CHAPTER 7

MODEL SOLUTION AND EVALUATION

7.1 INTRODUCTION

One of the primary purposes of macroeconometric modelling is to explain the behaviour of endogenous variables in the context of the system. Apart from the statistical and theoretical criteria that must be satisfied by the individual behavioural equations, the overall simultaneous solution of the model is one of the ways of testing the reliability of the model by allowing it to replicate the economy for which it is designed (Ghartey and Rao 1990). This chapter seeks to present the solution of the model and evaluate its performance in terms of how well it tracks the time paths of the endogenous variables. In addition, the model is evaluated on the basis of its ability to forecast values of the endogenous variables beyond the sample period. The forecast accuracy of the model is evaluated on the basis of the four different criteria. The model is first solved for the period starting 1984 to 2000 and then from 1984 to 2001 to produce out-of-sample forecasts for 2001. The former is taken as the base or control run and is regarded as the benchmark against which other scenarios are compared.

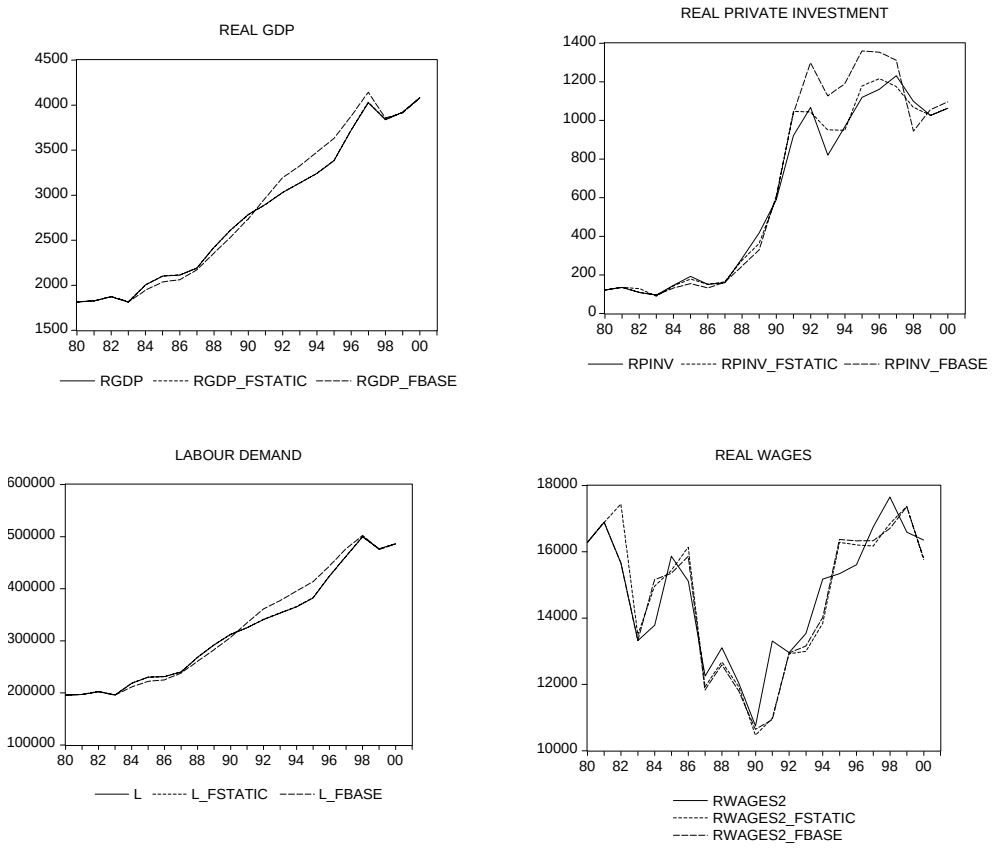
7.2 TRACKING PERFORMANCE OF THE MODEL

This section assesses the performance of the model in terms of tracking the actual series and hence the actual developments in the economy of Lesotho. This exercise is presented in two ways. First, a graphical presentation of this assessment is made both for in-sample and out-of-sample experiments. This is followed by a presentation and comparison of the figures of the actual with those of the static and dynamic solutions of the model.

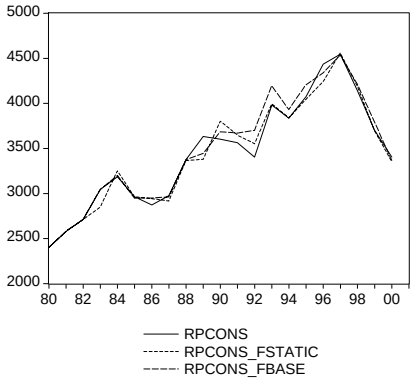
7.2.1 In-sample tracking performance

The simulation results of the model are also presented by way of graphical illustrations of the actual versus fitted values of the stochastic variables. This serves to indicate the ability of the model to track historical data. Figure 7.1 below shows the static (coded _FSTATIC) and dynamic solutions (coded _FBASE) and actual values for 25 endogenous variables determined stochastically in the model to assess the within-sample tracking performance of the model.

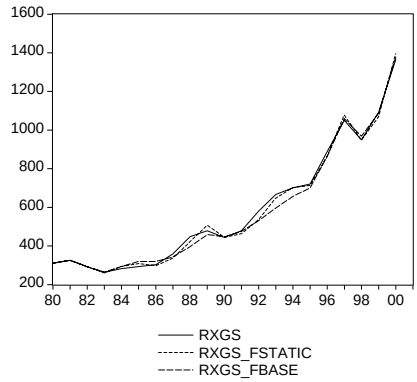
Figure 7.1 Dynamic simulation properties of the model (Within-sample, 1984-2000)



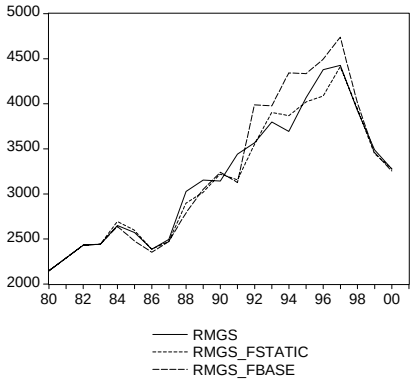
REAL PRIVATE CONSUMPTION



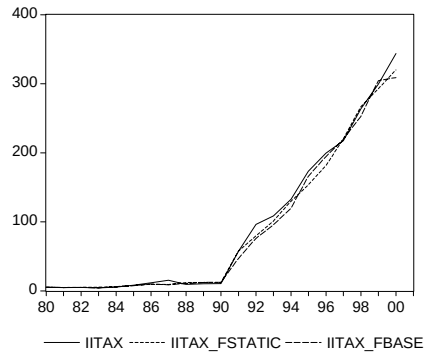
REAL EXPORTS OF GOODS AND SERVICES



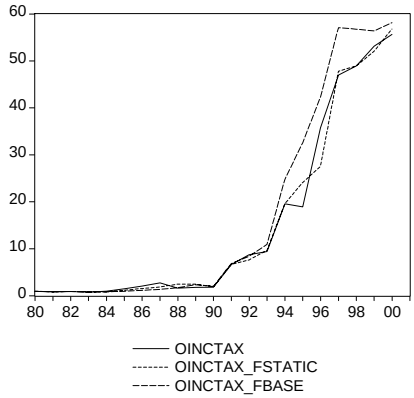
REAL IMPORTS OF GOODS AND SERVICES



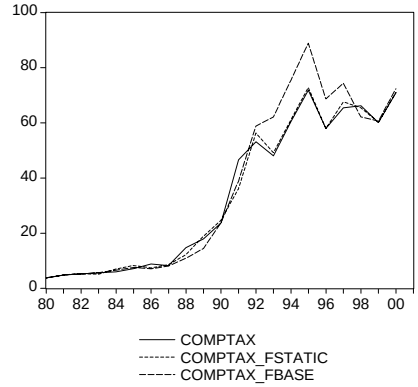
INDIVIDUAL INCOME TAX



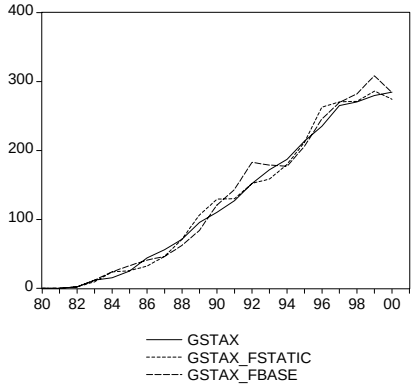
OTHER INCOME TAXES



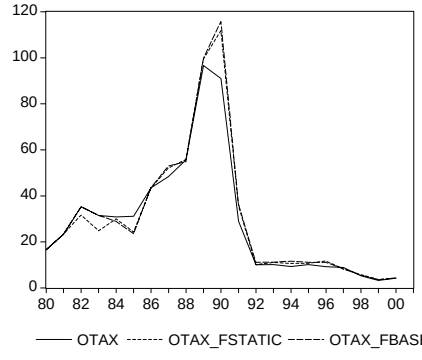
COMPANY TAX



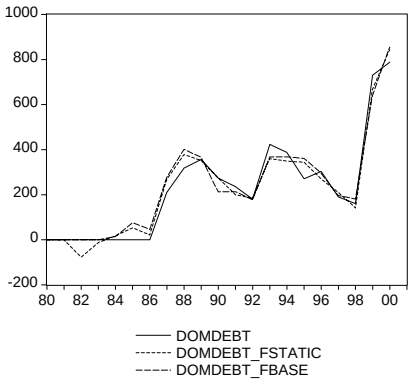
GOODS AND SERVICES TAX



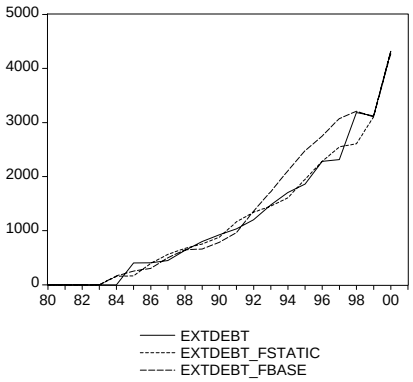
OTHER TAXES



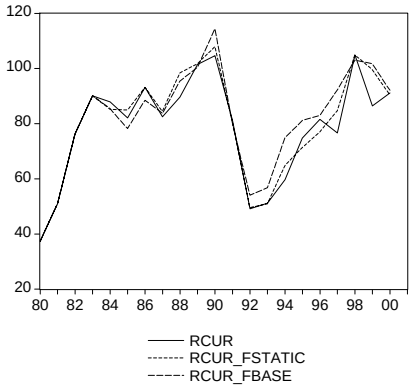
GOVERNMENT DOMESTIC DEBT



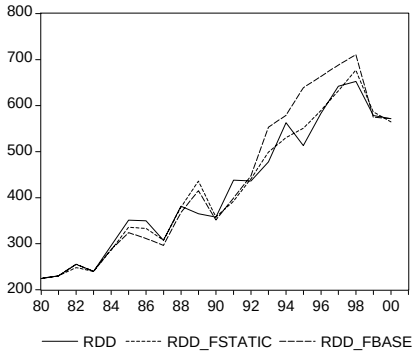
GOVERNMENT EXTERNAL DEBT



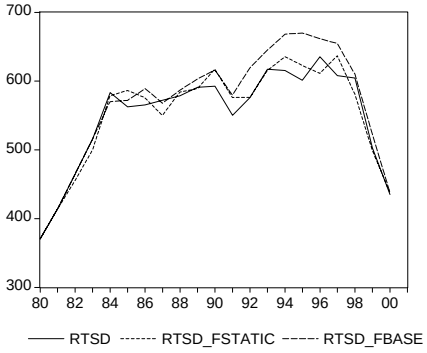
REAL CURRENCY DEMAND



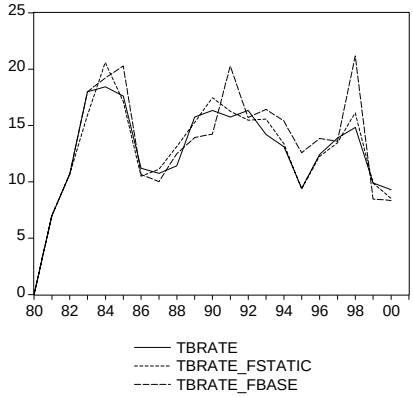
REAL DEMAND DEPOSITS DEMAND



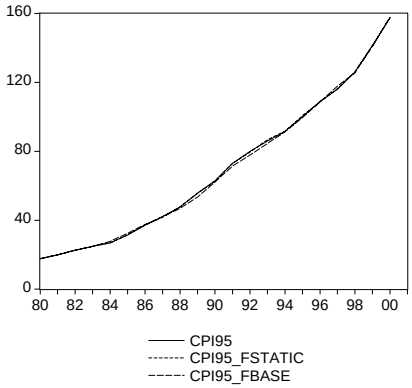
REAL TIME AND SAVING DEPOSITS DEMAND



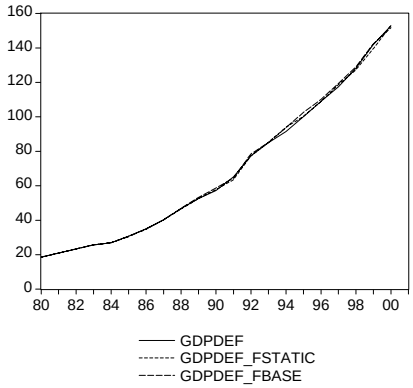
NOMINAL TREASURY BILL RATE

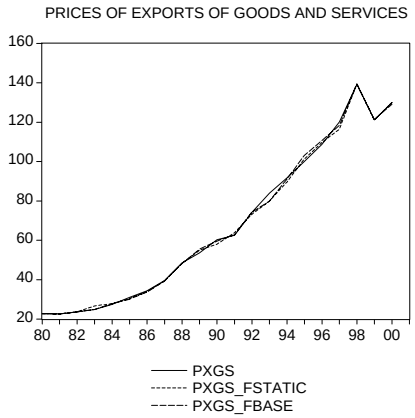


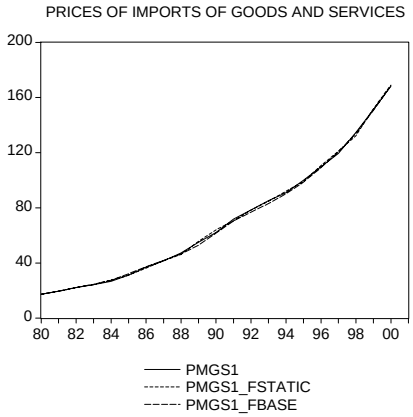
CONSUMER PRICE INDEX



PRODUCER PRICES





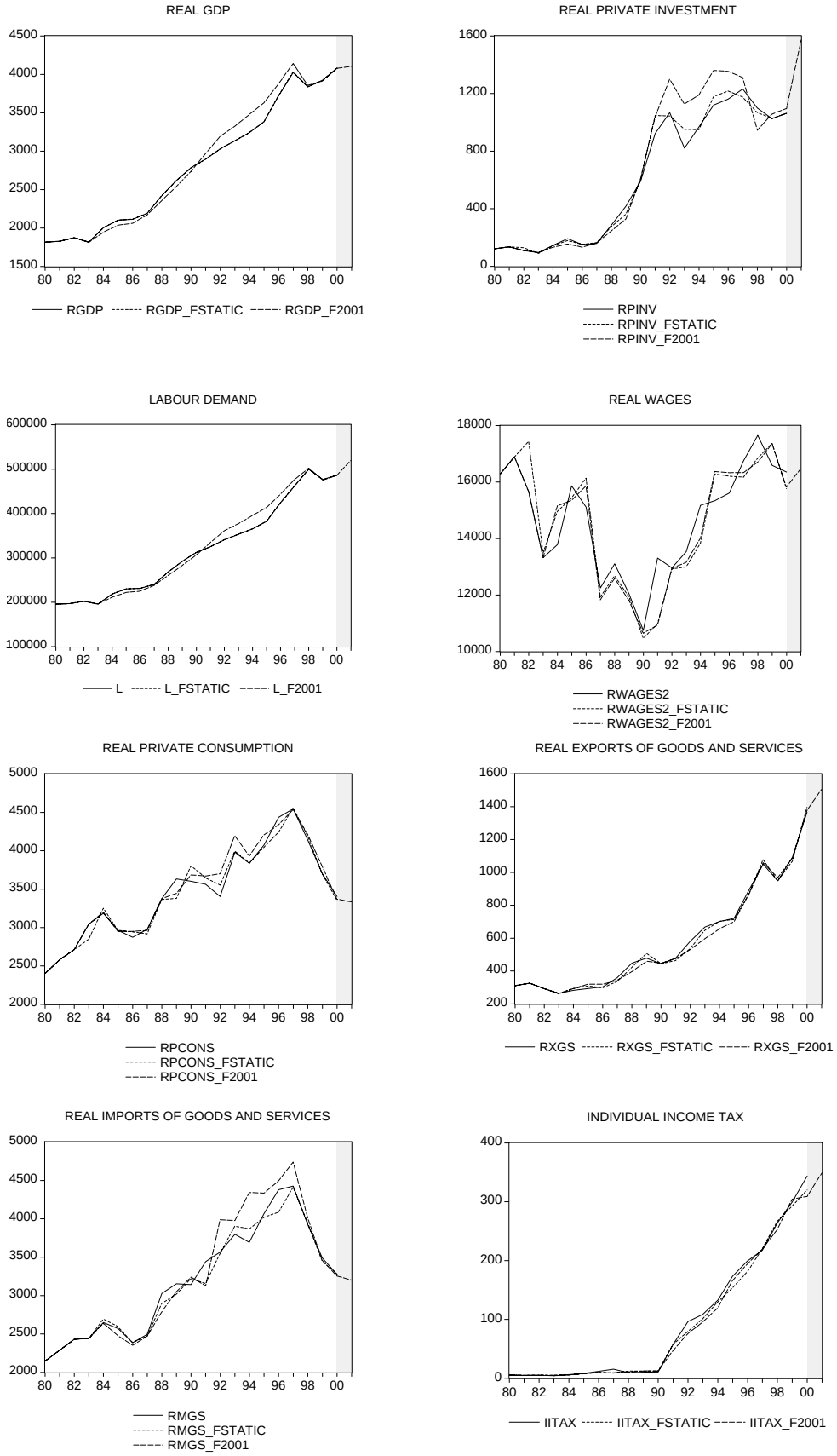


The actual series is plotted together with the static solution of the model as presented in the estimations of the individual behavioural equations in Chapter six, and the dynamic solution values of the model as a complete coherent system with closure rules as presented in section 5.5 of Chapter five. The dynamic system was solved from 1984 to 2000 because of the presence of lagged variables up to the second order in the ECMs. The graphical representations show that the tracking performance of the model within-sample was relatively satisfactory in general terms, indicating a good fit. Both the static and dynamic solution values tend to track the actual time paths of the variables closely. This is particularly true for real exports of goods and services, the consumer price index, the GDP deflator, export prices and import prices.

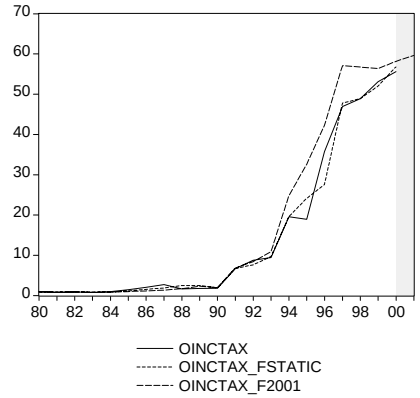
7.2.2 Out-of-sample tracking performance of the model

Figure 7.2 shows static and dynamic simulation experiments that assess the post-sample tracking performance of the model. In this experiment the model is made to produce values for one period ahead of the model sample, by assigning out-of-sample values for the extra period to all exogenous variables in the model. This exercise was performed by computing the average growth for each of the exogenous variables for the last five years of the sample period, from 1996 to 2000. The 2001 values were then computed by letting the last value in the sample grow by the five-year average growth rate. The model is then solved from 1984 to 2001 to solve for the 2001 values for endogenous variables.

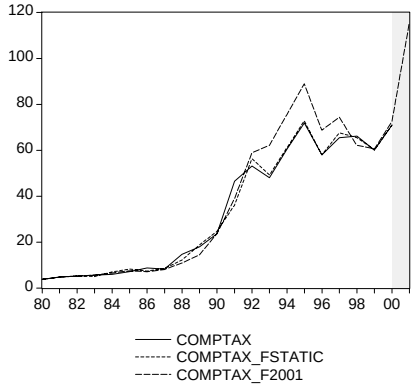
Figure 7.2 Dynamic simulation properties of the model (Post-sample, 1984-2001)



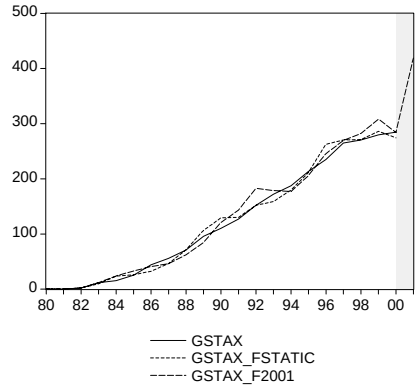
OTHER INCOME TAX



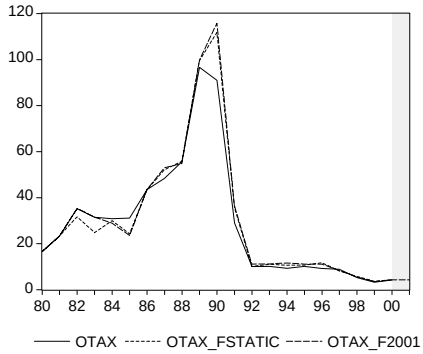
COMPANY TAX



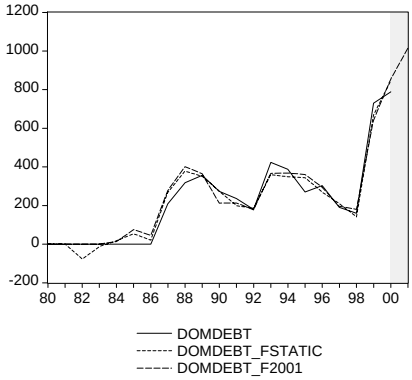
GOODS AND SERVICES TAX



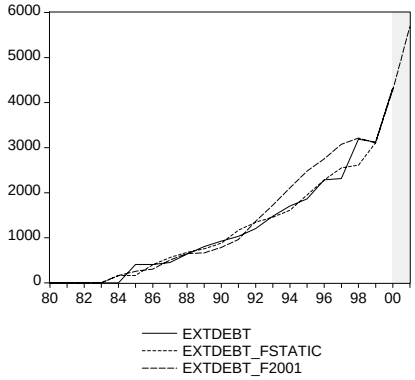
OTHER TAXES



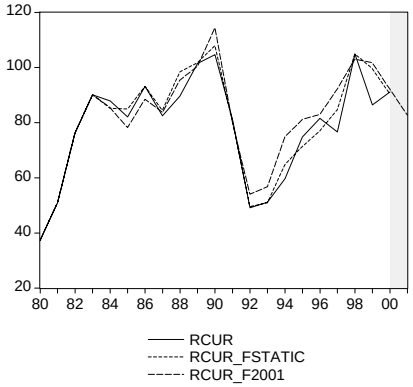
GOVERNMENT DOMESTIC DEBT



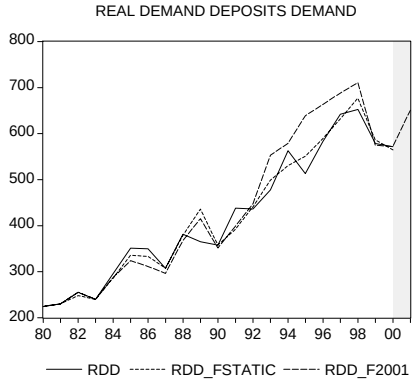
GOVERNMENT EXTERNAL DEBT

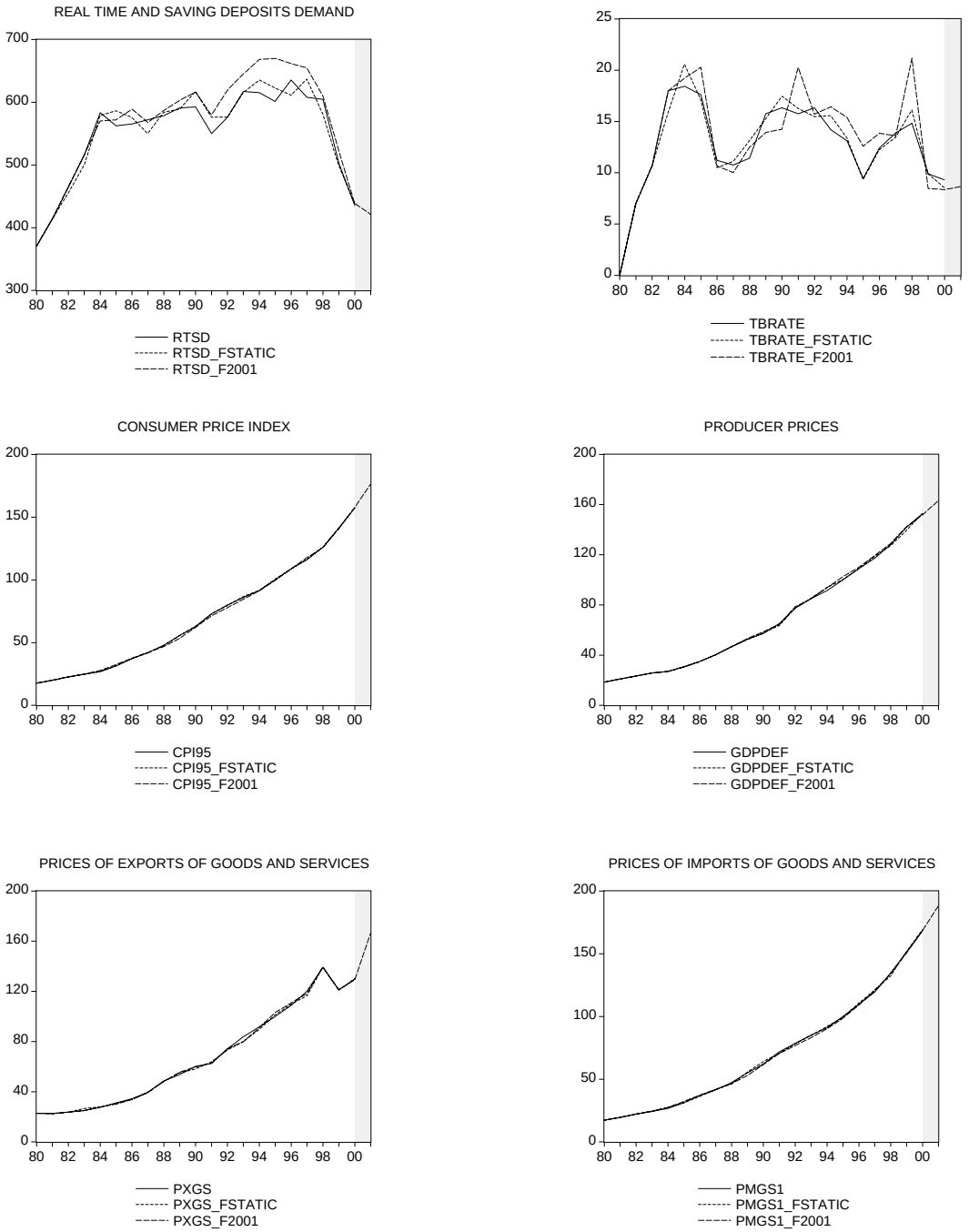


REAL CURRENCY DEMAND



REAL DEMAND DEPOSITS DEMAND





The forecasted period (2001) values are indicated by the shaded region, for each of the stochastic variables. The post-sample values seem to portray a smooth transition from 2000 into 2001 for almost all variables except for a few. This indicates a satisfactory predictive power of the model for at least a single period ahead. Formal tests for the forecasting performance of the model are carried out and reported in section 7.3 below.

The simulated results are compared against the actual and static values in table 7.1 below for 25 stochastically determined endogenous variables.

Table 7.1 Comparison of the actual values of the variables with the static and dynamic solutions of the model within and out-of-sample

Variable	Year	Actual	Static	Dynamic
RGDP	1996	3720.7	3724.7	3876.5
	1997	4023.8	4030.5	4142.2
	1998	3837.0	3839.1	3856.5
	1999	3920.3	3920.3	3912.7
	2000	4081.1	4081.1	4077.7
	2001			4102.9
L	1996	424323.2	423893.1	443599.4
	1997	461963	461776.7	476227.8
	1998	499602.8	499602.8	502077.6
	1999	476427.3	476427.3	475248.6
	2000	486072.6	486072.6	485578.4
	2001			519341.6
RPCONS	1996	4434.5	4240.7	4338.5
	1997	4544.3	4558.6	4536.3
	1998	4139.9	4189.4	4210.9
	1999	3700.2	3697.6	3793.1
	2000	3403.6	3356.0	3368.7
	2001			3331.9
RPINV	1996	1160.8	1216.6	1353.2
	1997	1231.1	1174.6	1310.8
	1998	1098.9	1067.5	944.7
	1999	1026.4	1026.4	1056.3
	2000	1062.7	1062.7	1095.5
	2001			1571.7
RXGS	1996	888.4	865.1	862.7
	1997	1050.2	1078.5	1059.2
	1998	948.7	948.7	968.8
	1999	1091.5	1068.2	1086.0
	2000	1365.9	1395.7	1377.2
	2001			1505.3
RMGS	1996	4378.2	4086.3	4494.9
	1997	4425.4	4410.3	4738.1
	1998	3923.8	3943.0	4008.2
	1999	3488.7	3445.2	3458.5
	2000	3276.1	3276.1	3253.9
	2001			3198.2
IITAX	1996	199.5	181.0	194.3
	1997	217.4	220.1	219.5
	1998	263.9	266.9	252.6
	1999	299.4	292.7	304.4
	2000	343.9	319.8	308.9

	2001			349.2
OINCTAX	1996	35.8	27.6	42.3
	1997	46.9	47.8	57.1
	1998	48.9	48.9	56.7
	1999	53.1	52.1	56.4
	2000	55.6	56.8	58.2
	2001			59.6
COMPTAX	1996	57.9	57.9	68.7
	1997	65.4	67.6	74.4
	1998	66.2	65.5	62.1
	1999	60.1	60.1	60.6
	2000	70.9	70.9	72.4
	2001			115.6
GSTAX	1996	235.2	262.6	245.6
	1997	264.9	270.5	269.7
	1998	270.5	271.1	282.1
	1999	279.8	286.0	308.2
	2000	284.6	274.2	283.9
	2001			418.9
OTAX	1996	9.2	11.6	11.1
	1997	8.9	8.1	8.2
	1998	5.2	5.7	5.4
	1999	3.3	3.6	3.5
	2000	4.2	4.3	4.2
	2001			4.2
DOMDEBT	1996	303.3	268.9	294.5
	1997	190.2	209.4	195.8
	1998	160.1	140.9	180.5
	1999	730.2	668.4	641.6
	2000	788.5	844.0	854.6
	2001			1014.0
EXTDEBT	1996	2283.6	2280.4	2749.9
	1997	2313.4	2548.4	3072.9
	1998	3185.1	2607.5	3209.9
	1999	3121.9	3103.6	3102.2
	2000	4319.6	4319.6	4270.6
	2001			5685.7
RCUR	1996	81.6	77.1	82.9
	1997	76.7	84.7	92.2
	1998	104.8	104.9	103.0
	1999	86.4	99.9	101.7
	2000	91.1	90.5	92.1
	2001			82.7
RDD	1996	582.9	589.0	663.3
	1997	641.9	631.8	688.0
	1998	652.6	676.9	710.8
	1999	578.6	586.1	574.4
	2000	571.6	564.2	571.6

	2001			650.7
RTSD	1996	635.4	611.1	661.5
	1997	607.8	636.8	654.5
	1998	604.5	581.1	610.2
	1999	503.1	499.1	522.7
	2000	434.9	438.4	438.5
	2001			420.9
TBRATE	1996	12.4	12.2	13.9
	1997	13.9	13.5	13.6
	1998	14.8	16.1	21.2
	1999	9.9	9.9	8.5
	2000	9.3	8.5	8.3
	2001			8.7
RWAGES2	1996	15611.9	16203.4	16328.9
	1997	16752.9	16173.2	16333.3
	1998	17649.6	16846.6	16704.0
	1999	16589.8	17366.5	17355.9
	2000	16353.7	15769.6	15821.8
	2001			16471.0
CPI95	1996	108.8	108.5	108.6
	1997	116.1	116.3	117.8
	1998	126.0	126.0	125.6
	1999	141.5	141.1	140.8
	2000	157.3	157.6	157.8
	2001			176.3
GDPDEF	1996	108.9	108.8	110.0
	1997	117.3	118.7	119.4
	1998	128.2	127.2	129.0
	1999	141.9	139.5	142.4
	2000	152.9	152.9	151.7
	2001			163.2
PXGS	1996	108.7	109.7	110.7
	1997	120.1	116.5	118.4
	1998	139.2	139.2	139.4
	1999	121.2	121.2	121.4
	2000	130.0	130.0	129.1
	2001			166.5
PMGS1	1996	109.7	110.6	109.0
	1997	119.5	121.1	120.7
	1998	134.7	132.3	133.9
	1999	151.2	151.8	150.6
	2000	168.3	169.1	168.5
	2001			188.1

A casual inspection of the simulated values of key macroeconomic variables such as GDP and its components shows that these variables are close to the observed values for both the static and dynamic solutions. This indicates that the model tracks historical

trends of these variables with a reasonable degree of accuracy. Major deviations are observed for individual income tax and goods and services tax from the 2000 to 2001 values. These are taken to reflect changes in the tax structures and administration resulting from the reforms enacted at that time.

7.3 EVALUATION OF THE FORECASTING PERFORMANCE OF THE MODEL

Four measures of forecast accuracy were used to evaluate the performance of the model over the entire sample period. These are the root mean square error (RMSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE) and the Theil inequality coefficient (U).[89] According to these statistics, better performance is indicated by smaller values.

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[89] $RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_t - Y_t)^2}$

$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{Y}_t - Y_t\right|$

$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\hat{Y}_t - Y_t}{Y_t}\right|$

$U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_t - Y_t)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_t)^2}\sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y_t)^2}}$

where $\hat{Y}_t$ is the simulated value in period *t*, Y_t is the actual value in period *t* and *T* is the sample size (Pindyck and Rubinfeld 1991:338).

Table 7.2 Dynamic simulation accuracy of stochastic variables (1984-2000)

Stochastic variables		Simulation error statistics			
Variable name	Logarithmic form	Root Mean Square Error (RMSE)	Mean Absolute Error (MAE)	Mean Absolute Percentage Error (MAPE)	Theil Inequality Coefficient (U)
Real GDP	LRGDP	0.00.196	0.001560	0.019319	0.000202
Labour Demand	LL	0.003917	0.002219	0.017392	0.000155
Real private consumption	LRPCONS	0.030334	0.024695	0.301456	0.001861
Real private investment	LRPINV	0.132673	0.105053	1.798096	0.010850
Real exports of goods and services	LRXGS	0.215620	0.141625	2.383285	0.017224
Real imports of goods and services	LRMGS	0.028004	0.024240	0.300652	0.001741
Individual income tax	LIITAX	0.175840	0.125203	5.118650	0.022855
Other income tax	LOINCTAX	0.223185	0.159107	64.97430	0.049266
Company tax	LCOMPTAX	0.145090	0.103131	5.273880	0.022486
Goods and services tax	LGSTAX	0.621530	0.437233	22.88485	0.067762
Other taxes	LOTAX	0.293930	0.210570	7.627085	0.047642
Government domestic debt		45.59232	34.22939	11.55054	0.069940
Government external debt		270.4660	212.3313	17.28135	0.074910
Real currency	LRCUR	0.159279	0.114381	2.722212	0.018352
Real demand deposits	LRDD	0.064985	0.050059	0.846871	0.005397
Real time and saving deposits	LRTSD	0.031097	0.024869	0.392545	0.002460
Nominal treasury bill rate	LTBRATE	0.126440	0.092662	3.693430	0.024611
Real wages	LRWAGES2	0.056632	0.045487	0.474175	0.002954
Consumer price index	LCPI95	0.005158	0.004476	0.102349	0.000630
Producer prices	LGDPDEF	0.149249	0.113045	2.940956	0.018293
Export prices	LPXGS	0.031705	0.024786	0.656493	0.003887
Import prices	LPMGS1	0.098620	0.081300	2.083890	0.012026

All 25 endogenous variables have a Theil's U statistic less than 0.1. Only four variables have MAPE of higher than 10 per cent. In general, the performance of the model is poor according to the RMSE, MAE and MAPE for both government domestic and external debt; other income taxes, goods and services taxes in particular. Nevertheless, since the

proportions of these variables to national output and other key macroeconomic variables are small, their poor performance is assumed to affect the forecasts of the rest of the model only marginally. Overall, the statistics indicate that the model is able to track historical developments in the economy reasonably well with the exception of a few variables.

7.4 SUMMARY OF THE PERFORMANCE OF THE MODEL

The results of the dynamic solution of the model show that both the dynamic solutions capture the direction of the actual values of the endogenous variables in the model. In addition, dynamically solved values of the variables in the model do not deviate much from the values solved for in static scenarios. The evaluation of the forecast performance of the model using the MAE, MAPE, RMSE and the Theil inequality coefficient also affirm the relatively good performance of the model.