REVIEW

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Reasoning and proof in mathematics education: a systematic literature review

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Abstract
This study presents a systematic literature review of 80 publications on reasoning and proof in mathematics education from 2019 to 2024. This review examined the distribution of the studies across mathematics content areas, educational levels, year of publication, and countries. In addition, it examined the instructional strategies and interventions that promote students' reasoning and proof, the common challenges students encounter in understanding and engaging with reasoning and proof in mathematics, and teachers' knowledge, beliefs, and attitudes towards reasoning and proof. The results show that most studies focused on geometry, with an apparent increase in research activity between 2019 and 2023. There were studies from 29 nations, thus signifying that reasoning and proof in mathematics education are internationally important. Türkiye, the United States, and Indonesia were the top three countries where the studies were carried out. Most of the studies were at the pre-service teacher education level, followed by the secondary school education level. The Guide-redirecting Warrant Construction instructional approach was found to be effective in improving students' reasoning and proof. Furthermore, students have difficulty with reasoning and proof, which is aggravated by teachers' low proficiency in these areas. The study identified important gaps in the literature and offered recommendations to address them.

Keywords Argumentation, Critical thinking, Mathematical justification, Proof, Reasoning

1 Introduction

Reasoning and proof are the processes of making logical arguments and establishing mathematical truths using formal proofs. Reasoning and proof are crucial aspects of mathematics as they enable students to have a thorough understanding of the content of mathematics [1] as well as develop other mathematical skills such as "critical thinking, communication and collaborative skills, and characteristics such as ownership of one's learning, curiosity, courage and flexibility" [2, p. 1] state that current educational reform documents globally have emphasised reasoning and proof as one of the most crucial objectives for students. Thus, the importance of reasoning and proof in mathematics education is evident in the inclusion of these skills in curriculum documents globally

[3, 4]. noted that the incorporation of reasoning and proof into standardised worldwide testing further underscores their significance.

According to [2], reasoning and proof entail formulating claims, offering evidence to support them, and assessing that evidence to determine their validity [3]. define proof as the theoretical support of a claim that is followed by logical conjunctions.

The goal of doing mathematics is to develop critical thinkers who can solve problems in various situations. Mathematics aims to develop "mathematical mind habits," that is, the ability to think of mathematical ideas in a manner that corresponds to that of mathematicians; the process of reasoning and proof works well for fostering critical thinking and communication abilities [5]. Similarly [6], noted that the mathematical ability that most closely reflects the methods used by mathematicians in their profession is reasoning and proof. Thus, reasoning and proof aid in the development of mathematical thinkers.

[7] assert that if students are to comprehend mathematics as more than a collection of facts and methods, they must be able to reason and prove. In reasoning and proof, students take ownership of their mathematical knowledge from the teacher or textbook through reasoning and proof [6].

Over the past decade, numerous educational methods, paradigms, and techniques have been examined through extensive and diverse research on reasoning and proof in mathematics education. Researchers have investigated the importance, inclusion, and use of reasoning and proof in mathematics education, focusing on teachers' knowledge, students' understanding, and the quality of reasoning and proof.

This systematic literature review aims to compile the substantial research conducted over the past six years on reasoning and proof in mathematics education. In addition to providing an overview of how educators and students understand reasoning and proof, the review aims to highlight key findings and gaps in the existing literature. It also identifies instructional strategies and interventions that can support the teaching and learning of reasoning and proof. This evaluation aims to enlighten educators in practice and guide future research endeavours.

2 Research questions

The following research questions guided the systematic literature review:

1) How are the publications on reasoning and proof distributed among mathematical content areas?
2) How are the studies distributed based on the year of publication?
3) What is the distribution of the publications across different countries?
4) How are the publications distributed across various educational levels?
5) What instructional strategies foster reasoning and proof skills in mathematics education?
6) What common challenges do students face in understanding and engaging with reasoning and proof in Mathematics?
7) What knowledge, beliefs, and attitudes do mathematics teachers hold about reasoning and proof?

3 Theoretical background

3.1 Current state of systematic literature reviews on reasoning and proof

In recent years, several systematic literature reviews have emerged focusing on reasoning and proof in mathematics education. Some of the reviews focused on the broad perfectives of reasoning and proof in mathematics education, spanning from school to university contexts [7–11]. While reviews that focus on broad reasoning and proof contexts provide valuable insights, there are relatively few of them. Other reviews have concentrated on specific domains of reasoning, such as mathematical argumentation [12], argumentative writing [13], semiotic reasoning [14], or the use of Toulmin's model for analysing proof [15].

Several reviews narrowed their scope to specific educational levels, contexts or populations. Reviews have focused on specific countries, such as Indonesia [16, 17], and educational levels such as the elementary level [18, 19], specific components of reasoning and proof, such as justification [20], support strategies [21], and types of reasoning addressed in empirical studies [22]. Other studies focused on specific educational resources, such as textbooks [23] and technology incorporation [24, 25].

The growing interest in reasoning and proof is evident in the expanding body of literature. However, the reviews remain focused on specific domains, such as thematic, contextual, or resource-driven. A noticeable gap in the literature is the lack of a comprehensive review that provides a general understanding of reasoning and proof, combining these specific domains. This review partially addresses this gap by providing insight into reasoning and proof in mathematics education across pedagogy, student understanding, and teachers' knowledge and beliefs.

This review contributes to expanding knowledge beyond the existing literature by providing a comprehensive synthesis of scattered findings into a clear, integrated perspective, thereby highlighting connections and patterns not emphasised in the existing literature.

3.2 Reasoning and proof

Every aspect of mathematics involves reasoning and proof. According to [26], there are two types of reasoning: geometric and algebraic. While geometrical reasoning uses geometric forms and objects like points, lines, angles, and actual objects, algebraic reasoning uses words, equations, diagrams, and symbols to convey mathematical information. Thus, reasoning and proof are part of mathematics in all facets and are not limited to a single mathematics component.

[27] argue that the goal of reasoning and proof is to persuade individuals regarding the truth or falsity of a proposition using a combination of rhetorical devices. Therefore, confirming the validity of mathematical claims and creating new knowledge is the primary goal of reasoning and proof.

According to [1], reasoning and proof are difficult mathematical task that involves finding a proof and developing an argument with its logical, conceptual, social, and reasoning-based problem-solving components [28]. argues that reasoning and proof are a series of mathematical arguments that persuasively show, with justification, whether a claim is true or false. According to [29], several ways in which reasoning and proof might emerge include disagreement, convincing others, inquiry, and negotiation. In reasoning and proof procedures, students debate with one another, support their mathematical positions with mathematical justifications, and, if someone else challenges their viewpoints, support their positions with mathematical evidence [27]. From the definitions of reasoning and proof, one may infer that reasoning and proof have four components for analysis: premise, conclusion, mathematical knowledge, and reasoning [30].

According to [31], argumentation and reasoning relate to the same mathematical process: reasoning when constructing an argument and developing an argument when reasoning. Therefore, in this study, the terms reasoning, proof, and argumentation are used interchangeably.

3.3 The role of reasoning and proof in mathematical thinking and learning

Reasoning and proof are fundamental aspects of mathematics teaching and learning. Reasoning and proof cannot only increase students' understanding of mathematics but can also be utilised to validate the truth of a mathematical statement.

As stated previously, reasoning and proof are skill that most closely represents the way mathematicians construct knowledge. As [6] argues that the construction of mathematical knowledge is quite similar to the process of reasoning and proof. Thus, reasoning and proof help students construct mathematical knowledge. This can be due to students finding it easier to understand the evidence of a conjecture when they participate in the process of reasoning and proof [32].

The process of reasoning and proof as an aid in validating the truth of a mathematical statement is not the only important factor in reasoning and proof. As [33] argues that the process by which students consider, communicate, and reflect on their own thought and reasoning, as well as assess the reasoning of others, is more significant than the outcomes of reasoning and proof. Furthermore, reasoning and proof are skill that goes beyond the classroom. As [2] state, one of the primary processes linked to the competencies that might assist students in navigating the demanding world more effectively is reasoning and proof.

Not only do reasoning and proof help students understand mathematics, but they also aid teachers in teaching mathematics. According to [30], the ability to reason and prove is crucial for demonstrating how students use a method or formula to connect provided data to a mathematical issue [34]. argues that one of the most significant indicators of students' mathematical thinking is their ability to justify assertions. Therefore, reasoning and proof can be used to assess students' mathematical thinking and support them in developing their mathematical thinking.

4 Methodology

A systematic literature review is an approach to collecting relevant publications on a particular topic that satisfies pre-established eligibility requirements [35]. Thus, a systematic literature review is an extensive search and analysis of a body of research on a specific topic to provide insight into the current research themes. To provide an overview of current research topics, a systematic literature review procedure includes obtaining publications from databases using specific search keywords, selecting publications based on preset criteria, and synthesising the results [24].

The review employed the Preferred Reporting Items for Systematic Reviews and Meta-analyses (PRISMA) guideline to analyse the collected publications [35]. claim that PRISMA created a technique that uses guideline checklists to enhance the calibre of a

Table 1 List of synonyms of the predetermined keywords

Mathematics	Education	Reasoning and proof
Math	Teaching	Argumentation
Maths	Learning	Justification
Mathematical		Proof
		Justify

Table 2 Inclusion and exclusion criteria

Inclusion	Exclusion
Publications between 2019 and 2024	< 2019
Peer reviewed	Non-Peer reviewed
Full text	Summaries
Language: English	Non-English
Mathematics	Other educational subjects not related to mathematics.
Education	General publications not related to education.

systematic literature review's revision process. The four phases of the PRISMA approach are identification, screening, eligibility, and inclusion. The subsections provide a detailed discussion of the procedures. This study applied the PRISMA guidelines because they offer a clear and systematic process for conducting literature reviews. The framework's four stages — identification, screening, eligibility, and inclusion — help ensure that study selection is transparent, consistent, and replicable.

4.1 Identification

We used our university's library services to identify the databases most appropriate for educational research and determined which to employ for the review. The databases ERIC, Taylor & Francis, Science Direct, and Emerald Insight were included in the evaluation. The selected four databases are well-established and publish quality studies, while providing comprehensive coverage of mathematics education research relevant to the review focus. Thus, these databases were chosen over other available databases. Additionally, it was decided to include only studies published in the last 6 years. Therefore, this evaluation only covered recent research studies.

Keywords used to search the databases were determined by the review. The review's keywords were reasoning, proof, mathematics, and education. To ensure that no publications were overlooked, the review also included a list of the acronyms for the keywords (Table 1).

Using the keywords and acronyms in different combinations, 4,374 publications were identified. The 4,374 publications were analysed, and 485 were identified as duplicates. Thus, 3,889 publications remained.

4.2 Screening

The review identified predetermined inclusion and exclusion criteria to ensure that only publications meeting the requirements of this review were included (Table 2).

The 3889 publications were subjected to further analysis after meeting the inclusion criteria. Publications completed before 2019 were excluded from our analysis, whereas papers that included at least one reference to reasoning and proof, or any synonym, were included. The review focused on studies published between 2019 and 2024. This six-year range was selected to include the most recent developments and empirical trends in mathematics education, particularly following the increasing integration of inquiry-based and reasoning-oriented pedagogies during this period. Previous reviews have already synthesised earlier literature, so this study aimed to build on that foundation by providing an updated synthesis of contemporary research. To ensure no seminal works were overlooked, we also examined studies published before 2019 and included them in the background to maintain conceptual continuity.

Following the screening stage, 478 publications were selected for further analysis, whereas 3,411 were excluded because they did not meet the inclusion criteria.

4.3 Eligibility

By reviewing the study titles, the remaining 478 publications were further examined. At this stage, 39 publications were identified as eligible for inclusion in the review. The abstracts, methods, and conclusions of the remaining publications were thoroughly examined. Thirty-one additional publications were found and added to the review. As a result, 70 publications were included in this evaluation stage.

4.4 Inclusion

To ensure that no publications were overlooked in the review, an ancestral analysis was conducted by examining the references of 70 articles to identify other publications that might be of interest. As a result, 10 publications were identified and included in the review. Thus, a total of 80 publications were included in the review. The 80 publications were analysed according to the inclusion criteria, and all were deemed suitable for the objective of this review. The PRISMA diagram in Fig. 1 below illustrates the analysis process.

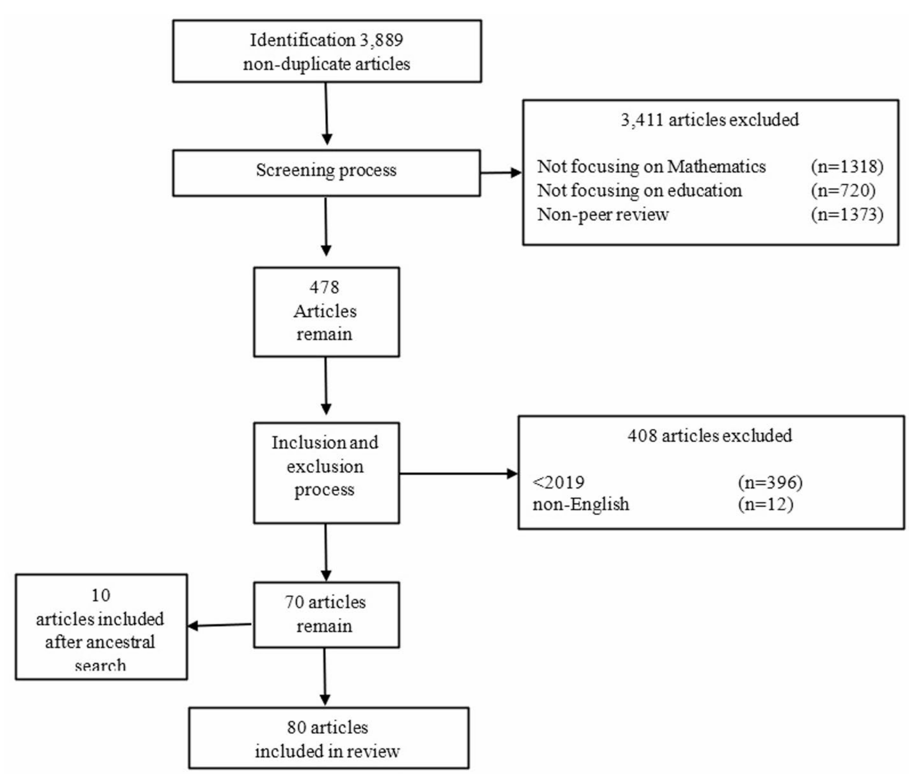

Fig. 1 PRISMA Diagram of the Inclusion and Exclusion Process

4.5 Data analysis

To analyse and synthesise the publications, the publications were read in full and coded according to a rigorous coding scheme. The coding scheme included classifying publications by year of publication, study country, participant level, and mathematical content area. The publications were further grouped according to common themes.

The first theme focused on effective instructional strategies that foster reasoning and proof in mathematics education; the second theme addressed the common challenges students face in understanding and engaging with reasoning and proof; and the last theme examined teachers' knowledge, beliefs, and attitudes toward reasoning and proof in mathematics education.

4.6 Methodological quality and risk of bias

The systematic literature review took several steps during the selection and evaluation process to ensure the rigour and reliability of the review. Both authors independently reviewed the titles and abstracts of the studies identified in the databases. The inclusion and exclusion criteria were rigorously followed and applied consistently throughout; disagreements among authors were resolved through discussion until consensus was reached. This collaboration minimised the likelihood of overlooking possible studies and strengthened the validity of the final set of studies included in the review.

In addition, a pilot review was conducted to ensure that all relevant coding options were included in the review form. Thereafter, the primary coding process was conducted jointly by both authors. Each author applied the inclusion and exclusion criteria independently, then cross-checked their decisions. This process served as a form of inter-rater reliability, as the coding of the studies was compared, and any discrepancies were discussed and mutually agreed upon.

A risk of bias assessment was conducted to further ensure the quality of the review. Each study was examined individually for potential bias, and when discrepancies arose, the authors revisited the studies together to reach consensus. These steps ensured that the review was reduced in subjectivity and enhanced in trustworthiness.

5 Results

The systematic literature review results, comprising 80 publications included in this study, are presented below.

5.1 Distribution of content areas

The 80 publications (see appendix Table) included in the review were coded according to the mathematical content area focus of the study. A total of 10 mathematical content areas were identified across the 80 publications, as shown in Table 3.

As seen in Table 3, the most common mathematical content area of the studies was mathematics in general (45%). These studies analysed reasoning and proof in mathematics education in general and were not explicitly related to a particular content area. These publications concluded in a broader sense of reasoning and proof in mathematics. Geometrical reasoning and proof were the second-most-common mathematical content area among the publications (32%). Although reasoning and proof are integral to every aspect of mathematics, they are especially crucial in geometry, as it is essential to use evidence to demonstrate the properties and relationships both inside and between

Table 3 Distribution of content areas

Mathematical Content Area	Percentage of Publications %
General	46
Geometry	32
Algebra	9
Probability	3
Functions	3
Fractions	3
Statistics	1
Complex numbers	1
Triangular numbers	1
Mathematical play	1

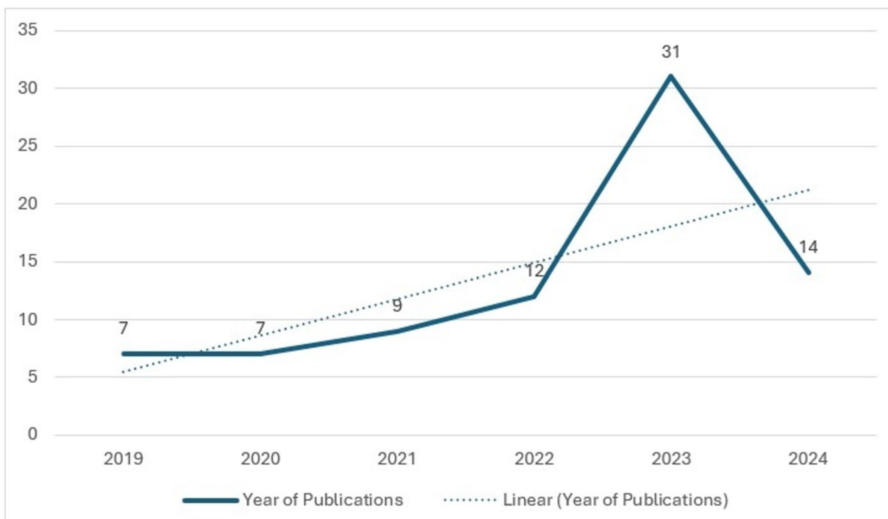

Fig. 2 Distribution of Publications across Different Years

figures [36]. Therefore, as expected, a high percentage of the publications focused on geometry. The other eight content areas constituted 23% of the publications.

5.2 Distribution of the studies based on the year of publication

The review found a noticeable rise in reasoning and proof in mathematics education publications over the past six years. This further demonstrates the growing significance of reasoning and proof in mathematics education. Figure 2 depicts the distribution of publications across different years.

As shown in Fig. 2, a general growth in reasoning and proof in mathematics education was observed across all years, despite a slight decline in publications from 7 in 2019 to 6 in 2020. With 31 publications, 2023 saw a significant increase, indicating a notable surge in research on reasoning and proof in mathematics education. This could be attributed to developments in the fields of reasoning and proof. Although there was a decline in 2024, it was not notable, as the number of publications remained higher than in 2019–2022, indicating that the emphasis on reasoning and proof in mathematics education remains.

Table 4 Distribution of publications across different countries

Country	Percentage of Publications %
Türkiye	21
USA	14
Indonesia	13
Australia	6
Norway	6
Romania	5
Israel	5
South Africa	4
Germany	4
Italy	3
Portugal	3
Chile	3
Spain	3
Netherlands	3
Colombia	3
England	1
Japan	1
Canada	1
China	1
Greece	1
Latvia	1
Barcelona	1
Peru	1
Mexico	1
Istanbul	1
Taiwan	1
Morocco	1
Zambia	1
Uganda	1
Rwanda	1
Switzerland	1
Sweden	1

5.3 Distribution of publications across different countries

Publications on reasoning and proof in mathematics education have been conducted in 29 countries over the last six years, indicating a global interest in the field.

It is crucial to remember that although specific publications were conducted in many nations, the distribution percentages do not sum to 100%. Three countries stood out among the nations that have contributed most to the subject of reasoning and proof in mathematics education. In Türkiye, the most significant contribution was observed. There is considerable interest in reasoning and proof in mathematics education in Türkiye, as evidenced by 17 papers published on the topic over the previous six years. With 11 and 10 publications, respectively, the USA and Indonesia are not far behind, making noteworthy contributions that are far more significant than those of the other nations. As a result, research on reasoning and proof has been especially active in Türkiye, the United States, and Indonesia during the reviewed period. Despite their modest contributions, the remaining nations made contributions to the field. Over the six years, publications from the remaining nations varied from 6% to 1%. Even if three nations made more significant contributions to the area, the fact that 32 nations have done so demonstrates a global focus on mathematical reasoning and proof, underscoring the topic's significance for mathematics education.

5.4 Distribution of publications across educational levels

Whether the focus is on students, teachers, university-level pre-service teachers, or curriculum documents, the educational levels of the publications may provide insight into where the emphasis on reasoning and proof in mathematics education lies. The educational levels include the following age groups: Early Childhood Education (1 to 6 years), Primary (7 to 13 years), Secondary (14 to 18 years), Pre-service Teachers (University students), and In-service Teachers (Teachers with a degree in education) (Fig. 3).

Table 3 illustrates the emphasis on reasoning and proof in mathematics education for pre-service and in-service teachers, with 39 of the 80 publications focusing on teachers. The findings suggest that teaching reasoning and proof has become an increasingly important focus in mathematics education worldwide. These publications included a variety of topics, including university courses, effective interventions, educational expertise, and instructional strategies. The evidence shows that researchers often emphasise teachers' role in supporting students' reasoning and proof skills.

Most of the school-level publications reviewed focused on secondary education. This could be because secondary school is when reasoning and proof are most common. Secondary school accounted for 21 publications, whereas middle school and early childhood received just 8 and 1 publications, respectively. Other educational levels were the focus of specific publications, such as curriculum documents (one publication) and general educational levels (ten publications).

5.5 Instructional strategies and interventions that foster reasoning and proof skills in mathematics education

As previously stated, a great emphasis was placed on publications focusing on teachers, whether in-service or pre-service. Thus, it is reasonable to assume that instructional strategies and interventions that foster reasoning and proof are implemented and developed by teachers. Across the literature, three central themes emerged: the importance of classroom culture, the design of mathematical tasks, and the teacher's role in mediating

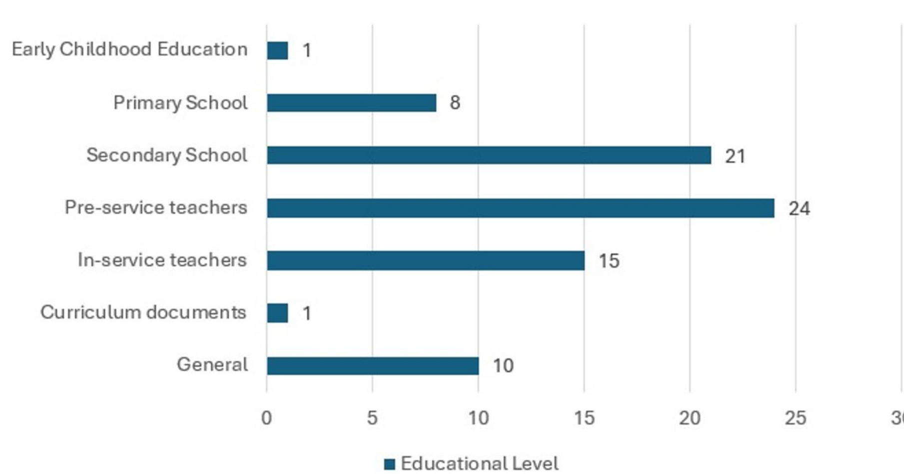

Fig. 3 Distribution of Publications across Educational Levels

proof-related thinking. Additional contributions were found from real-world applications that foster students' understanding of reasoning and proof.

Classroom culture consistently influences how students engage in reasoning and proof. A consistent pattern emerged showing that student-centred, inquiry-oriented classrooms were significantly more effective than traditional, teacher-led approaches. In teacher-centred classrooms, studies reported that these approaches limit students' reasoning and proof abilities, with only a few successful proof-related discussions observed [33]. By contrast, in classrooms that promote more personal conversations, the classroom environment improves, and students in these classes are more willing to participate in discussions involving reasoning and proof [3, 33]. In addition, the studies suggest that inquiry-oriented classrooms, which focus on conjecture formulation, expose students to more valuable information on a particular mathematical topic [3]. Thus, these findings indicate that open and supportive classroom environments significantly boost students' motivation to engage in conversations involving reasoning and proof.

Task design also plays an important role in fostering reasoning and proof. The studies demonstrated that open-ended, non-routine mathematical tasks shifted students from memorisation to analytical and creative thinking [2]. By encouraging students to explore multiple solution pathways, such tasks fostered argument construction and critical reflection. In contrast, narrowly defined procedural problems tended to restrict opportunities for reasoning.

The teacher's role as mediator was another crucial factor. Studies showed that guide-redirecting warrant construction, in which teachers indirectly intervene by making remarks that encourage students to formulate arguments, is an effective intervention technique. Studies have discovered that utilising the guide-redirecting warrant construction to foster reasoning and proof significantly improved students' post-test scores [37]. A valuable strategy for guide-directed instruction is to provide students with a table with two columns and prompt them to write two arguments in the two columns, one for the statement and the other for the reason. Similarly, the use of "how" and "why" questions during class discussions promotes reasoning and proof in the classroom by requiring students to explain and defend their claims [18, 33]. Together, these studies highlight that teachers who provided structured scaffolding, such as using two-column argument tables or promoting peer discussion, created conditions for deeper engagement with proof.

Beyond classroom tasks and talks, instructional frameworks structured approaches to include reasoning and proof in the mathematics classroom. The key practice framework, for instance, integrates reasoning and proof as a collection of critical thinking, writing, and reading abilities that need to be coordinated to meet specific literacy objectives [4]. Thus, by implementing the framework, teachers gain valuable insight and can utilise it to address the difficulties students experience when reasoning and proof are not sufficiently integrated into the classroom.

Finally, some research demonstrates that real-world application is sufficient for effective interventions that promote reasoning and proof in mathematics education. When mathematical concepts are linked to real-world applications, students are better able to construct, understand and communicate arguments [18, 33]. Therefore, when students have firsthand experience with the topic, they are more likely to reason and support their claims.

Overall, the evidence suggests that multifaceted instructional strategies, which combine an inquiry-based culture, cognitively demanding tasks, and guided teacher mediation, are the most effective in promoting reasoning and proof. Thus, teachers cannot rely on a single approach, but rather on a combination of approaches to collectively promote students' reasoning and proof abilities. However, the literature reveals several gaps: few studies evaluated the long-term impact of these strategies or compared their relative effectiveness across education levels. Moreover, most interventions focused on small classroom settings, limiting the generalisability of findings. Future research should therefore investigate how integrated, sustained instructional frameworks operate across diverse educational contexts.

5.6 Common challenges students face in understanding and engaging with reasoning and proof in mathematics

The reviewed studies reveal that students' difficulties with reasoning and proof are complex and related to various potential factors that may affect their ability to do so. Over the past decade, students have had significant difficulty with reasoning and proof. Their low performance in mathematical disciplines that heavily rely on reasoning and proof reflects this difficulty.

A consistent pattern across studies is students' focus on results rather than reasoning processes. Students often focus on the answer to the proof rather than on the process. It is evident in the study of [6] found that only 2% of the students included in the study focused on all the aspects of reasoning and proof. This behaviour reflects a procedural rather than a conceptual understanding of mathematics. Relatedly, several studies identified insufficient foundational and higher-order thinking skills as significant barriers. As Students who lacked conceptual understanding of mathematical principles struggled to construct and justify proofs [5, 29]. These findings suggest that reasoning and proof cannot develop in isolation but depend on students' broader mathematical fluency.

The second challenge outlined by the reviewed studies is the limited development of foundational and higher-order thinking skills. Several studies linked challenges in reasoning and proving to gaps in students' overall mathematical abilities [26]. argue that students' difficulties in other mathematical abilities directly affect their ability to reason and prove, while [4] noted that students struggle with reasoning and proof because these tasks require higher-order thinking. Students struggle with tasks that require higher-order thinking because they are accustomed to rote learning. Similarly, a study by [39] showed that students could find the correct solutions to mathematical problems but were unable to adequately justify their responses. Therefore, teaching and learning of reasoning and proof will not be effective if teachers or students lack the necessary mathematical skills. Thus, to address students' difficulties with reasoning and proof, students' overall mathematical abilities need to be addressed.

Another persistent challenge involves students' tendency toward rote and visual reasoning [26]. reported that students have difficulty with reasoning and proof, as they are prone to visual thinking and base their conclusions on shape rather than its properties, while [32] noted that the process of reasoning and proof is used in the classroom as an understanding and replicating skill rather than a process. Such approaches may support surface-level understanding but impede logical reasoning.

Teachers' knowledge and pedagogy further contribute to students' challenges with reasoning and proof. Studies have shown that teachers frequently feel unqualified to teach reasoning and proof, making it extremely difficult for them to help students develop these skills [40]. reported that teachers' lack of confidence in reasoning and proof instruction directly affected students' development, while [39] discovered that students' difficulties stem from their lack of fundamental mathematical knowledge. When reasoning and proof are taught in a teacher-led classroom, as [1] observed, students recite the information as facts and avoid engaging in mathematical problems.

Finally, affective factors also shape students' engagement with reasoning and proof [3]. noted that students are more likely to be unwilling to engage in the proof construction process when the conjecture construction is too challenging for them [41]. further highlighted that students' emotional and mental health can impact their ability to reason and prove. Thus, when reasoning and proof are incorporated into lessons as a challenging skill, students' ability to develop this skill can be affected. These findings suggest that while challenging tasks can enhance reasoning and problem-solving abilities, poorly planned challenging tasks may have the opposite effect.

In summary, these findings indicate that improving students' reasoning and proof abilities requires a dual focus: enhancing conceptual understanding and addressing motivational and affective factors. Teachers need to design learning environments that emphasise reasoning as a process of exploration rather than answer production. However, the literature remains limited in its exploration of emotional and metacognitive dimensions of reasoning and proof. Future research should therefore examine how students' beliefs, attitudes, and self-efficacy interact with cognitive and instructional factors to influence reasoning performance.

5.7 Teachers' knowledge, beliefs, and attitudes of reasoning and proof

The successful implementation of reasoning and proof in mathematics education depends heavily on teachers' knowledge, beliefs, and instructional strategies. The literature consistently shows that teachers' understanding of reasoning and proof, as well as their instructional decisions, are decisive factors influencing student outcomes. While teachers may hold strong mathematical content knowledge, this does not necessarily translate into practice. Across the studies, a recurring pattern was that teachers possessed adequate mathematical content knowledge but insufficient pedagogical content knowledge for teaching reasoning and proof—for instance [40], found that teacher knowledge and its application in the classroom are unrelated to reasoning and proof knowledge and competency, and [1] noted that teachers who lack sufficient understanding of reasoning and proof are reluctant to include it in the lessons and instead concentrate on fact-driven teaching. Similarly [32], found that teachers themselves struggle with these skills, indicating a lack of knowledge in reasoning and proof, which highlights a systematic gap in teacher preparation.

Beyond knowledge, teachers' participation is crucial for the development of reasoning and proof [34]. argued that teachers' mathematical knowledge should go far deeper than what the average student in mathematics learns, while [37] showed that while some aspects of reasoning and proof, such as constructing an argument, come readily to students, the process will not be sufficiently developed in the absence of a teacher. Similarly [42], emphasised that students are more likely to acquire more complex and clearly explored mathematical concepts and reasoning when they receive adult teaching. Teachers' participation in the process of reasoning and proof is therefore crucial.

However, studies indicated that teachers are unwilling to incorporate and engage with reasoning and proof [43]. found that 38% of the teachers in the study reported using reasoning and proof in their teaching sparingly, and 20% reported having no prior exposure to these concepts in mathematics. Similarly [44], observed that teachers utilised only a portion of the reasoning and proof process, leading them to conclude that their reasoning and proof abilities were weak. These findings suggest that teachers do not feel confident or capable of incorporating reasoning and proving processes in their mathematical teaching.

Finally, teacher beliefs emerged as another crucial factor. Teachers' beliefs influence their incorporation of reasoning and proof into their mathematical teaching [45]. reported that 33% of all teachers stated that not all students should be taught reasoning and proof, which directly influenced their willingness to incorporate it into their teaching. Kasa et al. [46], demonstrated that teachers' opinions and beliefs directly impacted their instructional strategies. These findings suggest that even when knowledge exists, negative or limited beliefs may be a barrier to implementation.

In sum, the literature suggests that teachers' knowledge, beliefs, and participation are crucial to the successful implementation of reasoning and proof in mathematics teaching. Professional development should therefore prioritise deep conceptual understanding of proof, opportunities for reflective practice, and strategies for facilitating reasoning discourse. Despite these insights, there remains a lack of longitudinal studies tracking changes in teachers' beliefs over time and systemic research into how curricular and policy contexts support or constrain proof instruction. Addressing these gaps could significantly advance the integration of reasoning and proof across educational systems.

6 Discussion

The results of the systematic literature review shed light on the significant role of reasoning and proof in mathematics education, as well as the various factors that influence their integration into teaching and learning. A key finding of this review is the strong emphasis on geometry-focused reasoning and proof in the publications. This finding was expected as [36] argue that reasoning and proof are most prevalent in the field of geometry, which could explain the 32% of publications dedicated to this content area. The publications that focused on general mathematical content areas reinforce the importance of reasoning and proof not only in geometry but in mathematics as a whole.

The review highlighted the growing global interest in reasoning and proof in mathematics education, as evidenced by the increase in publications on the topic since 2019. This continued increase in publications can be attributed to the growing awareness of the importance of reasoning and proof, as well as the ability to foster critical thinking and problem-solving in mathematics education. The review found that Türkiye, the United States, and Indonesia are at the forefront of publication contributions. This suggests that these nations are placing a great emphasis on the quality and incorporation of reasoning and proof in mathematics education.

Another key finding is the significant focus on teacher-focused publications, with nearly half of the studies focusing on reasoning and proof in teacher knowledge, beliefs, and instructional strategies. This highlights the crucial role that teachers play in the successful integration of reasoning and proof. A key observation in the review is teachers' limited knowledge of reasoning and proof, which affects their willingness to incorporate and develop these skills in mathematics education. As [1] found teachers' lack of reasoning and proof skills led them to avoid incorporating these skills into the classroom. Furthermore, a recurring theme in the literature is that teachers' lack of understanding of reasoning and proof directly affects their ability to engage students in the reasoning and proof process [40].

The emphasis on secondary school education in the review aligns with students' developmental stage, as those at this level are expected to engage more deeply with reasoning and proof. However, the low level of publications focusing on middle school and early childhood education may indicate that reasoning and proof are not central in the curriculum documents of these levels, although [42] argues that reasoning and proof are central to all mathematical levels.

Students' difficulties in the reasoning and proof process were a key theme in the review. Common difficulties students face with reasoning and proof include their inability to think critically and their tendency to rely on rote thinking [26]. argue that students' constant engagement with fact-driven tasks contributes to students' inability to think critically, which in turn affects students' ability to engage with reasoning and proof. According to [39], students' general mathematical abilities affect their ability to reason and prove. Thus, the difficulties students face in reasoning and proof are multifaceted, with multiple factors influencing their ability to reason and prove. It was found that these difficulties are exacerbated by teacher-led lessons which incorporate reasoning and proof as a fact-driven process,

To address these difficulties, several studies proposed intervention and instructional strategies, including question techniques and high-order problem-solving tasks that require students to think critically. Instructional strategies such as creating a supportive classroom environment that fosters reasoning and proof have been found effective in addressing reasoning and proof difficulties [34]. In addition, frameworks that can be incorporated into the mathematics classroom, such as the Key Practice framework [4], are beneficial for instruction. These frameworks can assist teachers in incorporating reasoning and proof.

Although there has been an overall emphasis on reasoning and proof in mathematics education over the last six years, several difficulties remain among students and teachers alike.

7 Conclusion

The review identified key instructional strategies and interventions that foster reasoning and proof in mathematics education. The strategies and interventions highlighted in the review include task designs, classroom environment, teacher reasoning and proof knowledge. The findings highlight the importance of a classroom environment that fosters a comfortable atmosphere where students and teachers alike feel free to engage in discussions about reasoning and proof. Studies reviewed suggest that transitioning from a teacher-centred approach to a learner-centred, interactive environment and incorporating higher-order and open-ended tasks can engage students in critical thinking and active participation, thereby fostering reasoning and proof.

The review highlighted several reasoning and proof challenges that students face, which may hinder their development of these skills. Students struggling with the complex nature of reasoning and proof were a key challenge in the studies. These challenges are often compounded by students' foundational knowledge of mathematics and teachers' limited understanding of reasoning and proof. The findings suggest that teachers' knowledge, beliefs and attitude towards reasoning and proof directly influence their ability to teach reasoning and proof. Which, in turn, reflects students' reasoning and proof skills.

The study revealed the importance of teachers in developing students' reasoning and proof skills. Therefore, emphasis should be placed on reasoning and proof in teacher training. In addition, teacher training should include the skills to develop a classroom environment that fosters interaction and conversation while guiding the conversations to include reasoning and proof. By implementing instructional strategies and interventions in practice, mathematics education can address current challenges faced by students and foster reasoning and proof in mathematics.

Future research is still necessary, even though there has been a noticeable rise in reasoning and proof in mathematics education publications. Future research might focus on the long-term effects of integrating reasoning and proof in teaching and on how to improve teachers' reasoning and proof skills. The success of integrating reasoning and proof in mathematics education can be enhanced by addressing teachers' grasp of these concepts.

7.1 Implications for practice

The systematic literature review's conclusions offer several recommendations for promoting reasoning and proof in mathematics instruction. First and foremost, integrating this ability requires establishing an educational environment that encourages reasoning and proof. Establishing a peaceful classroom atmosphere will enable students to participate in group discussions that the teacher can guide by incorporating reasoning and proof through "why" and "how" questions. Teachers can address the challenges of reasoning and proof as a fact-driven ability by integrating the process into its natural context through group discussions.

Additionally, incorporating problem-solving tasks into mathematics lessons can enhance students' critical thinking, enabling them to develop the mathematical skills necessary for reasoning and proof. Including real-world application tasks in mathematics lessons is one way to do this [38; 42]. The willingness of students to participate in familiar circumstances where they can apply their reasoning and proof skills can be increased by incorporating real-world application tasks into the classroom.

8 Limitations and recommendations

The study's inclusion and exclusion criteria presented several limitations. The scope of the study was restricted to studies published between 2019 and 2024. Although this focus highlights recent advancements in reasoning and proof in mathematics education, it inevitably can omit earlier studies that might have provided valuable background. Only peer-reviewed, full-text English studies were considered in the review. While peer-reviewed studies provided the study with reliability, other studies may have been omitted. Similarly, valuable studies published in another language may have been missed in the review due to the exclusion of non-English studies. The emphasis on mathematics education also narrows the findings, making them less applicable to other subject areas. Lastly, although explicit inclusion and exclusion criteria were applied, the review is still dependent on the availability and quality of studies published during the selected timeframe.

The review highlights several recommendations for future research on reasoning and proof in mathematics education. Future research can benefit from a broader timespan to examine how reasoning and proof in mathematics education have evolved over the years, while including Scopus as an additional journal. Including non-peer-reviewed studies in future reviews might enrich the body of evidence. Additionally, further research should examine how teachers' knowledge, beliefs, and attitudes toward reasoning and proof impact classroom practices, as these remain critical factors in shaping students' understanding and engagement.

Appendix

See Table 5.

Table 5 Publications and Sources

	Authors	Title	Source	Country	Mathematics content area
1	Akar, N., & Işıksal Bostan, M. (2024)	Nurturing Preservice Mathematics Teachers' Reasoning about Measures of Central Tendency and Variability	Taylor and Francis	Türkiye	Statistics
2	Aktas, F. N.	The Roles of Argumentation Structures for the Conviction of Proof Types	ERIC	Romania	General
3	Antonini, S.	Intuitive acceptance of proof by contradiction	ERIC	Norway	General
4	Anwar, L., Goedhart, M. J., & Mali, A.	Learning trajectory of geometry proof construction: Studying the emerging understanding of the structure of Euclidean proof	ERIC	Indonesia, Greece & Netherlands	Geometry
5	Arnesen, K. K., & Rø, K.	The complexity of supporting reasoning in a mathematics classroom of shared authority	Taylor and Francis	Norway	General
6	Ayalon, M., Wilkie, K. J., & Swaid, R.	Investigating students' arguments with real-life functional situations throughout a sequence of collaborative activities.	ERIC	Australia	Functions
7	Braithwaite, D. W.	Relations between geometric proof justification and probabilistic reasoning	ScienceDirect	USA	General
8	Can, Ö. S., & Isleyen, T.	The effect of probability instruction through argumentation approach on the achievement of preservice teachers and the permanence of their knowledge	ERIC	Türkiye	Probability
9	Castro, W. F., & Toro, J. A.	Mathematics teacher argumentation in a didactic perspective	ERIC	Colombia	General
10	Castro, W. F., Durango-Urrego, J. H., & Pino-Fan, L. R.	Preservice Teachers' Argumentation and Some Relationships to Didactic Mathematical Knowledge Features	ERIC	Italy	Geometry
11	Demir, M., & Zengin, Y.	The effect of a technology-enhanced collaborative learning environment on secondary school students' mathematical reasoning: A mixed method design	ERIC	Türkiye	Geometry
12	Demir, M., Zengin, Y., Özcan, Ş., Urhan, S., & Aksu, N.	Students' mathematical reasoning on the area of the circle:5E-based flipped classroom approach	Taylor and Francis	Türkiye	Geometry
13	Demiray, E., Işıksal, M., & Saygı, E.	Components of collective argumentation in geometric construction tasks	ERIC	Türkiye	Geometry
14	Demiray, E., Işıksal-Bostan, M., & Saygı, E.	How argumentation relates to formal proof process in geometry	ERIC	Türkiye	Geometry
15	Demircioğlu, H.	Preservice Mathematics Teachers' Proving Skills in an Incorrect Statement: Sums of Triangular Numbers	ERIC	Türkiye	Triangular numbers
16	Demircioğlu, H., & Hatip, K.	Examining Students' Proof Writing and Justification Skills in the Context of Sum of Measures of Polygons' Interior Angles	ERIC	Türkiye	Geometry
17	Dinçer, B.	Assessment of Students' Preferred Proof Schemes in the Context of the Analysis Course	ERIC	Türkiye	General

Table 5 (continued)

	Authors	Title	Source	Country	Mathematics content area
18	Erkek, O., & IŞIKSAL BOSTAN, M. İ. N. E.	A Different Look at the Reasoning Process of Prospective Middle School Mathematics Teachers: Global Argumentation Structures	ERIC	Türkiye	Geometry
19	Francisco, J. M.	Supporting argumentation in mathematics classrooms: The role of teachers' mathematical knowledge	ERIC	USA	General
20	Fredriksdotter, H., Norén, N., & Bråting, K.	Investigating grade-6 students' justifications during mathematical problem solving in small group interaction	ScienceDirect	Sweden	General
21	Gaita, R. C., Wilhelmi, M. R., Ugarte, F. J., & Gonzales, C. S.	Mathematical processes for the development of algebraic reasoning in geometrical situations with in-service secondary school teachers	ERIC	Peru & Spain	Algebra
22	Gaona, J., & Menares, R.	Argumentation of Prospective Mathematics Teachers in Fraction Tasks Mediated by an Online Assessment System With Automatic Feedback	ERIC	Chile	Fractions
23	Geçici, M. E., & Türnüklü, E.	Visual reasoning in mathematics education: A conceptual framework proposal	ERIC	Romania	General
24	Güler, C., & Güler, G.	Fifth-Grade Students' Argumentation Structures in the Pool of Geometric Shapes	ERIC	Türkiye	Geometry
25	Hacatrjana, L. & Dace N.	Breaking Down the Concept of Students' Thinking and Reasoning Skills for Implementation in the Classroom	ERIC	Latvia	General
26	Haj-Yahya, A.	Can a number of diagrams linked to a proof task in 3D geometry improve proving ability?	ERIC	Israel	Geometry
27	Haj-Yahya, A., Hershkowitz, R., & Dreyfus, T.	Investigating students' geometrical proofs through the lens of students' definitions	ERIC	Israel	Geometry
28	He, J., & An, T.	Pre-Service Secondary Mathematics Teachers' Opportunities to Learn Reasoning and Proof in Algebra	ERIC	USA	Algebra
29	Herbert, S., & Bragg, L. A.	Factors in a professional learning program to support a teacher's growth in mathematical reasoning and its pedagogy	ERIC	Australia	General
30	Herbert, S., & Williams, G.	Eliciting mathematical reasoning during early primary problem solving	ERIC	Australia	General
31	Hoffman, K., Williams, T. H., & Kephart, K.	The Use of Guided Reflections in Learning Proof Writing	ERIC	USA	General
32	Jablonski, S.	Real objects as a reason for mathematical reasoning – A comparison of different task settings	ERIC	Germany	Geometry

Table 5 (continued)

	Authors	Title	Source	Country	Mathematics content area
33	Joachin-Arizmendi, I., Locia-Espinoza, E., Morales-Carballo, A., & Reyna-Hernández, G.	Diagnostic study of mathematical reasoning in novice university students	ERIC	Mexico	General
34	Kartika, H., & Budiarto, M. T.	Assessing the Quality of Arguments in Students' Mathematical Problem Solving	ERIC	Indonesia	General
35	Kirsten, K., & Greefrath, G.	Proof construction and in-process validation – Validation activities of undergraduates in constructing mathematical proofs	ScienceDirect	Germany	General
36	Lee, T. N.	Justifying triangle shapes through their properties in argumentation	ERIC	Taiwan	Geometry
37	Mariotti, M. A., & Pedemonte, B.	Intuition and proof in the solution of conjecturing problems'	ERIC	Italy & USA	Geometry
38	Martins, M., Ponte, J. P. D., & Mata-Pereira, J.	Learning to promote students' mathematical reasoning: Lesson study contributions in initial teacher education	ERIC	Portugal	General
39	Miller, C., Menke, J., & Conner, A.	Collective Argumentation in Integrated Contexts: A Typology of Warrants Contributed in Mathematics and Coding Arguments	ERIC	USA	General
40	Mkhatshwa, T.	Problem Posing and Quantitative Reasoning Opportunities Provided by Mathematics Textbooks for Pre-service Elementary Teachers: A Qualitative Document Analysis	Taylor and Francis	USA	Fractions
41	Moralı, H. S. & Ahsen, F.	Incorrect theorems and proofs: An analysis of pre-service mathematics teachers' proof evaluation skills	ERIC	Türkiye	General
42	Muhtadi, D., Hermanto, R., & Sunendar, A.	How do students promote mathematical argumentation through guide-redirecting warrant construction?	ERIC	Indonesia	General
43	Mukuka, A., Balimuttajjo, S., & Mutarutinya, V.	Teacher efforts towards the development of students' mathematical reasoning skills	ScienceDirect	Zambia, Uganda & Rwanda	General
44	Negara, H. R. P., Santosa, F. H., & Siagian, M. D.	Overview of Student's Mathematics Reasoning Ability Based on Social Cognitive Learning and Mathematical Self-efficacy	ERIC	Indonesia	General
45	Nergård, B.	Preschool children's mathematical arguments in play-based activities	ERIC	Norway	Mathematical play
46	Nhiry, M., Abouhanifa, S., & El Khouzai, E. M.	The characterization of mathematical reasoning through an analysis of high school curricula and textbooks in Morocco	Taylor and Francis	Morocco	General

Table 5 (continued)

	Authors	Title	Source	Country	Mathematics content area
47	Otten, S., Wambua, M. M., & Govender, R.	Who Should Learn Proving and Why: An Examination of Secondary Mathematics Teachers' Perspectives	ERIC	South Africa & USA	General
48	Peng, X., Zhang, J., Chen, M., & Liu, S.	Self-evident Automated Geometric Theorem Proving Based on Complex Number Identity	ERIC	China	Complex numbers
49	Quaresma, P., & Graziani, P.	Measuring the Readability of Geometric Proofs: The Area Method Case	ERIC	Portugal & Italy	Geometry
50	Rahayu, P., & Cintamulya, I.	Scaffolding worksheets as a medium of student learning during the COVID-19 pandemic in constructing proof of group problems	ERIC	Indonesia	Algebra
51	Rahayu, P., & Cintamulya, I.	Scaffolding as a strategy to help student difficulties in proving group problems	ERIC	Indonesia	Algebra
52	Ramírez-Uclés, R., & Ruiz-Hidalgo, J. F.	Reasoning, Representing, and Generalizing in Geometric Proof Problems among 8th Grade Talented Students	ERIC	Spain	Geometry
53	Renninger, K. A., Gantt, A. L., & Lipman, D. A.	Comprehension of argumentation in mathematical text: what is the role of interest?	ERIC	USA	General
54	Rø, K., & Arnesen, K. K.	The opaque nature of generic examples: The structure of student teachers' arguments in multiplicative reasoning	ScienceDirect	Norway	Algebra
55	Rosita, C. D., Kusumah, Y. S., Suryadi, D., & Kartasasmita, B. G.	Implementation of cooperative integrated reading and composition (CIRC) to enhance mathematical argumentation ability of mathematics teacher students	ERIC	Indonesia	General
56	Salazar-Torres, J., Vera, M., Contreras, Y., Gelvez-Almeida, E., Valbuena, O., Barrera, D., & Rincon, O.	Mathematical argumentation in the classroom	ERIC	Colombia	General
57	Sarıbaş, H. M., & Ay, Z.	Investigation of Middle School Students' Probabilistic Reasoning Levels in Terms of Some Variables	ERIC	Türkiye	Probability
58	Schubert, S., Pekrun, R., & Ufer, S.	The role of epistemic emotions in undergraduate students' proof construction	ERIC	Germany, Australia & England	Geometry
59	Şen, C., & Güler, G.	Emerging Proof Productions of Freshmen in Euclidean Geometry Proof Tasks between Conjecturing and Proving	Taylor and Francis	Türkiye	Geometry
60	Sengül, S., & Satir Altinel, Ç.	The Examination of Argumentation Based Problem Solving Processes of 10th Grade Students in the Context of Quadratic Equations	ERIC	Istanbul	Algebra
61	Shimizu, Y.	Relation Between Mathematical Proof Problem Solving, Math Anxiety, Self-Efficacy, Learning Engagement, and Backward Reasoning	ERIC	Japan	Geometry

Table 5 (continued)

	Authors	Title	Source	Country	Mathematics content area
62	Shinno, Y., & Fujita, T.	Characterizing how and when a way of proving develops in a primary mathematics classroom: a commognitive approach	Taylor and Francis	UK	Algebra
63	Shongwe, B.	The Quality of Argumentation in an Euclidean Geometry Context: A Case Study	ERIC	South Africa	Geometry
64	Shongwe, B.	Grade 11 Students' Proof Construction Ability in Relation to Classroom Resources	ERIC	South Africa	Geometry
65	Smit, R., Hess, K., Taras, A., Bachmann, P., & Dober, H.	The role of interactive dialogue in students' learning of mathematical reasoning: A quantitative multi-method analysis of feedback episodes	ScienceDirect	Switzerland	General
66	Solar, H., Ortiz, A., Arriagada, V., & Deulofeu, J.	Argumentative orchestration in the mathematical modelling cycle in the classroom	ERIC	Chile & Barcelona	General
67	Song, Y., van Rijn, P., Deane, P., & Chao, S. F.	Assessing argumentation skills of middle school students: a learning progression approach	ERIC	Netherlands	General
68	Tchonang Youkap, P., Njomgang Ngansop, J., Tieudjo, D., & Nchia Ntam, L.	Influence of Drawing and Figures on Secondary School Students' Argumentation and Proof: An Investigation on Parallelogram	ERIC	Romania	Geometry
69	Thom, J. S., & Hallenbeck, T.	Spatial reasoning in mathematics: A cross-field perspective on deaf and general education research	Taylor and Francis	Canada & USA	General
70	Tristanti, L. B., & Nusantara, T.	Improving Students' Mathematical Argumentation Skill Through Infusion Learning Strategy	ERIC	Indonesia	General
71	Urhan, S., & Bülbül, A.	Analysis of mathematical proving in geometry based on Habermas' construct of rationality	ERIC	Türkiye	Geometry
72	Uygun-Eryurt, T.	Conception and Development of Inductive Reasoning and Mathematical Induction in the Context of Written Argumentations.	ERIC	Romania	Geometry
73	Uzun, A. G.	The development of mathematical argumentation: A case study on two mathematics classrooms	ERIC	Türkiye	General
74	Valenta, A., & Enge, O.	Teaching practices promoting meta-level learning in work on exploration-requiring proving tasks	ScienceDirect	Norway	General
75	Wihardjo, E., & Prayitno, S.	Male students' visual reasoning in solving mathematical problem	ERIC	Indonesia	General
76	Wilkie, K., & Ayalon, M.	Learning to argue while arguing to learn: Students' emotional experiences during argumentation for graphing real-life functions	ERIC	Israel & Australia	Functions
77	Winer, M. L., & Battista, M. T.	Investigating Students' Proof Reasoning: Analyzing Students' Oral Proof Explanations and their Written Proofs in High School Geometry	ERIC	USA	Geometry

Table 5 (continued)

	Authors	Title	Source	Country	Mathematics content area
78	Worowirastri Ekowati, D., Makbul, T., & Agus, D.	A Literature Review Of Multimodal Semiotic Reasoning In Mathematics	ERIC	Indonesia	General
79	Yerushalmy, M., & Olsher, S.	Online assessment of students' reasoning when solving example-eliciting tasks: using conjunction and disjunction to increase the power of examples	ERIC	Israel	General
80	Zengin, Ş., & Erdem, E.	A Detailed Examination of 8th Grade Students' Mathematical Reasoning Process	ERIC	Türkiye	General

Author contributions

Ugorji I. Ogbonnaya conceptualised the study and methodology; Both authors identified databases and reviewed the studies. Mary-Jane Lessing wrote the first draft. Both authors reviewed and edited the final draft.

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