Gordon Institute of Business Science
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Benford's Law and Electoral Integrity: A Forensic Analysis of African Elections

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Abstract

This study examines the applicability of Benford's Law (BL) as a forensic auditing tool for detecting anomalies in electoral results. BL, which predicts the expected distribution of leading digits in naturally occurring numerical datasets, has been widely employed in financial fraud detection and academic research validation. While prior studies have explored BL's potential in election forensics, its reliability in distinguishing between genuine fraud and natural statistical deviations remains inconclusive.

Using electoral data from Zimbabwe and Kenya, contrasted with benchmark cases from more stable democracies that are South Africa and Botswana, this research applies multiple BL digit tests (first-digit, second-digit, and first-two-digit analyses) alongside complementary statistical measures (chi-square, Kolmogorov-Smirnov, Mean Absolute Deviation, and p-value tests). The results indicate that while BL can flag irregularities in election data, its limitations as a standalone tool necessitate caution. False positives may arise due to legitimate data quirks, and contextual factors can distort digit distribution patterns.

The study concludes that BL should serve as a preliminary screening mechanism rather than definitive proof of electoral manipulation. To enhance election integrity, future forensic audits should integrate BL with advanced statistical techniques or machine learning models. These findings contribute to methodological debates in election forensics and provide practical recommendations for strengthening post-election audit frameworks.

Keywords

Benford’s Law

Elections

Election Forensics

Abbreviations

ANC	African National Congress
AP	Alliance for Progressives
BCP	Botswana Congress Party
BDP	Botswana Democratic Party
CCC	Citizens For Coalition for Change
COPE	Congress of the People
DA	Democratic Alliance
EFF	Economic Freedom Fighters
IFP	Inkatha Freedom Party
JP	Jubilee Party
MDC-A	Movement for Democratic Change - Alliance
MDC-T	Movement for Democratic Change - Tsvangirai
MK	uMkhonto weSizwe
UDC	Umbrella for Democratic Change
WDM-K	Wiper Democratic Movement - Kenya
ZANU-PF	Zimbabwe African National Union (Patriotic Front)
BL	Benford's Law

Declaration

I declare that this research project is my own work. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the Gordon Institute of Business Science, University of Pretoria. It has not been submitted before for any degree or examination in any other University. I further declare that I have obtained the necessary authorization and consent to carry out this research.

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Contents

List of Figures

List of Tables

1. Introduction to the Research Problem.

1.1 Introduction

Throughout the world, the change and handover of governments through the democratic process is of paramount importance. Elections provide the people the opportunity to elect their government and representatives in the government (Hicken & Mebane, 2017), and as such free and fair elections are a cornerstone of the democratic process. During the elections, the subsequent vote count and declaration, the integrity of the entire process is important to ensure that the outcome of the elections is credible and reflects the will of the voters. According to Hicken and Mebane (2017), when elections are perceived as free and fair, the possibility of candidates taking actions that will undermine the democratic foundations with destabilizing effects will be reduced.

The above highlights the need to hold free and fair, credible elections by governments. These elections should also be audited forensically, meaning there is a need to find a statistical forensic tool to audit the voting results to ensure there is no error or fraud in the declared results. This research will investigate the use and usefulness of Benford's Law to detect election fraud and error in the counting and declaration of votes.

1.2 Background to the Problem

People compiling reports like financial statements, tax reports and those reporting on the elections have numerous reasons to manipulate the data they report. Gorenc (2019) puts these reasons in three categories called the "Fraud Triangle". The motives for fraud according to Gorenc are perceived pressure, perceived opportunities, and the ability to justify fraudulent actions. In (Eutsler, Harris, Williams & Cornejo, 2023), researchers identified the reason for misrepresentation of reported Covid-19 cases in the United States (henceforth US) to partisan biases. This view is supported by Capalbo, Galati, Lupi and Smarra (2023) in their study of financial reporting of municipality owned entities. Researchers in the study found there was manipulation of financial statements during the election seasons by the municipality owned companies. They found that during regular periods, however, the data conformed to Benford's Law. The two studies above prove that there were perceived incentives to commit fraud in Covid-19 and financial reporting which require a definitive tool to conduct forensic analysis of reported data.

Transparency in the electoral process, including communication about the electoral processes and procedures, candidates, voting machines (where used) and voting mechanisms, must be always ensured. Accurate counting and capturing of the votes, election fraud and undue influence also form part of transparency. When not respected, the elections cannot be declared free and fair, and their credibility will be in question (Hicken, Mebane, 2017).

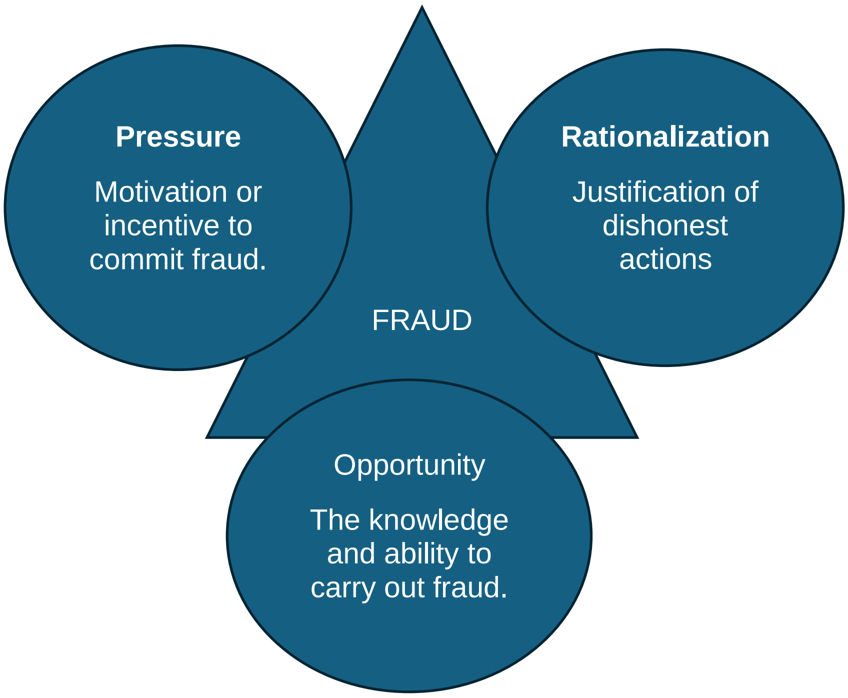
In many countries, election vote counts are usually contested, with the losing political party objecting to the results and claiming there were malfeasants in the vote count. Contested voting results are usually because of either mistrust in the electoral authorities, suspected ballot box stuffing or vote rigging through misstating the number of votes parties obtained in the elections. Lack of transparency by governments and electoral authorities also leads to communities doubting the credibility of these institutions. When election processes are viewed as flawed and results are suspected to be flawed or rigged, the benefits of elections evaporate, resulting in elections being corrosive and destabilizing and the support for democratic institutions declining (Hicken, Mebane, 2017).

In Africa, elections have not been run properly in most cases. With many countries experiencing pre-election and post-election violence most of the time. In Zimbabwe, as recent as in the 2023 national elections, the government and partisan government agencies were responsible for denying people of their rights to elect representatives. A Human Rights Watch report (Nassah, 2023) found that the basic rights of communities were imperilled. The main opposition party, the Citizens Coalition for Change, was on the receiving end of violence, harassment, and intimidation. According to Nassah, freedom of expression, assembly and association were denied making an environment for a free and fair election not possible. Coups have also been the order of the day when it comes to changes in political leadership. Zimbabwe experienced its coup d'etat in November 2017 when the then sitting president, President Robert Gabriel Mugabe, was removed from office by the army (Tendi, 2020), and the army chief, Emmerson Dambudzo Mnangagwa, took over the country's presidency.

Election observers look at the environment before and during the voting process, but not at the vote counting and results declarations. They assess if the elections are in line with international standards (Abutudu, n.d.). Election observers can give the government and electoral authorities recommendations for improvement and provide proof to the international communities of the genuine democratic election process. Observers also enhance the acceptability of the election outcome and legitimacy of the government thus formed (Abutudu, n. d.) as the result of said elections.

The need for a mathematical or statistical tool to conduct election forensics cannot be overstated. Manipulated numbers follow a different pattern than that of random or valid numbers (Gorenc, 2019). This different pattern is the sign that there may have been manipulation of the data, it is not, however, proof that there was fraud committed (Badal-Valero, et. al., 2018). According to Eutsler, et. Al. (2023), "Benford's Law possesses the ability to identify possible manipulation in information datasets". "It uncovers anomalies in certain digit patterns", (Galati, 2023).

Figure 1: Creesey's Fraud Triangle



Source: Adopted from Gorenc (2019)

Benford's Law has been used to detect fraud academic publications (Horton, Kumar & Wood, 2020), (Lazebnik & Gorlitsky, 2023), detecting money laundering (Badal-Valero, Alvarez-Jareño & Pavía, 2018) and financial fraud (Mehta & Bhavani, 2017). Most recently, Benford's Law was used to detect the manipulation of reported Covid-19 cases in the US (Eutsler et. al., 2023). According to Eutsler et. al. (2023), both the Democratic and Republican political parties engaged in misreporting of the Covid-19 cases to support their parties' decision making and to gain influence from the public.

1.3 Problem

This research aims to explore the applicability of statistical and mathematical tools, particularly Benford's Law, in auditing declared voting results to detect potential fraud or errors. The study will evaluate the effectiveness of Benford's Law as a diagnostic tool for identifying anomalies in vote counts and will also incorporate other statistical methods to enhance the robustness of the audit process.

To ensure comprehensive analysis, the research will examine election data from a diverse range of countries, including both contested elections - where results were challenged by losing parties or candidates - and elections that were conducted smoothly without reported irregularities. By comparing these datasets, the study seeks to determine whether Benford's Law and complementary statistical tools can reliably distinguish between legitimate and potentially manipulated election outcomes.

The ultimate goal is to assess the utility of these mathematical approaches in providing transparent, objective, and evidence-based methods for verifying the integrity of declared voting results.

1.4 Purpose statement.

The purpose of the research is to investigate if Benford's Law can be used to detect election fraud and error. The aim is to find out if Benford's Law can be used as a mathematical or statistical method to prove election fraud or error. This will be done using Benford's first- digit, second digit and first two-digit analysis to evaluate if the data conforms to the Benford's Law data distribution.

The scope of this research will be the declared election results where widespread allegations of election fraud and vote stealing were reported to evaluate for Benford's Law conformity. Other results where no allegations of fraud were made will be used as control.

1.5 Academic Use

The use of Benford's Law as a forensic tool has been studied extensively in various fields over the years. Benford's Law can be used in many types of numerical data (Barabesi, Cerioli & Perrotta, 2021). This law can be used for data such as baseball statistics, death rates and areas of lakes (Barabesi, et.al., 2021). Benford's Law can also be used to detect fraud in scientific research publications (Tošić & Vičič, 2021), (Eckhartt & Ruxton, 2023), and in applications for forensic accounting, fraud detecting and auditing (Nigrini, 2012). The theory has attracted the attention of mathematicians and applied scientists (Barabesi, et.al., 2021). In academia, BL is an attractive rule in Probability Theory (Rad, Amiri, Ranjbar, Salari, 2021). With academic fraud ever increasing, and the benefits of such fraud being plenty (Horton, et. al., 2020), this behaviour will not stop until methods of detecting such questionable research practices before publication have been devised. This shows that Benford's Law can be useful in academic studies as a verification tool to audit scientific publications.

1.6 Need in Business

The significance and importance of auditing cannot be understated. Benford's Law will be useful in conducting forensic auditing in financial statements, tax declarations or evasion and in money laundering (Nigrini, 2012).

BL has also been used successfully to by law enforcement in the Spanish court against a company and its numerous suppliers (Badal-Valero, et. al., 2018). The case involved money laundering between a company and a group of its suppliers, and the authors collaborated with the authorities as forensic data experts. (Gorenc, 2019) found that BL was useful in detecting possibility of fraud in financial statements of Mercator d.d. company. The use of Benford's Law as an auditing or forensic tool can save shareholders from losses resulting from fraudulent activities within the companies. The cases of Steinhoff and Tongaat Hullet prove that there is a need for thorough auditing of financial statements by credible, independent external auditors. The use of the word credible here is to highlight that although there were external auditors in both cases, the external auditors were not credible and brought the auditing profession into disrepute in both cases as it was later proven that there were extensive manipulation of reported data. Numerous other studies that have been conducted to test the usefulness of BL in forensic auditing of data shows that there is extensive use of BL in business, though they all state the need to refine the method as it not able to state where fraud has been committed, but only shows the need for further scrutiny of the reported data (Kuruppu, 2019; Nigrini, 2012). It is not confined to one field of study, like mathematics, but can be used even in elections. Since all sectors of business and other activities require auditing and validation of data, studies and further development of Benford's Law should be encouraged.

1.7 South African Context

With South Africa's post-apartheid ruling party constantly losing voters and facing constant challenges from opposition parties, there is a need to audit the voting results even before they are declared. This was evident in the elections held on the 29th of May 2024 in South Africa. During the last elections, numerous claims of vote rigging were forwarded to the Independent Electoral Commission of South Africa (IEC).

1.8 Conclusion

Credibility and transparency in the election process is of paramount importance in any democracy. This study aims to find a mathematical or statistical method or tool to conduct election forensics on declared election results. Chapter 2 reviews existing literature on Benford's Law, looking at various fields where it is applied and the level of research conducted to date. The research question is presented in Chapter 3 together with the hypothesis that will be tested to answer the research question.

Chapter 4 introduces the research methodology and design together with the statistical tools that will be used to test the null hypothesis. In Chapter 5 the findings are presented, which are then discussed in Chapter 6 and compared to the literature reviewed in Chapter 2. In Chapter 7 the conclusions are presented with recommendations for future studies.

2. Literature review.

2.1 Introduction

This section provides the review of available literature on the use of Benford's Law to detect fraud and error in reported data and scientific research papers. This section starts by introducing Benford's Law and its properties. The use of Benford's Law in various areas of forensics will then be discussed and at the end the use of Benford's Law in detecting election fraud will be discussed.

Researchers have shown that Benford's Law can be used to detect error and fraud in financial results reported by companies, academic and scientific publications and in election fraud detection. Nigrini (2012) classifies Benford's Law tests as (i) primary, (ii) secondary and (iii) associated tests. The first two-digit test is considered the primary test as it provides more information. It is also useful for findings biases and detecting invented data. The first digit and second-digit tests are high level tests. These are used to check the general reasonableness of data and to identify obvious anomalies (Mehta & Bhavani, 2017), requiring substantial amounts of data as input.

2.2 Benford's Law

Benford's Law (BL from here onwards) was first observed by Simon Newcomb in 1881 before being reported by Frank Benford in 1938 (Barabesi, et. al., 2021). Frank Benford was not aware of the discovery made by Newcomb when he made the observation himself. BL relates to the distribution of the first digit of numbers of large data (Mir, 2018). According to BL, the distribution of the first significant digits follows a decreasing logarithmic distribution. Benford, (1938) noticed that the first pages of a table of logarithms showed more wear in the first pages than in the last pages, indicating that ones and twos were used more than 8's and 9's. The basic conditions of the law are that the data range should not be restricted, secondly, where the decimal point or zero comes first, it is ignored. Naturally occurring data conforms to BL (Nigrini, 2012). Whereas where there is human intervention, fraudulent data does not conform to BL with leading digits being evenly distributed (Horton, et. al., 2020).

According to BL, the digit one will appear with the highest percentage as the leading digit than the highest number. That is to say, the number one will appear as the first digit 30% of the time, with the proportions gradually decreasing as the numbers increase to nine, with nine having a 4.6% chance of being the leading digit (Miller, 2015). In this research paper, a digit refers to a natural number from one (1) to nine (9), and a number is composed of two or more digits with zero (0) in the second or later position (Benford, 1938).

In naturally occurring datasets, digits one to nine do not appear uniformly (Barabesi, et. al., 2021). In number theory, the Weyl's Equidistribution Theorem and the Fibonacci sequence follow Benford's Law distribution (Barabesi, et. al., 2021). According to BL, the probability of the digit $d_1 \in \{1, 2, \ldots, 9\}$ to occur in the first index is given by:

$$Prob(D_1 = d_1) = \log\left(1 + \frac{1}{d_1}\right) \quad \ldots (1)$$

The probability frequency of the second digit is given by:

$$Prob(D_2 = d_2) = \sum_{d_1}^{9} \log\left(1 + \frac{1}{d_1 d_2}\right) \quad \ldots (2)$$

Where $d_2 \in \{0, 1, 2, \ldots, 9\}$. The frequency of the first two numbers is:

$$Prob(D_1 D_2 = d_1 d_2) = \log\left(1 + \frac{1}{d_1 d_2}\right) \quad \ldots (3)$$

Table 1: First, Second, Third- and Fourth-Digit Proportions of Benford's Law

	Position in Number			
Digit	**1st**	**2nd**	**3rd**	**4th**
0	-	0.11968	0.10178	0.10018
1	0.30103	0.11389	0.10138	0.10014
2	0.17609	0.10882	0.10097	0.1001
3	0.12494	0.10433	0.10057	0.10006
4	0.09691	0.10031	0.10018	0.10002
5	0.07918	0.09668	0.09979	0.09998
6	0.06695	0.09337	0.0994	0.09994
7	0.05799	0.09035	0.09902	0.0999
8	0.05115	0.08757	0.09864	0.09986
9	0.04576	0.085	0.09827	0.09982

Source: Nigrini (2012)

Not all data is suitable to be used with Benford's Law as some data, salaries, or bank account numbers, do not follow the law (Durtschi, Hillison & Pacini, 2004). It should not be identification numbers or flight numbers (da Silva, Gonçalves, Gava & de Mesquita, 2021). According to the authors, data should have more small numbers than large numbers and not clustered around the mean. Table 2 below shows when Benford's law is useful and when it is not.

Table 2: Data Useful and not useful for Benford's Law

When Benford Analysis Is or Is Not Likely Useful	
When Benford Analysis Is Likely Useful	**Examples**
Sets of numbers that result from mathematical combination of numbers - Result comes from two distributions	Accounts receivable (number sold * price), Accounts payable (number bought * price)
Transaction-level data - No need to sample	Disbursements, sales, expenses
On large data sets - The more observations, the better	Full year's transactions
Accounts that appear to conform - When the mean of a set of numbers is greater than the median and the skewness is positive	Most sets of accounting numbers
When Benford Analysis Is Not Likely Useful	**Examples**
Data set is comprised of assigned numbers	Check numbers, invoice numbers, zip codes
Numbers that are influenced by human thought	Prices set at psychological thresholds ($1.99), ATM withdrawals
Accounts with a large number of firm-specific numbers	An account specifically set up to record $100 refunds
Accounts with a built in minimum or maximum	Set of assets that must meet a threshold to be recorded
Where no transaction is recorded	Thefts, kickbacks, contract rigging

Source: Durtschi, Hillison & Pacini (2004)

Before using Benford's Law for data analysis, it should be noted that the data must be distributed over several orders of magnitude (Miller, 2015). Nigrini (2012) states that the dataset should be reasonably large to use BL. However, reasonably large is not explained, but the author suggests that with at least 1000 records the dataset can be expected to conform to BL. If the dataset has less than 1000 records, the tests can still be run, however large deviations can be expected Nigrini (2012).

Wang and Ma (2024) provided proof of BL using the Riemann integrable probability density functions, proposing a criterion to determine whether a distribution adheres to Benford's Law. This contribution is significant because it provides a more accessible understanding of the law, which has broad applications in fields such as fraud detection, physics, and data science. The authors also extend their discussion to the law's scale and base invariance, providing insights into its robustness across different contexts. They use of the Riemann integrable functions for the proof encompasses a wide range of distributions, unlike other studies before it. Wang and Ma (2024) rigorous approach also demonstrate the that the law can be universally applied to datasets that span multiple orders of multitude. However, it lacks empirical validation from which it could have benefited through contrasting and aligning with various fields. Although proof is bounded by assumptions, it offers important theoretical advancements in understanding Benford’s Law.

2.3 Benford’s Law in Financial Fraud

Extensive work has been done regarding using BL to detect financial fraud and anomalies in reported financial statements. The increasing number of companies and individuals committing financial fraud and the ever-improving methods used to commit this fraud necessitates developing advanced tools for conducting forensic audits. Auditors do not have the skills to detect manipulated financial statements, they also do not have knowledge of the characteristics of fraudulent transactions and managers of companies always come up with new ways of committing fraud (Mehta & Bhavani, 2017). According to Mehta & Bhavani, (2017), to overcome this shortcoming, auditors should use more than one forensic tool to detect fraudulent activities in financial statement.

To this extent, Mark Nigrini advocated for the use of Benford’s Law to audit financial statements (Nigrini, 2012; Durtschi, Hillison, & Pacini, 2004), using Benford’s Law digit analysis to conduct forensic audits on financial statements. Its utilization in picking up errors and fraudulent activities has gained interest due to its ability to identify anomalies that could otherwise not be noticed Antonio (2023). Mehta & Bhavani (2017) used Benford’s Law with two other forensic tools in his investigation of Toshiba Corporation. The aim of the authors’ study was to assess the efficacy of the tools in the study in detecting fraudulent financial statements and to compare the results to determine the most useful (Mehta & Bhavani, 2017). The study suggested that although Benford’s Law can detect fraud and errors in financial statements, it should be used with other forensic tools to increase chances of detecting fraud. In their conclusion, they found that although BL can indicate possible red flags in the data. However, the method cannot pinpoint the where exactly fraud took place. The authors also state that BL should be used on day-to-day transactions and not on financial statements (Mehta & Bhavani, 2017). This study, although consistent with other studies before it on

Toshiba, could have benefitted from more case studies to make it robust by testing BL with other cases.

In his study of the financial statements of two companies, Antonio (2023), concluded that Benford's Law was useful in detecting errors and fraud based on the observation of significantly large deviations from expected proportions to observed proportions. Where these significant deviations occurred, manual audit was conducted and found there were errors due to manual input of prices and manipulation of prices by salesmen for personal gain in the case of the distributor company from two of its branches. The study found the use of Benford's Law compelling and efficient in detecting fraud. The study noted two limitations of BL as the need for large dataset and the fact that some fraud may not deviate from BL expected distribution. More case studies could have assisted in making the study robust. Also, the study only employed the first digit test for analysis, which according to Nigrini (2012) does not tell much about the dataset.

da Silva Azevedo, Gonçalves, & de Mesquita Spinola, (2021) used the first digit and the first-two digit tests together with statistical tests of conformity which included the Z-statistic, the chi-square test and the MAD in the case of four Brazilian welfare program to detect fraud in their payment's accounts. They found the first digit test failed to conform to BL, including the chi-square test. However, when using the aggregated data grouped into municipalities the data conformed to BL tests for the first-two digit test and the statistical tests. This result is in concurrence with Nigrini who advocates for the use of aggregated data for BL tests. This method of using BL and statistical testing is similar to that used by Druică et. al., (2018) in their analysis of the banking accounts.

Druică, Oancea & Vâlsan, (2018) used BL first-two digit test and six other statistical methods to analyse Romanian bank deposits for a period of 13 years, from bank branches across the country. For their study, there was no prior expectation on where the results should point. The results did not conform to BL while the MAD returned marginal conformity and excess MAD also returning marginal nonconformity. The six null hypothesis significance tests all rejected the null hypothesis of conformity for the first-two digit test. This study is robust for two main reasons; (i) it uses longitudinal data spanning 13 years from many branches across the country, (ii) six different NHST methods are used to confirm the results of Benford's tests. The researchers conclude that results are either inconclusive or unreliable.

In contrast to the cases above, research by Rad, Amiri, Ranjbar, & Salari, (2021) found that Benford's Law cannot detect fraud as the law cannot separate between companies with fraud-risk and those without. Their study used data of 50 companies, 25 with fraud-risk (experimental group) and 25 without fraud-risk (control group). Their findings did not detect anomalies when evaluating the data of the two groups against BL, with all data showing conformance, concluding that BL "has zero prediction accuracy" (Rad, et. al., 2021). However, the use of BL first digit tests is not recommended by Nigrini (2012) for forensic audit purposes. Nigrini states that the first digit test is a high-level test that does not provide much information and might show conformance even when the data has issues. This could be the reason the authors could not find nonconformance in their study, the other factor to consider is that the data used is large and therefore will tend to conform to BL proportions (Nigrini, 2012).

The consensus is that Benford's Law can be used to highlight possible fraud and errors in financial statements. Numerous authors have concluded in their studies that the law must be used with other forensic tools to ensure reliability of the results, as BL does not confirm fraud or error, it only highlights possible areas where further investigation must be conducted (Nigrini, 2012; Mehta & Bhavani, 2017; Gorenc, 2019). The study by Rad et., al., (2021), had little deviations from BL expected proportions for both the experimental and control groups, which was the reason why BL could not detect anomalies. Another reason for their finding could have been that there was no fraud committed even in the experimental group, not to conclude that BL cannot detect fraud and error. The conclusion by Rad et. al., (2021) goes against findings from numerous volumes of articles of research that have proven that BL can be used to at least detect the possibility of errors and fraud in financial statements. Their conclusion should be revised to only state that they could not find anomalies in the research they conducted.

An investigation of municipal owned entities' pre-tax income financial statements for errors or fraud, especially during an election period, was conducted by Capalbo et. al. (2023) using BL first digit tests. The authors conducted a study of the financial statements over a period of ten years spanning over two election cycles. The samples were chosen and split into four subsamples of varying sizes, then split to a period before the local elections and a period during ordinary times. Their findings reveal possible manipulations or errors during the pre-election periods, and the data conformed to BL during the ordinary period. The data during the pre-elections time deviated from BL proportions while that of ordinary time conformed (Capalbo, et. al., 2023). The result was the same when they came to the chi-square statistic and the p-value, in that the chi-square statistic for the pre-election time rejected the null hypothesis while the data outside election time conformed to BL. The data for the period during normal time, that is, outside the elections period, failed to reject the null hypothesis for both the chi-square and the p-value (Capalbo, et. al., 2023). The study, however, did not check the MAD values and instead used the adjusted MAD, as the authors state the MAD is unfit for this type of study. They used the adjusted MAD developed by Cerqueti and Lupi (2021), who claim that the MAD as used by Nigrini (2012) does depend on sample size despite Nigrini stating it does not. Cerqueti and Lupi posit that Nigrini's statement on the sample size would only be valid if the relative frequencies were provided and not estimated from the observed data. The study by Capalbo et. al., (2023) is robust especially their use of adjusted MAD.

2.4 Use in Academic and Scientific Publishing

Due to an increasing number of unfounded claims in academic publications which cast doubts on their validity, Lazebnik & Gorlitsky (2023), conducted a study to identify potential manipulation of research manuscript results. The authors used Benford's Law on the aggregated data in the manuscripts studied instead of the raw data used to produce the manuscripts and found that BL can identify data manipulation. Their findings proved that BL can be used to identify manipulated manuscripts. However, there were limitations where the method produced some false positives, where manipulation free manuscripts were identified as manipulated (Lazebnik & Gorlitsky, 2023). The method did not produce a convincing result on manuscripts with known manipulation and only 18 out of 37 were positively identified. The conclusion was that the method should be used only as an alerting system (Lazebnik & Gorlitsky, 2023). The authors also found that the larger the data was available, the better the conformance to BL and the more manipulated the data, the easier it is to detect fraud. This study shows three of the important factors to consider when using BL, that is the sample size, whether data is aggregated or not and the effect on results of invented numbers.

Tošić & Vičič (2021) studied the use of BL application to analyse scientific research collaboration networks. Their focus was on the Slovenian scientific publications and the networks from the country's research database, where the publications were stored. In their research, where the data produced skewed results, these skewed observations were treated as suspect and was used as entry points for further detailed analysis to try and explain them. The study, however, did find that most fields conformed to BL expected proportions with the exception of papers in social sciences and conference papers in biotechnology. The research was to test the maturity of the research community in Slovenia, it concluded that BL can be used to test for non-isolated cases of deviations. It was also found that the number of manuscripts per researcher conformed with BL. With the discrepancies in humanities and social sciences, a refined method is required to allow these fields to be used with BL.

In Horton, et. al., (2020), researchers investigated the use of BL to differentiate between retracted academic papers that used fraudulent or manipulated data and academic papers that have not been retracted. They used a case of Professor James Hunton, whose 37 papers were retracted because of grave concerns that mis-stated or fabricated datasets were used. The research investigated if Hunton's digits in his retracted papers deviated significantly from BL distribution compared to non-retracted papers (Horton, et. al., 2020). The study applied BL to academic publications, specifically looking at numerical data in research outputs to detect potential fraud or manipulation. By analysing the frequency distribution of first digits in Professor Hunton's published data, the authors found 70% of retracted articles contained anomalous data while 61% of articles not retracted contained possible misstated data. They posit that BL can be used to detect anomalies in research publications. In the study however, the authors state that due to limitations of BL, they cannot conclusively state that the control group data is either free from anomalies or not, the lack of criteria to judge deviations or threshold of acceptability, the fact that not all datasets conform to BL in which case there could be false positives and false negatives and other questionable research practices that may be involved.

Since BL applies to population data such as census data, death and birth rates (Nigrini, 2012), reporting on COVID-19 infection rates must follow BL (Eutsler, et. al., 2023). In studying potential misreporting of COVID-19 reported data in the United States of America, Eutsler et. al., (2023), used the first and second digit tests together with other statistical methods and found there was misreporting of infection rates based on party lines. The study found that there were nonconformances with BL for the first and second digit tests at counties run by the Republican party and those run by the Democratic party conformed with BL, which was suggestive of underreporting in the Republican counties as there was more deviations from BL than in the Democratic counties. The study could have benefitted from the use of the first-two digit tests which provides more information than the first and second digit tests (Nigrini, 2012).

Kruger & Yadavalli (2017) explored the use of Benford's Law, including its theoretical foundation, practical applications, and limitations in detecting data anomalies and manipulation, their study presented a balanced examination of the statistical phenomenon called BL. They applied Benford's Law to practical cases, such as the Fisher-Mendel controversy and scientific publications, demonstrating the law's utility in diverse fields. Their exploration of engineering journal data was particularly innovative, as it extended BL beyond traditional forensic accounting and finance. The study investigated the data in six published papers in the engineering journal using Benford's first digit distribution together with the chi-square and p-value tests. In their study, papers two, four and five did not perform well to Benford's test and the chi-square tests, this was attributed to outliers in the dataset. Papers one, three and six passed all tests and were considered conforming to BL. The authors posit that the reasons for deviations from BL do not signal proof of existence of data tempering. The results should be interpreted with care, and they indicate where further investigation is required.

The authors acknowledge the law's limitations, such as its inapplicability to certain datasets. The statistical and graphical analysis used in evaluating datasets in their study is well-documented and aligns with best practices in the field. The application of statistical tests like chi-square and proportionality tests provides a robust framework for evaluating dataset conformity to BL, with the investigation into exceptions, such as outliers and limited sample sizes, highlights the complexity of applying Benford's Law in practice.

The study, however, analysed a small sample of datasets of six papers from one journal and Mendel's data and could have benefitted from a broader dataset selection, perhaps across multiple fields or regions, could have strengthened the generalizability of findings. While the paper highlights Benford's Law as a useful tool for identifying anomalies. The study does not provide a clear guidance on how practitioners should integrate BL into real-world processes, such as peer review or regulatory audits. Kruger and Yadavalli (2017) relied heavily on graphical evidence such as histograms and frequency plots. This heavy reliance on visual interpretation could reduce objectivity. A more quantitative focus, supported by confidence intervals or robustness checks, would have enhanced reliability of their study.

Kruger and Yadavalli (2017) provide an insightful and well-rounded examination of Benford's Law, highlighting the method's strengths as a diagnostic tool while acknowledging its limitations. However, their study could have benefited from a broader scope, deeper analysis of exceptions, and more practical recommendations for real-world applications. Despite these shortcomings, the paper succeeds in demonstrating the value of Benford's Law in detecting anomalies and fostering transparency in data integrity.

2.5 Benford's Law in Election Fraud Detection

Several research papers have been published that have investigated the use of BL in election forensic auditing. The researchers have approached the use of BL from various angles, others have used the first digit (Kukeli & Karunaratne, 2020), the second digit (Agyemang, et. al., 2023; Mebane & Kalinin, 2009), the first-two digits (Agyemang, et. al., 2023). Mebane (2013) used both the second digit and the last digit in the analysis of Russian elections between 2004 and 2012, while authors Pericchi and Torres (2011) used the first and second digits in their analysis of electoral data. The analysis using BL is always supported by statistical tests of conformity for a robust analysis. It is expected that the digit distribution proportions will deviate from BL digit distribution proportions where data has errors or has been tempered with.

Mebane (2006) studied the Florida vote counts in 2004 and the Mexican presidential votes of 2006 to illustrate BL second digit tests. The aim of the study was to show that the second digit test can show when there are fraud patterns in the data. Mebane used the precinct level votes and the voting machine votes. The author's findings state that although BL second digit tests are promising to become a standard tool to detect election fraud, the test cannot detect all kinds of fraud and there is a possibility for false positives. Mebane (2006) also states that the tests are not suitable for machine level vote counts, concurring with Pericchi, & Torres, (2011) on their study in the USA, Puerto Rico and Venezuela where the electronic votes returned mixed results when tested BL. The study could have benefitted from more real world data instead of simulations to validate its findings. It also does not deal with the problem of false positives and negatives.

In their investigation of Sri Lanka's 2010 presidential and general elections, Kukeli & Karunaratne (2020) used BL first digit tests and found that the data did not conform to BL expected proportions. The researchers also studied the mean, median, standard deviation and skewness of the data. Their conclusion was that since the actual proportions did not match BL expected proportions, there must have been some manipulation of the election results. This study could have benefitted from more elections being studied, instead of one. There is no comparison done from other election results to support the conclusion and make the study robust. The use of BL first digit is also not suggested for election forensic as it is a high-level test (Nigrini, 2012).

Agyemang, Nortey, Minkah & Asah-Asante (2023), used BL first-two digit test and a dip test to investigate anomalies in the Ghana 2012 and 2020 elections, with a focus on the parties' strongholds and swing regions where elections are normally decided. These regions were selected based on allegations of over-voting and cheating. The authors detected anomalies in the 2012 elections in the swing regions and in the stronghold regions of the two main parties and were able to quantify the suspected anomalies in both the 2012 and 2020 Ghanian elections. However, the use of Benford's Law is proposed only as baseline test to determine if the results require further investigation of fraud (Agyemang, et. al., 2023). Druică, Oancea & Vălsan (2018), also urge caution on the use of null hypothesis significance testing methods for the detection of fraud and error in auditing.

In their investigation of the 2015 and 2019 elections in Nigeria which were alleged to be fraudulent due to reports of underage voting, undue influence, missing and fake results sheets, ballot box snatching and violation of electoral rules among others, Tunmibi and Olatokun (2021), found anomalies in the reported results. The authors used BL second-digit tests to investigate the declared results and the voter turnout numbers. Their findings showed that there were deviations of the second-digit distributions in the vote counts of two political parties from the expected BL second-digit distribution. It was also found that the second-digit distribution of registered voters and total vote counts showed different patterns from those of Benford's Law distribution of the second digit (Tunmibi & Olatokun, 2021), a possible case of invented numbers. However, this study had shortcomings in that it only used the second digit analysis with the Spearman Rank Correlation tests for their analysis. It could have benefitted from using other Benford's digit tests like the first-two digit test. According to Nigrini (2012), the second digit test is a high-level test of reasonableness and cannot be of much use in forensics. Tunmibi and Olatokun (2021) other shortcoming from their findings is the expectation that they seem to ignore the voter turnout numbers and their relation to vote numbers. Since the voter turnout numbers do not conform to BL, it should be expected that the vote counts will also not conform to BL.

The US presidential elections were investigated by several researchers for conformity to BL. This was a result of numerous allegations of fraud in the declared results that were challenged even in courts. Shanaev, Shuraeva, & Ghimire (2020) used the first digit and first-two digit tests to assess conformity of the declared results to BL proportions. They eliminate three issues that Deckert et. al., stated as possibilities of data not conforming to BL. Issues such as (i) strategic voting – for this they stated the decentralised electoral system of the US and the fact that it is unlikely in established democracies to employ such malicious manipulation; (ii) coordinated manipulation of data – the data from the 2008, 2012 and 2016 elections was used was used as baseline to check for robustness of the 2020 findings; and (iii) the problem of the high power of the chi-square test for large samples – the Kuiper test and the Monte Carlo simulations were used to counter the chi-square problem. The authors found that there were anomalies with the Republican vote counts in blue states, all states won by the Democratic candidate and Democratic vote counts in swing states. Swing states won by the Republican candidate had no anomalies and swing states won by the Democratic candidate had anomalies.

However, these findings are disputed (Mebane, 2020; Groharing & McCune, 2020; Eggers, Garro & Grimmer, 2021). Mebane (2020) states that the use of first digit at precinct level is not useful for detecting anomalies. Mebane states that the purported anomaly in the Trump/Pence is a result of low vote counts which were in the single digits whereas the Biden/Harris votes had high vote counts. Groharing and McCune (2020) found there was nonconformance in the Biden's chi-square in Pennsylvania but concluded that it is not an indication of fraud. The researchers in this case seem to be arguing with their own findings in that they see nonconformance but try to argue that it is not a problem. A Reuters Fact Check article (Reuters, 15 November 2020) concurs with Groharing and McCune in that deviation from BL does not prove fraud.

According to Eggers et. al. (2021), the reported anomalous issue is either not fact or not anomalous. The authors state that the fact that Biden/Harris received more votes while winning much less counties, whereas Trump won much more counties and received lesser votes in his 2016 victory, does not point to anomaly, but the fact was Biden/Harris won counties with big voter numbers. Groharing and McCune also state that there was no fraud in the elections. Precinct level vote counts should not necessarily conform to BL as "precinct-level votes generally do not span multiple orders of magnitude" (Groharing & McCune, 2020). However, this argument fails as in the same article the authors admit that Trump's vote count conforms to BL. The authors posit that only county votes should follow Benford. However, from the analysis conducted for the Pennsylvania votes, Trump's chi-square is less than the critical value whereas Biden's is greater than the critical value, but the authors conclude there are no anomalies. Groharing and McCune seem to have made a conclusion in their paper and are trying to defend it. This is based on how they try to compare Biden's assessment in one county with Trump's assessment in different county and then make their findings. The 2020 US presidential elections require a more detailed study due to anomalies stated by all researchers and the lack of consensus on their conclusions. It is clear that there are issues with the Democratic votes declared that some researchers do not want to admit due to bias, whereas they admit to fraud in other areas where similar research is done.

In the 2004 Ukraine elections, Deckert et. al. (2011), called what took place during elections "as close to a controlled experiment we are likely to find in the social sciences". The elections went through three rounds with the first round not producing a clear winner. The second round produced a winner; however, the results were later nullified by the courts. In the second round re-elections, voter turnout fit BL whereas in the succeeding third round voter turnout did not. The second digit mean in the second round was 4.22 and in the third was 4.39. With BL second digit mean being 4.187, the second round seems more conforming than the third round. However, there were about 1,5 million stuffed ballot papers in the second round (Deckert, et. al., 2011). The 2007 elections were also marred by fraud, and the analysis using BL second digit mean showed that there was no fraud in regions where fraud was proven. This shows the law's propensity to commit both type 1 and type 2 errors (Deckert, et. al., 2011). Based on these observations and similar observations in Russia, the issue of false positives and negatives is made, concluding that BL is not useful in election forensics as a standalone tool. In reply to the research by Deckert and colleagues, Mebane (2011) argues that the researchers used incorrect data when they used aggregated data at precinct level and that the deviation from the second digit mean does not have a clear meaning in text.

The Russian elections have been studied extensively for their reported or perceived fraud. Here we look at some of them and the conclusions that were reached for each. Mebane (2013) conducted a Benford's analysis of Russia's elections held in the years 2004 to 2012 to investigate if there was any fraud in those elections. The tests were run at precinct level for the party and for the candidates at each voting cycle, using the second digit and last digit tests. The author found that for the 2012 elections, vote counts on areas where there was a turnout divisible by five, there were significant artificial vote counts. This suggests invented numbers. However, the findings suggested that the results where Putin was involved and those of his party, United Russia, were not manipulated. This result went against the expectation that the Russian elections during those years were riddled with fraud. The one election where results pointed to manipulation were the results where Medvedev was running for president.

The 2016 Russian elections were preceded by numerous regulations being passed. From limitation on domestic observers, changing the election season so that most people critical of the government were not available to anti-protest laws were passed (Mebane, 2017). The United Russia won by 54.2%, securing 343 out of 450 seats in the Duma – the Russian parliament.

In their article, Deckert, Myagkov & Ordeshook (2011) do not agree with the use of BL in detecting fraud in elections. They posit that the method is problematic at best and misleading at worst when used as an electoral forensic tool. The authors also state the propensity of the law to commit Type 1 and Type 2 errors and compare its success rate to a toss of a coin (Deckert, et. al., 2011). Of significance in their argument, the authors investigated the second digit distribution proportions of votes for each of the two candidates and the voter turnout against second digit BL distribution proportions in the 2004 and 2007 Ukraine elections and Russia's 2008 elections. Mebane (2009) also found anomalies with the Russian Duma elections of 2003 and 2007 and the presidential elections of 2004 and 2008. He found that the anomalies were getting worse as the period progressed, with voter turnout numbers being inflated in most areas.

The cases above of Ukraine and Russia could have benefited more with the use of goodness-of-fit tests and the use of MAD tests. The researchers' use of BL digit means as the only analysis tool is not sufficient. Also, Nigrini (2012) states that the first-two digit tests are the preferred tests for forensic analysis, which the authors did not use. BL should also be used as one of the tools not as a standalone tool. Mebane (2013) study findings yielded contradictory implications, such as instances where Putin's votes conformed to the digit tests while other candidates' votes do not, and lack of clarity in the interpretation of results which weakens the overall argument. The study also provides limited guidance for practical use on how to implement the findings in real-world electoral monitoring.

Pericchi and Torres (2011) used the first digit and the second digit to investigate the data from the 2004 US, Puerto Rico (1996, 2000 and 2004), 2000 Venezuela elections and the 2004 Venezuelan presidential recall referendum for both manual and electronic elections. For statistical tests, the authors used a modified p-value calculation based on Robust Bayesian Factor. Their findings showed that the data was a good fit to BL for most of the data with the nonconformity coming from the electronic NO votes in the recall referendum.

In investigating irregularities of the Bundestag of the unified Germany's 1990 to 2005 elections, Breunig & Goerres (2011) used BL second digit analysis to check for conformity. A chi-square goodness of fit test was used to confirm the results obtained from BL analysis. The analysis was divided into candidate per district level, party at district level and party at regional level. Four anomalies were identified at candidate level, nine for party at district level and 51 at national level. A deeper investigation revealed an elevated level of nonconformances in West Germany than in the East. The authors concluded that the anomalies were because of systematic fraud by the parties involved was because of partisan volunteers working as electoral officers.

Figueiredo Filho, Silva and Carvalho (2022) analysed the vote counts in the 2018 Brazilian presidential elections using BL second digit, last digit mean and other statistical tools. The study followed President Bolsonaro’s claims of electoral fraud. The second digit tests for the three top candidates did not show any nonconformances with each candidates’ proportions deviating only slightly from BL expected proportions with no suspicious spikes noted, finding no systematic fraud in the counts. The last digit mean did not show anomalies with all the candidates’ actual proportions having means close to the theoretical mean of BL last digit mean (Figueiredo Filho, et. al., 2022). The study provides a robust framework for using BL to detect errors and fraud by using cross-verification of data aggregation between polling stations and municipalities. The joint application of diverse techniques is a methodological innovation, showcasing the interplay between digit-focused and regression-based methods and demonstrates the utility of statistical tools in countering unfounded claims of electoral fraud. Examination of previous elections in the country could have provided a more robust conclusion as this could have shown trends in elections and prove that all elections are similar. The study provides a strong rebuttal to claims of fraud in the elections.

A study of the relationship between political campaign donations and election outcomes was conducted using BL first digit tests, chi-square, and regression analysis. The authors found significant deviations from BL expected proportions which meant there may have been manipulations in the donations. Their findings also found there was a direct relation between the amount of money a candidate received and the chances of winning an election. This study focused more on regression analysis than on BL. However, BL first digit tests did show anomalies in the data. The study, like the 2018 Brazilian election votes investigation above, could have benefitted more from using more data. For instance, using data from the period outside the election period like the study done by Capalbo et. al. (2023) could have made the study results more robust.

Hicken & Mebane (2017) conducted studies of three elections in two countries, South Africa in 2014 and Bangladesh in 2001, using BL second digit tests and other statistical tests. In the South African case, the authors found the ANC numbers did not differ from the no-fraud values while the DA and EFF did differ. In Bangladesh the authors suggest there may have been fraud or strategic voting in the elections, leading to nonconformance with the second digit tests.

Ringsholm (2022) investigated of the 2022 Hungarian elections using BL first and second digit tests. In their study, the author used the Denmark's 2019 elections as the control group as these had "really low probability for election fraud" Ringsholm (2022). In testing the Danish elections, the author found the data conformed to BL. The Hungarian elections first digit test did not conform to BL while the second digit test conformed. The author states that the reason for the first digit not conforming to BL is the fact that there were a lot of digits starting with twenty-thousands to forty-thousands, leading to spikes in 2s, 3s and 4s. This study did not use any other statistical methods and shows that the simplification of using only the graphical representation of comparison between BL and data being analysed can lead to incorrect conclusions. BL tests should be used with other statistical methods to reach conclusions.

Table 3 Summary of Literature Review

Author	Method Used	Findings
Agyemang, Nortey, Minkah & Asah-Asante (2023)	first-two digits	detected anomalies in the 2012 elections and 2020
Antonio (2023)	first digit test	Benford's Law was useful in detecting errors and fraud
Breunig & Goerres (2011)	second digit	elevated level of nonconformances in West Germany than in the East
Capalbo et. al. (2023)	BL first digit tests	findings reveal possible manipulations or errors during the pre-election periods, and the data conformed to BL during the ordinary period
da Silva Azevedo, Gonçalves, & de Mesquita Spinola, (2021)	first digit and the first-two digit tests	They found the first digit test failed to conform to BL,
Deckert, Myagkov & Ordeshook (2011)	second digit	BL is not useful in election forensics as a standalone tool, propensity to commit both type 1 and type 2 errors
Druică, Oancea & Vâlsan, (2018)	first-two digit	The results did not conform to BL while the MAD returned marginal conformity. The six null hypothesis significance tests all rejected the null hypothesis of conformity for the first-two digit test
Durtschi, Hillison, & Pacini, 2004	first digit	useful in identifying suspect accounts
Eutsler, Harris, Williams, Cornejo, (2023)	first and second digit tests	nonconformances with BL for the first and second digit tests at counties run by the Republican

Author	Method Used	Findings
Figueiredo Filho, Silva and Carvalho (2022)	second digit, last digit mean	second digit tests for the three top candidates did not show any nonconformances. The last digit mean did not show anomalies with all the candidates' actual proportions having means close to the theoretical mean of BL last digit mean
Groharing & McCune, 2020		nonconformance in the Biden's chi-square in Pennsylvania but concluded that it is not an indication of fraud
Hicken & Mebane (2017)	first and last digit	ANC conforms while the DA and EFF did not.
Horton, Kumar, Wood, (2020)	first digits	BL can be used to detect anomalies in research publications
Kruger and Yadavalli (2017)	first digit	Papers one, three and six passed all tests and were considered conforming to BL. The authors posit that the reasons for deviations from BL do not signal proof of existence of data tempering
Kukeli & Karunaratne, 2020	first digit	data did not conform to BL expected proportions
Lazebnik & Gorlitsky (2023)		Their findings proved that BL can be used to identify manipulated manuscripts
Mebane (2006)	second digit	cannot detect all kinds of electoral fraud
Mebane (2013)	second digit and the last digit	results where Putin was involved and those of his party, United Russia, were not manipulated and conformed to BL
Mebane (2020)	First digit	pricint level vote do not follow BL
Mebane, Alvarez, Hall, Hyde, (2008)	second digit	Some pricincts follow BL while others do not conform to BL.
Mehta & Bhavani, 2017	first and second digit	BL can indicate possible red flags in the data
Pericchi and Torres (2011)	first and second digits	cannot detect all kinds of electoral fraud
Rad, Amiri, Ranjbar, & Salari, (2021)	BL first digit tests	Their findings did not detect anomalies when evaluating the data of the two groups against BL

Author	Method Used	Findings
Ringsholm (2022)	first and second digit	first digit did not conform while the second digit conformed.
Shanaev, Shuraeva, & Ghimire (2020)	first-two digit tests	there were anomalies with the Republican vote counts in blue states, all states won by the Democratic candidate and Democratic vote counts in swing states
Tošić & Vičič (2021)		most fields conformed to BL expected proportions
Tunmibi and Olatokun (2021),	BL second-digit tests	found anomalies in the reported results. deviations of the second-digit distributions

2.6 Conclusion

BL has the potential to detect fraud and error in financial statements, election results and academic publications. Mehta & Bhavani, (2017), conclude that BL is useful to detecting digit fraud and is useful for day-to-day transactions. The method cannot accurately detect where fraud took place, but by identifying anomalies, it directs auditors on where to pay more attention (Mehta & Bhavani, 2017). In his research, Walker (2022) advises that researchers should “curb some enthusiasm” for the detection of material misstatements in financial statements. This is based on the nature of findings of BL, in that BL does not always detect fraud and misstatements. Instead, using BL can help point out if there are anomalies in the data, however this is not proof of fraud. The method has the propensity of producing Type 1 and Type 2 errors, in that it detects fraud where there is none or it does not detect fraud where fraud took place.

Caution is repeated by Druică, Oancea & Vălsan (2018). They found the null hypothesis was rejected by the null hypothesis significance tests while the MAD results showed marginal conformity to BL. According to Antonio (2023), shortcomings with BL highlighted include the need for large datasets for statistical significance, some types of fraud may not exhibit deviations from Benford. Agyemang, et. al., (2023) also posit that Benford’s Law should be used as a “dip test” and progressively dig deeper with other statistical and machine learning tests to prove fraud in election data. The need for machine learning algorithms and the development of automated tools and software as an additional tool to Benford’s Law is also emphasised by Antonio (2023). All the studies conducted have acknowledged the problems such as sample size, false positives and false negatives, the interpretation of the results. However, none have proposed methods to deal with the limitations of BL tests.

3. Research Question and Hypothesis

In the study of whether Benford's Law can be useful in detecting fraud and error in declared election results, the research will use two countries' where fraud was alleged in the elections. The Kenyan elections were nullified by the Supreme Court of Kenya, ordering a rerun of the elections (Data Report of 2017 Elections, IECB, 2017). The Zimbabwean elections were marred by pre-election intimidation and violence and in 2018 the opposition rejected the results and unsuccessfully challenged the results at the Constitutional Court (Election Situation Room, 2018; European Union Election Observer Mission, 2018).

The question this research aim to answer is:

(i) To what extent can Benford's Law, combined with statistical tests, reliably detect electoral fraud in different electoral contexts?

To answer the research question, hypothesis testing was employed. The following null and alternative hypothesis were assessed in the research:

The null hypothesis:

H_0: "The digit frequency distribution proportions in election data conforms to Benford's Law frequency distribution proportions".

Alternative hypothesis:

H_1: "The digit frequency distribution proportions in election data does not conform to Benford's Law frequency distribution proportions".

A single null hypothesis testing will be conducted for two cases, firstly where there were allegations of fraud in the declared election results, and secondly where there was no suspected fraud. The second case will be used as a control case to evaluate our results.

4. Research Methodology and Design

4.1 Introduction

This section details the choice of research methodology that was used in the research, the statistical tools used to test the null hypothesis and the timeline.

4.2 Research Methodology and Design

An exploratory study will be conducted to investigate the if there is a statistical or mathematical tool that can be used to detect fraud or error in the declared election results. The goal of the study is to provide a tentative answer to the research question through hypothesis testing. The case study method will be used in the research. By employing a case study method, the research will seek to explain why there are anomalies in the data collected and what is the cause of such anomalies. Each of the countries to be studied will be treated as a separate case, meaning there will be four cases in this research to be studied. The goal of the research is finding a tool or method of conducting forensic audits on declared election results that can be used by authorities, observers, and business. The research philosophy used was pragmatism. Pragmatism is used as it contributes practical solutions. The research will be guided by the research questions and objectives to guide the philosophy of the research (Saunders & Lewis, 2nd Ed., 2018, page 111). A pragmatist researcher focuses on finding what works in practical situations.

A deductive approach was used in the research. The research evaluated the Benford's Law theory of the first, second and first-two digit tests in detecting election fraud and misstatement or error by testing a hypothesis to answer the research questions (Saunders & Lewis, 2nd Ed., 2018, page 112). A deductive approach was utilised since it starts by stating Benford's Law (established theory) and then through a mathematical and statistical analysis, it proves or disproves the theory using hypothesis testing.

A case study strategy, mono quantitative method was used in the research and employed cross-sectional secondary data gathering. The cross-sectional data was used to also conduct a longitudinal study over time in Zimbabwe, Botswana, and South Africa. Data from various past election results was used in the research. The case study strategy is suitable for exploratory research (Saunders & Lewis, 2nd Ed., 2018, page 122).

4.3 Population

In various countries, there have usually been complains about the election results whenever these were released by the electoral commissions of countries. Election observers do not audit the declared votes. They observe the voting process at voting stations, the vote counting, and declaration (Independent Electoral Commission of South Africa, n. d.). Election observers help legitimise the elections and give the public the confidence in the results when they are announced. These observer missions, however, do not conduct forensic audits on the results.

The population for the research will be the previously declared election results from elections that were conducted in each country selected. From the population the data will be split into two categories. The first category is the election results where there were allegations of fraud in the declared election results and the second category will be election results where there was no fraud suspected or alleged.

An empirical study was done on the Zimbabwe elections from the 2013 elections to the last elections of 2023. Zimbabwe has become notorious for political violence, especially during the election periods, with elections being disputed by the opposition parties and supporters, and various complains about voter rights suppression (Crush Them Like Lice, 2023). The South African election data was gathered from the first elections held in 1994 including the last elections of the year 2024. However, the election data from 1994 and 1999 elections was too aggregated for the study and the researcher decided to use the election data from 2004 to 2024 instead. The Botswana democratic elections from the 2014 to the 2019 elections was used, and the Kenyan democratic elections from the year 2017 elections in which were either the declared results were disputed or where there were allegations or suspected of fraud. The control group is added following Diekmann and Jann (2010) paper. According to Diekmann & Jann (2010), it must be demonstrated that the non-manipulated data conforms to BL, while the manipulated data conforms to some other distribution. This approach will therefore ascertain that BL is a valid tool for fraud detection. The South African and Botswana election data is used as the control group as these are perceived as working democracies to the researcher.

4.4 Unit of analysis

The declared election results that are publicly available are the researcher's unit of analysis. The research evaluated whether Benford's Law can be used as a statistical or mathematical tool to conduct forensic auditing of the declared election results to detect fraud and error.

4.5 Sampling method and size

A non-probability sampling method was used to select the sample, a purposive sampling method. In this sampling method, each member or unit of the population does not have a specific probability of being selected. Judgement was used to select the samples. This type of sampling was used in the research since the researcher knew the attributes and characteristics (Sharma, 2017) of the sample chosen for the research (judgement), which is error or fraud in the declared election results. There have been several election results in countries such as Zimbabwe where the opposition parties and sometimes election observers have alleged fraud in the results declared by the electoral commission. These declared results were evaluated against Benford's Law to investigate their conformity to Benford's Law digit distribution frequency. As a control, the declared results from South Africa and Botswana elections from the previous national elections were used. According to Aguboshim (2021), it is difficult to define data saturation, assessment and reporting of data saturation is inconsistent as it has different meaning to different researchers.

For this research, samples were chosen from the democratic elections held in Zimbabwe from the 2013, 2018 and 2023 elections and from Kenya the 2017 elections were used. For the control groups, the South African sample was chosen from 2014 to the 2024 elections and for Botswana, the 2014 and 2029 results were used.

The research brief did not state the size of the sample required. For the research, election data from democratic, multi-party elections was used. The samples were divided into two categories. The first category was elections where fraud has been alleged, where the sample included Zimbabwe and Kenya. The second category was elections where no fraud was alleged or suspected, this group includes South Africa and Botswana.

4.6 Measurement instrument

In this study, to investigate conformance of the declared voting data with the Benford's Law, digit tests were conducted and then battery of five null hypothesis significance testing (NHST) was conducted. Conformity of the election data digit distribution proportions was evaluated against Benford's Law first digit, second digit and first-two digit distribution proportions. The goodness-of-fit tests were then conducted, which included the chi-square, Kolmogorov-Smirnov, the Z-Statistics test and the p-value test statistics (Druică, et. al., 2018; da Silva, et. al., 2021). The following sequence of tests was conducted:

4.6.1 Benford Benford's Law Tests

a) The first digit test was conducted to establish if the voting data conforms with BL. This is a high-level test used when there are a few records (Nigrini, 2012). The first digit test does not convey much information.

b) The second digit test is also a high level test of reasonableness. However, in his research on the US and Russian elections, Mebane (2007; 2013) used the second digit analysis as part of his research. This researcher would use the second digit test as a definite conclusion is sought.

c) The first-two digit tests are recommended by Nigrini (2012). According to Nigrini (2012) it detects abnormalities and biases, and it contains the information than the graphs of the first and second digits.

4.6.2 Mean Absolute Deviation (MAD) will be used as the first test of deviations of the observed proportions from BL expected proportions. The MAD results will be compared with the limits published in Nigrini (2012).

4.6.3 Z-Statistics Test is used to assess digit-by-digit the difference between BL expected proportions and observed proportions. A critical value of 1.96 will be used to assess each digit's conformance.

4.6.4 Pearson's Chi-squared Test was used to assess for goodness-of-fit for each group of tests conducted. A chi-square critical value will be calculated and used as a limit above which conformance fails.

4.6.5 P-value Test Statistic assessed the probability of the null hypothesis being accepted. The p-value is compared to alpha equal to 0,05.

4.6.6 Kolmogorov-Smirnov Test (K-S Test) is used only for the first-two digits to confirm conformity with BL using the K-S critical value.

The tests will be run on the Excel software using a spreadsheet adapted from Nigrini (2012) called NigriniCycle.xlsx.

4.7 Data gathering process.

Data for the research was collected from the respective country's election authorities. The data is considered secondary as it was collected, gathered, and recorded for election vote counts as a primary purpose, and the researcher used that data for this research without collecting it themselves. The South African data was collected from the Independent Electoral Commission (https://results.elections.org.za/home/) of South Africa for 2004 until 2024. Zimbabwe dataset was downloaded from the Zimbabwe Electoral Commission (https://www.zec.org.zw/download-category/election-results/) for the elections held in 2013 until 2023. The other electoral authorities from which data was downloaded are the

Independent Electoral Commission of Botswana (https://www.iec.gov.bw/index.php/election-results/) for the in 2014 and 2019, Kenya's Independent Electoral and Boundaries Commission (https://www.iebc.or.ke/media/?press) for the 2017 elections.

The South African and Botswana election results were used as the control group in the study. Since there has not been a case of alleged fraud in the declared South African election results and the Botswana elections, it is expected that these results will follow Benford's probability distribution proportions. The choice of South Africa and Botswana as control group is supported by the general peaceful nature during the campaign season and the fact that the results are generally declared free and fair. A report by the Electoral Integrity Project scores the election integrity in South Africa at 69 points and Botswana at 59 points in the Perceptions of Electoral Integrity index (PEI) for a period from 2012 to 2022 (Garnett, James, MacGregor, & Caal-Lam, 2023). Kenya has the PEI index of 56 and Zimbabwe a low 41 on the index.

After the data was gathered, it was prepared by "cleansing" and sorted for it to be usable. All the rows with missing digits and zeros were deleted (Antonio, 2023). The rows with zeros do not add to the analysis and those with missing values skew the study (Antonio, 2023). The data collected or downloaded from the electoral authorities is considered "clean", requiring minimum intervention, however there is sorting and arranging that will be required.

4.8 Data preparation

The data must be prepared before it is used. The totals and rejected votes cast were deleted from all the lists as they are not useful. Data was then separated into the winning party, the runner-up party and either the rest of the parties and independent candidates were grouped together, forming Others group (Zimbabwe and Kenya), or the third largest party was used (Botswana and South Africa. For 2004 until 2024 in South Africa, only the ANC was used as it is the one party's digit performance that was considered important for the study. In all the cases, data will be separated into three groups for assessment. Although not used in the study directly, candidate or party percentages achieved were not deleted as these were used to determine the parties or candidates to include in the study. Further details on which parties or candidates were chosen is presented in Chapter 5.

In South Africa, we investigate the data from the 2004 elections until the 2024 elections recently held. The election data from the 1994 and 1999 elections was not sufficient for the type of analysis that was run in the study, full data, only the totals per province and nationally. This results only nine data points to run per party and will not provide any information. Where available, the number of registered voters was analysed to see if it followed Benford's distribution frequency for the first digit.

4.9 Analysis approach

The datasets were evaluated to see if it conforms to Benford's Law using digit analysis. First digit, second digit and first two-digit analysis will be conducted in the research. Data will be analysed using the Microsoft Excel adapted from Nigrini (2012). Differences between observed frequencies and expected frequencies will be evaluated to check for conformance of the dataset to Benford's Law. The first digit and the second digit tests are high level tests. The first digit test does not provide much information, and the second digit test can detect biases in data rounding, but it also does not provide much information about the data (Nigrini, 2012).

The first-two digit test is the preferred primary BL test as it provides more detail than the high level tests above. The analysis will use inferential statistics to compare the differences between the frequency distributions of the election data and Benford's Law frequency distribution. This study will use the three BL digit tests as stated above as well as the statistical tests detailed in 4.9.1 to 4.9.5 below. Table 1 below illustrates the probability distributions of the digits from one (1) to nine (9) based Benford's Law.

Table 4: Frequency of Digits

Digit	First Place	Second Place
0	0.000	0.120
1	0.301	0.114
2	0.176	0.108
3	0.125	0.104
4	0.097	0.100
5	0.079	0.097
6	0.067	0.093
7	0.058	0.090
8	0.051	0.088
9	0.046	0.085

Source: Frank Benford (1938)

4.9.1 Pearson's chi-squared (χ^2) Goodness-of-fit test will be used in the research. The chi-squared statistic is used to assess for "statistically significant", differences in the observed data distribution compared to the expected data distribution (Antonio, 2023). The test is based on the frequency count, comparing observed against expected frequency of the null hypothesis (Antonio, 2023). The "Goodness-of-fit" test allows us to assess if the voting data follows the Benford distribution pattern.

$$\chi^2 = \sum_{i=1}^{K} \left(\frac{(AP - EP)^2}{EP} \right) \quad \ldots (4)$$

(Where: AP = actual proportion, EP = expected proportion, K = number of bins (or degrees of freedom) which is eight (8) for the first digit test, nine (9) for the second digit test and 89 for the first-two digits.)

The null hypothesis is rejected if the calculated chi-square statistic is greater than the chi-square critical value, concluding that our data does not conform to BL. A chi-square statistic lower than the calculated chi-square critical value results in a conformity conclusion. (Nigrini, 2012, p.153-154).

4.9.2 Kolmogorov-Smirnoff Test (K-S Test) was used to assess if the probability distribution of the of the vote data differs from that of the Benford Law probability distribution. Kaiser, (2019) expresses the K-S test as:

$$D_n = sup_x \, |F_n(x) - F(x))| \quad \ldots (5)$$

(Where: $F_n(x) = Pr_n(x \leq j)$; $F(x) = p(x \leq j)$)

The K-S test uses maximum absolute difference between the cumulative observed data distribution and the cumulative control data distribution, Benford's distribution in this case. A K-S critical value is calculated and if the supremum of the cumulative differences is higher than the critical value, we reject the null hypothesis. If the supremum is less than critical value, the conclusion is that there is not enough evidence to reject the null hypothesis. The equation for the critical value is calculated using the following equation:

$$KS \; Critical \; value = 1.36/\sqrt{N} \quad \ldots (6)$$

The K-S tests will be conducted on the first-two digits only.

4.9.3 The adjusted **Mean Absolute Deviation (MAD)** will be used to measure the average difference between observed and theoretical frequency distributions (Capalbo, Galati, Lupi, & Smarra, 2023; Horton, et. al., 2020). It is the measure of conformity to Benford's Law (Horton, et. al., 2020; Nigrini, 2012)

$$MAD = \frac{\sum_{i=1}^{K} |AP - EP|}{K} \qquad \dots (7)$$

(Where: AP = actual proportion, EP = expected proportion, K = number of bins)

The number of bins (K) from ten to 99 is 90, therefore K = 90 for the study. The MAD measure ignores the number of records and measures accuracy in the same units as the data (Nigrini, 2012). The higher the calculated MAD, the higher the difference between expected and actual proportions (Mehta & Bhavani, 2017).

Table 5: MAD Critical Values

Critical Values and Conclusions for Various MAD Values		
Digits	**Range**	**Conclusion**
First Digits	0,000 to 0,006 0,006 to 0,012 0,012 to 0,015 Above 0,015	Close Conformity Acceptable Conformity Marginally Acceptable Conformity Nonconformity
Second Digits	0,0000 to 0,008 0,008 to 0,010 0,010 to 0,012 Above 0,012	Close Conformity Acceptable Conformity Marginally Acceptable Conformity Nonconformity
First-Two Digits	0,0000 to 0,0012 0,0012 to 0,0018 0,0018 to 0,0022 Above 0,0022	Close Conformity Acceptable Conformity Marginally Acceptable Conformity Nonconformity
First-Three Digits	0,00000 to 0,00036 0,00036 to 0,00044 0,00044 to 0,00050 Above 0,00050	Close Conformity Acceptable Conformity Marginally Acceptable Conformity Nonconformity

Source: Nigrini, 2012

4.9.4 The **Z-Statistics** will be used to assess conformity of individual digits. It assesses for if individual digit differs significantly from the expected Benford's Law distribution (Nigrini, 2012). The number of records is considered in the calculation of the Z-statistic. A significance level of 5% is used and any Z-statistic greater than 1.96 means the actual proportion is significantly different from the Benford expected proportion (Nigrini, 2012, p.82).

$$Z = \frac{\left(|AP - EP| - \left(\frac{1}{2N}\right)\right)}{\sqrt{\frac{EP(1-EP)}{N}}} \quad \ldots (8)$$

(Where: AP = actual proportion, EP = expected proportion and N = number of records)

In assessing conformity to Benford's Law using the Z-statistic, the upper and lower bounds will be calculated using the following equations:

$$Upper\ Bound = EP + \left(1.96 * \sqrt{\frac{EP(1-EP)}{N}}\right) + \left(\frac{1}{2N}\right) \quad \ldots (9)$$

$$Lower\ Bound = EP - \left(1.96 * \sqrt{\frac{EP(1-EP)}{N}}\right) - \left(\frac{1}{2N}\right) \quad \ldots (10)$$

The upper bound is above the BL and the lower bound is below BL. Spikes that are above the upper bound or below the lower bound are significant at the 5% significance level.

4.9.5 P-value test statistic

The p-value was used to confirm the probability of observing chi-square test statistic that is as extreme as, or more extreme than the computed value. The calculated p-value was then compared to the alpha value selected for the study to make a decision of whether to accept or reject the null hypothesis being assessed.

4.10 Quality controls

The datasets to be used for the research were collected from the electoral bodies or authorities. This data is considered reliable and trusted, even though in some cases, there were opposition parties that may have refuted the data. The data is secondary in nature and is the only data relevant for this study.

Quality controls will include a) Accurate capturing and formatting, b) Checking data for validity and integrity, c) Ensuring data consistency and standardization, and d) Checking data for completeness. Data authenticity was conducted by running pre-Benford tests. These will include doing periodic graphs and histogram, to ensure data is complete and error free.

4.11 Limitations

As each research has limitations, the following limitations were identified in this research.

Analysis of data may flag existence of irregularities. However, these may be due to unintentional mismanagement of the voting process (Lacasa, Fernández-Gracia, 2018) and not fraud. It is therefore unclear if Benford's Law can differentiate between manipulated and unmanipulated data. The method cannot be used to prove or disprove fraud or error in elections but can only be used as a screening tool (Agyemang, et. al., 2023).

According to Druică, et. al., (2018), the null hypothesis significant tests are misleading for large sample sizes. But to get a statistically significant result, a large dataset is required (Antonio, 2023). Linear transformations of Benford sequence cannot be detected by the analytic methods described in the research (Druică, et. al., 2018). The other limitation is the availability of high-quality journal articles for the research.

BL method has the propensity of producing type 1 and type 2 errors, in that it detects fraud where there is none or it does not detect fraud where fraud took place (Deckert, et. al., 2011). The lack of peer reviewed studies about BL is a shortcoming in the method's advancement and improvement. Deckert, et. al. (2011) highlights the need for peer review and literature that link BL to elections. Mebane (2011) agrees on the need for the peer review of studies and articles on BL.

5. Findings

5.1 Introduction

This section presents the findings of the results of the analysis from the various tests conducted. For the analysis, the voting results were grouped into the winning party, the runner-up and the rest of the parties and independent candidates were grouped into one with their votes added together forming a group Others in each case (Agyemang, et. Al., 2023). For Botswana the top three parties were used, with the combined 2014 and 2019 the two last parties combined. In South Africa, for 2004 the top three parties are used and then the ruling party was used on its own to compare the 2014, 2019 and 2024 performance. These methods of analysis were considered effective by the researcher.

5.2 Analysis Results

Elections from Zimbabwe have been plagued by violence and allegations of fraud for many years. This has been a similar case with the Kenyan elections, especially in 2017, where lives were lost post the elections when the losing candidates protested the results, and the courts annulled the results and ordered a rerun of the elections. The results in the study are classified into two groups, in the first group are elections where fraud has been alleged. In the second group are election results from South Africa and Botswana were, where no fraud or election tempering was alleged.

5.3 Election Results where Fraud was Alleged

5.3.1 2013 Zimbabwe Results

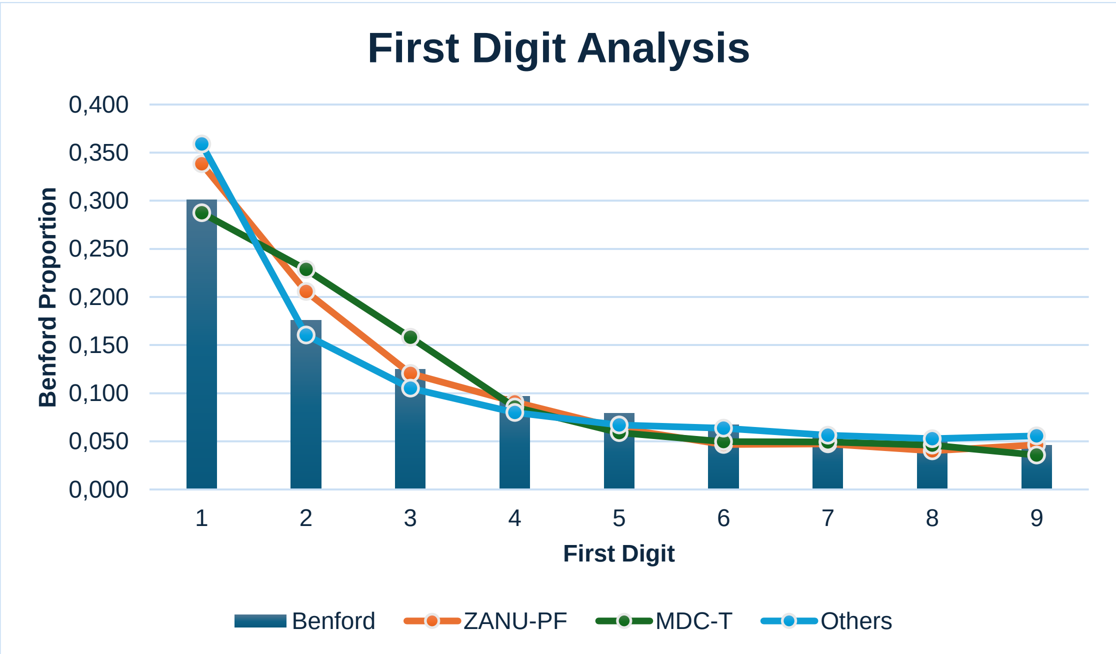
In Zimbabwe, for elections held before 2013 there are gaps in the data as some of the provinces' data was not available in the electoral commission's website. The tests were run using available data which provided enough information. The summarized data was used for analysing combined election data. The election data for Zimbabwe was grouped into three categories: the results for the wining party, Zimbabwe African National Union – Patriotic Front (ZANU-PF), the official opposition party (MDC-T in 2013 and MDC-A in 2018), and all other parties and independent candidates, Others group. The opposition party from the year 2000 until 2008 was the Movement for Democratic Change (MDC), which split into two factions in 2013 and changed the name to Movement for Democratic Change - Tsvangirai (MDC-T), which was the official opposition in that year, and the other faction was called the MDC. In 2018 the factions came back together and contested the elections as the MDC-Alliance MDC-

A) and were the official opposition. The main opposition in the last elections of 2023 was the Citizens Coalition for Change (CCC).

5.3.1.1 Benford's Law tests

The first digit test results for the three groups are presented in Figure 2 below. ZANU-PF and Others are above the expected proportions for the digit one. For the digit two ZANU-PF and the MDC-T are above the expected proportions with the Others group below the expected proportions. The MDC-T continues to be above the BL proportions for the digit three and then from the digit four all groups are below the expected proportions and at digits eight and nine Others group is above BL. The graph shows that the three groups do not conform to BL expected proportions for the first digit.

Figure 2: 2013 First Digit Graph

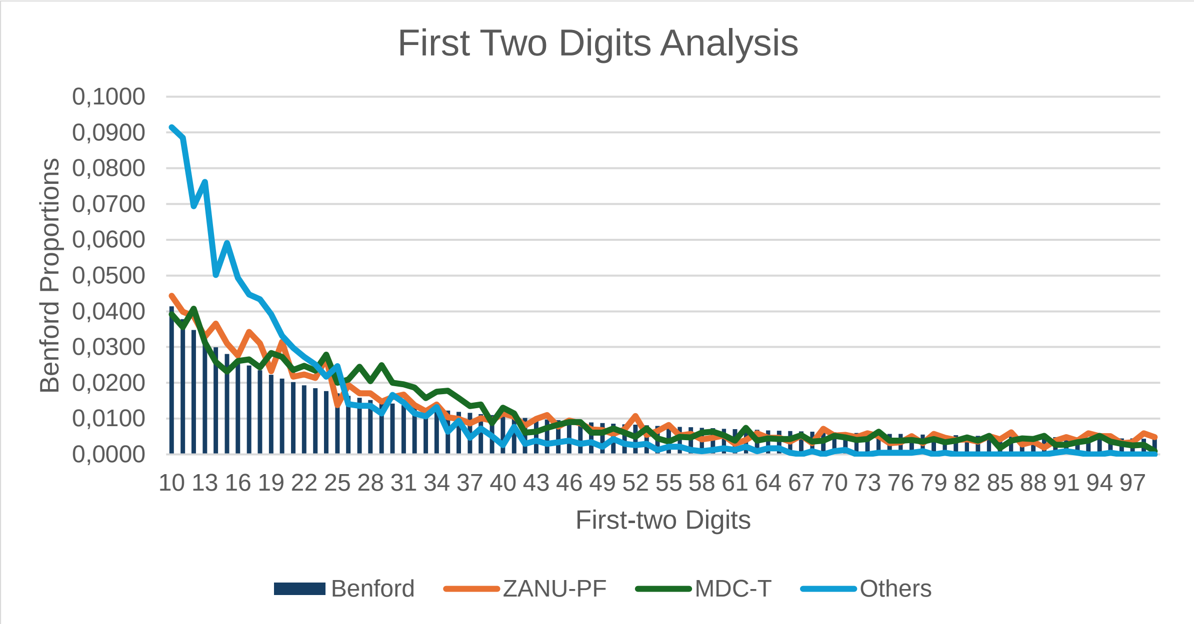


The second digit test shows ZANU-PF and the MDC-T closely tracking the expected proportions apart from spikes at digits zero and four. However, the two groups are a better fit to BL. The Others group has high spikes at digits zero to three and again slightly above the expected proportions at digit five. ZANU-PF and the NDC-T show close conformity to BL and Others does not conform. See Appendix 1.

The first-two digits test results show that the Others group does not conform to BL expected proportions. There are high spikes above BL expected proportions from the digit ten reaching above 0.090 and then from the digit 26 the group is below the expected proportions. ZANU-

PF and the MDC-T close to the expected proportions, with ZANU-PF performing better than the MDC-T. See Figure 3 below.

Figure 3: 2013 Zimbabwe First-Two Digit Graph



5.3.1.2 Other Statistical tests results

a) Mean Absolute Deviation

The MAD results show that for the first digit, the three groups do not comply with BL. ZANU-PF and the MDC-T have close conformity for the second digit whereas the Others group has nonconformity. The ZANU-PF has marginally acceptable conformity for the first-two digits MAD test and the MDC-T and Others group achieved nonconformity. See Appendix 2.

b) Z-Statistic Test Results

The first digit Z-statistics test results show that none of the three groups comply with BL statistically. ZANU-PF and the Others groups have only three of the nine digits below the critical value and the MDC-T has only one digit below the critical value. The second digit Z-statistic test has the ZANU-PF and the MDC-T complying with BL and have one digit each that statistically differs from BL. The Others group is nonconforming as it has three digits below the critical value. For the first-two digits ZANU-PF and MDC-T are conforming to BL expected proportions with no digit above the critical value. The Others group is nonconforming to BL with only two digits below the critical value. See Appendix 3, Appendix 4 and Appendix 5.

c) Chi-Squared Results

There is nonconformance with BL first digit chi-squared test for all three groups with their calculated chi-squared statistic above the chi-squared critical value. For the second digit, MDC-T conforms with BL with the calculated chi-squared statistic below the chi-squared critical value. ZANU-PF and the Others groups do not conform to BL with the calculated chi-squared statistic higher than the critical value. The first-two digits for all the three groups do not conform with BL. See Appendix 6.

d) The p-value statistic test

The first digit and the first-two digit p-values for the three groups are lower than alpha (p-value ≤ α), resulting in the null hypothesis being rejected. The second digit p-values for the ZANU-PF and Others groups are less than alpha, rejecting the null hypothesis, while the p-value for the MDC-T is greater than alpha resulting in a conclusion that there is not enough evidence to reject the null hypothesis. See Appendix 7.

e) Kolmogorov-Smirnov Tests

The K-S test statistic for all three parties or groups show that the voting data cumulative proportions does not conform to BL cumulative proportions. The calculated K-S critical values for all three groups are lower than the supremum for all three groups. The supremum for ZANU-PF is at the digit 34 where the highest deviation occurs. The MDC-T has the highest deviation at the digit 41 and the Others group has the deviation at digit 25.

5.3.2 2018 Zimbabwe Results

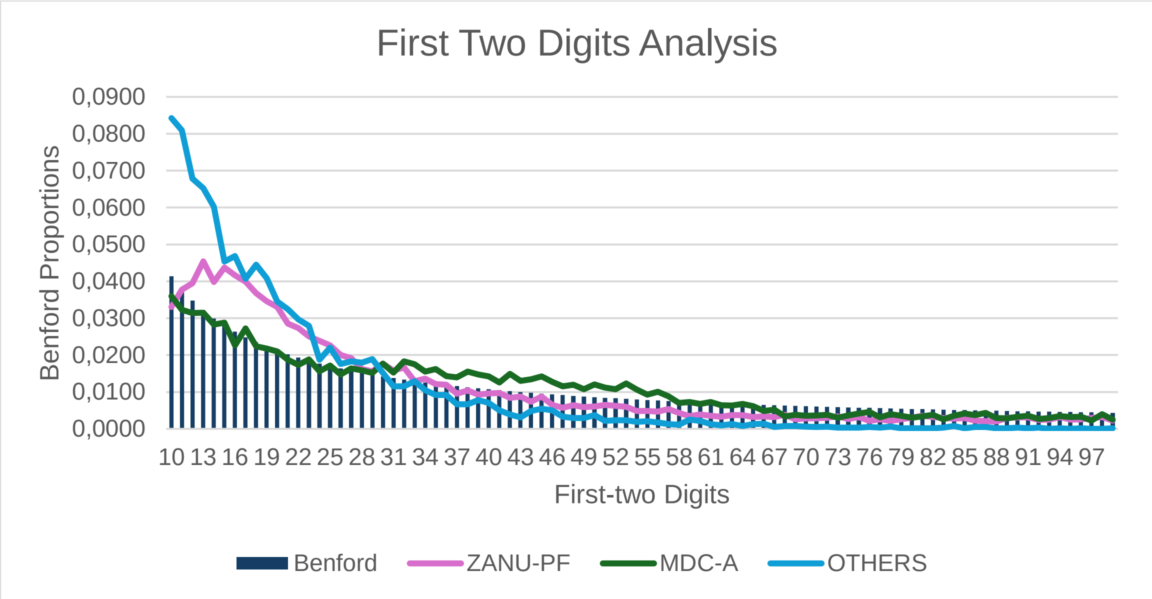
5.3.2.1 Benford’s Law test results

The national elections data for ZANU-PF first digit did not follow BL. The digit one for the party was above the expected proportion, at 0,391. similarly, the Others was also above the expected proportion at 0,369 and the MDC-A was below the expected at 0,276. The data for Others shows a better fit from the digit two than the other digits though. See Appendix 8.

The second digit analysis shows that the ZANU-PF and the MDC-A having close conformity to BL expected proportions. There were higher than expected proportions for digits three and five for both parties however and lower than expected proportions for digits zero and nine. The proportions of Others for digits zero, one, two and three were much higher than expected proportions and then from five to nine they were lower than expected proportions with digit seven much lower than expected proportions. See Appendix 8.

The first-two digits chart is presented in Figure 4 below. The ZANU-PF first-two digits show high spikes between digits 12 and 32 and then remains below the expected proportions from 37 until 99. The data does not conform to BL. The MDC-A shows close conformity from 13 until 27 and then it is above the expected proportions until digit 58 before going below the BL expected proportions from 66 until 99. The Others shows a high spike from 10 until 30, from where it goes below the expected proportions from 34 with most of the digits recording no entries. For the first-two digits test none of the parties conforms to BL.

Figure 4: 2018 Zimbabwe First-Two Digits Analysis



5.3.2.2 Other Statistical tests results

a) Mean Absolute Deviation Results

The MAD results for the first digits shows nonconformity to BL for all three groups. For the second digits we have the ZANU-PF and MDC-A closely conforming to Benford's Law and Others not conforming. The first-two digits MAD for MDC-A is marginally conforming to the expected proportions, and the ZANU-PF and Others group are nonconforming to BL. See Appendix 9.

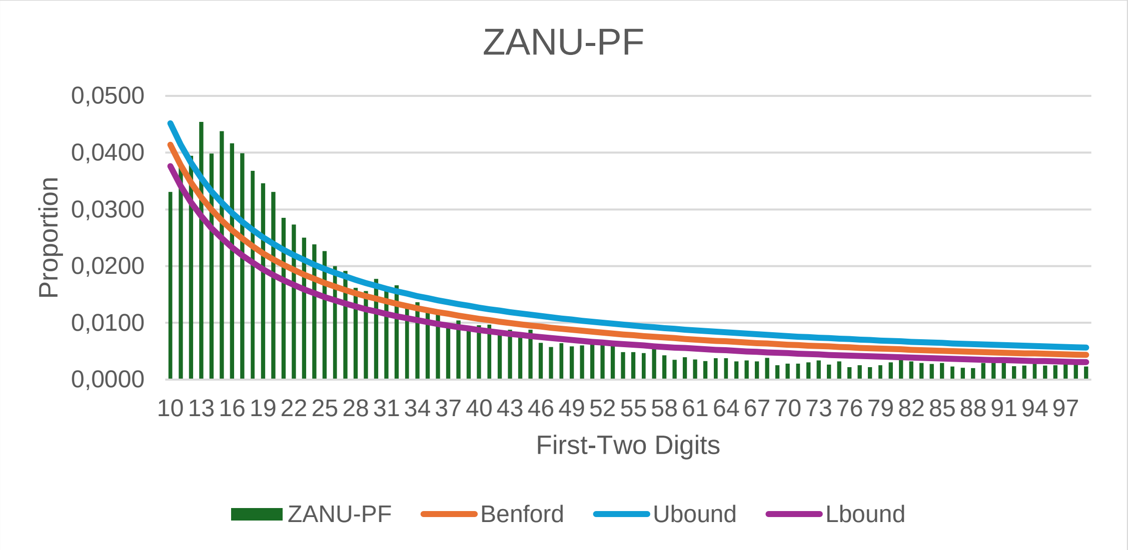
b) Z-Statistic test results

The Z-Statistic test results show that for the first digit all three groups do not conform to BL, with only the digit three for ZANU-PF and digits one and eight for the Others group below the critical value and all the first digits for the MDC-A group being above the critical value. The second digit Z-Statistic test results show that the MDC-A conforms to BL with only digit five above the critical value. ZANU-PF has four out of nine digits (zero, three, five and nine) above the critical value and the Others group has all but the digit four above the critical value. See Appendix 10.

The Z-Statistic test results for the first-two digits are found in Appendix 11. The results show that the first-two digits for all three groups do not conform to BL digit proportions.

Figure 5 shows the ZANU-PF first-two digits results for Z-Statistics test. The digits are either below the lower bound or above the upper bound as calculated using equations 9 and 10 in Chapter 4. The digits from 12 to 27 are above the upper bound and from 46 are below the lower bound.

Figure 5: 2018 Z-Statistic Results for First-Two Digits



c) Chi-squared

The results of the chi-squared tests for the first digits show that the data does not conform to BL expected proportions for all three groups, see Table 3 below. The chi-squared critical value calculated was 15,507 and the Others group had the lowest chi-squared result of 306,716. The chi-squared test results confirmed the Z-statistic results which stated that all three groups did not conform to BL proportions.

The second digit chi-squared results reveal nonconformance for ZANU and Others while the MDC-A conformed to BL expected proportions. The calculated chi-squared statistic is lower than the critical value for the MDC-A, also confirming Z-Statistic results above.

The first-two digits chi-squared test results gave a nonconformance outcome as all results were higher than the critical value.

Table 6: Chi-squared test results

Chi-Square Tests			
	ZANU	MDC-A	Others
1st Digit	1166,636	444,981	306,716
Chi-Sq_CritVal	**15,507**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
2n Digit	21,465	12,203	136,972
Chi-Sq_CritVal	**16,919**		
Result	**NonConformance**	**Conforms**	**NonConformance**
FT Digits	1555,530	590,497	3571,391
Chi-Sq_CritVal	**112,022**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
Last 2 Digits	114,103	362,218	3,241
Chi-Sq_CritVal	**123,225**		
Result	**Conforms**	**NonConformance**	**Conforms**
Last Digit	5,709	12,311	398,237
Chi-Sq_CritVal	**16,919**		
Result	**Conforms**	**Conforms**	**NonConformance**

d) The p-value statistic test

The first digit and first-two digit p-values for all three groups are less than alpha, leading to the null hypothesis being rejected. The second digit p-values for ZANU-PF and the Others groups reject the null hypothesis, while the p-value for the MDC-T is greater than alpha leading to a conclusion that there is not enough evidence to reject the null hypothesis. See Appendix 13.

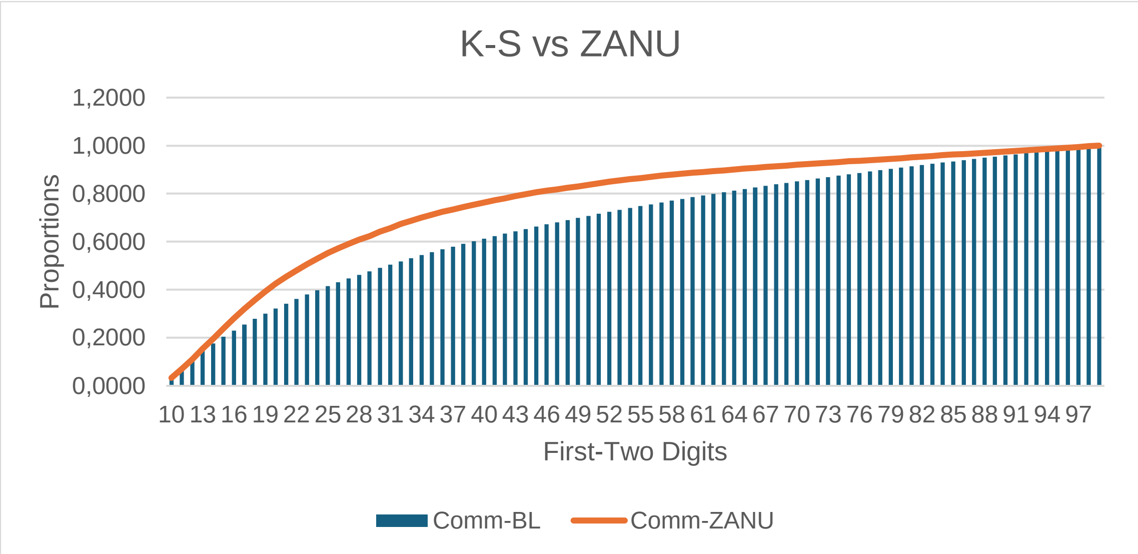
e) Kolmogorov-Smirnov Test Results

The K-S test results reveal that of the three groups, none conforms to BL expected distribution proportions. The calculated suprema of the three groups are higher than the calculated K-S Critical values for each group resulting in the nonconformity conclusion. Figure 6 below shows the cumulative proportions against the actual cumulative proportions for ZANU-PF. The actual cumulative proportions are above the expected, leading to the conclusion that the data does not conform to BL. The highest deviations take place for the digit 30 to the digit 40, with digit 34 having the largest deviation. See Figure 6.

The MDC-A K-S test results show that the actual cumulative proportions start below the expected cumulative proportions and then from the digit 36 they deviate upwards from the expected cumulative proportions. The highest deviation is at digit 57.

All the data for Others is outside the expected proportions with the highest deviation at digit 30. None of the actual digit proportions track the expected proportions and hence the data does not conform to BL. See Appendix 14.

Figure 6: 2018 K-S Chart for ZANU-PF First-Two Digits



5.3.3 2023 Zimbabwe Elections

The 2023 election data is not available per municipality per province. It is made available per constituency per province. This type of data does not provide enough information due to data being aggregated.

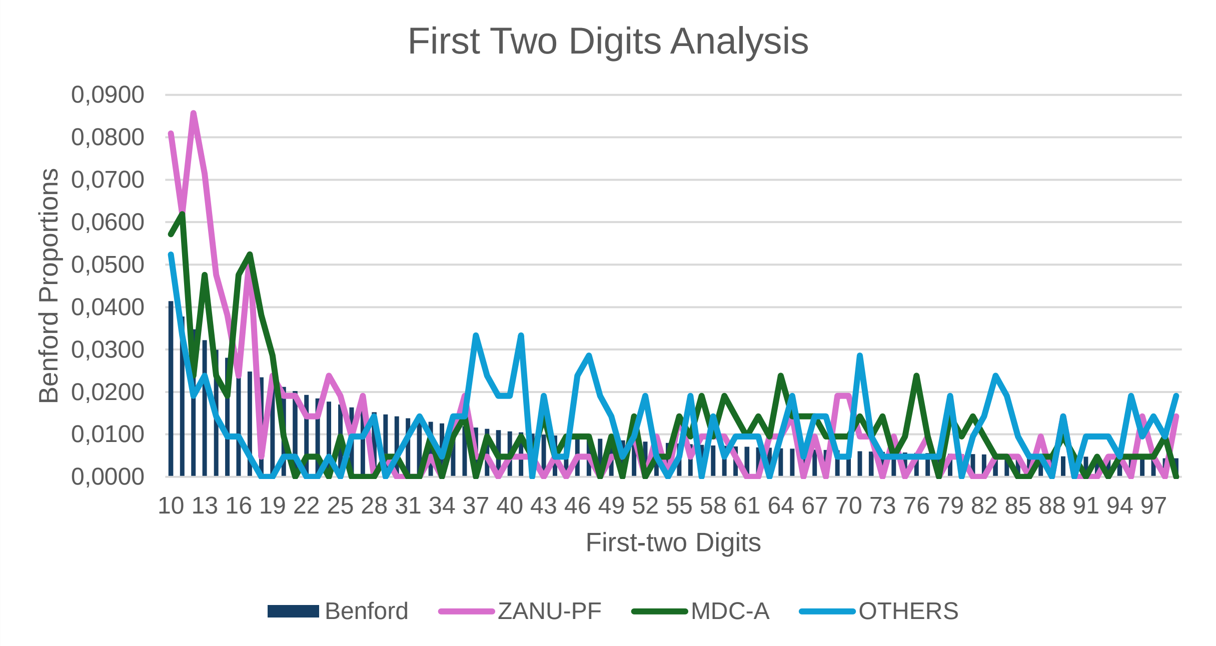
5.3.3.1 Benford's Law Test Results

The first digit shows high spikes for the digit one for ZANU-PF and the CCC, with the ZANU-PF almost reaching 0,490 and the CCC at 0,400. The ZANU-PF proportions are then lower than the expected proportions from the digit two to the digit five and then stays within the proportions. The CCC proportions are below the expected proportions from the digit two and then gradually rise to be above from the digit five and then peak at the digit six. The Others group data starts below the expected proportions at the digits one and two then go above from the digit three. The data does not conform to BL proportions. See Appendix 15.

The second digit actual proportions do not conform to BL. The proportions move up and down away from the expected proportions with high spikes at the digit two from the Others groups and the ZANU-PF dipping well below BL at the digit eight. See Appendix 15.

The first-two digit test reveals the ZANU-PF has high spikes at the digits 10 and 12, Figure 7 below. The none of the parties' proportions follow the expected proportions, with the actual proportions moving up and down away from the BL proportions.

Figure 7: 2023 First-Two Digit Zimbabwe



5.3.3.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

The MAD test shows that all three parties do not conform to BL for the first digit, second digit and first-two digit test, Table 4 below. This is the expected result based on the data analysed above.

Table 7: 2023 MAD Test Results

MAD Outcomes			
	ZANU	MDC-A	Others
1st Digit Outcome	0,0440 Nonconformity	0,0541 Nonconformity	0,0584 Nonconformity
2n Digit Outcome	0,0180 Nonconformity	0,0169 Nonconformity	0,0174 Nonconformity
FT Digits Outcome	0,0074 Nonconformity	0,0072 Nonconformity	0,0084 Nonconformity

b) The Z-Statistics Test Results

The Z-statistic test results show all three parties not conforming to BL. The ZANU-PF has three digits above the Z-statistic critical value, while the CCC and the Others group have five digits each above the Z-statistic critical value. The second digit Z-statistic test results show all three parties conforming with BL. ZANU-PF has one digit above the critical value whereas the CCC and the Others group have no digit above the critical value. See Appendix 16, Appendix 17 and Appendix 18.

c) Pearson's chi-squared Test Results

The chi-square results reveal nonconformity with BL for the first digit and the first-two digit tests. The second digit and the last-two digit test result in conformity with BL expected proportions. See Appendix 19.

d) The p-value Test Statistic Results

The first digit and the first-two digit p-value tests reject the null hypothesis, with the p-values less than alpha for all three parties. The second digit tests result in p-values greater than alpha, concluding that there is not enough evidence to reject the null hypothesis. See Appendix 20.

e) Kolmogorov-Smirnov Test (K-S Test)

The K-S tests for the three parties conclude that the second digit tests do not conform to BL cumulative proportions. The calculated suprema of the three parties are greater than the respective K-S critical values.

The greatest deviations are at the digit 17 for the ZANU-PF, at the digit 54 for the CCC and at the digit 34 for Others group. The charts of the K-S cumulative proportions against the actual proportions show that each group's respective actual proportions move away from the BL proportions. See Appendix 21.

5.3.4 2013 – 2023 Zimbabwe Elections

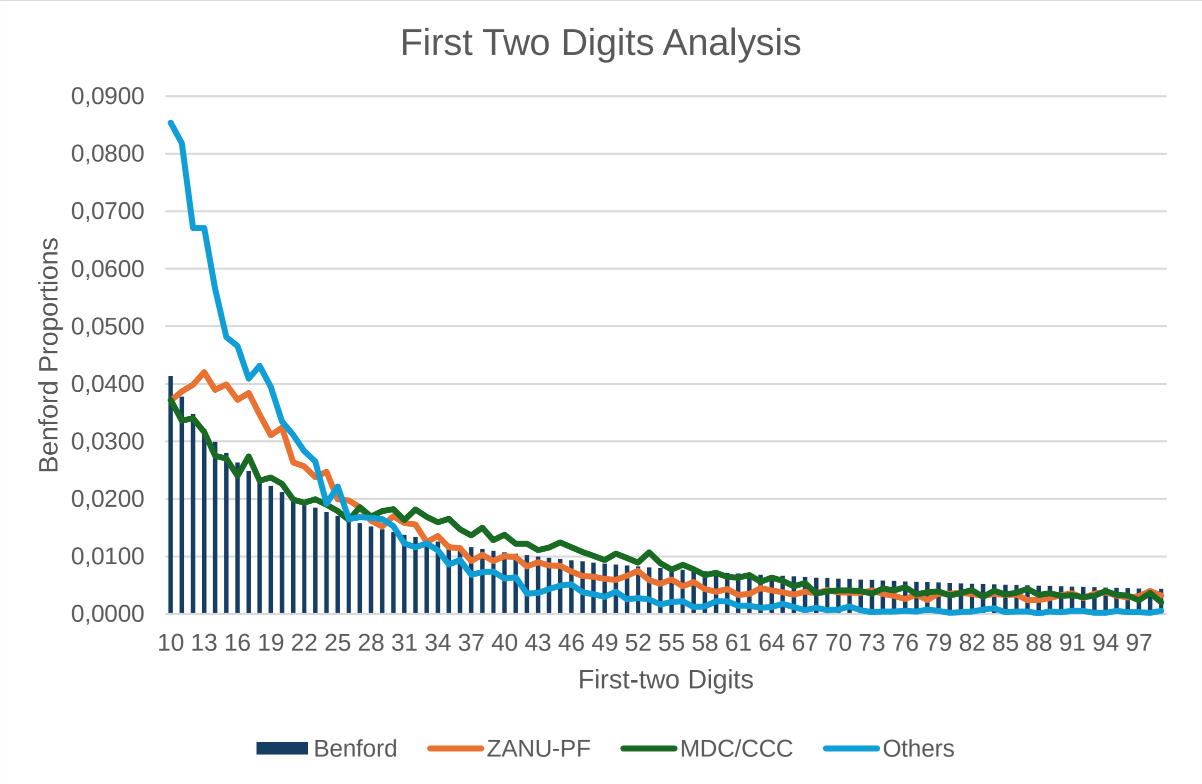
5.3.4.1 Benford's Law Test Results

The first digit test shows the ZANU-PF and the Others groups with high spikes for the digit one, ZANU-PF continues to be above the expected proportions for the digit two before being compliant at the digit three and then it is below the BL proportions for the rest of the first digits. The Others group fits the BL curve better than the two parties, with the MDC/CCC above the BL curve from digit two to five. See Appendix 22 for the first and second digit results.

For the second digit test, the ZANU-PF and the MDC/CCC fit the BL expected proportions better and are within close range to each other. The Others group has high spikes at digits zero and one above BL proportions and low spikes at digits seven and eight. The ZANU-PF and the MDC/CCC comply with BL whereas the Others group does not.

The first-two digit test reveal the Others group has high spikes above the expected proportions from the digit 10 to 25, then the achieved proportions are below the expected from the digit 34 to 99. The ZANU-PF and the MDC/CCC for the curve better despite some high proportions. See Figure 8 below.

Figure 8: 2013-2023 First-Two Digit Graph



5.3.4.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

The first digit MAD test results show ZANU-PF and the Others achieving nonconformity and the MDC/CCC marginally acceptable conformity with BL proportions. The second digit results show ZANU-PF, and the MDC-A have close conformity while the Others achieved nonconformity with BL. For the first-two digit test, the MDC/CCC achieved acceptable conformity and ZANU and Others are not conforming. See Table12 below.

Table 8: 2013-2023 Zimbabwe MAD Results

MAD Outcomes			
	ZANU-PF	MDC/CCC	Others
1st Digit Outcome	0,0273 Nonconformity	0,0148 Marginally acceptable conformity	0,0156 Nonconformity
2n Digit Outcome	0,0028 Close conformity	0,0017 Close conformity	0,0139 Nonconformity
FT Digits Outcome	0,0030 Nonconformity	0,0017 Acceptable conformity	0,0073 Nonconformity

b) The Z-Statistics Test Results

For the first digit and the first-two digits all three groups did not comply with BL due to a high number of digits above the critical value. The second digit MAD has the ZANU-PF with two out of 10 digits above the critical value and the Others group with nine digits above the critical value, the two groups do not conform to BL. The MDC/CCC conforms to BL as none of its second digits are above the critical value. See Appendix 23, Appendix 24 and Appendix 25.

c) Pearson's chi-squared Test Results

The chi-square test statistic results in nonconformance for the three groups for the first digit and the first-two digit tests. For the second digit chi-square tests there is nonconformance for the ZANU-PF and Others while the MDC/CCC conforms. See Appendix 26.

d) The p-value Test Statistic Results

The first digit and first-two digit p-values are less than alpha, rejecting the null hypothesis. The second digit p-values for ZANU-PF and Others are less than alpha rejection the null hypothesis, while the p-value for the MDC/CCC is greater than alpha, leading to a conclusion that there is not enough evidence to reject the null hypothesis. See Appendix 27.

e) Kolmogorov-Smirnov Test (K-S Test)

The three parties do not conform to the BL distribution proportions as per the K-S test results. The calculated suprema for the three parties were greater than the calculated respective K-S critical values. The greatest deviations were at the digit 34 for ZANU-PF, at the digit 57 for the MDC/CCC and at the digit 30 for the Others group. The K-S charts in Appendix G show that the MDC/CCC cumulative proportions are much closer to BL cumulative proportions that those of the other two parties. See Appendix 28.

5.3.5 2017 Kenya Elections

The Kenyan elections of 2017 were marred with violence and protest. The elections had to be taken for the second time after the constitutional court of Kenya ruled against electoral commission after it declared election results. The analysis that follows is based on the county assembly results. These results are used as they include all the counties and wards from which the national results are developed.

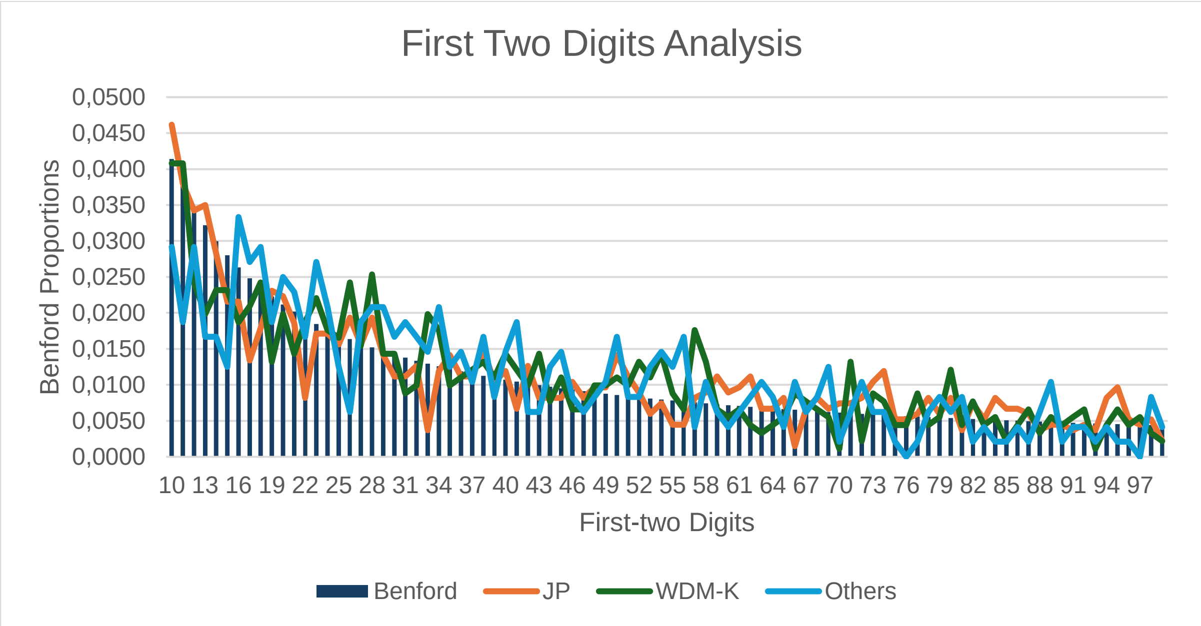
5.3.5.1 Benford's Law Test Results

The first digit results shows that the Jubilee Party (JP) show that for the digits one, two, three and four the actual proportion is below the expected proportions and for the digits five to nine the party achieved proportions above the expected. The Wiper Democratic Movement-Kenya (OMD) actual proportion for the digit one is below the expected proportions and from the digit two to five and seven and eight it is above BL proportions with the digits six and nine below the expected proportions. The Others group is below BL proportions for the digits one and seven to nine with the digits two to six above the expected proportions. The JP is the better fit to BL of the three parties. First digit shown in Appendix 29.

For the second digit the three parties achieved mixed outcomes with the JP having a spike above BL at the digits zero four and eight, the WDM-K spiking at seven and eight and the Others group spiking at the digits eight and nine. There are spikes below the expected proportions for the Others at digits five and seven and for the JP at the digit nine. The averages of the proportions should smooth the proportions to some conformity. See Appendix 29.

The first-two digit test has the three parties achieving proportions moving up and down away from the BL expected proportions, Figure 9 below. There is no party that is a better fit to the BL expected proportions with all parties having spikes above and below the BL.

Figure 9: 2017 Kenya First-Two Digit



5.3.5.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

The first digit MAD analysis has the JP party achieving acceptable conformity, the WDM-K shows marginally acceptable conformity and the Others group does not conform. For the second digit MAD test the JP and WDM-K have close conformity to BL and the Others group achieved marginally acceptable conformity. The JP party achieved marginally acceptable results for the first-two digit MAD test and both the WDM-K and Others groups achieved nonconformity. See Appendix 30.

b) The Z-Statistic Test Results

The first digit Z-statistic results show the JP party with only one digits, seven, that is above the critical value. The WDM-K and the Others group have two digits above the critical value. For the three parties the conclusion is that they all conform to BL. The JP and WDM-K Z-statistic results confirm the MAD results above. See Appendix 31.

All parties conform with BL proportions for the second digit as the JP and WDM-K have all the digits below the critical value, the Others group has only one digit above the critical value, also confirming the MAD results. See Appendix 31.

The Z-statistic results for the first-two digits show the JP having eight digits above the critical value. With eight out of 90 digits not complying with BL, the first-two digits for the JP party are noncompliant at five percent level of significance, Appendix 32. The WDM-K party has six of its digits above the critical value, which is over five percent, and therefore noncompliant with BL expected proportions. See Appendix 33.

The graph of the first-two digit Z-statistic test, Appendix 33, shows the JP party having two digits above the upper bound and four digits below the lower bound. This reduces the nonconforming digits to six from the eight out 90 from paragraph (c) above. The Z-statistic test graph of the WDM-K is found in Appendix 33. Four digits are above the upper bound and no digit is below the lower bound. From this result the first-two digit test conforms to BL expected proportions as now four percent of the digits do not conform. The first-two digit Z-statistic test graph of the Others group is found in Appendix J. The graph shows that the Others group has only the digit 11 below the lower bound, concluding that the first-two digits of the Others group conforms to BL expected proportions.

c) Pearson's chi-square Test Results

The chi-squared test statistic results for the first digit shows all the parties nonconforming with BL. The calculated chi-squared statistics for the three parties are above the calculated chi-squared critical value, with the JP slightly above the critical value. The chi-squared test statistics for all the parties for the second digit and the first-two digit tests all conform with the BL expected proportions, with all the calculated chi-squared statistic below the chi-squared critical value. See Appendix 34.

d) The p-value test statistic results

The p-value is calculated for each party and for each digit test and compared with the alpha ($\alpha = 0.05$) to test the null hypothesis that the voting data digit distribution proportions conform to the Benford's Law digit distribution proportions. See Appendix 35.

The calculated p-values for the first digit for the JP, WDM-K and the Others groups are less than alpha, p-value $< \alpha$, with the conclusion that the null hypothesis is rejected. The second digit p-values for all three groups are greater than alpha, p-value $> \alpha$, leading to the conclusion that there is not enough evidence to reject the null hypothesis.

The first-two digit p-values for the JP and the WDM-K groups are greater than alpha, with the conclusion that there is not enough evidence to reject the null hypothesis.

e) Kolmogorov-Smirnov Test (K-S Test)

The first-two digit K-S test statistic results for all three groups resulted in nonconformity with BL expected proportions. The calculated suprema for the three groups were greater that the calculated K-S critical values for all three groups.

The graphs of the K-S cumulative proportions against the parties' cumulative proportions are found in Appendix 36. The graph of the JP party shows the JP cumulative proportions are slightly above the BL cumulative proportions for digits 10 to 14 and then below the BL from the digit 16 to 94 and then from the digit 95 they are on par with BL cumulative proportions. The largest deviation happens at the digit 45.

The WDM-K graph shows the WDM-K cumulative proportions are on par with digits 10 and 11 and then from 12 they are below and picking up at the digit 57 and are mostly on par with BL cumulative proportions with minor deviations. The largest deviation is at the digit 22 for the WDM-K. Appendix 36.

The Others group graph shows the group's cumulative proportions are below the BL cumulative proportions from the digit 10 until the digit 53 and the from the digit 55 the proportions are above the BL proportions, getting back on par from the digit 96. The largest deviation is at the digit 15. Appendix 36.

The 2022 Kenyan results were highly aggregated, showing totals for the 48 voting districts. Analysis of candidates, William Ruto and Raila Odinga was done. Comparisons of the candidates' proportions against BL proportions showed high deviations for the first digit, second-digit and first-two digits. The MAD statistics results returned nonconformity for both candidates and voter turnout. However, the other statistical tests resulted in conformity. The Z-statistic tests for the first digit returned conformity for both candidate on all digits. For the second-digit Z-statistic tests only the digit eight for Ruto did not conform and the rest conformed for both. The first-two digits also conformed to BL. The chi-square results conformed, with the exception of the first-two digits for Odinga, while all p-value tests all returned conformity. Tests for the K-S also returned conformity, with the calculated critical higher than the supremum for both candidates. Appendix 37 – Appendix 42.

5.4 Election Results where no Fraud was Suspected.

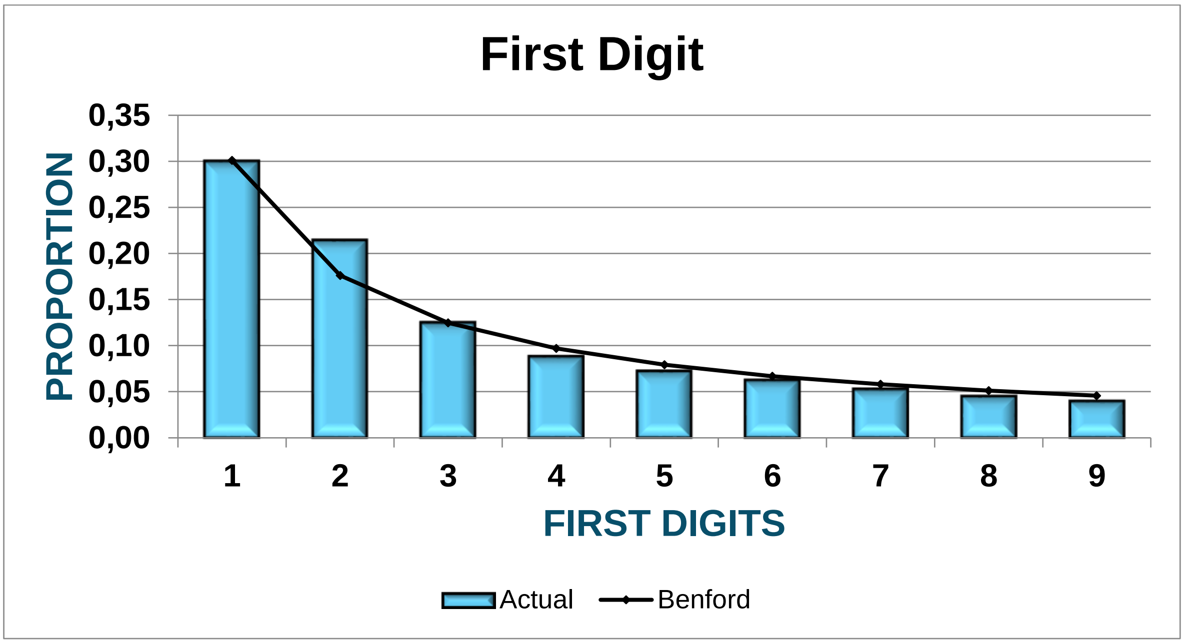
In this section the results of the elections in Botswana and South Africa are presented. These are the elections where no fraud had been alleged or suspected and there is no expectation that there may have been fraud in these elections.

5.4.1 2004 South African Elections

Starting with the 2004 elections, the parties that were considered were the African National Congress (ANC), the Democratic Alliance (DA) and the Inkatha Freedom Party (IFP). The researcher started looking at the behaviour of the registered voter population to investigate the population behaviour against Benford's Law. According to Nigrini (2012), if the registered voter population data conforms to Benford's Law, then the election data should follow BL expected distribution proportions.

Figure 10 below shows the first digit proportions of the registered voters in South Africa in 2004. The voter population is shown in bars and Benford's Law proportions are shown in a black line. Despite the digit two showing a spike and being above the expected proportions, the data fit very well with BL.

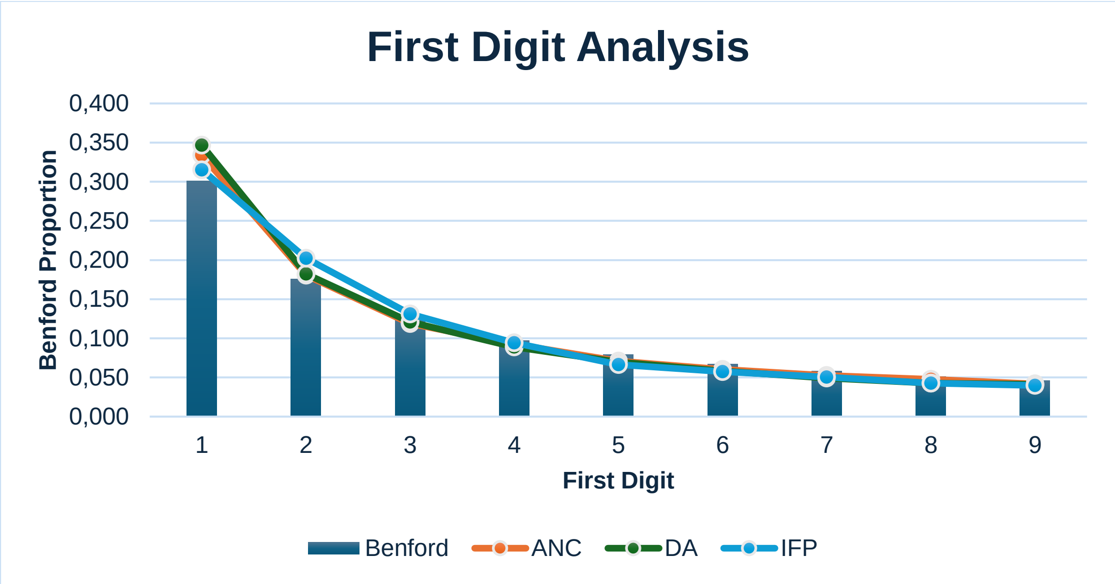
Figure 10: 2004 SA Registered Voters



5.4.1.1 Benford's Law Tests Results

Figure 11 below shows the observed proportions of the 2004 vote data for the top three parties in the elections. All the parties' proportions are above the expected proportions for the digits one and two. However, the data is a good fit against BL expected proportions.

Figure 11: 2004 First Digit Analysis



The second digit test shows that the IFP and the DA are above the expected proportions for the digits zero to three and then they go up and down but there are no spikes in their observed proportions, Appendix 42. The ANC proportions are slightly above the BL proportions at digits four and eight and the rest of the digits are within the expected proportions with the digit one being slightly below the expected proportions. The ANC data is a better fit among the three parties.

The first-two digits proportions for the ANC are above the expected proportions for the digits 12 to 21and then they are within the expected proportions for the rest of the proportions. The DA and IFP have high spikes for the digits 10 to 32 and then their data is below the expected proportions and that of the ANC. The ANC is a better fit for the first-two digits when compared to the other two parties. See Appendix 42.

5.4.1.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

The MAD calculations show that the three parties have acceptable conformity for the first digit analysis with a deviation of one percent on either side for all three parties, the ANC showing even less that a percent either above or below the expected proportions.

All three parties achieved closed conformity for the second digit analysis with the ANC performing better than the two parties. The MAD results for the first-two digit analysis shows the ANC achieving close conformity to BL expected proportions and the DA and IFP are nonconforming. See Appendix 43.

b) The Z-Statistics Test Results

The Z-statistic test results for the first digit have all the parties nonconforming to BL expected proportions, with the ANC having only three digits below the Z-statistic critical value. The DA has two digits out of nine below the critical value and the IFP has one digit below the critical value. The second digit Z-statistic results have all three parties conforming to expected proportions. The ANC has all digits below the critical value, the DA has one digit above the critical value and the IFP has two digits above the critical. The three parties performed better for the second digit Z-statistic tests. See Appendix 44.

The three parties do not conform to BL expected proportions for the first-two digit Z-statistic test with the ANC having 20 of the 90 digits above the critical value and the DA and IFP performing even worse than the ANC.

There are nine digits above the upper bound and nine below the lower bound which are significant at five percent level of confidence. There are 15 significant spikes above the upper bound and all the digits after the digit 54 are below the lower bound. The chart shows significant spikes above the upper bound from the digit 10 to the digit 24 and again at digits 26, 28 and 30. From the digit 49 the digits are below the lower bound apart from the digit 68 which is above the lower bound. See Appendix 45 and Appendix 46.

c) Pearson's Chi-squared Test Results

The chi-squared test statistic results show that all three parties are nonconforming to BL for the first digit and first-two digits. The ANC conforms with the expected proportions for the second digit and the DA and IFP are nonconforming with expected proportions, with the IFP slightly above the chi-squared critical value. Appendix 47.

d) P-value Test Statistic Results

The calculated p-values for the first and first-two digits show the three parties achieving p-values below alpha, leading to the conclusion that the null hypothesis is rejected. The second digit p-value for the ANC is greater than alpha, concluding that there is not enough evidence to reject the null hypothesis. The second digit p-values for the DA and IFP are less than alpha, rejecting the null hypothesis. See Appendix 48.

e) Kolmogorov-Smirnov Test (K-S Test) Results

The K-S test statistic shows that the three parties fail to conform to BL expected proportions. The supremum for the ANC 0,0423 which is higher than the critical value of 0,0104. The highest deviation is seen at the digit 25. Figure 14 below shows the plot of the K-S cumulative proportions against the ANC cumulative proportions. The line representing the ANC is above the K-S bars from the digit 16 until the digit 88, from where the two plots are aligned. See Appendix 49.

The K-S critical value calculated was 0.0140 and the supremum achieved was 0.1456. the highest deviation was at the digit 31. The cumulative proportions of the DA deviated further away from the BL cumulative proportions.

The IFP's K-S critical value was calculated to be 0.0129 and the supremum was 0.1252, which is higher than the critical value and showing nonconformance. Similar to the DA chart above, the IFP chart shows that the IFP cumulative proportions do not align with the BL cumulative proportions from the digit 10 until at the digit 98 when they align with the BL cumulative proportions. The highest deviation is at the digit 39.

5.4.2 2009 - 2024 South African Elections

The results from the year 2009 until 2024 for three leading parties in South Africa are presented below in summary. The 2024 results include the ANC, DA and the EFF/MK combined.

5.4.2.1 Benford's Law Test Results

BL first digit results showed the all the parties above the first-digit proportions in 2009 and the ANC and COPE better aligning for the rest of the first-digits with the DA below the expected proportions from the digit three. From 2014 all parties showed a similar trend with a better fit to BL. The second-digit results showed the ANC resulting in a better fit, with slight deviations, for all but the 2024 results. The opposition parties have bigger deviations for the second-digit proportions for all the assessed years. The first-two digits also showed the ANC performing better against BL expected proportions with the opposition parties having high deviations from digits 10 to 25 and then levelling down. See Appendix 50.

5.4.2.2 Other statistical tests

The MAD test statistic results have the DA nonconforming with the first digit in the 2009 elections, while in all other results the MAD resulted in either close conformity, acceptable conformity or marginal acceptable conformity for all the years under analysis, Appendix 51.

The Z-statistic test results showed the ANC nonconforming in the 2009 and 2024 for the first-digit. Nonconformances were noted for the DA and COPE in 2009, the DA and EFF in 2014, the EFF in 2019 and 2024. The second-digit Z-statistic resulted in the DA only conforming in 2024 and the EFF conforming in 2019 and 2024 as EFF/MK combination. The ANC conformed for all the years. The first-two digit tests showed all parties nonconforming. See Appendix 53.

The chi-square analysis resulted in mostly nonconformance for all the parties, which was a similar case with the p-value tests. The Kolmogorov-Smirnov tests also resulted in nonconformances for all the parties. See Appendix 50 to Appendix 71.

5.4.3 2014 - 2024 Elections ANC

In breaking with the traditions from the previous analysis in the document, the researcher analysed the ruling party's votes for the years 2014, 2019 and 2024. The logic is that should there be any vote manipulation or misstatement, the ruling party is the only candidate that can do it as it stands to benefit more than others by staying in power. It was evident in the pre-analysis the data of the opposition does not change, in terms of behaviour, throughout the voting years. Hence the analysis of the opposition parties will not affect the outcomes of the research.

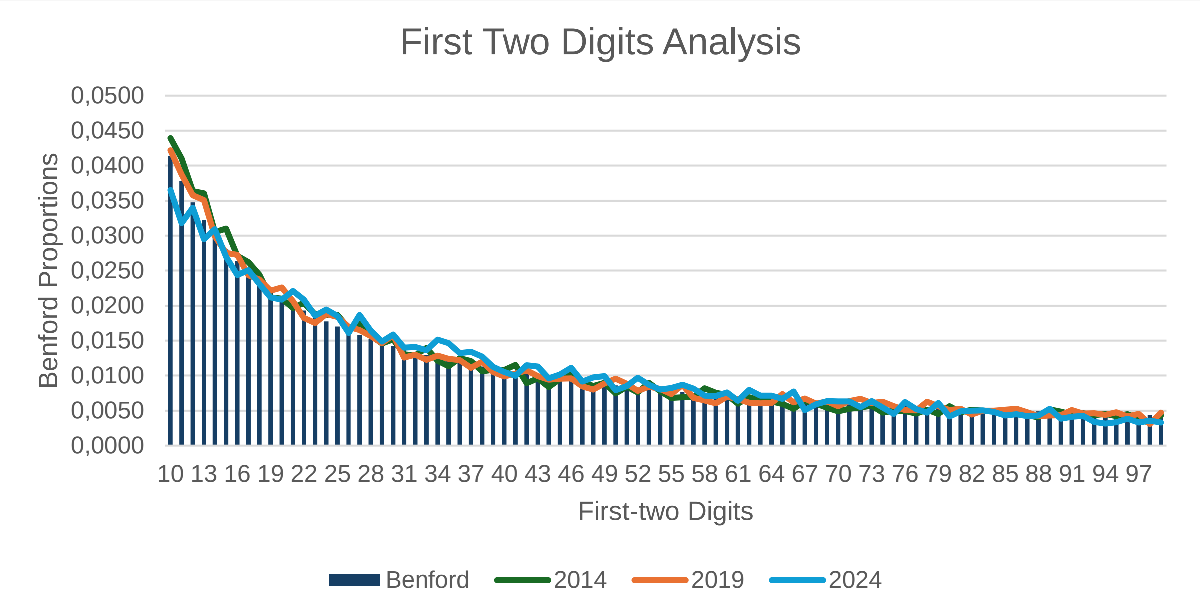
5.4.3.1 Benford's Law Test Results

The first digit test is slightly above the expected proportions for the digit one in 2014 and below the expected proportions for the digits six and seven. The 2019 proportions are like BL proportions with the digits one and two slightly above the expected proportions. The 2024 actual proportions are below the expected proportions for the digits one and nine, with the digits two, three, four, five and six above the expected proportions.

The second digit test shows the 2014 and 2019 actual proportions very close to the expected proportions, except for the digit 3 being higher for 2014 and the digit four lower. The 2024 proportions move up and down from the digit zero to four, then it is in line with the other years for the rest of the digits.

The first-two digit graph shows the data following the BL expected proportions, although moving up and down, with the general conclusion that the data conforms to BL expected proportions. See Figure 12 below.

Figure 12: ANC First-Two Digit Graph



5.4.3.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

The results of the first digit MAD assessment show close conformity for the 2014 and 2019 result and the 2024 results achieved acceptable conformity. The second digit and the first-two digit MAD assessment resulted in close conformity for all the results. Table C3

b) The Z-Statistics Test Results

The first digit Z-statistic test results had the 2014 and 2024 data not conforming to BL expected proportions, with three and six digits respectively greater than the Z-statistic critical value. The 2019 data conform with BL, with no digit greater than the critical value.

The second digit Z-statistic test resulted in the 2014 and 2019 data conforming and the 2024 data not conforming due to two digits above the critical value. The first-two digits for 2014 and 2024 did not conform to BL proportions with more than five digits above the critical value. The 2019 data conform to BL with only three out 90 digits with statistically significant deviations.

The 2014 second digit proportions have three digits above the upper bound and one digit below the lower bound. The 2019 data have one digit above the upper bound and one digit below the lower bound and the 2024 data has five digits above the upper bound and six digits below the lower bound.

c) Pearson's chi-squared Test Results

The chi-square test results show the 2024 data does not conform in all the digit tests. The 2014 data conform with the second digit chi-square test statistics and not conforming with the first digit chi-square test statistics and the first-two digit chi-square test statistics. The chi-square test statistics did not reject the 2019 data.

d) P-value Test Statistic Results

For the 2014 data the null hypothesis is rejected for the first digit and the first-two digit, with the p-value is less than alpha. The second-digit test results in the p-value greater than alpha, concluding that there is not enough evidence to reject the null hypothesis.

There is not enough evidence to reject the null hypothesis for the first digit, the second digit and the first-two digit for the 2019 data, with the calculated p-value greater than alpha. The 2024 data have all the calculated p-values less than alpha, rejecting the null hypothesis.

e) Kolmogorov-Smirnov Test (K-S Test) Results

The K-S test statistic results show that for the three elections, none conform to BL as the respective calculated suprema were greater than the respective K-S critical values. The greatest deviations for the 2014 and 2019 elections were at the digit 30 for both elections and for the 2024 elections it was at the digit 20.

The 2014 graph of the cumulative proportions show a good fit to the BL expected cumulative proportions, with the actual proportions moving above the BL proportions from the digit 15 and then back in line with BL cumulative proportions at the digit 84.

The 2019 cumulative proportions are a better fit to BL with very small deviations. The 2024 cumulative proportions are below the BL cumulative proportions from the digit 10 to the digit 33 and then from the digit 37 the 2024 data is above BL cumulative proportions until the digit 94.

5.4.4 2014 Botswana Elections

The Botswana election analysis starts with the 2014 elections and then the 2019 elections. There are two reasons for this selection; (i) no data is available for the elections before 1999 and, (ii) the 1999 data is highly aggregated, showing only 40 data points. The limited data results in high deviations from BL expected proportions, rendering the analysis unusable and unreliable.

5.4.4.1 Benford's Law test results

The first digit analysis shows the three parties not conforming to BL expected proportions. The Botswana Democratic Party's (BDP) digit one is below the expected proportions. There are high spikes at the digits four, five, six and seven, with the spikes getting lower at digits seven to nine. The BDP proportions do not conform to BL proportions. The actual proportions for the Botswana Congress Party (BCP) and the Umbrella for Democratic Change (UDC) does not conform to BL proportions, starting below BL proportions for the digits one and two and then above the proportions from the digit three to seven and then again at the digit nine for the UDC.

The second digit BL test shows all three parties not conforming to BL expected proportions with the parties' actual proportions spiking upwards and downwards throughout the digits. The BDP has an exceptionally high spike at the digit two and all three parties have high spikes at the digit nine. The BCP and UDC parties' proportions are better than those of the BDP.

The first-two digit test results show the BDP with high spikes at the digits 10 and 52, there are digits where the party does not have a single digit, like at digits 20, 22 and 23 and at 26. The BCP and UDC proportions also move up and down throughout the graph, but the parties perform better than the BDP. See Appendix 72.

5.4.4.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

The MAD results show that the three parties do not conform to BL digit tests for the first digit and the first-two digit test. BDP has marginally acceptable conformity for the second digit MAD test and the UDC and BCP are nonconforming for the second digit MAD analysis. See Appendix 73.

b) The Z-Statistics Test Results

For the first digit Z-statistics test analysis the three parties do not conform to BL expected proportions. The BDP has seven first digits above the critical value with the digit one performing worse than the other eight digits. The UDC and BCP have four out of nine digits each above the critical value with digit one performing worse statistically for both parties. See Appendix 74.

The second digit Z-statistic test shows all three parties conforming to BL expected proportions. The BDP has all the digits below the critical value, while UDC and the BCP have only the digit nine above the critical.

The first-two digit test shows the BDP and the BCP having 29 and nine digits respectively above the critical value. The UDC has eight digits above the critical value. To conform to BL proportions there must be no more than five out of 90 digits above the critical value for the first-two digit.

The BDP has 16 digits above the upper bound and nine below the lower bound. The UDC has five digits above the upper bound and two digits below the lower bound. The results of the BCP shows there are five digits above the upper bound and three below the lower bound.

c) Pearson's Chi-squared Test Results

The first digit chi-square test results show the chi-square statistics of each of the three parties greater than the chi-square critical value, resulting in all three parties nonconforming with BL expected proportions. The three parties conform to BL expected proportions for the second digit chi-square tests. Each of the parties achieved chi-square statistic lower than the chi-square critical value. See Appendix 75 for chi-square results.

The first-two digit chi-square test resulted in all three parties achieving nonconformance results.

d) The P-value Test Statistic Results

The first digit and first-two digit tests the p-values achieved by each of the parties are lower than alpha, resulting in the null hypothesis in Chapter 3 being rejected. The second digit p-values for each of the three parties were greater than alpha, concluding that there is not enough evidence to reject the null hypothesis. See Appendix 76.

e) Kolmogorov-Smirnov Test (K-S Test)

The test results show each of the three parties' cumulative proportions do not conform to BL cumulative proportions, with their respective supremum greater than their respective K-S critical values. The BDP has the greatest deviation at digit 35, the UDC's biggest deviation is at the digit 25 and the BCP's greatest deviation is at digit 28.

The BDP cumulative proportions are above the BL proportions for the digit 10 and then below the BL proportions from the digit 11 to the digit 94, from where they closely follow BL proportions. The UDC cumulative proportions are below the BL proportions from the digit 10 to the digit 98. However, the UDC's proportions try to follow the BL proportions with deviations better than those of the BDP. The BCP's cumulative proportions are below the BL cumulative proportions from the digits 10 to 68, from where they are above the BL proportions to the digit 99. Where the actual cumulative proportions are above BL proportions, they are closer to the expected proportions than where they are below.

5.4.5 2019 Botswana Elections

5.4.5.1 Benford's Law Tests Results

The first digit results show the BDP actual proportions below the BL expected proportions for the digits one to three and then above the expected proportions from the digit four to nine, with high spikes at digits five to eight. The UDC is also below the expected proportions from digit one to digit two, then the proportions are above the expected proportions from the digit three to the digit nine. High spikes are observed from the digit four to the digit eight. The AP data is slightly below the BL expected proportions at digit one, the proportions then go either below or above the expected proportions from the digit two to digit nine. There are no high spikes for the AP data, and it is a better fit to BL than the two other parties.

The second digit depiction show the BDP starting above the BL expected proportions at digits zero and one and then it is below, with this below and above trend continuing to the last digit nine. The UDC data starts below the BL expected proportions and then it goes above and below the BL proportions with high spikes above BL proportions at digits five, six and seven before dipping below BL proportions. The AP data also goes up and down against the BL expected proportions and has high spikes above expected proportions at the digits two, five, six, seven and eight before it goes slightly below BL proportions at the digit nine. The BDP and UDC data fit better to BL than the AP data.

The first-two digit test shows all the data spiking upwards and below the expected proportions. None of the parties' data fit well with BL expected data and therefore the data is nonconforming with BL proportions.

5.4.5.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

The first digit MAD test results show the BDP and the UDC are nonconforming to BL and the AP has acceptable conformity. The second digit test results have the BDP and the UDC achieving acceptable conformity with BL and the AP party is nonconforming. All three parties achieved nonconformity with BL for the first-two digit MAD test analysis.

b) The Z-Statistics Test Results

The Z-statistic test results for the first digit BL show the BDP and the UDC nonconforming, with only two digits each below the Z-statistic critical value. The AP conforms with the Z-statistic test with all its digits below the Z-statistic critical value. The second digit Z-statistic test shows all three parties conforming with BL, with the BDP and the UDC having all their digits below the Z-statistic critical value and the AP having only the digit one above the critical value.

The first-two digit Z-statistic test has the BDP and the UDC nonconforming with BL with more than ten digits each above the Z-statistic critical value. The AP is conforming, with only one out of 90 digits above the critical value.

c) Pearson’s Chi-squared Test Results

The chi-square test results for the first digit and the first-two digit show the BDP and the UDC are nonconforming with BL with their calculated chi-square statistic above the chi-square critical value. The AP is conforming with BL with the calculated chi-square statistic below the critical value. For the second digit chi-square test, all three parties conform with BL, with their respective chi-square statistic less than the critical value.

d) P-value Test Statistic Results

We use the p-value to test if the null hypothesis that the voting data digit distribution proportions conform with BL digit distribution proportions. The first digit p-values for the BDP and the UDC are less than alpha (p-value < α), with α = 0.05, concluding that the null hypothesis is rejected for both parties. The p-value of AP is greater than alpha, concluding that there is not enough evidence to reject the null hypothesis.

The second digit p-values of all three partied are greater than alpha, therefore there is not enough evidence to reject the null hypothesis. The first-two digit results reject the null hypothesis as all three p-values are less than alpha.

e) Kolmogorov-Smirnov Test (K-S Test) Results

The K-S tests have the BDP and the UDC suprema greater than the respective calculated K-S critical values, concluding that the data does not conform to BL. The supremum for the AP party is less than the K-S critical value, with the conclusion that the AP voter data conforms to BL proportions.

The graph showing the cumulative proportions of the BDP against BL cumulative proportions shows that the BDP proportions align with BL proportions for the digit 10 to 12, then it is below the BL proportions until the digit 97 where it aligns again. The biggest deviation for BDP takes place at the digit 41. The biggest deviation is at the digit 41. The UDC data follows the similar trend as that of the BDP, with the biggest deviation taking place at the digit 33. The graph of the AP party shows that the cumulative proportions of the AP party are closely aligned with BL cumulative proportions. The largest deviation of the AP party takes place at the digit 47.

5.4.6 2014 and 2019 Combined Botswana Elections

The combined data from the two election results in 2014 and 2019 was tested against Benford's Law and the hypothesis tests that were. This was done to see if the data produced similar results or if there were noticeable differences in the results. The findings will be summaries next.

5.4.6.1 Benford's Law Tests Results

Appendix F shows the figures and tables detailed in this section. The Benford's Law digit analysis showed that there was not much difference in the combined data against the two years results individually. The combined data resulted in averaging the two previous results. The first digit chart shows the data getting closed to BL expected proportions than in the previous test results. The BDP data has high spikes below BL proportions at digits one and two, then from the digit four there is a high spike above the BL proportions and peaking at the digit 5 and the spikes continue going down until at the digit nine where it is not as alarming. The UDC and BCP/AP data have no extremely varying ups and downs. The high spikes are at the digit four for the UDC and at the digit six for the BCP/AP parties.

The second digit results show the BCP/AP with highly varying ups and downs with high spikes above BL at the digits two, five and nine. The BDP and UDC proportions are better aligning with BL proportions, although not same. The first-two digit test produced similar results as the 2014 and 2019 individually. See Appendix 77.

5.4.6.2 Other Statistical tests results

a) Mean Absolute Deviation (MAD) Test Results

Appendix 78 shows the MAD analysis resulted in nonconformity for the first digit and first-two digit tests for all three parties. The second digit test results show the BDP and the UDC achieving close conformity with BL and the BCP/AP combination achieving nonconformity.

b) **The Z-Statistics Test Results**

The first digit Z-statistic test results show all three parties not conforming to BL. In contrast, there is conformance with the second digit Z-statistics tests for all three parties. There was also nonconformance for the first-two digit Z-statistics for the three groups. See Appendix 81.

c) **Pearson's Chi-squared Test Results**

Nonconformance was achieved for the first digit and first-two digit chi-squared test statistics for all three groups. For the second digit all three groups conformed with BL proportions, Appendix 79.

d) **P-value Test Statistic Results**

The first digit and first-two digit test resulted in the p-values for all three groups being less than alpha, rejecting the null hypothesis. The second digit test resulted in p-values greater than alpha, concluding that there was not enough evidence to reject the null hypothesis. See Appendix 80.

e) **Kolmogorov-Smirnov Test (K-S Test) Results**

The K-S test resulted in all three groups not conforming with BL proportions with their respective suprema greater than the respective K-S critical values. The K-S graphs show that the cumulative proportions for all three groups are below the BL cumulative proportions, with the BCP/AP cumulative proportions aligning with BL from the digit 68.

5.5 Conclusion

The findings have produced mixed results for the various tests conducted. To determine whether the null hypothesis is accepted or not, the researcher will follow on the footsteps and warning of previous researchers. Nigrini (2012) states that the use of the chi-square test statistic may produce a higher value than the critical value when the sample size is large, due to its excess power problem. The chi-square test problem is also highlighted by Miller (2015), who states that it cannot be used with large datasets due to its sensitivity to small deviations. The Z-statistic also has a problem because it considers the number of records into the calculation (Nigrini, 2012). Nigrini states that the chi-square, the Z-statistic and the Kolmogorov-Smirnov tests become sensitive with the increase in the sample size. He posits that they are not of practical use in the real world (Nigrini, 2012).

In support of Nigrini's point, Druică, et. al. (2018) found the null hypothesis statistic tests (NHST) rejecting the null hypothesis and the MAD and Excess-MAD were used to reach conclusions in their study. From the results presented in this chapter, there are conflicting outcomes when it comes to the MAD test statistic and the null hypothesis test statistics conducted. For the problems stated above regarding the tests statistics, this study will rely on the BL tests and the MAD statistic results to reach conclusions. The conclusions are presented in the next chapter.

6. Discussion of Findings

6.1 Introduction

This chapter discusses the research findings presented in Chapter 5 and the data in the appendix. The results of Benford's Law first digit, second digit and first-two digit tests conducted together with various hypothesis testing tools presented in Chapter 4 are discussed in trying to answer the research question and hypothesis presented in Chapter 3 and then compared with literature review presented in Chapter 2.

Two goodness if fit tests, chi-squared test and the Kolmogorov-Smirnov test, were used to test how good the observed data fit with Benford's Law. The Mean Absolute Deviation (MAD) was used to test for conformity, on a test-by-test basis, of the election data with Benford's Law and the Z-statistic test was used to measure the statistically significant deviation of each digit proportion to Benford's Law.

The aim of the study was to investigate if there is a mathematical or statistical tool that can be used to conduct forensic audit on declared elections results. The research question (RQ) the study aimed to answer was:

RQ: To what extent can Benford's Law, combined with statistical tests, reliably detect electoral fraud in different electoral contexts?

To answer the research question, the following null hypothesis and alternative hypothesis were tested:

The null hypothesis:

H_0: "The digit frequency distribution proportions in election data conforms to Benford's Law frequency distribution proportions".

Alternative hypothesis:

H_1: "The digit frequency distribution proportions in election data does not conform to Benford's Law frequency distribution proportions".

6.2 Summary of findings - Elections where Fraud was Alleged

6.2.1 Benford's First Digit Tests Analysis

i) Digit-digit results analysis

The Z-statistic was a primary statistical tool used to assess conformity of the digit-by-digit tests. A high number of the first digit Z-statistic results resulted in the null hypothesis being rejected. The only exceptions were the Kenya JP party in 2017. The researcher noted it that none of the assessed Zimbabwe votes conformed with the first digit Z-statistic test. The reasons for nonconformity for the first digit were either the digits deviated by large margins individually from the expected proportions as was the case with the Zimbabwe proportions, or that there were two or more proportions above the critical value, or a combination of the two. The first digit tests for this group results in nonconformity. These results are similar to those in the study of the Nigerian elections (Tunmibi & Olatokun, 2021). In the results presentation in Chapter 5, nonconformity decision was taken in cases where more than one out of nine first digits' Z-statistic was above the critical value.

ii) First digit group results analysis

The MAD statistic was the first group test conducted to analyse the first digit groups results. MAD statistic is a good measure for real-life data as it does not take the size of data into account (Nigrini, 2012), making it less sensitive to changes in data size (Agyemang, et. Al., 2023). The results show that most of the MAD results reject the null hypothesis. The 2017 Kenyan results show the Others group rejecting the null hypothesis, while in the Zimbabwe and Kenyan 2022 elections all MAD statistics rejected the null hypothesis except for the combined 2013-2023 Zimbabwe results for the MDC. First digit MAD result is a sign of close conformity of data according to Nigrini (2012).

The chi-square goodness of fit tests rejected the null hypothesis for all but the 2022 Kenyan results. The calculated chi-square statistic was not too high in the case of Kenya in 2017, whereas in Zimbabwe it was significantly high. The 2022 Kenyan results accepted the null hypothesis. The results prove the excess power problem of the chi-square test statistic (Nigrini, 2012). A similar picture was achieved with the p-value calculations, with all the results rejecting the null hypothesis, again the Kenya 2022 results being an exception. A picture that emerged from the chi-square and p-value results was the agreement between the two tests in all the cases.

It is therefore concluded that the first digit analysis for Kenya 2017 and Zimbabwe do not conform with BL. However, as stated by Nigrini (2012), the first digit tests do not provide enough information about the elections. Also to be noted is that deviations or nonconformity with BL expected proportions is not an indication of fraud or error.

6.2.2 Benford's Second Digit Tests Analysis

i) Digit-digit results analysis

In contrast with the first digit results, the second digit Z-statistic tests resulted in most of the election data proportions conforming with Benford's Law expected proportions. The ZANU-PF had mixed results when it comes to conformity in the three results and the combined result, while the MDC results were generally closely conforming with BL. All the Kenya 2017 data conform with BL expected proportions, while the 2022 Kenyan results did not. With the Kenyan 2017 results the researcher found a similarity with the findings made by Mebane (2013) in the Russian elections where the results where Putin was standing for a seat were not manipulated, going against the expectation of the researchers. The Kenyan 2017 results were fraudulent, resulting in the superior court issuing a rerun of the elections (Data Report of 2017 Elections, IECB, 2017).

ii) Second digit group results analysis

Kenyan 2017 MAD tests resulted in conformity for all parties and the 2022 results did not conform, while in the 2023 Zimbabwe elections all results did not conform. This mixed result from the MAD statistic is confusing considering all the results should not conform if there was fraud alleged. The chi-square goodness of fit tests shows most of the results conforming to expected proportions. The p-value results followed the similar pattern as the chi-square results, showing most results conforming. The MDC results from the three separate elections and the combined results all conform to BL. This is expected for a party that has no power on electoral body as has been suggested for the ZANU-PF. These mixed results concur with Deckert et. al. (2011) about BL propensity to produce false positive and false negatives.

Overall, Kenyan data conforms to BL while Zimbabwe reached close conformity especially with all the 2023 data conforming.

6.2.3 Benford's First-two Digit Tests Analysis

i) Digit-digit results analysis

The Z-statistic tests for the first-two digit resulted in mostly nonconformity, with only Others in Kenya 2017, both Kenyan candidates in 2022, ZANU-PF and MDC in Zimbabwe in 2023 conforming.

ii) First-two digit group results analysis

The Z-statistic tests revealed numerous nonconformities in the data, with Kenyan 2022 data all conforming. Odinga in Kenya 2022 results achieved five digits higher than the Z-statistic critical value. However, this is considered a conformity considering the number of digits under review (Nigrini, 2012). The first-two digit Z-statistic used the upper bound and lower bound to refine the single digit Z-statistic test. Proportions above the upper bound and those below the lower bound are nonconforming as these bounds are based on five percent above and five percent below the expected proportions. When this method is used for the Kenyan 2022 results, Odinga achieves five spikes above the upper bound and three below the lower bound, resulting in nonconformity. The high number of nonconformities is partially attributed to the fact that the conformity decision is based on having not more than five out of 90 single digits should be above the critical value (Nigrini, 2012). The MAD tests resulted in mostly nonconformity with BL, with a few cases where data conformed.

Chi-square tests results show Kenya conformed with BL except for Odinga in 2022, while Zimbabwe did not conform in all elections. The p-value results mostly confirmed the chi-square test results, except in Kenya 2017 for Others where the data that conformed on the chi-square test had the p-value rejecting the null hypothesis. Also, Odinga in 2022 did not accept the null hypothesis. The reason the two results do not agree may be caused by the high chi-square value achieved. In terms of Odinga in 2022, the higher chi-square value and the rejection of the null hypothesis by the p-value aligns with the Z-statistic results - where eight digits were nonconforming.

Across 2013, 2018, and 2023, nonconformance to BL remains consistent in the Zimbabwe election results, with ZANU-PF and MDC-T occasionally showing closer conformity in specific tests. The Others group in Zimbabwe consistently performs poorly, with significant deviations from BL proportions. This can be attributed to the fact that this group is composed of smaller parties that made a small percentage of the election results, nothing significant will be read in the results. There is no significant improvements or reforms in the electoral data's alignment with BL over time. The 2017 Kenyan elections demonstrated limited adherence to BL, with the

JP performing slightly better than WDM-K and Others. Despite this, significant deviations across all tests suggest widespread irregularities or data inconsistencies in Kenya 2017 results. The 2022 Kenyan results are not impressive graphically and both candidates do not conform with the MAD tests, all other tests, with the exception of the chi-square, returned conformity results.

Both Zimbabwe and Kenya 2017 exhibit electoral data patterns inconsistent with BL, potentially signalling irregularities in vote distributions or a result of voter behaviour patterns. Kenya's 2022 results show improvement from the 2017 data in that single digit tests and group tests all returned consistent results. However, Odinga's overall results do not conform to the expected BL proportions due to multiple tests they failed. The results above are similar to the findings by Rad, Amiri, Ranjbar, & Salari, (2021) who found BL cannot detect fraud between companies with fraud-risk and those without fraud-risk. They could not detect fraud concluding that the law has zero accuracy. Similarly, Lazebnik & Gorlitsky (2023) found that BL produced false positives in manipulation free manuscripts and did not produce convincing results in manipulated manuscripts that were withdrawn. da Silva Azevedo, Gonçalves, & de Mesquita Spinola, (2021) found the first digit tests did not conform, however the second digit used with aggregated data conformed together with the statistical tests.

6.3 Summary of findings - Elections where no Fraud was Alleged

6.3.1 Benford's First Digit Tests Analysis

i) Digit-digit results analysis

The Z-statistic was a primary statistical tool used to assess conformity of the digit-by-digit tests. A high number of the first digit Z-statistic tests resulted in the null hypothesis being rejected. The exceptions were the Botswana AP party in 2019 and the South African ANC in 2019 (2014-2024 results). The reasons for nonconformity with the Z-statistic test for the first digit were either the digits deviated by large margins individually from the expected proportions as was the case in most cases or, as in case of the South African data, small deviations from large datasets become amplified. The South African case confirms Nigrini (2012) on the effect of a large dataset on small deviations from expected proportions. Nigrini (2012) states that small deviations are likely to be flagged as significant when dealing with large datasets (Nigrini, 2012, p.151). In the results, nonconformity decision was taken in cases where more than one out of nine first digits' Z-statistic was above the critical value.

iii) First digit group results analysis

The MAD statistic was the first group test conducted to analyse the first digit groups results. MAD statistic is a good measure for real-life data as it does not take the size of data into account (Nigrini, 2012), making it less sensitive to changes in data size (Agyemang, et. Al., 2023). The results show that most of Botswana MAD results reject the null hypothesis, except for AP party in 2019. All the South African MAD results accepted the null hypothesis with varying levels of conformity. First digit MAD is a sign of close conformity of data (Nigrini, 2012).

The chi-square goodness of fit tests rejected the null hypothesis for all the results except for Botswana's AP in 2019 and the ANC in 2019. For the South African 2024 results, the DA and EFF/MK conformed with the chi-square. A similar picture was achieved with the p-value calculations, with many of them rejecting the null hypothesis. The chi-square and p-value results agreed in all the cases.

General nonconformity was observed for the first digit test in Botswana and the South African results conformed overall.

6.3.2 Benford's Second Digit Tests Analysis

iii) Digit-digit results analysis

In contrast with the first digit results, the second digit Z-statistic tests resulted in most of the election data conforming with Benford's Law expected proportions. All the Botswana data conforms with BL expected proportions while in South Africa the IFP in 2004 and the ANC in 2024 did not conform. The opposition parties in South Africa did not display good conformance in all elections except the 2024 elections.

iv) Second digit group results analysis

Kenyan and South African MAD tests resulted in conformity throughout. The other statistical tests results had mixed outcomes. The chi-square goodness of fit tests shows most of the results conforming to expected proportions, with all the Botswana single and combined data conforming with BL. The p-value results followed the similar pattern as the chi-square results. These findings are similar to those by da Silva Azevedo et. al. (2021) where the first digit tests did not conform while the second digit tests showed conformance.

6.3.3 Benford's First-two Digit Tests Analysis

i) Digit-digit results analysis

The Z-statistic for the first-two digit tests resulted in mostly nonconformity, with only six of the cases conforming. The high number of nonconformities is partially attributed to the fact that the conformity decision is based on having not more than five out of 90 single digits should be above the critical value (Nigrini, 2023).

ii) First-two digit group results analysis

The Z-statistic tests revealed numerous nonconformities in the data. The first-two digit Z-statistic used the upper bound and lower bound to refine the single digit Z-statistic test. Proportions above the upper bound and those below the lower bound are nonconforming as these bounds are based on five percent above and five percent below the expected proportions. Mostly nonconformance was observed for the Z-statistic tests except for the AP party in 2019 Botswana election results and the ANC in 2014 and 2019 in the South African results. The MAD tests showed the ANC in 2004 2014-2024 conformed with BL and the opposition parties did not conform.

Chi-square tests results show the Botswana 2019 for the AP party and South Africa's ANC in 2019 conforming to BL expected proportions, while the rest of the parties mostly did not conform. The p-value results mostly confirmed the chi-square test results. The reason the two results do not agree is caused by the high chi-square values achieved. The AP party results accepted the null hypothesis when tested with the K-S test. The first-two digit tests mostly rejected the null hypothesis.

Elections from 2014 to 2019 in Botswana exhibit consistent nonconformance to BL, with minor improvements for AP compared to BDP and UDC. Combining the data slightly smoothed out extreme deviations but did not lead to overall conformity. No significant trend of improvement was observed, with persistent issues across multiple tests.

The initial elections in South Africa studied here in 2004 showed better BL alignment for ANC compared to other parties. The 2014–2024 trend reveals fluctuating conformity, with 2019 showing the best alignment with BL expected proportions, but 2024 data indicated a regression with significant nonconformance for the ANC. The opposition parties in South Africa achieved nonconformance against BL expected proportion throughout the period. Overall, the conformity trend is inconsistent, with no clear improvement over time. Both countries' elections show inconsistent adherence to BL, with no sustained improvement over time and the ANC showing significant deviations in the 2024 results - where the party achieved its lowest overall margin, raising potential concerns about the integrity or reporting of electoral data.

6.4 The null hypothesis

In testing the null hypothesis, "The digit frequency distribution proportions in election data conforms to Benford's Law frequency distribution proportions", the researcher used Benford's Law first digit, second digit and first-two digits analysis together with the statistical tools such as the MAD, Z-statistics, Kolmogorov-Smirnov tests, the chi-squared tests and the p-value tests statistics. The test statistics used produced mixed results that rendered them unusable to some extent. The researcher decided to follow what Nigrini (2012) stated, that is the chi-squared test suffers from excess power problem and should not be used with large data samples. Excess power problem means that when the number of records (*N*) is large, the chi-square statistic will be larger than the calculated chi-square critical value. Miller (2015) agrees with Nigrini in that the chi-squared test is sensitive when used with large sample size. Due to its reliance on the results of the chi-squared test statistic, the p-value test statistic was also then ignored in reaching the conclusion in this research.

The BL tests, MAD test statistics, the Z-statistic and the Kolmogorov-Smirnov test statistics results were used to reach the conclusion of the hypothesis test. The Kolmogorov-Smirnov test was used only for the first-two digit test whereas the other test statistics were used on all digit tests performed.

In terms of the Zimbabwe election results, BL tests for the first digit and first-two digits produced results that showed close conformity to BL expected proportions whereas the second digit tests produced results that were not conforming to BL due to large deviations. The Kenyan results showed large deviations from BL expected proportions for the second and first-two digits while the first digit tests showed close conformance for the JP party. Antonio (2023) states that where significantly large deviations from expected proportions are observed, manual audit should be conducted, in this case a deeper investigation into the voting patterns and voter behaviour should be analysed.

The Botswana elections produced results that were nonconforming to BL expected proportions for all the digit tests conducted. However, the South African election results showed better conformance to BL in all the elections analysed despite small deviations.

To answer the null hypothesis, the election data does follow BL expected proportions. This happens when a sufficiently large sample size is used for the tests. The South African data used had large sample sizes and as such it was a better fit to BL.

The Mean Absolute Deviation test statistics showed conformity for the South African data on all elections, which supported the BL test results. Conformity varied from marginal conformity to conformity with a few cases of nonconformity only for the first-two digits tests for the opposition parties. The Botswana results showed mostly nonconformity in all elections apart from a few cases for the second digit tests. Kenyan elections showed mostly conformity with the MAD statistic tests for all with a few nonconformities. The Zimbabwe data resulted in mostly nonconformity with BL for all digit tests.

The overall result from the MAD test statistics is the null hypothesis is accepted that the data follows BL expected proportions.

The Z-statistic test for the first, second and first-two digit tests in the South African elections showed that the winning party conformed to BL whereas the opposition parties did not conform. The Botswana results showed nonconformity for the first and first-two digits while the second digit tests showed conformity with BL. The Zimbabwe results mostly rejected the null hypothesis for most of the results in the periods tested, whereas the Kenyan results conformed with BL for the first and second digit tests and the first-two digits tests rejected the null hypothesis. The Z-statistic did not reject the null hypothesis.

6.5 In Summary

The South African election results, particularly their fluctuating levels of conformity to Benford's Law (BL), are like several studies that have explored BL in the context of election data and registered voter populations. Mark Nigrini (2012) has applied BL to various datasets, including elections, to detect irregularities. His findings often show that registered voter populations tend to conform to BL better than actual election results. This is shown in South Africa's 2004 elections where registered voter population data aligned better with BL than vote counts. This is consistent with Nigrini's assertion that vote distributions are more susceptible to manipulation or natural deviations due to regional and demographic factors (Nigrini, 2012).

Walter Mebane has extensively studied BL in the context of election results, finding that deviations from BL can indicate potential irregularities. However, he emphasizes that not all deviations signify fraud, as natural political dynamics or systematic reporting processes can explain some discrepancies. The South African results in 2019 are showing better conformity than 2004 or 2024 election data. This reflects Mebane’s finding that deviations can vary over time, influenced by factors like election administration, voter behaviour, or data aggregation (Mebane, 2017).

Research applying BL to elections in developing nations, such as in Russia (Mebane, 2013), Ghana (Agyemang, et. al., 2023), and Nigeria (Tunmibi & Olatokun, 2021), often highlights mixed conformity to BL due to regional disparities, voter turnout variations, or incomplete reporting. As an example, a study on Kenyan elections showed better alignment for ruling parties than opposition groups, a trend like the ANC outperforming DA and IFP in South Africa. Research on Nigeria's elections (Tunmibi & Olatokun, 2021) identified high deviations from BL in specific regions, like deviations observed in South Africa's DA and IFP results.

Comparative studies of BL done in this study in mature democracies like the U.S. and Germany versus those emerging democracies like South Africa, Kenya and Russia show that all elections deviate from BL expected proportions. Elections in emerging democracies can deviate from BL due to systemic challenges, including weaker institutions, voter intimidation, or irregularities in vote reporting. The voter behaviour cannot be ignored in Africa, where tribalism is still an issue even in the best African democracy such as South Africa, where a strong performance of a new party, Umkhonto Wesizwe, in the Kwa-Zulu Natal was blamed on tribalism (Faizel Patel, The Citizen, 31 May 2024). South Africa's fluctuating conformity to BL over time mirrors findings in emerging democracies where electoral reforms or changes in governance temporarily improve alignment with BL like the 2019 elections but this may not sustain over time as shown in the 2024 elections. Although this research did not analyse the distribution of the votes and voters per district or region to investigate the patterns and voter behaviour, a further investigation of the regional voting patterns may reveal why there are anomalies in the distributions.

Research has shown that BL conformity often varies based on local or regional dynamics, like rural against urban turnout and voter distribution (Agyemang, et. al., 2023). This aligns with observations in South Africa, Botswana, Zimbabwe, and Kenya where deviations were likely influenced by demographic and regional variations in voter behaviour like regional loyalties, ethnic identities (Deckert, et. al., 2011). The elections in this research reflect the impact of localized factors on BL conformity, like findings in decentralized voting systems like Brazil (Filho, et. al., 2022) and in the unified Germany (Breunig & Goerres, 2011).

The election results analysis done in this research align with global trends in studies of elections using Benford's Law. Particularly relevant are research findings from emerging democracies, studies emphasizing regional dynamics, and work by experts like Nigrini and Mebane. These studies suggest that while BL can highlight irregularities, its results must be interpreted cautiously (Kruger & Yadavalli, 2017), accounting for systemic, regional, and procedural influences on election data (Deckert, et. al., 2011). BL should be used with other forensic tools to increase chances of detecting possible fraud, as it can only indicate possible red flags in the data and not where fraud was committed (Mehta & Bhavani, 2017).

6.6 Conclusion

This research was conducted to investigate whether Benford's Law could be a useful mathematical or statistical to conduct forensic audit of declared election results by electoral authorities. The study employed the use of Benford's Law first digit, second digit and first-two digit analysis to investigate conformity of declared results. The study also employed statistical goodness-of-fit tests to confirm the results obtained from the digit analysis.

The study showed that BL cannot be reliably used as a mathematical or statistical tool to conduct forensic audits, and the author concurs with previous researchers that stated that the method should be used as a high-level tool to investigate the possibility of anomalies in data. The method has shown that it can return possible erroneous results in the form of Type 1 and Type 2 errors. The unreliability of the statistical tests results also plays a key role in reaching the conclusion about the methodology. These findings concur with Deckert et. al (2011) in that Benford's Law is not reliable to conduct election forensic auditing, "the law is not universally applicable magic box into which we plug election statistics and out comes an assessment of an election legitimacy" (Deckert, et. al., 2011). A straightforward and uncomplicated application of digit tests for detecting fraud appears to be questionable (Mebane, 2013). This assertion by Mebane is in line with the findings of this research, in that the deviations from expected BL proportions are not an indication of fraud.

7. Conclusion

7.1 Introduction

In this study, the researcher investigated the usefulness of Benford's Law as a mathematical or statistical tool for election fraud detection. The study was influenced by the vast research currently out which does not seem to have a conclusive answer to the question of whether Benford's Law can be used for forensic audit of election data. Benford's Law has applications across fields such as auditing, fraud detection, data analysis, and the validation of large datasets, making it an invaluable tool for ensuring the integrity and authenticity of numerical information. However, researchers are not at all in agreement with its use. Chapter 1 introduced the problem statement as stated above.

In Chapter 2 a review of existing literature was presented. The literature showed some of the many areas where BL is used. However, there are limitations to its use due to inconsistent results. In Chapter 3 the research question and the null hypothesis were presented. The results of the tests were then presented in Chapter 5, the analysis of which used the methodology that was presented in Chapter 4.

The context of this research centres on Benford's Law's utility in forensic data analysis and electoral results forensic auditing. As organizations grow increasingly reliant on large datasets for decision-making, detecting anomalies and verifying data authenticity has become essential. By understanding the contexts in which Benford's Law can accurately signal irregularities, we can significantly enhance data integrity, prevent fraud, and support more informed decision-making. This is particularly important in business, finance, and public administration, where discrepancies in reported data can have significant consequences.

What Did We Already Know

Prior to this research, scholars had established the theoretical basis of Benford's Law and its prevalence across a variety of natural datasets. However, its practical applicability, especially in distinguishing between genuine anomalies and benign deviations, was less understood. Questions remained about its reliability across diverse data sources and its sensitivity in detecting specific types of fraud, especially as datasets grow more complex in structure and composition. BL has been widely used to detect anomalies in financial data such as tax returns, expense reports and financial statements, where fabricated numbers often deviate from Benford's distribution (Horton, et. al., 2020). It has been applied to vote counts to investigate electoral fraud by checking if the reported numbers align with Benford's expected distribution. It has also been used to identify fabricated or manipulated experimental results in research studies.

The BL method works across a wide range of datasets, including stock prices, population numbers, and energy consumption. Using the method requires only basic statistical tools and can be applied without detailed knowledge of statistics. Deviations from the expected distribution can be a red flag for further investigation, however, they are not proof of either fraud or error (Mehta & Bhavani, 2017).

Not all datasets are appropriate for Benford's analysis (Durtschi, Hillison & Pacini, 2004). For example, data with a narrow range or artificial constraints often fail to follow Benford’s distribution. The boundary conditions for effective application are not universally agreed upon. Datasets should span multiple orders of magnitude, (Miller, 2015), to be usable and numbers should not have arbitrary cutoffs or constraints such as fixed ranges or human-imposed limits. Data should represent measurements or natural growth, rather than assigned or fabricated numbers like account numbers.

Deviation from Benford’s Law is not definitive evidence of fraud (Nigrini, 2012; Deckert, et. al., 2011; Mehta & Bhavani, 2017). Natural datasets can deviate from the expected distribution for legitimate reasons such as sampling bias, cultural or systemic factors. Fraudulent datasets can sometimes mimic Benford’s distribution (Rad, et. al., 2021), evading detection. The influence of cultural, demographic, or geographic factors on deviations from Benford's Law remains underexplored in many fields (Deckert, et. al., 2011). How specific voting systems, tax regulations, or organizational practices impact compliance with Benford's expectations is not well understood. Deviation from Benford's distribution does not always imply data manipulation. Distinguishing between natural deviations and intentional fraud requires supplementary analysis. Lack of a standardized threshold or guidelines for interpreting deviations complicates its forensic application. Establishing statistical thresholds for deviations that justify suspicion is not well-defined in practice (Deckert, et. al., 2011). Researchers and analysts often use arbitrary cutoffs.

The research aimed to answer the research question presented in Chapter 3, “To what extent can Benford’s Law, combined with statistical tests, reliably detect electoral fraud in different electoral contexts?”. To answer the research question Benford’s Law digit tests were employed together with null hypothesis testing which included the chi-square tests, the Kolmogorov-Smirnov tests, the Mean Absolute Deviation tests, the Z-statistic tests and the p-value tests.

7.2 Main findings

In this section the researcher presents the main findings per category outlined in Chapter 5 and the discussion in Chapter 6.

7.2.1 Elections where Fraud was Alleged.

The digit tests for the Kenyan elections returned an unexpected mixed result. With the information that the results were nullified by the courts due to fraud and other issues, it was not expected to have them conforming to any of the digit tests. Since the first-two digit test provides more information than the first and second digit (Nigrini, 2012), it was expected that the first-two digits tests would provide a conclusive answer to the research question.

On the strength of most of the tests returning conformance in Kenya it can be concluded that the data conforms with BL and there is no reason to believe there was fraud. A result that concurs with the Russian elections (Mebane, 2013) where Putin’s results conformed to BL while there was evidence of fraud.

The Zimbabwe data on the other hand returned a complete set of nonconformance across the three elections under study. However, the nonconformance is neither a sign of fraud nor error. Further investigation may uncover the reasons for nonconformance.

7.2.2 Elections where no Fraud was Alleged.

The Botswana findings returned nonconformance for the first digit, except for the AP party in 2019. When concentrating on the MAD results it shows significant nonconformance in the Botswana election data. Due to the statement in the summary of Chapter 5 which stated that the MAD will be used to conclude, the Botswana elections are nonconforming with BL. This is an unexpected result, considering we expect these results to conform. However, reasons for the nonconformances cannot be known until further investigations are conducted, with data size being one of the limiting issues.

The South African results for the ANC are conforming with BL based on the MAD results for the first and second digit tests. The first-two digit test shows the MAD returns nonconformance for the DA and the IFP in the 2004 results. The outcome for South Africa is the data does conform with BL in most cases.

Due to the mixed results achieved in the study, the researcher concludes that the use of BL to conduct forensic audit of election results is not recommended. The results may have produced false positives especially in the case of the Kenyan elections where the elections were later annulled by the courts (Data Report of 2017 Elections, IECB, 2017). There is also a case to be made for the South African results where the opposition parties' data did not conform to BL expected proportions while these elections are known to be free and fair with no fraud alleged. This shows that there are inconsistencies in the use of BL to conduct election results forensic auditing, supporting the results by Deckert et. al (2011) that BL cannot be used for election results auditing.

This research contributes to the debate on the usefulness of BL in conducting election auditing by stating that there needs to be more than just caution when using the method for election results auditing, the research concludes that more studies are required to develop a proper theory and statistical methods to conduct auditing of election results using BL. The research adds to the voices that say BL is not suitable for election forensic auditing. Although the method has proven to be useful in accounting forensic auditing (Nigrini, 2012; Mehta & Bhavani, 2017), it has no practical relevance in election forensic auditing due to the prevalence of Type 1 and Type 2 errors.

7.3 Recommendations

The usefulness of Benford's Law in conducting election forensic auditing is not reliable due to the mixed results obtained, rendering them unusable and inconclusive at best. The author recommends that more research be conducted in the search for a reliable accurate tool to conduct election forensics. The research can concentrate on developing the basics of BL and integrate BL with some statistical tools that will provide reliable results. The proper use and calculation of the MAD for various sample sizes should be investigated and improvement of the other statistical tools should be properly studied and those statistical methods that work best with BL be implemented as part of the audit process.

A development of an election forensic audit toolkit would be a valuable addition to electoral authorities, parties and election observers. The toolkit should include Benford's Law digits tests, statistical tests and machine learning techniques (Agyemang, et. al., 2023). The digit tests should include BL first digit and second digit tests to provide a high level analysis and level of conformity of the data (Nigrini, 2012), first-two digit test which provides a more detailed analysis and biases in the data. The statistical tests should be able to eliminate the problems like the high power of the chi-square. The toolkit should also be able to deal with various sample sizes and the MAD conformity levels should be adjustable to deal with datasets of various magnitude.

7.4 Limitations

As each research has limitations, the following limitations were identified for the research.
Analysis of data may flag existence of irregularities. However, these may be due to unintentional mismanagement of the voting process (Lacasa, Fernández-Gracia, 2018) and not fraud. It is therefore unclear if Benford's Law can differentiate between manipulated and unmanipulated data. The method cannot be utilised to prove or disprove fraud or error in elections but can only be used as a screening tool (Agyemang, et. al., 2023).

According to Druică, et. al., (2018), the null hypothesis significant tests are misleading for large sample sizes. But to get a statistically significant result, a large dataset is required (Antonio, 2023). Linear transformations of Benford sequence cannot be detected by the analytic methods described in the research (Druică, et. al., 2018). The propensity of BL tests to return false positives and false negatives may result in incorrect conclusions being made is another limitation. BL cannot detect highly sophisticated forms of fraud or data manipulation. The other limitation is the availability of high-quality journal articles for the research.

BL method has the propensity of producing type 1 and type 2 errors, in that it detects fraud where there is none or it does not detect fraud where fraud took place (Deckert, et. al., 2011). The lack of peer reviewed studies about BL is a shortcoming in the method's advancement and improvement. Deckert, et. al. (2011) highlights the need for peer review and literature that link BL to elections. Mebane (2011) agrees on the need for the peer review of studies and articles on BL.

7.5 Recommendations for future research

There is a need to develop hybrid approaches that combine Benford's Law with other statistical or forensic techniques to reduce false positives and negatives. The developed model should be tailored to account for domain-specific factors such as election systems, financial audits and other areas like academic publications, which could improve accuracy. The models should also be built into software programmes that integrate Benford's analysis with contextual data to simplify and standardize its application in forensic investigations.

There is also a need to conduct large-scale empirical studies across different domains to better understand the conditions under which Benford's Law is most reliable and to develop statistical ranges to determine the acceptable levels of error into the system. Benford's Law is a powerful tool for forensic analysis, but its effectiveness depends on context, data quality, and complementary analytical methods.

7.6 Conclusion

In answering the question of whether there is a statistical tool for election forensic, hypothesis testing was conducted which resulted in conflicting results. The analysis just performed resulted in some of the statistical tools giving inconsistent results. The results are not reliable due to the Type 1 and Type 2 errors. The issue of using aggregated or unaggregated data brings forward yet another problem in using BL as a forensic tool. Benford's Law has proven effective in fields like accounting (Nigrini, 2012), but its reliability in elections, research fraud, and other domains is debatable. Studies show mixed results when applying it to election data, leading to questions about its consistency in identifying fraud in political processes. It should also be noted that BL does not confirm fraud or error, it only highlights possible areas where further investigation must be conducted (Nigrini, 2012; Mehta & Bhavani, 2017; Gorenc, 2019; Lazebnik & Gorlitsky, 2023).

To answer the research question of whether Benford's Law can be used as a mathematical or statistical tool to conduct forensic audit of election results, the researcher employed a quantitative analysis and hypothesis testing of election data from declared election results with known fraud and without fraud. The researcher therefore concludes that Benford's law cannot be used as a forensic tool in election results forensic audit. This conclusion aligns with other research done previously that could not reach a definitive conclusion on the usefulness of BL to conduct forensic audit (Rad, et. al., 2021; Shanaev, Shuraeva, & Ghimire, 2020). Horton, et. al. (2020) state the need for criteria to judge deviations or the threshold to accept the deviations, which aligns with Deckert et. al (2011) who state that there is a need for theory that can link BL to electoral fraud as the law cannot be used for electoral fraud forensic auditing.

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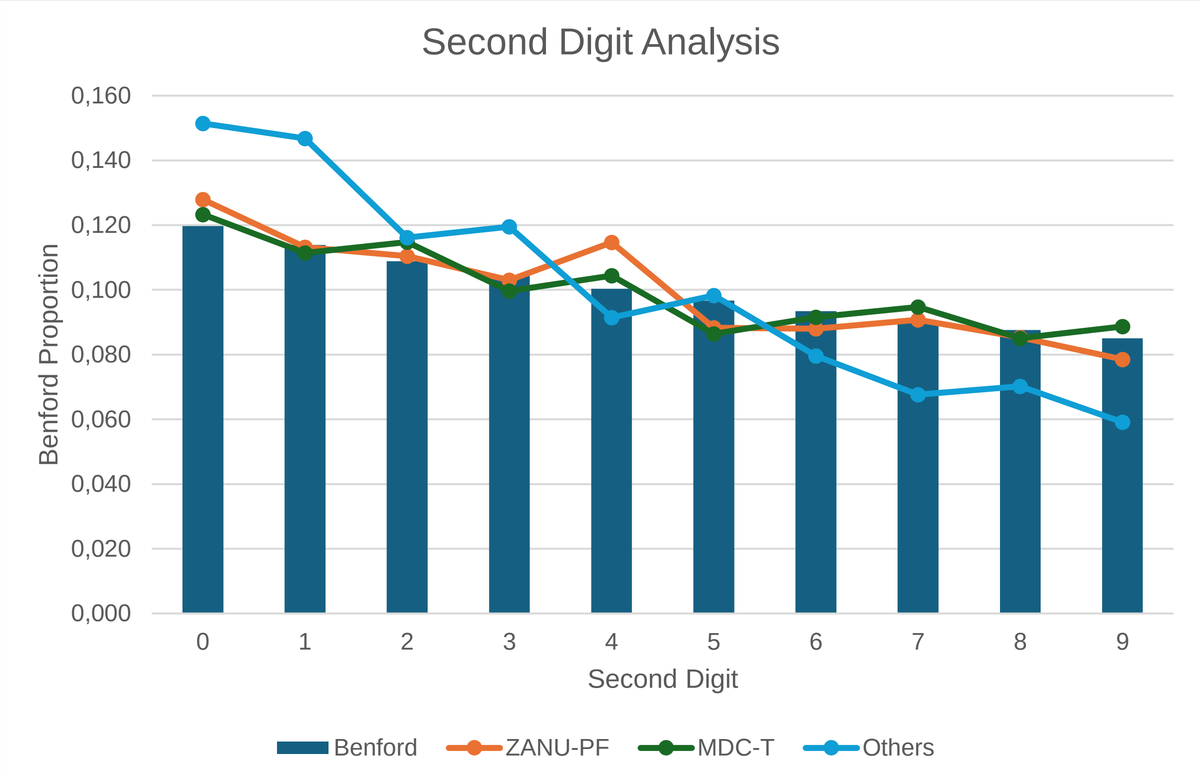
Appendices

Appendix 1: 2013 Zimbabwe Second Digit Benford's results



Appendix 2: 2013 Zimbabwe MAD results

MAD Outcomes			
	ZANU-PF	MDC-T	Others
1st Digit Outcome	0,01505 **Nonconformity**	0,01905 **Nonconformity**	0,01543 **Nonconformity**
2n Digit Outcome	0,00492 **Close conformity**	0,00434 **Close conformity**	0,01773 **Nonconformity**
FT Digits Outcome	0,00199 **Marginally acceptable conformity**	0,00267 **Nonconformity**	0,00803 **Nonconformity**

Appendix 3: 2013 Zimbabwe First and Second Digit Z-statistics results

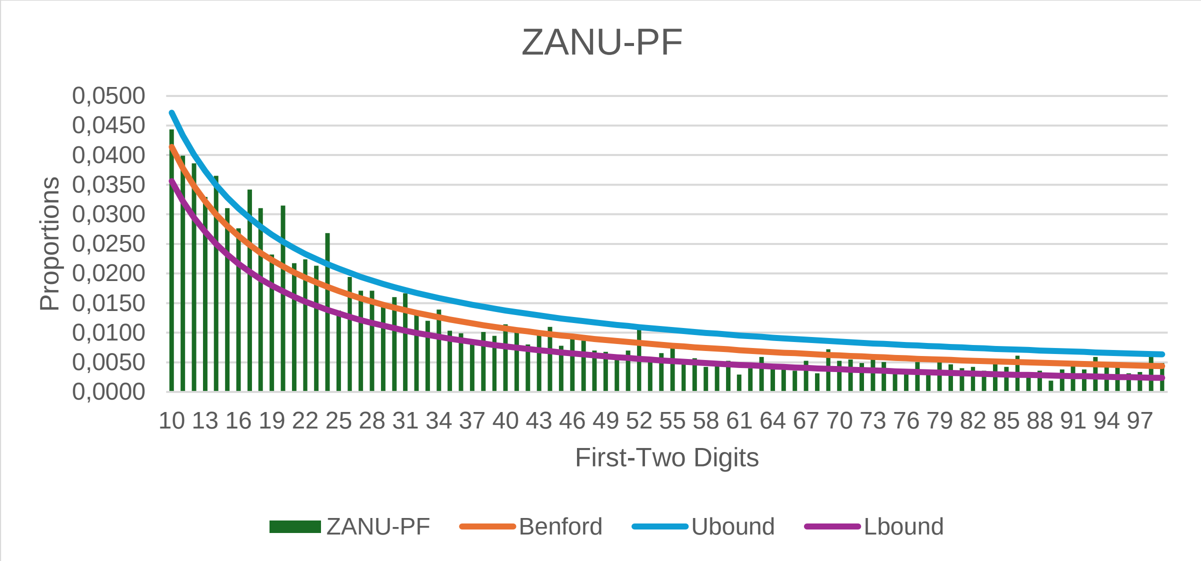
Z-Statistics for First Digit			
Digit	ZANU-PF	MDC-T	Others
1	5,6079	2,0144	8,5172
2	5,3285	9,4970	2,7790
3	0,8815	6,8586	3,9823
4	1,3692	2,5681	3,8514
5	3,9590	5,0778	3,0164
6	5,5481	4,6778	0,8404
7	3,1043	2,5511	0,4611
8	3,3924	1,5599	0,4668
9	0,2148	3,2779	3,1558
Z-Statistics for Second Digit			
Digit	ZANU-PF	MDC-T	Others
0	1,7268	0,7229	4,7103
1	0,1371	0,5018	4,9824
2	0,3275	1,2478	1,1036
3	0,2715	0,9884	2,3762
4	3,2565	0,8849	1,3956
5	1,9407	2,2916	0,2237
6	1,2381	0,3834	2,2692
7	0,0766	0,9934	3,8066
8	0,5304	0,5658	2,9460
9	1,5705	0,8477	4,4619

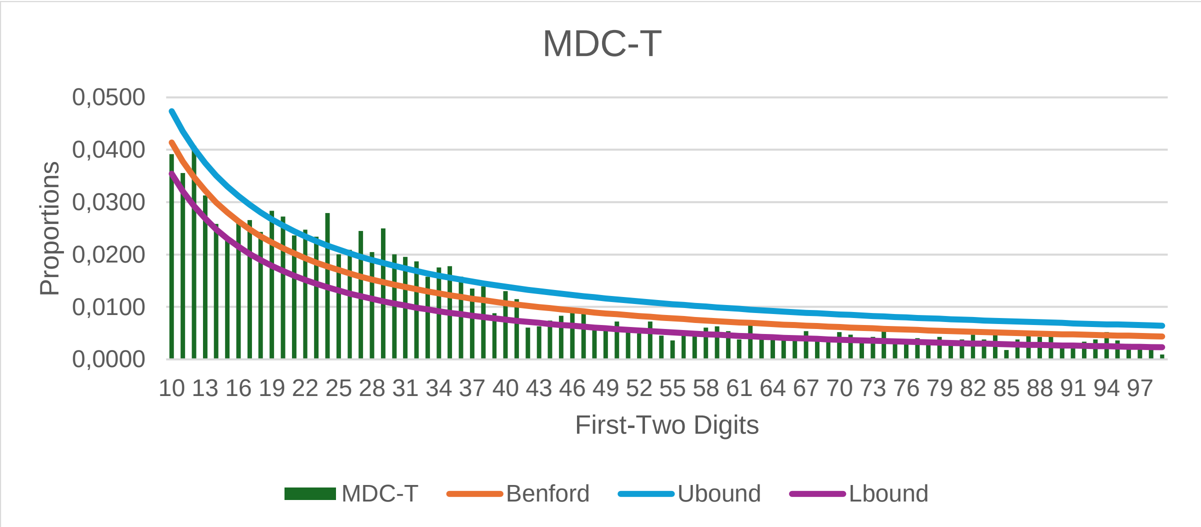
Appendix 4: 2013 Zimbabwe First-Two Digit Z-statistics Results

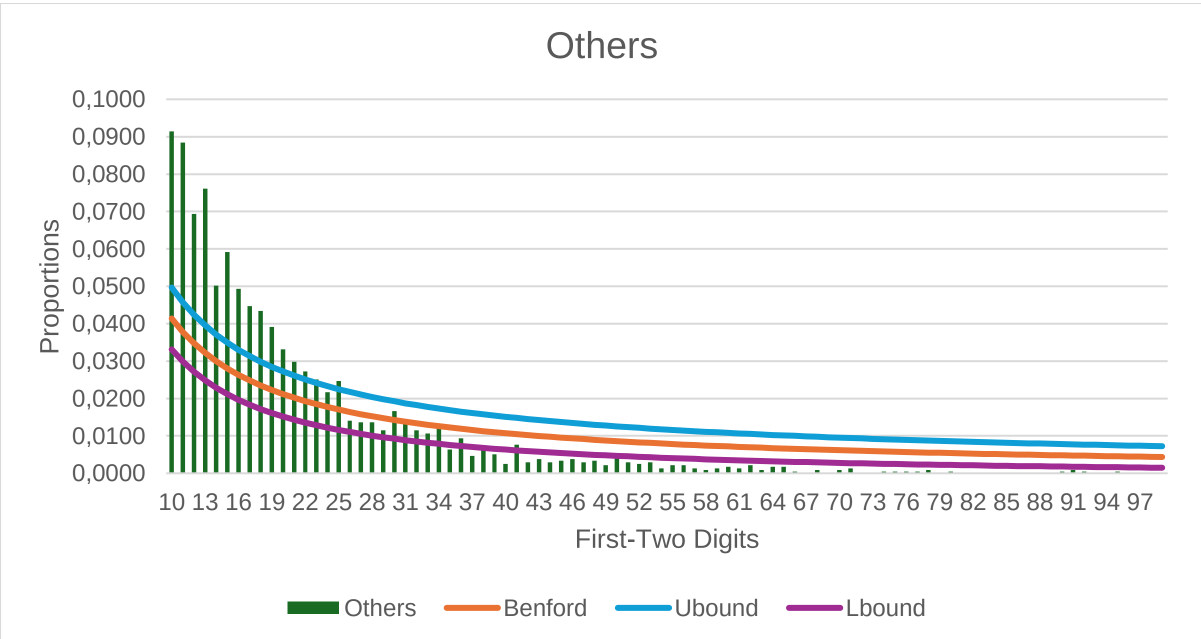
Z-Statistics for First-Two Digits							
Digit	ZANU-PF	MDC-T	Others	Digit	ZANU-PF	MDC-T	Others
10	0,9791	0,7116	12,1330	55	0,2360	3,1113	3,0188
11	0,7235	0,7420	12,8339	56	1,8154	2,0027	2,9687
12	1,4144	2,1306	9,0945	57	1,3894	2,0906	3,3962
13	0,2504	0,2999	12,0168	58	2,4826	0,9598	3,5927
14	2,6048	1,5536	5,6925	59	2,0613	0,6939	3,3097
15	1,2084	1,9140	9,0721	60	1,4637	1,3152	3,0236
16	0,5245	0,0474	6,9043	61	3,2884	2,4870	3,2270
17	4,1006	0,6926	6,1161	62	2,3467	0,2924	2,6905
18	3,3842	0,3121	6,3055	63	0,6872	2,3469	3,3981
19	0,3915	2,6939	5,4681	64	1,6693	1,7285	2,8573
20	4,8554	2,7431	3,9646	65	1,5950	1,8421	2,8178
21	0,7019	1,5690	3,2252	66	2,4237	1,7735	3,5472
22	1,4838	2,5847	2,7150	67	0,9047	0,7679	3,7729
23	1,3963	2,3789	2,3038	68	2,6604	2,2066	3,2235
24	4,6814	5,0827	1,3784	69	0,7188	1,9550	3,7141

Z-Statistics for First-Two Digits							
Digit	ZANU-PF	MDC-T	Others	Digit	ZANU-PF	MDC-T	Others
25	1,7053	1,4843	2,7820	70	0,6836	0,7432	3,1585
26	1,5858	2,3228	0,8177	71	0,4251	1,0606	2,8614
27	0,6622	4,6093	0,7663	72	0,9182	1,5787	3,6304
28	0,9854	2,7886	0,5605	73	0,0018	1,3229	3,6037
29	0,0309	5,6133	1,2182	74	0,5944	0,3140	3,3065
30	0,9863	3,1925	0,8740	75	2,2572	1,5994	3,2790
31	1,6428	3,2449	0,1917	76	2,0098	1,5432	3,2520
32	0,1512	3,0191	0,7038	77	0,3981	1,2868	3,2255
33	0,5029	1,5758	0,9080	78	2,0974	1,6354	2,9214
34	0,7643	2,9000	0,1670	79	0,1227	0,9722	3,4537
35	1,1174	3,2927	2,4883	80	0,6062	1,7356	3,1487
36	1,1881	2,2992	1,0414	81	1,1461	1,2736	3,4073
37	1,8147	1,1258	3,0319	82	0,8908	0,3927	3,3848
38	0,6794	1,6146	1,7618	83	1,4418	1,1708	3,3626
39	0,9175	1,3469	2,6403	84	0,0704	0,0334	3,3409
40	0,3810	1,4335	3,7468	85	0,7279	2,9697	3,3195
41	0,0607	0,5884	1,2371	86	0,9696	1,0214	3,2984
42	1,4314	2,6719	3,3888	87	2,0699	0,3324	3,2778
43	0,0431	2,3945	2,8986	88	1,1947	0,4955	3,2574
44	0,7785	1,5065	3,2403	89	2,8198	0,2019	3,2374
45	1,1530	0,7589	2,9570	90	0,8898	1,9159	2,9193
46	0,0387	0,1571	2,6712	91	0,0034	1,8755	2,5983
47	0,2766	0,0210	3,0326	92	0,7951	1,1773	2,8777
48	1,3756	1,9578	2,7481	93	1,1746	0,6932	3,1606
49	1,4119	1,8484	3,3455	94	0,3716	0,4605	3,1422
50	2,0832	0,9291	2,1708	95	0,4228	0,8271	2,8174
51	1,0245	1,6367	2,7801	96	1,2631	1,4568	3,1061
52	1,8147	2,3622	2,9484	97	1,0037	1,8686	3,0884
53	1,9356	0,5978	2,6627	98	1,4504	1,6062	3,0711
54	1,0211	2,5162	3,5339	99	0,4020	3,3899	3,0540

Appendix 5: 2013 Zimbabwe First-two Digit Z-statistics results







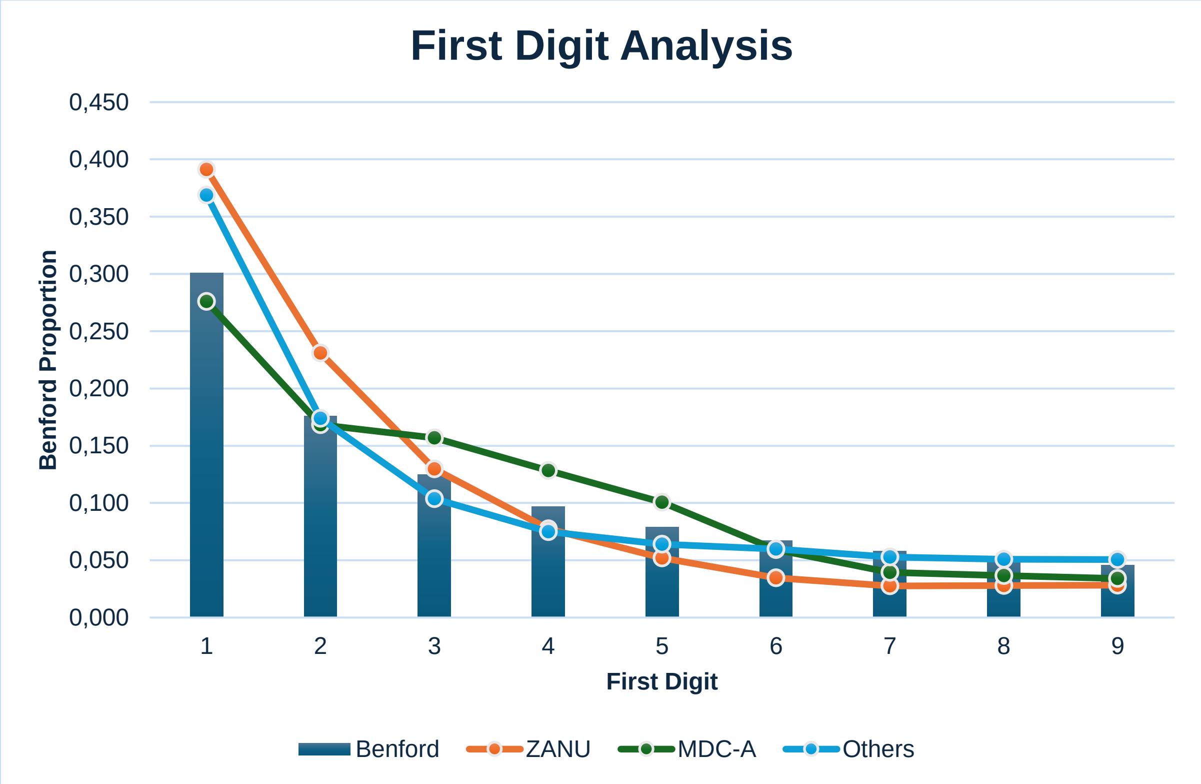
Appendix 6: 2013 Zimbabwe Chi-square results

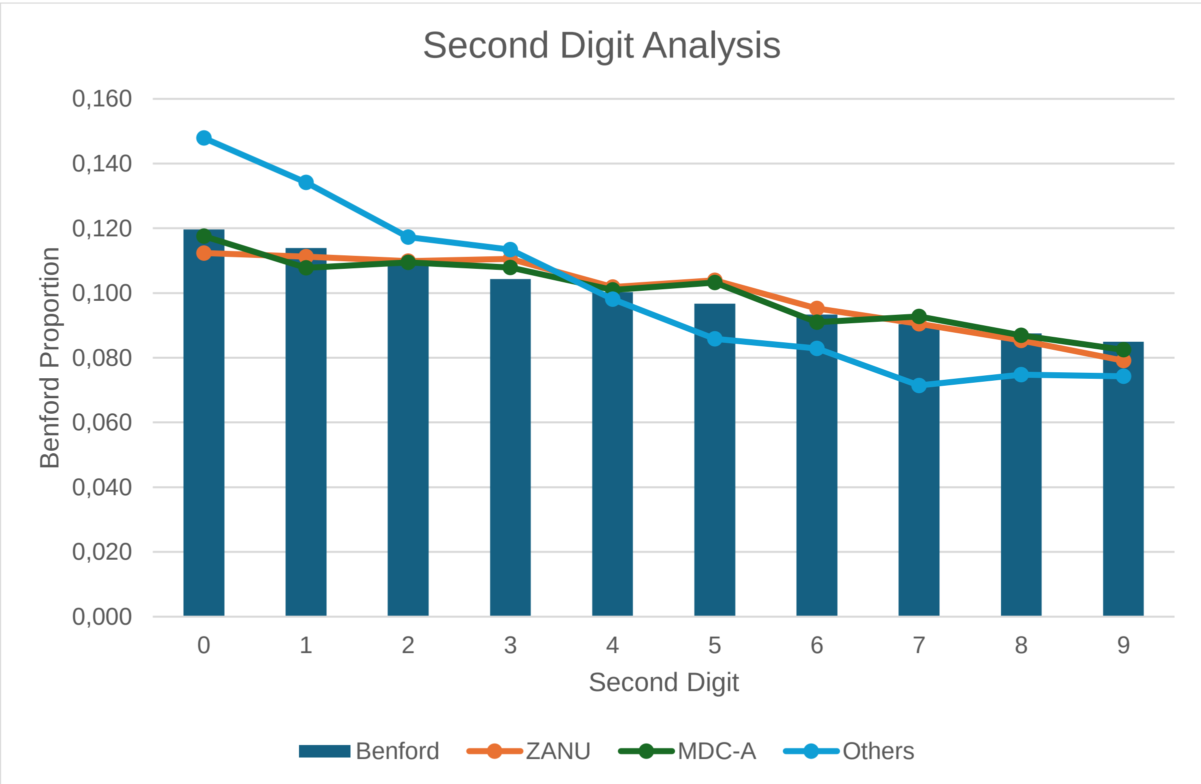
Chi-Square Tests			
	ZANU-PF	MDC-T	Others
1st Digit	112,238	188,814	104,375
Chi-Sq_CritVal	**15,507**		
Result	NonConformance	NonConformance	NonConformance
2n Digit	20,146	10,807	95,217
Chi-Sq_CritVal	**16,919**		
Result	NonConformance	Conforms	NonConformance
FT Digits	242,939	373,304	1517,098
Chi-Sq_CritVal	**112,022**		
Result	NonConformance	NonConformance	NonConformance

Appendix 7: 2013 Zimbabwe P-value results

P-Value Calculations			
	ZANU-PF	MDC-T	Others
1st Digit	1,31926E-20	1,44616E-36	5,43172E-19
2n Digit	0,017030406	0,289175194	1,45432E-16
FT Digits	3,22586E-16	1,74073E-36	1,8608E-258

Appendix 8: 2018 Zimbabwe Benford's results





Appendix 9: 2018 Zimbabwe MAD results

MAD Outcomes			
	ZANU-PF	MDC-A	Others
1st Digit	0,03332	0,01889	0,01618
Outcome	Nonconformity	Nonconformity	Nonconformity
2n Digit	0,00362	0,00276	0,01321
Outcome	Close conformity	Close conformity	Nonconformity
FT Digits	0,00366	0,00206	0,00752
Outcome	Nonconformity	Marginally acceptable conformity	Nonconformity

Appendix 10: 2018 Zimbabwe First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	ZANU-PF	MDC-A	Others
1	20,5880	5,7105	15,3503
2	15,1042	2,0892	0,6037
3	1,5057	10,1241	6,6350
4	6,8461	11,1331	7,6430
5	10,4839	8,3216	5,8392
6	13,4942	3,1563	2,8865
7	13,5993	8,2683	2,1987
8	11,0450	6,9080	0,1155
9	8,8050	5,8158	2,4174

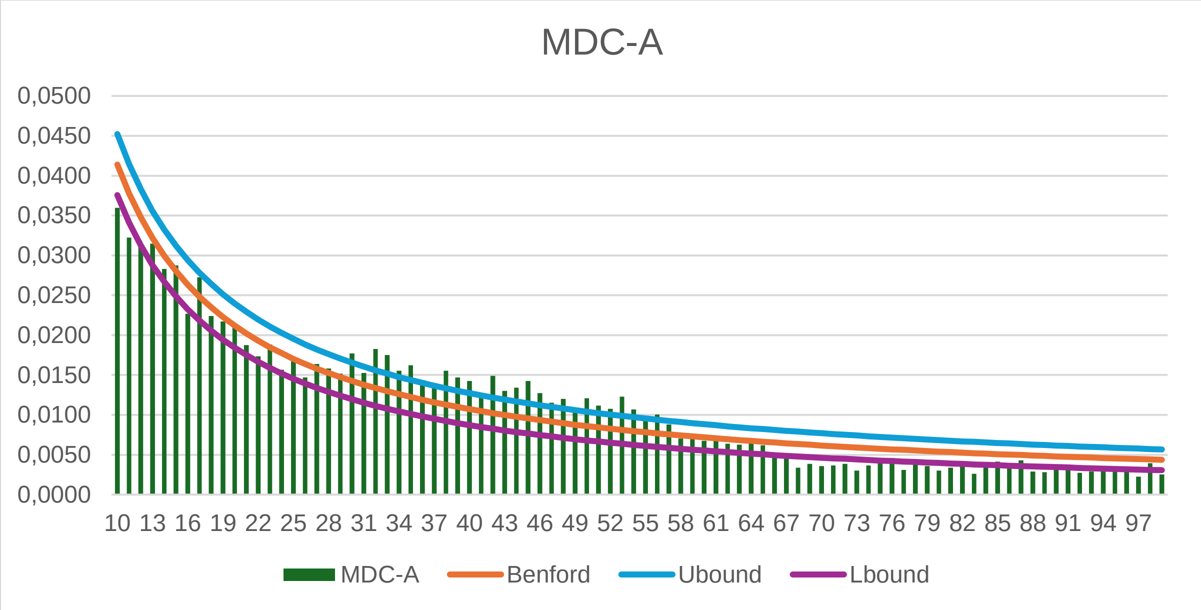
Z-Statistics for Second Digit			
Digit	ZANU-PF	MDC-A	Others
0	2,3450	0,6585	6,9400
1	0,8500	1,9583	5,0902
2	0,3452	0,1817	2,1644
3	2,1073	1,1961	2,3594
4	0,5220	0,2156	0,5608
5	2,5597	2,2743	2,9255
6	0,6366	0,8391	2,8763
7	0,0480	0,8445	5,2562
8	0,8159	0,2160	3,6155
9	2,1932	0,9327	3,0612

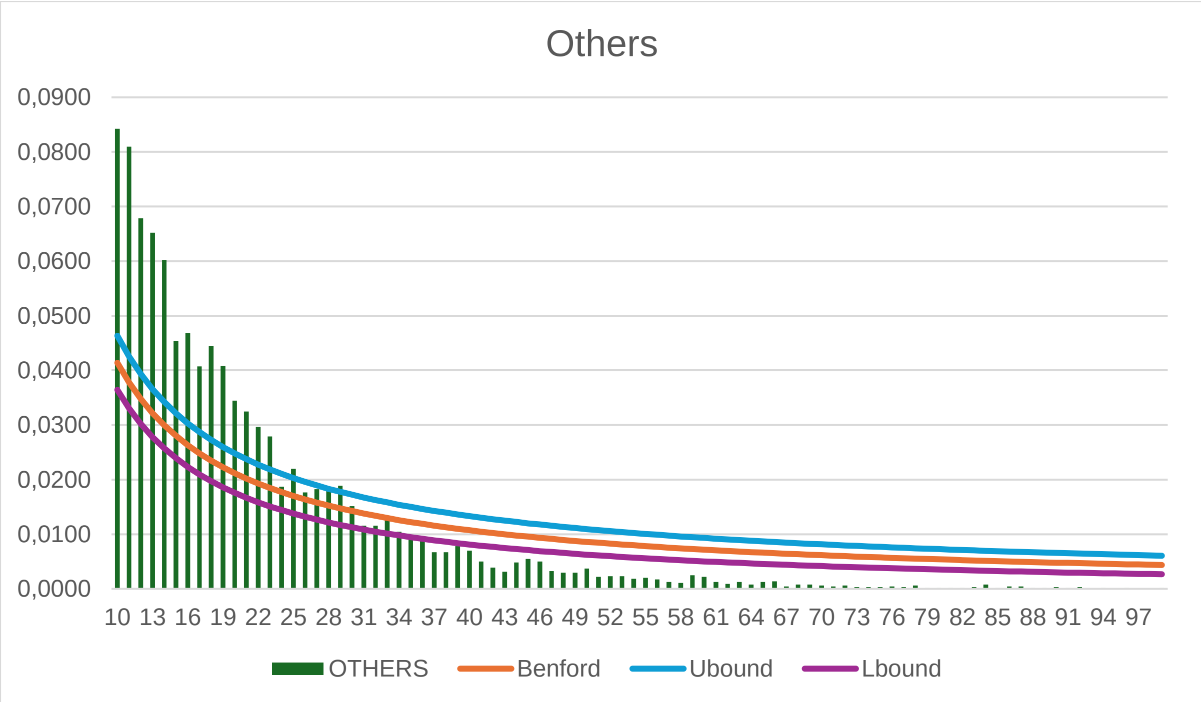
Appendix 11: 2018 Zimbabwe First-two Digit Z-statistics results

Z-Statistics for First-Two Digits							
Digit	ZANU-PF	MDC-A	Others	Digit	ZANU-PF	MDC-A	Others
10	4,3483	2,7874	17,1915	55	3,4888	1,6437	5,1966
11	0,0086	2,9893	18,0966	56	3,5728	2,7088	5,4019

Digit	ZANU-PF	MDC-A	Others	Digit	ZANU-PF	MDC-A	Others
12	2,6589	1,8803	14,4334	57	2,5593	1,4388	5,7583
13	7,8076	0,3878	14,9464	58	3,7600	0,4230	5,8329
14	6,0262	0,9854	14,1716	59	4,6489	0,0092	4,4442
15	9,9407	0,4277	8,3873	60	3,9720	0,4736	4,6626
16	9,9854	2,3310	10,1983	61	4,3151	0,2444	5,4841
17	10,1209	1,5868	8,1387	62	4,5527	0,6617	5,7200
18	9,1807	0,7134	11,0518	63	3,8699	0,6472	5,3558
19	8,7196	0,3476	10,0467	64	3,7644	0,0143	5,7519
20	8,5971	0,1140	7,3439	65	4,3674	0,5106	5,2329
21	6,1365	1,0436	6,9241	66	4,0335	2,0688	5,0183
22	6,0548	1,4478	5,9688	67	4,1760	1,5960	5,8960
23	5,0497	0,2292	5,5659	68	3,2399	3,8023	5,5298
24	4,8108	1,5956	0,5545	69	4,8402	3,0960	5,4771
25	4,5084	0,0500	3,0229	70	4,3897	3,3721	5,5850
26	2,9413	1,3315	0,7316	71	4,3045	3,1567	5,6962
27	2,8057	0,4577	1,5286	72	3,9730	2,8155	5,4869
28	0,7543	0,4593	1,7141	73	3,3900	3,8614	5,7652
29	0,7399	0,3479	2,7091	74	4,3083	2,8898	5,7206
30	3,0339	2,9803	0,5502	75	3,4718	2,1637	5,6769
31	1,7671	1,2707	1,4870	76	4,7892	1,4304	5,4678
32	2,9303	4,3695	1,2142	77	4,2054	3,4127	5,5920
33	0,0361	4,1153	0,0116	78	4,6470	2,1607	5,2139
34	0,9161	2,7007	1,4783	79	4,0589	2,6021	5,6796
35	0,0374	3,6888	2,1500	80	3,3349	3,3153	5,6408
36	0,0211	2,2740	1,9322	81	2,4726	2,7100	5,6028
37	1,9909	2,2492	3,5885	82	2,9226	1,9656	5,5654
38	0,8139	4,1309	3,4074	83	3,2471	3,6360	5,3550
39	1,7292	3,6313	2,3932	84	3,4436	2,2151	4,7939
40	1,1037	3,4790	2,8182	85	3,1064	1,3239	5,4571
41	0,7575	2,0716	4,2447	86	3,9841	1,7935	5,0688
42	1,8407	4,7551	4,9683	87	4,1935	0,8931	5,0325
43	1,2302	3,1073	5,4648	88	4,2701	2,8928	5,3543
44	2,4637	3,7721	3,9463	89	2,8361	2,9667	5,3211
45	0,7856	4,9376	3,2995	90	2,4940	2,2028	5,1078
46	3,0549	3,6017	3,5539	91	2,4279	1,7135	5,2564
47	3,6744	2,5314	4,8696	92	3,4810	2,9195	5,0421
48	2,7929	3,2781	5,0251	93	3,2819	2,5742	5,3774
49	3,2333	2,1627	4,9208	94	2,0928	1,6542	5,1631
50	2,8616	3,8461	4,1428	95	3,1653	2,0205	5,3186
51	2,1759	3,0140	5,4030	96	2,9653	1,8137	5,1033
52	2,3277	2,7987	5,1749	97	2,3340	3,3502	5,2616
53	2,4888	4,7288	5,0855	98	1,8417	0,6672	5,2337
54	3,6265	3,0950	5,4198	99	3,2305	2,8034	5,0168

Appendix 12: 2018 Zimbabwe First-two Digit Z-statistics results

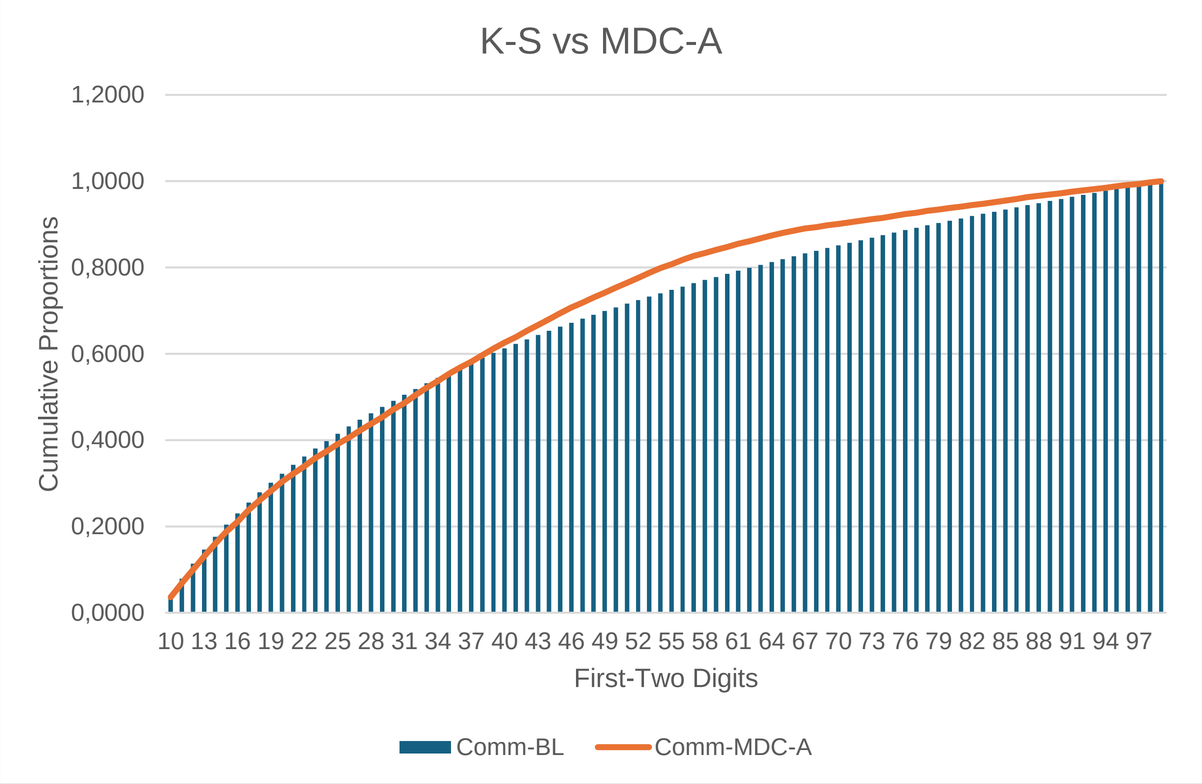


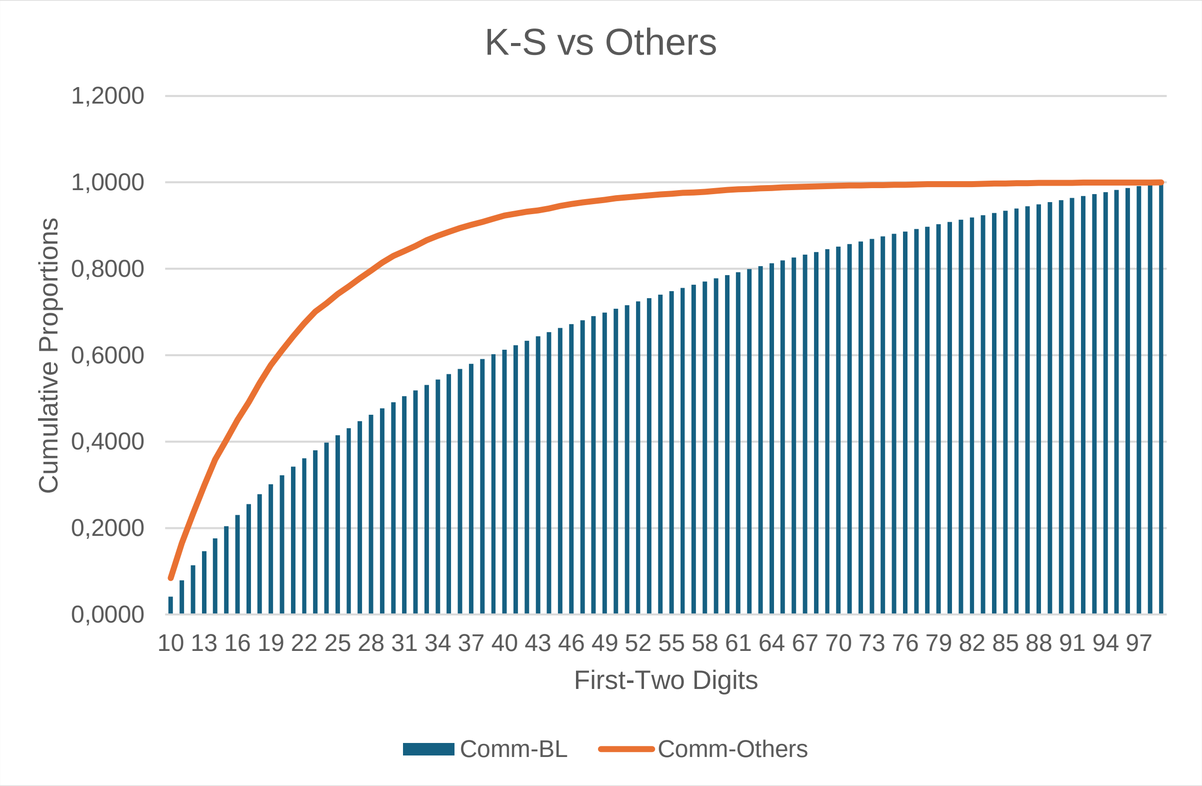


Appendix 13: 2018 Zimbabwe P-value results

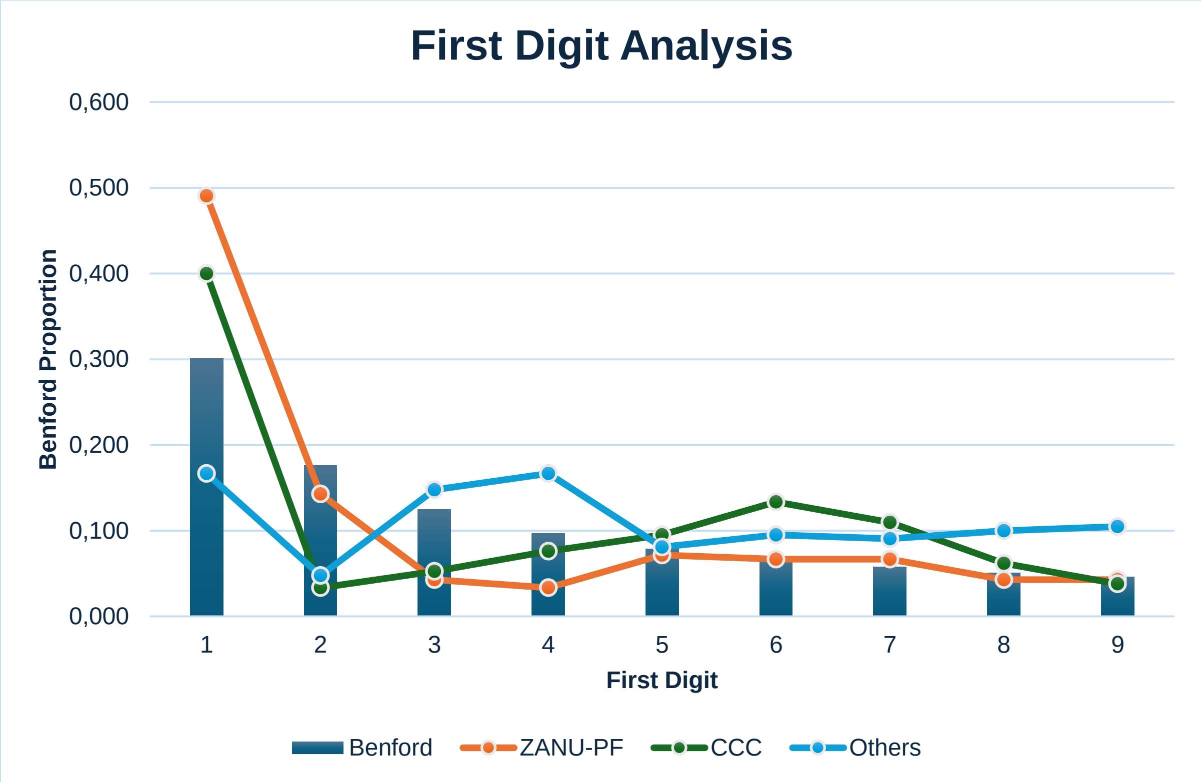
P-Value Calculations			
	ZANU	MDC-A	Others
1st Digit	1,5487E-246	4,39829E-91	1,53099E-61
2n Digit	0,010739728	0,202101525	4,34762E-25
FT Digits	2,4904E-266	4,96137E-75	0

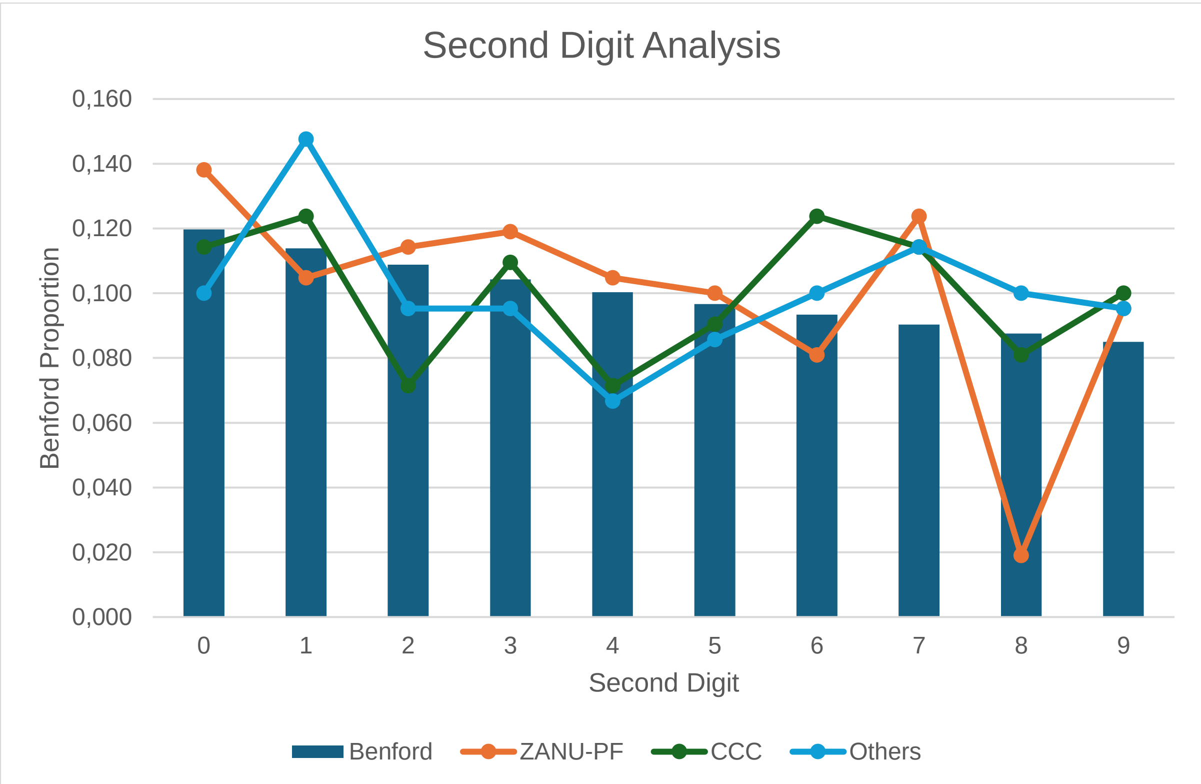
Appendix 14: 2018 Zimbabwe K-S results





Appendix 15: 2023 Zimbabwe Benford's results





Appendix 16: 2023 Zimbabwe First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	ZANU-PF	CCC	Others
1	5,9097	3,0514	4,1696
2	1,1738	5,3407	4,7971
3	3,4931	3,0757	0,8896
4	2,9976	0,8983	3,3004
5	0,2882	0,7340	0,0951
6	0,0164	3,7109	1,5021
7	0,3903	3,0476	1,8666
8	0,3889	0,5508	3,0567
9	0,0362	0,3664	3,9266

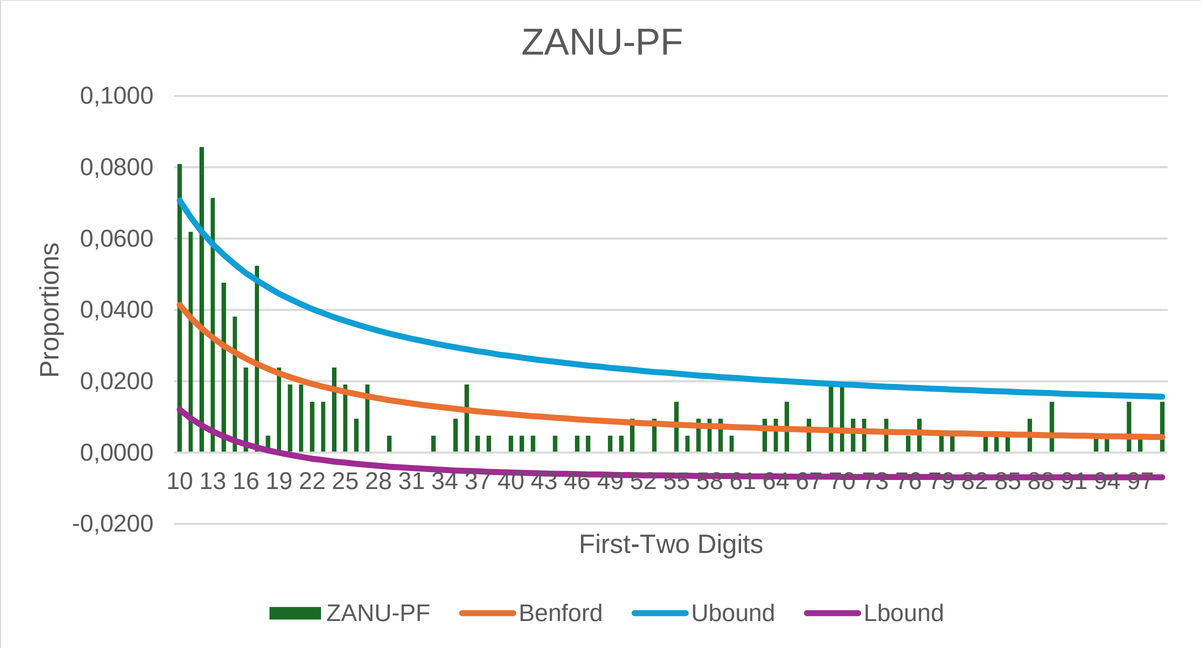
Z-Statistics for Second Digit			
Digit	ZANU-PF	CCC	Others
0	0,7159	0,1345	0,7723
1	0,3078	0,3439	1,4300
2	0,1435	1,6292	0,5212
3	0,5848	0,1333	0,3181
4	0,0999	1,2783	1,5080
5	0,0460	0,1875	0,4210
6	0,4999	1,3975	0,2116
7	1,5710	1,0896	1,0896
8	3,3908	0,2172	0,5152
9	0,4083	0,6557	0,4083

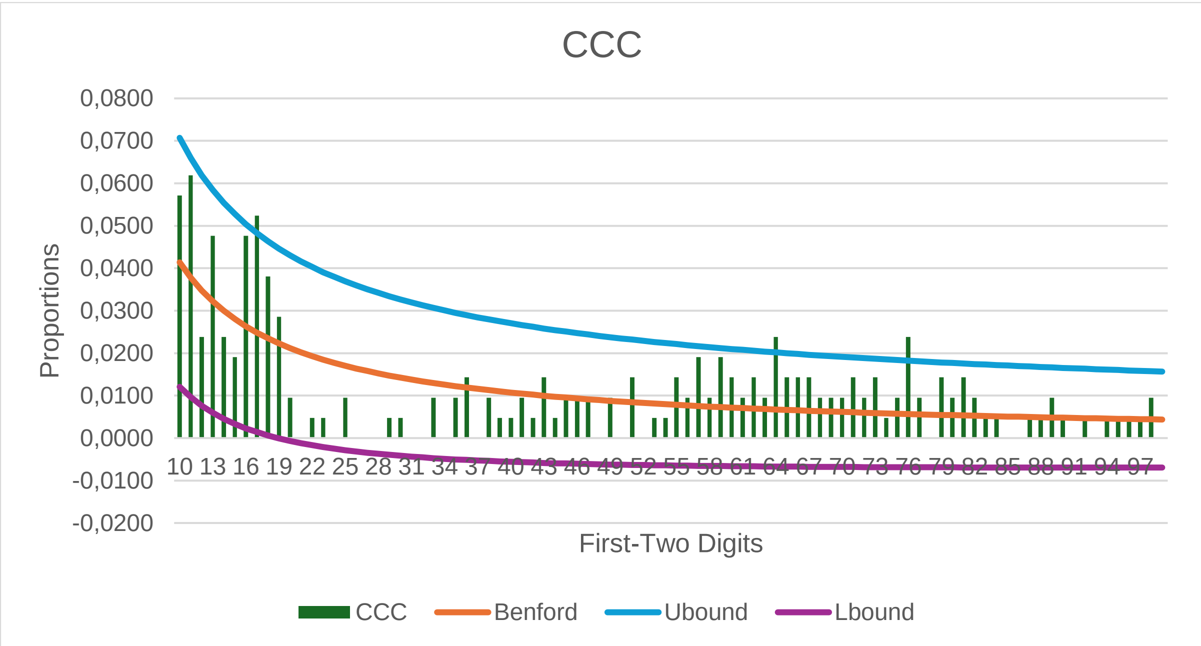
Appendix 17: 2023 Zimbabwe First-two Digit Z-statistics results

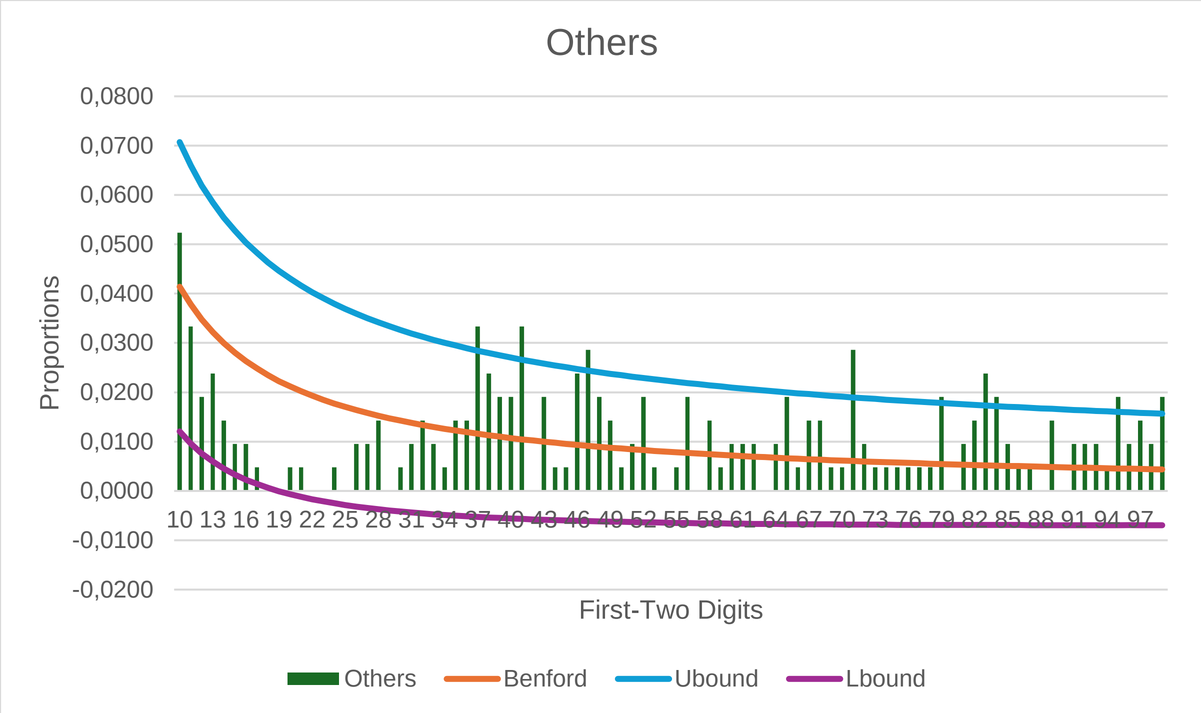
Z-Statistics for First-Two Digits							
Digit	ZANU-PF	CCC	Others	Digit	ZANU-PF	CCC	Others
10	2,7047	0,9726	0,6262	55	0,6709	0,6709	0,1122
11	1,6518	1,6518	0,1576	56	0,0903	0,3048	1,4900
12	3,8425	0,6781	1,0548	57	0,3298	1,5254	0,8657
13	3,0268	1,0718	0,4922	58	0,3545	0,3545	0,7564
14	1,2984	0,3207	1,1302	59	0,3787	1,5947	0,0266
15	0,6748	0,5795	1,4156	60	0,0061	0,8113	0,4026
16	0,0125	1,7114	1,3055	61	0,8101	0,0140	0,0140
17	2,3449	2,3449	1,6468	62	0,7969	0,8645	0,0338
18	1,5636	1,1707	2,0193	63	0,0534	0,0534	0,7839
19	0,1505	0,3843	1,9536	64	0,0726	2,6040	0,0726
20	0,2155	0,9342	1,4134	65	0,9417	0,9417	1,7920
21	0,1190	1,8357	1,3452	66	0,7466	0,9668	0,3182
22	0,2779	1,2809	1,7824	67	0,1285	0,9915	0,9915
23	0,1955	1,2201	1,7325	68	0,7229	0,1465	1,0160
24	0,4063	1,6854	1,1625	69	1,9158	0,1644	0,2735

Digit	ZANU-PF	CCC	Others	Digit	ZANU-PF	CCC	Others
25	0,2256	0,5744	1,6410	70	1,9458	0,1820	0,2590
26	0,5120	1,5989	0,5120	71	0,1993	1,0874	3,7518
27	0,1014	1,5590	0,4521	72	0,2164	0,2164	0,2164
28	1,5211	1,5211	0,1129	73	0,6671	1,1337	0,2169
29	0,9121	0,9121	1,4850	74	0,2500	0,2032	0,2032
30	1,4505	0,8681	0,8681	75	0,6460	0,2665	0,1898
31	1,4176	1,4176	0,2341	76	0,1765	3,0381	0,1765
32	1,3861	1,3861	0,1163	77	0,2988	0,2988	0,1634
33	0,7458	0,1358	0,1358	78	0,6157	0,6157	0,1505
34	1,3268	1,3268	0,7079	79	0,1378	1,2665	2,2027
35	0,0435	0,0435	0,2704	80	0,1253	0,3458	0,5963
36	0,6371	0,0007	0,0007	81	0,5868	1,3089	0,3611
37	0,6012	1,2462	2,6236	82	0,5774	0,3762	1,3298
38	0,5678	0,2411	1,3924	83	0,0885	0,0885	3,2692
39	1,1971	0,5354	0,7881	84	0,0766	0,0766	2,3360
40	0,5038	0,5038	0,8361	85	0,0647	0,5501	0,4206
41	0,4731	0,1341	2,9174	86	0,5412	0,5412	0,0531
42	0,4433	0,4433	1,1294	87	0,4494	0,0416	0,0416
43	1,1082	0,2799	0,9740	88	0,5239	0,0302	0,5239
44	0,3858	0,3858	0,3858	89	1,4707	0,4776	1,4707
45	1,0678	0,0032	0,3581	90	0,5070	0,0078	0,5070
46	0,3310	0,0277	1,8212	91	0,4987	0,4987	0,5053
47	0,3046	0,0579	2,5954	92	0,4906	0,0142	0,5189
48	1,0112	1,0112	1,1863	93	0,0249	0,4825	0,5324
49	0,2535	0,1165	0,4865	94	0,0356	0,0356	0,0356
50	0,2287	0,9760	0,2287	95	0,4667	0,0462	2,6102
51	0,1728	0,5502	0,1728	96	1,6030	0,0566	0,5721
52	0,9426	0,9426	1,3430	97	0,0669	0,0669	1,6213
53	0,2270	0,1575	0,1575	98	0,4436	0,5979	0,5979
54	0,9108	0,1346	0,9108	99	1,6575	0,4361	2,7043

Appendix 18: 2023 Zimbabwe First-two Digit Z-statistics results







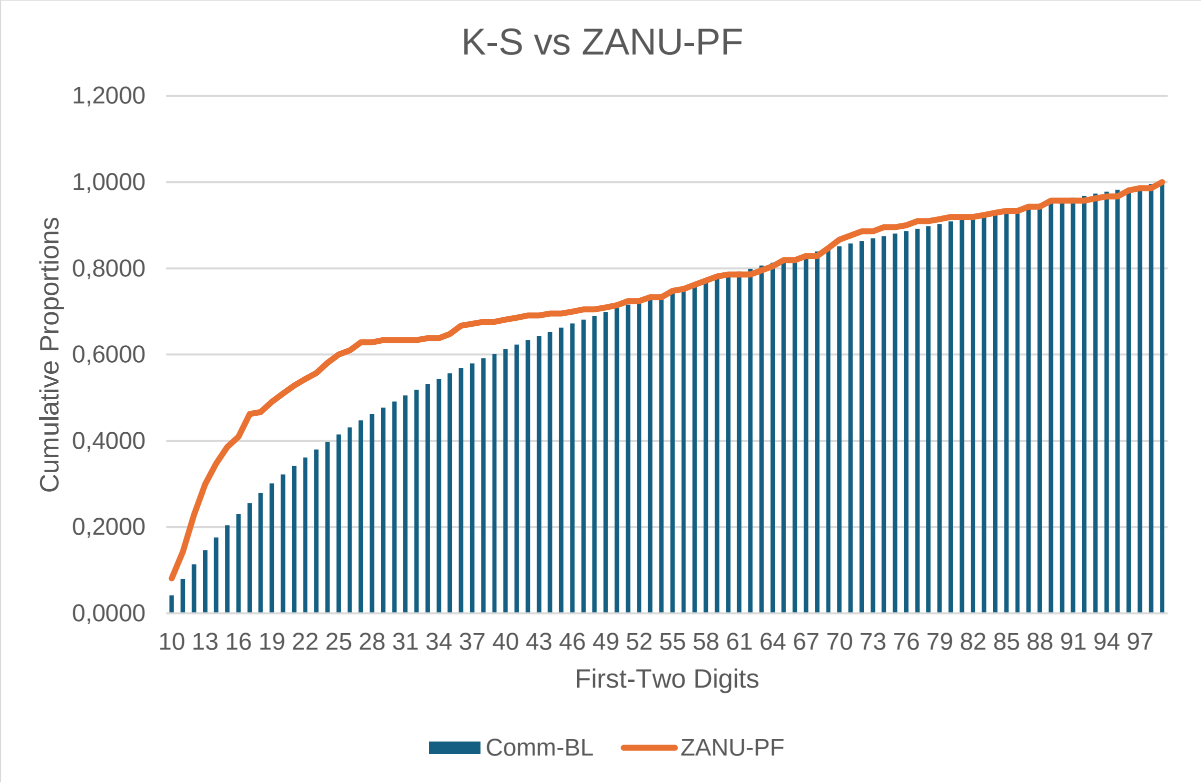
Appendix 19: 2023 Zimbabwe Chi-square results

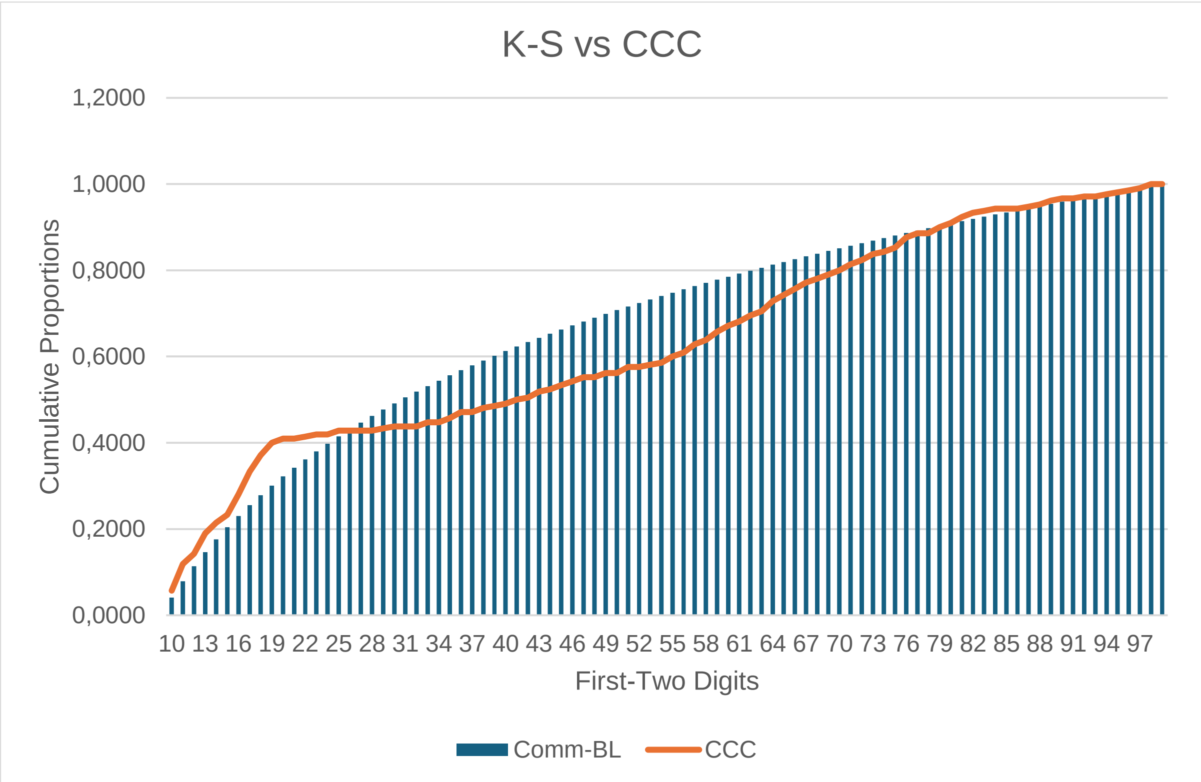
Chi-Square Tests			
	ZANU-PF	CCC	Others
1st Digit	47,191	65,785	75,799
Chi-Sq_CritVal	15,507		
Result	NonConformance	NonConformance	NonConformance
2n Digit	15,775	8,891	7,990
Chi-Sq_CritVal	16,919		
Result	Conforms	Conforms	Conforms
FT Digits	136,962	131,173	206,837
Chi-Sq_CritVal	112,022		
Result	NonConformance	NonConformance	NonConformance

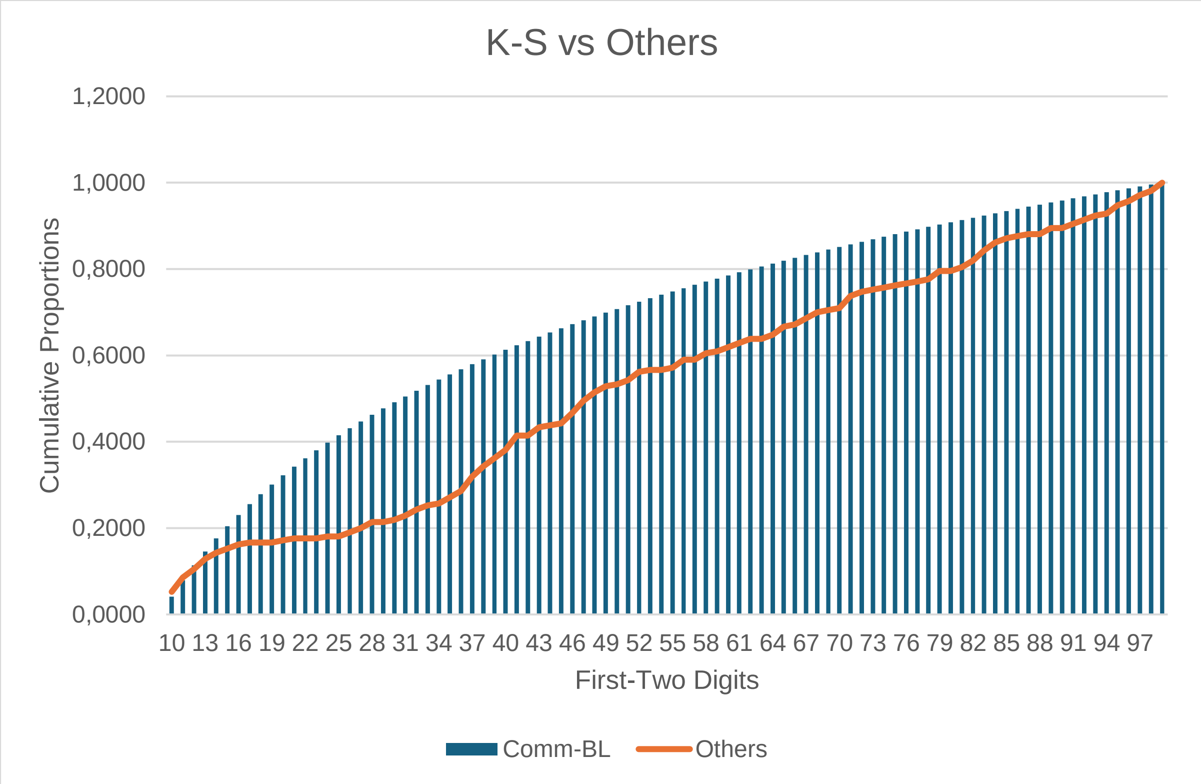
Appendix 20: 2023 Zimbabwe P-value results

P-Value Calculations			
	ZANU-PF	CCC	Others
1st Digit	1,41019E-07	3,37569E-11	3,41124E-13
2n Digit	0,071721853	0,447375203	0,535112943
FT Digits	0,000827701	0,00244312	2,24567E-11

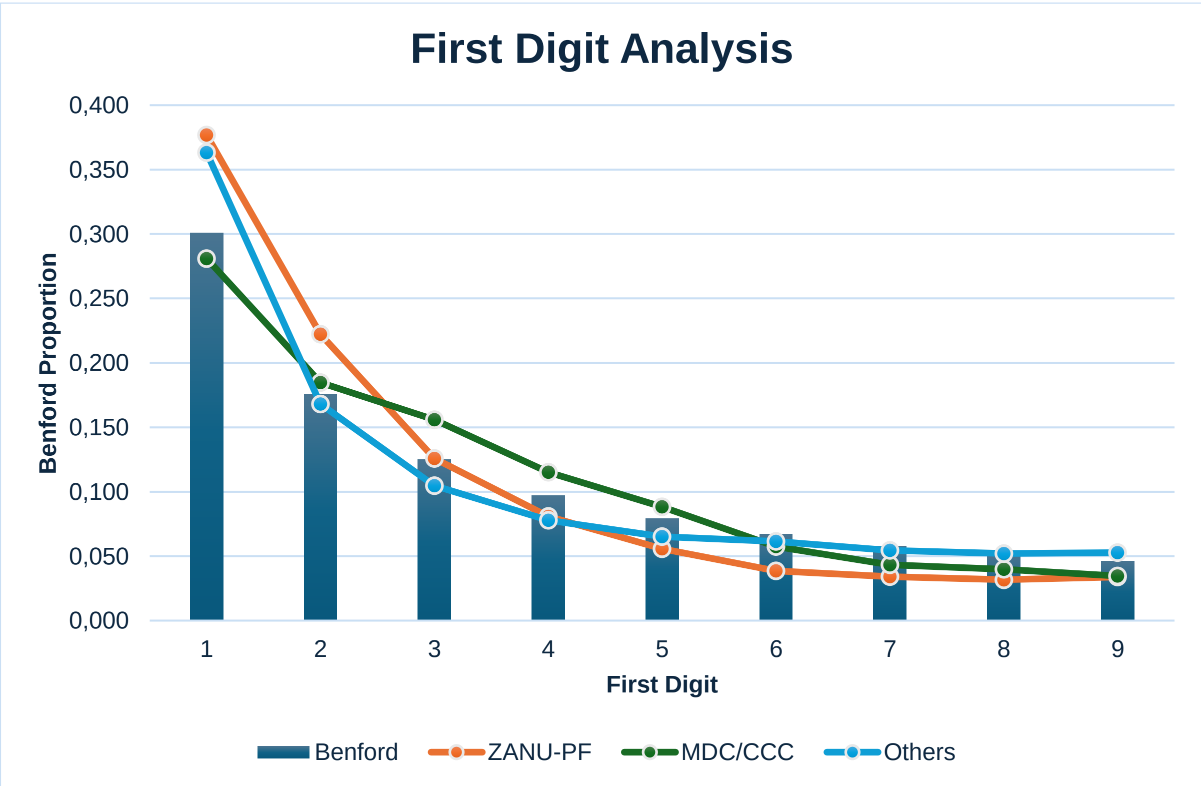
Appendix 21: 2023 Zimbabwe K-S results

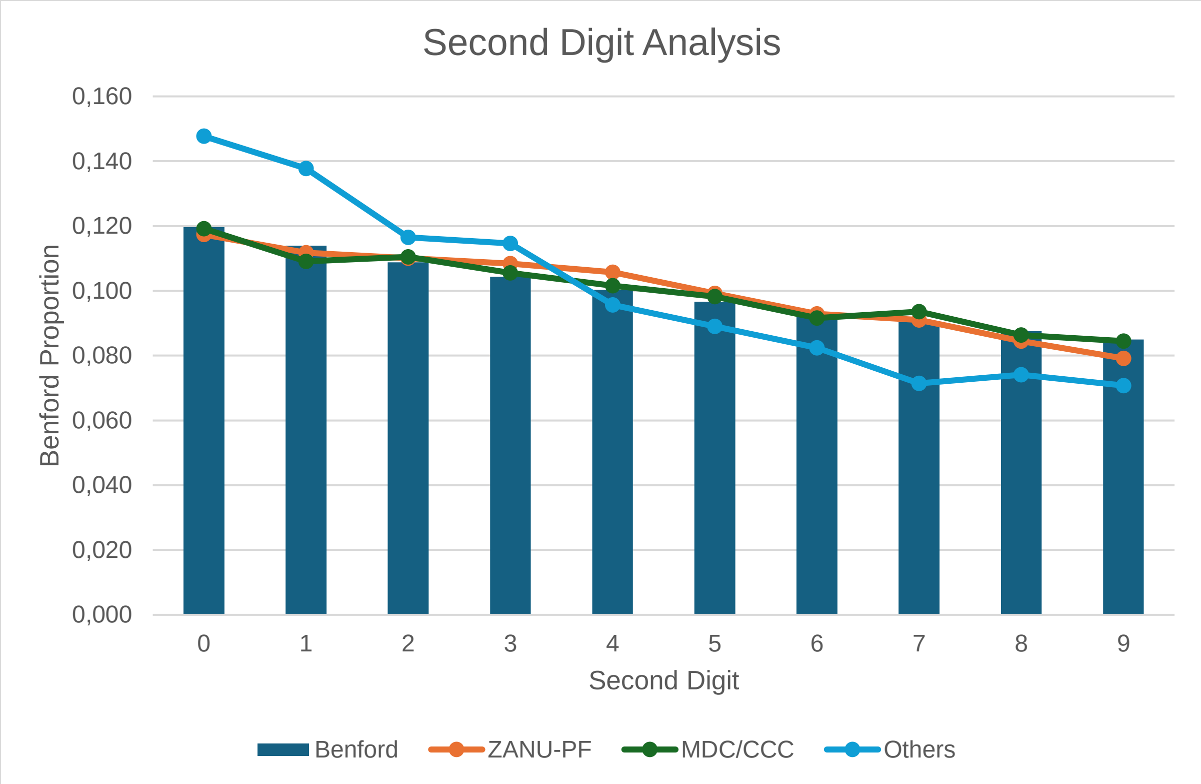






Appendix 22: 2013-2023 Zimbabwe Benford's results





Appendix 23: 2013-2023 Zimbabwe First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	ZANU-PF	MDC/CCC	Others
1	20,8432	5,4892	16,9081
2	15,3094	2,8117	2,5869
3	0,3433	11,7930	7,5774
4	6,7999	7,7118	8,0654
5	10,9245	4,2230	6,4987
6	14,2456	4,7457	2,6835
7	12,9360	7,9058	1,8631
8	11,0989	6,5226	0,5120
9	7,1940	6,6964	4,2169

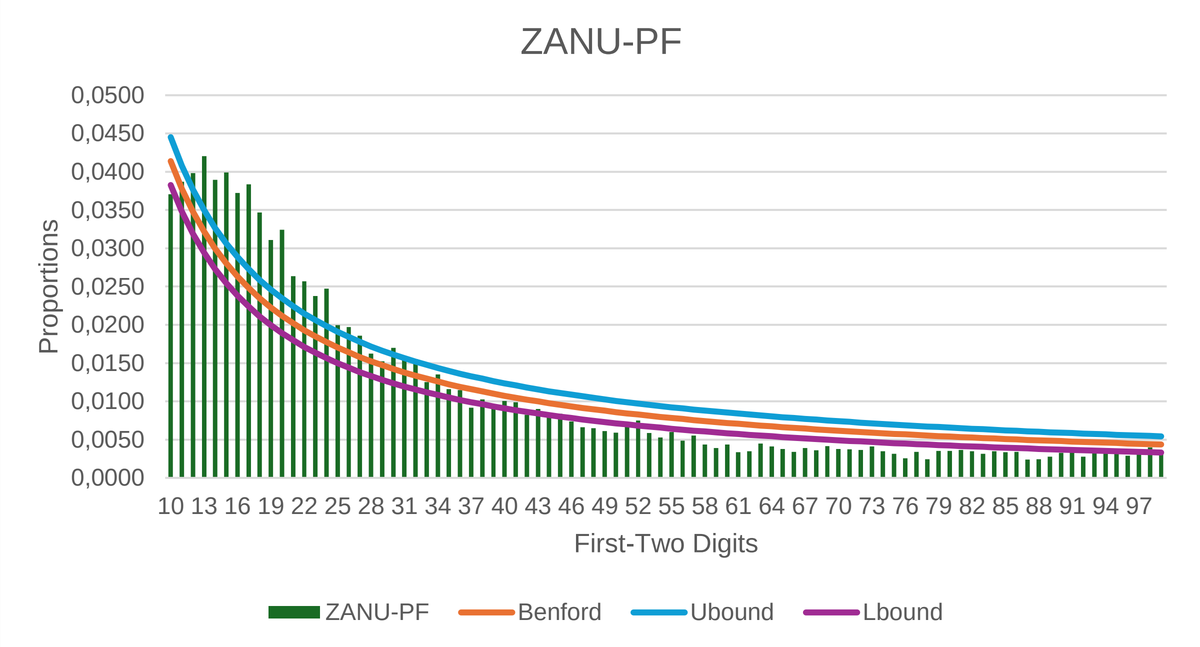
Z-Statistics for Second Digit			
Digit	ZANU-PF	MDC/CCC	Others
0	0,8967	0,1760	8,1595
1	0,8406	1,8641	7,1054
2	0,5073	0,6328	2,3148
3	1,6679	0,4816	3,1622
4	2,2489	0,5067	1,4543
5	1,0701	0,6147	2,4228
6	0,2051	0,7431	3,5606
7	0,2899	1,3953	6,2250
8	1,3845	0,5389	4,4855
9	2,6306	0,2162	4,8093

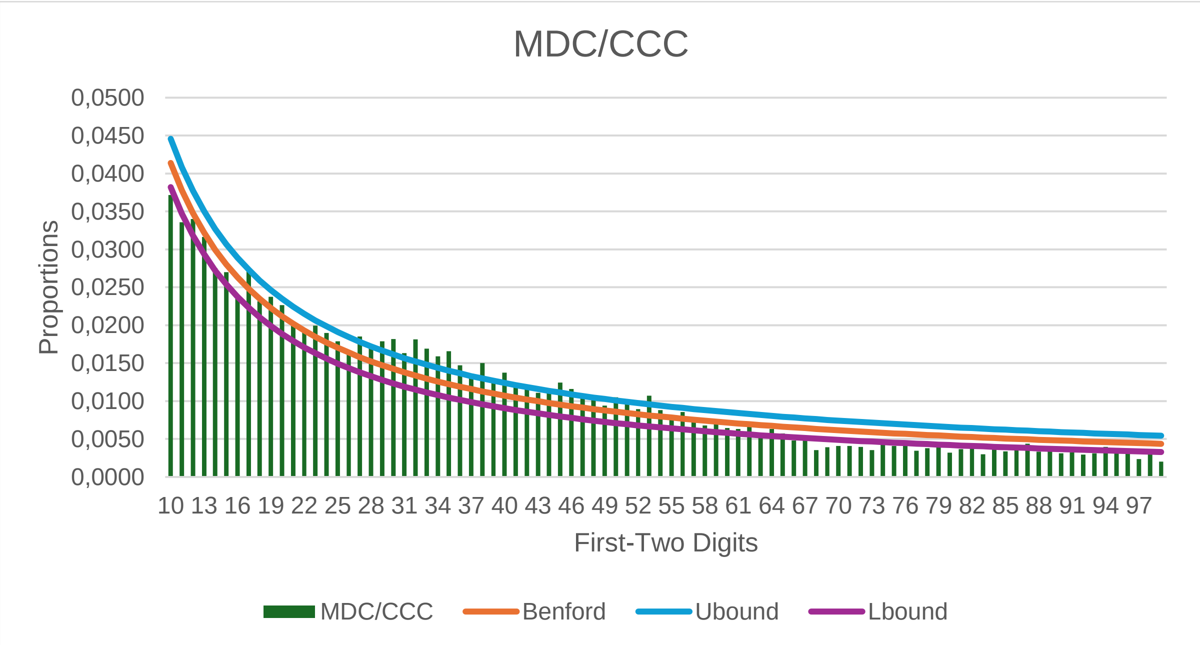
Appendix 24: 2013-2023 Zimbabwe First-two Digit Z-statistics results

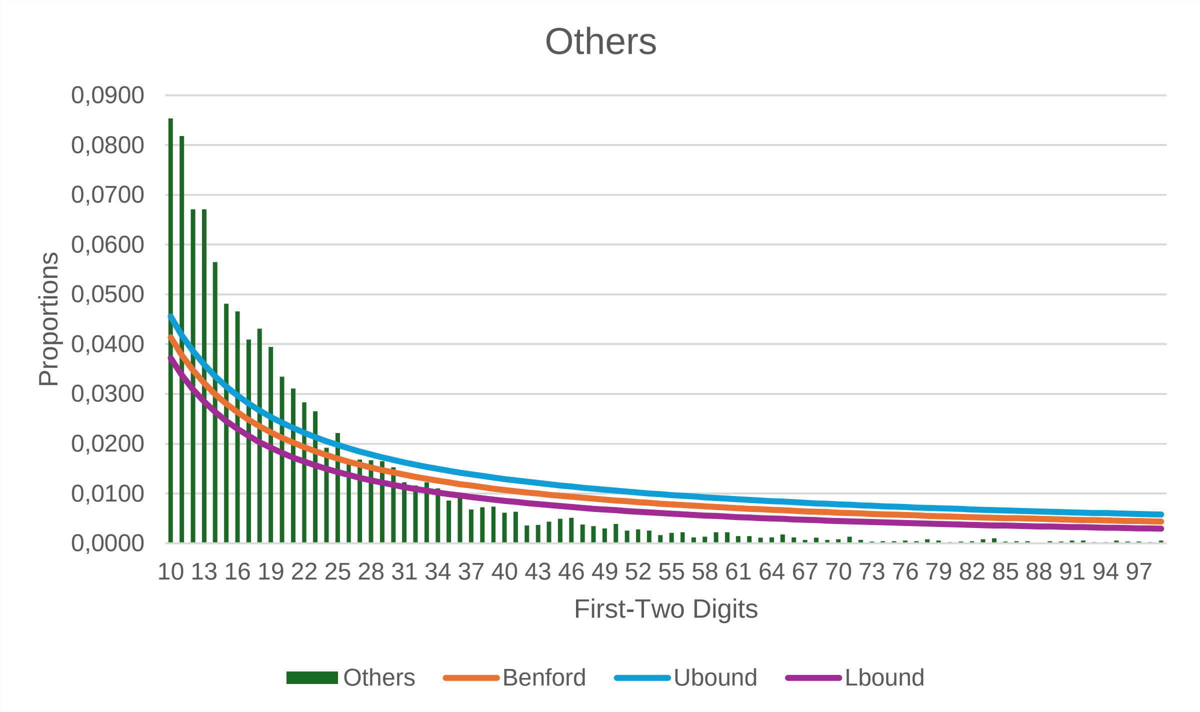
Z-Statistics for First-Two Digits							
Digit	ZANU-PF	MDC/CCC	Others	Digit	ZANU-PF	MDC/CCC	Others
10	2,7235	2,5954	20,8918	55	2,5994	0,1333	6,0751
11	0,5777	2,7007	21,8429	56	4,0574	1,1716	5,8580
12	3,4638	0,5013	16,6948	57	2,8904	0,2536	6,8605
13	7,0092	0,3597	18,7105	58	4,4813	0,8753	6,6540
14	6,6190	1,7437	14,7204	59	4,9866	0,1319	5,5791
15	9,0352	0,7415	11,5174	60	4,1901	1,0084	5,4901
16	8,5719	1,7704	11,9553	61	5,5642	1,0370	6,2855
17	10,9586	2,0235	9,7586	62	5,2465	0,1963	6,2072
18	9,2869	0,2369	12,2610	63	3,5808	1,7975	6,5149
19	7,4944	1,2054	10,9927	64	4,0271	0,5645	6,3137
20	9,7883	1,2164	8,0211	65	4,3870	1,3077	5,5917
21	5,4904	0,2410	7,2982	66	4,8549	2,5681	6,1721
22	5,8030	0,0042	6,1626	67	3,9448	1,6293	6,7636
23	4,9306	1,3293	5,6191	68	4,3242	4,3440	6,1691
24	6,6550	1,1467	0,9965	69	3,3036	3,6170	6,6399
25	2,8059	0,7738	3,7284	70	3,7926	3,1929	6,4450
26	3,2479	0,0476	0,0384	71	3,7815	3,0780	5,7058
27	2,7653	2,6813	0,7460	72	3,7736	3,1742	6,4634
28	0,9891	1,7067	1,1016	73	2,9408	3,8020	6,8201
29	0,4932	3,2059	1,3515	74	3,8712	2,2133	6,6287
30	2,8896	4,1044	0,7796	75	4,2927	2,6356	6,5761
31	2,2004	2,6417	1,1946	76	5,1448	1,6689	6,3840
32	2,3551	5,1097	1,4149	77	3,6731	3,5058	6,4738
33	0,4604	4,2668	0,5421	78	5,1793	2,8646	5,9970
34	1,0251	3,6599	1,2724	79	3,2636	2,5467	6,2319
35	0,7183	4,8461	3,0979	80	3,1671	3,6607	6,6154
36	0,4847	3,2102	2,1663	81	2,8541	2,7941	6,4250
37	2,7862	2,3372	4,1840	82	3,0878	1,8077	6,2337
38	1,1867	4,3266	3,5691	83	3,5472	3,7311	5,7478
39	2,1496	2,1649	3,2540	84	2,9051	1,8395	5,4051
40	0,7659	3,6161	4,1723	85	3,0387	2,8817	6,2478
41	0,6888	2,0696	3,7750	86	2,8394	2,2242	6,0560
42	2,4389	2,3973	6,2122	87	4,5586	0,9866	6,0134
43	1,2118	1,3365	5,9542	88	4,3698	2,7421	6,4247
44	1,6640	2,2094	5,1610	89	3,7241	2,1949	5,9302
45	1,4858	3,6699	4,4662	90	2,7287	2,9291	6,0424
46	2,5522	2,8823	4,0929	91	2,0704	2,6157	5,6960
47	3,3188	2,0655	5,2716	92	3,4954	3,1277	5,6558
48	3,2693	1,4773	5,4730	93	2,0212	2,6967	6,0820
49	3,5683	0,7799	5,7977	94	1,4712	1,0686	6,0455
50	3,6243	2,5076	4,7625	95	2,4498	2,1861	5,5388

51	2,3886	1,6983	6,0216	96	2,9663	2,3516	5,8163
52	1,0494	0,8641	5,6782	97	2,5368	3,8546	5,7806
53	3,1403	3,5159	5,8036	98	0,7876	1,3516	5,9047
54	3,7610	1,1214	6,6485	99	2,1507	4,3382	5,3904

Appendix 25: 2013-2023 Zimbabwe First-two Digit Z-statistics results







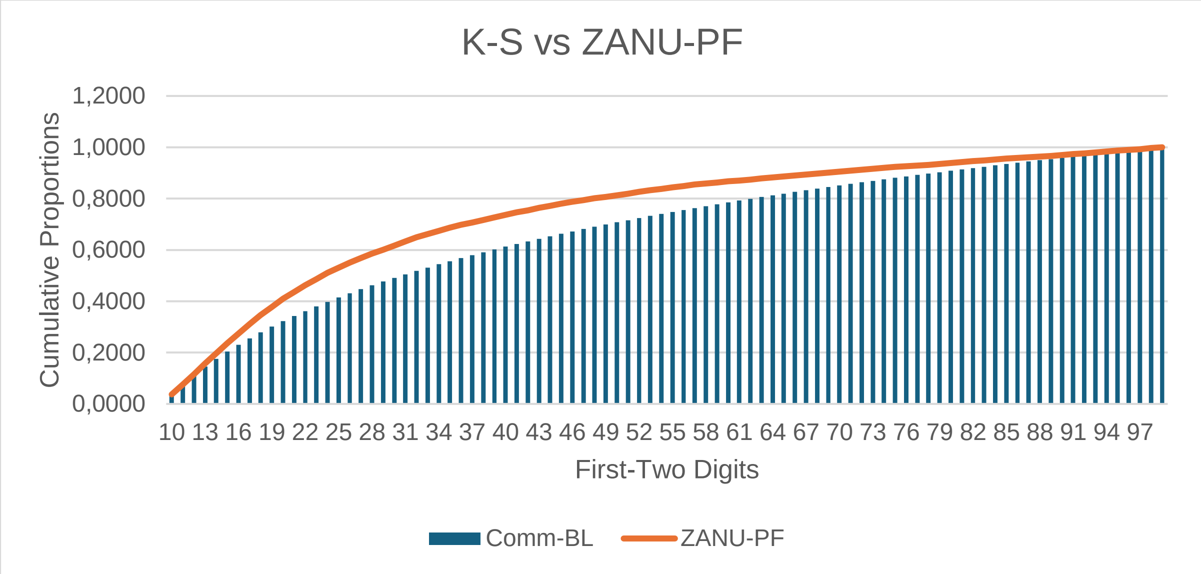
Appendix 26: 2013-2023 Zimbabwe Chi-square results

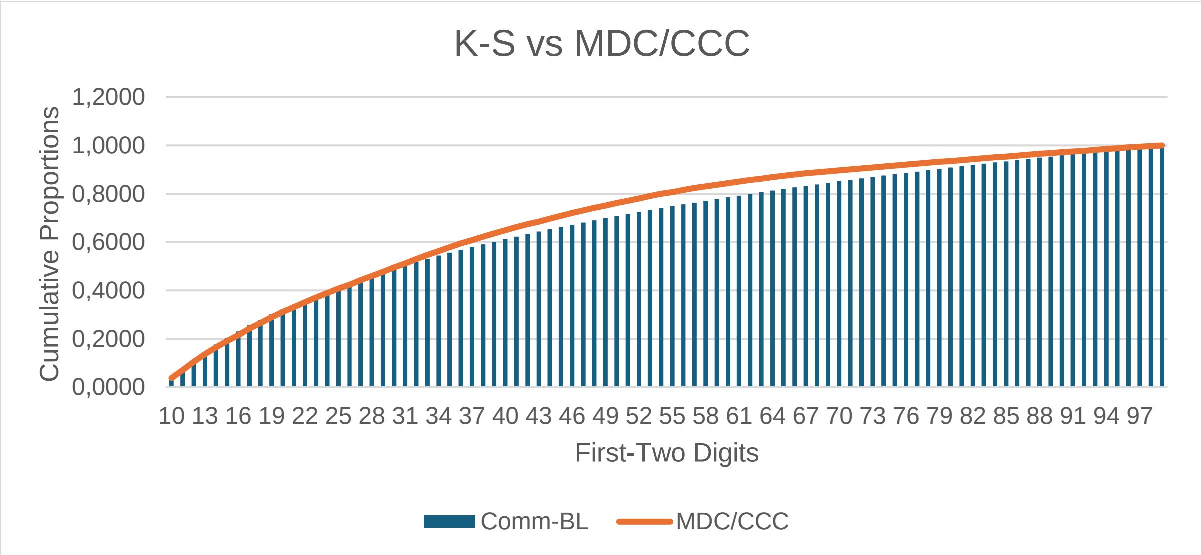
Chi-Square Tests			
	ZANU-PF	**MDC/CCC**	**Others**
1st Digit	1164,234	383,976	381,526
Chi-Sq_CritVal	**15,507**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
2n Digit	18,120	6,998	211,931
Chi-Sq_CritVal	**16,919**		
Result	**NonConformance**	**Conforms**	**NonConformance**
FT Digits	1529,449	575,579	4759,566
Chi-Sq_CritVal	**112,022**		
Result	**NonConformance**	**NonConformance**	**NonConformance**

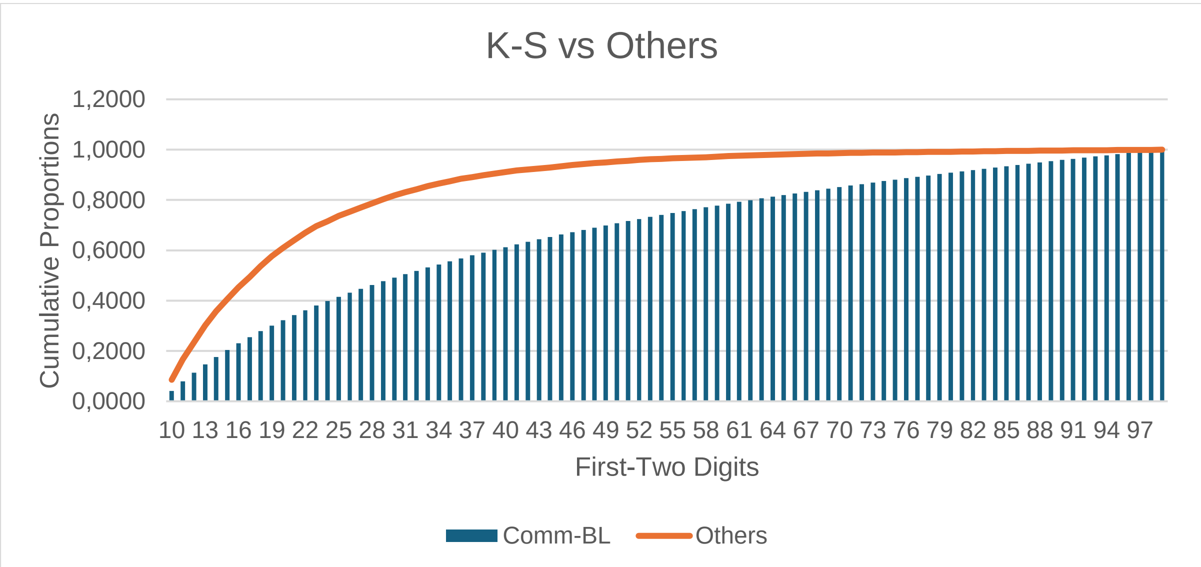
Appendix 27: 2013-2023 Zimbabwe P-value results

P-Value Calculations			
	ZANU-PF	**MDC/CCC**	**Others**
1st Digit	5,1168E-246	5,00126E-78	1,67046E-77
2n Digit	0,033802211	0,637324635	1,0391E-40
FT Digits	5,504E-261	2,84229E-72	0

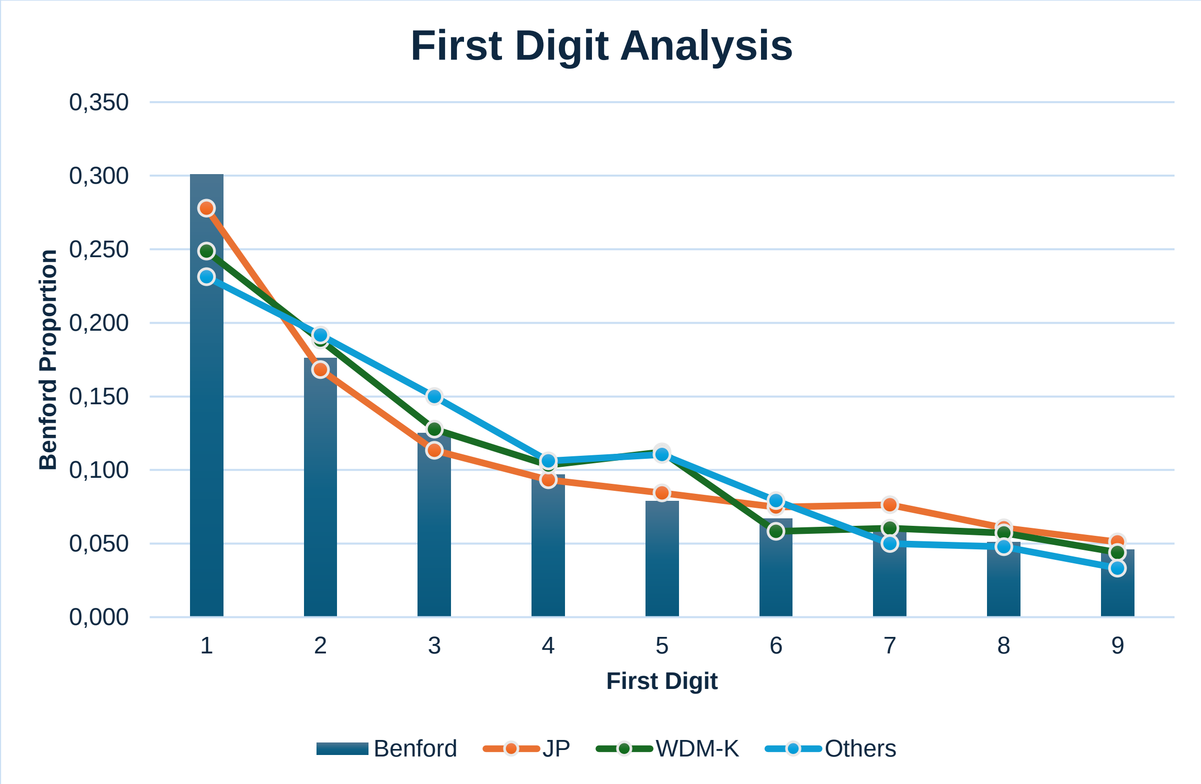
Appendix 28: 2013-2023 Zimbabwe First and Second Digit Z-statistics Results

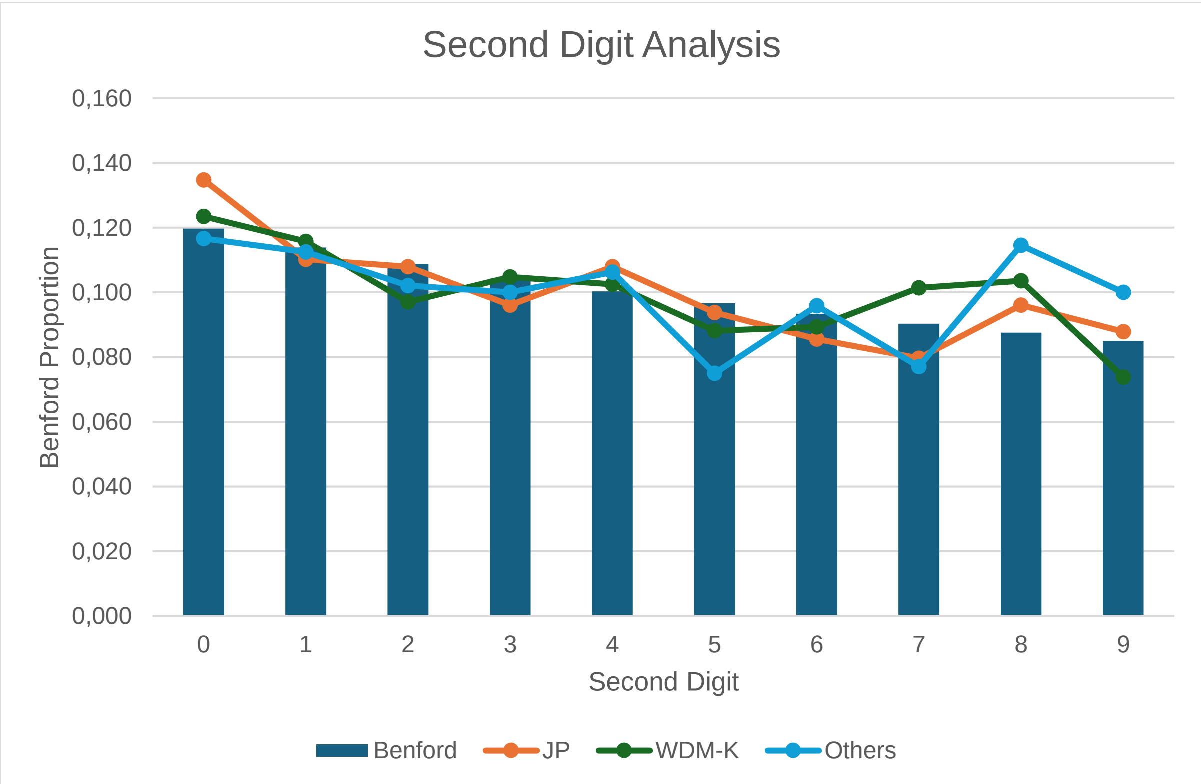






Appendix 29: 2017 Kenya First and Second Digits Analysis





Appendix 30: 2017 Kenya MAD results

MAD Outcomes			
	JP	WDM-K	Others
1st Digit	0,01031	0,01396	0,02076
Outcome	Acceptable conformity	Marginally acceptable conformity	Nonconformity
2n Digit	0,00682	0,00709	0,01008
Outcome	Close conformity	Close conformity	Marginally acceptable conformity
FT Digits	0,00218	0,00278	0,00392
Outcome	Marginally acceptable conformity	Nonconformity	Nonconformity

Appendix 31: 2017 Kenya First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	JP	WDM-K	Others
1	1,8328	3,4083	3,2831
2	0,7304	0,9086	0,8360
3	1,2486	0,1936	1,5915
4	0,3982	0,6064	0,6146
5	0,6660	3,6268	2,4500
6	1,1017	0,9764	0,9796
7	2,8196	0,2536	0,6513
8	1,5378	0,7535	0,2180
9	0,8758	0,1739	1,1937

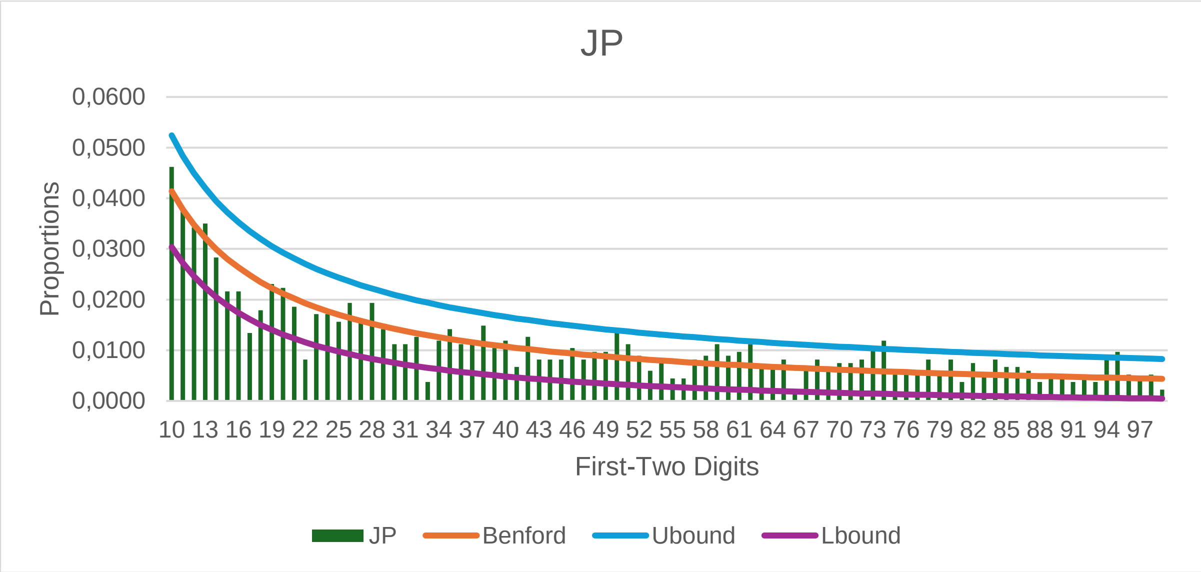
Z-Statistics for Second Digit			
Digit	JP	WDM-K	Others
0	1,6620	0,3018	0,1331
1	0,3826	0,1256	0,0240
2	0,0565	1,0875	0,4007
3	0,9476	0,0405	0,2357
4	0,8887	0,1679	0,3572
5	0,3085	0,8077	1,5301
6	0,9281	0,3637	0,1071
7	1,3173	1,1064	0,9343
8	1,0516	1,6532	2,0130
9	0,3273	1,1424	1,0966

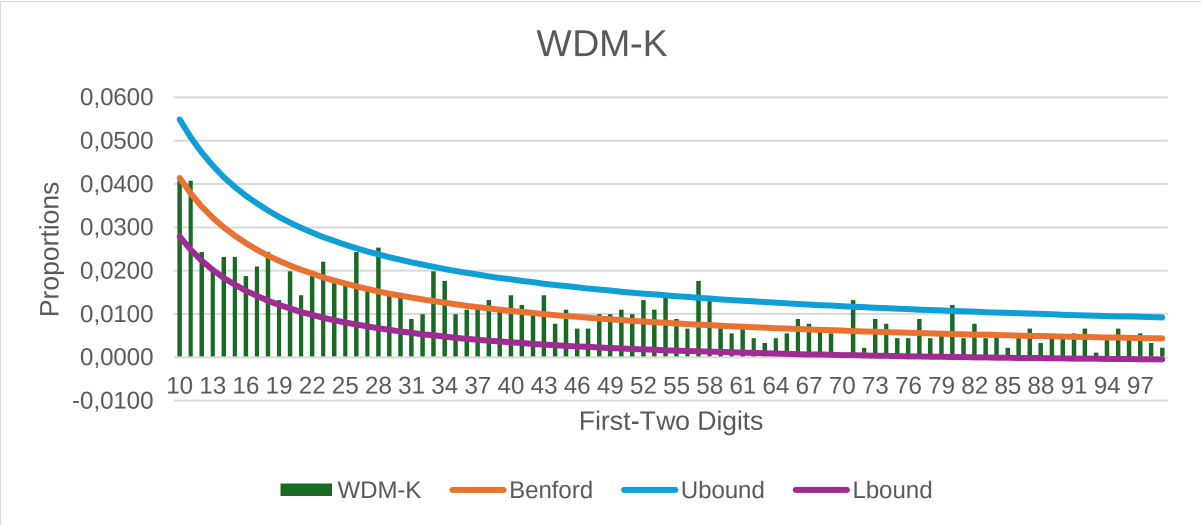
Appendix 32: 2017 Kenya First-two Digit Z-statistics results

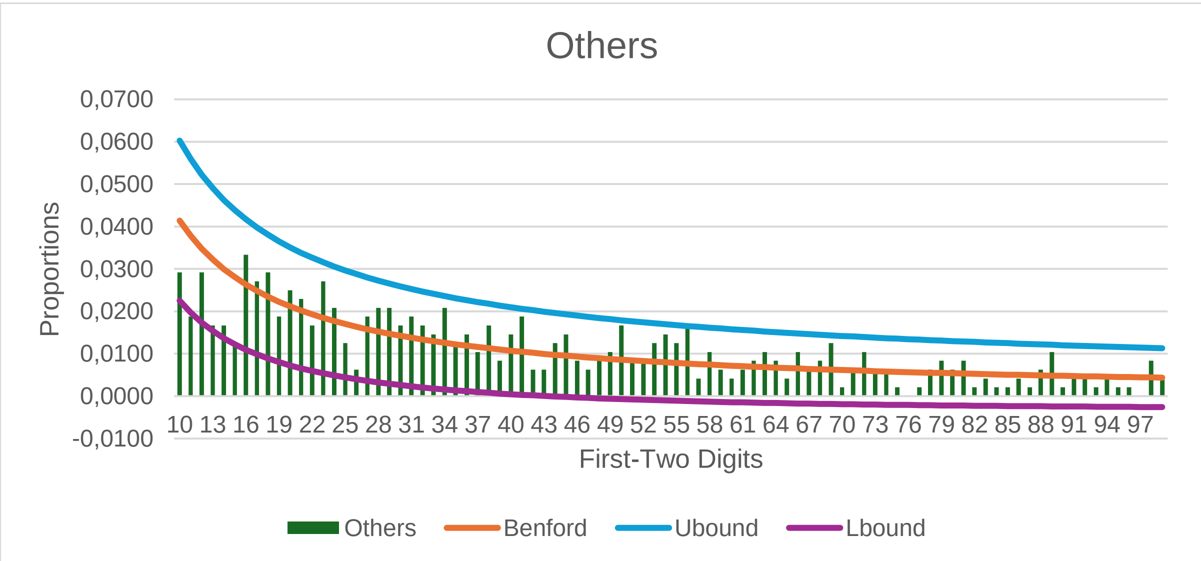
Z-Statistics for First-Two Digits							
Digit	JP	WDM-K	Others	Digit	JP	WDM-K	Others
10	0,8095	0,0072	1,2301	55	1,2416	0,1516	0,9033
11	0,0358	0,3876	2,0678	56	1,1946	0,1794	1,9913

12	0,0276	1,6367	0,5447	57	0,1122	3,3171	0,5933
13	0,5065	2,0115	1,7970	58	0,4862	1,8437	0,4979
14	0,2786	1,1056	1,5749	59	1,5057	0,0470	0,0019
15	1,3462	0,7890	1,9230	60	0,6009	0,3976	0,5113
16	0,9987	1,3232	0,8159	61	0,9828	0,1606	0,2124
17	2,6023	0,6434	0,1715	62	1,6975	0,7205	0,0904
18	1,2677	0,0444	0,6719	63	0,0614	1,0891	0,6740
19	0,1078	1,7335	0,3689	64	0,0143	0,6525	0,1496
20	0,1976	0,1657	0,4213	65	0,5363	0,2103	0,3839
21	0,3167	1,1386	0,2603	66	2,1244	0,6499	0,7736
22	2,8611	0,0024	0,2543	67	0,1225	0,2759	0,0504
23	0,2681	0,6744	1,2294	68	0,6825	0,1044	0,2626
24	0,0640	0,0201	0,3425	69	0,0373	0,0707	1,4483
25	0,2901	0,1153	0,5912	70	0,4278	1,7345	0,8499
26	0,7495	1,7348	1,5700	71	0,4714	2,5601	0,0496
27	0,0463	0,0867	0,3363	72	0,8681	1,2622	0,9610
28	1,1210	2,3519	0,8140	73	1,9811	0,9274	0,0975
29	0,0619	0,0976	0,9220	74	2,7496	0,5289	0,1210
30	0,8349	0,0235	0,2560	75	0,0813	0,3150	0,7611
31	0,7061	1,1407	0,7365	76	0,0452	0,2869	1,3517
32	0,1064	0,7579	0,4314	77	0,1733	1,0752	0,7275
33	2,8734	1,6851	0,1117	78	1,1293	0,2319	0,2119
34	0,0997	1,2156	1,4153	79	0,0605	0,0203	0,5436
35	0,5136	0,4823	0,0529	80	1,2123	2,5415	0,2557
36	0,1210	0,0896	0,3319	81	0,6209	0,1520	0,5907
37	0,0139	0,1537	0,0253	82	0,9164	0,7917	0,6477
38	1,1238	0,3987	0,9012	83	0,0056	0,1004	0,3151
39	0,0698	0,0087	0,3404	84	1,3728	0,1571	0,6173
40	0,2909	0,8941	0,5994	85	0,6442	0,9842	0,6024
41	1,2214	0,3289	1,5593	86	0,6784	0,0253	0,2648
42	0,7531	0,0887	0,6377	87	0,3239	0,4716	0,5731
43	0,5239	1,1503	0,5933	88	0,4258	0,4519	0,0944
44	0,4462	0,4567	0,3785	89	0,0066	0,0472	1,4258
45	0,3703	0,2877	0,9005	90	0,1757	0,1694	0,5307
46	0,2713	0,6805	0,2293	91	0,3472	0,0942	0,1848
47	0,2235	0,6255	0,4262	92	0,1220	0,6030	0,1694
48	0,1372	0,1332	0,1445	93	0,2962	1,3248	0,4897
49	0,2097	0,1930	0,1412	94	1,7461	0,0826	0,1390
50	2,0538	0,6112	1,6668	95	2,5926	0,6787	0,4632
51	0,9472	0,3091	0,0239	96	0,1858	0,0407	0,4502
52	0,1175	1,4652	0,0147	97	0,0073	0,2293	1,1228
53	0,7305	0,7908	0,8156	98	0,2383	0,2501	0,9532
54	0,0621	1,9689	1,3732	99	0,9777	0,7348	0,0659

Appendix 33: 2017 Kenya First-two Digit Z-statistics results







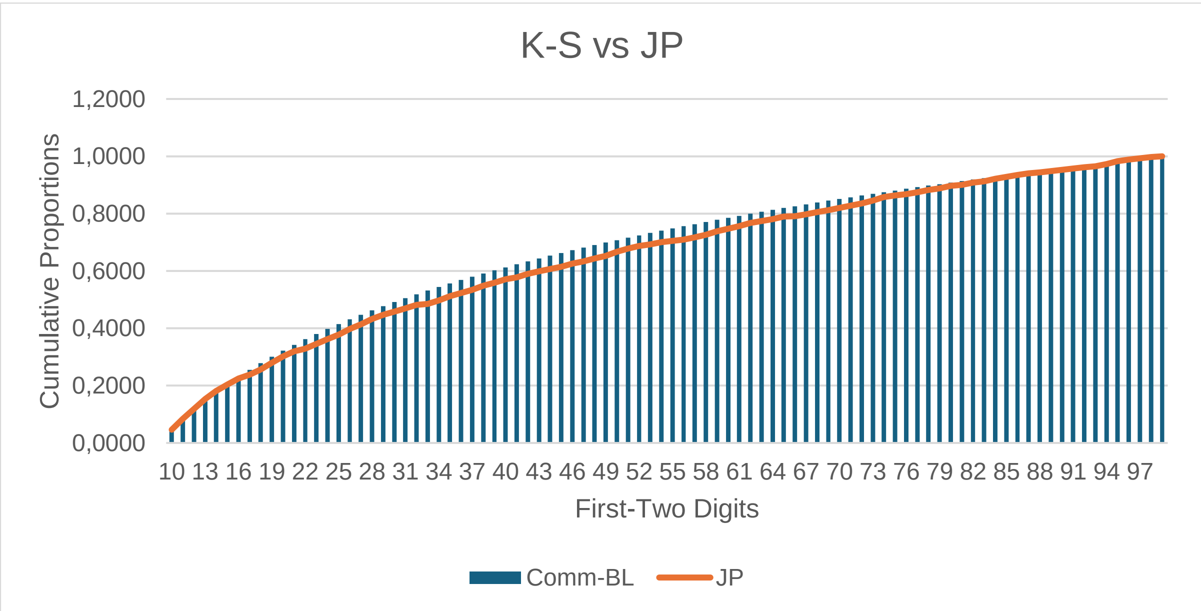
Appendix 34: 2017 Kenya Chi-square results

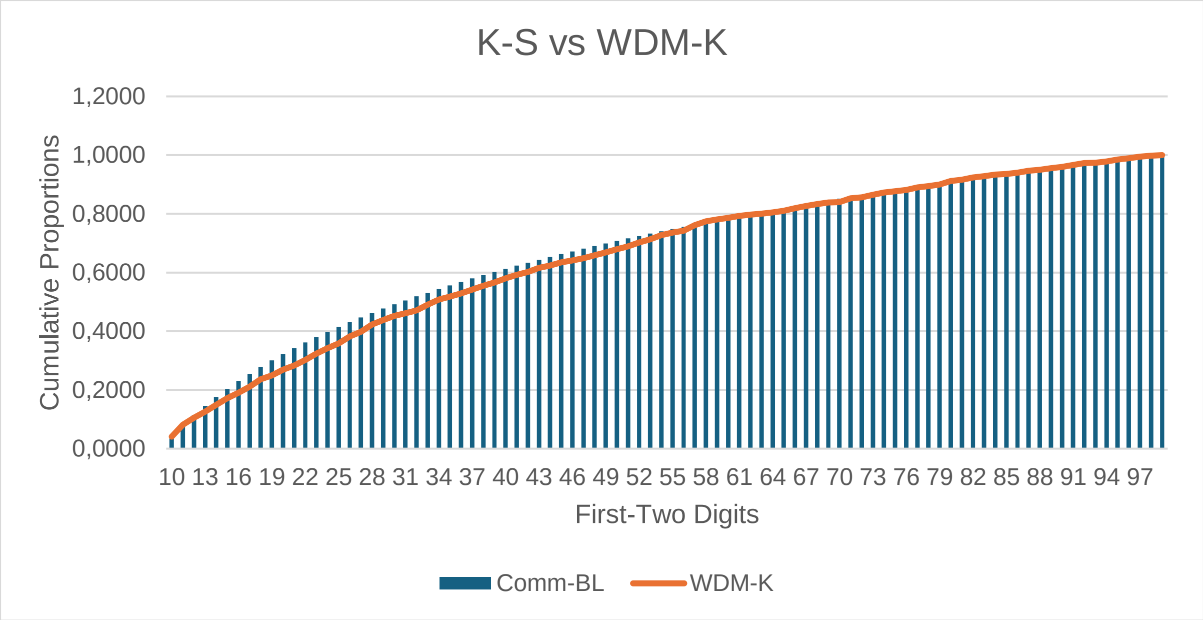
Chi-Square Tests			
	JP	WDM-K	Others
1st Digit	17,336	23,840	20,502
Chi-Sq_CritVal	**15,507**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
2n Digit	8,296	7,408	9,070
Chi-Sq_CritVal	**16,919**		
Result	**Conforms**	**Conforms**	**Conforms**
FT Digits	106,458	106,453	86,480
Chi-Sq_CritVal	**112,022**		
Result	**Conforms**	**Conforms**	**Conforms**

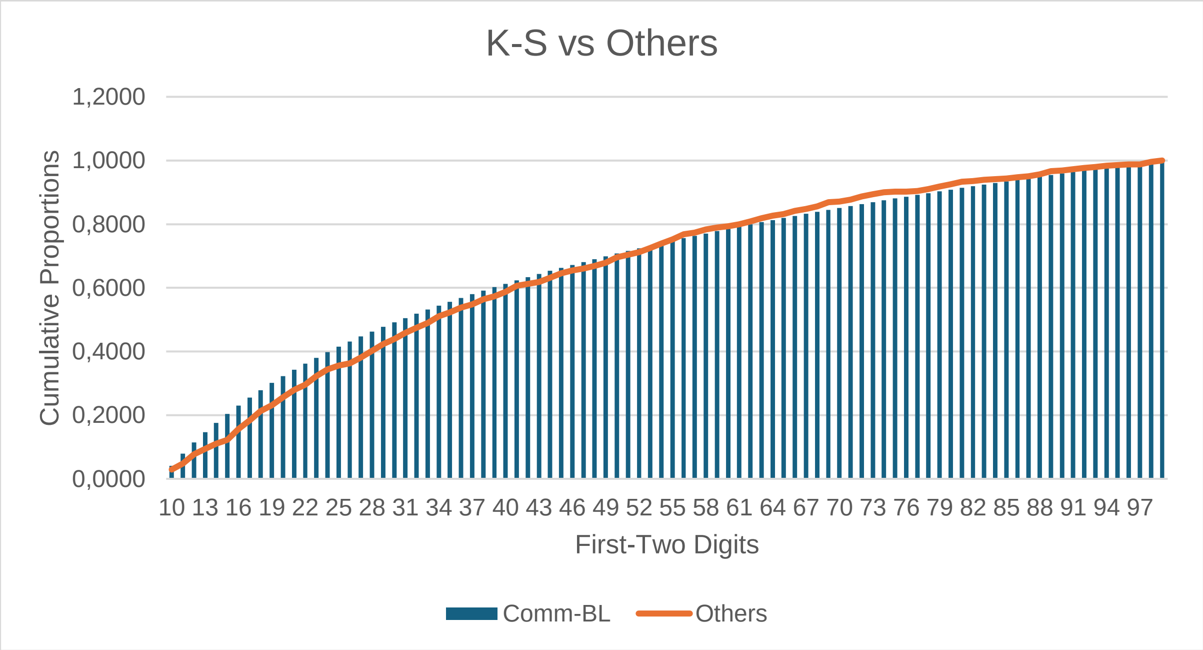
Appendix 35: 2017 Kenya P-value results

P-Value Calculations			
	JP	WDM-K	Others
1st Digit	0,0267958	0,002437403	0,008595851
2n Digit	0,504618091	0,59474379	0,430829122
FT Digits	0,100132623	0,100190737	0,555886634

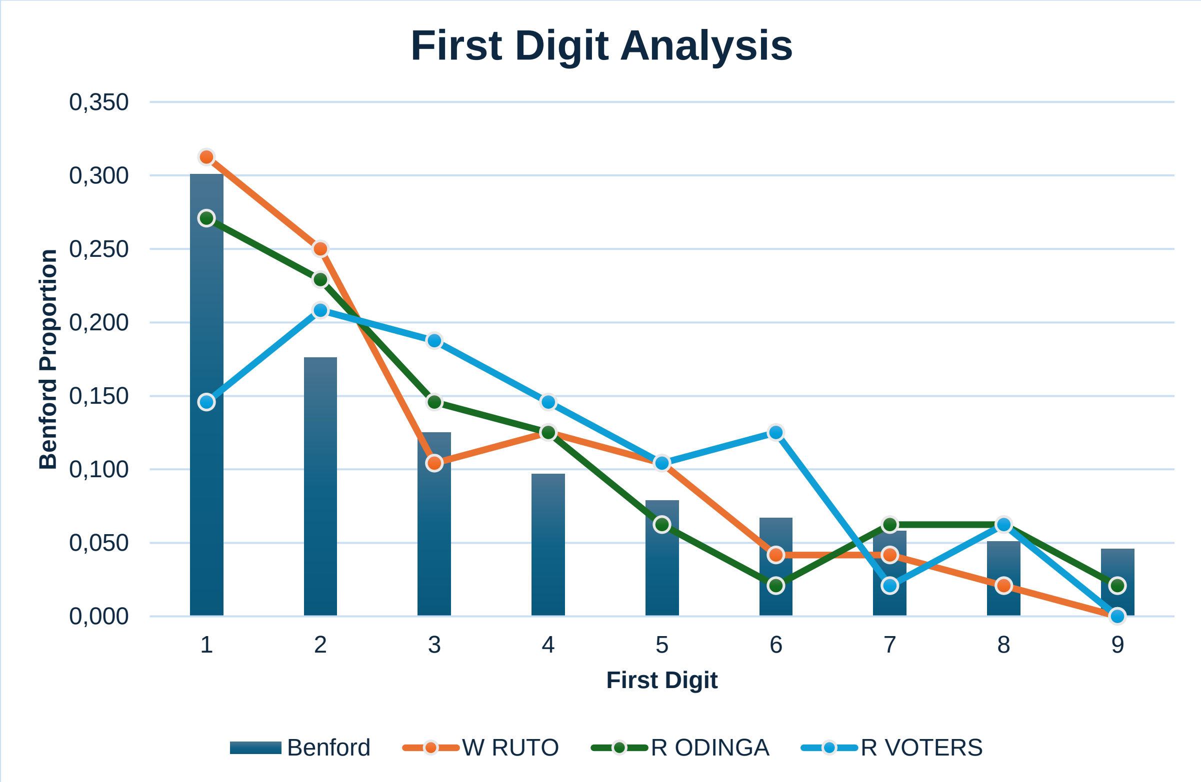
Appendix 36: 2017 Kenya K-S test results

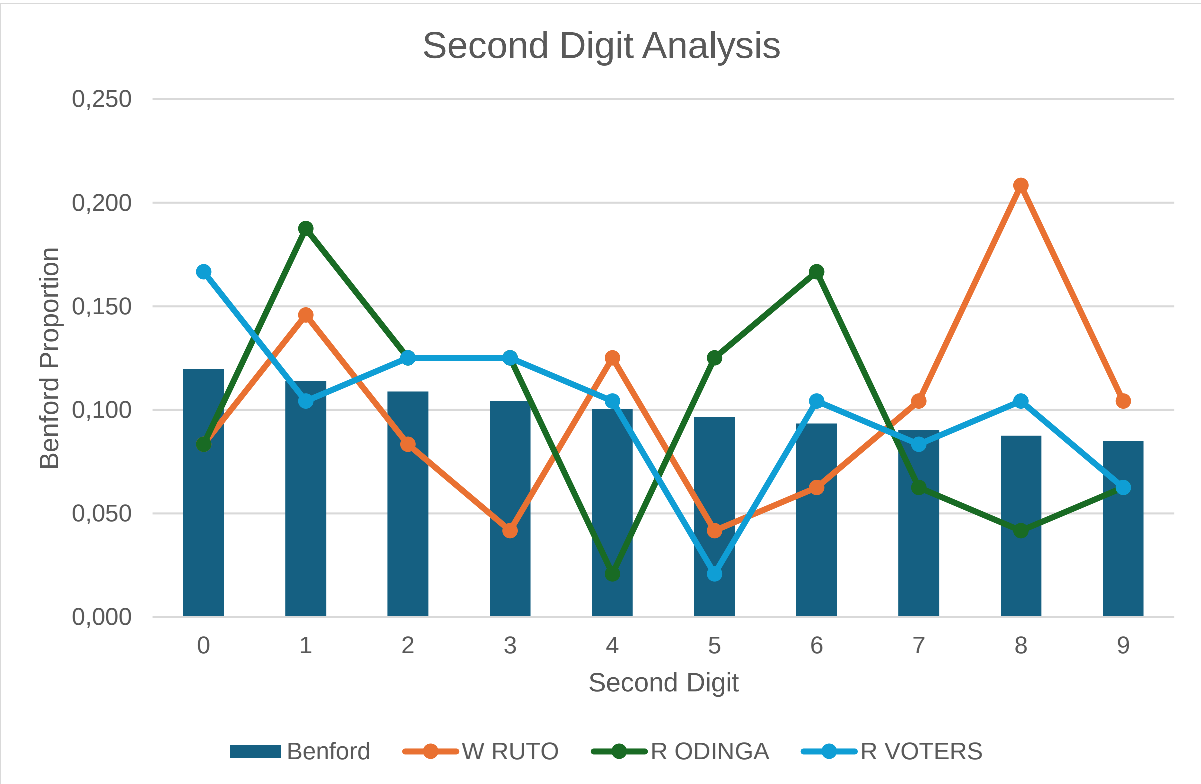


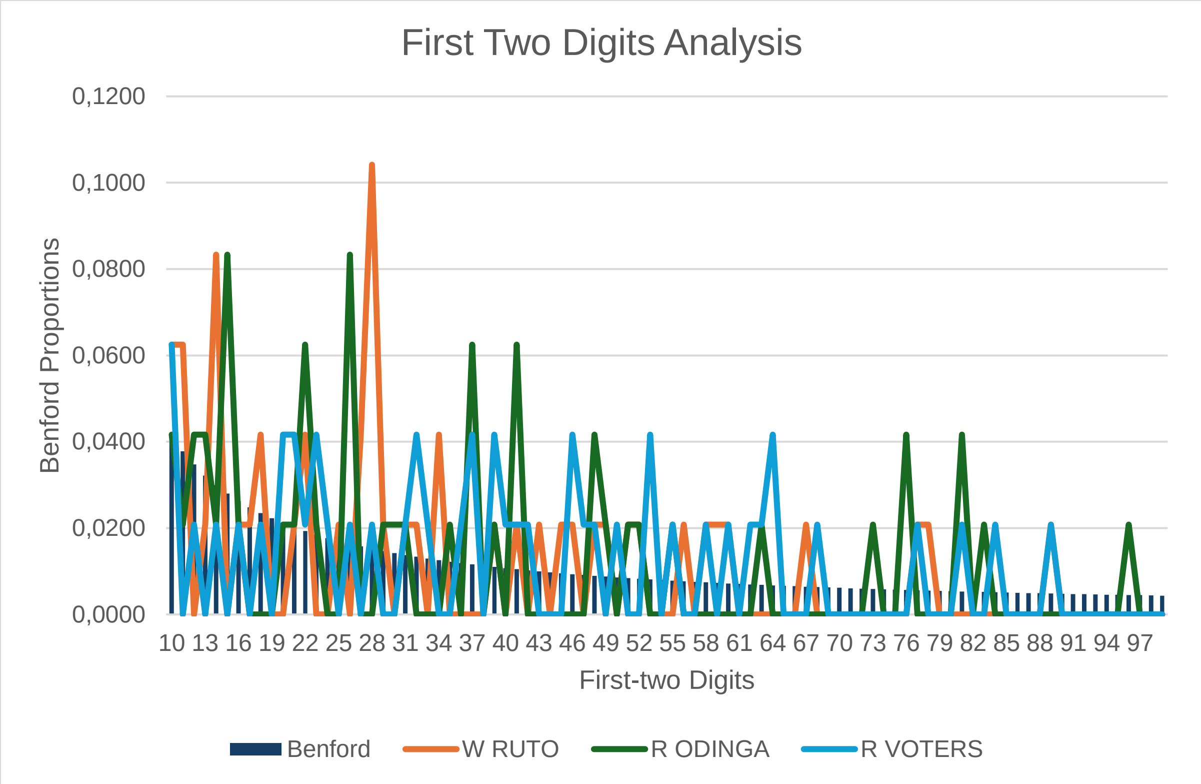

K-S vs WDM-K
1,2000
1,0000
0,8000
0,6000
0,4000
0,2000
0,0000
Cumulative Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-Two Digits
Comm-BL
WDM-K


K-S vs Others
1,2000
1,0000
0,8000
0,6000
0,4000
0,2000
0,0000
Cumulative Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-Two Digits
Comm-BL
Others

Appendix 37: 2022 Kenya Benford's Analysis







Appendix 38: 2022 Kenya MAD results

MAD Outcomes			
	W RUTO	R ODINGA	R VOTERS
1st Digit Outcome	0,03077 Nonconformity	0,02620 Nonconformity	0,05291 Nonconformity
2n Digit Outcome	0,04208 Nonconformity	0,04242 Nonconformity	0,02302 Nonconformity
FT Digits Outcome	0,01169 Nonconformity	0,01205 Nonconformity	0,01170 Nonconformity

Appendix 39: 2022 Kenya First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	W RUTO	R ODINGA	R VOTERS
1	0,0159	0,2988	2,1867
2	1,1549	0,7760	0,3970
3	0,2170	0,2195	1,0926
4	0,4139	0,4139	0,9018
5	0,3738	0,1607	0,3738
6	0,4121	0,9896	1,3204
7	0,1751	0,1337	0,7926
8	0,6258	0,0294	0,0294
9	1,1718	0,4811	1,1718

Z-Statistics for Second Digit			
Digit	W RUTO	R ODINGA	R VOTERS
0	0,5535	0,5535	0,7806
1	0,4695	1,3782	0,2121
2	0,3353	0,1282	0,1282
3	1,1841	0,2324	0,2324
4	0,3292	1,5927	0,0889
5	1,0455	0,4197	1,5339
6	0,4870	1,4973	0,0090
7	0,0822	0,4213	0,1696
8	2,7046	0,8698	0,1515
9	0,2174	0,3002	0,3002

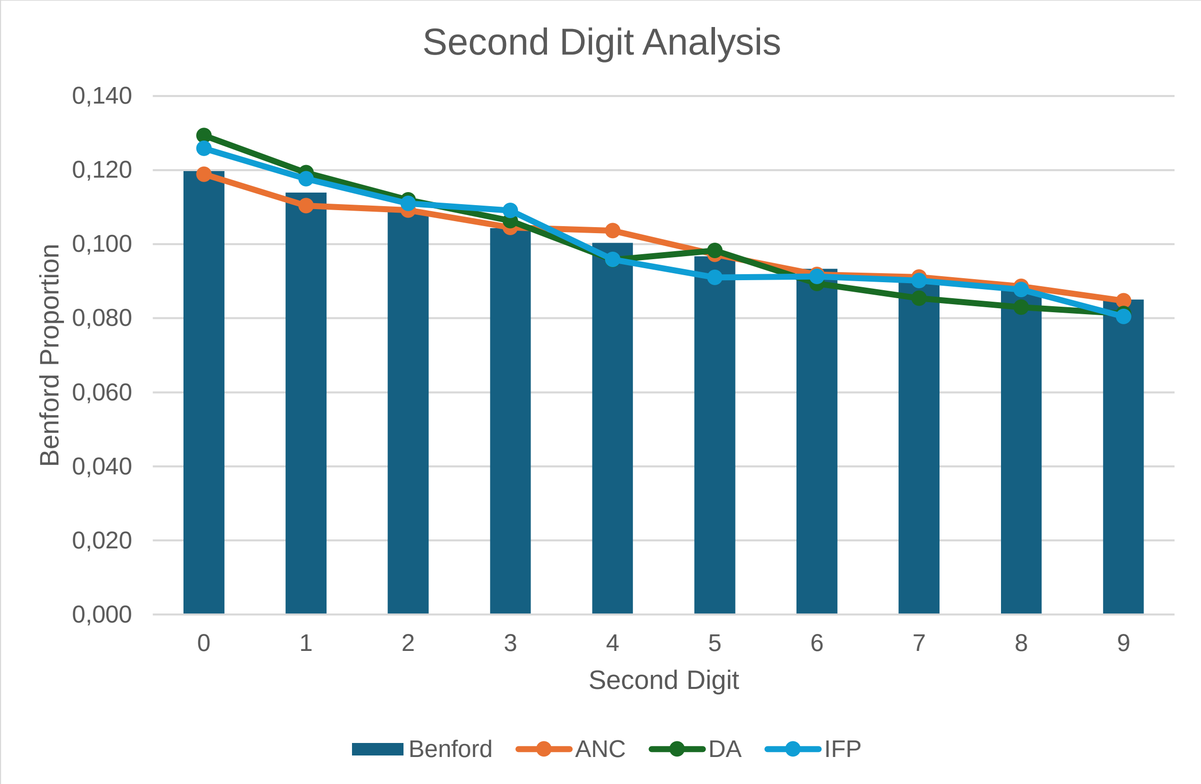
Appendix 40: 2022 Kenya First-two Digit Chi-square results

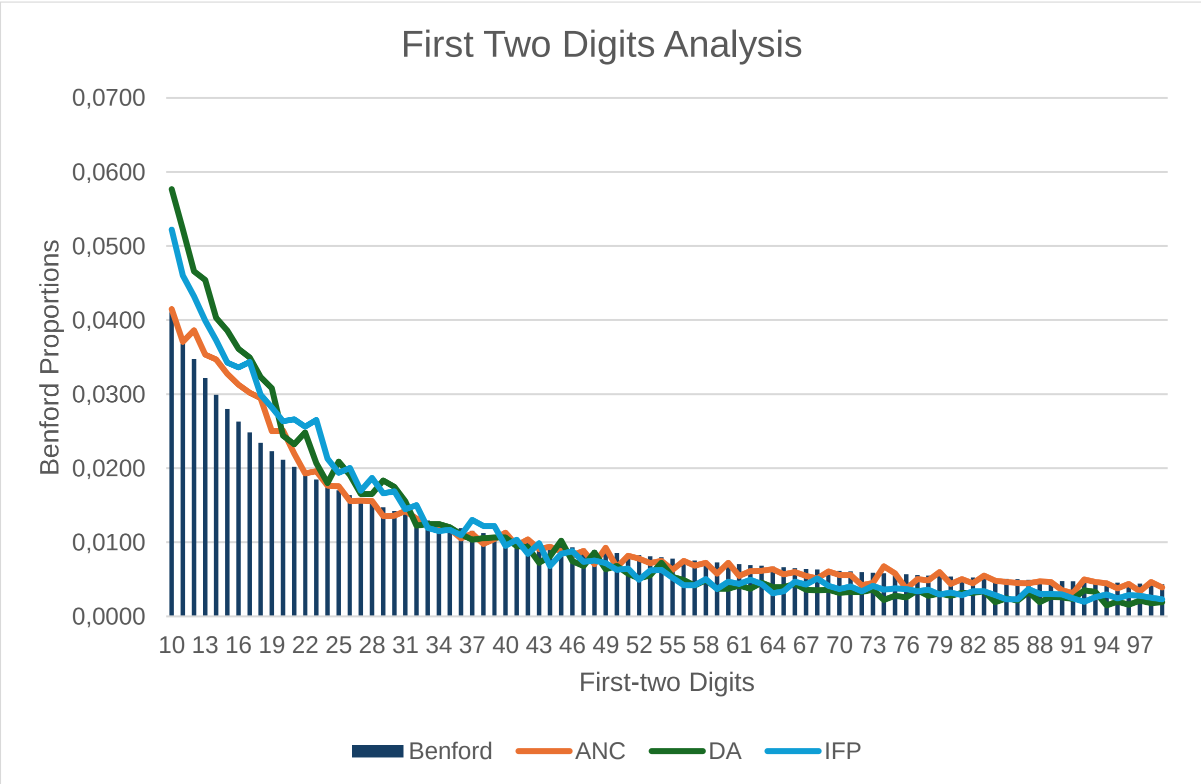
Chi-Square Tests			
	W RUTO	R ODINGA	R VOTERS
1st Digit	6,183	3,955	13,068
Chi-Sq_CritVal	**15,507**		
Result	**Conforms**	**Conforms**	**Conforms**
2n Digit	13,640	11,161	4,624
Chi-Sq_CritVal	**16,919**		
Result	**Conforms**	**Conforms**	**Conforms**
FT Digits	81,932	113,068	81,078
Chi-Sq_CritVal	**112,022**		
Result	**Conforms**	**NonConformance**	**Conforms**

Appendix 41: 2022 Kenya P-value results

P-Value Calculations			
	W RUTO	R ODINGA	R VOTERS
1st Digit	0,626745367	0,861182095	0,109541612
2n Digit	0,135723019	0,264828747	0,865813826
FT Digits	0,689390622	0,043453315	0,71294973

Appendix 42: 2004 South African Second Digit Analysis





Appendix 43: 2004 South African MAD results

MAD Outcomes			
	ANC	DA	IFP
1st Digit Outcome	0,00833 Acceptable conformity	0,01156 Acceptable conformity	0,01038 Acceptable conformity
2n Digit Outcome	0,00123 Close conformity	0,00437 Close conformity	0,00341 Close conformity
FT Digits Outcome	0,00106 Close conformity	0,00325 Nonconformity	0,00286 Nonconformity

Appendix 44: 2004 South Africa First and Second Digit Z-statistics results

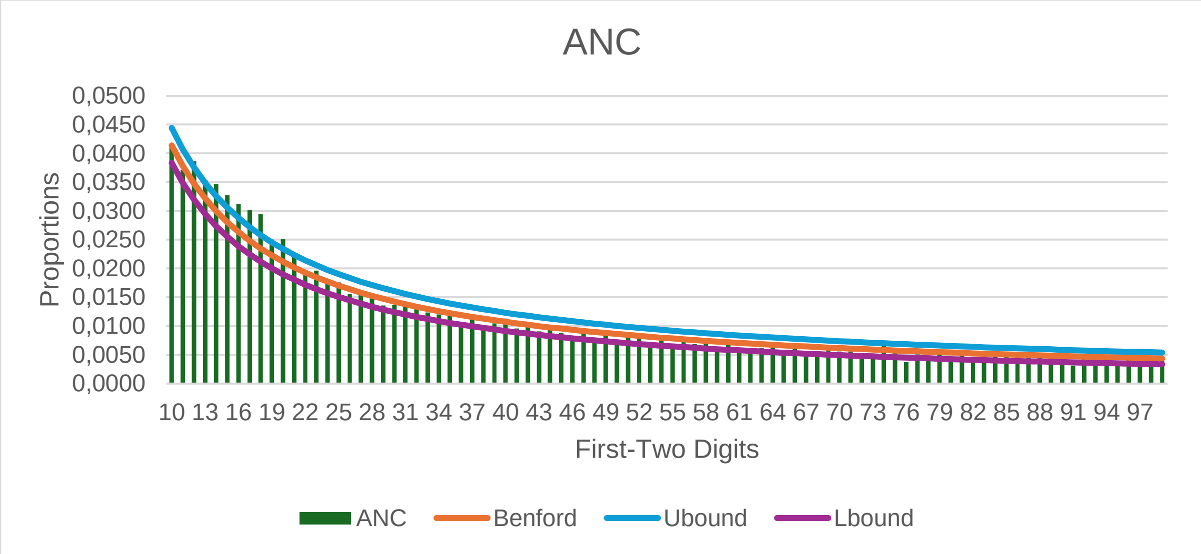
Z-Statistics for First Digit			
Digit	ANC	DA	IFP
1	9,3308	12,0913	3,9713
2	1,5655	1,9587	8,7248
3	2,3159	1,4728	2,4180
4	1,8502	3,2830	1,0344
5	3,8829	4,2331	6,0279
6	3,4440	4,2603	4,7256
7	2,9279	4,3973	4,1556
8	1,9555	4,6511	4,9685
9	2,4869	2,6479	3,5990
Z-Statistics for Second Digit			
Digit	ANC	DA	IFP
0	0,3030	2,8677	1,9916
1	1,4164	1,6396	1,2412
2	0,1221	0,9612	0,7303
3	0,0621	0,6112	1,6108
4	1,4394	1,4508	1,5575
5	0,2387	0,5184	2,0202
6	0,6935	1,3176	0,7543
7	0,3128	1,6498	0,0514
8	0,4473	1,5610	0,0460
9	0,1208	1,2823	1,7012

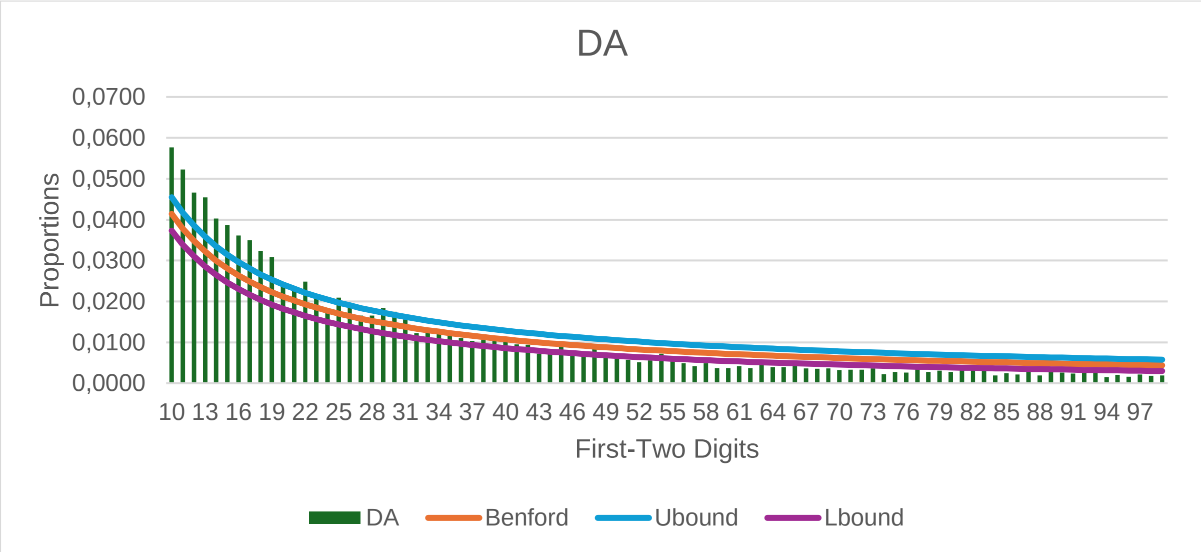
Appendix 45: 2004 South African First-two Digit Z-statistics results

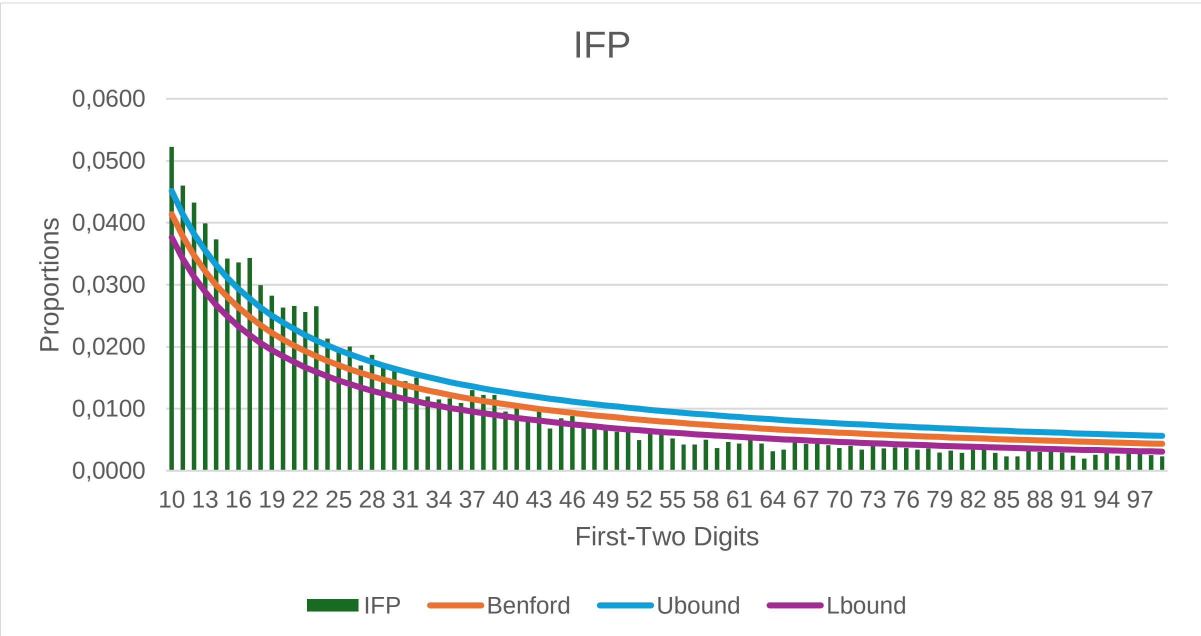
Z-Statistics for First-Two Digits							
Digit	ANC	DA	IFP	Digit	ANC	DA	IFP
10	0,0461	7,8962	5,7140	55	2,1996	2,6821	3,0720
11	0,4520	7,3172	4,5317	56	0,2479	3,0253	4,1262

Digit	ANC	DA	IFP	Digit	ANC	DA	IFP
12	2,7246	6,2293	4,8555	57	1,0293	3,7371	3,9994
13	2,3192	7,2359	4,5920	58	0,2144	2,7815	2,8811
14	3,5935	5,8453	4,5152	59	2,2905	3,9974	4,4216
15	3,6903	6,1716	3,9469	60	0,0678	3,8922	3,0722
16	3,9773	5,9069	4,7739	61	2,4211	3,2958	3,2899
17	4,4559	6,2855	6,4207	62	1,2431	3,6878	2,4881
18	5,1172	5,6120	4,4641	63	0,9872	2,5861	3,0580
19	2,3910	5,5691	4,2207	64	0,5439	3,2382	4,5681
20	3,4828	2,1358	3,7388	65	1,4288	3,1404	4,1191
21	1,6242	2,0574	4,7683	66	0,8004	2,5315	2,3717
22	0,0139	3,8617	4,8063	67	1,5173	3,3372	2,7361
23	1,0631	1,5458	6,2573	68	1,9578	3,3771	1,5563
24	0,0428	0,1753	2,8226	69	0,2563	3,1584	2,7689
25	0,5342	2,8539	1,9067	70	0,8995	3,5991	3,2750
26	0,8027	2,0162	2,9998	71	0,7620	3,3841	2,6930
27	0,1355	0,5285	0,9737	72	2,9246	3,3024	3,4572
28	0,3231	0,9761	2,9387	73	2,3036	2,8177	2,3790
29	1,2275	2,8658	1,6310	74	1,5633	4,4992	3,0324
30	0,6542	2,6106	2,3279	75	0,0749	3,7475	2,6941
31	0,4994	1,4340	0,5788	76	3,2839	3,9501	2,7324
32	0,1550	0,8838	1,4730	77	0,9056	2,6357	3,0277
33	0,6541	0,3729	0,8987	78	1,0984	3,5338	2,6899
34	0,6414	0,0519	0,9815	79	0,8999	3,0444	3,5077
35	0,4424	0,1160	0,4827	80	1,7109	3,3966	3,0435
36	1,6119	0,6753	0,8628	81	0,4386	2,7621	3,4873
37	0,4562	1,0727	1,3880	82	1,3910	2,6925	2,6282
38	1,7718	0,6153	0,8987	83	0,5365	2,3367	2,5516
39	0,6258	0,2589	1,1990	84	0,5347	4,2889	3,2715
40	0,6814	0,0069	1,1776	85	0,7535	3,5059	4,0018
41	1,1023	0,8772	0,0855	86	0,8683	3,8841	3,9374
42	0,1988	0,7532	1,8080	87	0,8771	2,2110	1,8501
43	1,1210	2,6103	0,0538	88	0,2263	4,0668	2,7257
44	0,4455	1,5785	3,0928	89	0,3471	2,9730	2,6578
45	0,9572	0,6355	1,1391	90	2,3682	3,0638	2,7279
46	1,4121	1,8347	0,5321	91	2,9553	3,3072	3,4900
47	0,3491	2,3050	1,9136	92	0,5096	1,5908	4,1233
48	2,5485	0,2717	1,5211	93	0,0155	1,8315	3,0914
49	0,6536	2,4119	1,7386	94	0,2063	4,3663	2,4704
50	2,6863	1,8064	2,5843	95	1,3751	3,5526	3,2535
51	0,2835	2,7764	2,2096	96	0,1391	4,1200	2,4879
52	0,6521	3,3153	3,8232	97	2,0182	3,2987	2,5700
53	1,2939	2,7193	2,1981	98	0,3875	3,7170	2,9404
54	0,6550	0,7239	1,9351	99	0,8099	3,5140	3,1719

Appendix 46: 2004 South Africa First-two Digit Z-statistics results







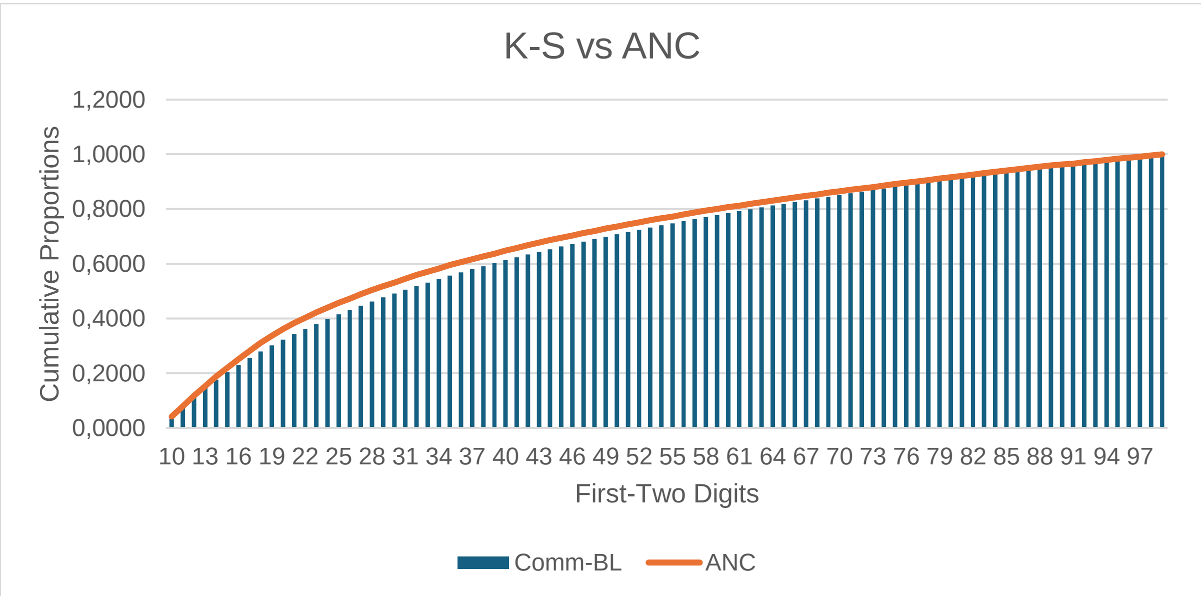
Appendix 47: 2004 South Africa Chi-square results

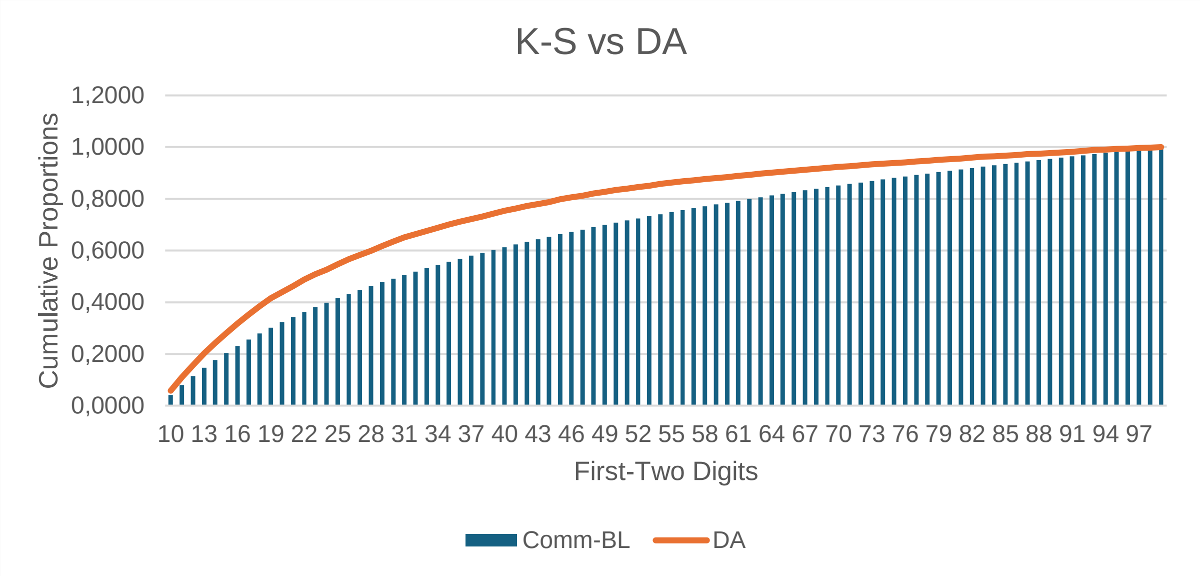
Chi-Square Tests			
	ANC	DA	IFP
1st Digit	113,885	196,814	187,169
Chi-Sq_CritVal	**15,507**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
2n Digit	4,631	21,124	17,025
Chi-Sq_CritVal	**16,919**		
Result	**Conforms**	**NonConformance**	**NonConformance**
FT Digits	250,862	1044,342	899,596
Chi-Sq_CritVal	**112,022**		
Result	**NonConformance**	**NonConformance**	**NonConformance**

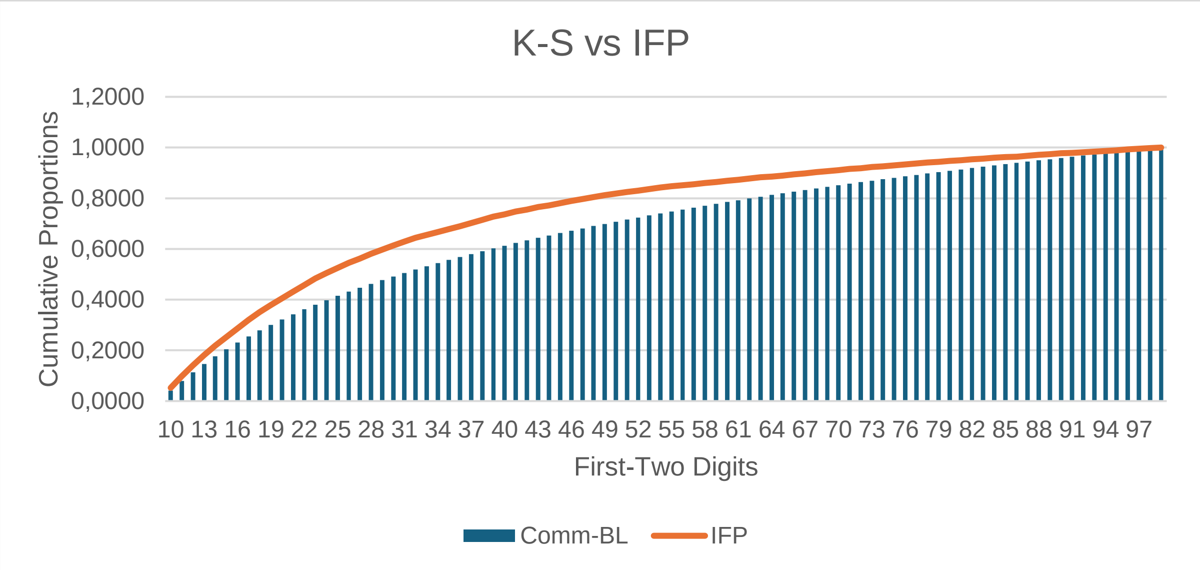
Appendix 48: 2004 South Africa P-value results

P-Value Calculations			
	ANC	DA	IFP
1st Digit	6,04518E-21	2,99649E-38	3,20795E-36
2n Digit	0,86518043	0,012111044	0,04831916
FT Digits	2,43943E-17	7,6708E-163	3,1909E-134

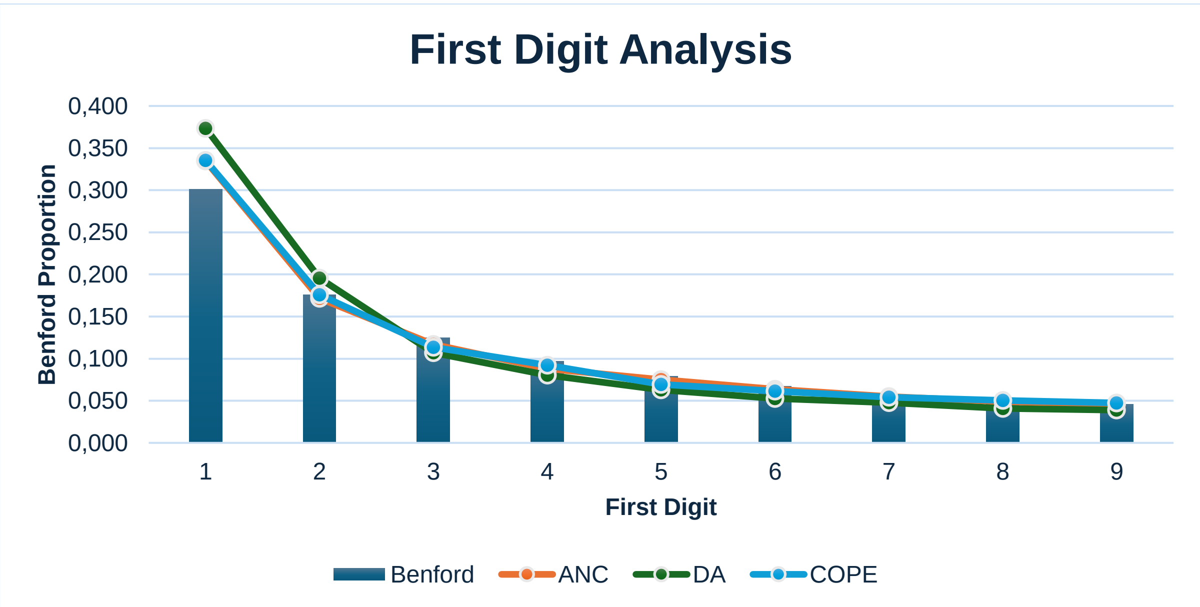
Appendix 49: 2004 South Africa K-S results

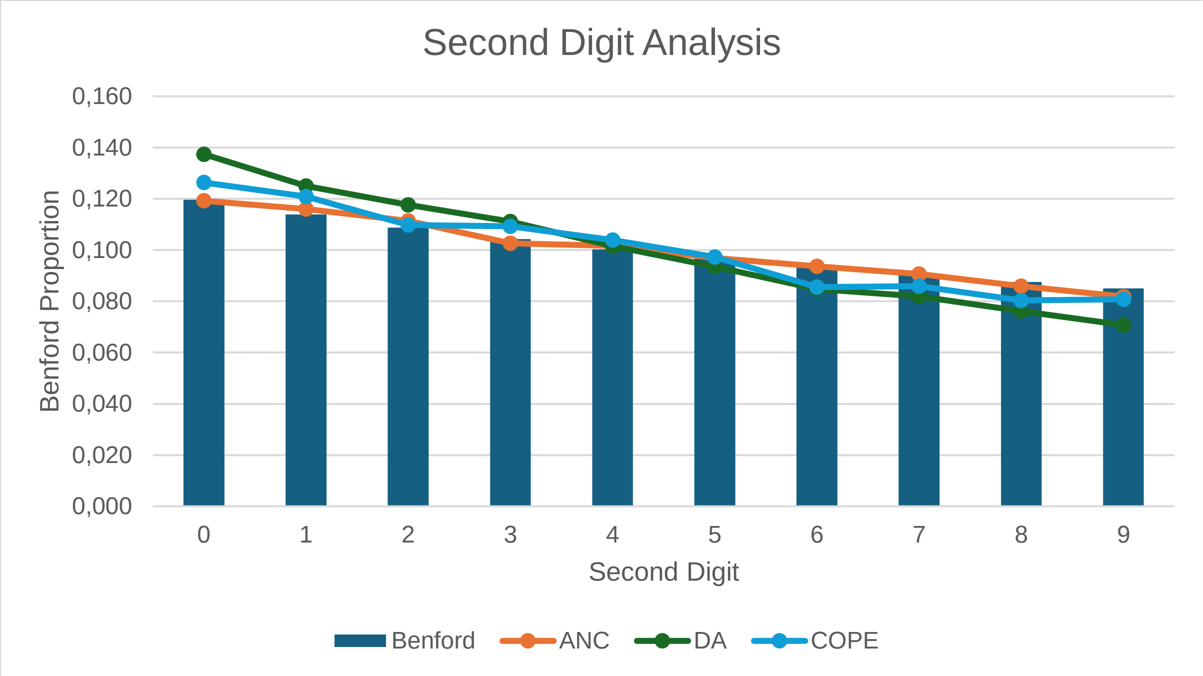


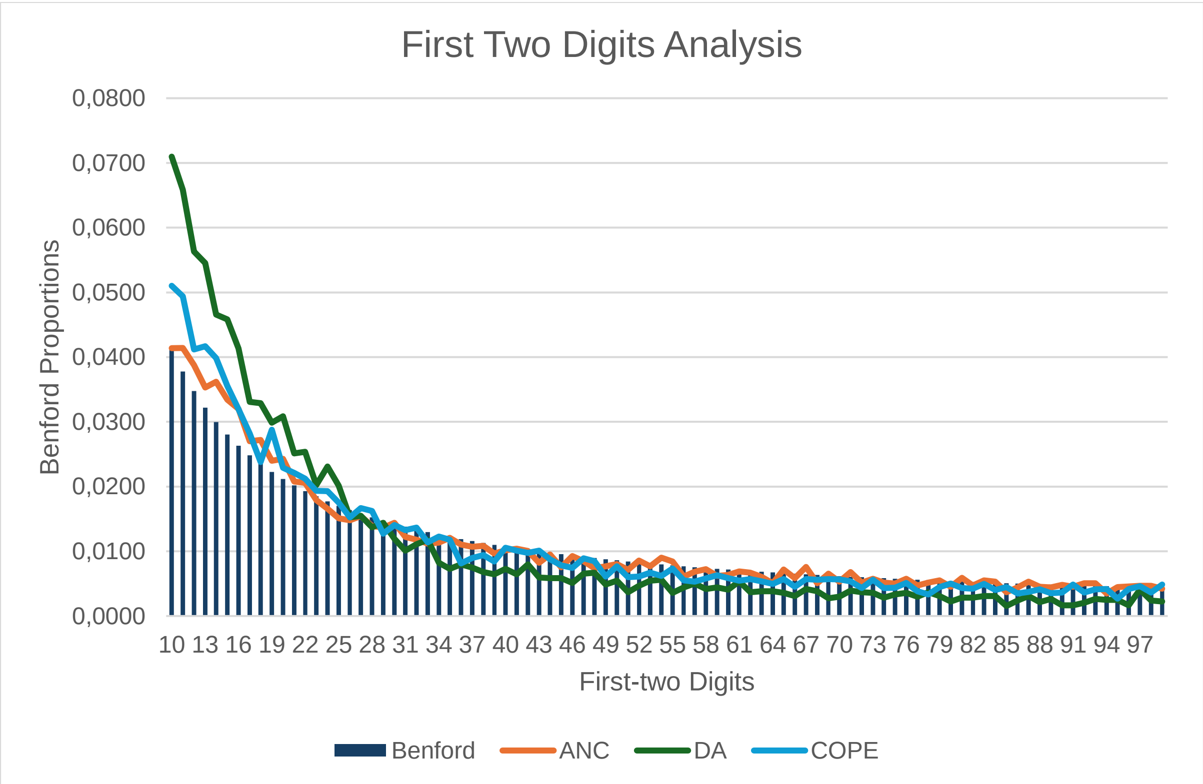




Appendix 50: 2009 South Africa Benford's Analysis




Second Digit Analysis
0,160
0,140
0,120
0,100
0,080
0,060
0,040
0,020
0,000
Benford Proportion
0
1
2
3
4
5
6
7
8
9
Second Digit
Benford
ANC
DA
COPE


First Two Digits Analysis
0,0800
0,0700
0,0600
0,0500
0,0400
0,0300
0,0200
0,0100
0,0000
Benford Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-two Digits
Benford
ANC
DA
COPE

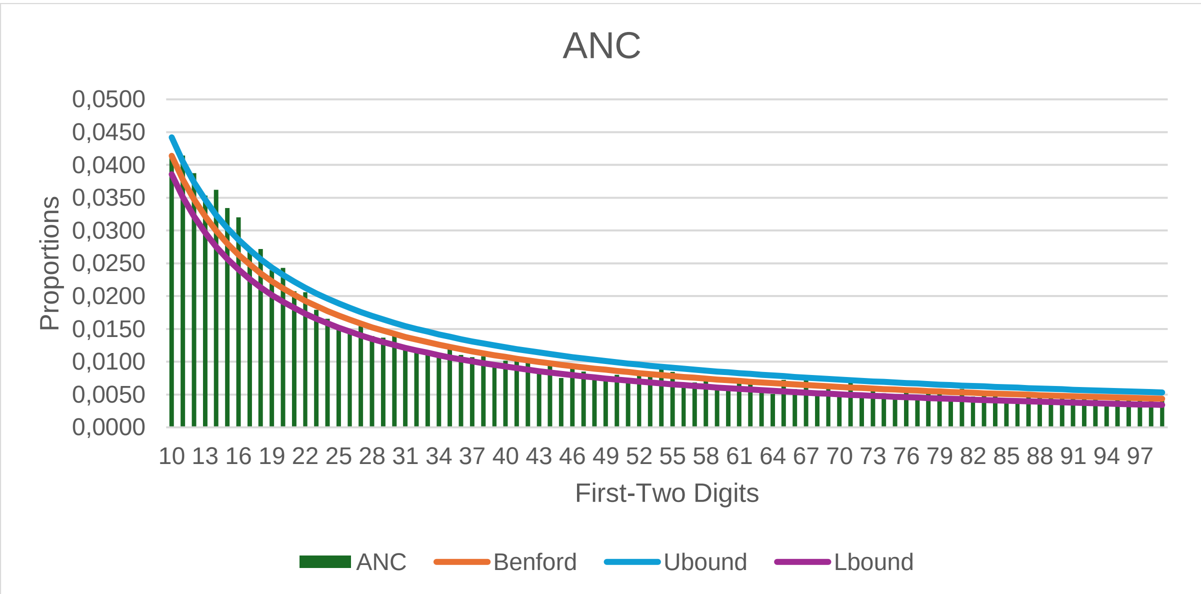
Appendix 51: 2009 South Africa MAD results

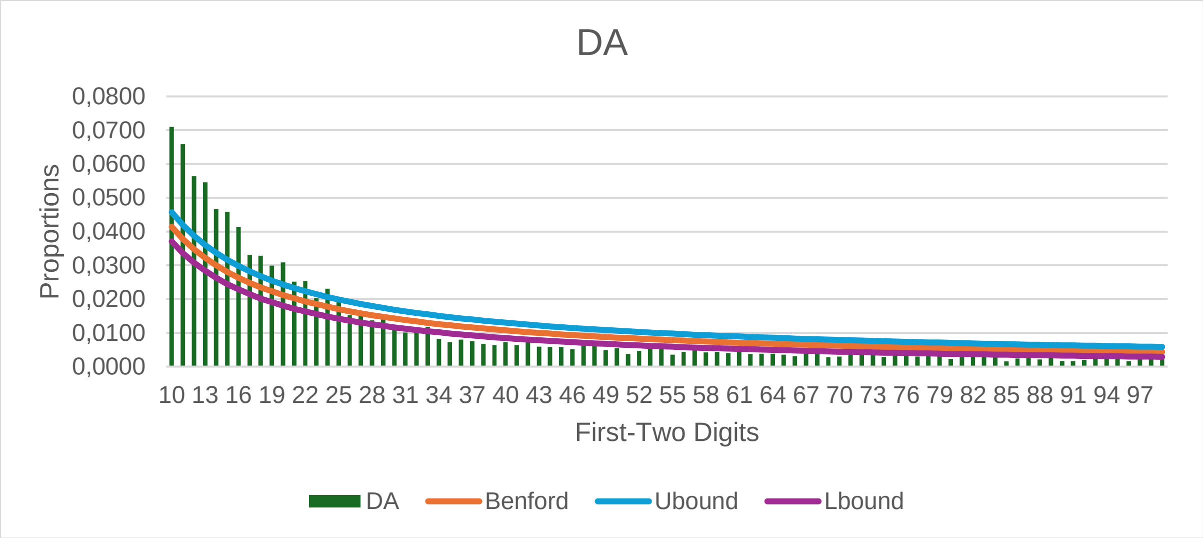
MAD Outcomes			
	ANC	DA	COPE
1st Digit	0,00752	0,02043	0,00802
Outcome	Acceptable conformity	Nonconformity	Acceptable conformity
2n Digit	0,00138	0,00915	0,00474
Outcome	Close conformity	Acceptable conformity	Close conformity
FT Digits	0,00107	0,00460	0,00182
Outcome	Close conformity	Nonconformity	Marginally acceptable conformity

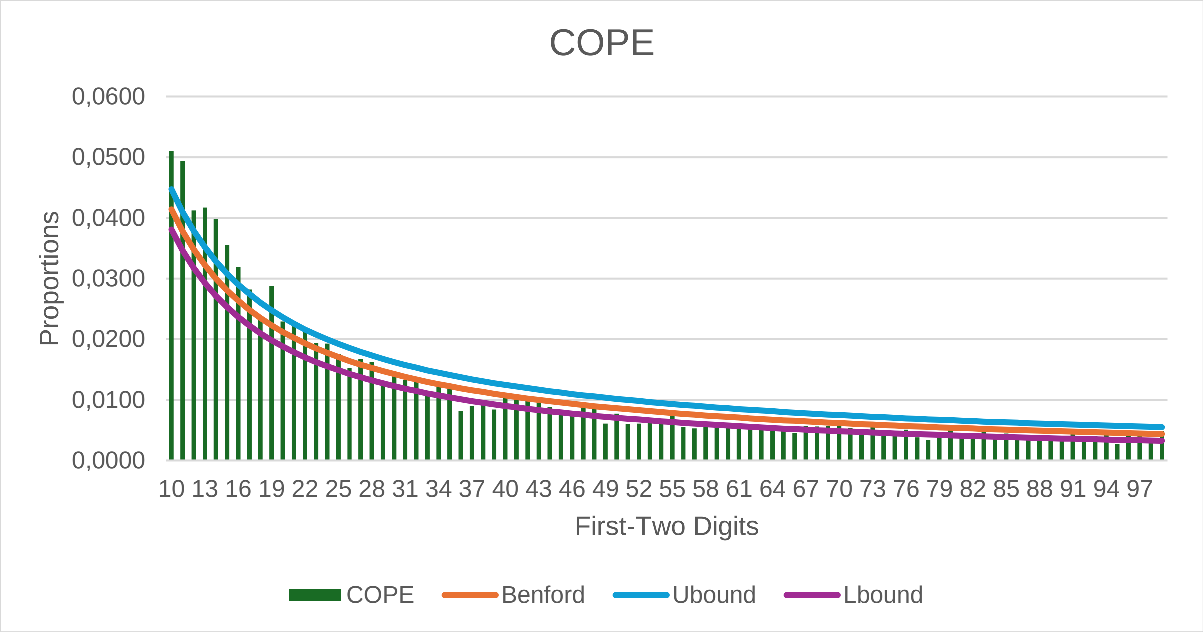
Appendix 52: 2009 South Africa Chi-square results

Chi-Square Tests			
	ANC	DA	COPE
1st Digit	114,077	590,905	132,614
Chi-Sq_CritVal	**15,507**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
2n Digit	5,766	87,606	40,372
Chi-Sq_CritVal	**16,919**		
Result	**Conforms**	**NonConformance**	**NonConformance**
FT Digits	259,230	1730,473	531,386
Chi-Sq_CritVal	**112,022**		
Result	**NonConformance**	**NonConformance**	**NonConformance**

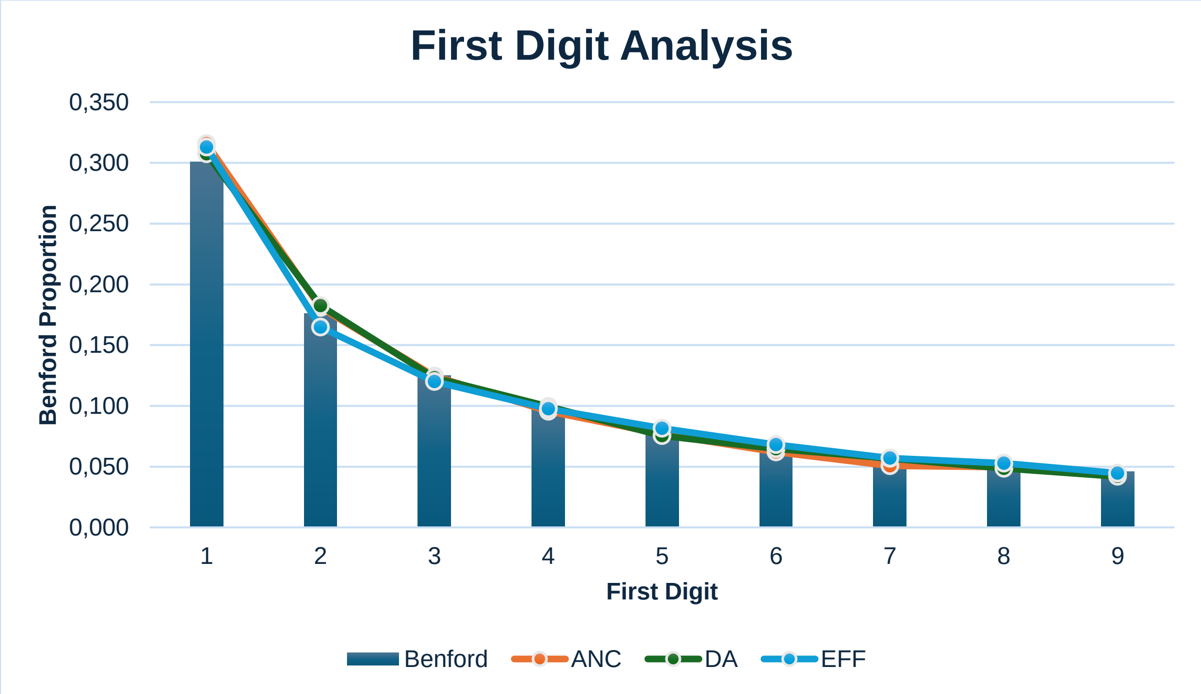
Appendix 53: 2009 South Africa First-two Digit Z-statistics results

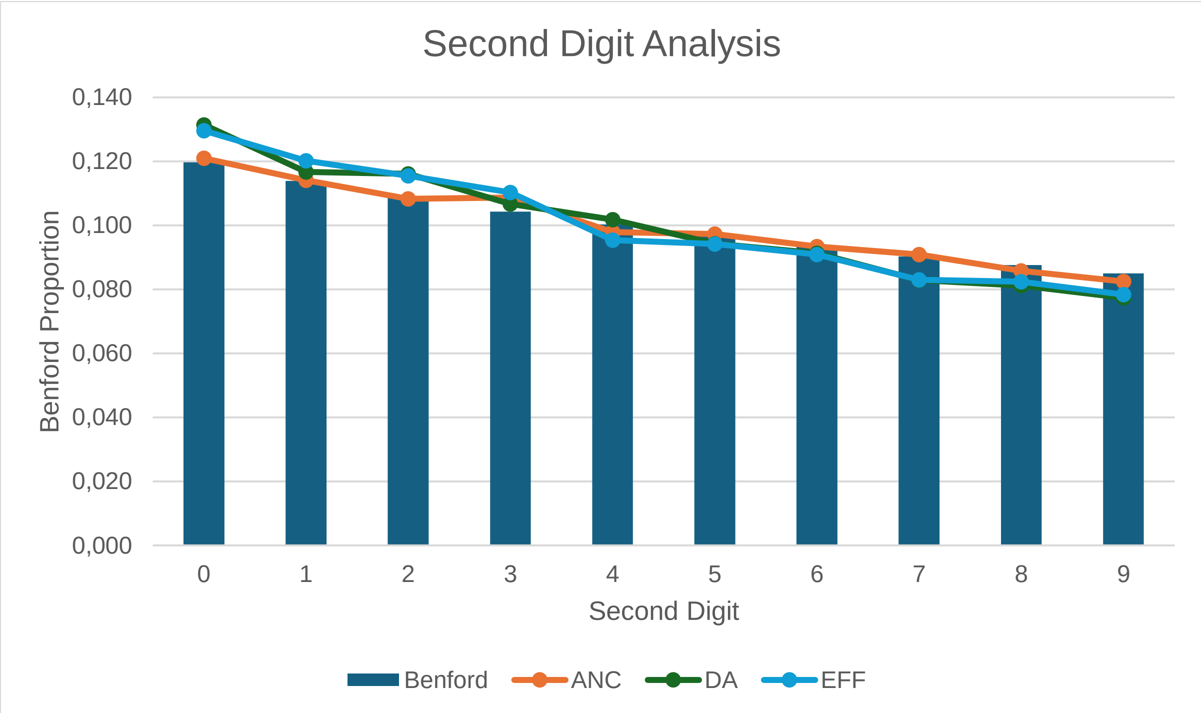


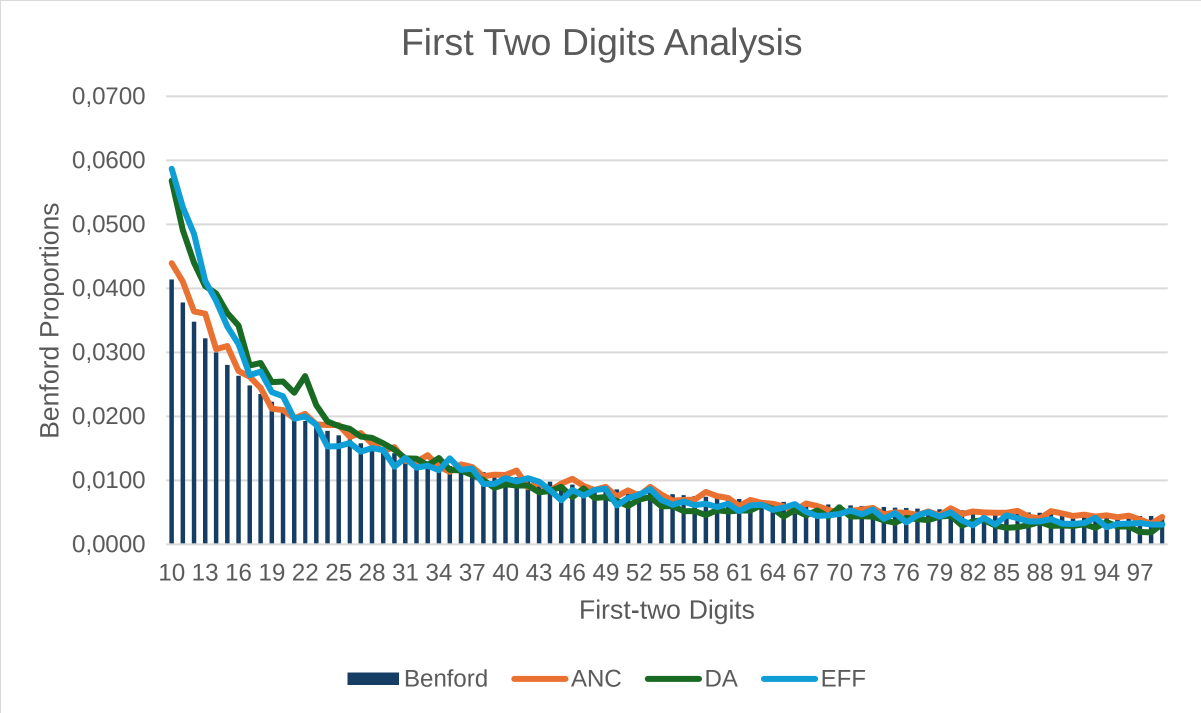

DA
0,0800
0,0700
0,0600
0,0500
0,0400
0,0300
0,0200
0,0100
0,0000
Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-Two Digits
DA
Benford
Ubound
Lbound


COPE
0,0600
0,0500
0,0400
0,0300
0,0200
0,0100
0,0000
Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-Two Digits
COPE
Benford
Ubound
Lbound

Appendix 54: 2014 South Africa Benford's Analysis







Appendix 55: 2014 South African MAD results

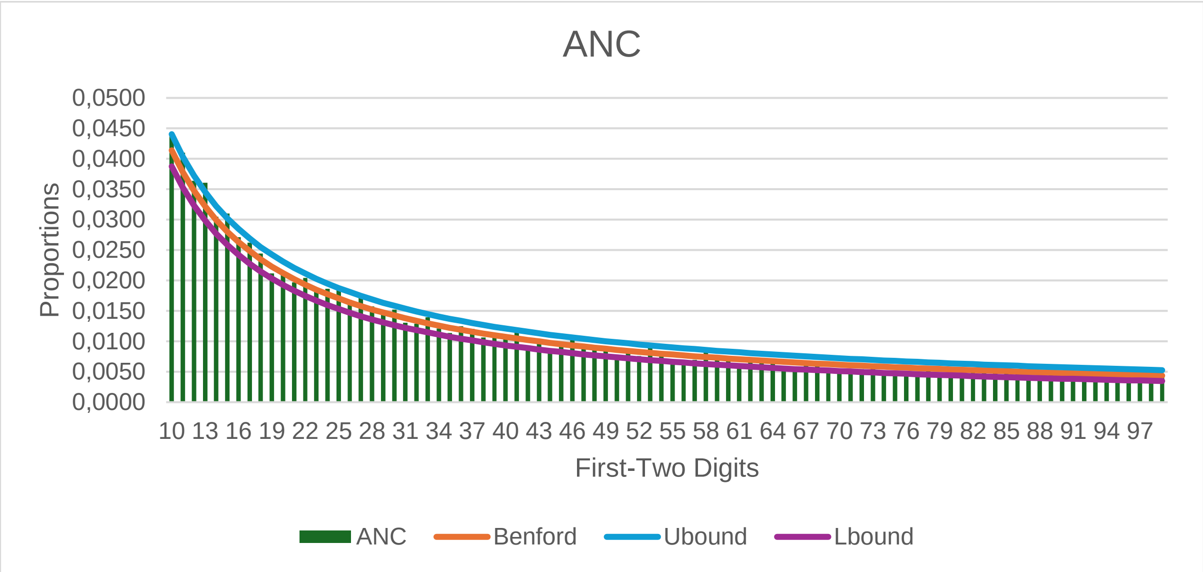
MAD Outcomes			
	ANC	DA	EFF
1st Digit Outcome	0,00437 Close conformity	0,00344 Close conformity	0,00398 Close conformity
2n Digit Outcome	0,00145 Close conformity	0,00517 Close conformity	0,00579 Close conformity
FT Digits Outcome	0,00071 Close conformity	0,00239 Nonconformity	0,00190 Marginally acceptable conformity

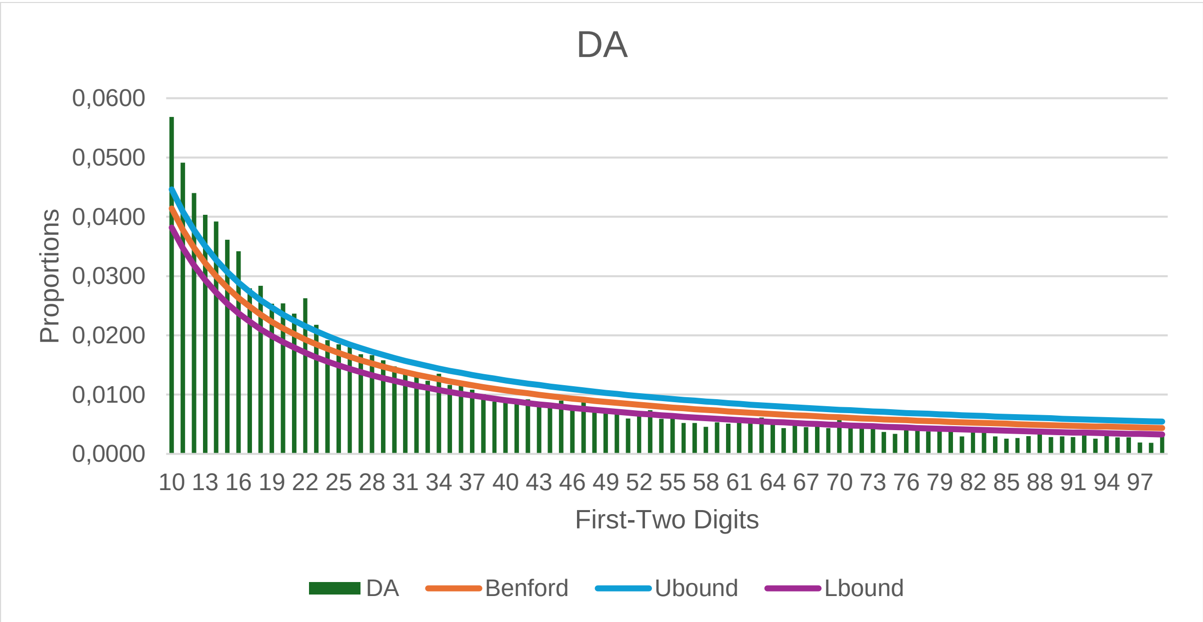
Appendix 56: 2014 South Africa First and Second Digit Z-statistics results

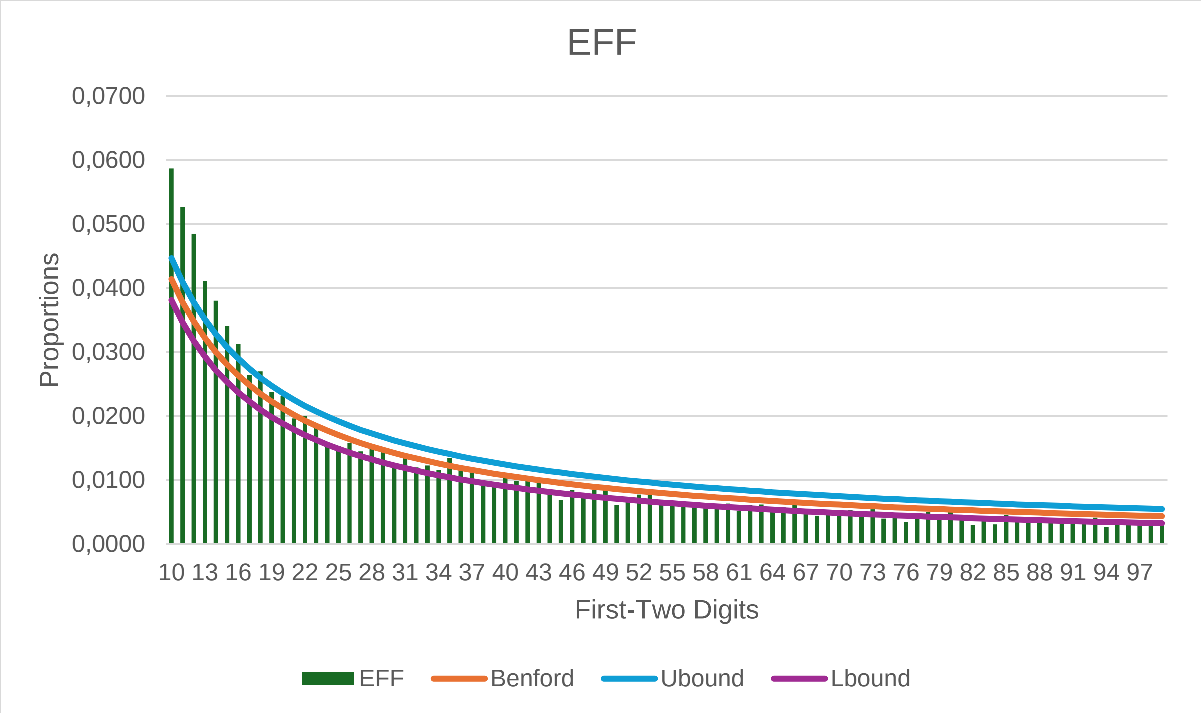
Z-Statistics for First Digit			
Digit	ANC	DA	EFF
1	4,8315	2,0223	3,7758
2	1,8568	2,4771	4,2431
3	0,1503	0,7381	2,1276
4	0,6968	1,3218	0,3720
5	1,2726	2,0528	1,3322
6	2,8737	1,3010	0,6115
7	4,5830	0,7457	0,5026
8	0,9612	1,7666	1,0516
9	1,4649	2,7036	0,7300

Z-Statistics for Second Digit			
Digit	ANC	DA	EFF
0	0,5822	4,4375	3,6720
1	0,0863	1,0943	2,3955
2	0,2561	2,8766	2,5669
3	2,1429	0,9423	2,3489
4	1,1811	0,5903	1,9365
5	0,3297	0,9577	0,9938
6	0,0150	0,8675	1,0076
7	0,2910	3,1738	3,0531
8	0,9077	2,7345	2,2049
9	1,3347	3,3381	2,8502

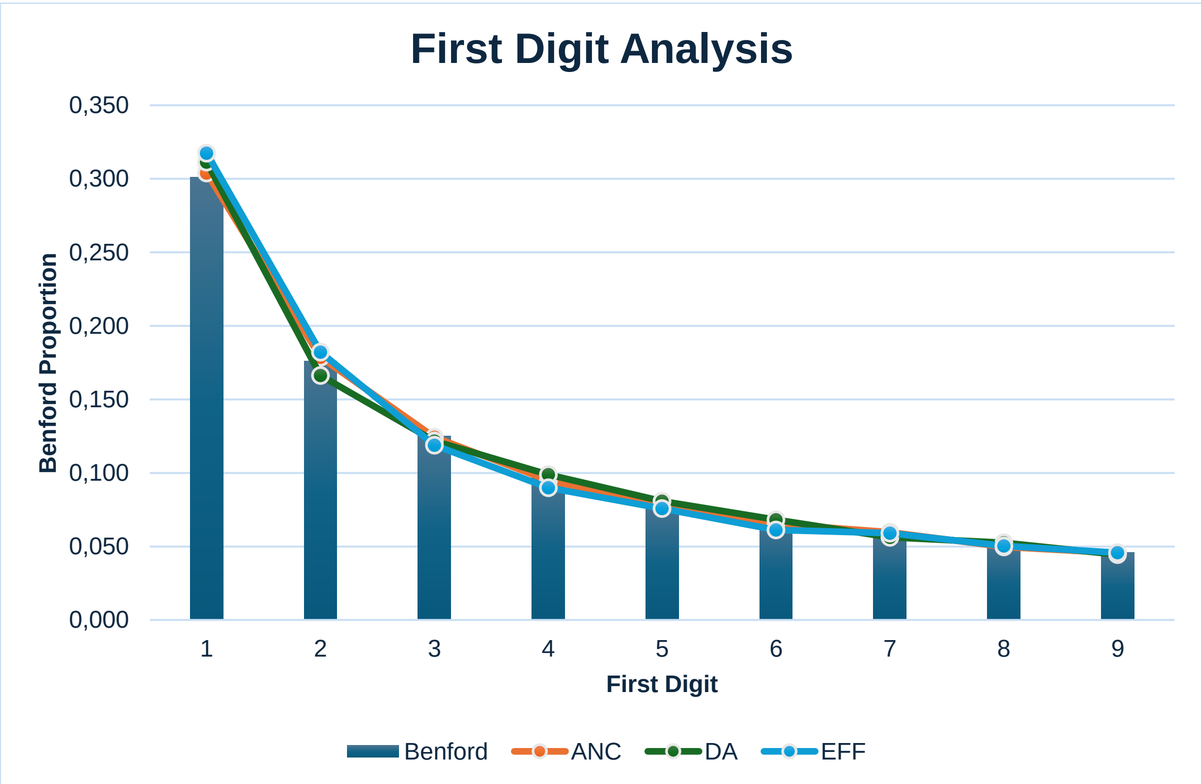
Appendix 57: 2014 South Africa First-two Digit Z-statistics results

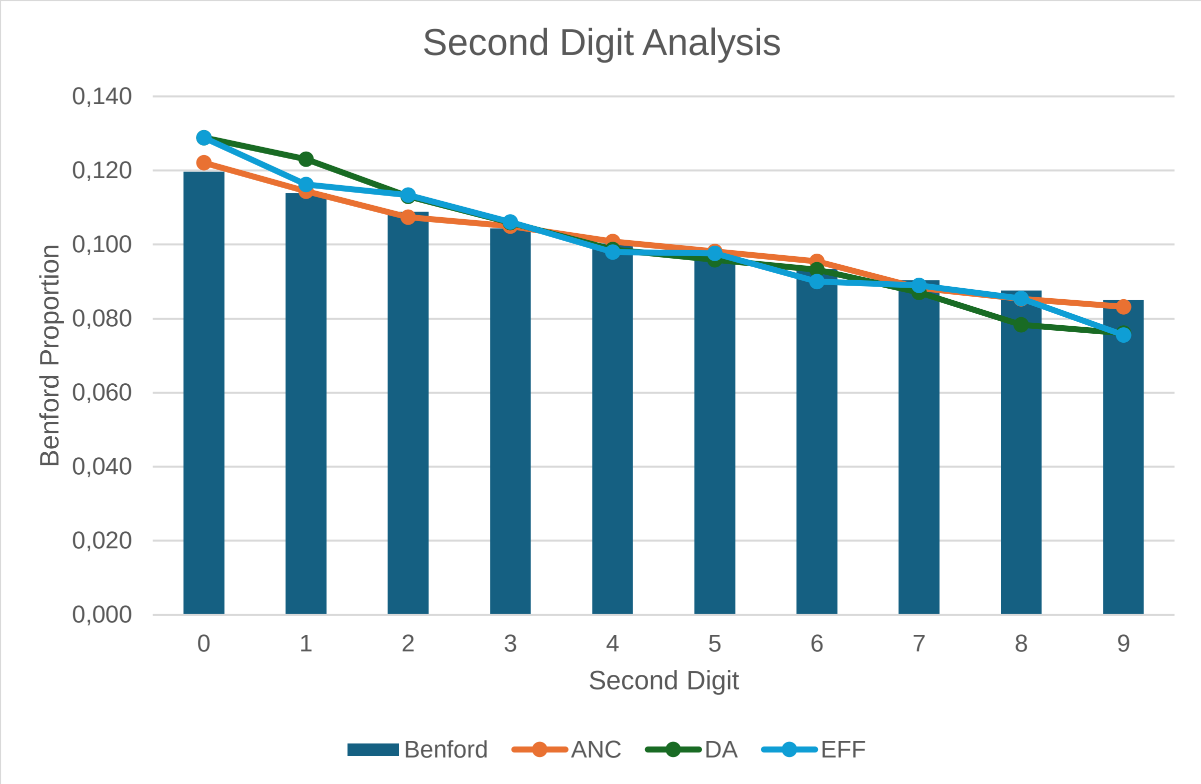


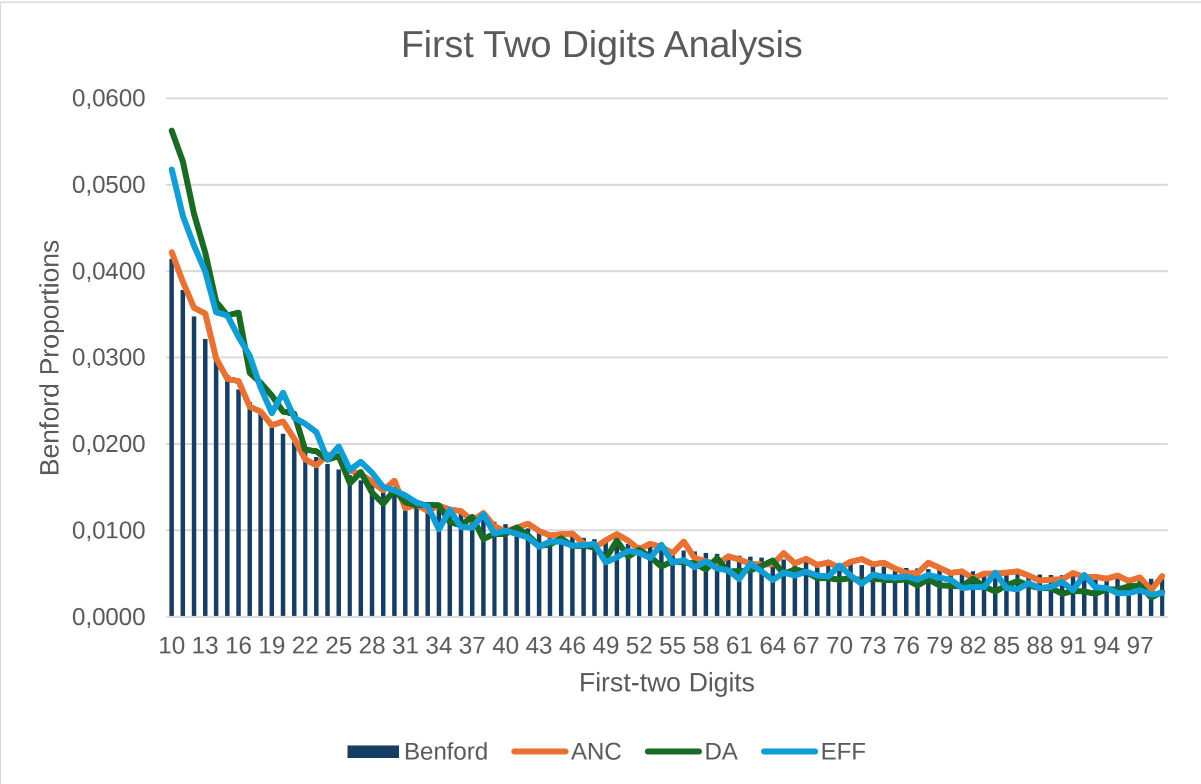




Appendix 58: 2019 South Africa Benford's Analysis




Second Digit Analysis
0,140
0,120
0,100
0,080
0,060
0,040
0,020
0,000
Benford Proportion
0 1 2 3 4 5 6 7 8 9
Second Digit
Benford ANC DA EFF


First Two Digits Analysis
0,0600
0,0500
0,0400
0,0300
0,0200
0,0100
0,0000
Benford Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-two Digits
Benford ANC DA EFF

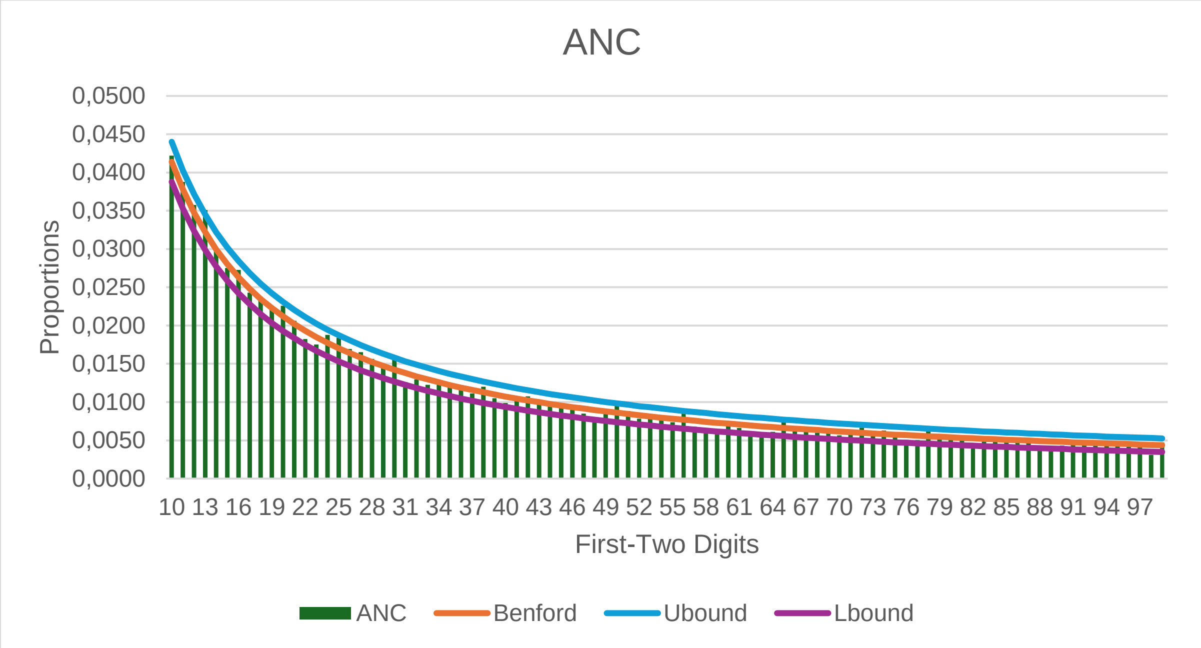
Appendix 59: 2019 South Africa MAD results

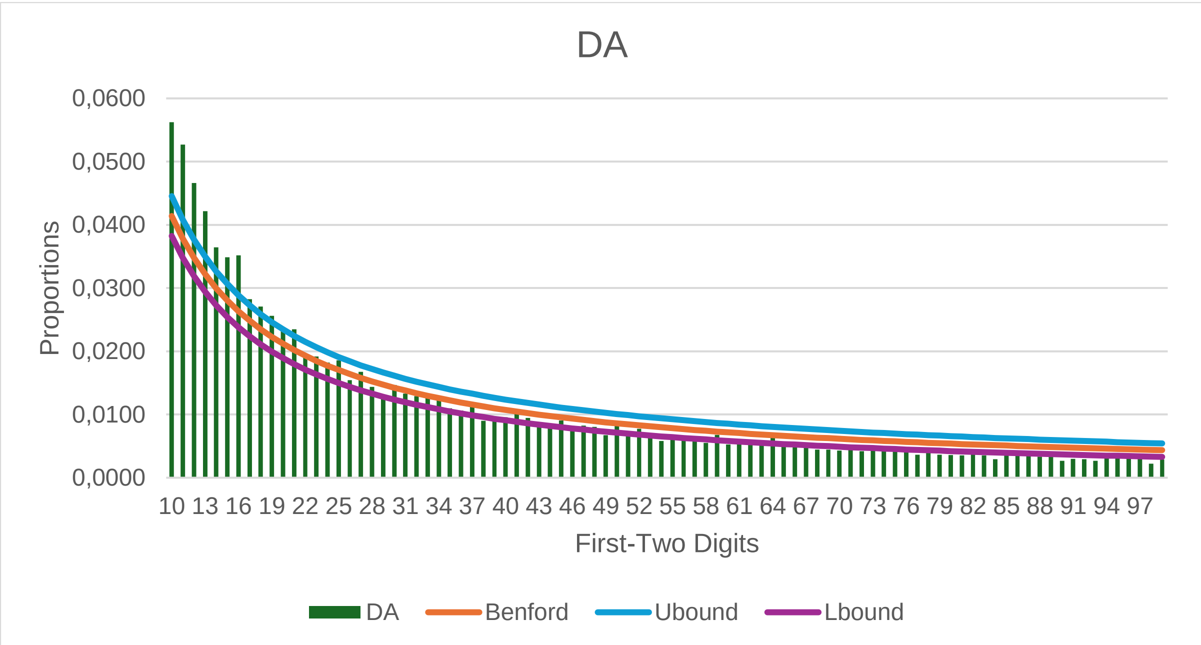
MAD Outcomes			
	ANC	DA	EFF
1st Digit Outcome	0,00160 Close conformity	0,00363 Close conformity	0,00518 Close conformity
2n Digit Outcome	0,00151 Close conformity	0,00481 Close conformity	0,00375 Close conformity
FT Digits Outcome	0,00054 Close conformity	0,00210 Marginally acceptable conformity	0,00191 Marginally acceptable conformity

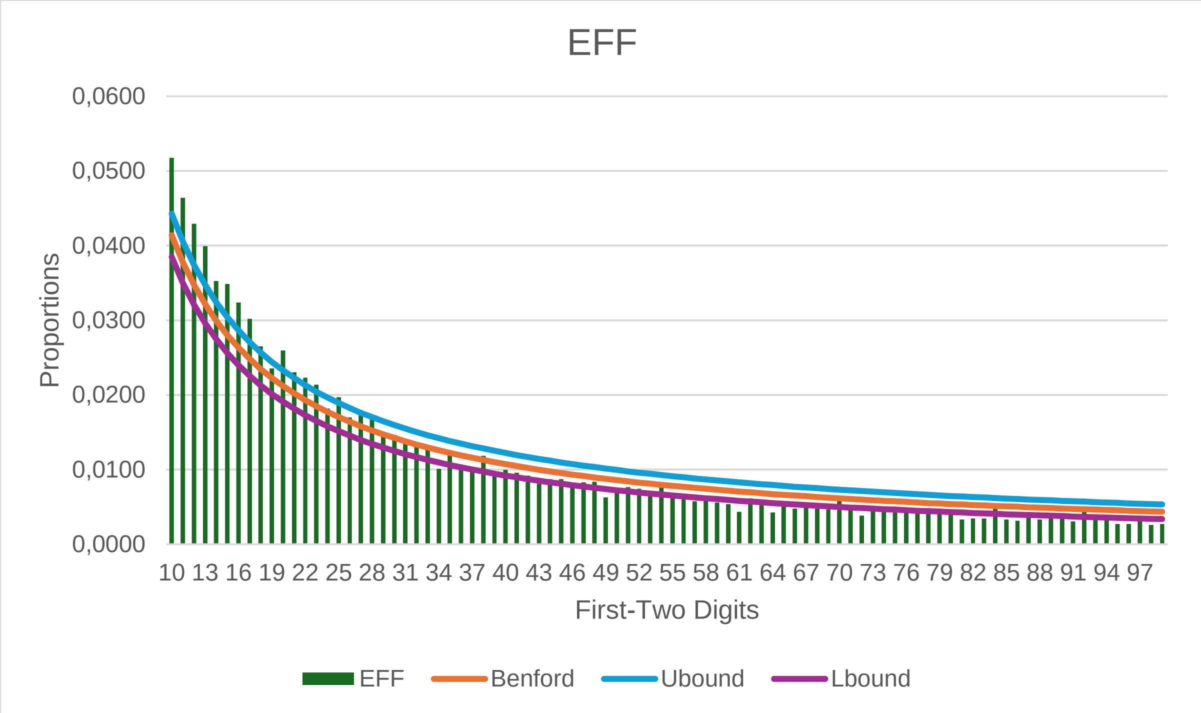
Appendix 60: 2019 South Africa First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	ANC	DA	EFF
1	0,9368	3,3079	5,3362
2	1,0285	3,8334	2,3317
3	0,3348	1,4634	2,7724
4	1,0290	0,9636	3,5747
5	0,5181	0,8994	1,8910
6	1,0098	0,6543	3,4495
7	1,1010	1,1724	0,6365
8	1,0043	0,9281	0,5668
9	0,1924	0,9504	0,0134
Z-Statistics for Second Digit			
Digit	ANC	DA	EFF
0	1,1009	3,5313	3,8332
1	0,2087	3,5997	0,9975
2	0,6857	1,6548	1,9884
3	0,3171	0,6221	0,7641
4	0,2361	0,6718	1,0613
5	0,7402	0,3086	0,4218
6	1,0486	0,0709	1,5759
7	1,0810	1,4297	0,6495
8	1,1672	4,0642	1,0310
9	0,9631	3,9978	4,6025

Appendix 61: 2019 South Africa First-two Digit Z-statistics results







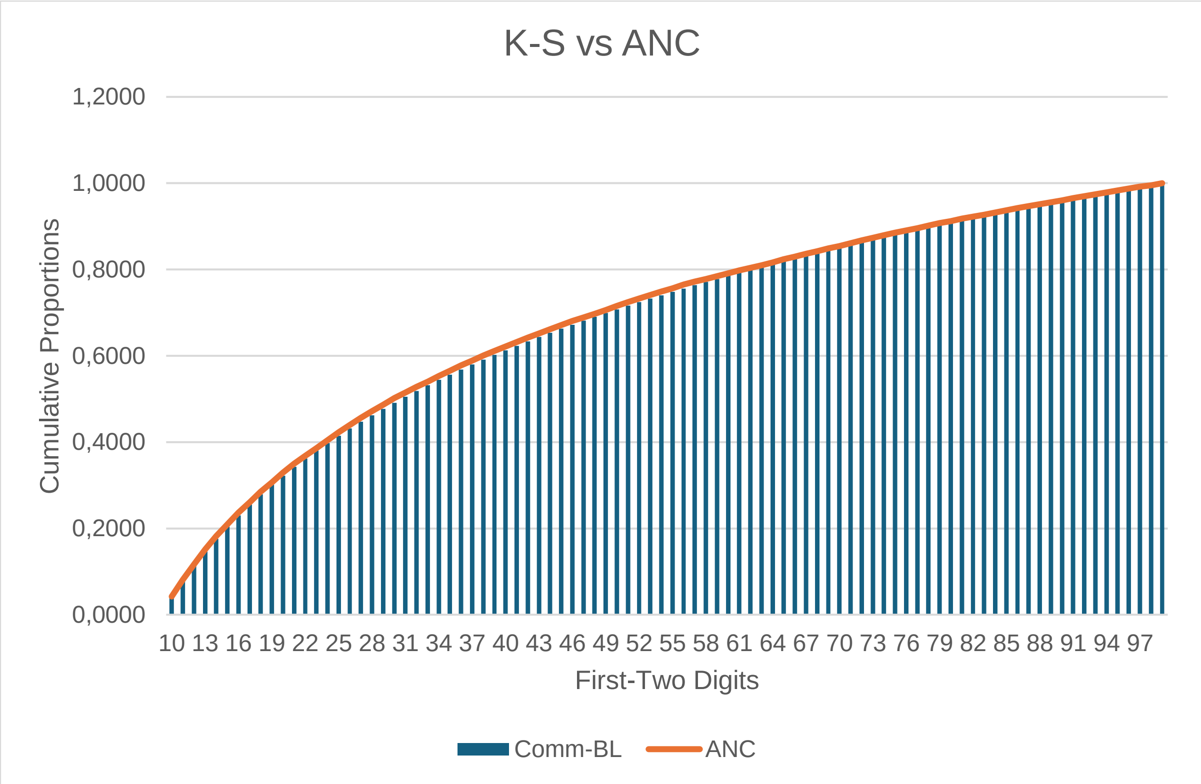
Appendix 62: 2019 South Africa Chi-square results

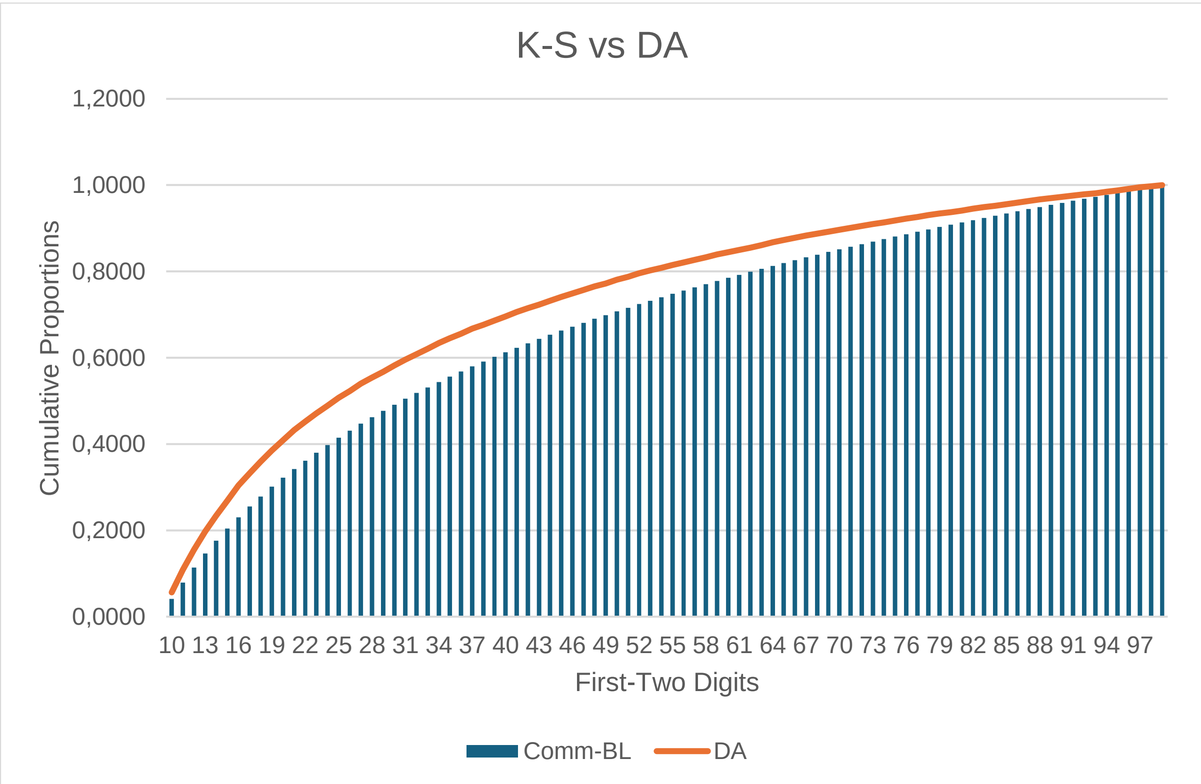
Chi-Square Tests			
	ANC	**DA**	**EFF**
1st Digit	6,025	26,848	58,104
Chi-Sq_CritVal	**15,507**		
Result	**Conforms**	**NonConformance**	**NonConformance**
2n Digit	6,466	57,779	42,400
Chi-Sq_CritVal	**16,919**		
Result	**Conforms**	**NonConformance**	**NonConformance**
FT Digits	90,116	794,504	714,876
Chi-Sq_CritVal	**112,022**		
Result	**Conforms**	**NonConformance**	**NonConformance**

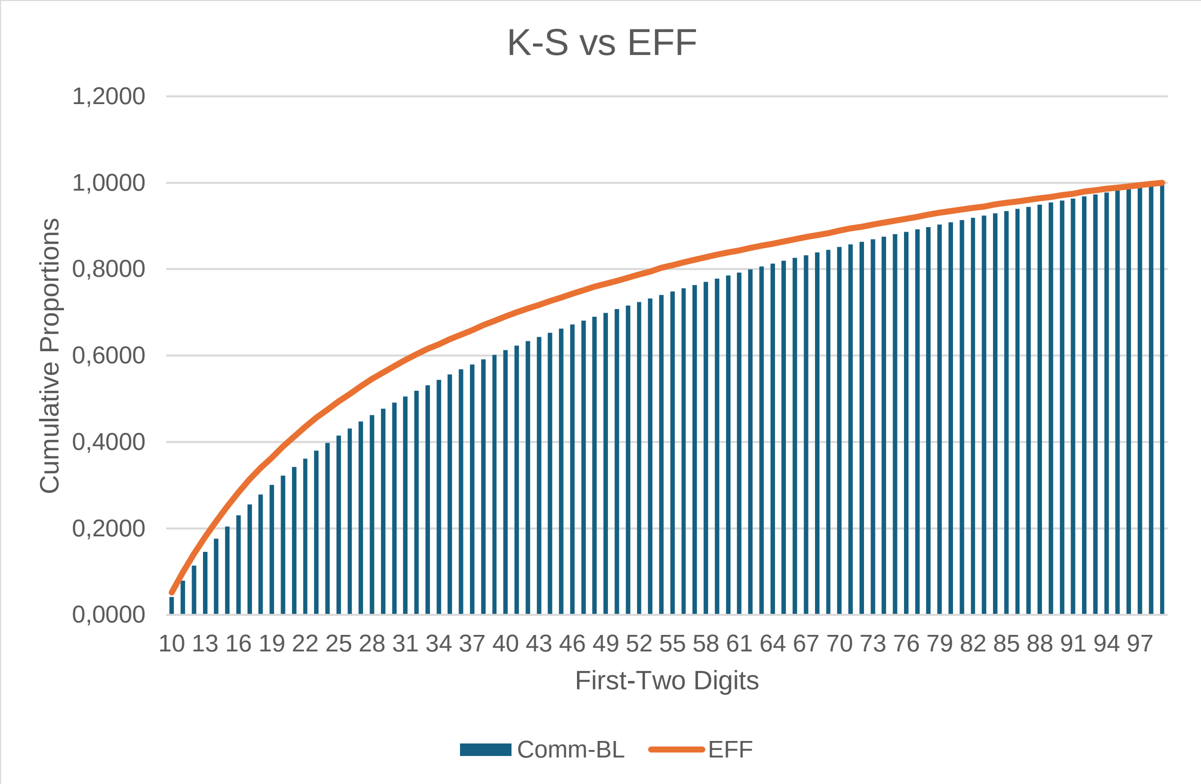
Appendix 63: 2019 South Africa P-value results

P-Value Calculations			
	ANC	**DA**	**EFF**
1st Digit	0,644485441	0,00075091	1,09606E-09
2n Digit	0,69256806	3,5829E-09	2,77655E-06
FT Digits	0,447010105	9,6302E-114	1,92948E-98

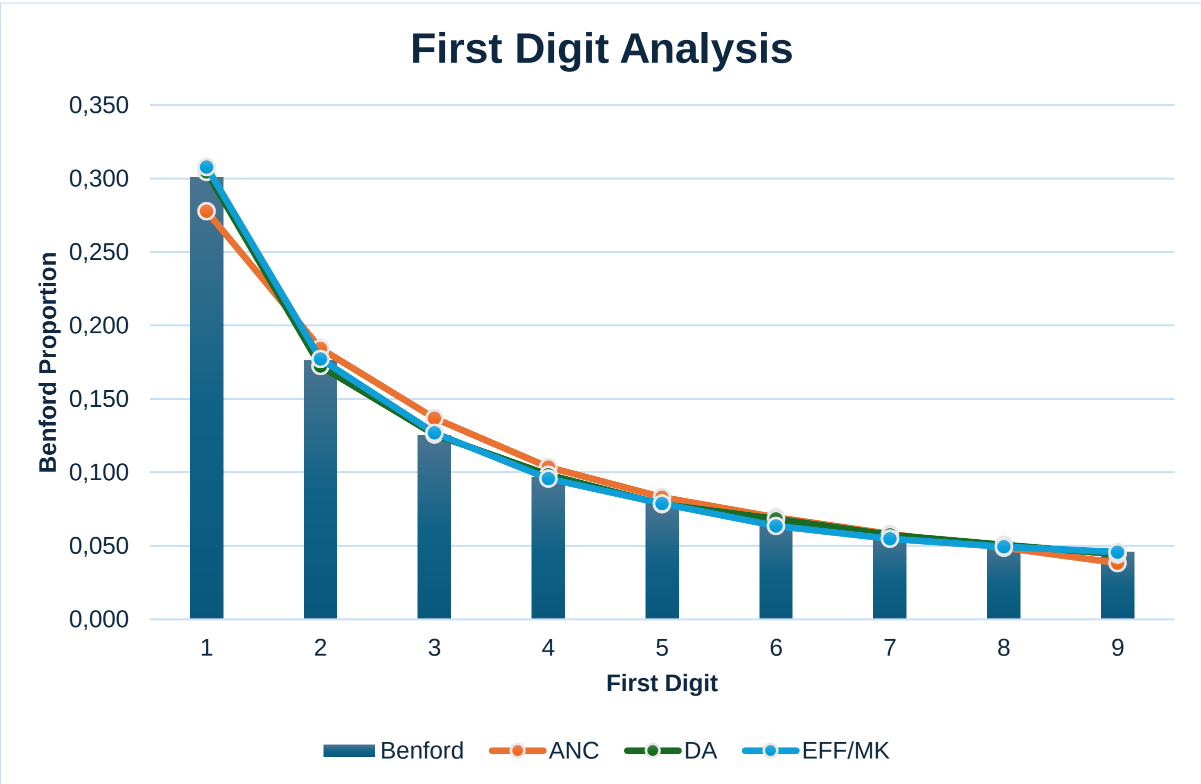
Appendix 64: 2019 South Africa K-S results

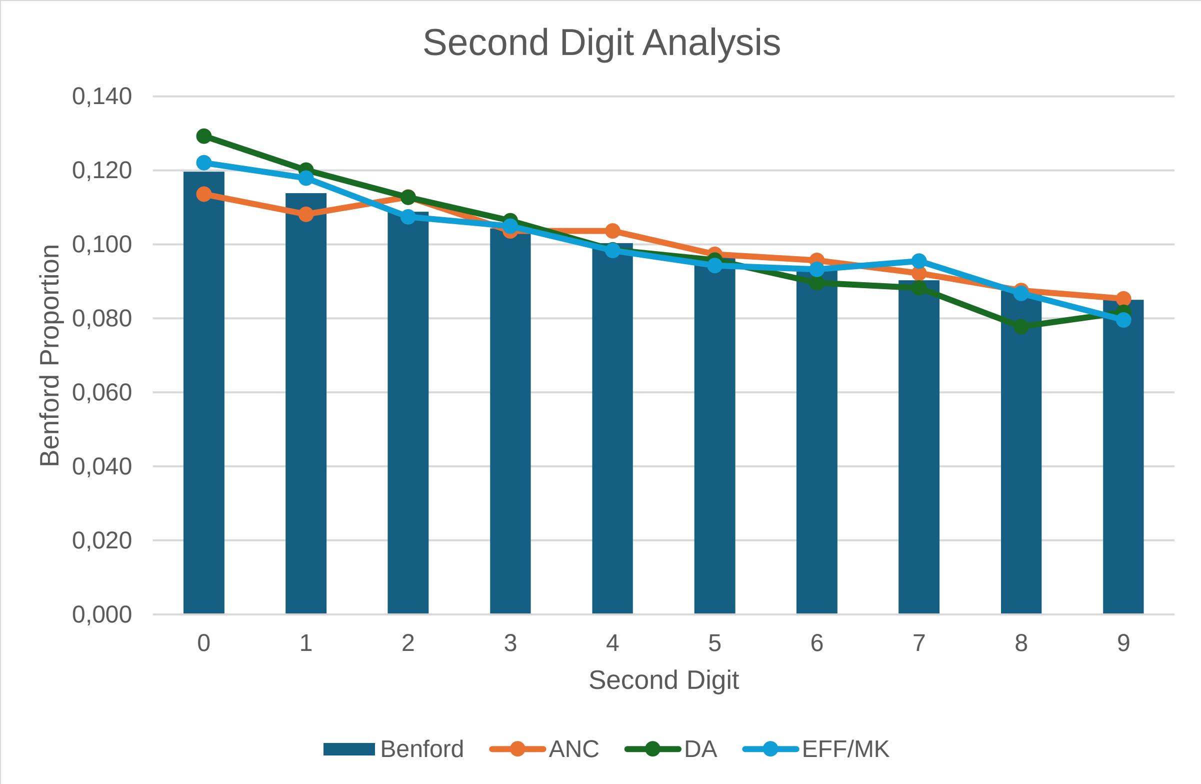


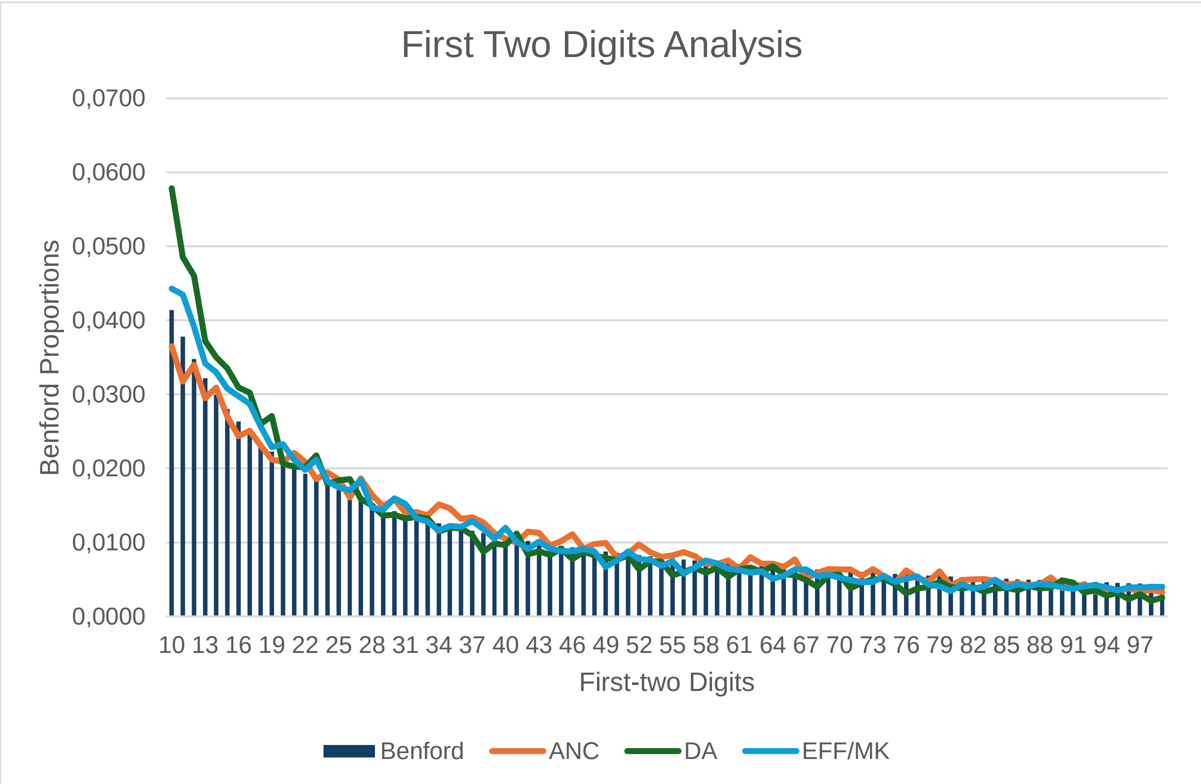




Appendix 65: 2024 South Africa Benford's Analysis




Second Digit Analysis
0,140
0,120
0,100
0,080
0,060
0,040
0,020
0,000
Benford Proportion
0 1 2 3 4 5 6 7 8 9
Second Digit
Benford ANC DA EFF/MK


First Two Digits Analysis
0,0700
0,0600
0,0500
0,0400
0,0300
0,0200
0,0100
0,0000
Benford Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-two Digits
Benford ANC DA EFF/MK

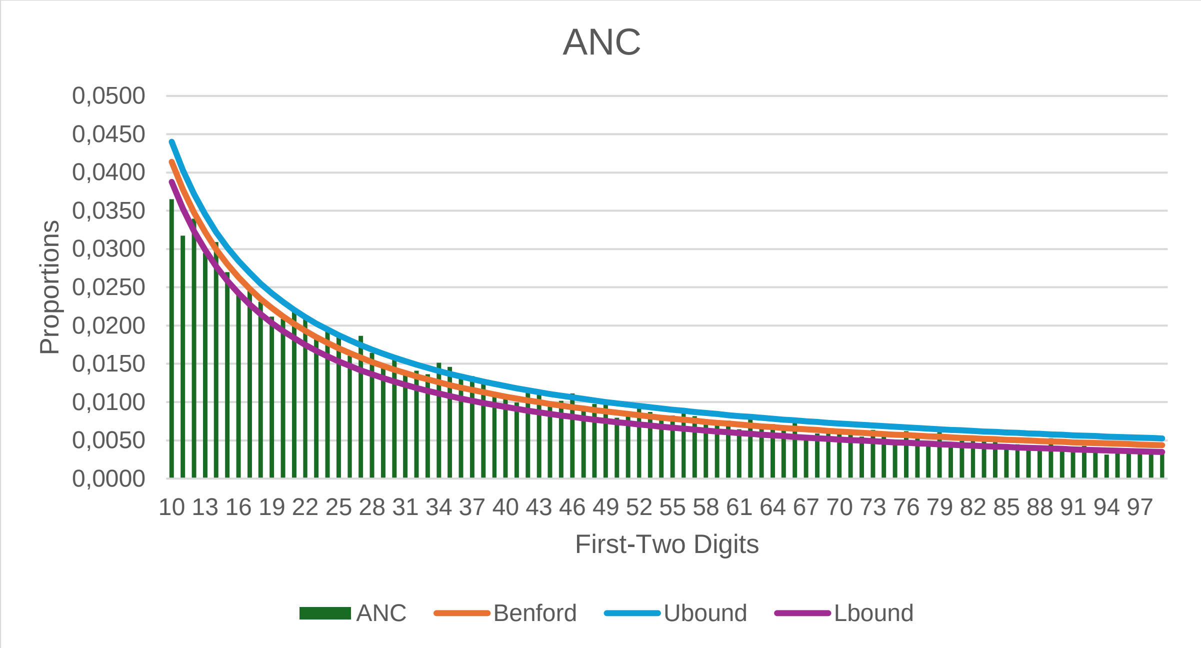
Appendix 66: 2024 South Africa MAD results

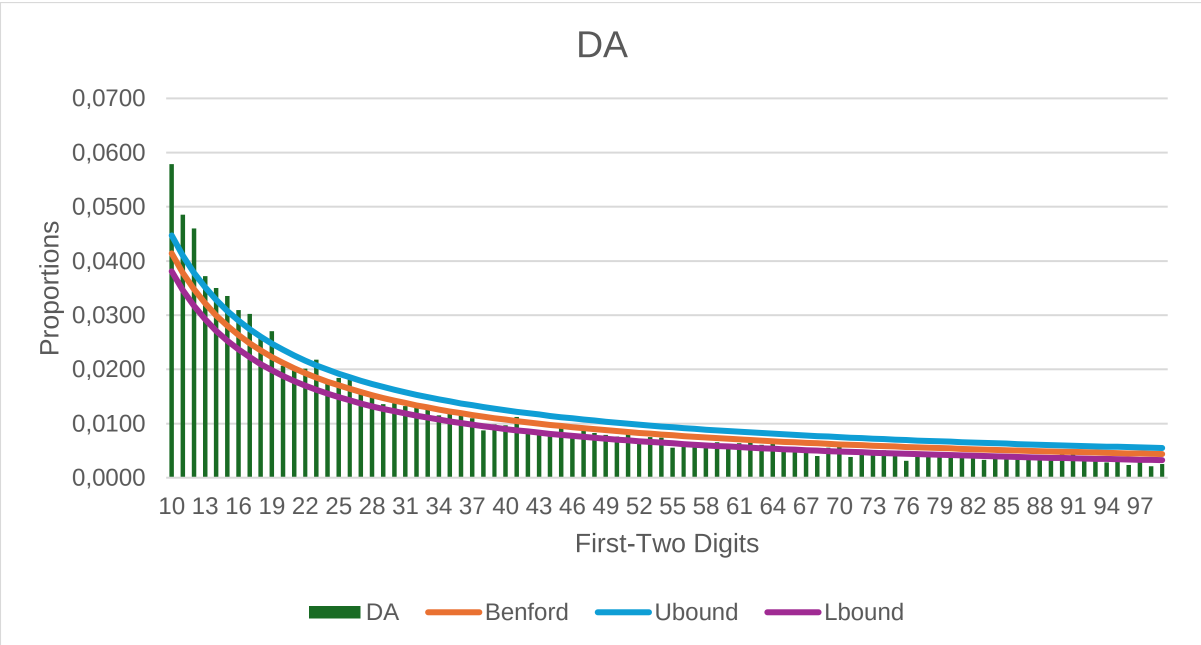
MAD Outcomes			
	ANC	DA	EFF/MK
1st Digit	0,00736	0,00157	0,00221
Outcome	Acceptable conformity	Close conformity	Close conformity
2n Digit	0,00251	0,00435	0,00242
Outcome	Close conformity	Close conformity	Close conformity
FT Digits	0,00094	0,00179	0,00108
Outcome	Close conformity	Acceptable conformity	Close conformity

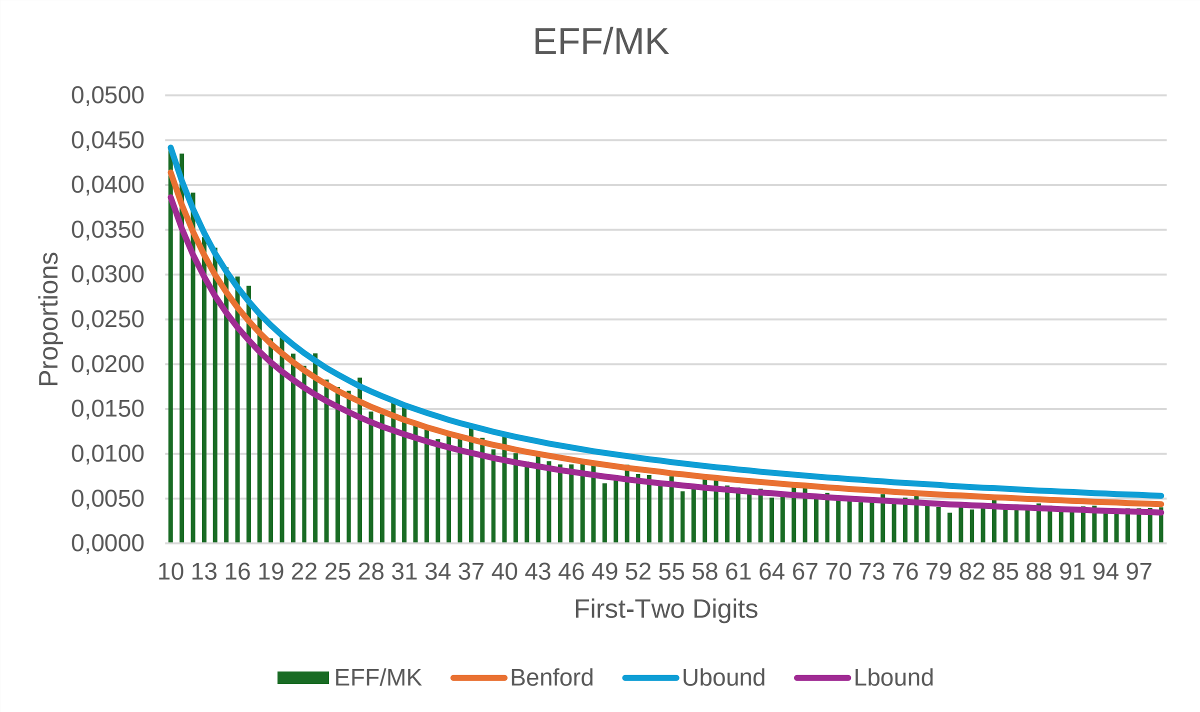
Appendix 67: 2024 South Africa First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	ANC	DA	EFF/MK
1	7,7270	1,1028	2,2342
2	3,2758	1,4744	0,4334
3	5,4866	0,3313	0,9117
4	3,3729	0,6825	0,5655
5	2,1774	0,4083	0,2495
6	1,5089	0,8555	2,0371
7	0,0122	0,3736	2,0020
8	1,6623	0,2349	1,2639
9	5,3926	1,0376	0,0102
Z-Statistics for Second Digit			
Digit	ANC	DA	EFF/MK
0	2,8100	3,5139	1,0252
1	2,7099	2,3003	1,7762
2	1,9259	1,4830	0,6207
3	0,3384	0,7893	0,2574
4	1,6662	0,7041	0,9057
5	0,3185	0,3601	1,1282
6	1,1863	1,5204	0,0616
7	0,9758	0,8736	2,5390
8	0,0066	4,1092	0,4096
9	0,1622	1,4118	2,7398

Appendix 68: 2024 South Africa First-two Digit Z-statistics results







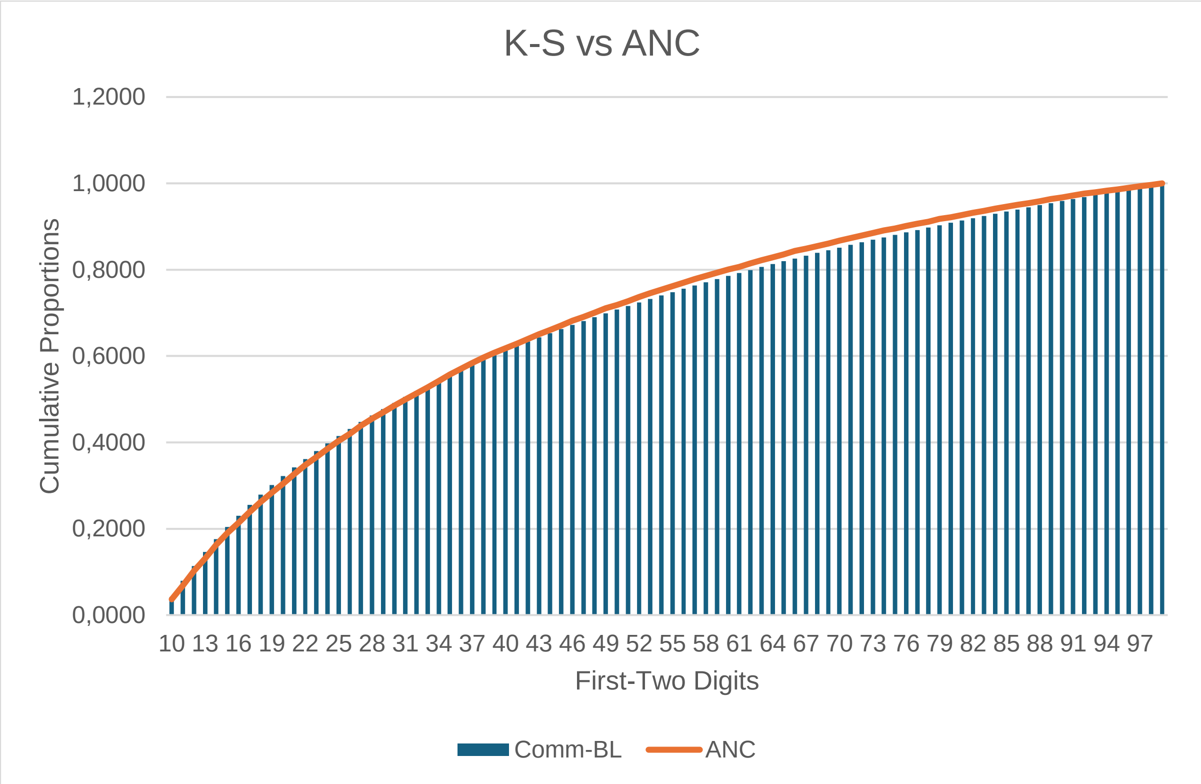
Appendix 69: 2024 South Africa Chi-square results

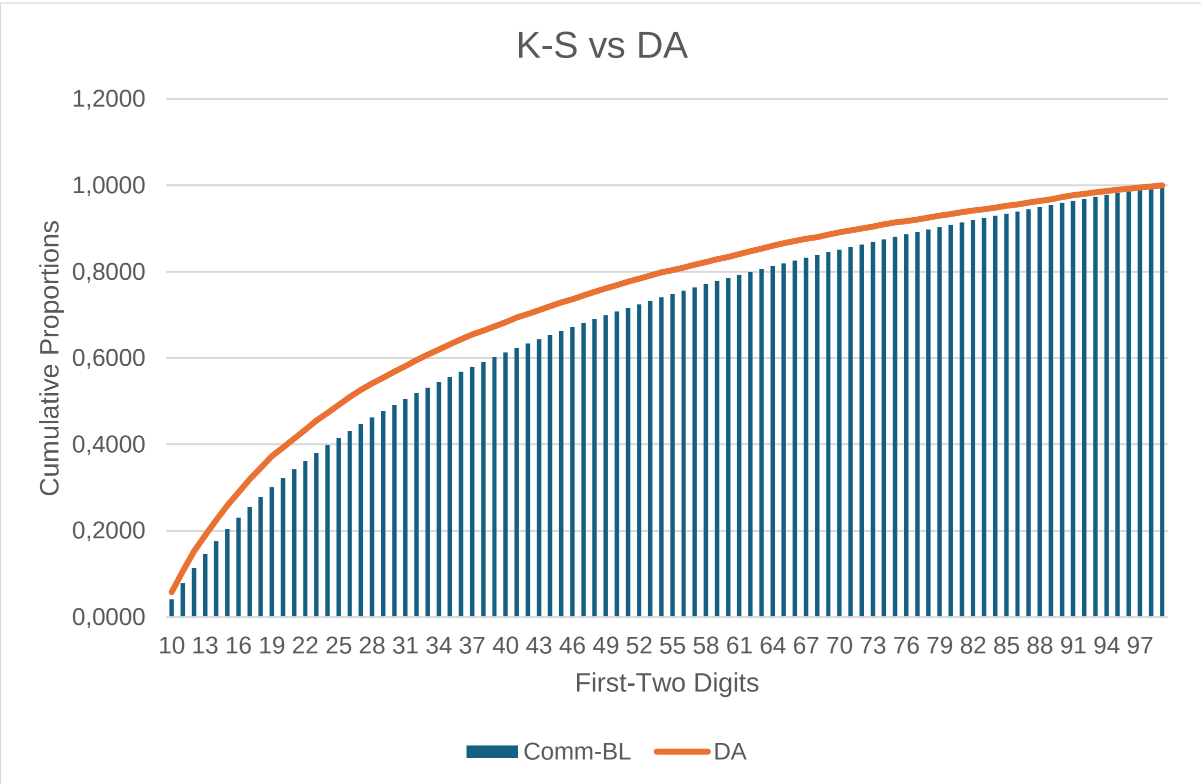
Chi-Square Tests			
	ANC	DA	EFF/MK
1st Digit	124,633	5,340	14,083
Chi-Sq_CritVal	**15,507**		
Result	**NonConformance**	**Conforms**	**Conforms**
2n Digit	21,855	39,092	19,148
Chi-Sq_CritVal	**16,919**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
FT Digits	251,323	565,654	276,200
Chi-Sq_CritVal	**112,022**		
Result	**NonConformance**	**NonConformance**	**NonConformance**

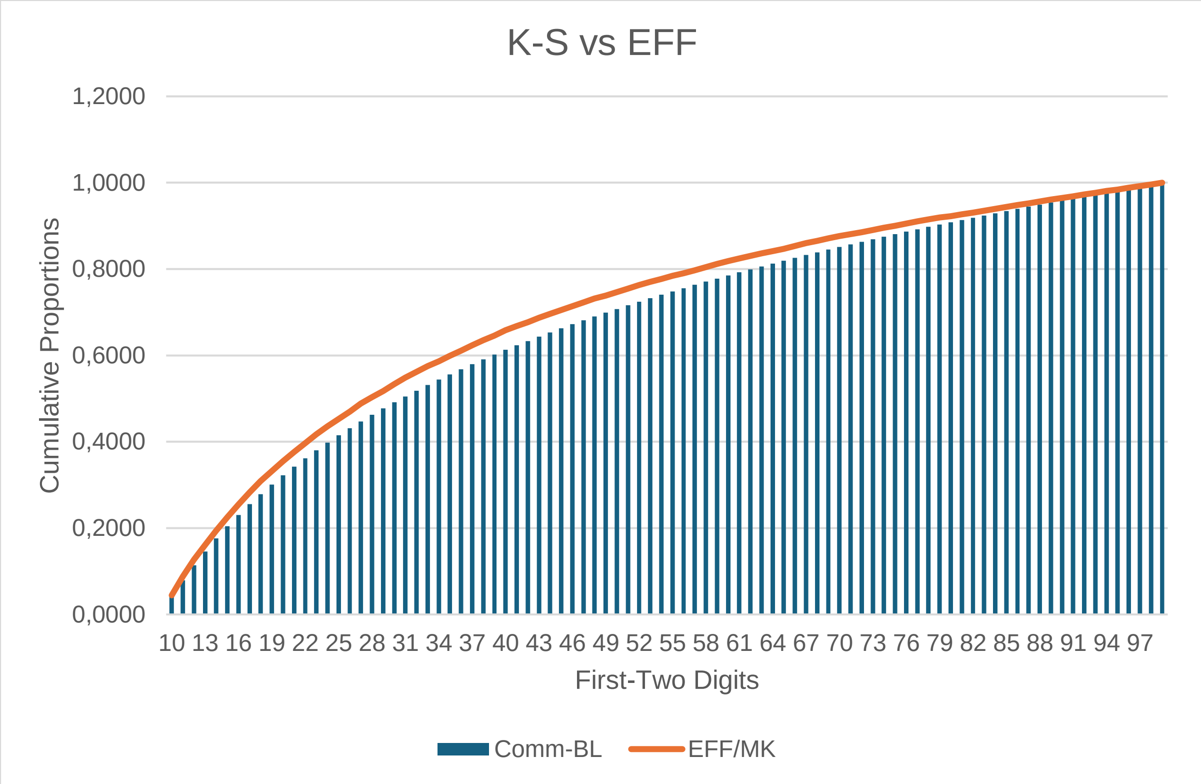
Appendix 70: 2024 South Africa P-value results

P-Value Calculations			
	ANC	DA	EFF/MK
1st Digit	3,65589E-23	0,72069563	0,079617017
2n Digit	0,009348591	1,10905E-05	0,023967144
FT Digits	2,09622E-17	1,91253E-70	4,81983E-21

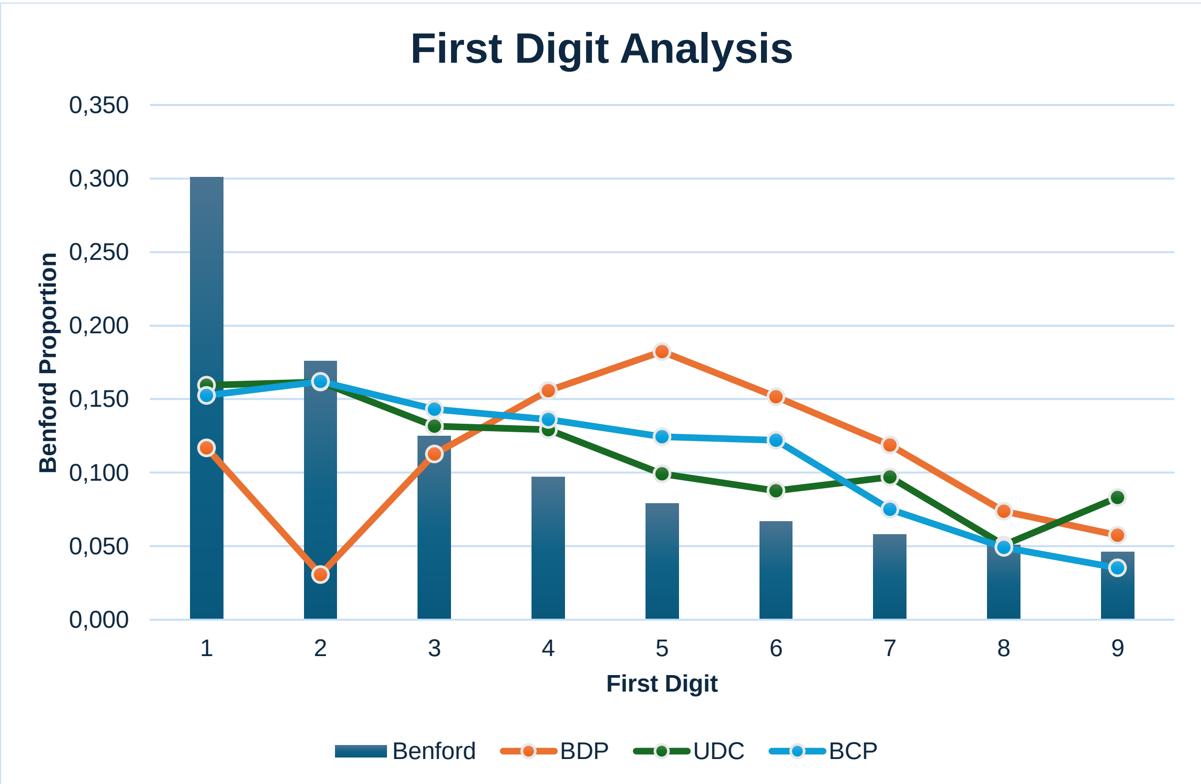
Appendix 71: 2024 South Africa K-S results

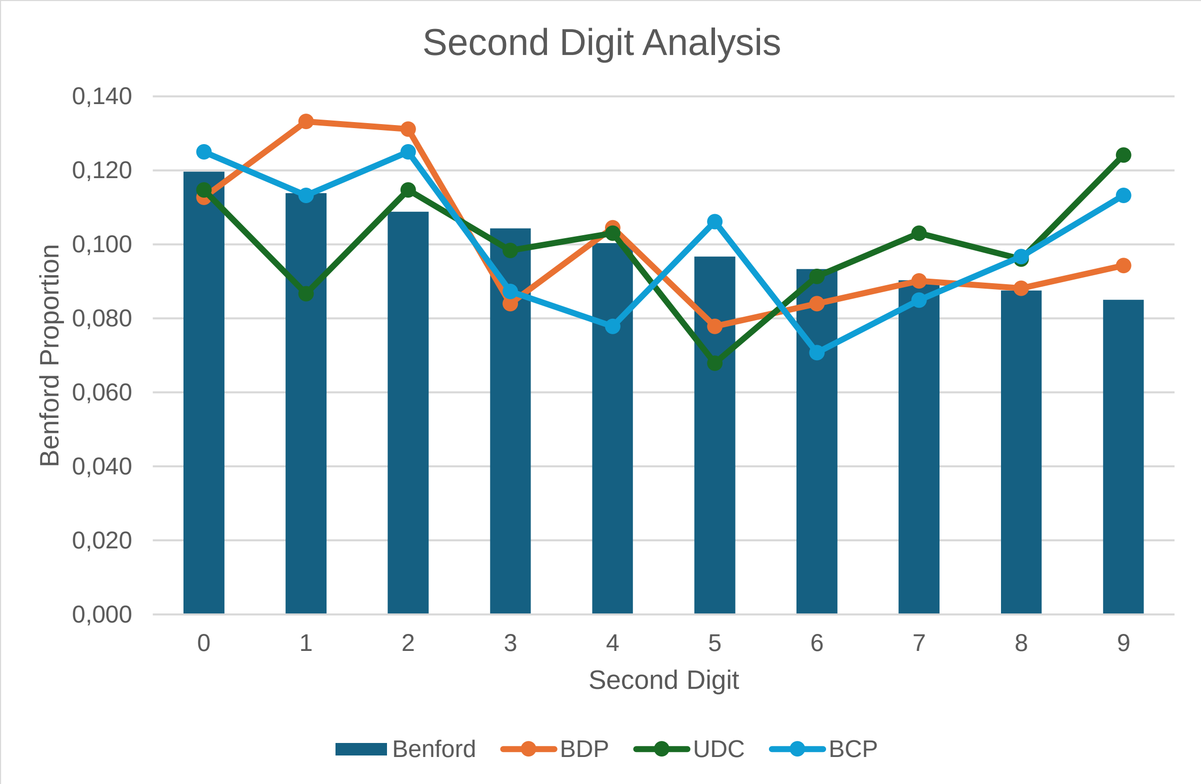


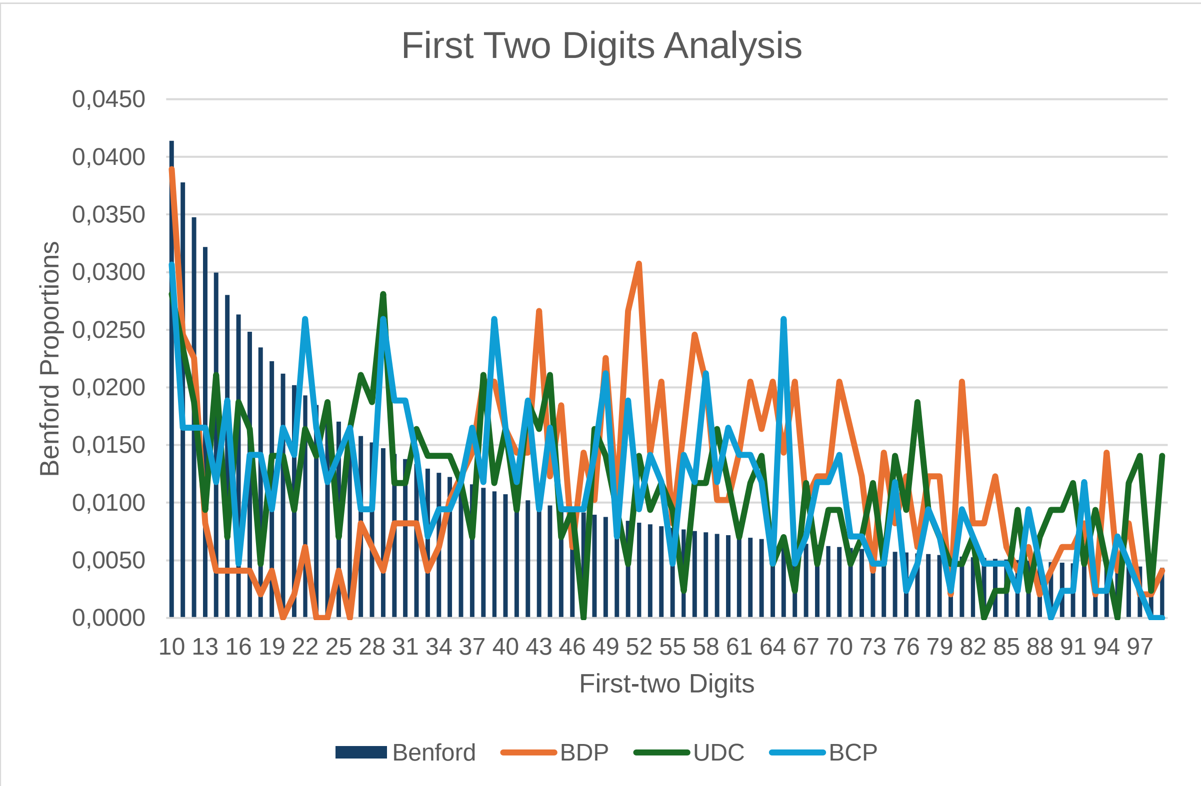




Appendix 72: 2014 Botswana Benford's Analysis




Second Digit Analysis
0,140
0,120
0,100
0,080
0,060
0,040
0,020
0,000
Benford Proportion
0 1 2 3 4 5 6 7 8 9
Second Digit
Benford BDP UDC BCP


First Two Digits Analysis
0,0450
0,0400
0,0350
0,0300
0,0250
0,0200
0,0150
0,0100
0,0050
0,0000
Benford Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-two Digits
Benford BDP UDC BCP

Appendix 73: 2014 Botswana MAD results

MAD Outcomes			
	BDP	UDC	BCP
1st Digit Outcome	0,07596 Nonconformity	0,03477 Nonconformity	0,03888 Nonconformity
2n Digit Outcome	0,01113 Marginally acceptable conformity	0,01379 Nonconformity	0,01366 Nonconformity
FT Digits Outcome	0,00855 Nonconformity	0,00528 Nonconformity	0,00536 Nonconformity

Appendix 74: 2014 Botswana First and Second Digit Z-statistics results

Z-Statistics for First Digit			
Digit	BDP	UDC	BCP
1	8,8228	6,3746	6,6267
2	8,3705	0,7251	0,7014
3	0,7490	0,3490	1,0661
4	4,3163	2,1992	2,6558
5	8,3589	1,4621	3,3678
6	7,3948	1,6364	4,4546
7	5,6556	3,3701	1,4088
8	2,1655	0,0323	0,0638
9	1,1198	3,6074	0,9260
Z-Statistics for Second Digit			
Digit	BDP	UDC	BCP
0	0,4050	0,2390	0,2627
1	1,2713	1,6956	0,0442
2	1,5112	0,3161	0,9919
3	1,3939	0,3244	1,0701
4	0,2334	0,1075	1,4600
5	1,3296	1,9294	0,5764
6	0,6324	0,0614	1,5171
7	0,0143	0,8306	0,3063
8	0,0426	0,5320	0,5790
9	0,6525	2,8120	1,9956

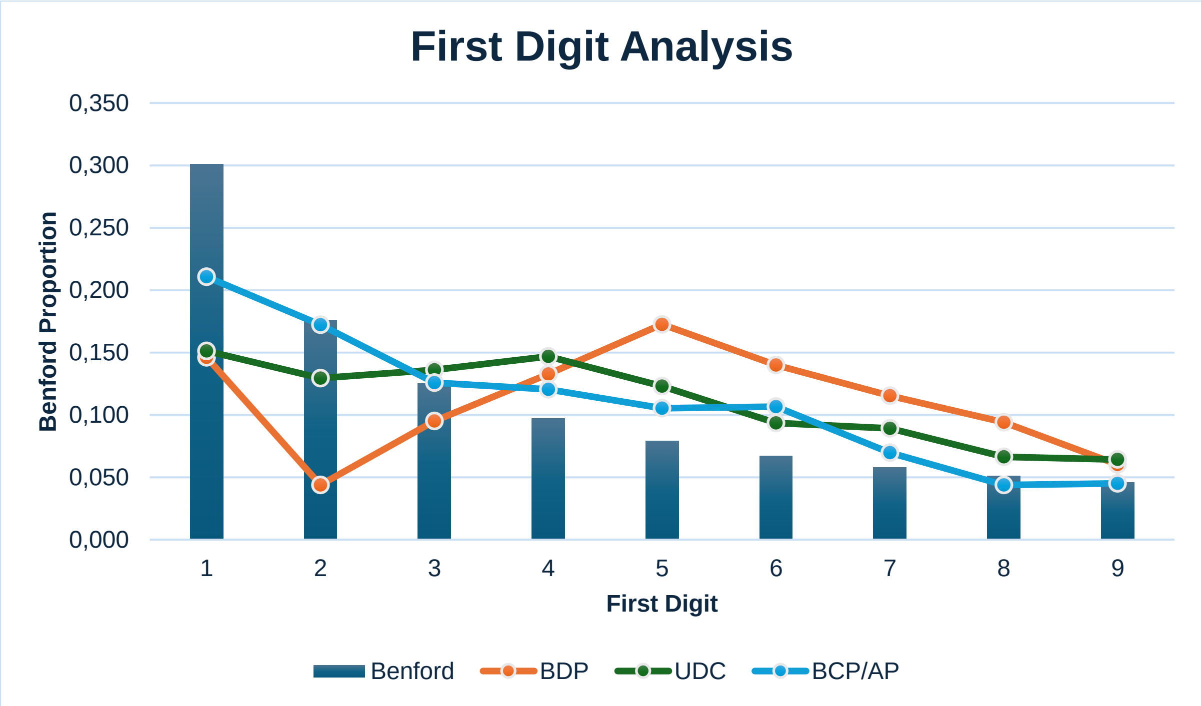
Appendix 75: 2014 Botswana Chi-square results

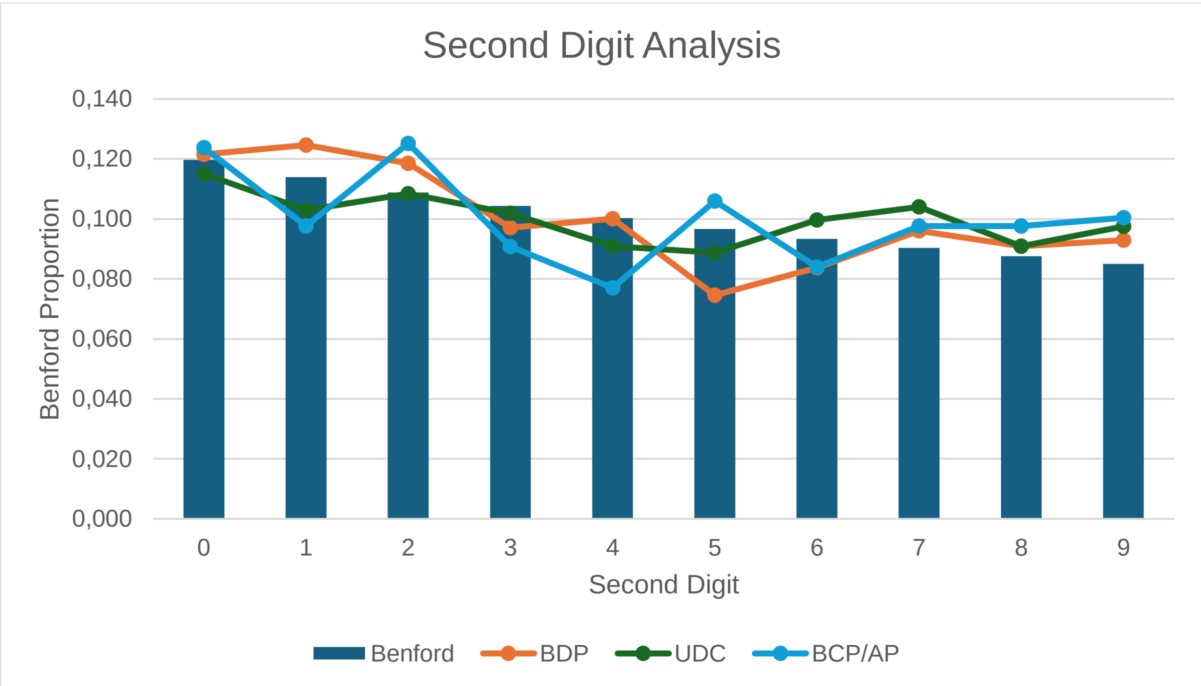
Chi-Square Tests			
	BDP	**UDC**	**BCP**
1st Digit	286,989	63,836	73,128
Chi-Sq_CritVal	**15,507**		
Result	**NonConformance**	**NonConformance**	**NonConformance**
2n Digit	8,785	15,656	11,668
Chi-Sq_CritVal	**16,919**		
Result	**Conforms**	**Conforms**	**Conforms**
FT Digits	430,642	157,779	163,796
Chi-Sq_CritVal	**112,022**		
Result	**NonConformance**	**NonConformance**	**NonConformance**

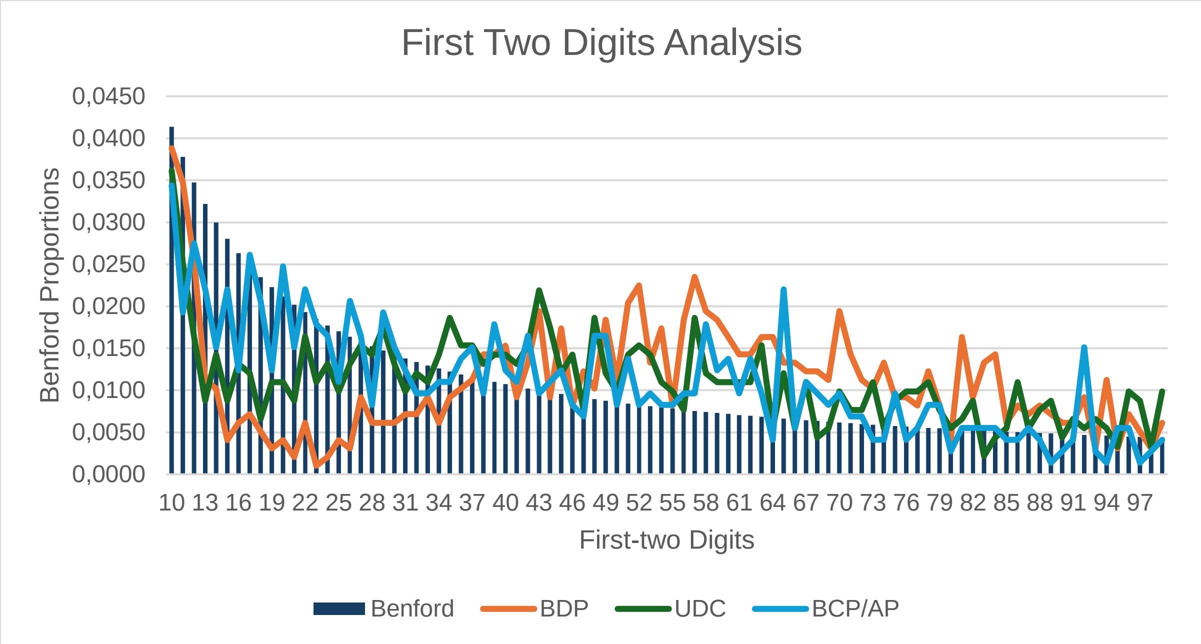
Appendix 76: 2014 Botswana P-value results

P-Value Calculations			
	BDP	**UDC**	**BCP**
1st Digit	2,41378E-57	8,19468E-11	1,16806E-12
2n Digit	0,457372944	0,074430313	0,232707149
FT Digits	2,96694E-46	9,82213E-06	2,56996E-31

Appendix 77: 2014-2019 Botswana Benford's Analysis







Appendix 78: 2014-2019 Botswana MAD results

MAD Outcomes			
	BDP	UDC	BCP/AP
1st Digit Outcome	0,07046 Nonconformity	0,04364 Nonconformity	0,02268 Nonconformity
2n Digit Outcome	0,00785 Close conformity	0,00716 Close conformity	0,01250 Nonconformity
FT Digits Outcome	0,00738 Nonconformity	0,00487 Nonconformity	0,00351 Nonconformity

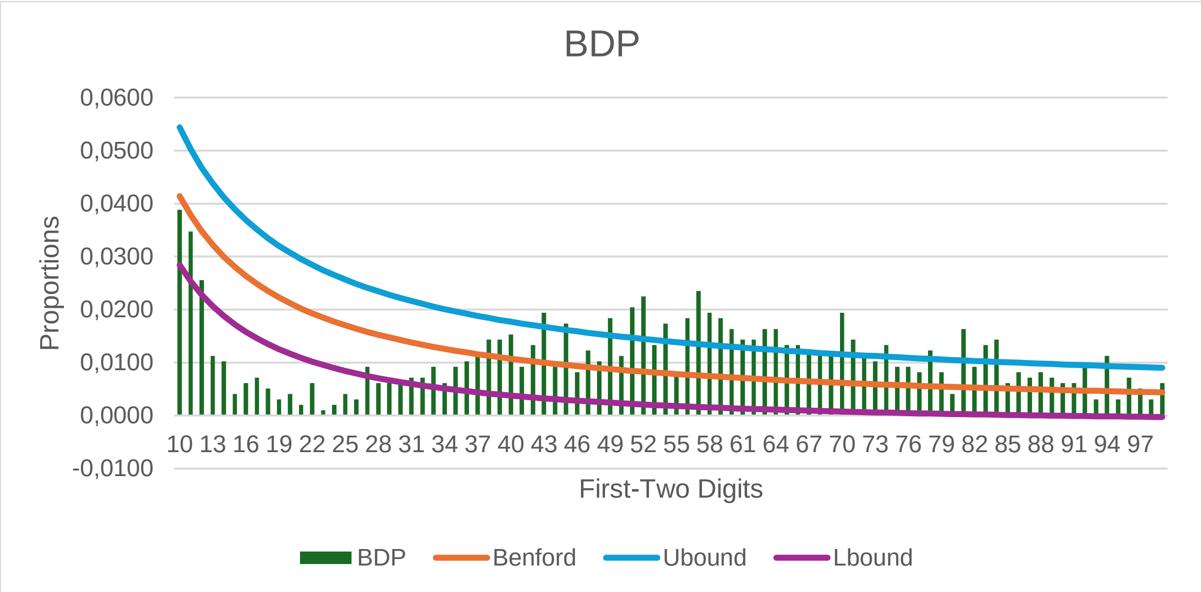
Appendix 79: 2014-2019 Botswana Chi-square results

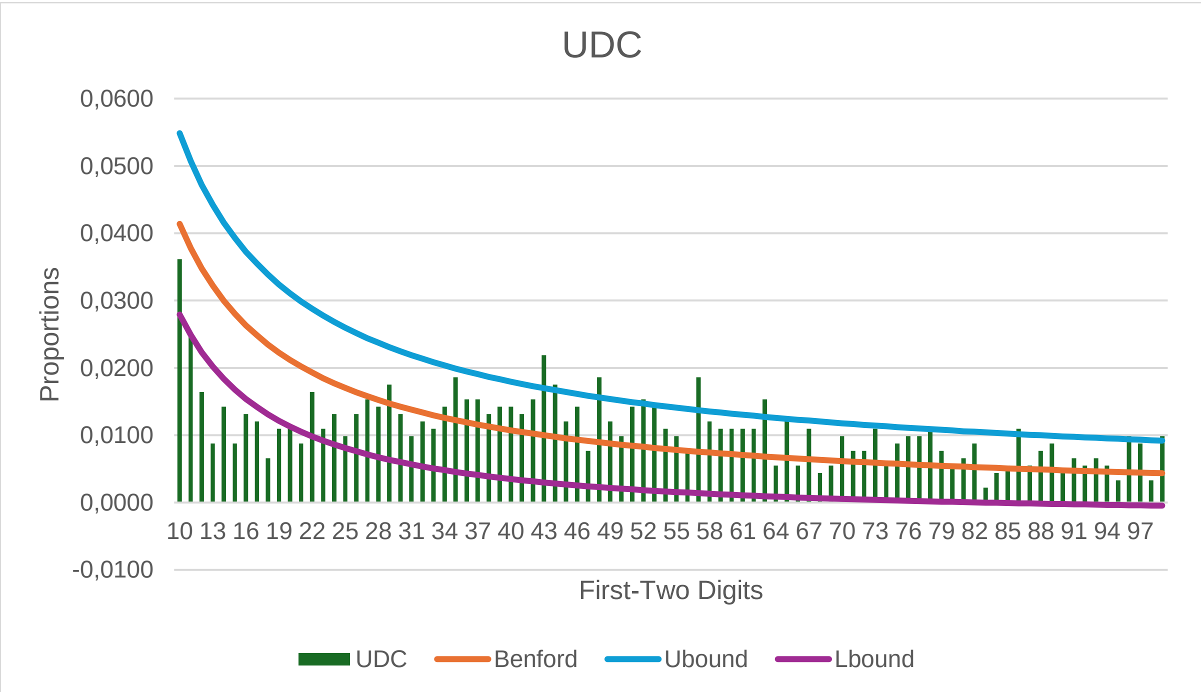
Chi-Square Tests			
	BDP	UDC	BCP/AP
1st Digit	476,399	162,858	50,148
Chi-Sq_CritVal	15,507		
Result	NonConformance	NonConformance	NonConformance
2n Digit	9,480	6,657	13,423
Chi-Sq_CritVal	16,919		
Result	Conforms	Conforms	Conforms
FT Digits	622,400	250,123	143,524
Chi-Sq_CritVal	112,022		
Result	NonConformance	NonConformance	NonConformance

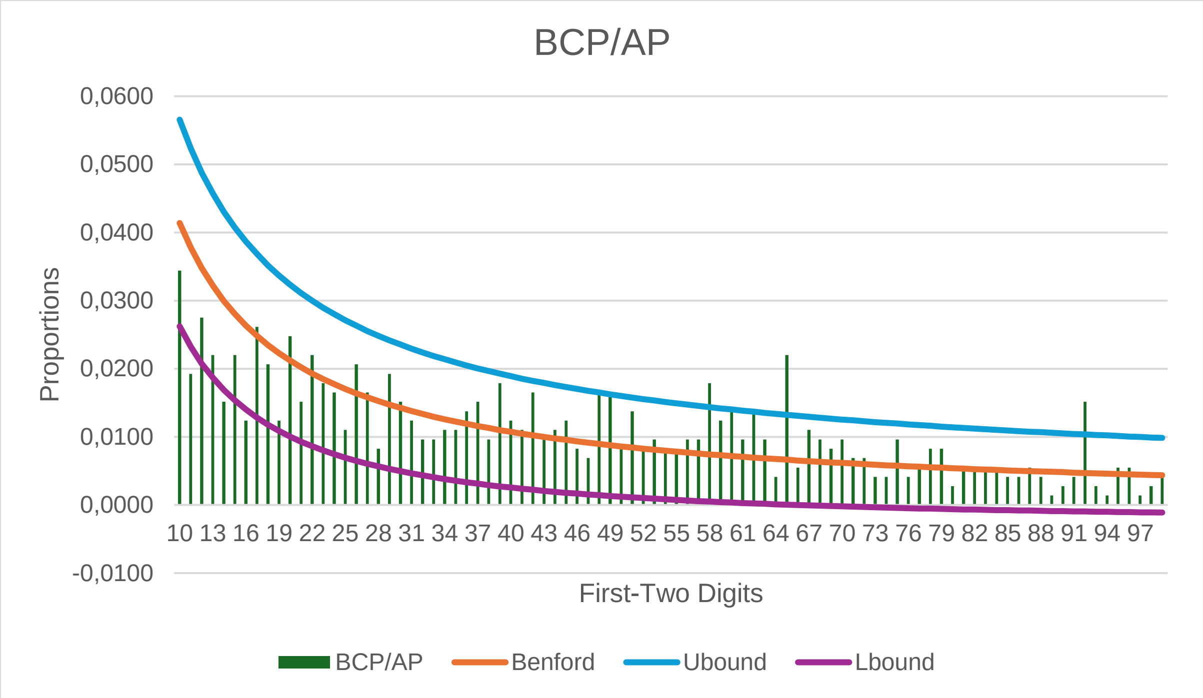
Appendix 80: 2014-2019 Botswana P-value results

P-Value Calculations			
	BDP	UDC	BCP/AP
1st Digit	8,11759E-98	4,03687E-31	3,82811E-08
2n Digit	0,39422821	0,672737666	0,144371092
FT Digits	5,73126E-81	3,10861E-17	4,38471E-27

Appendix 81: 2014-2019 Botswana First-two digit results






BCP/AP
0,0600
0,0500
0,0400
0,0300
0,0200
0,0100
0,0000
-0,0100
Proportions
10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97
First-Two Digits
BCP/AP
Benford
Ubound
Lbound