literature study briefly discusses damage models and the formulation of damage and fragility curves from experimental data.

2.1 EARTHQUAKES

An earthquake is a naturally occurring phenomenon that typically does not pose a threat to humans; however, earthquakes become a hazard when considered in relation to structures and infrastructure due to the structure being subjected to the seismic excitation (Penelis & Kappos, 2010). Earthquakes have a severe impact on economic, social, psychological and political effects due to deaths and damage to infrastructure.

Earthquakes are ground vibrations that predominantly occur along plate boundary zones where one tectonic plate slides relative to another or subducts beneath the other or due to the fracturing of the crust (Penelis & Kappos, 2010). A schematic representation of the origin of earthquakes is shown in Figure 2-1.

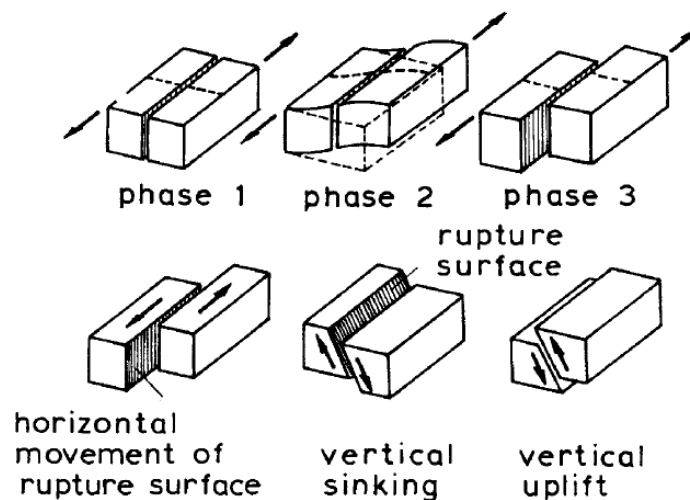


Figure 2-1 Origins of earthquakes (Penelis & Kappos, 2010)

In areas where plates move relative to one another, large amounts of energy build up over a relatively small area. As a result, these regions are responsible for more frequent and severe earthquakes. However, regions located within the boundaries of tectonic plates are generally rigid, which requires a much longer time to deform and build up energy (Kijko et al., 2015). South Africa is located within the boundaries of the African plate and is characterised as an area with moderate seismicity. Further information on the seismic hazard in South Africa and the tectonic makeup can be found in Brandt (2011).

Continents consist of vast geological histories that can comprise of up to 4 billion years. Earthquakes that occur in plate interiors are infrequent due to the slow speed of internal deformation, which can be associated with asperities in the mantle or faults in geological formations. However, Davies and Kijko (2003) indicate that the seismic risk faced by South Africa is non-negligible and therefore the risk of seismic activity needs to be considered when designing a structure. More alarming is that Kijko et al. (2015) conclude that structures designed to SANS Standard 10160 (SANS 10160-4, 2017) for a seismic load of 0.1 g are at significant seismic risk. Therefore, seismic hazard is an issue in South Africa that needs to be considered by the insurance industry and disaster management agencies as a probable threat to infrastructure and life (Kijko et al., 2015).

2.1.1 EARTHQUAKE MAGNITUDE AND INTENSITY

The magnitude of an earthquake is a measure of the amount of energy released at its point of origin in the form of seismic waves and is measured on the Richter scale. The largest earthquake to have been recorded had a magnitude of 8.9 on the Richter scale (Colombia-Ecuador, 1906; Japan, 1933) and is taken to be the largest earthquake to have ever occurred (Penelis & Kappos, 2010).

The damage caused by an earthquake is partly related to magnitude but is also related to several other factors such as the focal depth of the earthquake, the distance to the epicentre, and the geology and mechanical properties that make up the structures. The intensity provides a measure of quantifying the consequences that the earthquake will have on people and the structures in which they reside. Due to the complexity of the problem, empirical intensity scales have been formulated to quantify damage qualitatively. One of the most common intensity scales is the modified Mercalli (I_{MM}) scale, which has 12 intensity grades and can be related to peak ground acceleration and is shown in Table 2-1. Strong ground motions that are of interest



to structural engineers are recorded using accelerograms and record the acceleration of the ground as a function of time. An example of a strong motion record shown in Figure 2-2.

Table 2-1 Modified Mercalli scale (Penelis & Kappos, 2010)

	Ground acceleration a	
	cm/sec	$\frac{a}{g}$
I Not felt except by a very few under especially favourable circumstances		
II Felt only by a few persons at rest, especially on upper floors of buildings. Delicately suspended objects may swing	2 3	
III Felt quite noticeably indoors, especially on upper floors of buildings, but many people do not recognize it as an earthquake. Standing motor cars may rock slightly. Vibration like passing truck. Duration estimated	4 5 6	0.005 g
IV During the day felt indoors by many, outdoors by few. At night some awakened. Dishes, windows, doors disturbed; walls make creaking sound. Sensation like heavy truck striking building. Standing motor cars rocked noticeably	7 8 9 10	0.01 g
V Felt by nearly everyone: many awakened. Some dishes, windows, etc., broken; a few instances of cracked plaster; unstable objects overturned. Disturbances of trees, poles and other tall objects sometimes noticed. Pendulum clocks may stop	20 30	
VI Felt by all; many frightened and run outdoors. Some heavy furniture moved; a few instances of fallen plaster or damaged chimneys. Damage slight	40 50 60	0.05 g
VII Everybody runs outdoors. Damage negligible in buildings of good design and construction: slight to moderate in well-built ordinary structures: considerable in poorly built or badly designed structures; some chimneys broken. Noticed by persons driving motor cars	70 80 90 100	0.1 g
VIII Damage slight in specially designed structures: considerable in ordinary substantial buildings, with partial collapse; great in poorly built structures. Panel walls thrown out of frame structures. Fall of chimneys, factory stacks, columns, monuments, walls. Heavy furniture overturned. Sand and mud ejected in small amounts. Changes in well water. Disturbs persons driving motor cars	200 300	
IX Damage considerable in specially designed structures: well-designed frame structures thrown out of plumb; great in substantial buildings, with partial collapse. Buildings shifted off foundations. Ground cracked conspicuously. Underground pipes broken	400 500 600	0.5 g
X Some well-built, wooden structures destroyed: most masonry and frame structures destroyed with foundations; ground badly cracked. Rails bent. Landslides considerable from river banks and steep slopes. Shifted sand and mud. Water splashed over banks	700 800 900 1000	1 g
XI Few, if any masonry structures remain standing. Bridges destroyed. Broad fissures in ground. Underground, pipelines completely out of service. Earth slumps and landslips in soft ground. Rails bent greatly	2000 3000	
XII Damage total. Waves seen on ground surfaces. Lines of sight and level distorted. Objects thrown upward into the air	4000 5000 6000	5 g

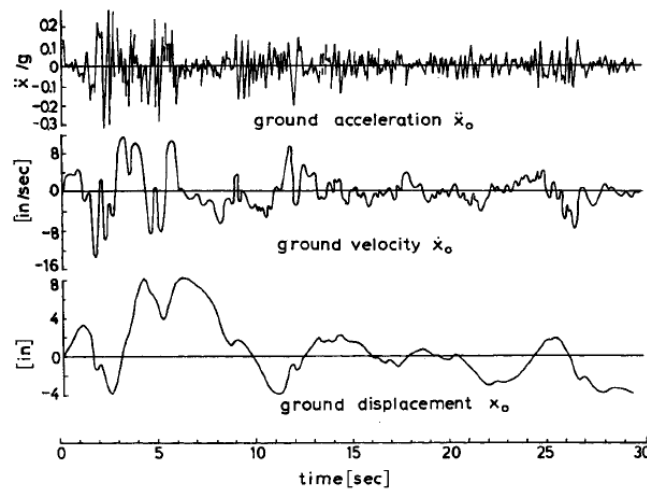


Figure 2-2 Example of a strong ground motion accelerogram of the El Centro earthquake on 18 May 1940 (N-S component) with the calculated velocity and displacement diagrams (Penelis & Kappos, 2010)

Intensity provides a single means of describing the effect that an earthquake has on human-made structures. The reason for using intensity (I_{MM}) to describe an earthquake as opposed to just using peak ground acceleration is that it considers the local condition inherent to a site, which is not possible when expressing an earthquake in terms of peak ground acceleration (Kijko et al., 2015). Many empirical relationships have been developed to correlate intensity with peak ground acceleration. Figure 2-4 shows various conversions that have been developed by various authors to relate intensity with maximum earthquake accelerations.

Discussions with Prof. A Kijko at the University of Pretoria's Natural Hazard Centre indicated that the correlation between peak ground acceleration and intensity developed by Ambraseys (1974) provided the best correlation for South African conditions. Figure 2-3 shows the maximum horizontal acceleration and maximum vertical acceleration against reported intensity between 1967 and 1974 during the study by Ambraseys (1974).

The solid dots shown in Figure 2-3 indicate the maximum horizontal acceleration and the open dots represent the maximum vertical acceleration. The stars indicate points obtained on hard ground, and the dots represent points on soft ground. Utilising the results from Figure 2-3 Ambraseys (1974) developed a first-order approximation for relating maximum horizontal acceleration and intensity, which is shown by Equation 2.1. The conclusions drawn by Ambraseys (1974) indicate that:

- There is a weak correlation between intensity and maximum ground accelerations due to the considerable variability in geologic conditions, foundation conditions, earthquake mechanisms and the type of accelerograms that are used;

- The maximum horizontal accelerations are in the order of 1.5 to 2.5 times greater than the vertical accelerations; and
- Accelerations on hard ground are more significant than those on soft ground.

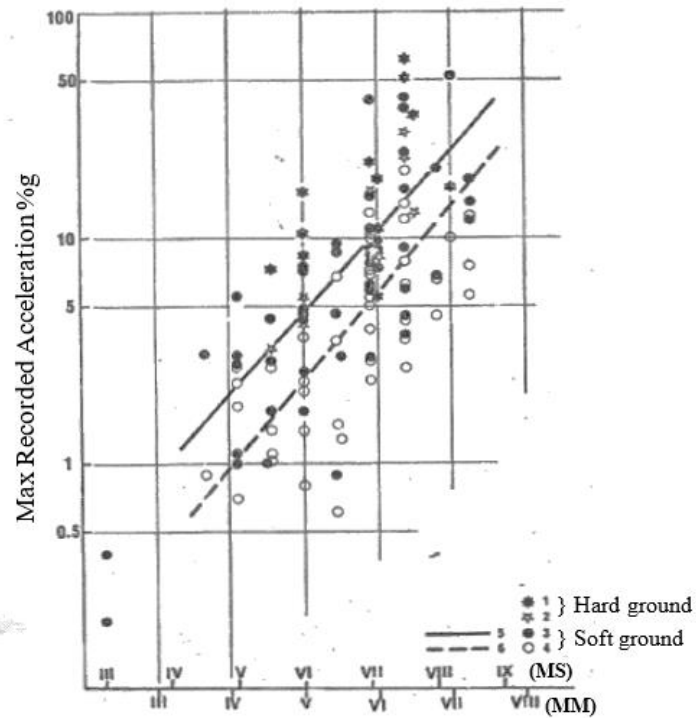


Figure 2-3 Plot of the maximum recorded acceleration versus reported intensity (Ambraseys, 1974)

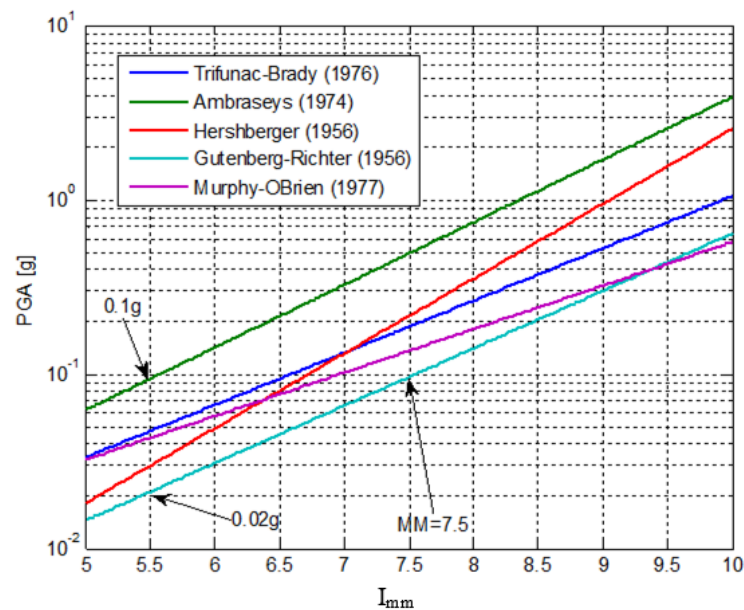


Figure 2-4 Peak ground accelerations conversions to intensity (IMM) relationships from different studies (Kijko, 2008)

$$\log(a_h) = -0.16 + 0.36(I_{MM}) \quad (2.1)$$

2.1.2 SEISMIC RISK IN SOUTH AFRICA

Two significant groups of seismicity occur in South Africa, namely natural seismicity and mining-related seismicity. The first recorded earthquake in South Africa occurred on Robben Island in 1620 and was recorded by the early Dutch settlers (Kijko et al., 2015). The 20th century predominately comprised of tectonic induced seismicity, with an example being the magnitude 6.0 to 6.5 seismic event that occurred on 31 December 1932 off the coast of Cape St Lucia (Kijko et al., 2015). Another example of a large magnitude earthquake occurring in South Africa is the Tulbagh-Ceres earthquake of magnitude 6.3 that occurred in 1969. The Tulbagh-Ceres earthquake resulted in extensive damage with examples of damage shown in Figure 2-5. The Tulbagh-Ceres earthquake was felt across the Western Cape and several buildings suffered severe damage, which ranged from large cracks to complete destruction. Twelve people lost their lives due to the earthquake, and the event produced an insured loss of US \$ 7.4 million. However, the uninsured loss was estimated to be approximately 3.5 times higher than the insured damage (Kijko et al., 2015).

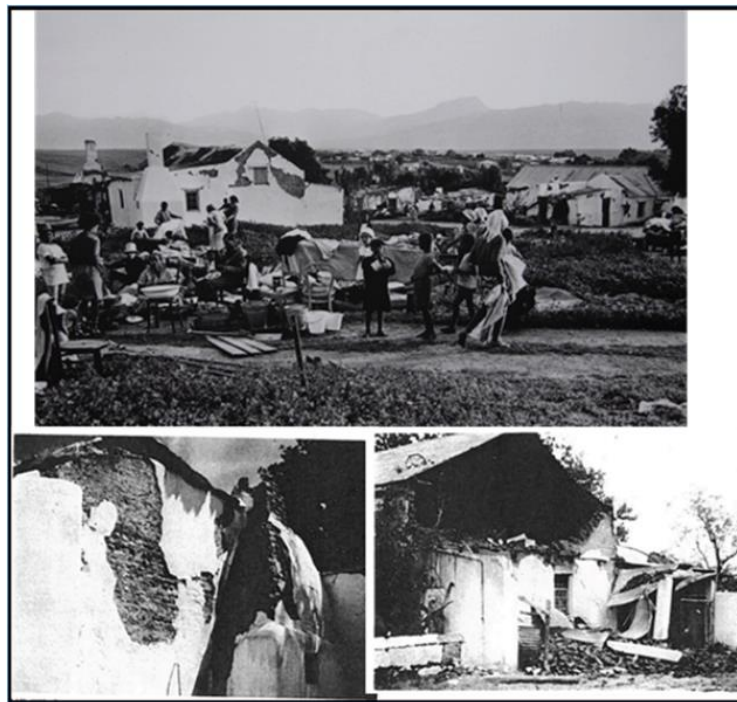


Figure 2-5 Resultant damage at Tulbagh after the Ceres earthquake of 29 September 1969 (Kijko et al., 2015)

Other examples of seismic events in South Africa include the Welkom earthquake in 1976 that resulted in the collapse of a six-story-high block of flats and incurred a total insured cost of R4.5 million. Figure 2-6 shows the collapse of the six-story building during the Welkom earthquake event. A seismic event that occurred in Anglovaal in 1998 resulted in an issued loss of R23 million (Kijko et al., 2015).



Figure 2-6 Collapse of a six-storey high block of flats due to the 1976 Welkom seismic event (Kijko et.al, 2015)

Figure 2-7 shows historical epicentral locations of seismicity in Southern Africa. Belt-like zones of seismicity characterise Southern Africa, which is surrounded by regions of low seismicity. The belt-like zones of seismicity extend from the African Rift Valley down into South Africa along the South African and Mozambique north-south border, which extends southwards into Kwa-Zulu Natal. Another belt of seismic activity occurs from east to west through southern KwaZulu Natal, Lesotho and the southern parts of the Free State (Brandt, 2011).

Due to limited information of historical earthquakes in South Africa, a probabilistic seismic hazard analysis is undertaken to produce a map showing the expected peak ground accelerations as a factor of gravitational acceleration ($g = 9.81 \text{ m/s}^2$) with a 10% probability of being exceeded within the next 50 years, which is shown in Figure 2-8. As noted in Kijko et al. (2015), due to the slow rate of deformation of the tectonic plates within South Africa, the repeat times of earthquakes range from thousands to tens of thousands of years. Due to this, instrumental and historical records cannot provide a comprehensive overview of the potential risk (Kijko et al., 2015).

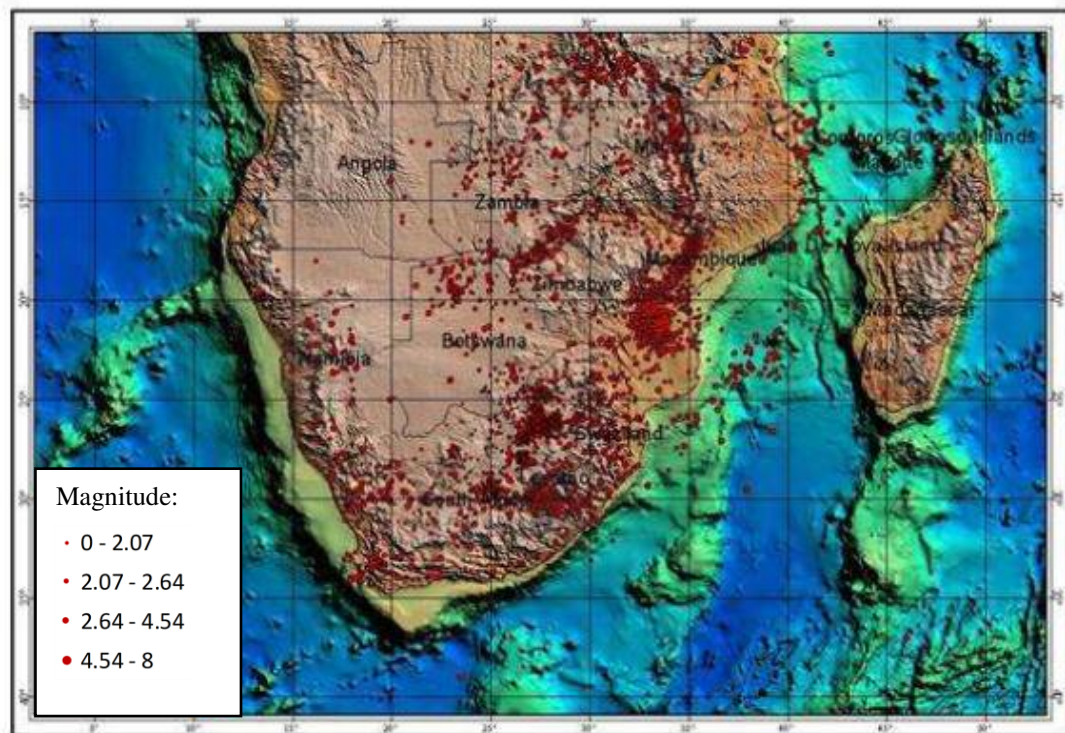


Figure 2-7 Seismic map of Southern Africa during the period of 1620 to 2010 (Brandt, 2011)

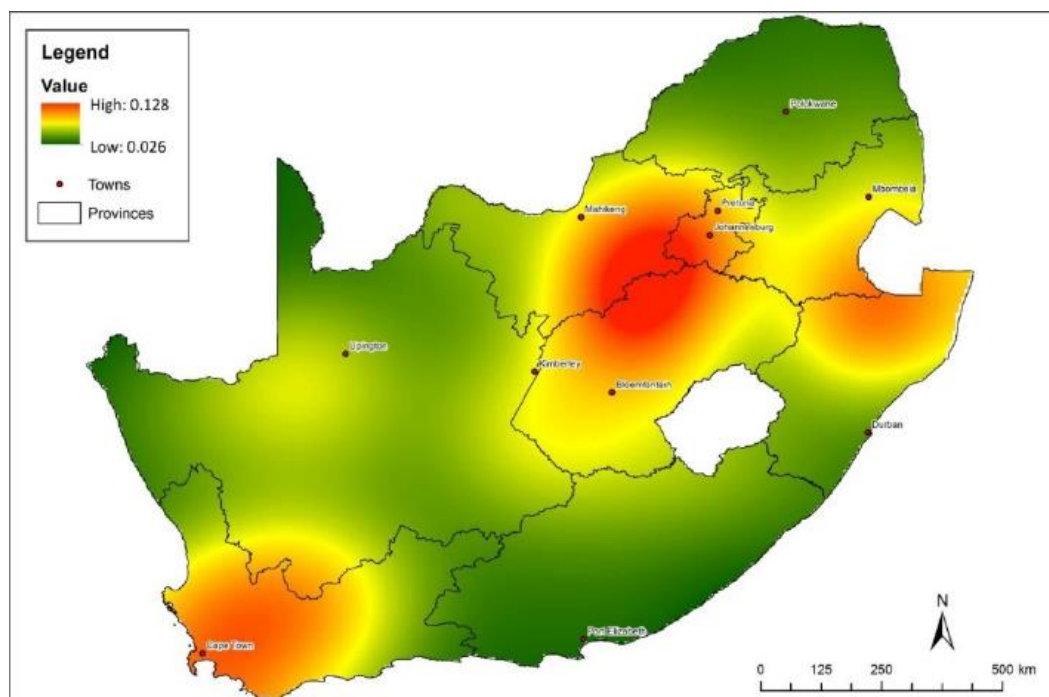


Figure 2-8 Expected peak ground acceleration (PGA) with a 10% probability of being exceeded at least once in a 50-year period (Kijko et. al. 2015)

2.1.3 SEISMICITY DUE TO MINING ACTIVITY

Many earthquakes that occur in Gauteng are due to mining activity within the region (Linzer et al., 2007). The largest seismic event that occurred in South Africa due to mining-related activity occurred in Stilfontein and happened on 9 March 2005. The seismic event recorded a 5.3 on the local Richter magnitude scale. Figure 2-9 shows some of the damage to buildings due to the seismic tremor in Stilfontein. Closure of the mine due to the earthquake resulted in large-scale socio-economic consequences for the people and the community that relied on the mine for their livelihood. Seismic events in mining regions will continue to happen if mining operations continue with the risk of seismic events continuing even after the mines have been decommissioned (Linzer et al. 2007). With increased development in regions of previous mining activity, the risk of damage to structures and infrastructure due to seismic activity is increasing. Also, due to the ever-increasing frequency of large-scale seismic activity within mining districts, the demand for more accurate identification and recording of epicentral locations in mining regions is required (Linzer et al., 2007). An earthquake of magnitude 5.5 occurred on 5 August 2014 in Orkney, South Africa, which is located at around 150 km southwest of Johannesburg. The Orkney earthquake is the largest recorded event to have occurred near a mining town and resulted in one death and damage to private houses (Manzunzu et al., 2017).



Figure 2-9 Structural damage due to the Stilfontein tremor of ML = 5.3 on 9 March 2005 (Linzer et al., 2007)

2.2 PSEUDO-DYNAMIC EXPERIMENTATION

In this section, a brief discussion is presented on the pseudo-dynamic testing technique and its background. Numerous studies have been conducted using the pseudo-dynamic testing method; however, this section only presents essential contributions and provides the relevant workflows and numerical time-stepping algorithms used during the research.

2.2.1 PSEUDO-DYNAMIC METHOD

The pseudo-dynamic testing technique is a computer controlled experimental method whereby the dynamic behaviour of the structure is mathematically calculated on a computer with the resultant displacement being statically imposed on a test specimen of the structure using servo-controlled actuators in an on-line procedure (Mosalam et al., 1997; Xing et al., 2007). Pseudo-dynamic testing emerged as an alternative to shake table testing in the 1960s and 1970s from the research done by Takanashi et al. (1975) as it produced a more controlled testing environment for large and heavy test specimens.

The pseudo-dynamic testing technique uses the same equipment as conventional quasi-static tests; however, the analysis is controlled by a closed loop system comprising of computer software that is integrated and runs in tandem with the quasi-static experiment (Kurt, 2010). Due to the inertia forces being modelled numerically, the test procedure does not need to be undertaken in real time (Takanashi et al., 1987; Pinto et al., 2004; Mosalam et al., 1997). The pseudo-dynamic method utilises well-established step-by-step time integration methods, whereby the calculated deformation is applied to the test specimen at any given time step at a common degree of freedom between the numerical model and the test specimen and the restoring force is measured using a load cell. The force obtained from the load cell is fed back into the computational model, which is used in successive iterations to determine the new deformation. The method utilises the same numerical approach generally undertaken in nonlinear structural dynamics; however, the structural restoring force is based on experimental feedback from load cells as opposed to an idealised hysteretic model (Shing & Mahin, 1984). Typical computational time domain analysis requires the idealisation of the non-linear response of the structure; however, pseudo-dynamic testing enables the true material response of the structure to be directly obtained from a physical model for the duration of the analysis (Mosalam et al., 1997).

The benefit of pseudo-dynamic testing is that it provides a better understanding of the seismic performance of a structure as it incorporates the overall response of the structure, the non-linear

behaviour of the structural component, and the seismic excitation. Thereby, structural damage can be related to earthquake intensity, which is not possible during traditional quasi-static testing. The measured quantities from the actuators, calibrated displacement transducers and load cells are utilised in subsequent calculations, which enables both dynamic effects and progressive damage of the specimen to be observed for the duration of the experiment (Mosalam et al., 1997).

The pseudo-dynamic method comprises of two cycles as shown in Figure 2-10. The first cycle comprises of the calculation cycle, which requires the relevant software and hardware for solving the equation of motion using time-integration numerical methods. The second cycle of the pseudo-dynamic experiment comprises of the loading cycle, which requires the control system consisting of a servo-controlled hydraulic actuator for applying the calculated displacement to the test specimen and a load cell for reading the resultant restoring force (Takanashi et al., 1987).

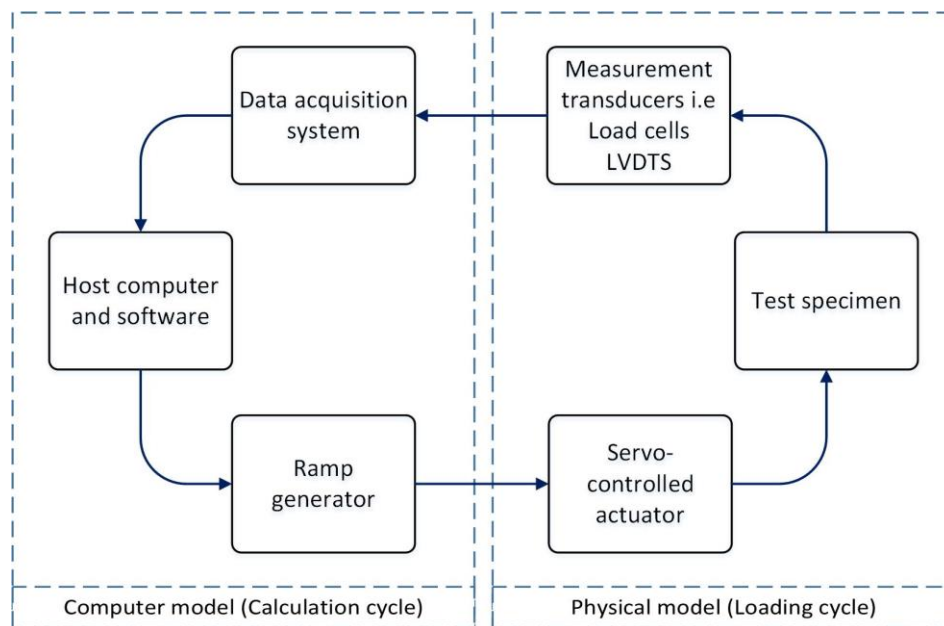


Figure 2-10 Pseudo-dynamic test loop adapted from Mosalam et al. (1997)

The advantages of pseudo-dynamic testing are indicated by Takanashi et al. (1975) as follows:

- The true non-linear restoring force characteristics of the structure with displacement can be considered without the need to assume a non-linear hysteretic model for the member;
- Testing can be performed on large structures including one to one scale structures or substructures with the use of electro-hydraulic actuators and load cells;

- The experiment can be analysed at each time-step, and the failure mechanism can be identified during the analysis. The test can be stopped at any point to inspect the structure and collect data; and
- Gravity loads can be incorporated into the analysis by utilising an actuator that applies the load before the commencement of the test to simulate the true stresses in the member such as axial stress in a column or bending stresses in a beam.

2.2.2 BACKGROUND TO THE PSEUDO-DYNAMIC METHOD

Pseudo-dynamic testing originated approximately forty years ago as an alternative to shake table testing with Takanashi et al. (1975) seen as one of the pioneers of the testing method (Kurt, 2010). Pseudo-dynamic testing is especially efficient when having to test structures that are too heavy or too large to be practically tested on available shake tables (Thewalt & Mahin, 1987). Pioneering work in pseudo-dynamic testing was done by Hakuno in 1969 (as cited by Takanashi et al., 1975) where he tested cantilever beams using an on-line system that comprised of an analogue computer and an electromagnetic actuator. However, the results produced by the test were rather poor due to the limitations of the available hardware. Following from the work that was done on cantilever beams, Takanashi et al. (1975) did substantial work in establishing the pseudo-dynamic technique by replacing the analogue computer with a more accurate digital computer. Modifications done by Takanashi et al. (1975) enabled the procedure to not have to operate in real time, thus producing a procedure that could be subjected to slow loading and pausing. The first pseudo-dynamic tests were restricted to planar test specimens that were subjected to a single horizontal component of base excitation (Takanashi et al., 1975; Shing & Mahin, 1984; Takanashi & Nakashima, 1987). However, the method can be easily adapted to be used to analyse three-dimensional response of structures with several components of base excitation (Thewalt & Mahin, 1987). Multi-degree of freedom testing was done by Chang (2009) whereby he subjected a one-storey frame to bidirectional loading.

The first pseudo-dynamic tests showed that the results were susceptible to measurement and control errors and subsequently resulted in research into error analysis (Takanashi et al., 1975). Numerous pseudo-dynamic experiments were done in Japan with 27 on-line tests being summarised in the journal paper by Takanashi & Nakashima (1987).

Pseudo-dynamic excitation enables the investigation of geometric nonlinearities, three dimensional and multi-support excitations and soils structure interaction all while subjecting the structure to an input earthquake excitation (Mahin et al., 1989). The practicability of pseudo-

dynamic testing was shown in a report published by Mahin and Shing in 1985. Mahin and Shing (1985) demonstrated the pseudo-dynamic test by analysing a cantilever column as a single degree of freedom system with further tests being conducted on a strengthened steel structure. The tests undertaken by Mahin and Shing (1985) comprised of both shake table tests and pseudo-dynamic tests and confirmed that pseudo-dynamic is viable if conducted using well established analytical techniques combined with calibrated and precise loading and recording instruments (Kurt 2010).

Mahin et al. (1989) indicated that a series of investigations were undertaken through coordinated cooperation between the United States and Japan as part of the U.S.-Japan Cooperative Earthquake Research Program to investigate the limitations of the pseudo-dynamic testing method. Udagawa and Mimura (1991) investigated the seismic behaviour of frames with composite beams by using the pseudo-dynamic testing method and using the El Centro 1940 earthquake ground motion. The tests involved performing both cyclic load testing on the frames under constant displacement amplitudes to examine the safety of the frames. They indicated that it is not feasible to perform large-scale true time dynamic analysis using a large-scale servo-controlled hydraulic actuator, although Udagawa et al. (1984), and Takanashi and Udagawa (1989) said that displacement rate could not be neglected.

Mosalam et al. (1998) investigated the response of masonry infill frames using the pseudo-dynamic method and indicated that the pseudo-dynamic method provides an acceptable approximation of the dynamic response of a structure that is subjected to earthquake excitation.

Wang et al. (2006) investigated the response of a base isolated eight-storey building to a large earthquake using an on-line hybrid procedure. The analysis involved using ABAQUS (2003) finite element analysis software to analyse the overall response of an eight-storey superstructure and a physical model comprising of the base isolation layer and surrounding retaining walls. The resultant restoring force obtained from the physical test was input back into ABAQUS (2003) software at each cycle to solve for the resultant displacement.

The explicit Newmark's method was used by Wang et al. (2006) to solve for the unknown displacements at each time-step and was chosen due to its simplicity. The test used sub-structuring whereby part of the structure was modelled numerically with the remaining part of the structure tested in parallel within the laboratory. The test comprised of two models, the numerical model used to simulate the structural dynamics and the other model to obtain the restoring force from the physical model. The on-line hybrid system used by Wang et al. (2006) is shown in Figure 2-11 with their experimental test setup shown in Figure 2-12.

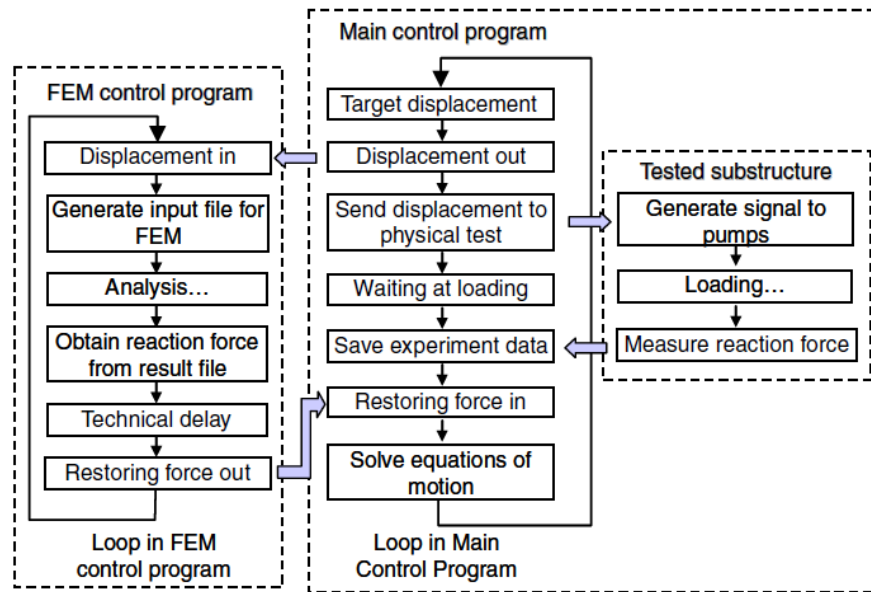


Figure 2-11 On-line hybrid system (Wang et al., 2006)

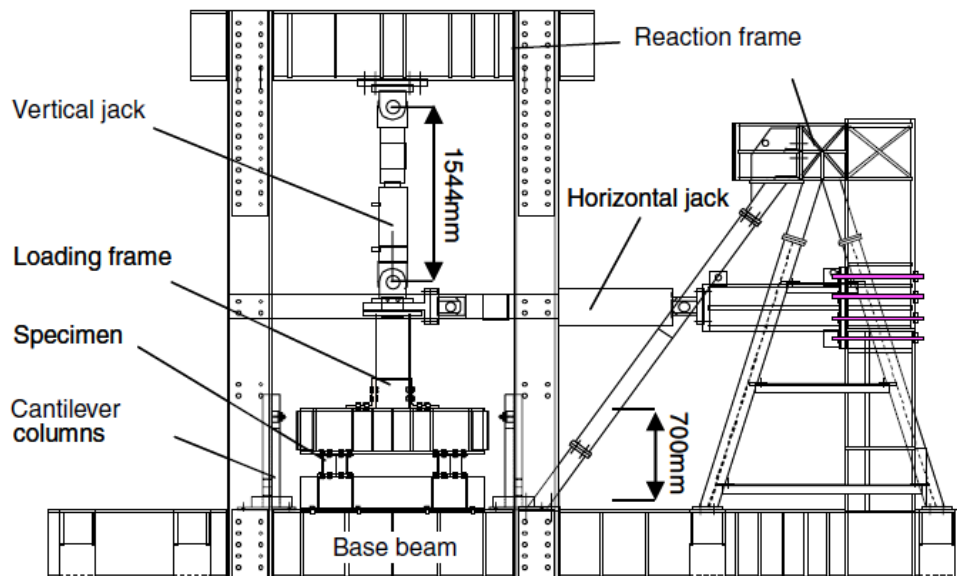


Figure 2-12 Loading system (Wang et al., 2006)

A multi-site hybrid simulation framework was developed at the University of Illinois at Urbana Champaign (Spencer et al., 2007) for piers of a bridge structure and is shown in Figure 2-13. The Ui-SimCor Hybrid Simulation Framework developed by Spencer et al. (2007) showed that the method provides a flexible and powerful method for using the pseudo-dynamic method to evaluate several components of an overall single structure utilising laboratories at three

universities distributed across the United States of America. Spencer et al. (2007) indicated that hybrid-simulation can be a gruelling task as it requires detailed knowledge of both numerical modelling and physical modelling tools, and the programming requirements to integrate the two methods. They indicate that it is necessary to use both numerical modelling and physical modelling to investigate the complex behaviour of reinforced concrete and the influence it has on the overall response of the structure.

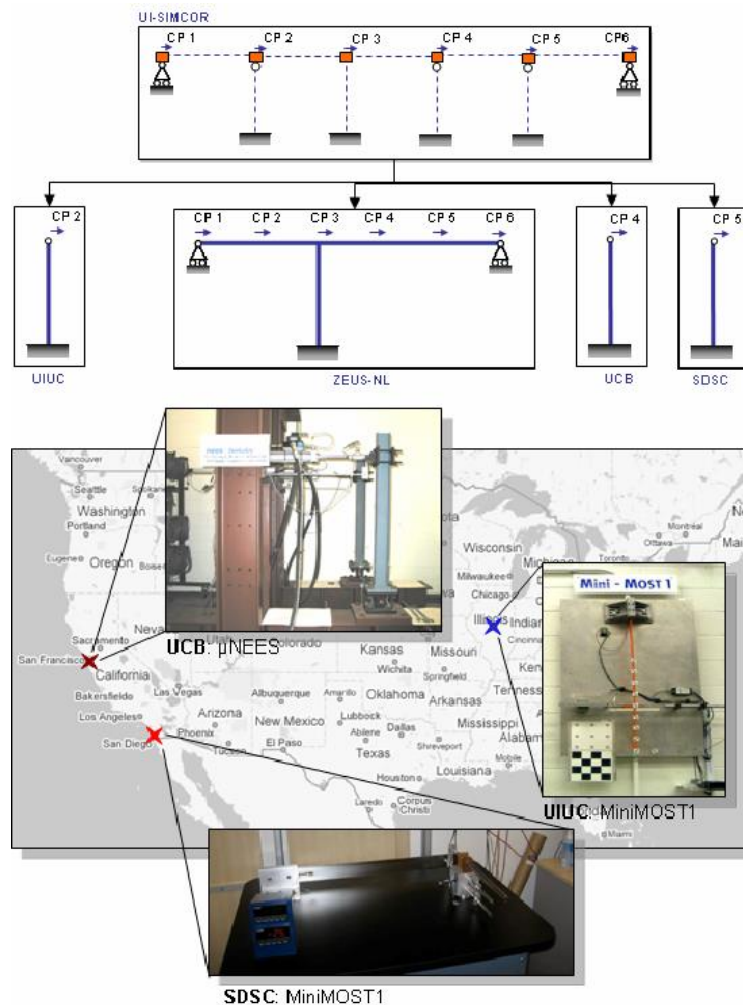


Figure 2-13 Three site hybrid simulation of piers of a bridge (Spencer et al., 2007)

Xing et al. (2017) performed pseudo-dynamic tests on concrete columns under earthquake loading. The column formed part of a four-bay, four storey prototype reinforced concrete planar frame structure and had a fundamental period of vibration of 0.8 s. Rayleigh damping was used to model the energy loss within the frame structure and a damping ratio of 5% was used with the first two modes of vibration. They concluded that the damage encountered by the reinforced

concrete column is dependent on the displacement history of the structure, repeated load cycles, and the maximum displacement experienced by the structural member.

Li et al. (2019) used pseudo-dynamic tests on a two-story, two-bay reinforced concrete frame structures to investigate and compare the damage sustained by the moment resisting frame structure with and without the presence of an Energy-Dissipative Rocking Column (EDRC). The pseudo-dynamic tests were undertaken by solving the governing equation of motion using the central difference time integration method and used a damping ratio of 5%. There experiments showed that cracks occurred with minor concrete spalling at the column base due to the application of Shifang ground motion record produced during the 2008 Wenchuan Earthquake in China with a PGA of 0.55 g. The research undertaken by Li et al. (2019) shows that the pseudo-dynamic method provides an accurate method to relate the damage sustained by a structure with earthquake intensity.

2.2.3 FORMULATION OF PSEUDO-DYNAMIC METHOD

The pseudo-dynamic method was formulated from the time-discretised equation of motion for each time step, i , as shown by Equation 2.2. During pseudo-dynamic experimentation, Equation 2.2 is solved using numerical methods in a stepwise procedure with the restoring force, $\{R_i\}$, measured directly from the test specimen using a load cell (Kurt, 2010). The solution of the second order differential equation can either be solved using implicit or explicit numerical methods to solve for the displacements at each time step.

$$[M]\{\ddot{u}_i\} + [C]\{\dot{u}_i\} + [K]\{u_i\} + \{R_i\} = -[M]\{I\}\ddot{u}_{gi} \quad (2.2)$$

Where:

$[M]$:	Mass matrix
$[C]$:	Viscous damping matrix
$[K]$:	Overall structural stiffness matrix
$\{R_i\}$:	Measured restoring force vector from the test specimen
$\{\ddot{u}_i\}, \{\dot{u}_i\}, \{u_i\}$:	Nodal acceleration, velocities and displacements at time step i
$\ddot{u}_{gi}$:	Ground acceleration at time i
$\{I\}$:	Influence vector

The initial stiffness of the physical structure is generally chosen to be close to the maximum achievable stiffness of the test structure, which is generally the elastic stiffness of the structure. However, to ensure stability, the selection of the initial stiffness needs to be greater or equal to the maximum achievable tangent stiffness of the physical test structure (Pegan & Pinto, 2000).

Takanashi et al. (1975) indicated that the assessment of R_i is the most critical aspect in the analysis to achieve a satisfactory accuracy during the pseudo-dynamic testing. An appropriate numerical analysis time stepping method of enough accuracy is required to predict the incremental restoring force at the current time step utilising the data obtained from the preceding time step. Several numerical schemes are available to formulate a time-stepping approximation to the governing equation of motion as previously shown in Equation 2.2. The numerical schemes can be separated into purely explicit numerical methods such as the central difference method, which relies entirely on the results of the previous time-step, or purely implicit methods such as the Newmark's method, which relies on information of the previous time-step as well as information in the present time-step. Solving the equation of motion using explicit numerical integration methods can result in stability issues but was traditionally preferred over implicit methods as it eliminated the need for iteration (Mosalam et al., 1997). A mixture of implicit and explicit methods is also available, with an example being the operator splitting method (Mosalam et al., 1997). Thewalt and Mahin (1987) proposed a hybrid approach that uses the available experimental data, which includes both a digital computer and analogue voltage signals and summing amplifiers to solve the equation of motion.

In early applications of the pseudo-dynamic method, the difference equation was solved using the linear acceleration method. However, the method only produced reasonable results for flexible and straightforward structural systems (Takanashi & Nakashima, 1987). The linear acceleration method had issues in producing accurate results for stiff systems and systems with rapid changes in stiffness when estimating the instantaneous stiffness due to the accuracy limits of the measurement instruments. Takanashi et al. (1975) solved these issues by employing the central difference method, which mitigated the need for measuring the tangent stiffness at each increment and having to not solve the equation of motion in incremental form.

The central difference method enabled the direct input of the force reading produced by the load cell to be used in the equation of motion. Figure 2-14 shows the basic routine followed by Takanashi and Nakashima (1987) during the pseudo-dynamic method whereby the computed displacement is converted from a digital signal to an analogue signal using a digital to analogue converter (D/A). The analogue signal is then used to apply the displacement onto the structure and a force reading is taken using the load cell. The reading from the load cell is taken in the

form of an analogue signal and is converted to a digital signal using an analogue to digital converter (A/D), which is subsequently fed back into the computer and used in subsequent calculations.

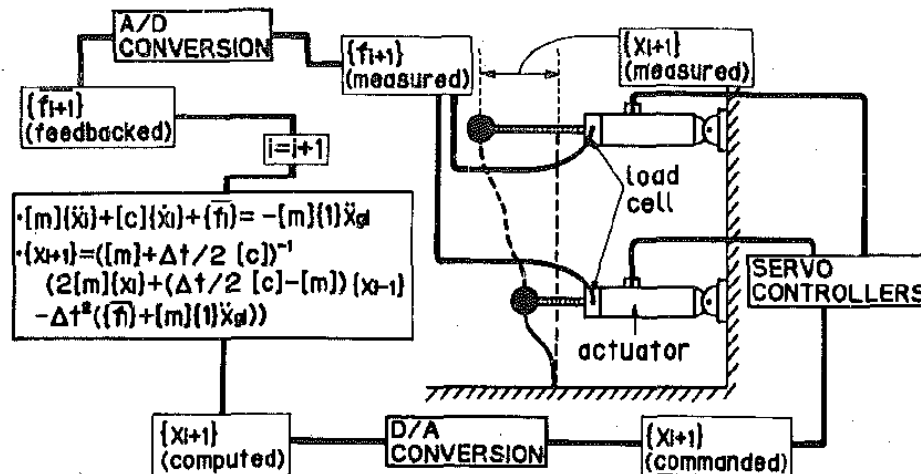


Figure 2-14 On-line algorithm adopted by Takanashi and Nakashima (1987)

Bursi and Shing (1996) discuss various implicit time stepping methods for non-linear problems, which includes the modified Newton iteration method. The modified Newton's method requires that the predictor stiffness $[K]$ used in the analysis to be greater than the actual stiffness of the structure. Mosalam et al. (1997) implemented the predictor-corrector algorithm, which produced an adequate control of experimental error propagation. The method enabled careful inspection and documentation of the complex cracking of infill masonry walls.

2.2.4 INHERENT LIMITATIONS AND ERRORS

Every test method involves a series of inherent limitations, and therefore several assumptions need to be made when performing pseudo-dynamic tests (Kurt, 2010). The first assumption that needs to be made is whether the structure can be accurately and precisely solved using the equation of motion and the resultant displacements can be applied to the structure with enough accuracy (Mahin et al., 1989). Mahin et al. (1989) described several limitations inherent to pseudo-dynamic testing that relates to the way the structure is idealised, damping effects within the structure and strain rate effects.

Assumptions need to be made on how the structure is discretised into a finite number of degrees of freedom and how the mass is subsequently lumped at the degrees of freedom. Damping effects are incorporated into the structure by assuming viscous damping, however, how energy is dissipated within the structure is complex (Mahin et al., 1989).

The rate of the load applied to the structure during a pseudo-dynamic test is much slower than real time dynamic tests and excludes time-dependent material effects, such as strain rate effects in the reinforcing steel. The time it takes for each time step can average at around 1 second resulting in tests being 100 times longer than the actual earthquake record, which can influence the response of structures subjected to impulse loading and short period structures (Mahin et al., 1989). Shing and Mahin (1988) investigated the rate of loading effects during pseudo-dynamic tests by comparing the response of an elastic/viscoplastic system with rate dependent characteristics with that of previous studies on the dynamic yield strength of mild steel. They showed that the dynamic response of mild steel had a 30% higher yield strength in comparison to that obtained during the pseudo-dynamic test.

Shing and Mahin (1990) investigated the experimental error effect in pseudo-dynamic testing by evaluating the error propagation characteristics of several algorithms. They found that the extent of error propagation depends on the numerical properties of the algorithm and the frequency characteristics of the test specimen. Response errors during the pseudo-dynamic testing are due to inherent errors in the displacement control system and the measurement system (Udagawa & Mimura, 1991).

The results obtained from the pseudo-dynamic experimentation are susceptible to the accuracy of the recording instruments and the numerical method used to solve the equation of motion (Mosalam et al., 1997). Several researchers investigated the reliability of the pseudo-dynamic method due to the potential of cumulative errors developing for the duration of the earthquake record. Calibration of the instruments needs to be ensured due to the sensitivity of the analysis to experimental error (Mosalam et al., 1997). The procedure employed during the pseudo-dynamic method can produce three significant sources of error, as shown by Shing & Mahin (1984):

- The reliability of the analytical techniques employed. The analogy of a discrete system does not necessarily account for the actual dynamic response of the continuous system. The prescribed damping may be an overly idealised energy dissipating mechanism;

- The numerical method employed can only produce an approximate solution to the equation of motion. The introduced numerical errors may result in the distortion of the actual dynamic response of the system; and
- Feedback errors from the experimental equipment. The errors introduced into the analysis through displacement control and restoring force feedback are inherently cumulative.

Despite the prospect of errors being introduced into the analysis, experiments done at the University of California Berkeley indicated that the pseudo-dynamic test method could be as reliable and realistic as shake table testing (Shing & Mahin, 1984). Mosalam et al. (1997) indicate that the recent trend in pseudo-dynamic testing is to use implicit numerical methods, such as Newmark's method, because of its superior stability properties.

2.3 IMPLICIT NEWMARK TIME INTEGRATION METHOD

The preference in the past has traditionally been towards explicit numerical methods because of the disinclination to numerical iteration at each time step. However, advancements in computational power and the increase in the resolution of computers have made using implicit numerical methods more favourable due to the superior stability properties it provides (Mosalam et al., 1997). Newmark's implicit method has been widely adopted in finite element analysis software to solve non-linear problems as it can be unconditionally stable for any time increment (Chopra, 2012). Therefore, the selection of the time increment only influences the accuracy of the solution and not the stability.

Newmark's method was developed in 1959 by N.M. Newmark and formulated from Equation 2.3 and Equation 2.4 (Chopra, 2012). The selection of the parameters γ and β determine the accuracy and stability of the solution. The average acceleration method has factors $\gamma = 1/2$ and $\beta = 1/4$ and results in an implicit and unconditionally stable solution (Chopra, 2012). A constant acceleration within the time interval $t \in [t_i, t_{i+1}]$ is presumed and is graphically shown in Figure 2-15.

$$\dot{u}_{i+1} = \dot{u}_i + [(1 - \gamma)\Delta t]\ddot{u}_i + (\gamma\Delta t)\ddot{u}_{i+1} \quad (2.3)$$

$$u_{i+1} = u_i + (\Delta t)\dot{u}_i + [(0.5 - \beta)(\Delta t)^2]\ddot{u}_i + [\beta(\Delta t)^2]\ddot{u}_{i+1} \quad (2.4)$$

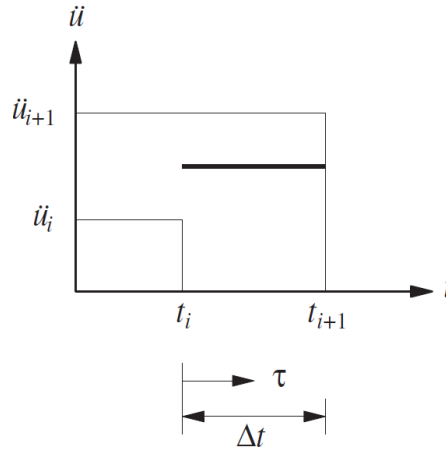


Figure 2-15 Newmark's constant average acceleration method (Chopra, 2012)

Newmark's method is an implicit numerical method, as it determines the solution at time $i + 1$ from the equilibrium condition at the same time $i + 1$ and therefore produces an equation whereby the resisting force $(f_s)_{i+1}$ is an unknown function of the unknown displacement u_{i+1} . As a result, Newton-Raphson iteration method is used to solve for the unknown force $(f_s)_{i+1}$ at time $i + 1$. The derivation of the Newton-Raphson method is obtained by expanding the resisting force $(f_s)^{j+1}$ in Taylor series about the known estimate $u^{(j)}$, with the full derivation shown in Chopra (2012).

The objective of the Newton-Raphson method is to solve for Equation 2.5 by reducing the force $R^{(j)}$ to zero, which is shown in Equation 2.6. The tangent stiffness at $u^{(j)}$ is given by $k_T^{(j)} = \frac{\partial f_s}{\partial u} \Big|_{u^{(j)}}$ and by solving Equation 2.6 provides the $\Delta u^{(j)}$, which produces a better estimate of the resultant displacement in Equation 2.7. The process continues by incrementally increasing j within the time step i until the residual R is less than a specified tolerance at which point the next time step $i + 1$ is initialised (Chopra, 2012). Figure 2-16 shows graphically the Newton-Raphson iteration procedure that iterates within each time step i until convergence to a solution.

$$f_s(u) = P \quad (2.5)$$

$$k_T^{(j)} \Delta u^{(j)} = P - f_s^{(j)} = R^{(j)} \quad (2.6)$$

$$u^{(j+1)} = u^{(j)} + \Delta u^{(j)} \quad (2.7)$$

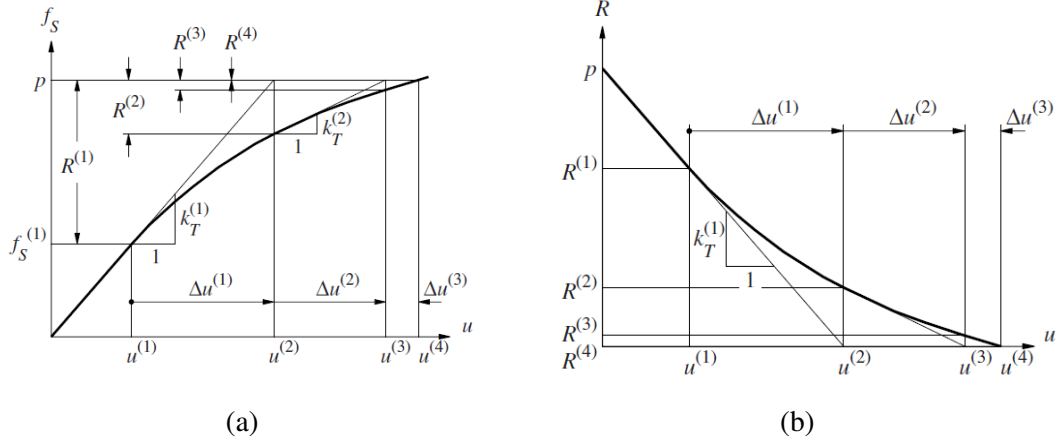


Figure 2-16 Newton-Raphson iteration procedure for time step i showing (a) convergence to the resultant force and (b) the residual force converging to zero (Chopra, 2012)

Newmark's method extends the Newton-Raphson method from a static problem to a dynamic problem by setting the resulting force, as previously shown in Equation 2.2, equal to the summation of the forces due to inertia, damping and stiffness within the structure and results in the formation of Equation 2.8 and Equation 2.9. The tangent stiffness $k_T^{(j)}$ for the nonlinear equilibrium of the dynamic problem is given by Equation 2.10 with the full derivation of the dynamic tangent stiffness provided in Chopra (2012). The resultant Newton-Raphson iteration method for a dynamic system is given by Equation 2.11 with the residual force vector being calculated using Equation 2.12.

$$(\hat{f}_s)_{i+1} = p_{i+1} \quad (2.8)$$

$$(\hat{f}_s)_{i+1} = m\ddot{u}_{i+1} + c\dot{u}_{i+1} + (f_s)_{i+1} \quad (2.9)$$

$$(\hat{k}_T)_{i+1}^{(j)} \equiv \frac{\partial \hat{f}_s}{\partial u_{i+1}} = (k_T)_{i+1}^{(j)} + \frac{\gamma}{\beta \Delta t} c + \frac{1}{\beta (\Delta t)^2} m \quad (2.10)$$

$$(\hat{k}_T)_{i+1}^{(j)} \Delta u^{(j)} = p_{i+1} - (\hat{f}_s)_{i+1}^{(j)} = \hat{R}^{(j)}_{i+1} \quad (2.11)$$

$$\begin{aligned} \hat{R}^{(j)}_{i+1} = & p_{i+1} - (f_s)^{(j)}_{i+1} - \left[\frac{1}{\beta (\Delta t)^2} m + \frac{\gamma}{\beta (\Delta t)} c \right] ((f_s)^{(j)}_{i+1} - u_i) + \\ & \left[\frac{1}{\beta (\Delta t)} m + \left[\frac{\gamma}{\beta} - 1 \right] c \right] \dot{u}_i + \left[\left(\frac{1}{2\beta} - 1 \right) m + \Delta t \left(\frac{\gamma}{2\beta} + 1 \right) c \right] \ddot{u}_i \end{aligned} \quad (2.12)$$

The adapted iteration technique using the Newton-Raphson method is similar in form to the static solution except the damping and inertia terms are included in both the residual force, $\hat{R}$, and the tangent stiffness k_T . Table 2-2, adopted from Chopra (2012), summarises the Newmark's method for the solution of the non-linear system. Step 3.6 in Table 2-2 requires the input of a hysteretic model that relates the calculated displacement $u_{i+1}^{(j+1)}$ to resultant force $(f_s)_{i+1}^{(j+1)}$, however, as previously discussed in Section 2.2, the calculated displacement $u_{i+1}^{(j+1)}$ in Step 3.5 is applied directly to the structure using a servo-controlled hydraulic actuator with the resultant force read from a calibrated load cell and fed back into the numerical model in Step 3.6. This circumvents the need to assume a hysteretic model and structural stiffness (Takanashi & Nakashima, 1987).

Table 2-2 Newmark's method for the solution of nonlinear systems (Chopra, 2012)

Special cases	
(1)	Average acceleration method ($\gamma = \frac{1}{2}, \beta = \frac{1}{4}$)
1.0	Initial conditions
1.1	State determination $(f_s)_0$ and $(k_T)_0$
1.2	$\ddot{u}_0 = \frac{p_0 - c\dot{u}_0 - (f_s)_0}{m}$
1.3	Select Δt
1.4	$a_1 = \frac{1}{\beta(\Delta t)^2}m + \frac{\gamma}{\beta\Delta t}c; \quad a_2 = \frac{1}{\beta\Delta t}m + \left(\frac{\gamma}{\beta} - 1\right); \quad a_3 = \left(\frac{1}{2\beta} - 1\right)m + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)c$
2.0	Calculations for each time instant $i = 0, 1, 2, \dots$
2.1	Initialise $j = 1, u_{i+1}^{(j)} = u_i, (f_s)_{i+1}^{(j)} = (f_s)_i$, and $(k_T)_{i+1}^{(j)} = (k_T)_i$
2.2	$\hat{p}_{i+1} = p_{i+1} + a_1 u_i + a_2 \dot{u}_i + a_3 \ddot{u}_i$
3.0	For each iteration, $j = 1, 2, 3 \dots$
3.1	$3.1 \hat{R}_{i+1}^{(j)} = \hat{p}_{i+1} - (f_s)_{i+1}^{(j)} - a_1 u_{i+1}^{(j)}$
3.2	3.2 Check convergence: If the acceptance criteria are not met, implement steps 3.3 to 3.7, otherwise, skip these steps and go to step 4.0
3.3	$(\hat{k}_T)_{i+1}^{(j)} = (k_T)_{i+1}^{(j)} + a_1$
3.4	$\Delta u^{(j)} = \hat{R}_{i+1}^{(j)} \div (\hat{k}_T)_{i+1}^{(j)}$
3.5	$u_{i+1}^{(j+1)} = u_{i+1}^{(j)} + \Delta u^{(j)}$
3.6	State determination: $(f_s)_{i+1}^{(j+1)}$ and $(k_T)_{i+1}^{(j+1)}$ (Pseudo-dynamic loading cycle onto the test specimen) Replace j by $j + 1$ and repeat steps 3.1 to 3.6; denote final value as u_{i+1}
4.0	Calculations for velocity and acceleration
4.1	$\dot{u}_{i+1} = \frac{\gamma}{\beta\Delta t}(u_{i+1} - u_i) + \left(1 - \frac{\gamma}{\beta}\right)\dot{u}_i + \Delta t\left(1 - \frac{\gamma}{2\beta}\right)\ddot{u}_i$
4.2	$\ddot{u}_{i+1} = \frac{1}{\beta(\Delta t)^2}(u_{i+1} - u_i) - \frac{1}{\beta\Delta t}\dot{u}_i - \left(\frac{1}{2\beta} - 1\right)\ddot{u}_i$
5.0	Repetition for the next time step. Replace i by $i + 1$ and implement steps 2.0 to 4.0 for the next time step.

2.4 FUNDAMENTAL PERIOD OF VIBRATION OF A STRUCTURE

The response of a structure to a given ground motion excitation is dependent on several characteristics inherent to the structure. The performance of the structure during an earthquake depends on the ability of the structure to absorb and dissipate energy including the frequency characteristics of the earthquake that correspond with the frequency characteristics of the structure (Chopra, 2012).

By solving Equation 2.13, with the stiffness matrix and mass matrix known, the scalar values ω_n^2 and vector Φ_n can be obtained. This produces n homogeneous algebraic equations for n modes of vibration. The natural period of vibration is determined by solving the eigenvalues for the non-trivial solution in Equation 2.13 by setting the determinate equal to zero as shown in Equation 2.14. The n roots, ω_n^2 , are the natural frequencies of vibration of the structure and when ordered from smallest to largest, the smallest value is known as the fundamental frequency of vibration. The fundamental period of vibration is the typical vibration mode that is excited during a seismic event (Chopra, 2012).

$$[K] - \omega_n^2[M]\Phi_n = \{0\} \quad (2.13)$$

$$\det[K - \omega_n^2[M]] = 0 \quad (2.14)$$

2.4.1 BUILDING PERIOD FORMULAS

The fundamental period of vibration of structures appears in design codes for the calculation of the design base shear and lateral forces. Before completing the design of the structure, the fundamental period of vibration is unknown and therefore to circumvent this predicament, building codes provide empirical formulas that enable the estimation of the fundamental period of vibration for various building types and structural materials. Figure 2-17 provides historical fundamental periods of vibration from a database of 2622 Chilean buildings and shows that most fundamental periods for buildings show a linear relationship with increasing structural height (Lagos & Kupfer, 2012). Regression analysis of the database of the recorded fundamental period was used to develop empirical formulas for reinforced concrete moment resisting frame structures and steel moment resisting frame structures.

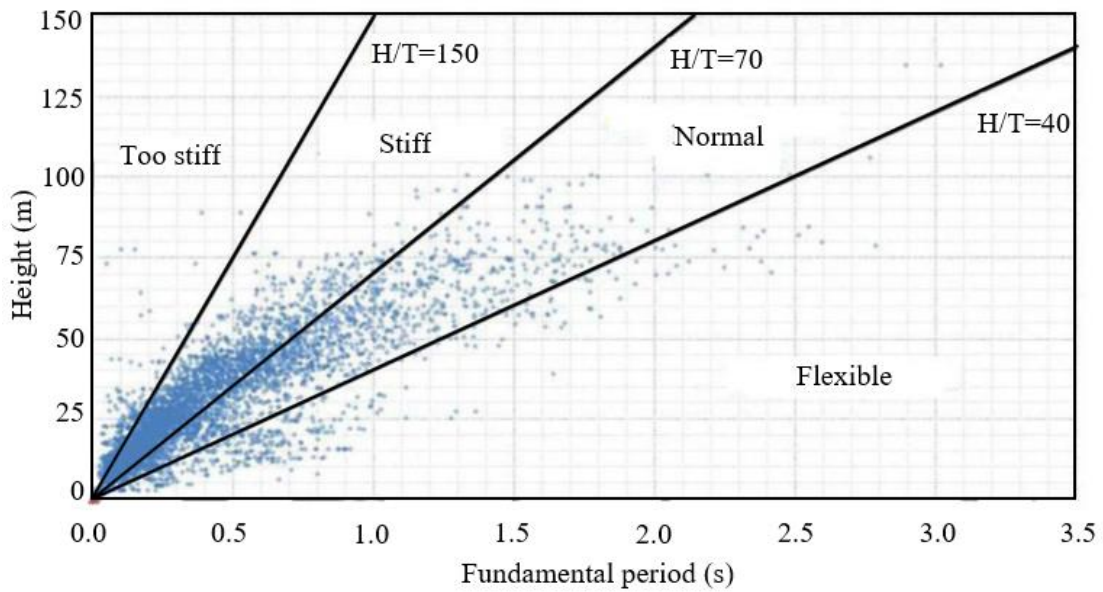


Figure 2-17 Comparison between fundamental periods and height of structures H for 2622 Chilean Buildings (Lagos & Kupfer, 2012)

SANS 10160-4:2017 provides building period formulas that can be used to estimate the fundamental period of vibration of a steel frame building, reinforced concrete moment resisting frame structures and other building types. Equation 2.15 enables the estimation of fundamental period of vibration of a steel frame moment resisting structures and Equation 2.16 provides the estimation of the fundamental period of vibration of reinforced concrete moment resisting frame structures as a function of the height of the structure. However, Goel and Chopra (1996) concluded that code formulas for concrete and moment resisting frame structures typically produce lower period values in comparison to measured periods.

$$T = 0.085 \times h_t^{3/4} \quad (2.15)$$

$$T = 0.075 \times h_t^{3/4} \quad (2.16)$$

2.5 RAYLEIGH DAMPING

Rayleigh damping provides a method of dissipating energy during a linear elastic structural analysis that is subjected to seismic loads (Hall, 2006) and can be used to account for energy loss in the linear computational portion of the pseudo-dynamic analysis. Numerical models used to solve vibrating structures consider three sources of energy dissipation, which includes energy dissipated through hysteretic non-linear material behaviour, energy radiation and damping in the structure (Hall, 2006). Incorporating damping into a linear elastic analysis is

necessary as it accounts for complex non-linear behaviour of the structure that would otherwise be neglected. The mechanism by which the energy of a vibrating structure can be dissipated in a linear system is accounted for by equivalent viscous damping (Chopra, 2012). The viscous damping matrix is dependent on the distribution of stiffness within the structure, the mass of the structure and the natural modes of vibration of the structure (Park & Hashash, 2004).

Rayleigh damping is used to account for energy dissipation in a multi-degree of freedom system during a linear analysis and was proposed by Rayleigh and Lindsay (as cited in Park & Hashash, 2004). The damping matrix is assumed to be proportional to the mass matrix $[M]$ and the stiffness matrix $[K]$, using constants a_0 and a_1 as shown in Equation 2.17.

$$[C] = a_0[M] + a_1[K] \quad (2.17)$$

The constants a_0 and a_1 are calculated using Equation 2.18 and have units s^{-1} and s , respectively. Two significant modes are selected to solve for the constants and are assigned a damping coefficient. The damping coefficient assigned to each mode of vibration is equal as the damping coefficient is known to be frequency independent (Park and Hashash, 2004). The damping ratio is given by ζ at mode i with the damping ratio being the ratio of the mode's damping to critical damping (Hall, 2006). The natural frequency at a mode is given by ω_i . As can be seen from Figure 2-18, the damping ratio is only true at the two selected frequencies, with the damping typically being greater at frequencies less than ω_i and greater than ω_j . Frequencies between ω_i and ω_j tend to result in damping ratios less than the selected damping ratio ζ .

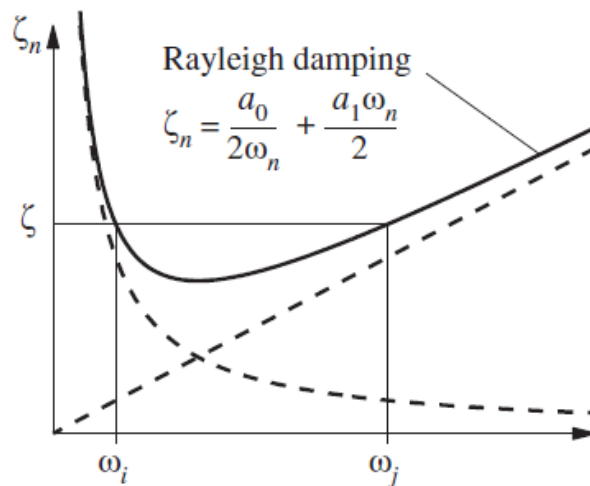


Figure 2-18 Rayleigh damping (Chopra, 2012)

$$\begin{bmatrix} \zeta \\ \zeta \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} \quad (2.18)$$

2.6 HYSTERETIC BEHAVIOUR OF REINFORCED CONCRETE COLUMNS

With the increase in world population and the subsequent increase urbanisation, more people and properties are at risk of impending natural hazards. As a result, the performance of reinforced concrete structures to natural hazard has gained research popularity amongst engineers to mitigate the devastating economic results and loss of life due to natural hazards such as earthquakes. Therefore, the prediction of the hysteresis behaviour of reinforced concrete is critical to understanding the performance of it during a seismic event (Sengupta & Li, 2017).

Over the last 50 years, hundreds of laboratory tests have been undertaken to determine the hysteretic behaviour of structural components for earthquake conditions (Chopra, 2012). Figure 2-19 shows the cyclic response of reinforced concrete and shows that the initial loading provides three stages of stiffness degradation for reinforced concrete. The figure also shows pinching effect upon unloading, which is evident by the reduction in stiffness as the load approaches zero. The monotonic loading response of reinforced concrete column forms the upper bound during a strong seismic excitation (Penelis & Kappos, 2010).

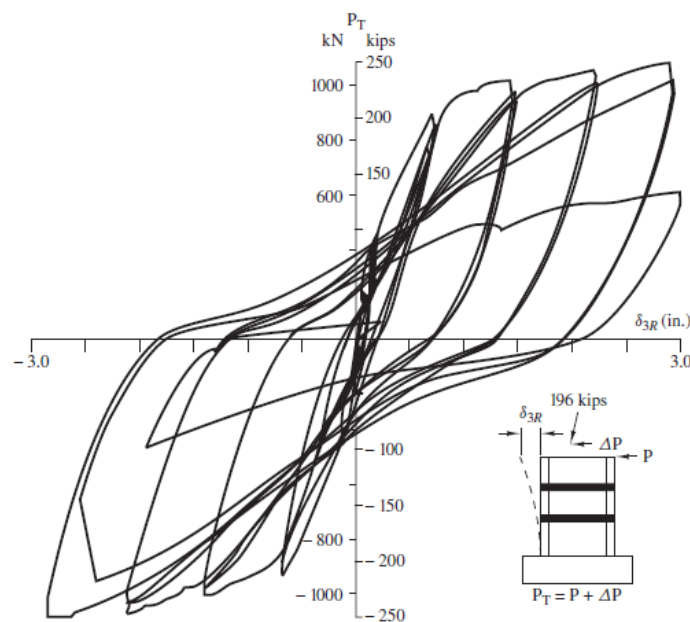


Figure 2-19 Force-deformation relationship for reinforced concrete (Chopra, 2012)

Low and Moehle (1987) did experimental studies on reinforced concrete columns subjected to multi-axial cyclic loading. Their study involved testing cantilever columns projecting from stiff foundation blocks and subjecting them to uniaxial cyclic lateral load histories with a constant axial load on the columns. The load-deflection curve for the constant axial load using imperial units is shown in Figure 2-20. From Figure 2-20, three regions of varying stiffness can be observed with the first corresponding to loading before flexural cracking, the second region being before reinforcement yielding but post the onset of cracking, and the final region is the yielding of the reinforcing bars. The hysteretic response as shown in Figure 2-20 is typical for reinforced concrete columns that are subjected to axial loads and do not encounter significant shear or anchorage weakening (Low & Moehle, 1987).

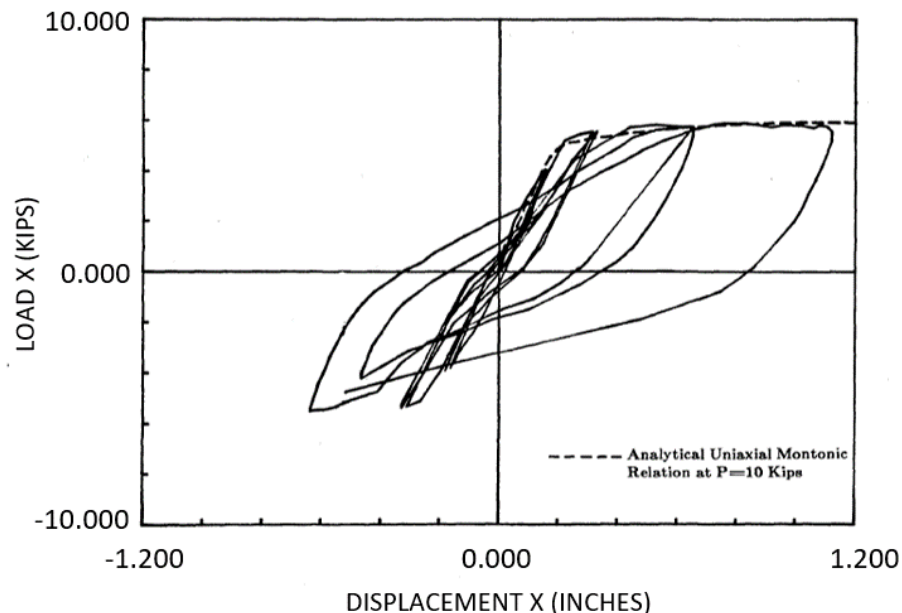


Figure 2-20 Lateral load versus lateral displacement for a column subjected to constant axial load (Low & Moehle, 1987)

The degradation response of reinforced concrete columns subjected to reversed cyclic loading is influenced by the applied axial loading on the column, which may or may not be favourable (Penelis & Kappos, 2010). Axial loading has the benefit of closing flexural and shear cracks in the concrete column. Penelis & Kappos (2010) indicated that the increase in axial load results in an increase in column stiffness and hysteresis loops with larger widths as shown in Figure 2-21. The flexural strength of a column varies substantially with the magnitude of the axial load and is related to the M-N interaction diagram for the section under consideration.

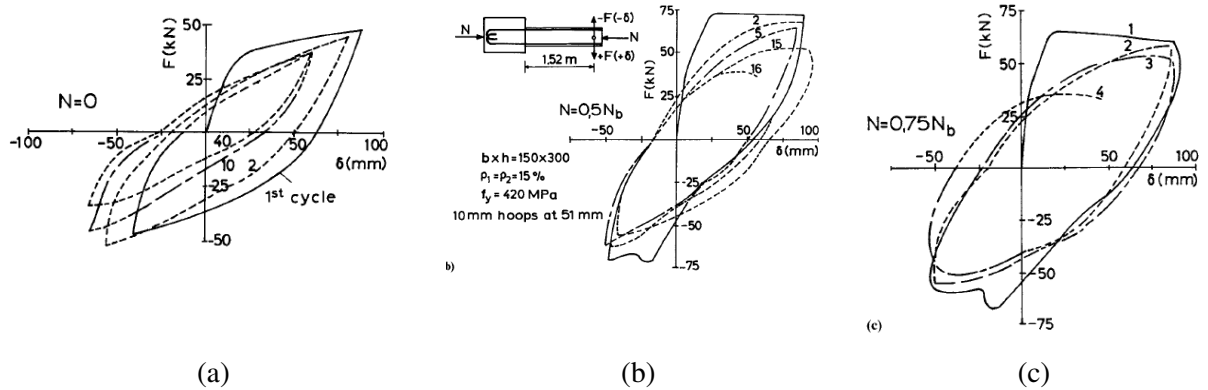


Figure 2-21 Hysteretic behaviour of elements with different levels of axial load (Penelis & Kappos, 2010)

2.6.1 CONFINEMENT AND DUCTILITY OF REINFORCED CONCRETE

The ability of reinforced concrete to undergo extensive plastic deformation enables the structural member to absorb more energy without a sudden and catastrophic collapse. Seismic design of reinforced concrete structures typically relies on the yielding of the steel reinforcement to provide for energy dissipation due to large deformations of the structure. Energy absorption and dissipation are fundamental for a structure to survive an earthquake and therefore ductility is critical in ensuring that a structure can sustain large plastic deformations before failure (Robbarts & Marshall, 2010; Elmenhawawi & Brown, 2009; Penelis & Kappos, 2010). However, uncertainty exists between the amount of ductility prescribed in design codes and actual ductility experienced during seismic events (Xing et al., 2017).

Figure 2-22 shows the stress-strain behaviour of reinforced concrete with different levels of confinement. Figure 2-23 shows the influence that rectangular hoops have on the concrete confinement response and the resultant stress concentrations that develop at the corners of the rectangular hoops (Penelis & Kappos, 2010). Parts of the concrete section produce zero confinement due to the outward deflection at the centre between bends of the rectangular hoop legs, as can be seen in Figure 2-23. Reinforced concrete columns with adequate longitudinal and lateral reinforcement can develop large amounts of ductility with a subsequent increase in strength (Cusson & Paultre, 1994; Penelis & Kappos, 2010). Confinement is initiated by the formation of internal bond cracks between the aggregates and the mortar, which increases the volume of the element (Kent and Park, 1971). Confinement has two critical advantages as it increases the strength of the concrete and increases the ductility of the concrete to strain values exceeding 0.35%, which is accepted as the maximum strain before failure in most codes (Penelis & Kappos, 2010).

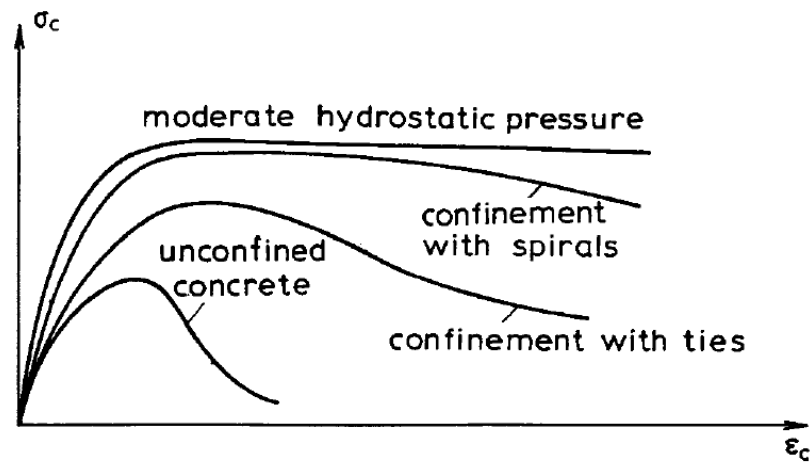


Figure 2-22 Stress-strain diagrams for concrete with different types of confinement (Penelis & Kappos, 2010)

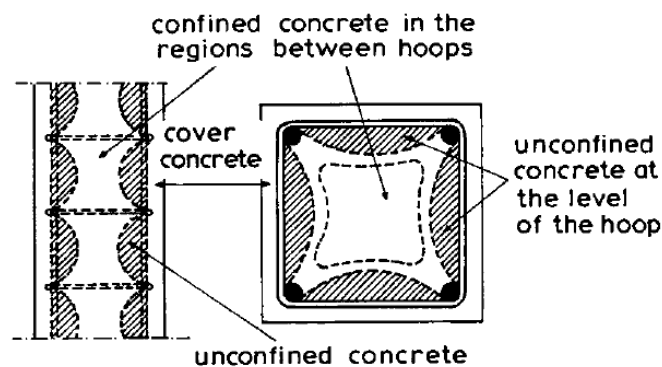


Figure 2-23 Influence of rectangular hoops on concrete confinement (Penelis & Kappos, 2010)

Kent and Park (1971) developed one of the first models for confined reinforced concrete models that only considered the increase in ductility due to rectangular confining steel without considering the increase in concrete strength. Figure 2-24 shows the results produced by Kent and Park (1971) for confined reinforced concrete with rectangular hoops. Passive confinement is provided to concrete in the form of closely spaced spirals or hoops and results in the concrete becoming confined once the stresses in the concrete approaches the uniaxial strength (Kent and Park, 1971). The Kent and Park (1971) model was modified by Scott et al. (1982) and Park et al. (1982) to account for concrete strength and ductility by considering confinement and the effect of strain rate. Mander et al. (1988) developed a model for confined concrete subjected to uniaxial compressive loading and confined by stirrups. The Mander et al. (1988) model considers various means of confining concrete and allows for cyclic loading and strain rate

effect. The main parameters that influence the response of confinement in concrete are (Penelis & Kappos, 2010):

- The ratio of transverse reinforcement or volumetric ratio, ρ_w , defined as the ratio between the volume of hoops to the volume of the confined core;
- The yield strength of the transverse reinforcement, which results in an increase in confinement with an increase in strength;
- The compressive strength of concrete, with higher strength concrete being less ductile than lower strength concrete;
- The spacing of the hoops, with an increase in confinement resulting due to a reduction hoop spacing;
- The hoop pattern; and
- The longitudinal reinforcement.

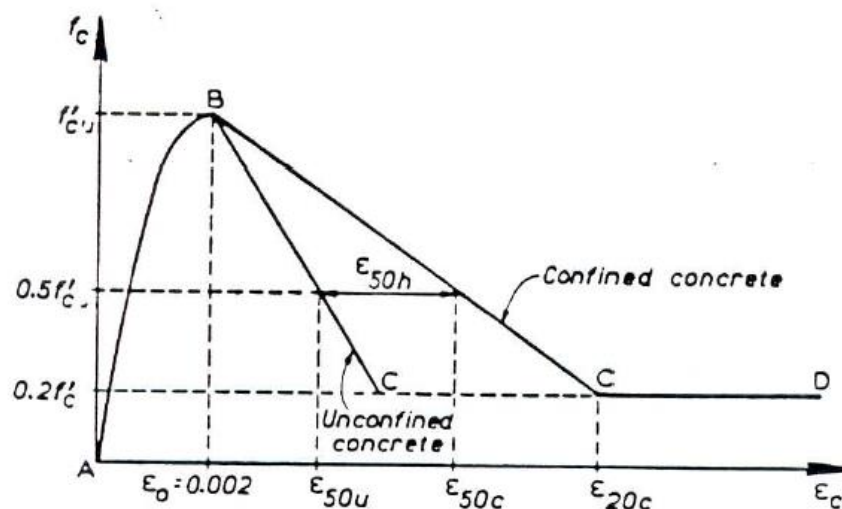


Figure 2-24 Proposed stress strain relationship for confined and unconfined concrete (Kent and Park, 1971)

Stress-strain models for confined concrete have been developed by Saatcioglu and Razvi (1992) and were based on a series of experimental tests on reinforced concrete columns. The proposed stress-strain relationship is given in Figure 2-25 with one of the experimental results shown in Figure 2-26. Concrete without any reinforcement that is subjected to uniaxial compressive load shows a brittle failure mechanism, however, confined concrete shows a substantial improvement in deformability (Cusson & Paultre, 1994). From Figure 2-26 the column can undergo a significant increase in deformation before failure with failure occurring at a strain of 4%, however, the decrease in strength with increase in strain is an indication that the concrete

has spalled. The 4% failure strain, as shown in Figure 2-26, is significantly more than the 0.35% ultimate compressive strain as indicated in the South African standard for the design of structural concrete (SANS 10100-1, 2000), which indicates that a structure with adequate confining reinforcement can sustain much larger deformations before collapse. During inelastic load reversals, the ductility capacity of a structural element generally reduces due to an increase in shear deformation and bond deterioration (Park et al., 1984).

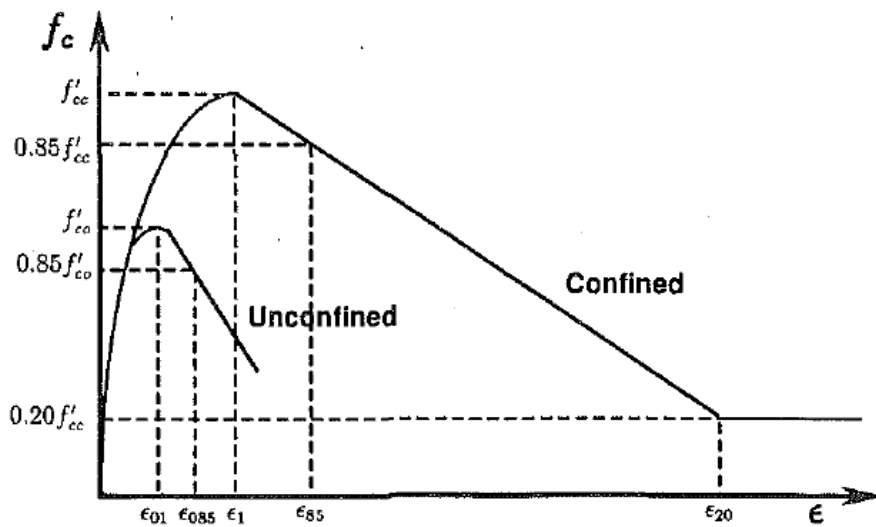


Figure 2-25 Proposed stress-strain relationship as given by Saatcioglu and Razvi (1992)

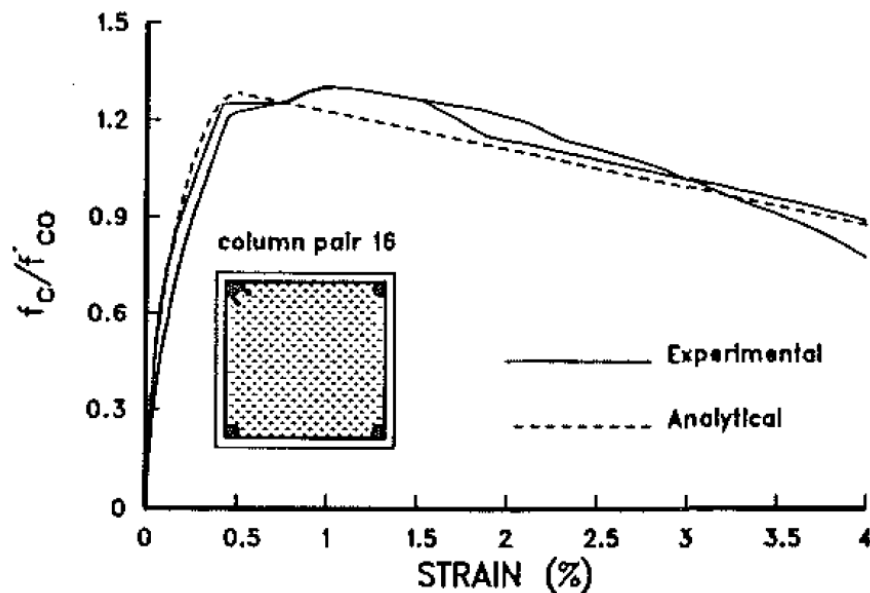


Figure 2-26 Normalised stress-strain diagram of a square column as tested by Razvi & Saatcioglu (1989)

2.6.2 PINCHING EFFECT IN REINFORCED CONCRETE

The pinching effect is commonly observed in the hysteretic response of reinforced concrete structures during the application of repeated cyclic loading, which is common during an earthquake (Yu et al., 2016). The pinching effect shown in the hysteretic loops is related to crack closure, shear crack sliding, shear lock, delayed closure of two cracked surfaces, slippage of reinforcement embedded in the concrete and concrete crushing to name a few. The pinching effect has been shown in experimental studies to result due to previous loadings and is revealed by a reduction in stiffness during reloading (Yu et al., 2016). Figure 2-27 shows the pinching effect in reinforced concrete and is characterised by the large reduction in stiffness with unloading and the subsequent increase in stiffness upon reloading once the cracks have closed in the direction of reloading.

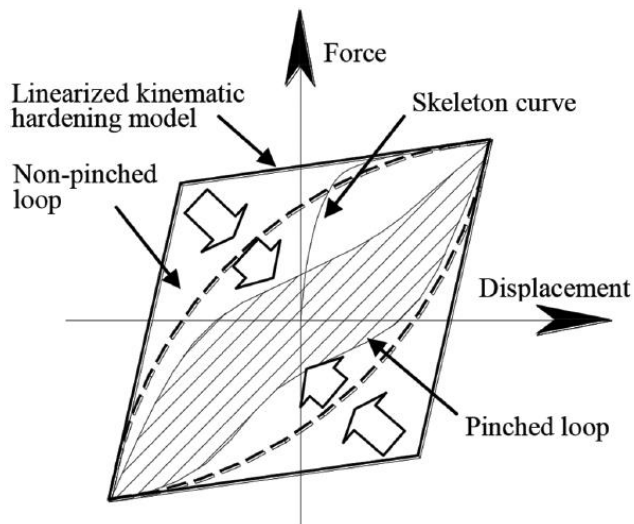


Figure 2-27 Pinching in reinforced concrete under cyclic loading (Yu et al., 2016)

Yu et al. (2016) undertook a combination of experimental tests and finite element modelling to investigate the mechanism of pinching in reinforced concrete columns. The contribution of the rebar and concrete to the pinching effect was analysed and found that the concrete results in a larger contribution to pinching than the reinforcement. Yu et al. (2016) attributes the delay in crack closure as one of the main contributions to pinching. The counteraction between material behaviour between concrete and reinforcement also plays a role in the formation of the pinching effect. During the unloading stage during cyclic loading, the forces in the steel may vary rapidly from tensile yielding to compressive yielding, which often occurs whilst the crack is still open. Reinforcing steel consists of a high elastic modulus and shows the Bauschinger effect after

yielding, which is the property whereby the steel's yield strength reduces in the reverse direction to the initial loading (Yu et al., 2016).

2.6.3 BUCKLING OF LONGITUDINAL BARS

Figure 2-28 shows a flexural failure of a reinforced concrete member due to crushing and spalling of the concrete cover and subsequent buckling of the longitudinal bars. This failure mechanism typically occurs in members with high ductility that results in the member absorbing a significant amount of hysteretic energy. When the member reloads in the opposite direction to the initial loading and the reinforcing bar has been permanently deformed, the reloading occurs at a slope less than that of the unloading branch and decreases with increasing deformation, which contributes to the pinching effect.

Longitudinal bars that have undergone permanent elongation in the initial direction of loading prevents the cracks from closing during load reversal and reloading in the opposite direction. Provided the elongated bars do not yield in compression during load reversal, a force couple occurs between the longitudinal bars on either face. Buckling can occur in the longitudinal bars subjected to compression loading in areas where concrete spalling has occurred (Penelis & Kappos, 2010). In the book by Penelis & Kappos (2010) various analytical methods to determine the required stirrup spacing to prevent premature buckling have been formulated, but the methods are rather onerous.

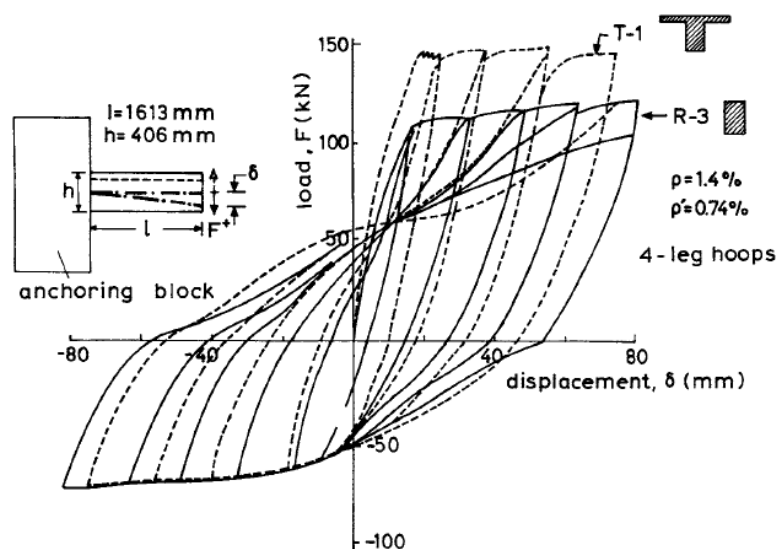


Figure 2-28 Reinforced concrete hysteresis loops subjected to cyclic loading (Penelis & Kappos, 2010)

Figure 2-29 shows various modes of longitudinal reinforcement buckling that can occur in reinforced concrete members subjected to cyclic loading. Determining the buckling length of a longitudinal bar is complicated as it depends on several factors such as the stirrup spacing, the stiffness of the stirrups and the extent of permanent elongation of the reinforcement when subjected to a compression load (Penelis & Kappos, 2010).

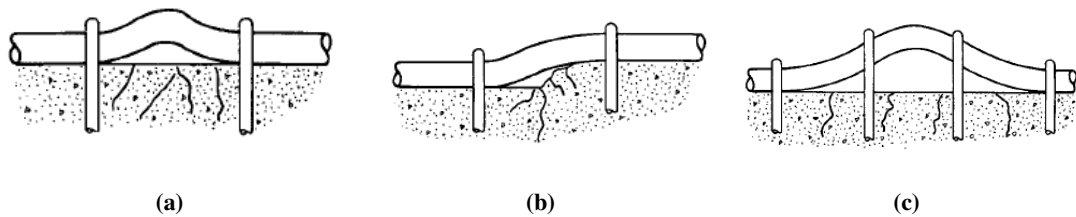


Figure 2-29 Different modes of longitudinal reinforcement buckling (Penelis & Kappos, 2010)

2.6.4 REINFORCED CONCRETE HYSTERETIC MODELS

Non-linear hysteretic models, such as bi-linear and tri-linear models, have typically been used to model the earthquake response of structures (Takanashi et al., 1975). However, the need to develop more realistic models that can account for stiffness and strength degradation of structural components is necessary to accurately determine the performance and resultant damage to structures at various earthquake intensities. The need for more accurate models stems from the inability of simplified analytical models to accurately account for non-linear behaviour of the structure due to uncertainty related to the material behaviour, local failure mechanisms, loss of stability and pinching effects in reinforced concrete members. Several analytical models have been proposed for reinforced concrete to mitigate the uncertainty of simplified models with some of the earliest models provided in Takanashi et al. (1975). Sengupta and Li (2017) undertook an extensive literature study on hysteresis models for reinforced concrete that were developed by various researchers. They performed comparative studies of the various hysteretic models by performing quasi static-cyclic load tests. Sengupta and Li (2017) concluded that elasto-plastic and degrading bilinear models do not capture the realistic hysteretic behaviour of reinforced concrete and does not incorporate the pinching effect experienced in reinforced concrete.

Ozcebe & Saatcioglu (1989) developed a hysteretic shear model for reinforced concrete members subjected to constant axial loads. Their purpose of the model is to predict the strength,

stiffness and ductility response of reinforced concrete under cyclic loading. The model comprises of series of rules that describe the path of loading and unloading branches for the duration of the cyclic load or seismic loading. The model comprises of a primary curve (or backbone curve) that is considered as the force-displacement response under monotonic loading and is used as the envelope of the unloading and loading branches within the hysteretic model. The material model was derived from experimental data and statistical analysis with the comparison between the experimental and analytical model results presented in Ozcebe & Saatcioglu (1989). Comparing the hysteretic behaviour as shown in Figure 2-19 and the hysteretic model produced by Ozcebe & Saatcioglu (1989), the model closely approximates the hysteretic response of reinforced concrete. However, the unloading and reloading branches in the Ozcebe & Saatcioglu (1989) model follow a straight line to defined points of stiffness transition as shown in Figure 2-30. A full explanation of the hysteretic material model rules is provided in Ozcebe & Saatcioglu (1989). The material model is shown to predict the observed response of reinforced concrete under constant axial loads. From Figure 2-30 the model utilises the same cloverleaf hysteretic pattern as previously observed in Figure 2-19 and Figure 2-20.

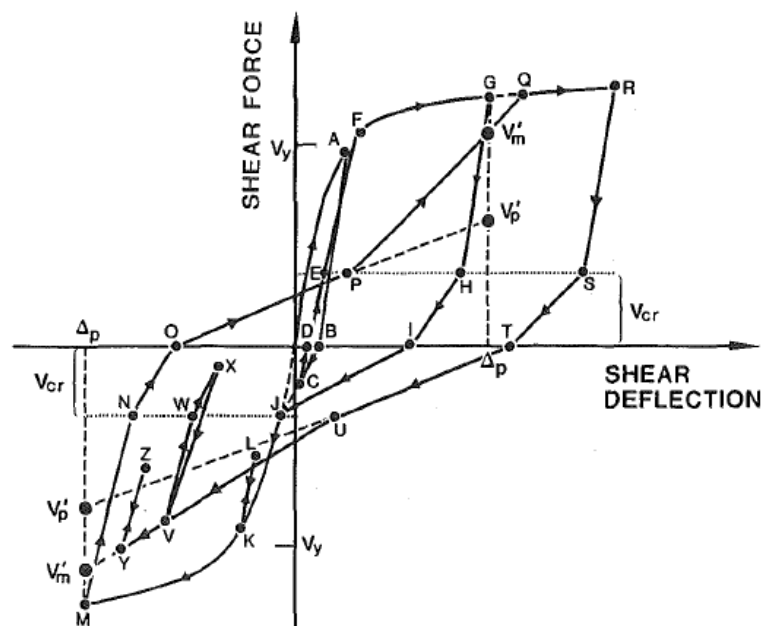


Figure 2-30 Hysteretic shear model developed by Ozcebe and Saatcioglu (1989)

Ibarra et al., (2005) used the data from Sezen (2000) to calibrate the hysteretic reinforced concrete model. Sezen (2000) undertook tests on columns with underprovided transverse reinforcement, and the column was connected to a rigid bottom beam and top beam to ensure double curvature bending. The tests were undertaken by subjecting the columns to a stepwise

increasing cycle load with the experimental results from Sezen (2000) and analytical results from Ibarra et al. (2005) shown in Figure 2-31

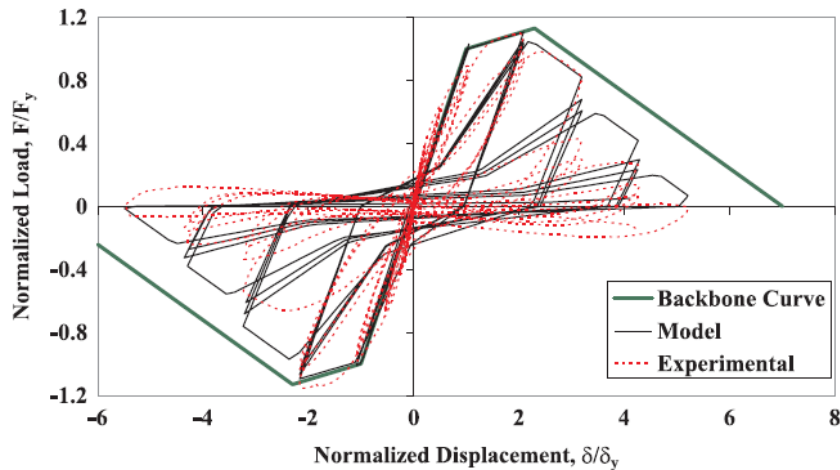


Figure 2-31 Hysteretic reinforced concrete model (Sezen, 2000; Ibarra et. al, 2005)

2.7 ENERGY AND HYSTERETIC ENERGY LOSS

Under serviceability limit states, structures are designed to absorb the energy within the structure without incurring damage upon loading and unloading. Upon unloading the structure should return to its initial position, which is achieved by ensuring that the structure remains in the elastic region. However, when a structure is subjected to large ground motions that result in substantial amounts of energy being imparted to the structure, a portion of the energy is temporarily stored as either kinetic or strain energy (Zahrah & Hall, 1984). The remainder of the energy is dissipated due to damping within the structure and the plastic deformation of structural components that make up the structure (Mosalam et al., 1997). Structures that are correctly designed should be able to sustain the imparted energy to the structure with minimal damage (Zahrah & Hall, 1984).

When the structure undergoes large deformations resulting in the structure being plastically deformed, energy is dissipated through hysteresis. Energy can be absorbed in the system by either recoverable energy or dissipated energy (Elmenschawi & Brown, 2009). The recoverable energy results in the structure returning to its original state under elastic deformation or recovering a portion of the plastic deformation upon unloading as shown in Figure 2-32. The ability to mitigate the earthquake effect due to the inelastic response of the structure is shown by the area enclosed by the hysteretic loop in Figure 2-32.

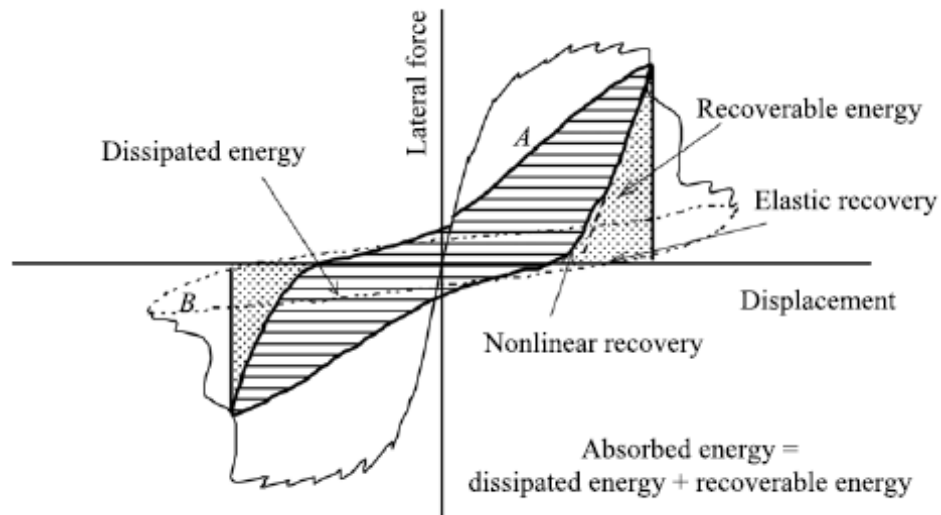


Figure 2-32 Recoverable and dissipated energy in a structural element (Elmenschawi & Brown, 2009)

The dissipated energy is due to the inelastic behaviour of the reinforcing steel, which results in excessive cracking of the concrete and permanent deformation of the structure. The energy dissipation capacity of a structural element is a critical factor during seismic design as the performance of the structure is significantly improved with an increase in the energy dissipation capacity of the structure before collapse (Elmenschawi & Brown, 2009).

The various energy terms can be deduced by integrating the equation of motion of an inelastic system as given in Equation 2.19 with respect to the change in displacement (Chopra, 2012). Energy being a scalar quantity allows the total energy input into the system to be quantified, thus enabling the overall response of the system to be assessed. The energy assessment in this section provides the response of the structure whereby the mass is acted on by a force calculated according to Equation 2.20. The kinetic energy in the system represents the energy imparted to the structure relative to the base of the structure and not due to the overall ground motion. It is the relative displacement and velocity that results in forces in the structure and subsequent damage. Therefore, it is more meaningful to quantify an energy expression in relative motion in respect of the base as opposed to the overall motion of the structure (Chopra, 2012).

$$\int_0^u [M]\{\ddot{u}(t)\}\{du\} + \int_0^u [C]\{\dot{u}(t)\}\{du\} + \int_0^u \{f_s(u)\}\{du\} = - \int_0^u [M]\{I\}\ddot{u}_g(t)\{du\} \quad (2.19)$$

$$\{P_{eff}\} = -[M]\{I\}\ddot{u}_g(t) \quad (2.20)$$

The total energy (E_T) input into the system is determined by integrating $\{P_{eff}\}$ with respect to the change in displacement. The total energy, as given by Equation 2.21, must equal the summation of kinetic, damping and stiffness energy. The distribution of the energy between the terms is a function of the structure's stiffness and mass characteristics (Chopra, 2012).

$$E_T(t) = - \int_0^u [M]\{I\}\ddot{u}_g(t) \{du\} \quad (2.21)$$

The kinetic energy (E_M), within the structures depends on the mass of the structure and its relative motion with respect to the ground. Equation 2.22 gives the kinetic energy as follows.

$$E_M(t) = \int_0^u [M]\{\ddot{u}(t)\} \{du\} = \int_0^{\dot{u}} [M]\{\dot{u}(t)\} \{d\dot{u}\} = \frac{m\dot{u}^2}{2} \quad (2.22)$$

The viscous damping energy (E_C) within the structure, which is determined as a function of the mass and stiffness matrices when considering Rayleigh damping, is given in Equation 2.23 (Chopra, 2012).

$$E_C(t) = \int_0^u [C]\{\dot{u}(t)\} \{du\} \quad (2.23)$$

The third term in Equation 2.19 is the strain energy in the structure due to the overall structural stiffness. The strain energy (E_S) can be separated into two terms, namely linear and non-linear/hysteretic strain energy as given by Equation 2.24. When analysing the structure, the structure can consist of regions of linear behaviour and regions of non-linear behaviour. Energy lost in the non-linear regions of the structure are determined by integrating the force-displacement relationship of the material at each time step.

$$E_S = E_K + E_H \quad (2.24)$$

The hysteretic energy absorbed within the structure is determined by integrating the force-displacement curve of the material between zero and the resultant displacement. Depending on the complexity of the material model, numerical methods need to be used, whereby the resultant change in energy is determined due to the change in displacement and force, which is then added to the overall energy. The energy absorbed due to the overall structural stiffness is given by the first term in Equation 2.25 and the hysteretic energy absorbed by a single non-linear element in the structure is given by the second term in the Equation 2.25.

$$E_S = \int_0^u [K]\{u(t)\} \{du\} + \int_0^u F_H(u) du \quad (2.25)$$

By studying the time history response of structural systems during seismic loading, valuable information can be obtained about the number of yield reversals, the displacement ductility of the structure and the duration of the earthquake record that is undergoing plastic deformation. Figure 2-33 shows the energy versus time response for a high-frequency, low-period structure subjected to the El Centro earthquake record. Figure 2-33 shows that energy stored incorporates a small proportion of the overall energy imparted to the structure with most of the imparted energy being dissipated almost immediately through damping and non-linear hysteretic behaviour.

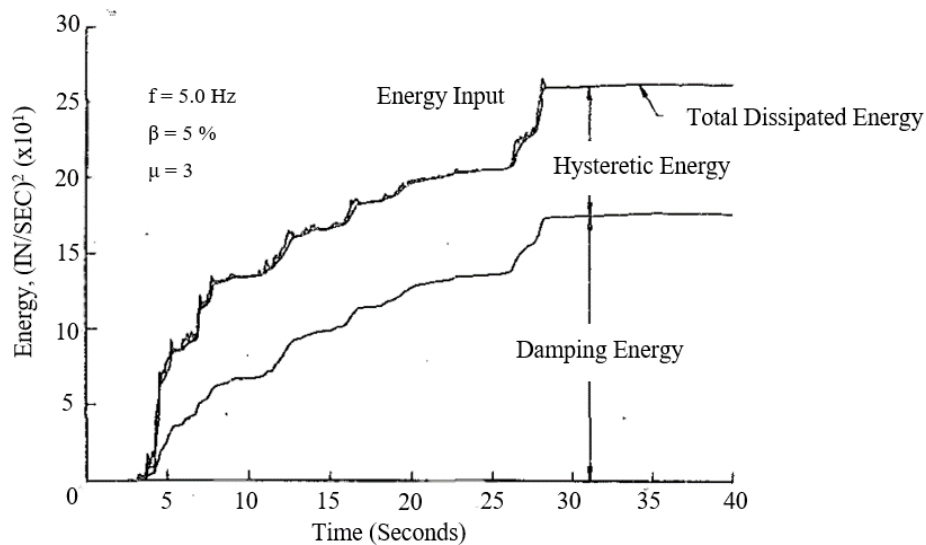


Figure 2-33 Energy versus Time for a structure subjected to the El Centro Earthquake Ground Motion (Zahrah & Hall, 1984)

Zahrah and Hall (1984) studied the earthquake energy absorption in a single degree of freedom structures. Their study focused on the amount of energy imparted to a structure and the amount of energy absorbed by each of the terms including inelastic deformations and damping. The research showed that peak ground acceleration is not a great measure of energy imparted to a structure and found that large amplitude and high-frequency components of acceleration do not correlate well with peak ground acceleration. Zahrah and Hall (1984) used the Newmark's method with linear acceleration in their experiments.

2.8 DAMAGE ASSESSMENT OF BUILDINGS

Seismic risk describes the probability of damage that an area can expect following a seismic event (Nielson, 2005). Most injuries and loss of life occur because of partial collapse or complete collapse of structures during a seismic event (FEMA P-58-1, 2012). Historically, moderate to severe seismic events have resulted in the collapse of reinforced concrete structures that were only designed for gravity loads with inadequate transverse reinforcement in columns (Jeon et al., 2014). It is necessary to determine the probability of collapse of a structure as a function of a ground motion intensity parameter and the type of failure that is being investigated to quantify the potential of structural collapse during an earthquake (FEMA P-58-1, 2012). Freeman (as cited by Davies & Kijko, 2003) undertook the first comprehensive study in 1932 to evaluate structural damage due to the applied forces caused by an earthquake. The study incorporated the impact that an earthquake would have on direct economic losses on essential facilities and infrastructure.

A study was undertaken after the 1971 San Fernando earthquake to investigate methods of correlating ground motion parameters to damage and losses with the result being the formation of damage probability matrices (Davies & Kijko, 2003). The extent of damage due to an earthquake, which ranges from none to collapse, was placed into various damage states. The damage states describe the extent of physical damage in words (Davies & Kijko, 2003).

2.8.1 PARK AND ANG DAMAGE INDEX

Performance based earthquake engineering evaluates the response of a structural component at various seismic hazard levels and therefore to undertake a comprehensive performance assessment the true response leading to the collapse of the structure needs to be considered (Ibarra et al., 2005). Performance based seismic design aims to accurately account for seismic structural damage and the associated risk (Ghosh et al., 2011). To approximate damage caused

by a seismic event, a relationship between a ground motion parameter and damage to a structure is required (Rajabi et al., 2012). The amount of damage sustained by a structure can be represented by the maximum deformation of the structure, the number of yield cycles, and the amount of energy absorbed by the structure or structural component in the form hysteretic energy. A reinforced concrete member subjected to cyclic loading during a seismic event results in a reduction in the reinforced concrete columns ductility and ability to dissipate energy (Penelis & Kappos, 2010).

Seismic design philosophy and performance based seismic design involve the dissipation of energy by controlling the damage to the structure at moderate to strong earthquakes with the objective of preventing structural collapse (Ghosh et al., 2011). A damage index allows for the quantification of seismic damage to a structure and produces a dimensionless parameter that ranges between 0 for undamaged structures and 1 for structures that have collapsed. Intermediate values of damage indicate the extent of damage that the structure has incurred.

The most common approach to performance based seismic design entails designing a structure for a specified inelastic displacement and ductility demand. Assessment of the structure's performance often entails the use of the same parameters to determine the structure's various performance levels and limit states. Displacement of the structure in both the elastic and inelastic range can be used to quantify the extent of damage to the structure.

During an earthquake, reinforced concrete structures are generally damaged due to repeated stress reversals and large deformations. A damage index is a parameter that specifies the ratio between the demand placed on the member and the capacity of the member before failure (Rajabi et al., 2012). The relationship between damage and the structure's vulnerability is complex and depends on structural strength, ductility, seismic intensity and vibration characteristics of the earthquake. Damage indices are typically classified as either being cumulative or non-cumulative.

Non-cumulative damage indices only account for the maximum deformation of the structure, whereas cumulative damage accounts for both the maximum deformation and the cumulative hysteretic energy that is absorbed by the structure (Rajabi et al., 2012). The first damage indices were established in the early 1970s when non-linear analysis models were used to study the response of structures. Whitman (as referenced in Rajabi et al., 2012) produced one of the first models that related ground motion intensity with earthquake magnitude. The damage index formulated by Whitman was expressed using the modified Mercalli intensity criteria that concerned the cost to repair the structure in comparison to the reconstruction cost. The history of various damage indices is discussed in Rajabi et al. (2012).

The Park and Ang damage index is used to quantify the damage sustained by the structure and is considered as one of the most realistic measures of structural damage for reinforced concrete structures (Ghosh et al., 2011). The Park and Ang damage index is shown by Equation 2.26 (Park & Ang, 1985) and combines the cumulative energy demand with the ductility demand and has been supported by several researchers (Ghosh et al., 2011). The advantage of the Park and Ang damage index is its simplicity. Park and Ang (1985) proposed that a limiting damage index of 0.4 be used to separate the feasibility between repairing the structure and having to reconstruct the structure (the total loss of asset).

$$DI = \frac{d_m}{d_u} + \frac{\beta}{V_y d_u} \int dE_h \leq 1 \quad (2.26)$$

Where:

- d_m : The absolute value of the maximum deformation applied to the member under dynamic loading
- d_u : The ultimate deformation that the structure can sustain before failure under monotonic loading
- dE_h : Cumulative value of hysteretic energy
- V_y : The yield strength of the member
- β : A non-dimensional, non-negative parameter

The selection of the non-dimensional non-negative constant β could have a considerable influence on the damage index. The Park and Ang damage index incorporates β into the definition to account for the response of the structure under cyclic load reversals. Empirical formulas have been proposed to calculate β by various authors, including Park and Ang (1985), which is given by Equation 2.27. Using a low value of β reduces the influence of low cycle fatigue and results in the damage being governed by the maximum displacement. A high value of β typically represents a structure that was poorly designed and detailed. Since the advent of the Park and Ang model in 1985, researchers have proposed various values for β for reinforced concrete columns that ranges from 0.05 to 0.24 (Ghosh et al., 2011).

Park et al., (1984) showed a negative correlation between β and the confining ratio ρ_w and a positive correlation between β and the shear span ratio $\frac{l}{d}$, longitudinal steel ratio p_t and axial

stress n_0 through the numerous experiments that were conducted on reinforced concrete members. Park and Ang compared the experimentally determined β values with the results produced using Equation 2.27. Figure 2-34 shows the Park and Ang comparison between the analytical model and the experimental results and shows substantial dispersion of the results (Rajabi et al., 2012).

$$\beta = \left(-0.447 + 0.073 \frac{l}{d} + 0.24n_0 + 0.314p_t \right) \times 0.7^{\rho_w} \quad (2.27)$$

With:

$\frac{l}{d}$: Shear span ratio (with $\frac{l}{d} = 1.7$ if $\frac{l}{d} \leq 1.7$)

n_0 : Normalised axial stress (with $n_0 = 0.2$ if $n_0 < 0.2$)

p_t : Longitudinal steel ratio as a percentage (with $p_t = 0.75\%$ if $p_t < 0.75\%$)

ρ_w : Confinement ratio

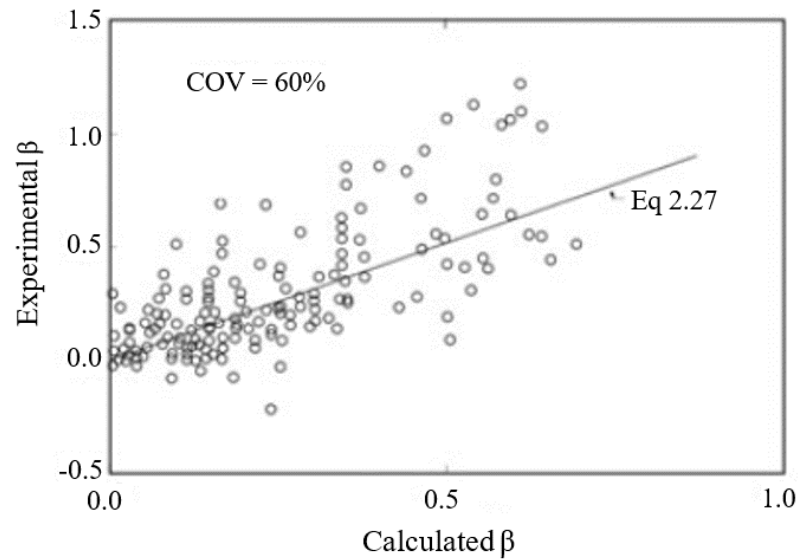


Figure 2-34 Park-Ang analytical and experimental comparison of β (Rajabi et al., 2012)

The ductility of the structure is given by μ , which is the ratio between the ultimate deformation (d_u) and the deformation at which the member yields (d_y), as shown in Equation 2.28.

$$\mu = \frac{d_u}{d_y} \quad (2.28)$$

Park et al. (1984) described three damage states for a structure in the following ranges: the structure is reparable when the damage index is less than 0.4, the structure is beyond repair when the damage index is greater than 0.4, but less than 1; and structure has collapsed when damage index is greater than 1. Ghosh et al. (2011) used the Park and Ang damage index in their work as it provides one of the most realistic measures of structural damage. Kunnath and Jenne (as cited by Ghosh et al., 2011) compared various damage indices with experimental observations and concluded that the Park and Ang damage index correlated the best with experimental observations and laboratory results. Therefore, due to its effectiveness at representing damage to reinforced concrete structures and its ability to incorporate hysteretic behaviour makes the Park and Ang (1985) damage model the most preferable when assessing the damage to a reinforced concrete structure.

2.8.2 FRAGILITY CURVES

Fragility functions are statistical distributions that quantify the probability of reaching prescribed damage states as a function of an earthquake intensity parameter (Jeon et al., 2014; FEMA P-58-1, 2012). Fragility functions provide a means for the quick assessment of the level of risk to a structure and can be generated using numerical models of structures by varying the loading intensity. There are several ways of determining fragility functions that include expert-based fragility functions, empirical fragility functions and analytical fragility functions with further details of each method given in Nielson (2005). Fragility functions are in the form of lognormal cumulative distribution functions with a median value of θ and a logarithmic standard deviation or dispersion β . Equation 2.29 provides a means to calculate fragility curves, and Figure 2-35 shows a typical lognormal fragility function.

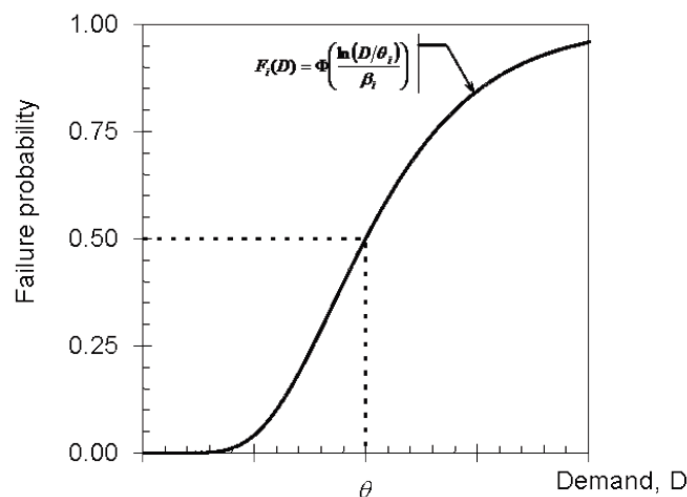


Figure 2-35 Typical lognormal fragility function (FEMA P-58-1, 2012)

$$F_i(D) = \Phi\left(\frac{\ln(D/\theta_i)}{\beta_i}\right) \quad (2.29)$$

Where:

$F_i(D)$: Conditional probability that a structure will incur damage at damage state i

Φ : Standard normal (Gaussian) cumulative distribution function

D : Demand parameter

θ_i : Median value

β_i : Logarithmic standard deviation

FEMA P-58-1 (2012) prescribes a method for determining the fragility functions from actual demand data and provides Equation 2.30 to calculate the median θ and Equation 2.31 being used to calculate the random dispersion β_r . The total dispersion, β , is calculated using Equation 2.32.

$$\theta = e^{\left(\frac{1}{M} \sum_{i=1}^M \ln(d_i)\right)} \quad (2.30)$$

$$\beta_r = \sqrt{\frac{1}{M-1} \sum_{i=1}^M \left(\ln\left(d_i/\theta\right)\right)^2} \quad (2.31)$$

Where:

M : Total number of tested specimens

d_i : Demand in test “i” at which the damage state first occurred

$$\beta = \sqrt{\beta_r^2 + \beta_u^2} \quad (2.32)$$

Where:

β_r : Random variability observed in the test data

β_u : Represents uncertainty in test data from real life conditions

Nielson (2005) used analytical methods during his doctoral studies to generate fragility curves for bridges from 3-D analytical models. The bridges were subjected to a range of ground motions, and probabilistic seismic demand models (PDSM) were generated for the analysis. Nielson (2005) indicates that one of the critical links during seismic hazard assessment is to determine the damage to critical structural components.

Jeon et al. (2014) developed fragility curves for non-ductile concrete frames using numerical models to investigate the failure of rigid beam-column joints by evaluating the nonlinear joint shear response, bond slip response, and column failure due to shear. Jeon et al. (2014) indicate that fragility functions should be developed using appropriate simulation techniques and a good understanding of how the material responds during an earthquake. The process for developing fragility function entails the following (Jeon et. al, 2014):

1. Choose response mechanisms to be considered;
2. Select a suite of N ground motions;
3. Perform nonlinear dynamic analysis for each of the N assigned ground motion pairs;
4. Record demand parameters in relation to the ground motion intensity at which the demand parameter was exceeded; and
5. Develop fragility curves from the results.

2.9 CONCLUSION OF THE LITERATURE STUDY

In this chapter, literature was presented on the prevalence of seismic activity within South Africa, the pseudo-dynamic experimentation technique and the determination of damage and fragility curves. Firstly, a brief overview of seismic activity in South Africa was presented with South Africa being considered a region of moderate seismicity, and it was shown that seismic risk in South Africa is non-negligible and should be considered when designing a structure.

The experimental technique known as pseudo-dynamic experimentation was discussed and it was shown how it enables the correlation between damage and peak ground acceleration to be determined all while accounting for the actual hysteretic response of the test specimen. When evaluating the damage to structures during an earthquake, several parameters need to be assessed to accurately quantify damage with increasing peak ground acceleration.

The pseudo-dynamic testing method is a two-stage procedure that contains a numerical model, which considers the response due to mass and damping of the overall structure, and the physical model under investigation. The calculated displacement at a shared degree of freedom between

the numerical model and the physical test specimen is applied to the test specimen using a servo-controlled hydraulic actuator. The resultant force measured using a load cell is then fed back into the numerical model and used in subsequent calculations. The inherent limitations and errors associated with the method were investigated and shown that the method can be prone to cumulative errors, which can be mitigated by accurately calibrating the instruments and using high-resolution servo-controllers and data acquisition systems. Various numerical models can be used to perform the pseudo-dynamic experimentation, including both implicit and explicit numerical models. The implicit Newmark's method was discussed in further detail as it forms the basis of the pseudo-dynamic experimentation undertaken during this research.

The literature study goes on to discuss the hysteretic response of reinforced concrete, showing that the response of reinforced concrete is highly non-linear and subject to the pinching effect. The behaviour of reinforced concrete is dependent on whether the concrete has cracked, the reinforcement has yielded and concrete spalling. The response of reinforced concrete under monotonic loading results in a trilinear behaviour and forms the envelope to the unloading and reloading cycles.

A discussion on the Park and Ang damage model was presented and shows that the damage index incorporates both instantaneous damages due to the maximum deformation of the structure as well as cumulative damage due to hysteretic energy loss. The Park and Ang damage index is regarded as the most suitable model for analysing damage to a reinforced concrete structure. The literature study concludes by discussing the formulation of fragility curves using the results obtained from the experimental tests.

3 EXPERIMENTAL TEST SETUP AND ANALYSIS METHODS

3.1 GENERAL

This chapter describes the experimental program developed to study the resultant damage to an axially loaded reinforced concrete footing due to increased peak ground acceleration. The pseudo-dynamic experimentation, as discussed in Chapter 2, was used to correlate the resultant damage incurred by reinforced concrete footing, which forms part of a two-bay, two-storey unbraced frame structure with masonry infill walls, to increasing earthquake intensity. The focus of this research is the reinforced concrete footing as shown in Figure 3-1 that forms part of a hypothetical frame structure. A constant axial load from the structure occurs on the reinforced concrete footing for the duration of the earthquake record due to the symmetry of the structure. The procedure followed to design and analyse both the reinforced concrete footing and the conceptual frame structure is discussed in this chapter. The design of the frame structure and reinforced concrete footing only considered the gravity loads and the wind loads. Earthquake loading was not considered during the design of the frame structure and the reinforced concrete footing. The construction of the reinforced concrete footings and experimental procedure are also discussed in this chapter.

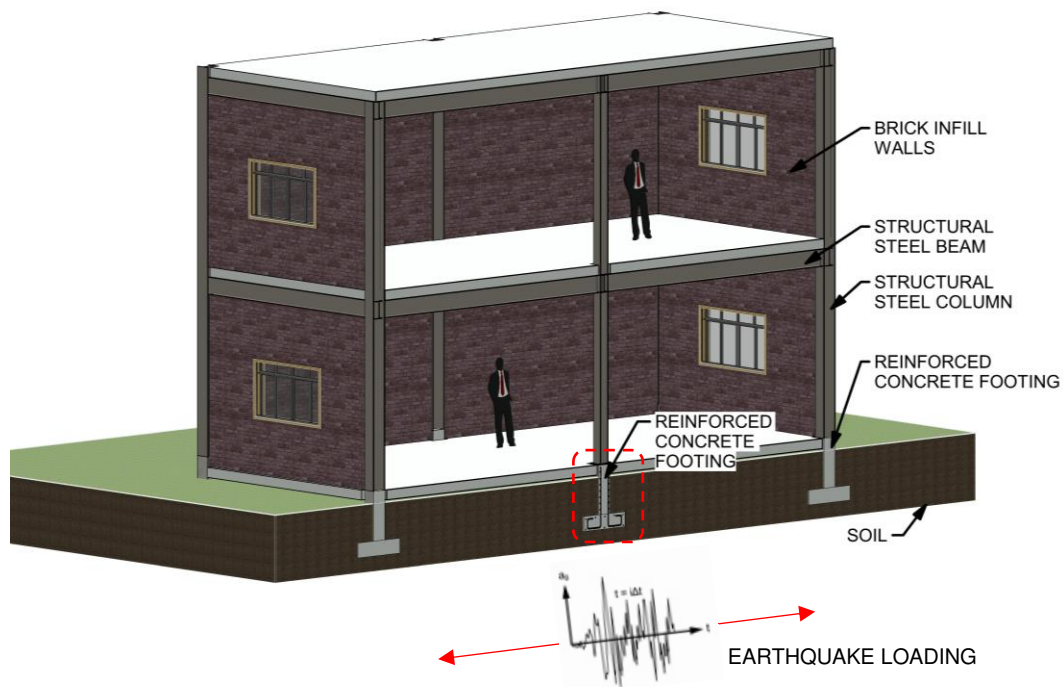


Figure 3-1 Conceptual model of the overall structure

The pseudo-dynamic analysis procedure will comprise of two parts, with the first part being the experimental test setup of the reinforced concrete footing in the laboratory, which contains the physical component of the pseudo-dynamic tests. The second part of the pseudo-dynamic test is the formulation of the numerical model using the Newmark's implicit time-stepping method with static condensation to eliminate the zero mass degrees of freedom and accounts for the overall structure that is placed on the reinforced concrete footing.

3.2 CHARACTERISTICS OF THE TESTED STRUCTURE

The structural element investigated during this research is a reinforced concrete footing under constant axial load that forms part of a hypothetical two-bay, two-storey linear elastic moment resisting frame structure founded on the footing. Figure 3-2 and Figure 3-3 illustrate the geometry of the frame structure used during the pseudo-dynamic tests. Due to limitations imposed on the project, the external columns were pin supported in the numerical model and therefore their contribution to the overall response of the structure was not considered. Only the non-linear lateral response of an internally located footing under constant axial load was considered in this research.

The axial load applied to the reinforced concrete footing had to be limited due to the capacity of the press frame that was available in the laboratory, which also limited the size of tests specimens that could be tested successfully in the laboratory. The mass distribution within the frame structure was selected to result in a 300 kN axial load being placed on the reinforced concrete footing. Most structures in South Africa are small to moderate size structures consisting of one to three stories and are generally the most critical structures. This is because they tend to be stiffer, thus producing lower periods of vibration and more substantial shear stresses at the base of the structure. The larger shear forces placed on the foundations result in more damage to the foundations.

The steel frame structure was divided into three segments, namely the internal columns, external columns and beams. The three segments enable a series of combinations of three steel sections that could be used to construct the steel frame structure. The width of each bay was selected to be 6 m with the storey heights selected to be 4 m as shown in Figure 3-3. The details of the structure used in the research are as follows:

- Consists of a steel moment resisting frame structure;
- The steel structure was founded on pad foundations that only allowed for the transfer of shear and axial load (i.e. pin connection);

- A 250 mm reinforced concrete slab spans in the transverse direction between the steel frames for each of the floors, which was selected to produce a 300 kN axial load on the central footing;
- The occupation of the structure is commercial/residential;
- Masonry infill panels with a thickness of 230 mm are used for the external walls; and
- The transverse span length between the frames was taken as 2.6 m, which is less than the typical span length used in buildings. A limitation had to be placed on the maximum axial load and the shear load placed on the foundations due to the available capacity of the actuators in the laboratory.

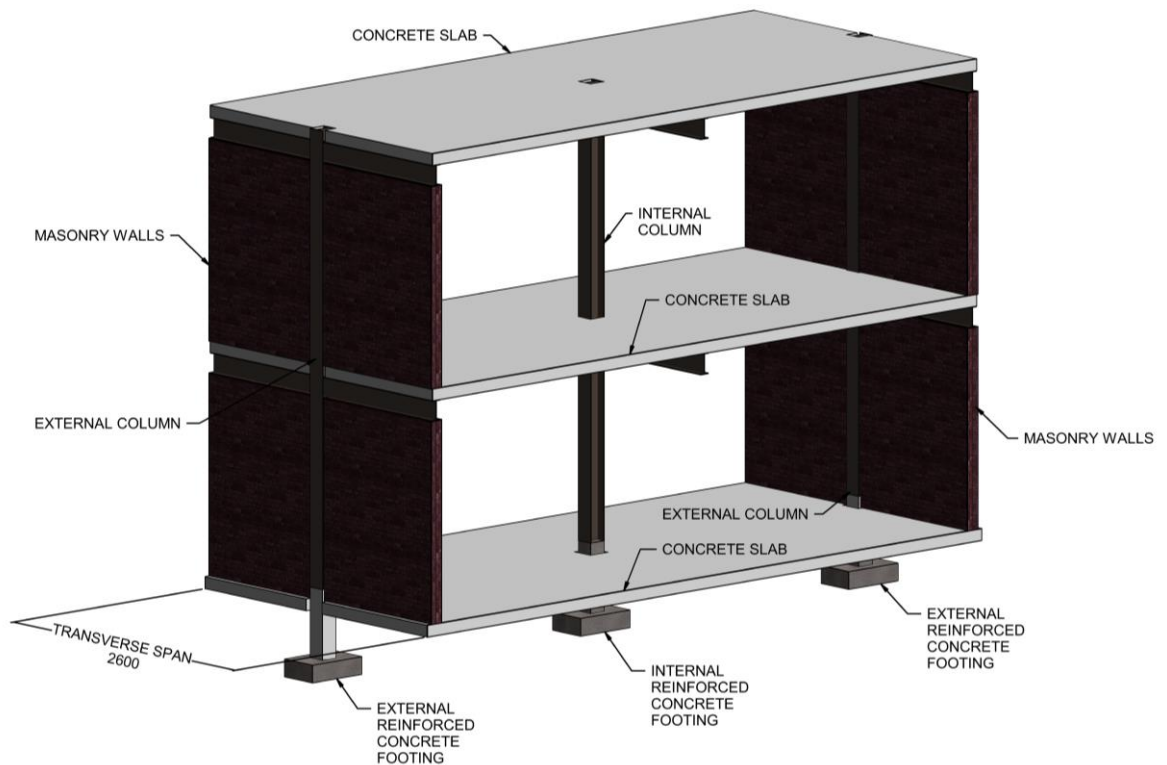


Figure 3-2 Characteristic 3D model of the frame structure

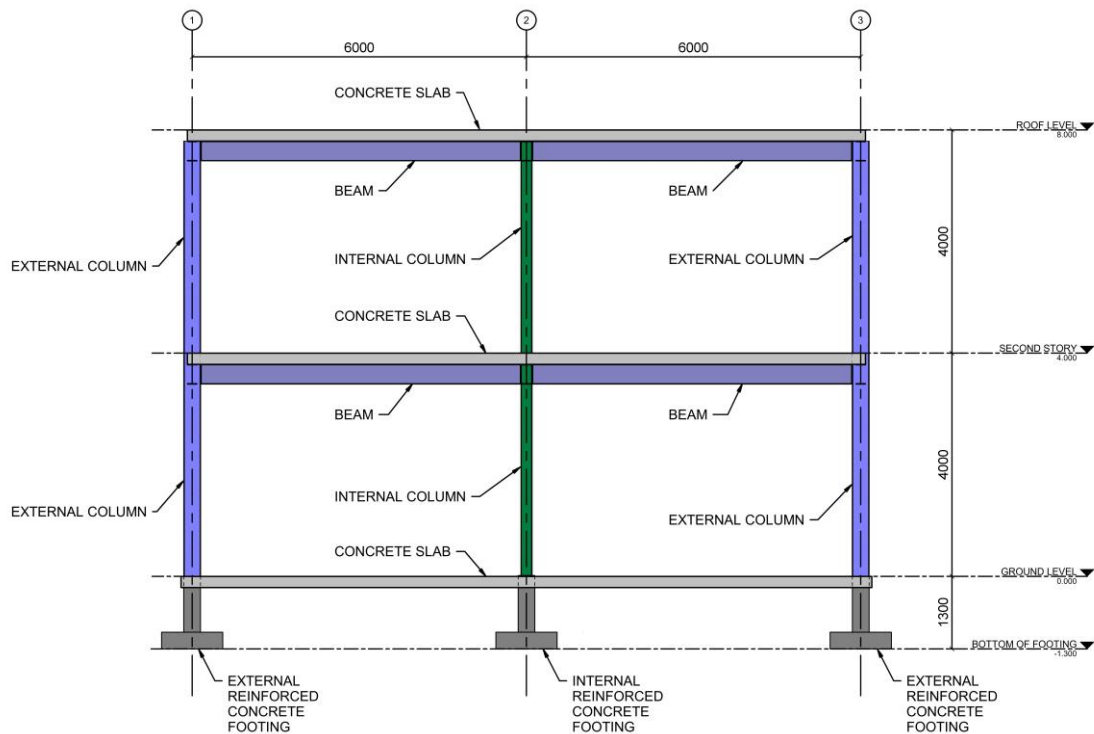


Figure 3-3 Characteristic section of the frame structure

3.3 LOADING ON THE TESTED STRUCTURE

This section describes the loading applied to the structure during the static analysis and pseudo-dynamic experiments. The design of the structure only considers gravity loading and wind loading with a description of the earthquake loading that is only applied to the structure during the pseudo-dynamic tests. The applied loads on the structure were determined using the following codes of practice:

- SANS 10160-1: 2011 Basis of structural design and actions for buildings and industrial structures – Part 1: Basis of structural design;
- SANS 10160-2: 2011 Basis of structural design and actions for buildings and industrial structures – Part 2: Self-weight and imposed loads; and
- SANS 10160-3:2011 Basis of structural design and actions for buildings and industrial structures – Part 3: Wind actions.

3.3.1 GRAVITY LOADS ON THE STRUCTURE

The static loads on the structure, which consists of the dead and live loads, were obtained from SANS 10160-2:2011 and remained constant for each of the pseudo-dynamic experiments. The dead loads (DL) on the structure comprised of the self-weight of the frame structure (W_{DL} and P_{DL}), the superimposed dead load consisted of the reinforced concrete slabs (W_{SDL}) and the masonry walls (P_m), and finally the live load that accounts for people that use the structure (W_{LL}). Distributed loads are indicated with a (W), and point loads are represented with a (P). Figure 3-4 shows static loading on the frame structure with the weight of the beams, concrete slab and live load being applied to the structure as a distributed load and the masonry walls and columns being applied to the structure as point loads. Table 3-1 shows the material properties used for the steel frame structure, concrete slab and masonry walls.

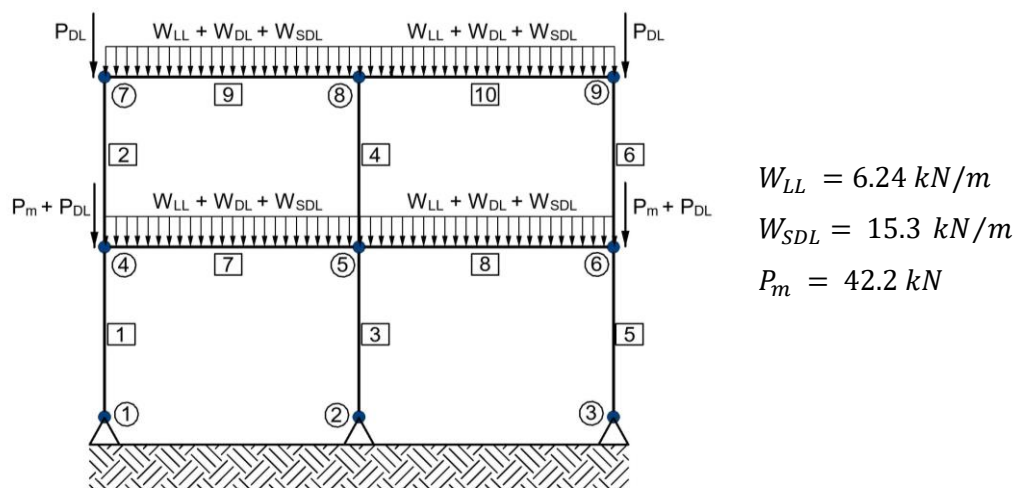


Figure 3-4 Static loading applied to the structure

Table 3-1 Material densities and applied loading used in design

Load Type	Value	Reference
Live loading	LL = 2.5 kN/m ²	Category B1 Table 1 SANS 10160-2:2011
Dead load - Self weight		
Concrete	$\rho_c = 2400 \text{ kg/m}^3$	
Steel	$\rho_{st} = 7850 \text{ kg/m}^3$	
Superimposed dead load		
Masonry	$\rho_m = 1850 \text{ kg/m}^3$	

3.3.2 WIND LOADS ON THE STRUCTURE

The design wind loading on the structure was calculated using SANS 10160-3:2011 with the resultant unfactored design wind pressures on the structure shown in Figure 3-5. The wind loads were obtained by selecting Johannesburg, South Africa as the location of the structure with a regular cover of buildings surrounding the structure.

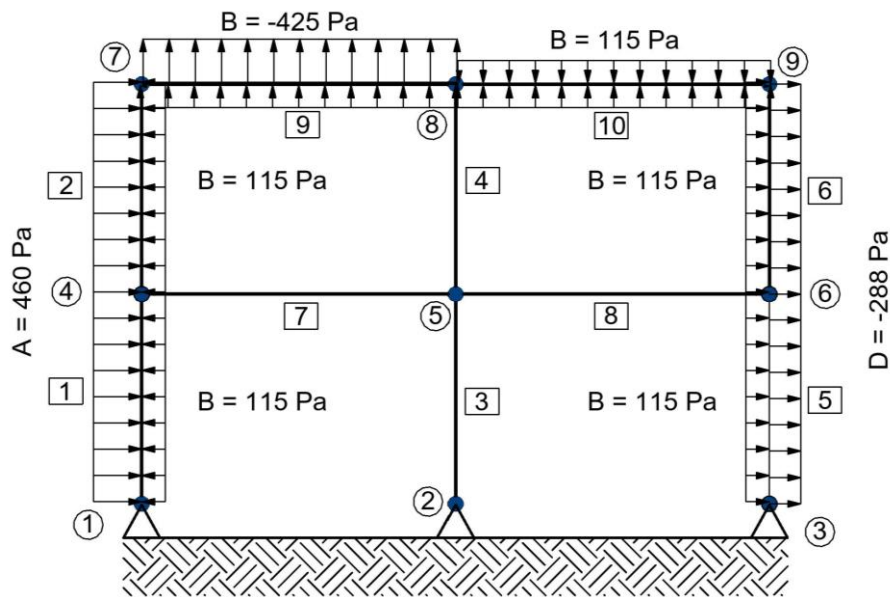


Figure 3-5 Wind pressure loading calculated from SANS 10160:3 2011

3.3.3 EARTHQUAKE LOADING ON THE STRUCTURE

The El Centro earthquake record was selected as the input ground motion for the pseudo-dynamic analysis. Figure 3-6 shows the El Centro, S00E of the event at Imperial Valley, California on May 18, 1940, and represents a strong ground shaking. Mosalam et al. (1997) indicate that the El Centro earthquake record is typical of the North American west coast earthquakes. The maximum acceleration during the ground motion record occurred at a time of 2.14 seconds. For each of the pseudo-dynamic tests, the peak ground acceleration was increased by scaling the amplitude of the El Centro earthquake.

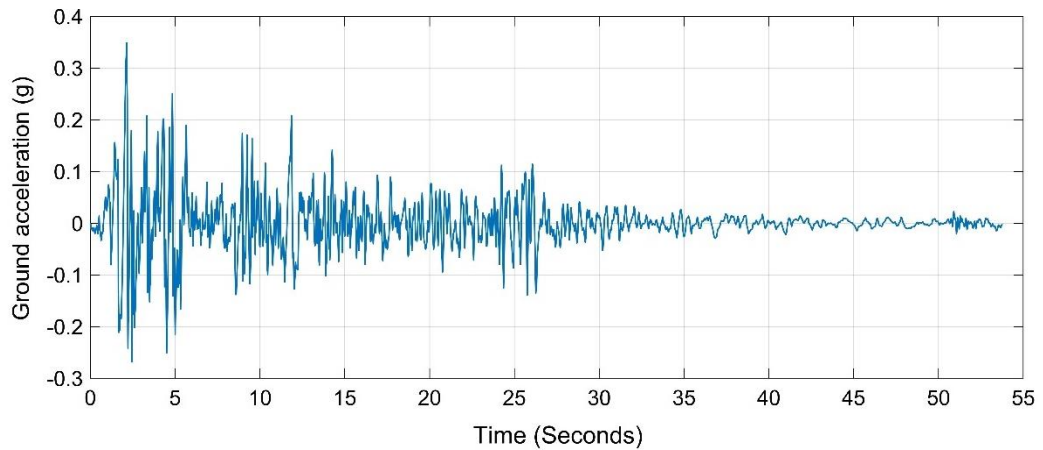


Figure 3-6 Acceleration record of the El Centro Earthquake

The elastic response spectrum of the El Centro earthquake is plotted in Figure 3-7 using a damping ratio of 5%. SANS 10160-4:2017 specifies four ground types for South Africa when determining the design elastic response spectrum. The ground type ranges from rock to sand and gravel, to clayey material. The objective during the analysis was to select a representative earthquake that closely follows the design response spectrum as provided in SANS 10160-4:2017. Figure 3-7 shows the elastic response spectrum of the El Centro ground motion record superimposed on the design elastic response spectra as specified in SANS 10160-4:2017. From Figure 3-7, it can be observed that the El Centro elastic response spectrum closely follows the design response spectra. Because of this, the El Centro earthquake was selected as the base earthquake for this research.

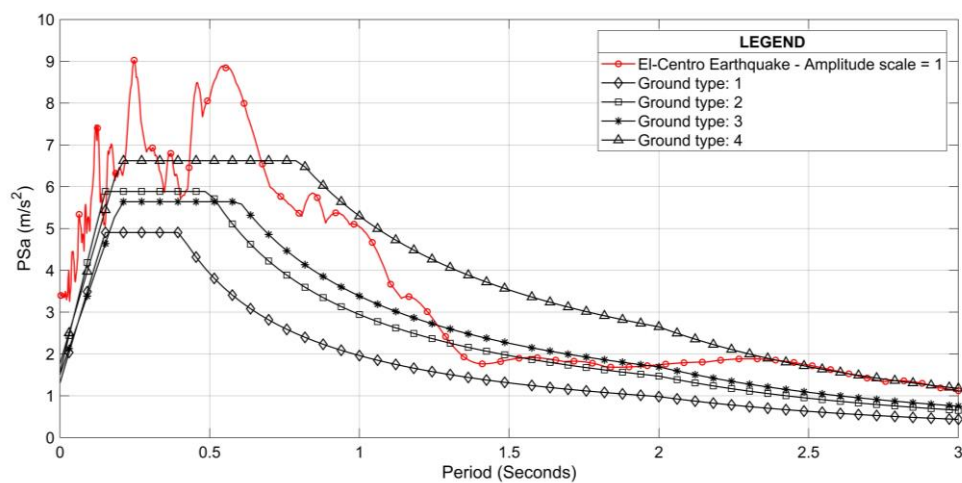


Figure 3-7 Elastic response spectrum of the El Centro earthquake with the design response spectra for the four ground types as specified in SANS 10160-4:2017

3.3.4 DESIGN LOAD COMBINATIONS

The critical load combinations acting on the structure are shown in Table 3-2 and were determined from SANS 10160-1:2011 for live load, dead load and wind load. For serviceability limit state, two load combinations, as shown in Table 3-2, are found to be critical. The first load combination governs the deflection of the beams and the second load combination determines the maximum horizontal deflection of the frame structure.

Table 3-2 Design load combinations

Serviceability Limit States
$STR1_s = 1.1DL + 1.0LL$
$STR2_{s_i} = 0.9DL + 0LL + 0.6WL_i$
Ultimate limit state
$STR1 = 1.2DL + 1.6LL$
$STR2_i = 1.2DL + 1.6LL + 1.3WL_i$
$STR3_i = 0.9DL + 0LL + 1.3WL_i$
$STRP1 = 1.35DL + 1LL$
$STRP2_i = 1.35DL + 1LL + 1WL_i$

3.4 ANALYSIS AND DESIGN OF THE FRAME STRUCTURE

This section describes the results of the analysis and design of the planar steel frame structure with reinforced concrete floor slabs and masonry infill panels on the external columns. The structure was analysed using all prescribed sections available in the Southern African Steel and Construction Handbook (SAISC, 2010), with the most practical sections selected for the analysis. The frame structure was divided into three segments, as previously shown in Figure 3-3, with separate steel sections used for the beams, external structural columns and the internal structural columns.

During the analysis of the frame structure, only I-Sections are used for the beams, with H-Sections being used for the columns. The reinforced concrete floor slabs are assumed not to contribute to the stiffness and strength of the beams. The analysis and design of the sections are

carried out using a script written in MATLAB (2017), which analyses the structure to determine the forces within each of the members with the actual stiffness properties of the design members. Once the analysis was completed, SANS 10162-1:2011 was used to design the members. The MATLAB (2017) program analysed the structure for each member combination of beam, internal column and external columns using the steel members obtained from Southern African Steel and Construction Handbook (SAISC, 2010). The analysis and design comprised of two steps: ultimate limit state (ULS) design, to ensure enough strength capacity and stability of the members, and serviceability limit state (SLS) design, to prevent excessive deflections during the day to day operations of the structure.

3.4.1 ULTIMATE LIMIT STATE DESIGN

The structure was analysed for each of the member combinations as previously shown in Figure 3-3 and the load combinations as specified in Table 3-2. The design of the structure considered the following during the design:

- All beams and columns are designed by incorporating both axial and uniaxial bending loads;
- The class of the section was selected to ensure local stability of the members;
- The columns are braced for out of plane buckling at storey heights;
- The beams are continuously braced for lateral torsional buckling due to the presence of the concrete slab along the top flange of the beam; and
- The beams and columns are analysed against the following interactions equations obtained from SANS 10162-1 (2011):
 - Cross-sectional strength;
 - Overall member strength; and
 - Lateral torsional buckling strength

From the analysis, regarding Table 3-2, it was found that load combination two governed the design of columns, whereas the load combination one dictated the design of the beams. Considering all the sections that satisfied the interaction equations as specified in SANS 10162-1 (2011), there were a total of 4095 steel member combinations for the external columns, interior columns and beam members that satisfy the ultimate limit state.

3.4.2 SERVICEABILITY LIMIT STATE

The second step in the design of the frame structure consisted of analysing the structure using the load combinations specified for the serviceability limit state loads, which are shown previously in Table 3-2. The analysis took all the members that satisfied the ultimate limit state design requirements and calculated the beam deflections and horizontal deflections for each of the storeys. The maximum allowed inter-storey deflection and overall structural deflection was obtained from SANS 10160-1:2011 and Figure 3-8 shows the points where the deflections were calculated and the design criteria that was used to limit the deflection. The maximum inter-storey deflections (u_{1i} & u_{2i}), the overall structural deflection (u_g) and the deflection of the beams (u_{bi}) were determined for each of the load combinations.

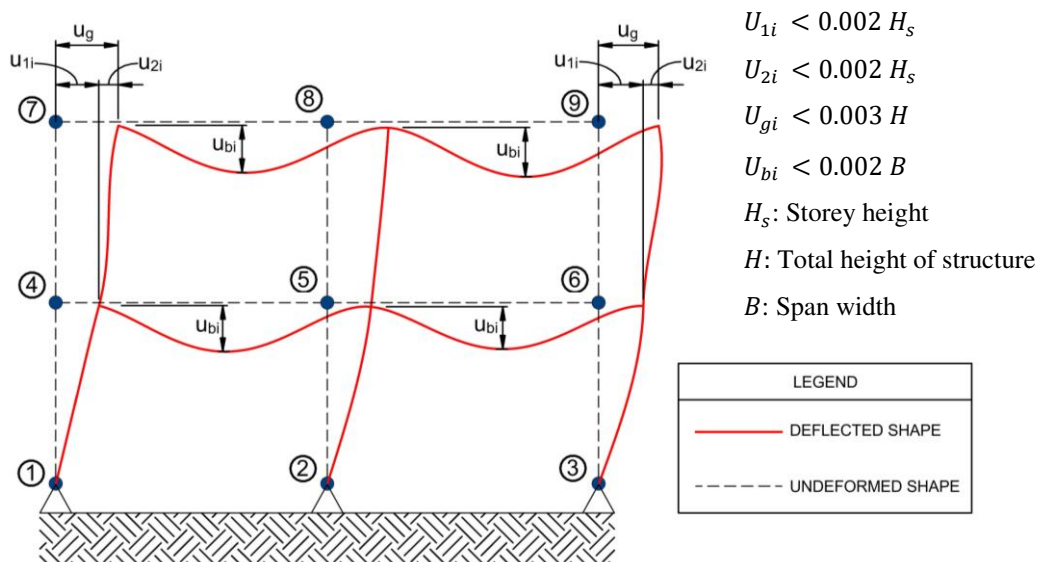


Figure 3-8 Deflection points on the frame structure for serviceability limit state design

The calculated deflections for each member combinations were compared to the maximum allowable deflection and disregarded if the member combination failed the design criteria. The total mass of the steel frame structure for each of the member combinations was calculated and sorted from the lowest mass to the largest mass, which is shown in Figure 3-9. Of the 4095 sections that satisfied the ultimate limit state criteria, 2272 combinations satisfied the serviceability limit state criteria.

As part of the serviceability limit state analysis, the fundamental period of vibration was calculated for each of the member combinations that satisfied the SLS and ULS designed requirements. The fundamental period of vibration for each frame member combination is

plotted from lowest mass to the largest mass in Figure 3-10 and on average the natural period of vibration reduced with an increase in the mass of the steel frame. The variation in the natural period of vibration with an increase in mass was due to the variation in section properties, as the mass of the section does not increase as quickly as the stiffness of the section. Considering all possible steel section combinations that satisfy ultimate limit state and serviceability limit state conditions, the natural period of vibration of the frame structure under the given design conditions varied between 0.72 s and 1.28 s, with the average period of vibration equalling 1.05 s. However, considering the practical design and economic considerations, a frame structure producing the lowest mass that satisfies both the SLS and ULS design requirements would typically be used for the design of the steel frame structure.

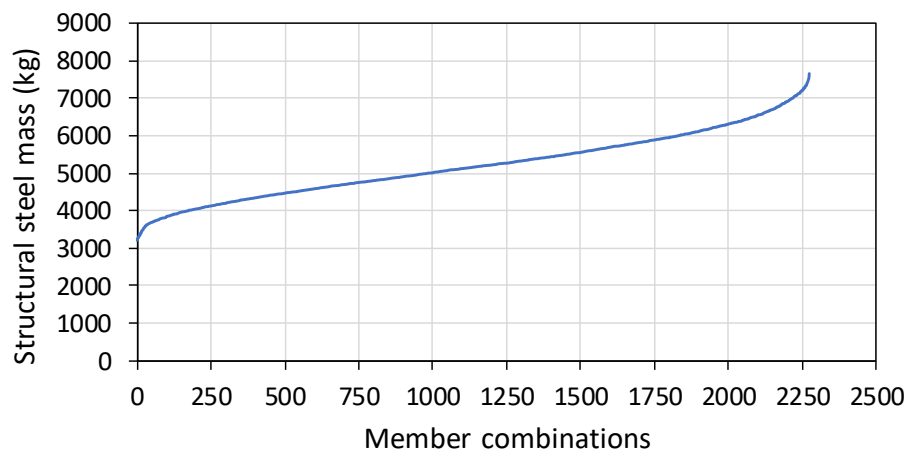


Figure 3-9 Mass distribution for all the member combinations satisfying SLS and ULS design ordered from lowest mass to largest mass

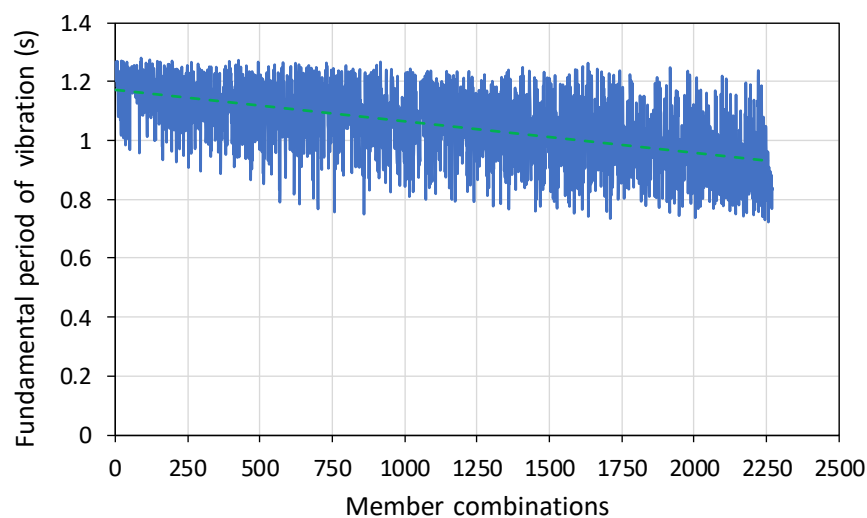


Figure 3-10 Fundamental period of vibration for each of the member combinations satisfying ULS and SLS design

The final design selected for the steel moment resisting frame structure to be used during the pseudo-dynamic experiments is shown in Figure 3-11 and corresponds to an overall natural period of vibration of 0.86 s. To optimise the design a separate steel section member was used for the internal column and external columns with the ratio of the moment of inertia between the internal column to the external column equalling 0.2.

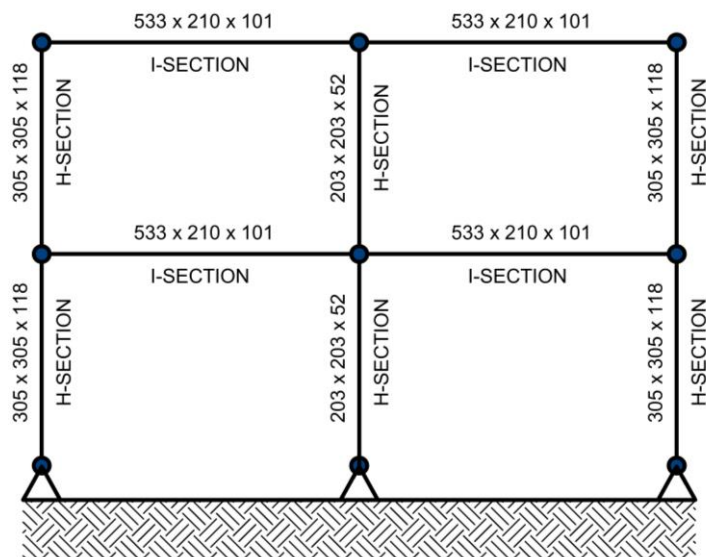


Figure 3-11 Member design for the steel moment resisting frame structure

3.5 TEST SPECIMENS – REINFORCED CONCRETE FOOTINGS

The reinforced concrete footings used in this research were designed using SANS 10100-1:2000 to satisfy the ultimate limit state, serviceability limit state and practical design considerations. The footings are considered representative of those that would typically be used in low to moderately tall structures of one to three stories that only carry axial load.

3.5.1 DESIGN OF THE FOOTINGS

Eight reinforced concrete footings were constructed at the University of Pretoria's experimental farm with a base size of 1100 x 700 x 300 mm and column size of 300 x 300 x 1000 mm. Four M16 L-Shape Class 8.8 holding down bolts were fixed within the reinforcement cage before the concrete was cast to allow for a shear connection between the base plate of the steel column and the top of the reinforced concrete footing.

The characteristic design yield stress of 450 MPa and a Young's Modulus of 200 GPa was used for the design of the reinforcement. The characteristic compressive strength of concrete specified and used during the design was 30 MPa. Using the concrete and reinforcing steel design parameters resulted in the reinforced concrete column only required minimum reinforcement to resist gravity and wind loads and comprised of 4 x Y12 reinforcing bars, with one bar placed at each of the column corners. The reinforced concrete footings all had the same design and configuration as shown in Figure 3-12.

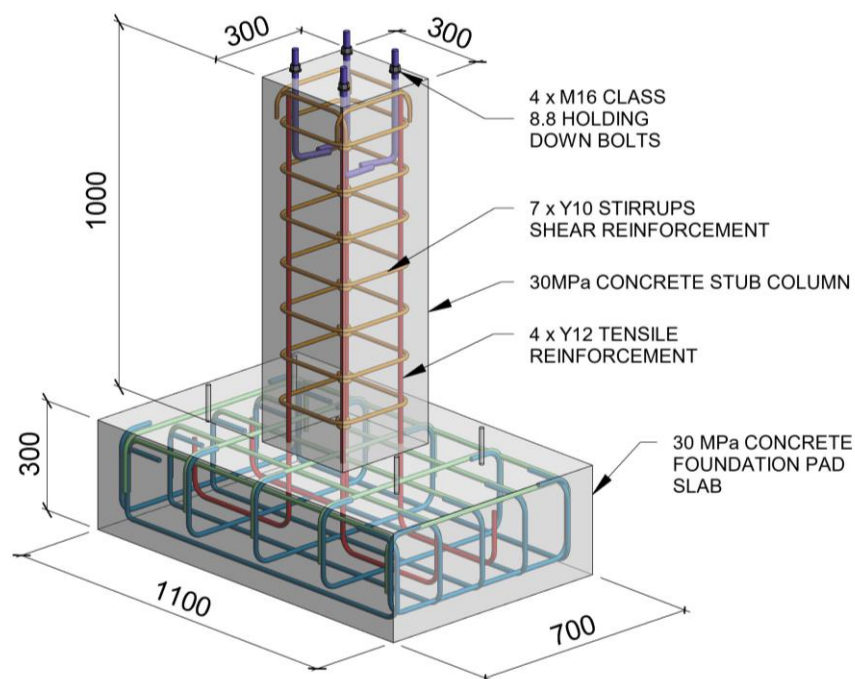


Figure 3-12 Three-dimensional visualisation of the reinforced concrete footing (Units in mm)

Shear reinforcement consisted of seven Y10 reinforcing stirrups placed at 150 mm centres within the column. The reinforced concrete base slab comprised of five Y12 bars spaced at 150 mm centre to centre in both the long and short spans. Both top and bottom reinforcement are provided, and a concrete cover of 30 mm was used for all the reinforcement.

The footings were designed to sustain the reactions for each of the ultimate limit state load cases determined at node number 2, as shown previously in Figure 3-8, during the analysis and design of the frame. The internal reinforced concrete footing is only considered during this study and will not be subjected to a varying axial load for the duration of the applied earthquake excitation due to the symmetry of the frame structure. The structural supports during the frame analysis were modelled as pin supports and therefore only axial and shear loads are applied to the

footing, with the moment being equal to zero. The internal footing will have zero shear load applied to it under only gravity loading due to the symmetry of the frame structure.

The design of the reinforced concrete footing also needed to consider the practical design considerations by ensuring that there was sufficient space for the connection of the base plate of the steel column to the reinforced concrete stub column. The resultant axial load and bending moment at the critical section for design is shown in Figure 3-13 and occurs at the interface between the column and the concrete base slab. The maximum moment was calculated by multiplying the shear force applied to the HD-bolts by the height of the column. The concrete stub column was considered stocky according to SANS 10100-1:2000, and therefore the required tensile reinforcement for the column could be obtained by only using the MN-interaction diagram shown in Figure 3-14.

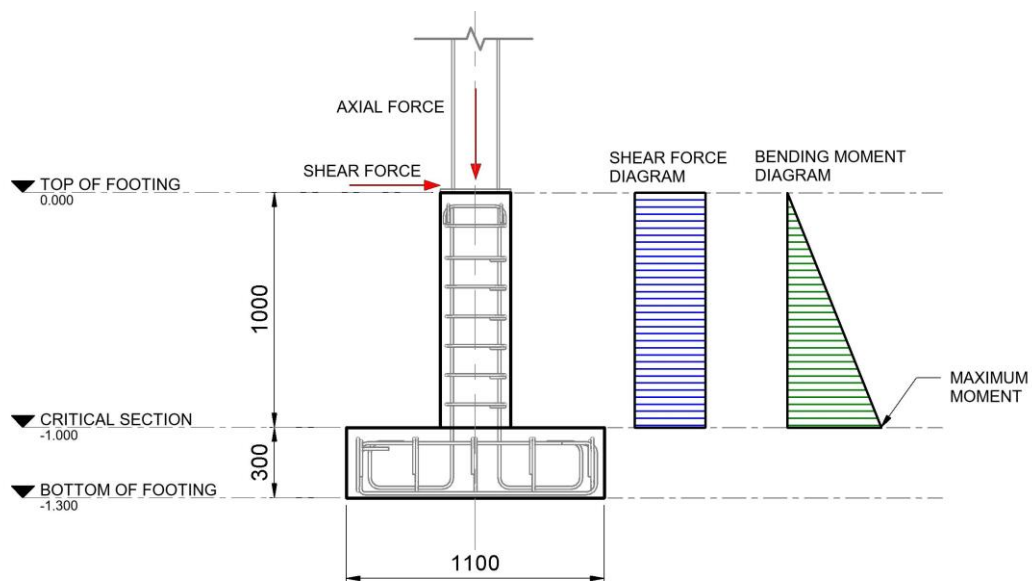


Figure 3-13 Analysis of the reinforced concrete footing

Only minimum reinforcement, with a reinforcement ratio of 0.4% according to SANS 10100-1:2000, was required for the given loading and column dimensions. The MN-interaction diagram and reinforcement layout for the reinforced concrete section with a 300 x 300 mm column and 4Y12 reinforcing bars is shown in Figure 3-15. As can be seen from Figure 3-15, the applied static loads on the footing due to gravity loads and wind loads are much lower than the available capacity of the footing. Therefore, the footing has additional capacity to carry larger shear loads.

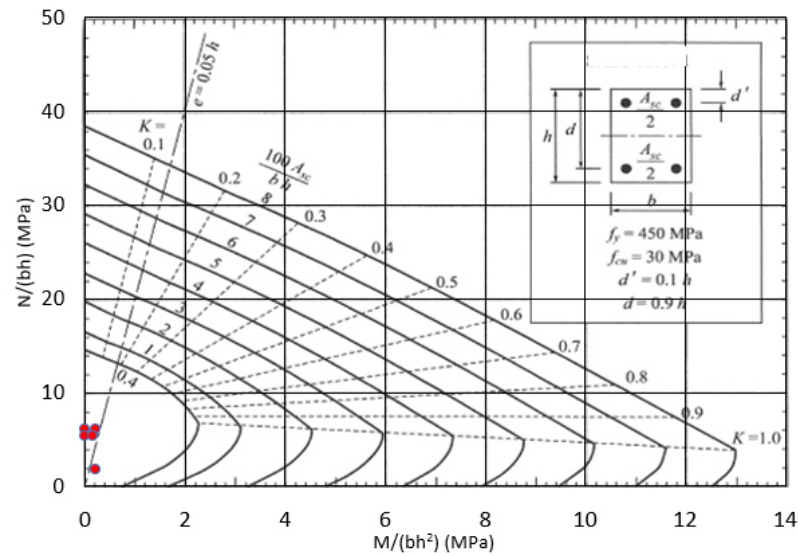
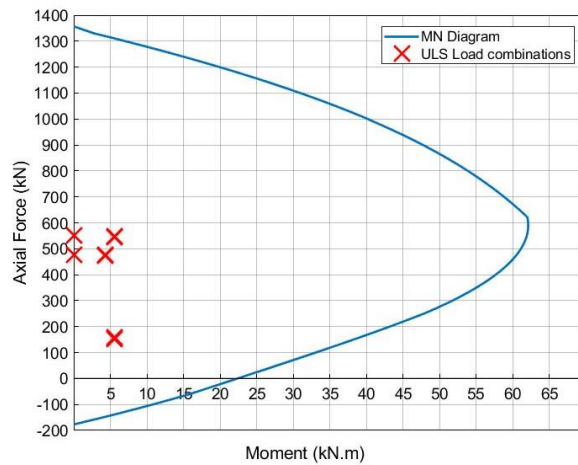
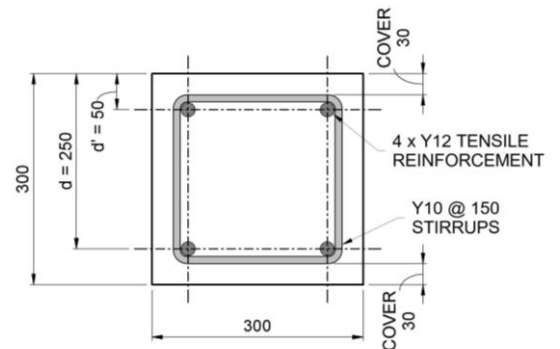


Figure 3-14 Design of reinforced concrete column for the ULS load cases (SANS 10100.1:2000)



(a)



(b)

Figure 3-15 Design of the reinforced concrete column showing (a) the MN-Interaction diagram for the column of the footing at ULS and (b) a section through the reinforced concrete column of the footing

3.5.2 MATERIALS AND CONSTRUCTION OF THE FOOTINGS

The reinforced concrete footings were constructed at the University of Pretoria's experimental farm and were cast monolithically in an upright position. The footings were cast monolithically to prevent a cold joint from forming between the base slab and the stub column. A lid on the base slab shuttering was provided to allow for a single concrete pour of the base slab and column. Figure 3-16 shows the shuttering for the reinforced concrete footings and the base slab

lid. Figure 3-17 shows the reinforcing cages for the footings with four Y12 tensile reinforcement bars and seven Y10 stirrups. Threaded rods were placed at each of base slab corners to enable the lid to be bolted down to prevent uplift during the casting of the concrete. The threaded rods were also used to lift and transport the footings from the experimental farm to the laboratory to ensure that the integrity of the concrete stub column was not compromised before testing. Figure 3-18 shows the holding down bolts and the templates that were used to position the bolts within the concrete column. The holding down bolts consisted of four M16 Class 8.8 L-Shape bolts placed on a 180 mm x 180 mm grid.



Figure 3-16 Reinforced concrete footing shuttering with base slab lid



Figure 3-17 Reinforcement cages for the concrete footings



Figure 3-18 Holding down bolts and template

Concrete was obtained from a commercial company to ensure that the concrete used in each of the footings remained consistent and was representative of concrete that would typically be used in industry. A concrete cube strength at 28 days of 30 MPa was specified and used to cast the footings, and Figure 3-19 shows the casting of the reinforced concrete footings. The footings were left to cure for 28 days before removing the shuttering. Concrete cube tests were done to determine the strength of the concrete after 28 days with the results shown in Table 3-3. The concrete cube tests show that an average cube strength of 32 MPa was obtained and therefore the specified cube strength of 30 MPa was reached at 28 days.

Table 3-3 Concrete cube tests at 28 days

Cube	Weight in air (grams)	Weight in water (grams)	Applied load (kN)	Concrete cube strength (F_{cu}) (MPa)
1	1486	2476	332	33.2
2	1502	2496	298	29.8
3	1486	2478	328	32.8
Average:				32.0

Tension coupon tests were undertaken on the reinforcing bars to determine the strength of the bars used to construct the reinforcement cages. Figure 3-20 shows the stress-strain curve for a Y12 reinforcement bar taken from the same batch of steel used to construct the reinforcement

cages for the footings. The reinforcement yielded at a stress of 545 MPa, which corresponds to a strain of 0.3%.



Figure 3-19 Concrete casting of the reinforced concrete footings

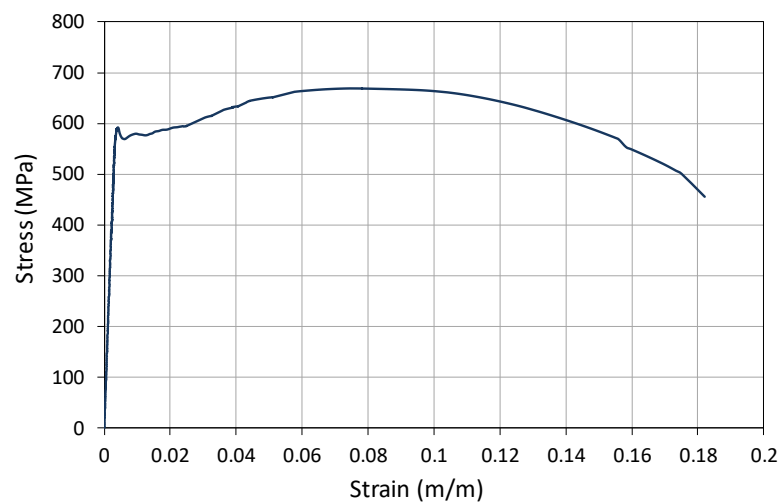


Figure 3-20 Stress-Strain curves for reinforcement

3.6 PSEUDO-DYNAMIC EXPERIMENTAL METHOD

The formulation of the numerical model used to perform the pseudo-dynamic experiment is presented in this section using the implicit Newmark's method with static condensation to remove the zero rotational mass degrees of freedom. The procedure followed in this study comprised of replacing the internal pin support of the frame structure at the location of the reinforced concrete footing with a mass-spring system as shown in Figure 3-21. The dynamic response of a linear multi-degree of freedom system with a single non-linear translational degree of freedom was solved using Equation 3.1.

$$\underbrace{[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\}}_{\text{Linear component}} + \underbrace{[K_s(t)]\{u(t)\}}_{\text{Non-linear component}} = \underbrace{\{P\}}_{\text{Static load}} - \underbrace{[M]\{I\}\ddot{u}_g(t)}_{\text{Earthquake loading}} \quad (3.1)$$

Where:

$[M]$:	The lumped mass matrix of the frame structure including the mass of the reinforced concrete footing
$[C]$:	Rayleigh damping matrix for the elastic part of the frame structure
$[K]$:	Linear elastic stiffness matrix of the frame structure
$\{\ddot{u}(t)\}, \{\dot{u}(t)\}, \{u(t)\}$:	Acceleration, velocity and displacement vector respectively
$[K_s(t)]$:	Lateral non-linear spring stiffness matrix of the reinforced concrete footing
$\{I\}$:	Influence vector that accounts for the horizontal direction of the earthquake loading
$\ddot{u}_g(t)$:	Ground acceleration

Figure 3-21(a) shows the discretised numerical model used in the pseudo-dynamic analysis with the equivalent mass-spring system. The boundary conditions comprised of two pin supports at the external columns with the internal reinforced concrete footing idealised as a single degree of freedom consisting of a constant lumped mass that was supported laterally by a massless non-linear spring of stiffness $k_s(t)$. The horizontal displacement was taken at the top of the column where the steel column of the frame structure connects to the footing using a base plate and holding down bolts. The degree of freedom $\{u_s = u_1\}$ shown in Figure 3-21 couples the calculation cycle with the physical test specimen. The total mass of the reinforced concrete

footing was lumped together into a single mass with dynamic effect considered in the numerical model.

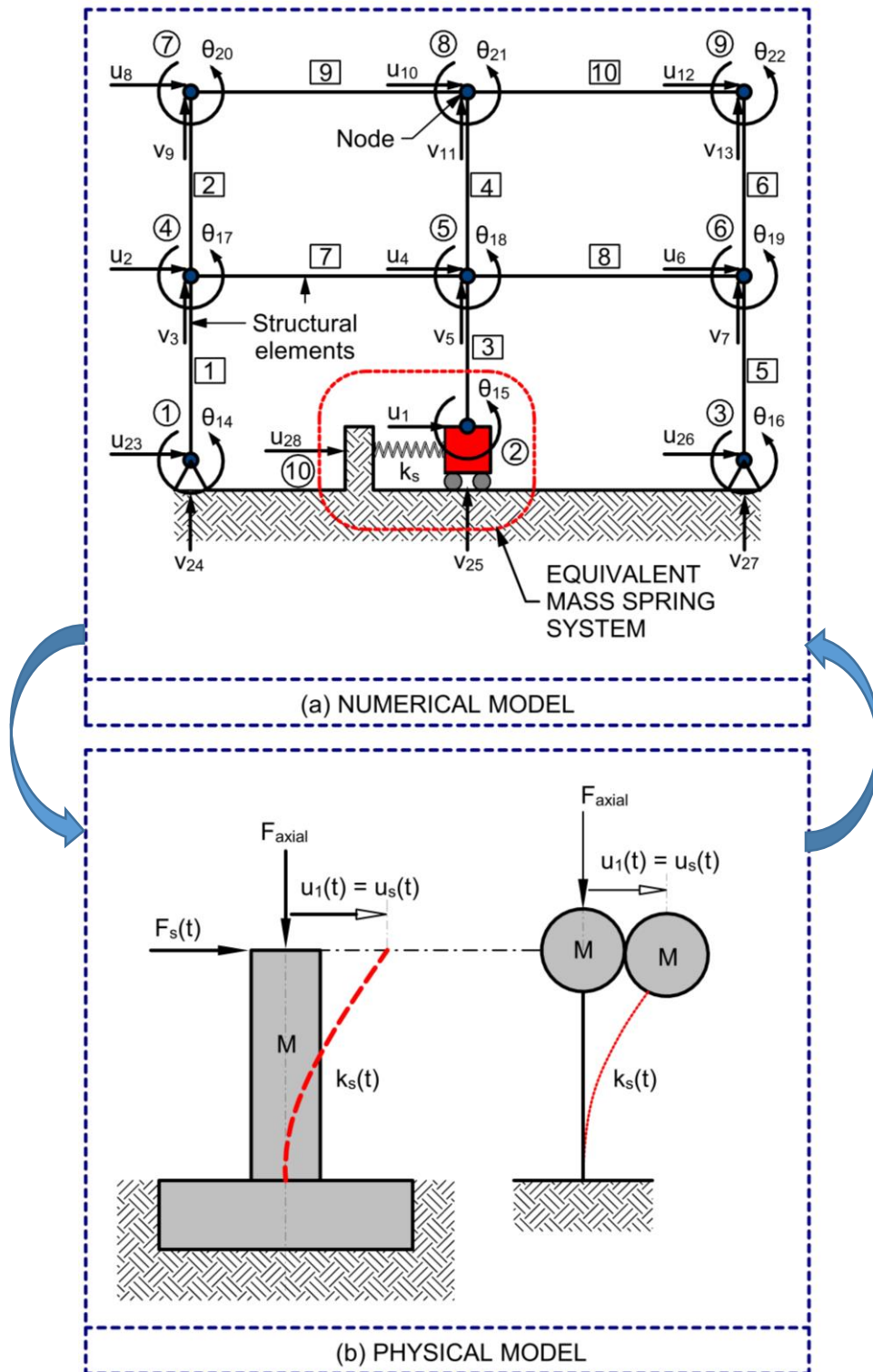


Figure 3-21 Pseudo-dynamic numerical model and physical model

3.6.1 PSEUDO-DYNAMIC ANALYSIS ASSUMPTIONS

To limit the scope of the project several assumptions had to be made regarding the distribution of stiffness and mass within the structure. The assumptions made during the derivation and formulation of the numerical model are as follows:

- The pseudo-dynamic analysis was run considering only a single design for the steel frame structure and the reinforced concrete footing. Therefore, the analysis only considers a single natural period of vibration, which would otherwise vary depending on the selection of the members used to construct the frame;
- The steel frame was modelled using beam elements with 6 degrees of freedom, with two translational and one rotational degree of freedom per node. Because the frame structure will remain linear elastic for the duration of the earthquake record, energy loss due to the non-linear behaviour that would otherwise occur during an earthquake was simulated using Rayleigh damping. Only the first two natural modes of vibration are used to determine the Rayleigh damping coefficients. Failure to incorporate energy loss within the rest of the structure by assuming zero damping will give an unrealistic view of the damage sustained by the footing, and therefore damping needs to be incorporated into the overall structural model that may otherwise experience non-linear behaviour. The downside to not incorporating nonlinearity into the rest of the structure means that the influence that the degradation of stiffness and subsequent increase in the natural period of vibration is not considered when studying the response of the reinforced concrete footing;
- The single lumped mass was used for the reinforced concrete footing, and therefore discrete material responses within the foundation are not considered;
- The only nonlinearity incorporated into the analysis was the lateral deformation of the footing, and therefore any reduction in stiffness of the overall structure will only be due to the stiffness degradation of the footing; and
- During the pseudo-dynamic tests, a constant axial load was applied to the footing with only the horizontal load varying. The constant loads comprised of the static loads on the structure under serviceability limit state loading, which only includes the gravity loads due to the dead weight of the structure and a nominal live load.

3.6.2 LOADING CYCLE PHYSICAL TEST SETUP

A conceptual model of the experimental test setup is shown in Figure 3-22 and the setup within the laboratory is shown in Figure 3-23. The schematic layout showing the front and back view of the experimental test setup with the test setup dimensions and each of the components is shown in Figure 3-24 and Figure 3-25. The test setup comprised of two actuators and two load cells with the vertical actuator being used to simulate the 300 kN constant axial load applied by the overall structure on the footing, which corresponds to the degree of freedom v_{25} in Figure 3-21. The horizontal actuator was used to simulate the horizontal shear load on the footing during the earthquake loading due to the overall dynamic response of the frame structure. The displacement u_s calculated in the computer model was applied directly to the footing using the horizontal actuator with the corresponding force F_s measured using the horizontal load cell [LC1]. The horizontal force was subsequently fed back into the numerical model at which time the numerical model computation continued.

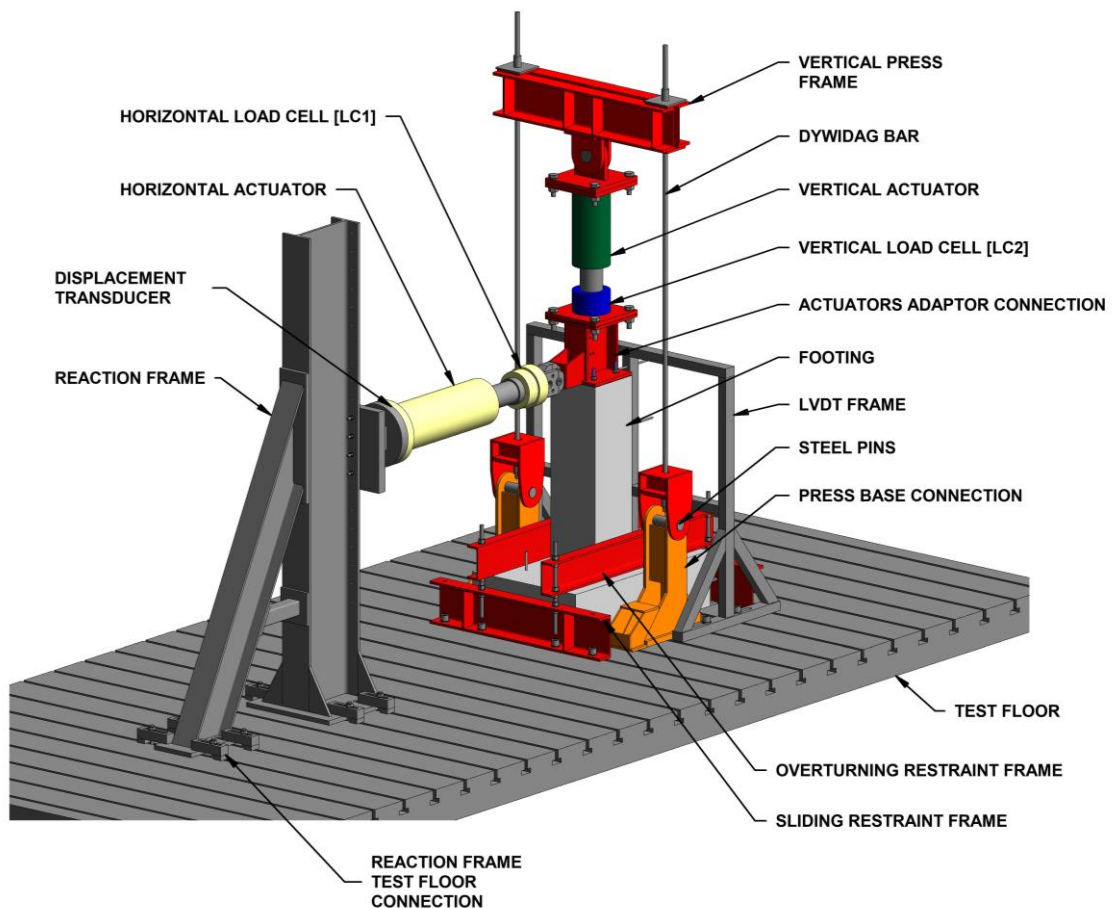


Figure 3-22 Conceptual experimental test setup



Figure 3-23 Experimental test setup in the University of Pretoria's Sasol Laboratory

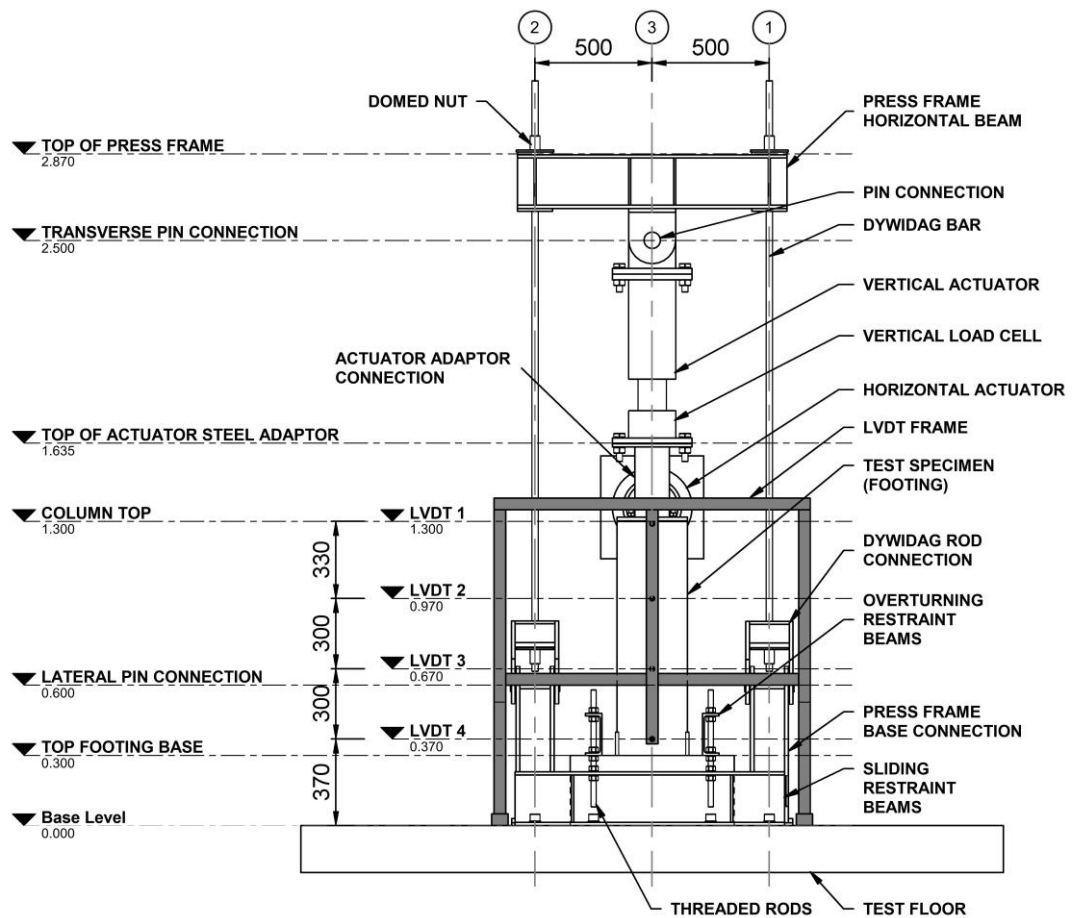


Figure 3-24 Back elevation of the pseudo-dynamic experimental test setup

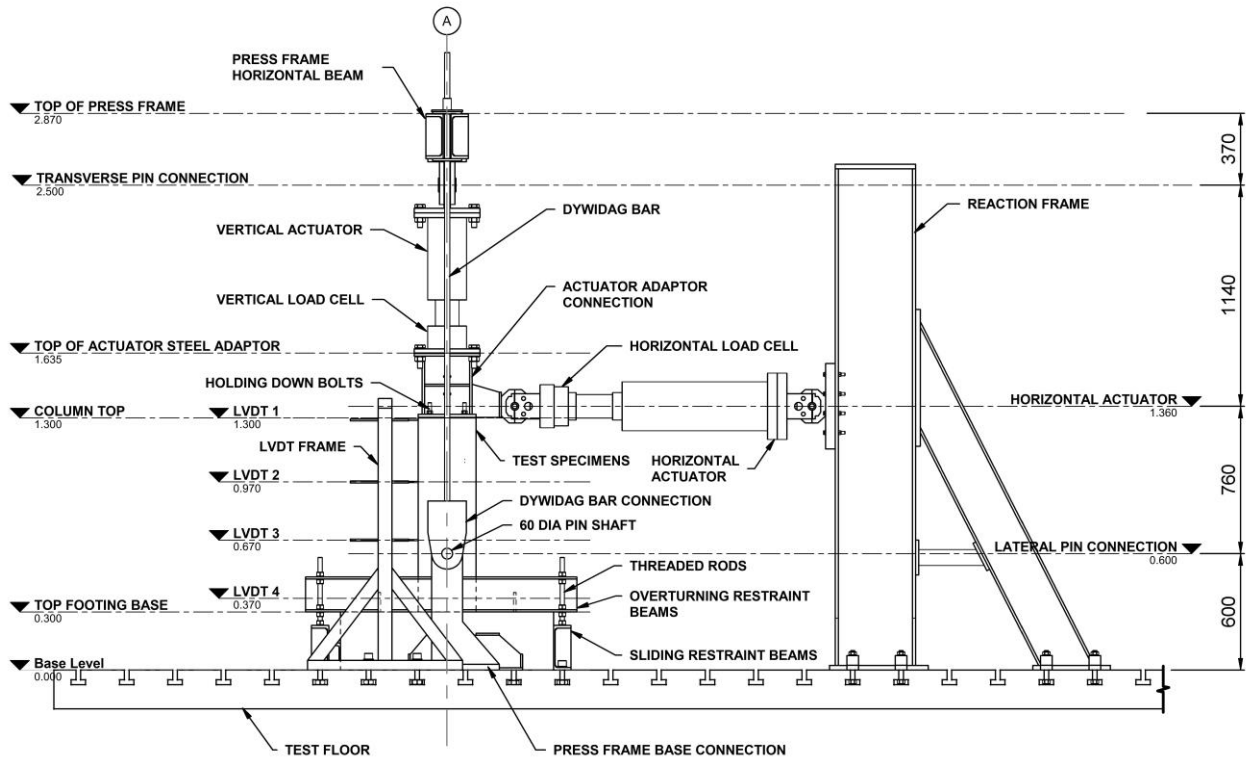


Figure 3-25 Side elevation of the pseudo-dynamic experimental test setup

Figure 3-26 shows the hardware and software that was used to integrate the calculation cycle with the loading cycle. Two control loops were used in the experimental tests with the first control loop being used under force control to maintain the vertical axial load on the reinforced concrete footing. The second control loop was used under displacement control to apply the horizontal displacement calculated in the numerical model onto the reinforced concrete footing at each iteration within each time step. A linear ramp function was used to apply the load incrementally and delay the rate of lateral load applied onto the footing. The objective of the ramp function was to mitigate dynamic effects during the analysis due to large changes in calculated displacements between iterations.

Figure 3-26 also shows a third path that comprised of additional instruments attached to the experimental test setup. Strain gauges were attached to two of the four longitudinal reinforcing bars, with a reinforcing bar with strain gauge being placed on each column face in the direction of loading. The strain gauges were applied at the base of the column where the maximum moment was expected. Four additional linear displacement transducers were placed at equal increments over the length of the column with the aim to compare the results with the internal displacement transducer in the horizontal actuator.

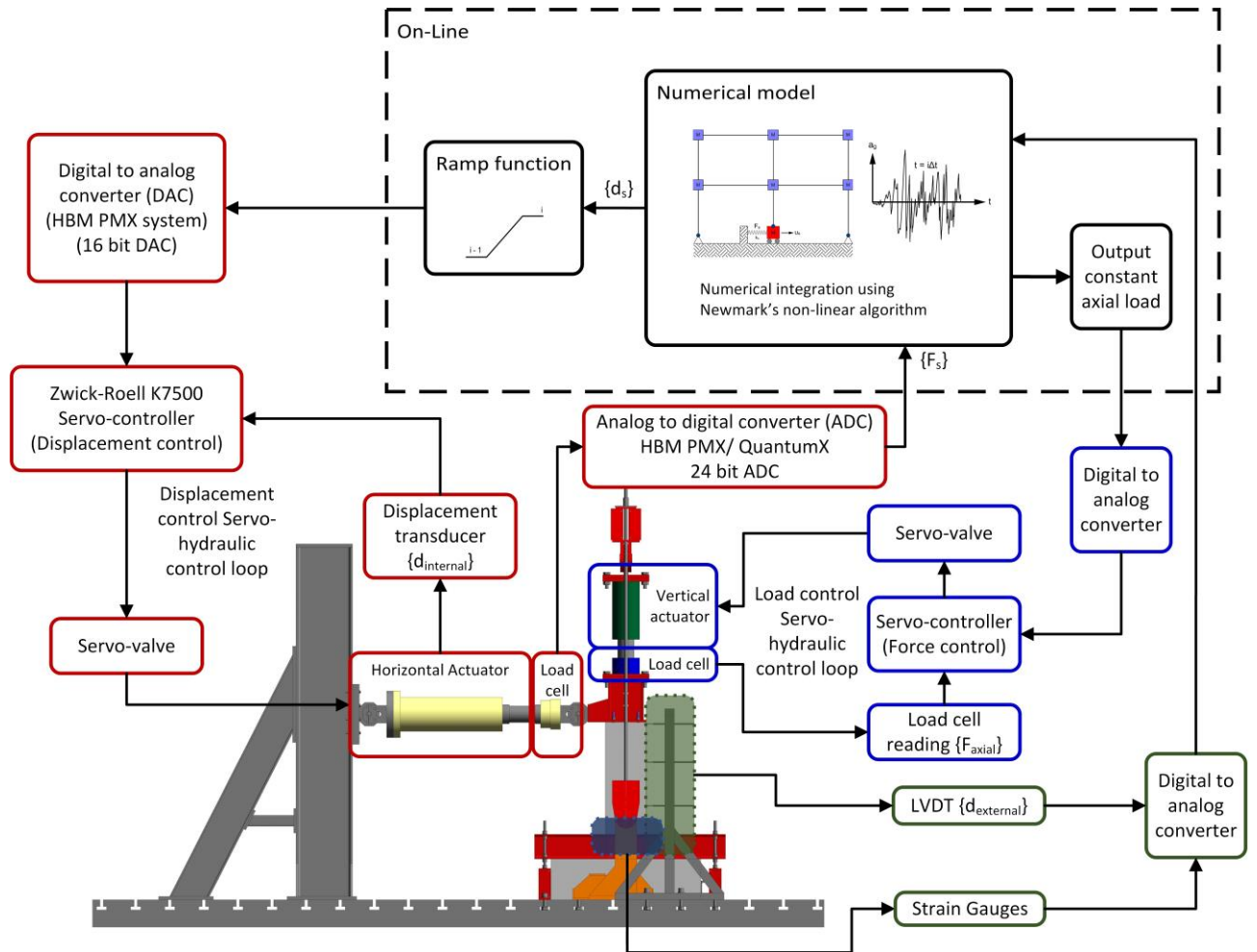


Figure 3-26 Schematic illustration of external components of the pseudo-dynamic experiment

3.6.3 CALCULATION CYCLE NUMERICAL MODEL

The non-linear Newmark's implicit method using the average acceleration method was used to perform the pseudo-dynamic experiments, and Figure 3-27 shows the pseudo-dynamic algorithm used to run the analysis. The script that was developed to run the pseudo-dynamic analysis is given in Appendix A. The algorithm was adapted to incorporate static condensation to eliminate the rotational free degrees of freedom with zero mass within the overall frame structure. The unknown displacements and forces within the structure using the initial state of the structure are solved at each time-step:

$$i = 1, 2, 3 \dots n \quad \text{with } t = i\Delta t$$

With n being the total number of time steps in the earthquake record and t being the time in seconds at any point within the analysis. The average acceleration method produces an unconditionally stable solution and therefore the selection of the time step only influenced the accuracy of the solution and not the stability of the solution. For each time step " i " within the earthquake record, iteration, " j ", was required due to the implicit relationship between the restoring force matrix $(f_s^*)_i$ and the unknown displacement of the structure at $(u)_i$ as shown in Step 15 in Figure 3-27. Therefore, the stiffness of the footing must be assumed before the calculation at each time step can be initialised. The initial stiffness of the footing had to be determined before the pseudo-dynamic analysis could commence. The selection of the initial lateral stiffness of the footing had to be considered carefully because assuming a stiffness that was lower than the maximum true elastic stiffness of the footing would produce premature damage to the footing at the start of each time step. Selecting a stiffness lower than the elastic stiffness of the footing can also produce instability problems in the convergence of the solution.

Reinforced concrete is a highly non-linear material, and with the application of the applied loading at the beginning of each time step, an initial out of balance residual force vector was produced. As a result, the structure was no longer in force or energy equilibrium. In general, the displacement applied to the structure results in a restoring force that differs from that of the restoring force that was calculated using the initially maximum stiffness of the structure. Because of this, the stiffness of the footing (k_s) between the previous time step and the current time step was updated at each iteration until there was convergence to a solution. Once the solution converged, and equilibrium within the structure was achieved, the next time step " i " can commence. Force and energy equilibrium had to be ensured within a certain level of accuracy by specifying a convergence criterion at the start of the pseudo-dynamic analysis, which was less than the resolution of the data acquisition system.

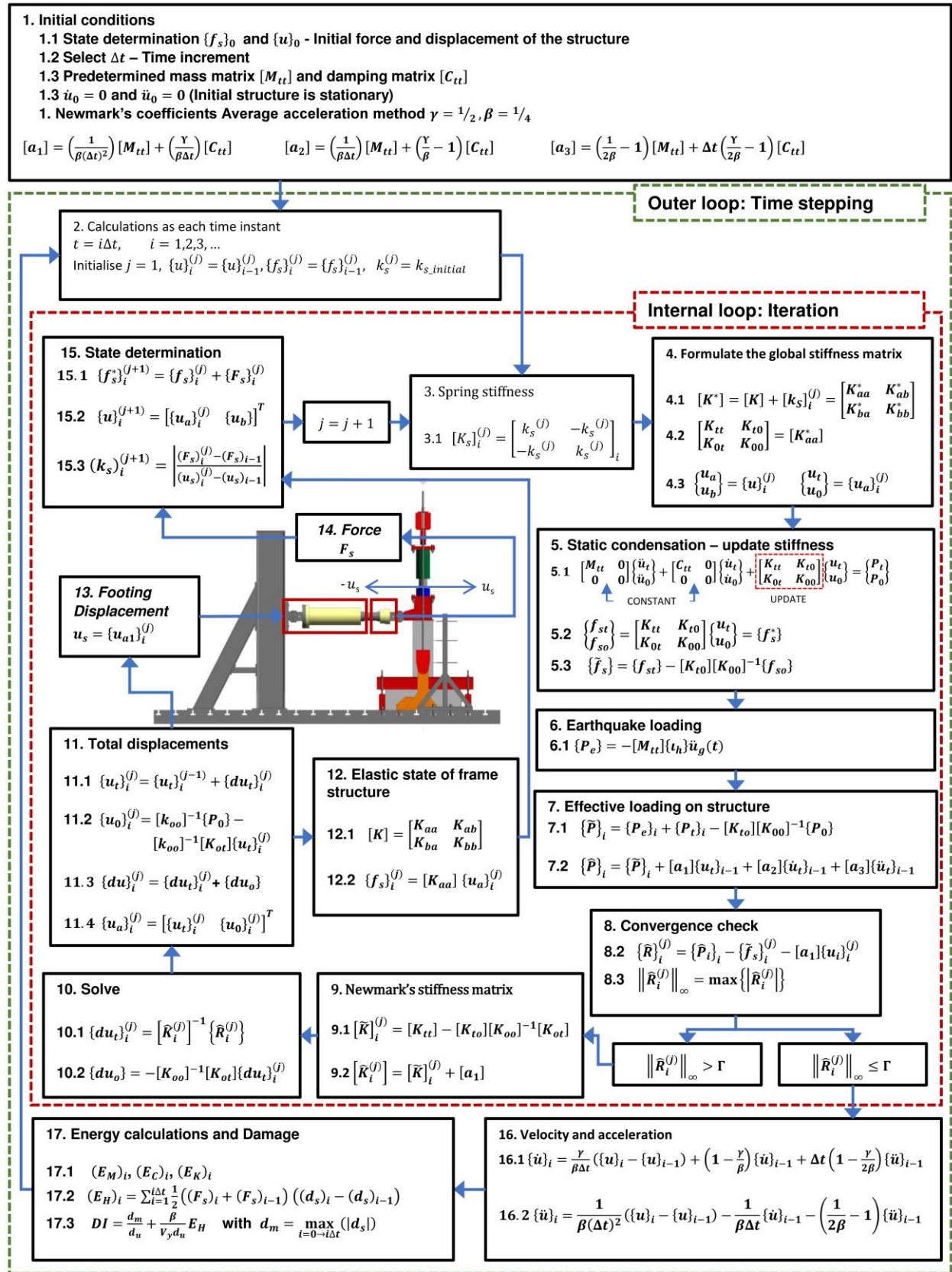


Figure 3-27 Pseudo-dynamic numerical model algorithm

The initial calculations consisted of analysing the structure under static loading to determine the initial state of the structure before the application of the earthquake excitation. Equation 3.2 needs to be solved for the unknown displacements, using the applied gravity loads, to determine the static distribution of load in the structure. Equation 3.3 is the initial stiffness matrix of a spring element that was used for the lateral displacement of the footing and includes the position within the overall global stiffness matrix.

$$[K]\{u\}_0 + [K_s]_0\{u\}_0 = [K^*]_0\{u\}_0 = \{P\} \quad (3.2)$$

$$[K_s]_0 = \begin{bmatrix} u_1 & \dots & u_{28} \\ k_{s0} & \dots & -k_{s0} \\ \vdots & \ddots & \vdots \\ -k_{s0} & \dots & k_{s0} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_{28} \end{bmatrix} \quad \text{with } u_s = u_1 \quad (3.3)$$

Where $\{P\}$ is the statically applied point loads on the structure and $[K]$ represents the overall global elastic stiffness matrix of the frame structure that was formulated using beam elements. The matrix $[K_s]_0$ is the initial stiffness matrix of the reinforced concrete footing with k_{s0} the initial stiffness of the spring element at time equal to zero. The vector given by $\{u\}_0$ was the initial displacements of the structure before the earthquake loading was applied to the structure.

Beam elements were used to model the frame structure with three degrees of freedom associated with each node to reduce computation demand. Regarding Figure 3-21(a), the structure was discretised into ten beam elements with ten nodes. Nodes one through nine of the frame structure comprised of three degrees of freedom (two translational and one rotational degree of freedom) whereas node ten only consisted of a single translational degree of freedom. Node ten only provides a horizontal translational boundary condition for the non-linear idealised spring element used to model the lateral displacement of the footing. This results in 28 degrees of freedom.

The degrees of freedom of the frame structure was numbered such that the translational free degrees of freedom were numbered first followed by the rotational free degrees of freedom and finally the support degrees of freedom, which have zero displacement. Figure 3-28 shows the degrees of freedom for the frame structure and the order in which they were numbered. During the static analysis, the numbering of the free degrees of freedom is not critical. However, during the dynamic analysis, only the translational free degrees of freedom are considered to contain mass with the inertia effects due to rotational degrees of freedom being neglected. Therefore, the static condensation method was used to eliminate the rotational free degrees of freedom

from the dynamic analysis. The order that the degrees of freedom were numbered was done to ameliorate the formulation of the statically condensed mass and stiffness matrices.

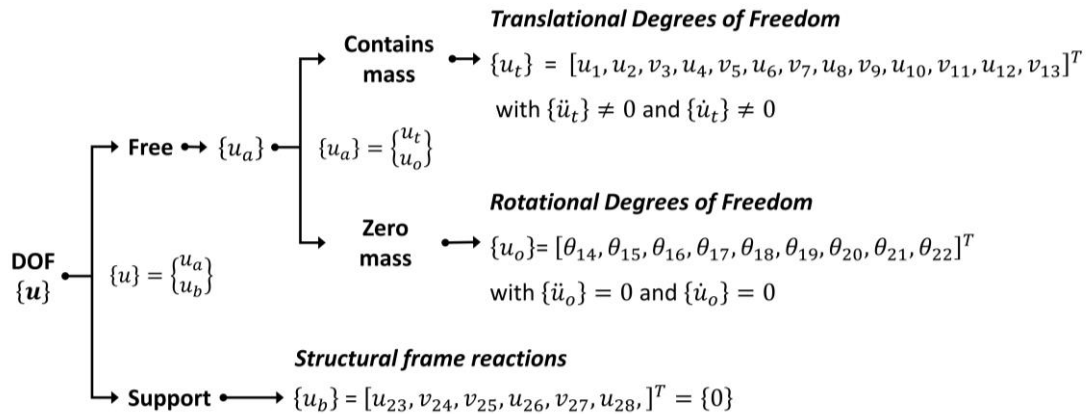


Figure 3-28 Degrees of freedom numbering

Before the structure was subjected to the earthquake loading, the structure would have initial displacements and forces in the members due to gravity loading from the dead load of the structure and live load under serviceability limit states. The same loadings as presented in Section 3.3.1 were used with the applied loading on the numerical model shown in Figure 3-29 to ensure an axial load of 300 kN was produced on the central footing. As the calculations are only done at the nodes, the point loads and moments at the nodes due to the distributed loading was determined using the fixed end reactions as shown in Figure 3-30.

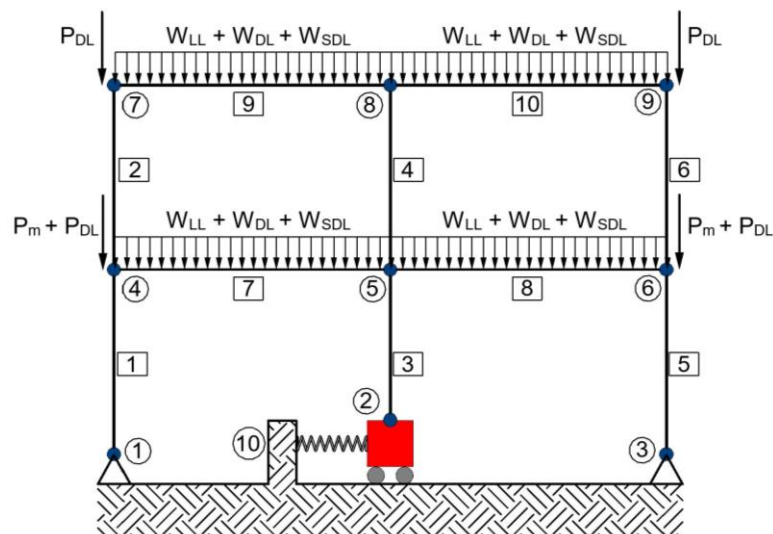


Figure 3-29 Static loading applied to the numerical model

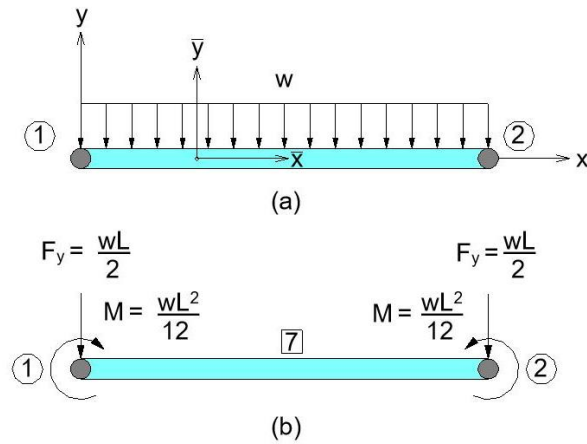


Figure 3-30 Fixed-end reactions due to a uniformly distributed load

The primary objective of the static analysis was to obtain the initial condition or state of the structure before applying the earthquake loading. The overall stiffness matrix $[K^*]$ consists of both the stiffness components from the frame structure and spring element as shown in Equation 3.4. In Equation 3.4 the subscript "b" indicates components of the stiffness matrix relating to the supports and subscript "a" relating to components associated with the free degrees of freedom as previously alluded to in Figure 3-28. The first subscript in the stiffness matrix in Equation 3.5 relates to the force component of a degree of freedom and the second subscript relates to the displacement component.

$$[K^*] = \underbrace{[K]}_{\substack{\text{Frame} \\ \text{Stiffness} \\ \text{Matrix} \\ \text{(Linear)}}} + \underbrace{[K_s]}_{\substack{\text{Spring} \\ \text{Stiffness} \\ \text{Matrix} \\ \text{(Non-Linear)}}} = \begin{bmatrix} K_{aa}^* & K_{ab}^* \\ K_{ba}^* & K_{bb}^* \end{bmatrix} \quad (3.4)$$

$$\begin{Bmatrix} P_a \\ P_b \end{Bmatrix} = \begin{bmatrix} K_{aa}^* & K_{ab}^* \\ K_{ba}^* & K_{bb}^* \end{bmatrix} \begin{Bmatrix} u_a \\ u_b \end{Bmatrix} \quad \text{Where } \{u_b\} = \{0\} \quad (3.5)$$

Using Equation 3.5, the unknown displacements $\{u_a\}$ was obtained by solving Equation 3.6 with the reactions determined using Equation 3.8. The initial forces at each of the degrees of freedom $\{f_s^*\}$ under static conditions were obtained from the applied loads on the structure in global coordinates as shown in Equation 3.9.

$$\{u_a\} = [K_{aa}^*]^{-1}\{P_a\} \quad (3.6)$$

$$\{u_{s0}\} = \{u_a\} \quad (3.7)$$

$$\{P_b\} = [K_{ba}^*]\{u_a\} \quad (3.8)$$

$$\{f_s^*\} = [K_{aa}^*]\{u_a\} = \{P_a\} \quad (3.9)$$

The mass matrix $[M]$ was formulated by lumping the mass of the beams, columns and floor slab equally amongst the corresponding nodes as shown in Figure 3-31. All the mass associated with the masonry walls was lumped at the base node as it was assumed that the mass would not be attached to the top beam. Inertial effects due to the mass rotation have not been considered in this analysis. Therefore, the masses associated with rotational free degrees of freedom are equal to zero. The mass matrix is formulated from the following masses within the structure:

- Self-weight of the columns and beams;
- Dead load from the concrete floor slabs;
- Nominal live load from occupants and other non-structural items; and
- Mass of the masonry infill panels.

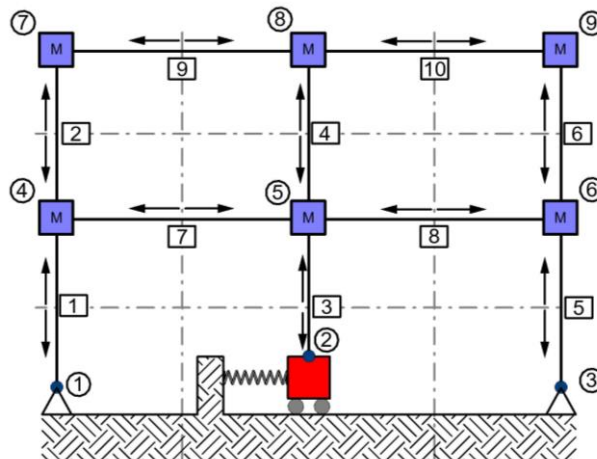


Figure 3-31 Distribution of mass within the structure to the nodes

The rotational degrees of freedom that contain zero mass were eliminated from the dynamic analysis using static condensation. It is therefore assumed that energy dissipation only occurs in the mass degrees of freedom. Therefore, the diagonal elements in the mass matrix $[M]$ that

contain zero entry values need to be eliminated from the dynamic analysis during the time stepping analysis. The process entails ordering the mass matrix in such a manner that the first diagonal entries within the mass matrix correspond to the degrees of freedom that contain mass and the subsequent diagonal entries containing zero mass. The ordering of the degrees of freedom was shown previously in Figure 3-21(a) when the degrees of freedom were numbered.

Numbering the translational free degrees of freedom followed by the rotational degrees of freedom results in the formulation of Equation 3.10 without having to reorder the matrix $[K_{aa}^*]$. The degrees of freedom $\{u_t\}$ corresponds to free degrees of freedom that contain mass (free translational degrees of freedom) and $\{u_0\}$ corresponds to free degrees of freedom that contain zero mass (rotational degrees of freedom). The statically condensed equation of motion, which includes damping, containing all the degrees of freedom is written in partitioned form is shown in Equation 3.10. Equation 3.11 shows the statically condensed form that excludes the support degrees of freedom.

$$\begin{bmatrix} \mathbf{M}_{tt} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_t \\ \ddot{u}_0 \\ 0 \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{tt} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_t \\ \dot{u}_0 \\ 0 \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{tt} & \mathbf{K}_{t0} & K_{ab} \\ \mathbf{K}_{0t} & \mathbf{K}_{00} & K_{bb} \\ K_{ba} & & K_{bb} \end{bmatrix} \begin{Bmatrix} u_t \\ u_0 \\ u_b \end{Bmatrix} = \begin{Bmatrix} P_t \\ P_0 \\ P_b \end{Bmatrix} \quad (3.10)$$

$$\begin{bmatrix} M_{tt} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_t \\ \ddot{u}_0 \end{Bmatrix} + \begin{bmatrix} C_{tt} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_t \\ \dot{u}_0 \end{Bmatrix} + \begin{bmatrix} K_{tt} & K_{t0} \\ K_{0t} & K_{00} \end{bmatrix} \begin{Bmatrix} u_t \\ u_0 \end{Bmatrix} = \begin{Bmatrix} P_t \\ P_0 \end{Bmatrix} \quad (3.11)$$

Where:

$$\begin{bmatrix} K_{tt} & K_{t0} \\ K_{0t} & K_{00} \end{bmatrix} = [K_{aa}^*], \quad [u_a] = \begin{Bmatrix} u_t \\ u_0 \end{Bmatrix}, \quad P_a = \begin{Bmatrix} P_t \\ P_0 \end{Bmatrix}$$

And:

- $[M_{tt}]$: Mass matrix that corresponds to the free translational degrees of freedom
- $[C_{tt}]$: Rayleigh damping matrix that is determined from $[M_{tt}]$ and $[K_{tt}]$
- $[K_{tt}]$: Stiffness matrix consisting of free translational degrees of freedom with assigned mass
- $\{\ddot{u}_t\}$: Accelerations of the translational degrees of freedom with mass
- $\{\ddot{u}_0\}$: Accelerations of the free rotational degrees of freedom
- $\{\dot{u}_t\}$: Velocities of the free translational degrees of freedom
- $\{\dot{u}_0\}$: Velocities of the free rotational degrees of freedom
- $\{u_t\}$: Displacements of the free translational degrees of freedom
- $\{u_0\}$: Displacements of the free rotational degrees of freedom

Multiplying through with Equation 3.11 produces Equation 3.12 and Equation 3.13. Equation 3.13 allows for the static relationship between $\{u_0\}$ and $\{u_t\}$ to exist due to the lack of presence of inertia and damping terms and is reordered in Equation 3.14. Any static loads applied to degrees of freedom with zero mass is also incorporated into Equation 3.13.

$$[M_{tt}]\{\ddot{u}_t\} + [C_{tt}]\{\dot{u}_t\} + [K_{tt}]\{u_t\} + [K_{t0}]\{u_0\} = \{P_t\} \quad (3.12)$$

$$[K_{0t}]\{u_t\} + [K_{00}]\{u_0\} = \{P_0\} \quad (3.13)$$

$$\{u_0\} = [K_{00}]^{-1}\{P_0\} - [K_{00}]^{-1}[K_{0t}]\{u_t\} \quad (3.14)$$

Equation 3.14 is substituted into Equation 3.12 with the resultant expression shown in Equation 3.15.

$$\begin{aligned} [M_{tt}]\{\ddot{u}_t\} + [C_{tt}]\{\dot{u}_t\} + [[K_{tt}] - [K_{t0}][K_{00}]^{-1}[K_{0t}]]\{u_t\} \\ = \{P_t\} - [K_{t0}][K_{00}]^{-1}[K_{0t}]\{P_0\} \end{aligned} \quad (3.15)$$

Simplifying Equation 3.15 results in the following expressions:

$$[M_{tt}]\{\ddot{u}_t\} + [C_{tt}]\{\dot{u}_t\} + [\tilde{K}]\{u_t\} = \{\tilde{P}\} \quad (3.16)$$

$$[\tilde{K}] = [K_{tt}] - [K_{t0}][K_{00}]^{-1}[K_{0t}] \quad (3.17)$$

$$\{\tilde{P}\} = \{P_t\} - [K_{t0}][K_{00}]^{-1}[K_{0t}]\{P_0\} \quad (3.18)$$

$$\{P_t\} = \{P_g\} + \{P_f\} = -[M_{tt}]\{I\}\ddot{u}_g + \{P_f\} \quad (3.19)$$

The static loads in the structure or state of the structure $\{f_s^*\}$ at each time increment was separated into two components as shown in Equation 3.20 with $\{f_{st}\}$ representing the translational degrees of freedom and $\{f_{s0}\}$ representing the rotational degrees of freedom. The state of the structure due to static condensation $\{\tilde{f}_s\}$ is defined in Equation 3.21 and derived by multiplying out Equation 3.20 and reordering the terms such that Equation 3.22 is produced.

$$\{f_s^*\} = \begin{Bmatrix} f_{st} \\ f_{s0} \end{Bmatrix} = \begin{bmatrix} K_{tt} & K_{t0} \\ K_{0t} & K_{00} \end{bmatrix} \begin{Bmatrix} u_t \\ u_0 \end{Bmatrix} \quad (3.20)$$

The derivation of $\{\tilde{f}_s\}$ using Equation 3.20 is as follows:

$$\{f_{st}\} = [K_{tt}]\{u_t\} + [K_{t0}]\{u_0\}$$

$$\{u_0\} = [K_{00}]^{-1}\{f_{s0}\} - [K_{00}]^{-1}[K_{0t}]\{u_t\}$$

$$([K_{tt}] - [K_{t0}][K_{00}]^{-1}[K_{0t}])\{u_t\} = \{f_{st}\} - [K_{t0}][K_{00}]^{-1}\{f_{s0}\}$$

$$[\tilde{K}]\{u_t\} = \{f_{st}\} - [K_{t0}][K_{00}]^{-1}\{f_{s0}\}$$

$$\{\tilde{f}_s\} = [\tilde{K}]\{u_t\} \quad (3.21)$$

$$\{\tilde{f}_s\} = \{f_{st}\} - [K_{t0}][K_{00}]^{-1}\{f_{s0}\} \quad (3.22)$$

To produce the damping matrix $[C_{tt}]$ required the calculation of the first two natural periods of vibration of the structure by solving the determinant for Equation 3.23 for the non-trivial solution. Equation 3.24 was used to solve for the Rayleigh damping coefficients a_0 and a_1 using a 5% ($\xi = 0.05$) damping ratio and the natural periods of vibration of the structure calculated using Equation 3.23. The first two modes of vibration contribute significantly to the overall response of the structure, with both modes of vibration being assigned the same damping value. SANS 10160-4:2017 traditionally uses a damping value of 5%, as the design elastic response spectra within the code are derived using a 5% damping value. Upon solving for the statically condensed mass and stiffness matrices, the damping matrix $[C_{tt}]$ can be calculated using Equation 3.25.

$$([\tilde{K}] - \omega_n^2[M_{tt}])\varphi_n = \{0\} \quad (3.23)$$

$$\frac{1}{2} \begin{bmatrix} 1/\omega_1 & \omega_1 \\ 1/\omega_2 & \omega_2 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} \xi \\ \xi \end{Bmatrix} \quad (3.24)$$

$$[C_{tt}] = a_0[M_{tt}] + a_1[K_{tt}] \quad (3.25)$$

The only non-linear component in the structure was due to the lateral stiffness ($k_s(t)$) of the reinforced concrete footing. Therefore, the stiffness matrix of the frame structure remains constant for the duration of the experiment. Within each iteration within a timestep, the spring stiffness $k_s(t)$ was updated in Equation 3.26, which was then incorporated back into the overall global stiffness matrix.

$$[K_s]_i^{(j)} = \begin{bmatrix} u_1 & \dots & u_{28} \\ k_s^{(j)} & \dots & -k_s^{(j)} \\ \vdots & \ddots & \vdots \\ -k_s^{(j)} & \dots & k_s^{(j)} \end{bmatrix} \begin{matrix} u_1 \\ \vdots \\ u_{28} \end{matrix} \text{ with } u_s = u_1 \quad (3.26)$$

The earthquake loading on the frame structure was determined by using Equation 3.27 with the influence vector $\{I\}$ only including the horizontal translational free degrees of freedom. The effective loading $\{\tilde{P}\}_i$ was calculated in Equation 3.28 and accounts for the static load components due to static condensation of the frame structure and the earthquake loading. Equation 3.28 is then substituted into the familiar Newmark equation, which is shown in Equation 3.29.

$$\{P_e\}_i = -[M_{tt}]\{I\}(\ddot{u}_g)_i \quad (3.27)$$

$$\{\tilde{P}\}_i = \{P_e\}_i + \{P_t\}_i - [K_{to}][K_{00}]^{-1}\{P_0\} \quad (3.28)$$

$$\{\hat{P}\}_i = \{\tilde{P}\}_i + [a_1]\{u\}_{i-1} + [a_2]\{\dot{u}\}_{i-1} + [a_3]\{\ddot{u}\}_{i-1} \quad (3.29)$$

The unbalanced load on the structure for each degree of freedom was determined using Equation 3.30. The infinity norm of a vector was used to determine whether the residual vector had converged to a solution within the specified convergence criteria. Once the convergence criterion has been achieved, the next time step was initialised. The infinity norm determines the maximum absolute value within the residual vector produced in Equation 3.30. The stop criteria in the research was selected to be 5×10^{-5} and was selected to be less than the resolution of the data acquisition system, which had a resolution of 0.003.

$$\{\hat{R}\}_i^{(j)} = \{\hat{P}\}_i - \{\tilde{f}_s\}_i^{(j)} - [a_1]\{u\}_i^{(j)} \quad (3.30)$$

$$\|\hat{R}_i^{(j)}\|_{\infty} = \max\{|\hat{R}_i^{(j)}|\} \quad (3.31)$$

If the stop criterion was not met, the following calculations were performed to update the displacements and internal forces within the structure. The statically condensed stiffness matrix was determined using Equation 3.32 and was added to the Newmark's coefficient matrix $[a_1]$ to produce the Newmark's stiffness matrix $[\hat{K}_i^{(j)}]$ as shown in Equation 3.33.

$$[\tilde{K}]^{(j)} = [K_{tt}] - [K_{to}][K_{oo}]^{-1}[K_{ot}] \quad (3.32)$$

$$[\hat{K}_i^{(j)}] = [\tilde{K}]^{(j)} + [a_1] \quad (3.33)$$

The change in displacement due to the unbalanced residual vector is calculated indicatively using Equation 3.34. The change in displacement was added to the current displacement of the structure at the degrees of freedom with mass from the previous iteration, which is indicated in Equation 3.35.

$$\{du_t\}_i^{(j)} = [\hat{K}_i^{(j)}]^{-1}\{\hat{R}\}_i^{(j)} \quad (3.34)$$

$$\{u_t\}_i^{(j)} = \{u_t\}_i^{(j-1)} + \{du_t\}_i^{(j)} \quad (3.35)$$

Equation 3.36 is solved to determine the change in deflection of the zero-mass degrees of freedom. Equation 3.37 is used to determine the overall displacement of the zero-mass degrees of freedom, which requires the displacements for static rotational degrees of freedom. The overall displacement of the structure was subsequently combined into a single vector $\{du\}_i^{(j)}$ that was ordered according to the degrees of freedom as defined in Figure 3-21(a).

$$\{du_o\} = -[K_{oo}]^{-1}[K_{ot}]\{du_t\}_i^{(j)} \quad (3.36)$$

$$\{u_0\}_i^{(j)} = [k_{oo}]^{-1}\{P_0\} - [K_{oo}]^{-1}[K_{ot}]\{u_t\}_i^{(j)} \quad (3.37)$$

Equation 3.38 produces the total displacement of the structure that includes both the zero-mass and mass degrees of freedom while the change in displacement is given by Equation 3.39. For Equation 3.38 and Equation 3.39, the values within the vectors are ordered consecutively according to the numbering of the degrees of freedom shown previously in Figure 3-21(a).

$$\{u_a\}_i^{(j)} = [\{u_t\}_i^{(j)}, \{u_0\}_i^{(j)}]^T \quad (3.38)$$

$$\{du_a\}_i^{(j)} = [\{du_t\}_i^{(j)}, \{du_o\}_i^{(j)}]^T \quad (3.39)$$

The state of the structure $\{f_s^*\}$ and $\{u_s\}$ was subsequently updated within a given iteration "j" in each timestep "i". Because the only non-linearity in the system was the lateral displacement of the reinforced concrete footing, the force and displacements components due to the frame structure can be calculated using Equation 3.40 and Equation 3.41.

$$\{f_s\}_i^{(j)} = [K_{aa}]\{u_a\}_i^{(j)} \quad (3.40)$$

$$\{u_s\}_i^{(j)} = \{u_a\}_i^{(j)} \quad (3.41)$$

The force component F_s from the lateral displacement of the reinforced concrete footing was read from the horizontal load cell from the physical test setup and added to the respective degree of freedom in $\{f_s^*\}$. The updated $\{f_s^*\}$ is shown in Equation 3.42. The stiffness of the footing can be updated once the restoring force of the reinforced concrete footing was fed back into the numerical model from the horizontal load cell. The stiffness of the footing was determined by using the previous time steps displacement and force components and the calculated total displacement and measured restoring force in the current time step and iteration. The stiffness of the reinforced concrete footing was calculated using Equation 3.43.

$$\{f_s^*\}_i^{(j)} = \{f_s\}_i^{(j)} + \{F_s\}_i^{(j)} \quad (3.42)$$

$$(k_s)_i^{(j+1)} = \left| \frac{(F_s)_i^{(j)} - (F_s)_{i-1}}{(u_s)_i^{(j)} - (u_s)_{i-1}} \right| \quad (3.43)$$

Figure 3-32 and Figure 3-33 shows a hypothetical example of a change in slope from positive to negative that was run for two cases whereby the absolute value of the calculated stiffness is used and when it is not used. Using the absolute value of the slope, as shown in Equation 3.43, results in the rate of convergence to the solution being slower than that produced by not taking the absolute value. In both cases, the solution converged to the same point on the negative slope and the only benefit with using the absolute value is that the load increments that are applied by the horizontal load actuator to the structure are much smaller. The smaller load increments in combination with the ramp function, as described in Figure 3-26, ensures a slow rate of loading onto the test specimen thus preventing dynamic effects being imparted on the footing.

Figure 3-32 and Figure 3-33 show that the point $(u_s)_{i-1}$ does not necessarily coincide with the inflection/yield point of the analytical model used to describe the reinforced concrete section. During the pseudo-dynamic experiment on the physical test specimen, the reinforced concrete cyclic behavior is not known, and the actual F_s values are obtained from the horizontal load cell [LC1] as shown previously in Figure 3-22. The error at the inflection points in the material response is remedied by reducing the time increments.

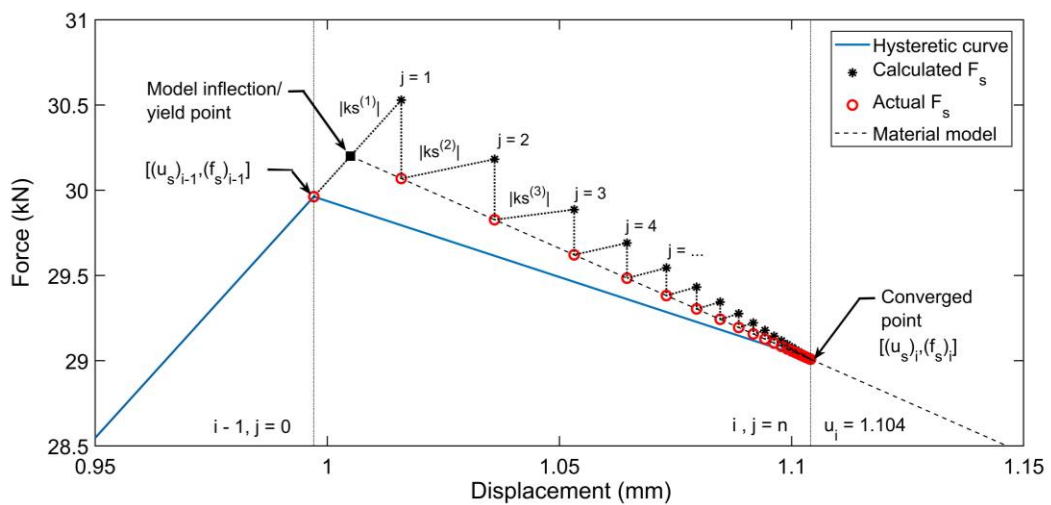


Figure 3-32 Newton-Raphson iteration at an inflection point where the slope changes from positive to negative with the absolute value used for the computational slope k_s

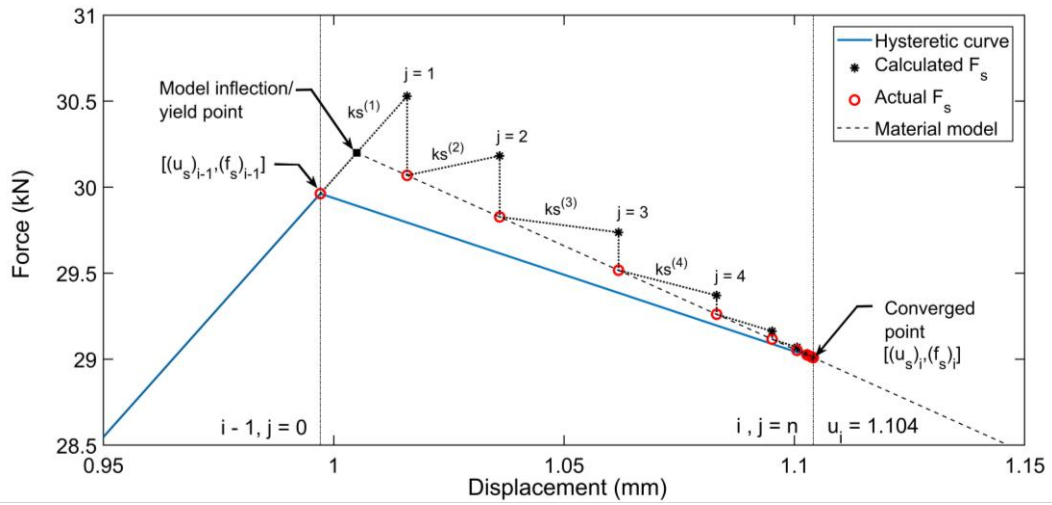


Figure 3-33 Newton-Raphson iteration at an inflection point where the model slope changes from positive to negative and the computation slope k_s being either positive or negative

Figure 3-34 and Figure 3-35 shows the Newton-Raphson iteration on a negative slope. Figure 3-34 shows the iteration in a time step whereby the absolute value of the stiffness is used for each iteration and therefore produces more iterations with smaller displacement increments. Figure 3-35 shows the iteration that produces fewer iteration steps as the stiffness can either be positive or negative. However, the displacement steps produced at each iteration are much larger. The stiffness $k_s^{(1)}$ is equal to the initial stiffness of the structure and corresponds to the maximum stiffness that the test specimen can achieve.

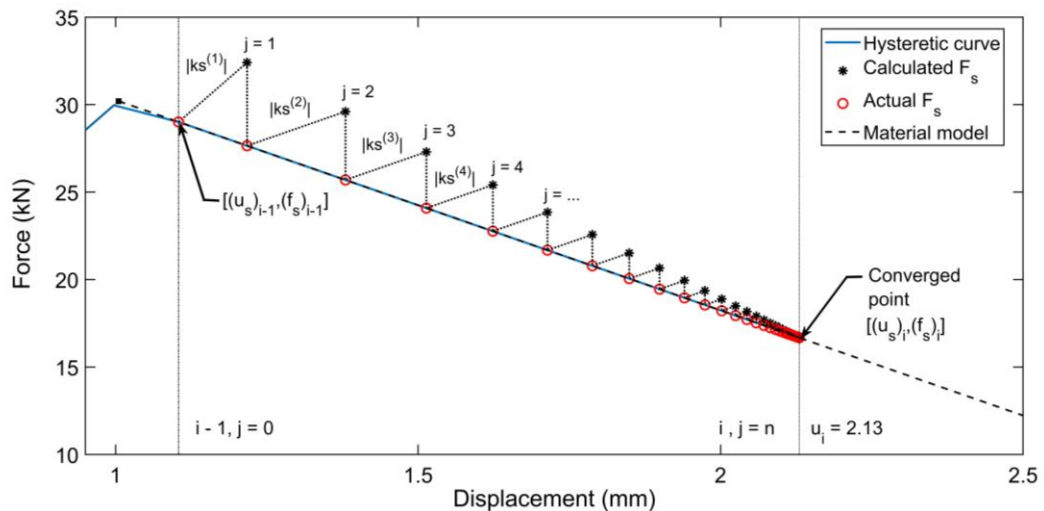


Figure 3-34 Newton-Raphson iteration on a negative slope with the computational slope k_s taken as only positive

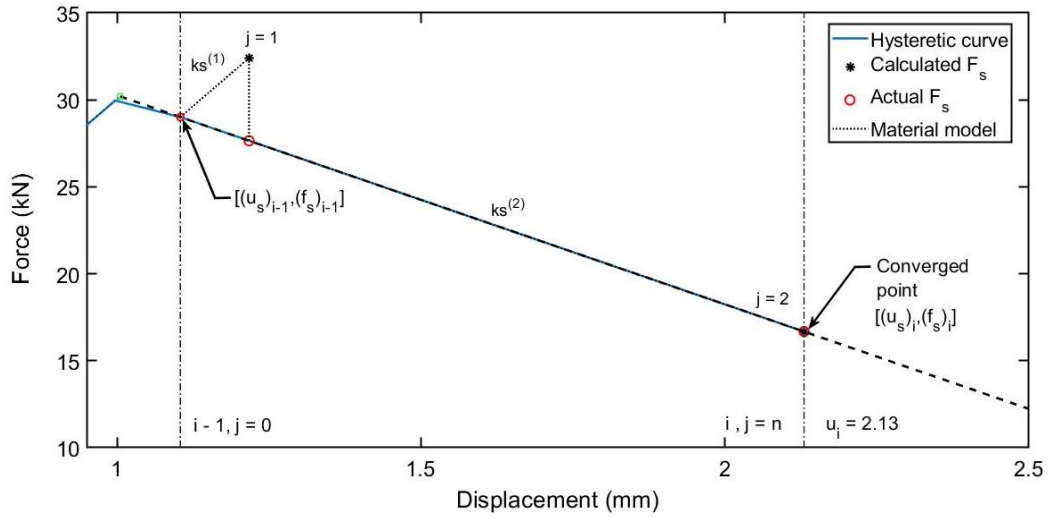


Figure 3-35 Newton-Raphson iteration on a negative slope with the computation slope k_s being either positive or negative

3.6.4 ENERGY AND DAMAGE CALCULATIONS

The expressions for the various energy terms in this study were determined numerically at each time increment and were determined by cumulatively summing the change in energy at each time increment for the duration of the experiment. The initial energy expressions at the start of the analysis are equal to the work done by the statically applied loads, which is equal to the energy absorbed by the structure as shown in Equation 3.44. At the start of the analysis the structure was stationary, and therefore the kinetic energy and damping energy were equal to zero. Because the structure is symmetrical, the lateral displacement of the internal footing was equal to zero under gravity loads, which results in the hysteretic energy being equal to zero before the analysis starts.

$$(E_k)_0 = \frac{1}{2} \{P\}^T \{u\}_0 = \frac{1}{2} [[K^*]_0 \{u\}_0]^T \{u\}_0 \quad (3.44)$$

Energy (E_T) was imparted to the structure once the structure was subjected to the earthquake excitation. During the analysis, a portion of the energy was temporally stored as kinetic energy (E_M) and strain energy (E_K), with the rest being dissipated in the form of damping energy (E_C), and hysteretic energy (E_H). The total energy was calculated using Equation 3.45 and Equation 3.46. The expressions for each of the energy terms is given by Equation 3.47 to Equation 3.50.

$$\{P^*\}_i = \{P\} - \{P_e\}_i \quad (3.45)$$

$$(E_T)_i = (E_k)_0 + \sum_{i=1}^i \frac{1}{2} [\{P^*\}_i + \{P^*\}_{i-1}]^T [\{u\}_i + \{u\}_{i-1}] \quad (3.46)$$

$$(E_M)_i = \sum_{i=1}^i \frac{1}{2} [M_{tt}] \{\ddot{u}_t\}_i + [M_{tt}] \{\ddot{u}_t\}_{i-1}]^T [\{u_t\}_i - \{u_t\}_{i-1}] \quad (3.47)$$

$$(E_C)_i = \sum_{i=1}^i \frac{1}{2} [C_{tt}] \{\dot{u}_t\}_i + [C_{tt}] \{\dot{u}_t\}_{i-1}]^T [\{u_t\}_i - \{u_t\}_{i-1}] \quad (3.48)$$

$$(E_K)_i = \frac{1}{2} [[K^*]_0 \{u\}_0]^T \{u\}_0 + \sum_{i=1}^i \frac{1}{2} [[K] \{u\}_i + [K] \{u\}_{i-1}]^T [\{u\}_i - \{u\}_{i-1}] \quad (3.49)$$

$$(E_H)_i = \sum_{i=1}^i \frac{1}{2} ((F_s)_i + (F_s)_{i-1}) ((d_s)_i - (d_s)_{i-1}) \quad (3.50)$$

Summing the energy terms results in Equation 3.51

$$(E_M)_i + (E_C)_i + (E_K)_i + (E_H)_i = (E_T)_i \quad (3.51)$$

The damage to the structure was determined by using the Park and Ang (1985) damage index that is shown in Equation 3.52 with the terms defined previously. During the pseudo-dynamic tests, the maximum displacement $\{d_u\}$ and shear force $\{V_y\}$ is not known, therefore needs to be assumed and compared to the result once the experiment is done. At which point the maximum displacement and shear force can be updated.

$$DI = \frac{d_m}{d_u} + \frac{\beta}{V_y d_u} (E_H)_i \leq 1 \quad \text{with} \quad d_m = \max_{i=0 \rightarrow n} (|d_s|) \quad (3.52)$$

3.6.5 TESTING INSTRUMENTATION

The pseudo-dynamic analysis algorithm was written in CatmanAP (2016) software, which uses Visual Basic Programming Language. HBM CatmanAP (2016) software enabled the integration between the hardware and software, which allowed the script to calculate the displacement of the footing and transfer the calculated displacement value as a calibrated output voltage ($\pm 10\text{V}$) to the servo-controller. Once the actuator had reached its final position and stabilised, the resultant restoring force was read from the load cell back into the script, and the script continued running.

Figure 3-36 shows the interface that was created with CatmanAP (2016) to monitor the progress of the pseudo-dynamic experiment. The interface enabled the real-time visualisation and interpretation of the hysteretic response and energy characteristics of the structure to be compared with the resultant damage encountered by the footing as the earthquake progressed. The interface also enabled the monitoring of the convergence of the iterations and the number of iterations required before satisfying the stop criterion.

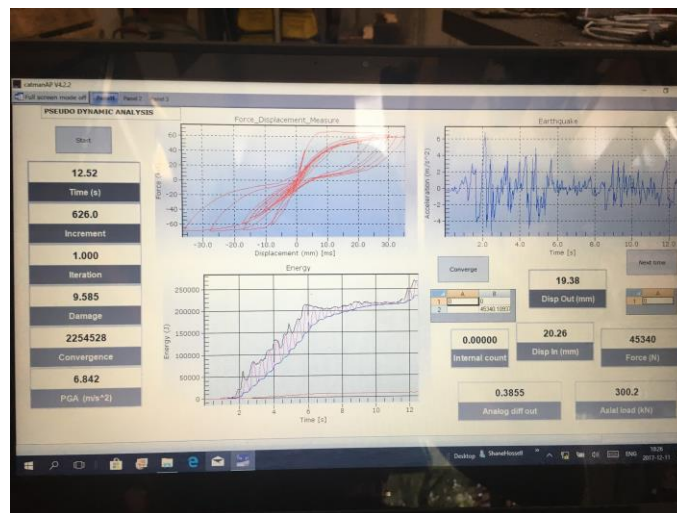


Figure 3-36 Pseudo-dynamic analysis computer interface

The calculated displacement determined from the numerical model was converted from a digital signal to an analogue output signal (DAC) that ranged between $\pm 10\text{ V}$. The HBM PMX data acquisition system shown in Figure 3-37 was used to integrate the computer software with the servo-controller by converting the digital signal produced by the computer to an analogue output voltage that is used to control the position of the horizontal actuator. The data acquisition

system has a 16-bit digital to analogue converter (DAC) and produced an output resolution of 0.003 mm for the horizontal actuator with a stroke length of 200 mm.

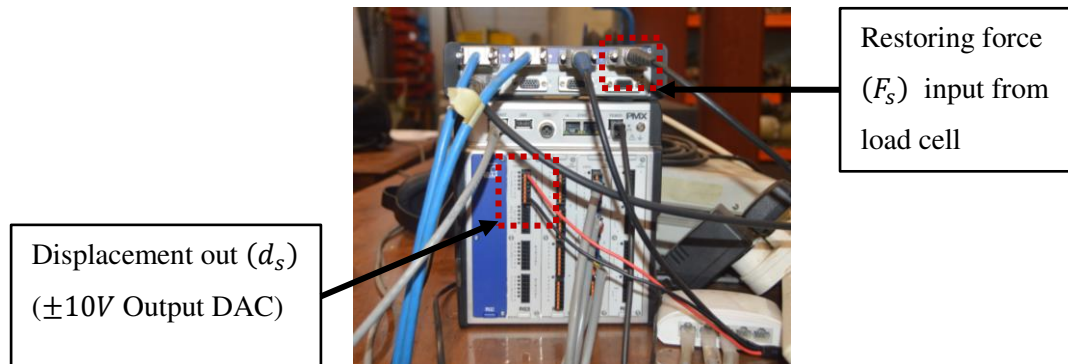


Figure 3-37 HBM Quantum X and PMX Data acquisition system with analogue output

Two Zwick-Roell K7500 servo-controllers are shown in Figure 3-38 and were used during the pseudo-dynamic analysis to control the servo-valves and actuators. The first Zwick-Roell servo-controller was used under displacement control with an input voltage from the HBM PMX system. The constant axial load of 300 kN applied to the footing was controlled by using the second Zwick-Roell Servo-controller under force control.



Figure 3-38 Zwick-Roell K7500 Servo-controllers

A 100 kN hydraulic servo-controlled actuator is shown in Figure 3-40 and was used to place the horizontal load on the reinforced concrete footing. Once the actuator had reached its final position at each iteration within each time step, and the load reading had stabilised, a reading was taken from the horizontal load cell and input back into the computer using a Quantum X

MX840B amplifier as was shown previously in Figure 3-37. The load reading was calibrated such that $1 \text{ V} = 10 \text{ kN}$ as is shown in Figure 3-39(b).

The axial load that was placed on the footing from the frame structure was simulated using a vertical actuator and load cell as shown in Figure 3-41. The axial compression load that was placed on the footing was equilibrated using a custom press frame that only allowed for the transfer of tension forces and not bending moments due to the application of the lateral load. Free rotation of the vertical actuator was achieved by using a pin connection as shown in Figure 3-41 and Figure 3-42.

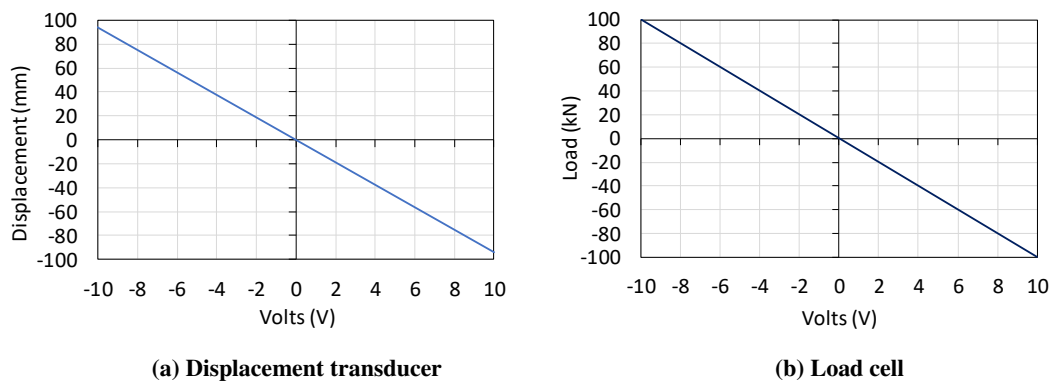


Figure 3-39 Relationship between voltage and (a) Displacement of the actuator and (b) Input force from the horizontal load cell



Figure 3-40 Horizontal servo-controlled hydraulic actuator

The reinforced concrete footings were connected to the test floor using steel channel sections and threaded rods as shown in Figure 3-42. Sliding was prevented by placing a channel section on either side of the footing in the direction of the applied lateral load and bolted to the test floor. However, if the static friction coefficient of 0.57 as indicated Rabbat & Russell (1985) was used between concrete and steel with an applied axial load of 300 kN, the shear resistance at the base of the footing due to friction was approximately 170 kN, which is larger than the capacity of the horizontal actuator. Therefore, the channel sections provided additional safety against sliding.

Overturning of the footing was prevented by placing two channel sections across the top of the footing base slab and securing them to the test floor using threaded rods, which is also shown in Figure 3-42. The overturning restraint beams were designed to sustain a maximum overturning moment of 150 kN.m. Figure 3-42 also shows the base connection of the press frame to the test floor with the pin connection from the Dywidag threaded rods. For each of the press frame base connections, 4 x M24 Class 10.9 bolts were used to fix the press frame base steel component to the test floor. Figure 3-43 shows the steel actuator connection to the reinforced concrete footing with the base plate connected to the top of the reinforced concrete footing using four M16 Class 8.8 holding down bolts.



Figure 3-41 Vertical servo-controlled hydraulic actuator with press frame

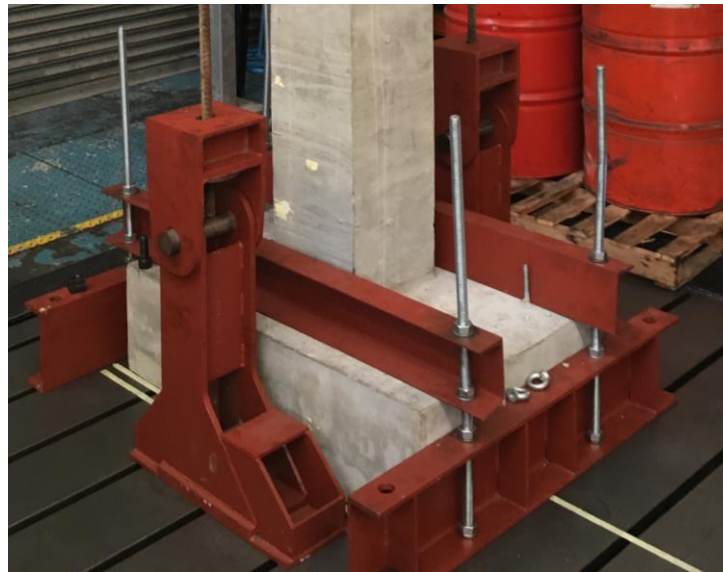


Figure 3-42 Sliding and overturning restraint beams, and the vertical press frame connection to the test floor



Figure 3-43 Steel actuator adaptor connection to the footing with the base plate and holding down bolts

The strain gauges, which were attached to the reinforcing bars, were placed in a half bridge configuration with both strain gauges active and at 90 degrees to each other. The half bridge configuration ensured that temperature effects were negated. The characteristics of the strain gauges are shown in Table 3-4. Figure 3-44 shows the position of the strain gauges on the reinforcing bars, which were cast into the reinforced concrete footings.

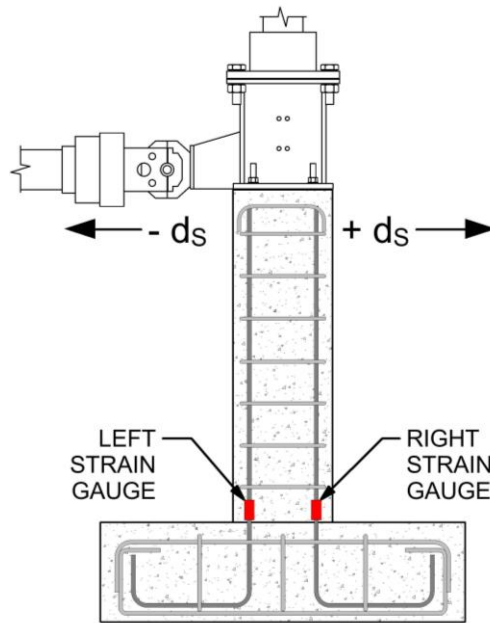


Figure 3-44 Strain gauges placement position on tensile reinforcement

The Y12 tensile bars had to be modified slightly to provide a clean surface for the strain gauges to be attached, with Figure 3-45 showing the strain gauges attached to the reinforcing bars. To ensure that a smooth surface was provided on the bars, the ribs on the Y12 bars had to be removed at the localised position where the strain gauges were going to be placed. The strain gauges on the reinforcement were waterproofed by first painting a sealant over them and their connecting wires and then using shrink tubing to ensure the strain gauges were watertight during the casting of the concrete. Figure 3-36 shows the sealed strain gauges on the reinforcement within the reinforcement cages. The strain gauges cables were loosely fastened to the bars to allow for movement during the casting of the concrete.

Table 3-4 Characteristics of the strain gauges used in the footings

Parameter	Value
Gauge type	TML-YEFLA-5-3LJC
Gauge length	5 mm
Gauge factor	$2.14 \pm 1\%$
Gauge resistance	$119.5 \pm 0.5 \Omega$



Figure 3-45 Strain gauge attachment to the reinforcing bars



Figure 3-46 Strain gauges fixed to reinforcing bars

3.6.6 TESTING PROCEDURE AND SEQUENCE

Five pseudo-dynamic tests were conducted on the reinforced concrete footings to determine the extent of damage with increasing peak ground acceleration, with the final experimental setup shown in Figure 3-47. The initial horizontal stiffness of the reinforced concrete footing was required before the pseudo-dynamic analysis could commence and to enable the elastic natural period of vibration of the structure to be determined. The first two footings were subjected to cyclic loading to get an indication of the initial stiffness of the footings and to test the displacement transducer within the horizontal actuator.

Once the initial stiffness had been determined, the pseudo-dynamic tests could commence. For each test, the frame structure and static loading were kept constant with the only variable being the peak ground acceleration, which was determined by scaling the El Centro ground motion record. Each pseudo-dynamic experiment commenced by initialising the actuators and ensuring the load on the footing was equal to zero. The 300 kN axial load was then applied to the footing,

and once the axial load had reached 300 kN, the horizontal actuator was initialised on the servo-controller, and the pseudo-dynamic test could commence. The initial calculation within the software ensured that all the instruments were zeroed, and the static analysis was performed before commencing the pseudo-dynamic analysis. Time stepping, and iteration commenced upon completion of the initial calculations. The peak ground accelerations used for the pseudo-dynamic experiments, which were obtained by scaling the amplitude of the El Centro earthquake record, are as follows:

- Specimen 1: A maximum peak ground acceleration of 0.34 g;
- Specimen 2: A maximum peak ground acceleration of 0.68 g;
- Specimen 3: A maximum peak ground acceleration of 0.78 g;
- Specimen 4: A maximum peak ground acceleration of 1 g; and
- Specimen 5: A maximum peak ground acceleration of 2 g.



Figure 3-47 Pseudo-dynamic analysis control system

4 EXPERIMENTAL RESULTS

In this chapter, the results obtained from the cyclic load tests and pseudo-dynamic tests are presented, analysed and discussed. The evolution of damage to the reinforced concrete footing with increasing earthquake intensity and cycles of vibration are discussed, compared, and conclusions are drawn. The hysteretic response of the footings and the energy characteristics of the structure are presented due to the applied scaled El Centro ground motion record for each of the pseudo-dynamic experiments.

4.1 CYCLIC LOAD TESTING

Cyclic load tests were undertaken to determine the maximum horizontal elastic stiffness of the reinforced concrete footing under an applied axial load of 300 kN. The maximum horizontal stiffness produced from the cyclic load tests was required as an input into the pseudo-dynamic experiments. Figure 4-1 shows the two cyclic load tests that were undertaken under displacement control whereby the lateral displacement was incrementally increased by 1 mm with each cycle of vibration. Appendix B shows the algorithm used to perform the cyclic load test.

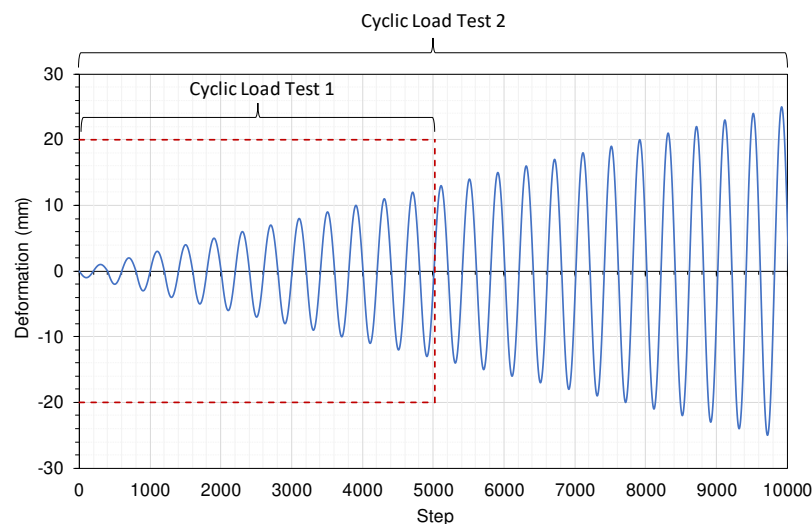


Figure 4-1 Incrementally applied cyclic load independent of time

The first cyclic load test was undertaken to evaluate the test setup and hardware and to provide an initial understanding of the lateral behaviour of the reinforced concrete footing. Figure 4-2 shows the lateral hysteretic response of the reinforced concrete footing with an applied axial load of 300 kN. Cracking of the concrete was evident by the reduction in lateral stiffness at a

deformation of 1 mm and at a corresponding restoring force of 21 kN, which can be observed by the reduction in lateral stiffness that is shown in Figure 4-2. Therefore, a maximum elastic stiffness of 21000 N/mm is produced for the section. A single horizontal crack opened at the base of the column at the interface between the concrete slab and the column. With each cycle of vibration, the horizontal crack increased in size and became more apparent without the formation of additional cracks.

However, during the cyclic load test, the experimental test rig experienced unexpected vibrations when the footing was displacement in the positive direction (extension of the hydraulic actuator piston) from its initial position, which can be seen in Figure 4-2. The test was stopped at a maximum lateral deflection of 13 mm due to the vibrations of the press frame. The vibrations were due to the single top pin connection between the vertical actuator and the top press beam not having a flush connection, which resulted in the top beam rotating relative to the vertical actuator causing an unbalanced load and the subsequent vibrations. To mitigate the vibrations in the test frame, 5 mm steel packing plates were placed in the gap between at the pin connection, which is shown in Figure 4-3.

As the cyclic load test was stopped at a maximum displacement of 13 mm, a static load test was done on the same specimen by applying an incrementally increasing negative lateral deformation (retraction of the actuator) to the footing with an applied axial load of 300 kN, which is also shown in Figure 4-2. The static load test was terminated at a maximum deformation of 20 mm with the formation of a large horizontal crack at the base of the column and the spalling of the concrete at the top of the column at the holding down bolts. Figure 4-4 shows the damage to the reinforced concrete column at the end of the cyclic load test and the static load test.

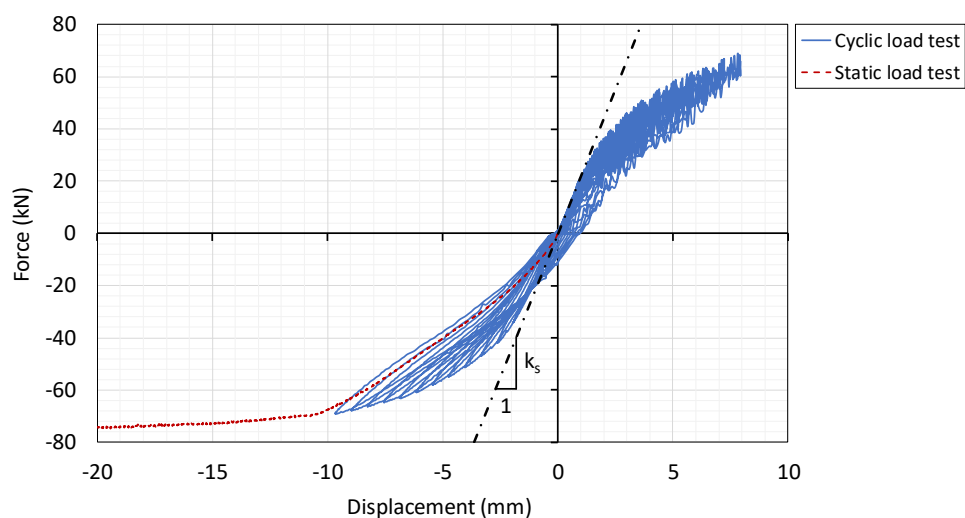


Figure 4-2 Hysteretic response for cyclic load test one



Figure 4-3 Top beam connection stiffening of the press frame

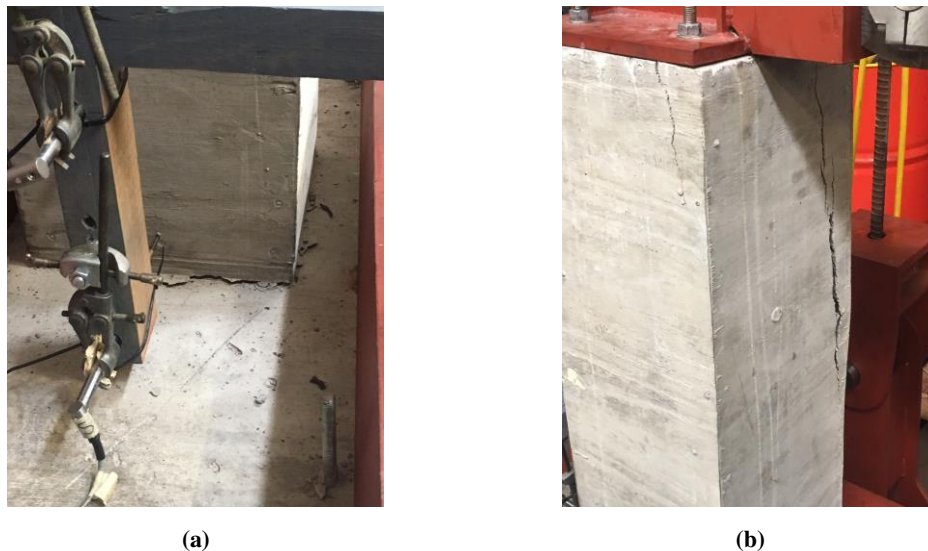


Figure 4-4 Cracking during the first cyclic load test

Figure 4-5 shows the hysteretic response for the second cyclic load test. The cyclic load test was undertaken by incrementally increasing the lateral cyclic displacement by 1 mm after each vibration cycle up to a maximum lateral displacement of ± 25 mm. The backbone curve is also shown in Figure 4-5 and forms the envelope to the unloading and reloading branches. The maximum stiffness of the footing (k_s) was obtained from the backbone curve before the concrete started to crack and produced a lateral stiffness of 14536 N/mm. The resultant stiffness from the cyclic load tests is less than the calculated flexural stiffness from first principles for a cantilever member. The stiffness of the footing is not only governed by the flexural stiffness of the reinforced concrete but also influenced by the connection between the base plate and the concrete column and the connection of the footing with the test floor.

The hysteretic response shows a distinct change in stiffness in the backbone curve at 2 mm with a corresponding shear force of 29 kN, which indicates the onset of concrete cracking. A further increase in the lateral displacement of the footing resulted in the stiffness tending to zero, which occurs at an applied deformation of 8 mm and at a corresponding shear force of 66 kN. The flattening of the backbone curves indicates that the lateral capacity of the footing had been reached under the applied axial load and therefore the maximum horizontal force that can be transmitted from the ground into the structure had been reached. Visible pinching can be observed with each cycle of applied load and is likely due to the incompatibility between the material behaviour of reinforcing steel and concrete and the delayed closure of the crack upon load reversal. Upon load reversal the plastically elongated reinforcing steel is first mobilised in compression prior to the closure of the crack in the direction of unloading and reloading. The vertical axial load applied to the footing most likely also contributed to the pinching effect because as the horizontal load is removed, the vertical load overcomes the overturning moment produced by the horizontal force and attempts to stabilise the section by returning the footing back to its original vertical position as can be observed in Figure 4-5.

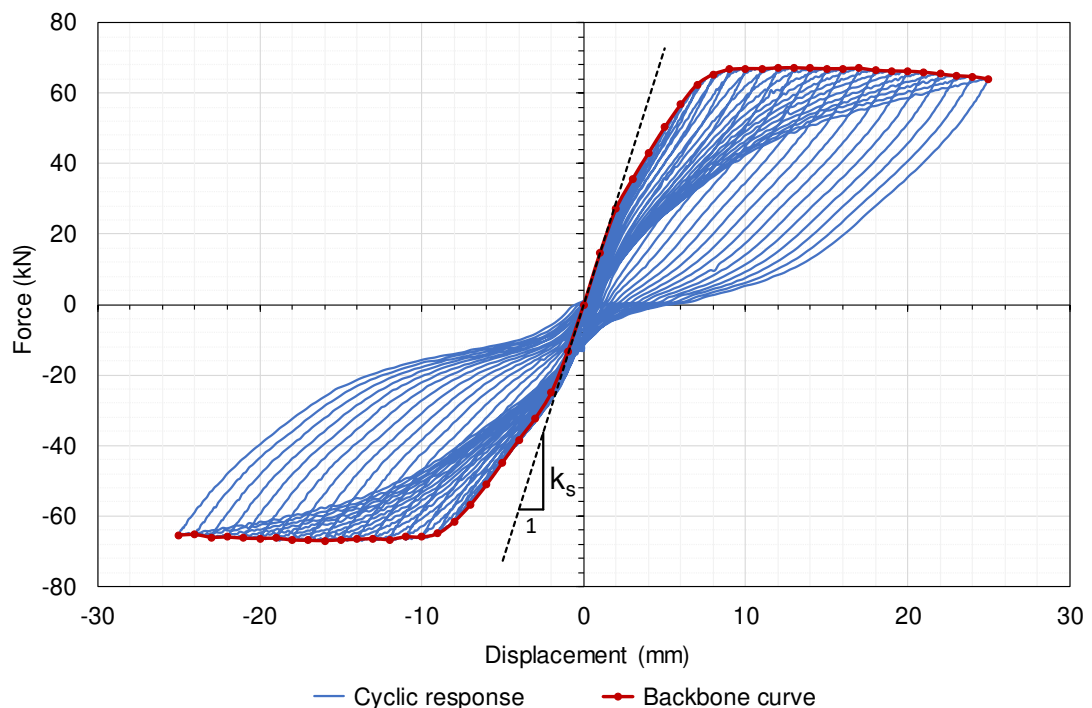


Figure 4-5 Hysteresis curves for cyclic load test two

Figure 4-6 shows that cracking occurred at approximately 40 mm from the base of the column with a single horizontal flexural crack occurring on each face of the column in the direction of the applied shear load. The cracks on either side of the footing were less than 1 mm in width before the reinforcement yielded. However, upon yielding the size of the cracks increased substantially and with a further increase in the displacement amplitude and with each cycle of vibration, the extent of spalling of the concrete increased. The spalling of concrete was found to be exacerbated with repeated cyclic loading of the footing and became more severe post yielding of the reinforcement. Therefore, because of the spalling of the concrete, a slight decrease in the maximum shear capacity of the footing can be observed with an increase in lateral deformation. At the maximum applied displacement of ± 25 mm the test was stopped at which point a 5 mm crack had opened at the base of the concrete column, which can be seen in Figure 4-6.



Figure 4-6 Cracking at the base of the reinforced concrete column during the cyclic load testing

Figure 4-7 shows the strain gauge readings for the cyclic load test up until yielding of the reinforcement. Once the tensile strain in the reinforcement exceeded the proportional limit, there is a reduction in the change in strain between corresponding cycles of vibration. This is contrary to what is expected as the strain should increase with a reduction in lateral stiffness of the reinforced concrete footing.

The observed compression strains in Figure 4-7 differed between the two bars, indicating that the position of the actuator connection to the test specimen and the loading direction could have influenced the response of the footing. As the left reinforcing bar approaches the maximum tensile strain, the compressive strain tends to reduce in the right reinforcing bar, indicating that the neutral axis was closer to the compression reinforcement at the maximum strain in the left

reinforcing bar. This can also justify why the hysteretic curve is unsymmetrical in Figure 4-5. Figure 4-8 shows the lateral shear force applied at the top of the column with each load step. The shear capacity of the column is reached once the strain in the reinforcement exceeds the proportional limit and the yield stress of the longitudinal reinforcement. The results obtained from the cyclic load tests were considered when determining the initial lateral stiffness, k_{s0} , for the reinforced concrete footings during the pseudo-dynamic tests.

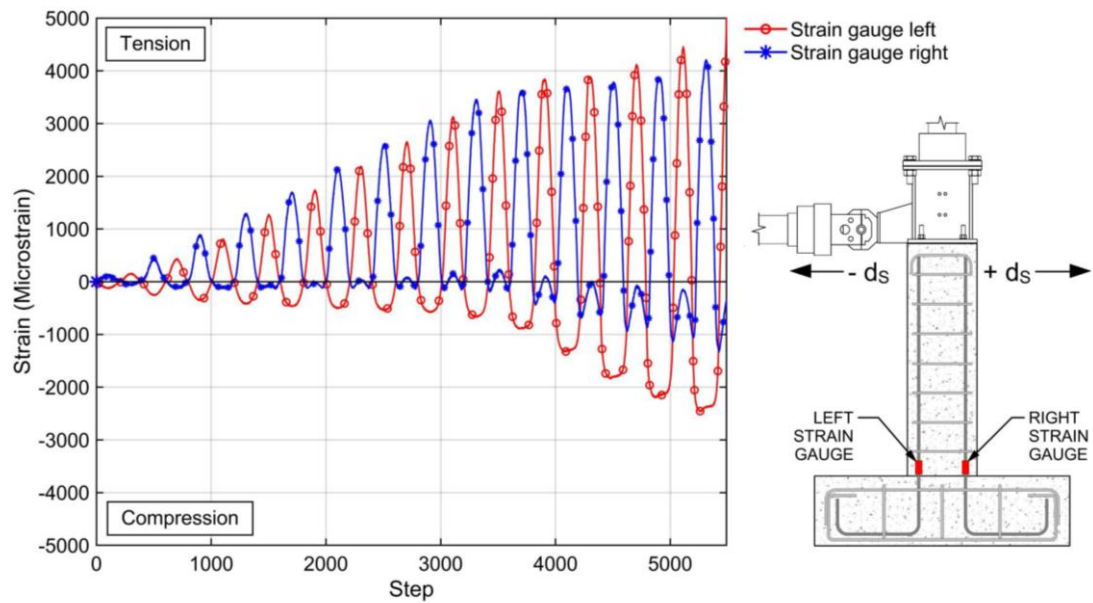


Figure 4-7 Cyclic test 2 strain gauge readings

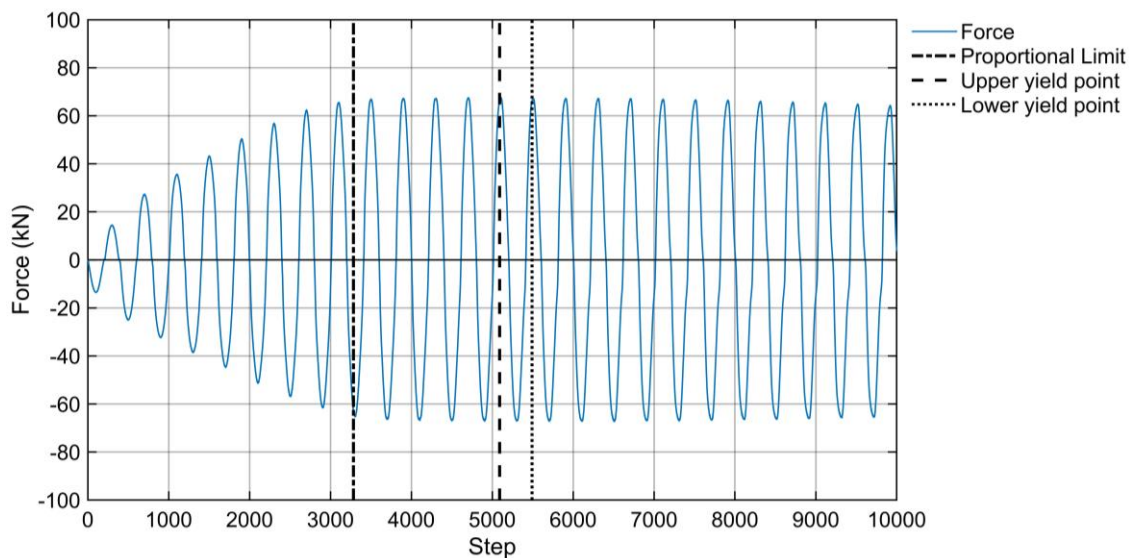


Figure 4-8 Lateral shear force capacity of the reinforced concrete footing

4.2 NATURAL FREQUENCY AND DAMPING PROPERTIES

Rayleigh damping was used to account for energy loss within the overall frame structure that remained linear elastic for the duration of the applied earthquake excitation. Ignoring damping in the frame structure would overpredict the amount of damage incurred by the reinforced concrete footing as the footing would provide the only mechanism of energy loss in the analysis. A damping ratio of 5% was used from SANS 10160-4:2017 for each of the pseudo-dynamic experiments. Figure 4-9 shows the variation of the damping ratio with increasing modal frequencies for the frame structure used during the pseudo-dynamic experiments. As can be seen from Figure 4-9, the damping ratio of 5% is only true at the first two modes and produces higher damping ratios at frequencies less than ω_1 and greater than ω_2 . However, at intermittent frequencies between ω_1 and ω_2 , the damping ratio is less than the specified 5%.

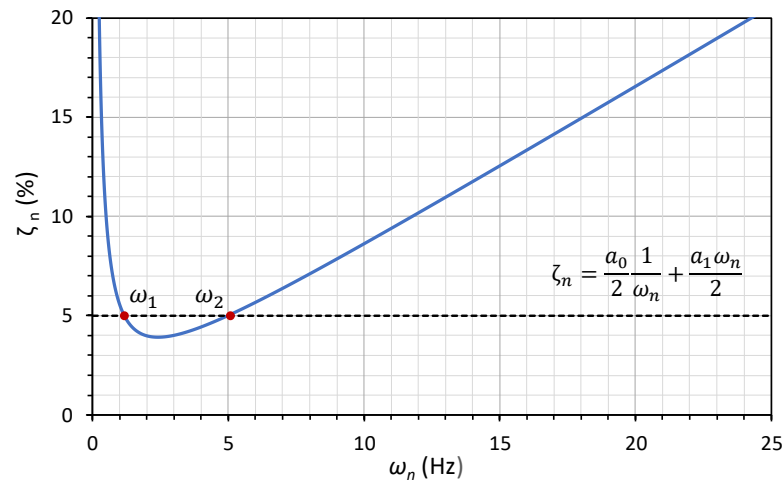


Figure 4-9 Effect of frequency on the damping ratio

Because of the change in the lateral stiffness of the reinforced concrete footing during the pseudo-dynamic experiments, a sensitivity analysis was undertaken to determine the impact that the lateral stiffness of the footing would have on the Rayleigh damping coefficients. The aim was to determine whether the Rayleigh damping coefficients would have to be updated at each time increment within the analysis or whether just calculating the Rayleigh damping coefficients at the start of the analysis would be enough. Having to calculate the damping coefficients at each time increment would result in a significant increase in the computational time that results from the time it takes solve numerically for the eigenvalues in the pseudo-dynamic algorithm as shown in Appendix A.

Assuming an initial lateral stiffness of the footing that is less than the actual elastic stiffness of the footing would result in premature damage being incurred to the reinforced concrete footing as the analysis progresses and would thus produce an unrealistic response. To ensure that the correct response was obtained during the pseudo-dynamic tests, it was considered conservative to select an initial lateral stiffness value greater than that calculated from the cyclic load tests to account for potential variability between the test samples. Thus, the sensitivity analysis would also provide insight into the impact of selecting a larger lateral stiffness value would have on the Rayleigh damping coefficients.

Figure 4-10 shows the results of varying the initial elastic stiffness of the footing on the damping coefficients a_0 and a_1 using 2%, 5% and 10% damping. The Rayleigh damping coefficients are used to calculate the damping matrix $[C]$ from the mass $[M]$ and the stiffness matrices $[K]$. Selecting a larger initial stiffness of the footing to perform the pseudo-dynamic analysis has no significant influence on the damping within the structure. However, the selection of the damping ratio has a significant influence on the overall energy lost, which is expected. Reducing the damping ratio will result in more damage to the footing because more energy needs to be absorbed by the footing with a concomitant reduction in energy loss due to damping.

Figure 4-10 shows that as the lateral stiffness of the footing approaches zero, the Rayleigh damping coefficient a_0 reduces and the coefficient a_1 increases. There is a reduction in the influence of the mass component on the Rayleigh damping matrix $[C]$ as the stiffness of the footing reduces with a subsequent increase in the influence in the stiffness component on the Rayleigh damping matrix $[C]$.

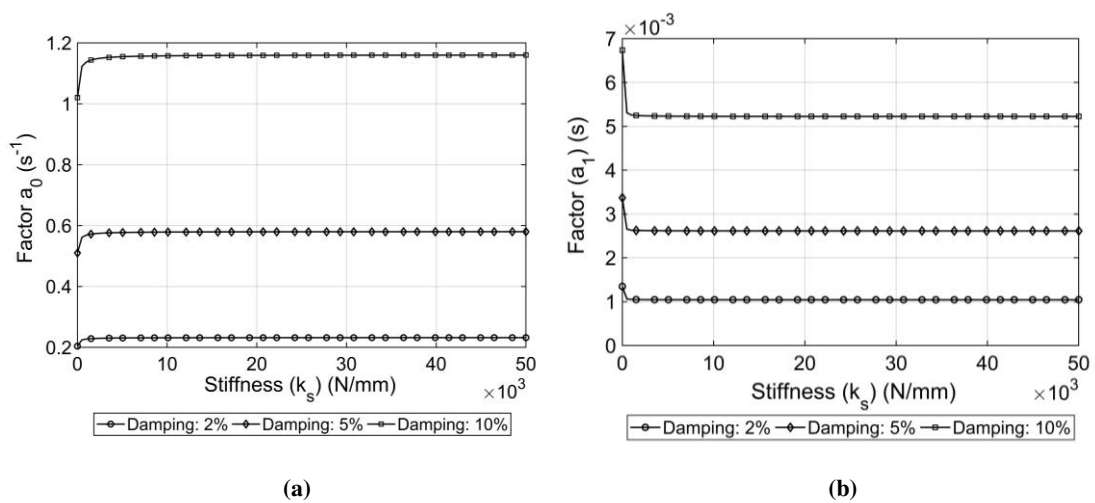


Figure 4-10 Variation of Rayleigh damping coefficients versus the initial elastic horizontal stiffness of the reinforced concrete footing

The selection of the initial stiffness of the footing that was used for the pseudo-dynamic experiments was not critical, particularly when selecting an initial stiffness that is greater than the predicted stiffness of the reinforced concrete footing. However, the selection of damping ratio has a significant influence on the amount of energy absorbed by the frame structure and the subsequent damage suffered by the reinforced concrete footing. Using this information and the initial stiffness obtained from the cyclic load tests, the Rayleigh damping coefficients with 5% damping was calculated as follows:

$$a_0 = 0.59 \text{ s}^{-1} \text{ and } a_1 = 0.0256 \text{ s}$$

4.3 PSEUDO-DYNAMIC ANALYSIS HYSTERETIC TEST RESULTS

The hysteretic curves for each of the pseudo-dynamic experiments are discussed in this section. Figure 4-11 shows the five linear elastic response spectra for each of the scaled El Centro ground motion records that were used to perform the pseudo-dynamic experiments. The fundamental period of vibration of the structure used during the pseudo-dynamic tests was equal to 0.86 s and calculated using the initial elastic stiffness of the footing. However, the building period formula given by Equation 2.15 produces an approximate fundamental period of vibration equal to 0.40 s for a two-storey steel structure, which is lower than the calculated value determined in Chapter 3. The disparity is likely due to the simple configuration of the frame structure as well as ignoring the stiffness contribution of the concrete slabs in the frame structure and the stiffness of the masonry infill panels when calculating the structural stiffness matrix. However, similar pseudo-accelerations (PSa) are produced by using either fundamental period of vibration from the response spectrum determined using the El Centro ground motion record.

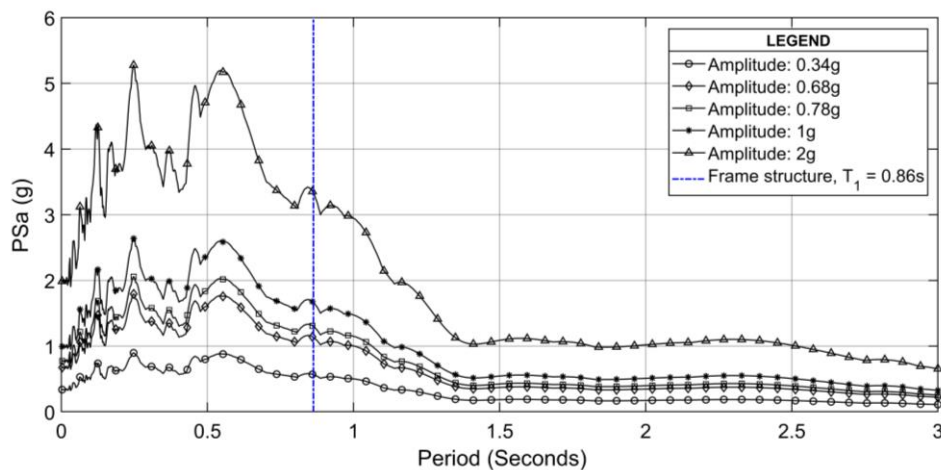


Figure 4-11 Elastic response spectra for each of the scaled El Centro earthquake records

Figure 4-11 also indicates that the fundamental period of vibration of the structure has a considerable influence on the overall response of the structure. A larger fundamental period of vibration of the structure will result in a lower shear force being applied to the top of the footing with a concomitant increase in the lateral deflection of the structure. Reducing the fundamental period of vibration indicates a stiffer structure that generally results in larger shear forces being placed on the footing and a subsequent increase in damage suffered by the footings.

4.3.1 SPECIMEN 1 – 0.34 G PEAK GROUND ACCELERATION

The first pseudo-dynamic test was undertaken at a maximum peak ground acceleration of 0.34 g and continued for the full duration of the El Centro earthquake record. The experiment took 5 hours and 8 minutes to run the full 53.76 s of the amplified earthquake record. Figure 4-12 shows the hysteretic response of the reinforced concrete column that was produced from the pseudo-dynamic experiment and shows that slippage occurred between the base plate and the top of the footing at the start of the applied load. The hysteretic curve of the footing showed a decrease in stiffness at a displacement of 2 mm, which indicates the onset of concrete cracking. Figure 4-13 shows the displacement versus time graph and Figure 4-14 shows the force versus time graph for the duration of the earthquake record. The profiles that are produced by both the displacement versus time graph and the force versus time graph each follow a similar profile of the El Centro ground motion record.

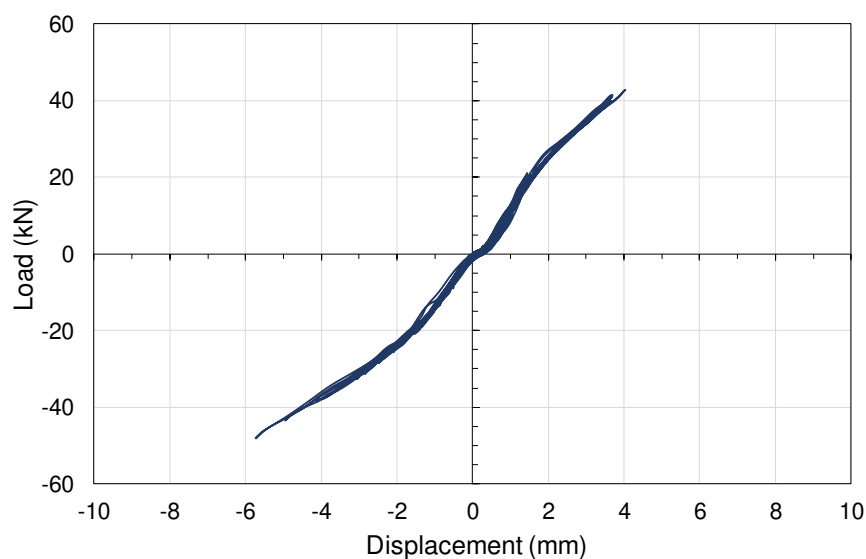


Figure 4-12 Hysteretic response under the El Centro Earthquake scaled to 0.34 g

Figure 4-15 shows a 1 mm horizontal crack opened at the maximum applied lateral displacement on either face of the column in the direction of the applied shear load. Upon load reversal, the cracks closed with some spalling of concrete occurring at the corners of the columns. The response of the footing remains predominantly elastic-perfectly plastic as there was no significant permanent deformation upon unloading. Yielding of the reinforcement did not occur, and none of the reinforcement become exposed by the end of the experiment.

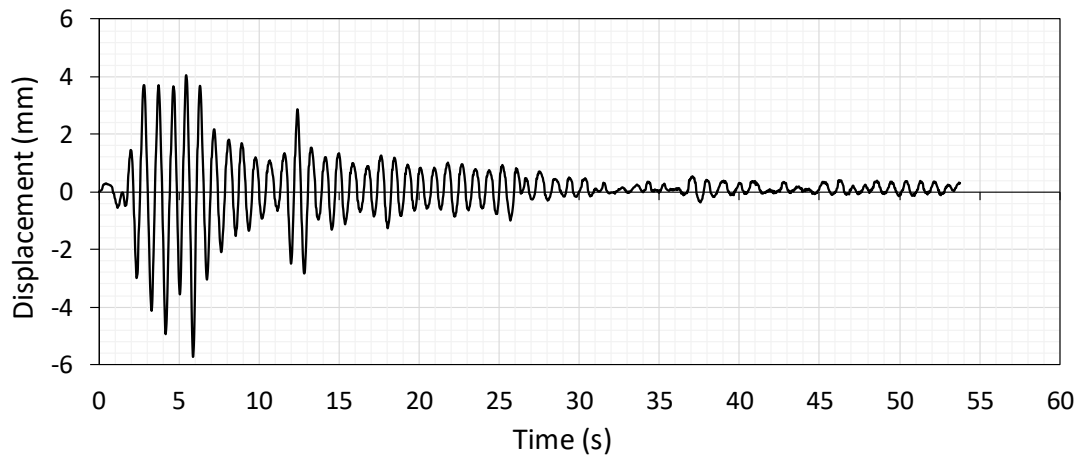


Figure 4-13 Displacement vs time for the 0.34 g experiment

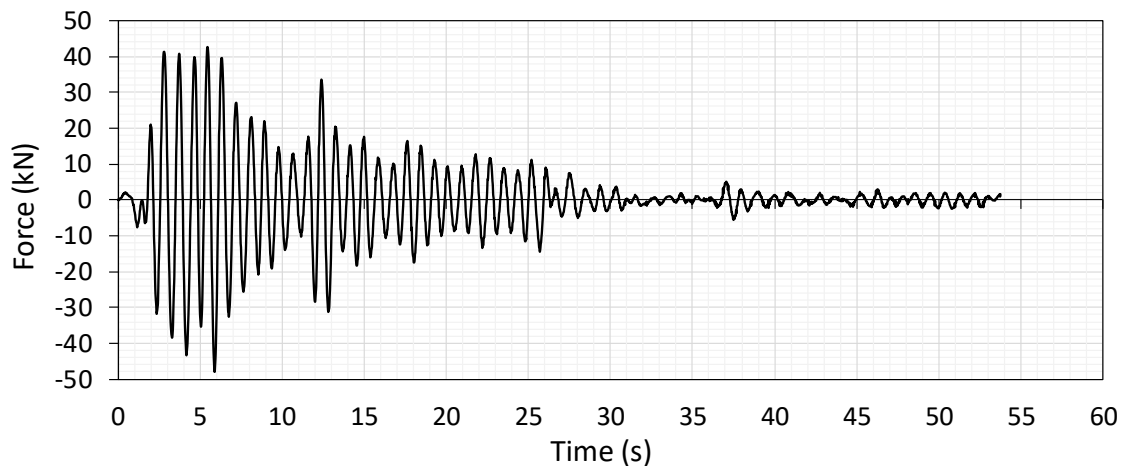


Figure 4-14 Force vs time graph for the 0.34 g experiment

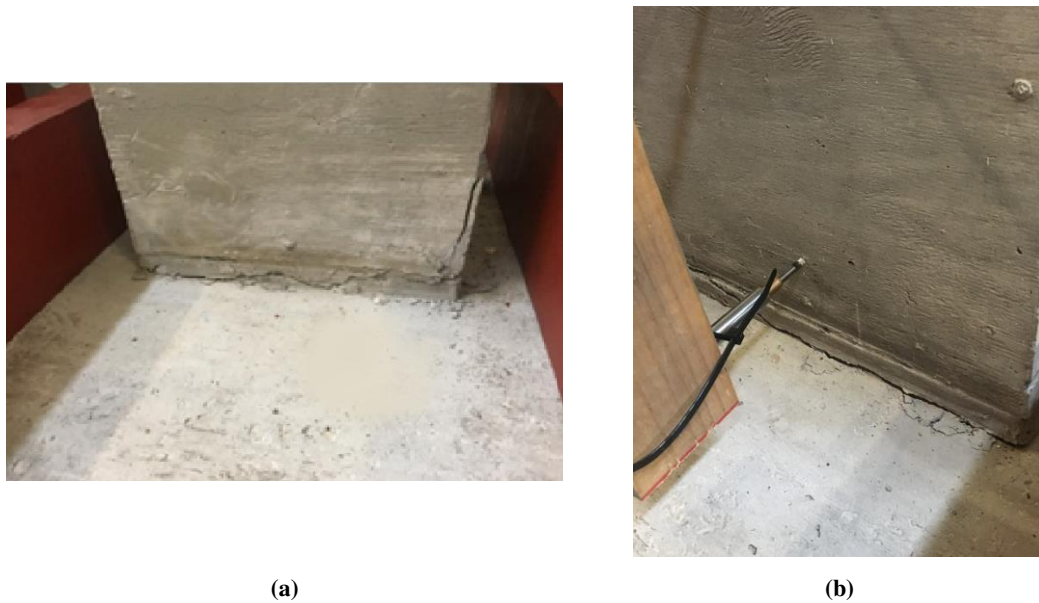


Figure 4-15 Crack patterns at the base of the column at the end of the experiment with (a) the left face of the column and (b) the right face of the column

4.3.2 SPECIMEN 2 – 0.68 G PEAK GROUND ACCELERATION

The second pseudo-dynamic experiment was conducted by amplifying the El Centro earthquake to produce a maximum peak ground acceleration of 0.68 g, with the hysteretic response shown in Figure 4-16. The experiment took 6 hours and 52 minutes to run the full 53.76 s of the amplified earthquake record. Figure 4-17 shows the displacement versus time graph and Figure 4-18 shows the force versus time graph and both follow the profile of the El Centro earthquake ground motion record. The footing did not collapse during the earthquake record, and therefore the full duration of the earthquake record was applied to the structure.

From the hysteretic response, the footing underwent significant deformation resulting in the formation of large cracks on either face of the column in the direction of loading. The reinforcement yielded on either face of the footing, and therefore the maximum shear capacity of the footing was reached. Figure 4-19 shows a crack of 5 mm opening on the left face (the face to which the actuator was connected) of the column at a time of 3.8 s. Figure 4-20 shows visible crushing of the concrete that occurred after 4.8 s on the right face of the column. Although the concrete showed visible crushing and spalling, the footing was still able to maintain the applied axial load of 300 kN by the end of the earthquake record.

The hysteretic response shows that a permanent degradation of lateral stiffness of the footing occurs with an increase in concrete cracking and yielding of the reinforcement. The pinching

effect of reinforced concrete was observed. The load path followed upon unloading from the backbone curve shows an initial increase in stiffness at the unloading point from the backbone curve, which approximates to the initial elastic stiffness of the footing. With further reduction in the lateral load, there is an associated reduction in the lateral stiffness of the footing. Upon unloading, the displacement at which the shear force is equal to zero occurs at approximately the same displacement at which the reinforcement first started yielding. Therefore, the displacement at which yielding first occurred can provide an indication of the maximum displacement the structure will suffer by the end of the earthquake provided the structure does not collapse.

It appears that the maximum deformation encountered in either direction of the applied load dictates the extent of stiffness degradation upon unloading and reloading in the same direction. The reloading of the footing in either direction tends to form a parabolic curve with the stiffness decreasing with increasing displacement and the reloading branch tends to the maximum displacement and force that had occurred previously in the direction of loading.

Figure 4-16 shows the cyclic loading that occurred within the bounds of the maximum displacement that had previously being reached and indicates that the same reloading path is followed up until the maximum displacement in the reloading direction is exceeded. Therefore, indicating that the reloading path is governed by the maximum displacement reached in the reloading direction.

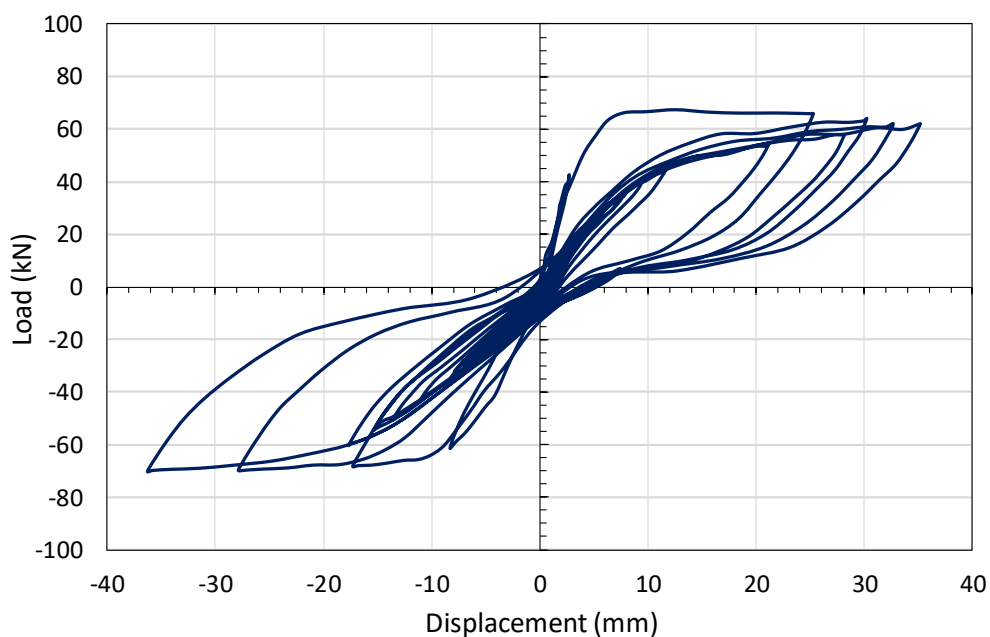


Figure 4-16 Hysteretic response under the El Centro Earthquake scaled to 0.68 g

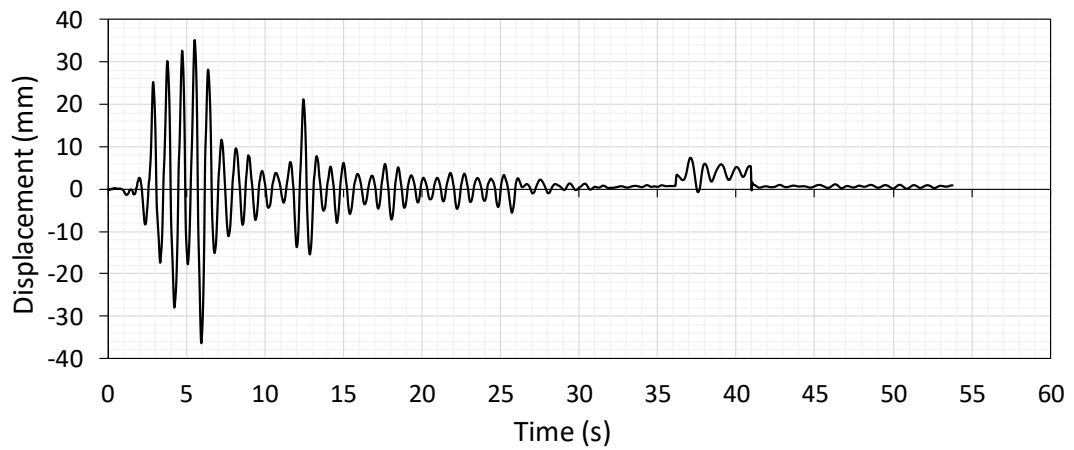


Figure 4-17 Displacement vs time for the 0.68 g experiment

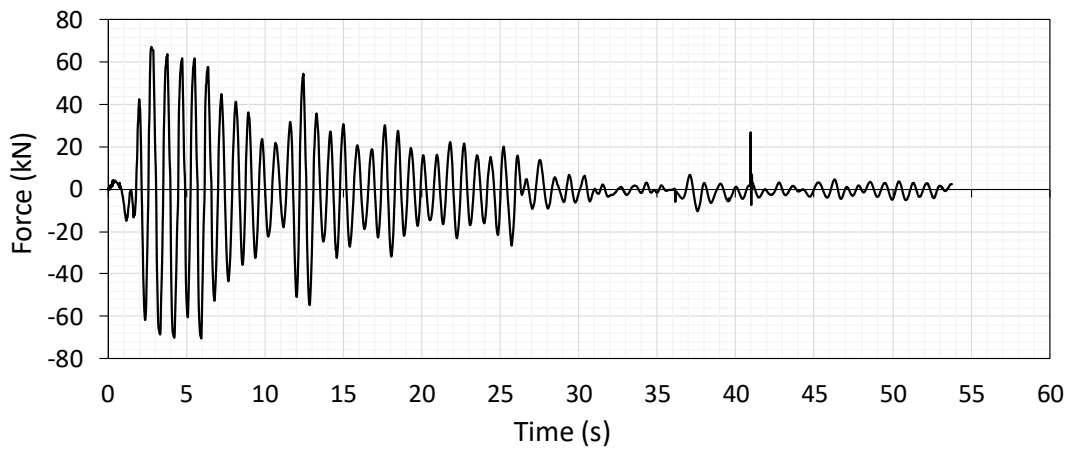


Figure 4-18 Force vs time graph for the 0.68 g experiment



Figure 4-19 Cracking of concrete after the maximum acceleration had been applied to the footing during the El Centro earthquake at 0.68 g



Figure 4-20 Crushing of the concrete after the maximum acceleration had been applied to the footing during the El Centro earthquake at 0.68 g

4.3.3 SPECIMEN 3 – 0.78 G PEAK GROUND ACCELERATION

The El Centro ground motion record was scaled to produce a maximum peak ground acceleration of 0.78 g. The experiment took 45 minutes to run the 5.88 s of the amplified earthquake record. Figure 4-21 shows the resultant hysteretic response of the footing obtained from the pseudo-dynamic experiment. Figure 4-22 shows the displacement versus time graph and Figure 4-23 shows the force versus time graph, which both show that the displacement and force follow a similar profile to the El Centro earthquake ground motion record. Upon reaching the maximum earthquake intensity, large deformations of the column had occurred resulting in the reinforcement yielding and the maximum shear capacity of the footing having been reached.

Figure 4-24(a) shows the initial crack patterns on the left face (the side of the actuator) and Figure 4-24(b) shows the initial crack patterns and spalling of the concrete on the right face of the columns (opposite face to the actuator) before exceeding a lateral deflection of ± 30 mm. A reduction in the shear capacity occurred primarily at a lateral displacement exceeding approximately 30 mm. Significant concrete spalling occurred on each face of the column due to the buckling of the tensile reinforcement during load reversal from tension to compression, which had previously undergone significant permanent plastic elongation. The pronounced pinching effect is likely due to the buckling of the reinforcement, which becomes more pronounced as the horizontal load is reduced and the axial load is distributed to the reinforcement prior to closing the crack in the concrete. The loss of compatibility between the permanent elongation of the ductile reinforcement and the cracking of the brittle concrete

resulted in the plastically elongated reinforcement carrying all the compression force upon load reversal. For the concrete to become mobilised in compression, the reinforcement would need to buckle, which results in significant spalling of the concrete and the reinforcement becoming exposed. Thus, producing significant observed damage. Most of the concrete spalling occurred on either face of the column due to the buckling of the reinforcement due to the cyclic behaviour and not at the maximum lateral deformation of the column. The points at which significant concrete spalling occurred are shown in the unloading branch in Figure 4-21 where the lateral force approximates -10 kN and shows a substantial reduction in lateral displacement with a relatively small reduction in lateral load. This indicates that the vertical axial load governs the reduction in displacement as the horizontal load is removed, contributing to the pinching effect.

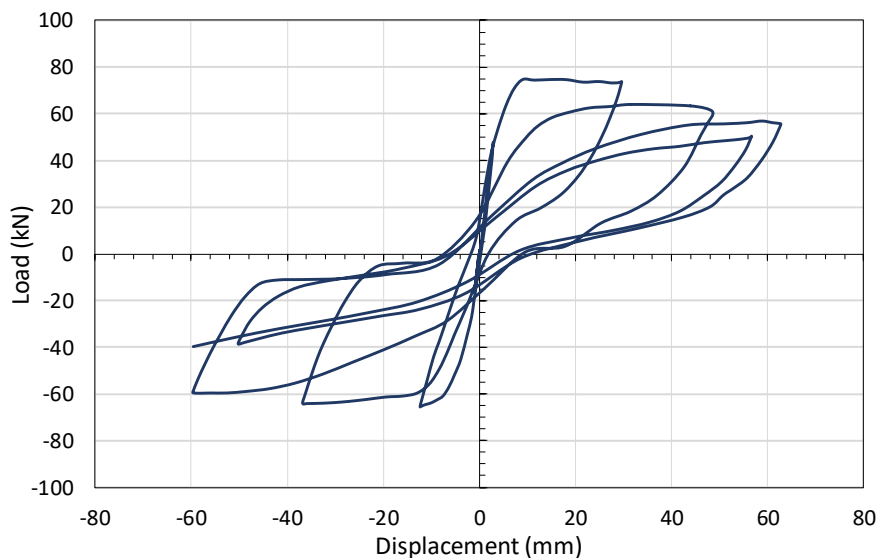


Figure 4-21 Hysteretic response under the El Centro Earthquake scaled to 0.78 g

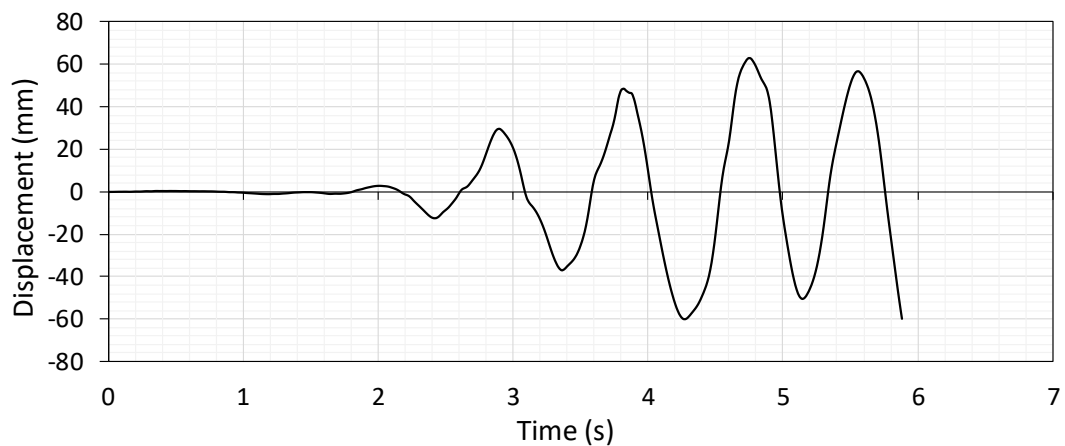


Figure 4-22 Displacement vs time for the 0.78 g experiment

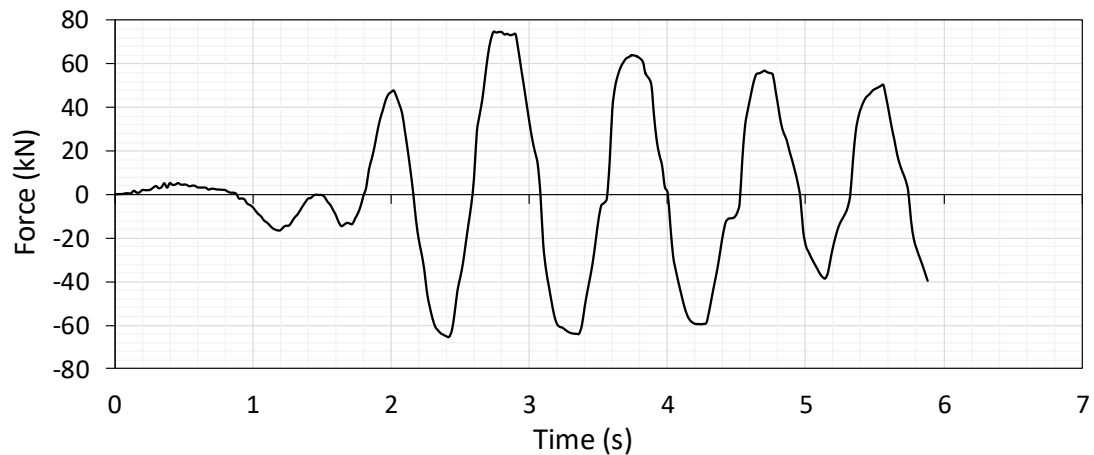


Figure 4-23 Force vs time graph for the 0.78 g experiment

The spalling of the concrete resulted in a reduction in the gross cross-sectional area of the column and the concomitant reduction in the shear capacity of the footing. The lateral resistance of the footing degraded due to the footing being subjected to repeated cyclic loading at large deformations until collapse occurred due to the fracturing of the tensile reinforcement. The column continued to carry the axial load until the cross-sectional area had reduced substantially, resulting in the axial capacity of the footing being exceeded and the subsequent collapse of the structure. Figure 4-25 shows the buckled and fractured reinforcement and Figure 4-26 shows the formation of the plastic hinge at the point of collapse.

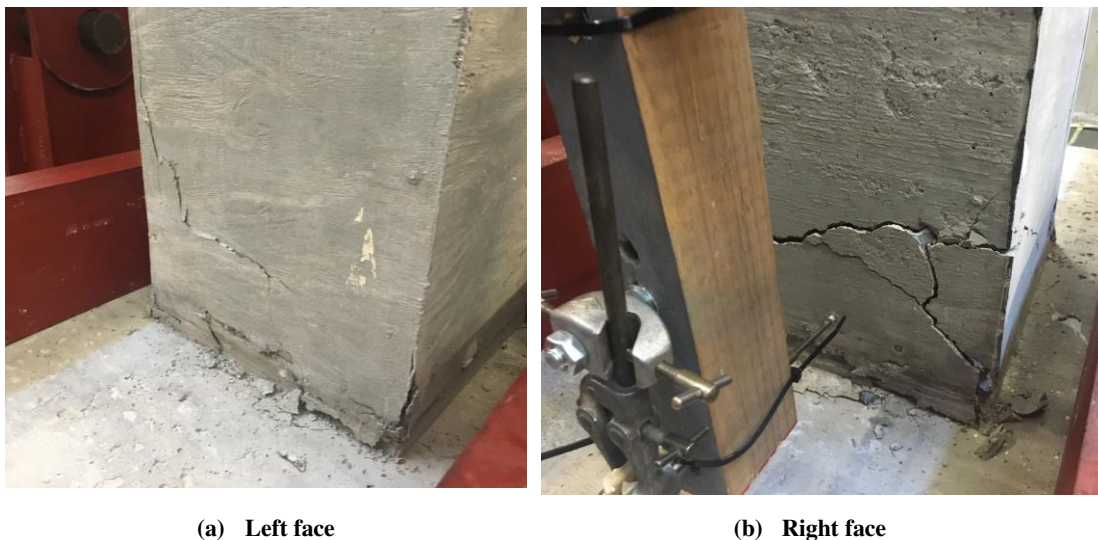


Figure 4-24 Initial crack patterns and concrete spalling on (a) left face and (b) right face of the column during the 0.78 g test



(a)



(b)

Figure 4-25 Resultant damage to the reinforced concrete footing with (a) outward buckling of the reinforcement and (b) reinforcement fracturing during the 0.78 g test



(a)



(b)

Figure 4-26 Plastic hinge formation at the base of the column at collapse during the 0.78 g test

4.3.4 SPECIMEN 4 – 1 G PEAK GROUND ACCELERATION

The El Centro ground motion record was scaled to produce a maximum peak ground acceleration of 1 g, and Figure 4-27 shows the hysteretic response produced from the pseudo-dynamic experiment. Figure 4-28 shows the displacement versus time graph and Figure 4-29 shows the force versus time graph and both follow a similar profile to that produced by the El Centro ground motion record. The structure could only sustain a maximum of 2.8 s of the applied earthquake load before failure occurred, which took 37 minutes to run the pseudo-dynamic experiment. However, the maximum peak ground acceleration of 1 g was reached before failure.

The structural failure occurred due to the fracturing of the tensile reinforcement and crushing of the concrete at a horizontal displacement of 62 mm. Figure 4-30(a) shows the horizontal cracking that occurred on the left face at the base of the column and Figure 4-30(b) shows the horizontal cracking that occurred on the right face at the base of the column without any significant spalling of the concrete occurring at the failure displacement. There was no loss in shear capacity before the structure failed, which indicates that the degradation of shear capacity of the footing was dominated by the repeated cyclic loading that is placed on the footing, which results in an increase in concrete spalling. Figure 4-31 shows the crushing of the concrete at the end of the 1 g experiment.

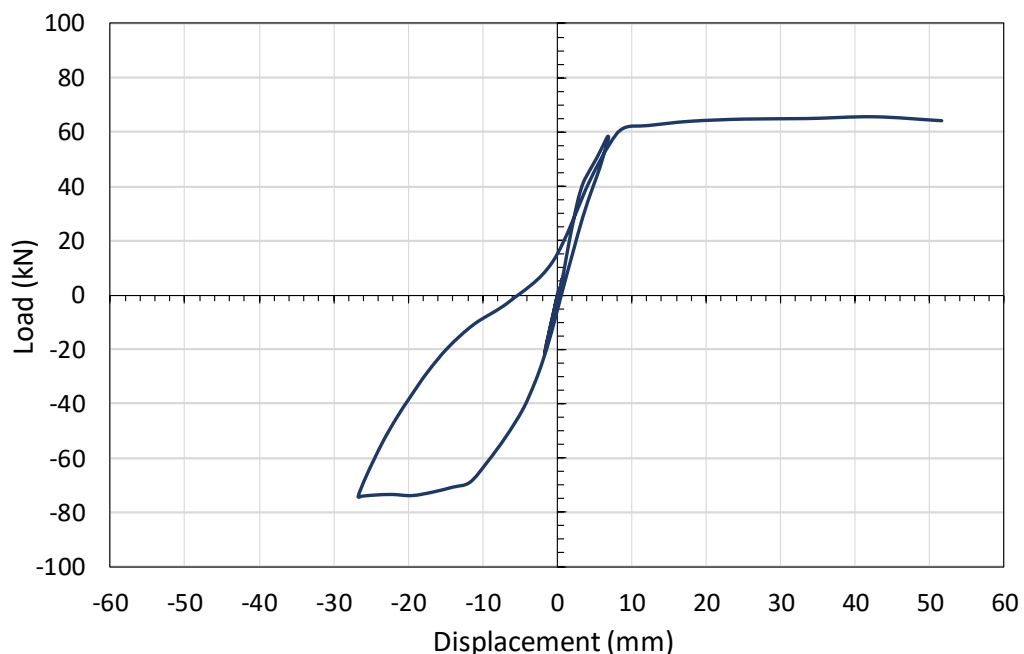


Figure 4-27 Hysteretic response under the El Centro Earthquake scaled to 1 g

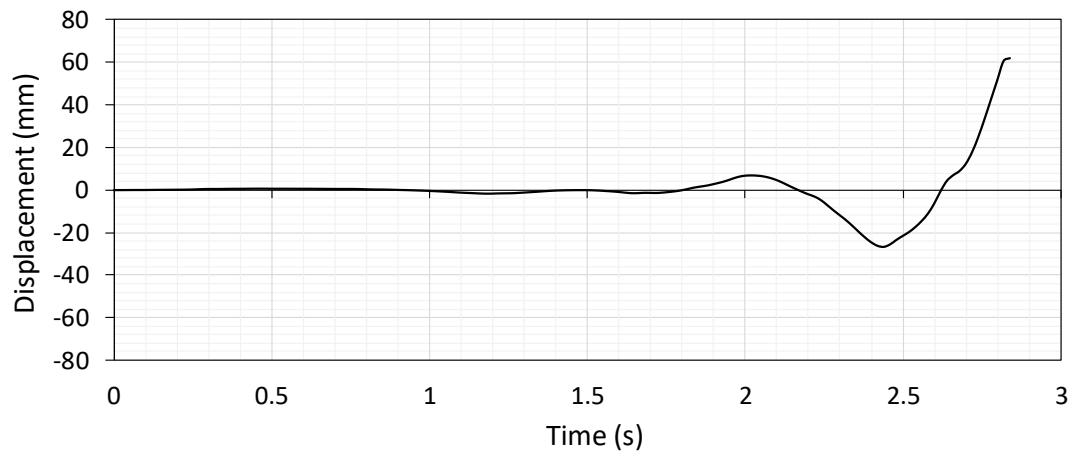


Figure 4-28 Displacement vs time graph for the 1 g experiment

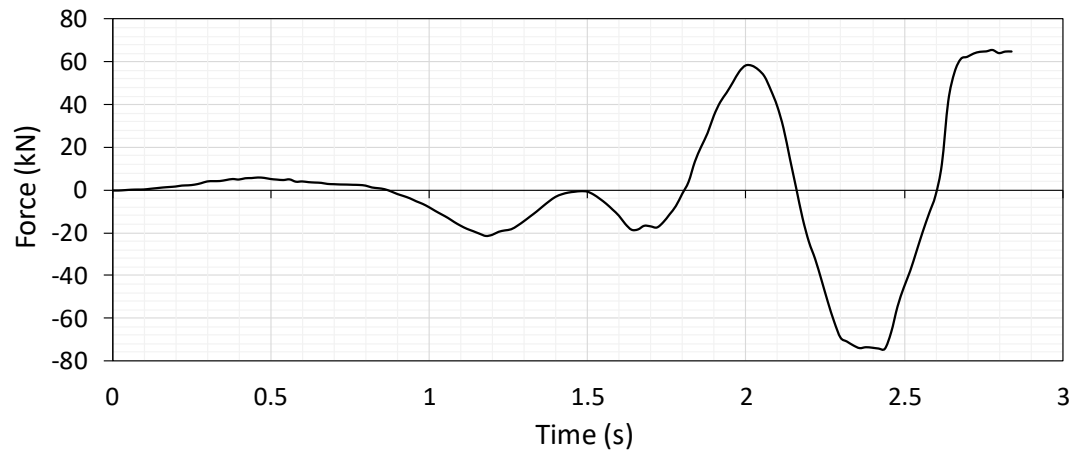


Figure 4-29 Force vs time graph for the 1 g experiment



Figure 4-30 Damage and cracking to the footing during the 1 g experiment



Figure 4-31 Crushing of the concrete during the 1 g experiment

4.3.5 SPECIMEN 5 – 2 G PEAK GROUND ACCELERATION

The final test was undertaken by scaling the El Centro earthquake record to produce a maximum peak ground acceleration of 2 g, and Figure 4-32 shows the hysteretic response produced during the pseudo-dynamic experiment. The experiment took 33 minutes to run the 2.06 s of the amplified earthquake record. Figure 4-33 shows the displacement versus time graph and Figure 4-34 shows the force versus time graph and both produce a similar profile to the El Centro ground motion record. The footing could only sustain the applied earthquake record for 2.02 s and a maximum peak ground acceleration of 1.21 g before the reinforcement fractured and the footing failed. Therefore, the structure did not achieve a peak ground acceleration of 2 g. The footing remained in the linear region before drifting to the right and achieving a maximum displacement of 62 mm before the reinforcement fractured.

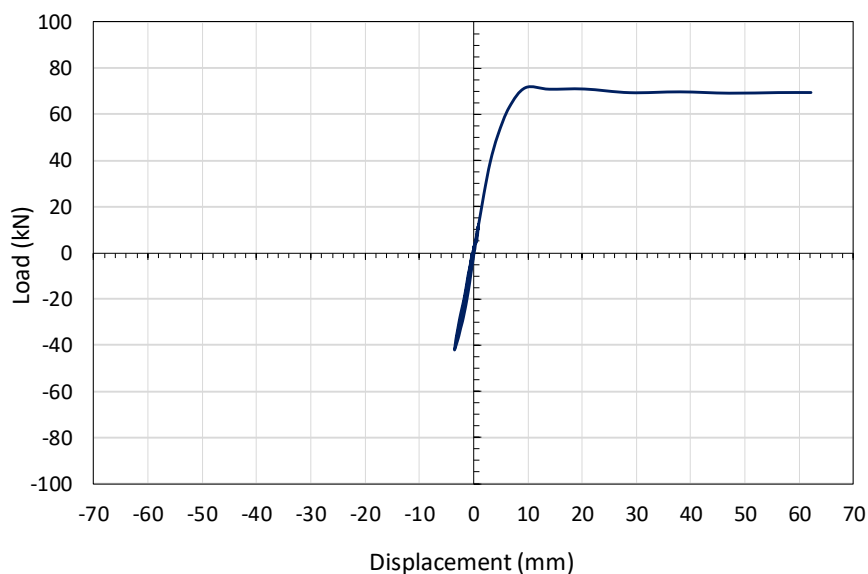


Figure 4-32 Hysteretic response under the El Centro Earthquake scaled to 2 g

Figure 4-35 shows the cracking that occurred on the left face of the reinforced concrete column, and unlike the previous tests, the cracks occurred at approximately 110 mm from the base of the concrete column. Figure 4-36 shows the crushing of the concrete at structural failure and similar to the test carried out at a PGA of 1 g, no significant degradation in shear capacity was observed at the failure displacement.

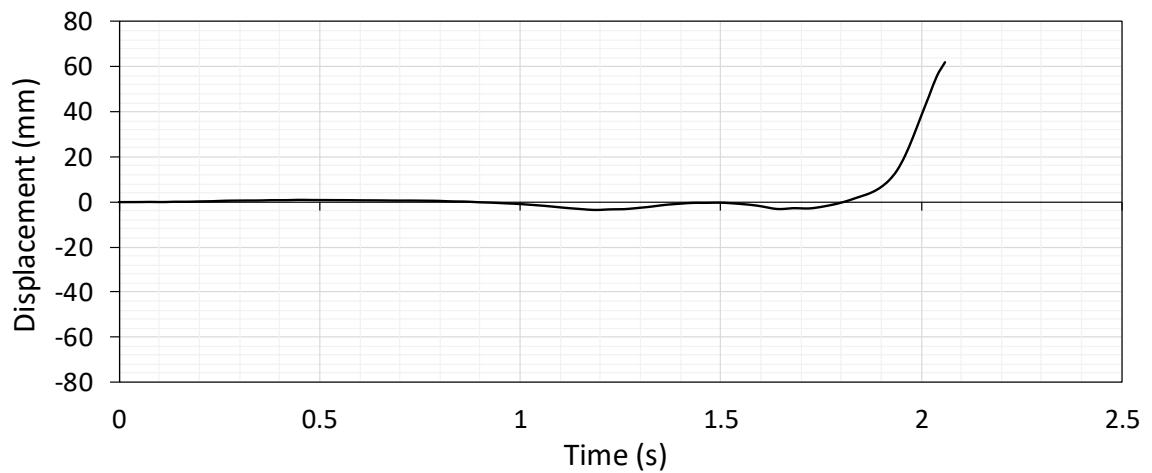


Figure 4-33 Displacement vs time graph for the 2 g experiment

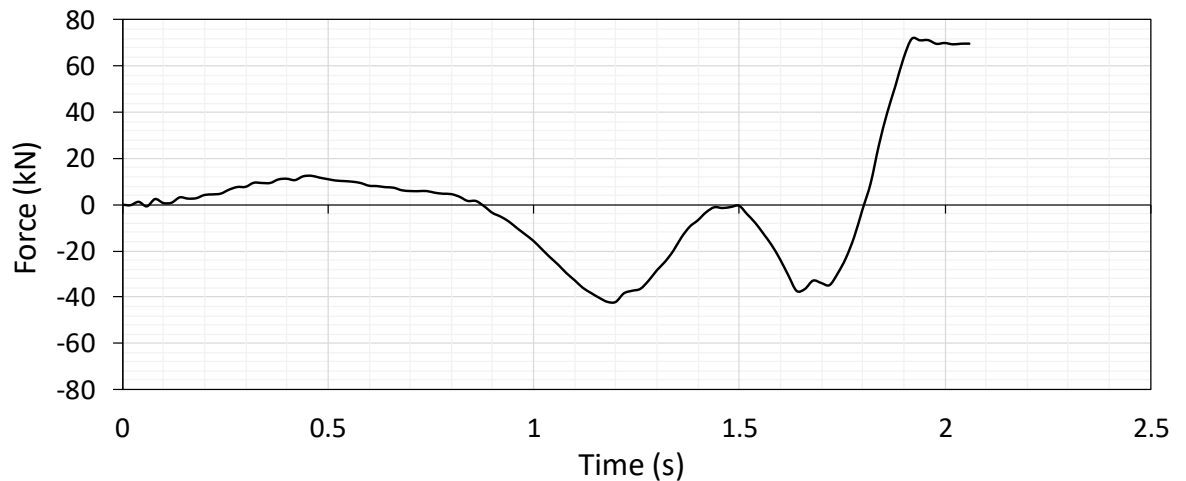


Figure 4-34 Force vs time graph for the 2 g experiment



(a)



(b)

Figure 4-35 Cracking of the concrete during the 2 g experiment



Figure 4-36 Concrete crushing during the 2 g experiment

4.3.6 PSEUDO-DYNAMIC ANALYSIS REINFORCEMENT STRAIN RESULTS

The aim of using the strain gauges in the research was to determine the points at which the concrete cracks and the reinforcement yields, and to relate the data obtained from the strain gauges with the hysteretic curves produced during the pseudo-dynamic tests. The strain gauges performed well at the low peak ground accelerations (PGA) as the strain in the reinforcement remained in the elastic region. At high PGAs, the strain gauges failed soon after the strain exceeded the yield strength of the reinforcement, despite having used high post yield strain gauges with a maximum strain of 10 % to 15 %. Because the strain gauges failed soon after the yield capacity of the reinforcement was reached, the results produced by the strain gauges were not meaningful for the tests at a PGA of 0.68 g and greater. The failure of the strain gauges can be presumed to be caused by the failure of the connecting wires to the strain gauges.

Figure 4-37 shows the strain gauge results for the El Centro earthquake scaled to a peak ground acceleration of 0.34 g. The recorded strain at the base of the column indicates that the yield strain was not reached and that the strain results followed a similar profile to the applied ground motion record. The maximum strain occurred in the right strain gauge, with the maximum strain point lagging the maximum PGA that occurs at 2.14 s during the El Centro ground motion record.

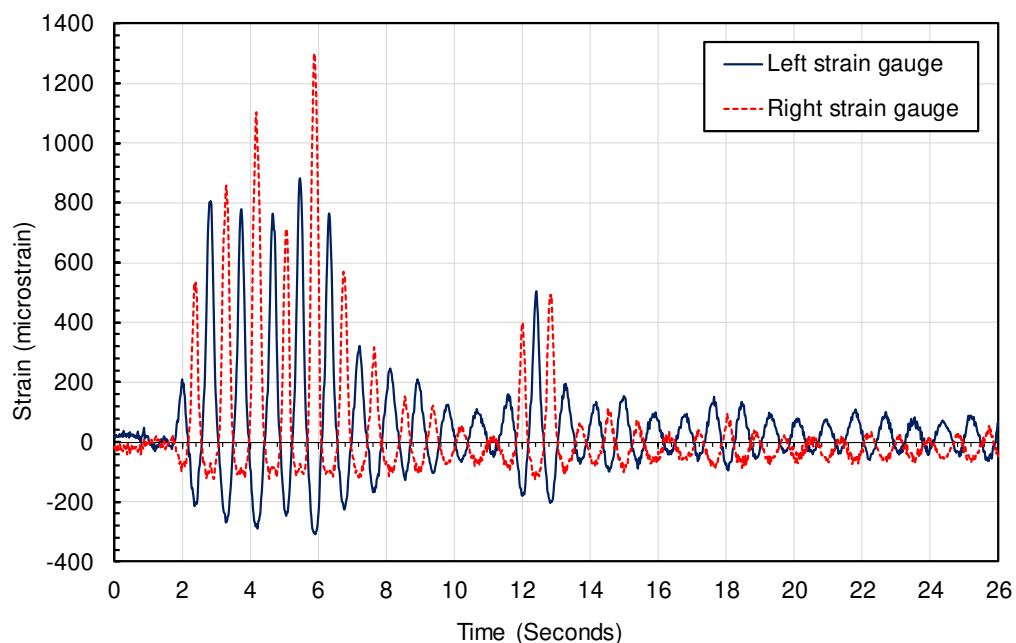


Figure 4-37 Reinforcement strain measurement under the El Centro Earthquake record scaled to 0.34 g

4.3.7 ENERGY-RELATED RESULTS

The amount of energy imparted to the structure due to the earthquake is distributed between the kinetic energy (E_M), damping energy (E_C), strain energy (E_K) and hysteretic energy (E_H). This section shows the distribution of energy within the structure for each of the pseudo-dynamic experiments. Figure 4-38 shows the time history of the total energy imparted to the structure for the duration of each of the scaled earthquake ground motion records during the pseudo-dynamic experiments and is either shown for the entire duration of the earthquake record or until structural failure of the footing. The experiments conducted at 0.34 g and 0.68 g ran for the full duration of the earthquake, whereas the 0.78 g, 1 g and 2 g experiments all failed before completion of the earthquake record. The maximum energy imparted to the structure occurred during the experiment with a maximum peak ground acceleration of 0.68 g and the tests conducted at higher peak ground accelerations showed a reduction in the total energy imparted to the structure before failure of the reinforced concrete footing occurred.

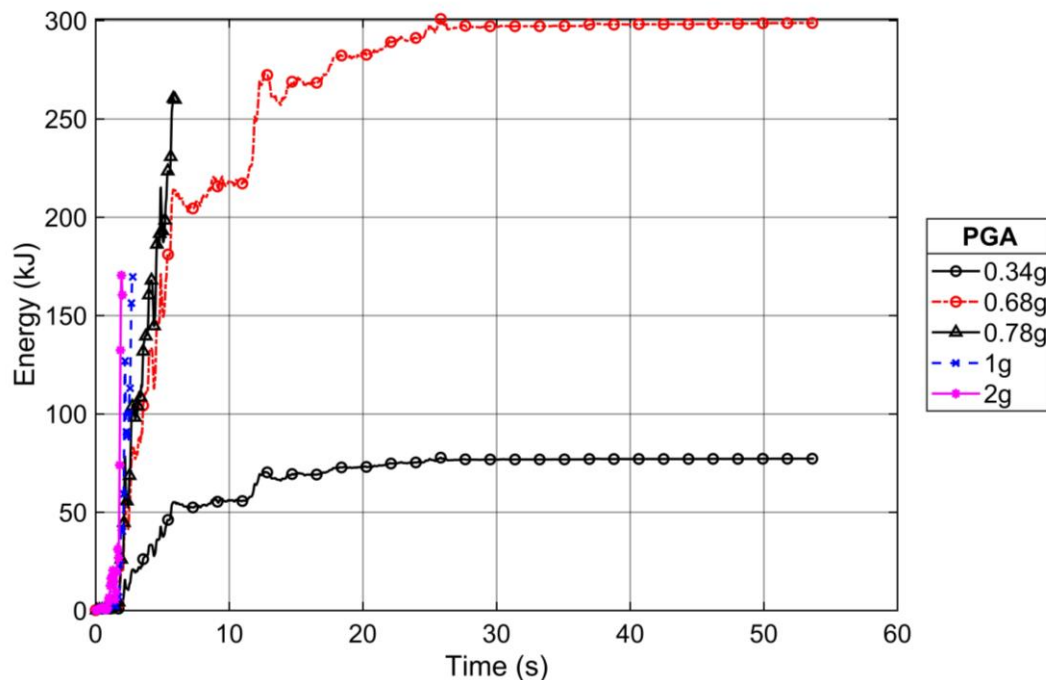


Figure 4-38 Total energy imparted to the structure at the scaled peak ground accelerations (PGAs)

Figure 4-39 shows the total hysteretic energy absorbed as time progressed with the application of the scaled El Centro ground motion record for each of the pseudo-dynamic experiments. The footings that showed more substantial observed damage during the experiments absorbed the

largest quantity of energy. The 0.78 g ground motion record resulted in the most observed damage and absorbed the greatest quantity of energy, which can be seen in Figure 4-39. Even though the 1 g and 2 g ground motion records resulted in the failing of the structure, the amount of energy absorbed by the footing is lower than that absorbed by the 0.68 g and 0.78 g PGA earthquake tests.

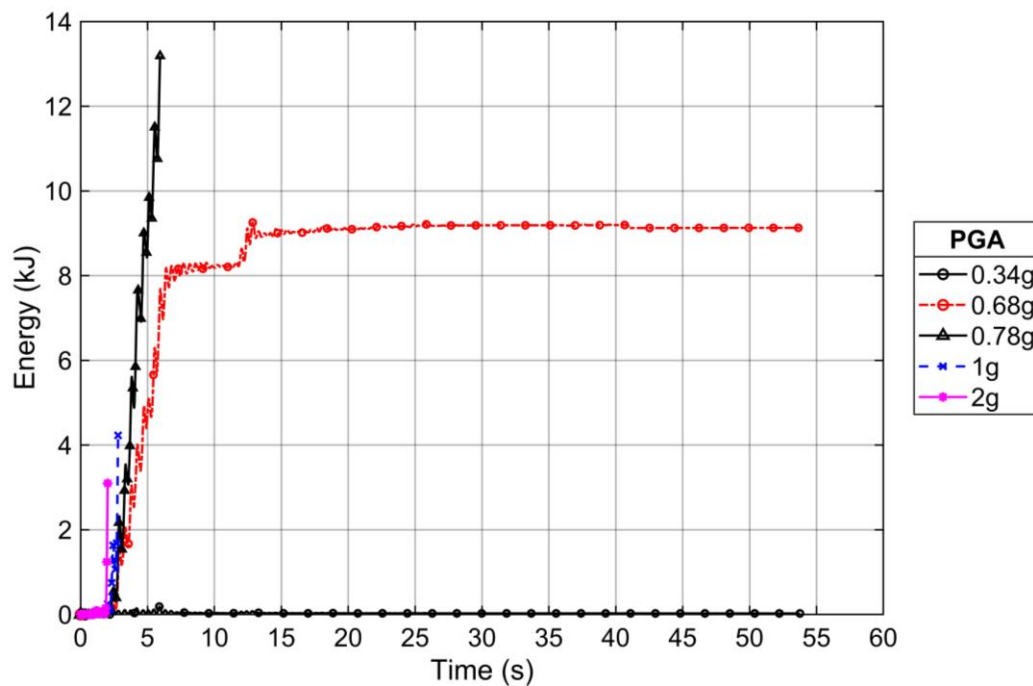


Figure 4-39 Hysteretic energy of the reinforced concrete footings during the pseudo-dynamic tests

Table 4-1 shows the distribution of energy within the structure for each of the scaled amplitudes of the ground motion records, which is either recorded at the end of the earthquake motion record or at the point of failure of the footing during the applied earthquake. For each of the pseudo-dynamic experiments, the hysteretic energy absorbed by the footing was determined as a percentage of the total energy imparted to the structure. The 0.78 g earthquake absorbed the largest percentage of the total energy imparted to the structure by the earthquake during the pseudo-dynamic experiment, which correlates with the observed damage regarding cracking and spalling of the concrete. Although the observed damage to the footing subjected to a PGA of 0.68 g was greater when considering cracking and spalling of concrete than that observed by the 1 g and 2 g experiments, the 1 g and 2 g reinforced concrete footings still failed due to the fracturing of the reinforcement and crushing of the concrete. The hysteretic energy absorbed by the footing during the induced ground motion record is a good indicator of the damage incurred

by the reinforced concrete member; however, it does not necessarily indicate whether the structure has failed.

Table 4-1 Distribution of energy at the end of the ground motion record or at failure

PGA (g)	Total energy imparted (kJ)	Hysteretic (E_H) (kJ)	Inertia (E_M) (kJ)	Damping (E_C) (kJ)	Strain (E_K) (kJ)	Percentage energy absorbed (%)
0.34	77.3	0.02	0.0	77.0	0.3	0.03
0.68	298.8	9.1	0.0	289.4	0.3	3.1
0.78	259.9	13.2	5.3	182.4	59.0	5.1
1	169.6	4.2	1.1	56.3	108.0	2.5
2 (1.21*)	158.6	3.1	0.2	33.0	122.3	2.0

* The maximum acceleration achieved by the structure before failure

Figure 4-40 and Figure 4-41 show the energy components for the frame structure for the entire duration of the ground motion record scaled to 0.34 g and 0.68 g peak ground acceleration. The structure subjected to the 0.34 g peak ground acceleration resulted in damping absorbing all the energy imparted to the structure. The 0.68 g peak ground acceleration resulted in the footing absorbing hysteretic energy and therefore indicates a correlation between the number of cycles of vibration and the resultant damage in terms of observed concrete spalling, yielding of the reinforcement and buckling of the reinforcement. The ductility provided by the reinforcement has a significant influence on the amount of energy that can be absorbed by the footing before the failure displacement is exceeded. The energy-time histories show that with an increase in the number of cycles of vibration the amount of energy absorbed by the footing is increased as expected. The test at 0.34 g did not absorb a large amount of energy as the footing did not undergo significant lateral deformation and the reinforcement did not yield.

Figure 4-42 to Figure 4-44 show the energy components of the 0.78 g, 1 g and 2 g pseudo-dynamic tests that all failed before the full duration of the amplified El Centro record could be applied to the structure. For the tests at peak ground accelerations of 0.68 g and 0.78 g, the footing underwent a larger number of cycles of vibration in the plastic region of the reinforcement, which consequently resulted in more damage to the footing due to the spalling of the concrete cover. The spalling of the concrete cover occurred predominately due to the buckling of the permanently elongated reinforcement upon load reversal from tension to compression and to a lesser extent due to compression capacity of the concrete being exceeded. The test carried out a 1 g and 2 g had fewer or no cycles of vibration, as shown in Figure 4-43

and Figure 4-44 respectively, before the reinforcement fractured and visually did not experience the same amount of damage in terms of concrete crushing and spalling than that observed from the tests conducted at the lower peak ground accelerations.

It is evident that most of the energy absorbed in the structure is due to Rayleigh damping, with only a small percentage of the overall energy being absorbed by the reinforced concrete footing. The hysteretic energy correlates well with the cumulative damage to the reinforced concrete footing and the observed damage experienced by the specimens during the pseudo-dynamic experiments.

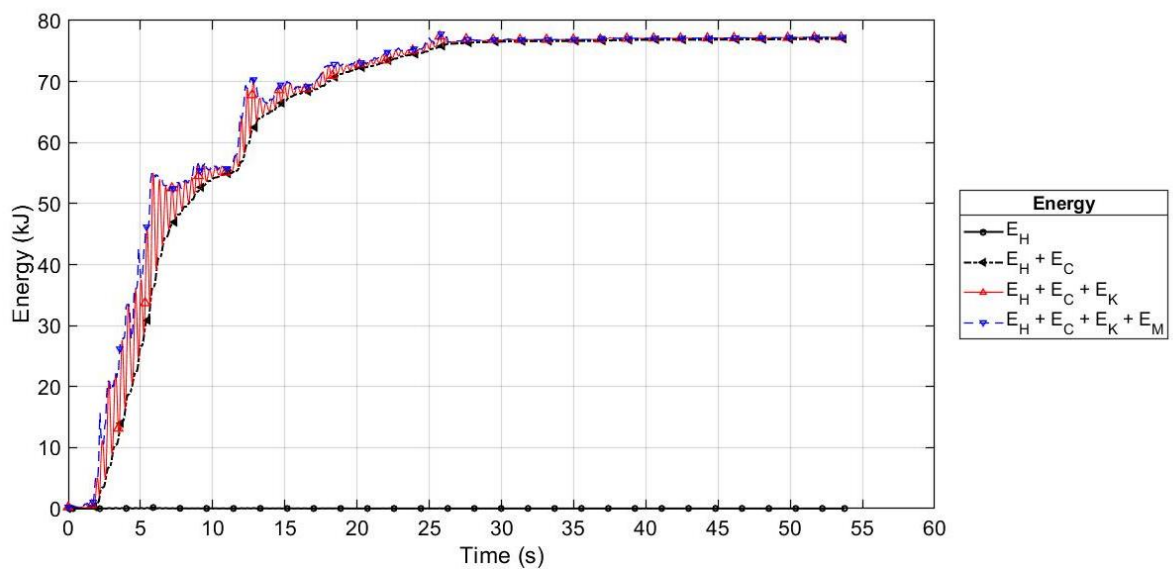


Figure 4-40 Time histories for energy terms during the El Centro earthquake scaled to a PGA of 0.34 g

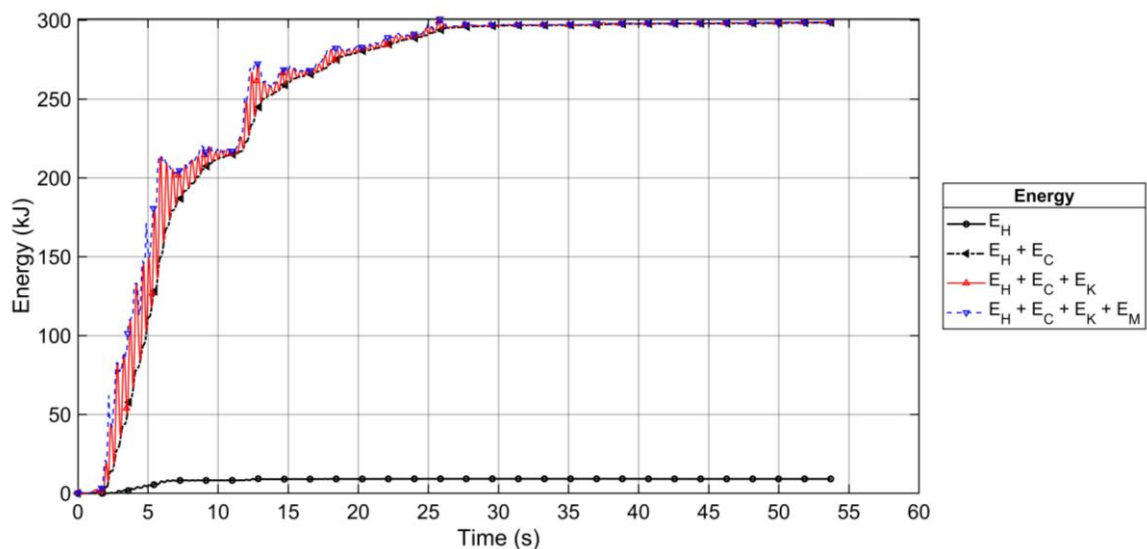


Figure 4-41 Time histories for energy terms during the El Centro earthquake scaled to a PGA of 0.68 g

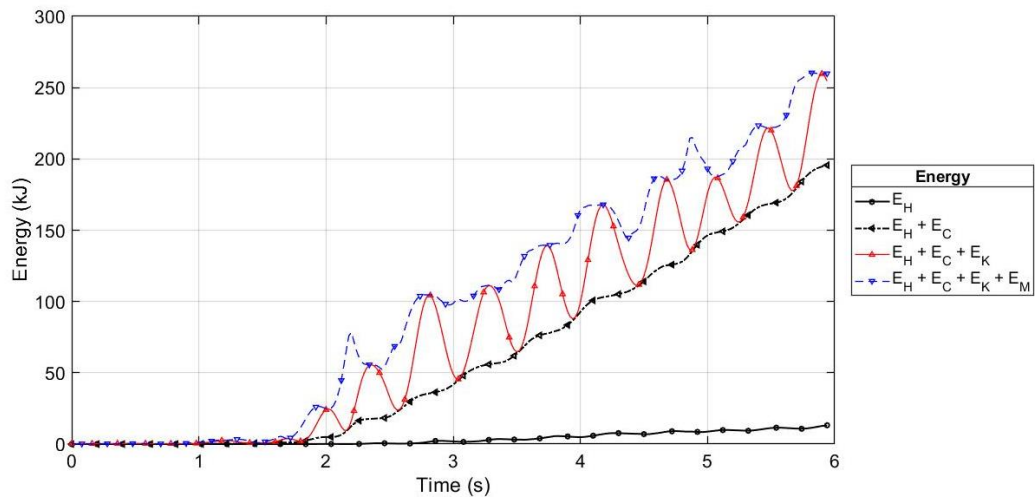


Figure 4-42 Time histories for energy terms during the El Centro earthquake scaled to a PGA of 0.78 g

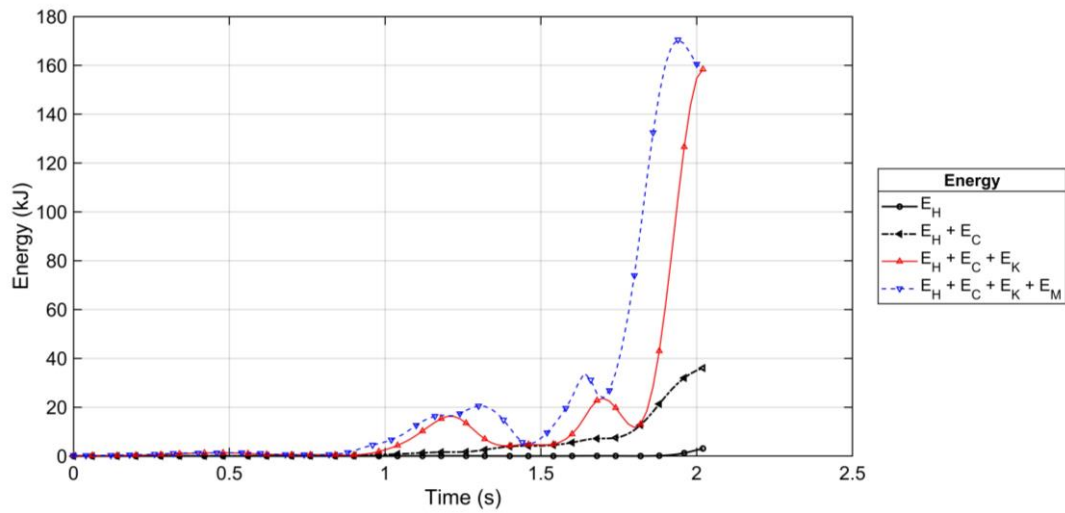


Figure 4-43 Time histories for energy terms during the El Centro earthquake scaled to a PGA of 1 g

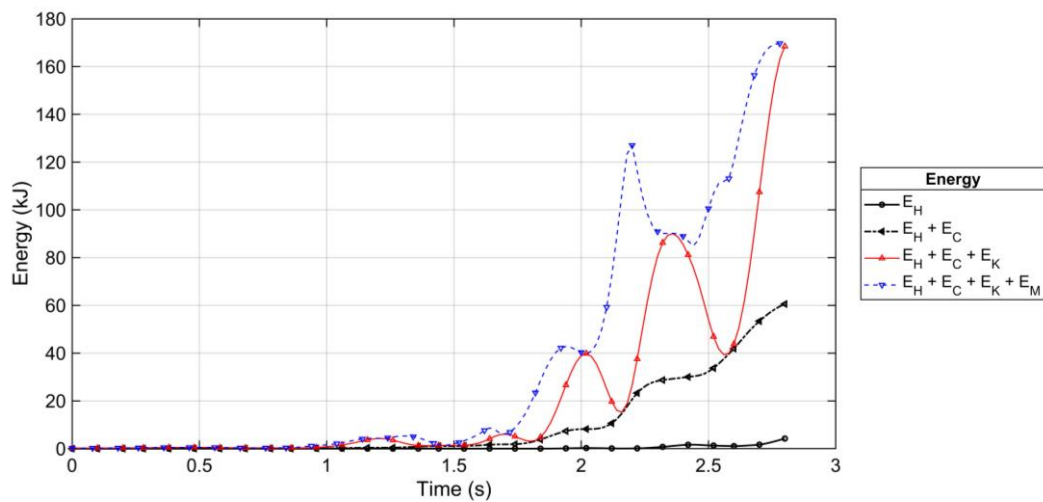


Figure 4-44 Time histories for energy terms during the El Centro earthquake scaled to a PGA of 2 g

4.4 PSEUDO-DYNAMIC ANALYSIS OVERALL STRUCTURE RESPONSE

The response of the overall frame structure is discussed in this section with the results presented for the maximum structural deflections with its corresponding bending moment and shear force diagrams. The resultant axial loads applied to each of the supports are also presented. Table 4-2 shows the moment capacities for the overall frame structure used during the pseudo-dynamic experiments and the strong axis moment of inertia for each of the members.

Table 4-2 Members moment capacities for the steel frame moment resisting structure

Member	Moment capacity	Moment of inertia
	M_r (kN.m)	I_{xx} (mm ⁴)
305 x 305 118 H-Section (External columns)	586	276×10^6
203 x 203 x 52 H-Section (Internal columns)	185	52.5×10^6
533 x 210 x 101 I-Section (Beams)	476	616×10^6

Figure 4-45 shows the initial deflection of the structure under static loads for all the pseudo-dynamic tests. Under the initial conditions, the horizontal deflection at node 2 was equal to zero due to the symmetry of the loading. Figure 4-46 shows the initial shear force diagram and bending moment diagram of the structure before the earthquake load was applied. The bending moments in each of the members are less than the capacities of the members.

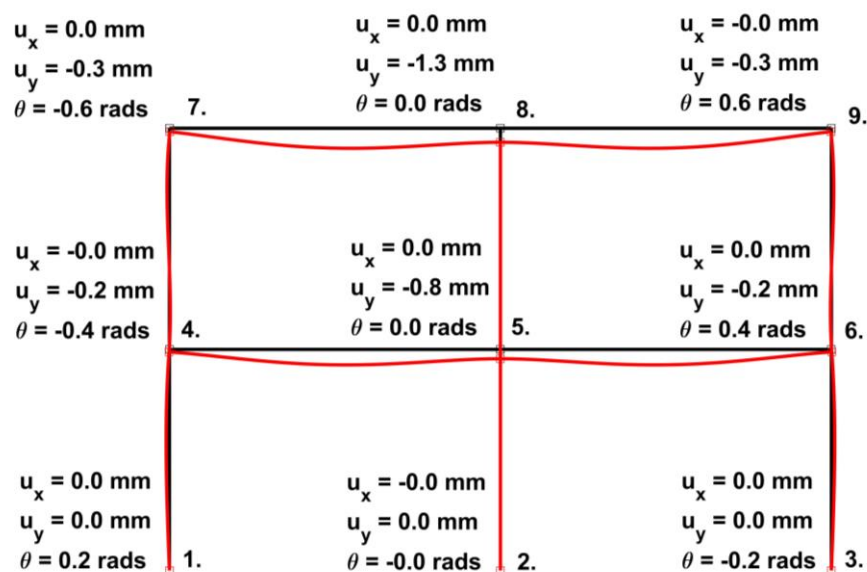


Figure 4-45 Initial deflection of the structure under static loads

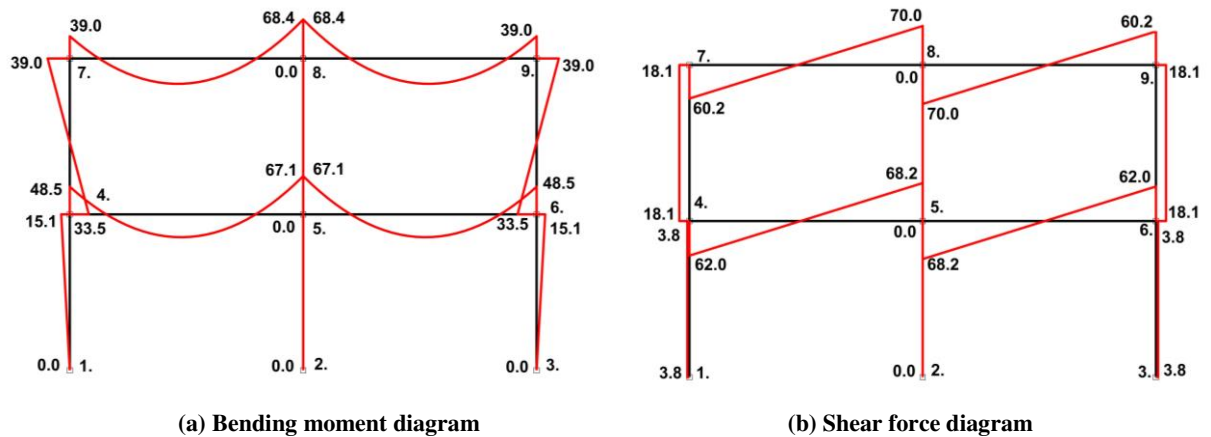


Figure 4-46 The initial state of the structure before earthquake loading

Figure 4-48 to Figure 4-56 shows the response of the structure at the maximum displacement reached during the 0.34 g to 2 g peak ground acceleration pseudo-dynamic experiments. As can be seen from Figure 4-48(b), the maximum bending moment reached in the external column and beam exceeds the capacity of the members, and therefore the beam would have failed before the external column due to the beam having a lower moment capacity than the external columns. The maximum bending moment that was reached by the internal column only marginally exceeded the steel member's capacity, and therefore a plastic hinge would have formed resulting in the loss of structural stability with the combination of steel members used to perform the pseudo-dynamic tests.

At a peak ground acceleration of 0.68 g and greater, the maximum bending moment reached in all the steel members exceeded the capacity of the steel members, and therefore the structure would have failed before the footing failed. However, a variation in the design of the frame and the selection of the structural members and connections all influence the behaviour of the overall structure.

Figure 4-57 to Figure 4-61 shows the axial load in each of the supports for the duration of the applied earthquake record. The axial load is shown either for the entire duration of the earthquake loading or until failure of the reinforced concrete footing during the applied earthquake loading. For each of the experiments, the axial load in the centre support (Node 2) is constant, whereas the external footings are subjected to varying axial load for the duration of the earthquake record. The pseudo-dynamic experiment undertaken at a maximum peak ground acceleration of 0.34 g resulted in zero tensile forces in the external columns for the duration of the applied earthquake record. However, the footings subjected to a PGA of 0.68 g and greater

all resulted in tensile forces and uplift being experienced by the external footings. The variation in axial load for the duration of the earthquake in combination with the lateral load will result in the performance of the reinforced concrete footing varying substantially. Therefore, it is recommended that future studies focus on the response of the footing that is subjected to both a varying axial and shear load and to investigate the effect this would have on the performance of the foundation. Future studies can also focus on the influence that progressive failure within the superstructure would have on the fragility of the footing as this study only assumed a linear elastic frame structure with 5 % damping.

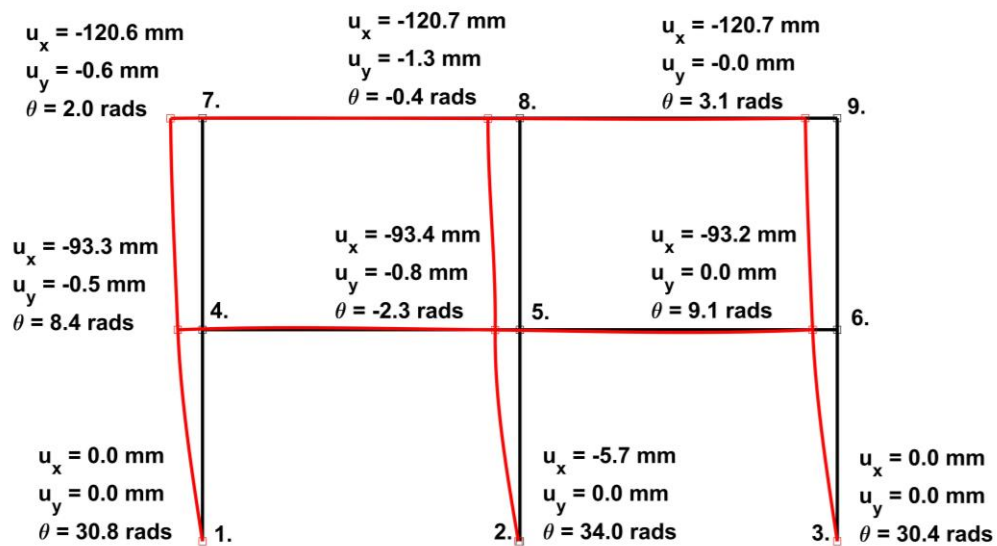


Figure 4-47 Maximum deflection of the overall frame structure at the maximum lateral displacement of the footing at a PGA of 0.34 g

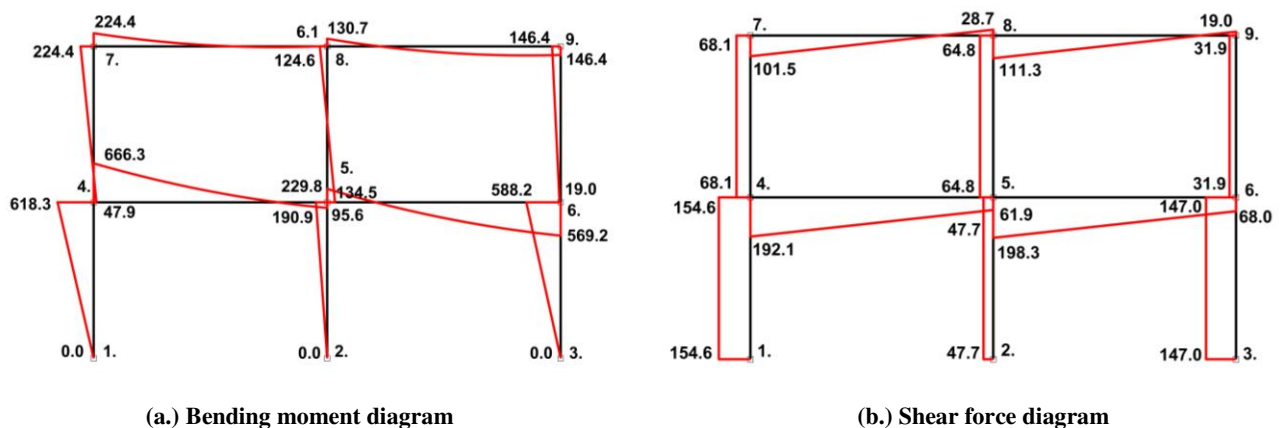


Figure 4-48 Bending moment diagram and shear force diagram at the maximum displacement during the 0.34 g PGA experiment

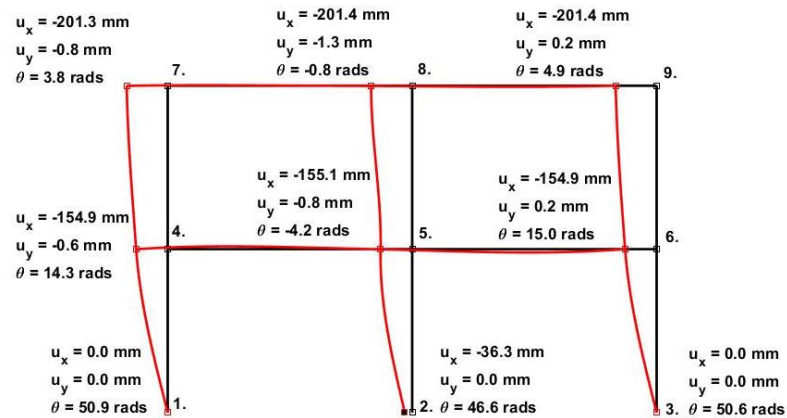


Figure 4-49 Maximum deflection of the overall frame structure at the maximum lateral displacement of the footing at a PGA of 0.68 g

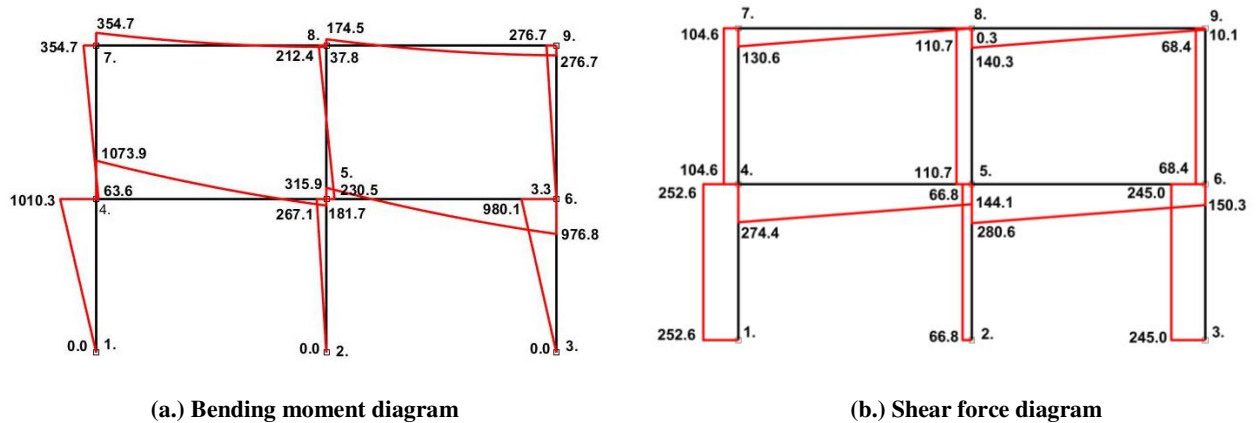


Figure 4-50 Bending moment diagram and shear force diagram at the maximum displacement during the 0.68 g PGA experiment

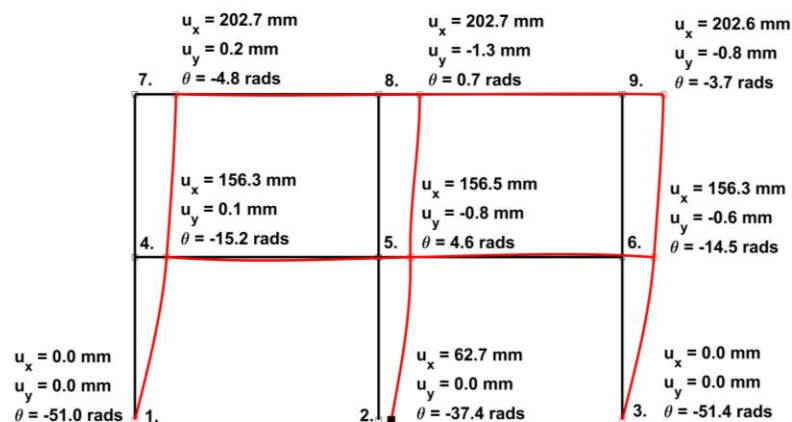
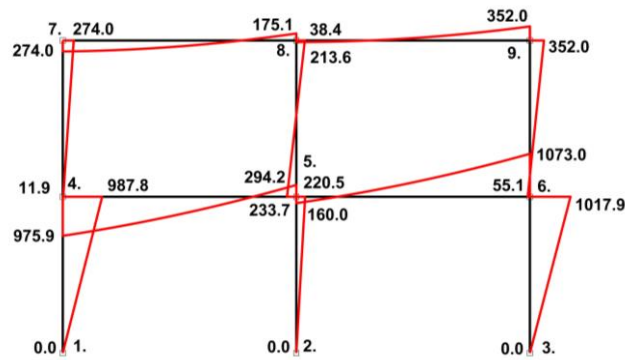
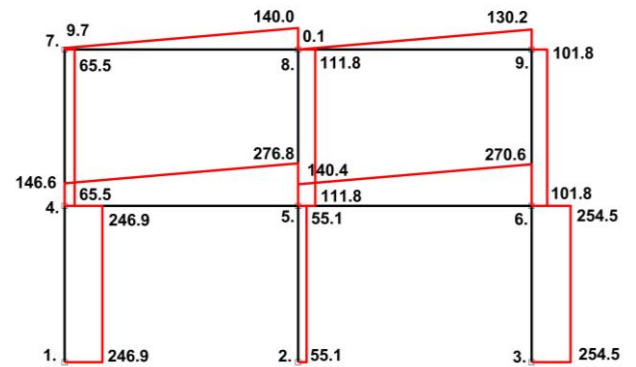


Figure 4-51 Maximum deflection of the overall frame structure at the maximum lateral displacement of the footing at a PGA of 0.78 g



(b.) Bending moment diagram



(c.) Shear force diagram

Figure 4-52 Bending moment diagram and shear force diagram at the maximum displacement during the 0.78 g PGA experiment

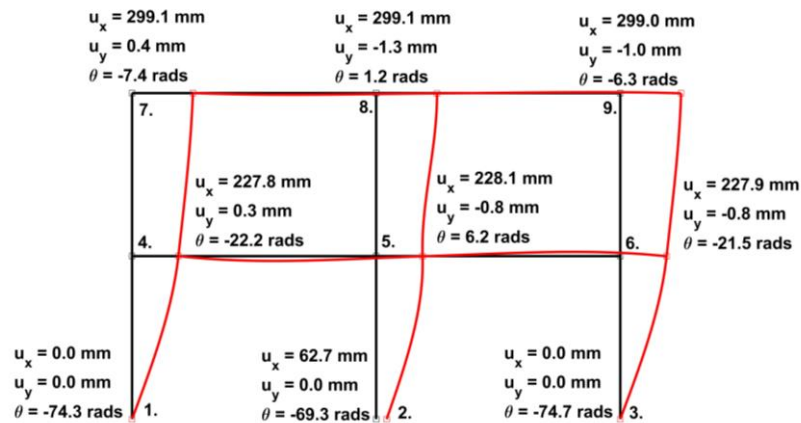
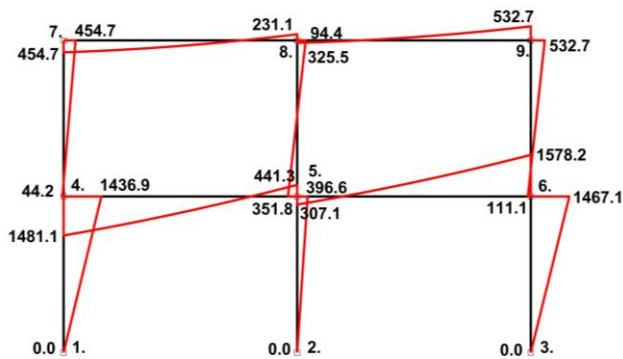
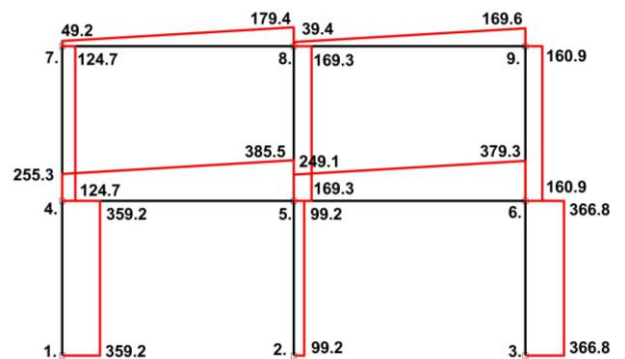


Figure 4-53 Maximum deflection of the overall frame structure at the maximum lateral displacement of the footing at a PGA of 1 g



(b.) Bending moment diagram



(c.) Shear force diagram

Figure 4-54 Bending moment diagram and shear force diagram at the maximum displacement during the 1 g PGA experiment

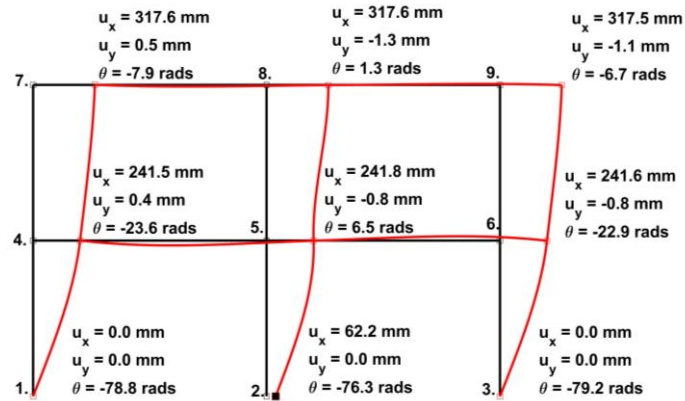
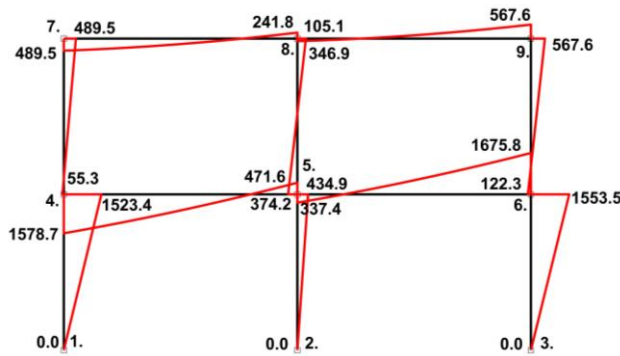
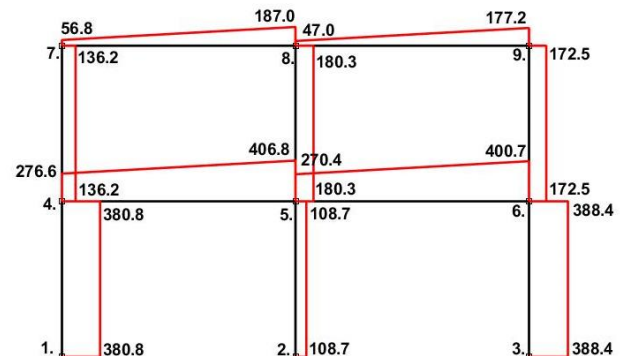


Figure 4-55 Maximum deflection of the overall frame structure at the maximum lateral displacement of the footing at a PGA of 2 g



(b.) Bending moment diagram



(c.) Shear force diagram

Figure 4-56 Bending moment diagram and shear force diagram at the maximum displacement during the 2 g PGA experiment

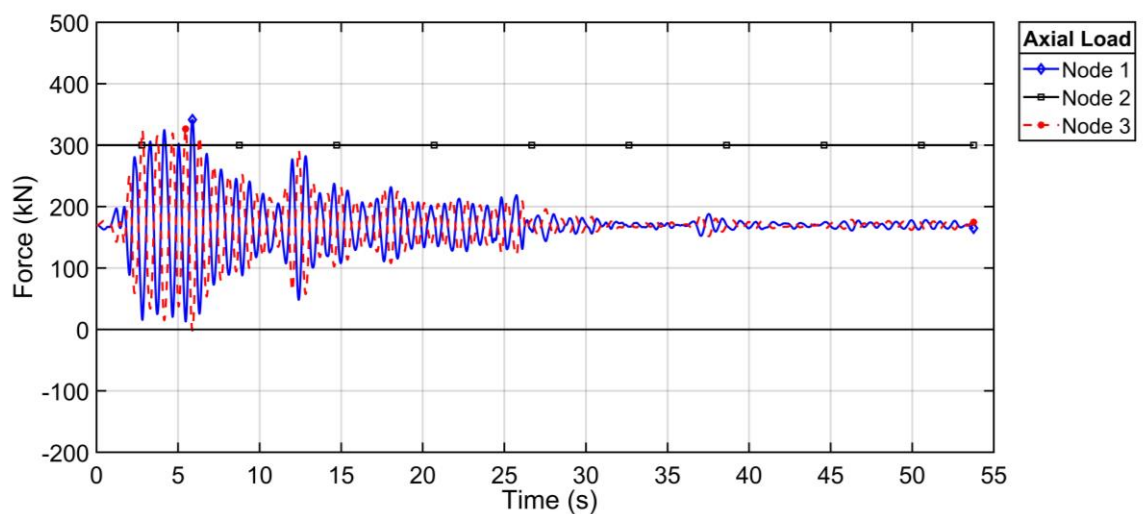


Figure 4-57 Axial force reactions in each of the columns for the duration of the 0.34 g experiment

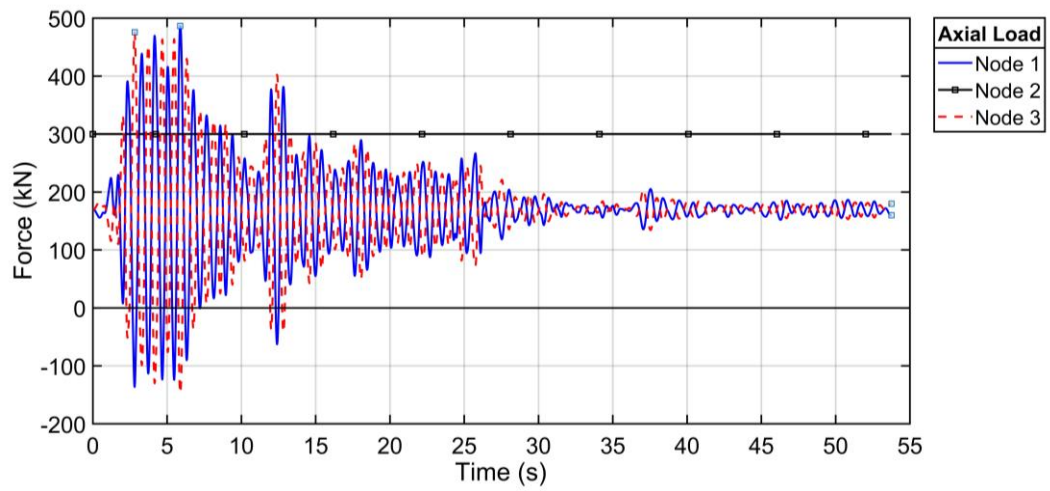


Figure 4-58 Axial force reactions in each of the columns for the duration of the 0.68 g experiment

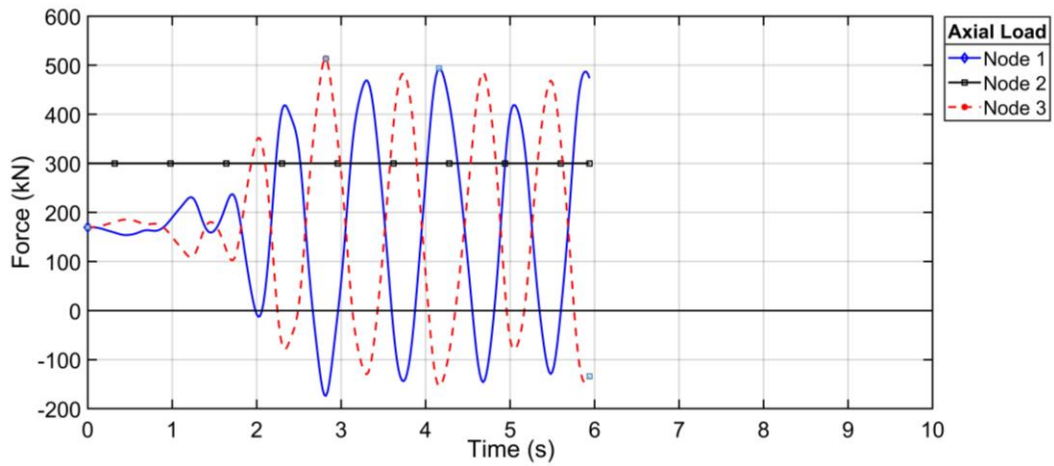


Figure 4-59 Axial force reactions in each of the columns for the duration of the 0.78 g experiment

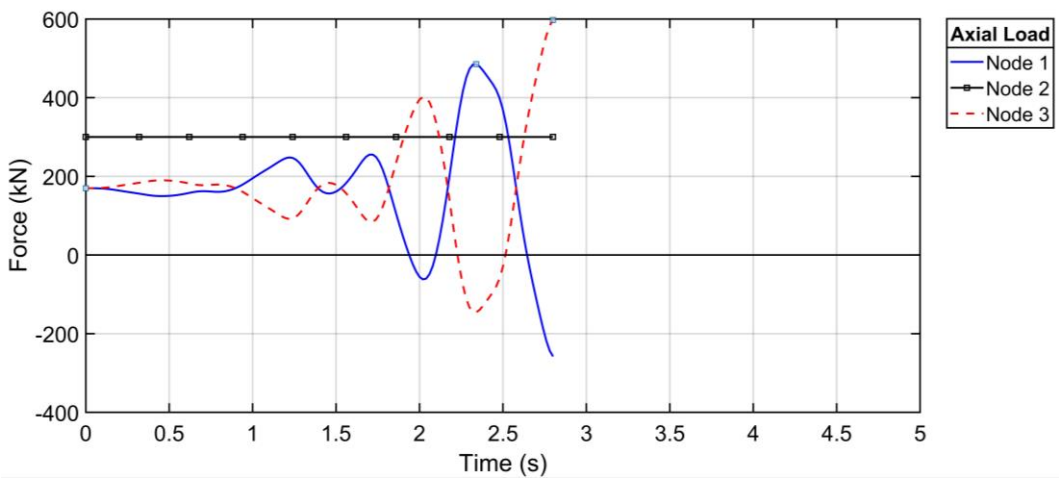


Figure 4-60 Axial force reactions in each of the columns for the duration of the 1 g experiment

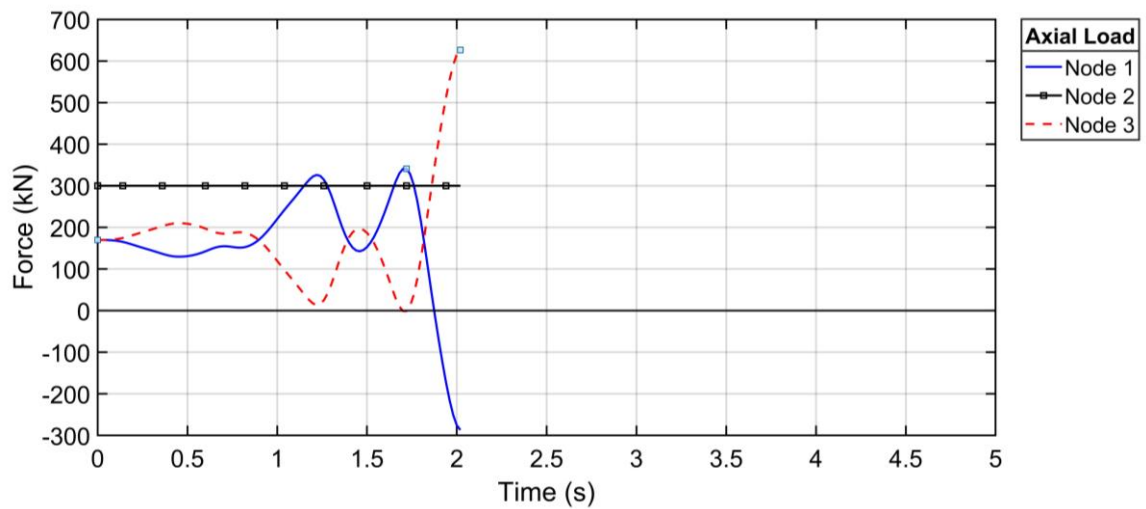


Figure 4-61 Axial force reactions in each of the columns for the duration of the 2 g experiment

4.5 CONCLUSION AND SUMMARY

Pseudo-dynamic experiments were undertaken on five reinforced concrete footing at different peak ground accelerations, which were obtained by amplifying the El Centro earthquake ground motion record. The peak ground accelerations (PGA) ranged from 0.34 g to 2 g. Only minimal cracking resulted at a PGA of 0.34 g, with complete failure having occurred at a PGA of 2 g. Table 4-3 summarises the results obtained during the pseudo-dynamic experiments for the following critical points: the minimum cracking force (F_c) and cracking deformation (u_c), the minimum reinforcement yielding force (F_y) and deformation (u_y), and the maximum achieved force (F_m) and deformation (u_m). The following observations were made during the pseudo-dynamic experiments:

- Increasing the amplitude of the El Centro ground motion record showed that there is a maximum PGA that can be sustained by the footing before failure occurs. For example, the pseudo-dynamic test that was undertaken by amplifying the El Centro ground motion record to produce a PGA of 2 g, only managed to achieve a maximum PGA of 1.21 g before failure occurred;
- An increase in the applied lateral deformation to the reinforced concrete footing results in cracking and yielding of the reinforcement, which in turn results in a reduction in the lateral stiffness of the footing;
- The lateral capacity of the reinforced concrete footing and subsequent damage is predominately controlled by the yield strength and ductility of the reinforcement;

- The ductility of the reinforced concrete footing is controlled by the tensile reinforcement and the shear reinforcement;
- Cracks occurred near the base of the column where the maximum moment was expected, however, the cracks do not always open at the interface between the base of the column and the top of the footing;
- The number of cycles of vibration increases the damage to the reinforced concrete footing, particularly when the load reverses after the reinforcement has yielded in tension;
- Spalling of concrete occurs due to the buckling of reinforcement during load reversal from tensile loading to compression loading on either face of the concrete column in the direction of loading. This occurs due to the incompatibility between the brittle concrete material and the ductile reinforcement. Upon load reversal, the permanently elongated reinforcement is first mobilised in compression before the crack that has formed in the concrete can close and mobilise in compression. To overcome this incompatibility, the reinforcement buckles, resulting in the spalling of the concrete. The spalling of the concrete subsequently results in a reduction in the gross cross-sectional area of the column, which in turn results in a decrease in the axial and shear capacity of the footing;
- The axial load applied to the footing contributes to pinching effect, which was observed in the pseudo-dynamic hysteretic curves. As the horizontal load is reduced the axial load stabilises the column by causing the column to pivot back to its original vertical position and closing the cracks in the concrete;
- The unloading stiffness from the backbone curve of the footing is greater than the reloading stiffness into the backbone curve, which indicates that the structure is absorbing energy and incurring damage;
- Before the reinforcement yields, the response under cyclic loading remains predominantly perfectly plastic without any significant permanent deformation;
- The hysteretic energy absorbed by the footing gives a good indication of damage incurred by the footing under repeated cyclic loading but does not indicate structural failure at large deformations with few or no loading cycles. Therefore, the results correlate with the Park and Ang damage index, as discussed in Chapter 2, that incorporates both damage due to excessive deformation and repeated cyclic loading; and

- The loss of moment capacity due to the formation of a plastic hinge is governed by the repeated cycling of the footing at a displacement less than the failure displacement of the footing and greater than the yielding displacement of the reinforcement.
- Plastic hinges and progressive failure within the frame structure were not considered in the numerical model. However, depending on the capacity of the members used within the frame structure, plastic hinges would have formed, which could have resulted in the frame structure failing before the reinforced concrete footing. The progressive failure of the structure could have altered the response of the footing with increasing earthquake intensity.

Table 4-3 Results from pseudo-dynamic analysis at 5% damping and a natural period of vibration of 0.86 s

Intensity (MMI)	PGA (g)	Cracking ⁽¹⁾		Yielding ⁽¹⁾		Maximum		Damage state
		F _c (kN)	u _c (mm)	F _y (kN)	u _y (mm)	F _m (kN)	u _m (mm)	
7.45	0.34	22.1	1.68	-	-	48	5.72	Onset of cracking, still serviceable
8.29	0.68	40.5	2.48	67.4	6.85	70.3	36.3	Large cracks, extensive damage
8.45	0.78	45.8	2.75	74.6	7.21	74.6	62.7	Collapse
8.75	1	40.84	3.37	62.0	7.41	74	62.7	Collapse
9.59	2 (1.21 ⁽²⁾)	35.8	2.47	71.5	7.81	71.6	62.2	Collapse

(1) Minimum lateral force and displacement that results in cracking of the concrete and yielding of the reinforcement

(2) The maximum acceleration achieved by the structure before failure

5 DAMAGE FORMULATION WITH EARTHQUAKE INTENSITY

The aim of this chapter is to show the procedure that was followed to formulate the damage and fragility curves for the analysed reinforced concrete footing, which forms part of an overall linear elastic moment resisting frame structure, by utilising the results obtained from the laboratory experiments in the previous chapter. The results obtained from the cyclic load tests and pseudo-dynamic tests were used to formulate an analytical hysteretic model for the reinforced concrete footing under the constant axial load of 300 kN, which was then used to replace the laboratory test setup. The results produced during the cyclic load tests and pseudo-dynamic experiments were utilised to formulate the hysteretic model to enable the requisite degradation of stiffness and pinching effect of reinforced concrete to be incorporated into the damage formulation at peak ground accelerations and fundamental period of vibration that were not undertaken during the pseudo-dynamic laboratory experiments.

The force value produced from the analytical hysteretic model is used to circumvent the force reading from the load cell in the pseudo-dynamic experiment. The damage to the reinforced concrete footing could be interpolated at amplified peak ground accelerations and overall structural fundamental periods of vibration that were not undertaken during the laboratory experiments by using the pseudo-dynamic testing algorithm in combination with the developed analytical hysteretic model described in this chapter.

5.1 ANALYTICAL HYSTERETIC MODEL

Figure 5-1 shows the numerical hysteretic shear model that was formulated using the limited results produced during the cyclic load tests and the pseudo-dynamic tests to determine the damage states of the footing over a range of earthquake intensities, structural stiffnesses and damping ratios that were not undertaken during the laboratory work. The numerical model was predominately formulated from the results from the second cyclic load test and the trilinear model that is typically used to model reinforced concrete.

The same numerical analysis model that was used to perform the pseudo-dynamic experiments, as previously shown in Figure 3-27, was used to perform the analysis using the formulated hysteretic model. The displacement calculated using pseudo-dynamic computational algorithm was input into the hysteretic model, instead of being applied directly to the test specimen, which then calculates the resultant force that is based on the historical response of the footing at previous time increments.

The hysteretic model for cyclic loading was purely based on the observed results from the experimental analysis that only considered an axial load of 300 kN and the minimally reinforced concrete footing. The accuracy of the model under different axial loads and different structural configurations has not been verified. Three curves can be deduced from the observations made during the cyclic load tests and are shown in Figure 5-1 and described below:

- **Primary Curves (PC):** Is the backbone curve of the analytical model and produces the same response under monotonic loading. The backbone curve provides the envelope to which the unloading and reloading curves are confined. Two points are defined on the primary curve and represent the maximum displacement points reached in either direction of loading from the initial vertical position of the footing. The maximum deformation in either direction governs the shape of the unloading and reloading curves. The value $u_{\max_neg}$ is the maximum negative lateral deformation achieved by the footing from the start of ground motion load application, and $u_{\max_pos}$ is the maximum positive deformation achieved by the footing since the start of the earthquake.
- **Secondary Curves (SC):** Is the response of the footing due to a change in direction from the initial direction of loading along the backbone curve and accounts for the softening of the reinforced concrete due to concrete cracking, yielding of the reinforcement and the pinching effect. The hysteretic analytical model is driven by the maximum positive and negative deformation points ($u_{\max_neg}$ and $u_{\max_pos}$) along the backbone curve for each time increment as defined in Figure 5-1. The point u_{UL} indicates a point where the path changes from the primary curve to the secondary curve and the point u_{RL} indicates the point where there is a transition from the secondary curve into the primary curve. If the footing is unloading from the positive direction along the backbone curve, then the unloading point $u_{UL} = u_{\max_pos}$ and the reloading point is $u_{RL} = u_{\max_neg}$, otherwise, if the footing is unloading from the backbone curve in the negative direction, then the unloading point $u_{UL} = u_{\max_neg}$ and the reloading point is $u_{RL} = u_{\max_pos}$. The response of the footing travels down the path of unloading until it reaches the origin and starts reloading in the opposite direction.
- **Tertiary curves (TC):** When reloading and unloading occur in the same quadrant without a load direction reversal. The tertiary curve is driven by the point where there is a change in direction from the secondary curve and the initial unloading point from the primary curve.

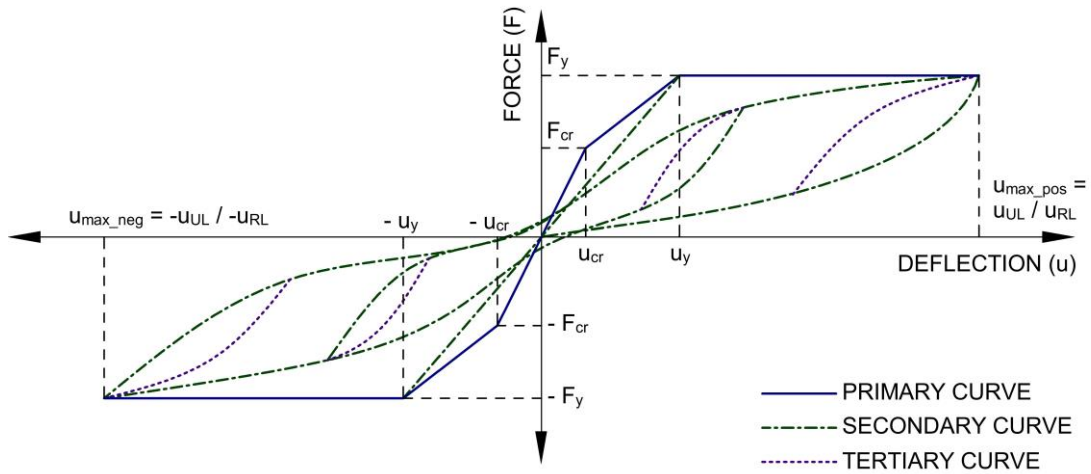


Figure 5-1 Numerical hysteretic shear model showing the primary, secondary, and tertiary curves

Figure 5-2 shows the calculation model used to drive the hysteretic model, which depends on the change in sign of displacement from the final displacement from the previous time increment and the calculated displacement at the current time increment. Using Figure 5-2, a positive change in displacement will result in the path remaining in the same direction as the previous time increment. However, if the change in displacement is negative, it indicates a load reversal whereby the path changes from Path 1 to Path 2. Only the force (F_s) produced from the calculated displacement u_i in the current iteration and time step is used further in the analysis. However, the model also outputs the maximum positive and negative displacement of the footing and the current load path, which is then used in the next iteration or time increment.

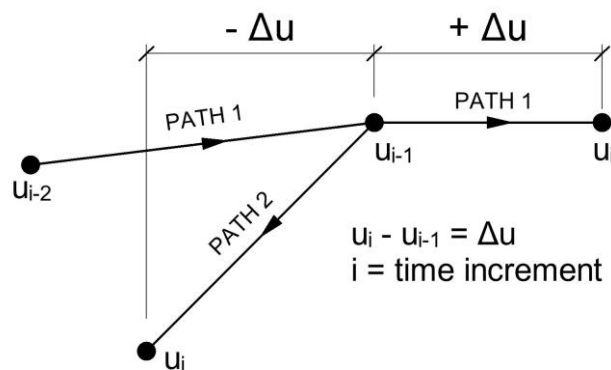


Figure 5-2 Basis of the analytical numerical hysteretic model formulation

5.1.1 PRIMARY BRANCHES

Figure 5-3 shows the primary curve of the hysteretic model, which is the same as the backbone curve produced during monotonic load testing. The primary curve is initialised at the start of the analysis and consists of three regions using a trilinear model:

1. Before cracking, $|u_i| \leq |\pm u_{cr}|$, Region O – A or O – D: the model follows a linear elastic response model.
2. Concrete cracking, $|\pm u_{cr}| \leq |u_i| \leq |\pm u_y|$, Region A – B or Region D – E: after cracking of the concrete in either direction there is a reduction in the stiffness of the cross section. With a continued increase in the load applied to the footing, the loading path follows Path A – B or D – E. However, upon unloading, the path follows region O – A' or O – D' and therefore there is a permanent reduction in the stiffness of the footing due to the cracking of the concrete. Point A' and Point D' then becomes the new reloading point into the backbone curve. If the displacement exceeds the new u_{cr} point, the path will continue along the backbone curve (Path 1). From the cyclic load tests and pseudo-dynamic tests, it was observed that upon unloading, and before the reinforcement yields, the displacement returns to the original position without any permanent deformation. Further experimentation will be needed to verify this, however, for this model the unloading curve follows a linear line from the unloading point on the backbone curve to the origin before the reinforcement yields.
3. Yielding of the reinforcement, $|u_i| \geq |\pm u_y|$, Region B – C or Region E – G: once the displacement of the footing exceeds the yield strength of the reinforcement, the stiffness of the backbone curve becomes equal to zero with a further increase in the displacement. The secondary path is calculated using the maximum displacement achieved on the primary curve and used to interpolate the unloading and reloading curves obtained from the second cyclic load test. If the change in displacement from the current time step and the previous time step is positive ($|u_{i,j}| - |u_{i-1}| \geq 0$), where i indicates the time increment and j the iteration step, then the path continues along the backbone curve. However, if the change in displacement is negative ($|u_{i,j}| - |u_{i-1}| < 0$), then the unloading displacement follows the secondary curve. The point of unloading along the backbone curve then governs the point of reloading upon load reversal.

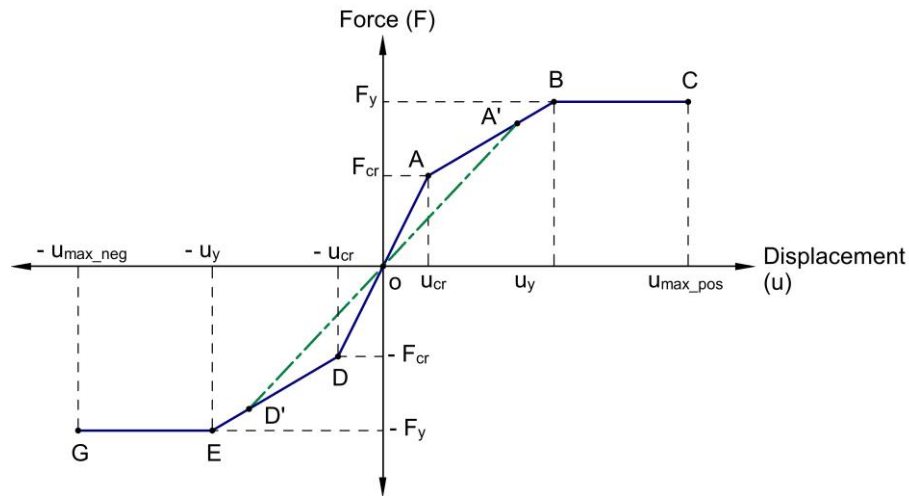


Figure 5-3 Primary curve

5.1.2 SECONDARY BRANCHES: UNLOADING AND RELOADING CURVES

Figure 5-4 shows the unloading and reloading branches of the secondary curves for the analytical hysteretic material, which are driven by the maximum deformations ($u_{\max_pos}$ and $u_{\max_neg}$) that have been reached in both directions along the primary curve. The secondary curves account for the degradation in stiffness with each cycle of loading due to the cracking and yielding of the reinforcement.

The unloading branch is defined as a path that is formed upon load reversal whereby the path changes from the primary curve to the secondary curve and tends to have a concave up shape. The maximum slope along the unloading curve occurs at the unloading point along the primary curve and decreases to the minimum slope at the point of zero shear force. The reloading branch is defined as the path that transitions from the secondary curve into the primary curve and tends to have a concave down shape with the maximum slope occurring at the point of zero shear force and the minimum slope occurring at the transition point (u_{RL}) back into the primary curve.

The unloading curves are shown in Figure 5-6 and are tabulated in Table 5-1 and the reloading curves are shown in Figure 5-7 and tabulated in Table 5-2. The unloading and reloading curves were interpolated from the second cyclic load test, as previously shown in Figure 4-5, and compared to the results produced during the pseudo-dynamic experiments. The unloading point along the backbone curve, which occurs due to load reversal, is used to calculate the unloading curve by interpolating the values given in Table 5-1. The reloading point along the primary curve is used to calculate the reloading curve by interpolating the values in Table 5-2.

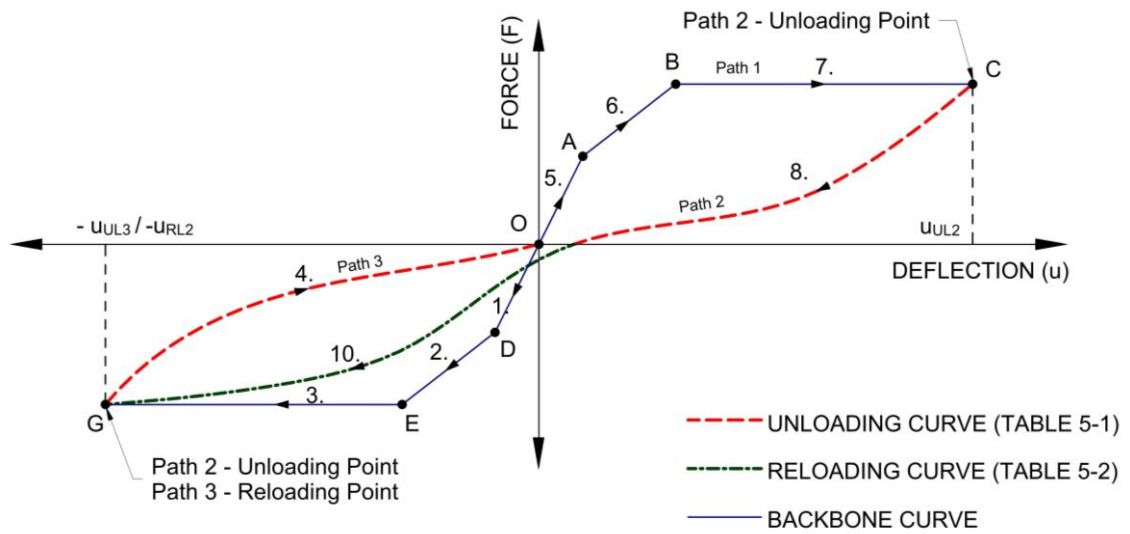


Figure 5-4 Unloading and reloading curves

The problem that arises when separately interpolating the unloading and reloading curves is that the points of intersection produced by the unloading and reloading curves at the x-axis typically does not intercept the axis at the same point. To overcome the incompatibility at the x-axis intercept, the average between the two x-axis points is determined between the unloading curve and the reloading curve. The two curves are then scaled equally until the points of intersection along the x-axis occurs at the same point. Figure 5-5 shows the method followed to produce the final secondary curve that is used upon unloading from the primary curve and reloading back into the primary curve in the opposite direction from the unloading point. Further experimentation will be needed to verify this.

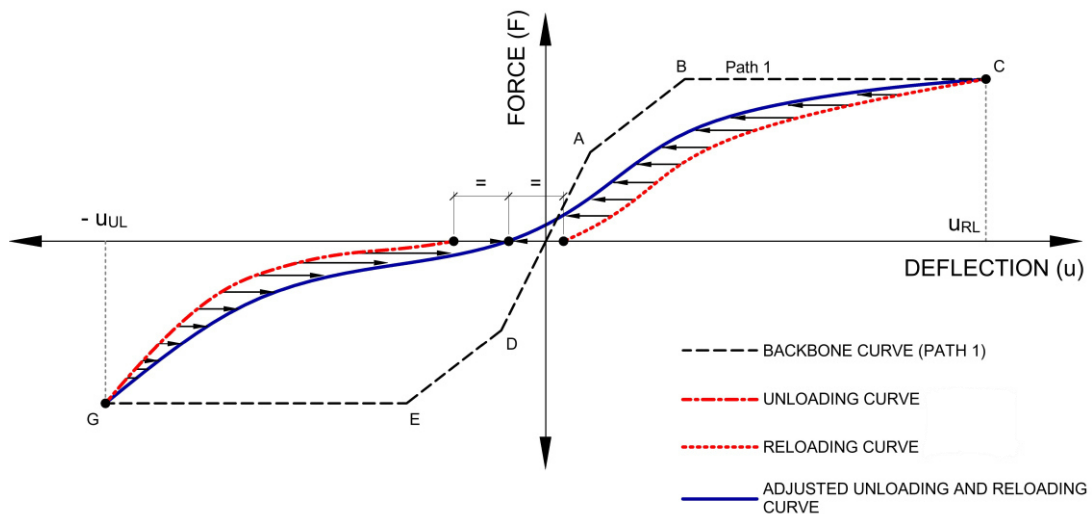


Figure 5-5 Adjusted unloading and reloading curves from the interpolated data

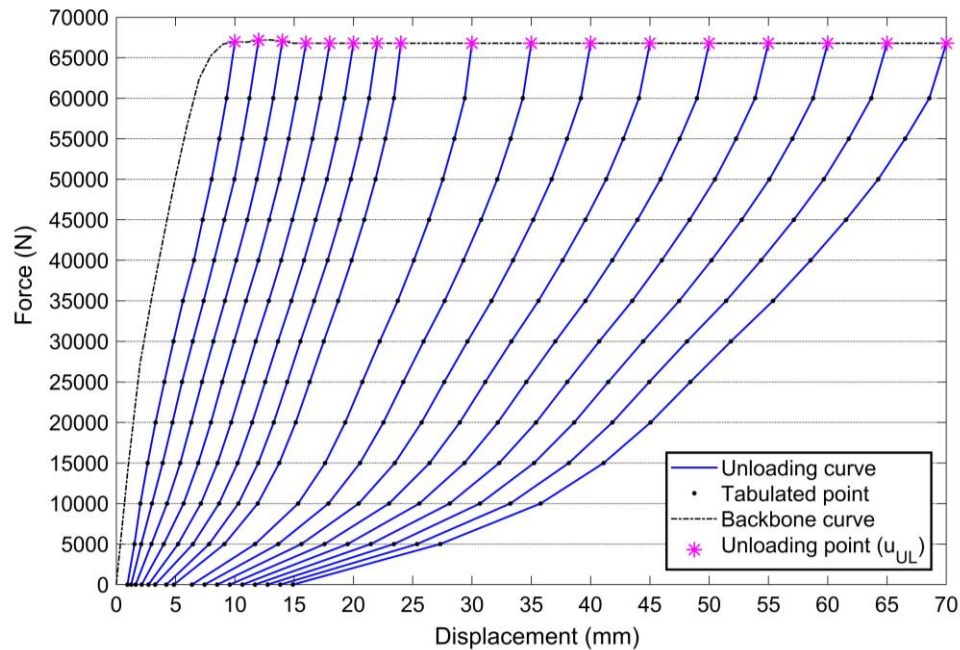


Figure 5-6 Unloading curves as a function of the point of unload along the primary curve

Table 5-1 Tabulated displacement points along the unloading curve as a function of the point of unloading along the primary curve

		Backbone curve force (N)	Force (N)												
			0	5000	10000	15000	20000	25000	30000	35000	40000	45000	50000	55000	60000
Backbone curve unloading displacement (u_{UL}) (mm)	0	0	0.00	0.00	0.00	-	-	-	-	-	-	-	-	-	-
	1	14523	0.11	0.53	0.78	1.09	1.38	1.68	-	-	-	-	-	-	-
	2	27215	0.16	0.61	0.88	1.16	1.48	1.83	2.28	2.70	-	-	-	-	-
	3	35497	0.21	0.67	0.95	1.24	1.58	1.98	2.45	2.95	3.41	-	-	-	-
	4	42976	0.30	0.73	1.02	1.34	1.67	2.09	2.63	3.20	3.71	4.13	4.61	-	-
	5	50278	0.37	0.81	1.09	1.40	1.76	2.20	2.76	3.38	4.01	4.49	4.99	5.41	-
	6	56819	0.45	0.90	1.20	1.51	1.92	2.35	2.92	3.60	4.25	4.85	5.37	5.87	6.12
	7	62439	0.57	1.01	1.32	1.64	2.05	2.52	3.16	3.82	4.53	5.16	5.77	6.32	6.82
	8	65307	0.68	1.13	1.47	1.87	2.30	2.86	3.51	4.25	4.99	5.67	6.39	6.96	7.52
	9	66677	0.81	1.31	1.70	2.18	2.73	3.38	4.10	4.86	5.66	6.40	7.11	7.78	8.34
	10	66960	0.94	1.53	2.03	2.65	3.31	4.05	4.82	5.62	6.55	7.28	8.04	8.69	9.30
	11	66921	1.12	1.79	2.51	3.28	4.02	4.79	5.61	6.49	7.38	8.21	8.98	9.66	10.30
	12	67146	1.26	2.11	3.01	3.91	4.74	5.55	6.45	7.37	8.25	9.11	9.96	10.62	11.23
	13	67199	1.40	2.45	3.60	4.60	5.49	6.34	7.27	8.21	9.13	10.00	10.86	11.58	12.19
	14	67026	1.64	2.97	4.28	5.40	6.32	7.23	8.18	9.16	10.12	11.04	11.85	12.59	13.24
	15	66774	1.92	3.50	4.97	6.17	7.14	8.08	9.06	10.07	11.04	11.96	12.81	13.57	14.21
	16	66774	2.14	4.01	5.67	6.91	7.99	8.94	9.93	10.94	11.98	12.90	13.77	14.53	15.19
	17	66774	2.42	4.60	6.37	7.74	8.81	9.82	10.86	11.89	12.91	13.89	14.78	15.53	16.20
	18	66774	2.71	5.21	7.13	8.54	9.69	10.73	11.76	12.86	13.91	14.85	15.75	16.55	17.22
	19	66774	2.96	5.84	8.14	9.37	10.57	11.62	12.68	13.80	14.86	15.84	16.74	17.53	18.22
	20	66774	3.28	6.47	8.66	10.24	11.46	12.56	13.64	14.76	15.84	16.81	17.76	18.56	19.25
	21	66774	3.78	7.12	9.47	11.11	12.36	13.49	14.60	15.72	16.83	17.84	18.79	19.57	20.28
	22	66774	4.22	7.83	10.30	11.97	13.28	14.42	15.53	16.69	17.82	18.84	19.78	20.61	21.32
	23	66774	4.61	8.50	11.09	12.88	14.20	15.35	16.52	17.69	18.81	19.86	20.79	21.65	22.33
	24	66774	4.86	9.12	11.93	13.75	15.13	16.32	17.50	18.71	19.85	20.91	21.86	22.69	23.42
	25	66774	5.32	9.76	12.78	14.68	16.09	17.29	18.51	19.78	20.91	21.98	22.95	23.76	24.49
	30	66774	6.38	11.71	15.34	17.62	19.31	20.75	22.21	23.73	25.09	26.37	27.54	28.51	29.39
	35	66774	7.45	13.66	17.89	20.55	22.53	24.20	25.92	27.69	29.28	30.77	32.13	33.26	34.28
	40	66774	8.51	15.61	20.45	23.49	25.74	27.66	29.62	31.65	33.46	35.16	36.72	38.01	39.18
	45	66774	9.57	17.56	23.00	26.42	28.96	31.12	33.32	35.60	37.64	39.56	41.31	42.76	44.08
	50	66774	10.64	19.51	25.56	29.36	32.18	34.58	37.02	39.56	41.82	43.95	45.90	47.51	48.98
	55	66774	11.70	21.46	28.12	32.30	35.40	38.04	40.73	43.51	46.01	48.35	50.49	52.27	53.87
	60	66774	12.77	23.41	30.67	35.23	38.62	41.49	44.43	47.47	50.19	52.74	55.08	57.02	58.77
	65	66774	13.83	25.37	33.23	38.17	41.83	44.95	48.13	51.43	54.37	57.14	59.67	61.77	63.67
	70	66774	14.89	27.32	35.78	41.10	45.05	48.41	51.83	55.38	58.55	61.53	64.26	66.52	68.57

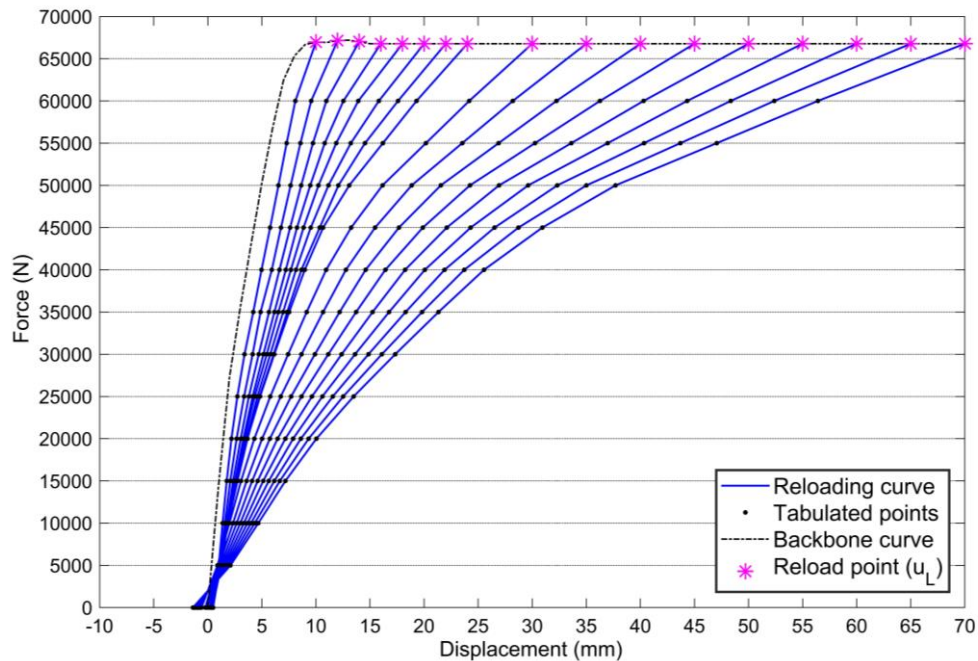


Figure 5-7 Reloading curves as a function of the point of reload along the primary curve

Table 5-2 Tabulated displacement points along the reloading curve as a function of the point of reloading along the primary curve

		Backbone curve force (N)	Force (N)												
			0	5000	10000	15000	20000	25000	30000	35000	40000	45000	50000	55000	60000
Backbone curve reloading displacement (u_L) (mm)	0	0	0.00	0.00	0.00	-	-	-	-	-	-	-	-	-	-
	1	14523	0.02	0.45	0.73	1.02	1.35	1.73	-	-	-	-	-	-	-
	2	27215	0.12	0.54	0.81	1.10	1.43	1.80	2.27	2.83	-	-	-	-	-
	3	35497	0.14	0.60	0.88	1.17	1.50	1.87	2.37	2.94	3.47	-	-	-	-
	4	42976	0.20	0.68	0.96	1.25	1.57	1.97	2.47	3.04	3.62	4.14	4.80	-	-
	5	50278	0.26	0.75	1.03	1.33	1.66	2.05	2.57	3.15	3.77	4.33	4.95	5.57	-
	6	56819	0.38	0.82	1.09	1.42	1.76	2.17	2.69	3.30	3.92	4.52	5.11	5.77	6.33
	7	62439	0.38	0.86	1.18	1.49	1.83	2.26	2.78	3.43	4.06	4.66	5.32	5.97	6.62
	8	65307	0.41	0.92	1.23	1.55	1.90	2.35	2.90	3.55	4.23	4.92	5.58	6.25	6.91
	9	66677	0.42	0.96	1.28	1.64	2.01	2.50	3.10	3.79	4.52	5.24	6.01	6.70	7.42
	10	66960	0.42	1.01	1.37	1.75	2.19	2.73	3.40	4.21	4.97	5.77	6.54	7.29	8.10
	11	66921	0.45	1.07	1.47	1.91	2.41	3.05	3.79	4.65	5.46	6.27	7.13	7.96	8.84
	12	67146	0.49	1.14	1.58	2.08	2.67	3.36	4.15	4.88	5.92	6.77	7.66	8.57	9.56
	13	67199	0.49	1.19	1.66	2.22	2.86	3.59	4.42	5.36	6.29	7.18	8.14	9.17	10.27
	14	67026	0.44	1.21	1.74	2.34	3.04	3.80	4.71	5.66	6.63	7.61	8.64	9.72	10.97
	15	66774	0.42	1.23	1.78	2.42	3.15	3.99	4.93	5.95	6.91	7.97	9.07	10.26	11.71
	16	66774	0.43	1.25	1.85	2.52	3.29	4.15	5.14	6.16	7.22	8.25	9.47	10.82	12.54
	17	66774	0.39	1.25	1.88	2.58	3.38	4.26	5.31	6.38	7.41	8.62	9.84	11.31	13.22
	18	66774	0.30	1.21	1.86	2.58	3.40	4.34	5.45	6.54	7.68	8.85	10.24	11.87	13.92
	19	66774	0.21	1.14	1.86	2.61	3.47	4.42	5.57	6.75	7.89	9.20	10.60	12.39	14.89
	20	66774	0.17	1.16	1.89	2.69	3.58	4.58	5.78	6.97	8.15	9.52	11.16	13.23	15.82
	21	66774	-0.01	1.09	1.89	2.73	3.62	4.67	5.92	7.13	8.41	9.81	11.69	13.99	16.58
	22	66774	-0.03	1.06	1.89	2.75	3.68	4.79	6.09	7.34	8.66	10.35	12.08	14.54	17.59
	23	66774	-0.05	1.06	1.89	2.74	3.69	4.82	6.12	7.40	8.89	10.51	12.52	15.07	18.79
	24	66774	-0.23	0.93	1.79	2.66	3.66	4.84	6.15	7.53	8.97	10.67	13.08	16.19	19.30
	25	66774	-0.49	0.75	1.67	2.56	3.59	4.82	6.19	7.62	9.12	11.05	13.47	16.81	20.15
	30	66774	-0.59	0.90	2.00	3.08	4.30	5.78	7.43	9.14	10.95	13.26	16.16	20.17	24.18
	35	66774	-0.69	1.05	2.34	3.59	5.02	6.74	8.67	10.67	12.77	15.47	18.85	23.53	28.21
	40	66774	-0.78	1.20	2.67	4.10	5.74	7.71	9.90	12.19	14.59	17.68	21.54	26.89	32.24
	45	66774	-0.88	1.35	3.00	4.61	6.46	8.67	11.14	13.72	16.42	19.89	24.24	30.25	36.26
	50	66774	-0.98	1.50	3.34	5.13	7.17	9.63	12.38	15.24	18.24	22.10	26.93	33.61	40.29
	55	66774	-1.08	1.65	3.67	5.64	7.89	10.60	13.62	16.76	20.07	24.31	29.62	36.97	44.32
	60	66774	-1.17	1.80	4.00	6.15	8.61	11.56	14.86	18.29	21.89	26.52	32.32	40.33	48.35
	65	66774	-1.27	1.95	4.34	6.66	9.33	12.52	16.09	19.81	23.72	28.72	35.01	43.70	52.38
	70	66774	-1.37	2.10	4.67	7.18	10.04	13.49	17.33	21.33	25.54	30.93	37.70	47.06	56.41

Another path that needs to be considered during the analysis is a change in direction from a secondary path before it transitions back into the backbone curve. This typically occurs when the footing has previously been displaced to its maximum position in the direction of loading and load reversal occurs at a displacement that is less than the maximum displacement reached on the primary curve. Because of this, the cyclic behaviour of the footing and load reversals occurs within the maximum displacements achieved on the primary curve. Figure 5-8 shows a load reversal that has occurred along a secondary curve before reaching the maximum displacement $u_{\max_pos}$. Figure 5-9 shows diagrammatically the method used to determine the unloading path that occurs from a reloading path along the secondary curve. The unloading path is determined by adjusting and scaling the unloading curve that would otherwise have been calculated from the maximum point $u_{\max_pos}$ along the primary curve.

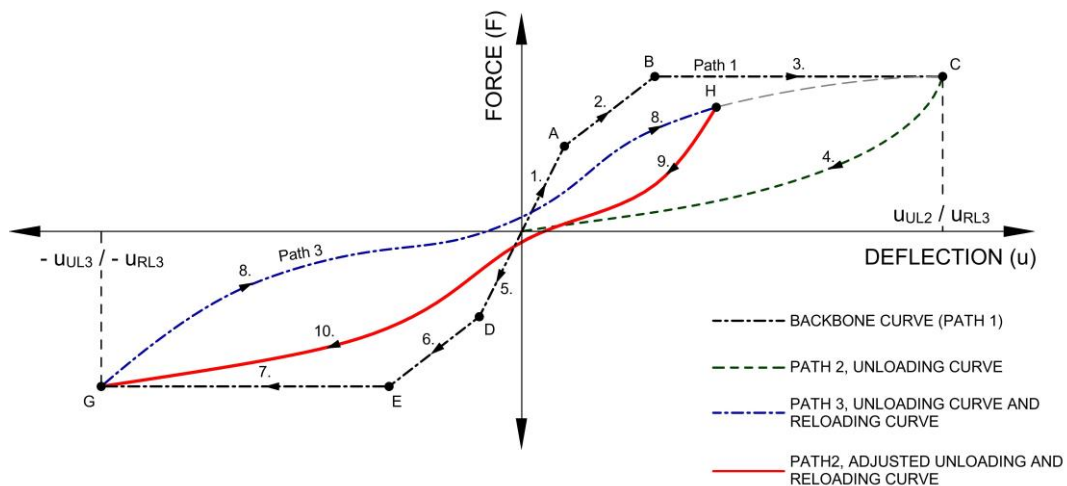


Figure 5-8 Adjusted unloading from a secondary reloading curve

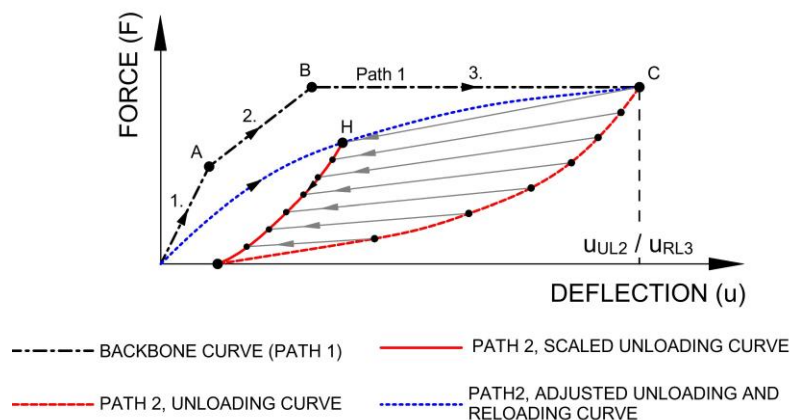


Figure 5-9 Scaling of the unloading curve to produce the adjusted unload curve

5.1.3 TERTIARY CURVES: INTERMEDIATE CURVES

Tertiary curves account for unloading and reloading behaviour that occurs within the same quadrant without crossing the origin as shown in Figure 5-10. The tertiary curves are determined by mirroring the secondary curve about a straight line drawn from the unloading point (Point C in Figure 5-10) along the backbone curve and the point of load reversal (Point D in Figure 5-10) along the secondary curve. The new curve is then mirrored about a line perpendicular and equidistant along the line drawn between Point D and Point C, which is shown by Line D'-C' in Figure 5-10, to produce the final tertiary curve. The same procedure is followed for each cycle of vibration within the quadrant. Double mirroring of the curve about two perpendicular lines ensures that the slope at point D and Point F is greater than the slope at point C and Point E as shown in Figure 5-10.

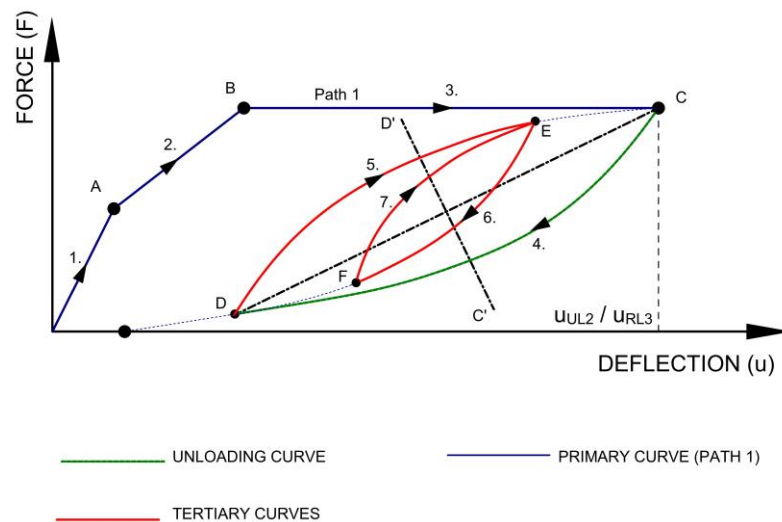


Figure 5-10 Tertiary curves

5.2 ANALYTICAL MODEL HYSTERETIC RESULTS

The hysteretic curves produced using the analytical hysteretic model for cyclic loading are compared to the hysteretic curves that were generated during the pseudo-dynamic experiments. The comparisons for the five tests are shown in Figure 5-11 to Figure 5-15. The numerical model was run using the analytical hysteretic material with the El Centro earthquake and was scaled to a peak ground acceleration of 0.34 g and Figure 5-11 shows the comparison of the two hysteretic curves. The hysteretic response produced using the analytical model resulted in

more hysteresis during the unloading cycle than that observed during the pseudo-dynamic laboratory test. The slight disparity between the formulated hysteretic model analysis and the result produced during the laboratory test is due to the unloading rule specified in the hysteretic model before the reinforcement had yielded. However, the developed analytical hysteretic model provides a maximum shear force and backbone curve that is similar to the result produced during the pseudo-dynamic experiments.

Figure 5-12 shows the comparison between the result produced using the analytical hysteretic model and pseudo-dynamic experiment at a PGA of 0.68 g. The analytical hysteretic model for cyclic loading provides a similar hysteretic response to the result produced during the pseudo-dynamic experiments. The response is particularly evident by the maximum shear force and displacement being almost the same for both the hysteretic curves. Figure 5-13 shows the comparison between the analytical model and experimental tests at a PGA of 0.78 g. The hysteretic response produced from the analytical hysteretic model shows that the unloading and reloading branches differ from that generated from the experimental result. The difference in response was most likely due to the reduction in cross section due to spalling of the concrete not being incorporated into the model. The pinching effect is captured by the analytical hysteretic model as shown in Figure 5-12 and Figure 5-13.

Figure 5-14 and Figure 5-15 shows the comparison between the results generated using the analytical hysteretic model and the experimental tests at a PGA of 1 g and 2 g respectively. The analytical material model provided a similar response to the results produced during the pseudo-dynamic experiments for both the 1 g and 2 g. For the 1 g experiment, the analytical hysteretic model produces a single loop in the negative direction before drifting off in the positive direction and failing. The test that was undertaken at a PGA of 2 g resulted in a similar hysteretic curve being formed between the analytical hysteretic model and the laboratory test, with only a linear-elastic displacement of the footing in the negative direction before drifting off to the right and failing.

The comparison between the experimental test results and the results produced using the analytical hysteretic model indicate that the formulated analytical hysteretic model provides a relatively accurate means to predict damage at peak ground accelerations and structural fundamental periods of vibration that were not undertaken during the laboratory experiments. Cognisance should be taken that the analytical hysteretic model was produced using the limited set of results for the given reinforced concrete footing design. However, the model provides a better method to interpolate the damage sustained by the given reinforced concrete footing at various peak ground accelerations and overall structural fundamental periods of vibration.

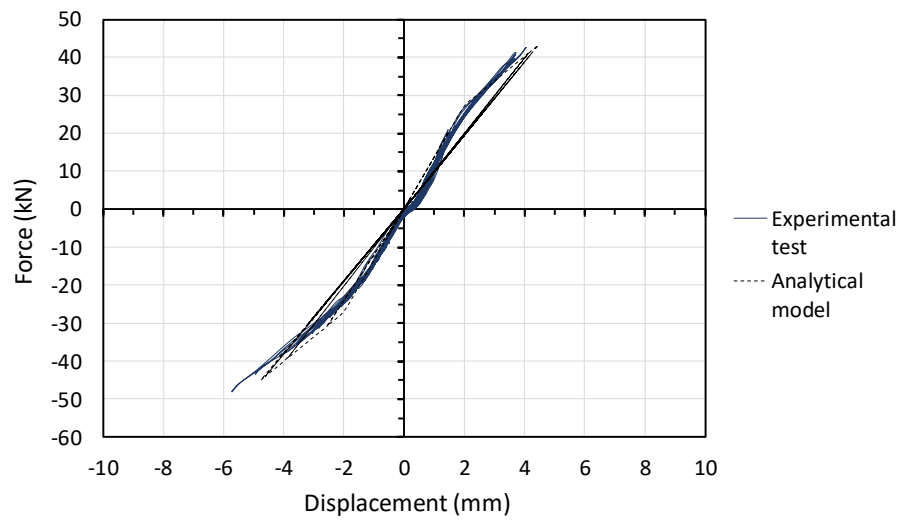


Figure 5-11 Analytical hysteretic model comparison with experimental results at a PGA of 0.34 g

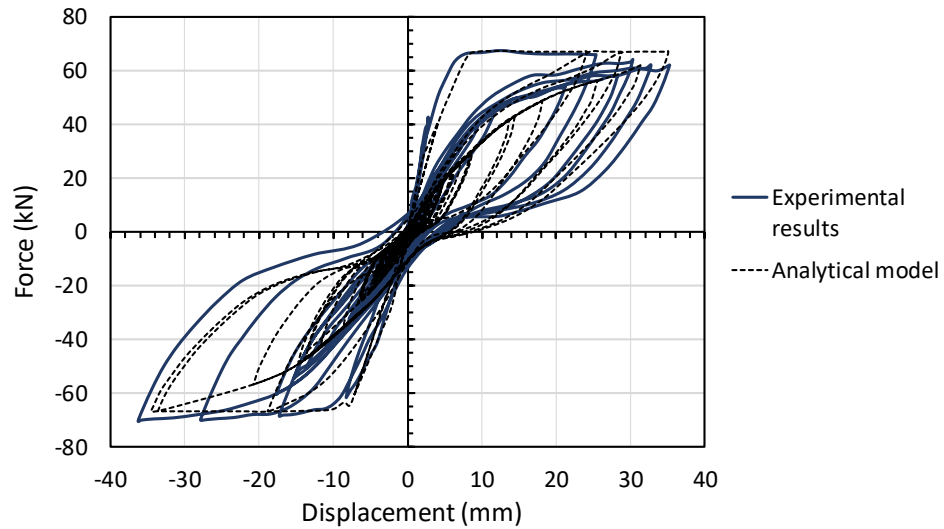


Figure 5-12 Analytical hysteretic model comparison with experimental results at a PGA of 0.68 g

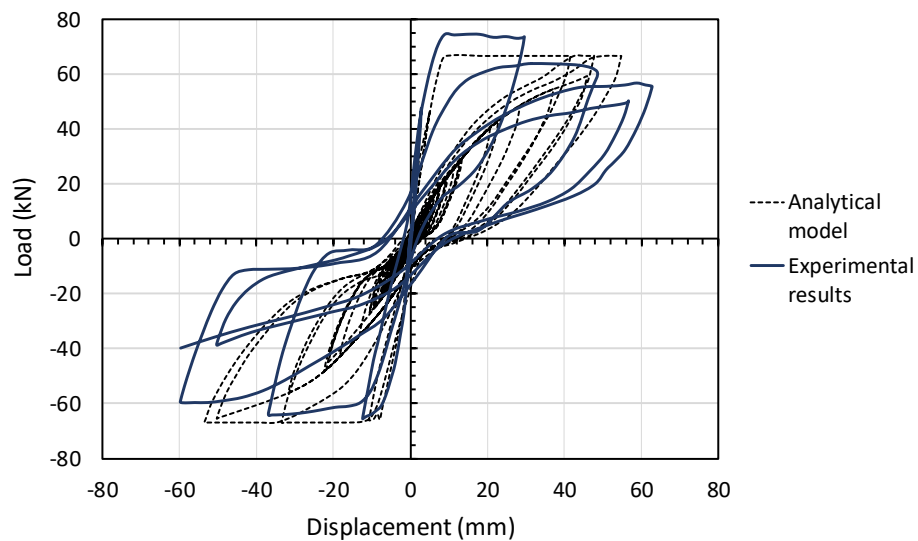


Figure 5-13 Analytical hysteretic model comparison with experimental results at a PGA of 0.78 g

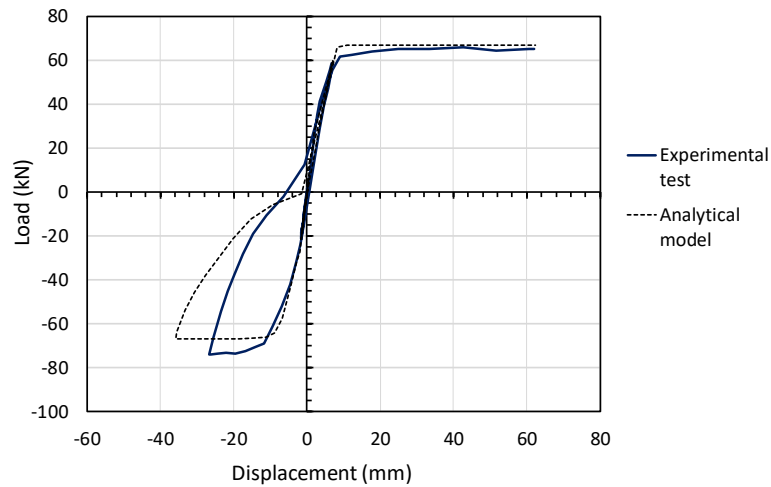


Figure 5-14 Analytical hysteretic model comparison with experimental results at a PGA of 1 g

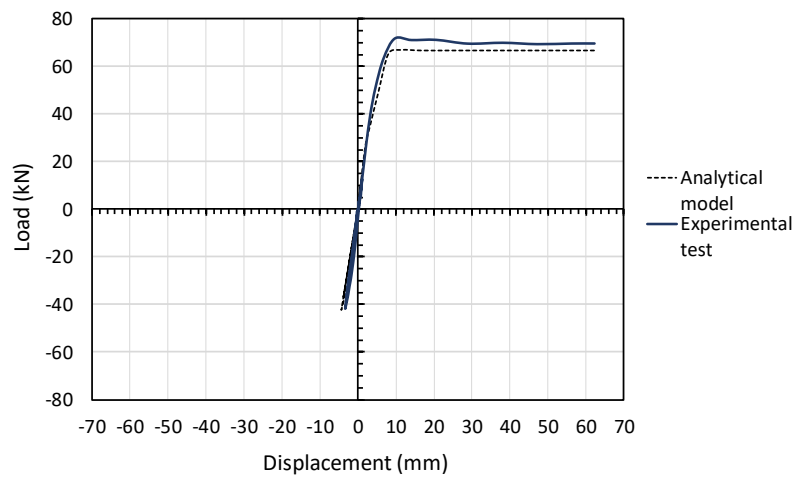


Figure 5-15 Analytical hysteretic model comparison with experimental results at a PGA of 2 g

5.3 DAMAGE AND FRAGILITY ANALYSIS

The numerical model was run using the analytical hysteretic model that was developed in the previous section over a range of scaled peak ground accelerations using the El Centro ground motion record to evaluate the extent of damage sustained by the reinforced concrete footing. The same frame structure that was used during the pseudo-dynamic experiments, with a fundamental period of vibration of 0.86 s was used to run the computational model with the developed analytical hysteretic model for the reinforced concrete footing. The conversion between PGA and intensity was done using Equation 2.1 in Section 2.1.1, which was developed by Ambraseys (1974).

A damage index was formulated using the Park and Ang (1985) damage model by utilising the results produced during the pseudo-dynamic experiments and the results produced using the analytical hysteretic model. Equation 4.1 shows the Park and Ang (1985) damage index formula for brevity and the values used within the formula for each of the analytical hysteretic model analysis. The values used within the Park and Ang damage model were obtained from the cyclic load tests and the pseudo-dynamic tests, which were summarised in Table 4-3. The maximum shear capacity of the footing was determined by averaging the absolute values of the maximum shear values obtained from both the negative and positive displacements from each of the pseudo-dynamic tests and cyclic load tests. The damage index ranges from 0 to 1, with 0 indicating an undamaged structure and 1 indicating a complete collapse of the structure.

$$DI = \frac{d_m}{d_u} + \frac{\beta}{V_y d_u} \int dE_h \leq 1 \quad (4.1)$$

With:

$$\begin{aligned} d_u &= 0.062 \text{ m} && \text{(maximum displacement before collapse, Table 4-3)} \\ V_y &= 66775 \text{ N} && \text{(shear capacity at yielding averaged from the experiments)} \\ \beta &= 0.1 && \text{(equation 2.27 in section 2.8.1 for the analysed footing)} \\ E_h &= \text{Hysteretic energy absorbed by the reinforced concrete footing} \end{aligned}$$

The value of β represents the effect that cyclic loading has on the cumulative structural damage incurred by the reinforced concrete footing for the duration of the earthquake record. The value of β was calculated using the recommended formula provided by Park and Ang (1985) using the characteristics of the reinforced concrete used during the research.

Figure 5-16 and Figure 5-17 shows the five damage index values determined from the pseudo-dynamic experiments superimposed on the damage index values produced using the analytical hysteretic model that was used to run the analysis over a range of peak ground accelerations. Figure 5-16 is given in terms of PGA and Figure 5-17 is given in terms of earthquake intensity. The damage index values calculated from the numerical hysteretic model for each of the five tested specimens compares well with the calculated values determined from the pseudo-dynamic experiments. By running the analysis over a range of peak ground accelerations, two distinct points can be observed where there is a change in the rate of damage with increasing peak ground acceleration. The first point occurs at a damage index of 0.03, which indicates a

point of softening of the reinforced concrete footing and relates to the onset of concrete cracking. The second point occurs at a damage index of 0.13 and corresponds to the point at which the reinforcement starts to yield. The first two points are pronounced and consistent with the cracking and the yielding of the reinforcement.

A third point can be characterised due to concrete spalling, which is directly related to the number of cycles of vibration and the resultant reduction in shear capacity. From the results and observations made during the pseudo-dynamic tests, the third point occurs at a damage index of approximately 0.4, which correlates with the value produced in literature. A loss of shear capacity with increased cycles of vibration was observed to occur at a damage index exceeding 0.4 during the pseudo-dynamic experiments undertaken at peak ground accelerations of 0.68 g and 0.78 g.

Comparing the damage index values with the Modified Mercalli intensity scale, as previously shown in Table 2-1, indicates that there is a correlation between the observed damage to the footings and the descriptions given by the Modified Mercalli Intensity scale. At an intensity of between VIII and IX, the damage to the reinforced concrete footing varies between being slightly damaged to considerably damaged in ordinary buildings. At an intensity of IX, the footings had failed, which corresponds with the description given in Table 2-1 that indicates buildings having shifted off of their foundations.

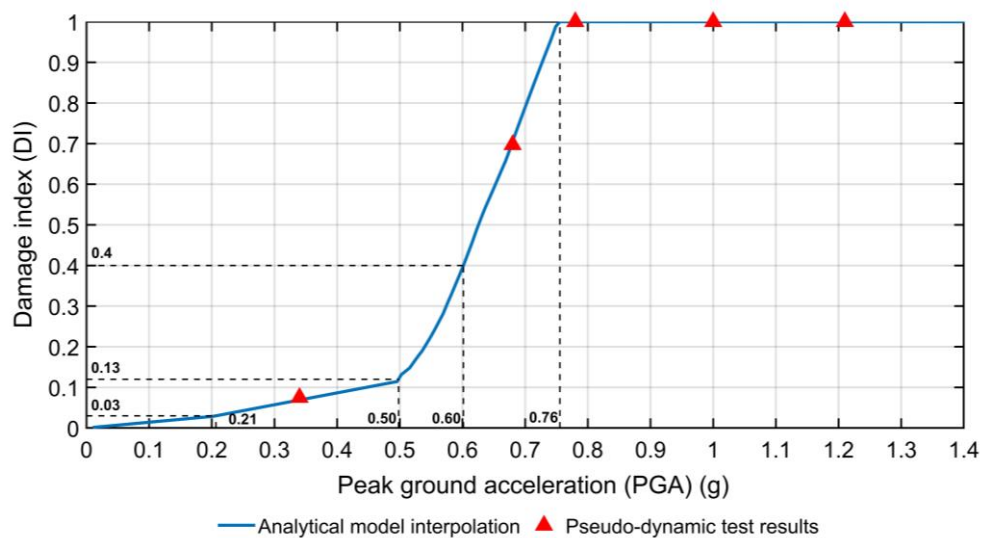


Figure 5-16 PGA vs Damage Index of a reinforced concrete footing using the derived analytical hysteretic model and the laboratory results with 5% structural damping and a linear structural natural period of vibration of 0.86 s

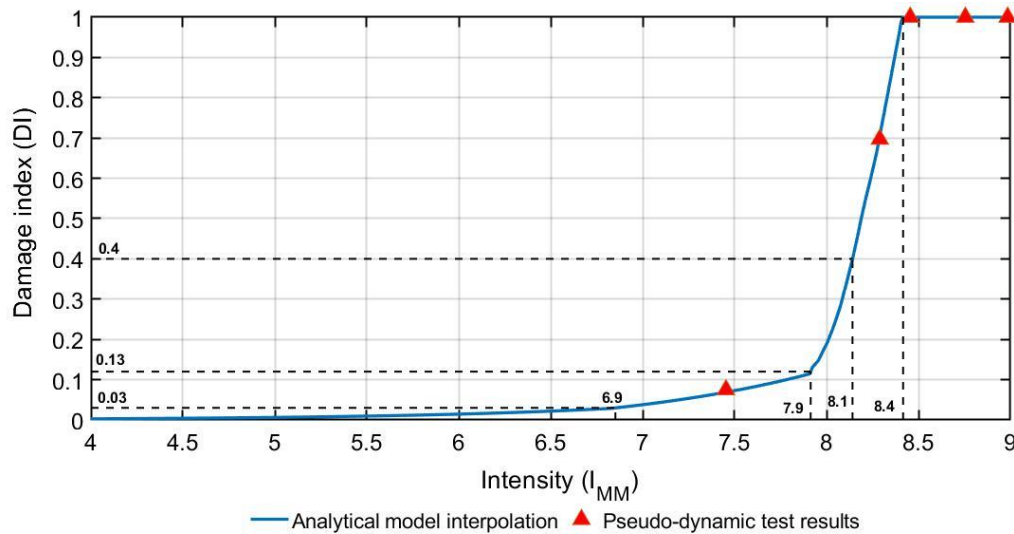


Figure 5-17 Earthquake Intensity vs Damage Index of a reinforced concrete footing using the derived analytical hysteretic model and the laboratory results with 5% structural damping and a linear structural natural period of vibration of 0.86 s

Figure 5-18 compares the influence that the structural damping ratio has on the damage incurred by the reinforced concrete footing. The same natural period of vibration of 0.86 s was used for each of the tests, and the damping ratio was varied for each of the series of tests. The series of tests comprised of analysing the structure over a range of peak ground accelerations and determining the extent of damage at each of the peak ground accelerations. Figure 5-18 shows that the damping ratio is independent of the damage index value due to concrete cracking and reinforcement yielding. However, the damping ratio has a significant influence on the amount of damage incurred by the reinforced concrete footing at any given peak ground acceleration. The damage sustained by the footing decreases with a concomitant increase in the damping ratio used in the overall frame structure. The large difference in damage incurred by the reinforced concrete footing at the different damping ratios is most likely due to the reinforced concrete footing only forming a small component within the overall frame structure and therefore does not consider the extent of damage sustained by the elements that comprise the overall superstructure. Therefore, the distribution of damage within the overall structure has a significant influence on the damage to the footing.

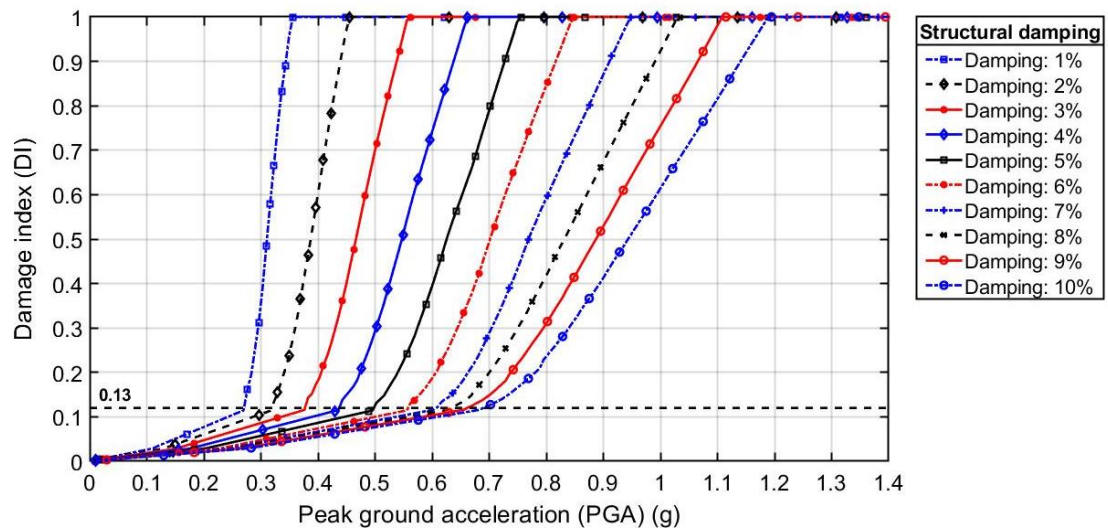


Figure 5-18 Earthquake Intensity vs Damage Index of a reinforced concrete footing at various damping ratios with linear structural natural period of vibration of 0.86 s

5.3.1 DAMAGE STATES

Five categories of damage were deduced from the results and observations made during the experimental tests and the numerical analysis run using the analytical hysteretic model. The first three damage states were obtained from plotting the damage index with increasing intensity and identifying the points at which the rate of damage increased with increasing earthquake intensity. The fourth damage state was obtained from visual observations made during the pseudo-dynamic tests and from literature. The damage states are as follows:

- 1) Undamaged (Damage Index < 0.03): The maximum load applied to the footing does not exceed the tensile capacity of the concrete and therefore cracking of the concrete does not occur.
- 2) Minor damage ($0.03 < \text{Damage Index} \leq 0.13$): Formation of cracks in the concrete and opening of existing cracks. Occurs when the maximum deformation does not exceed the yield displacement of the footing. The maximum shear load capacity of the footing has not been reached. Basic visual inspection advised with limited and isolated remedial work.
- 3) Moderate damage ($0.13 < \text{Damage Index} \leq 0.4$): Occurs when the reinforcement yields, which results in a significant loss of stiffness within the footing and the subsequent increase in the overall displacement of the structure. The loss of concrete cover is not observed, and the reinforcement has not become exposed. The footing can still carry the axial load from the structure above and therefore allows occupants time to evacuate. Repair and retrofitting

of the foundations will be required post-earthquake to ensure that the structure conforms to relevant design codes of practice.

- 4) Extensive damage ($0.40 < \text{Damage Index} < 1$): Large lateral displacement of the footing resulting in the visible crushing of the concrete and the loss of concrete cover to the reinforcement. Buckling of the exposed reinforcement and significant reduction in the load carrying capacity of the footing. The structure is still standing but is at risk of collapse and not safe for occupation. The structure is subsequently condemned, requiring demolition.
- 5) Collapse (Damage Index = 1): The capacity of the footing has been reduced significantly resulting in the applied load exceeding the carrying capacity of the footing and the subsequent collapse of the structure.

5.3.2 DAMAGE CURVES

The analytical hysteretic model was used in conjunction with the pseudo-dynamic algorithm, as shown in Figure 3-27, to determine damage curves and fragility curves for the various damage states described in Section 5.3.1 for the minimally reinforced concrete footing. The response of the footing was analysed over a range of earthquake intensities and fundamental periods of vibration using the scaled El Centro ground motion record and the formulated hysteretic model.

The majority of the building classes, specifically low to medium rise buildings, are placed on reinforced concrete footings, and one of the main structural characteristics that separate the response of different buildings is the fundamental period of vibration of the structure. Many seismic codes distinguish the extent of seismic loading on the structure by the structure's resultant fundamental period of vibration in the form of response spectra, and therefore the damage curves were produced in terms of the overall structural fundamental period of vibration.

Utilising the same frame structure that was used during the pseudo-dynamic experiments, the stiffness of the column and beam elements were increased by changing the moment inertia of each of the members by a constant value to ensure a similar stiffness distribution within the frame. Figure 5-19 shows the damage contour plot in terms of peak ground acceleration (PGA) and the fundamental period of vibration of the structure and shows that the fundamental period of vibration of the overall structure has a significant influence on the damage incurred by the reinforced concrete footing. The reinforced concrete footing that will incur the most damage at lower earthquake intensities are those that have an overall structural fundamental period of vibration in the order of 0.53 s.

Figure 5-20 shows the damage contour plot between earthquake intensity and overall structural fundamental period of vibration. The variation in damage defined at each of the Modified Mercalli intensity values is most likely due to structures with different fundamental periods of vibration experiencing different levels of damage for a given peak ground acceleration. At a Modified Mercalli intensity of VIII, the extent of damage to the structure varies between being slight to considerable. Structures with very low and very high fundamental periods of vibration will incur slight to moderate damage at an intensity of VIII when subjected to the scaled El Centro earthquake record. However, structures with a fundamental period in the order of 0.53 s will suffer considerable damage and even collapse, which indicates that the Modified Mercalli intensity scale can be subjective and dependent on the type of structure, and the mass and stiffness distribution within the structure.

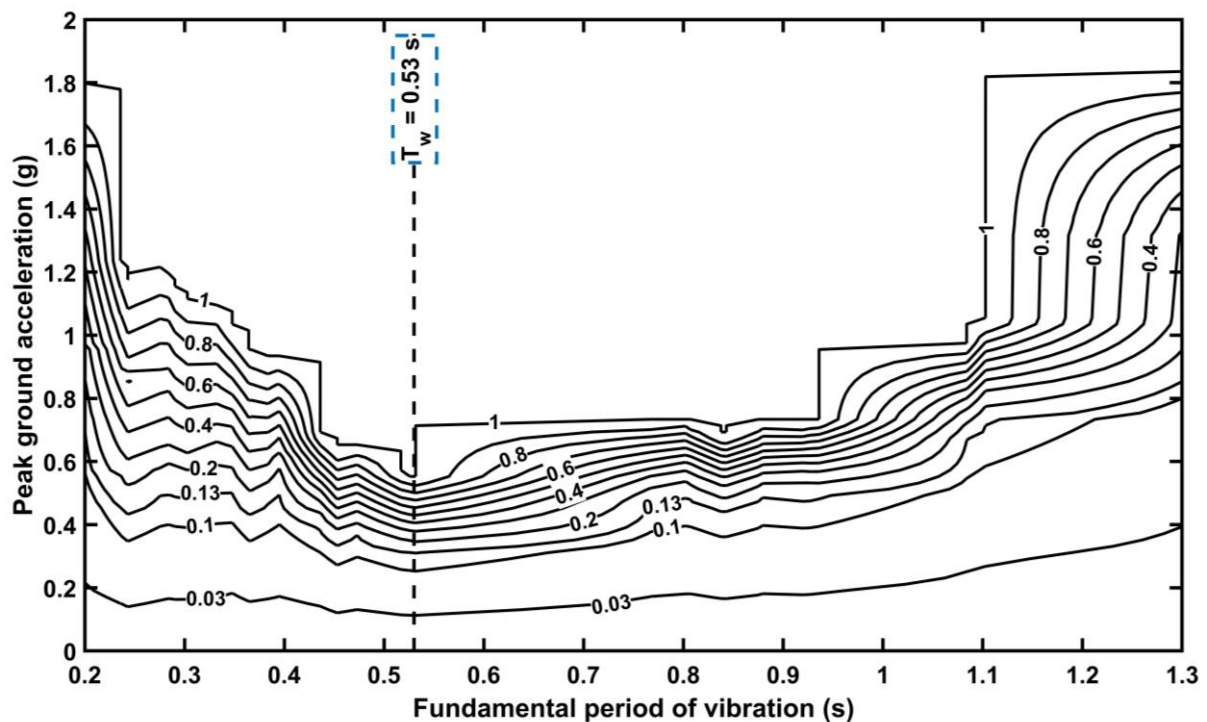


Figure 5-19 Damage contour plot in terms of peak ground acceleration at 5% damping

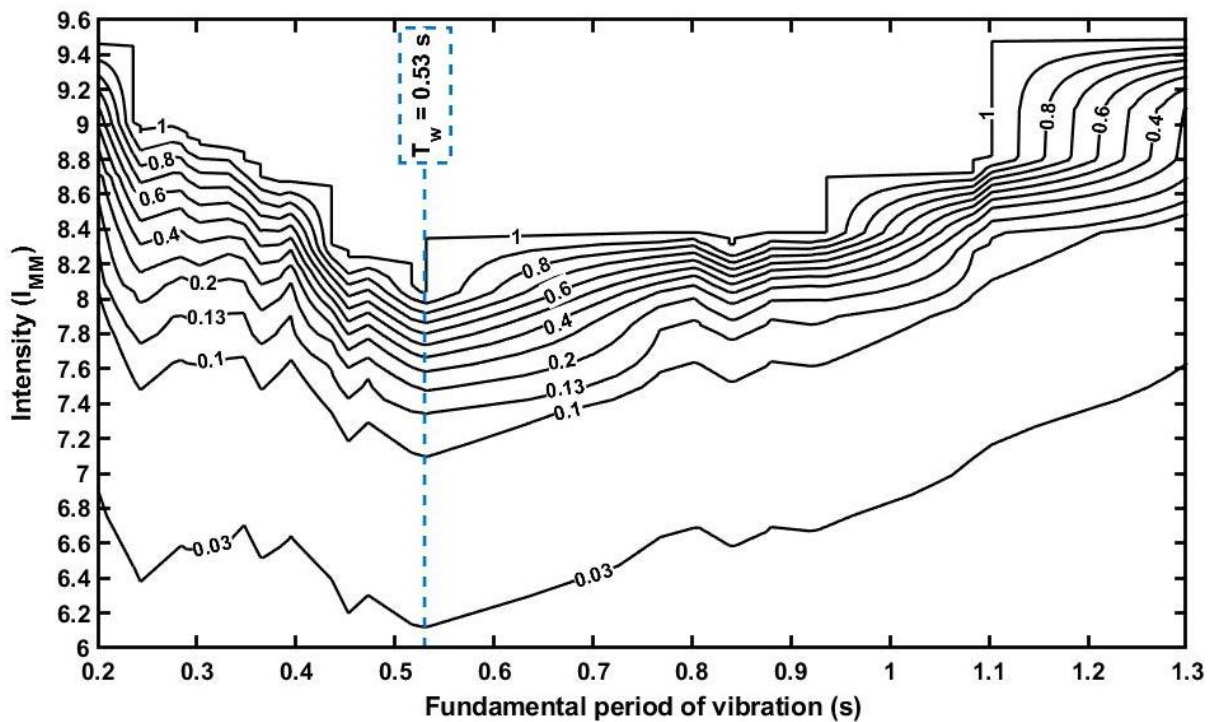


Figure 5-20 Damage contour plot in terms of intensity at 5% damping

Cognisance should be given to the fact that the analysis that was undertaken during this research only considered a single reinforced concrete foundation design and a single design for the moment resisting frame structure that was analysed using a single earthquake ground motion record. Therefore, the contour plot of damage could substantially differ in Figure 5-19 and Figure 5-20 if a different earthquake ground motion record was used or a different combination of structural steel members was used. Therefore, when predicting the extent of damage at different earthquake intensities, it would be conservative to assume a damage curve plotted for the worst-case fundamental period of vibration. In this case, under the analysed conditions, the fundamental period of vibration of 0.53 s produced the worst-case damage curve and is shown in Figure 5-21 in terms of PGA and Figure 5-22 in terms of intensity. Under the worse-case fundamental period of vibration for the reinforced concrete footing analysed using the El Centro ground motion record, moderate damage can be expected in reinforced concrete structures that are subjected to earthquake intensities greater than 0.31 g. Extensive damage can be expected in reinforced concrete structures subjected to earthquake intensities exceeding 0.41 g, thus indicating that some footings will require demolition and therefore a total loss of the asset. Earthquakes producing intensities exceeding 0.55 g will result in the total collapse of footings and therefore collapse of the overall structure.

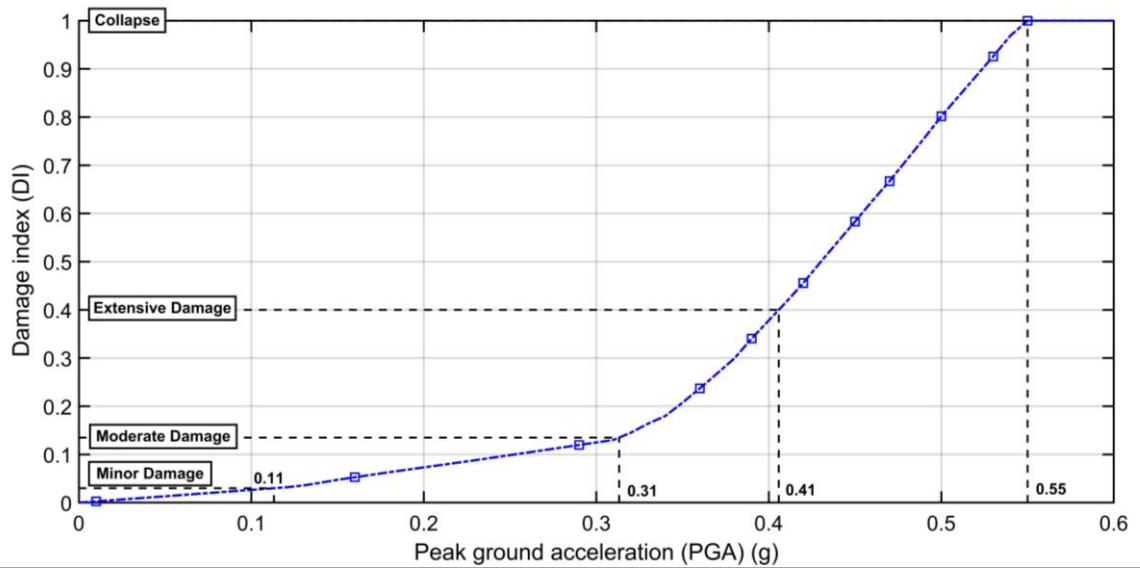


Figure 5-21 Worst case damage curve for the analysed nominally reinforced concrete footing in terms of PGA

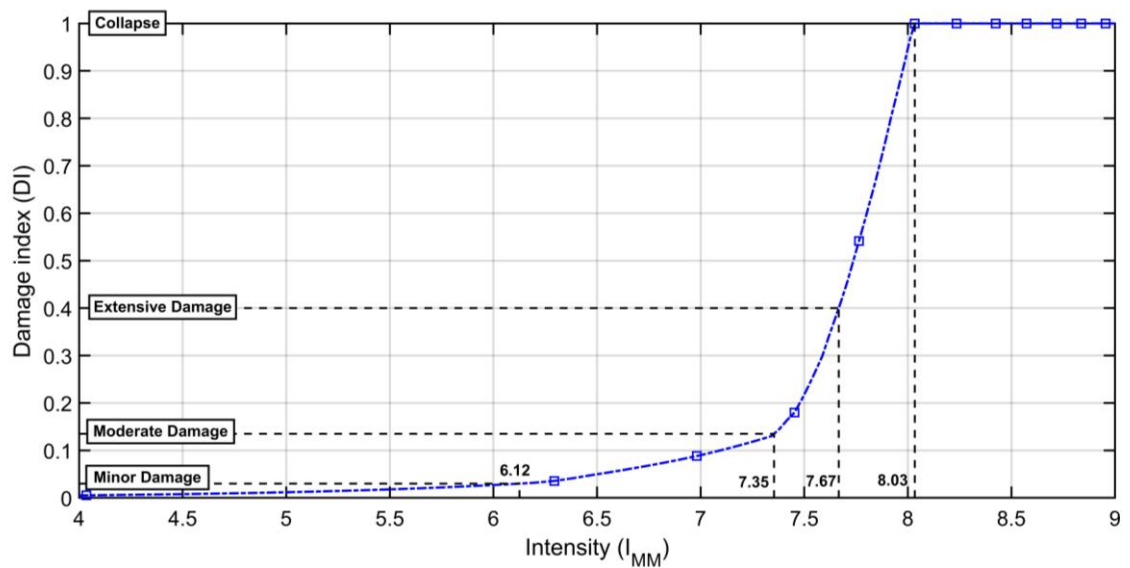


Figure 5-22 Worst case damage curve for the analysed nominally reinforced concrete footing in terms of intensity

5.3.3 FRAGILITY CURVES

A method of producing fragility curves is proposed using the formulated damage contour plots produced in the previous section. The fragility curves are calculated for each damage state over a range of fundamental periods of vibration that are deemed to be representative of low-rise structures from the building period formulas specified in SANS 10160-4:2017. The fragility curves were developed by considering the following assumptions and limitations:

- The fragility curves were determined using the procedure given in FEMA P-58-1 (2012) that results in a lognormal distribution from the demand data obtained from results produced by running the pseudo-dynamic algorithm using the formulated analytical hysteretic model;
- The range of fundamental periods of vibration used to develop the fragility curves were determined by using the building period formulas given by Equations 2.15 and Equation 2.16 for steel and reinforced concrete moment resisting frame structures. The lowest fundamental period of vibration and the maximum period of vibration for one storey to three storey high structures, which range between 4 m to 12 m in height, was used to develop the fragility curves;
- A continuous uniform distribution of fundamental periods of vibration was determined as shown in Figure 5-23. Therefore, the probability of each period occurring with the range of fundamental period of vibrations is equal. The exact distribution of structures that could reasonably be placed on the footing is unknown, and therefore a uniform distribution is assumed when calculating the median value, θ , and logarithmic standard deviation, β .

Table 5-3 shows the fundamental period of vibration of low-rise structures between one-storey and three stories and produces a minimum fundamental period of vibration of 0.21 s for a one-storey reinforced concrete moment resisting frame structure with a 4 m height, and a maximum fundamental period of vibration of 0.55 s for a three-storey steel frame moment resisting structure. Selecting structures between one storey and three stories account for the variability in fundamental periods of vibrations and compare with the distribution of fundamental periods of vibration as previously shown in Figure 2-17, which shows the fundamental period of vibration for a range of 2622 Chilean buildings. The range of fundamental periods of vibration is deemed to satisfy all strength and deformation requirements as specified in the South African structural design codes of practice.

Table 5-3 Fundamental period of vibration for 1 storey to 3 storey structures calculated using the building period formulas in SANS 10160:4-2017

	$C_T^{(1)}$	Fundamental period of vibration ⁽¹⁾ (s)		
		1 storey ⁽²⁾	2 storey ⁽²⁾	3 storey ⁽²⁾
		$h_t = 4m$	$h_t = 8m$	$h_t = 12m$
Steel frame structures (Equation 2.15)	0.085	0.24	0.40	0.55
Reinforced concrete moment resisting frame structures (Equation 2.16)	0.075	0.21	0.36	0.48

(1) Building period formula $T = C_T \times h_t^{3/4}$ from SANS 10160-4:2017

(2) A storey is taken as 4 m

Figure 5-23 shows the contour lines representing each of the damage states that are extracted from Figure 5-19 with the continuous uniform distribution used to determine the median value, θ , and logarithmic standard deviation, β . Using the equations given in Section 2.8.2 and the produced median value, θ , and standard deviation, β , for each damage state, the fragility curves were determined. Figure 5-24 shows the fragility curves for the minimally reinforced concrete footing subjected to the El Centro ground motion record for structures with periods that range between 0.21 s and 0.55 s. At a peak ground acceleration of 0.128 g or earthquake intensity of 6.3, only minor damage is expected in approximately 30% of low-rise structure's reinforced concrete footings with a period in the range of 0.21 s to 0.55 s when subjected to the El Centro ground motion record.

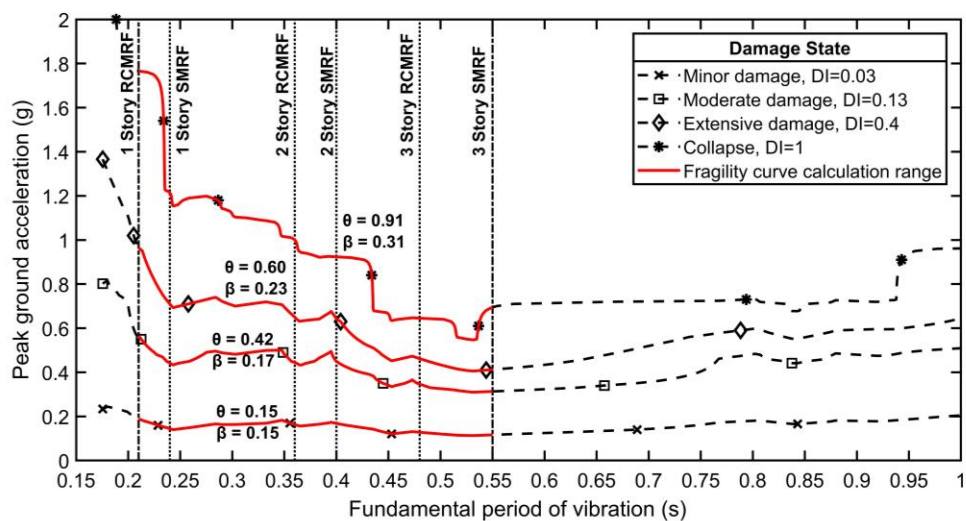
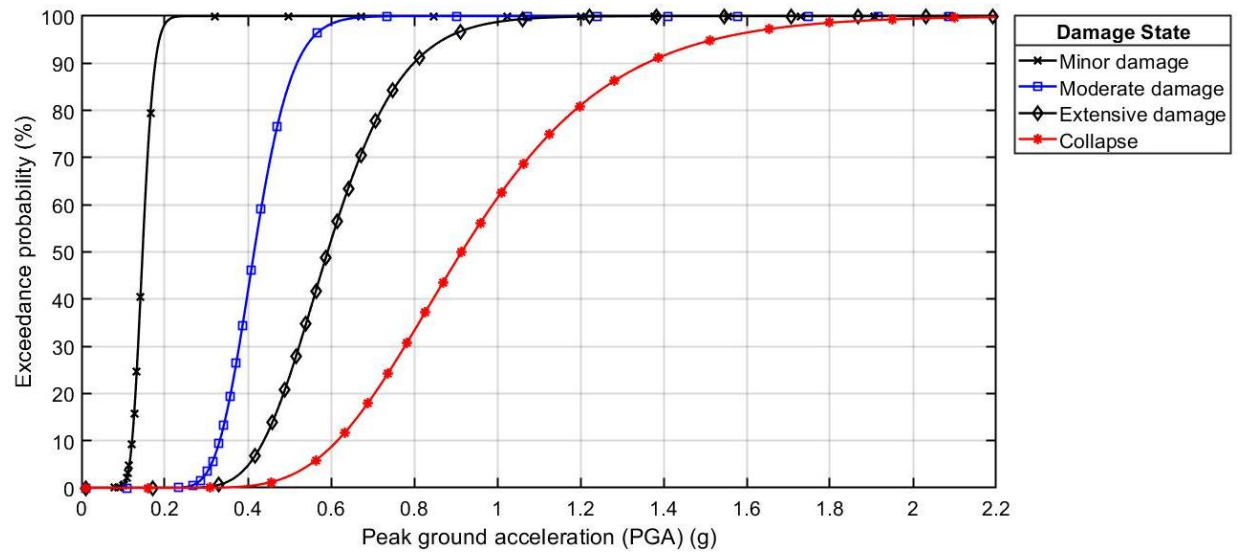
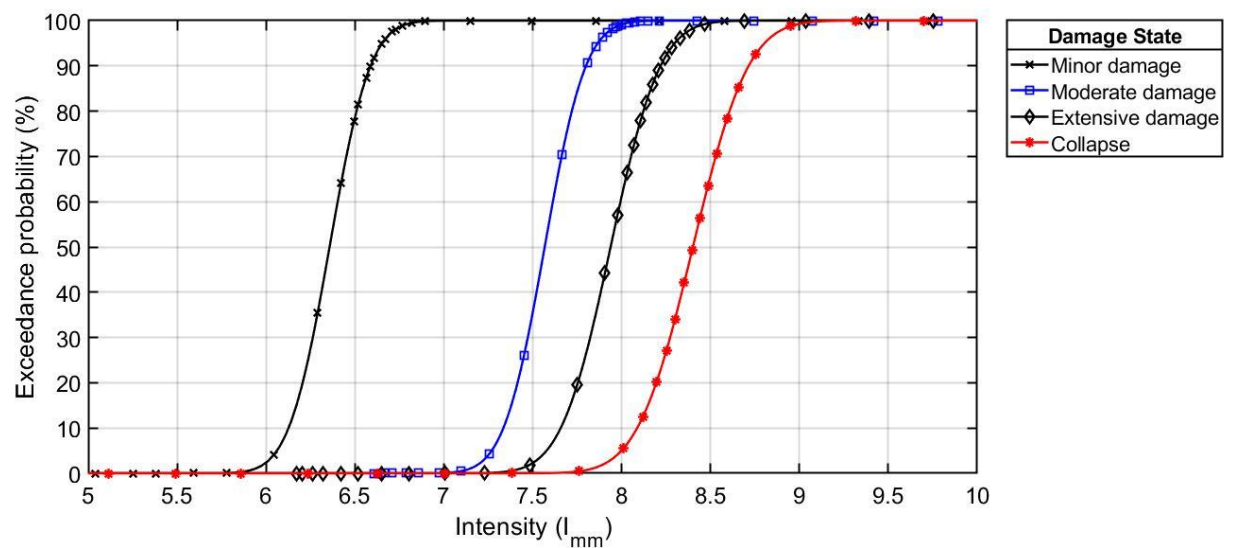


Figure 5-23 Continuous range for each of the damage states to formulate the fragility curves



(a)



(b)

Figure 5-24 Fragility curves for minimally reinforced concrete footing of fundamental period range of 0.21 s to 0.55 s

5.4 CONCLUSION

This chapter described a method to approximate the damage curves and fragility curves for a structural component by utilising the results produced during the pseudo-dynamic tests and cyclic load tests. To interpolate damage at PGAs that were not undertaken during the laboratory tests, an analytical hysteretic model was developed to replace the physical test setup in the laboratory that was used to perform the pseudo-dynamic tests. Running the analysis over a range of peak ground accelerations and overall structural fundamental periods of vibration showed that the rate of damage increased at distinct points, which were independent of the damping ratio used for the frame structure. Damage states were formulated using the results produced during the analysis using the analytical hysteretic model and the results and observations made during the laboratory tests.

The produced damage contour plots, which show the amount of damage as a function of the overall fundamental period of vibration and the PGA, indicated the extent of damage sustained by the reinforced concrete footing correlates with the descriptions given by the Modified Mercalli intensity scale. However, the variation in damage expected at any given intensity within the Modified Mercalli intensity scale is likely due to the influence that the fundamental period of vibration has on the extent of damage sustained by the reinforced concrete footing. The fundamental period of vibration is a characteristic that relies on the mass and stiffness distribution of the structure, the type of structure, and the composition of the structure that is placed on the footing. Therefore, knowing the fundamental period of vibration of the structure will better enable the risk of damage at various earthquake intensities to be quantified. This chapter shows that quantifying the expected damage to a structure and individual structural components is complex and dependant on several variables that define the structure.

6 CONCLUSIONS AND RECOMMENDATIONS

In the following section conclusions are made regarding the viability of using the pseudo-dynamic experimental method to relate structural damage with increasing earthquake intensity for an axially loaded reinforced concrete footing, which forms part of a two-bay, two-storey moment resisting frame structure. A total of five reinforced concrete footings were tested using the pseudo-dynamic experimentation method and two reinforced concrete footings were cyclic load tested. The results produced during the experiments were used to formulate an analytical hysteretic model to interpolate damage at peak ground accelerations and overall structural fundamental periods of vibration that were not undertaken during the pseudo-dynamic experiments. The results produced during the pseudo-dynamic tests and those produced using the analytical hysteretic model aided in the development of a damage index and the formulation of damage curves and fragility curves for the analysed reinforced concrete footing. Furthermore, this section provides recommendations for further work.

6.1 CONCLUSIONS FROM THE STUDY

The pseudo-dynamic experimentation method provides a viable approach to correlate the damage that has occurred to a physical model of a reinforced concrete footing, which forms part of an overall linear elastic structure, with increasing earthquake intensity. The use of the implicit Newmark's method provides a stable and accurate algorithm to quantify the damaged incurred to the reinforced concrete footing due to the overall response of the structure that has been subjected to an applied seismic excitation. The initial stiffness that is used within in the implicit Newmark's algorithm must be greater than the maximum achievable stiffness of the specimen being tested to ensure the stability of the analysis and to prevent premature damage.

By using the pseudo-dynamic method, the hysteretic response of the footings could be related to earthquake intensity and observations could be made on the extent of damage incurred by the footing due to the applied earthquake loading. The time history of energy terms and hysteretic energy dissipation characteristics could be determined for each time increment during the application of the scaled El Centro earthquake record, which enabled the amount of energy dissipated by the footing to be calculated as a percentage of the total energy imparted to the structure. The reinforced concrete footing only dissipates a small percentage of the overall energy imparted to the structure in the form of hysteretic energy, with the remainder of the energy being dissipated due to damping within the frame structure. The pseudo-dynamic method enables the determination of the energy dissipation potential for individual structural

components during an applied earthquake record to the overall structure, which is comprised of many structural elements.

With an increase in the hysteretic energy dissipated by the reinforced concrete footing there is an associated increase in the observed damage to the footing in the form of concrete cracking, reinforcement yielding and spalling of the concrete. The repeated cyclic loading at deformations exceeding the yield strength of the reinforcement but not exceeding the failure deformation results in the largest quantity of hysteretic energy being absorbed. At large PGAs, the hysteretic energy does not give an indication of whether the footing has failed, therefore demonstrating the applicability of the Park and Ang damage index that consists of two terms that depends on the maximum deformation of the structure and the cumulative hysteretic energy. At large PGAs the structure tends to fail due to excessive deformation before undergoing cyclic loading and at lower PGAs the absorbed hysteretic energy tends to govern the damage and failure of the footing due to the larger number of cycles of vibration.

The research showed that by using the pseudo-dynamic method to analyse a single component of a much larger structure, which is assumed to be linear elastic for the duration of the analysis, does have its limitations. The capacity of structural members and connections within the overall frame structure is not accounted for during the analysis due to the linear elastic assumption, and therefore, the formation of plastic hinges and the resulted loss of stiffness within the overall structure is not considered in the response of the reinforced concrete footing.

Five damage states can be deduced for the analysed reinforced concrete footing, with the damaged states being linked to the material characteristics of the concrete and reinforcement. The extent of damage to the reinforced concrete footing is governed predominately by the yield strength of the reinforcement, which results in the rate of damage increasing substantially once the reinforcement had yielded. The mechanisms that resulted in the failure of the reinforced concrete footing due to increasing earthquake intensity are as follows:

- The shear capacity of the reinforced concrete footing was reached soon after the reinforcement yielded, and large cracks had formed at the interface between the base of the column and the top of the concrete base slab. Upon reaching the maximum shear strength, the stiffness of the footing approached zero with a subsequent increase in the horizontal displacement at the top of the reinforced concrete footing;
- Spalling of the concrete is predominately governed due to the buckling of the yielded and elongated reinforcement upon load reversal from tension to compression;

- The pinching effect was observed in the pseudo-dynamic and cyclic load tests and is due to the presence of the axial load. With a reduction in the horizontal load, the vertical load overcomes the overturning moment produced by the horizontal shear force and attempts to stabilise the member by returning the footing back to its original vertical position. Upon load reversal the plastically elongated reinforcing steel is first mobilised in compression by the axial load prior to the closure of the crack resulting in a reduction in the lateral displacement that is independent of the applied horizontal load;
- An increase in the number of cycles of vibration results in an increase in the degree of concrete spalling, which governs the failure of the footing at lower earthquake intensities;
- Significant spalling of the concrete resulted in the reduction and loss of axial capacity and shear capacity of the footing, which ultimately resulted in the failure of the reinforced concrete footing's column; and
- A reduction in shear capacity was not observed at tests conducted at large earthquake intensities and at large deformations. The fracturing of the reinforcement and loss of stability of the structure governed the failure of the test specimen at large PGAs.

Although pseudo-dynamic testing between computer modelling and laboratory testing provides an alternative and more cost-effective solution to full scale testing, the cost and time required to produce a single result under a single set of structural conditions limits the amount of data that can be produced to formulate complete set of damage curves and fragility curves. Therefore, to develop damage curves and fragility curves, an analytical hysteretic plastic hinge model had to be formulated for the analysed footing from the results produced during the pseudo-dynamic tests and cyclic load tests to interpolate damage at PGAs and overall structural fundamental periods of vibration that were not undertaken during the laboratory tests.

By performing the pseudo-dynamic tests by scaling the El Centro ground motion record showed that the footing could only sustain a maximum PGA with further increase in the amplitude of the ground motion record. The pseudo-dynamic test that was undertaken by amplifying the El Centro ground motion record to a PGA of 2 g, only managed to achieve a maximum PGA of 1.21 g before failure. This observation was further clarified by the damage contour plots and curves that were produced during the computational analysis using the analytical hysteretic model derived from the cyclic load tests and pseudo-dynamic tests. The analysis shows that with increasing amplitude of the ground motion record, the reinforced concrete footing is only able to endure a maximum peak ground acceleration before failure. The level of damage

incurred by the reinforced concrete footing at a given earthquake intensity is dependent on the fundamental period of vibration of the structure and the frequency characteristics of the earthquake record to which the structure is subjected. The damage contour plot, in terms of earthquake intensity and fundamental period of vibration tends to follow the profile of the linear response spectrum that is produced using the same earthquake record.

The investigation shows that the resultant damage to the reinforced concrete footing is complex and it depends on several characteristics that relates to the overall super structure. The structural configuration, the distributed loading within the structure and the initial stress state of the structural component all plays a critical role in the resultant damage to the reinforced concrete footing. The results obtained from this research enabled a better understanding of the performance of a reinforced concrete footing that accounts for the overall response of a moment resisting frame structure subjected to an earthquake excitation. The research shows how the damage to the reinforced concrete footing manifests at different peak ground accelerations when subjected to the El Centro earthquake excitations and can be used to better understand the material response of reinforced concrete and improve the selection of PGAs for future pseudo-dynamic tests.

6.2 RECOMMENDATIONS FOR FURTHER WORK

Cognisance should be given to the fact that the analysis undertaken during this research only considered a single reinforced concrete foundation design that was analysed using a single earthquake ground motion record and the overall superstructure remained linear elastic for the duration of the earthquake record. Therefore, recommendation for future work would include the following:

- Analysing the structure by varying the longitudinal and shear reinforcement to determine the influence it has on the damage to the footing;
- Perform pseudo-dynamic tests on reinforced concrete footings by incorporating non-linear behaviour within the overall structure and determining the influence that the reduction in stiffness within the overall frame structure has on the performance of the footing;
- Investigating reinforced concrete footings that are designed to the seismic structural design code of practice (SANS 10160-4:2017) and comparing their capacity with results produced in this research. Testing reinforced concrete footings with closed seismic stirrups and comparing the capacity with the results produced using traditional stirrups;

- Performing pseudo-dynamic experiments that account for both a varying shear and axial loading as could be experienced by foundations placed on the exterior of a building;
- Soil structure interaction is one of the most important factors that influences the overall response of the foundation system. Pseudo-dynamic experiments can be performed by embedding footings within soil to determine the response and resultant damage to the reinforced concrete footing at various earthquake intensities;
- Developing a hysteretic model from a series of cyclic load tests that can account for a variation in both horizontal and vertical load for the duration of the earthquake record for a range of reinforced concrete footing designs will be beneficial in evaluating the damage encountered to number of different structural types;
- Account for several elements within the structure during a pseudo-dynamic experiment to investigate the progressive failure of the structure and determine the contribution of each element to the overall energy dissipation and fragility of the structure;
- Investigation can be done to determine the distribution of fundamental periods of vibration of all structures that form part of the insured portfolio as this will improve the formulation of the fragility curves;
- Investigation into improving the experimental test setup to reduce external noise experienced during the pseudo-dynamic experiments. Base isolation of the experimental test setup is recommended;
- Performing pseudo-dynamic experiments that enables the analysis of the reinforced concrete footing under biaxial bending and shear; and
- Use hybrid testing methods combined with machine learning algorithms to analyse a large sample of footings or reinforced concrete members by varying all significant parameters to develop a hysteretic model that can be used during earthquake loading and cyclic loading. The model can be developed to account for cyclic behaviour, degradation in strength due to reinforcement buckling and spalling, and varying axial loads on the section. The material can be further developed to account for variability in material properties and reinforcement configuration. Such a hysteretic model can be programmed into finite element analyse software packages to evaluate the progression of damage under different earthquake loadings for different structural configurations.

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APPENDIX A

PSEUDO-DYNAMIC EXPERIMENTATION SCRIPT

```

1 Sub Main
2 'EA_Job.Start (Job1)
3
4 'Activating plots in the panel and setting them to zero
5 EA_Graph.RemovePlot (Panel1, "Graph1", 1)
6 EA_Graph.Refresh (Panel1, "Graph1")
7 EA_Graph.RemovePlot (Panel1, "Graph2", 1)
8 EA_Graph.Refresh (Panel1, "Graph2")
9 EA_Graph.RemovePlot (Panel1, "Energy_plot", 1)
10 EA_Graph.Refresh (Panel1, "Energy_plot")
11
12 EA_Graph.RemovePlot (Panel2, "Strain_out_disp", 1)
13 EA_Graph.Refresh (Panel2, "Strain_out_disp")
14 EA_Graph.RemovePlot (Panel2, "Disp_read_LVDT", 1)
15 EA_Graph.Refresh (Panel2, "Disp_read_LVDT")
16
17 EA_Panel.SetValue (Panel1, "Increment_out", 0)
18 EA_Panel.SetValue (Panel1, "Time_out", 0)
19 EA_Panel.SetValue (Panel1, "PGA", 0)
20 EA_Panel.SetValue (Panel1, "Iteration_out", 0)
21 EA_Panel.SetValue (Panel1, "Convergence_out", 0)
22 EA_Panel.SetValue (Panel1, "Damage_out", 0)
23 EA_Panel.SetValue (Panel1, "DispRead_out", 0)
24 EA_Panel.SetValue (Panel1, "DispRead_in", 0)
25 EA_Panel.SetValue (Panel1, "Interal_counter", 0)
26
27 'Sets analog output to zero volts
28 EA_IO.SetAnalogOut ("PMX_1 CH 9", 1, 0, OperMode)
29
30 'Zeros input data
31
32 EA_IO.ZeroBalanceControl ("MX840_SR", 1) 'Zeros right strain gauge
33 EA_IO.ZeroBalanceControl ("MX840_SL", 1) 'Zeros left strain gauge
34
35 EA_IO.ZeroBalanceControl ("PMX_LVDT1", 1) 'Zeros LVDT1
36 EA_IO.ZeroBalanceControl ("PMX_LVDT2", 1) 'Zeros LVDT2
37 EA_IO.ZeroBalanceControl ("PMX_LVDT3", 1) 'Zeros LVDT3
38 EA_IO.ZeroBalanceControl ("PMX_LVDT4", 1) 'Zeros LVDT4
39
40 EA_IO.ZeroBalanceControl ("Displacement_Hor", 1) 'Zeros Horizontal displacement
41 EA_IO.ZeroBalanceControl ("Force_Hor", 1) 'Zeros Horizontal force
42
43 End Sub
44 Sub Axial_load
45 'Runs data logging during axial load application
46 Dim Analysis As Variant 'Input to change for the number of analysis being performed
47 Analysis=1
48
49 Dim objXls_a As Object
50 Set objXls_a = CreateObject ("Excel.Application")
51 objXls_a.Workbooks.Add
52 objXls_a.Worksheets(1).Name = "Linear results"
53 objXls_a.Workbooks(1).SaveAs
54 "C:\Octave\Dynamics\Pseudo\Pseudo_test_"+CStr(Analysis)+".xlsx" '
55 "C:\Octave\Dynamics\Pseudo\OutputDispOutput4.xls"
56
57 objXls_a.Worksheets(1).Cells(1,1).Value ="Counter"
58 objXls_a.Worksheets(1).Cells(1,2).Value ="Axial load (kN) "
59 objXls_a.Worksheets(1).Cells(1,3).Value ="Strain gauge left (micro)"
60 objXls_a.Worksheets(1).Cells(1,4).Value ="Strain gauge right (micro)"
61
62 Dim Force_axial As Double
63 Dim Strain_Left As Double
64 Dim Strain_right As Double
65
66 Dim Axial_load_applied() As Double
67 Dim Strain_Left_disp() As Double
68 Dim Strain_right_disp() As Double
69 Dim StepInc() As Double
70
71 Dim Button2 As Variant
72 Button2=0
73
74 Dim counter As Variant

```

```

72 counter=0
73
74 EA_Panel.SetCell(Panel2,"Axial_stop_table",1,1,0)
75
76 Do While Button2=0 'Convergence at each time step
77
78 EA_Panel.GetCell(Panel2,"Axial_stop_table",1,1,Button2)
79
80 ReDim Preserve StepInc(counter+1)
81 ReDim Preserve Axial_load_applied(counter+1)
82 ReDim Preserve Strain_Left_disp(counter+1)
83 ReDim Preserve Strain_right_disp(counter+1)
84
85 StepInc(counter)=CDBl(counter) 'Counter to array
86 objXls_a.Worksheets(1).Cells(counter+2,1).Value =counter
87
88 'Read out axial force from servo controller
89 EA_IO.Measure("Load cell",Force_axial,1) 'Axial force read out
90 objXls_a.Worksheets(1).Cells(counter+2,2).Value =Force_axial 'Saves axial
    force to excel spreadsheet
91 Axial_load_applied(counter)=CDBl(Force_axial) 'Saves axial force to a array
92
93 'Read out Strain gauge Left from servo controller
94 EA_IO.Measure("MX840_SL",Strain_Left,1) 'Strain gauge left read out
95 objXls_a.Worksheets(1).Cells(counter+2,3).Value =Strain_Left 'Saves strain
    gauge left to excel spreadsheet
96 Strain_Left_disp(counter)=CDBl(Strain_Left) 'Saves Strain gauge left to array
97
98 'Read out Strain gauge Right from servo controller
99 EA_IO.Measure("MX840_SR",Strain_right,1) 'Strain gauge right to read out
100 objXls_a.Worksheets(1).Cells(counter+2,4).Value =Strain_right 'Saves strain
    gauge right to excel spreadsheet
101 Strain_right_disp(counter)=CDBl(Strain_right) 'Strain gauge right to array
102
103 'Axial load out (Plotting)
104 EA_Graph.PlotArrayXY(Panel3,"Axial_load_initial",1,counter+1, StepInc(),
    Axial_load_applied())
105 EA_Graph.SetPlotProperty(Panel3,"Axial_load_initial",1,2,vbRed)
106 EA_Graph.Refresh(Panel3,"Axial_load_initial")
107
108 'Plots Strain values (Plotting)
109 EA_Graph.PlotArrayXY(Panel3,"Strain_gauge_axial",1,counter+1, StepInc(),
    Strain_right_disp())
110 EA_Graph.SetPlotProperty(Panel3,"SStrain_gauge_axial",1,2,vbBlue)
111 EA_Graph.SetPlotProperty(Panel3,"Strain_gauge_axial",1,5,0)
112
113 counter=counter+1 'Step counter
114 objXls_a.Workbooks(1).Save
115 Loop
116 objXls_a.Workbooks(1).Save
117 objXls_a.Quit
118
119 End Sub
120
121 Sub Pseudo_analysis 'Pseudo analysis algorithm
122 'Performs a single degree of freedom pseudo dynamic analysis on multi degree of
    freedom system
123
124 Dim Analysis As Variant 'Input to change for the number of analysis being performed
125 Analysis=1
126
127 'In SI Units kg, N , m, otherwise indicated
128
129 'Earthquake record time step
130 Dim dt As Variant
131 dt=0.02 'Seconds
132
133 'Analog output
134 Dim analog_out As Variant
135 analog_out=0
136 EA_IO.SetAnalogOut("PMX_1 CH 9",1,analog_out,OperMode)
137
138 'Convergence criteria

```

```

139 Dim ConvergeC As Variant
140 ConvergeC=0.0005
141
142 'If the analysis has to be restarted during the test
143 Dim Restart As Variant
144 Restart=0 'Restart = 0 (Start new analysis) 'Restart=1 (Continue existing analysis)
145 Dim time_restart As Variant
146 time_restart=0 'Last time recorded
147 Dim i_Lres As Variant
148 i_Lres=time_restart/dt
149 'Amplify record
150 Dim AeR As Variant
151 AeR=1 'Amplify the import earthquake record
152
153 Dim Load_axial As Variant
154
155 Dim objXls_u As Object
156 Set objXls_u = CreateObject("Excel.Application")
157
158 If Restart=0 Then
159 'Data files - Excel output of the data
160 objXls_u.Workbooks.Add
161 objXls_u.Worksheets.Add
162 objXls_u.Worksheets.Add
163 objXls_u.Worksheets.Add
164 objXls_u.Worksheets.Add
165 objXls_u.Worksheets.Add
166 objXls_u.Worksheets.Add
167 objXls_u.Worksheets.Add
168 objXls_u.Worksheets.Add
169 objXls_u.Worksheets.Add
170 objXls_u.Worksheets.Add
171 objXls_u.Worksheets.Add
172 objXls_u.Worksheets.Add
173 objXls_u.Worksheets.Add
174 objXls_u.Worksheets.Add
175
176 objXls_u.Worksheets(1).Name = "Input_and_loading"
177 objXls_u.Worksheets(2).Name = "Earthquake"
178 objXls_u.Worksheets(3).Name = "Displacement"
179 objXls_u.Worksheets(4).Name = "Velocity"
180 objXls_u.Worksheets(5).Name = "Acceleration"
181 objXls_u.Worksheets(6).Name = "Energy"
182 objXls_u.Worksheets(7).Name = "Stiffness"
183 objXls_u.Worksheets(8).Name = "Mass"
184 objXls_u.Worksheets(9).Name = "Local_matrices"
185 objXls_u.Worksheets(10).Name = "Spring_DF"
186 objXls_u.Worksheets(11).Name = "Force_FDOF"
187 objXls_u.Worksheets(12).Name = "Force_M"
188 objXls_u.Worksheets(13).Name = "Force_C"
189 objXls_u.Worksheets(14).Name = "It_disp"
190 objXls_u.Worksheets(15).Name = "It_force"
191
192 objXls_u.Workbooks(1).SaveAs
193 "C:\Octave\Dynamics\Pseudo\Pseudo_Test_analysis1112_"+CStr(Analysis)+".xlsx" '
194 "C:\Octave\Dynamics\Pseudo\OutputDispOutput4.xls"
195
196 ElseIf Restart=1 Then
197
198 objXls_u.Workbooks.Open
199 "C:\Octave\Dynamics\Pseudo\Pseudo_Test_analysis1112_"+CStr(Analysis)+".xlsx" '
200 "C:\Octave\Dynamics\Pseudo\OutputDispOutput4.xls"
201
202 End If
203
204 'Structural frame input
205 Dim Bays As Variant
206 Bays=2 'Number of bays
207 objXls_u.Worksheets(1).Cells(2,1).Value ="Number of bays ="
208 objXls_u.Worksheets(1).Cells(2,2).Value =Bays
209 Dim Stories As Variant
210 Stories = 2 'Number of stories
211 objXls_u.Worksheets(1).Cells(3,1).Value ="Number of stories ="

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208 objXls_u.Worksheets(1).Cells(3,2).Value =Stories
209 Dim h As Variant
210 h=4 'm (Height of the story)
211 objXls_u.Worksheets(1).Cells(4,1).Value ="Story height (h) ="
212 objXls_u.Worksheets(1).Cells(4,2).Value =h
213 objXls_u.Worksheets(1).Cells(4,3).Value ="m"
214 Dim b As Variant
215 b=6 'm (Width of a bay)
216 objXls_u.Worksheets(1).Cells(5,1).Value ="Bay width (b) ="
217 objXls_u.Worksheets(1).Cells(5,2).Value =b
218 objXls_u.Worksheets(1).Cells(5,3).Value ="m"
219
220 'Steel sections
221 Dim nmat As Variant 'Number of material properties
222 nmat= 3 'Number
223 Dim fy As Variant 'Structural steel strength
224 fy=350 'MPa
225
226 'Material 1 (Internal Columns)
227 objXls_u.Worksheets(1).Cells(7,1).Value ="Material properties"
228 objXls_u.Worksheets(1).Cells(8,1).Value ="Material 1 (Internal columns)"
229
230 objXls_u.Worksheets(1).Cells(8,2).Value ="203.00 x 203.00 x 52.00"
231 Dim E1 As Variant 'E1 of material type 1
232 E1=200*10^9 'Pa
233 objXls_u.Worksheets(1).Cells(9,1).Value ="E1 ="
234 objXls_u.Worksheets(1).Cells(9,2).Value =E1
235 objXls_u.Worksheets(1).Cells(9,3).Value ="Pa"
236
237 Dim I1 As Variant 'I1 of material type 1
238 I1=52500000/1000^4 'm^4
239 objXls_u.Worksheets(1).Cells(10,1).Value ="I1 ="
240 objXls_u.Worksheets(1).Cells(10,2).Value =I1
241 objXls_u.Worksheets(1).Cells(10,3).Value ="m^4"
242
243 Dim A1 As Variant 'A1 of material type 1
244 A1=6640/1000^2 'm^2
245 objXls_u.Worksheets(1).Cells(11,1).Value ="A1 ="
246 objXls_u.Worksheets(1).Cells(11,2).Value =A1
247 objXls_u.Worksheets(1).Cells(11,3).Value ="m^2"
248
249 Dim Den1 As Variant 'Den1 of material type 1
250 Den1=7850 'kg/m^3
251 objXls_u.Worksheets(1).Cells(12,1).Value ="Material density ="
252 objXls_u.Worksheets(1).Cells(12,2).Value =Den1
253 objXls_u.Worksheets(1).Cells(12,3).Value ="kg/m^3"
254
255 'Material 2 (Beams)
256 objXls_u.Worksheets(1).Cells(8,5).Value ="Material 2 (Beams)"
257 objXls_u.Worksheets(1).Cells(8,6).Value ="533.00 x 210.00 x 101.00"
258 Dim E2 As Variant 'E2 of material type 2
259 E2=200*10^9 'Pa
260 objXls_u.Worksheets(1).Cells(9,5).Value ="E2 ="
261 objXls_u.Worksheets(1).Cells(9,6).Value =E2
262 objXls_u.Worksheets(1).Cells(9,7).Value ="Pa"
263
264 Dim I2 As Variant 'I2 of material type 2
265 I2=616000000/1000^4 'm^4
266 objXls_u.Worksheets(1).Cells(10,5).Value ="I2 ="
267 objXls_u.Worksheets(1).Cells(10,6).Value =I2
268 objXls_u.Worksheets(1).Cells(10,7).Value ="m^4"
269
270 Dim A2 As Variant 'A2 of material type 2
271 A2=12900/1000^2 'm^2
272 objXls_u.Worksheets(1).Cells(11,5).Value ="A2 ="
273 objXls_u.Worksheets(1).Cells(11,6).Value =A2
274 objXls_u.Worksheets(1).Cells(11,7).Value ="m^2"
275
276 Dim Den2 As Variant 'Den2 of material type 2
277 Den2=7850 'kg/m^3
278 objXls_u.Worksheets(1).Cells(12,5).Value ="Material density ="
279 objXls_u.Worksheets(1).Cells(12,6).Value =Den2
280 objXls_u.Worksheets(1).Cells(12,7).Value ="kg/m^3"

```

```

281
282 'Material 3 (External Columns)
283 objXls_u.Worksheets(1).Cells(8,9).Value ="Material 3 (External columns)"
284 objXls_u.Worksheets(1).Cells(8,10).Value ="305.00 x 305.00 x 118.00"
285 Dim E3 As Variant
286 E3=200*10^9 'Pa
287 objXls_u.Worksheets(1).Cells(9,9).Value ="E3 ="
288 objXls_u.Worksheets(1).Cells(9,10).Value =E3
289 objXls_u.Worksheets(1).Cells(9,11).Value ="Pa"
290
291 Dim I3 As Variant
292 I3=2760000000/1000^4 'm^4
293 objXls_u.Worksheets(1).Cells(10,9).Value ="I3 ="
294 objXls_u.Worksheets(1).Cells(10,10).Value =I3
295 objXls_u.Worksheets(1).Cells(10,11).Value ="m^4"
296
297 Dim a3 As Variant
298 a3=15000/1000^2 'm^2
299 objXls_u.Worksheets(1).Cells(11,9).Value ="A3 ="
300 objXls_u.Worksheets(1).Cells(11,10).Value =a3
301 objXls_u.Worksheets(1).Cells(11,11).Value ="m^2"
302
303 Dim Den3 As Variant
304 Den3=7850 'kg/m^3
305 objXls_u.Worksheets(1).Cells(12,9).Value ="Material density ="
306 objXls_u.Worksheets(1).Cells(12,10).Value =Den3
307 objXls_u.Worksheets(1).Cells(12,11).Value ="kg/m^3"
308
309 'Get earthquake record
310 Dim nt As Variant
311 nt=2689 'Number of rows (Data points) of earthquake record
312 Dim uppe() As Variant
313 ReDim Preserve uppe(nt-1,1)
314 uppe=Import_data(nt)
315
316 objXls_u.Worksheets(2).Cells(1,1).Value ="Time (s)"
317 objXls_u.Worksheets(2).Cells(1,2).Value ="Accleration (m/s^2)"
318 For i=0 To nt-1
319     objXls_u.Worksheets(2).Cells(i+2,1).Value =uppe(i,0)
320     objXls_u.Worksheets(2).Cells(i+2,2).Value =uppe(i,1)
321 Next
322
323 'Dead load Is imposed On the model by a concrete slab that Is one way
324 'spaning between the frames
325 objXls_u.Worksheets(1).Cells(13,1).Value ="Loading on structure"
326 objXls_u.Worksheets(1).Cells(14,1).Value ="Dead loading"
327 Dim Span As Variant
328 Span=2.5 'm (Transverse span distance between the frames)
329 objXls_u.Worksheets(1).Cells(15,1).Value ="Transverse span ="
330 objXls_u.Worksheets(1).Cells(15,2).Value =Span
331 objXls_u.Worksheets(1).Cells(15,3).Value ="m"
332
333 'Concrete slab
334 Dim tc As Variant
335 tc=0.25 'm (Thickness of the slab)
336 objXls_u.Worksheets(1).Cells(16,1).Value ="Slab thickness ="
337 objXls_u.Worksheets(1).Cells(16,2).Value =tc
338 objXls_u.Worksheets(1).Cells(16,3).Value ="m"
339
340 Dim Den_conc As Variant
341 Den_conc=2400 'kg/m^3 (Density of the concrete)
342 objXls_u.Worksheets(1).Cells(17,1).Value ="Concrete density ="
343 objXls_u.Worksheets(1).Cells(17,2).Value =Den_conc
344 objXls_u.Worksheets(1).Cells(17,3).Value ="kg/m^3"
345
346 'Live load
347 objXls_u.Worksheets(1).Cells(19,1).Value ="Live loading"
348 Dim LL As Variant
349 LL=2400 'N/m^2 (Live load imposed On the frame structure)
350 objXls_u.Worksheets(1).Cells(19,1).Value ="Live load ="
351 objXls_u.Worksheets(1).Cells(19,2).Value =LL
352 objXls_u.Worksheets(1).Cells(19,3).Value ="N/m^2"
353

```

```

354 'Masonry walls
355 Dim Den_mas As Variant
356 Den_mas=1800 'kg/m3
357 Dim t_wall As Variant
358 t_wall=0.23 'm
359 Dim M_mas As Variant
360 M_mas=Den_mas*t_wall*h*Span 'kg
361 Dim W_mas As Variant
362 W_mas=M_mas*9.81 'N
363
364 'Dynamic properties
365 objXls_u.Worksheets(1).Cells(21,1).Value ="Dynamic properties"
366 Dim Dp As Variant
367 Dp=0.05 '(Damping ratio)
368 objXls_u.Worksheets(1).Cells(22,1).Value ="Damping ratio ="
369 objXls_u.Worksheets(1).Cells(22,2).Value =Dp
370
371 'Footing placement And initial stiffness
372 objXls_u.Worksheets(1).Cells(23,1).Value ="Footing initial stiffness"
373 Dim Column_hinge As Variant
374 Column_hinge=2
375 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
376 Dim kinitial As Variant
377 kinitial=30000000 'N/m
378
379 objXls_u.Worksheets(1).Cells(24,1).Value ="ki ="
380 objXls_u.Worksheets(1).Cells(24,2).Value =kinitial
381 objXls_u.Worksheets(1).Cells(24,3).Value ="N/m"
382
383 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
384
385 Dim Prop(3,2) As Variant 'Table of material properties
386
387 Prop(0,0)=E1
388 Prop(2,0)=I1
389 Prop(1,0)=A1
390 Prop(3,0)=Den1
391
392 Prop(0,1)=E2
393 Prop(2,1)=I2
394 Prop(1,1)=A2
395 Prop(3,1)=Den2
396
397 Prop(0,2)=E3
398 Prop(2,2)=I3
399 Prop(1,2)=a3
400 Prop(3,2)=Den3
401
402 'Preprocessor
403 Dim nnode As Variant
404 nnode=(Bays+1)*(Stories+1)
405
406 'Coordinates of the nodes
407 Dim NodeNumber() As Variant
408 ReDim Preserve NodeNumber(Stories,Bays)
409 Dim Node_x() As Variant
410 ReDim Preserve Node_x(Stories,Bays)
411 Dim Node_y() As Variant
412 ReDim Preserve Node_y(Stories,Bays)
413 Dim coord() As Variant 'Dim coord(nnode,2)
414 ReDim Preserve coord(1,nnode-1)
415 Dim N_mas As Variant
416 N_mas=(Stories-1)*2
417 Dim Nodes_mas() As Variant
418 ReDim Preserve Nodes_mas(N_mas-1,0)
419
420 'Node Numbers
421 Dim counter As Variant
422 counter=0
423 Dim count_mas As Variant
424 count_mas=-1
425
426 For i=0 To Stories

```

```

427 Dim width As Variant
428 width = 0
429 For j=0 To Bays
430 counter=counter+1
431 NodeNumber(-i+2,j)=counter
432 If j=0 Then
433 width =0
434 Node_x(i,j)=width
435 Else
436 width=width+b
437 Node_x(i,j)=width
438 End If
439 Node_y(i,j)=(-i+2)*h
440 If j=0 Or j=Bays Then
441 If i>0 And i<Stories Then
442 count_mas=count_mas+1
443 Nodes_mas(count_mas,0)=counter
444 End If
445 End If
446 Next
447 Next
448
449 For i=0 To Stories
450 For j=0 To Bays
451 coord(0,NodeNumber(i,j)-1)=Node_x(i,j)
452 coord(1,NodeNumber(i,j)-1)=Node_y(i,j)
453 Next
454 Next
455
456 Dim N_columns As Variant
457 N_columns=Stories*(Bays+1)
458 Dim N_beams As Variant
459 N_beams=Stories*Bays
460 Dim nbc As Variant
461 nbc=N_columns+N_beams 'Number of beam elements
462
463 'Assign the columns first To the idbc matrix
464 Dim idbc() As Variant 'Dim idbc(nbc-1,2)
465 ReDim Preserve idbc(nbc-1,2)
466
467 counter=-1
468 For i=0 To Bays
469 If i=0 Or i=Bays Then
470 For j=0 To Stories-1
471 counter=counter+1
472 idbc(counter,0)=NodeNumber(-j+2,i)
473 idbc(counter,1)=NodeNumber(-j+1,i)
474 idbc(counter,2)=3
475 Next
476 Else
477 For j=0 To Stories-1
478 counter=counter+1
479 idbc(counter,0)=NodeNumber(-j+2,i)
480 idbc(counter,1)=NodeNumber(-j+1,i)
481 idbc(counter,2)=1
482 Next
483 End If
484 Next
485
486 'Assign the beams Second To the idbc matrix
487 For i=1 To 2
488 For j=0 To 1
489 counter=counter+1
490 idbc(counter,0)=NodeNumber(-i+2,j)
491 idbc(counter,1)=NodeNumber(-i+2,j+1)
492 idbc(counter,2)=2
493 Next
494 Next
495
496 'Supports of the frame structure
497 Dim support() As Variant
498 ReDim Preserve support(3,Bays)
499

```

```

500 For i=0 To Bays
501     If i=Column_hinge-1 Then      'Nonlinear support
502         support(0,i)=NodeNumber(Stories,i)
503         support(1,i)=0
504         support(2,i)=1
505         support(3,i)=0
506     Else                          'Rest of the supports In the model
507         support(0,i)=NodeNumber(Stories,i)
508         support(1,i)=1
509         support(2,i)=1
510         support(3,i)=0
511     End If
512
513 Next
514
515 'Loading On the structure that includes
516 'Point loads And beam distributed loads On the structure
517 'Wind loading Not included
518
519 Dim loading() As Variant 'loading=zeros(4,nnode);
520 ReDim Preserve loading(3,nnode-1)
521
522 For i=0 To nnode-1
523     loading(0,i)=i+1
524 Next
525
526 For i=0 To nbc-1
527     If idbc(i,2)=1 Then
528         loading(2,idbc(i,0)-1)=loading(2,idbc(i,0)-1)-Prop(3,0)*Prop(1,0)*9.81*h/2
529         loading(2,idbc(i,1)-1)=loading(2,idbc(i,1)-1)-Prop(3,0)*Prop(1,0)*9.81*h/2
530     ElseIf idbc(i,2)=3 Then
531         loading(2,idbc(i,0)-1)=loading(2,idbc(i,0)-1)-Prop(3,2)*Prop(1,2)*9.81*h/2
532         loading(2,idbc(i,1)-1)=loading(2,idbc(i,1)-1)-Prop(3,2)*Prop(1,2)*9.81*h/2
533     End If
534 Next
535
536 'Add masonry loading
537 For i=0 To N_mas-1
538     loading(2,Nodes_mas(i,0)-1)=loading(2,Nodes_mas(i,0)-1)-W_mas
539 Next
540
541 'Distributed loading columns And beams
542 Dim Distributed_loads(2,1) As Variant      'Distributed_loads=zeros(3,2);
543
544 Distributed_loads(0,0)=LL*Span              'Beams live loads
545 Distributed_loads(0,1)=0                    'Columns live loads
546 Distributed_loads(1,0)=Den_conc*Span*tc*9.81 'Beams dead load
547 Distributed_loads(1,1)=0                    'Columns dead load
548 Distributed_loads(2,0)=Prop(3,1)* Prop(1,1)*9.81 'Beams dead load
549 Distributed_loads(2,1)=0                    'Columns dead load
550
551 Dim loading_beam() 'loading_beam=zeros(nbc,2);
552 ReDim Preserve loading_beam(nbc-1,1)
553
554 For i=0 To nbc-1
555     If idbc(i,2)=2 Then
556
557         loading_beam(i,1)=Distributed_loads(0,0)+Distributed_loads(1,0)+Distributed_loads(
558             2,0)
559         loading_beam(i,0)=i+1
560     Else
561         loading_beam(i,1)=Distributed_loads(0,1)+Distributed_loads(1,1)+Distributed_loads(
562             2,1)
563         loading_beam(i,0)=i+1
564     End If
565 Next
566
567 'Static calculations
568 '

```

```

566
567 Dim supp() As Variant

```

```

568 ReDim Preserve supp(2,nnode-1)
569
570 For i=0 To 2
571     For j=0 To 8
572         supp(i,j)=0
573     Next
574 Next
575
576 For i=1 To 3 'Dim support(3,2) As Double
577     For j=0 To 2
578         supp(i-1,support(0,j)-1)=support(i,j)
579     Next
580 Next
581
582 'Determines the dof of freedom
583 Dim dof() As Variant
584 ReDim Preserve dof(2,nnode-1)
585 counter=0
586
587 'First assigns the free dof numbers
588
589 For i=0 To 8 'Number of nodes-1
590     For j=0 To 2
591         If supp(j,i)=0 Then
592             counter=counter+1
593             dof(j,i)=counter
594         End If
595     Next
596 Next
597 Dim nfdof As Variant
598 nfdof=counter
599
600 'Assigns the fixed dof numbers
601
602 For i=0 To 8 'Number of nodes-1
603     For j=0 To 2
604         If supp(j,i)=1 Then
605             counter=counter+1
606             dof(j,i)=counter
607         End If
608     Next
609 Next
610 Dim tdof As Double
611 tdof=counter+1 'Total degrees of freedom + DOF of the footing hinge
612
613 'Calculates the point loads on the nodes
614 Dim ptfof() As Variant
615 ReDim Preserve ptfof(tdof-1,0) 'ptfof=zeros(tdof,1);
616 Dim aL As Variant
617
618 For i=0 To nnode-1
619     For j=0 To 2
620         If dof(j,loading(0,i)-1)<=tdof Then
621             ptfof(dof(j,loading(0,i)-1)-1,0)=loading(j+1,i)
622         End If
623     Next
624 Next
625
626 Dim P() As Variant
627 ReDim Preserve P(nfdof-1,0)
628
629 For i=0 To nfdof-1
630     P(i,0)=ptfof(i,0)
631 Next
632
633 'Calculates the member information
634 Dim mem_info() As Variant
635 ReDim Preserve mem_info(1,nbc-1) 'mem_info=zeros(2,nbc);
636
637 'Calculates the length of Each of the members And places it In mem_info(1,i)
638 For i=0 To nbc-1
639     mem_info(0,i)=((coord(0,idbc(i,1)-1)-coord(0,idbc(i,0)-1))^2+(coord(1,idbc(i,1)-1)-coo
rd(1,idbc(i,0)-1))^2)^(1/2)

```

```

640 Next
641
642 'Calculates the rotation of the member where anticlockwise Is positive
643 Dim x1 As Variant
644 Dim y1 As Variant
645 Dim x2 As Variant
646 Dim y2 As Variant
647 Dim pi As Variant
648 pi=4*Atn(1)
649
650 For i=0 To nbc-1
651 'Obtain the nodal coordinates from the coord Array
652 x1 = coord (0,idbc(i,0)-1)
653 y1 = coord (1,idbc(i,0)-1)
654 x2 = coord (0,idbc(i,1)-1)
655 y2 = coord (1,idbc(i,1)-1)
656 'Calculate the rotation angle
657 If (x2-x1)=0 Then
658 mem_info(1,i)=pi/2
659 Else
660 mem_info(1,i)=Atn((y2-y1)/(x2-x1))
661 End If
662 Next
663
664 'Calculates the member degrees of freedom
665 Dim mdof() As Variant
666 ReDim Preserve mdof(5,nbc-1) 'mdof=zeros(6,nbc)
667
668 For i=0 To nbc-1
669 counter=-1
670 For K=0 To 1
671 For j=0 To 2
672 counter=counter+1
673 mdof(counter,i)=dof(j,idbc(i,K)-1)
674 Next
675 Next
676 Next
677
678 Dim estiff_local() As Variant
679 ReDim Preserve estiff_local(5,5,nbc-1) 'estiff_local{1,nbc}=[]
680 Dim etran_local() As Variant
681 ReDim Preserve etran_local(5,5,nbc-1) 'estiff_local{1,nbc}=[]
682 Dim etranT_local() As Variant
683 ReDim Preserve etranT_local(5,5,nbc-1) 'estiff_local{1,nbc}=[]
684 Dim K_beam_local() As Variant
685 ReDim Preserve K_beam_local(5,5,nbc-1) 'estiff_local{1,nbc}=[]
686
687 Dim global_stiffness() As Variant
688 ReDim Preserve global_stiffness(tdof-1,tdof-1)
689 Dim KSpring() As Variant
690 ReDim Preserve KSpring(1,1)
691 Dim Smdof() As Variant
692 ReDim Preserve Smdof(1,0)
693
694 'Zeros the global stiffness matrix
695 For i=0 To tdof-1
696 For j=0 To tdof-1
697 global_stiffness(i,j)=0
698 Next
699 Next
700
701 assemble_local_stiffness_beam(estiff_local,etran_local,K_beam_local,etranT_local,nbc,P
rop,idbc,mem_info)
702 assemble_beam_stiffness(global_stiffness,tdof,K_beam_local,nbc,mdof)
703 KSpring=Spring_stiff(global_stiffness,Column_hinge,dof,tdof,kinitial,Smdof)
704
705 Dim Pw_local() As Variant
706 ReDim Preserve Pw_local(5,nbc-1)
707 Dim DOF_c() As Variant
708 ReDim Preserve DOF_c(5,nbc-1)
709 Dim Pwout() As Variant
710 ReDim Preserve Pw(nfdof-1,0)
711 Dim Pwtdof() As Variant

```

```

712 ReDim Preserve Pwtdof (tdof-1,0)
713
714 For i=0 To tdof-1
715     Pwtdof(i,0)=0
716 Next
717
718 Pw=beam_loads (loading_beam,nbc,etranT_local,dof,mem_info,idbc,tdof,nfdof,Pw_local,DOF_
c,Pwtdof)
719
720 R=Matrix_addition (P,Pw,nfdof,1)
721
722 Disp=solve (global_stiffness,R,nfdof)
723
724 Dim Rs() As Variant
725 Rs=reactions (global_stiffness,Disp,nfdof,tdof)
726
727 If Restart=0 Then
728     objXls_u.Worksheets(3).Cells(1,1).Value =0
729     objXls_u.Worksheets(11).Cells(1,1).Value =0
730 For i = 0 To nfdof-1
731     objXls_u.Worksheets(3).Cells(i+2,1).Value =Disp(i,0)
732     objXls_u.Worksheets(11).Cells(i+2,1).Value =R(i,0)
733 Next
734 objXls_u.Workbooks(1).Save
735 End If
736
737 Dim IMF_L() As Variant
738 ReDim Preserve IMF_L(5,nbc-1)
739 Dim IMD_L() As Variant
740 ReDim Preserve IMD_L(5,nbc-1)
741 Dim IMD_G() As Variant
742 ReDim Preserve IMD_G(5,nbc-1)
743 Dim IMF_G() As Variant
744 ReDim Preserve IMF_G(5,nbc-1)
745 Dim fso() As Variant
746 ReDim Preserve fso(5,nbc-1)
747
748 memf (IMF_L,IMD_L,IMD_G,IMF_G,fso,supp,nnode,Disp,nbc,idbc,etran_local,etranT_local,est
iff_local,Pw_local)
749
750 Dim ds() As Variant
751 ReDim Preserve ds(1,0)
752 Dim IMF_s() As Variant
753 ReDim Preserve IMF_s(1,0)
754 memf_spring (KSpring,Disp,Smdof,ds,IMF_s)
755
756 'Dynamic calculations
757 '


---


758
759 Dim fyspring As Variant
760 Dim k2 As Variant
761 Dim Mf As Variant
762
763 fyspring=37830.3235480421 'N
764 k2=0.146706193729911*kinitial 'N/m
765 Mf=770.3 'kg
766
767 'Damage index
768 Dim Dpa As Variant
769 Dpa=0
770 Dim di As Variant
771 di=0
772 Dim dumax As Variant
773 dumax=0.00504481947024698
774 Dim bPa As Variant
775 bPa=0.05
776
777 'Initial calculations
778 Dim Fsil As Variant
779 ReDim Preserve Fsil(tdof-1,0)
780
781 Dim Uil As Variant

```

```

782 ReDim Preserve Uil(tdof-1,0)
783 For i=0 To tdof-1
784   Uil(i,0)=0
785 Next
786
787 If Restart=1 Then
788   For i=0 To 5
789     For j=0 To nbc-1
790       IMD_G(i,j)=objXls_u.Worksheets(9).Cells(6*i_Lres+2+i,j+2).Value
791       fso(i,j)=objXls_u.Worksheets(9).Cells(6*i_Lres+2+i,j+nbc+4).Value
792     Next
793   Next
794
795   For i=0 To 1
796     IMFs(i,0)=objXls_u.Worksheets(10).Cells(i+2,i_Lres+1).Value
797     ds(i,0)=objXls_u.Worksheets(10).Cells(i+4,i_Lres+1).Value
798   Next
799   Dpa=objXls_u.Worksheets(2).Cells(i_Lres+2,5).Value
800
801 End If
802
803 initial_conditions_static(Fsil,Uil,fso,IMD_G,mdof,tdof,IMFs,Smdof,nbc)
804
805 Dim global_mass() As Variant
806 ReDim global_mass(tdof-1,tdof-1)
807
808 'Zeros the global mass matrix
809 For i=0 To tdof-1
810   For j=0 To tdof-1
811     global_mass(i,j)=0
812   Next
813 Next
814
815 global_mass_self=assemble_mass_self(global_mass,nbc,Prop,idbc,mem_info,mdof,tdof)
816 'Self weight
817 global_mass_DL=assemble_mass_DL(global_mass,nbc,idbc,mem_info,mdof,tdof,tc,Span,Den_co
nc) 'Dead load (Concrete slab)
818 global_mass_LL=assemble_mass_LL(global_mass,nbc,idbc,mem_info,mdof,tdof,Span,LL)
819 'Live load
820 global_mass_Point=assemble_mass_Point(global_mass,dof,tdof,N_mas,Nodes_mas,M_mas,mdof)
821 'Mass due to masonry walls load
822
823 'Foundation mass
824 global_mass(Smdof(1,0)-1,Smdof(1,0)-1)=global_mass(Smdof(1,0)-1,Smdof(1,0)-1)+Mf
825
826 Dim tmdof As Variant
827 Dim kttv() As Variant
828 Dim Mttf() As Variant
829 Dim Uto() As Variant
830 ReDim Preserve Uto(nfdof-1,0)
831 Dim eig() As Variant
832 ReDim Preserve eig(1,0)
833 Dim Lm() As Variant
834 ReDim Preserve Lm(nfdof-1,0)
835
836 static_condensation_initial(Lm,kttv,Uto,tmdof,Mttf,global_stiffness,global_mass,mdof,n
fdof,Fsil,Uil,nbc)
837 eig=eigenvalues(Mttf,kttv,tmdof)
838
839 For i=0 To tmdof-1
840   For j=0 To tmdof-1
841     objXls_u.Worksheets(7).Cells(i+1,j+1).Value =kttv(i,j)
842     objXls_u.Worksheets(8).Cells(i+1,j+1).Value =Mttf(i,j)
843   Next
844 Next
845
846 If Restart=0 Then
847   objXls_u.Worksheets(1).Cells(26,1).Value ="Eigenvalues"
848   objXls_u.Worksheets(1).Cells(27,1).Value ="Eig1 = "
849   objXls_u.Worksheets(1).Cells(28,1).Value ="Eig2 = "
850   objXls_u.Worksheets(1).Cells(27,2).Value =eig(0,0)
851   objXls_u.Worksheets(1).Cells(28,2).Value =eig(1,0)
852 End If

```

```

850
851 Dim Cff() As Variant
852 'ReDim Preserve Damp(1,0)
853
854 Cff=Rayleigh_damping_nonlinear(eig,Mttf,kttv,Dp,tmdof)
855
856 Dim ui() As Variant
857 ReDim Preserve ui(tmdof-1,1)
858 Dim upi() As Variant
859 ReDim Preserve upi(tmdof-1,1)
860 Dim uppi() As Variant
861 ReDim Preserve uppi(tmdof-1,1)
862 Dim fsi() As Variant
863 ReDim Preserve fsi(tmdof-1,0)
864
865 If Restart=0 Then
866     For I=0 To tmdof-1
867         ui(i,1)=Uto(i,0)
868         upi(i,1)=0
869         uppi(i,1)=0
870     Next
871
872 ElseIf Restart=1 Then
873
874     For I=0 To tmdof-1
875         ui(i,1)=Uto(i,0)
876         upi(i,1)=objXls_u.Worksheets(4).Cells(i+2,i_Lres+2).Value
877         uppi(i,1)=objXls_u.Worksheets(5).Cells(i+2,i_Lres+2).Value
878     Next
879 End If
880
881 fsi=Matrix_multiplication(kttv,ui,tmdof,tmdof,tmdof,1)
882
883 Dim IMF_Li() As Variant
884 ReDim Preserve IMF_Li(5,nbc-1,1)
885 Dim IMD_Li() As Variant
886 ReDim Preserve IMD_Li(5,nbc-1,1)
887 Dim IMD_Gi() As Variant
888 ReDim Preserve IMD_Gi(5,nbc-1,1)
889 Dim IMF_Gi() As Variant
890 ReDim Preserve IMF_Gi(5,nbc-1,1)
891 Dim fsiE() As Variant
892 ReDim Preserve fsiE(5,nbc-1,1)
893
894 For i=0 To 5
895     For j=0 To nbc-1
896         IMD_Gi(i,j,0)=0
897         IMD_Gi(i,j,1)=1
898         fsiE(i,j,0)=0
899         fsiE(i,j,1)=1
900     Next
901 Next
902
903 Sort_matrix(IMF_Li,IMF_L,6,nbc)
904 Sort_matrix(IMD_Li,IMD_L,6,nbc)
905 Sort_matrix(IMD_Gi,IMD_G,6,nbc)
906 Sort_matrix(IMF_Gi,IMF_G,6,nbc)
907 Sort_matrix(fsiE,fso,6,nbc)
908
909 If Retart=0 Then
910 'Saves local matrices to output file
911 objXls_u.Worksheets(9).Cells(1,2).Value ="IMD_Gi"
912 objXls_u.Worksheets(9).Cells(1,nbc+4).Value ="fsiE"
913 objXls_u.Worksheets(9).Cells(1,1).Value ="Time (s)"
914 objXls_u.Worksheets(9).Cells(1,nbc+3).Value ="Time (s)"
915 objXls_u.Worksheets(9).Cells(2,1).Value =0
916 objXls_u.Worksheets(9).Cells(2,nbc+3).Value =0
917
918     For i=0 To 5
919         For j=0 To nbc-1
920             objXls_u.Worksheets(9).Cells(i+2,j+2).Value =IMD_G(i,j)
921             objXls_u.Worksheets(9).Cells(i+2,j+nbc+4).Value =fso(i,j)
922         Next

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```

923     Next
924 End If
925
926 Dim IMDsi() As Variant
927 ReDim Preserve IMDsi(1,nt-1)
928 Dim IMFsi() As Variant
929 ReDim Preserve IMFsi(1,nt-1)
930
931 If Restart=0 Then
932 vector_sort(IMFsi,IMFs,2,0)
933 vector_sort(IMDsi,ds,2,0)
934 'Saves force matrix
935 objXls_u.Worksheets(10).Cells(1,1).Value =0
936     For i=0 To 1
937         objXls_u.Worksheets(10).Cells(i+2,1).Value =IMFs(i,0)
938         objXls_u.Worksheets(10).Cells(i+4,1).Value =ds(i,0)
939     Next
940 ElseIf Restart=1 Then
941     For i=0 To i_Lres
942         For j=0 To 1
943             IMFsi(j,i)=objXls_u.Worksheets(10).Cells(j+2,i+1).Value
944             IMDsi(j,i)=objXls_u.Worksheets(10).Cells(j+4,i+1).Value
945         Next
946     Next
947 End If
948
949 'Energy calculations
950 Dim Energy_Ps As Variant
951 Dim Energy_Ms() As Variant
952 ReDim Preserve Energy_Ms(nt-1,0)
953 Dim Energy-Cs() As Variant
954 ReDim Preserve Energy-Cs(nt-1,0)
955 Dim Energy_Ks() As Variant
956 ReDim Preserve Energy_Ks(nt-1,0)
957 Dim Energy_Hs() As Variant
958 ReDim Preserve Energy_Hs(nt-1,0)
959
960 Dim Energy_K As Variant
961 Dim Energy_H As Variant
962 Dim Energy_M As Variant
963 Dim Energy_C As Variant
964
965 'Internal energy
966 Dim Energy_Total() As Double
967 ReDim Preserve Energy_Total(nt-1)
968 Dim Energy_Stiffness() As Double
969 ReDim Preserve Energy_Stiffness(nt-1)
970 Dim Energy_Damping() As Double
971 ReDim Preserve Energy_Damping(nt-1)
972 Dim Energy_Hys() As Double
973 ReDim Preserve Energy_Hys(nt-1)
974
975 'Load cell, strain gauges and LVDT readings
976
977 Dim Strain_Left As Double
978 Dim Strain_right As Double
979
980 Dim Strain_Left_disp() As Double
981 ReDim Preserve Strain_Left_disp(nt-1)
982 Dim Strain_right_disp() As Double
983 ReDim Preserve Strain_right_disp(nt-1)
984
985 Dim LVDT1 As Double
986 Dim LVDT2 As Double
987 Dim LVDT3 As Double
988 Dim LVDT4 As Double
989
990 Dim LVDT1_disp() As Double
991 ReDim Preserve LVDT1_disp(nt-1)
992 Dim LVDT2_disp() As Double
993 ReDim Preserve LVDT2_disp(nt-1)
994 Dim LVDT3_disp() As Double
995 ReDim Preserve LVDT3_disp(nt-1)

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```

996 Dim LVDT4_disp() As Double
997 ReDim Preserve LVDT4_disp(nt-1)
998
999 'External energy
1000 Dim Disp_P() As Variant
1001 ReDim Preserve Disp_P(nfdof-1,1)
1002 Dim Force_P() As Variant
1003 ReDim Preserve Force_P(nfdof-1,1)
1004
1005 If Restart= 0 Then
1006     Energy_K=0
1007     Energy_H=0
1008     Energy_M=0
1009     Energy_C=0
1010
1011     For i=0 To 5
1012         For j=0 To nbc-1
1013             Energy_K=Energy_K+0.5*IMD_G(i,j)*fso(i,j)
1014         Next
1015     Next
1016     Energy_H=0.5*IMDsi(1,0)*IMFsi(1,0)
1017
1018     Energy_Ms(0,0)=0
1019     Energy-Cs(0,0)=0
1020     Energy_Ks(0,0)=Energy_K
1021     Energy_Hs(0,0)=Energy_H
1022
1023     Energy_Ps=0
1024     For i=0 To nfdof-1
1025         Energy_Ps=Energy_Ps+0.5*R(i,0)*Disp(i,0)
1026     Next
1027
1028     'External energy
1029
1030     For i=0 To nfdof-1
1031         Disp_P(i,1)=Disp(i,0)
1032         Force_P(i,1)=R(i,0)
1033     Next
1034
1035     Energy_Total(0)=Energy_Ms(0,0)+Energy-Cs(0,0)+Energy_Ks(0,0)+Energy_Hs(0,0)
1036     Energy_Stiffness(0)=Energy_Ks(0,0)+Energy_Hs(0,0)+Energy-Cs(0,0)
1037     Energy_Damping(0)=Energy_Hs(0,0)+Energy-Cs(0,0)
1038     Energy_Hys(0)=Energy_Hs(0,0)
1039
1040     'Readings from LVDTs, load cells and strain gauges
1041     EA_IO.Measure("MX840_SL",Strain_Left,1)
1042     EA_IO.Measure("MX840_SR",Strain_right,1)
1043     EA_IO.Measure("Load_cell",Load_axial,1)
1044
1045     objXls_u.Worksheets(2).Cells(2,8).Value =Load_axial
1046
1047     objXls_u.Worksheets(2).Cells(2,10).Value =Strain_Left
1048     objXls_u.Worksheets(2).Cells(2,11).Value =Strain_right
1049
1050     Strain_Left_disp(0)=Strain_Left
1051     Strain_right_disp(0)=Strain_right
1052
1053     EA_IO.Measure("PMX_LVDT1",LVDT1,1)
1054     EA_IO.Measure("PMX_LVDT2",LVDT2,1)
1055     EA_IO.Measure("PMX_LVDT3",LVDT3,1)
1056     EA_IO.Measure("PMX_LVDT4",LVDT4,1)
1057
1058     objXls_u.Worksheets(2).Cells(2,13).Value =LVDT1
1059     objXls_u.Worksheets(2).Cells(2,14).Value =LVDT2
1060     objXls_u.Worksheets(2).Cells(2,15).Value =LVDT3
1061     objXls_u.Worksheets(2).Cells(2,16).Value =LVDT4
1062
1063     LVDT1_disp(0)=LVDT1
1064     LVDT2_disp(0)=LVDT2
1065     LVDT3_disp(0)=LVDT3
1066     LVDT4_disp(0)=LVDT4
1067
1068 ElseIf Restart=1 Then

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```

1069 Energy_K=objXls_u.Worksheets(6).Cells(i_Lres+2,4).Value
1070 Energy_H=objXls_u.Worksheets(6).Cells(i_Lres+2,5).Value
1071 Energy_M=objXls_u.Worksheets(6).Cells(i_Lres+2,2).Value
1072 Energy_C=objXls_u.Worksheets(6).Cells(i_Lres+2,3).Value
1073 Energy_Ps=objXls_u.Worksheets(6).Cells(i_Lres+2,6).Value
1074
1075 For i=0 To i_Lres
1076     Energy_Ms(i,0)=objXls_u.Worksheets(6).Cells(i+2,2).Value
1077     Energy-Cs(i,0)=objXls_u.Worksheets(6).Cells(i+2,3).Value
1078     Energy_Ks(i,0)=objXls_u.Worksheets(6).Cells(i+2,4).Value
1079     Energy_Hs(i,0)=objXls_u.Worksheets(6).Cells(i+2,5).Value
1080
1081     Energy_Total(i)=CDBl(Energy_Ms(i,0)+Energy-Cs(i,0)+Energy_Ks(i,0)+Energy_Hs(i,
1082     0))
1083     Energy_Stiffness(i)=CDBl(Energy_Ks(i,0)+Energy_Hs(i,0)+Energy-Cs(i,0))
1084     Energy_Damping(i)=CDBl(Energy_Hs(i,0)+Energy-Cs(i,0))
1085     Energy_Hys(i)=CDBl(Energy_Hs(i,0))
1086     Strain_Left_disp(i)=objXls_u.Worksheets(2).Cells(i+2,10).Value
1087     Strain_right_disp(i)=objXls_u.Worksheets(2).Cells(i+2,11).Value
1088     LVDT1_disp(i)=objXls_u.Worksheets(2).Cells(i+2,13).Value
1089     LVDT2_disp(i)=objXls_u.Worksheets(2).Cells(i+2,14).Value
1090     LVDT3_disp(i)=objXls_u.Worksheets(2).Cells(i+2,15).Value
1091     LVDT4_disp(i)=objXls_u.Worksheets(2).Cells(i+2,16).Value
1092
1093 Next
1094
1095 For i=0 To nfdof-1
1096     Disp_P(i,1)=objXls_u.Worksheets(3).Cells(i+2,i_Lres+1).Value
1097     Force_P(i,1)=objXls_u.Worksheets(11).Cells(i+2,i_Lres+1).Value
1098 Next
1099
1100 End If
1101
1102 Dim Force_M() As Variant
1103 ReDim Preserve Force_M(tmdof-1,1)
1104 Dim Force_C() As Variant
1105 ReDim Preserve Force_C(tmdof-1,1)
1106
1107 If Restart=0 Then
1108     For i=0 To tmdof-1
1109         For j=0 To 1
1110             Force_M(i,j)=0
1111             Force_C(i,j)=0
1112         Next
1113         objXls_u.Worksheets(12).Cells(i+2,1).Value =0
1114         objXls_u.Worksheets(13).Cells(i+2,1).Value =0
1115     Next
1116
1117 ElseIf Restart=1 Then
1118     For i=0 To tmdof-1
1119         Force_M(i,0)=objXls_u.Worksheets(12).Cells(i+2,i_Lres+1).Value
1120         Force_C(i,0)=objXls_u.Worksheets(13).Cells(i+2,i_Lres+1).Value
1121     Next
1122 End If
1123
1124 '-----
1125 'Output data initial
1126 If Restart=0 Then
1127     objXls_u.Worksheets(4).Cells(1,1).Value ="DOF \ t(s) "
1128     objXls_u.Worksheets(5).Cells(1,1).Value ="DOF \ t(s) "
1129     For i = 0 To tmdof-1
1130         objXls_u.Worksheets(4).Cells(i+2,1).Value =Lm(i,0)+1
1131         objXls_u.Worksheets(5).Cells(i+2,1).Value =Lm(i,0)+1
1132     Next
1133
1134     objXls_u.Worksheets(4).Cells(1,2).Value =0
1135     objXls_u.Worksheets(5).Cells(1,2).Value =0
1136     For i = 0 To tmdof-1
1137         objXls_u.Worksheets(4).Cells(i+2,2).Value =0
1138         objXls_u.Worksheets(5).Cells(i+2,2).Value =0
1139     Next

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1140 'Energy data
1141 objXls_u.Worksheets(6).Cells(1,1).Value ="Time (s) "
1142 objXls_u.Worksheets(6).Cells(1,2).Value ="Mass energy (J) "
1143 objXls_u.Worksheets(6).Cells(1,3).Value ="Damping (J) "
1144 objXls_u.Worksheets(6).Cells(1,4).Value ="Stiffness (J) "
1145 objXls_u.Worksheets(6).Cells(1,5).Value ="Hysteretic (J) "
1146 objXls_u.Worksheets(6).Cells(1,6).Value ="Input Energy (J) "
1147
1148 objXls_u.Worksheets(6).Cells(1,8).Value ="H"
1149 objXls_u.Worksheets(6).Cells(1,9).Value ="H+D"
1150 objXls_u.Worksheets(6).Cells(1,10).Value ="H+D+S"
1151 objXls_u.Worksheets(6).Cells(1,11).Value ="H+D+S+M"
1152
1153 objXls_u.Worksheets(6).Cells(2,1).Value =0
1154 objXls_u.Worksheets(6).Cells(2,2).Value =0           'Mass or inertia energy /
kinetic energy
1155 objXls_u.Worksheets(6).Cells(2,3).Value =0           'Damping energy
1156 objXls_u.Worksheets(6).Cells(2,4).Value =Energy_K   'Stiffness energy/potential energy
1157 objXls_u.Worksheets(6).Cells(2,5).Value =Energy_H   'Hysteretic energy of the footing
1158 objXls_u.Worksheets(6).Cells(2,6).Value =Energy_Ps  'Input energy
1159 objXls_u.Worksheets(6).Cells(2,8).Value =Energy_H
1160 objXls_u.Worksheets(6).Cells(2,9).Value =Energy_H+0
1161 objXls_u.Worksheets(6).Cells(2,10).Value =Energy_H+0+Energy_K
1162 objXls_u.Worksheets(6).Cells(2,11).Value =Energy_H+0+Energy_K+0
1163
1164 'Earthquake and numerical results output
1165 objXls_u.Worksheets(2).Cells(1,3).Value ="Displacement (mm) "
1166 objXls_u.Worksheets(2).Cells(1,4).Value ="Force (kN) "
1167 objXls_u.Worksheets(2).Cells(1,5).Value ="Damage Index"
1168 objXls_u.Worksheets(2).Cells(1,6).Value ="Convergence"
1169 objXls_u.Worksheets(2).Cells(1,7).Value ="Iterations"
1170 objXls_u.Worksheets(2).Cells(1,8).Value ="Axial Load (kN) "
1171 objXls_u.Worksheets(2).Cells(2,3).Value =ds(1,0)
1172 objXls_u.Worksheets(2).Cells(2,4).Value =IMFs(1,0)
1173 objXls_u.Worksheets(2).Cells(2,5).Value =0
1174
1175 'Strain gauges output data
1176 objXls_u.Worksheets(2).Cells(1,10).Value ="Strain gauge left (micro) "
1177 objXls_u.Worksheets(2).Cells(1,11).Value ="Strain gauge right (micro) "
1178
1179 'LVDTs output data
1180 objXls_u.Worksheets(2).Cells(1,13).Value ="LVDT1 (mm) "
1181 objXls_u.Worksheets(2).Cells(1,14).Value ="LVDT2 (mm) "
1182 objXls_u.Worksheets(2).Cells(1,15).Value ="LVDT3 (mm) "
1183 objXls_u.Worksheets(2).Cells(1,16).Value ="LVDT4 (mm) "
1184
1185 End If
1186
1187 Dim count As Variant
1188 Dim num As Variant
1189 Dim ks As Variant
1190
1191 EA_Graph.ClearPlots(Panell,"Graph1")
1192 EA_Graph.Refresh(Panell,"Graph1")
1193 Dim xSpring() As Double
1194 ReDim Preserve xSpring(nt)
1195 Dim ySpring() As Double
1196 ReDim Preserve ySpring(nt)
1197 xSpring(0)=0
1198 ySpring(0)=0
1199
1200 EA_Graph.PlotArrayXY(Panell,"Graph1",1,nt+1, xSpring(), ySpring())
1201 EA_Graph.SetPlotProperty(Panell,"Graph1",1,2,vbRed)
1202 EA_Graph.Refresh(Panell,"Graph1")
1203
1204 EA_Graph.ClearPlots(Panell,"Graph2")
1205 EA_Graph.Refresh(Panell,"Graph2")
1206 Dim xEarth() As Double
1207 ReDim Preserve xEarth(nt)
1208 Dim yEarth() As Double
1209 ReDim Preserve yEarth(nt)
1210 xEarth(0)=0
1211 yEarth(0)=0

```

```

1212
1213 EA_Graph.PlotArrayXY(Panell1,"Graph2",1,nt+1, xEarth(), yEarth())
1214 EA_Graph.SetPlotProperty(Panell1,"Graph2",1,2,vbBlue)
1215 EA_Graph.Refresh(Panell1,"Graph2")
1216 Dim LVDT_counter As Variant
1217 LVDT_counter=0
1218 Dim PGAmay As Variant
1219 PGAmay=0
1220 Dim Disp_read_servo As Double
1221 Dim Force_read_servo As Double
1222
1223 EA_IO.ZeroBalanceControl("Displacement_Hor",1) 'Zeros Horizontal displacement
1224 EA_IO.ZeroBalanceControl("Force_Hor",1) 'Zeros Horizontal force
1225
1226 Dim ncpts As Variant
1227 ncpts=50
1228 Dim Fsavg As Variant
1229 Dim Disp_i_1 As Variant
1230 Disp_i_1=0
1231 Dim axial_read As Double
1232 'Start of time stepping
1233 '-----
1234 For i_Load=i_Lres+1 To nt-1 'Starts the earthquake record
1235 EA_Panel.SetValue(Panell1,"Increment_out",i_Load)
1236 EA_Panel.SetValue(Panell1,"Time_out",uppe(i_Load,0))
1237 objXls_u.Worksheets(14).Cells(1,i_Load).Value =uppe(i_Load,0)
1238 objXls_u.Worksheets(15).Cells(1,i_Load).Value =uppe(i_Load,0)
1239
1240 EA_IO.Measure("Load cell",axial_read,1)
1241 EA_Panel.SetValue(Panell1,"Axial_load",CVar(axial_read))
1242 objXls_u.Worksheets(2).Cells(i_Load+2,8).Value =CVar(axial_read)
1243
1244 If Abs(uppe(i_Load,1))>PGAmay Then
1245 PGAmay=Abs(uppe(i_Load,1))
1246 End If
1247 EA_Panel.SetValue(Panell1,"PGA",PGAmay)
1248
1249 Initial_sort_vec(ui,tmdof,1,0)
1250 Initial_sort_vec(upi,tmdof,1,0)
1251 Initial_sort_vec(uppi,tmdof,1,0)
1252
1253 Initial_sort_vec(Disp_P,nfdof,1,0)
1254 Initial_sort_vec(Force_P,nfdof,1,0)
1255
1256 'Initial_sort_mat(IMF_Li,6,nbc,1,0)
1257 'Initial_sort_mat(IMD_Li,6,nbc,1,0)
1258 Initial_sort_mat(IMD_Gi,6,nbc,1,0)
1259 'Initial_sort_mat(IMF_Gi,6,nbc,1,0)
1260 Initial_sort_mat(fsiE,6,nbc,1,0)
1261
1262 For i=0 To tdof-1
1263     For j=0 To 0
1264         EA_Panel.SetCell(Panell2,"PropTable",j+1,i+1,Fsil(i,j))
1265     Next
1266 Next
1267
1268 Initial_sort_vec(IMFsi,2,i_Load-1,i_Load)
1269 Initial_sort_vec(IMDsi,2,i_Load-1,i_Load)
1270 count=count+1
1271 num=0
1272 Fsavg=0
1273 EA_Panel.SetCell(Panell1,"Converge_table",2,1,0)
1274
1275 ks=kinitial
1276
1277 xEarth(i_Load)=CDBl(uppe(i_Load,0))
1278 yEarth(i_Load)=CDBl(AeR*uppe(i_Load,1))
1279 EA_Graph.PlotArrayXY(Panell1,"Graph2",1,nt+1, xEarth(), yEarth())
1280 EA_Graph.SetPlotProperty(Panell1,"Graph2",1,2,vbBlue)
1281 EA_Graph.Refresh(Panell1,"Graph2")
1282
1283 Dim converge_time_step As Variant
1284 converge_time_step=0

```

```

1285 EA_Panel.SetCell(Panell1,"Converge_time_table",1,1,0)
1286
1287 Do While num<5000 'Convergence at each time step
1288 EA_Panel.GetCell(Panell1,"Converge_time_table",1,1,converge_time_step)
1289
1290 'Zeros the global stiffness matrix
1291 For i=0 To tdof-1
1292     For j=0 To tdof-1
1293         global_stiffness(i,j)=0
1294     Next
1295 Next
1296 assemble_beam_stiffness(global_stiffness,tdof,K_beam_local,nbc,mdof)
1297 KSpring=Spring_stiff(global_stiffness,Column_hinge,dof,tdof,ks,Sm dof)
1298
1299 Dim Mfft() As Variant
1300 Dim lh() As Variant
1301 Dim Ps() As Variant
1302 Dim koof() As Variant
1303 Dim ktof() As Variant
1304 Dim kotf() As Variant
1305
1306
1307 static_condensation(Lm,kotf,kttv,fsi,ktof,koof,Ps,lh,Mttf,global_stiffness,mdo
1308 f,nfdof,Fsil,Uil,global_mass,Pwtdof,ptfof,i_Load,nbc,tdof)
1309
1310 Dim a_1() As Variant
1311 Dim a_2() As Variant
1312 Dim a_3() As Variant
1313
1314 Dynamic_analysis_coefficients(a_1,a_2,a_3,dt,Mttf,Cff,tmdof)
1315 'P=-Mttf*lh*uppe(i_Load,2);
1316 Dim PEarth() As Variant
1317 ReDim Preserve PEarth(tmdof-1,0)
1318
1319 PEarth=Matrix_multiplcation_constant(-AeR*uppe(i_Load,1),Matrix_multiplication
1320 (Mttf,lh,tmdof,tmdof,tmdof,1),tmdof,1)
1321 'Pa=P+Ps(1:tmdof,1)-ktof*(koof\Ps(tmdof+1:nfdof,1))
1322 Pa_calc(tmdof,nfdof,Ps,PEarth,ktof,koof)
1323 Dim Pa() As Variant
1324 ReDim Preserve Pa(tmdof-1,0)
1325 Pa=Pa_calc(tmdof,nfdof,Ps,PEarth,ktof,koof)
1326 'Pp=Pa+a1*ui(:,i_Load-1)+a2*upi(:,i_Load-1)+a3*uppi(:,i_Load-1)
1327 Dim Pp() As Variant
1328 ReDim Preserve Pp(tmdof-1,0)
1329 Pp=Pp_calcs(Pa,tmdof,ui,upi,uppi,a_1,a_2,a_3)
1330 'R=Pp-fsi(:,i_Load)-a1*ui(:,i_Load)
1331 Dim Residual() As Variant
1332 ReDim Preserve Residual(tmdof-1,0)
1333 Residual=R_calc(Pp,fsi,a_1,ui,tmdof)
1334 'Norm_max=max(abs(R))
1335 Dim Norm_max As Variant
1336 Norm_max=Norm_Residual(Residual,tmdof)
1337 num=num+1
1338
1339 EA_Panel.SetValue(Panell1,"Iteration_out",num)
1340 EA_Panel.SetValue(Panell1,"Convergence_out",Norm_max)
1341
1342 If Norm_max<ConvergeC Then Exit Do
1343
1344 If converge_time_step=1 Then Exit Do
1345
1346 'Calculates the Newmarks pseudo stiffness
1347 'ktp=kttv+A1
1348 Dim ktp() As Variant
1349 ReDim Preserve ktp(tmdof-1,tmdof-1)
1350 ktp=Matrix_addition(kttv,a_1,tmdof,tmdof)
1351 Dim du() As Variant
1352 ReDim Preserve du(tmdof-1,0)
1353 du=Matrix_solver(ktp,Residual,tmdof)
1354 'ui(:,i_Load)=ui(:,i_Load)+du
1355 For i= 0 To tmdof-1
1356     ui(i,1)=ui(i,1)+du(i,0)
1357 Next

```

```

1354
1355 Dim uig() As Variant
1356 Dim uigdu() As Variant
1357 Dim Puig() As Variant
1358
1359 static_condensation_out(uig(),uigdu(),Puig(),ui,kotf,koof,Ps,Lm,i_Load,tmdof,nfdof
1360 ,tdof,du,PEarth)
1361 Dim Fsi2() As Variant
1362
1363 internal_dynamic_loads(IMD_G,IMD_L,IMF_L,IMF_G,Fsi2,nbc,uig,mdof,etran_local,etran
1364 T_local,estiff_local,Pw_local)
1365 Dim Fs As Variant
1366 memf_spring_dynamic(Fs,IMFsi,IMDsi,KSspring,uig,Smdof,uigdu,i_Load)
1367
1368 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
1369 'Sends displacement reading to actuator
1370 'Reads from load cell and LVDTs
1371
1372 'EA_IO.SetAnalogOut(ByVal Channel As Variant, ByVal Connector As Integer, ByVal
1373 Voltage As Double, ByVal OperMode As Long, Optional ByVal OutputNumber As Integer)
1374 As Long
1375 'Dim LVDT_read As Double
1376 'EA_IO.Measure("MX410_1_CH 1",LVDT_read,1)
1377
1378 If num<=ncpts Then
1379
1380 'Analog output
1381 Dim anolog_step As Variant
1382 Dim time_step_analog As Variant
1383 Dim change_disp As Variant
1384
1385 change_disp=Abs(uig(Smdof(1,0)-1,0)*1000-Disp_i_1)
1386 anolog_step=Int(2*change_disp)+1
1387 time_step_analog=(uig(Smdof(1,0)-1,0)*1000-Disp_i_1)/anolog_step
1388
1389 For i_as=1 To anolog_step
1390 analog_out=(1/-6.5025)*(Disp_i_1+i_as*time_step_analog) 'volts from mm
1391 EA_Panel.SetValue(Panell1,"DispRead_out",(Disp_i_1+i_as*time_step_analog)) 'mm
1392 EA_IO.SetAnalogOut("PMX_1 CH 9",1,analog_out,1)
1393 Wait 0.05
1394 Next
1395
1396 End If
1397 Disp_i_1=uig(Smdof(1,0)-1,0)*1000
1398
1399 analog_out=(1/-6.5025)*uig(Smdof(1,0)-1,0)*1000 'volts from mm
1400 EA_IO.SetAnalogOut("PMX_1 CH 9",1,analog_out,1)
1401 EA_Panel.SetValue(Panell1,"DispRead_out",uig(Smdof(1,0)-1,0)*1000) 'mm
1402
1403 objXls_u.Worksheets(2).Cells(i_Load+2,18).Value =analog_out 'Volts
1404 objXls_u.Worksheets(14).Cells(num+1,i_Load).Value =uig(Smdof(1,0)-1,0) 'm
1405
1406 Dim diff_servo As Variant
1407 Dim Disp_check_in As Variant
1408
1409 EA_IO.Measure("Displacement_Hor",Disp_read_servo,1) 'mm from servo controller
1410 Disp_check_in=CVar(Disp_read_servo)
1411 EA_Panel.SetValue(Panell1,"DispRead_in",Disp_check_in)
1412
1413 diff_servo=Abs(Disp_check_in-uig(Smdof(1,0)-1,0)*1000)
1414 EA_Panel.SetValue(Panell1,"Analog_diff",diff_servo)
1415
1416 Dim Internal_loop As Variant
1417 Internal_loop=0
1418 EA_Panel.SetValue(Panell1,"Interal_counter",0)
1419 EA_Panel.SetCell(Panell1,"Converge_table",1,1,0)
1420 Dim Internal_count1 As Variant
1421 Internal_count1=0
1422
1423 Do While diff_servo>20 'Determines the difference between output and input
1424 voltage
1425 Internal_count1=Internal_count1+1
1426

```

```

1420 EA_Panel.SetValue(Panell1,"Internal_counter",Internal_count1)
1421
1422 EA_Panel.GetCell(Panell1,"Converge_table",1,1,Internal_loop)
1423 If Internal_loop=1 Then Exit Do
1424 EA_IO.Measure("Displacement_Hor",Disp_read_servo,1)
1425 EA_Panel.SetValue(Panell1,"DispRead_in",Disp_read_servo)
1426 Disp_check_in=CVar(Disp_read_servo)
1427 diff_servo=Abs(Disp_check_in-uig(Smdof(1,0)-1,0)*1000)
1428 EA_Panel.SetValue(Panell1,"Analog_diff",diff_servo)
1429 If diff_servo<20 Then Exit Do
1430 Loop
1431
1432 EA_IO.Measure("Force_Hor",Force_read_servo,1)
1433
1434 If num<=ncpts Then
1435 Wait 0.1 'Waits for displacement to be applied and to settle
1436 Fs=CVar(Force_read_servo) 'N
1437 Fsavg=Fs
1438 ElseIf num>ncpts Then
1439 Fs=Fsavg
1440 EA_Panel.SetCell(Panell1,"Converge_table",2,1,1)
1441 EA_Panel.SetCell(Panell1,"Converge_table",2,2,Fs)
1442 End If
1443
1444 EA_Panel.SetValue(Panell1,"Force_servo_in",Fs)
1445
1446
1447
1448 objXls_u.Worksheets(2).Cells(i_Load+2,19).Value =Fs
1449 objXls_u.Worksheets(15).Cells(num+1,i_Load).Value =Fs
1450
1451
1452 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
1453
1454 '[Fsi1,ks,IMFsi]=Post_calcs_nonlinear_dynamic(Fsi2,Fs,mdof,Smdof,tdof,IMFsi,IMDsi,i_Load);
1455
1456 Post_calcs_nonlinear_dynamic(Fsi1,ks,IMFsi,Fsi2,Fs,mdof,Smdof,tdof,IMDsi,i_Load,nb
1457 c)
1458 'objXls_u.Workbooks(1).Save
1459 Loop
1460
1461 '[upi,uppi]=velocity_acceleration(ui,upi,uppi,i_Load,dt);
1462 velocity_acceleration(ui,upi,uppi,dt,tmdof)
1463 'Calculates the reactions
1464 Rs=reactions(global_stiffness,Disp,nfdof,tdof)
1465
1466 'Reads strain gauge data
1467 EA_IO.Measure("MX840_SL",Strain_Left,1)
1468 EA_IO.Measure("MX840_SR",Strain_right,1)
1469
1470 objXls_u.Worksheets(2).Cells(i_Load+2,10).Value =Strain_Left
1471 objXls_u.Worksheets(2).Cells(i_Load+2,11).Value =Strain_right
1472
1473 Strain_Left_disp(i_Load)=Strain_Left
1474 Strain_right_disp(i_Load)=Strain_right
1475
1476 EA_IO.Measure("PMX_LVDT1",LVDT1,1)
1477 EA_IO.Measure("PMX_LVDT2",LVDT2,1)
1478 EA_IO.Measure("PMX_LVDT3",LVDT3,1)
1479 EA_IO.Measure("PMX_LVDT4",LVDT4,1)
1480
1481 objXls_u.Worksheets(2).Cells(i_Load+2,13).Value =LVDT1
1482 objXls_u.Worksheets(2).Cells(i_Load+2,14).Value =LVDT2
1483 objXls_u.Worksheets(2).Cells(i_Load+2,15).Value =LVDT3
1484 objXls_u.Worksheets(2).Cells(i_Load+2,16).Value =LVDT4
1485
1486 LVDT1_disp(i_Load)=LVDT1
1487 LVDT2_disp(i_Load)=LVDT2
1488 LVDT3_disp(i_Load)=LVDT3
1489 LVDT4_disp(i_Load)=LVDT4
1490
1491 'Saves local stiffness matrices

```

```

1490
1491 objXls_u.Worksheets(9).Cells(6*i_Load+2,1).Value =uppe(i_Load,0)
1492 objXls_u.Worksheets(9).Cells(6*i_Load+2,nbc+3).Value =uppe(i_Load,0)
1493 For i=0 To 5
1494     For j=0 To nbc-1
1495         objXls_u.Worksheets(9).Cells(6*i_Load+2+i,j+2).Value =IMD_G(i,j)
1496         objXls_u.Worksheets(9).Cells(6*i_Load+2+i,j+nbc+4).Value =Fsi2(i,j)
1497     Next
1498 Next
1499
1500 'Saves force matrix
1501 objXls_u.Worksheets(10).Cells(1,i_Load+1).Value =uppe(i_Load,0)
1502 For i=0 To 1
1503     objXls_u.Worksheets(10).Cells(i+2,i_Load+1).Value =IMFsi(i,i_Load)
1504     objXls_u.Worksheets(10).Cells(i+4,i_Load+1).Value =IMDsi(i,i_Load)
1505 Next
1506
1507 'LVDT_counter=LVDT_counter+1
1508 'EA_Panel.SetValue(Panel2,"DIGIT_1",LVDT_read)
1509
1510 'Energy calculations
1511 'Force_M(:,2)=Mttf*uppi(:,i_Load);
1512
1513 Dim upiE() As Variant
1514 ReDim Preserve upiE(tmdof-1,0)
1515
1516 Dim uppiE() As Variant
1517 ReDim Preserve uppiE(tmdof-1,0)
1518
1519 For i=0 To 5
1520     For j=0 To nbc-1
1521         fsiE(i,j,1)=Fsi2(i,j)
1522         IMD_Gi(i,j,1)=IMD_G(i,j)
1523     Next
1524 Next
1525
1526 For i=0 To tmdof-1
1527     upiE(i,0)=upi(i,1)
1528     uppiE(i,0)=uppi(i,1)
1529 Next
1530
1531 Dim FeM() As Variant
1532 ReDim Preserve FeM(tmdof-1,0)
1533 FeM=Matrix_multiplication(Mttf,uppiE,tmdof,tmdof,tmdof,1)
1534
1535 Dim FeC() As Variant
1536 ReDim Preserve FeC(tmdof-1,0)
1537 FeC=Matrix_multiplication(Cff,upiE,tmdof,tmdof,tmdof,1)
1538
1539 For i=0 To tmdof-1
1540     Force_M(i,1)=FeM(i,0)
1541     Force_C(i,1)=FeC(i,0)
1542     objXls_u.Worksheets(12).Cells(i+2,i_Load+1).Value =FeM(i,0)
1543     objXls_u.Worksheets(13).Cells(i+2,i_Load+1).Value =FeC(i,0)
1544 Next
1545
1546 For i=0 To nfdof-1
1547     Disp_P(i,1)=uig(i,0)
1548     Force_P(i,1)=Puig(i,0)
1549 Next
1550
1551 For i=0 To nfdof-1
1552     Energy_Ps=Energy_Ps+0.5*(Disp_P(i,1)-Disp_P(i,0))*(Force_P(i,1)+Force_P(i,0))
1553 Next
1554
1555 For i=0 To tmdof-1
1556     Energy_M=Energy_M+0.5*(ui(i,1)-ui(i,0))*(Force_M(i,1)+Force_M(i,0))
1557     Energy_C=Energy_C+0.5*(ui(i,1)-ui(i,0))*(Force_C(i,1)+Force_C(i,0))
1558 Next
1559
1560 For i=0 To 5
1561     For j=0 To nbc-1
1562         Energy_K=Energy_K+0.5*(IMD_Gi(i,j,1)-IMD_Gi(i,j,0))*(fsiE(i,j,1)+fsiE(i,j,0))

```

```

1563     Next
1564 Next
1565
1566 Energy_H=Energy_H+0.5*(IMDsi(1,i_Load)-IMDsi(1,i_Load-1))*(IMFsi(1,i_Load)+IMFsi(1,i_Load-1))
1567
1568 Energy_Ms(i_Load,0)=Energy_M
1569 Energy-Cs(i_Load,0)=Energy_C
1570 Energy_Ks(i_Load,0)=Energy_K
1571 Energy_Hs(i_Load,0)=Energy_H
1572
1573 For i=0 To tmdof-1
1574 Force_M(i,0)=Force_M(i,1)
1575 Force_C(i,0)=Force_C(i,1)
1576 Next
1577
1578 Energy_Total(i_Load)=CDBl(Energy_Ms(i_Load,0)+Energy-Cs(i_Load,0)+Energy_Ks(i_Load,0)+Energy_Hs(i_Load,0))
1579 Energy_Stiffness(i_Load)=CDBl(Energy_Ks(i_Load,0)+Energy_Hs(i_Load,0)+Energy-Cs(i_Load,0))
1580 Energy_Damping(i_Load)=CDBl(Energy_Hs(i_Load,0)+Energy-Cs(i_Load,0))
1581 Energy_Hys(i_Load)=CDBl(Energy_Hs(i_Load,0))
1582
1583 'Plots Total Energy
1584 EA_Graph.PlotArrayXY(Panell,"Energy_plot",1,nt+1, xEarth(), Energy_Total())
1585 EA_Graph.SetPlotProperty(Panell,"Energy_plot",1,2,vbBlack)
1586 EA_Graph.SetPlotProperty(Panell,"Energy_plot",1,5,0)
1587
1588 EA_Graph.PlotArrayXY(Panell,"Energy_plot",2,nt+1, xEarth(), Energy_Stiffness())
1589 EA_Graph.SetPlotProperty(Panell,"Energy_plot",2,2,vbMagenta)
1590 EA_Graph.SetPlotProperty(Panell,"Energy_plot",2,5,0)
1591
1592 EA_Graph.PlotArrayXY(Panell,"Energy_plot",3,nt+1, xEarth(), Energy_Damping())
1593 EA_Graph.SetPlotProperty(Panell,"Energy_plot",3,2,vbBlue)
1594 EA_Graph.SetPlotProperty(Panell,"Energy_plot",3,5,0)
1595
1596 EA_Graph.PlotArrayXY(Panell,"Energy_plot",4,nt+1, xEarth(), Energy_Hys())
1597 EA_Graph.SetPlotProperty(Panell,"Energy_plot",4,2,vbRed)
1598 EA_Graph.SetPlotProperty(Panell,"Energy_plot",4,5,0)
1599
1600 EA_Graph.Refresh(Panell,"Energy_plot")
1601
1602 'Plots Strain values
1603 EA_Graph.PlotArrayXY(Panell2,"Strain_out_disp",1,nt+1, xEarth(), Strain_Left_disp())
1604 EA_Graph.SetPlotProperty(Panell2,"Strain_out_disp",1,2,vbBlue)
1605 EA_Graph.SetPlotProperty(Panell2,"Strain_out_disp",1,5,0)
1606
1607 EA_Graph.PlotArrayXY(Panell2,"Strain_out_disp",2,nt+1, xEarth(), Strain_right_disp())
1608 EA_Graph.SetPlotProperty(Panell2,"Strain_out_disp",2,2,vbRed)
1609 EA_Graph.SetPlotProperty(Panell2,"Strain_out_disp",2,5,0)
1610
1611 EA_Graph.Refresh(Panell2,"Strain_out_disp")
1612
1613 'Plots LVDT values out
1614 EA_Graph.PlotArrayXY(Panell2,"Disp_read_LVDT",1,nt+1, xEarth(), LVDT1_disp())
1615 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",1,2,vbBlack)
1616 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",1,5,0)
1617
1618 EA_Graph.PlotArrayXY(Panell2,"Disp_read_LVDT",2,nt+1, xEarth(), LVDT2_disp())
1619 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",2,2,vbRed)
1620 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",2,5,0)
1621
1622 EA_Graph.PlotArrayXY(Panell2,"Disp_read_LVDT",3,nt+1, xEarth(), LVDT3_disp())
1623 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",3,2,vbBlue)
1624 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",3,5,0)
1625
1626 EA_Graph.PlotArrayXY(Panell2,"Disp_read_LVDT",4,nt+1, xEarth(), LVDT4_disp())
1627 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",4,2,vbMagenta)
1628 EA_Graph.SetPlotProperty(Panell2,"Disp_read_LVDT",4,5,0)
1629
1630 EA_Graph.Refresh(Panell2,"Disp_read_LVDT")
1631
1632 'Calculates the Park and Ang damage intensity

```

```

1633 If Abs(IMDsi(1,i_Load))>di Then
1634     di=Abs(IMDsi(1,i_Load))
1635 End If
1636
1637 If (di/dumax)+(bPa/(fyspring*dumax))*Energy_H>Dpa Then
1638     Dpa=(di/dumax)+(bPa/(fyspring*dumax))*Energy_H
1639 End If
1640 EA_Panel.SetValue(Panel,"Damage_out",Dpa)
1641
1642 'Saves data to output file
1643 objXls_u.Worksheets(3).Cells(1,i_Load+1).Value =uppe(i_Load,0)
1644 objXls_u.Worksheets(11).Cells(1,i_Load+1).Value =uppe(i_Load,0)
1645 For i = 0 To nfdof-1
1646     objXls_u.Worksheets(3).Cells(i+2,i_Load+1).Value =uig(i,0)
1647     objXls_u.Worksheets(11).Cells(i+2,i_Load+1).Value =Puig(i,0)
1648 Next
1649
1650 'Saves the velocity and acceleration data
1651 objXls_u.Worksheets(4).Cells(1,i_Load+2).Value =uppe(i_Load,0)
1652 objXls_u.Worksheets(5).Cells(1,i_Load+2).Value =uppe(i_Load,0)
1653 For i = 0 To tmdof-1
1654     objXls_u.Worksheets(4).Cells(i+2,i_Load+2).Value =upi(i,1)
1655     objXls_u.Worksheets(5).Cells(i+2,i_Load+2).Value =uppi(i,1)
1656 Next
1657
1658 'Saves the energy data
1659 objXls_u.Worksheets(6).Cells(i_Load+2,1).Value =uppe(i_Load,0)
1660 objXls_u.Worksheets(6).Cells(i_Load+2,2).Value =Energy_M 'Mass or inertia energy /
kinetic energy
1661 objXls_u.Worksheets(6).Cells(i_Load+2,3).Value =Energy_C 'Damping energy
1662 objXls_u.Worksheets(6).Cells(i_Load+2,4).Value =Energy_K 'Stiffness
energy/potential energy
1663 objXls_u.Worksheets(6).Cells(i_Load+2,5).Value =Energy_H 'Hysteretic energy of the
footing
1664 objXls_u.Worksheets(6).Cells(i_Load+2,6).Value =Energy_Ps 'External energy input
1665
1666 objXls_u.Worksheets(6).Cells(i_Load+2,8).Value =Energy_H
1667 objXls_u.Worksheets(6).Cells(i_Load+2,9).Value =Energy_H+Energy_C
1668 objXls_u.Worksheets(6).Cells(i_Load+2,10).Value =Energy_H+Energy_C+Energy_K
1669 objXls_u.Worksheets(6).Cells(i_Load+2,11).Value =Energy_H+Energy_C+Energy_K+Energy_M
1670
1671 xSpring(i_Load)=CDbl(IMDsi(1,i_Load)*1000)
1672 ySpring(i_Load)=CDbl(IMFsi(1,i_Load)/1000)
1673 EA_Graph.PlotArrayXY(Panell,"Graph1",1,nt+1, xSpring(), ySpring())
1674 EA_Graph.SetPlotProperty(Panell,"Graph1",1,2,vbRed)
1675 EA_Graph.Refresh(Panell,"Graph1")
1676
1677 'Output for foundation and numerical procedure
1678
1679 objXls_u.Worksheets(2).Cells(i_Load+2,3).Value =IMDsi(1,i_Load)*1000
1680 objXls_u.Worksheets(2).Cells(i_Load+2,4).Value =IMFsi(1,i_Load)/1000
1681
1682 objXls_u.Worksheets(2).Cells(i_Load+2,5).Value =Dpa
1683 objXls_u.Worksheets(2).Cells(i_Load+2,6).Value =Norm_max
1684 objXls_u.Worksheets(2).Cells(i_Load+2,7).Value =num
1685 'objXls_u.Worksheets(2).Cells(i_Load+2,8).Value =Load_axial
1686
1687
1688 objXls_u.Workbooks(1).Save
1689 Next
1690
1691 objXls_u.Quit
1692
1693 End Sub
1694
1695 Sub static_condensation(ByRef Lm() As Variant,ByRef kotf() As Variant,ByRef kttv()
As Variant,ByRef fsi() As Variant,ByRef ktotf() As Variant,ByRef koof() As
Variant,ByRef Ps() As Variant,ByRef lh() As Variant,ByRef Mttf() As Variant,ByVal K
As Variant,ByVal mdof As Variant,ByVal nfdof As Variant,ByVal fso As Variant,ByVal
Uio As Variant,ByVal global_mass As Variant,ByVal Pwtdof As Variant,ByVal ptfof As
Variant,ByVal i_Load As Variant,ByVal nbc As Variant,ByVal tdof As Variant)
1696
1697 Dim Lmi() As Variant

```

```

1698 ReDim Preserve Lmi(nfdof-1,0)
1699 Dim Lhi() As Variant
1700 ReDim Preserve Lhi(nfdof-1,0)
1701 Dim Lvi() As Variant
1702 ReDim Preserve Lvi(nfdof-1,0)
1703
1704 For i=0 To nfdof-1
1705     Lmi(i,0)=0
1706     Lhi(i,0)=0
1707     Lvi(i,0)=0
1708 Next
1709
1710 For i=0 To 5
1711     For j=0 To nbc-1
1712         If i<>2 And i<>5 Then
1713             If mdof(i,j)<=nfdof Then
1714                 Lmi(mdof(i,j)-1,0)=1
1715             End If
1716         End If
1717         If i=0 Or i=3 Then
1718             If mdof(i,j)<=nfdof Then
1719                 Lhi(mdof(i,j)-1,0)=1
1720             End If
1721         End If
1722         If i=1 Or i=4 Then
1723             If mdof(i,j)<=nfdof Then
1724                 Lvi(mdof(i,j)-1,0)=1
1725             End If
1726         End If
1727     Next
1728 Next
1729
1730 Dim Lms As Variant
1731 Lms=Matrix_sum(Lmi,nfdof,1)
1732
1733 ReDim Preserve Lm(nfdof-1,0)
1734
1735 Dim counter As Variant
1736 counter=-1
1737 For i=0 To nfdof-1
1738     If Lmi(i,0)<>0 Then
1739         counter=counter+1
1740         Lm(counter,0)=i
1741     End If
1742 Next
1743 tmdof=counter+1
1744
1745 For i=0 To nfdof-1
1746     If Lmi(i,0)=0 Then
1747         counter=counter+1
1748         Lm(counter,0)=i
1749     End If
1750 Next
1751
1752 Dim psi() As Variant
1753 ReDim Preserve psi(tdof-1,0)
1754 psi=Matrix_addition(Pwtdof,ptfof,tdof,1)
1755
1756 Dim Kff() As Variant
1757 ReDim Preserve Kff(nfdof-1,nfdof-1)
1758
1759 Dim Mff() As Variant
1760 ReDim Preserve Mff(nfdof-1,nfdof-1)
1761
1762 For i=0 To nfdof-1
1763     For j=0 To nfdof-1
1764         Kff(i,j)=K(i,j)
1765         Mff(i,j)=global_mass(i,j)
1766     Next
1767 Next
1768
1769 Dim Kfft() As Variant
1770 ReDim Preserve Kfft(nfdof-1,nfdof-1)

```

```

1771 ReDim Preserve Mfft(nfdof-1,nfdof-1)
1772
1773 For i=0 To nfdof-1
1774     For j=0 To nfdof-1
1775         Kfft(i,j)=0
1776         Mfft(i,j)=0
1777     Next
1778 Next
1779
1780 Dim Fsto() As Variant
1781 ReDim Preserve Fsto(nfdof-1,0)
1782
1783 ReDim Preserve Ps(nfdof-1,0)
1784
1785 For i=0 To nfdof-1
1786     Fsto(i,0)=0
1787     Ps(i,0)=0
1788 Next
1789
1790 Dim lv() As Variant
1791 ReDim Preserve lv(tmdof-1,0)
1792
1793 ReDim Preserve lh(tmdof-1,0)
1794
1795 For i=0 To tmdof-1
1796     lv(i,0)=0
1797     lh(i,0)=0
1798 Next
1799
1800 For i=0 To nfdof-1
1801     Fsto(i,0)=fso(Lm(i,0),0)
1802     Ps(i,0)=psi(Lm(i,0),0)
1803     For j=0 To nfdof-1
1804         Kfft(i,j)=Kff(Lm(i,0),Lm(j,0))
1805         Mfft(i,j)=Mff(Lm(i,0),Lm(j,0))
1806     Next
1807 Next
1808
1809 Dim kttf() As Variant
1810 ReDim Preserve kttf(tmdof-1,tmdof-1)
1811
1812 For i=0 To tmdof-1
1813     For j=0 To tmdof-1
1814         kttf(i,j)=Kfft(i,j)
1815     Next
1816 Next
1817
1818 ReDim Preserve ktof(tmdof-1,nfdof-tmdof-1)
1819
1820 For i=0 To tmdof-1
1821     For j=0 To nfdof-tmdof-1
1822         ktof(i,j)=Kfft(i,j+tmdof)
1823     Next
1824 Next
1825
1826 ReDim Preserve kotf(nfdof-tmdof-1,tmdof-1)
1827
1828 For i=0 To nfdof-tmdof-1
1829     For j=0 To tmdof-1
1830         kotf(i,j)=Kfft(i+tmdof,j)
1831     Next
1832 Next
1833
1834 ReDim Preserve koof(nfdof-tmdof-1,nfdof-tmdof-1)
1835
1836 For i=0 To nfdof-tmdof-1
1837     For j=0 To nfdof-tmdof-1
1838         koof(i,j)=Kfft(i+tmdof,j+tmdof)
1839     Next
1840 Next
1841
1842 Dim koofinv() As Variant
1843 ReDim Preserve koofinv(nfdof-tmdof-1,nfdof-tmdof-1)

```

```

1844 koofinv=Matrix_inverse(koof,nfdof-tmdof,nfdof-tmdof)
1845
1846 Dim kttvi() As Variant
1847
1848 'kttv=kttf-ktof*inv(koof)*kotf;
1849 kttvi=Matrix_multiplication(ktof,koofinv,tmdof,nfdof-tmdof,nfdof-tmdof,nfdof-tmdof)
1850 kttvi=Matrix_multiplication(kttvi,kotf,tmdof,nfdof-tmdof,nfdof-tmdof,tmdof)
1851 ReDim kttv(tmdof,tmdof)
1852 kttv=Matrix_subtraction(kttf,kttvi,tmdof,tmdof)
1853
1854 'Mttf=Mfft(1:tmdof,1:tmdof)
1855 ReDim Mttf(tmdof-1,tmdof-1)
1856 For i=0 To tmdof-1
1857     For j=0 To tmdof-1
1858         Mttf(i,j)=Mfft(i,j)
1859     Next
1860 Next
1861
1862 'fsi(:,i_Load)=Fsto(1:tmdof,1)-ktof*inv(koof)*Fsto(tmdof+1:nfdof,1);
1863 ReDim Preserve fsi(tmdof-1,0)
1864
1865 Dim fsiinv() As Variant
1866
1867 Dim Fstol() As Variant
1868 ReDim Preserve Fstol(tmdof-1,0)
1869 For i=0 To tmdof-1
1870     Fstol(i,0)=Fsto(i,0)
1871 Next
1872
1873 Dim Fsto2() As Variant
1874 ReDim Preserve Fsto2(nfdof-tmdof-1,0)
1875
1876 For i=0 To nfdof-tmdof-1
1877     Fsto2(i,0)=Fsto(i+tmdof,0)
1878 Next
1879
1880 fsiinv=Matrix_multiplication(ktof,koofinv,tmdof,nfdof-tmdof,nfdof-tmdof,nfdof-tmdof)
1881 fsiinv=Matrix_multiplication(fsiinv,Fsto2,tmdof,nfdof-tmdof,nfdof-tmdof,1)
1882 fsi=Matrix_subtraction(Fstol,fsiinv,tmdof,1)
1883
1884 For i=0 To tmdof-1
1885     lv(i,0)=Lvi(Lm(i,0),0)
1886     lh(i,0)=Lhi(Lm(i,0),0)
1887 Next
1888
1889 End Sub
1890
1891 Private Sub PrintMatrix(ByRef Prop As Variant) ' Keeps giving a ) error, I need to
1892 pass a multidimensional array into a sub
1893 'The code works when I run it In visual studio
1894     For i = 0 To 3
1895         For j = 0 To 2
1896             EA_Panel.SetCell(Panell,"PropTable",i+1,j+1,Prop(i,j))
1897         Next
1898     Next
1899 End Sub
1900
1901 Function assemble_beam_stiffness(ByRef global_stiffness As Variant,ByVal tdof As
1902 Variant,ByVal K_beam_local As Variant,ByVal nbc As Variant,ByVal mdof As Variant) As
1903 Variant
1904     For i = 0 To tdof-1
1905         For j = 0 To tdof-1
1906             global_stiffness(i,j)=0
1907         Next
1908     Next
1909
1910 'Assembles the Global stiffness matrix
1911 For i=0 To nbc-1
1912     For j=0 To 5
1913         For K=0 To 5
1914
1915             global_stiffness(mdof(j,i)-1,mdof(K,i)-1)=global_stiffness(mdof(j,i)-1,mdof(K,

```

```

        i)-1)+K_beam_local(j,K,i)
1913     Next
1914     Next
1915     Next
1916
1917 End Function
1918
1919 Sub assemble_local_stiffness_beam(ByRef estiff_local As Variant,ByRef etran_local As
Variant,ByRef K_beam_local As Variant,ByRef etranT_local As Variant,ByVal nbc As
Variant,ByVal Prop As Variant,ByVal idbc As Variant,ByVal mem_info As Variant)
1920
1921 For j=0 To nbc-1
1922
1923     Dim local_stiff() As Variant
1924     Dim local_etran() As Variant
1925     Dim local_etranTrans() As Variant
1926     Dim local_Kbeam() As Variant
1927
1928     phi=mem_info(1,j)
1929     local_stiff=estiff_BEAM(Prop(1,idbc(j,2)-1),Prop(0,idbc(j,2)-1),Prop(2,idbc(j,2)-1),me
m_info(0,j))
1930     local_etran=etran(phi)
1931     local_etranTrans=eTransT(phi)
1932
1933     'M_beam_local{1,j}=transpose(gamma)*mass*gamma
1934     local_Kbeam=Matrix_multiplication(local_etranTrans,local_stiff,6,6,6,6)
1935     local_Kbeam=Matrix_multiplication(local_Kbeam,local_etran,6,6,6,6)
1936
1937     For i=0 To 5
1938         For K= 0 To 5
1939             estiff_local(i,K,j)=local_stiff(i,K)
1940             etran_local(i,K,j)=local_etran(i,K)
1941             etranT_local(i,K,j)=local_etranTrans(i,K)
1942             K_beam_local(i,K,j)=local_Kbeam(i,K)
1943         Next
1944     Next
1945 Next
1946
1947 End Sub
1948
1949 Function estiff_BEAM(ByVal a As Variant,ByVal e As Variant,ByVal i As Variant,ByVal
l As Variant ) As Variant
1950
1951 Dim elk() As Variant
1952 ReDim Preserve elk(5,5)
1953
1954 elk(0,0)=(a*e)/1
1955 elk(0,1)=0
1956 elk(0,2)=0
1957 elk(0,3)=(-a*e)/1
1958 elk(0,4)=0
1959 elk(0,5)=0
1960
1961 elk(1,0)=0
1962 elk(1,1)=(12*e*i)/1^3
1963 elk(1,2)=(6*e*i)/1^2
1964 elk(1,3)=0
1965 elk(1,4)=(-12*e*i)/1^3
1966 elk(1,5)=(6*e*i)/1^2
1967
1968 elk(2,0)=0
1969 elk(2,1)=(6*e*i)/1^2
1970 elk(2,2)=(4*e*i)/1
1971 elk(2,3)=0
1972 elk(2,4)=(-6*e*i)/1^2
1973 elk(2,5)=(2*e*i)/1
1974
1975
1976 elk(3,0)=(-a*e)/1
1977 elk(3,1)=0
1978 elk(3,2)=0
1979 elk(3,3)=(a*e)/1
1980 elk(3,4)=0

```

```

1981     elk(3,5)=0
1982
1983     elk(4,0)=0
1984     elk(4,1)=(-12*e*i)/1^3
1985     elk(4,2)=(-6*e*i)/1^2
1986     elk(4,3)=0
1987     elk(4,4)=(12*e*i)/1^3
1988     elk(4,5)=(-6*e*i)/1^2
1989
1990     elk(5,0)=0
1991     elk(5,1)=(6*e*i)/1^2
1992     elk(5,2)=(2*e*i)/1
1993     elk(5,3)=0
1994     elk(5,4)=(-6*e*i)/1^2
1995     elk(5,5)=(4*e*i)/1
1996
1997     estiff_BEAM=elk
1998
1999     End Function
2000
2001     Function etrans(ByVal phi As Variant) As Variant
2002
2003     Dim Gamma() As Variant
2004     ReDim Preserve Gamma(5,5)
2005
2006     For i=0 To 5
2007         For j=0 To 5
2008             Gamma(i,j)=0
2009         Next
2010     Next
2011
2012     Gamma(0,0)=Cos(phi)
2013     Gamma(0,1)=Sin(phi)
2014     Gamma(1,0)=-Sin(phi)
2015     Gamma(1,1)=Cos(phi)
2016     Gamma(2,2)=1
2017     Gamma(3,3)=Cos(phi)
2018     Gamma(3,4)=Sin(phi)
2019     Gamma(4,3)=-Sin(phi)
2020     Gamma(4,4)=Cos(phi)
2021     Gamma(5,5)=1
2022
2023     etrans=Gamma
2024
2025     End Function
2026
2027     Function eTransT(ByVal phi As Variant) As Variant
2028     Dim Gamma() As Variant
2029     ReDim Preserve Gamma(5,5)
2030
2031     For i=0 To 5
2032         For j=0 To 5
2033             Gamma(i,j)=0
2034         Next
2035     Next
2036
2037     Gamma(0,0)=Cos(phi)
2038     Gamma(0,1)=-Sin(phi)
2039     Gamma(1,0)=Sin(phi)
2040     Gamma(1,1)=Cos(phi)
2041     Gamma(2,2)=1
2042     Gamma(3,3)=Cos(phi)
2043     Gamma(3,4)=-Sin(phi)
2044     Gamma(4,3)=Sin(phi)
2045     Gamma(4,4)=Cos(phi)
2046     Gamma(5,5)=1
2047
2048     eTransT=Gamma
2049
2050     End Function
2051
2052     Function Matrix_multiplication(ByVal matrix1 As Variant, ByVal Matrix2 As
Variant,ByVal m1 As Variant,ByVal n1 As Variant, ByVal m2 As Variant, ByVal n2 As

```

```

Variant) As Variant
2053
2054 Dim result() As Variant
2055 ReDim Preserve result(m1-1,n2-1)
2056 'EA_Panel.SetCell(Panell1,"PropTable",1,1,m1)
2057 'EA_Panel.SetCell(Panell1,"PropTable",1,2,n2)
2058
2059 For i =0 To m1-1
2060     For j = 0 To n2-1
2061         For K=0 To n1-1
2062             result(i,j)=result(i,j)+matrix1(i,K)*Matrix2(K,j)
2063         Next
2064     Next
2065 Next
2066 Matrix_multiplication=result
2067
2068 End Function
2069
2070 Function Matrix_solver(ByVal a As Variant,ByVal b As Variant,ByVal n As Variant) As
Variant
2071
2072 'To reduce the coefficient matrix To the upper triangular form And Then solve With
backward substitution
2073 'Reduce the coefficient matrix To upper triangular form
2074 Dim m As Variant
2075 Dim pt() As Variant
2076 Dim pivot As Variant
2077 Dim counter As Variant
2078
2079 For K=0 To n-2
2080     For i=K+1 To n-1
2081         If a(K,K)<>0 Then
2082             m=a(i,K)/a(K,K)
2083         ElseIf a(K,K)=0 Then
2084             pt=eye(n)
2085             pivot=a(K,K)
2086             counter=K
2087             While pivot =0 And counter<n-1
2088                 counter=counter+1
2089                 pivot=a(counter,K)
2090             Wend
2091             pt(K,K)=0
2092             pt(K,counter)=1
2093             pt(counter,K)=1
2094             pt(counter,counter)=0
2095             a=Matrix_multiplication(pt,a,n,n,n,n)
2096             b=Matrix_multiplication(pt,b,n,n,n,1)
2097             m=a(i,K)/a(K,K)
2098         End If
2099         For j=K+1 To n-1
2100             a(i,K)=0
2101             a(i,j)=a(i,j)-m*a(K,j)
2102         Next
2103         b(i,0)=b(i,0)-m*b(K,0)
2104     Next
2105 Next
2106
2107 'Solve With backward substitution
2108 Dim x() As Variant 'x=zeros(n,1);
2109 ReDim Preserve x(n-1,0)
2110 For i=0 To n-1
2111     x(i,0)=0
2112 Next
2113 Dim s As Variant
2114
2115 For K=n-1 To 0 Step -1
2116     s=0
2117     For j=K To n-1
2118         If K<>j Then
2119             s=s+a(K,j)*x(j,0)
2120         End If
2121     Next
2122     x(K,0)=(b(K,0)-s)/a(K,K)

```

```

2123 Next
2124 Matrix_solver=x
2125
2126
2127 End Function
2128
2129 Function eye(ByVal n As Variant) As Variant
2130
2131 Dim a() As Variant
2132 ReDim Preserve a(n-1,n-1)
2133
2134 For i=0 To n-1
2135     For j=0 To n-1
2136         If i=j Then
2137             a(i,j)=1
2138         Else
2139             a(i,j)=0
2140         End If
2141     Next
2142 Next
2143 eye=a
2144
2145 End Function
2146
2147
2148 Function Spring_stiff(ByRef global_stiffness As Variant ,ByVal Column_hinge As
Variant,ByVal dof As Variant,ByVal tdof As Variant,ByVal ks As Variant,ByRef Smdof
As Variant)
2149
2150 'Adds the spring stiffness In the Global matrix
2151
2152 Smdof(0,0)= tdof
2153 Smdof(1,0)= dof(0,Column_hinge-1)
2154
2155 Dim KSpring() As Variant
2156 ReDim Preserve KSpring(1,1)
2157
2158 KSpring(0,0)=ks
2159 KSpring(0,1)=-ks
2160 KSpring(1,0)=-ks
2161 KSpring(1,1)=ks
2162
2163 'Assembles the Global stiffness matrix
2164 For j=0 To 1
2165     For K=0 To 1
2166
2167         global_stiffness(Smdof(j,0)-1,Smdof(K,0)-1)=global_stiffness(Smdof(j,0)-1,Smdof(K,
0)-1)+KSpring(j,K)
2167     Next
2168 Next
2169
2170 Spring_stiff=KSpring
2171
2172 End Function
2173
2174 Function beam_loads(ByVal loading_beam As Variant,ByVal nbc As Variant,ByVal
etranT_local As Variant,ByVal dof As Variant,ByVal mem_info As Variant,ByVal idbc As
Variant,ByVal tdof As Variant,ByVal nfdof As Variant,ByRef Pw_local As Variant,ByRef
DOF_c As Variant,ByRef Pwtdof() As Variant) As Variant
2175
2176 For i=0 To nbc-1
2177     For j=0 To 5
2178         Pw_local(j,i)=0
2179         DOF_c(j,i)=0
2180     Next
2181 Next
2182
2183 Dim Wbeam() As Variant
2184 ReDim Preserve Wbeam(nbc-1,0)
2185
2186 For i=0 To nbc-1
2187     Pw_local(0,loading_beam(i,0)-1)=0
2188

```

```

Pw_local(1,loading_beam(i,0)-1)=- (loading_beam(i,1)*mem_info(0,loading_beam(i,0)-1
2189 ))/2
Pw_local(2,loading_beam(i,0)-1)=- (loading_beam(i,1)*mem_info(0,loading_beam(i,0)-1
2190 )^2)/12
2191 Pw_local(3,loading_beam(i,0)-1)=0
Pw_local(4,loading_beam(i,0)-1)=- (loading_beam(i,1)*mem_info(0,loading_beam(i,0)-1
2192 ))/2
Pw_local(5,loading_beam(i,0)-1)=(loading_beam(i,1)*mem_info(0,loading_beam(i,0)-1
2193 )^2)/12
2194 Wbeam(loading_beam(i,0)-1,0)=loading_beam(i,1)
2195 Next
2196 Dim Pw_global() As Variant
2197 ReDim Preserve Pw_global(5,nbc-1)
2198
2199 For i=0 To nbc-1
2200     Dim trans() As Variant
2201     ReDim Preserve trans(5,5) As Variant
2202     Dim LocalPW() As Variant
2203     ReDim Preserve LocalPW(5,5) As Variant
2204
2205     For j=0 To 5
2206         For K=0 To 5
2207             trans(j,K)=etranT_local(j,K,i)
2208         Next
2209         LocalPW(j,0)=Pw_local(j,i)
2210     Next
2211
2212     local_Kbeam=Matrix_multiplication(trans,LocalPW,6,6,6,1)
2213
2214     For K=0 To 5
2215         Pw_global(K,i)=local_Kbeam(K,0)
2216     Next
2217
2218     For K=0 To 2
2219         DOF_c(K,i)=dof(K,idbc(i,0)-1)
2220         DOF_c(K+3,i)=dof(K,idbc(i,1)-1)
2221     Next
2222 Next
2223
2224 Dim Pw() As Variant
2225 ReDim Preserve Pw(nfdof-1,0)
2226
2227 For i=0 To nfdof-1
2228     Pw(i,0)=0
2229 Next
2230
2231 For i=0 To nbc-1
2232     For j=0 To 5
2233         Pwtdof(DOF_c(j,i)-1,0)=Pwtdof(DOF_c(j,i)-1,0)+Pw_global(j,i)
2234     Next
2235 Next
2236
2237 For i=0 To nfdof-1
2238     Pw(i,0)=Pwtdof(i,0)
2239 Next
2240
2241 beam_loads=Pw
2242
2243 End Function
2244
2245 Function solve(ByVal global_stiffness As Variant,ByVal R As Variant,ByVal nfdof As
Variant) As Variant
2246
2247 Dim Kff() As Variant
2248 ReDim Preserve Kff(nfdof-1,nfdof-1)
2249
2250 For i=0 To nfdof-1
2251     For j=0 To nfdof-1
2252         Kff(i,j)=global_stiffness(i,j)

```

```

2253     Next
2254 Next
2255
2256 solve=Matrix_solver(Kff,R,nfdof)
2257
2258 End Function
2259
2260 Function Matrix_addition(ByVal a As Variant,ByVal b As Variant,ByVal m1 As
Variant,ByVal n1 As Variant) As Variant
2261
2262 Dim Out() As Variant
2263 ReDim Preserve Out(m1-1,n1-1)
2264
2265 For i=0 To m1-1
2266     For j=0 To n1-1
2267         Out(i,j)=a(i,j)+b(i,j)
2268     Next
2269 Next
2270 Matrix_addition=Out
2271
2272 End Function
2273
2274 Function Matrix_subtraction(ByVal a As Variant,ByVal b As Variant,ByVal m1 As
Variant,ByVal n1 As Variant) As Variant
2275
2276 Dim Out() As Variant
2277 ReDim Preserve Out(m1-1,n1-1)
2278
2279 For i=0 To m1-1
2280     For j=0 To n1-1
2281         Out(i,j)=a(i,j)-b(i,j)
2282     Next
2283 Next
2284 Matrix_subtraction=Out
2285
2286 End Function
2287
2288 Sub memf(ByRef IMF_L As Variant,ByRef IMD_L As Variant,ByRef IMD_G As Variant,ByRef
IMF_G As Variant,ByRef fso As Variant,ByVal supp As Variant,ByVal nnode As
Variant,ByVal Disp As Variant,ByVal nbc As Variant,ByVal idbc As Variant,ByVal
etran_local As Variant,ByVal etranT_local As Variant,ByVal estiff_local As
Variant,ByVal Pw_local As Variant)
2289 '[IMF_L,IMD_L,IMD_G,IMF_G,fso,coord_out]=memf(supp,Disp,estiff_local,etran_local,idbc,
nbc,coord,nnode,Pw_local)
2290
2291 Dim displacement_out() As Variant
2292 ReDim Preserve displacement_out(2,nnode-1)
2293 'displacement_out=zeros(m,n)
2294 Dim count As Variant
2295 count=-1
2296
2297 For j=0 To nnode-1
2298     For i=0 To 2
2299         If supp(i,j)=1 Then
2300             displacement_out(i,j)=0
2301         ElseIf supp(i,j)=0 Then
2302             count=count+1
2303             displacement_out(i,j)=Disp(count,0)
2304         End If
2305     Next
2306 Next
2307
2308 'Transform {displacement_out} from Global To local coordinates by multiplying With
member local stiffness matrix
2309 'And transformation matrix
2310
2311 For i=0 To nbc-1
2312
2313     For K=0 To 2
2314         IMD_G(K,i)=displacement_out(K,idbc(i,0)-1)
2315         IMD_G(K+3,i)=displacement_out(K,idbc(i,1)-1)
2316     Next
2317

```

```

2318 Dim IMDLi() As Variant
2319 ReDim Preserve IMDLi(5,0)
2320 Dim IMFLi() As Variant
2321 ReDim Preserve IMFLi(5,0)
2322 Dim IMFGi() As Variant
2323 ReDim Preserve IMFGi(5,0)
2324 Dim fsoi() As Variant
2325 ReDim Preserve fsoi(5,0)
2326 Dim PwL() As Variant
2327 ReDim Preserve PwL(5,0)
2328
2329 Dim trans As Variant
2330 ReDim Preserve trans(5,5)
2331 Dim transT As Variant
2332 ReDim Preserve transT(5,5)
2333 Dim estiff As Variant
2334 ReDim Preserve estiff(5,5)
2335
2336 For j=0 To 5
2337     For K=0 To 5
2338         trans(j,K)=etran_local(j,K,i)
2339         transT(j,K)=etranT_local(j,K,i)
2340         estiff(j,K)=estiff_local(j,K,i)
2341     Next
2342     IMDLi(j,0)=IMD_G(j,i)
2343     PwL(j,0)=Pw_local(j,i)
2344 Next
2345
2346 IMDLi=Matrix_multiplication(trans,IMDLi,6,6,6,1)
2347 IMFLi=Matrix_multiplication(estiff,IMDLi,6,6,6,1)
2348 fsoi=Matrix_multiplication(transT,IMFLi,6,6,6,1)
2349 IMFLi=Matrix_subtraction(IMFLi,PwL,6,1)
2350 IMFGi=Matrix_multiplication(transT,IMFLi,6,6,6,1)
2351
2352 For j=0 To 5
2353     IMD_L(j,i)=IMDLi(j,0)
2354     IMF_L(j,i)=IMFLi(j,0)
2355     IMF_G(j,i)=IMFGi(j,0)
2356     fso(j,i)=fsoi(j,0)
2357 Next
2358
2359 Next
2360
2361 End Sub
2362
2363 Sub memf_spring(ByVal KSpring As Variant,ByVal Disp As Variant,ByVal Smdof As
Variant,ByRef ds As Variant,ByRef IMFs As Variant)
2364 'Calculates the New spring stiffness
2365
2366 ds(0,0)=0
2367 ds(1,0)=Disp(Smdof(1,0)-1,0)
2368 Dim IMFsi() As Variant
2369 IMFsi=Matrix_multiplication(KSpring,ds,2,2,2,1)
2370
2371 For i=0 To 1
2372     IMFs(i,0)=IMFsi(i,0)
2373 Next
2374
2375 End Sub
2376
2377 Sub initial_conditions_static(ByRef Fsil As Variant,ByRef Uil As Variant,ByVal fso
As Variant,ByVal IMD_G As Variant,ByVal mdof As Variant,ByVal tdof As Variant,ByVal
IMFs As Variant,ByVal Smdof As Variant,ByVal nbc As Variant)
2378
2379 For i=0 To nbc-1
2380     For j=0 To 5
2381         Fsil(mdof(j,i)-1,0)=Fsil(mdof(j,i)-1,0)+fso(j,i)
2382         Uil(mdof(j,i)-1,0)=IMD_G(j,i)
2383     Next
2384 Next
2385
2386 Fsil(Smdof(0,0)-1,0)=Fsil(Smdof(0,0)-1,0)+IMFs(0,0)
2387 Fsil(Smdof(1,0)-1,0)=Fsil(Smdof(1,0)-1,0)+IMFs(1,0)

```

```

2388
2389 End Sub
2390
2391 Function assemble_mass_self(ByRef global_mass As Variant,ByVal nbc As Variant,ByVal
Prop As Variant,ByVal idbc As Variant,ByVal mem_info As Variant,ByVal mdof As
Variant,ByVal tdof As Variant)
2392
2393 Dim M_beam_local As Variant
2394 ReDim Preserve M_beam_local(5,5,nbc-1)
2395
2396 For j=0 To nbc-1
2397
2398 Dim local_mass() As Variant
2399 Dim local_etran() As Variant
2400 Dim local_etranTrans() As Variant
2401 Dim local_Mself() As Variant
2402
2403 'e=Prop(0,idbc(j,2)-1)
2404 'a=Prop(1,idbc(j,2)-1)
2405 'i=Prop(2,idbc(j,2)-1)
2406 'l=mem_info(0,j)
2407 'den=Prop(3,idbc(j,2)-1)
2408
2409 phi=mem_info(1,j)
2410
2411 local_mass=mass_local(Prop(1,idbc(j,2)-1),mem_info(0,j),Prop(3,idbc(j,2)-1))
2412 local_etran=etran(phi)
2413 local_etranTrans=eTransT(phi)
2414
2415 'M_beam_local{1,j}=transpose(gamma)*mass*gamma
2416 local_Mself=Matrix_multiplication(local_etranTrans,local_mass,6,6,6,6)
2417 local_Mself=Matrix_multiplication(local_Mself,local_etran,6,6,6,6)
2418
2419 For i=0 To 5
2420 For K= 0 To 5
2421 M_beam_local(i,K,j)=local_Mself(i,K)
2422 Next
2423 Next
2424 Next
2425
2426 assemble_mass_self=M_beam_local
2427
2428 For i=0 To nbc-1
2429 For j=0 To 5
2430 For K=0 To 5
2431
2432 global_mass(mdof(j,i)-1,mdof(K,i)-1)=global_mass(mdof(j,i)-1,mdof(K,i)-1)+M_be
am_local(j,K,i)
2433 Next
2434 Next
2435
2436 End Function
2437
2438 Function mass_local(ByVal a As Variant,ByVal l As Variant,ByVal den As Variant) As
Variant
2439
2440 Dim Mass As Variant
2441 ReDim Preserve Mass(5,5)
2442
2443 For i=0 To 5
2444 For j= 0 To 5
2445 Mass(i,j)=0
2446 Next
2447 Next
2448
2449 Mass(0,0)=(a*1*den)/2
2450 Mass(1,1)=(a*1*den)/2
2451 Mass(3,3)=(a*1*den)/2
2452 Mass(4,4)=(a*1*den)/2
2453
2454 mass_local=Mass
2455

```

```

2456 End Function
2457
2458 Function assemble_mass_DL(ByRef global_mass As Variant,ByVal nbc As Variant,ByVal
idbc As Variant,ByVal mem_info As Variant,ByVal mdof As Variant,ByVal tdof As
Variant,ByVal tc As Variant,ByVal Span As Variant,ByVal Den_conc As Variant)
2459
2460 Dim a As Variant
2461 a=tc*Span
2462
2463 Dim M_beam_local As Variant
2464 ReDim Preserve M_beam_local(5,5,nbc-1)
2465
2466 For j=0 To nbc-1
2467
2468     If idbc(j,2)=2 Then
2469         Dim local_mass() As Variant
2470         Dim local_etran() As Variant
2471         Dim local_etranTrans() As Variant
2472         Dim local_MDL() As Variant
2473
2474         phi=mem_info(1,j)
2475
2476         local_mass=mass_local(a,mem_info(0,j),Den_conc)
2477         local_etran=etran(phi)
2478         local_etranTrans=eTransT(phi)
2479
2480         'M_beam_local{1,j}=transpose(gamma)*mass*gamma
2481         local_MDL=Matrix_multiplication(local_etranTrans,local_mass,6,6,6,6)
2482         local_MDL=Matrix_multiplication(local_MDL,local_etran,6,6,6,6)
2483
2484         For i=0 To 5
2485             For K= 0 To 5
2486                 M_beam_local(i,K,j)=local_MDL(i,K)
2487             Next
2488         Next
2489
2490     Else
2491         For i=0 To 5
2492             For K= 0 To 5
2493                 M_beam_local(i,K,j)=0
2494             Next
2495         Next
2496     End If
2497
2498 Next
2499
2500 assemble_mass_DL=M_beam_local
2501
2502 For i=0 To nbc-1
2503     For j=0 To 5
2504         For K=0 To 5
2505
2506             global_mass(mdof(j,i)-1,mdof(K,i)-1)=global_mass(mdof(j,i)-1,mdof(K,i)-1)+M_be
am_local(j,K,i)
2507         Next
2508     Next
2509
2510
2511 End Function
2512
2513 Function assemble_mass_LL(ByRef global_mass As Variant,ByVal nbc As Variant,ByVal
idbc As Variant,ByVal mem_info As Variant,ByVal mdof As Variant,ByVal tdof As
Variant,ByVal Span As Variant,ByVal LL As Variant)
2514
2515 Dim a As Variant
2516 a=Span
2517 Dim den As Variant
2518 den=LL/9.81
2519
2520 Dim M_beam_local As Variant
2521 ReDim Preserve M_beam_local(5,5,nbc-1)
2522

```

```

2523 For j=0 To nbc-1
2524
2525     If idbc(j,2)=2 Then
2526         Dim local_mass() As Variant
2527         Dim local_etran() As Variant
2528         Dim local_etranTrans() As Variant
2529         Dim local_MLL() As Variant
2530
2531         'e=Prop(0,idbc(j,2)-1)
2532         'a=Prop(1,idbc(j,2)-1)
2533         'i=Prop(2,idbc(j,2)-1)
2534         'l=mem_info(0,j)
2535         'den=Prop(3,idbc(j,2)-1)
2536
2537         phi=mem_info(1,j)
2538
2539         local_mass=mass_local(a,mem_info(0,j),den)
2540         local_etran=etran(phi)
2541         local_etranTrans=eTransT(phi)
2542
2543         'M_beam_local{1,j}=transpose(gamma)*mass*gamma
2544         local_MLL=Matrix_multiplication(local_etranTrans,local_mass,6,6,6,6)
2545         local_MLL=Matrix_multiplication(local_MLL,local_etran,6,6,6,6)
2546
2547         For i=0 To 5
2548             For K= 0 To 5
2549                 M_beam_local(i,K,j)=local_MLL(i,K)
2550             Next
2551         Next
2552     Else
2553
2554         For i=0 To 5
2555             For K= 0 To 5
2556                 M_beam_local(i,K,j)=0
2557             Next
2558         Next
2559
2560     End If
2561 Next
2562
2563 assemble_mass_LL=M_beam_local
2564
2565 For i=0 To nbc-1
2566     For j=0 To 5
2567         For K=0 To 5
2568
2569             global_mass(mdof(j,i)-1,mdof(K,i)-1)=global_mass(mdof(j,i)-1,mdof(K,i)-1)+M_be
2570             am_local(j,K,i)
2571         Next
2572     Next
2573
2574
2575
2576 End Function
2577
2578 Function Matrix_inverse(ByVal a As Variant,ByVal m1 As Variant,ByVal n1 As Variant)
2579
2580 Dim m As Variant
2581 Dim pt() As Variant
2582 Dim pivot As Variant
2583 Dim counter As Variant
2584 Dim b() As Variant
2585 b=eye(n1)
2586
2587 For K=0 To n1-2
2588     For i=K+1 To n1-1
2589         If a(K,K)<>0 Then
2590             m=a(i,K)/a(K,K)
2591
2592             ElseIf a(K,K)=0 Then
2593                 pt=eye(n1)

```

```

2594         pivot=a(K,K)
2595         counter=K
2596         While pivot =0 And counter<n1-1
2597             counter=counter+1
2598             pivot=a(counter,K)
2599         Wend
2600         pt(K,K)=0
2601         pt(K,counter)=1
2602         pt(counter,K)=1
2603         pt(counter,counter)=0
2604         a=Matrix_multiplication(pt,a,n1,n1,n1,n1)
2605         b=Matrix_multiplication(pt,b,n1,n1,n1,n1)
2606         m=a(i,K)/a(K,K)
2607         End If
2608         For j=K+1 To n1-1
2609             a(i,K)=0
2610             a(i,j)=a(i,j)-m*a(K,j)
2611         Next
2612         For j=0 To n1-1
2613             b(i,j)=b(i,j)-m*b(K,j)
2614         Next
2615     Next
2616 Next
2617
2618 For K=n1-1 To 1 Step -1
2619     For i=K-1 To 0 Step -1
2620
2621         m=a(i,K)/a(K,K)
2622
2623         For j=K-1 To 0 Step -1
2624             a(i,K)=0
2625             a(i,j)=a(i,j)-m*a(K,j)
2626         Next
2627
2628         For j=0 To n1-1
2629             b(i,j)=b(i,j)-m*b(K,j)
2630         Next
2631     Next
2632 Next
2633
2634 For i=0 To n1-1
2635     For j=0 To n1-1
2636         b(i,j)=b(i,j)/a(i,i)
2637     Next
2638 Next
2639
2640 Matrix_inverse=b
2641
2642 End Function
2643
2644 Sub static_condensation_initial(ByRef Lm As Variant,ByRef kttv() As Variant,ByRef
Uto As Variant,ByRef tmdof As Variant,ByRef Mttf() As Variant,ByVal K As
Variant,ByVal global_mass As Variant,ByVal mdof As Variant,ByVal nfdof As
Variant,ByVal fso As Variant,ByVal Uio As Variant,ByVal nbc As Variant)
2645 'The model incorporates Static condensation of the frame structure To only
2646 'include certain degrees of freedom where the mass will act
2647 'The model accounts For only horizontal mass And vertical mass response And
2648 'ignores the rotational mass moment of inertia
2649 'The Function reorders the matrixes And solves For the equivalent
2650 'statically condensed stiffnes matix
2651 '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2652
2653 Dim Lmi() As Variant
2654 ReDim Preserve Lmi(nfdof-1,0)
2655     For i=0 To nfdof-1
2656         Lmi(i,0)=0
2657     Next
2658
2659 For i=0 To 5
2660     For j=0 To nbc-1
2661         If i<>2 And i<>5 Then
2662             If mdof(i,j)<=nfdof Then
2663                 Lmi(mdof(i,j)-1,0)=1

```

```

2664         End If
2665     End If
2666     Next
2667 Next
2668
2669 Dim Lms As Variant
2670 Lms=Matrix_sum(Lmi,nfdof,1)
2671
2672 Dim counter As Variant
2673 counter=-1
2674 For i=0 To nfdof-1
2675     If Lmi(i,0)<>0 Then
2676         counter=counter+1
2677         Lm(counter,0)=i
2678     End If
2679 Next
2680 tmdof=counter+1
2681
2682 For i=0 To nfdof-1
2683     If Lmi(i,0)=0 Then
2684         counter=counter+1
2685         Lm(counter,0)=i
2686     End If
2687 Next
2688
2689 Dim Kff() As Variant
2690 ReDim Preserve Kff(nfdof-1,nfdof-1)
2691
2692 Dim Mff() As Variant
2693 ReDim Preserve Mff(nfdof-1,nfdof-1)
2694
2695 For i=0 To nfdof-1
2696     For j=0 To nfdof-1
2697         Kff(i,j)=K(i,j)
2698         Mff(i,j)=global_mass(i,j)
2699     Next
2700 Next
2701
2702 Dim Kfft() As Variant
2703 ReDim Preserve Kfft(nfdof-1,nfdof-1)
2704
2705 Dim Mfft() As Variant
2706 ReDim Preserve Mfft(nfdof-1,nfdof-1)
2707
2708 For i=0 To nfdof-1
2709     For j=0 To nfdof-1
2710         Kfft(i,j)=0
2711         Mfft(i,j)=0
2712     Next
2713 Next
2714
2715 Dim Fsto() As Variant
2716 ReDim Preserve Fsto(nfdof-1,0)
2717
2718 For i=0 To nfdof-1
2719     Uto(i,0)=0
2720     Fsto(i,0)=0
2721 Next
2722
2723 For i=0 To nfdof-1
2724     Fsto(i,0)=fso(Lm(i,0),0)
2725     Uto(i,0)=Uio(Lm(i,0),0)
2726     For j=0 To nfdof-1
2727         Kfft(i,j)=Kff(Lm(i,0),Lm(j,0))
2728         Mfft(i,j)=Mff(Lm(i,0),Lm(j,0))
2729     Next
2730 Next
2731
2732 Dim kttf() As Variant
2733 ReDim Preserve kttf(tmdof-1,tmdof-1)
2734
2735 For i=0 To tmdof-1
2736     For j=0 To tmdof-1

```

```

2737         kttf(i,j)=Kfft(i,j)
2738     Next
2739 Next
2740
2741 Dim ktof() As Variant
2742 ReDim Preserve ktof(tmdof-1,nfdof-tmdof-1)
2743
2744 For i=0 To tmdof-1
2745     For j=0 To nfdof-tmdof-1
2746         ktof(i,j)=Kfft(i,j+tmdof)
2747     Next
2748 Next
2749
2750 Dim kotf() As Variant
2751 ReDim Preserve kotf(nfdof-tmdof-1,tmdof-1)
2752
2753 For i=0 To nfdof-tmdof-1
2754     For j=0 To tmdof-1
2755         kotf(i,j)=Kfft(i+tmdof,j)
2756     Next
2757 Next
2758
2759 Dim koof() As Variant
2760 ReDim Preserve koof(nfdof-tmdof-1,nfdof-tmdof-1)
2761
2762 For i=0 To nfdof-tmdof-1
2763     For j=0 To nfdof-tmdof-1
2764         koof(i,j)=Kfft(i+tmdof,j+tmdof)
2765     Next
2766 Next
2767
2768 Dim koofinv() As Variant
2769 ReDim Preserve koofinv(nfdof-tmdof-1,nfdof-tmdof-1)
2770 koofinv=Matrix_inverse(koof,nfdof-tmdof,nfdof-tmdof)
2771
2772 Dim kttvi() As Variant
2773
2774 'kttv=kttf-ktof*inv(koof)*kotf;
2775 kttvi=Matrix_multiplication(ktof,koofinv,tmdof,nfdof-tmdof,nfdof-tmdof,nfdof-tmdof)
2776 kttvi=Matrix_multiplication(kttvi,kotf,tmdof,nfdof-tmdof,nfdof-tmdof,tmdof)
2777 ReDim kttv(tmdof,tmdof)
2778 kttv=Matrix_subtraction(kttf,kttvi,tmdof,tmdof)
2779
2780 'Mttf=Mfft(1:tmdof,1:tmdof)
2781 ReDim Mttf(tmdof-1,tmdof-1)
2782 For i=0 To tmdof-1
2783     For j=0 To tmdof-1
2784         Mttf(i,j)=Mfft(i,j)
2785     Next
2786 Next
2787
2788 End Sub
2789
2790 Function Matrix_sum(ByVal a As Variant,ByVal m As Variant,ByVal n As Variant) As Variant
2791
2792 Dim Sumi As Variant
2793 Sumi=0
2794
2795 For i=0 To m-1
2796     For j=0 To n-1
2797         Sumi=Sumi+a(i,j)
2798     Next
2799 Next
2800
2801 Matrix_sum=Sumi
2802
2803 End Function
2804
2805 Function Rayleigh_damping_nonlinear(ByVal eig As Variant,ByVal m As Variant,ByVal K As Variant,ByVal Dp As Variant,ByVal tmdof As Variant)
2806 'Determine the Constants ao And al
2807 'Rayleigh damping (2 modes)

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```

2808 ' [A]{a}=2[D]
2809 'Construction of the [A]
2810
2811 Dim a() As Variant
2812 ReDim Preserve a(1,1)
2813 Dim D() As Variant
2814 ReDim Preserve D(1,0)
2815 Dim R() As Variant
2816 ReDim Preserve R(1,0)
2817 Dim Damp() As Variant
2818 ReDim Preserve Damp(1,0)
2819
2820 a(0,0)=1/eig(0,0)
2821 a(0,1)=eig(0,0)
2822 a(1,0)=1/eig(1,0)
2823 a(1,1)=eig(1,0)
2824
2825 D(0,0)=2*Dp
2826 D(1,0)=2*Dp
2827
2828 R=Matrix_solver(a,D,2)
2829
2830 Dim mi() As Variant
2831 ReDim Preserve mi(tmdof-1,tmdof-1)
2832 Dim ki() As Variant
2833 ReDim Preserve ki(tmdof-1,tmdof-1)
2834
2835 mi=Matrix_multiplication_constant(R(0,0),m,tmdof,tmdof)
2836 ki=Matrix_multiplication_constant(R(1,0),K,tmdof,tmdof)
2837
2838 Rayleigh_damping_nonlinear=Matrix_addition(mi,ki,tmdof,tmdof)
2839
2840 End Function
2841
2842
2843 Function Matrix_multiplication_constant(ByVal c As Variant,ByVal matrixi As
Variant,ByVal m As Variant,ByVal n As Variant) As Variant
2844
2845 Dim mati() As Variant
2846 ReDim Preserve mati(m-1,n-1)
2847
2848 For i=0 To m-1
2849     For j=0 To n-1
2850         mati(i,j)=c*matrixi(i,j)
2851     Next
2852 Next
2853
2854 Matrix_multiplication_constant=mati
2855
2856
2857 End Function
2858
2859 Function eigenvalues(ByVal Mass As Variant,ByVal Kstiff As Variant,ByVal n As Variant)
2860
2861 'Finds the first two eigenvalues  $\det(K-\omega n^2 M)=0$ 
2862
2863 Dim rangew As Variant
2864 rangew=6000
2865 Dim y() As Variant
2866 ReDim Preserve y(rangew-1,0)
2867 Dim x() As Variant
2868 ReDim Preserve x(rangew-1,0)
2869
2870 x=linspace(0,200000,rangew)
2871
2872 For ii=0 To rangew-1
2873     wi=x(ii,0)
2874
2875     Dim mi() As Variant
2876     ReDim Preserve mi(n-1,n-1)
2877     Dim a() As Variant
2878     ReDim Preserve a(n-1,n-1)
2879

```

```

2880 mi=Matrix_multiplication_constant(wi,Mass,n,n)
2881 a=Matrix_subtraction(Kstiff,mi,n,n)
2882
2883 Dim m As Variant
2884 Dim pt() As Variant
2885 Dim pivot As Variant
2886 Dim counter As Variant
2887
2888 For K=0 To n-2
2889     For i=K+1 To n-1
2890         If a(K,K)<>0 Then
2891             m=a(i,K)/a(K,K)
2892         ElseIf a(K,K)=0 Then
2893             pt=eye(n)
2894             pivot=a(K,K)
2895             counter=K
2896             While pivot =0 And counter<n-1
2897                 counter=counter+1
2898                 pivot=a(counter,K)
2899             Wend
2900             pt(K,K)=0
2901             pt(K,counter)=1
2902             pt(counter,K)=1
2903             pt(counter,counter)=0
2904             a=Matrix_multiplication(pt,a,n,n,n,n)
2905             m=a(i,K)/a(K,K)
2906         End If
2907         For j=K+1 To n-1
2908             a(i,K)=0
2909             a(i,j)=a(i,j)-m*a(K,j)
2910         Next
2911     Next
2912 Next
2913
2914 Dim deti As Variant
2915 deti=1
2916 For i=0 To n-1
2917     deti=deti*a(i,i)
2918 Next
2919 y(ii,0)=deti
2920 Next
2921
2922 Dim counter2 As Variant
2923 counter2=0
2924 Dim A1 As Variant
2925 Dim B1 As Variant
2926 Dim c1 As Variant
2927 Dim D1 As Variant
2928 Dim det As Variant
2929 Dim Eig_values() As Variant
2930 ReDim Preserve Eig_values(1,0)
2931 i=0
2932 Do Until counter2=2
2933     i=i+1
2934     If y(i,0)=0 Then
2935         counter2=counter2+1
2936         A1=y(i-1,0)
2937         B1=y(i,0)
2938         c1=x(i-1,0)
2939         D1=x(i,0)
2940         det=(((-A1)*(D1-c1)/(B1-A1))+c1
2941         Eig_values(counter2-1,0)=Sqr(det)
2942     ElseIf (y(i-1,0)/y(i,0))<0 Then
2943         counter2=counter2+1
2944         A1=y(i-1,0)
2945         B1=y(i,0)
2946         c1=x(i-1,0)
2947         D1=x(i,0)
2948         det=(((-A1)*(D1-c1)/(B1-A1))+c1
2949         Eig_values(counter2-1,0)=Sqr(det)
2950     End If
2951 Loop
2952

```

```

2953     eigenvalues=Eig_values
2954
2955 End Function
2956
2957 Function linspace(ByVal min As Variant,ByVal max As Variant,ByVal n As Variant) As
Variant
2958
2959 Dim part() As Variant
2960 ReDim Preserve part(n-1,0)
2961
2962 For I=0 To n-1
2963     part(I,0)=((max-min)/(n-1))*I+min
2964 Next
2965
2966     linspace=part
2967
2968 End Function
2969
2970 Function Import_data(ByVal nt As Variant ) As Variant
2971
2972 Dim Earthquake() As Variant
2973 ReDim Preserve Data(nt-1,1)
2974 Dim objXls As Object
2975 Set objXls = CreateObject("Excel.Application")
2976 objXls.Workbooks.Open "C:\Octave\Dynamics\Pseudo\ElCentro0.87g.xlsx"
2977 For i=0 To nt-1
2978     Data(i,0)=objXls.Worksheets(1).Cells(i+1,1).Value
2979     Data(i,1)=objXls.Worksheets(1).Cells(i+1,2).Value
2980 Next
2981 objXls.Quit
2982
2983 Import_data=Data
2984
2985 End Function
2986
2987 Sub Sort_matrix(ByRef mati As Variant,ByRef mat As Variant,ByVal m As Variant,ByVal
n As Variant)
2988
2989 For i=0 To m-1
2990     For j=0 To n-1
2991         mati(i,j,1)=mat(i,j)
2992     Next
2993 Next
2994
2995 End Sub
2996
2997 Function vector_sort(ByRef veci As Variant,ByRef vec As Variant,ByVal m As
Variant,ByVal col As Variant)
2998
2999 For i=0 To m-1
3000     veci(i,col)=vec(i,0)
3001 Next
3002
3003 End Function
3004
3005 Sub Initial_sort_mat(ByRef mati As Variant,ByVal m As Variant,ByVal n As
Variant,ByVal matfrom As Variant,ByVal matto As Variant)
3006
3007 For i=0 To m-1
3008     For j=0 To n-1
3009         mati(i,j,matto)=mati(i,j,matfrom)
3010     Next
3011 Next
3012
3013 End Sub
3014
3015 Sub Initial_sort_vec(ByRef veci As Variant,ByVal m As Variant,ByVal vecfrom As
Variant,ByVal vecto As Variant)
3016
3017 For i=0 To m-1
3018     veci(i,vecto)=veci(i,vecfrom)
3019 Next
3020

```

```

3021 End Sub
3022
3023 Sub Dynamic_analysis_coefficients(ByRef a_1() As Variant,ByRef a_2() As
Variant,ByRef a_3() As Variant,ByVal dt As Variant,ByVal Mttf As Variant,ByVal Cff
As Variant,ByVal tmdof As Variant )
3024 'a_1=(1/(Beta*dt^2))*Mttf+(Gamma/(Beta*dt))*Cff
3025 'a_2=(1/(Beta*dt))*Mttf+((Gamma/Beta)-1)*Cff
3026 'a_3=((1/(2*Beta))-1)*Mttf+dt*((Gamma/(2*Beta))-1)*Cff
3027
3028 Dim Beta As Variant
3029 Beta=0.25
3030 Dim Gamma As Variant
3031 Gamma=0.5
3032
3033 ReDim Preserve a_1(tmdof-1,tmdof-1)
3034 ReDim Preserve a_2(tmdof-1,tmdof-1)
3035 ReDim Preserve a_3(tmdof-1,tmdof-1)
3036
3037 a_1=Matrix_addition(Matrix_multiplication_constant(1/(Beta*dt^2),Mttf,tmdof,tmdof),Matr
ix_multiplication_constant(Gamma/(Beta*dt),Cff,tmdof,tmdof),tmdof,tmdof)
3038 a_2=Matrix_addition(Matrix_multiplication_constant(1/(Beta*dt),Mttf,tmdof,tmdof),Matrix
_multiplication_constant((Gamma/Beta)-1,Cff,tmdof,tmdof),tmdof,tmdof)
3039 a_3=Matrix_addition(Matrix_multiplication_constant((1/(2*Beta))-1,Mttf,tmdof,tmdof),Mat
rix_multiplication_constant(dt*((Gamma/(2*Beta))-1),Cff,tmdof,tmdof),tmdof,tmdof)
3040
3041 End Sub
3042
3043 Function Pa_calc(ByVal tmdof As Variant,ByVal nfdof As Variant,ByVal Ps As
Variant,ByVal PEarth As Variant,ByVal ktof As Variant,ByVal koof As Variant) As
Variant
3044 'Pa=P+Ps(1:tmdof,1)-ktof*(koof\Ps(tmdof+1:nfdof,1))
3045 Dim Pa() As Variant
3046 ReDim Preserve Pa(tmdof-1,0)
3047
3048 Dim ps1() As Variant
3049 ReDim Preserve ps1(tmdof-1,0)
3050 Dim ps2() As Variant
3051 ReDim Preserve ps2(nfdof-tmdof-1,0)
3052
3053 For i=0 To tmdof-1
3054 ps1(i,0)=Ps(i,0)
3055 Next
3056
3057 For i=0 To nfdof-tmdof-1
3058 ps2(i,0)=Ps(i+tmdof,0)
3059 Next
3060
3061 Dim koofs() As Variant
3062 ReDim Preserve koofs(nfdof-tmdof-1,0)
3063 Pa=Matrix_subtraction(Matrix_addition(PEarth,ps1,tmdof,1),Matrix_multiplication(ktof,M
atrix_solver(koof,ps2,nfdof-tmdof),tmdof,nfdof-tmdof,nfdof-tmdof,1),tmdof,1)
3064 Pa_calc=Pa
3065
3066 End Function
3067
3068 Function Pp_calcs(ByVal Pa As Variant,ByVal tmdof As Variant,ByVal ui As
Variant,ByVal upi As Variant,ByVal uppi As Variant,ByVal a_1 As Variant,ByVal a_2 As
Variant,ByVal a_3 As Variant) As Variant
3069
3070 Dim Pp As Variant
3071 ReDim Preserve Pp(tmdof-1,0)
3072
3073 Dim Ppdis As Variant
3074 ReDim Preserve Ppdis(tmdof-1,0)
3075 Dim Ppvel As Variant
3076 ReDim Preserve Ppvel(tmdof-1,0)
3077 Dim Ppac As Variant
3078 ReDim Preserve Ppac(tmdof-1,0)
3079 Dim uipp() As Variant
3080 ReDim Preserve uipp(tmdof-1,0)
3081 Dim upipp() As Variant
3082 ReDim Preserve upipp(tmdof-1,0)
3083 Dim uppipp() As Variant

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```

3084 ReDim Preserve uppipp(tmdof-1,0)
3085
3086 For i=0 To tmdof-1
3087     uipp(i,0)=ui(i,0)
3088     upipp(i,0)=upi(i,0)
3089     uppipp(i,0)=uppi(i,0)
3090 Next
3091
3092 Ppdis=Matrix_multiplication(a_1,uipp,tmdof,tmdof,tmdof,1)
3093 Ppvel=Matrix_multiplication(a_2,upipp,tmdof,tmdof,tmdof,1)
3094 Ppac=Matrix_multiplication(a_3,uppipp,tmdof,tmdof,tmdof,1)
3095
3096 Pp=Matrix_addition(Pa,Ppdis,tmdof,1)
3097 Pp=Matrix_addition(Pp,Ppvel,tmdof,1)
3098 Pp=Matrix_addition(Pp,Ppac,tmdof,1)
3099 Pp_calcs=Pp
3100
3101 End Function
3102
3103 Function R_calc(ByVal Pp As Variant,ByVal fsi As Variant,ByVal a_1 As Variant,ByVal
ui As Variant,ByVal tmdof As Variant)
3104 'R=Pp-fsi(:,i_Load)-a1*ui(:,i_Load)
3105 Dim R() As Variant
3106 ReDim Preserve R(tmdof-1,0)
3107
3108 Dim uiR() As Variant
3109 ReDim Preserve uiR(tmdof-1,0)
3110 For i=0 To tmdof-1
3111     uiR(i,0)=ui(i,1)
3112 Next
3113
3114 Dim R2() As Variant
3115 ReDim Preserve R2(tmdof-1,0)
3116
3117 R2=Matrix_multiplication(a_1,uiR,tmdof,tmdof,tmdof,1)
3118 R=Matrix_subtraction(Pp,fsi,tmdof,1)
3119 R=Matrix_subtraction(R,R2,tmdof,1)
3120 R_calc=R
3121
3122 End Function
3123
3124 Function Norm_Residual(ByVal Residual As Variant, ByVal tmdof As Variant) As Variant
3125 'Calculates the norm of the array
3126 Dim max As Variant
3127 max=Abs(Residual(0,0))
3128
3129 For i=1 To tmdof-1
3130     If Abs(Residual(i,0))>max Then
3131         max=Abs(Residual(i,0))
3132     End If
3133 Next
3134 Norm_Residual=max
3135
3136 End Function
3137
3138 Sub static_condensation_out(ByRef uig() As Variant,ByRef uigdu() As Variant,ByRef
Puig() As Variant,ByVal ui As Variant,ByVal kotf As Variant,ByVal koof As
Variant,ByVal Ps As Variant,ByVal Lm As Variant,ByVal i_Load As Variant,ByVal tmdof
As Variant,ByVal nfdof As Variant,ByVal tdof As Variant,ByVal du As Variant,ByVal
PEarth As Variant)
3139
3140 'Reverses the static condensation
3141 Dim ps1() As Variant
3142 ReDim Preserve ps1(tmdof-1,0)
3143 Dim ps2() As Variant
3144 ReDim Preserve ps2(nfdof-tmdof-1,0)
3145
3146 For i=0 To tmdof-1
3147     ps1(i,0)=Ps(i,0)
3148 Next
3149
3150 For i=0 To nfdof-tmdof-1
3151     ps2(i,0)=Ps(i+tmdof,0)

```

```

3152 Next
3153
3154 'Pload=p+Ps(1:tmdof,1);
3155 Dim Pload() As Variant
3156 ReDim Preserve Pload(tmdof-1,0)
3157 Pload=Matrix_addition(PEarth,ps1,tmdof,1)
3158 Dim uiR() As Variant
3159 ReDim Preserve uiR(tmdof-1,0)
3160 For i=0 To tmdof-1
3161   uiR(i,0)=ui(i,1)
3162 Next
3163
3164 'uo=koof\Ps(tmdof+1:nfdof,1)-koof\(kotf*ui(:,i_Load))
3165 Dim uo1() As Variant
3166 ReDim Preserve uo1(nfdof-tmdof-1,0)
3167 Dim uo2() As Variant
3168 ReDim Preserve uo2(tmdof-1,0)
3169 Dim uo() As Variant
3170 ReDim Preserve uo(nfdof-tmdof-1,0)
3171
3172 uo1=Matrix_solver(koof,ps2,nfdof-tmdof)
3173
3174 uo2=Matrix_solver(koof,Matrix_multiplication(kotf,uiR,nfdof-tmdof,tmdof,tmdof,1),nfdof
-tmdof)
3175 uo=Matrix_subtraction(uo1,uo2,nfdof-tmdof,1)
3176
3177 'uodu=-koof\(kotf*du(:,1))
3178 Dim uodu() As Variant
3179 ReDim Preserve uodu(nfdof-tmdof,0)
3180 uodu=Matrix_multiplication_constant(-1,Matrix_solver(koof,Matrix_multiplication(kotf,du
,nfdof-tmdof,tmdof,tmdof,1),nfdof-tmdof),nfdof-tmdof,1)
3181
3182 'uoi=[ui(:,i_Load);uo]
3183 'uoidu=[du(:,1);uodu];
3184 Dim uoi() As Variant
3185 ReDim Preserve uoi(nfdof-1,0)
3186 Dim uoidu() As Variant
3187 ReDim Preserve uoidu(nfdof-1,0)
3188 'Pload=[Pload;Ps(tmdof+1:nfdof,1)];
3189 Dim counter As Variant
3190 Dim Pload1() As Variant
3191 ReDim Preserve Pload1(nfdof-1,0)
3192
3193 counter=-1
3194 For i=0 To tmdof-1
3195   counter=counter+1
3196   uoi(counter,0)=uiR(i,0)
3197   uoidu(counter,0)=du(i,0)
3198   Pload1(counter,0)=Pload(i,0)
3199 Next
3200 'Pload=[Pload;Ps(tmdof+1:nfdof,1)];
3201
3202 For i=0 To nfdof-tmdof-1
3203   counter=counter+1
3204   uoi(counter,0)=uo(i,0)
3205   uoidu(counter,0)=uodu(i,0)
3206   Pload1(counter,0)=ps2(i,0)
3207 Next
3208
3209 ReDim Preserve uig(tmdof-1,0)
3210 ReDim Preserve uigdu(tmdof-1,0)
3211 ReDim Preserve Puig(tmdof-1,0)
3212
3213 For i=0 To tmdof-1
3214   uig(i,0)=0
3215   uigdu(i,0)=0
3216   Puig(i,0)=0
3217 Next
3218
3219 For i=0 To nfdof-1
3220   uig(Lm(i,0),0)=uoi(i,0)
3221   uigdu(Lm(i,0),0)=uoidu(i,0)
3222   Puig(Lm(i,0),0)=Pload1(i,0)

```

```

3223 Next
3224 End Sub
3225
3226 Sub internal_dynamic_loads(ByRef IMD_G() As Variant,ByRef IMD_L() As Variant,ByRef
IMF_L() As Variant,ByRef IMF_G() As Variant,ByRef Fsi2() As Variant,ByVal nbc As
Variant,ByVal uig As Variant,ByVal mdof As Variant,ByVal etran_local As
Variant,ByVal etranT_local As Variant,ByVal estiff_local As Variant,ByVal Pw_local
As Variant)
3227
3228 'duv=zeros(6,nbc)
3229 Dim duv() As Variant
3230 ReDim Preserve duv(5,nbc-1)
3231
3232 For i=0 To nbc-1
3233     For j=0 To 5
3234         duv(j,i)=uig(mdof(j,i)-1,0)
3235     Next
3236 Next
3237
3238 'Calculate the internal member force at the time
3239
3240 'Transform {displacement_out} from Global To local coordinates by multiplying With
member local stiffness matrix
3241 'And transformation matrix
3242 ReDim Preserve Fsi2(5,nbc-1)
3243 For i=0 To nbc-1
3244     For K=0 To 5
3245         IMD_G(K,i)=duv(K,i)
3246     Next
3247
3248     Dim IMDLi() As Variant
3249     ReDim Preserve IMDLi(5,0)
3250     Dim IMFLi() As Variant
3251     ReDim Preserve IMFLi(5,0)
3252     Dim IMFGi() As Variant
3253     ReDim Preserve IMFGi(5,0)
3254     Dim fsoi() As Variant
3255     ReDim Preserve fsoi(5,0)
3256     Dim PwL() As Variant
3257     ReDim Preserve PwL(5,0)
3258
3259     Dim trans As Variant
3260     ReDim Preserve trans(5,5)
3261     Dim transT As Variant
3262     ReDim Preserve transT(5,5)
3263     Dim estiff As Variant
3264     ReDim Preserve estiff(5,5)
3265
3266     For j=0 To 5
3267         For K=0 To 5
3268             trans(j,K)=etran_local(j,K,i)
3269             transT(j,K)=etranT_local(j,K,i)
3270             estiff(j,K)=estiff_local(j,K,i)
3271         Next
3272         IMDLi(j,0)=IMD_G(j,i)
3273         PwL(j,0)=Pw_local(j,i)
3274     Next
3275     IMDLi=Matrix_multiplication(trans,IMDLi,6,6,6,1)
3276     IMFLi=Matrix_multiplication(estiff,IMDLi,6,6,6,1)
3277     fsoi=Matrix_multiplication(transT,IMFLi,6,6,6,1)
3278     IMFLi=Matrix_subtraction(IMFLi,PwL,6,1)
3279     IMFGi=Matrix_multiplication(transT,IMFLi,6,6,6,1)
3280
3281     For j=0 To 5
3282         IMD_L(j,i)=IMDLi(j,0)
3283         IMF_L(j,i)=IMFLi(j,0)
3284         IMF_G(j,i)=IMFGi(j,0)
3285         Fsi2(j,i)=fsoi(j,0)
3286     Next
3287
3288 Next
3289 End Sub
3290

```

```

3291 Sub memf_spring_dynamic(ByRef Fs As Variant,ByRef IMFsi As Variant,ByRef IMDsi As
Variant,ByVal KSpring As Variant,ByVal uig As Variant,ByVal Smdof As Variant,ByVal
uigdu As Variant,ByVal i_Load As Variant)
3292
3293 Dim ds() As Variant
3294 ReDim Preserve ds(1,0)
3295 Dim dsidu() As Variant
3296 ReDim Preserve dsidu(1,0)
3297
3298 For I=0 To 1
3299     ds(I,0)=0
3300     dsidu(I,0)=0
3301 Next
3302
3303 ds(1,0)=uig(Smdof(1,0)-1,0)
3304 dsidu(1,0)=uigdu(Smdof(1,0)-1,0)
3305
3306 'IMFs=KSpring*dsidu
3307 Dim IMFs() As Variant
3308 ReDim Preserve IMFs(1,0)
3309 IMFs=Matrix_multiplication(KSpring,dsidu,2,2,2,1)
3310
3311 'IMFsi{i_Load}=IMFsi{i_Load}+IMFs;
3312 IMFsi(0,i_Load)=IMFsi(0,i_Load)+IMFs(0,0)
3313 IMFsi(1,i_Load)=IMFsi(1,i_Load)+IMFs(1,0)
3314
3315 'IMDsi{i_Load}=IMDsi{i_Load}+dsidu;
3316 IMDsi(0,i_Load)=IMDsi(0,i_Load)+dsidu(0,0)
3317 IMDsi(1,i_Load)=IMDsi(1,i_Load)+dsidu(1,0)
3318
3319 'Fs=IMFsi{i_Load}(2,1)
3320 Fs=IMFsi(1,i_Load)
3321
3322 End Sub
3323
3324 Sub Post_calcs_nonlinear_dynamic(ByRef Fsil As Variant,ByRef ks As Variant,ByRef
IMFsi As Variant,ByVal Fsi2 As Variant,ByVal Fs As Variant,ByVal mdof As
Variant,ByVal Smdof As Variant,ByVal tdof As Variant,ByVal IMDsi As Variant,ByVal
i_Load As Variant,ByVal nbc As Variant)
3325
3326 For i=0 To tdof-1
3327     Fsil(i,0)=0
3328 Next
3329
3330 For i=0 To nbc-1
3331     For j=0 To 5
3332         Fsil(mdof(j,i)-1,0)=Fsil(mdof(j,i)-1,0)+Fsi2(j,i)
3333     Next
3334 Next
3335
3336 Fsil(Smdof(0,0)-1,0)=Fsil(Smdof(0,0)-1,0)-Fs
3337 Fsil(Smdof(1,0)-1,0)=Fsil(Smdof(1,0)-1,0)+Fs
3338
3339 IMFsi(1,i_Load)=Fs
3340 IMFsi(0,i_Load)=-Fs
3341
3342 If Abs((IMDsi(1,i_Load)-IMDsi(1,i_Load-1)))*1000<0.0001 Then
3343     EA_Panel.GetCell(Panell1,"Converge_time_table",1,1,converge_time_step)
3344 ElseIf Abs((IMDsi(1,i_Load)-IMDsi(1,i_Load-1)))*1000>=0.0001 Then
3345     ks=Abs((IMFsi(1,i_Load)-IMFsi(1,i_Load-1))/(IMDsi(1,i_Load)-IMDsi(1,i_Load-1)))
3346 End If
3347 End Sub
3348
3349 Sub velocity_acceleration(ByVal ui As Variant,ByRef upi As Variant,ByRef uppi As
Variant,ByVal dt As Variant,ByVal tmdof As Variant)
3350
3351 Dim Gamma As Variant
3352 Dim Beta As Variant
3353 Gamma=0.5
3354 Beta=0.25
3355
3356 For i= 0 To tmdof-1
3357     upi(i,1)=(Gamma/(Beta*dt))* (ui(i,1)-ui(i,0))+(1-Gamma/Beta)*upi(i,0)+dt*(1-Gamma/(2*Be

```

```

3358     ta)) *uppi(i, 0)
3359     uppi(i, 1) = (1 / (Beta * dt ^ 2)) * (ui(i, 1) - ui(i, 0)) - (1 / (Beta * dt)) * upi(i, 0) - ((1 / (2 * Beta)) - 1) * up
3360     pi(i, 0)
3361     Next
3362
3363     End Sub
3364
3365     Function reactions(ByVal global_stiffness As Variant, ByVal Disp As Variant, ByVal
3366     nfdof As Variant, ByVal tdof As Variant) As Variant
3367
3368     Dim Rs() As Variant
3369     Dim Ksf() As Variant
3370     ReDim Preserve Ksf(tdof - nfdof - 1, nfdof - 1)
3371     Dim fixdof As Variant
3372     fixdof = tdof - nfdof
3373
3374     For i = 0 To fixdof - 1
3375         For j = 0 To nfdof - 1
3376             Ksf(i, j) = global_stiffness(i + nfdof, j)
3377         Next
3378     Next
3379
3380     Rs = Matrix_multiplication(Ksf, Disp, tdof - nfdof, nfdof, nfdof, 1)
3381     reactions = Rs
3382
3383     End Function
3384
3385     Function assemble_mass_Point(ByRef global_mass As Variant, ByVal dof As Variant, ByVal
3386     tdof As Variant, ByVal N_mas As Variant, ByVal Nodes_mas As Variant, ByVal M_mas As
3387     Variant, ByVal mdof As Variant) As Variant
3388     'Assembles the mass as a result of the masonry walls
3389
3390     Dim global_mass_Point() As Variant
3391     ReDim Preserve global_mass_Point(tdof - 1, tdof - 1)
3392     For i = 0 To tdof - 1
3393         For j = 0 To tdof - 1
3394             global_mass_Point(i, j) = 0
3395         Next
3396     Next
3397
3398     For i = 0 To N_mas - 1
3399         For j = 0 To 1
3400             global_mass_Point(dof(j, Nodes_mas(i, 0) - 1) - 1, dof(j, Nodes_mas(i, 0) - 1) - 1) = global_mass
3401             _Point(dof(j, Nodes_mas(i, 0) - 1) - 1, dof(j, Nodes_mas(i, 0) - 1) - 1) + M_mas
3402         Next
3403     Next
3404     assemble_mass_Point = global_mass_Point
3405
3406     End Function
3407
3408     Sub Axial_stop_OnClick
3409     'Automatically created procedure: Do not change or delete name or signature!
3410     EA_Panel.SetCell(Pane13, "Axial_stop_table", 1, 1, 1)
3411
3412     End Sub
3413
3414     Sub Converge_OnClick
3415     'Automatically created procedure: Do not change or delete name or signature!
3416     EA_Panel.SetCell(Pane11, "Converge_table", 1, 1, 1)
3417
3418     End Sub
3419
3420     Sub Converge_time_OnClick
3421     'Automatically created procedure: Do not change or delete name or signature!
3422     EA_Panel.SetCell(Pane11, "Converge_time_table", 1, 1, 1)
3423
3424     End Sub

```



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APPENDIX B

CYCLIC LOAD EXPERIMENTATION SCRIPT

```

1 Sub Main
2 'Performs a cyclic load test
3 EA_IO.ZeroBalanceControl("Displacement_Hor",1)
4 EA_IO.ZeroBalanceControl("Force_Hor",1)
5 EA_IO.SetAnalogOut("PMX_1 CH 9",1,0,1)
6
7 'Activating plots In the panel And setting them To zero
8 EA_Graph.RemovePlot(Panell,"Graph1",1)
9 EA_Graph.Refresh(Panell,"Graph1")
10 EA_Graph.RemovePlot(Panell,"Displacement_out",1)
11 EA_Graph.Refresh(Panell,"Displacement_out")
12 EA_Graph.RemovePlot(Panell,"LVDT_plot",1)
13 EA_Graph.Refresh(Panell,"LVDT_plot")
14 EA_Graph.RemovePlot(Panell,"Strain_out_disp",1)
15 EA_Graph.Refresh(Panell,"Strain_out_disp")
16 'EA_IO.ZeroBalanceControl("MX840_SR",1)
17 'EA_IO.ZeroBalanceControl("MX840_SL",1)
18 EA_IO.ZeroBalanceControl("PMX_LVDT1",1)
19 EA_IO.ZeroBalanceControl("PMX_LVDT2",1)
20 EA_IO.ZeroBalanceControl("PMX_LVDT3",1)
21 EA_IO.ZeroBalanceControl("PMX_LVDT4",1)
22 EA_IO.ZeroBalanceControl("Displacement_Hor",1)
23 EA_IO.ZeroBalanceControl("Force_Hor",1)
24
25 End Sub
26
27 Sub Axial_load
28 'Runs data logging during axial load application
29 Dim Analysis As Variant 'Input to change for the number of analysis being performed
30 Analysis=2
31 Dim objXls_a As Object
32 Set objXls_a = CreateObject("Excel.Application")
33 objXls_a.Workbooks.Add
34 objXls_a.Worksheets(1).Name = "Linear results"
35 objXls_a.Workbooks(1).SaveAs
36 "C:\Octave\Dynamics\Pseudo\Axial_unload_test_pseudo1_linear"+CStr(Analysis)+".xlsx"
37 ' "C:\Octave\Dynamics\Pseudo\OutputDispOutput4.xls"
38
39 objXls_a.Worksheets(1).Cells(1,1).Value ="Counter"
40 objXls_a.Worksheets(1).Cells(1,2).Value ="Axial load (kN)"
41 objXls_a.Worksheets(1).Cells(1,3).Value ="Strain gauge left (micro)"
42 objXls_a.Worksheets(1).Cells(1,4).Value ="Strain gauge right (micro)"
43
44 Dim Force_axial As Double
45 Dim Strain_Left As Double
46 Dim Strain_right As Double
47 Dim Axial_load_applied() As Double
48 Dim Strain_Left_disp() As Double
49 Dim Strain_right_disp() As Double
50 Dim StepInc() As Double
51
52 Dim Button2 As Variant
53 Button2=0
54 Dim counter As Variant
55 counter=0
56
57 EA_Panel.SetCell(Panell,"Axial_stop_table",1,1,0)
58
59 Do While Button2=0 'Convergence at each time step
60
61 EA_Panel.GetCell(Panell,"Axial_stop_table",1,1,Button2)
62
63 ReDim Preserve StepInc(counter+1)
64 ReDim Preserve Axial_load_applied(counter+1)
65 ReDim Preserve Strain_Left_disp(counter+1)
66 ReDim Preserve Strain_right_disp(counter+1)
67 StepInc(counter)=CDBl(counter)
68 objXls_a.Worksheets(1).Cells(counter+2,1).Value =counter
69
70 'Read out axial force from servo controller
71 EA_IO.Measure("Load cell",Force_axial,1) 'Axial force from load cell
72 objXls_a.Worksheets(1).Cells(counter+2,2).Value =Force_axial 'Saves force to
73 excel spreadsheet

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```

71     Axial_load_applied(counter)=CDbl(Force_axial) 'Stores force in array for
       plotting
72
73     'Read out Strain gauge Left from servo controller
74     EA_IO.Measure("MX840_SL",Strain_Left,1) 'Left strain gauge reading from
       strain gauge
75     objXls_a.Worksheets(1).Cells(counter+2,3).Value =Strain_Left 'Saves left
       strain gauge reading to excel spreadsheet
76     Strain_Left_disp(counter)=CDbl(Strain_Left) 'Stores left strain gauge
       reading in array for plotting
77
78     'Read out Strain gauge Right from servo controller
79     EA_IO.Measure("MX840_SR",Strain_right,1) 'Right strain gauge reading from
       strain gauge
80     objXls_a.Worksheets(1).Cells(counter+2,4).Value =Strain_right 'Saves right
       strain gauge reading to excel spreadsheet
81     Strain_right_disp(counter)=CDbl(Strain_right) 'Stores right strain gauge
       reading in array for plotting
82
83     'Axial load out (Plotting)
84     EA_Graph.PlotArrayXY(Panel2,"Axial_load_initial",1,counter+1, StepInc(),
       Axial_load_applied())
85     EA_Graph.SetPlotProperty(Panel2,"Axial_load_initial",1,2,vbRed)
86     EA_Graph.Refresh(Panel2,"Axial_load_initial")
87
88     'Plots Strain values (Plotting)
89     EA_Graph.PlotArrayXY(Panel2,"Strain_gauge_axial",1,counter+1, StepInc(),
       Strain_right_disp())
90     EA_Graph.SetPlotProperty(Panel2,"SStrain_gauge_axial",1,2,vbBlue)
91     EA_Graph.SetPlotProperty(Panel2,"Strain_gauge_axial",1,5,0)
92
93     counter=counter+1 'Counter per loop
94     objXls_a.Workbooks(1).Save 'Save excel spreadsheet
95     Loop
96
97     objXls_a.Workbooks(1).Save 'Final save of excel spreadsheet
98     objXls_a.Quit 'Closes excel spreadsheet
99 End Sub
100
101 Sub Cyclic_test
102 'Runs a cyclic test on the specimum
103 'Pulling (Tension)      Negative
104 'Pushing (Compression) Positive
105 EA_IO.ZeroBalanceControl("Displacement_Hor",1)
106 EA_IO.ZeroBalanceControl("Force_Hor",1)
107
108 'Data files - Excel output of the data
109 Dim Analysis As Variant 'Input to change for the number of analysis being performed
110 Analysis=4
111 Dim objXls_u As Object
112 Set objXls_u = CreateObject("Excel.Application")
113 objXls_u.Workbooks.Add
114 objXls_u.Worksheets(1).Name = "Linear results"
115 objXls_u.Workbooks(1).SaveAs
       "C:\Octave\Dynamics\Pseudo\Pseudo1_load_test_0512"+CStr(Analysis)+".xlsx" '
       "C:\Octave\Dynamics\Pseudo\OutputDispOutput4.xls"
116 Dim n As Variant
117 Dim pi As Variant
118 pi=4*Atn(1)
119 Dim x() As Variant
120 x=Sine_increment(n)
121 Dim analog_out As Variant
122
123 objXls_u.Worksheets(1).Cells(1,1).Value ="Displacement (mm) "
124 objXls_u.Worksheets(1).Cells(1,2).Value ="Analog out (V) "
125 objXls_u.Worksheets(1).Cells(1,3).Value ="Displacement in (mm) "
126 objXls_u.Worksheets(1).Cells(1,4).Value ="Horizontal force (N) "
127 objXls_u.Worksheets(1).Cells(1,5).Value ="Vertical force (kN) "
128 objXls_u.Worksheets(1).Cells(1,7).Value ="LVDT1 (mm) "
129 objXls_u.Worksheets(1).Cells(1,8).Value ="LVDT2 (mm) "
130 objXls_u.Worksheets(1).Cells(1,9).Value ="LVDT3 (mm) "
131 objXls_u.Worksheets(1).Cells(1,10).Value ="LVDT4 (mm) "
132 objXls_u.Worksheets(1).Cells(1,12).Value ="Strain gauge left (micro) "

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```

133 objXls_u.Worksheets(1).Cells(1,13).Value ="Strain gauge right (micro)"
134
135 Dim Disp_read_servo As Double
136 Dim Force_servo_hor As Double
137 Dim Force_axial As Double
138 Dim LVDT1 As Double
139 Dim LVDT2 As Double
140 Dim LVDT3 As Double
141 Dim LVDT4 As Double
142 Dim StrainLeft As Double
143 Dim StrainRight As Double
144 objXls_u.Workbooks(1).Save
145
146 Dim Displacement_xOut() As Double
147 ReDim Preserve Displacement_xOut(n-1)
148 Dim Displacement_xIn() As Double
149 ReDim Preserve Displacement_xIn(n-1)
150 Dim Force_hor() As Double
151 ReDim Preserve Force_hor(n-1)
152 Dim Strain_Left As Double
153 Dim Strain_right As Double
154 Dim Strain_Left_disp() As Double
155 ReDim Preserve Strain_Left_disp(n-1)
156 Dim Strain_right_disp() As Double
157 ReDim Preserve Strain_right_disp(n-1)
158
159 Dim LVDT1_disp() As Double
160 ReDim Preserve LVDT1_disp(n-1)
161 Dim LVDT2_disp() As Double
162 ReDim Preserve LVDT2_disp(n-1)
163 Dim LVDT3_disp() As Double
164 ReDim Preserve LVDT3_disp(n-1)
165 Dim LVDT4_disp() As Double
166 ReDim Preserve LVDT4_disp(n-1)
167 Dim Axial_load() As Double
168 ReDim Preserve Axial_load(n-1)
169 Dim StepInc() As Double
170 ReDim Preserve StepInc(n-1)
171 Dim Button_stop As Variant
172 Button_stop=0
173
174 EA_Panel.SetCell(Panell1,"Linear_push_stop",1,1,0)
175
176 'Starts applying the horizontal load
177 For i_Load=0 To n-1
178
179 EA_Panel.SetValue(Panell1,"Counterlout",i_Load)
180
181 StepInc(i_Load)=CDbl(i_Load)
182
183 Dim xout As Variant
184 xout=x(i_Load,0)
185 objXls_u.Worksheets(1).Cells(i_Load+2,1).Value =xout 'Displacement
186
187 analog_out=(1/-6.6025)*xout 'volts from mm (Check to make sure of correct direction)
188 EA_IO.SetAnalogOut("PMX_1 CH 9",1,analog_out,1)
189 objXls_u.Worksheets(1).Cells(i_Load+2,2).Value =analog_out 'Analog out
190
191 'Read out disp from servo controller
192 EA_IO.Measure("Displacement_Hor",Disp_read_servo,1) 'Displacement from servo
controller
193 objXls_u.Worksheets(1).Cells(i_Load+2,3).Value =Disp_read_servo 'Saves horizontal
displacement to excel spreadsheet
194
195 'Read out horizontal force from servo controller
196 EA_IO.Measure("Force_Hor",Force_servo_hor,1) 'Horizontal force from servo controller
197 objXls_u.Worksheets(1).Cells(i_Load+2,4).Value =Force_servo_hor 'Saves horizontal
force to excel spreadsheet
198
199 'Read out axial force from servo controller
200 EA_IO.Measure("Load cell",Force_axial,1) 'Axial force from servo controller
201 objXls_u.Worksheets(1).Cells(i_Load+2,5).Value =Force_axial 'Saves axial force to
excel spreadsheet

```

```

202 EA_Panel.SetValue(Panell,"DIGIT_2_axial",Force_axial)
203
204 'Read out LVDT1 from servo controller
205 EA_IO.Measure("PMX_LVDT1",LVDT1,1) 'LVDT1 reading
206 objXls_u.Worksheets(1).Cells(i_Load+2,7).Value =LVDT1 'Saves LVDT1 reading to excel
spreadsheet
207
208 'Read out LVDT2 from servo controller
209 EA_IO.Measure("PMX_LVDT2",LVDT2,1) 'LVDT2 reading
210 objXls_u.Worksheets(1).Cells(i_Load+2,8).Value =LVDT2 'Saves LVDT2 reading to excel
spreadsheet
211
212 'Read out LVDT3 from servo controller
213 EA_IO.Measure("PMX_LVDT3",LVDT3,1) 'LVDT3 reading
214 objXls_u.Worksheets(1).Cells(i_Load+2,9).Value =LVDT3 'Saves LVDT3 reading to excel
spreadsheet
215
216 'Read out LVDT4 from servo controller
217 EA_IO.Measure("PMX_LVDT4",LVDT4,1) 'LVDT4 reading
218 objXls_u.Worksheets(1).Cells(i_Load+2,10).Value =LVDT4 'Saves LVDT4 reading to excel
spreadsheet
219
220 'Read out Strain gauge Left from servo controller
221 EA_IO.Measure("MX840_SL",StrainLeft,1) 'Strain gauge left reading
222 objXls_u.Worksheets(1).Cells(i_Load+2,12).Value =StrainLeft 'Saves strain left
reading to excel spreadsheet
223
224 'Read out Strain gauge Right from servo controller
225 EA_IO.Measure("MX840_SR",StrainRight,1) 'Strain gauge right reading
226 objXls_u.Worksheets(1).Cells(i_Load+2,13).Value =StrainRight 'Saves strain right
reading to excel spreadsheet
227
228 objXls_u.Workbooks(1).Save 'Saves excel spreadsheet data
229
230 Displacement_xOut(i_Load)=Cdbl(xout)'Output displacement array for plotting
231 Displacement_xIn(i_Load)=Cdbl(Disp_read_servo)'Input Displacement array for plotting
232
233 Force_hor(i_Load)=Cdbl(Force_servo_hor)
234
235 LVDT1_disp(i_Load)=Cdbl(LVDT1) 'LVDT1 array for plotting
236 LVDT2_disp(i_Load)=Cdbl(LVDT2) 'LVDT2 array for plotting
237 LVDT3_disp(i_Load)=Cdbl(LVDT3) 'LVDT3 array for plotting
238 LVDT4_disp(i_Load)=Cdbl(LVDT4) 'LVDT4 array for plotting
239
240 Strain_Left_disp(i_Load)=Cdbl(StrainLeft) 'Strain gauge left array for plotting
241 Strain_right_disp(i_Load)=Cdbl(StrainRight) 'Strain gauge right array for plotting
242
243 Axial_load(i_Load)=Cdbl(Force_axial) 'Axial load array for plotting
244
245 'Force displacement out (Plotting)
246 EA_Graph.PlotArrayXY(Panell,"Graph1",1,i_Load+1, Displacement_xOut(), Force_hor())
247 EA_Graph.SetPlotProperty(Panell,"Graph1",1,2,vbRed)
248 EA_Graph.Refresh(Panell,"Graph1")
249
250 'Displacement load out (Plotting)
251 EA_Graph.PlotArrayXY(Panell,"Displacement_out",1,i_Load+1, StepInc(),
Displacement_xOut())
252 EA_Graph.SetPlotProperty(Panell,"Displacement_out",1,2,vbRed)
253 EA_Graph.Refresh(Panell,"Displacement_out")
254
255 EA_Graph.PlotArrayXY(Panell,"Displacement_out",2,i_Load+1, StepInc(),
Displacement_xIn())
256 EA_Graph.SetPlotProperty(Panell,"Displacement_out",1,2,vbBlue)
257 EA_Graph.Refresh(Panell,"Displacement_out")
258
259 'Plots Strain values (Plotting)
260 EA_Graph.PlotArrayXY(Panell,"Strain_out_disp",1,i_Load+1, StepInc(),
Strain_right_disp())
261 EA_Graph.SetPlotProperty(Panell,"Strain_out_disp",1,2,vbBlue)
262 EA_Graph.SetPlotProperty(Panell,"Strain_out_disp",1,5,0)
263
264 'EA_Graph.PlotArrayXY(Panell,"Strain_out_disp",2,i_Load+1, StepInc(),
Strain_Left_disp())

```

```

265 'EA_Graph.SetPlotProperty(Panell,"Strain_out_disp",2,2,vbRed)
266 'EA_Graph.SetPlotProperty(Panell,"Strain_out_disp",2,5,0)
267 'EA_Graph.Refresh(Panell,"Strain_out_disp")
268
269 'Plots LVDT values out (Plotting)
270 EA_Graph.PlotArrayXY(Panell,"LVDT_plot",1,i_Load+1, StepInc(), LVDT1_disp())
271 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",1,2,vbBlack)
272 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",1,5,0)
273
274 EA_Graph.PlotArrayXY(Panell,"LVDT_plot",2,i_Load+1, StepInc(), LVDT2_disp())
275 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",2,2,vbRed)
276 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",2,5,0)
277
278 EA_Graph.PlotArrayXY(Panell,"LVDT_plot",3,i_Load+1, StepInc(), LVDT3_disp())
279 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",3,2,vbBlue)
280 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",3,5,0)
281
282 EA_Graph.PlotArrayXY(Panell,"LVDT_plot",4,i_Load+1, StepInc(), LVDT4_disp())
283 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",4,2,vbMagenta)
284 EA_Graph.SetPlotProperty(Panell,"LVDT_plot",4,5,0)
285 EA_Graph.Refresh(Panell,"LVDT_plot")
286
287 Wait 0.1
288
289 Next
290 objXls_u.Workbooks(1).Save
291 objXls_u.Quit
292
293 End Sub
294
295 Sub BUTTON_1_OnClick
296 'Automatically created procedure: Do not change or delete name or signature!
297 Dim Button1 As Double
298 Button1=1
299 EA_Panel.SetValue(Panell,"Button1out",Button1)
300 EA_Panel.SetCell(Panell,"TABLE_1",1,1,Button1)
301
302 End Sub
303
304 Sub Axial_stop_OnClick
305 'Automatically created procedure: Do not change or delete name or signature!
306 EA_Panel.SetCell(Panell2,"Axial_stop_table",1,1,1)
307 End Sub
308
309 Sub Linear_push_stop_OnClick
310 'Automatically created procedure: Do not change or delete name or signature!
311 EA_Panel.SetCell(Panell2,"Linear_push_stop",1,1,1)
312 End Sub
313
314 Function Sine_increment(ByRef n As Variant) As Variant
315 'Creates an incrementally increasing cyclic load
316 Dim increments As Variant
317 n1=100
318 Dim nmm As Variant
319 nmm=25
320 Dim x_inc() As Variant
321 ReDim Preserve x_inc(4*n1*nmm+nmm,0)
322 Dim pi As Variant
323 pi=4*Atn(1)
324 Dim counter As Variant
325 counter=0
326
327 For i=0 To nmm-1
328     For j=0 To 4*n1
329         x_inc(counter,0)=-1*(i+1)*Sin((pi/(2*n1))*j)
330         counter=counter+1
331     Next
332 Next
333 n=counter
334 Sine_increment=x_inc
335
336 End Function

```