

Micro-grid energy optimisation with demand management

by

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SUMMARY

MICRO-GRID ENERGY OPTIMISATION WITH DEMAND MANAGEMENT

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Pressure on the utility grid and aims to reduce carbon emissions have led to the development of micro-grids. Extensive research has been done to optimally utilize the various energy sources present therein. Demand management strategies have also been developed to minimize strain on the national utility grid. However, not much research has been done to consider demand management in a microgrid environment. This dissertation proposes the development of a microgrid for a small residential complex in South Africa. Heuristic optimisation techniques are investigated to ensure optimal energy source utilisation and load scheduling, while minimising inconvenience caused for consumers. It is expected that the system will offer significant long-term savings as well as a stable supply of electricity.

LIST OF ABBREVIATIONS

AC	alternating current
AMSL	above mean sea level
CTF	cycles-to-failure
DC	direct current
DE	differential evolution
DG	diesel generator
DM	demand management
DN	dynamic
DOD	depth-of-discharge
EoL	end of life
EUR	Euro
ER	electrolyser
ESS	energy storage system
FC	fuel cell
GA	genetic algorithms
GOT	Global Optimization Toolbox
HOMER	Hybrid Optimization of Multiple Energy Resources
ICs	initial conditions
LB	lower bound
LP	load profile
LEC	levelised energy cost
MG	micro-grid
MPPT	maximum power-point tracking
NG	nano-grid
NN	neural network
NRES	non-renewable energy sources
PEM	proton exchange membrane
PSO	particle swarm optimization
PV	photo-voltaic
RES	renewable energy sources

SAURAN	Southern African Universities Radiometric Network
SM	surrogate modelling
SO	surrogate optimization
SOC	state-of-charge
SS	steady-state
SOM	self organizing map
SR	sampling rate
STC	standard test conditions
STP	standard temperature and pressure
TLF	tank level factor
TOU	time of use
UB	upper bound
UG	utility grid
USD	United States Dollar
VRLA	valve-regulated lead-acid
WTG	wind turbine generator
ZAR	South African Rand

LIST OF SYMBOLS

A	area
A_{blade}	WTG blade sweep area
A_{FC}	FC active cell area
A_{floor}	floor area per house
a_{H_2O}	ER electrode and membrane water activity
A_{PV}	PV array area
A_{roof}^{tot}	total roof area per house
A_{roof}^{use}	usable roof area per house
A_{rot}	WTG rotational sweep area
b_n	ESS state
c_{bat}^{cap}	single battery capital cost
c_{bat}^{ref}	battery reference cycle cost
c_{bat}^{cyc}	battery discharging cycle cost
c_{bnk}^{cap}	battery bank capital cost
c_{ER}^{cap}	ER capital cost
c_{ER}^{cyc}	ER cycle cost
c_{ER}^{ref}	ER reference cost
C_{FC}	FC equivalent capacitance
c_{FC}^{cap}	FC capital cost
c_{FC}^{cyc}	FC cycle cost
C_{H_2}	Hydrogen concentration
C_{O_2}	Oxygen concentration
C_p	wind turbine power coefficient
c_{PV}^{cap}	total PV capital cost
c_{PV}^{sgl}	single PV module cost
C_{UG}^{pen}	UG emission penalty cost
C_{UG}^{tot}	UG total cost
C_{UG}^{TOU}	UG TOU cost
CL_k	communal load k
CTF	cycles-to-failure

$\overline{CTF}$	reference cycles to failure value
c_{WTG}^{cap}	total WTG capital cost
c_{WTG}^{sgl}	single WTG cost
$DOD\%$	depth of discharge (%)
$\overline{DOD}$	reference depth of discharge value
d_{rot}	WTG rotor diameter
e_{ER}	Electrolyser thermodynamic potential
e_{FC}	Fuel cell thermodynamic potential
e_{FC}^{del}	FC fuel and oxidant delay effect
E_{home}	Average household daily energy consumption
E_{LD}^{tot}	daily energy consumed by MG
$E_{through}$	lifetime energy throughput per battery
E_{wnd}	wind kinetic energy
F	Faraday's Constant
F_{bat}	battery lifetime de-rating factor
F_{CO}	CO emission factor
F_{CO_2}	CO ₂ emission factor
F_{SO_2}	SO ₂ emission factor
F_{NO_x}	NO _x emission factor
F_{RE}	renewable energy factor
F_{UG}	UG emission factor
g	gravitational acceleration
G_{sol}	solar irradiance
G_d	time delay transfer function
H	Diesel cost
H_{CO}	CO penalty cost
H_{CO_2}	CO ₂ penalty cost
h_{FC}	convective heat transfer coefficient for FC
H_{SO_2}	SO ₂ penalty cost
H_{NO_x}	NO _x penalty cost
H_{UG}	UG penalty cost
$HL_{h,\ell}$	household h load ℓ
I_{FC}	FC current

J_a	ER anode current density
J_a^0	ER anode exchange current density
J_c	ER cathode current density
J_c^0	ER cathode exchange current density
J_{FC}	FC operating current density
J_{FC}^{max}	FC maximum current density
K	DG maintenance cost coefficient
K_{ER}^{cell}	ER cell specific heat capacity
K_{FC}^{cell}	FC cell specific heat capacity
K_{ER}^{stack}	ER stack specific heat capacity
K_{FC}^{stack}	FC stack specific heat capacity
ℓ_{mem}	FC membrane thickness
m_{air}	air mass
M_{air}	air molar mass
m_{FC}^{stack}	FC stack mass
M_{TOU}	TOU matrix
n_e	Number of electrons transferred
N_{bat}^{MG}	number of batteries in MG
N_{bat}^{se}	number of series connected batteries
N_{bat}^{sh}	number of shunt connected batteries
N_{home}^{MG}	number of homes in MG
N_{PV}^h	number of PVs per home
$N_{PV}^{h,max}$	maximum number of PVs per home
$N_{PV}^{h,min}$	minimum number of PVs per home
N_{PV}^{MG}	total number of PVs installed in MG
$N_{PV}^{MG,max}$	maximum number of PVs in the MG
$N_{PV}^{MG,min}$	minimum number of PVs in the MG
N_{WTG}^{MG}	total number of WTGs installed in MG
p_{H_2}	Hydrogen partial pressure
P_{load}	load power
P_{LD}^h	single home load power
P_{LD}^{tot}	total MG load power
$P_{non-ren}$	power generated using NRES

p_{O_2}	Oxygen partial pressure
P_{PV}^{sgl}	PV power (single module)
P_{PV}^{tot}	PV power (total)
P_{ren}	power generated using RES
p_{STP}	STP pressure, 101.3 kPa
P_{sun}	sun power
P_{wnd}	wind power
P_{WTG}^{extr}	extracted wind power
P_{WTG}^{sgl}	power generated by single wind turbine
P_{WTG}^{tot}	power generated by all wind turbines
Q_{bat}	single battery capacity
Q_{ESS}	ESS capacity
$R_{act+con}$	FC activation and concentration over-voltage equivalent resistance
R_{blade}	WTG blade radius
R_e	Electron membrane resistance
R_p	Proton membrane resistance
R_{rot}	WTG rotor radius
R_{se}	series resistance
R_{sh}	shunt resistance
R_T	thermal resistance due to air convection
R_{uni}	Universal Gas Constant
$S_{h,\ell}$	state of household h load ℓ
S_k	state of communal load k
SF	safety factor
$SOC_{\%}$	state-of-charge (%)
SOC_0	initial state-of-charge (interval 0)
SOC_{48}	final state-of-charge (interval 48)
SOC_{max}	maximum state-of-charge
SOC_{min}	minimum state-of-charge
t	time
T_A	ambient temperature
T_C	PV cell temperature
T_{FC}	Fuel cell temperature

T_{nom}	WTG nominal STC temperature, 15 °C
T_{STC}	PV STC temperature, 25 °C
T_{STP}	STP temperature, 273.15 K
V_{batt}	single battery terminal voltage
V_{ER}^{act}	ER total activation voltage drop
$V_{ER}^{act,a}$	ER anode activation voltage drop
$V_{ER}^{act,c}$	ER cathode activation voltage drop
V_{ER}^{cell}	ER single cell voltage
V_{ER}^{con}	ER concentration voltage drop
V_{ER}^{ohm}	ER Ohmic voltage drop
V_{ER}^{stack}	ER stack voltage
v_f	wind speed measured well above PV array
V_{FC}^{act}	FC total activation voltage drop
$V_{FC}^{act,a}$	FC anode activation voltage drop
$V_{FC}^{act,c}$	FC cathode activation voltage drop
V_{FC}^{cell}	FC single cell voltage
V_{FC}^{con}	FC concentration voltage drop
V_{FC}^{ohm}	FC Ohmic voltage drop
V_{FC}^{stack}	FC stack voltage
v_w	wind speed measured close to PV array / average wind speed
x	thickness
Z_{EUR}	EUR to ZAR conversion factor
Z_{USD}	USD to ZAR conversion factor
z_{AMSL}	elevation above sea level
α	WTG constant 1
α_{PV}	PV de-rating coefficient
α_{STC}	PV power-temperature coefficient
β	WTG constant 2
β_{con}	FC concentration drop constant
γ	WTG constant 3
ΔG_{ER}	ER Gibbs free energy
ΔG_{FC}	FC Gibbs free energy
ΔH_{ER}	ER Gibbs enthalpy change

ΔH_{FC}	FC Gibbs enthalpy change
ΔS_{ER}	ER Gibbs entropy change
ΔS_{FC}	FC Gibbs entropy change
ΔT	temperature difference coefficient
Δt	integration period
ζ	number of days that the ESS should be able to meet the MG load demand
$\eta_{coulomb}$	ESS Coulombic efficiency
η_{ESS}	Overall ESS efficiency
η_{gear}	WTG gearbox efficiency
η_{gen}	WTG generator efficiency
η_{PV}	PV module efficiency
η_{STC}	PV module efficiency at STC
η_{volt}	ESS Voltage efficiency
η_{WTG}	WTG efficiency
θ	blade pitch angle
θ_{roof}	roof pitch angle
λ	tip speed ratio
λ_e	
μ	FC operating parameter
ξ_1	FC parametric coefficient 1
ξ_2	FC parametric coefficient 2
ξ_3	FC parametric coefficient 3
ξ_4	FC parametric coefficient 4
ρ_{air}	air density
ρ_{mem}	FC membrane resistivity
ρ_{nom}	WTG nominal air density for STC, 1.225 kg/m ³
τ_e	
τ_{FC}	FC time delay
ϕ	PV mounting coefficient
ω_B	WTG rotational speed
ω_R	WTG rotor speed

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CHAPTER 1 INTRODUCTION

1.1 PROBLEM STATEMENT

1.1.1 Context of the problem

The 21st century has been marked by a considerable increase in the human population. There is also an increase in the number of developing countries, e.g. in Africa, Asia and South America, resulting in higher electricity demand as more consumers are added to the electricity grid [13, 14]. In South Africa alone, the utility grid is under severe pressure. This led to the 2008 energy crisis where large-scale rolling blackouts had a tremendous effect on the economy [15].

Many countries are still mainly dependent on fossil fuels for electricity generation and there is a growing emphasis on reducing the resultant carbon emissions [16]. Solutions are required to reduce pressure on the utility grid, increase the reliability of supply and decrease dependence on fossil fuels, especially in South Africa.

1.1.2 Research gap

A micro-grid (MG) is considered to be one of the solutions and will most likely form an integral part of future electrical generation and supply. Research has previously been done to optimally control MGs with regards to the different energy sources present and many of these publications focus on rural electrification [17, 18], while some consider residential grid-tied environments [19].

To optimize the operation of an MG, heuristic methods such as particle swarm optimisation (PSO) and genetic algorithms (GA), are most frequently used [20, 21]. However, there are more than 37 suitable techniques that can be considered, depending on each unique scenario and MG requirements [22].

As a means to further optimize an MG, demand management (DM) is also considered [23]. However, the inconvenience caused by DM, especially for residential use, is not thoroughly investigated. A comparison is also required to evaluate whether the implementation of DM presents a significant amount of savings, or whether it can be neglected.

1.2 RESEARCH OBJECTIVE AND QUESTIONS

The research objective is to develop a hybrid energy solution that can minimize the impact of national electricity shortages, price hikes and environmental concerns on consumers. The solution will mainly focus on the needs of a small residential complex in South Africa, however, with minor adjustments, it could easily be customized to meet different requirements.

To minimize operating costs, different renewable energy sources will be utilized and DM will be implemented. Optimisation techniques are required to optimally tap power from the available sources as well as to schedule certain loads on the system. However, load scheduling will inevitably result in some form of inconvenience caused. It would thus be required to determine whether the cost-saving benefits exceed the drawback of the resulting disruption caused.

Furthermore, as most renewable energy sources rely on atmospheric conditions, it is extremely fluctuating by nature. It is therefore required for the controller to execute its optimisation algorithms in real time, using a reasonable amount of computing power.

Keeping these considerations in mind, the following research questions will be investigated:

- What are the typical loads contributing to the daily load profile of a small residential complex in South Africa, and which of these loads can be scheduled at a minimal inconvenience?
- How do the different optimisation techniques compare when determining the most efficient operating point of the MG, considering factors like accuracy, speed and complexity?
- What are the savings present, if any, when DM is considered to minimize the operating costs of an MG and is it worthwhile to implement DM on a daily basis when considering the possible inconvenience caused?

1.3 APPROACH

A renewable energy MG system will be developed for a small residential complex in South Africa. The MG will utilize a wind turbine generator (WTG), fuel cell (FC), solar photovoltaic (PV) cells, a

hybrid energy storage system (ESS) as well as the utility grid (UG), to meet the electricity demand. Optimisation techniques will then be applied to optimally tap electricity from the sources, given their availability and operational costs. These costs will typically consider expenses related to fuel, maintenance and/or emissions.

To further optimize the MG by implementing DM, an estimation of the typical loads present and their usage data would be required. Each load will then be classified according to its flexibility and the inconvenience caused when rescheduled. They will also be classified as being either household loads (HLs) or communal loads (CLs), with HLs including equipment used for cooking, temperature regulation, lighting and entertainment. CLs will typically include a swimming pool, water or irrigation pumps, as well as security systems. It is expected that CLs will be easier to schedule, presenting greater savings at lower drawbacks.

A multi-objective optimisation problem will thus be developed to minimize both the cost and inconvenience functions. Different optimisation techniques will be evaluated and compared in order to determine the most effective solution. This will be done in a simulation environment, comparing speed, accuracy and computational complexity. It is expected that GA would be most suitable, however, the implementation thereof would be more complex than PSO. To determine whether the savings presented by DM is in fact worthwhile, the MG will also be simulated without load scheduling. The results will then be compared with the initial set, and sufficient savings are expected.

1.4 RESEARCH GOALS

The research goals are to develop an MG solution for a small residential complex in South Africa that can aid in mitigating the current energy crisis. There will also be aimed at providing a comparison between different optimisation techniques, as well as the impact of DM, and whether significant savings are procured.

1.5 RESEARCH CONTRIBUTION

The research will contribute to the body of science by providing insight into the benefits of DM in a residential MG environment. Additionally, the research will also discuss and evaluate some of the most prevalent optimisation techniques that are often considered for MG-related applications.

1.6 RESEARCH OUTPUTS

The research outputs include this dissertation, as well as a journal publication. IET Renewable Power Generation will be the target journal.

1.7 OVERVIEW OF STUDY

Chapter 2 describes the review of the literature. In Chapter 3 the research methodology is discussed. Chapter 4 outlines numerical and experimental results. In Chapter 5 results are summarized and discussed. In Chapter 6 concluding remarks are provided.

CHAPTER 2 LITERATURE REVIEW

2.1 CHAPTER OVERVIEW

The literature study presented in this chapter is divided into the following parts: Micro-Grids (MGs) as a renewable energy solution in Section 2.2, Energy optimisation and Control in Section 2.3 as well as Demand Management (DM) in Section 2.4. The literature study is summarized in Section 2.5.

2.2 MICRO-GRIDS

2.2.1 Background

MGs as a solution to the future of electricity supply and generation was first proposed by R. H. Lasseter in 2002, who defined it as a "cluster of loads and micro-sources operating as a single controllable system" [24]. These micro-sources can vary greatly, depending on the location and requirements of each individual MG, and would typically have a generation capacity of less than 100 kW. More recently, nano-grids (NGs) are also being considered, which can be described as down-scaled, DC-only versions of MGs [25, 26].

MGs are often classified according to grid connection, integration method and integrated elements [27]. An MG can interact with the utility grid (UG) by either supplying or demanding electricity, and when operating in isolation from the UG, it is referred to as islanding mode. Integration methods consider how the different MG loads and sources are integrated, usually by either a DC or AC bus-bar, or both (hybrid approach); each with its own advantages and disadvantages. The term "integrated elements" refers to the loads and sources, albeit AC or DC, single or hybrid, renewable energy or not. A centralized controller ensures continuous and optimal operation of the MG. It should be able to effectively monitor and control real and reactive power, as well as voltage and frequency in a matter of milliseconds, with sufficient fault protection in place [24].

An MG developed in South Africa will be able to utilize both wind turbine generators (WTGs) and solar photovoltaic (PV) cells as renewable energy sources (RES) due to most parts of the country having adequate conditions therefor [16].

In urban areas, a residential MG will typically interact with the UG [19], whereas rural MGs are operated in islanding mode [28, 29]. MGs can also be used where the UG is unreliable due to theft and maintenance issues, however controlling such a system can become quite involved: A previously developed DC MG is able to automatically change between on-grid and off-grid modes, with the ability to sell energy back to the UG [30].

The performance of an MG can be evaluated by representing the environmental impact as a renewable energy factor (F_{RE}): A higher F_{RE} will indicate that more RES are being utilized and vice versa: $\sum P_{non-ren}$ is the sum of the non-renewable energy sources (NRES) and $\sum P_{ren}$ is the sum of the RES [31].

$$F_{RE} = \left(1 - \frac{\sum P_{non-ren}}{\sum P_{ren}} \right) \quad (2.1)$$

2.2.2 Load profiling

The first step in developing an MG would be to consider the electrical loads that need to be energized, whereafter the respective energy sources can be selected and sized accordingly [32]. In one study, an MG was developed for a small city consisting of 500 homes and 10 shops: The average load for a single house and shop was determined and then multiplied accordingly to determine the overall daily energy consumption [32]. For a "rural household micro-grid system" developed in China, researchers surveyed 20 households in order to determine their load requirements [33]. The number of active hours per day for four electrical loads (television, water heater, lights and a fan) was used to calculate the daily energy consumption of each home [33].

As the amount of energy generated by most RES are varying functions of time, more detailed load data is required: A 24-hour load profile (LP) depicting all the different loads and their durations can thus be used [34]. LPs can have sampling rates (SRs) between 15 and 60 minutes (low resolution) or even less than 60 seconds (high resolution) [35]. External factors, such as the rate of measurements available from weather stations or UG integration period can influence the choice of SR.

Two means for generating LPs are proposed by a study from 2012: Self Organizing Maps (SOMs) and

Neural Networks (NNs) [36]. SOMs are described as having the "capability to classify complex sets of patterns in an unsupervised way" which, in short, can be used to generate a load profile (LP) based on data from surveys [36]. This technique is useful during the MG design phase, whereas NNs will rather be implemented during day-to-day operation allowing for a forecasting horizon of up to 2 days ahead [36]. Both SOMs and NNs, however, require a fair amount of computational processes.

A publication from 2021 aimed at reviewing residential electricity LP models, evaluated more than 30 approaches based on method, SR, application and statistical procedures [35]. These models can be broadly classified as being either bottom-up or top-down, with the former using the energy consumption of different loads to determine the overall household requirements. Bottom-up models are also described as being "very well suited for studying energy optimisation techniques" on household LPs, with the drawback of being "computationally heavy" [35]. Top-down models, on the other hand, consider the overall energy consumption of the residential sector and uses macro-variables such as climate, housing construction and employment rates for LP estimations [37].

With DM considered, 15 suitable models were identified using the bottom-up method, two using the top-down and two using hybrid approaches; however, the latter is recommended [35]. These models had SRs varying from 1 second to 1 hour, and it should be emphasized that the number of hybrid approaches will likely increase in future works [35]. For this dissertation, a bottom-up model, consisting of survey data, will be used to construct daily LPs for individual households as well as the overall MG.

2.2.3 Source modelling

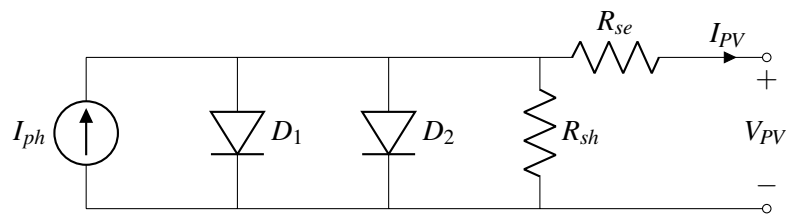
After developing load models, energy sources can be modelled and sized accordingly to balance operational costs with performance reliability. Different techniques are proposed therefor, depending on the RES present and whether the MG operates in islanding mode or not. Operational constraints can be placed on the number of WTGs, PVs and batteries installed, each being a function of the available installation area and estimated load [38].

In a stand-alone MG developed for a small village, the total daily load power demand was proportionally divided between the three energy sources present: 25% PV, 15% WTG and 60% DG [32]. Many publications also use the HOMER (Hybrid optimisation of Multiple Energy Resources) software package for MG-related problems [32, 39, 40].

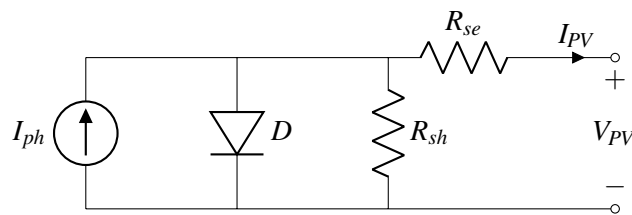
2.2.3.1 Solar PVs

Different solar PV models have been developed over the years, each with varying degrees of complexity and accuracy. Broadly, these models can be classified as either dynamic (DN) or steady-state (SS), with the choice of the model being guided by the available data and what the model is required to predict. A textbook dedicated to PV modelling identified two sets of steady-state models: One to describe the "functional current-voltage relationship at the PV terminals" and the other for long-term energy analysis [1].

To look at the current-voltage relationship, the equivalent circuit diagrams shown in Figure 2.1 can be used [1]. The double-diode circuit is the most accurate, but for simplicity, the single-diode approximation can be used with small errors occurring only when solar irradiance is low [1]. Variations on these diagrams also exist, where either the series (R_{se}) or shunt resistances (R_{sh}), or even both, are neglected.



(a) Double-diode equivalent circuit.



(b) Single-diode equivalent circuit.

Figure 2.1. Basic PV equivalent circuit diagrams. Adapted with permission [1].

Various models for long-term energy analysis have been identified and their accuracies evaluated [4]. Different environmental data and assumptions are considered for each model, which is summarized in Table 2.1. To estimate the output power delivered by a PV module system, the most accurate models will have to consider the effects of ambient temperature, solar irradiation and PV efficiency. Such information should be readily available from weather stations and PV manufacturers.

Table 2.1. Table summarizing different PV models and their respective accuracies. Adapted from [4], ©IEEE 2018.

Model name	Brief description	Accuracy level
Irradiance model	Only considers the effect of solar irradiance and PV ratings	Medium
Irradiance and temperature model	Considers the effect of solar irradiance, ambient temperature and PV ratings	High
Irradiance, temperature and weather model	In addition to irradiance and temperature, weather probabilities are also considered	Low
Irradiance, temperature and loss model	In addition to irradiance and temperature, a loss coefficient is introduced	High
Simplified irradiance model	Only considers efficiency, solar irradiance and PV module area	Medium

The irradiance and temperature model is highly accurate because the efficiency of a solar PV is directly influenced by the cell temperature, T_C . As T_C increases, the open-circuit voltage drops, yielding higher short-circuit currents and a lower efficiency [5]. Real-time information regarding the PV cell temperature is generally unknown and needs to be determined from available data, such as solar irradiation and ambient temperature. The wind speed, PV technology and installation technique can also influence the cell temperature [5, 41].

A simple approximation which relates the cell temperature to the ambient temperature, T_A , based on how the panels are mounted is shown in (2.2) [5]. Values for the difference coefficient – ΔT , which is dependent on the solar panel installation method – are shown in Table 2.2 [5]. (2.3) can also be used to determine the cell temperature as a function of wind speed (v_f), solar irradiation (G_{sol}) and ambient temperature by using a mounting coefficient, ϕ [6]. The latter is expected to be more accurate since wind speed is considered.

$$T_C = T_A + \Delta T \quad (2.2)$$

$$T_C = T_A + \left(\frac{0.32\phi}{8.91 + 2v_f} \times G_{sol} \right) \quad (2.3)$$

Table 2.2. Table showing the temperature difference values (ΔT) and mounting coefficients (ϕ) of different PV installation methods [5, 6].

PV mounting type	ΔT [$^{\circ}\text{C}$]	ϕ
Free standing	-	1.0
Flat roof (frame mounted)	18.4	1.2
Pitched roof (directly mounted)	21.5	1.8
Facade integrated (built-in)	25.4	2.4

2.2.3.2 Wind turbine generator

At first glance, the modelling of WTGs may sound fairly straightforward. However, this is not always the case, especially when the modelling of synchronous generators and power electronics are considered [42]. Throughout the literature, four different types of WTGs are often seen, each with a different combination of shunt capacitors, variable resistors and/or power converters:

- Fixed-speed induction motor WTG;
- Variable-speed induction motor WTG, with variable rotor resistance;
- Variable-speed, doubly-fed induction motor WTG; and
- Variable-speed WTG with a full power converter [43].

As wind passes through the turbine's blades, a rotational force is generated which turns a gearbox shaft. The gearbox then increases the speed of rotation fed to the generator, which can be of either induction, synchronous or DC type. Power electronics and/or transformers then modify the generated power before it is distributed. There are thus quite a few subsystems with varying degrees of efficiency present, which all require consideration.

Many publications use (2.4) – or slightly modified versions thereof – to estimate the WTG output power, P_{WTG}^{sgl} , as a function of the wind speed (v_{wnd}) and air density (ρ_{air}) [44, 45]. Other factors considered are the blade cross-sectional area (A_{blade}), power coefficient (C_p), as well as the gearbox

and generator efficiencies (η_{gear} and η_{gen}) calculated in (2.5).

$$P_{WTG}^{sgl} = \eta_{wtg} \times P_{extr} \quad (2.4)$$

$$= \eta_{wtg} \times C_p \times \frac{\rho_{air} A v_{wnd}^3}{2}$$

$$\eta_{wtg} = \eta_{gear} \times \eta_{gen} \quad (2.5)$$

Randomized values between zero and the WTG's rated power can be used to simulate the typical intermittent behaviour of wind power [46]. Such a technique is, however, more suitable when developing a control system, as it does not consider real-time atmospheric conditions. For the purpose of this dissertation, a hybrid approach considering both atmospheric conditions and manufacturers' data will be used.

As a WTG generator does not consume any fuel or pollute the atmosphere in any way, only the initial capital cost and periodic maintenance expenses will apply [21]. A value of 0.02 USD/kWh is suggested for the per unit maintenance cost of a 30 kW WTG system, with another publication suggesting a levelised energy cost (LEC) between 0.25 - 0.33 USD/kWh for a 10 kW WTG [21, 44]. However, the latter set of values is determined for a system with miscellaneous components included, such as an inverter and battery bank [44].

2.2.3.3 Energy storage system

Due to the intermittent behaviour of RES and their dependency on atmospheric conditions, an ESS is crucial for stable MG operation. Many different ESS technologies are available, including batteries, super-capacitors (SCs) and FCs. Batteries are still being most frequently used, but there have been significant advancements made in both SC and FC systems.

2.2.3.3.1 BATTERY STORAGE Valve-regulated lead-acid (VRLA) batteries are commonly used due to their affordability and reasonable performance characteristics [26]. Fundamental to these batteries are the depth-of-discharge (DOD) and state-of-charge (SOC) values, which are usually expressed as a percentage of the battery bank's rated capacity. These values protect the batteries from being fully discharged and (2.6) relates the SOC and DOD.

$$SOC_{\%} = 100 - DOD_{\%} \quad (2.6)$$

In an MG containing a WTG, hydrogen system and batteries, nominal SOC values often vary between 75% and 95% [47]. These values are, however, "case-specific" and the SOC can drop to as low as 40% for emergency use [47]. Additionally, the efficiency of VRLA batteries is described as being the product of both Coulombic and Voltaic efficiencies, which can be influenced by many external factors.

It will typically vary between 80 - 85% under normal conditions and can increase to 90 - 95% during winter months [48].

In some publications, the maintenance cost of battery ESS is calculated based on the number of charging cycles [49, 50]. Other works, however, suggest using a fixed annual maintenance expense independent of how the battery ESS is operated [51]. Assuming no maintenance costs, however, VRLA battery expenses can be directly linked to its lifetime, expressed as the cycles-to-failure (CTF) value. The CTF varies with respect to the DOD and can be determined using either the weighted or average Ah-throughput models, the Rainflow method, the UMass model or basic cycle counting [52, 53]. A novel method – which is easy to implement and only relies on the CTF versus DOD graphs – is also considered [52].

Most battery sizing methods consider how long the ESS should be able to sustain the MG in the absence of other sources. These so-called days of autonomy are typical of lengths 1 to 3, with fewer days being more suitable for MGs with multiple energy sources and vice versa [32]. Batteries are often sized using either a constant current or constant power technique: The former is more suitable for MG purposes, as different loads are switched on and off during the discharge cycle of the battery, causing power fluctuations.

Operational constraints should also be placed on the batteries' charging and discharging rates to prolong battery life [38]. Without a grid connection, a typical control scheme will result in the batteries being charged when the PV and WTG generate more energy than which the load consumes [38]. In turn, the battery will supply electricity when the RES are insufficient [38]. When interacting with the UG, energy is first drawn from the RES and battery bank before tapping from the UG, and surplus energy is fed back into the grid when the batteries are sufficiently charged [38]. This dissertation will follow a similar approach: Depending on the varying generation costs, energy will be tapped from each source accordingly.

2.2.3.3.2 HYDROGEN STORAGE In the past two decades, many advancements have been made regarding the use of hydrogen as a RES and are described by many as "the energy of the future" [54]. Typically, such a system will utilize FCs to generate electricity from hydrogen, albeit for vehicular or stationary applications (i.e. MGs) [55, 56]. Hydrogen, in turn, can either be supplied from manufacturers or generated on-site. The latter is frequented for stationary applications, especially where hydrogen infrastructure has not yet been developed [57, 56].

In MG environments, FCs are replacing DGs as an energy source. However, they can also be used as an ESS by using an ER which produces hydrogen from surplus electricity [57]. These processes can be either CO₂ neutral or CO₂ free, and five different types of FCs are often seen [58]:

- Proton exchange membrane FC;
- Direct methanol FC;
- Microbial FC;
- Solid oxide FC; and
- Intermediate-temperature solid oxide FC [58].

Recent advancements have made proton exchange membrane (PEM) technology a promising solution, given its wide range of operating temperatures compared to other types [57, 59]. Figure 2.2 shows a basic ER and FC system, with oxygen and water being either products or reactants.

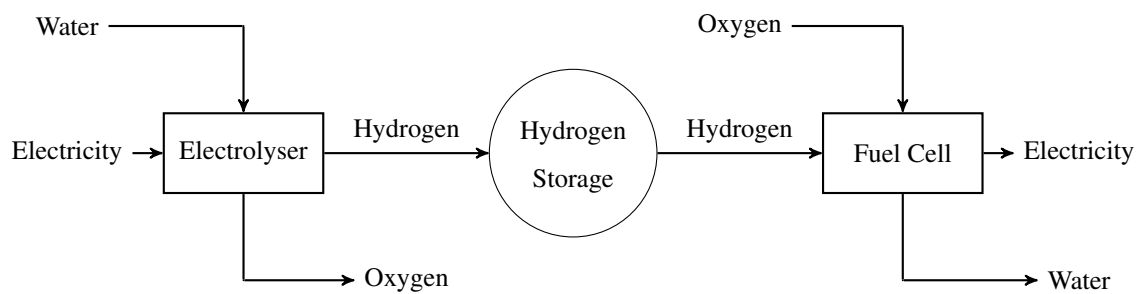


Figure 2.2. Simplified block diagram example of a renewable energy Electrolyser and Fuel Cell system. Adapted from [2], with permission.

The lifetime of a hydrogen ESS is dependent on many factors, including how the cells are operated. These operating regimes are either DN or SS, with the former most often associated with automotive uses [60]. A cell typically reaches its end of life (EoL) when the voltage drops by more than 10% and under SS, which can occur after 11 900 hours of continuous operation or in as little as 3 800 hours for automotive applications [60, 61]. A PEM FC operating under constant load lasted three years (26 300 hours) and was only interrupted during short intervals to perform diagnostics [62]. The cell's EoL failure of the membrane and the output voltage dropped from 0.65 to 0.54 Volts per cell (16.92%) [62].

Throughout literature, there are different means of storing hydrogen: Either as a pressurized gas at room temperature, in liquid form at cryogenic temperatures or in metal hydrides at room temperature and pressure [63]. Storing hydrogen as a compressed gas is most frequently used, but the use of metal hydrides is becoming more prominent given the safety concerns of the former [63]. Although liquid hydrogen takes up very little space, it is not preferred for everyday use due to the extremely low boiling temperature of the gas. Density issues are also noted to be of concern when considering liquid storage [63].

Nowadays, underground storage of hydrogen gas is being considered, with the benefit of utilizing existing geological structures [64]. This is, however, beyond the scope of the small MG that will be developed in this study and for simplicity, pressurized gas storage – above ground – will be considered.

2.3 ENERGY OPTIMISATION AND CONTROL

Throughout the literature, various strategies have been proposed to control power, voltage and frequency in an MG. These can broadly be classified as either centralized or decentralized, with the former utilizing a central controller making adjustments based on data from all the loads and sources [25]. It requires a reliable, high-bandwidth communication channel and a failure thereof will cause the entire system to collapse [25]. Decentralized control, on the other hand, consists of numerous independent control nodes: Each node is connected to a sensor on either a load or source and is overall more reliable than centralized control [25].

Each energy source has some form of operating cost or penalty, which can either be linked to the initial investment, fuel, maintenance or emissions. A typical control scheme for a grid-connected MG will thus aim to meet the load demand whilst minimizing operating costs: When RES fail to meet the demand, energy can be tapped from the ESS or UG. On the other hand, surplus power can be sold to the UG and/or used to charge the batteries [65].

Traditional optimisation methods such as linear programming are generally not applied to MGs because of their complexity, with preference given to heuristic techniques. A publication from 2019, however, used a mixed-integer, non-linear optimisation approach to develop a real-time MG operation strategy, with the aim of reducing the computational requirements of heuristic techniques [66].

As mentioned in Chapter 1, 37 heuristic algorithms have been in use between 1975 and 2016; with more still being developed today [22]. In thoroughly analysing all these possible techniques, GA, PSO and DE (Differential Evolution) are considered best for electrical engineering, scheduling and function optimisation [22]. The study also concluded that swarm intelligence means (PSO, firefly, artificial bee colony, ant colony optimisation, etc.) are best suitable to solve MG-related cost minimization problems [22]. In the following subsections, PSO, GA, DE, as well as Surrogate optimisation (SO), will be discussed.

2.3.1 Particle Swarm optimisation

PSO was first described in a joint publication from 1995 by Russell Eberhart and James Kennedy. This technique is developed to solve non-linear functions using swarm theory, and is related to both GA and evolution strategies [67]. PSO is further described as being simple to implement and not computationally heavy; the algorithm also has memory – unlike GA [68].

Minimizing function f using PSO can be explained by considering P particles moving through a search space and each particle, p , is initialized with a random position and velocity (x_p^k and v_p^k) [69]. The personal best of each particle at iteration k , b_p^k , is calculated by evaluating f and comparing it with the previous iteration's value. The global best, g_p^k , is then determined by comparing the personal best of each particle. Alternatively, a "local best" technique can also be used, where only selected neighbouring particles are compared to one another.

During each iteration, the position and velocity of each particle are updated using (2.7) and (2.8): w is an inertia factor and c_1 and c_2 are positive random values, both constrained by upper limits [69]. Like most heuristic optimisation techniques, PSO is not exempt from convergence issues. These can be rectified by managing the population size, and velocity as well as stopping criteria [69].

$$v_p^{k+1} = wv_p^k + c_1 \times (b_p^k - x_p^k) + c_2 \times (g_p^k - x_p^k) \quad (2.7)$$

$$x_p^{k+1} = x_p^k + v_p^{k+1} \quad (2.8)$$

A study from 2018 used a simplified PSO algorithm to minimize the cost function of an MG: Only fuel and maintenance costs were considered, and the Weibull distribution was used to predict solar and wind generation [20]. The operational costs of a micro-turbine and FC was directly related to fuel costs, whereas PV and WTG expenses were calculated using a per-unit energy maintenance factor. It

is unclear how these factors were determined, but the general cost minimization function is shown in (2.9), with the operating cost, C_{MG} minimized for T time intervals [20].

$$\text{Min } C = \sum_{t=1}^T C_{MG}(t) \quad (2.9)$$

Another publication considered a multi-objective PSO problem which maximizes the renewable energy factor and minimizes both the cost of electricity and loss of power probability [31]. This objective function is also used to size the MG – not only to control it.

MATLAB's Global optimisation Toolbox (GOT) contains a built-in PSO function which can only be used on unconstrained problems. This makes it unsuitable for this work and an independently developed constrained PSO algorithm – shown in (2.10) – will be used [70]. Input parameters required therefore are similar to the built-in GA function, with $\bar{x}$ being a vector containing the optimized variables based on the objective function, *fitfun*. The number of variables (*nvars*), linear inequality and equality matrices (*Aineq*, *bineq*, *Aeq* and *beq*), lower and upper bounds (*lb* and *ub*), non-linear constraint function (*confun*), integer constraint array (*intcon*) and non-default options (*opts*) are also relevant to the function.

$$\bar{x} = \text{pso}(\text{fitfun}, \text{nvars}, \text{Aineq}, \text{bineq}, \text{Aeq}, \text{beq}, \text{lb}, \text{ub}, \text{confun}, \text{opts}) \quad (2.10)$$

2.3.2 Genetic Algorithms

Older than PSO, GA forms part of Evolutionary Algorithms (EA) which also contains Evolution Strategies (ES) and Evolutionary Programming (EP) [71]. As the name suggests, these techniques are based on "natural, biological evolution" as described by Charles Darwin more than a century ago [71]. GA is somewhat more complex to implement than PSO; however MATLAB's built-in function, shown in (2.11), can be used. The input parameters are similar to those of (2.10).

$$\bar{x} = \text{ga}(\text{fitfun}, \text{nvars}, \text{Aineq}, \text{bineq}, \text{Aeq}, \text{beq}, \text{lb}, \text{ub}, \text{confun}, \text{intcon}, \text{opts}) \quad (2.11)$$

An objective function – constrained by generation limits and power balance equations – was minimized using GA, yielding positive results regarding optimal MG operation [72]. Choosing between GA and PSO can thus be quite challenging and a recent publication compared the two algorithms. During simulation, GA showed average savings of 11.31%, compared to 14.31% from PSO [73]. Furthermore, when comparing a modified PSO algorithm with MATLAB's built-in GA toolbox, the former was shown to achieve more optimal solutions in less time [74]. Instead of considering emissions as being

part of the cost function, a multi-objective problem was formulated using separate cost and emission functions.

2.3.3 Differential Evolution

In the year 1996 Rainer Storn and Kenneth Price authored several publications relating to the development of DE for optimizing "real-valued multi-modal objective functions" [75, 76, 77]. It is described as a "parallel direct search method" which is easy to understand and implement, with good convergence properties [75]. Similar to PSO, a fixed number of particles are initialized randomly whereafter parameter vectors are created by adding the weighted difference between two existing vectors. This is then compared to a population vector which will be replaced by the calculated vector if it yields a lower objective function value [75].

A publication using DE to optimally control an MG considered both operating and maintenance costs, emission penalties and a novel start-up cost [78]. Sources present in the MG were WTGs, PVs, batteries, DGs and a micro-turbine; only considering SUCs for the latter two. For the present work, start-up costs will only be applicable to the FCs and ERs. As shown in (2.12), SUCs can be calculated using an exponential cost curve, with SUC_{hot} and SUC_{cold} being the hot and cold SUC, T_{off} total off-time and τ_{cool} the cooling time constant [79].

$$SUC = SUC_{hot} + SUC_{cold} \left[1 - \exp \left(\frac{-T_{off}}{\tau_{cool}} \right) \right] \quad (2.12)$$

2.3.4 Surrogate optimisation

SO is not frequently seen in MG-related applications, and is not even discussed in a large meta-heuristic review paper from recent years [22]. This technique is based on Surrogate Modelling (SM) and is frequented for computationally expensive, highly dimensional problems which are typically found in aerospace systems [80]. It aims to speed up the optimisation process by making use of surrogates – simplified approximations of more complex models – for the objective and constraint functions [80].

The SO function's syntax is shown in (2.13), with parameters similar to PSO and GA, except for the objective function and non-linear constraints now being combined in a structure array, *objcon*.

$$\bar{x} = \text{surrogateopt}(\text{objcon}, lb, ub, \text{intcon}, A, b, Aeq, beq, \text{opts}) \quad (2.13)$$

2.4 DEMAND MANAGEMENT

Time-of-use (TOU) tariffs force consumers to reconsider when certain equipment is used [81]. This introduces the concept of DM, which can be either price or incentive-based and can avoid "under-utilization of electrical infrastructure" [49]. DM thus aims to reduce system costs while increasing reliability. As part of the implementation thereof, loads can be scheduled to operate at the most suitable time, aiming to reduce demand during peak hours. Non-essential loads can also be rejected when the system is severely constrained [82].

For loads to be managed, they need to be classified accordingly. Four such categories are proposed below [83]:

1. On/off: These appliances can be turned on and off; while drawing constant power when on (such as a standard light bulb or kettle);
2. Variable: Instead of drawing constant power when turned on, variable loads' energy consumption can vary depending on customizable settings. An example thereof is a stove, with different heat settings;
3. Multi-stage/Automatic: Loads in this category can also be turned on or off, but automatically goes through multiple stages during each on/off cycle, with varying energy consumption during each. Washing machines are a good example of such a load; and
4. Constant: Such appliances cannot be turned off and will draw a constant amount of power continuously. Day/night sensors and CCTV cameras are typically constant loads that will draw power continuously (albeit very little).

A previous study used multi-scenario, multi-objective optimisation and divided loads as being either critical, deferrable or ordinary [84]. Critical loads are to be powered at all times, whereas ordinary loads can be shed if needed. The operation of deferrable loads can be shifted to more suitable time slots, with water heating systems being an example thereof [84]. The authors used an adopted linear weighted sum method to solve the problem.

Another work categorized loads as being either controllable or not and used GA to optimize operations [49]. This is done by limiting the inconvenience caused by DM, which is modelled as a third-order polynomial function, shown in (2.14) [49]. The inconvenience caused when appliance one is shifted

for time period t_1 is denoted as I_1 . A_1 , B_1 and C_1 are parameters unique to each load. Important to note is that the inconvenience experienced will vary between different consumers as it is linked to their lifestyle patterns.

$$I_1(t_1) = A_1 t_1^3 + B_1 t_1^2 + C_1 t_1 \quad (2.14)$$

The DM problem can further be expanded by formulating three objectives: To minimize both generation cost and inconvenience, as well as to reduce the difference between peak and average demand [85]. Additionally, neglecting possible inconvenience, a recent publication aimed to optimally manage demand with an objective function aiming to maximize profit [86]. An example of the profit maximization equation used is shown in (2.15), with C_{sold} and C_{bought} being the selling and buying cost of electricity.

$$\max \text{profit} = \sum_{t=1}^T C_{sold}(t) - C_{bought}(t) \quad (2.15)$$

2.5 CHAPTER SUMMARY

This section concludes the literature review. The rest of the document is summarized as follows: Chapter 3 considers the methods followed in developing the MG, with the results summarized in Chapter 4. These results are then discussed in Chapter 5 and conclusions are drawn in Chapter 6.

CHAPTER 3 METHODS

3.1 CHAPTER OVERVIEW

Figure 3.1 shows a simplified block diagram of the grid-connected, renewable energy MG that will be developed for a small residential complex in South Africa. Each AC source supplies power to the AC bus bar, whereas the DC sources interface with their DC equivalent. A bi-directional AC/DC converter allows for power transfer between the two bars and control systems manage the power flow of each source and load.

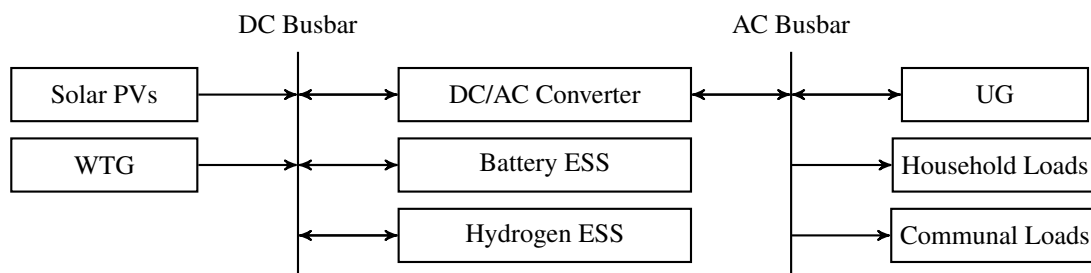


Figure 3.1. Simplified block diagram example of a grid-connected MG with RES.

The small residential complex consists of five homes and a communal area, with a basic layout shown in Figure 3.2. In a recent South African publication, the authors summarized 5 types of local households with different backgrounds [87]. Of interest are the lower and upper middle, with a floor area varying from 80 - 250 m² [87]. Each home in the MG will thus be assumed to have a floor area of 165 m², which is the average of the range mentioned above.

Throughout the course of this work, constant exchange rates will be used to convert from United States Dollar (USD) and Euro (EUR) to South African Rand (ZAR): 1 EUR = 17.48 ZAR and 1 USD = 15.33 ZAR.

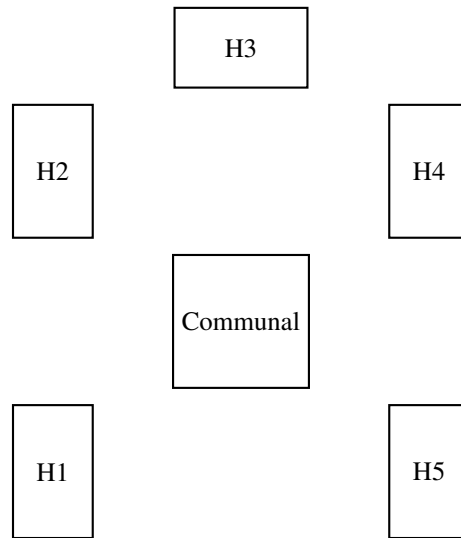


Figure 3.2. Porposed MG layout showing five homes and communal area.

Different phases of the MG design and development will also be discussed in the following sections: Data Gathering in Section 3.2, Load and Source Modelling in Sections 3.3 and 3.4 respectively and Energy optimisation with DM in Section 3.5.

3.2 DATA GATHERING AND MANAGEMENT

To develop and simulate an MG solution for a small residential complex in South Africa, a significant amount of data must be gathered and processed. This includes supply and demand data, with the former relating to the availability of electricity from different energy sources and the operating costs thereof. Such data will typically be obtained from online sources, albeit manufacturers' datasheets, previous research publications or weather station measurements.

Demand data, on the other hand, will be unique and dependent on the MG's load requirements – directly influenced by consumer habits and needs. This dissertation will use a survey to gather the required information from South African households. The survey will be in the form of a questionnaire, distributed as a Google Form which can be found in Appendix A. Respondents will then be required to provide basic TOU information for common household appliances, as well as an inconvenience rating for each, based on the nuisance caused when DM is implemented.

3.2.1 Supply data

With the amount of power generated by RES being dependent on weather conditions, historical measurements can be used to represent a typical day. Different seasons will have to be considered, as it influences consumer demand. For South Africa – which lies in the southern hemisphere – summer, autumn, winter and spring can be represented by the months of January, April, July and October. S. A. Klein defined "recommended average days" for each month in a review paper from 1976, with relevant days for each season shown in Table 3.1 [7].

Table 3.1. Table showing average days and months to represent each season in South Africa. Adapted from [7], with permission.

Month	Day	Season
January	17 th	Summer
April	15 th	Autumn
July	17 th	Winter
October	15 th	Spring

Historical weather data will be obtained from the Southern African Universities Radiometric Network (SAURAN). SAURAN was born from collaborations between the University of KwaZulu-Natal and Stellenbosch University, and "aims to make high-resolution, ground-based solar radiometric data available from stations located across the Southern African region" [88]. Data from the station based at the University of Pretoria, on the roof of the Engineering 2 building, will be used for this study. It can be downloaded from the SAURAN website as being either minutely, hourly or daily averaged values. The former will be used and processed in MATLAB to obtain 30-minute averaged readings.

Furthermore, data from the year 2020 will be used as it is the most recent year with viable data prior to the study's commencement. Relevant information extracted are the ambient temperature (T_A), solar irradiance (G_{sol}) and wind speed (v_w). Air density (ρ_{air}) is calculated based on the ambient temperature and elevation, z_{AMSL} which is 1 410 meters AMSL, as shown in (3.1) [88, 89].

$$\rho_{air}(t) = \left[\frac{353.049}{273 + T_A(t)} \right] \exp \left[\frac{-0.034 z_{AMSL}}{273 + T_A(t)} \right] \quad (3.1)$$

3.2.2 Demand data

As mentioned earlier, the questionnaire in Appendix A will be used to obtain demand data, which will aid in developing the MG model. The survey consists of two sections which aim to gather both TOU data and inconvenience ratings. Appliances are grouped into eight categories based on typical middle-class South African households. These categories, with their respective power ratings, are shown in Table 3.2. Additionally, each load is given an annotation that will be used for quick reference in upcoming sections: $HL_{h,\ell}$ refers to load ℓ of household h and CL_k is communal load k .

Table 3.2. Table showing different household loads and their respective power ratings.

Load	Power Rating [kW]	Annotation
Water heating (electric geyser, etc.)	3.00	$HL_{h,1}$
Cooking and baking (stove, microwave oven, etc.)	5.00	$HL_{h,2}$
Beverages (kettle, coffee machine, etc.)	2.00	$HL_{h,3}$
Clothing (washing machine, iron, tumble-dryer, etc.)	2.50	$HL_{h,4}$
Ambient comfort (heater, air conditioning, etc.)	2.50	$HL_{h,5}$
Miscellaneous electronics (television, computer, etc.)	0.50	$HL_{h,6}$
Irrigation (borehole pump, sprinkler system, etc.)	1.50	CL_1
Swimming pool (pump, heating, etc.)	1.00	CL_2

The total demand, E_{LD}^{tot} , is calculated using (3.2). Binary variables – $S_{h,\ell}$ and S_k – determine the state (on / off) of each household and communal load per interval of operation.

$$E_{LD}^{tot} = \sum_{h=1}^5 \sum_{\ell=1}^6 S_{h,\ell} \times HL_{h,\ell} + \sum_{k=1}^2 S_k \times CL_k \quad (3.2)$$

For the TOU data, questionnaire respondents will be required to indicate when, on average, their household utilizes each appliance group. This is done in 30-minute intervals for a 24-hour period, i.e. an average weekday. Although a lot of the appliances will likely be operational for less than 30 minutes, higher resolution intervals would significantly increase the time required for participants to complete the survey, as well as computational complexity during post-processing.

Lastly, inconvenience factors are gathered by requesting respondents to rate the inconvenience caused when each appliance's operation is to be rescheduled to minimize operating costs. These ratings are based on a scale from one to five, with higher values resulting in more inconvenience caused. Although the gathered data considers each household to have its own swimming pool and irrigation systems, these will form part of CLs and will not contribute to the developed household LP.

3.3 LOAD PROFILING

To develop and control the MG model, a fairly accurate representation of the daily load requirements is needed. This will provide the baseline for sizing the relevant energy sources in the Source Modelling section, which will in turn lead to the development of output power models and cost functions.

Data extracted from the SAURAN database is either daily, hourly or minutely averaged, thus limiting the LP sampling rate to 60 seconds [88]. When considering DM and appliance use, however, very few devices will be consuming electricity for less than 60 seconds at a time and more practical sampling rates will be between 5 to 15 minutes. To ease the burden of questionnaire respondents having to provide appliance usage data at 5-minute intervals, a 30-minute sampling rate was chosen. This is also in line with ESKOM's integrating period, and weather station readings can easily be averaged accordingly.

Using the questionnaire data in conjunction with Table 3.2, typical LPs of middle-class households can be obtained. For the electric water heating system, a 150 litre, 3 kW geyser is considered. This should be sufficient to meet the needs of a small family – the typical residents catered for. The cooking equipment is rated at 5 kW to consider the simultaneous use of multiple devices, and beverage-related appliances are rated to consume 2 kW. Given the households' size, it is unlikely that the washing machine, tumble dryer and iron will be used at the same time. Doing so will account for roughly 9 kW of electricity consumed, and only 2.5 kW (the rating of a typical washing machine or dryer) will be considered.

2.5 kW will be allocated for ambient comfort appliances, such as air conditioning, fans and electric heaters: This will allow for two small heating elements to be used during winter time as well as fans and air coolers in summer. By assuming the latest technology with regard to energy-saving light bulbs and modern electronics, a mere 500 W is set aside for such miscellaneous components.

Typical irrigation and borehole systems can vary from 0.34 to 4.00 kW, with the smaller sized values will more applicable to single households. A power rating of 1.5 kW is semi-randomly chosen for the purposes of this study. Lastly, the swimming pool pump and filtering system will be assumed to consume 1.0 kW and is considered to include no heating devices.

3.4 SOURCE MODELLING

Effective modelling of energy sources – especially those dependent on atmospheric conditions such as wind and sunshine – is crucial to estimate the amount of energy generated, which can change at a moment's notice. In the following subsections, modelling of PVs and WTGs are discussed (3.4.1 and 3.4.2), battery and hydrogen ESS (3.4.3 and 3.4.4) and lastly the UG (3.4.5).

3.4.1 Solar PVs

In its simplest form, the output power delivered by a single PV module (P_{PV}^{sgl}) is the product of the sun's power (P_{sun}) and the module efficiency (η_{PV}), as shown in (3.3). By substituting the power from the sun with the PV module area (A_{PV}) and solar irradiance (G_{sol}), the output power can be written as a function of solar irradiation.

$$\begin{aligned} P_{PV}^{sgl} &= \eta_{PV} \times P_{sun} \\ &= \eta_{PV} \times A_{PV} \times G_{sol} \end{aligned} \quad (3.3)$$

The PV efficiency is, however, not constant and can be written as a function of cell temperature, T_C , shown in (3.4) [5, 6]. Relevant to the calculation thereof is the power-temperature coefficient and efficiency under Standard Test Conditions (STC), respectively denoted as α_{STC} and η_{STC} . These values should be readily available from manufacturers' datasheets, with the STC temperature, T_{STC} , fixed at 25°C.

$$\eta_{PV} = \eta_{STC} [1 + \alpha_{STC}(T_C - T_{STC})] \quad (3.4)$$

To calculate the cell temperature, the effects of both wind and PV mounting type are considered using (3.5). Assuming the PVs to be mounted directly onto pitched roofs, the mounting coefficient's value – ϕ – is shown in Tables 2.2 and 3.3 [6, 90]. Additionally, the wind speed measured well above the PV array is denoted as v_f , whereas v_w denotes the wind speed close to the array, with some debate

regarding which value to use. For this work, it is assumed that the SAURAN weather station measures the wind speed close to the array and (3.6) transforms it accordingly [6, 91].

$$T_C = T_A + \left(\frac{0.32\phi}{8.91 + 2v_f} \times G_{sol} \right) \quad (3.5)$$

$$v_f = \frac{v_w + 0.5}{0.68} \quad (3.6)$$

Once the solar PV has been sized and modules selected, the total power generated from the solar PVs – P_{PV}^{tot} – can be calculated as shown in (3.7). N_{PV}^{MG} represents the total number of solar panel modules to be installed in the MG, constrained by the MGs energy requirements as well as the available roof space.

$$P_{PV}^{tot} = N_{PV}^{MG} \times P_{PV}^{sgl} \quad (3.7)$$

A publication from 2021 determined that a shed roof with a 25-degree tilt angle is the optimum design for PV installations in Perth, Australia [92]. This roof type is, however, fairly uncommon in older South African dwellings and hence gable-type roofs with the same pitch will be considered instead. Assuming all the homes are rectangular shaped and ignoring overhangs, the total roof area (A_{roof}^{tot}) can be calculated as a function of floor area and tilt angle (A_{floor} and θ_{roof}) using (3.8).

$$A_{roof}^{tot} = \frac{A_{floor}}{\cos \theta_{roof}} \quad (3.8)$$

However, only half thereof (A_{roof}^{use}) will be useful to install PVs given its alignment towards the sun and the equivalent usable roof area per house – with a floor area of 165 m^2 – is calculated in (3.9). Detailed derivations of these equations can be found in Appendix B.

$$\begin{aligned} A_{roof}^{use} &= \frac{A_{roof}^{tot}}{2} \\ &= \frac{A_{floor}}{2 \cos \theta_{roof}} \\ &= \frac{165}{2 \cos 25^\circ} \\ &= 91.03 \text{ [m}^2\text{]} \end{aligned} \quad (3.9)$$

For this study, a 420 W PV module from Canadian Solar will be used [93]. Relevant parameters from the datasheet are shown in Table 3.3 and – at the time of writing – a single unit can be purchased from the Solar Shop for R3 310 (c_{PV}^{sgl}). The maximum number of such modules that can be installed in each building ($N_{PV}^{h,max}$) is calculated using (3.10) [38]. A derating coefficient, α_{PV} , is introduced to compensate for possible shadowing on the available surface area and is taken as 80 % [38].

Table 3.3. Table showing relevant parameters of Canadian Solar’s 420 W PV module.

Parameter	Value	Unit
System voltage	48.0	[V]
Power	420	[W]
Cell type	poly-crystalline	
Module efficiency	19.0	[%]
Module area	2.21	[m ²]
Mounting coefficient	1.8	[-]
Temperature coefficient for power	-0.36	[%/°C]
Temperature coefficient for voltage	-0.28	[%/°C]
Temperature coefficient for current	0.05	[%/°C]
Life expectancy	25	[years]
N_{PV}^h	20	[modules]
N_{PV}^{MG}	120	[modules]

$$\begin{aligned}
 N_{PV}^{h,max} &= \left(\frac{A_{roof}^{use}}{A_{PV}} \right) \times \alpha_{PV} \\
 &= \left(\frac{91.03}{2.21} \right) \times 0.8 \\
 &\approx 32 [PVs]
 \end{aligned} \tag{3.10}$$

Now, the minimum amount of PVs per home, $N_{PV}^{h,min}$, can be calculated using (3.11). It is based on the load demand of a single house (P_{LD}^h) and the amount of power generated by a single PV during active hours, $t_{PV,i}$ to $t_{PV,f}$. Table 3.4 shows the total daily power generated by a single PV module for each month, with the average thereof used in (3.11). Because the number of PVs cannot be a fraction, the maximum value is rounded down and the minimum is rounded up, as summarized in (3.12).

$$\begin{aligned}
 N_{PV}^{h,min} &= \sum_{i=t_{PV,i}}^{t_{PV,f}} \frac{P_{LD,i}^h}{P_{PV,i}^{sg}} \\
 &= \sum_{i=07:00}^{17:00} \frac{P_{LD,i}^h}{P_{PV,i}^{sg}} \\
 &= \frac{35.68}{4.601} \\
 &\approx 8 [PVs]
 \end{aligned} \tag{3.11}$$

$$\begin{aligned}
 N_{PV}^{h,min} &\leq N_{PV}^h \leq N_{PV}^{h,max} \\
 7 &\leq N_{PV}^h \leq 32
 \end{aligned} \tag{3.12}$$

Table 3.4. Table showing total daily power generated by a single module for each relevant month.

Month	Value	Unit
January	5.447	[kW]
April	3.798	[kW]
July	3.505	[kW]
October	5.653	[kW]
Average	4.601	[kW]

Twenty PVs will be installed at each of the five homes in the complex and twenty on the communal area's roof. An even number is chosen to make it possible for parallel connection of the modules if needed. The total number of PVs forming part of the MG will thus equal 120 and the total PV investment cost – c_{PV}^{cap} – is calculated in (3.13).

$$\begin{aligned}
 c_{PV}^{cap} &= c_{PV}^{sgl} \times N_{PV}^{MG} \\
 &= 3\,310 \times 120 \\
 &= 397\,200 [ZAR]
 \end{aligned} \tag{3.13}$$

3.4.2 Wind turbine generator

A single WTG's output power – P_{WTG}^{sgl} – is calculated as the product of the turbine efficiency and power extracted from the wind – η_{wtg} and P_{WTG}^{extr} – as shown in (3.14). In turn, the extracted power is calculated using (3.15), with C_p denoting the power coefficient, ρ_{air} the air density, A_{rot} the rotational sweep area and v_w the wind speed. A detailed derivation of (3.15) can be found in Appendix B and the power generated by all the WTGs installed in the MG – P_{WTG}^{tot} – is calculated using (3.16), with N_{WTG}

the number of WTGs.

$$P_{WTG}^{sgl} = \eta_{WTG} \times P_{WTG}^{extr} \quad (3.14)$$

$$P_{WTG}^{extr} = C_p \times \frac{\rho_{air} A_{rot} v_w^3}{2} \quad (3.15)$$

$$P_{WTG}^{tot} = N_{WTG}^{MG} \times P_{WTG}^{sgl} \quad (3.16)$$

As with solar PVs, the maximum number of WTGs that can be installed is limited by spacial constraints [38]. To avoid shading the PVs, the WTGs can be installed on the opposite side of each dwelling's roof. Only four units will be installed per home to allow for sufficient spacing in between, resulting in 24 WTGs in the MG.

The WTG considered is the e300ⁱ model manufactured by EverReady-Kestrel, which can be bought online from "Being off the Grid" at R57 750 per unit. This module is chosen due to its low cut-in wind speed, and relevant parameters are shown in Table 3.5 [8]. The initial expense – c_{WTG}^{cap} – is calculated based on the single WTG cost, and the number of WTGs installed – c_{WTG}^{sgl} and N_{WTG}^{MG} – shown in (3.17).

Table 3.5. Table showing relevant parameters of the e300ⁱ WTG module [8].

Parameter	Value	Unit
Rated power	1000	[W]
Rated wind speed	10.5	[m/s]
Step-in wind speed	2.5	[m/s]
Rotor diameter	3.0	[m]
Number of WTGs	24	[-]

$$\begin{aligned}
c_{WTG}^{cap} &= c_{WTG}^{sgl} \times N_{WTG}^{MG} \\
&= 57\,750 \times 24 \\
&= 1\,386\,000 \text{ [ZAR]}
\end{aligned} \quad (3.17)$$

The WTG datasheet rating applies to nominal atmospheric conditions, which translates to an air density (ρ_{nom}) of 1.225 kg/m³ at sea level and temperature (T_{nom}) of 15 °C [94]. Using this information, the power-speed curve shown in Figure 3.3 is plotted and fitted in MATLAB [8]. The nominal output power of the WTG ($P_{WTG}^{extr,nom}$) is estimated using a piecewise function (3.18):

- When the wind speed is lower than the turbine's cut-in speed, no power is generated;
- When the wind speed is higher than the cut-in speed but lower than the rated speed, the output power is modelled by a quadratic function; and
- When the wind speed is higher than the rated speed and neglecting the maximum/cut-out speed, a power curve is followed.

Coefficient values $a_1 - c_2$ for (3.18) are shown in Table 3.6.

$$P_{WTG}^{extr,nom} = \begin{cases} 0 & v_w < v_{min} \\ a_1 v_w^2 + b_1 v_w + c_1 & v_{min} \leq v_w \leq v_{rtd} \\ a_2 v_w^{b_2} + c_2 & v_w > v_{rtd} \end{cases} \quad (3.18)$$

Table 3.6. Table showing coefficients for (3.18) as obtained in MATLAB.

Coefficient	Value	Coefficient	Value
a_1	13.26	a_2	-2.17×10^9
b_1	-50.69	b_2	-7.291
c_1	68.53	c_2	1 079

As the operating air density is dependent on numerous environmental conditions – such as air temperature and elevation – it can be approximated using (3.1) [89]. The power extracted from the WTG shown in Figure 3.3 should thus be derated, and by using the WTG data and nominal values, the power coefficient can be determined with (3.19). This equation is derived from (3.15) and the C_p values are also plotted in Figure 3.3.

$$C_p = P_{WTG}^{extr,nom} / (0.5 \rho_{nom} A_{rot} v_w^3) = \begin{cases} 0 & v_w < v_{min} \\ (a_1 v_w^2 + b_1 v_w + c_1) / (0.5 \rho_{nom} A_{rot} v_w^3) & v_{min} \leq v_w \leq v_{rtd} \\ (a_2 v_w^{b_2} + c_2) / (0.5 \rho_{nom} A_{rot} v_w^3) & v_w > v_{rtd} \end{cases} \quad (3.19)$$

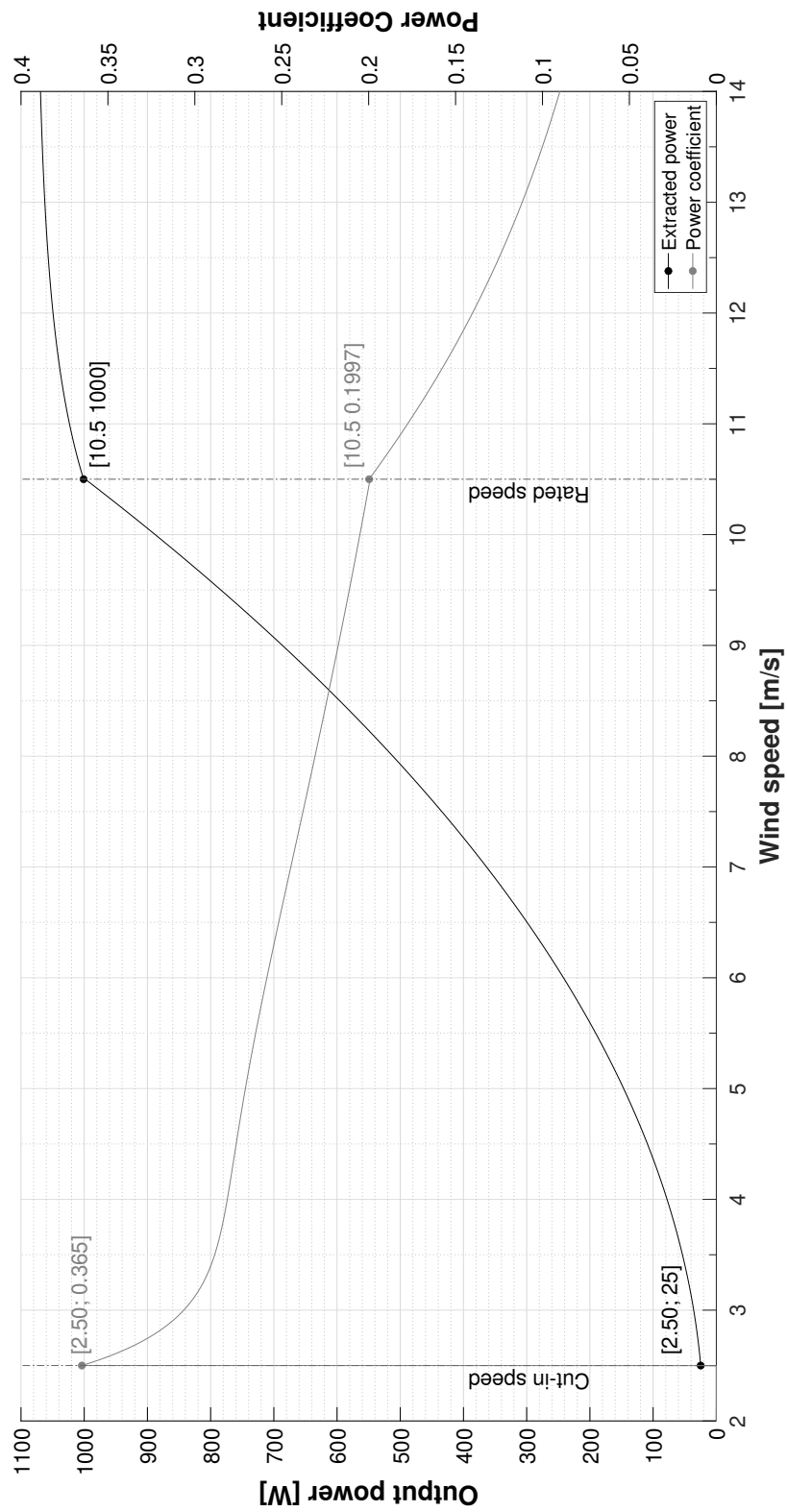


Figure 3.3. Plot showing power generated by the WTG and power coefficients at varying wind speeds.

The actual extracted power can now be calculated using (3.20), which is simply a modified version of (3.15) as well.

$$P_{WTG}^{extr} = C_p \times (0.5 \rho A_{rot} v_w^3) = \begin{cases} 0 & v_w < v_{min} \\ (a_1 v_w^2 + b_1 v_w + c_1) \times \left(\frac{\rho}{\rho_{nom}}\right) & v_{min} \leq v_w \leq v_{rtd} \\ (a_2 v_w^2 + c_2) \times \left(\frac{\rho}{\rho_{nom}}\right) & v_w > v_{rtd} \end{cases} \quad (3.20)$$

3.4.3 Battery ESS

VRLA batteries are used in the MG for short- to medium-term energy storage. The battery bank's SOC at interval i is calculated using (3.21). As shown in (3.22), E_{bnk}^{ch} will be positive, whereas E_{bnk}^{disch} will have a negative value. Furthermore, the SOC values are calculated here as a fraction of the battery bank's total capacity, E_{bnk}^{tot} .

$$SOC_{i+1} = SOC_i + \left(\frac{\eta_{bat} \times E_{bnk}^{ch}}{E_{bnk}^{tot}} \right) + \left(\frac{E_{bnk}^{disch}}{\eta_{bat} \times E_{bnk}^{tot}} \right) \quad (3.21)$$

$$\begin{aligned} E_{bnk}^{ch} &\geq 0 \\ E_{bnk}^{disch} &\leq 0 \end{aligned} \quad (3.22)$$

For simplicity, it will be assumed that the battery charging and discharging efficiencies will be the same, η_{bat} . It is calculated as the product of the Voltaic and Coulombic efficiencies in (3.23) with values from Table 3.7.

$$\begin{aligned} \eta_{bat} &= \eta_{coulomb} \times \eta_{volt} \\ &= 0.85 \times 0.85 \\ &= 0.75 \end{aligned} \quad (3.23)$$

To avoid the batteries being over-charged or over-discharged, upper and lower constraints are applied to the SOC values, shown in (3.24). The minimum SOC is determined by the maximum allowed DOD ($\overline{DOD}$), which is expressed as a percentage in (3.25) and the maximum SOC is shown in (3.26).

$$SOC_{min} \leq SOC_n \leq SOC_{max} \quad (3.24)$$

$$\begin{aligned} SOC_{min} &= (100 - \overline{DOD}) \times SOC_{max} \\ &= 50\% \end{aligned} \quad (3.25)$$

$$SOC_{max} = 100\% \quad (3.26)$$

The number of batteries that can be installed in the MG (N_{bat}^{MG}) is determined by the days of autonomy (ζ), the total daily energy consumption (E_{LD}^{tot}), the efficiency, single battery capacity (Q_{bat}), the battery voltage (V_{bat}) and the DOD.

Based on consumer usage data shown in Figure 4.10, each household consumes 40.25 kWh per day, on average. This value is multiplied by the number of homes in the MG (N_{home}^{MG}) and a 5% safety factor (SF), as shown in (3.27). The latter considers possible load fluctuations and CLs.

$$\begin{aligned} E_{LD}^{tot} &= E_{LD}^h \times N_{home}^{MG} \times SF \\ &= 40.25 \times 5 \times 1.05 \\ &\approx 210 [kWh] \end{aligned} \quad (3.27)$$

200 Ah, 12V lead-acid batteries from AC DC Dynamics will be considered, and relevant parameters required for calculations are shown in Table 3.7 [95]. For a system voltage of 48 V, parallel strings of four batteries per string will be used, with the minimum number of batteries calculated in (3.28) for a DOD of 50%. According to the datasheet, each battery is estimated to last 1 200 cycles ($\overline{CTF}$) at the set DOD [95]. The MG will thus be designed to include 240 batteries: 40 installed per home and 40 in the communal area.

$$\begin{aligned} N_{bat}^{MG} &= \frac{\zeta \times E_{LD}^{tot}}{\eta_{ESS} \times Q_{bat} \times V_{bat} \times DOD} \\ &= \frac{1 \times 210\,000}{0.72 \times 200 \times 12 \times 0.5} \\ &\approx 240 \end{aligned} \quad (3.28)$$

Table 3.7. Table showing parameters for the ESS calculations.

Coefficient	Value	Unit	Coefficient	Value	Unit
ζ	1	[day(s)]	E_{LD}^{tot}	210	[kWh]
$\eta_{coulomb}$	85	[%]	$\overline{CTF}$	1 200	[%]
η_{volt}	85	[%]	$\overline{DOD}$	50	[%]
η_{bat}	72	[%]	Q_{bat}	200	[Ah]
V_{bat}	12	[V]	E_{bat}	2.4	[kWh]
V_{bnk}	48	[V]	E_{bnk}	576	[kWh]
N_{bat}^{MG}	240	[batteries]	Q_{bnk}	12	[kAh]
N_{bat}^{se}	4	[batteries]	c_{bat}^{cap}	7 948	[ZAR]
N_{bat}^{sh}	60	[batteries]	c_{bnk}^{cap}	1 907 520	[ZAR]

The total capacity of the battery bank is calculated as 12 kAh in (Q_{bnk}). The bank's operating voltage and total energy (V_{bnk} and E_{bnk}) are respectively calculated in (3.30) and (3.31).

$$\begin{aligned} Q_{bnk} &= Q_{bat} \times N_{bat}^{sh} \\ &= 200 \times 60 \\ &= 12 \text{ [kAh]} \end{aligned} \quad (3.29)$$

$$\begin{aligned} V_{bnk} &= V_{bat} \times N_{bat}^{se} \\ &= 12 \times 4 \\ &= 48 \text{ [V]} \end{aligned} \quad (3.30)$$

$$\begin{aligned} E_{bnk}^{tot} &= Q_{bnk} \times V_{bnk} \\ &= 12\,000 \times 48 \\ &= 576 \text{ [kWh]} \end{aligned} \quad (3.31)$$

The "cost estimation method" will be used to estimate the battery lifetime and replacement cost based on its use [52]. For this method, the energy drawn from the battery bank during each cycle and the reference DOD energy ($E_{bnk_n}^{disch}$ and $E_{\overline{DOD}}$) is calculated in (3.32) and (3.33) [52].

$$E_{bnk_i}^{disch} = \Delta SOC_i \times E_{bnk}^{tot} \quad (3.32)$$

$$\begin{aligned} E_{\overline{DOD}} &= \overline{DOD} \times E_{bnk} \\ &= 0.5 \times 576\,000 \\ &= 288 \text{ [kWh]} \end{aligned} \quad (3.33)$$

The change in charge, ΔSOC , and battery stress factor, k_{SOC} , is calculated in (3.34) and (3.35), with the latter considering the effects of the final SOC during each cycle [52]. As the value of $\overline{CTF}$ is already known, it can be substituted into (3.35). An approximation of the value for CTF_i is still required, which will be derived from the manufacturer's data, as discussed below.

$$\Delta SOC_i = SOC_{i-1} - SOC_i \quad (3.34)$$

$$\begin{aligned} k_{SOC_i} &= \overline{CTF} \times CTF_i^{-1} \\ &= 1\,200 \times CTF_i^{-1} \end{aligned} \quad (3.35)$$

The SOC values are calculated from the manufacturer's DOD data and are plotted against CTF in Figure 3.4. Using MATLAB's curve fitting toolbox, the DOD and SOC can now be written as functions of CTF, using (3.36) and (3.37). Both are piecewise functions, consisting of a linear first-order polynomial

and reciprocal function, with coefficient values summarized in Table 3.8.

$$DOD_i = \begin{cases} \frac{a_1}{CTF_i + b_1} + c_1 & CTF_i \leq 2\,800 \\ d_1 \times CTF_i + e_1 & CTF_i > 2\,800 \end{cases} \quad (3.36)$$

$$SOC_i = \begin{cases} \frac{a_2}{CTF_i + b_2} + c_2 & CTF_i \leq 2\,800 \\ d_2 \times CTF_i + e_2 & CTF_i > 2\,800 \end{cases} \quad (3.37)$$

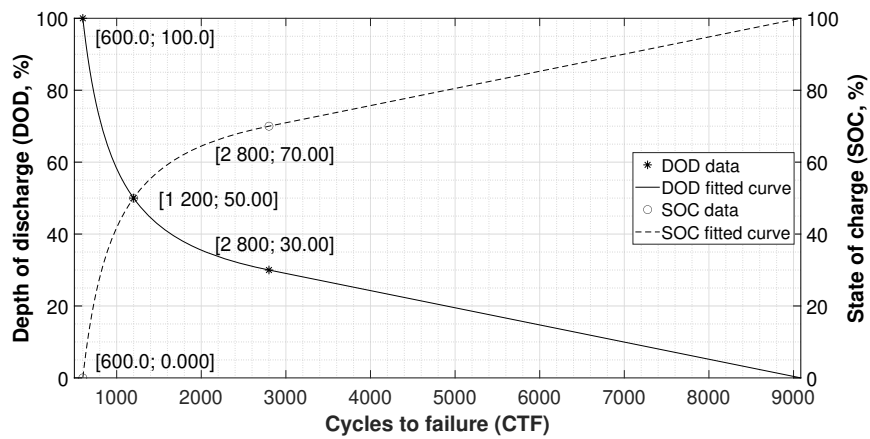
Taking the respective inverses of (3.36) and (3.37) yields (3.38) and (3.39): The CTF can now be calculated as a function of either DOD or SOC and substituted into (3.35) to calculate the stress factor.

$$CTF_i = \begin{cases} \left(\frac{a_1}{DOD_i - c_1} \right) - b_1 & DOD_i \geq 30 \\ (DOD_i - e_1) / d_1 & DOD_i < 30 \end{cases} \quad (3.38)$$

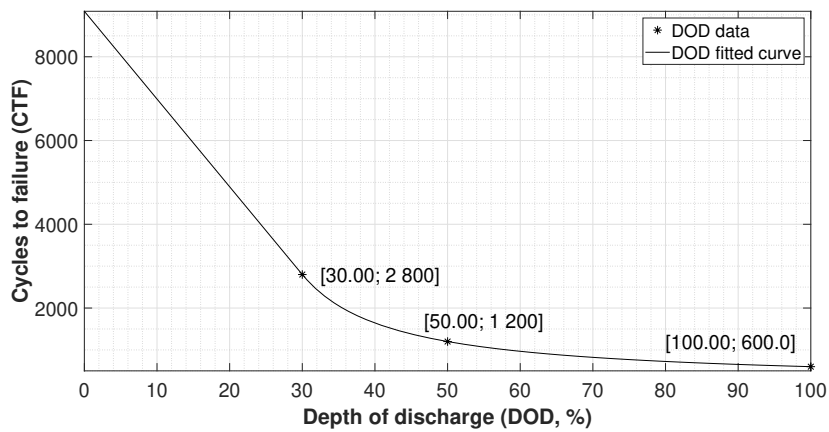
$$CTF_i = \begin{cases} \left(\frac{a_2}{SOC_i - c_2} \right) - b_2 & SOC_i \leq 70 \\ (SOC_i - e_2) / d_2 & SOC_i > 70 \end{cases} \quad (3.39)$$

Finally, the battery lifetime derating factor (F_{bat}) can be obtained, shown in (3.40).

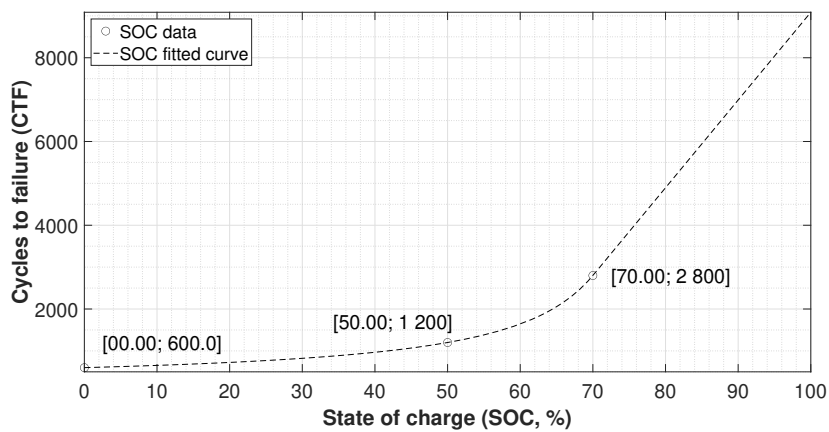
$$\begin{aligned} F_{bat_i} &= k_{SOC_i} \times \frac{E_{bnk_i}^{disch}}{E_{DOD}} \\ &= \left(\frac{CTF}{CTF_i} \right) \times \left(\frac{\Delta SOC_i}{DOD} \right) \\ &= \frac{1\,200}{50} \times \left(\frac{\Delta SOC_i}{CTF_i} \right) \\ &= 24 \times \left(\frac{\Delta SOC_i}{CTF_i} \right) \end{aligned} \quad (3.40)$$



(a) Graph showing CTF versus DOD and SOC for a 200 Ah VRLA battery.



(b) Graph showing DOD versus CTF for a 200 Ah VRLA battery.



(c) Graph showing SOC versus CTF for a 200 Ah VRLA battery.

Figure 3.4. Graphs relating DOD, SOC and CTF for a single 200 Ah VRLA battery from AC DC Dynamics.

At the time of writing, each battery is priced at R7 948 [95]. The cost of a reference cycle – c_{bat}^{ref} – is calculated in (3.41) using $\overline{DOD}$ and $\overline{CTF}$. The current discharging cycle's cost – c_{bat}^{cyc} – with varying DOD and CTF values, is calculated in (3.42), with (3.41) substituted accordingly.

$$\begin{aligned} c_{bat}^{ref} &= c_{bnk}^{cap} \times \overline{CTF}^{-1} \\ &= \frac{1\,907\,520}{1\,200} \\ &= 1\,589.60 \text{ ZAR/cycle} \end{aligned} \quad (3.41)$$

$$\begin{aligned} c_{bat}^{cyc} &= c_{bat}^{ref} \times F_{bat_n} \\ &= 1\,589.60 \times 24 \times \left(\frac{\Delta SOC_i}{CTF_i} \right) \\ &= 38\,150.40 \times \left(\frac{\Delta SOC_i}{CTF_i} \right) \end{aligned} \quad (3.42)$$

3.4.4 Hydrogen ESS

Hydrogen is used as a medium to long-term ESS within the developed MG. A PEM ER will generate hydrogen from surplus electricity, whereas a PEM FC will generate electricity from hydrogen. Such devices are generally purchased as stacks, each containing multiple cells connected in series. The number of cells per stack is fixed by the manufacturers; however, multiple stacks can be connected in parallel to increase system capacity.

In the following subsections, models of FC and ER systems will be developed. Relevant chemical formulae and the entropy, enthalpy and Gibbs free energy values are shown in Table 3.9. Calculations for it can be found in Appendix C, with relevant chemical constants shown in Table 3.10.

3.4.4.1 PEM FC

The output voltage of an FC stack (V_{FC}^{stack}) is calculated in (3.43), with N_{FC}^{cell} being the number of cells per stack and V_{FC}^{cell} being the single cell voltage. To simplify calculations, the equivalent circuit of an FC, as shown in Figure 3.5, can also be used [3]. A resistor represents each voltage drop, and the double-layer charging effect (to be discussed later) is modelled as a capacitor, C_{FC} .

$$V_{FC}^{stack} = N_{FC}^{cell} \times V_{FC}^{cell} \quad (3.43)$$

The single cell output voltage is calculated in (3.44), with e_{FC} the reversible potential discussed in Paragraph 3.4.4.1.1 [9]. Additionally, the activation, Ohmic and concentration voltage drops, as well

as fuel and oxidant delays, (V_{FC}^{act} , V_{FC}^{ohm} , V_{FC}^{con} and e_{FC}^{del}) are discussed in the paragraphs after that.

$$V_{FC}^{cell} = e_{FC} - V_{FC}^{act} - V_{FC}^{ohm} - V_{FC}^{con} - e_{FC}^{del} \quad (3.44)$$

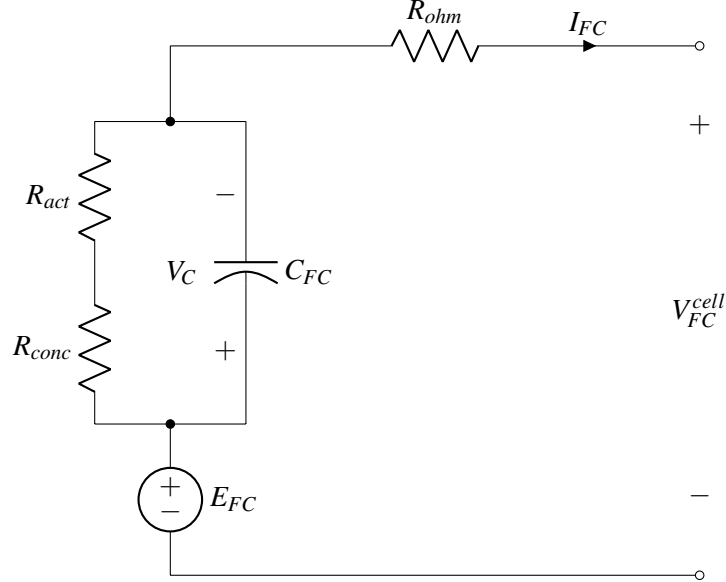


Figure 3.5. Equivalent circuit diagram used to model a FC [3], ©IEEE 2005.

3.4.4.1.1 REVERSIBLE POTENTIAL The FC reversible potential (also known as the open-circuit voltage) is calculated by looking at the chemical process occurring within the cell and is shown in (3.45) [9]. Relevant to the calculation thereof is Faraday's constant and the Universal Gas Constant (F and R_{uni}) as well entropy and Gibbs free energy (ΔS_{FC} and ΔG_{FC}), with values given in Table (3.10) [3].

$$e_{FC} = \frac{|\Delta G_{FC}|}{2F} - \frac{|\Delta S_{FC}|}{2F} \times (T_{FC} - T_{ref}) + \frac{R_{uni} \times T_{FC}}{2F} \times [\ln(p_{H_2}) + 0.5 \ln(p_{O_2})] \quad (3.45)$$

Substitution constant values and using logarithmic laws, (3.45) is simplified to (3.46). The thermodynamic potential is now a function of the FC temperature in Kelvins (T_{FC}) and partial pressures of Hydrogen and Oxygen (p_{H_2} and p_{O_2}) measured in atmospheres. The first term of (3.46) is also known as the standard state reference potential, which equals 1.229 V at 298.15 K and 1 atm [96].

$$\begin{aligned}
 e_{FC} &= \frac{237\,171}{2 \times 96\,487} - \frac{163.27}{2 \times 96\,487} \times (T_{FC} - 298.15) + \frac{8.3143 \times T_{FC}}{2 \times 96\,487} \times \ln(p_{H_2} \times p_{O_2}^{0.5}) \\
 &= 1.229 - [8.461 \times 10^{-4} \times (T_{FC} - 298.15)] + [4.309 \times 10^{-5} \times T_{FC} \times \ln(p_{H_2} \times p_{O_2}^{0.5})]
 \end{aligned} \quad (3.46)$$

3.4.4.1.2 ACTIVATION VOLTAGE DROP The activation voltage drop (activation over-voltage, V_{FC}^{act}) of an FC is caused by the "slowness of the reactions taking place on the surface of the electrodes" [59]. It is calculated as the sum of the anode and cathode's respective over-voltages ($V_{FC}^{act,a}$ and $V_{act,c}^{act}$) and can also be written in parametric form [96, 97]. Both formulae are shown in (3.47), with $\xi_1 - \xi_4$ being constant parametric coefficients unique to each type of FC stack.

$$\begin{aligned} V_{FC}^{act} &= V_{FC}^{act,a} + V_{FC}^{act,c} \\ &= \xi_1 + \xi_2 T_{FC} + [\xi_3 T_{FC} \times \ln(C_{O_2})] + [\xi_4 T_{FC} \times \ln(I_{FC})] \end{aligned} \quad (3.47)$$

I_{FC} is the current flowing through the cell, and the oxygen concentration at the cathode (C_{O_2}) is calculated using (3.48) [9]. The parametric coefficients' values for an SR-12 PEM FC stack are shown in Table 3.11.

$$C_{O_2} = \frac{P_{O_2}}{5.08 \times 10^6 \times \exp(-498/T_{FC})} \quad (3.48)$$

3.4.4.1.3 OHMIC VOLTAGE DROP The Ohmic (resistive) losses, which are linear and proportional to the current density, can be defined as the "resistance to the flow of electrons through all the relevant materials" [59]. Throughout the literature, many means are proposed for calculating this voltage drop (V_{FC}^{ohm}), some of which originate from older publications and are based on correlations with experimental results [98].

For this work, (3.49) will be used, with R_e and R_p respectively referring to the electron and proton membrane resistances [9]. The latter can be assumed constant, whereas the former is calculated in (3.50) [9]. A_{FC}^{cell} is the cell active area, ℓ_{FC}^{mem} the membrane thickness and ρ_{FC}^{mem} the specific resistivity of the electron flow membrane calculated in (3.51) [9].

$$V_{FC}^{ohm} = I_{FC} \times (R_e + R_p) \quad (3.49)$$

$$R_e = \frac{\rho_{FC}^{mem} \times \ell_{FC}^{mem}}{A_{FC}^{cell}} \quad (3.50)$$

$$\rho_{FC}^{mem} = \frac{181.6 \left[1 + 0.03 \left(\frac{I_{FC}}{A_{FC}^{cell}} \right) + 0.062 \left(\frac{T_{FC}}{303} \right)^2 \left(\frac{I_{FC}}{A_{FC}^{cell}} \right)^{2.5} \right]}{\left[\mu - 0.634 - 3 \left(\frac{I_{FC}}{A_{FC}^{cell}} \right) \right] \exp \left[4.18 \left(\frac{T_{FC} - 303}{T_{FC}} \right) \right]} \quad (3.51)$$

(3.51) above is derived from sets of published FC performance curves and considers the widely used Nafion membrane [97]. An adjustable parameter, μ , increases as the FC's operating conditions become

more ideal. A value of 14 can be considered for ideal conditions of 100% humidity, whereas values of up to 23 can be used when the FC is fully over-saturated [9, 97].

3.4.4.1.4 CONCENTRATION VOLTAGE DROP Also referred to as mass transport losses, the concentration over-voltage is caused by changes in the concentration of the reactants at the electrodes' surface due to the fuel being consumed [59]. This voltage drop – V_{FC}^{con} – can be calculated using one of two formulas: (3.52) and (3.53), with J and J_{max} referring to the operating and maximum current densities of the FC and n_e the number of electrons transferred [99].

The transferred electrons will differ between the reactants: For hydrogen, it will have a value of two, and for oxygen, it will be four [99]. One publication suggested that the anode concentration losses (linked to hydrogen) can be considered negligible, and hence the value of four is substituted in (3.52) [10].

$$\begin{aligned} V_{FC}^{con} &= \frac{R_{uni} \times T_{FC}}{n_e F} \ln \left(\frac{J_{max}}{J_{max} - J} \right) \\ &= \frac{R_{uni} \times T_{FC}}{4F} \ln \left(\frac{J_{max}}{J_{max} - J} \right) \end{aligned} \quad (3.52)$$

In (3.53) a parametric coefficient, β_{con} , is introduced [9]. The value depends on the FC operating state, and the $\ln$ -term is simplified using logarithmic rules.

$$V_{FC}^{con} = -\beta_{con} \ln \left(1 - \frac{J}{J_{max}} \right) \quad (3.53)$$

3.4.4.1.5 FC DYNAMICS A phenomenon called the "double-layer charging effect" occurs within FCs and is of great importance when modelling these devices [9]. It causes part of the FC to exhibit characteristics similar to an electrical capacitor and can also be modelled as such. Only the activation and concentration over-voltages will be affected, as can be seen by the placement of capacitor C_{FC} in Figure 3.5. Additionally, it can also be represented by a time delay transfer function, $G_d(s)$, is shown in (3.54) with the time constant, τ_{FC} , calculated in (3.55) [9].

$$G_d(s) = e^{-\tau_{FC}s} \quad (3.54)$$

$$\tau_{FC} = C_{FC} \times R_{act+con} \quad (3.55)$$

The equivalent capacitance, C_{FC} , and the equivalent resistance of the activation and concentration voltage drops, $R_{act+con}$, can be calculated using (3.56). A value of 4.8 Farads per cell is typically used as an approximation for C_{FC} , and as the cells are connected in series, the overall capacitance is

calculated using (3.57) [3].

$$R_{act+con} = \frac{V_{FC}^{act} + V_{FC}^{con}}{I_{FC}} \quad (3.56)$$

$$\begin{aligned} C_{FC}^{tot} &= \frac{C_{FC}^{cell}}{N_{FC}^{cell}} \\ &= 4.8/48 \\ &= 0.1 [F] \end{aligned} \quad (3.57)$$

Mention is also made of fuel and oxidant delays on the FC output voltage: It can be represented by a voltage – e_{FC}^{del} – which is subtracted from the total FC output voltage [3]. At SS, e_{FC}^{del} will equal to zero, and it can be modelled as a transfer function shown in (3.58) [3]. Values for the total flow delay – τ_e – and calculation factor – λ_e – of the SR-12 FC stack are taken as 80 seconds and 3.33 mΩ, respectively [3].

$$\begin{aligned} e_{FC}^{del}(s) &= I_{FC}(s) \times \left(\frac{\lambda_e \times \tau_e}{\tau_e s + 1} \right) \\ &= I_{FC}(s) \times \left(\frac{0.00333 \times 80}{80s + 1} \right) \\ &= I_{FC}(s) \times \left(\frac{0.00333}{s + 1/80} \right) \end{aligned} \quad (3.58)$$

3.4.4.1.6 THERMAL ANALYSIS As seen in some of the aforementioned equations, the FC stack temperature – T_{FC} – directly influences the output voltage and hence also the cell current and output power. The temperature is, in turn, also dependent on these operational values, resulting in a feedback loop. After choosing an initial temperature, the temperature of the next calculation period will be determined by the heat transfer equation, shown in (3.59) [3]. For simplicity, it is assumed that stack and cell temperatures (T_{FC}^{cell} and T_{FC}^{stack}) will be equal, as shown in (3.60).

$$m_{FC}^{stack} \times K_{FC}^{stack} \times \frac{dT_{FC}}{dt} = Q_{net} \quad (3.59)$$

$$T_{FC}^{cell} = T_{FC}^{stack} = T_{FC} \quad (3.60)$$

The overall stack weight is denoted as m_{FC}^{stack} and K_{FC}^{stack} is the specific heat capacity. Q_{FC}^{net} is the net heat generated by the reaction, with its different components shown in (3.61): The energy released by the chemical reaction (Q_{FC}^{chem}), electrical output power (Q_{FC}^{elec}), absorbed sensible and latent heat (Q_{FC}^{conv}) and heat loss absorbed by the atmosphere (Q_{FC}^{loss}).

$$Q_{FC}^{net} = Q_{FC}^{chem} - Q_{FC}^{elec} - Q_{FC}^{conv} - Q_{FC}^{loss} \quad (3.61)$$

To simplify the calculation of T_{FC} , the analogies between thermodynamic and electrical quantities can be used, as shown in Figure 3.6 [3]. The heat capacity is modelled as a capacitor of the same value,

and the "voltage" across it will equal the cell temperature. A voltage source is used to represent the ambient temperature, T_A .

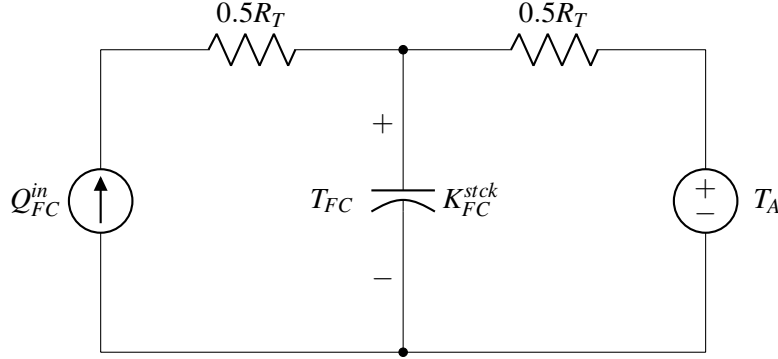


Figure 3.6. Equivalent circuit diagram of the thermodynamic properties of a FC [3], ©IEEE 2005.

The heat source – Q_{FC}^{in} , modelled as a current source in Figure 3.6 – is the power consumed by the activation, ohmic and concentration over-voltages as calculated in (3.62) [3]. The thermal resistance due to air convection, R_T , is calculated in (3.63) and the convective heat transfer coefficient, h_{FC} , can be determined empirically [3]. N_{FC}^{cell} is the number of fuel cells per stack, which equals 48 [3].

$$Q_{FC}^{in} = (E_{FC} - V_{FC}^{cell}) \times I_{FC} \quad (3.62)$$

$$R_T = (h_{FC} \times N_{FC}^{cell} \times A_{FC}^{cell})^{-1} \quad (3.63)$$

3.4.4.2 PEM ER

Analogous to FCs, the stack voltage required for the generation of hydrogen, V_{ER}^{stack} , is calculated in (3.64) as the product of the single cell voltage and number of cells per stack (V_{ER}^{cell} and N_{ER}^{cell}). The former is calculated in (3.65), with e_{ER} being the reversible, open-circuit voltage and V_{ER}^{act} , V_{ER}^{ohm} and V_{ER}^{con} the activation, ohmic and concentration over-voltages [100].

$$V_{ER}^{stack} = N_{ER}^{cell} \times V_{ER}^{cell} \quad (3.64)$$

$$V_{ER}^{cell} = e_{ER} + V_{ER}^{act} + V_{ER}^{ohm} + V_{ER}^{con} \quad (3.65)$$

3.4.4.2.1 REVERSIBLE POTENTIAL Once again, the ER reversible potential is calculated by using the Nernst equation in (3.66) [100]. The partial pressures of hydrogen and oxygen are denoted by p_{H_2} and p_{O_2} , whereas a_{H_2O} is the water activity between the electrode and the membrane [100]. For liquid water, the latter's value can be approximated as unity [100].

$$\begin{aligned}
 e_{ER} &= \frac{|\Delta G_{ER}|}{2F} - \frac{|\Delta S_{ER}|}{2F} \times (T_{ER} - T_{ref}) + \frac{R_{uni} \times T_{ER}}{2F} \times \ln \left(\frac{p_{H_2} \times p_{O_2}^{0.5}}{a_{H_2O}} \right) \\
 &= 1.229 - 8.461 \times 10^{-4} \times (T_{ER} - 298.15) + 4.309 \times 10^{-5} \times T_{ER} \times \ln \left(\frac{p_{H_2} \times p_{O_2}^{0.5}}{a_{H_2O}} \right)
 \end{aligned} \quad (3.66)$$

Many publications, however, assume that the effects of pressure can be neglected when calculating the reversible potential [11, 101]. Thus when considering only the effects of temperature and performing appropriate substitutions, e_{ER} is calculated in (3.67) [102].

$$e_{ER} = 1.5184 - (1.5421 \times 10^{-3} \times T_{ER}) + [9.523 \times 10^{-5} \times T_{ER} \times \ln(T_{ER})] + (9.84 \times 10^{-8} \times T_{ER}^2) \quad (3.67)$$

3.4.4.2.2 OVER VOLTAGES Similar to a PEM FC, ERs also suffer the effect of losses due to over-voltages V_{ER}^{act} , V_{ER}^{ohm} and V_{ER}^{con} [12]. The latter can, however, be ignored since the operating current densities are never high enough to consider the effect thereof [12]. V_{ER}^{act} is calculated as the sum of the anode and cathode activation drops – $V_{ER}^{act,a}$ and $V_{ER}^{act,c}$ – as shown in (3.68) [11].

$$V_{ER}^{act} = V_{ER}^{act,a} + V_{ER}^{act,c} \quad (3.68)$$

$V_{ER}^{act,a}$ and $V_{ER}^{act,c}$ are calculated in (3.69) and (3.70), with the charge coefficients for the anode and cathode reactions are denoted by α_a and α_c . Due to a lack of information available on their values, it is usually assumed that each will equal 0.5, as most reactions tend towards symmetry [11].

$$\begin{aligned}
 V_{ER}^{act,a} &= \frac{R_{uni} \times T_{ER}}{2\alpha_a \times F} \times \ln \left(J_a / J_a^0 \right) \\
 &= \frac{R_{uni} \times T_{ER}}{F} \times \ln \left(J_a / J_a^0 \right)
 \end{aligned} \quad (3.69)$$

$$\begin{aligned}
 V_{ER}^{act,c} &= -\frac{R_{uni} \times T_{ER}}{2\alpha_c \times F} \times \ln \left(J_c / J_c^0 \right) \\
 &= -\frac{R_{uni} \times T_{ER}}{F} \times \ln \left(J_c / J_c^0 \right)
 \end{aligned} \quad (3.70)$$

Assuming uniform current density at both the anode and cathode – J_a and J_c – (3.71) is used to calculate the densities based on the ER cell current and area – I_{ER} and A_{ER}^{cell} .

$$J_a = J_c = J_{ER} = \frac{I_{ER}}{A_{ER}^{cell}} \quad (3.71)$$

The ohmic over-voltage, V_{ER}^{ohm} , is calculated as shown in (3.72): r_{elec} is the electronic (open-circuit) resistance which can be measured at the stack terminals, and r_{memb} is the resistance to the flow of protons through the membrane [11, 12]. The electronic resistance is much smaller than the membrane resistance and can be neglected without significant errors, however, a previous study used a value of

0.0153 [11, 103]. Note that the unit for both these resistances are $\Omega \cdot cm^2$, as it is multiplied by the current density in (3.72).

$$V_{ER}^{ohm} = (r_{elec} + r_{memb}) \times J_{ER} \quad (3.72)$$

The membrane resistance is calculated as a function of its thickness, ℓ_{ER}^{mem} , and the membrane's conductivity, σ_{ER}^{mem} , as shown in (3.73) [100]. Additionally, σ_{ER}^{mem} is calculated in (3.74), as a function of the ER's operating temperature and water content, λ_{ER}^{mem} . The water content value can be as low as 0.5 under dry conditions, however for a completely hydrated PEM ER, a value of 22 is assumed [100].

$$r_{memb} = \frac{\ell_{ER}^{mem}}{\sigma_{ER}^{mem}} \quad (3.73)$$

$$\sigma_{ER}^{mem} = (5.139 \times \lambda_{ER}^{mem} - 3.26) \times 10^{-3} \times \exp\left(\frac{1268}{303} - \frac{1268}{T_{ER}}\right) \quad (3.74)$$

3.4.4.2.3 THERMAL ANALYSIS The operating temperature of the ER, T_{ER} , will directly affect its efficiency and can be approximated using a lumped thermal capacitance model [11]. Neglecting external cooling, (3.75) is used. The power dissipated by the chemical reaction inside the ER is denoted as Q_{ER}^{gen} , whereas the resulting heat flow of the ambient temperature is denoted as Q_{ER}^{loss} . These are respectfully calculated in (3.76) and (3.77) [11].

$$\frac{dT_{ER}}{dt} = \frac{Q_{ER}^{gen} - Q_{ER}^{loss}}{C_{th}} \quad (3.75)$$

$$Q_{ER}^{gen} = N_{ER}^{cell} \times (V_{ER}^{cell} - E_{tn}) \times I_{ER} \quad (3.76)$$

$$Q_{ER}^{loss} = \frac{T_{ER} - T_{amb}}{R_{th}} \quad (3.77)$$

Values for the thermal capacitance and thermal time constant (C_{th} and τ_{th}) were determined empirically in a previous publication and are shown in Table 3.12 [11]. The thermal resistance and thermoneutral voltage – R_{th} and E_{tn} – are calculated using (3.78) and (3.79) [11].

$$R_{th} = \frac{\tau_{th}}{C_{th}} \quad (3.78)$$

$$\begin{aligned} E_{tn} &= \frac{\Delta H_{ER}}{2F} \\ &= 1.4813 [V] \end{aligned} \quad (3.79)$$

To solve the first order differential equation in the thermal model, (3.76) - (3.78) is substituted into (3.75) and it is rewritten in the standard form, as shown in (3.80). The ambient temperature at time t is taken as the initial condition for the heat transfer.

$$\frac{dT_{ER}}{dt} = \left[\frac{N_{ER}^{cell} \times (V_{ER}^{cell} - E_{in}) \times I_{ER}}{C_{th}} + \frac{T_{amb}}{\tau_{th}} \right] - \frac{T_{ER}}{\tau_{th}} \quad (3.80)$$

3.4.4.3 Hydrogen production and storage

The amount of hydrogen consumed by an FC is a function of the cell current, as shown by Faraday's Law in (3.81), with $\dot{m}_{H_2}^{con}$ the mass flow rate in moles per second and M_{H_2} the molar mass of hydrogen [99]. Similarly, the amount of hydrogen generated by a PEM ER – $\dot{m}_{H_2}^{gen}$ – can be calculated using (3.82). Here, Faraday's efficiency (η_f) is also introduced, typically having a value of more than 99% [11].

$$\dot{m}_{H_2}^{con} = \frac{I_{FC} \times M_{H_2}}{2F} \times N_{FC}^{cell} \quad (3.81)$$

$$\dot{m}_{H_2}^{gen} = \frac{I_{ER} \times M_{H_2}}{2F} \times N_{ER}^{cell} \times \eta_f \quad (3.82)$$

For simplicity, hydrogen will be stored in a pressurized gas cylinder, sized to hold a certain mass thereof, regardless of ambient temperature and pressure fluctuations. The net mass of hydrogen flowing either in or out of the tank per interval – Δm_{H_2} – is calculated by integrating over the interval duration (t_i and t_f) using (3.83).

$$\Delta m_{H_2} = \int_{t_i}^{t_f} \dot{m}_{H_2}^{gen} - \dot{m}_{H_2}^{con} \quad (3.83)$$

Analogous to the SOC of the battery bank, a tank level factor (TLF) is introduced to monitor the amount of hydrogen stored. The TLF is presented as a fraction of the total tank capacity, with values between zero and one, as shown in (3.84).

$$TLF_{n+1} = TLF_n + \frac{\Delta m_{H_2}}{m_{tnk}} \quad (3.84)$$

The maximum demand in the MG (P_{MG}^{pk}) occurs during the evening peak times and is, on average, 4.788 kW per household (P_{hme}^{pk}). Multiplying this by the number of households and a safety factor, SF , the MG is estimated to peak at 30 kW as shown in (3.85). A relatively large SF value also considers the possibility of some CLs operating during peak conditions. Each SR-12 PEM GEN stack can generate up to 0.5 kW power (P_{FC}^{stck}), and to be able to meet this demand, 60 stacks are required, as calculated in (3.86).

$$\begin{aligned} P_{MG}^{pk} &= P_{hme}^{pk} \times 5 \times SF \\ &= 4.788 \times 5 \times 1.25 \\ &\approx 30 \text{ [kW]} \end{aligned} \quad (3.85)$$

$$\begin{aligned} N_{FC}^{stck} &= P_{MG}^{pk} / P_{FC}^{stck} \\ &= 30 / 0.5 \\ &= 60 \text{ [stacks]} \end{aligned} \quad (3.86)$$

Using simulations of the developed ER model, it is determined that a current of 43.31 A flows through each cell – and each stack – when operated at maximum power. Similarly, the peak current in the FCs, as per simulations, is determined to be 14.63 A. The maximum rate of hydrogen consumption by all 60 FC stacks – $\dot{m}_{H_2}^{con,max}$ – is calculated in (3.87).

$$\begin{aligned}\dot{m}_{H_2}^{con,max} &= \frac{I_{FC}^{max} \times M_{H_2}}{2F} \times N_{FC}^{cell} \times N_{FC}^{stack} \\ &= \frac{14.63 \times 2.016}{2 \times 96487} \times 48 \times 60 \\ &= 440.2 [mg/s]\end{aligned}\quad (3.87)$$

The maximum amount of hydrogen generated by a single ER stack – $\dot{m}_{H_2}^{gen,stack}$ – is calculated in (3.88). This is done to estimate the amount of ER stacks (N_{ER}^{stack}) required to match the hydrogen generation and consumption rates, as shown in (3.90).

$$\begin{aligned}\dot{m}_{H_2}^{gen,stack} &= \frac{I_{ER,max} \times M_{H_2}}{2F} \times N_{ER}^{cell} \times \eta_f \\ &= \frac{43.31 \times 2.016}{2 \times 96487} \times 12 \times 0.99 \\ &= 5.375 [mg/s]\end{aligned}\quad (3.88)$$

$$\begin{aligned}N_{ER}^{stack} &= \frac{\dot{m}_{H_2}^{con,tot}}{\dot{m}_{H_2}^{gen,stack}} \\ &= \frac{440.2}{5.375} \\ &\approx 82 [stacks]\end{aligned}\quad (3.89)$$

82 stacks will thus be required for an equal flow of hydrogen. For simplicity, this number is decreased to match the amount of FC stacks, as shown in (3.90). The maximum amount of hydrogen that can now be generated – $\dot{m}_{H_2}^{gen,max}$ – is calculated in (3.91).

$$\begin{aligned}N_{ER}^{stack} &= N_{FC}^{stack} \\ &= 60 stacks\end{aligned}\quad (3.90)$$

$$\begin{aligned}\dot{m}_{H_2}^{gen,max} &= \frac{I_{ER}^{max} \times M_{H_2}}{2F} \times N_{ER}^{cell} \times N_{ER}^{stack} \times \eta_f \\ &= \frac{43.31 \times 2.016}{2 \times 96487} \times 12 \times 60 \times 0.99 \\ &= 322.5 [mg/s]\end{aligned}\quad (3.91)$$

3.4.4.4 Model Simplification

Using the equations mentioned earlier, FC and ER models are developed in SIMULINK, with block diagrams shown in Appendix D. These models are, however, computationally heavy and would require some simplification to speed up the optimisation process. Using the expected operating conditions shown in (3.92) - (3.94) as set-points, the amount of hydrogen consumed and generated was recorded for each. This data was then plotted in Figure 3.7 and fitted to a surface equation using MATLAB's

curve-fitting functionality.

$$T_{cell,min} \leq T_{cell} \leq T_{cell,max} \quad (3.92)$$

$$270 \leq T_{cell} \leq 400$$

$$E_{ER,min} \leq E_{ER} \leq E_{ER,max} \quad (3.93)$$

$$3\,000 \leq E_{ER} \leq 30\,000$$

$$E_{FC,min} \leq E_{FC} \leq E_{FC,max} \quad (3.94)$$

$$1\,500 \leq E_{ER} \leq 15\,000$$

The standard form of the surface equation is shown in (3.95), with a , b and c coefficients determined by the surface-fitting algorithm. The operating temperature will be the x -parameter, and energy generated or consumed will be the y -parameter. (3.96) and (3.97) shows the modified versions of the standard form, with coefficient values summarized in Table 3.13.

$$f(x,y) = (ax^2 + bx + c) \times y \quad (3.95)$$

$$m_{H_2}^{gen} = (a_{gen}T_{ER}^2 + b_{gen}T_{cell} + c_{gen}) \times E_{ER} \quad (3.96)$$

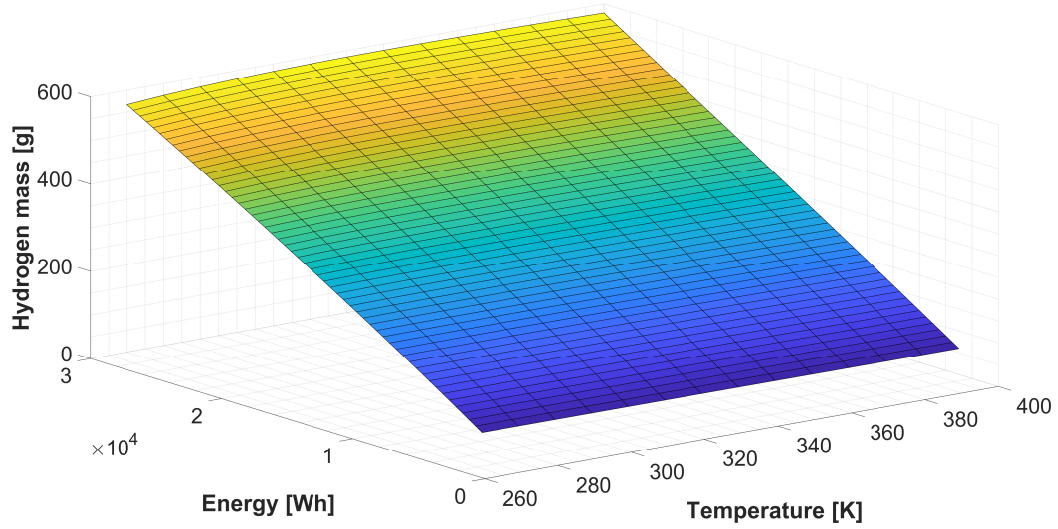
$$m_{H_2}^{con} = (a_{con}T_{FC}^2 + b_{con}T_{cell} + c_{con}) \times E_{FC} \quad (3.97)$$

3.4.4.5 Cost Analysis

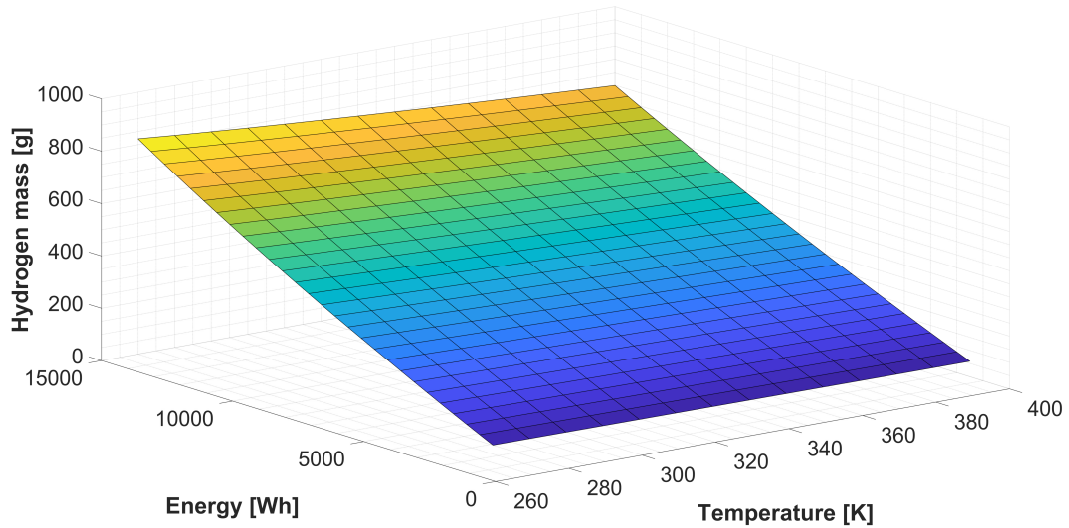
Based on average values, a stationary FC will have a life expectancy of 11 900 hours with a degradation rate of $5\,\mu V$ per hour [60]. However, PEM ER cells with the same degradation rate can be expected to last up to 60 000 hours – and even longer – before a 10% drop in efficiency occurs [104]. During each half-hour integration period, the output voltage of a single cell in both the ER and FC will drop $2.5\,\mu V$. The FC will thus last 23 800 such periods and the ER 120 000.

3.4.4.5.1 FC A publication from 2021 performed a techno-economic analysis using optimized sizing techniques for a hybrid PV and hydrogen storage system: A 1.5 kW PEM FC, priced at €702.23, was considered [56]. Using the required FC power (P_{FC}^{req}) and 1.5 kW unit for reference cost and power (c_{FC}^{ref} and P_{FC}^{ref}) the total FC cost (c_{FC}^{cap}) is estimated in (3.98). A conversion factor – Z_{EUR} – converts the cost from Euro to Rand.

$$\begin{aligned} c_{FC}^{cap} &= c_{FC}^{ref} \times \left(P_{FC}^{req} / P_{FC}^{ref} \right) \times Z_{EUR} \\ &= 702.23 \times \frac{30}{1.5} \times 17.48 \\ &= 245\,500 \text{ [ZAR]} \end{aligned} \quad (3.98)$$



(a) Surface plot relating ER operating temperature, energy and generated hydrogen mass.



(b) Surface plot relating FC operating temperature, energy and consumed hydrogen mass.

Figure 3.7. Surface plots relating ER and FC operating temperature, energy and hydrogen mass.

The cost per 30-minute interval – c_{FC}^{cyc} – is calculated in (3.99), with the estimated amount of cycles the FC can endure denoted by N_{FC}^{cyc} .

$$\begin{aligned}
 c_{FC}^{cyc} &= c_{FC}^{cap} / N_{FC}^{cyc} \\
 &= \frac{245\,500}{23\,800} \\
 &= 10.32 \text{ [ZAR/cycle]}
 \end{aligned} \tag{3.99}$$

3.4.4.5.2 ER Based on an article published in 2020, a PEM ER system which generates 1 Nm³ (normal cubic meter) per hour was priced at around 7 600 USD in 2018 [105]. Two years later, these prices have dropped as low as 4 900 USD for a similar unit [105].

The molar volume (v_m) for an ideal gas is calculated using standard temperature (T_{STP}) and pressure (p_{STP}) values of 273.15 K and 101.3 kPa, respectively, as shown in (3.100) [99]. From there, using the simulated ER current, the amount of hydrogen generated in standard m³ per hour (V_{H_2}) is calculated in (3.101) [99].

$$\begin{aligned} v_m &= R_{uni} T_{STP} / p_{STP} \\ &= \frac{8.314 \times 273.15}{101\,300} \\ &= 0.02242 \text{ [m}^3/\text{mol]} \end{aligned} \quad (3.100)$$

$$\begin{aligned} V_{H_2} &= N_{ER}^{cell} \times N_{ER}^{stck} \times v_m \times 60^2 \times \frac{I_{ER}}{2F} \\ &= 12 \times 60 \times 0.02242 \times 60^2 \times \frac{43.31}{2 \times 96487} \\ &= 13.04 \text{ [Nm}^3/\text{h]} \end{aligned} \quad (3.101)$$

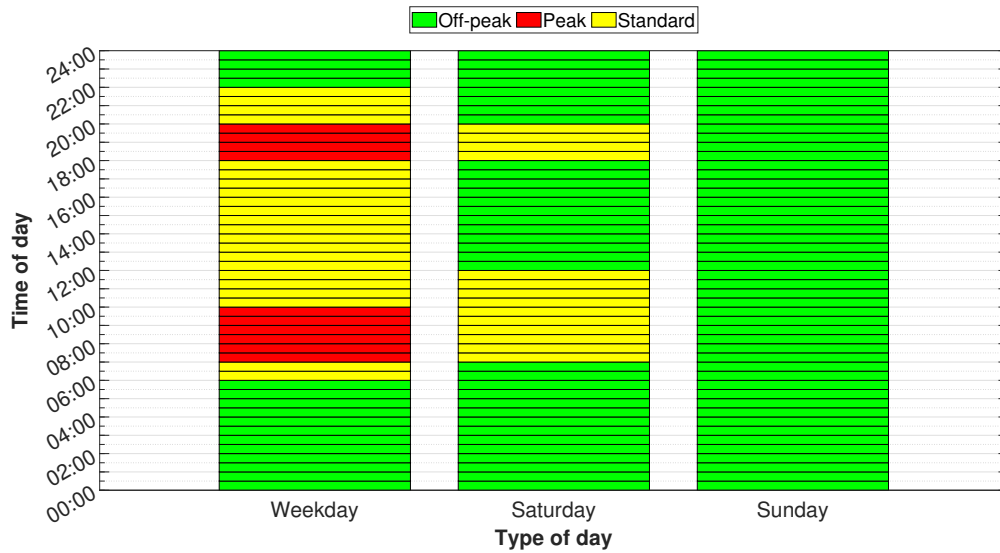
Taking the results from (3.100) and (3.101), the total ER cost is calculated in (3.102), with Z_{USD} converting from USD to ZAR. Assuming a life expectancy of 60 000 hours, the ER system will endure 120 000 30-minute intervals, with the total cost per interval, c_{ER}^{cyc} , calculated in (3.103).

$$\begin{aligned} c_{ER}^{cap} &= V_{H_2} \times c_{ER}^{unit} \times Z_{USD} \\ &= 13.04 \times 4\,900 \times 15.33 \\ &= 979\,526 \text{ [ZAR]} \end{aligned} \quad (3.102)$$

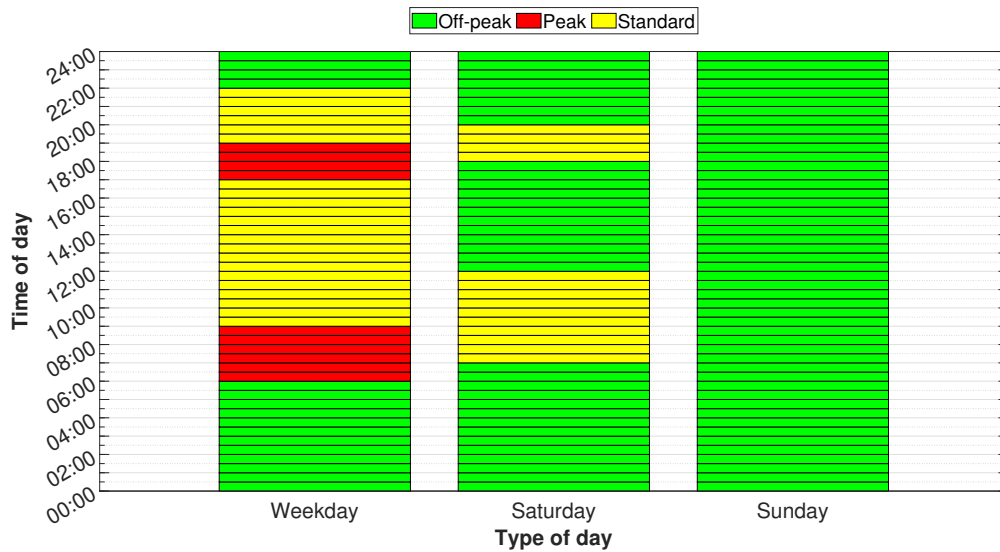
$$\begin{aligned} c_{ER}^{cyc} &= c_{ER}^{cap} / N_{cyc}^{ER} \\ &= \frac{979\,526}{120\,000} \\ &= 8.138 \text{ [ZAR/cycle]} \end{aligned} \quad (3.103)$$

3.4.5 Utility grid

TOU pricing for South Africa's power utility, ESKOM, is shown in Figure 3.8. These periods are applicable to power utility's "MiniFlex" package, which is aimed at urban consumers with a notified maximum demand between 25 kVA and 5 MVA [106]. Additionally, when considering active energy charges, local authority tariffs will be used, i.e. it will be assumed that the electricity is being supplied to the MG via a municipal connection. Using the rates from ESKOM's 2020/2021 Tariffs & Charges Booklet, relevant TOU pricing is shown in Table 3.14.



(a) TOU periods for low demand season.



(b) TOU periods for high demand season.

Figure 3.8. Plots showing different TOU periods as defined by ESKOM.

The low-demand season is defined as the TOU period from 1 September to 31 May, whereas the high-demand season runs from 1 June to 31 August [106]. Using the months from Table 3.1, only electricity consumed during July will be charged according to the high-demand season tariff. The TOU tariffs are placed in a matrix, M_{TOU} , where the rows (i) represent a time slot and the columns (j) are either high or low-demand seasons. The UG TOU cost over a 24-hour (c_{UG}^{TOU}) period is calculated in (3.104), with $E_{UG_{i,j}}$ being the energy bought from the UG at time interval i of season j .

$$c_{UG_j}^{TOU} = \sum_{i=1}^{48} M_{TOU_{i,j}} \times E_{UG_{i,j}} \quad (3.104)$$

As ESKOM mainly relies on fossil fuels (coal) for electricity generation, a carbon emission penalty factor is considered. In a publication from 2012, the authors used values of 0.259 ton/MWh and 40 USD/ton for the emission factor and penalty cost respectively (F_{UG} and H_{UG}) [107]. Ten years ago, 1 USD equalled 8.2115 ZAR, and the average inflation rate in South Africa since then is 5% [108]. This then translates to a penalty factor of 89.32 ZAR/MWh and the total UG penalty cost – c_{UG}^{pen} – is now calculated using (3.105).

$$\begin{aligned} c_{UG_j}^{pen} &= H_{UG} \times F_{UG} \times \sum_{i=1}^{48} E_{UG_{i,j}} \\ &= 8.932 \times 10^{-5} \times \sum_{i=1}^{48} E_{UG_{i,j}} \end{aligned} \quad (3.105)$$

The total costs relating to electricity bought from the UG – c_{UG}^{tot} – is now calculated using (3.106).

$$c_{UG_j}^{tot} = c_{UG_j}^{TOU} + c_{UG_j}^{pen} \quad (3.106)$$

3.5 ENERGY OPTIMISATION AND DEMAND MANAGEMENT

The MG energy optimisation problem aims to minimize operating costs by optimally utilizing different energy sources and scheduling loads. This includes managing whether the batteries are charged or discharged and whether hydrogen is consumed or generated. Two optimisation problems will be developed: A single-objective (SNGO) function to minimize cost and a multi-objective (MLTO) function that will also aim to reduce the inconvenience caused by load scheduling. Both algorithms will optimize the MG's operation over 24 hours, consisting of 48 30-minute intervals.

It is assumed that energy will only be bought from the UG and that the batteries only be charged. Discharging of the batteries will only be considered during emergency conditions when no other sources are available. All the relevant variables – per interval n – is written as a single vector, $\bar{E}_{tot}$, shown in (3.107). E_{UG} is the energy bought from the UG, E_{BAT}^{ch} is the battery bank's charging energy and E_{ER} and E_{FC} the energy of the ER and FC, respectively. The energy generated by the sun and the

wind are combined in a single variable denoted as E_{RE} , and E_{LD} is the load demand.

$$\bar{E}_{tot_n} = \begin{bmatrix} E_{UG_n} & E_{BAT_n}^{ch} & E_{ER_n} & E_{FC_n} & E_{RE_n} & E_{LD_n} \end{bmatrix} \quad (3.107)$$

The variables in (3.107) all have upper and lower bounds (UB and LB), with values shown in Table 3.15. These bounds for the UG and VRLA are implemented using (3.108), whereas (3.109) is used to represent the operating range of the ER and FC. The latter is modified because zero is not included in the allowed operating range of the hydrogen system, yet it is needed to indicate whether the FC or ER is turned off.

$$E_{UG_{min}} \leq E_{UG_n} \leq E_{UG_{max}} \quad (3.108)$$

$$E_{BAT_{min}}^{ch} \leq E_{BAT_n}^{ch} \leq E_{BAT_{max}}^{ch}$$

$$E_{ER_{min}} \leq E_{ER_n} \leq E_{ER_{max}}, \vee E_{ER_n} = 0 \quad (3.109)$$

$$E_{FC_{min}} \leq E_{FC_n} \leq E_{FC_{max}}, \vee E_{FC_n} = 0$$

3.5.1 Single-objective problem: Minimize cost

To minimize costs, the SNGO problem should be able to predict the optimal control actions on a daily basis. Using 48 30-minute intervals, the algorithm considers both day-ahead predictions for RES generation capacity and consumer behaviour. Vector $\bar{E}_{tot_n}$ contains six variables, and optimizing it for 48 intervals would result in 288 unknowns. Such high dimensionality will typically be computationally heavy and time-consuming; thus, some simplification is required.

Assuming maximum drawn energy from RES and a fixed load, the number of variables per interval can be decreased to four, resulting in 192 per optimisation period. An updated variable vector is shown in (3.110), with $\bar{E}_{fxd-l_d_n}$ containing the four variables to be optimized per interval. These variables are combined in a single vector, as shown in (3.111), with all 192 dimensions.

$$\bar{E}_{fxd-l_d_n} = \begin{bmatrix} E_{UG_n} & E_{BAT_n}^{ch} & E_{ER_n} & E_{FC_n} \end{bmatrix} \quad (3.110)$$

$$\bar{E}_{fxd-lt_{tot}} = \begin{bmatrix} E_{UG_1} \\ E_{BAT_1}^{ch} \\ E_{ER_1} \\ E_{FC_1} \\ E_{UG_2} \\ E_{BAT_2}^{ch} \\ E_{ER_2} \\ E_{FC_2} \\ \vdots \\ E_{UG_{48}} \\ E_{BAT_{48}}^{ch} \\ E_{ER_{48}} \\ E_{FC_{48}} \end{bmatrix}^T = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ \vdots \\ E_{48 \times 4} \end{bmatrix}^T = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ \vdots \\ E_{192} \end{bmatrix}^T \quad (3.111)$$

3.5.1.1 Cost function

The cost function to be minimized will only include expenses relating to the UG, ER and FC: c_{UG_n} , c_{ER_n} and c_{FC_n} . The objective function (c_{fxd-lt}) can thus be defined as shown in (3.112), with the cost of each component calculated using (3.113), (3.114) and (3.115). More detail regarding the TOU matrix, M_{TOU} , can be found in Section 3.4.5 and cost values is summarized in Table 3.16.

$$\text{Min } c_{fxd-lt} = \sum_{t=1}^{48} (c_{UG_n} + c_{ER_n} + c_{FC_n}) \quad (3.112)$$

$$c_{UG_n} = E_{UG_n} \times M_{TOU_n} \quad (3.113)$$

$$c_{ER_n} = \begin{cases} 0 & E_{ER_n} = 0 \\ 8.138 \text{ ZAR} & \text{otherwise} \end{cases} \quad (3.114)$$

$$c_{FC_n} = \begin{cases} 0 & E_{FC_n} = 0 \\ 10.32 \text{ ZAR} & \text{otherwise} \end{cases} \quad (3.115)$$

3.5.1.2 Lower and upper bounds

For the SNGO problem, all LB and UB values shown in Table 3.15 – except for E_{UG_n} – will apply. The latter's LB (LB_{UG_n}) is configured only to allow utilizing the UG during off-peak periods, as shown in (3.116).

$$LB_{UG_n} = \begin{cases} 0 & n \in [13, 44] \\ -50k & \text{otherwise} \end{cases} \quad (3.116)$$

3.5.1.3 Constraints

3.5.1.3.1 LINEAR CONSTRAINTS One of the most significant constraints of the optimisation problem is the energy balance equation shown in (3.117). This is crucial for the MG's stability, as the energy sources should always be able to meet the load demand. Using the passive sign convention shown in (3.118), loads will be assumed to have a positive energy (and power) value, whereas sources' values will be negative. (3.117) is re-written and expanded in the standard form for linear equality constraints, $A_{eq} \times \bar{E}_{fxd-Id_{tot}} = b_{eq}$, as shown in (3.119).

$$E_{LD_n} + E_{UG_n} + E_{BAT_n}^{ch} + E_{ER_n} + E_{FC_n} + E_{RE_n} = 0 \quad (3.117)$$

$$E_{sink} > 0 \quad (3.118)$$

$$E_{source} < 0$$

$$\begin{bmatrix} E_{UG_1} + E_{BAT_1}^{ch} + E_{ER_1} + E_{FC_1} \\ E_{UG_2} + E_{BAT_2}^{ch} + E_{ER_2} + E_{FC_2} \\ \vdots \\ E_{UG_{48}} + E_{BAT_{48}}^{ch} + E_{ER_{48}} + E_{FC_{48}} \end{bmatrix} = \begin{bmatrix} -(E_{LD_1} + E_{PV_1} + E_{WTG_1}) \\ -(E_{LD_2} + E_{PV_2} + E_{WTG_2}) \\ \vdots \\ -(E_{LD_{48}} + E_{PV_{48}} + E_{WTG_{48}}) \end{bmatrix} \quad (3.119)$$

Linear inequality constraints are used to restrict the SOC of the battery bank and TLF of the hydrogen system, shown in (3.120) and (3.121). This is to ensure sufficient battery capacity and hydrogen available to generate electricity and avoid either overcharging the batteries or generating hydrogen when the tank is full.

$$SOC_{min} \leq SOC \leq SOC_{max} \quad (3.120)$$

$$0.5 \leq SOC \leq 1$$

$$TLF_{min} \leq TLF \leq TLF_{max} \quad (3.121)$$

$$0.1 \leq TLF \leq 1$$

As only battery charging will be considered, the following assumptions can be made: The SOC of each interval will only depend on the current and historical E_{BAT}^{ch} values as well as the initial set-point, and the minimum SOC constraint need not be considered.

The first step in writing the SOC constraint in the standard form of $A_{ineq} \times \bar{E}_{fxd-Id_{tot}} \leq b_{ineq}$ would be to expand it mathematically, as shown in (3.122). To ease the readability of the equations, coefficient a is introduced as shown in (3.123). After some substitution, the desired form of the linear inequality

constraint is obtained in (3.124).

$$\begin{bmatrix} SOC_1 \\ SOC_2 \\ \vdots \\ SOC_{48} \end{bmatrix} = \begin{bmatrix} SOC_0 + a E_{BAT_1}^{ch} \\ SOC_1 + a E_{BAT_2}^{ch} \\ \vdots \\ SOC_{47} + a E_{BAT_{48}}^{ch} \end{bmatrix} \quad (3.122)$$

$$a = \frac{\eta_{bat}}{E_{bnk}} \quad (3.123)$$

$$\begin{bmatrix} a E_{BAT_1}^{ch} \\ a E_{BAT_1}^{ch} + a E_{BAT_2}^{ch} \\ \vdots \\ a E_{BAT_1}^{ch} + a E_{BAT_2}^{ch} + \dots + a E_{BAT_{48}}^{ch} \end{bmatrix} \leq \begin{bmatrix} SOC_{max} - SOC_0 \\ SOC_{max} - SOC_0 \\ \vdots \\ SOC_{max} - SOC_0 \end{bmatrix} \quad (3.124)$$

A similar approach is followed for the TLF of the hydrogen storage system: Coefficients b and c are introduced in (3.125) to simplify the expanded equation, shown in (3.126). These coefficients are based on the simplified ER and FC model shown in (3.96) and (3.97), discussed in Section 3.4.4.4.

$$\begin{aligned} b &= (-8.197 \times 10^{-8}) T_{ER}^2 + (5.475 \times 10^{-5}) T_{ER} + 1.065 \times 10^{-2} \\ c &= (8.784 \times 10^{-8}) T_{FC}^2 + (-1.161 \times 10^{-4}) T_{FC} + 7.731 \times 10^{-2} \end{aligned} \quad (3.125)$$

$$\begin{bmatrix} TLF_1 \\ TLF_2 \\ \vdots \\ TLF_{48} \end{bmatrix} = \begin{bmatrix} TLF_0 + (b E_{ER_1} + c E_{FC_1}) \\ TLF_1 + (b E_{ER_1} + c E_{FC_2}) \\ \vdots \\ TLF_{47} + (b E_{ER_{48}} + c E_{FC_{48}}) \end{bmatrix} \quad (3.126)$$

For the hydrogen system, both the ER and FC will be utilized during the 24-hour optimisation period and hence the maximum and minimum tank level constraints are considered. These are respectively shown in (3.127) and (3.128).

$$\begin{bmatrix} b E_{ER_1} + c E_{FC_1} \\ b E_{ER_1} + c E_{FC_1} + b E_{ER_2} + c E_{FC_2} \\ \vdots \\ b E_{ER_1} + c E_{FC_1} + \dots + b E_{ER_{48}} + c E_{FC_{48}} \end{bmatrix} \leq \begin{bmatrix} TLF_{max} - TLF_0 \\ TLF_{max} - TLF_0 \\ \vdots \\ TLF_{max} - TLF_0 \end{bmatrix} \quad (3.127)$$

$$\begin{bmatrix} -b E_{ER_1} - c E_{FC_1} \\ -b E_{ER_1} - c E_{FC_1} - b E_{ER_2} - c E_{FC_2} \\ \vdots \\ -b E_{ER_1} - c E_{FC_1} - \cdots - b E_{ER_{48}} - c E_{FC_{48}} \end{bmatrix} \leq \begin{bmatrix} TLF_0 - TLF_{min} \\ TLF_0 - TLF_{min} \\ \vdots \\ TLF_0 - TLF_{min} \end{bmatrix} \quad (3.128)$$

3.5.1.3.2 NON-LINEAR CONSTRAINTS The standard forms for writing non-linear constraints in MATLAB are $c_{eq}(\bar{E}_{fxd-l_{tot}}) = 0$ for equalities and $c_{ineq}(\bar{E}_{fxd-l_{tot}}) \leq 0$ for inequalities. Equality constraints are implemented to avoid simultaneous use of the ER and FC per interval. This is shown in (3.129) where at least one of the variables, E_{ER_n} or E_{FC_n} , should be equal to zero. The constraint for each interval can be combined for all 48 intervals, as shown in (3.130).

$$E_{ER_n} \times E_{FC_n} = 0 \quad (3.129)$$

$$E_{ER_1} E_{FC_1} + E_{ER_2} E_{FC_2} + \cdots + E_{ER_{48}} E_{FC_{48}} = 0 \quad (3.130)$$

As mentioned earlier, the operation of both ER and FC systems is confined by both lower and upper bounds for optimal performance. These bounds, however, do not include zero, which will be the energy output when either component is inactive. (3.109) is re-written as a polynomial inequality function as shown in (3.131) and (3.132) for the ER and FC, respectively.

$$-E_{ER_n}^2 + ER_{lwr} E_{ER_n} \leq 0 \quad (3.131)$$

$$-E_{FC_n}^2 + FC_{upr} E_{FC_n} \leq 0 \quad (3.132)$$

ER_{lwr} is the lower non-zero operating limit of the ER, and FC_{upr} is the upper non-zero operating limit of the FC. Rewriting the abovementioned equations in the standard form for non-linear inequality constraints, (3.133) is obtained.

$$\begin{bmatrix} \left(ER_{lwr} E_{ER_1} - E_{ER_1}^2 \right) + \left(FC_{upr} E_{FC_1} - E_{FC_1}^2 \right) \\ \left(ER_{lwr} E_{ER_2} - E_{ER_2}^2 \right) + \left(FC_{upr} E_{FC_2} - E_{FC_2}^2 \right) \\ \vdots \\ \left(ER_{lwr} E_{ER_{48}} - E_{ER_{48}}^2 \right) + \left(FC_{upr} E_{FC_{48}} - E_{FC_{48}}^2 \right) \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3.133)$$

3.5.1.3.3 INTEGER CONSTRAINTS Integer constraints are implemented to force each variable only to allow integer values. This is done in an effort to minimize computational strain, as there will now be fewer feasible points for each variable. To set these constraints, an incremental vector is

created, starting from one until the number of optimisation variables, as shown in (3.134).

$$\begin{aligned}\overline{intcon} &= \begin{bmatrix} 1 & 2 & 3 & \dots & nvars \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 3 & \dots & 192 \end{bmatrix}\end{aligned}\quad (3.134)$$

3.5.1.4 Initial conditions

Upon initial testing of the EO algorithms, it was found that GA could converge to a feasible point that meets all the constraints. The PSO algorithm, however, failed to find satisfying starting points, yielding incorrect results within an acceptable time frame. Random initial conditions (ICs) will be generated separately and provided as a starting point for the algorithms to combat this issue. These ICs are also required to meet all the constraints whilst being bounded by LBs and UBs.

These ICs took some computational power to generate; hence, only 250 points will be generated per day, limiting the PSO population size to the same value. Table 3.17 shows the time required to develop the ICs for each day.

3.5.2 Multi-objective problem: Minimize cost and inconvenience

An MLTO function will be developed using MATLAB's *gamultiobj*; with a syntax similar to the standard GA function. For the SNGO problem discussed earlier, each interval contained four integer variables to be optimised, which yielded 192 per day. When the load is considered variable, the number of variables will increase to 36 per interval and 1 728 per day. Although most thereof will now be binary, a problem of such high dimensionality will be very computationally heavy.

To combat this problem, the consumption of each HL – as well as the CLs – will be optimized individually before being combined. The basic structure that will be followed to optimize the MG operation with DM considered is shown in Figure 3.9.

The optimisation variables for home h at interval n is shown in (3.135): E_{BAT}^{ch} , E_{ER} and E_{FC} is the battery charging, ER and FC energies. $S_{L1_{h,n}} - S_{L6_{h,n}}$ are binary variables, indicating whether loads one to six of household h are powered at interval n . It is expanded for all intervals in (3.136). A similar approach is followed for the CLs, as shown in (3.137) and (3.138). S_{CL1_n} and S_{CL2_n} are binary variables indicating whether CL one or two is powered during interval n .

$$\overline{E}_{var-l_{d_{h,n}}} = \begin{bmatrix} E_{BAT_{h,n}}^{ch} & E_{ER_{h,n}} & E_{FC_{h,n}} & S_{L1_{h,n}} & \dots & S_{L6_{h,n}} \end{bmatrix} \quad (3.135)$$

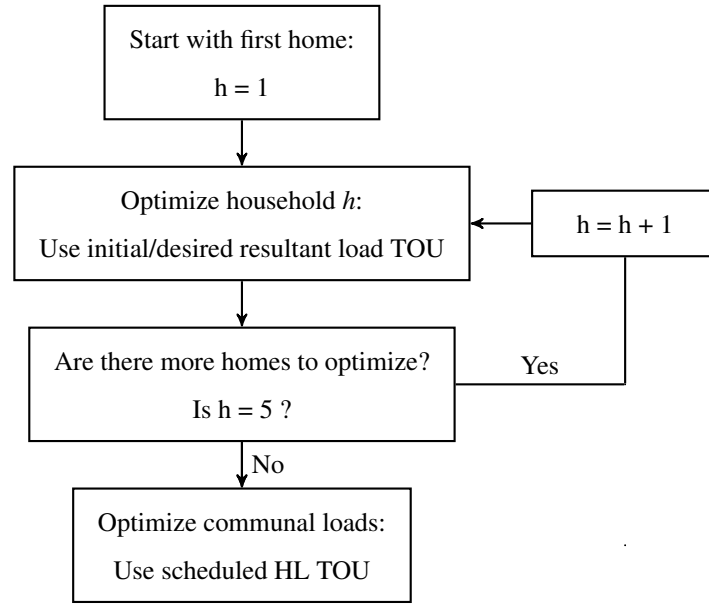


Figure 3.9. Block diagram showing how the different households (and communal loads) will be optimized when considering DM.

$$\begin{aligned}
 \bar{E}_{var-l_d_h} &= \begin{bmatrix} E_{BAT_{h,1}}^{ch} \\ E_{ER_{h,1}} \\ E_{FC_{h,1}} \\ S_{L1_{h,1}} \\ \vdots \\ S_{L6_{h,1}} \\ \vdots \\ E_{BAT_{h,48}}^{ch} \\ E_{ER_{h,48}} \\ E_{FC_{h,48}} \\ S_{L1_{h,48}} \\ \vdots \\ S_{L6_{h,48}} \end{bmatrix}^T = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \\ E_7 \\ E_8 \\ E_9 \\ \vdots \\ E_{48 \times 9} \end{bmatrix}^T = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \\ E_7 \\ E_8 \\ E_9 \\ \vdots \\ E_{432} \end{bmatrix}^T \quad (3.136)
 \end{aligned}$$

$$\bar{E}_{var-l_d_n} = \begin{bmatrix} E_{BAT_n}^{ch} & E_{ER_n} & E_{FC_n} & S_{CL1_n} & S_{CL2_n} \end{bmatrix} \quad (3.137)$$

$$\bar{E}_{var-ld_c} = \begin{bmatrix} E_{BAT_{c,1}}^{ch} \\ E_{ER_{c,1}} \\ E_{FC_{c,1}} \\ S_{CL1_1} \\ S_{CL2_1} \\ \vdots \\ E_{BAT_{h,48}}^{ch} \\ E_{ER_{h,48}} \\ E_{FC_{h,48}} \\ S_{CL1_{48}} \\ S_{CL2_{48}} \end{bmatrix}^T = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \\ E_7 \\ E_8 \\ E_9 \\ \vdots \\ E_{48 \times 9} \end{bmatrix}^T = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \\ E_7 \\ E_8 \\ E_9 \\ \vdots \\ E_{432} \end{bmatrix}^T \quad (3.138)$$

3.5.2.1 Objective functions

3.5.2.1.1 MINIMIZE COST The cost minimization part of the MLTO problem is similar to that of the SNGO problem, shown in (3.139). The individual component costs – c_{UG_n} , c_{ER_n} and c_{FC_n} – can be found in (3.113) - (3.115).

$$Min c_{var-ld} = \sum_{t=1}^{48} \left(c_{UG_n} + c_{ER_n} + c_{FC_n} \right) \quad (3.139)$$

3.5.2.1.2 MINIMIZE INCONVENIENCE Based on questionnaire feedback, each household is allocated an inconvenience vector – $\bar{I}_h$ – containing unique ratings for each appliance, shown in (3.140). These ratings can be any integer from one to five, with higher values translating to a higher inconvenience. This vector is then multiplied by the number of intervals a load is shifted (i.e. the time shift distance) to determine the overall inconvenience caused per appliance.

$$\bar{I}_h = \begin{bmatrix} I_{h,1} & I_{h,2} & I_{h,3} & I_{h,4} & I_{h,5} & I_{h,6} \end{bmatrix} \quad (3.140)$$

$$I_{h,l} \in \{1, 2, 3, 4, 5\}$$

Two binary matrices shown in (3.141) and (3.142) will respectively represent the desired and scheduled load TOU: $\bar{S}_{des}$ and $\bar{S}_{sch}$. The difference between the desired and scheduled loads, $\bar{S}_{diff}$, is calculated

in (3.143) and will be used to calculate the time shift distance.

$$\bar{S}_{des_h} = \begin{bmatrix} S_{des1,1} & S_{des1,2} & \dots & S_{des1,6} \\ S_{des2,1} & S_{des2,2} & \dots & S_{des2,6} \\ \vdots & \vdots & \ddots & \vdots \\ S_{des48,1} & S_{des48,2} & \dots & S_{des48,6} \end{bmatrix} \quad (3.141)$$

$$S_{des_{i,j}} \in \{0, 1\}$$

$$\bar{S}_{sch_h} = \begin{bmatrix} S_{sch1,1} & S_{sch1,2} & \dots & S_{sch1,6} \\ S_{sch2,1} & S_{sch2,2} & \dots & S_{sch2,6} \\ \vdots & \vdots & \ddots & \vdots \\ S_{sch48,1} & S_{sch48,2} & \dots & S_{sch48,6} \end{bmatrix} \quad (3.142)$$

$$S_{sch_{i,j}} \in \{0, 1\}$$

$S_{diff_{i,j}}$ will be zero when there is no difference between $\bar{S}_{des_{i,j}}$ and $\bar{S}_{sch_{i,j}}$, i.e. no load is scheduled; a value of -1 will indicate the new interval of operation for a given load and a value of +1 will indicate the initially desired interval. The distance between each -1 and +1 pair ($\bar{D}_{-1,+1}$) can thus be used to calculate the time shift, which is in turn multiplied by the inconvenience vector in (3.144).

$$\bar{S}_{diff_h} = \bar{S}_{des_h} - \bar{S}_{sch_h}$$

$$\bar{S}_{diff_h} = \begin{bmatrix} S_{diff1,1} & S_{diff1,2} & \dots & S_{diff1,6} \\ S_{diff2,1} & S_{diff2,2} & \dots & S_{diff2,6} \\ \vdots & \vdots & \ddots & \vdots \\ S_{diff48,1} & S_{diff48,2} & \dots & S_{diff48,6} \end{bmatrix} \quad (3.143)$$

$$S_{diff_{i,j}} \in \{-1, 0, 1\}$$

$$Min I_{var-ld} = \sum_{t=1}^{48} (\bar{D}_{-1,+1} \times \bar{I}_h) \quad (3.144)$$

3.5.2.2 Lower and upper bounds

Similar to the SNGO problem, values for the lower and upper bounds of the ER, FC and battery charging energy are shown in Table 3.15. The lower and upper limits of the loads are set to zero and one, respectively, to force these variables to be binary. All these bounds are vectorized for each HL, as well as the CLs, as shown in (3.145) - (3.148).

$$lb_{HL_n} = \begin{bmatrix} E_{BAT_{min}}^{ch} & E_{ER_{min}} & E_{FC_{min}} & S_{L1_{h,1}}^{min} & \dots & S_{L6_{h,1}}^{min} \\ 0 & 0 & -15\,000 & 0 & \dots & 0 \end{bmatrix} \quad (3.145)$$

$$\begin{aligned} ub_{HL_n} &= \begin{bmatrix} E_{BAT_{max}}^{ch} & E_{ER_{max}} & E_{FC_{max}} & S_{L1_h}^{max} & \dots & S_{L6_h}^{max} \end{bmatrix} \\ &= \begin{bmatrix} 106\ 000 & 30\ 000 & 0 & 1 & \dots & 1 \end{bmatrix} \end{aligned} \quad (3.146)$$

$$\begin{aligned} lb_{CL_n} &= \begin{bmatrix} E_{BAT_{min}}^{ch} & E_{ER_{min}} & E_{FC_{min}} & S_{L1_c}^{min} & S_{L2_c}^{min} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & -15\ 000 & 0 & 0 \end{bmatrix} \end{aligned} \quad (3.147)$$

$$\begin{aligned} ub_{CL_n} &= \begin{bmatrix} E_{BAT_{max}}^{ch} & E_{ER_{max}} & E_{FC_{max}} & S_{L1_c}^{max} & S_{L2_c}^{max} \end{bmatrix} \\ &= \begin{bmatrix} 106\ 000 & 30\ 000 & 0 & 1 & 1 \end{bmatrix} \end{aligned} \quad (3.148)$$

3.5.2.3 Constraints

3.5.2.3.1 LINEAR CONSTRAINTS The same linear constraints discussed in the previous section will apply, with an additional "on-time" constraint. This equality constraint is included to ensure that each appliance remains on for the same duration each day, before and after the implementation of DM. It is implemented using (3.149) and (3.150) for HLs and CLs, respectively, by simply summing the number of on-intervals per day and comparing it to the desired value, $t_{1,h}$, determined from the initial consumer TOU data.

$$\begin{bmatrix} S_{L1_{h,1}} & S_{L1_{h,2}} & \dots & S_{L1_{h,48}} \\ S_{L2_{h,1}} & S_{L2_{h,2}} & \dots & S_{L2_{h,48}} \\ \vdots & \vdots & \ddots & \vdots \\ S_{L6_{h,1}} & S_{L6_{h,2}} & \dots & S_{L6_{h,48}} \end{bmatrix} = \begin{bmatrix} t_{1,h} \\ t_{2,h} \\ \vdots \\ t_{6,h} \end{bmatrix} \quad (3.149)$$

$$\begin{bmatrix} S_{L1_{c,1}} & S_{L1_{c,2}} & \dots & S_{L1_{c,48}} \\ S_{L2_{c,1}} & S_{L2_{c,2}} & \dots & S_{L2_{c,48}} \end{bmatrix} = \begin{bmatrix} t_{1,h} \\ t_{2,h} \end{bmatrix} \quad (3.150)$$

3.5.2.3.2 NON-LINEAR CONSTRAINTS For individual household optimisation, the non-linear constraints will be neglected to speed up the process. However, the same non-linear constraints from the SNGO problem will still be present in the final CL optimisation to avoid simultaneous ER and FC use and force both to operate within optimal regions.

3.5.2.3.3 INTEGER CONSTRAINTS Integer constraints are implemented similarly to the SNGO problem. They are shown in (3.151) and (3.152) for HLs and CLs, with $nvars_{HL}$ and $nvars_{CL}$ the number of variables in the HL and CL problems, respectively.

$$\begin{aligned} \overline{intcon}_{HL} &= \begin{bmatrix} 1 & 2 & 3 & \dots & nvars_{HL} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 3 & \dots & 432 \end{bmatrix} \end{aligned} \quad (3.151)$$

$$\begin{aligned} \overline{intcon}_{CL} &= \begin{bmatrix} 1 & 2 & 3 & \dots & nvars_{CL} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 3 & \dots & 240 \end{bmatrix} \end{aligned} \quad (3.152)$$

3.6 CHAPTER SUMMARY

This concludes the Methodology section and the final MG considered for the case studies is shown in Figure 3.10. Results will be presented in Chapter 4, whereafter Discussions and Conclusions are provided in Chapters 5 and 6.

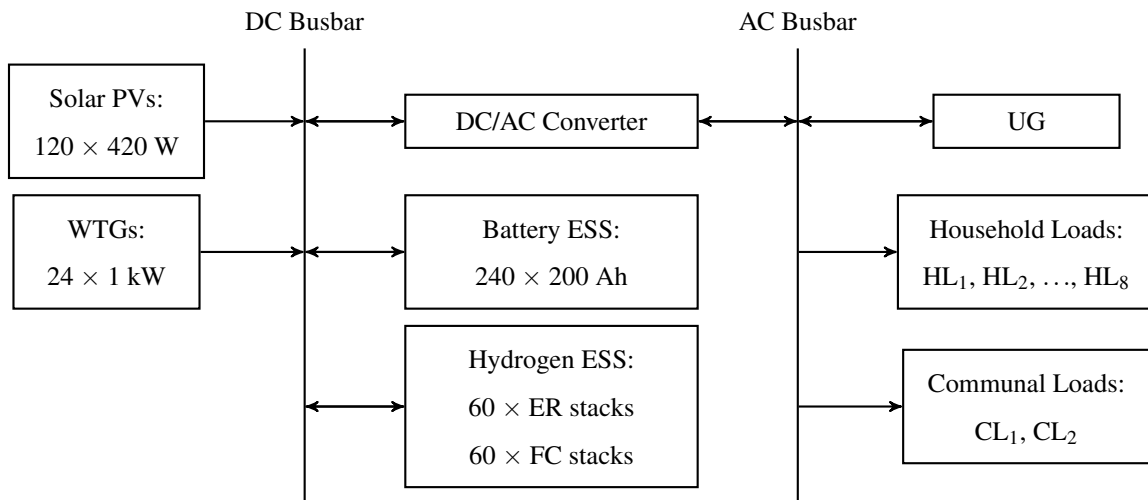


Figure 3.10. Detailed block diagram of the MG that will be considered in the case studies.

Table 3.8. Table showing SOC and DOD coefficient values.

Coefficient	Value	Coefficient	Value
a_1	31 972	a_2	-31 972
b_1	-211.76	b_2	-211.76
c_1	17.647	c_2	82.353
d_1	-0.0048	d_2	0.0048
e_1	43.364	e_2	56.636

Table 3.9. Table showing FC and ER chemical reactions and values.

	Electrolyser	Fuel Cell
Anode half-reaction	$H_2O \rightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$	$H_2 \rightarrow 2H^+ + 2e^-$
Cathode half-reaction	$2H^+ + 2e^- \rightarrow H_2$	$2H^+ + \frac{1}{2}O_2 + 2e^- \rightarrow H_2O$
Overall reaction	$H_2O \rightarrow H_2 + \frac{1}{2}O_2$	$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$
$\Delta S [J/mol.K]$	163.27	-163.27
$\Delta H [J/mol]$	285 850	-285 850
$\Delta G [J/mol]$	237 171	-237 171

Table 3.10. Table showing relevant chemical constants used for FC and ER calculations.

Symbol	Name	Value	Unit
F	Faraday's constant	96 487	$[C \cdot mol^{-1}]$
g	Gravitational acceleration	9.8066	$[m \cdot s^{-2}]$
M_{air}	Molar mass of air	28.97	$[g \cdot mol^{-1}]$
M_{H_2}	Molar mass of hydrogen	2.016	$[g \cdot mol^{-1}]$
p_0	Air pressure at sea level	1.000	$[atm]$
R_{uni}	Universal gas constant	8.3143	$[J \cdot (mol \cdot K)^{-1}]$
T_{ref}	Reference temperature	298.15	$[K]$

Table 3.11. Table showing parameters for the SR-12 PEM FC stack [3, 9, 10].

Symbol	Description	Value	Unit
ξ_1	V_{FC}^{act} parametric coefficient 1	0.948	-
ξ_2	V_{FC}^{act} parametric coefficient 2	-3.4×10^{-3}	-
ξ_3	V_{FC}^{act} parametric coefficient 3	-7.22×10^{-5}	-
ξ_4	V_{FC}^{act} parametric coefficient 4	1.06×10^{-4}	-
β_{con}	V_{FC}^{con} parametric coefficient	0.15	[V]
J_{max}	Maximum current density	672	[mA/cm ²]
ℓ_{mem}	Membrane thickness	0.0025	[cm]
A_{FC}^{cell}	FC active area	62.5	[cm ²]
R_p	Electron membrane resistance	3×10^{-4}	[Ω]

Table 3.12. Table showing parameters for 1 kW PEM ER stack [11, 12].

Symbol	Description	Value	Unit
A_{cell}	Single cell area	50	[cm ²]
C_{th}	Thermal capacitance	42 045	[J/K]
N_{ER}^{cell}	Number of cells per stack	12	[-]
R_{th}	Thermal resistance	8.1698×10^{-2}	[K/W]
r_{elec}	Electronic resistance	1.53×10^{-2}	[$\Omega \cdot cm^2$]
J_a^0	Anode exchange current density	10^{-12}	[A/cm ²]
J_c^0	Cathode exchange current density	10^{-3}	[A/cm ²]
α_a	Anode transfer coefficient	0.5	[-]
α_c	Cathode transfer coefficient	0.5	[-]
δ_{memb}	Membrane thickness	183	[μm]
τ_{th}	Thermal time constant	3 435	[s]
λ_{memb}	Membrane water content	22	[-]

Table 3.13. Table showing surface-fitting coefficients for hydrogen generation and consumption.

Coefficient	ER	FC
a	-8.197×10^{-8}	8.784×10^{-8}
b	5.475×10^{-5}	-1.161×10^{-4}
c	1.065×10^{-2}	7.731×10^{-2}

Table 3.14. Table showing ESKOM's TOU tariffs for the 2020/2021 year.

	Active energy charge [c/kWh]		
	Peak	Standard	Off-Peak
High demand season	426.71	129.85	70.850
Low demand season	139.70	96.420	61.460

Table 3.15. Table showing lower and upper limits for optimization variables.

Variable	Minimum	Maximum	Unit
E_{UG_n}	-50.0	0.00	[kWh]
$E_{BAT_n}^{ch}$	0.00	106	[kWh]
E_{ER_n}	3.00	30.0	[kWh]
E_{FC_n}	-15.0	-1.50	[kWh]

Table 3.16. Table showing expected duration and operating costs of relevant MG components.

Component	Life expectancy	Initial investment cost	Operating cost
30.0 kW FCs	5 [years]	245 500 [ZAR]	10.32 [ZAR/cycle]
60.0 kW ERs	12.5 [years]	979 526 [ZAR]	8.138 [ZAR/cycle]
50.4 kW PVs	25 [years]	397 200 [ZAR]	-
24.0 kW WTGs	25 [years]	1 386 000 [ZAR]	-
576 kWh VRLA Batteries	5 [years]	1 907 520 [ZAR]	1 590 [ZAR/cycle] ¹

¹ Based on a DOD of 50%.

Table 3.17. Table showing time elapsed during generation of 250 initial points.

	Day 1	Day 2	Day 3	Day 4	Average
Duration [hours]	4.7	5.2	5.0	4.7	4.9

CHAPTER 4 RESULTS

4.1 CHAPTER OVERVIEW

This chapter is divided into two main categories: Processed Data in Section 4.2 and optimisation Results in Section 4.3. The first part considers how the SAURAN and questionnaire data is analysed, which forms the basis of the optimisation results in the second part.

4.2 PROCESSED DATA

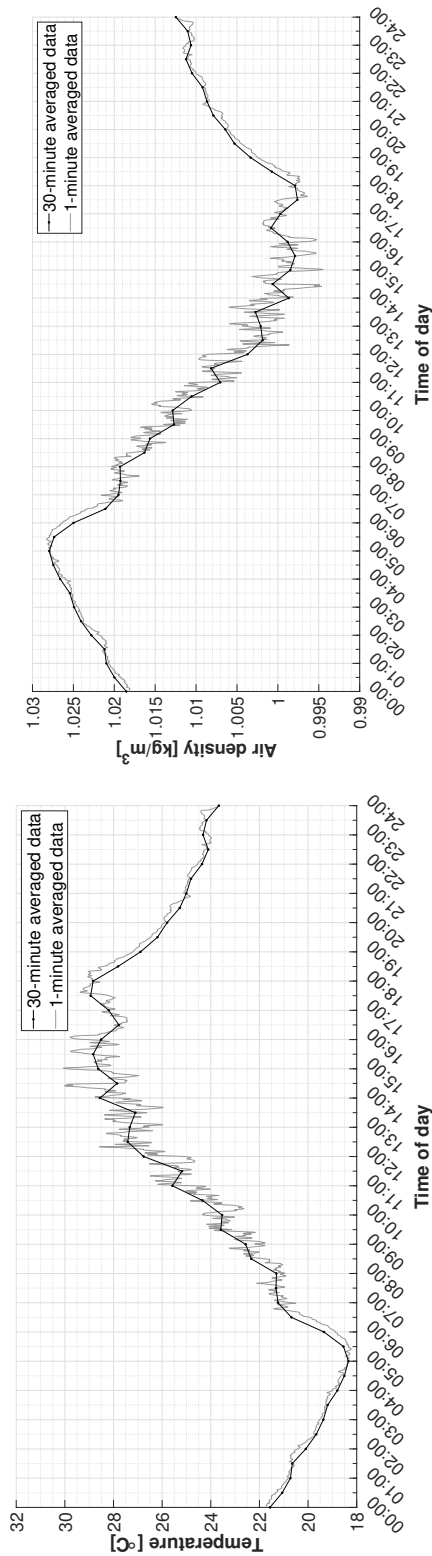
As part of this work, large amounts of data were processed, albeit from the SAURAN weather station (Subsection 4.2.1) or the research questionnaire (Subsection 4.2.2). The results of each are presented accordingly.

4.2.1 SAURAN Data

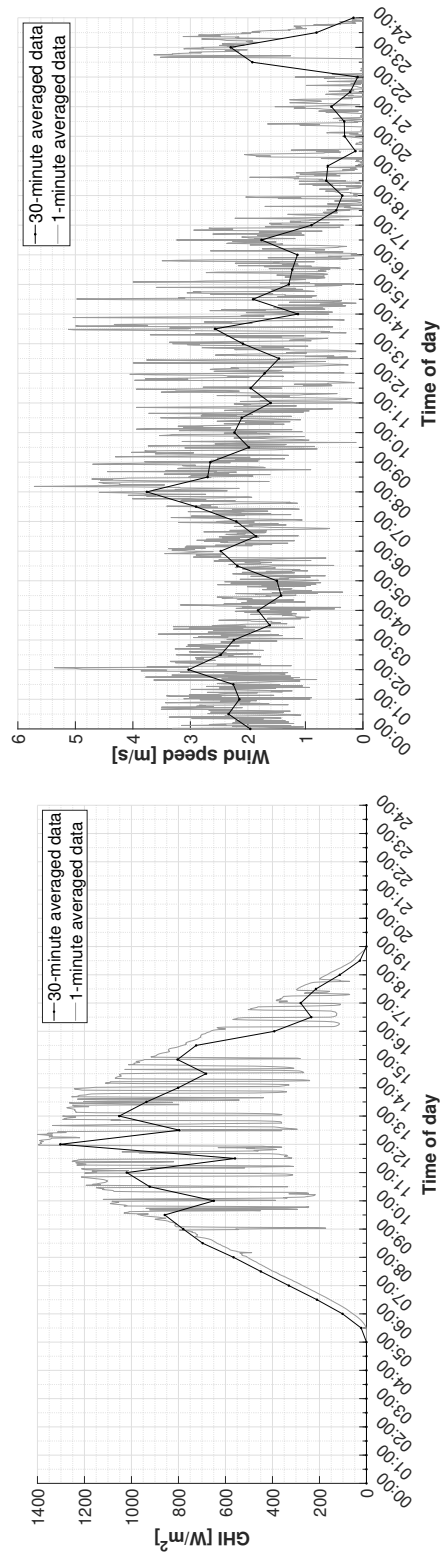
The data extracted from the SAURAN database had a resolution of 60 seconds. Not only was it required for the data to be averaged over 30-minute intervals, but error-checking also needed to be implemented. After finding no erroneous data, the relevant information was extracted and plotted in Figures 4.1 - 4.4 for each season considered. Using the relevant source modelling techniques discussed in Chapter 3, the output power of each RES is determined for the four relevant seasons. Figures 4.5 - 4.8 show the combined output power plots of a single PV and WTG per 30-minute interval.

4.2.2 Questionnaire Data

Forty individuals, each representing a household, completed the research survey providing TOU and inconvenience ratings for different appliances. Out of these 40, five were selected at random, with Figure 4.9 showing the appliances' TOU of these households. Based on ratings from Table 3.2, load profiles for each household and a load profile for the small residential complex are developed in Figure 4.10. Additionally, inconvenience ratings allocated to each appliance for the five selected households (as well as the average inconvenience) are shown in Figure 4.11.



(a) Plot showing temperature versus time for 17 January 2020.

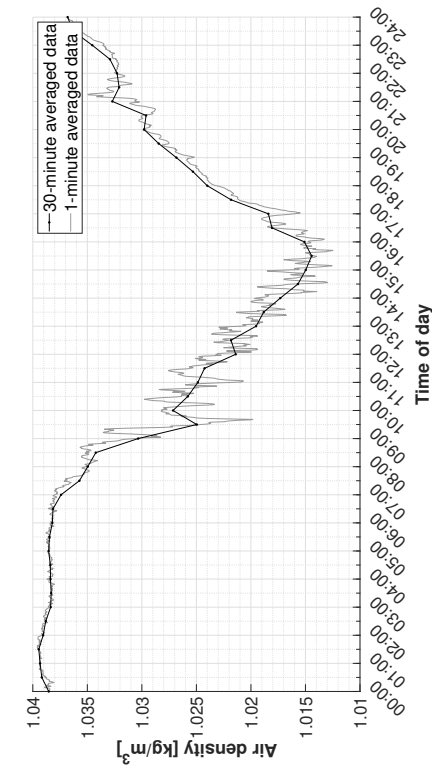


(b) Plot showing air density versus time for 17 January 2020.

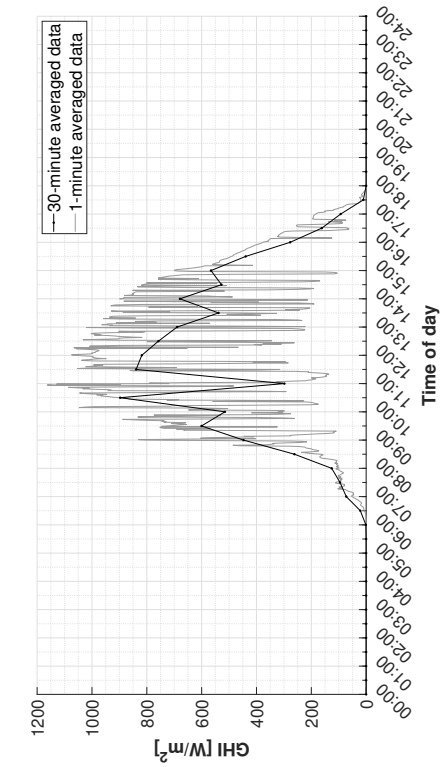
(c) Plot showing GHI versus time for 17 January 2020.

(d) Plot showing wind speed versus time for 17 January 2020.

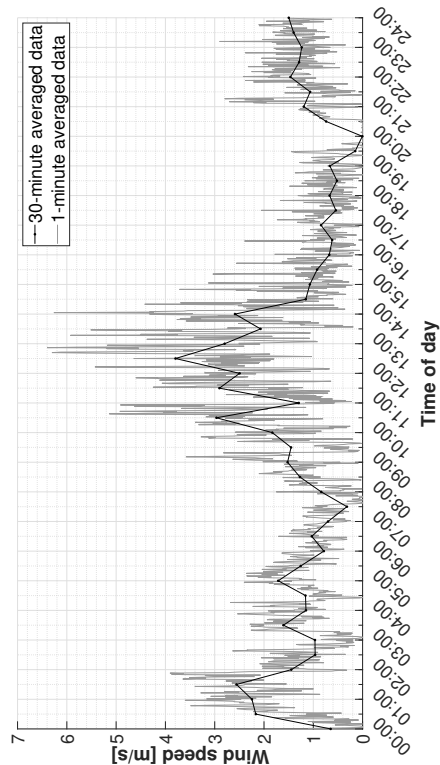
Figure 4.1. Plots showing atmospheric conditions on the 17th of January 2020 (summer).



(a) Plot showing temperature versus time for 15 April 2020.



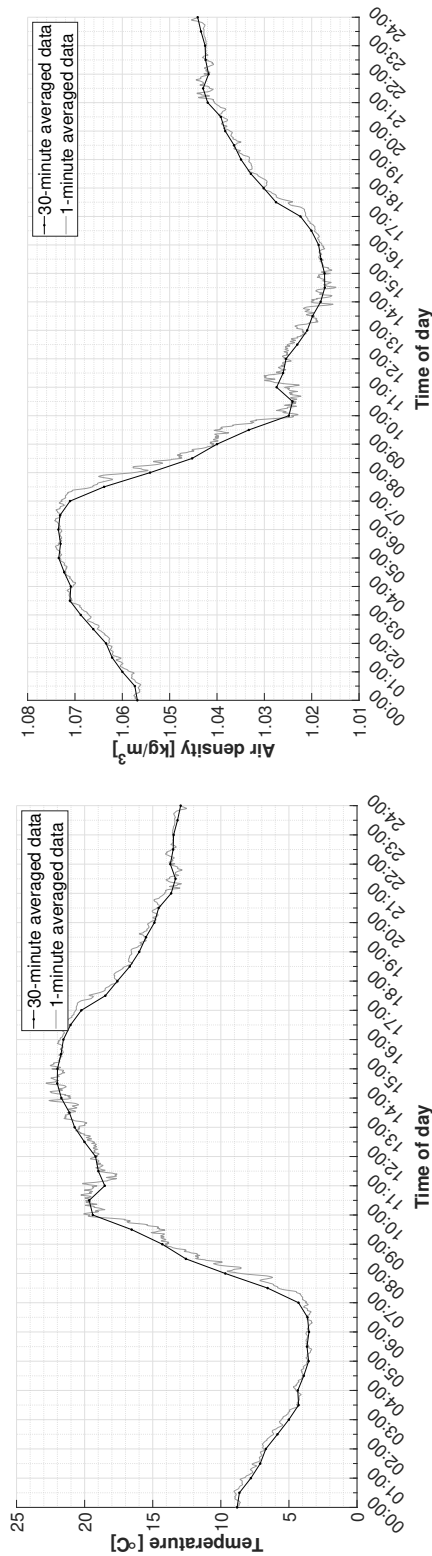
(c) Plot showing GHI versus time for 15 April 2020.



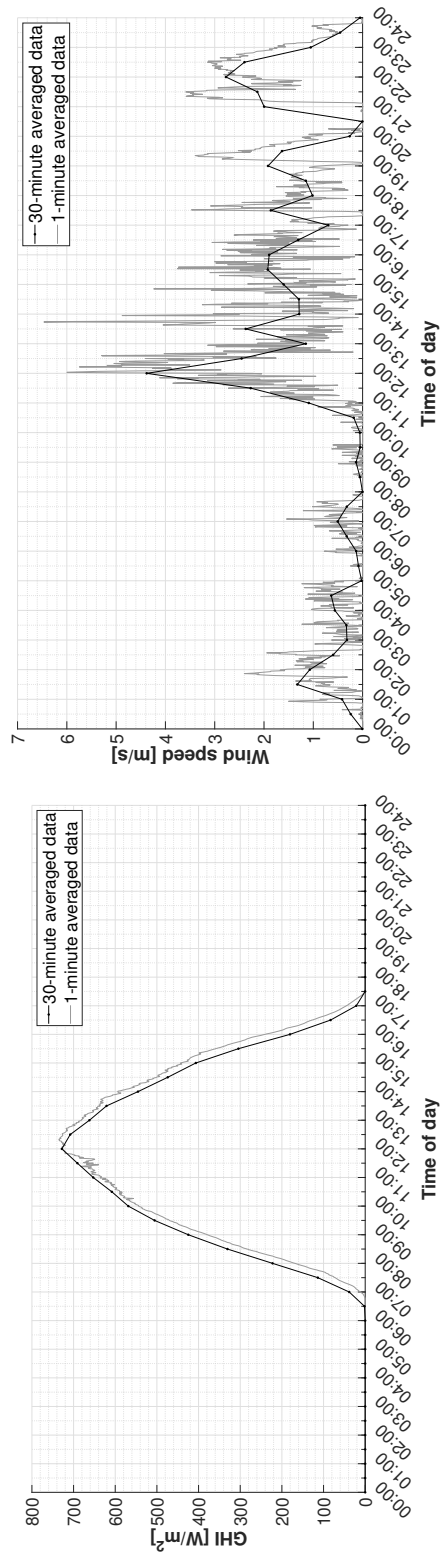
(b) Plot showing air density versus time for 15 April 2020.

(d) Plot showing wind speed versus time for 15 April 2020.

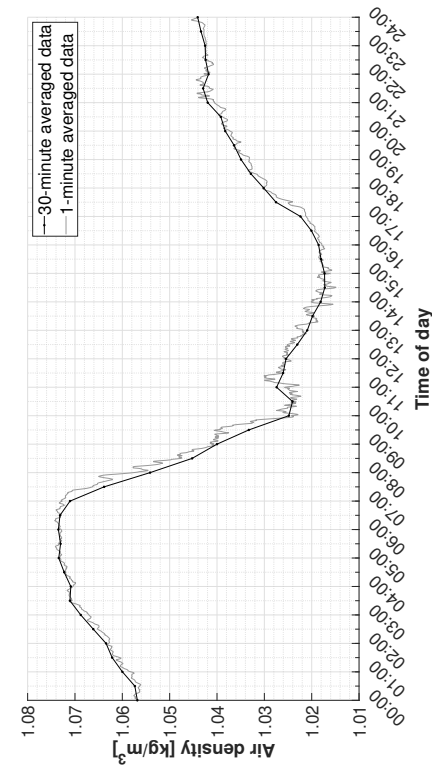
Figure 4.2. Plots showing atmospheric conditions on the 15th of April 2020 (autumn).



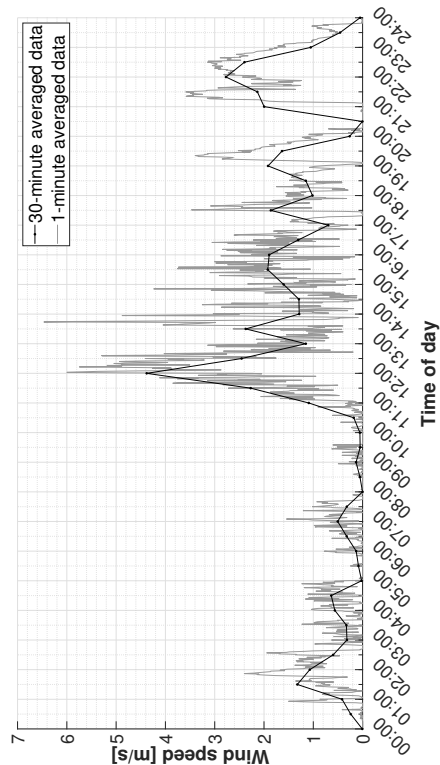
(a) Plot showing temperature versus time for 17 July 2020.



(c) Plot showing GHI versus time for 17 July 2020.

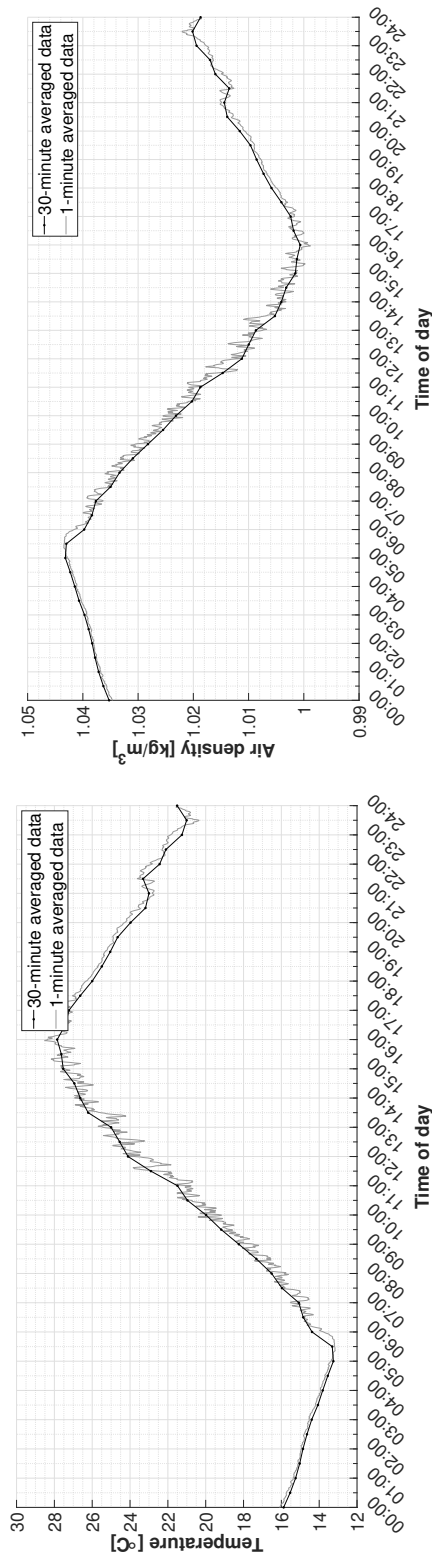


(b) Plot showing air density versus time for 17 July 2020.



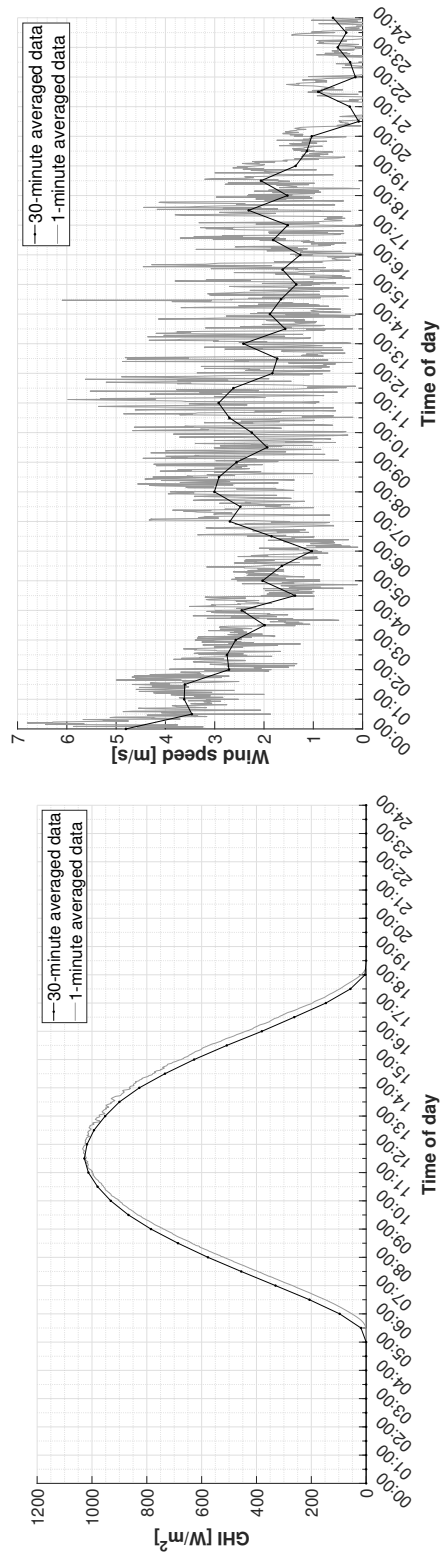
(d) Plot showing wind speed versus time for 17 July 2020.

Figure 4.3. Plots showing atmospheric conditions on the 17th of July 2020 (winter).

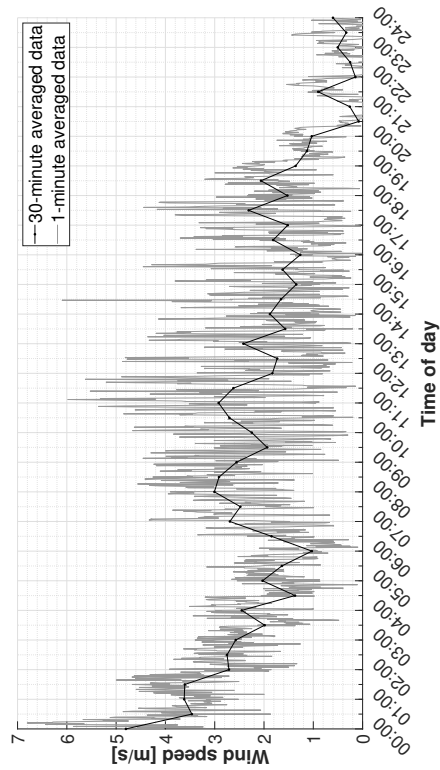


(a) Plot showing temperature versus time for 5 October 2020.

(b) Plot showing air density versus time for 5 October 2020.



(c) Plot showing GHI versus time for 5 October 2020.



(d) Plot showing wind speed versus time for 5 October 2020.

Figure 4.4. Plots showing atmospheric conditions on the 5th of October 2020 (spring).

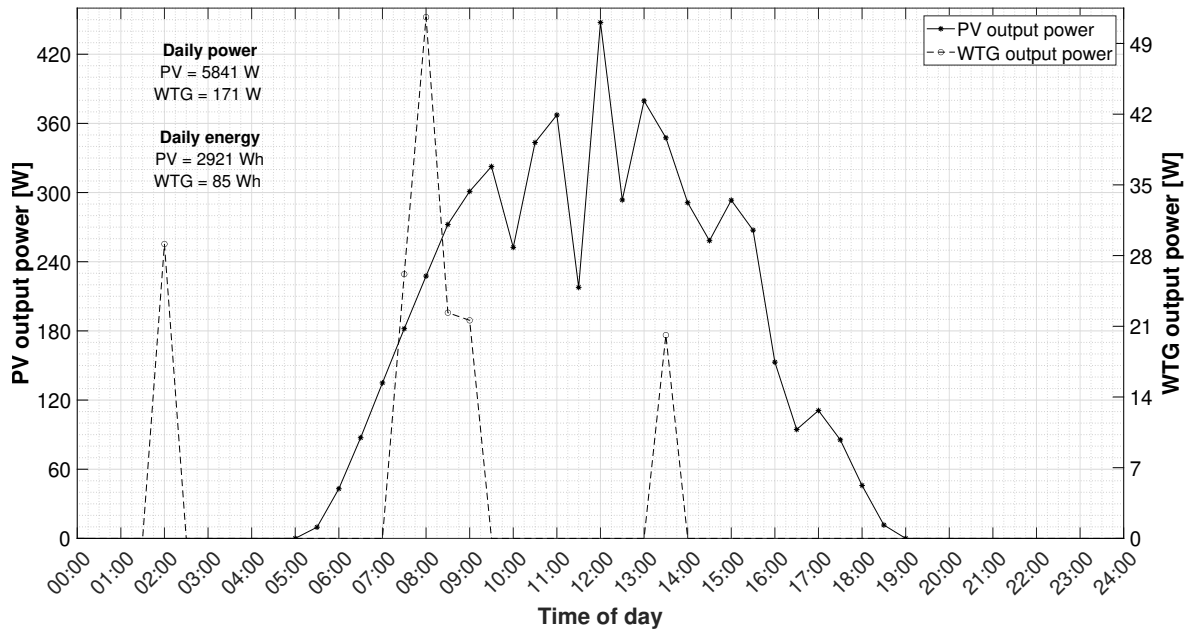


Figure 4.5. Plot showing power generated by a single solar PV and WTG on the 17th of January 2020 (summer).

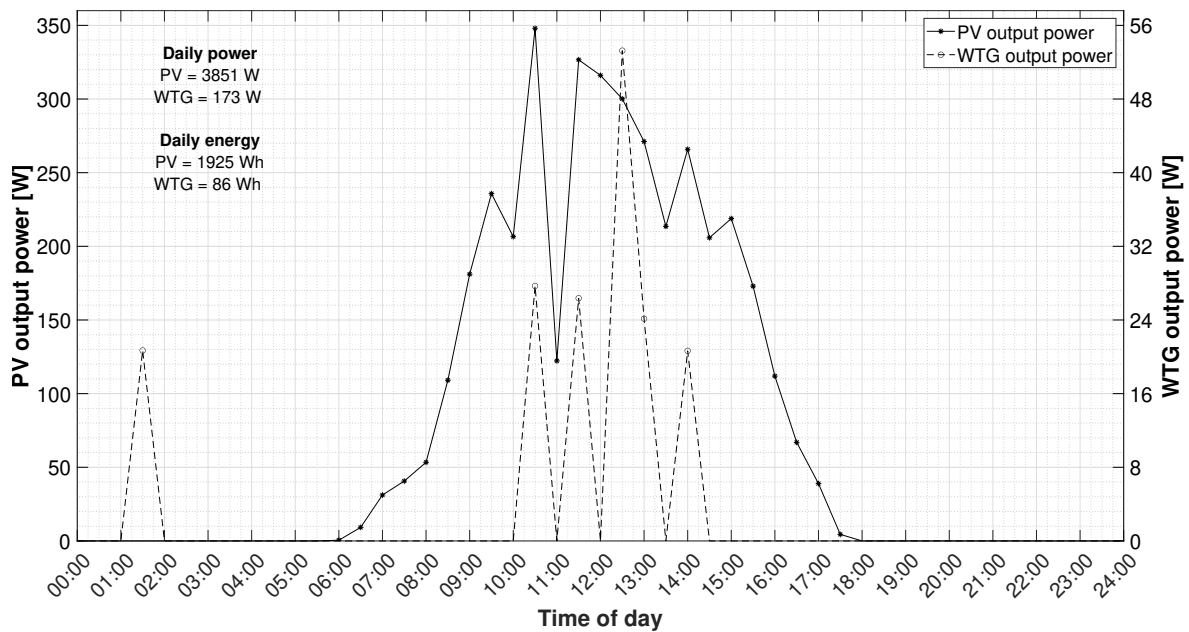


Figure 4.6. Plot showing power generated by a single solar PV and WTG on the 15th of April 2020 (autumn).

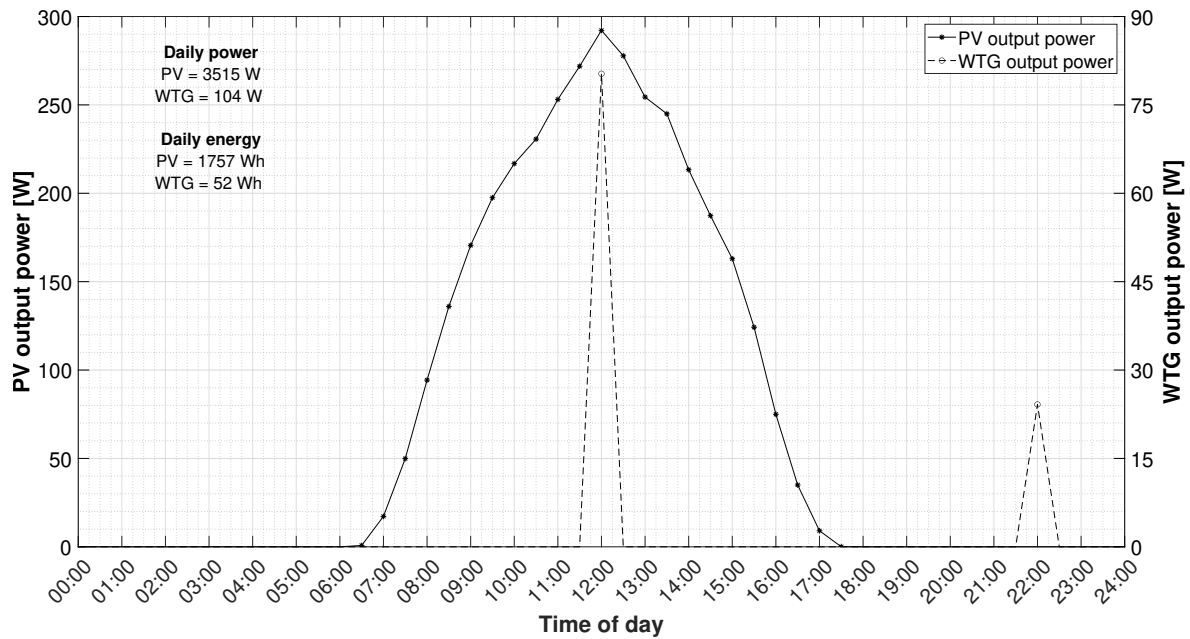


Figure 4.7. Plot showing power generated by a single solar PV and WTG on the 17th of July 2020 (winter).

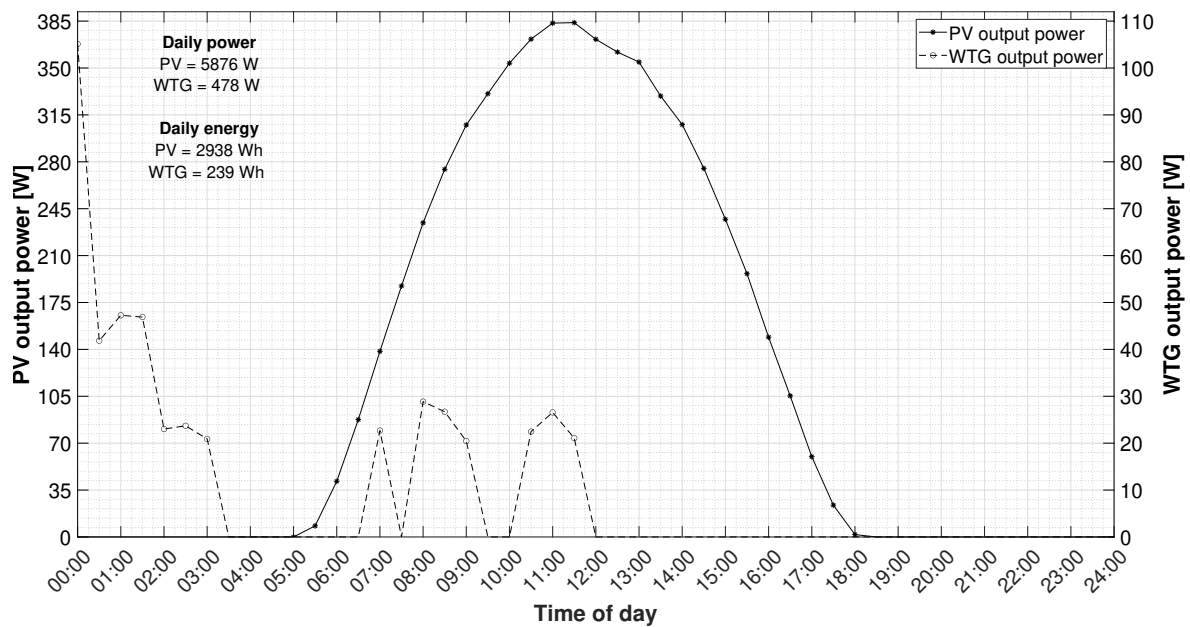


Figure 4.8. Plot showing power generated by a single solar PV and WTG on the 15th of October 2020 (spring).

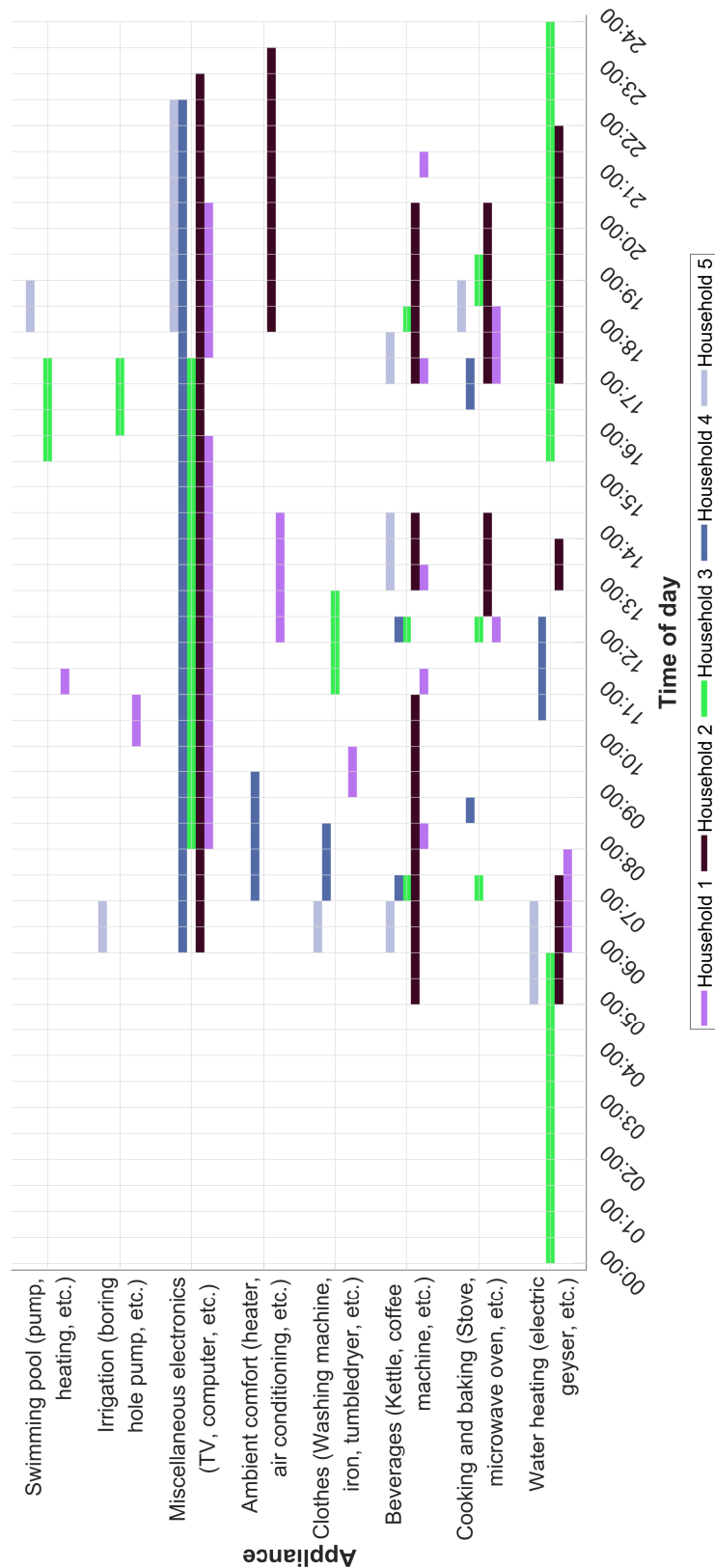


Figure 4.9. Plot showing different appliances' TOU of five randomly selected households.

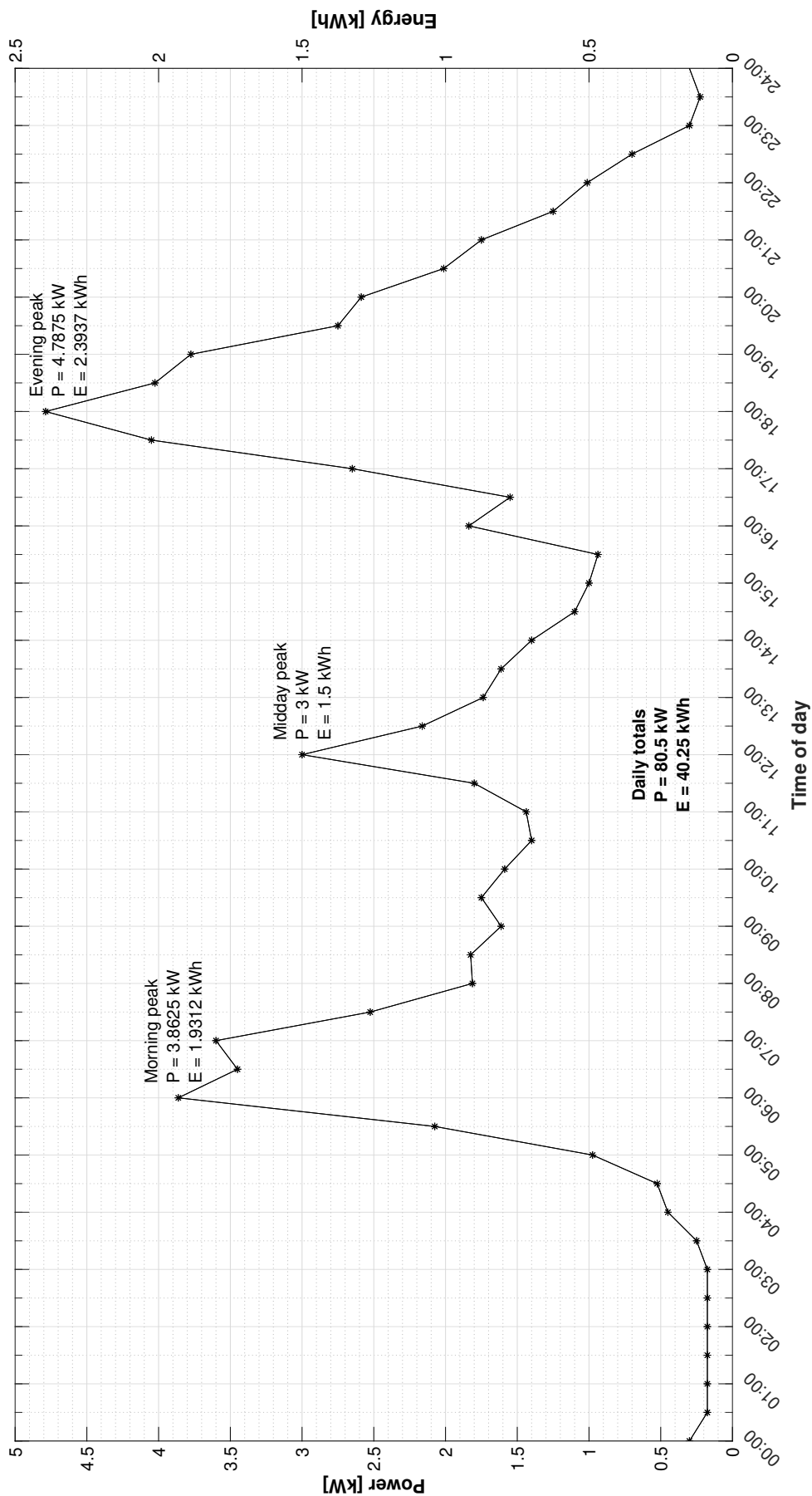


Figure 4.10. Plots showing the average daily load profile (energy and power) for all 40 households.

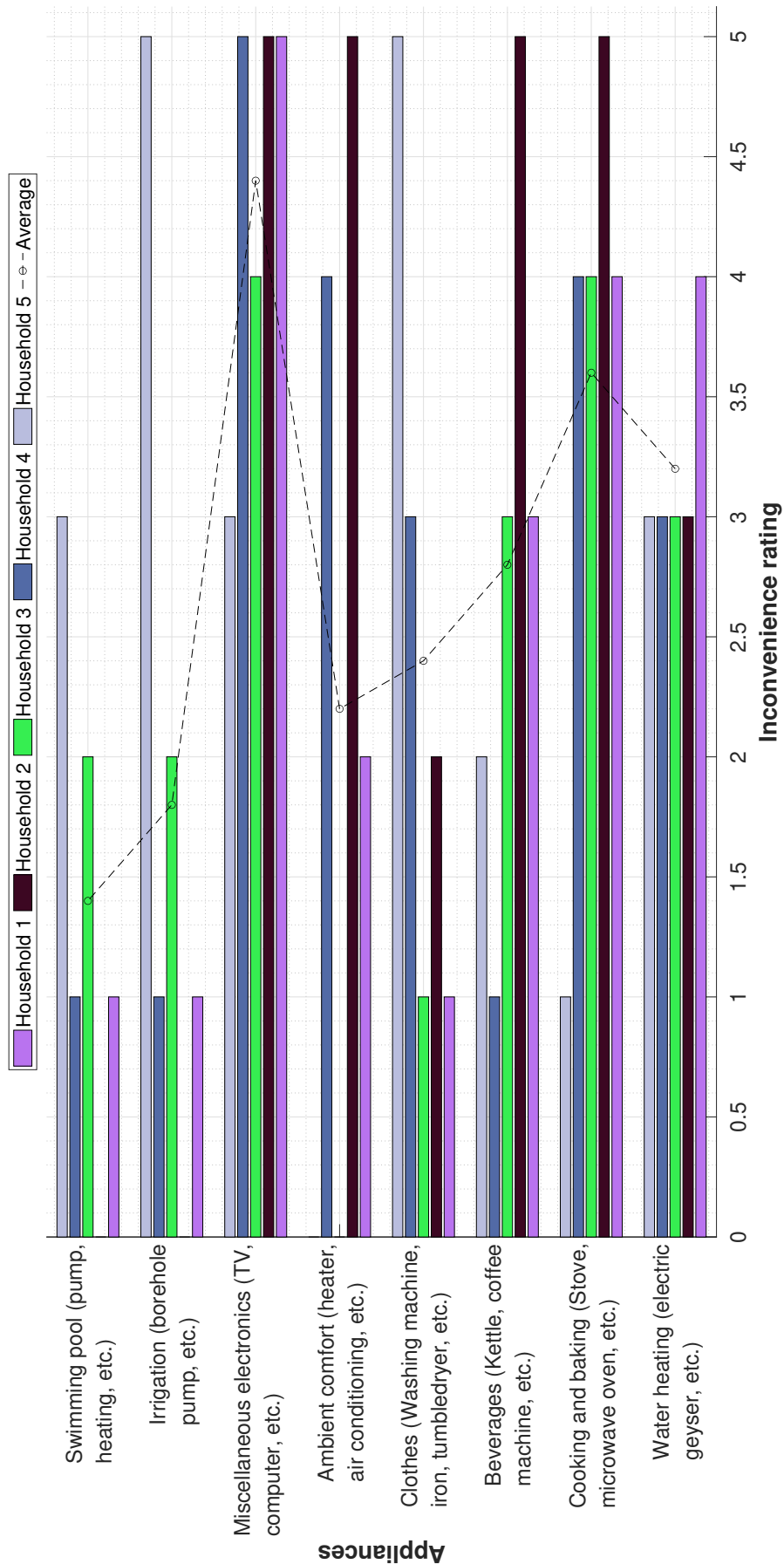


Figure 4.11. Graph showing the inconvenience per household (and an average) allocated to each appliance group.

4.3 OPTIMISATION RESULTS

This section deals with the optimisation results, starting with analysing the initial conditions in Subsection 4.3.1. After that, results for the fixed and variable-interval optimisation problems are shown in Subsections 4.3.2 and 4.3.3.

4.3.1 Initial Conditions

Generation of the initial conditions is discussed in Section 3.5.1.4 of the previous chapter. Figures 4.12 to 4.15 respectively show the distribution of the randomized starting values, both unconstrained and constrained by lower and upper bounds.

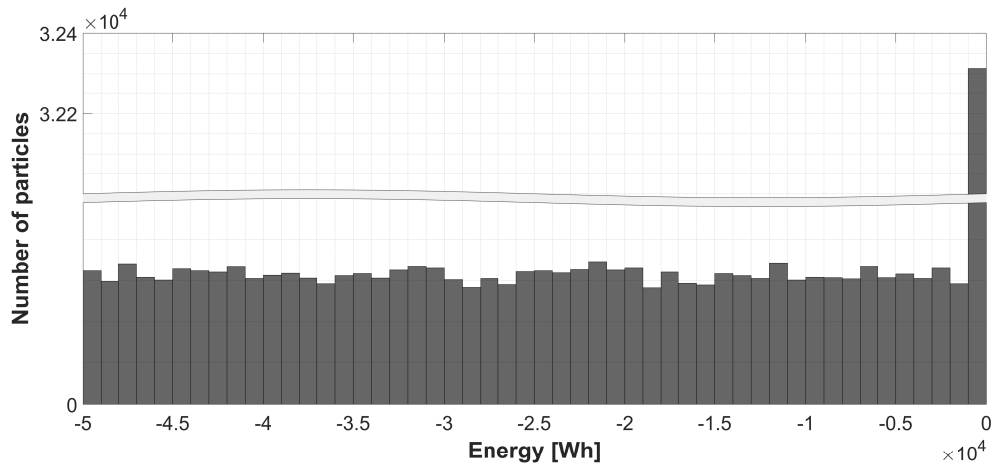
4.3.2 Fixed-Load optimisation Results

In this section, the results of the fixed-load optimisation problem is provided for each season and optimisation algorithm. Energy utilization plots for GA are shown in Figures 4.16 - 4.19, plots for PSO in Figures 4.20 - 4.23 and plots for SO in Figures 4.24 - 4.27. Additionally, the operating costs of each algorithm is plotted for each season respectively in Figures 4.28 - 4.31.

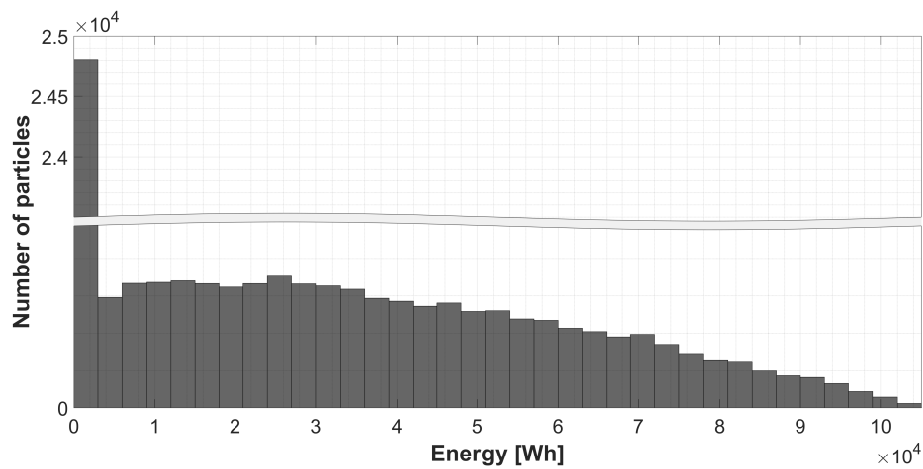
The computational time and operating cost differences between each EO algorithm are summarized in Table 4.1.

Table 4.1. Table showing optimization results for GA, PSO and SO.

Parameter		GA	PSO	SO
Computational time [s]	Jan	287.48	1.2967	349.22
	Apr	144.23	1.1362	259.49
	Jul	154.90	1.0880	240.94
	Oct	203.58	1.2417	370.36
	Avg	197.55	1.1906	305.00
Operating cost [ZAR]	Jan	582.52	375.94	366.34
	Apr	644.44	398.92	237.67
	Jul	661.01	403.18	283.30
	Oct	607.05	405.52	402.68
	Avg	623.76	395.89	322.57

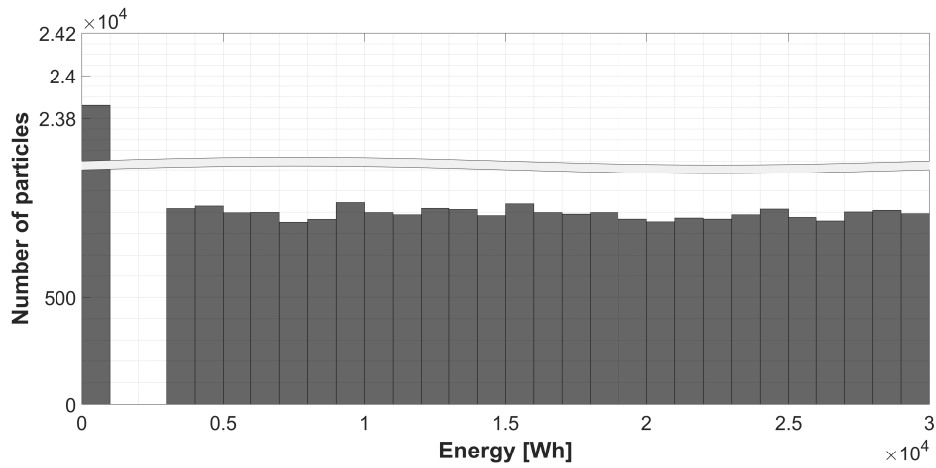


(a) Distribution of UG values.

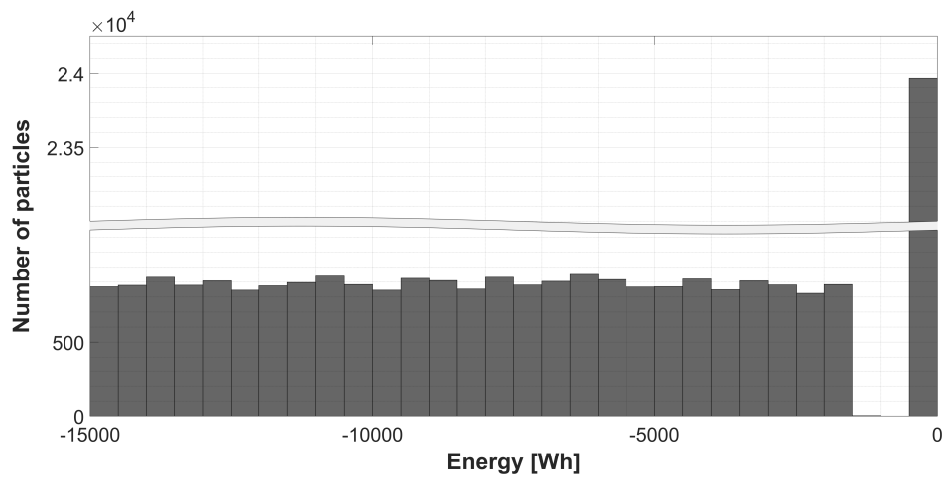


(b) Distribution of battery charging values.

Figure 4.12. Histograms showing the distribution of initial conditions for UG and battery variables, limited only by LB and UB.

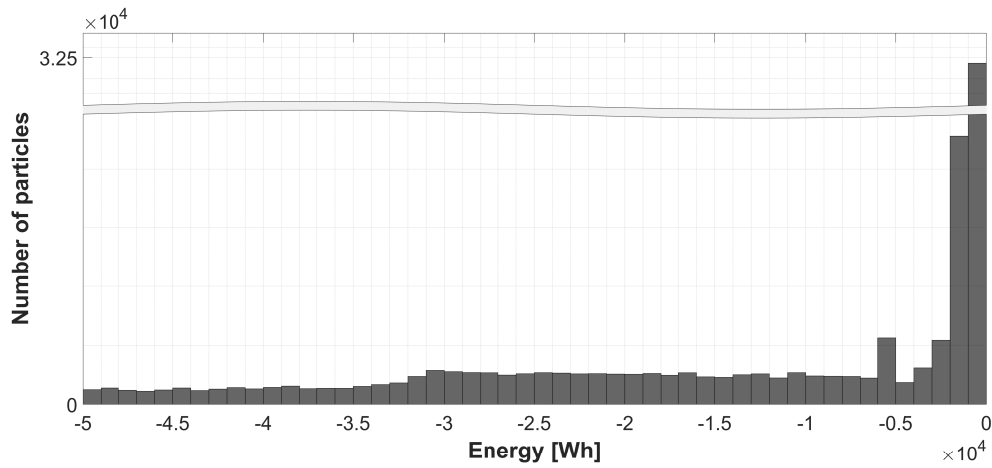


(a) Distribution of ER values.

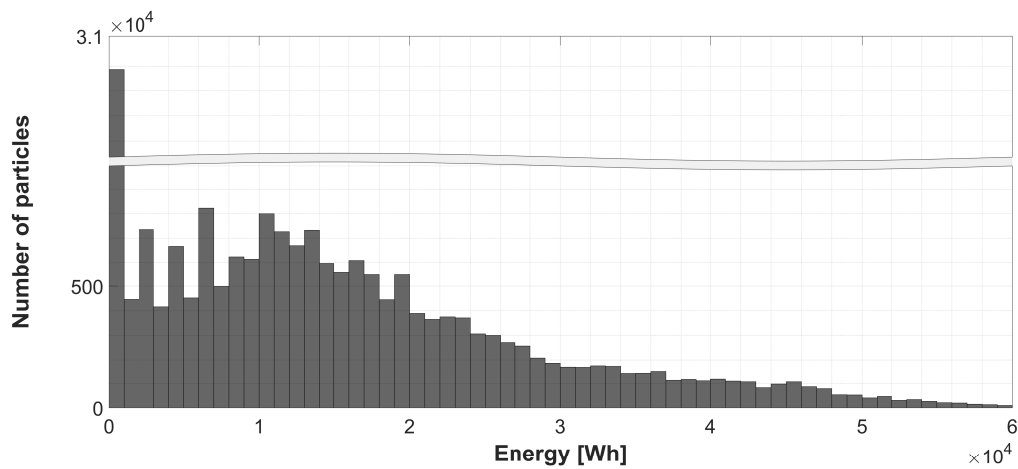


(b) Distribution of FC values.

Figure 4.13. Histograms showing the distribution of initial conditions for ER and FC variables, limited only by LB and UB.

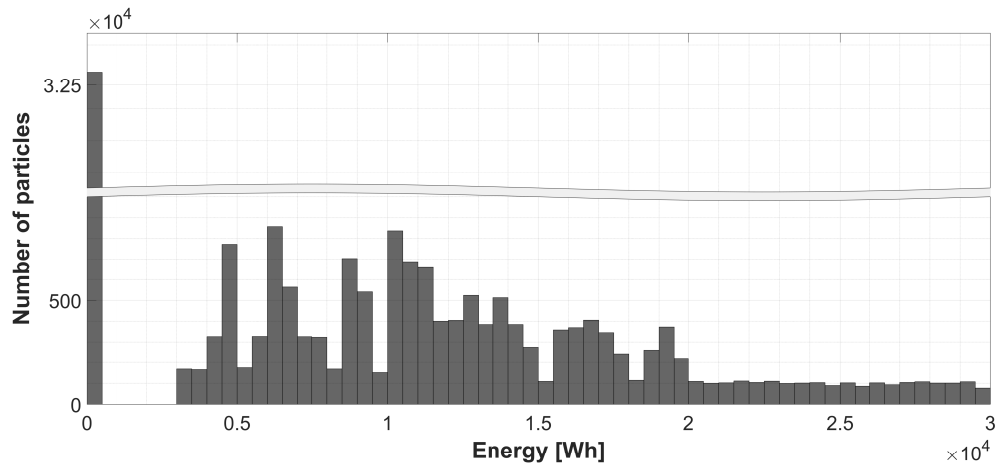


(a) Distribution of UG values.

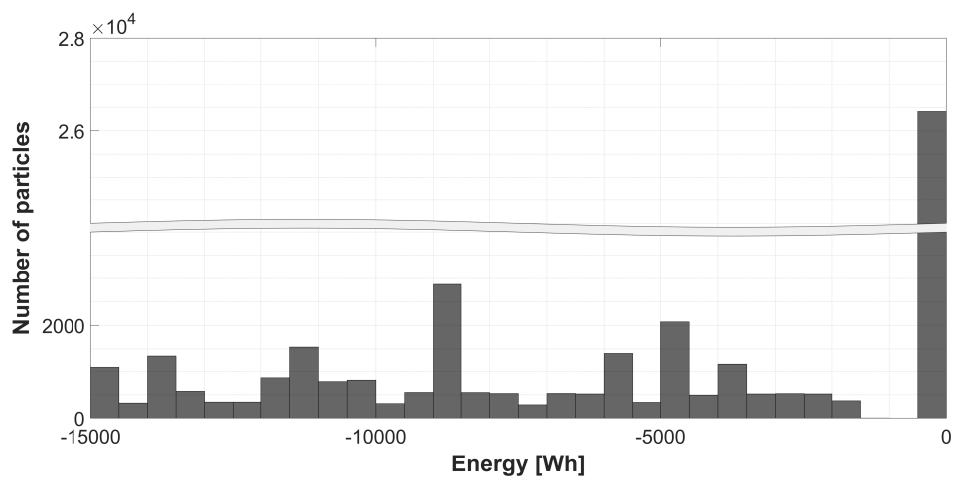


(b) Distribution of battery charging values.

Figure 4.14. Histograms showing the distribution of initial conditions for UG and battery variables, limited by LB, UB and constraints.



(a) Distribution of ER values.



(b) Distribution of FC values.

Figure 4.15. Histograms showing the distribution of initial conditions for ER and FC variables, limited by LB, UB and constraints.

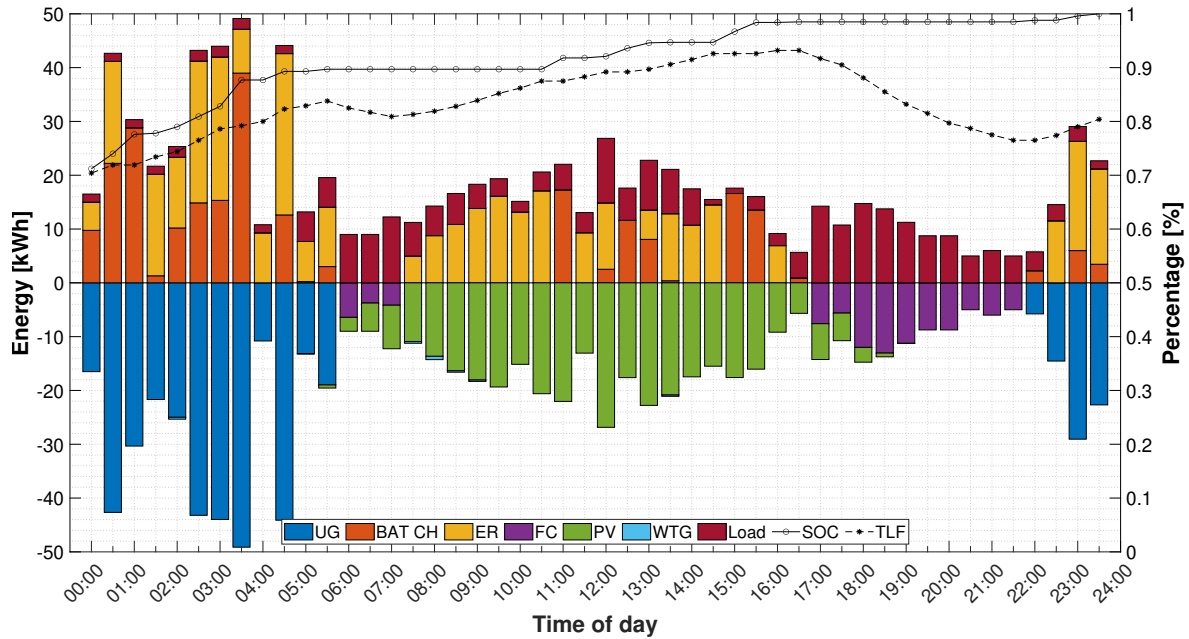


Figure 4.16. Plot showing the energy utilization for January using GA.

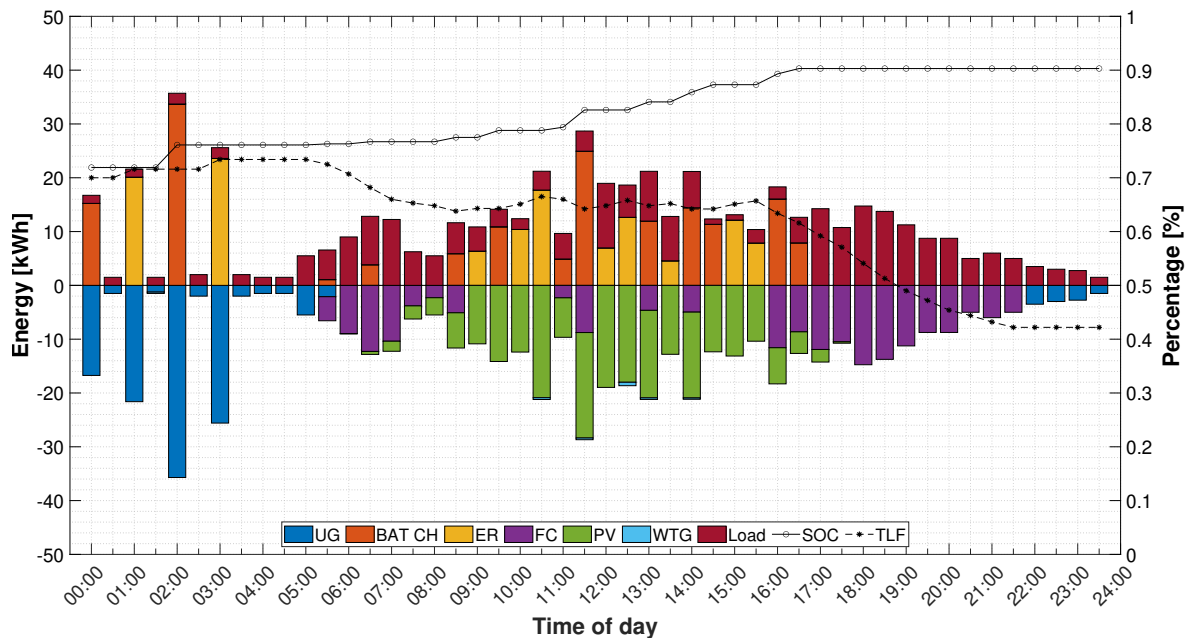


Figure 4.17. Plot showing the energy utilization for April using GA.

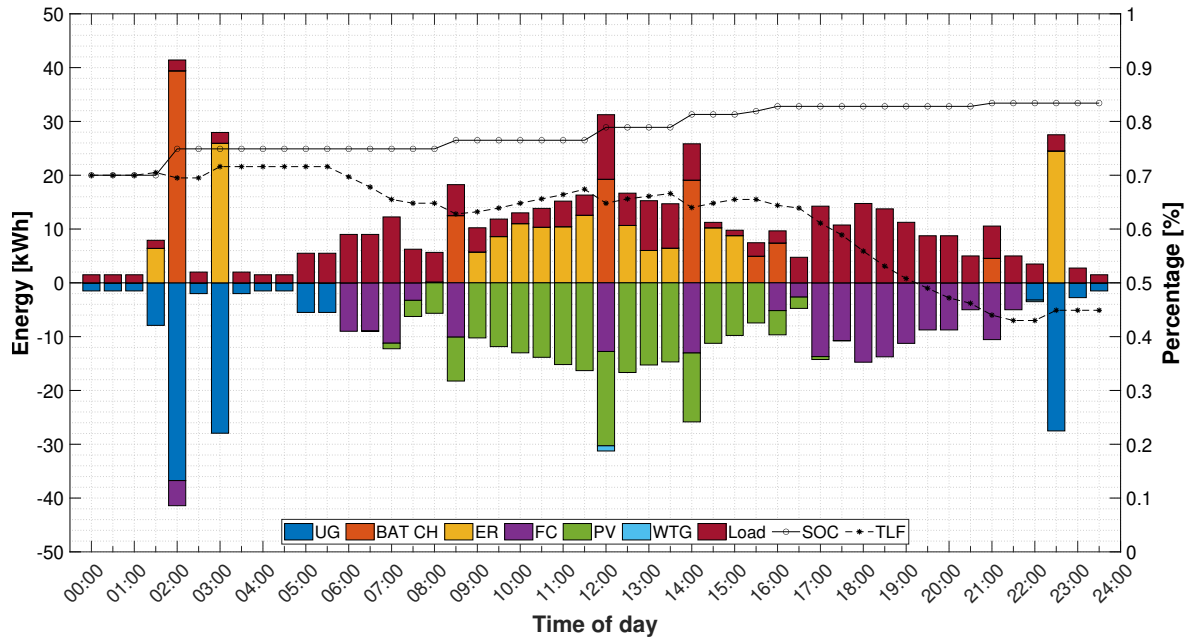


Figure 4.18. Plot showing the energy utilization for July using GA.

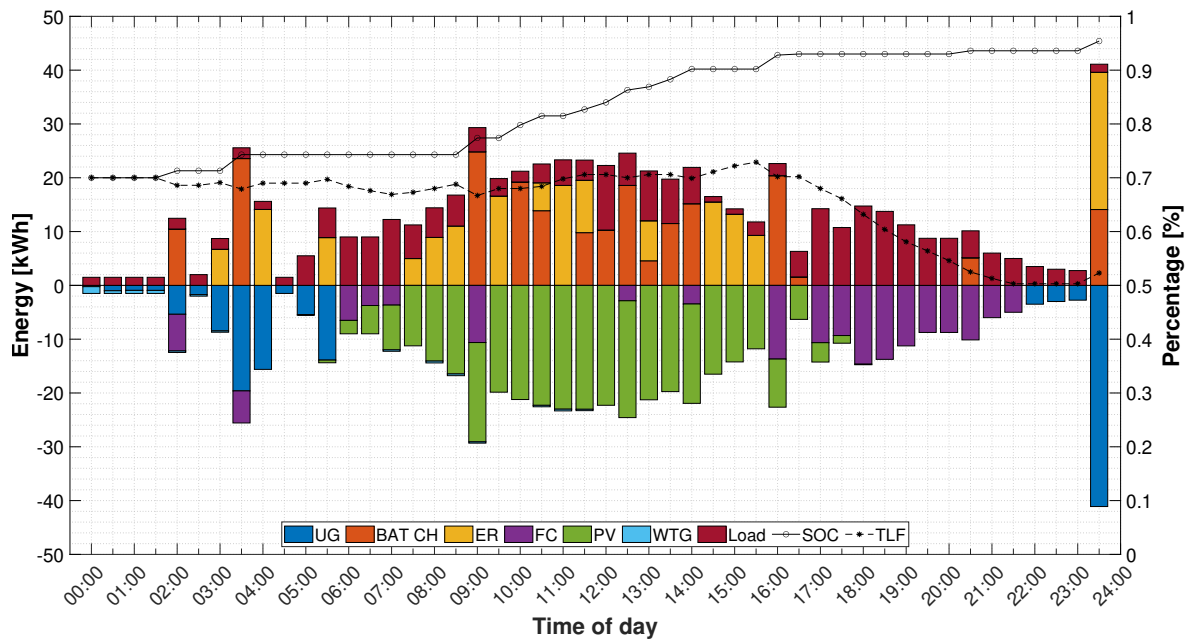


Figure 4.19. Plot showing the energy utilization for October using GA.

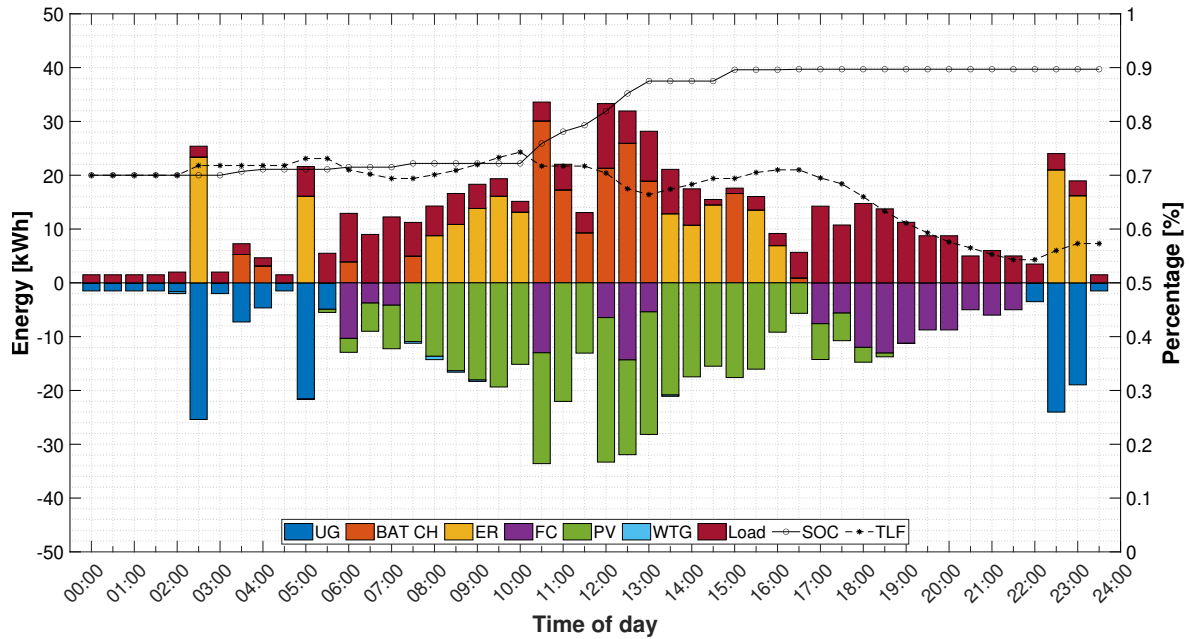


Figure 4.20. Plot showing the energy utilization for January using PSO.

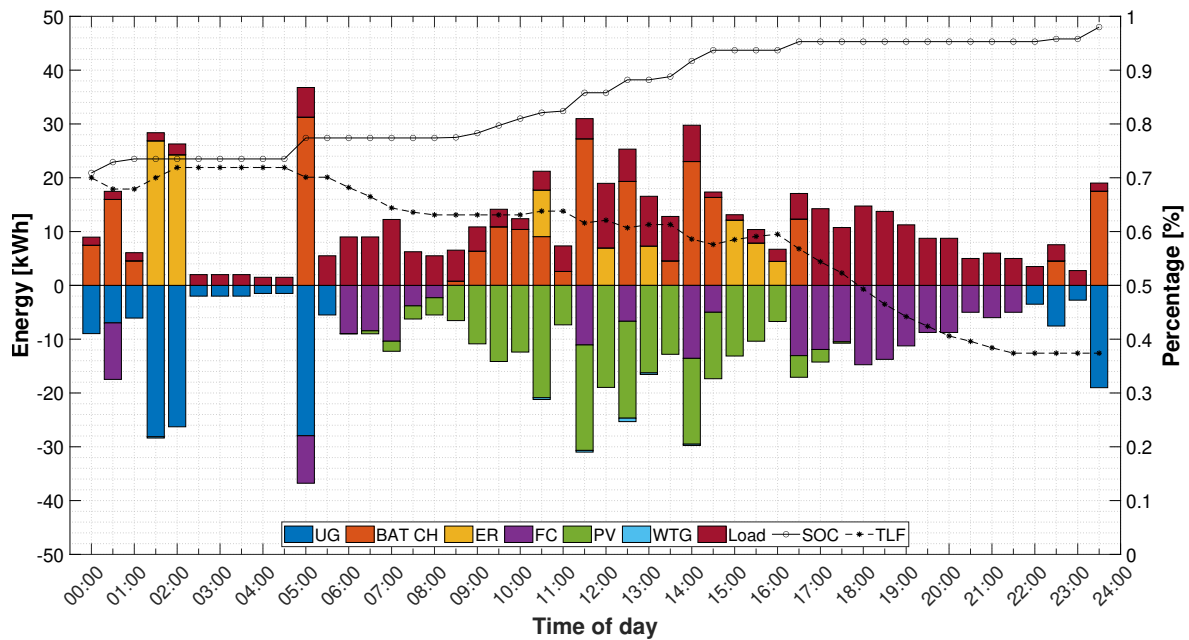


Figure 4.21. Plot showing the energy utilization for April using PSO.

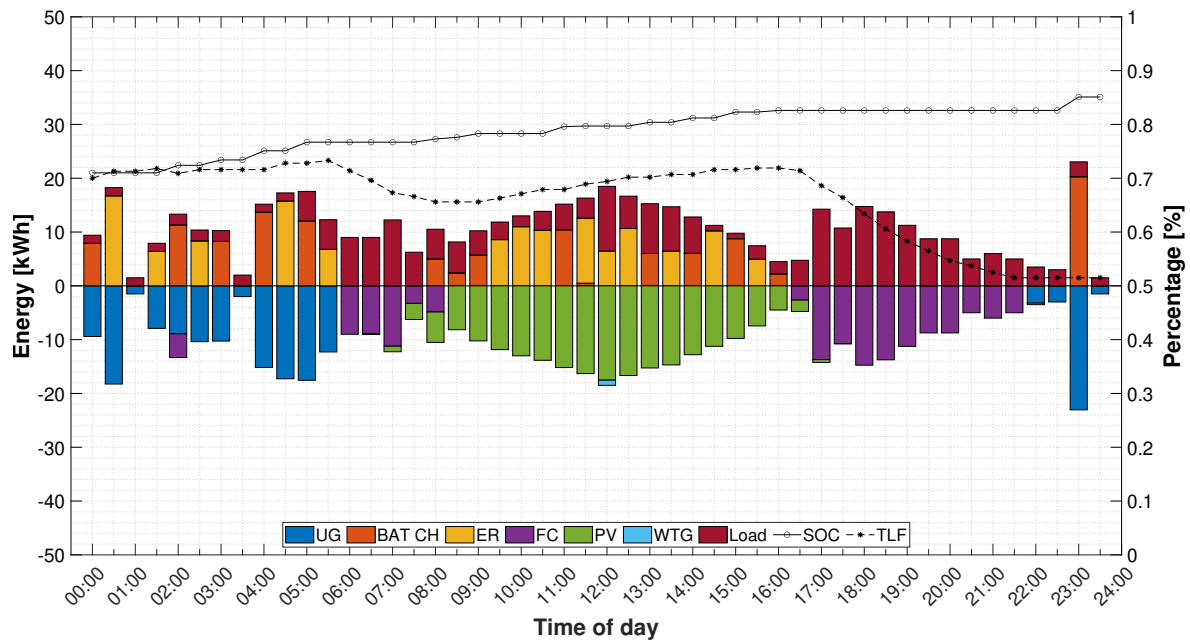


Figure 4.22. Plot showing the energy utilization for July using PSO.

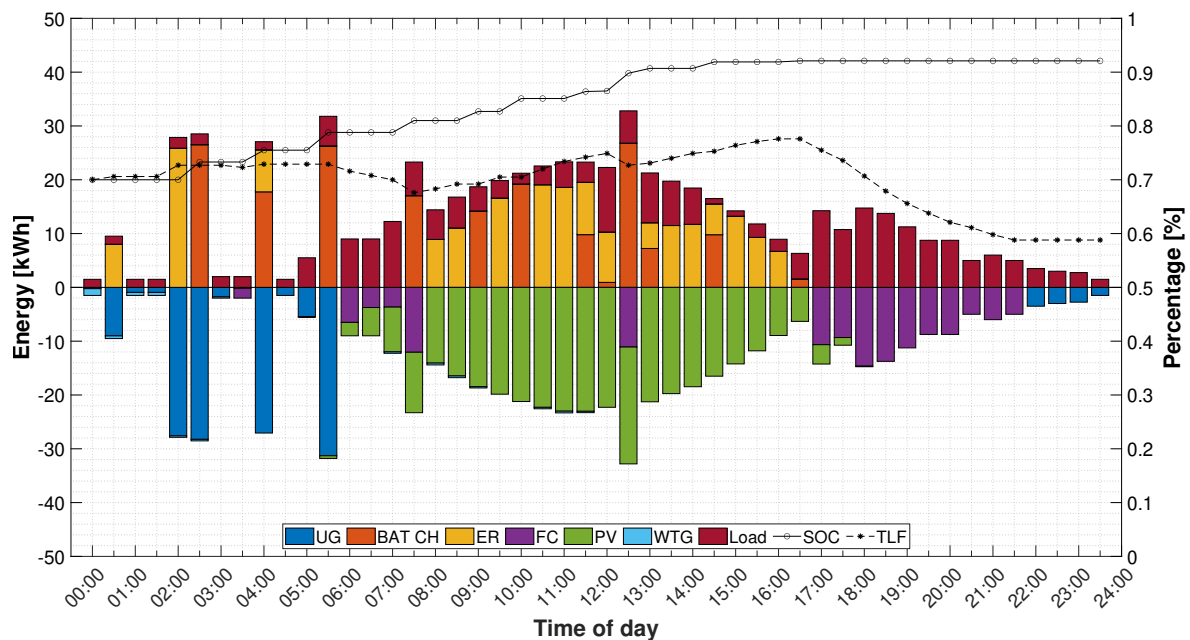


Figure 4.23. Plot showing the energy utilization for October using PSO.

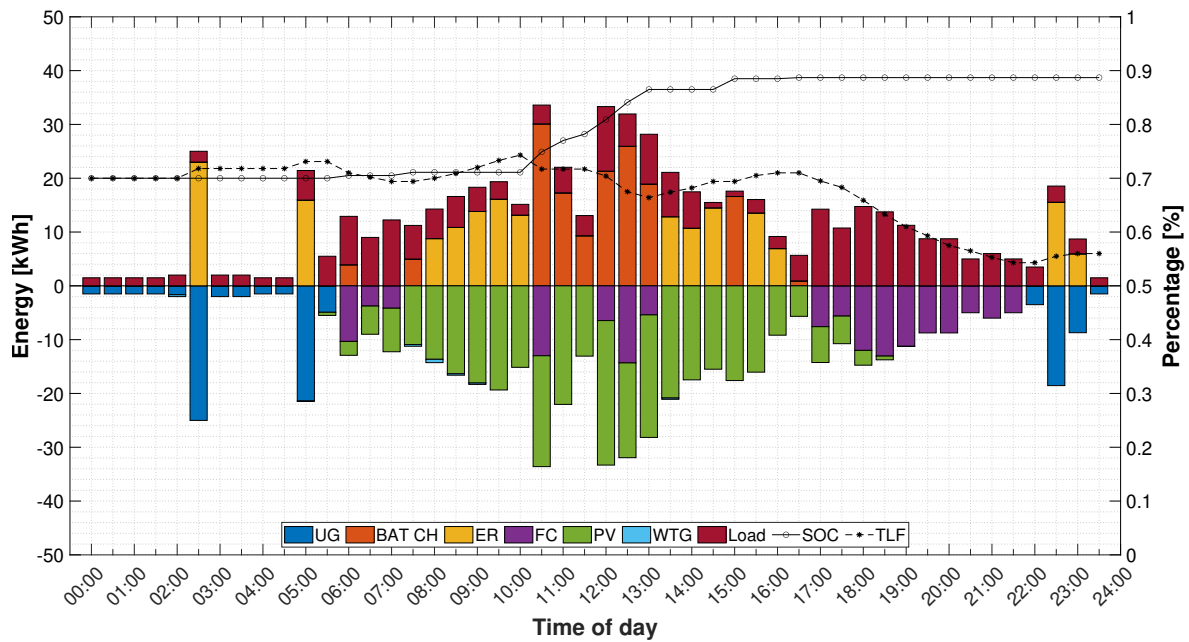


Figure 4.24. Plot showing the energy utilization for January using SO.

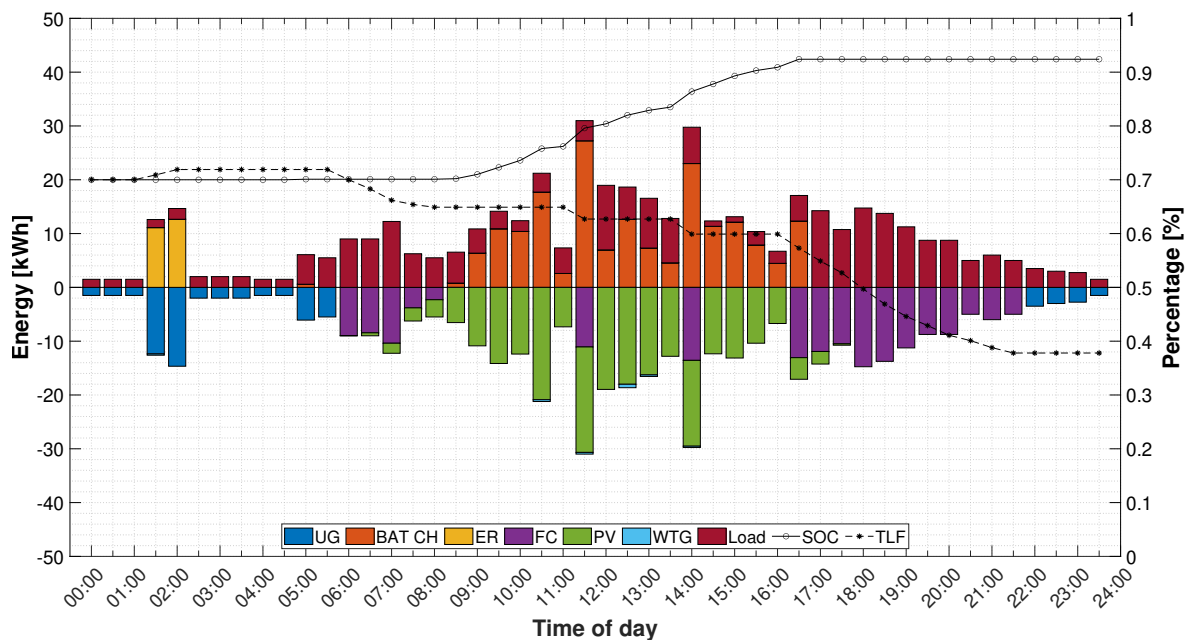


Figure 4.25. Plot showing the energy utilization for April using SO.

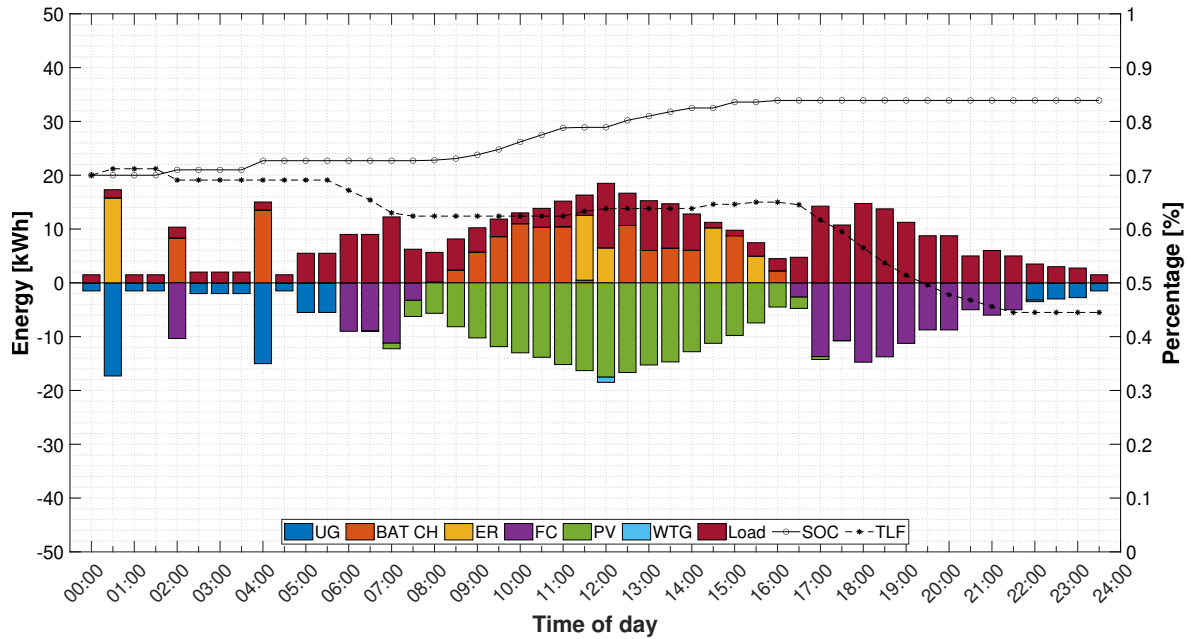


Figure 4.26. Plot showing the energy utilization for July using SO.

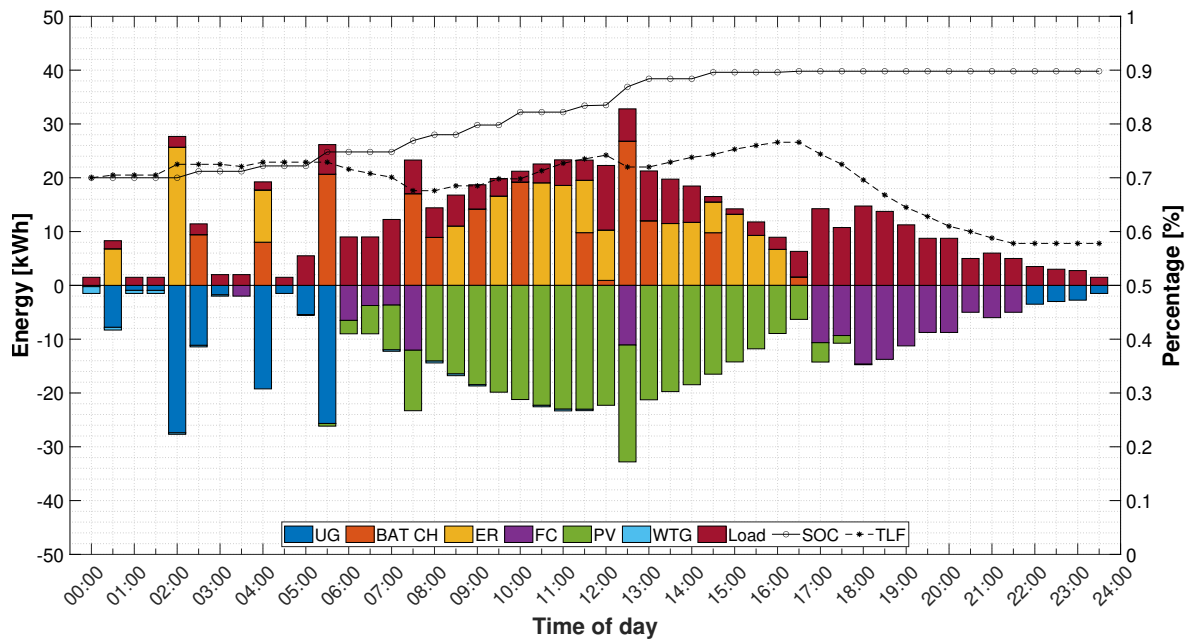
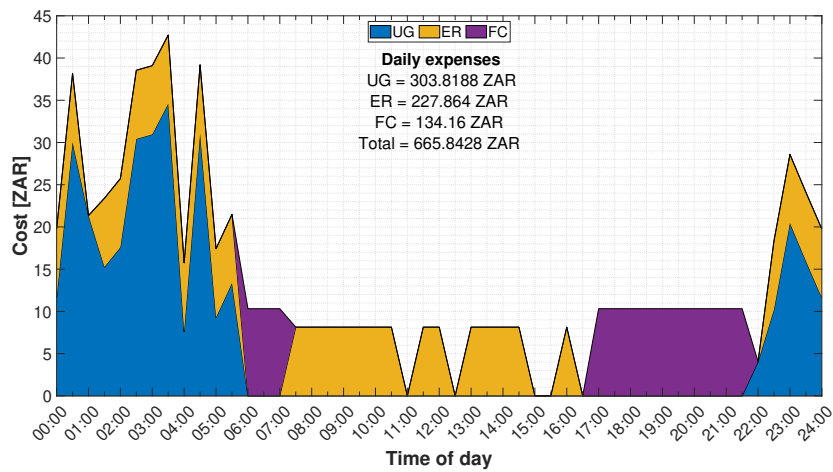
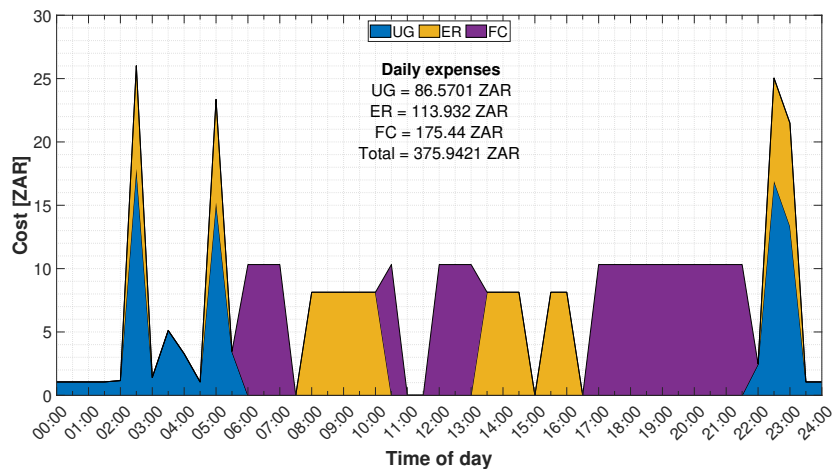


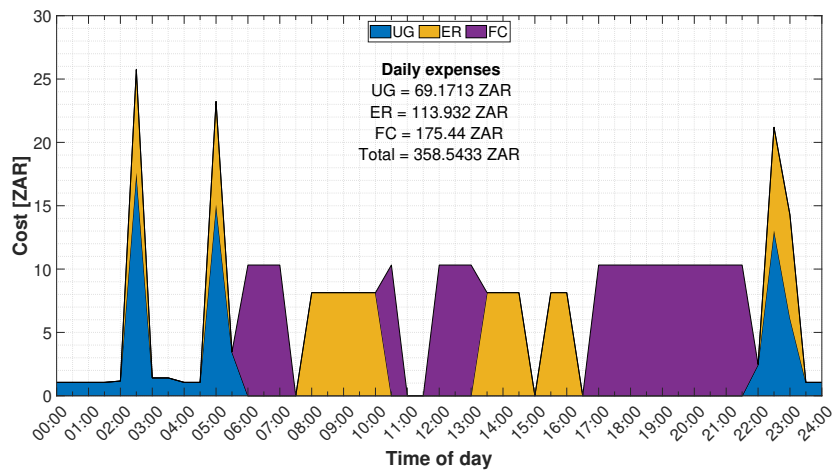
Figure 4.27. Plot showing the energy utilization for October using SO.



(a) Plot showing operating cost using GA.

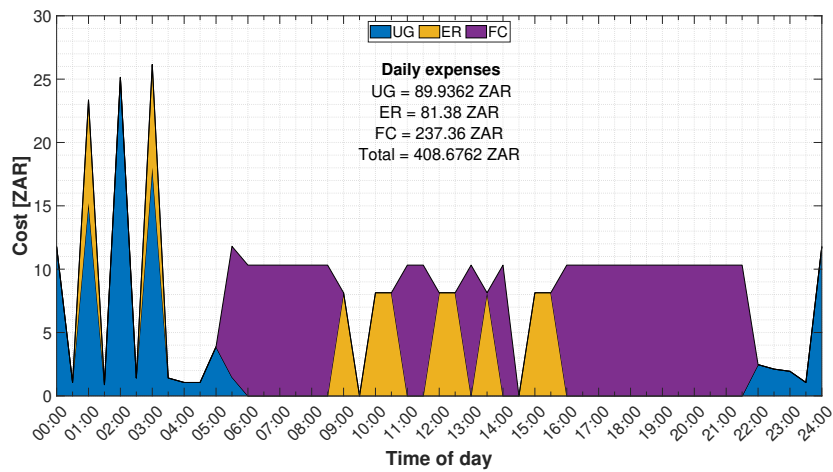


(b) Plot showing operating cost using PSO.

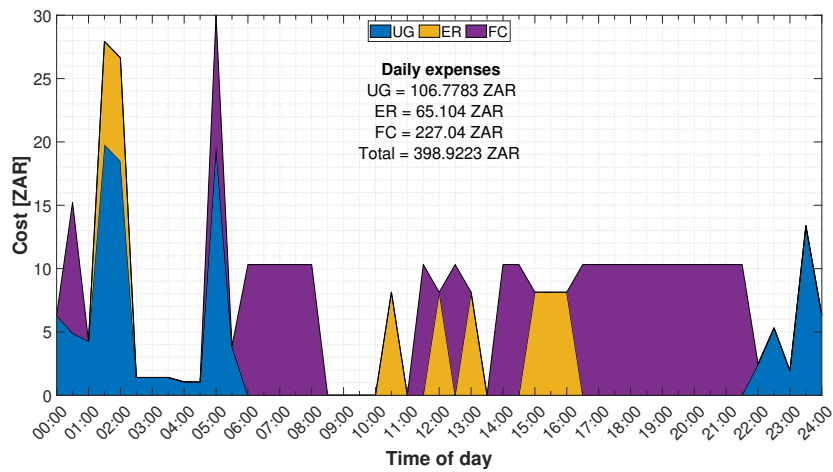


(c) Plot showing operating cost using SO.

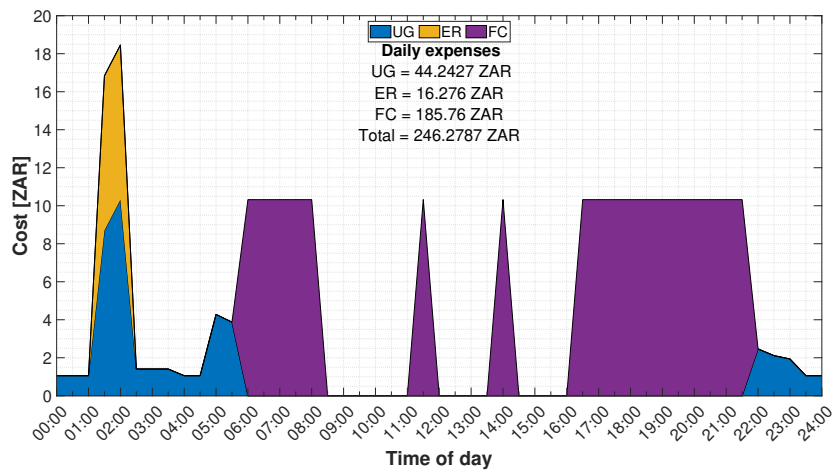
Figure 4.28. Plots showing the average daily MG operating cost for January using different algorithms.



(a) Plot showing operating cost using GA.

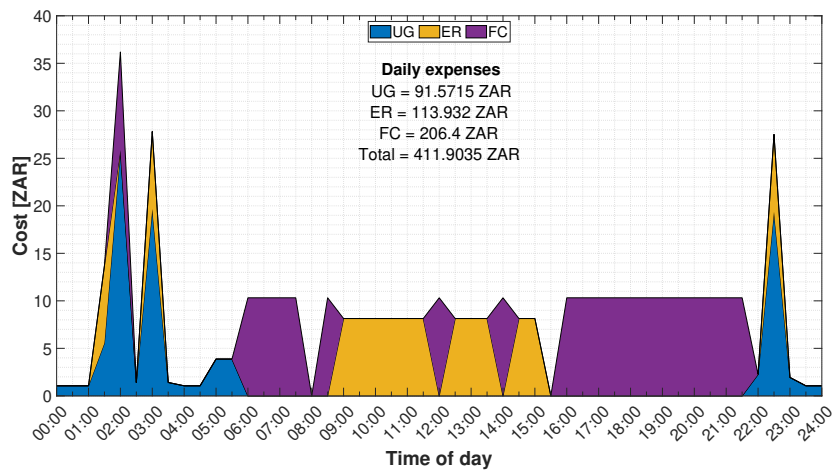


(b) Plot showing operating cost using PSO.

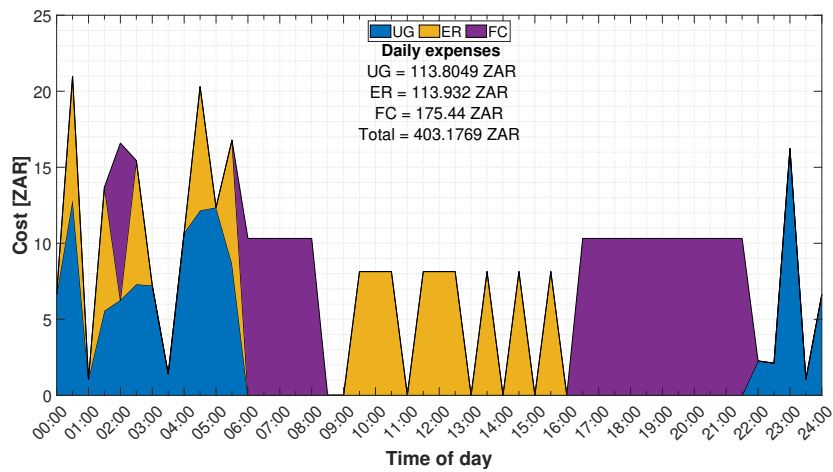


(c) Plot showing operating cost using SO.

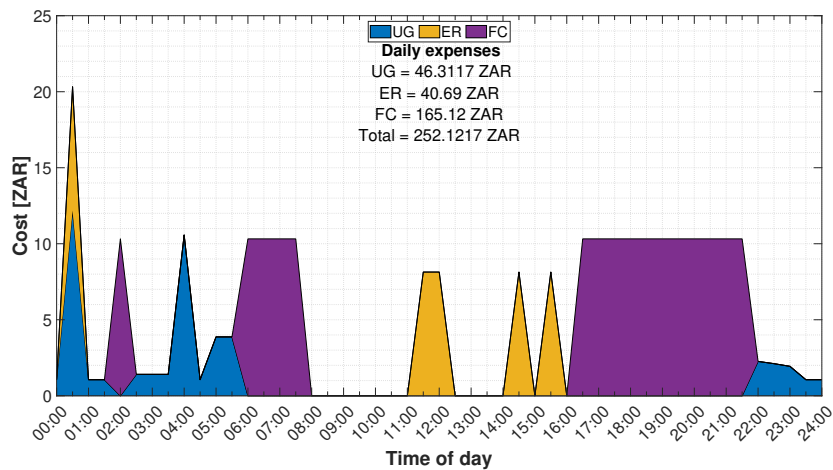
Figure 4.29. Plots showing the average daily MG operating cost for April using different algorithms.



(a) Plot showing operating cost using GA.

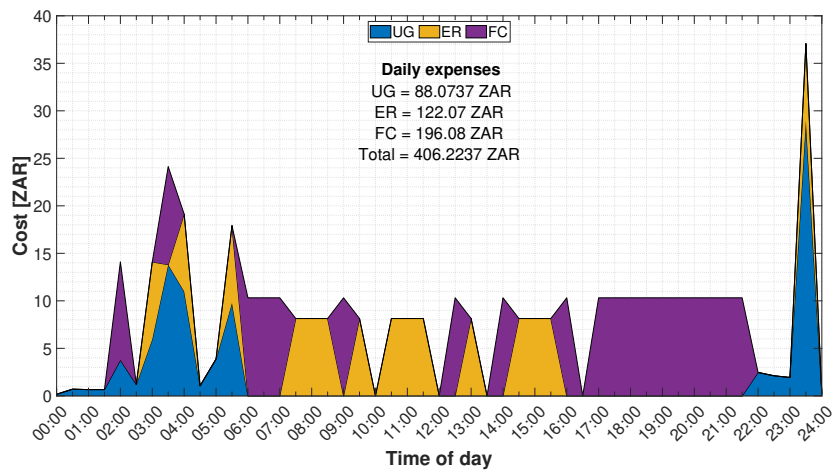


(b) Plot showing operating cost using PSO.

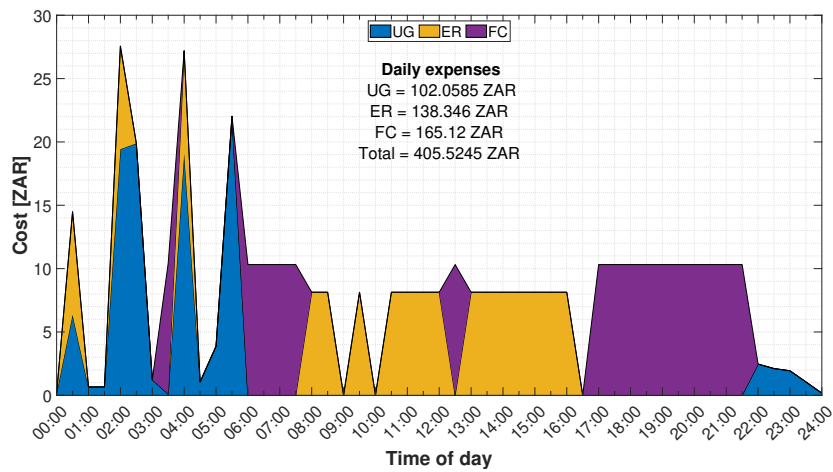


(c) Plot showing operating cost using SO.

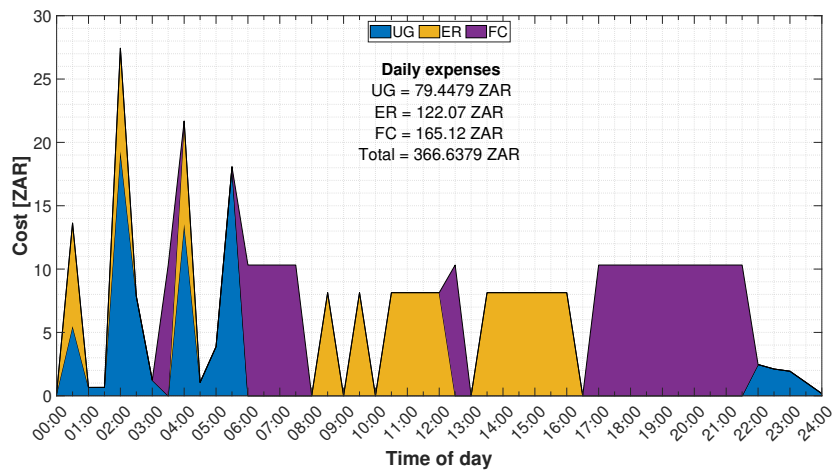
Figure 4.30. Plots showing the average daily MG operating cost for July using different algorithms.



(a) Plot showing operating cost using GA.



(b) Plot showing operating cost using PSO.



(c) Plot showing operating cost using SO.

Figure 4.31. Plots showing the average daily MG operating cost for October using different algorithms.

4.3.3 Variable-Load optimisation Results

Results for the variable-load (multi-objective) optimisation problem are summarized in this section. It is divided into two subsections, respectively containing load scheduling (Subsection 4.3.3.1) and source utilization results (Subsection 4.3.3.2).

4.3.3.1 Load scheduling

Results for DM of household loads are shown in Figures 4.32 - 4.35 for each season, respectively. These can be compared with Figure 4.9 which shows each household's appliance usage without DM implemented. The operation of communal loads – with and without DM – is shown in Figures 4.36 - 4.39 for each season.

4.3.3.2 Energy utilization and costs

Energy utilization plots, showing the different sources and loads' energy generation consumption during each interval, can be found in Figures 4.40 - 4.43. These plots also contain the variation of SOC and TLF of the ESS for each interval. Additionally, Figures 4.44 and 4.45 contain cost comparison plots for each respective season.

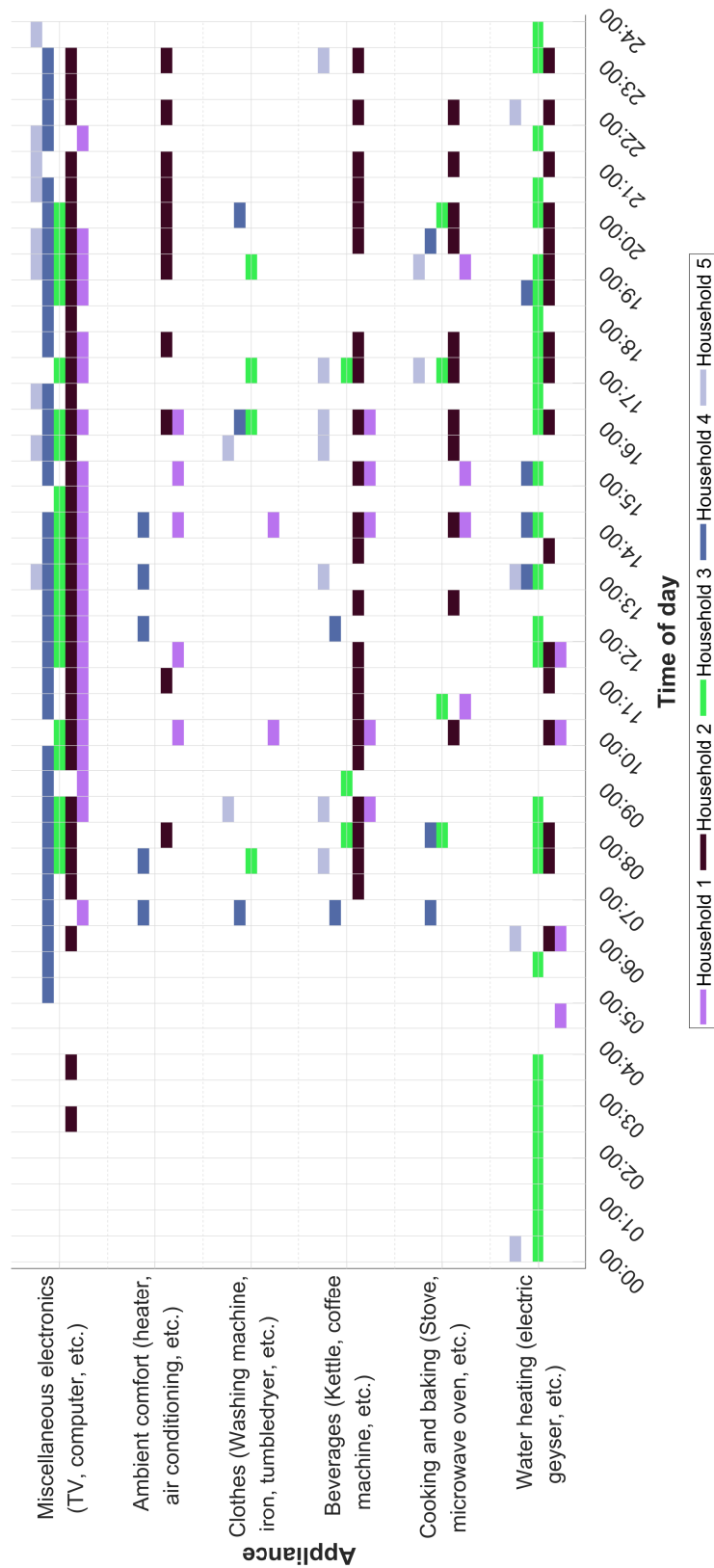


Figure 4.32. Plot showing daily appliance usage of the five households with DM implemented during summer.

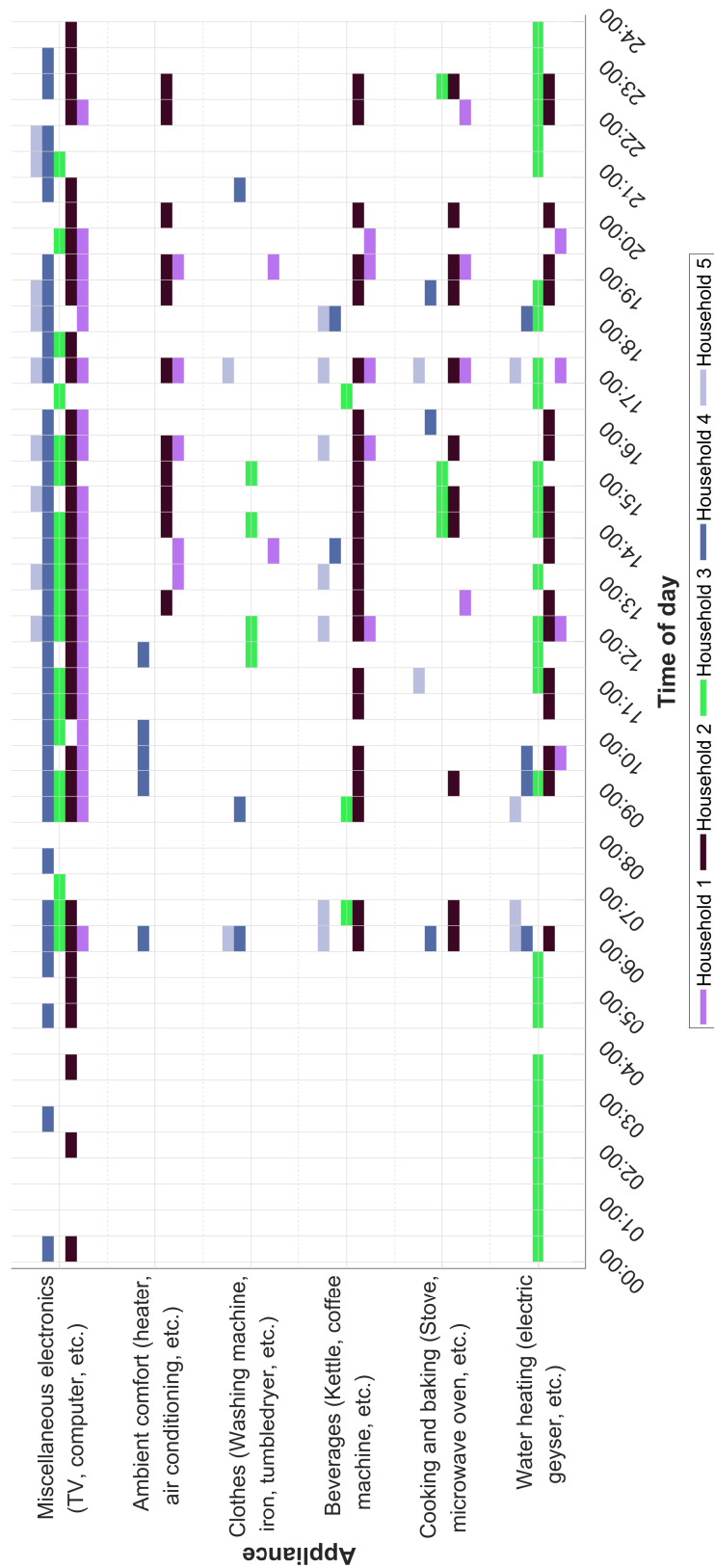


Figure 4.33. Plot showing daily appliance usage of the five households with DM implemented during autumn.

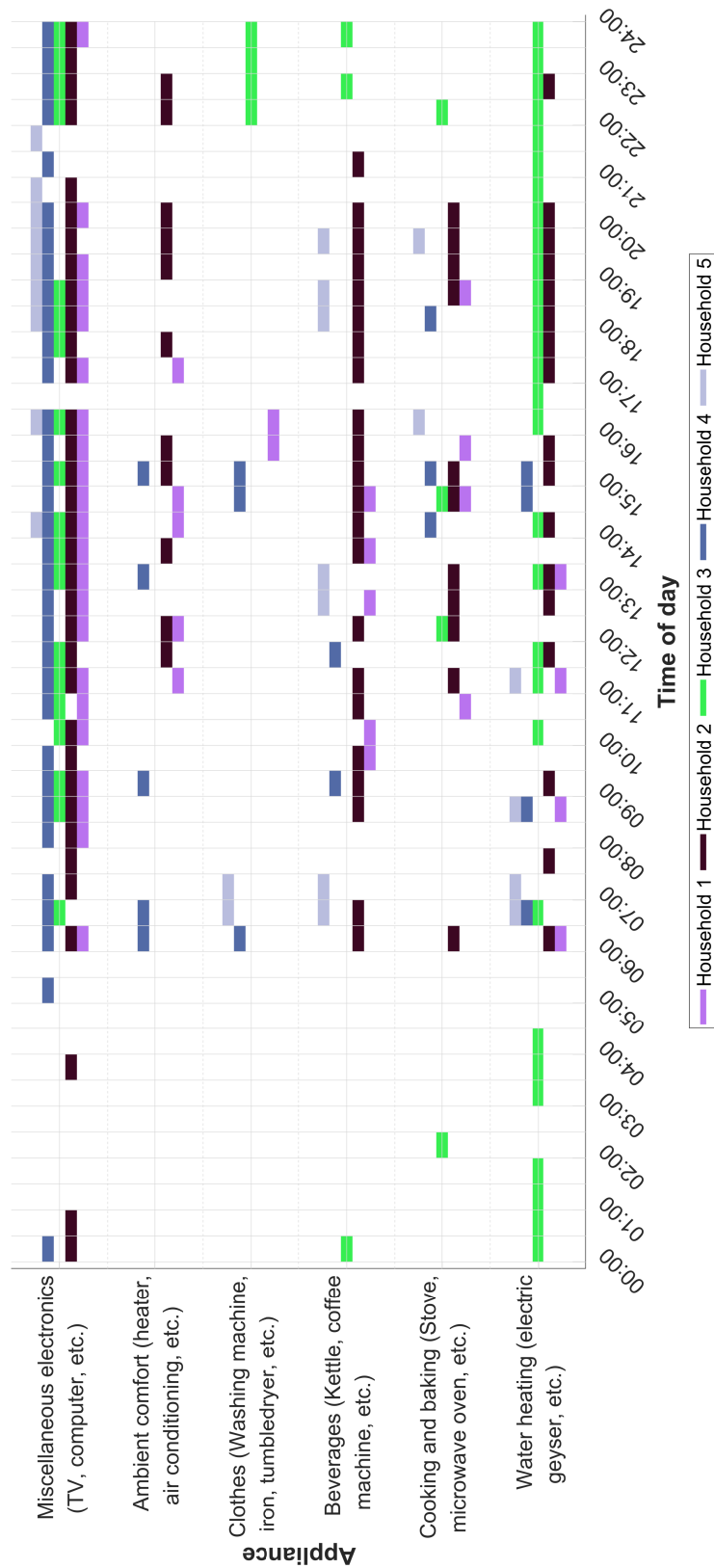


Figure 4.34. Plot showing daily appliance usage of the five households with DM implemented during winter.

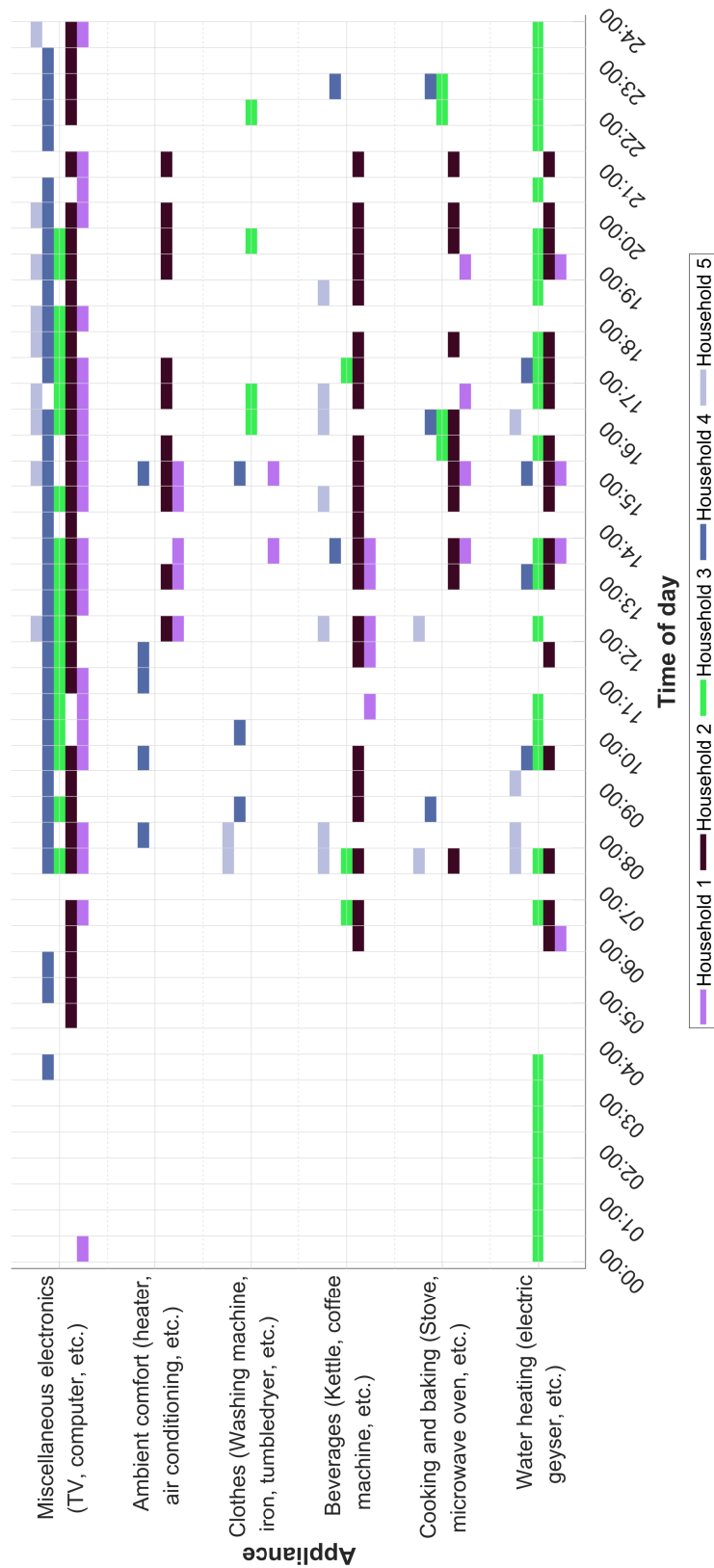


Figure 4.35. Plot showing daily appliance usage of the five households with DM implemented during spring.

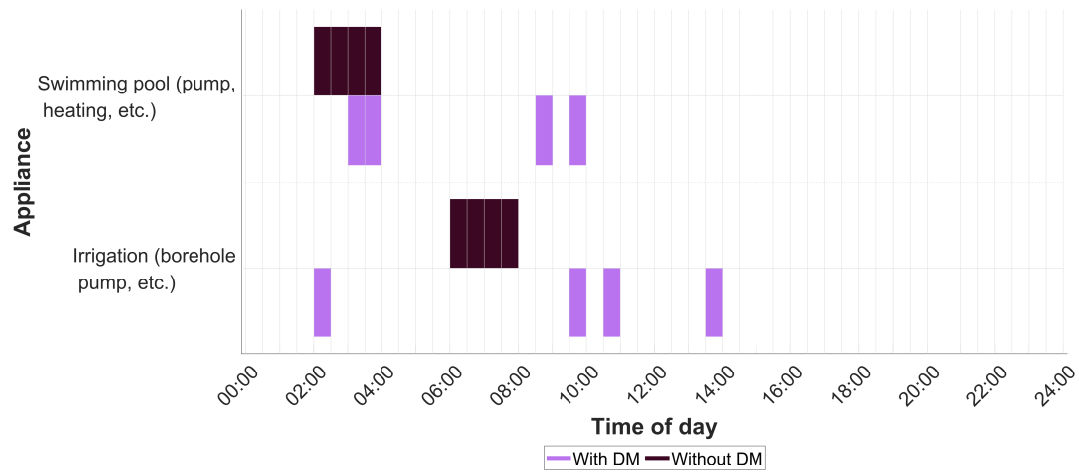


Figure 4.36. Plot showing daily utilization of communal loads DM implemented during summer.

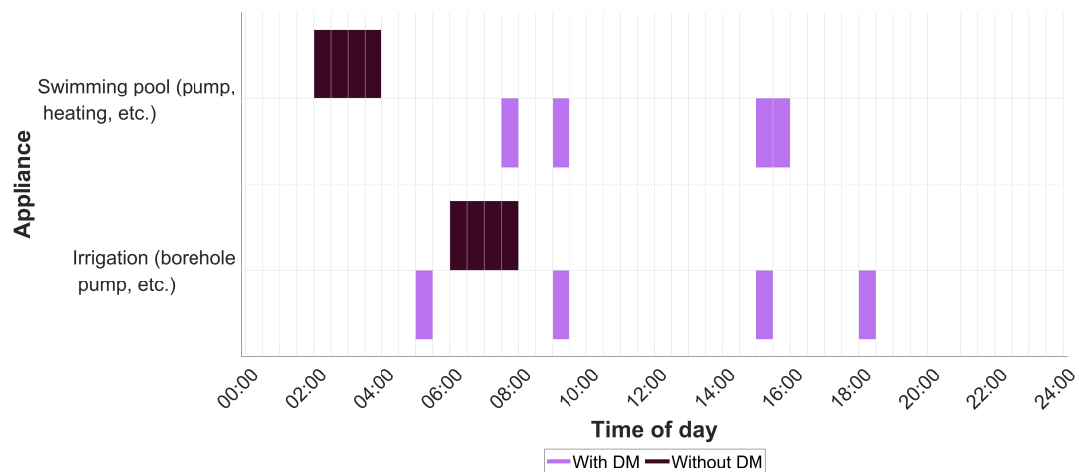


Figure 4.37. Plot showing daily utilization of communal loads DM implemented during autumn.

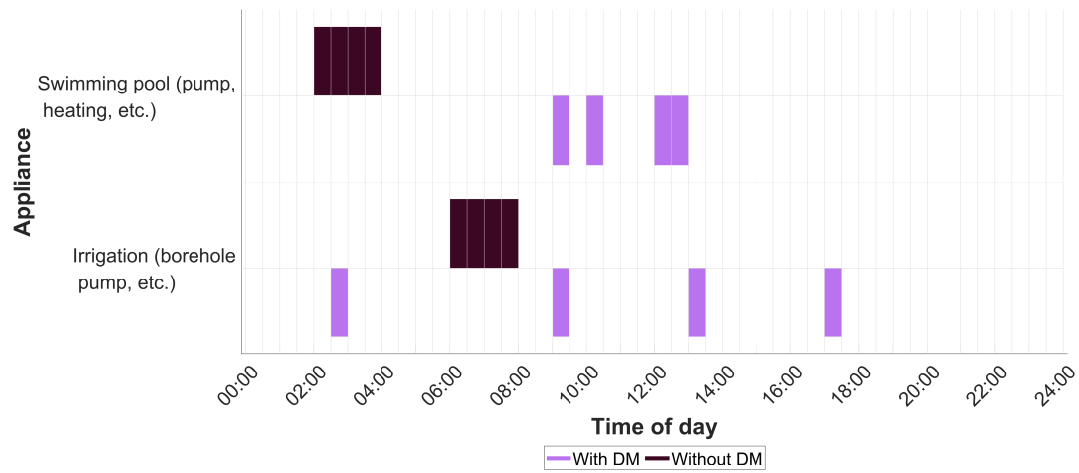


Figure 4.38. Plot showing daily utilization of communal loads DM implemented during winter.

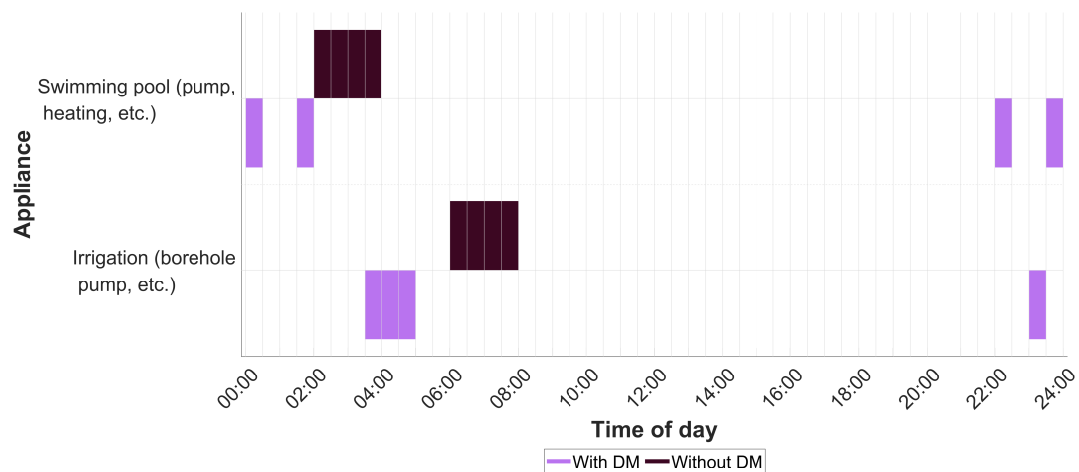


Figure 4.39. Plot showing daily utilization of communal loads DM implemented during spring.

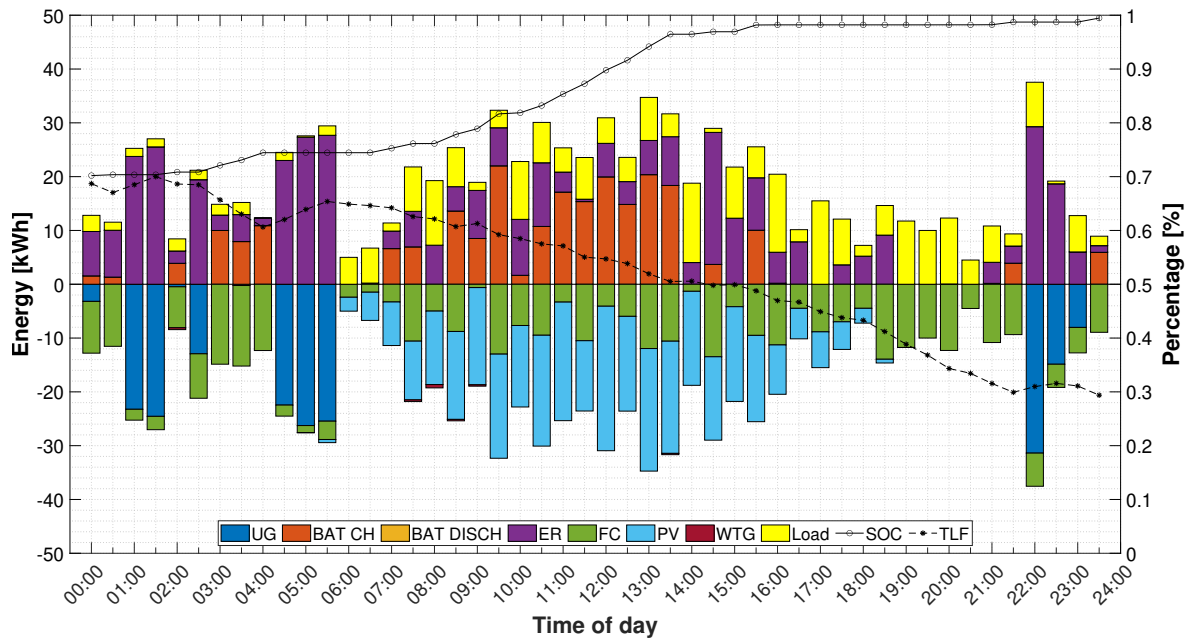


Figure 4.40. Plot showing the energy utilization for January with DM implemented.

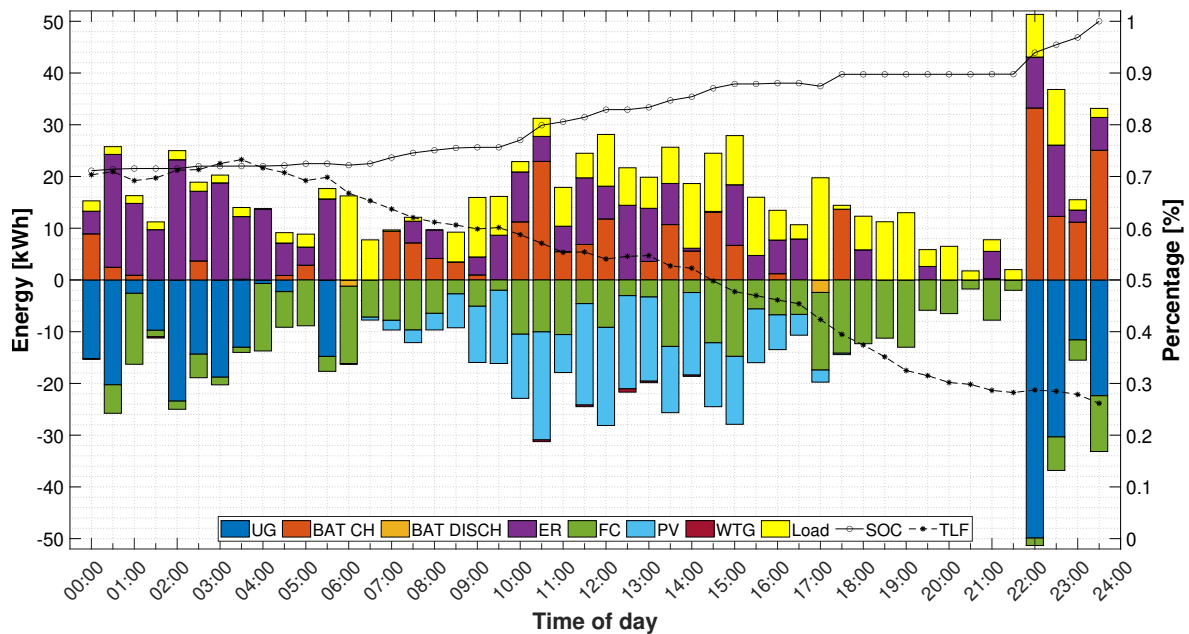


Figure 4.41. Plot showing the energy utilization for April with DM implemented.

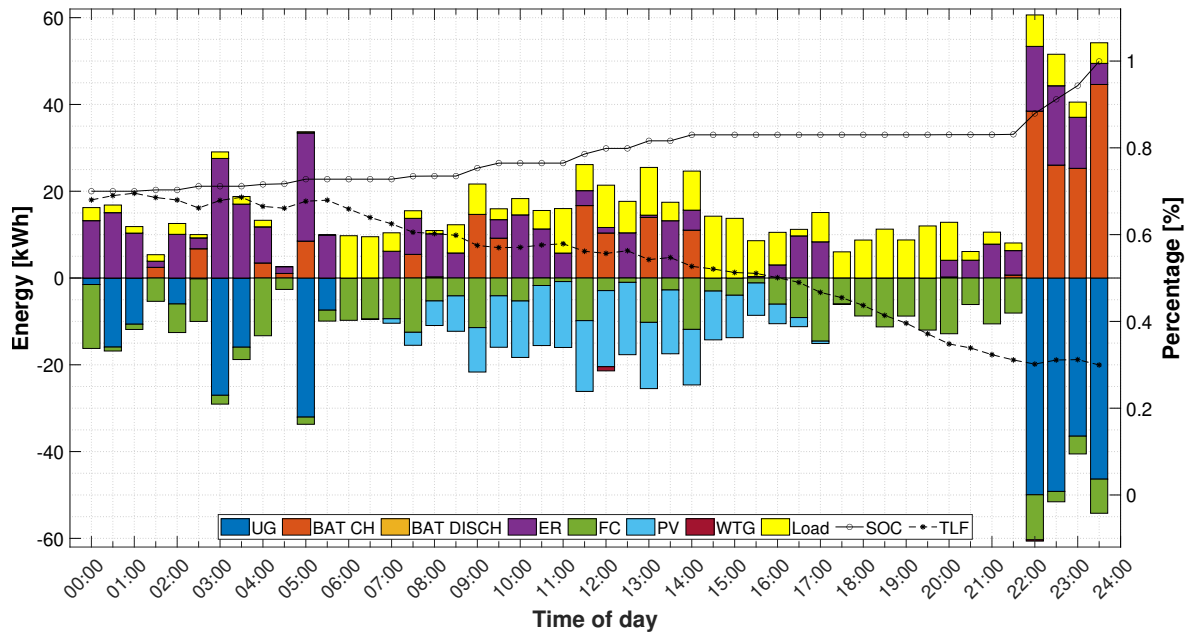


Figure 4.42. Plot showing the energy utilization for July with DM implemented.

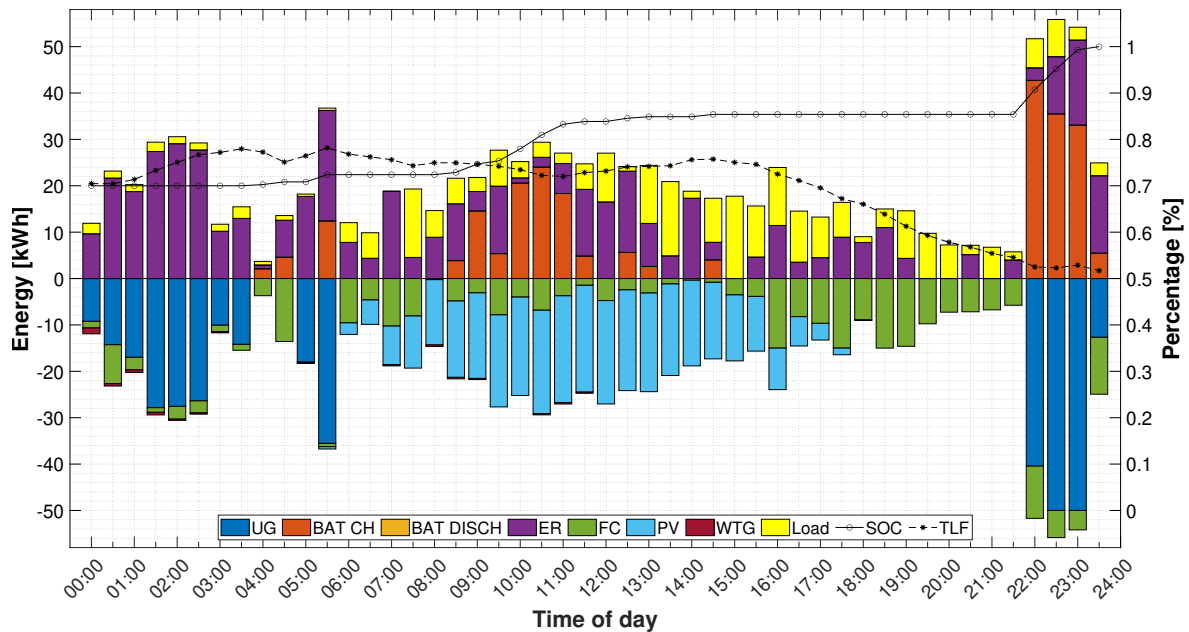
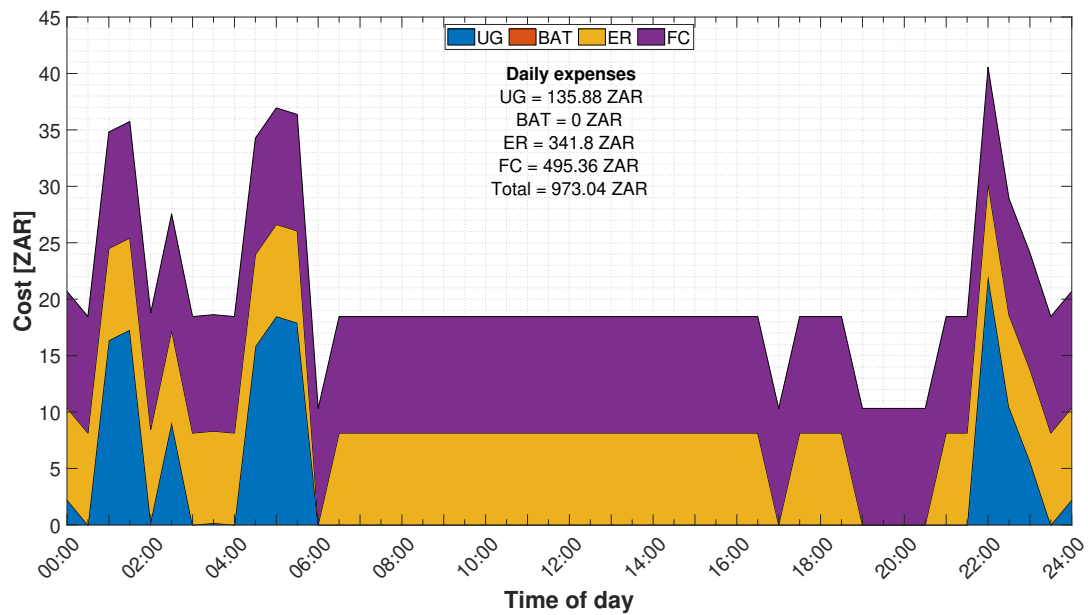
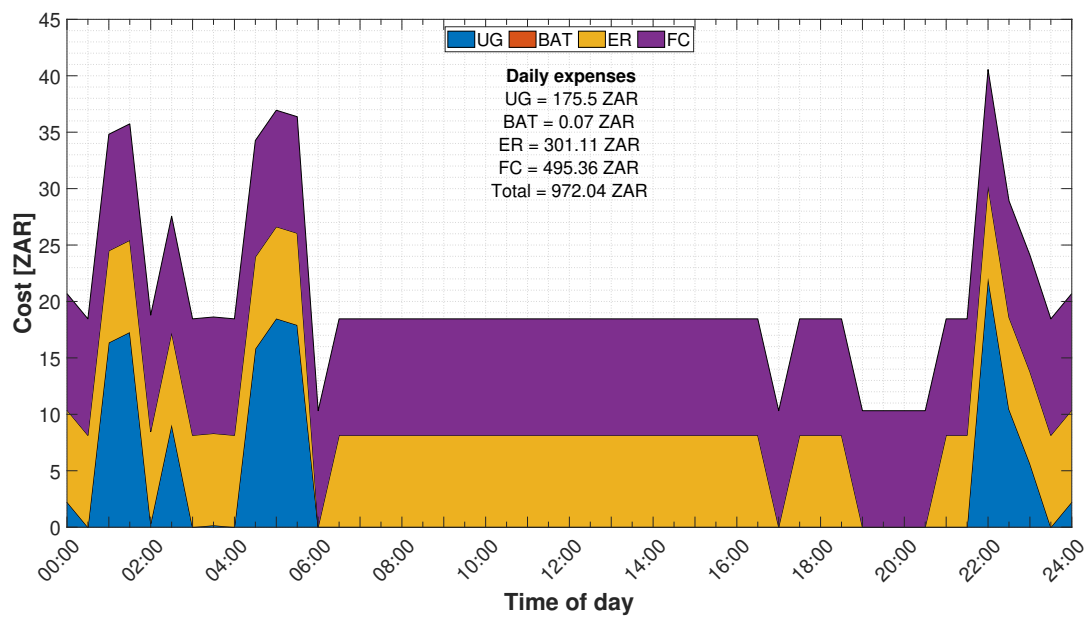


Figure 4.43. Plot showing the energy utilization for October with DM implemented.

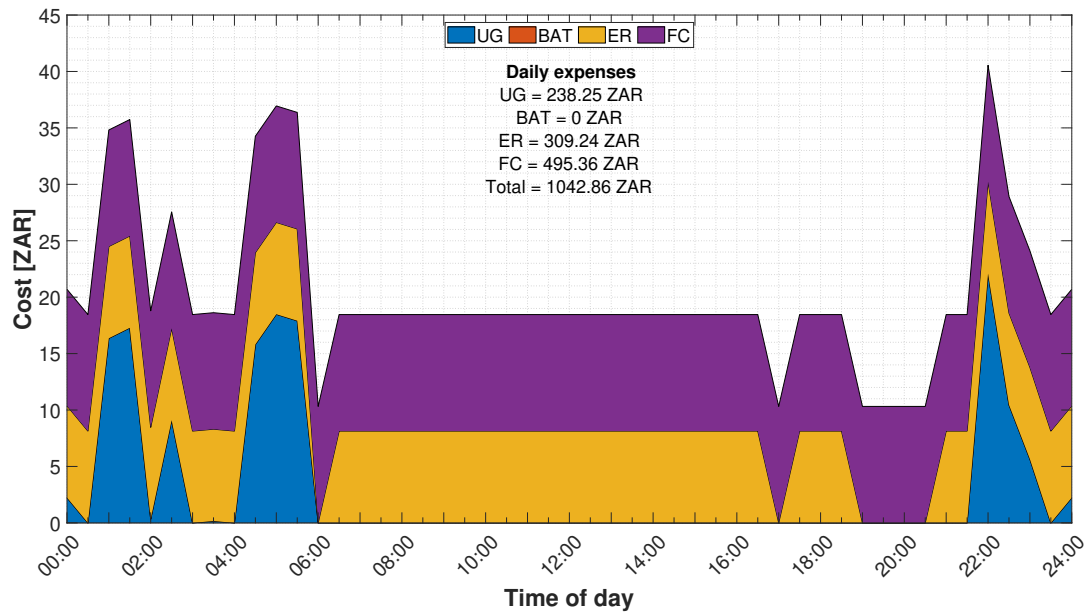


(a) Plot showing the average daily operating cost in summer.

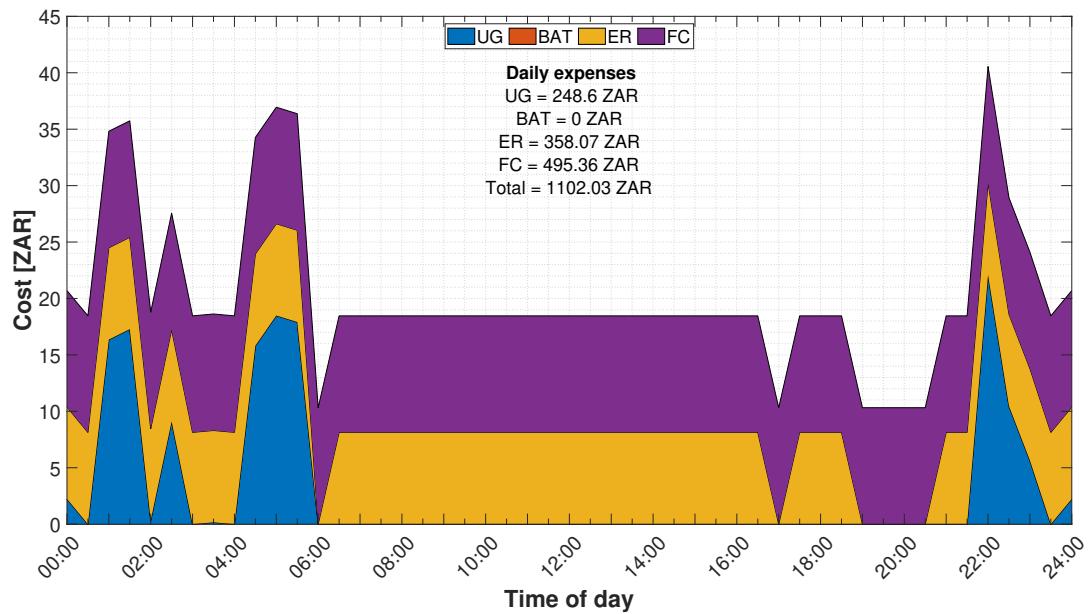


(b) Plot showing the average daily operating cost in autumn.

Figure 4.44. Plots showing the average daily MG operating cost for summer and autumn with DM considered.



(a) Plot showing the average daily operating cost in winter.



(b) Plot showing the average daily operating cost in spring.

Figure 4.45. Plots showing the average daily MG operating cost for winter and spring with DM considered.

CHAPTER 5 DISCUSSION

5.1 CHAPTER OVERVIEW

This chapter serves to discuss the results obtained in Chapter 4. It is divided into two main sections titled Processed Data and Models (Section 5.2) and Energy Optimisation (Section 5.3). The latter focuses on the ultimate goal of this dissertation: Energy optimisation with DM.

5.2 PROCESSED DATA AND MODELS

In this section, SAURAN, questionnaire and RES data will be looked at and discussed.

5.2.1 SAURAN Data

Wind speed, temperature and solar irradiation data were obtained – and air density calculated – from the SAURAN database. These are represented graphically in Figures 4.1 - 4.4, and although all these variables will influence the generated power, solar irradiation and wind speed are the most prevalent.

For all seasons, solar irradiation follows an expected parabolic trend: slowly rising in the mornings, peaking at midday and falling in the afternoon. During summer and autumn (Figures 4.1 and 4.2) some noise can be seen on the graphs. This is typically caused by cloud cover, as Pretoria is situated in a summer rainfall area. Additionally, during July (Figure 4.3) the solar irradiation only starts to increase at around 06:30 compared to 05:30 in January. This correlates with shorter days in winter and thus also less energy being generated by the PV modules.

The wind speed seems to be lower during summer, autumn and winter and higher during spring. However, the daily variation appears to be quite random without following a specific trend. This will make estimates of WTG generation capacity very difficult and possibly unreliable.

5.2.2 Questionnaire Data

The research questionnaire provided some interest in how different consumers will react to the implementation of DM. Based on Figure 4.11, there is a tendency for specific loads to have a high inconvenience rating throughout all the individuals surveyed: Water heating, food preparation and miscellaneous electronics. The inconvenience related to beverages, clothes washing, ambient comfort and irrigation showed significant variation among respondents, whereas scheduling of the swimming pool pump's operation will have minimal impact on all.

Additionally, the LP in Figure 4.10 correlates with expected tendencies: Low power consumption during the early morning hours and average consumption during the day, with a significant morning and evening peak. The latter is the highest, and the maximum demand per household is taken as 4.7875 kW (or 2.3937 kWh).

5.2.3 RES

As per Table 3.16, the PV modules and WTGs are expected to last 25 years. However, the former's capital cost (neglecting miscellaneous components such as inverters) is R397 200 compared to the latter of R1 386 000. The expected lifetime energy of each RES is calculated using the total daily generation per single RES shown in Figures 4.5 - 4.8, multiplied by the number of each source and the expected lifetime in days. Table 5.1 summarizes these values.

Table 5.1. Table comparing PV and WTG operational expenses.

RES	Life Expectancy	Capital Cost	Lifetime energy
PV	25 [years]	397 200 [ZAR]	2 619 [kWh]
WTG	25 [years]	1 386 000 [ZAR]	25.30 [kWh]

Even though there are more PV modules installed than WTGs, the capital cost of the PV is still significantly lower (one order of magnitude) than the cost of the WTGs. Although the cost of the PV is expected to increase slightly, given inverters and other components are not considered, the WTGs will require more maintenance because of moving parts. Additionally, the amount of power contributed to the MG by the PVs is two orders of magnitude higher than the WTGs' lifetime power contribution.

5.3 ENERGY OPTIMISATION

This section aims to discuss the energy optimisation part of the dissertation. The different algorithms are compared in Subsection 5.3.1, and the benefits of DM, if any, are evaluated in Subsection 5.3.2. The results summarized in Table 5.2 will form the basis of the comparison.

Table 5.2. Table comparing optimum points of the fixed and variable load problems.

Month	Fixed-load			Variable-load
	GA [ZAR]	PSO [ZAR]	SO [ZAR]	MOGA [ZAR]
Jan	582.52	375.94	366.34	973.04
Apr	644.44	398.92	237.67	972.04
Jul	661.01	403.18	283.30	1 042.86
Oct	607.05	405.52	402.68	1 102.03

5.3.1 Comparison between energy optimisation algorithms

In Section 4.3.2, results for the fixed-load optimisation problem are provided. These results mainly aim to provide a platform for comparing different algorithms, in this case, GA, PSO and SO. The main trade-off between the different optimisation algorithms is the computational time required and the accuracy (lowest cost) achieved. Other parameters, such as ease of implementation and simplicity, can also be considered but will not be discussed as MATLAB's built-in functions were used.

As summarized in Table 4.1, PSO achieved its optimal solution in the fastest time (1.19 seconds average), whereas SO achieved the best optimum point (R322.57 average). It should, however, be noted that SO took the longest to converge. Each of the algorithms was provided with the same set of ICs, whose computational time (Table 3.17) was not considered as part of the comparison.

5.3.2 Comparison between fixed and variable load optimisation

When looking at the variable-load optimisation results (Figures 4.40 - 4.43), it can quickly be seen that an optimum solution is not reached. The algorithm allows simultaneous usage of both FCs and ERs, resulting in wasted energy as the hydrogen generated will be consumed at the same time. For optimal operation of either component, minimum conditions are also implemented which are not always met by the algorithm.

Lastly, as a constant expense occurs per interval of usage for the ER and FC, each device should aim to operate at maximum capacity when powered on.

CHAPTER 6 CONCLUSION

Based on discussions in Section 5, conclusions can be made regarding the MG developed for a small residential complex in Pretoria, South Africa. The SAURAN weather stations provided seemingly accurate data for estimating the daily atmospheric conditions to allow for RES generation. Including a WTG in the system, however, corresponds to unnecessary expenses as it does not contribute to the total amount of generated energy when compared to PVs.

When comparing the different optimization algorithms, PSO was the most effective. Although not converging at the most optimal solution, the computational time was significantly lower than SO, whose solution was only slightly more optimal.

Both the fixed-load and variable-load optimisation problems took significant time to converge to an optimum solution: More than an hour per day, including the generation of initial conditions. Thus, it is unsuitable for real-time applications, and additional simplification would be required. A possible solution would be to formulate simpler mathematical models for the MG, allowing for using more traditional optimisation techniques, such as linear programming. Additionally, the process can be sped up by limiting the range of possible values per variable, either by statistical means or assumptions.

Future work can also look at a more detailed analysis of the questionnaire feedback, especially how different consumers respond to DM and the inconvenience caused. DM will also be better implemented in smaller intervals (i.e. five minutes) however, that will have a significant impact on computational time. Furthermore, there is being moved away from VRLA due to their limited DOD and short life expectancy when compared to alternatives, such as lithium batteries. The use of electric vehicles and super-capacitors should also be considered as possibilities for a hybrid ESS.

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APPENDIX A RESEARCH QUESTIONNAIRE

Table A.1. Table showing questionnaire input for first set of appliances' TOU.

	Water heating (electric geyser, etc.)	Cooking and baking (stove, oven, etc.)	Beverages (kettle, etc.)	Clothing (washing machine, etc.)
00:00 - 00:30	☐	☐	☐	☐
00:30 - 01:00	☐	☐	☐	☐
01:00 - 01:30	☐	☐	☐	☐
01:30 - 02:00	☐	☐	☐	☐
02:00 - 02:30	☐	☐	☐	☐
02:30 - 03:00	☐	☐	☐	☐
03:00 - 03:30	☐	☐	☐	☐
03:30 - 04:00	☐	☐	☐	☐
04:00 - 04:30	☐	☐	☐	☐
04:30 - 05:00	☐	☐	☐	☐
05:00 - 05:30	☐	☐	☐	☐
05:30 - 06:00	☐	☐	☐	☐
06:00 - 06:30	☐	☐	☐	☐
06:30 - 07:00	☐	☐	☐	☐
07:00 - 07:30	☐	☐	☐	☐
07:30 - 08:00	☐	☐	☐	☐
08:00 - 08:30	☐	☐	☐	☐
08:30 - 09:00	☐	☐	☐	☐
09:00 - 09:30	☐	☐	☐	☐
09:30 - 10:00	☐	☐	☐	☐
10:30 - 10:00	☐	☐	☐	☐
10:00 - 10:30	☐	☐	☐	☐
10:30 - 11:00	☐	☐	☐	☐
11:00 - 11:30	☐	☐	☐	☐
11:30 - 12:00	☐	☐	☐	☐

Table A.2. Table showing questionnaire input for first set of appliances' TOU (continued).

	Water heating (electric geyser, etc.)	Cooking and baking (stove, oven, etc.)	Beverages (kettle, etc.)	Clothing (washing machine, etc.)
12:00 - 12:30	☐	☐	☐	☐
12:30 - 13:00	☐	☐	☐	☐
13:00 - 13:30	☐	☐	☐	☐
13:30 - 14:00	☐	☐	☐	☐
14:00 - 14:30	☐	☐	☐	☐
14:30 - 15:00	☐	☐	☐	☐
15:00 - 15:30	☐	☐	☐	☐
15:30 - 16:00	☐	☐	☐	☐
16:00 - 16:30	☐	☐	☐	☐
16:30 - 17:00	☐	☐	☐	☐
17:00 - 17:30	☐	☐	☐	☐
17:30 - 18:00	☐	☐	☐	☐
18:00 - 18:30	☐	☐	☐	☐
18:30 - 19:00	☐	☐	☐	☐
19:00 - 19:30	☐	☐	☐	☐
19:30 - 20:00	☐	☐	☐	☐
20:00 - 20:30	☐	☐	☐	☐
20:30 - 21:00	☐	☐	☐	☐
21:00 - 21:30	☐	☐	☐	☐
21:30 - 22:00	☐	☐	☐	☐
22:00 - 22:30	☐	☐	☐	☐
22:30 - 23:00	☐	☐	☐	☐
23:00 - 23:30	☐	☐	☐	☐
23:30 - 24:00	☐	☐	☐	☐

Table A.3. Table showing questionnaire input for second set of appliances' TOU.

	Ambient comfort (air-conditioning, etc.)	Misc. electronics (TV, laptop, etc.)	Irrigation (borehole pump, etc.)	Swimming pool (pump, heating, etc.)
00:00 - 00:30	☐	☐	☐	☐
00:30 - 01:00	☐	☐	☐	☐
01:00 - 01:30	☐	☐	☐	☐
01:30 - 02:00	☐	☐	☐	☐
02:00 - 02:30	☐	☐	☐	☐
02:30 - 03:00	☐	☐	☐	☐
03:00 - 03:30	☐	☐	☐	☐
03:30 - 04:00	☐	☐	☐	☐
04:00 - 04:30	☐	☐	☐	☐
04:30 - 05:00	☐	☐	☐	☐
05:00 - 05:30	☐	☐	☐	☐
05:30 - 06:00	☐	☐	☐	☐
06:00 - 06:30	☐	☐	☐	☐
06:30 - 07:00	☐	☐	☐	☐
07:00 - 07:30	☐	☐	☐	☐
07:30 - 08:00	☐	☐	☐	☐
08:00 - 08:30	☐	☐	☐	☐
08:30 - 09:00	☐	☐	☐	☐
09:00 - 09:30	☐	☐	☐	☐
09:30 - 10:00	☐	☐	☐	☐
10:30 - 10:00	☐	☐	☐	☐
10:00 - 10:30	☐	☐	☐	☐
10:30 - 11:00	☐	☐	☐	☐
11:00 - 11:30	☐	☐	☐	☐
11:30 - 12:00	☐	☐	☐	☐

Table A.4. Table showing questionnaire input for second set of appliances' TOU (continued).

	Ambient comfort (air-conditioning, etc.)	Misc. electronics (TV, laptop, etc.)	Irrigation (borehole pump, etc.)	Swimming pool (pump, heating, etc.)
12:00 - 12:30	☐	☐	☐	☐
12:30 - 13:00	☐	☐	☐	☐
13:00 - 13:30	☐	☐	☐	☐
13:30 - 14:00	☐	☐	☐	☐
14:00 - 14:30	☐	☐	☐	☐
14:30 - 15:00	☐	☐	☐	☐
15:00 - 15:30	☐	☐	☐	☐
15:30 - 16:00	☐	☐	☐	☐
16:00 - 16:30	☐	☐	☐	☐
16:30 - 17:00	☐	☐	☐	☐
17:00 - 17:30	☐	☐	☐	☐
17:30 - 18:00	☐	☐	☐	☐
18:00 - 18:30	☐	☐	☐	☐
18:30 - 19:00	☐	☐	☐	☐
19:00 - 19:30	☐	☐	☐	☐
19:30 - 20:00	☐	☐	☐	☐
20:00 - 20:30	☐	☐	☐	☐
20:30 - 21:00	☐	☐	☐	☐
21:00 - 21:30	☐	☐	☐	☐
21:30 - 22:00	☐	☐	☐	☐
22:00 - 22:30	☐	☐	☐	☐
22:30 - 23:00	☐	☐	☐	☐
23:00 - 23:30	☐	☐	☐	☐
23:30 - 24:00	☐	☐	☐	☐

Table A.5. Table showing requested rescheduling inconvenience input for the first set of appliances.

	1	2	3	4	5
	(No inconvenience)	(Some inconvenience)			(Extreme inconvenience)
Water heating (electric geyser, etc.)	☐	☐	☐	☐	☐
Cooking and baking (stove, oven, etc.)	☐	☐	☐	☐	☐
Beverages (kettle, etc.)	☐	☐	☐	☐	☐
Clothing (washing machine, etc.)	☐	☐	☐	☐	☐

Table A.6. Table showing requested rescheduling inconvenience input for the second set of appliances.

	1	2	3	4	5
	(No inconvenience)	(Some inconvenience)			(Extreme inconvenience)
Ambient comfort (air-conditioning, etc.)	☐	☐	☐	☐	☐
Misc. electronics (TV, laptop, etc.)	☐	☐	☐	☐	☐
Irrigation (borehole pump, etc.)	☐	☐	☐	☐	☐
Swimming pool (pump, heating, etc.)	☐	☐	☐	☐	☐

APPENDIX B MATHEMATICAL DERIVATIONS

B.1 PV DERIVATIONS

The amount of PV modules that can be installed is limited by the available roof space of each house. Generally, houses are sized by the available floor space, and hence a formula is required to convert between the two areas, given a known pitch angle (θ_{roof}). Figure B.1 will be used to aid with the derivation process.

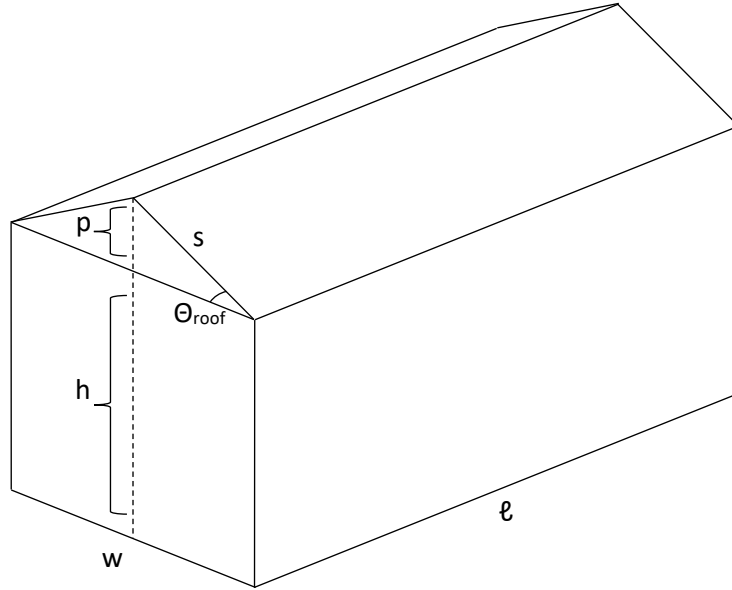


Figure B.1. Basic house design for calculating relationship between floor and roof areas.

The total floor and roof areas are calculated using (B.1) and (B.2). However, solar PVs will only be installed on one side of the roof in order to gain maximum solar irradiation. The total usable roof area is thus calculated in (B.3).

$$A_{floor} = w \times \ell \quad (B.1)$$

$$A_{roof,tot} = 2 \times s \times \ell \quad (B.2)$$

$$A_{roof,use} = s \times \ell \quad (B.3)$$

Substituting (B.3) into (B.1), (B.4) is obtained, with s being unknown:

$$A_{roof,use} = \frac{s}{w} \times A_{floor} \quad (B.4)$$

Using basic trigonometry, (B.5) is derived and rewritten as (B.6):

$$\cos \theta_{roof} = \frac{w}{2s} \quad (B.5)$$

$$s = \frac{w}{2 \cos \theta_{roof}} \quad (B.6)$$

Now, (B.6) is substituted into (B.4) and the total usable floor area can be calculated using (B.7).

$$A_{roof,use} = \frac{A_{floor}}{2 \cos \theta_{roof}} \quad (B.7)$$

B.2 WTG DERIVATIONS

The kinetic energy available in the wind, E_{wnd} , is shown in (B.8), with air mass, m_{air} , and average wind speed, v_{wnd} . When moving perpendicularly through an object with area A and thickness x , the equation is expanded, with the air density denoted as ρ . By taking the time derivative of the kinetic energy (B.8), the ideal wind power can be determined as shown in (B.9) [109].

$$\begin{aligned} E_{wnd}(t) &= \frac{m_{air}(t) \times v_{wnd}^2(t)}{2} \\ &= \frac{\rho(t)Ax(t) \times v_{wnd}^2(t)}{2} \end{aligned} \quad (B.8)$$

$$\begin{aligned} P_{wnd}(t) &= \frac{d}{dt} [E_{wnd}(t)] \\ &= \frac{dx}{dt} \left[\frac{\rho Ax \times v_{wnd}^2}{2} \right] \\ &= \frac{\rho(t)Av_{wnd}^3(t)}{2} \end{aligned} \quad (B.9)$$

While the above equations resemble the amount of power and energy the wind possesses, the power that can be extracted for use, P_{extr} , is significantly lower and determined by the power coefficient, C_p , as shown in (B.10). According to Betz law, there is an upper limit to the value of C_p , restricting the maximum amount usable power for even the best designed wind turbines, as shown in (B.11) [109].

$$\begin{aligned} P_{extr}(t) &= C_p(t) \times P_{wnd}(t) \\ &= C_p(t) \times \frac{\rho(t)Av_{wnd}^3(t)}{2} \end{aligned} \quad (B.10)$$

$$0 \leq C_p(t) \leq 0.593 \quad (B.11)$$

For a modern, single-rotor WTG, the area is calculated using the standard area formula for a circle, with either the rotor diameter, d_{rot} , or rotor radius, r_{rot} , as input variable (B.12). Additionally, the WTG efficiency is calculated as the product of the gearbox and generator efficiencies, η_{gear} and η_{gen} ,

as shown in (B.13) [21].

$$\begin{aligned} A &= A_{rot} \\ &= \pi \times R_{rot}^2 \\ &= \frac{\pi}{4} \times d_{rot}^2 \end{aligned} \tag{B.12}$$

$$\eta_{wtg} = \eta_{gear} \times \eta_{gen} \tag{B.13}$$

Substituting (B.12) into (B.10) and multiplying with the efficiency (B.13), the output power delivered by a single WTG is calculated using (B.14).

$$\begin{aligned} P_{wtg}(t) &= \eta_{wtg} \times P_{extr}(t) \\ &= \eta_{wtg} \times C_p(t) \times \frac{\rho(t) A v_{wnd}^3(t)}{2} \end{aligned} \tag{B.14}$$

APPENDIX C CHEMISTRY

Standard temperature and pressure (STP) values for entropy and enthalpy of relevant molecules are shown in Table C.1 [110]. Using these values, the entropy and enthalpy changes (ΔS and ΔH) for both a ER and FC are calculated in (C.1, C.2) and (C.4, C.5). The Gibbs free energies (ΔG) are calculated in (C.3) and (C.6), with T_{ref} as the STP temperature of 298.15 K, or 25°C.

Table C.1. Table showing STP values for entropy and enthalpy of relevant elements.

	Entropy [J/K.mol]	Enthalpy [J/mol]
	ΔS	ΔH
$H_2(g)$	130.59	0
$O_2(g)$	205.29	0
$H_2O(\ell)$	69.964	-285.85×10^3

C.1 ELECTROLYSER CHEMISTRY

$$\begin{aligned}
 \Delta S_{ER} &= \Delta S_{prod} - \Delta S_{reac} \\
 &= (\Delta S_{H_2(g)} + 0.5\Delta S_{O_2(g)}) - \Delta S_{H_2O(\ell)} \\
 &= (130.59 + 0.5 \times 205.29) - 69.964 \\
 &= 163.27 \text{ [J/mol.K]}
 \end{aligned} \tag{C.1}$$

$$\begin{aligned}
 \Delta H_{ER} &= \Delta H_{prod} - \Delta H_{reac} \\
 &= (\Delta H_{H_2(g)} + 0.5\Delta H_{O_2(g)}) - \Delta H_{H_2O(\ell)} \\
 &= (0 + 0) - (-285.85 \times 10^3) \\
 &= 285.85 \times 10^3 \text{ [J/mol]}
 \end{aligned} \tag{C.2}$$

$$\begin{aligned}
 \Delta G_{ER} &= \Delta H_{ER} - T_{ref} \times \Delta S_{ER} \\
 &= 285.85 \times 10^3 - 298.15 \times 163.27 \\
 &= 237\,171 \text{ [J/mol]}
 \end{aligned} \tag{C.3}$$

C.2 FUEL CELL CHEMISTRY

$$\begin{aligned}
 \Delta S_{FC} &= \Delta S_{prod} - \Delta S_{reac} \\
 &= \Delta S_{H_2O(\ell)} - (\Delta S_{H_2(g)} + 0.5\Delta S_{O_2(g)}) \\
 &= 69.964 - (130.59 + 0.5 \times 205.29) \\
 &= -163.27 \text{ [J/mol.K]}
 \end{aligned} \tag{C.4}$$

$$\begin{aligned}\Delta H_{FC} &= \Delta H_{prod} - \Delta H_{reac} \\ &= \Delta H_{H_2O(\ell)} - (\Delta H_{H_2(g)} + 0.5\Delta H_{O_2(g)}) \\ &= -285.85 \times 10^3 - (0 + 0) \\ &= -285.85 \times 10^3 [J/mol]\end{aligned}\tag{C.5}$$

$$\begin{aligned}\Delta G_{FC} &= \Delta H_{FC} - T_{ref} \times \Delta S_{FC} \\ &= -285.85 \times 10^3 - (-298.15 \times 163.27) \\ &= -237\,171 [J/mol]\end{aligned}\tag{C.6}$$

APPENDIX D SIMULINK MODELS

D.1 FC AND ER MODEL

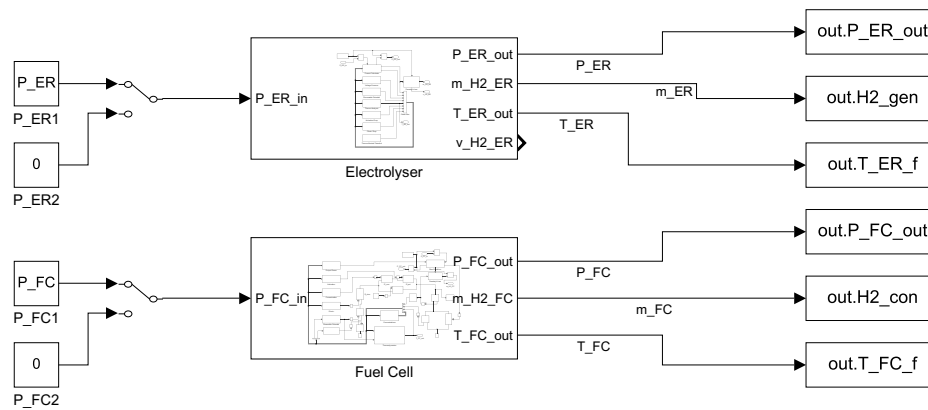


Figure D.1. FC and ER upper hierarchy block diagram.

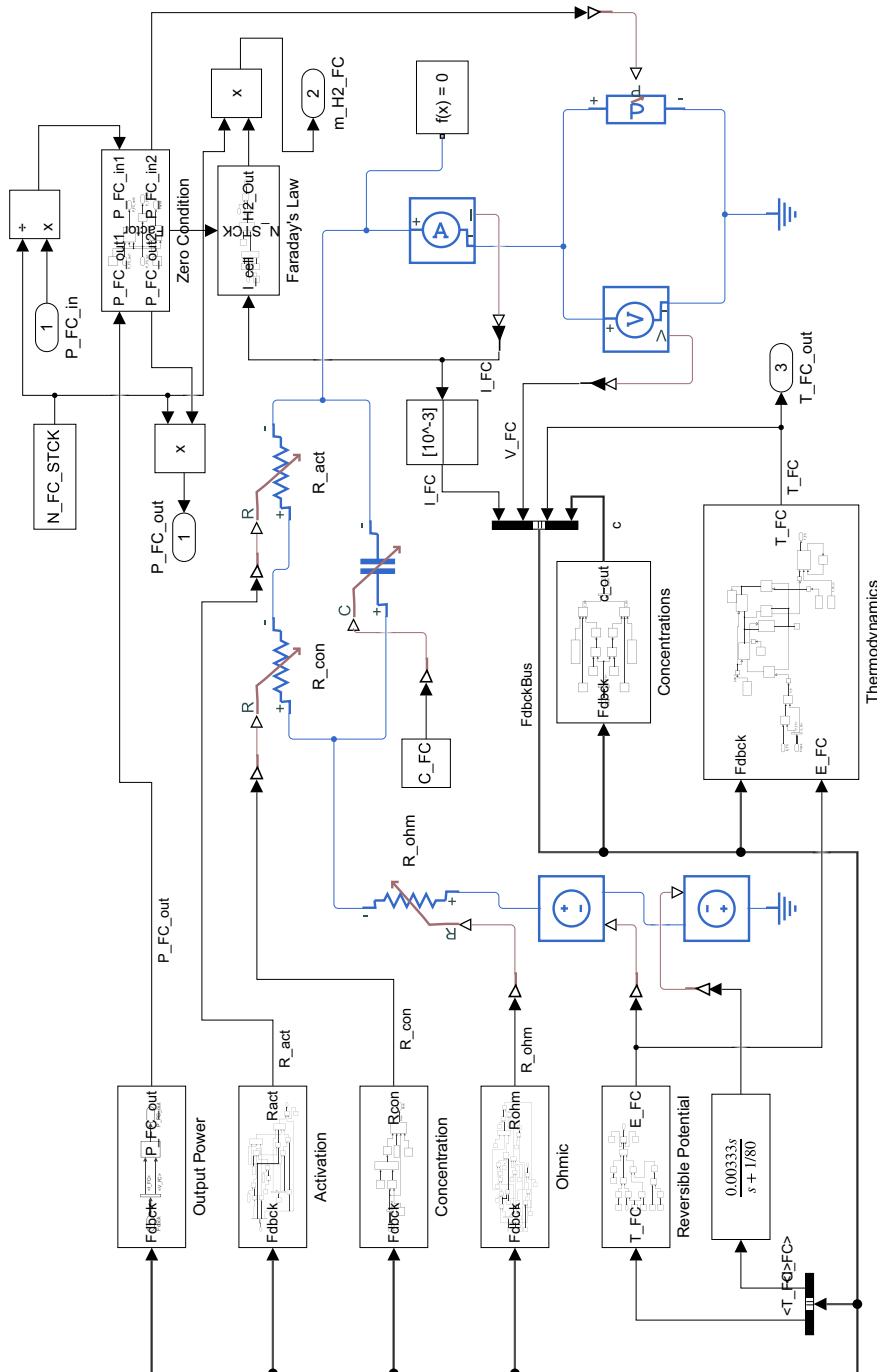


Figure D.2. FC model block and circuit diagram, as developed in SIMULINK.

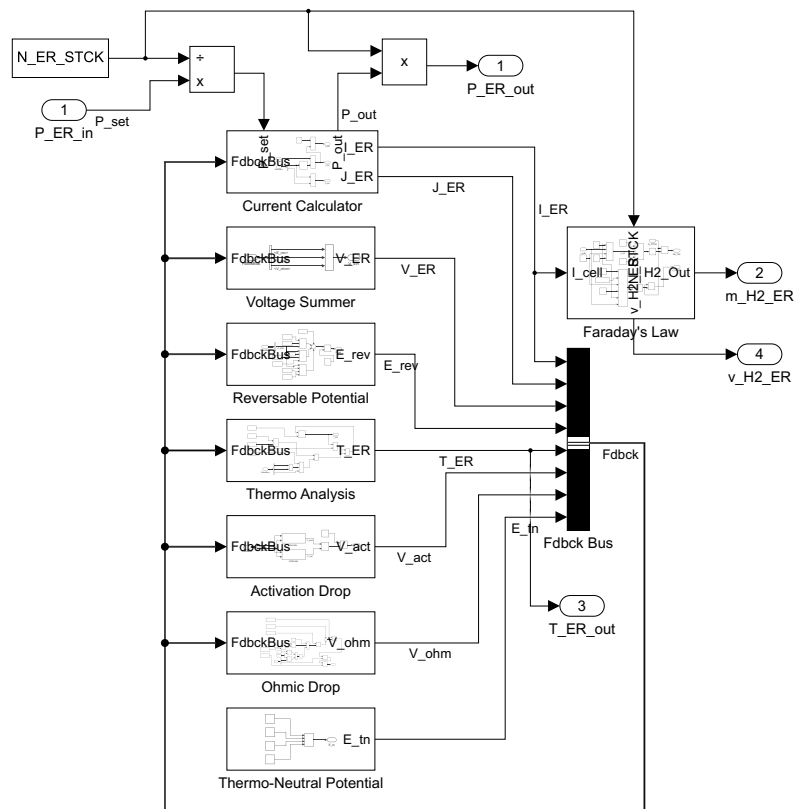


Figure D.3. ER model blockdiagram, as developed in SIMULINK.