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REVIEW

A Critical Review of the Modelling and Factors Considered for Permanent Deformation of the Unbound Granular Materials

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Abstract
The elastoplastic behavior of Unbound Granular Materials (UGMs) under repeated loading plays a critical role in the performance and longevity of flexible pavements. This paper presents a comprehensive review of the permanent deformation modelling and factors considered in UGMs. The factors influencing resistance to permanent deformation such as stress magnitude, moisture content, gradation, density, porosity, particle morphology, mineralogical composition, are discussed, emphasizing their complex interdependence and impact on material behaviour. The study also examines the development and evolution of mathematical and empirical models used to predict permanent deformation under cyclic loading, highlighting that most existing models, primarily derived from repeated load triaxial (RLT) tests, remain empirical in nature, lack physical interpretation, and do not capture the true three-dimensional stress state experienced in the field. Rutting, the predominant distress mode in flexible pavements, arises from the accumulation of both elastic and plastic deformations across all pavement layers. Although numerous predictive models express accumulated strain as a function of load repetitions and deviator stress, they often neglect critical influences such as environmental conditions and shear strength of the subgrade. This review identifies limitations in current design practices and modeling approaches, provides insights into modern concepts in rutting analysis, and outlines future research needs for developing more mechanistic and physically meaningful models to predict the permanent deformation behavior of UGMs in pavements.

Keywords Permanent deformation · Unbound granular materials · Modelling · Mechanistic empirical · Cyclic loading

1 Introduction

It is well established that in the Mechanistic-Empirical (M-E) pavement design approach, the mechanical behaviour of unbound granular materials (UGMs) commonly used in base, subbase, and subgrade layers is characterised primarily through permanent deformation (PD) testing [1–3]. These tests are typically performed using a repeated load triaxial (RLT) apparatus [4–7], which applies a large number of load cycles under a constant stress level to simulate the cyclic loading conditions experienced by pavement materials under vehicular traffic [6, 8, 9]

UGMs are continuously graded granular materials, generally composed of crushed rock particles [10, 11]. They often include a small percentage of fines (typically 4% to 10%) and are usually partially saturated due to the presence of moisture [10, 12]. These materials are primarily used in the base and subbase layers of roads, as well as in capping layers [10]. In pavement systems, the long-term performance of UGMs is largely governed by the accumulation

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of permanent deformations, which manifests as surface rutting a major distress mode affecting serviceability and structural integrity [10].

Despite the critical importance of accurately predicting rutting behavior in UGMs, considerable gaps persist in the scientific modeling of their permanent deformation characteristics [13, 14]. These gaps are particularly pronounced given the recent advancements in artificial intelligence (AI) technologies, which offer promising avenues for enhancing modeling accuracy. This study aims to provide a comprehensive review of existing literature on the characterisation and modeling of permanent deformation in UGMs. Furthermore, it seeks to identify current limitations in conventional modeling approaches and propose future research directions, with a focus on integrating AI-based techniques to improve the predictive capabilities of deformation models.

2 Cyclic Behaviour of Unbound Granular Materials

The cyclic triaxial test is among the most widely adopted laboratory methods for investigating the mechanical behaviour of unbound granular materials (UGMs) [15–17]. Its primary advantage lies in its ability to simulate cyclic loading conditions representative of those encountered in the field, thereby enabling a more realistic assessment of material performance under repeated traffic loads [18–20]. Cyclic triaxial testing systems are now extensively used across the globe [21, 22]; however, they vary considerably in their design specifications, particularly in terms of specimen dimensions, loading capabilities, and control systems [10, 23]. The fundamental operating principle of the cyclic triaxial test is illustrated in Fig. 1[10]. During testing with a repeated load triaxial (RLT) apparatus, the specimen is subjected to a cyclic axial stress (σ_1) and a cyclic confining stress (σ_3), both of which vary simultaneously and in phase [10, 24]. This loading configuration, referred to as variable confining pressure (VCP) loading, more accurately replicates the in-situ stress conditions experienced by pavement materials under moving vehicle loads. Alternatively, some triaxial systems are designed to apply only a cyclic axial stress while maintaining a constant confining pressure an approach known as constant confining pressure (CCP) loading. While simpler in implementation, CCP loading does not capture the full complexity of stress variations experienced in the field as effectively as VCP loading. Most UGM-related studies in pavement engineering rely on CCP-based RLT tests due to their standardised nature and ease of implementation. However, CCP testing presents certain limitations. It does not simulate horizontal cyclic stresses and often overemphasizes the influence of confining pressure. In contrast, VCP testing incorporates both vertical and horizontal cyclic stresses, offering a more realistic representation of traffic-induced loading, although it still excludes shear stresses [25]. Despite these advantages, VCP testing is less commonly adopted due to several practical constraints [25, 26]:

a) Limitations in current technologies restrict the loading frequency, making VCP tests time-consuming.
b) Cyclic confining pressures are often limited to low magnitudes.

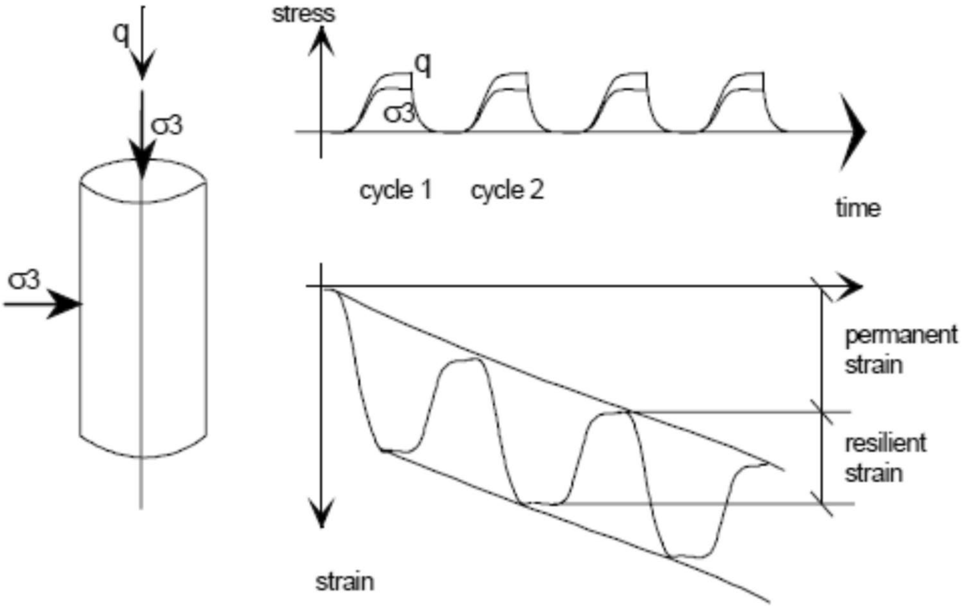

Fig. 1 Principle of cyclic loading [10]

c) Specialized equipment and instrumentation are required for both axial and radial strain measurements.
d) VCP equipment is not widely available in most laboratories.
e) Results from VCP tests may exhibit high variability.

Several comparative studies highlight the differences between CCP and VCP test results. Allen and Thompson [27] observed that values from CCP tests are typically similar to or slightly higher than those from VCP tests, with differences diminishing as mean normal stress (θ) increases. Brown and Hyde [28] also reported comparable values using both methods, though their study was limited to a single stress path. Figure 2 presents a representative example of stress–strain behavior observed during permanent deformation testing using RLT equipment. Despite variations in specimen size and loading configurations among different apparatuses, repeated load triaxial testing has consistently proven to be a reliable and effective method for characterizing the permanent deformation behavior of UGMs [29]. Numerous studies have confirmed its suitability for evaluating permanent deformation parameters in the laboratory, making it a preferred tool in mechanistic-empirical pavement design and material characterisation research [13, 24].

On other hand, determination of the stress levels to be used in permanent deformation has been raised questions by researchers. Predetermined stress level/ deviator stress which are presented in BS EN 13286–7 [30], are not generalised to be applicable on all type of materials, because these stress level were developed based on specific tested materials. The suggested appropriated way to determine deviator stress by considering a percentage of the maximum deviatoric stress of each soil sample obtained in the unconsolidated undrained triaxial tests [5, 6, 31]. This approach is to determine the shear strength (Mohr circles) of the material using static tests at a range of confining pressures prior to repeated load testing and use this as a basis for defining the stress regimes to be used for dynamic testing [6, 31, 32]. The primary objective of conducting static shear strength tests is to establish appropriate stress levels, expressed in terms of the shear stress ratio (SSR), for use in PD testing [31]. SSR is defined as the ratio of the induced shear stress to the shear strength of a given UGMs [33]. To determine shear strength parameters, monotonic triaxial tests can be performed using triaxial equipment, with confining stresses matching those anticipated in the subsequent resilient modulus testing. The analysis follows the fundamental state of stress based on the Mohr–Coulomb failure criterion. Accordingly, the representation of $\frac{\tau_f}{\tau_{max}}$ originates from the Mohr–Coulomb failure envelope as illustrated in Fig. 3.

For a certain combination of confining pressure (σ_3) and deviator stress (σ_d) applied during triaxial testing, the mobilized normal pressure and shearing resistance (represented by σ_f and τ_f, respectively) on the potential failure surface [32] (oriented at an angle of $45^\circ + \frac{\varnothing}{2}$ with the horizontal) can be computed. The applied stress states on the failure plane to compute shear stress ratios in Fig. 4 can be derived from the following Eqs. (1–3):

$$\text{Shear Stress Ratio (SSR)} = \frac{\text{Mobilised Shearing Resistance}}{\text{Shear Strength}} = \frac{\tau_f}{\tau_{max}} \tag{1}$$

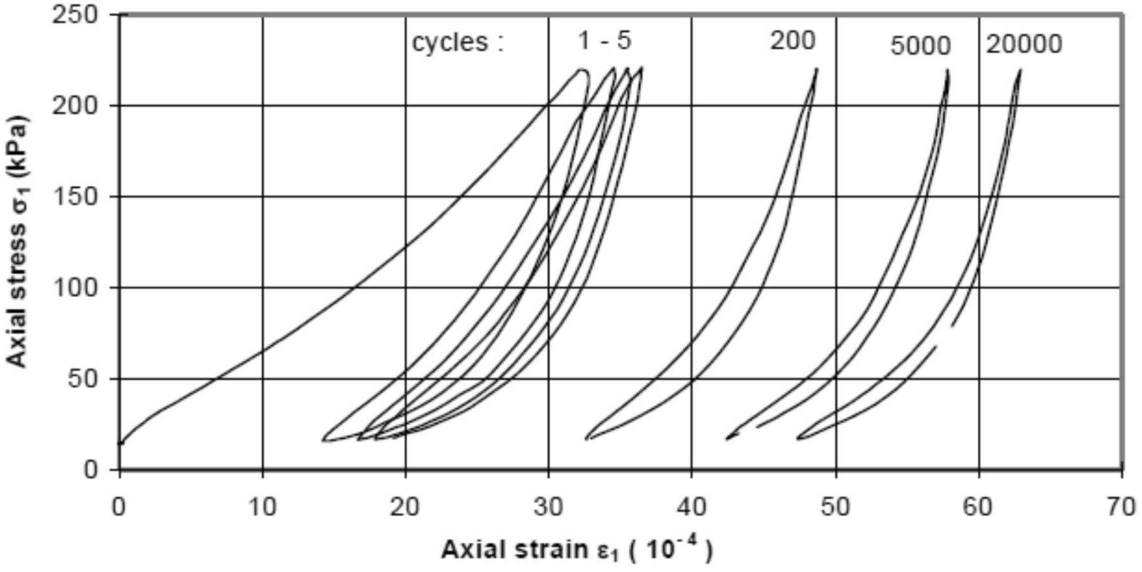

Fig. 2 Example of the stress–strain cycles obtained in repeated load triaxial test on UGMs [10]

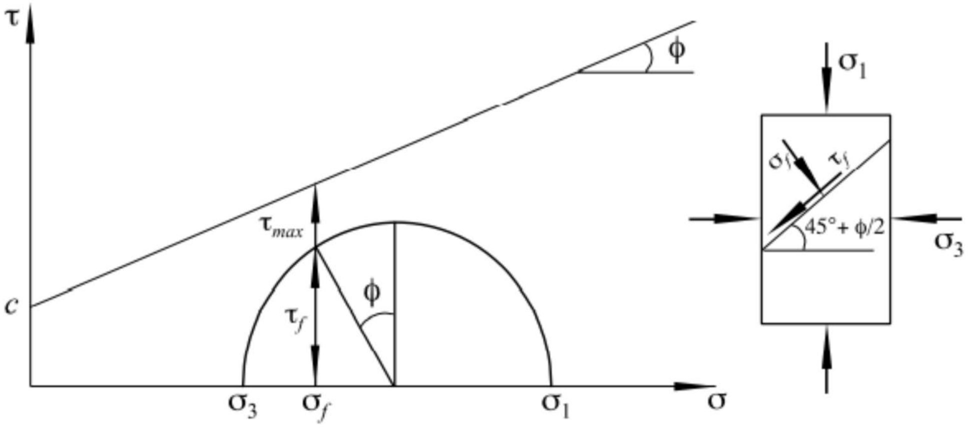

Fig. 3 Mohr–Coulomb representation of shear strength and applied stress [33]

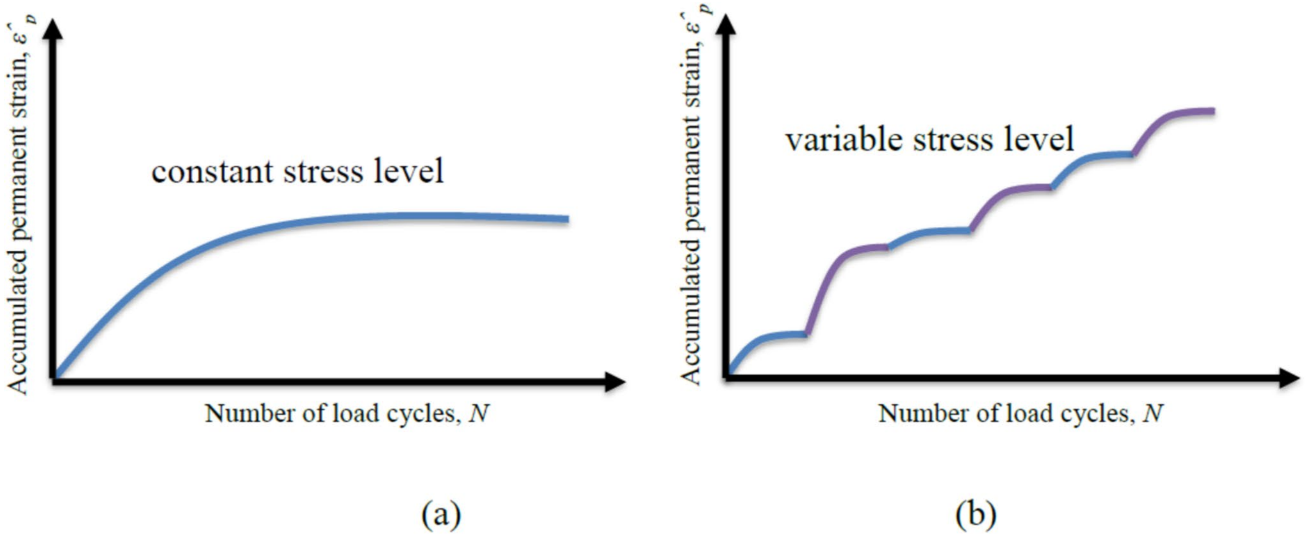

Fig. 4 Development of PD at constant versus multiple stress levels [2]

$$\sigma_f = \frac{2\sigma_3\left(1+\tan\varnothing^2\right)+\sigma_d\left(1+\tan\varnothing^2\right)-\sqrt{{\sigma_d}^2\tan\varnothing^2\left(1+\tan\varnothing^2\right)}}{2\left(1+\tan\varnothing^2\right)} \tag{2}$$

$$\tau_f = \sqrt{\left(\frac{\sigma_d}{2}\right)^2-\left[\sigma_f-\left(\sigma_3+\frac{\sigma_d}{2}\right)\right]^2} \tag{3}$$

where τ_f is the mobilised shearing resistance acting on failure plane; τ_{max} is shear strength obtained through Mohr–Coulomb failure criteria,$\tau_{max} = c + \sigma_f\tan\varnothing$; σ_f is normal stress acting on failure plane; σ_3 is minor principal stress or confining pressure in this case; σ_d is deviator stress,; and $\varnothing$ is internal friction angle determined from shear strength tests. The ratio between τ_f and shear strength of the material corresponding to that particular normal stress $(\tau_{max} = c + \sigma_f\tan\varnothing)$ is defined as the SSR. Lower SSR values essentially mean that the material is less likely to undergo bearing capacity type shear failure, whereas a unity value of SSR (SSR = 1.0) represents shear failure of the material.

Previous research has emphasised that obtaining accurate measurements of axial strain during laboratory testing requires a well-designed arrangement of Linear Variable Differential Transformers (LVDTs) [29]. In existing triaxial testing systems for cyclic loading, LVDTs are typically installed either externally or internally. However, this variation in placement often raises questions about the reliability and comparability of the test results. Some researchers have suggested equipping triaxial systems with locally mounted LVDTs to improve measurement accuracy. While this approach can enhance data quality, it also increases the complexity of the testing process and makes it more difficult to achieve consistent and repeatable results. Therefore, to establish a standardised and repeatable testing protocol, current studies should focus on developing an improved triaxial testing setup capable of accurately capturing the cyclic behavior of UGMs. This setup should be designed to maintain data reliability while remaining simple, cost-effective, and practical enough for routine use in the construction industry, rather than being limited to specialised research applications.

3 Factors Affecting Permanent Deformation Response

A substantial body of research has focused on identifying the factors that influence the PD of UGMs. According to existing studies, the PD of granular materials is affected by various parameters, including stress level, density, particle grading, fines content, maximum grain size, aggregate type, particle shape, moisture content, stress history, and the number of load applications [3, 16, 34–41]

3.1 Stress Levels

The literature available shows that stress level is one of the most important factors affecting the development of permanent deformation in granular materials [8, 34, 42]. Early repeated load triaxial tests, reported by Morgan [43], showed clearly that accumulation of axial permanent strain is directly related to deviator stress and inversely related to confining pressure. Since then, several researchers have reported that permanent deformation in granular materials is principally governed by some form of stress ratio consisting of both deviatoric and confining stresses [43–45]. Lashine et al. [46] conducted repeated load triaxial tests on a crushed stone in a partially saturated and drained condition, and found that the measured permanent axial strain settled down to a constant value directly related to the ratio of deviator stress to confining pressure. Similar results were reported by Brown and Hyde [28], who studied the response of crushed stone under repeated triaxial loading conditions with constant confinement. Brown and Hyde further stated that similar results are obtained in tests with variable confining pressure, if the mean value of the applied confining stress is used in the analysis. Other researchers [38, 47, 48] have attempted to explain permanent strain behavior under repeated loading using the ultimate shear strength of the material. In this approach, the static failure line is considered as a boundary for permanent strain under repeated loading. This has been questioned by Lekarp and Dawson [49] who argue that failure in granular materials under repeated loading is a gradual process and not a sudden collapse as in static failure tests. Therefore, ultimate shear strength and stress levels that cause sudden failure are of no great interest for analysis of material behavior when the increase in permanent strain is incremental. On other hand, Stress history also plays a critical role in PD behaviour [2, 6, 50]. Previous loading events tend to reduce further PD accumulation under similar subsequent load applications [50]. A typical PD progression under cyclic loading at a constant stress level is shown in Fig. 4 (Fig. 4a). However, in real pavement conditions, where stress levels vary during service, PD develops differently due to fluctuating loads and the influence of prior loading this behavior is shown in Fig. 4b. The constant stress case is typically simulated using a Single Stage (SS) Repeated Load Triaxial (RLT) test, while the varying stress scenario is captured by the Multi Stage (MS) RLT test [2]. The incorporation of static shear strength into permanent deformation (PD) modeling has long been a subject of debate in the literature [5, 49]. Recent developments, however, have seen the emergence of models that explicitly integrate static shear strength within PD frameworks [5, 33]. Despite these advancements, substantial gaps persist in understanding the extent to which static shear strength influences PD behavior. This issue warrants further rigorous and comprehensive investigation to establish a clearer mechanistic basis. Future research should therefore focus on examining the role of shear strength in governing the PD response of unbound granular materials (UGMs). Moreover, comparative analyses of long-term deformation behavior under multi-stage and single-stage loading conditions are essential to elucidate the influence of loading history on the cumulative deformation characteristics of UGMs.

3.2 Number Load Repetitions/Cycles and Loading Frequency

PD in UGMs accumulates progressively with the number of load applications [2, 3, 51]. The development of PD under repeated loading is a gradual process in which each load cycle contributes a small increment to the total accumulated strain [2]. Consequently, the number of load cycles is a critical factor in evaluating the long-term behaviour of these materials [2]. The importance of load repetitions has been emphasised by several researchers [5, 38, 39, 43, 52]. Although the PD induced during a single load cycle is small compared with the resilient response, its cumulative effect can become significant over time, potentially leading to rutting and eventual pavement failure as shown in Fig. 5 [2]. Early studies [5, 38, 39, 43, 52] reported a continuous increase in permanent strain under repeated loading. For example, Morgan [43] applied up to 2 million load cycles and observed that PD continued to increase throughout the testing period. Similarly, Barksdale [52] concluded that permanent axial strain in untreated granular materials accumulates linearly with the logarithm of the number of load cycles. His results further indicated that after a large number of load repetitions, the rate of plastic strain accumulation may suddenly increase. In contrast, Brown and Hyde [28], who studied a well-graded crushed granite, observed that an

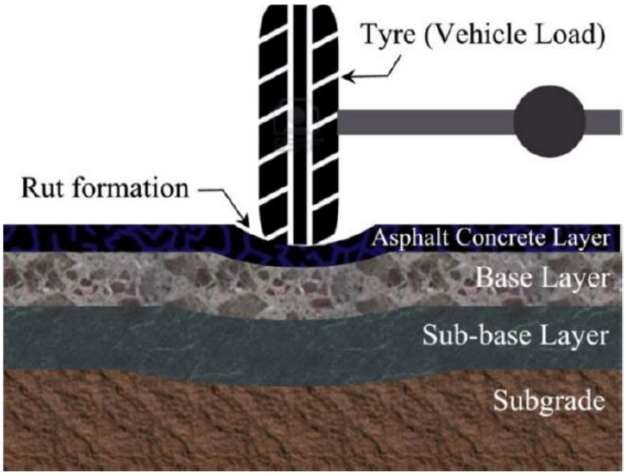

Fig. 5 Development of rutting due to PD of the UGMs [2]

equilibrium state was reached after approximately 1,000 load applications. Paute et al. [47] later suggested that the rate of increase in permanent strain decreases progressively, allowing the definition of a limiting value for PD accumulation. Lekarp and Dawson [49] also reported that stabilisation occurs only at low stress levels, while high stress conditions lead to continuous strain accumulation and gradual material deterioration. The appropriate number of load cycles to apply in PD testing remains a debated issue among researchers. There is no consensus on the number of cycles required to accurately determine the rate of PD or to rank material performance. Many studies have used a large number of cycles to simulate long-term strain accumulation; however, the use of such extended testing durations raises concerns regarding laboratory feasibility and time efficiency. Larew and Leonards [13], for instance, applied between 60,000 and 80,000 cycles in their repeated load tests on various UGMs, with some specimens subjected to over 400,000 cycles. Zhou et al.[53] noted that UGMs and subgrade soils may experience millions of load cycles under field conditions and, therefore, used 30,000 cycles per stage in multi-stage cyclic loading tests. Similarly, Chern Chow, Debakanta Mishra, and Tutumluer [54] applied 30,000 cycles per stage in their PD testing of different aggregates. Alnedawi et al.[55] conducted tests using three stress stages, each comprising 10,000 cycles, for a total of 30,000 cycles, following AG-PT/T053 specifications. Ma et al.[31] investigated open-graded UGMs and used a maximum of 9,000 cycles, assuming strain stabilization after a certain number of cycles. Cern et al. [56, 57] evaluated the influence of water content on PD up to 10,000 cycles, selecting this duration to balance laboratory time constraints with reliable material characterization; similar cycle ranges were adopted by Gu et al. [58]. Finally, Ma et al.[31] terminated cyclic loading tests either when the accumulated axial plastic strain reached 15% or when the strain stabilised, with a maximum limit of 50,000 load cycles. In summary, there remains inconsistency in the number of load cycles applied in PD testing across the literature, largely due to varying experimental objectives, material types, and practical laboratory constraints.

Youd (as cited in Rondón-Quintana et al. [59],) performed simple shear tests with cyclic loading on sand under drained conditions, using loading frequencies between 0.2 and 1.9 Hz, and concluded that the strain accumulation rate is independent of frequency [13]. Similar findings were reported by Shenton [60], who conducted RLT tests on ballast under drained conditions with loading frequencies ranging from 0.1 to 30 Hz. Thom and Brown [61] also noted that loading frequency has little influence on permanent deformation accumulation, though they observed a slight reduction in strain rate at higher frequencies. Wichtmann et al. [62] found that coarse sand (maximum particle size of 1 mm) continues to accumulate strain even after 2×10^6 loading cycles. In a subsequent study, Wichtmann [63] confirmed through RLT tests that loading frequency between 0.05 and 2.0 Hz does not affect permanent deformation. Similarly, Rondon-Quintana [25] and Rondon-Quintana et al. [26] conducted four RLT tests on a UGM two at 0.05 Hz and two at 1 Hz and found no significant frequency effect on permanent deformation. Li et al. [64] observed that increasing loading frequency (1, 5, 8, and 10 Hz) reduced permanent deformation in smaller gravels (4.75–9.50 mm), while no effect was seen for larger gravels. However, their results were based on single-sized gravel samples. Alnedawi et al. [55] performed RLT tests on two UGMs (basalt and granite) at frequencies of 0.20, 0.33, and 0.50 Hz, and reported negligible effects of frequency on deformation. A sensitivity analysis using artificial neural networks (ANN) further showed that frequency contributed only about 3% to deformation accumulation compared to other factors. Li et al. [65] reported similarly weak frequency dependence (0.5–2.0 Hz) in Monte Carlo and Discrete Element Method (DEM) simulations based on 50 random samples. Overall, the influence of loading frequency on the elastoplastic behavior of UGMs appears small. However, when fine content increases, frequency-dependent (rate-sensitive) behavior becomes more evident [66]. Regarding rest periods, limited research exists. Alnedawi et al. [34] studied their impact on two UGMs using both RLT tests and finite element method (FEM) simulations. They found that specimens subjected to rest periods exhibited greater permanent deformation than those without, possibly due to enhanced material segregation during rest. Nonetheless, further studies involving higher loading cycles (beyond N = 30,000) are needed to confirm these findings.

The determination of an appropriate number of load cycles to be applied in PD testing remains a topic of ongoing debate among researchers. Currently, there is no established consensus regarding the optimal number of cycles required to accurately capture the rate of PD accumulation or to reliably rank the performance of UGMs. Different studies have adopted a wide range of loading cycles, often extending to tens or even hundreds of thousands, with the aim of replicating long-term field conditions and cumulative strain behavior. While such extensive testing provides valuable insights into material durability and deformation trends, it also introduces significant challenges related to laboratory feasibility, equipment fatigue, and time efficiency. Consequently, the practical implementation of these prolonged tests is often limited, particularly when multiple specimens or test conditions need to be evaluated. To address these limitations, further research should aim to identify a feasible and representative number of load cycles that can effectively characterise the stabilising phase of PD behaviour without compromising test reliability. Establishing a standardised criterion for determining the “stabilising” or “steady-state” number of cycles would enhance test repeatability, improve comparability across studies, and support the development of more efficient laboratory testing protocols for assessing the cyclic performance of UGMs.

3.3 Moisture Content

The presence of moisture increases PD in UGMs [41, 56, 67]. According to Thom and Brown [61], PD may increase dramatically for a relatively small increase in moisture content [57, 58, 68, 69]. They have reported that the lubrication effect of moisture is the primary influencing factor [13, 32, 70]. Researchers generally agree that when there is a high degree of saturation and low permeability[15, 32, 71], PD increases with moisture due to an increase in pore water pressure, thus reducing the effective stress, which in turn, lowers the stiffness and deformation resistance of the material [2, 41, 49]. The literature available reveals that researchers who have studied the effect of water content in granular pavement layers in the laboratory and in the field believe that the combination of a high degree of saturation and low permeability, due to poor drainage, leads to high pore pressure, low effective stress and, consequently, low stiffness and low deformation resistance [3, 52, 61]. However, any well-established model to mathematically describe the influence of moisture on the PD behaviour of UGMs is still lacking.

3.4 Density, Fine Content and Gradation

Clearly, an increase of the degree of compaction of the material reduces the void ratio, and increases the number of contacts between the grains, thus increasing the resistance to permanent deformations [24, 41, 72]. More generally, the resistance to permanent deformations strongly depends on the void content [41, 73, 74]. For the same degree of compaction, a modification of the grading, reducing the void content, increases the resistance to permanent deformations [9, 41, 75]. However, relatively less study has been focused on the PD of a granular material with high fines content (greater than 10% by mass), especially when considering the changing moisture content. In the practical engineering projects, the fines very often contain a high fraction of clays, making the analysis of the PD behaviour of granular layers more complicated. On other hand the effect of density, as described by degree of compaction, has been regarded in previous studies as being significantly important for the long-term behaviour of granular materials [41, 76]. Resistance to permanent deformation in these materials under repetitive loading appears to be highly improved as a result of increased density [41]. Barksdale [52] studied the behaviour of several granular materials and observed an average of 185% more permanent axial strain when the material was compacted at 95% instead of 100% of maximum compactive density.

Previous research has clearly demonstrated that the fines content plays a significant role in influencing the PD behaviour of UGMs. The presence and proportion of fine particles within the granular matrix can substantially alter key mechanical properties such as stiffness, strength, and resistance to cyclic loading. Specifically, fines influence the inter-particle contact mechanics, pore pressure response, and overall load transfer mechanisms, thereby governing the rate and magnitude of PD accumulation under repeated traffic loading. Despite these well-recognised effects, most existing PD models tend to oversimplify or completely neglect the influence of fines content, gradation characteristics, and density state. Such simplifications limit the applicability of current models to a relatively narrow range of well-graded, coarse-grained materials. Moreover, there remains a notable lack of systematic studies investigating the PD behavior of UGMs containing higher fines contents, particularly those exceeding 15%. These materials, which are common in field conditions due to crushing, weathering, or the use of marginal aggregates, often exhibit distinct mechanical responses compared to cleaner granular mixtures. High-fines UGMs tend to exhibit increased moisture sensitivity, reduced drainage capacity, and more pronounced viscous and plastic deformation behavior under cyclic loading. Consequently, their long-term performance cannot be adequately represented by models calibrated for low-fines materials.

To bridge this gap, future research should focus on comprehensive experimental and numerical investigations that explore the PD characteristics of high-fines UGMs under controlled cyclic loading. Such studies should aim to quantify the combined effects of fines content, gradation, and compaction density on the evolution of permanent strain response. Additionally, the findings should be integrated into mechanistic-empirical (M-E) pavement design frameworks to enable more realistic performance predictions for granular layers composed of materials with varying fines contents. Developing refined PD models that explicitly account for these parameters would greatly enhance the reliability and generalizability of M-E design methods, thereby improving material selection and pavement design practices in the construction industry.

3.5 Mineralogical Nature of the Material

The mineralogical composition of UGMs plays a crucial role in determining their mechanical and deformation behavior, even when the grading and particle size distribution are similar [77]. The mineralogy of the parent rock governs fundamental properties such as particle shape, surface texture, angularity, hardness, and resistance to fragmentation, which in turn affect the material's overall performance under repeated loading conditions [78]. Different minerals exhibit varying degrees of hardness, brittleness, and weathering resistance. For instance, igneous rocks such as basalt and granite typically contain hard and durable minerals like quartz and feldspar, which promote angular particle shapes and rough surfaces after crushing [79–81]. These characteristics enhance interparticle friction and mechanical interlock, resulting in higher shear strength, greater stiffness, and reduced permanent deformation under cyclic loading. Conversely, sedimentary rocks such as limestone or sandstone often contain softer or more rounded grains, leading to smoother particle surfaces, lower frictional resistance, and increased susceptibility to crushing and plastic deformation. Mineralogy also influences the microscopic roughness and surface energy of the particles. Minerals with rough, irregular surfaces (e.g., feldspar, quartz) promote higher friction angles, whereas smoother minerals (e.g., calcite, clay-coated particles) tend to reduce interparticle friction. This directly affects the elastoplastic behavior of the material, as rougher surfaces resist relative movement and sliding, limiting strain accumulation during RLT testing. In addition, the quality of the fine fraction which often arises from the weathering or crushing of the parent rock is highly dependent on mineralogy. Fines derived from clay-rich or mica-bearing minerals tend to be sensitive to moisture, exhibiting water-induced softening, loss of suction, and reduced stiffness. These fines can fill the voids between coarse particles, changing the load transfer mechanism and leading to increased permanent deformation and reduced resilient modulus. In contrast, non-plastic fines originating from siliceous minerals typically have a less detrimental effect, maintaining stability even under variable moisture conditions. Moreover, the petrographic characteristics of the parent rock influence its long-term durability under cyclic loading. Rocks composed of minerals with variable hardness or pronounced cleavage planes may experience preferential breakage along weaker zones, resulting in particle degradation and a progressive change in grading during service life. This aggregate degradation alters the packing structure and fabric of the UGM, thereby modifying its deformation response and compaction behavior. Overall, the mineralogical composition of UGMs through its control on particle morphology, surface roughness, fine fraction characteristics, and water sensitivity has a decisive impact on their mechanical stability and deformation characteristics. Two aggregates with identical particle size distributions but different mineralogies can therefore exhibit markedly different responses to cyclic loading, highlighting the importance of considering mineralogical analysis alongside traditional gradation and strength tests in UGM characterisation.

In general, there remain significant gaps in understanding the relationship between the PD behaviour of UGMs and their underlying mineralogical characteristics. While numerous studies have focused on macroscopic parameters such as gradation, fines content, density, and stress conditions relatively few have explored how the intrinsic mineral composition and microstructural properties of the constituent particles influence the long-term deformation response. The mineralogical composition governs key material attributes such as particle hardness, shape, surface roughness, and susceptibility to crushing or weathering, all of which play a critical role in the accumulation of plastic strain under cyclic loading. To address this knowledge gap, future research should incorporate detailed microstructural and mineralogical analyses to better elucidate the mechanisms linking mineral composition to PD behaviour. Techniques such as Scanning Electron Microscopy (SEM) combined with Energy Dispersive X-ray Spectroscopy (EDS) can provide valuable insights into particle morphology, surface texture, and micro-cracking patterns before and after cyclic loading. These observations can be correlated with macroscopic deformation trends to establish a mechanistic understanding of how mineralogical variations influence particle rearrangement, contact degradation, and cumulative strain development. Furthermore, integrating mineralogical parameters into mechanistic and mechanistic-empirical (M–E) analysis frameworks would represent a major advancement in PD modeling. Incorporating quantitative descriptors of mineralogy such as mineral hardness indices, particle angularity factors, or micro-fracture susceptibility could significantly enhance the predictive capability of existing PD models. Such integration would allow for more accurate characterisation of material performance across different geological sources and environmental conditions, ultimately leading to improved material selection, design optimization, and long-term pavement performance prediction.

3.6 Permanent Deformation Modelling of UGMs

The deformation occurring in pavement layers consists of two parts, recoverable deformation (resilient behavior) and non-recoverable deformations (absorbing behavior) [10, 82, 83]. Several attempts have been made by researchers over the last few decades in order to develop permanent strain models for better characterisation of pavement materials from repeated load tests [84–86]. The accumulated permanent deformations in unbound granular materials in aggregate base and sub-base layers also causes significant amount of rutting at the surface, resulting in failure of pavement serviceability criterion by exceeding certain allowable rutting values which is measured in terms of surface roughness[87–90]. The total accumulated rut depth in pavement is due to the summation of plastic strain of different components in a multilayer pavement system [91] as given by Eq. 4.

$$\delta_p = \sum \varepsilon_{p,i} h_i \tag{4}$$

where δ_p is total rut depth, $\varepsilon_{p,i}$ is plastic strain of i^{th} layer, and h_i is thickness of i^{th} layer.

Several models are available in the literature for describing the PD behaviour of UGMs most of them presented on Table 1 [41, 92]. PD properties determined from repeated load triaxial tests are key parameters used to model pavement layers rutting potential in any M-E pavement design guide [93]. The earlier models can be divided into two types as (a) relationships describing the influence of the number of load applications on the permanent strain and (b) relationships describing the influence of applied stresses on the permanent strain [54]. Relatively recent models combine the effect of stress with the number of load cycles. However, the influence of other important factors such as moisture content, density, and grading are not directly taken into account by these models. The testing procedure for PD as mentioned before it includes either Single Stage (SS) in order to study the influence of different stress levels on the material, each time a new SS RLT test on a new specimen needs to be run. In this case, the effect of stress history on the material cannot be studied refer to Fig. 4. On the other hand, the Multi Stage (MS) RLT test applies several stress paths to a single specimen and thus (a) it includes the effect of stress history, (b) reduces the time and effort required to study the influence of several stress levels and (c) reduces the experimental scatter usually experienced in testing several specimens. Furthermore, it should be more reliable to evaluate the material parameters of a model from a large number of stress paths in MS RLT tests. Therefore, for a comprehensive evaluation of material behaviour, the multi-stage repeated load triaxial (MS-RLT) test represents a more practical and informative option compared to the traditional single-stage (SS-RLT) approach. The MS-RLT test allows for the assessment of material response under a wider range of stress conditions within a single specimen, thereby providing a more realistic representation of in-service loading scenarios. However, the modeling of PD behavior based on MS-RLT data becomes inherently more complex due to the nonlinear interactions between stress history, material densification, and cumulative strain effects.

It is noteworthy that most of the existing PD models have been developed and calibrated using results from single-stage RLT tests, which often fail to capture the complex stress-path dependency exhibited under multi-stage loading. Furthermore, the majority of these models summarized in Table 1 consider only a limited number of influencing factors, primarily stress level and moisture condition, while neglecting other critical parameters such as fines content, gradation, density, mineralogy, and loading frequency. This simplification restricts their applicability to a narrow range of material types and environmental conditions. To overcome these limitations, future modeling efforts should focus on developing more generalised and data-driven PD prediction frameworks capable of incorporating multiple interacting variables. In this regard, the integration of advanced computational techniques such as artificial neural networks (ANNs) and other artificial intelligence (AI) approaches offers a promising pathway. These methods can efficiently capture nonlinear and multivariate relationships within complex datasets, leading to the development of more robust, adaptive, and accurate PD models. Emphasising the application of AI-based modeling in PD research will not only enhance the predictive capability of current mechanistic-empirical design methods but also contribute to establishing a new generation of intelligent, performance-based models for UGMs.

Table 1 Computational Models for Permanent Strain Behavior

Reference	Model	Model definitions	
Lashine et al. [46]	$\varepsilon_p = 0.9\frac{q}{\sigma_3}$	q deviator stress σ_3 Confining pressure	5
Barksdale Model [52]	$\varepsilon_p = a + blogN$ $\varepsilon_p = \frac{q/a\sigma_3^b}{1-\left[\frac{R_{fq}/2(c\cos\phi+\sigma_3\sin\phi)}{(1-\sin\phi)}\right]}$	a, b = are regression model parameters ε_p Permanent Deformation N Number of Loading Cycles R_f = ratio of measured strength to ultimate hyperbolic strength ϕ = angle of internal friction C = apparent cohesion	6
Monismith et al. Model [94]	$\varepsilon_p = AN^b$ $\log\varepsilon_p = A + blogN$	A, a, b = are regression model parameters ε_p Permanent Deformation N Number of Loading Cycles	7
Pappin Model [38]	$\gamma_p = (fnN)\,L(\frac{q}{p})^{2.8}_{max}$	γ_p = permanent shear strain; (fn N) = the shape function, p = mean normal stress = $(\sigma_1+\sigma_2+\sigma_3)/3$; q = deviator stress = $(\sigma_1-\sigma_3)$; L = length of stress path = $(p^2+q^2)^{½}$; (q/p) max = maximum stress ratio	8
Veverka [118]	$\varepsilon_p = a\varepsilon_r N^b$	ε_r = resilient strain	9
Khedr [119]	$\frac{\varepsilon_p}{N} = A_1 N^{-b}$	Ditto	10
Sweere [120]	$\varepsilon_p = AN^b$	Ditto	11
Tseng and Lytton Model [121]	$\varepsilon_p = \varepsilon_0 e^{-\left(\frac{\rho}{N}\right)^\beta}$	Where ε_0, β and ρ are different material parameters, depending on material physical properties, moisture content and bulk stress of laboratory testing	12
Wolff Model [95]	$\varepsilon_p = (mN + a)\left(1 - e^{-bN}\right)$	a, b and m are model parameters, in which m has physical interpretation of permanent deformation accumulation indicating plastic shakedown (m = 0) and plastic creep response (m > 0)	13
Thompson and Nauman Model [122]	$\mathrm{RR} = \frac{\mathrm{RD}}{\mathrm{N}} = \frac{\mathrm{A}}{\mathrm{N^B}}$	RR and RD are rutting rate and rut depth in inches, respectively. A and B terms are developed from field calibration testing data	14
Van Niekerk and Huurman (1995)	$\varepsilon_p = a_1\left(\frac{\sigma_1}{\sigma_{1,f}}\right)^{a_2}\left(\frac{N}{1000}\right)^{b1\left(\frac{\sigma_1}{\sigma_{1,f}}\right)^{b2}}$	σ_1 is major principal stress; σ_{1f} is major principal stress at failure; a_1, a_2, b1 and b_2 are model parameters. The stress ratio $(\sigma_1/\sigma_{1,f})$	15
Paute et al. [47]	$\varepsilon_p = \frac{A\sqrt{N}}{B+\sqrt{N}}$ $\in_{p,100} = \mathrm{A}\left(1 - \left(\frac{N}{100}\right)^{-B}\right)$	Cycles $\epsilon_{p,100}$ is permanent deformation for number of load cycles after 100 cycles; parameter A and B in above equations are regression model parameters	16
Bonaquist & Witczak (1997)	$\varepsilon_p = \sum\varepsilon_N$	ε_N Sum of number of cycles	17
Huurman Model [123]	$\varepsilon_p = A\left(\frac{N}{1000}\right)^B + C\left(\left(e^{D\frac{N}{1000}}\right) - 1\right)$ X in the following equation: $X = x_1(\frac{\sigma_1}{\sigma_{1,f}})^{x_2}$	N is number of load cycles, σ_1 is major principal stress, σ_{1f} is major principal stress at failure, A, B, C and D are stress dependent model parameters	18
Ullidtz Model [124]	$\varepsilon_p = A\left(\frac{\sigma_d}{p_0}\right)^B N^C$	σ_d is deviator stress; p_0 is the normalizing reference stress (i.e. p_0 = 1 psi or 1 kPa); and A, B and C are parameters obtained from multiple regression analysis	19
Lekarp and Dawson [49]	$\frac{\in_{p(Nref)}}{\frac{L}{p_0}} = A\left(\frac{q}{p}\right)^B_{max}$	$\epsilon_{p\,(Nref)}$ is permanent strain at a given reference number of load cycles (N_{ref}), where $N_{ref} > 100$; L is the length of stress path; p is mean normal stress equals to $(\sigma_1+\sigma_2+\sigma_3)/3$; q is deviator stress = $(\sigma_1-\sigma_3)$; $(q/p)_{max}$ is maximum stress ratio; p_0 is the normalizing reference stress; and A and B are model parameters	20
Gidel et al. (2001)	$\varepsilon_p = \in_{p,0}\left[1 - \left(\frac{N}{N_0}\right)^{-B}\right]\left(\frac{L_{max}}{p_a}\right)^n\left(\frac{1}{m+\frac{s}{pmax} - \frac{qmax}{pmax}}\right)$	p^{max} is maximum normal stress; q^{max} is maximum cyclic deviator stress; L_{max} is stress path length, or $(p^{max2}+q^{max2})^{½}$; p_a is atmospheric pressure = 100 kPa; N_0 is reference number of cycles; and $\epsilon_{p,0}$, B and n are model parameters; m and s are parameters of the stress path, q = mp + s	21

Table 1 (continued)

Reference	Model	Model definitions	
Korkiala-Tanttu (2005)	$\varepsilon_{pi}(\mathrm{N})=C(N-N_{i-1}+N_i^{eq})^{\mathrm{b}}\frac{R_i}{A-R_i}$ Where $N_i^{eq}=\left(\frac{\varepsilon_{pi-1(A-Ri)}}{CRi}\right)^{1/b}$	$A, a,$ b = are regression model parameters $\in_P$ = permanet deformation N = number of loading cycles	22
Chow-Mishra-Tutumluer [33]	$\varepsilon_p\left(N\right)=AN^B{\sigma_d}^C\left(\frac{\tau_f}{\tau_{max}}\right)^D$ $\varepsilon_p\left(N\right)=AN^B{\sigma_d}^C\left(\frac{\tau_f}{c+\sigma f\tan\varnothing}\right)^D$	A, B, C, D = are regression model parameters $\in_P$ = permanet deformation N = number of loading cycles τ_{f} = Shear stress $\tau_{\max}$ = Maximum shear stress σ_{d} = Deviator stress	23
Rahman and Erlingsson, 2015b	$\varepsilon_{pi}(\mathrm{N})=C\left(N-N_{i-1}+N_i^{eq}\right)^{bs_{fi}}S_{fi}$ where; $N_i^{eq}=\left(\frac{\varepsilon_{pi-1}}{aS_{fi}}\right)$	all of, the subscript *i* refers to the i$^{\mathrm{th}}$ stress path	24

3.7 Modeling Permanent Strain with Respect to Number of Load Applications

Barksdale [52] conducted an extensive investigation into the behavior of various base course materials subjected to repeated loading using triaxial tests involving numerous load applications. His study revealed that, under a specified stress condition, the accumulation of permanent axial strain was proportional to the logarithm of the number of load cycles. He described this behavior using a log-normal model. Further research into the long-term behavior of granular materials was performed by Monismith et al. [94], who also used repeated load triaxial testing. However, the applicability of the log–log model was later questioned by Wolff and Visser [95] following a series of full-scale Heavy Vehicle Simulator tests with several million load repetitions. They identified two distinct phases in the buildup of permanent deformation. The first phase, lasting up to approximately 1.2 million load repetitions, was characterized by rapid development of permanent strain accompanied by a continuously decreasing rate of increase. In the subsequent phase, the accumulation of permanent strain slowed significantly, with the strain rate approaching a constant value. Because the log–log model was unable to accurately predict permanent strain at very high numbers of load cycles, Wolff and Visser proposed an alternative stress–strain model. Earlier, Paute et al. [47] suggested that permanent strain gradually increases toward an asymptotic value as the number of load cycles rises. They developed a relationship to describe the connection between permanent axial strain excluding the initial cycles and the number of loading repetitions. Lekarp and Dawson [49]reported that the model proposed by Paute et al.[47] was valid primarily under low applied stress levels, thus indicating limitations in its general applicability. Building on the theoretical framework of plasticity, Bonaquist and Witczak [96] developed a constitutive model based on the flow theory of plasticity. This incremental model separates the total strain increment into a reversible resilient strain component and an irreversible plastic strain component. The plastic strain increment is assumed to depend on the current states of stress and strain, as well as on the incremental changes in stress. The flow theory describes plastic behavior through three key concepts: the yield function, which distinguishes elastic from plastic material responses; the flow rule, which defines the incremental stress–strain relationship within the plastic region; and the hardening rule, which characterises how the yield surface evolves due to plastic deformation [13]. By combining the hierarchical approach introduced by Desai et al.[97] with the bounding surface concept proposed by Mroz et al.[98], Bonaquist and Witczak[99] formulated a novel permanent strain model suited for soils and granular materials under repeated loading. This hierarchical model first defines the magnitude of permanent strain during the initial load cycle. For subsequent cycles, it captures cyclic hardening behavior by expressing permanent strain as a power function of the strain developed in the first cycle. The total accumulated permanent strain is then calculated as the sum of permanent strains from all load cycles. However, modeling permanent deformation (PD) behavior solely as a function of the number of load applications has been increasingly questioned in the literature [5, 13]. Such simplified models, while useful for preliminary assessments, fail to account for the complex dependency of PD on the prevailing stress conditions and other influencing factors. In reality, the accumulation of permanent strain in unbound granular materials (UGMs) is highly sensitive to variations in applied stress levels, stress ratios, confinement pressures, and material characteristics such as gradation, fines content, and moisture state. Recognising these limitations, several researchers have refined early empirical models by incorporating both stress levels and the number of load applications into their formulations.

This approach enables a more realistic representation of the nonlinear relationship between load intensity, stress history, and deformation rate. The integration of stress-dependent parameters has therefore enhanced the predictive accuracy of PD models, as elaborated in the subsequent discussion.

3.8 Modeling Permanent Strain with Respect to Stress Condition

The previously discussed relationships describing how permanent deformations vary with the number of load cycles cannot be directly applied to predict permanent deformation in pavement structures, primarily because they do not account for the applied stress levels. To address this limitation, other researchers have pursued approaches that relate permanent deformation after a given number of load cycles to the applied stresses, typically focusing on the maximum stresses experienced [59, 100]. Several such relationships have been proposed, some of which attempt to incorporate the combined effects of both applied stresses and the number of load cycles, though these are relatively few all of these models presented on Table 1.

A review of existing studies reveals that current PD models primarily rely on the number of load cycles and applied stress levels as the dominant predictors of pavement performance. Although these parameters are undeniably important, their exclusive consideration results in an oversimplified representation of the complex mechanical behavior of UGMs. In reality, PD behavior is influenced by a combination of interacting factors, including fines content, particle gradation, density state, moisture condition, and mineralogical composition. The omission of these variables limits the accuracy and general applicability of existing models, particularly when predicting long-term pavement response under realistic field loading conditions. Therefore, it is evident that substantial gaps remain in fully understanding and modeling the PD behavior of UGMs under cyclic loading. To bridge these gaps, future research should aim to develop generalised and comprehensive PD models capable of capturing the combined influence of all key factors affecting deformation behaviour. The integration of advanced data-driven approaches particularly artificial intelligence (AI) and machine learning (ML) techniques offers a promising pathway toward achieving this goal. By leveraging AI's capability to model nonlinear, multivariate interactions, researchers can formulate robust, adaptive, and realistic PD prediction frameworks. Such advancements will enhance the mechanistic–empirical pavement design process, providing the construction industry with more accurate, reliable, and performance-oriented tools for material evaluation and pavement analysis.

3.9 Modeling Permanent Strain Using a Shake-Down Theory

Currently, the shakedown theory is being extensively used in recognizing different responses of UGMs under cyclic loading [41, 101]. Based on this theory, Werkmeister et al.[102] have identified that depending on stress levels, the evolution of PD in UGMs with the number of load cycles falls within the three shakedown ranges, shown in Fig. 6 [103, 104]. These ranges are described below in the order of ascending stress levels, where in each case, the stress level remains constant with the number of load applications: Shake-down theory describes the plastic strain behaviour of pavement materials subjected

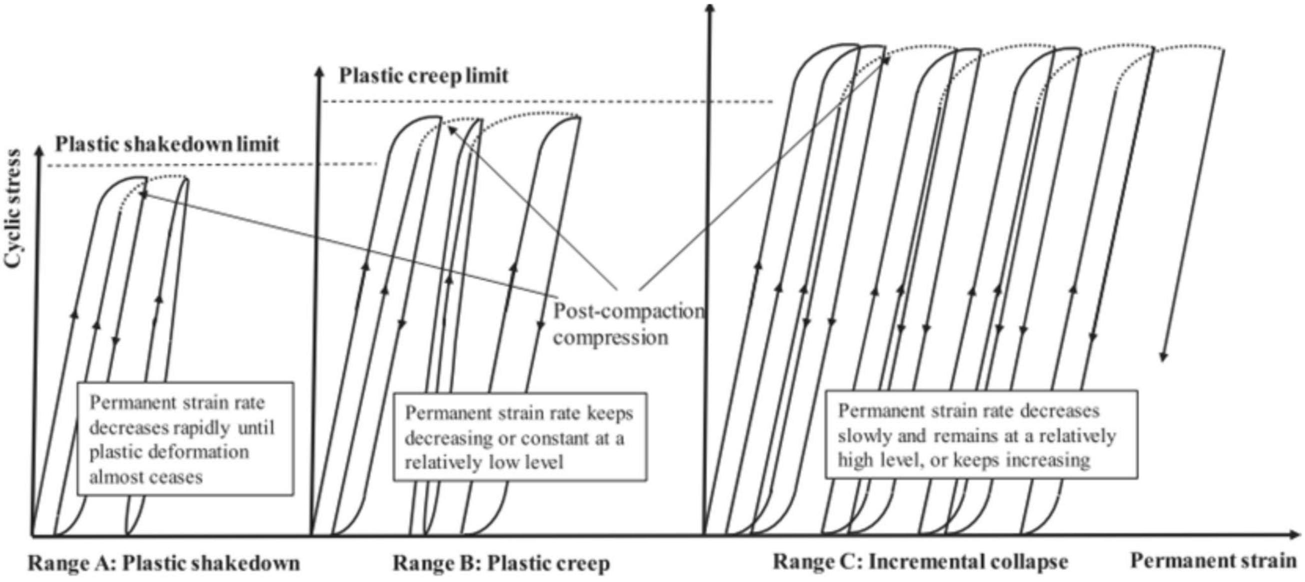

Fig. 6 Shakedown ranges classification for the UGMs

Table 2 Proposed boundaries of shakedown ranges

Shake-down criteria	Ranges	Boundaries of shakedown range
[125]	A	$PD_{5000} - PD_{3000} < 4.5 \times 10^{-5}$
	B	$4.5 \times 10^{-5} < PD_{5000} - PD_{3000} < 4.5 \times 10^{-4}$
	C	$PD_{5000} - PD_{3000} > 4.5 \times 10^{-4}$
[58]	A	$PD_{5000} - PD_{3000} < 6.0 \times 10^{-5}$
	B	$6.0 \times 10^{-5} < PD_{5000} - PD_{3000} < 6.0 \times 10^{-4}$
	C	$PD_{5000} - PD_{3000} > 6.0 \times 10^{-4}$
[36]	A	$0^0 <$ Range A $< 22.5^0$
	B	$22.5^0 <$ Range B $< 45^0$
	C	$45^0 <$ Range C $< 90^0$

to repeated loading. It makes a distinction between three types of plastic strain behaviour [24]. The three ranges of possible behaviour (A, B and C) apply to ascending stress levels respectively. Range A is termed the plastic shake-down range, range B is termed the plastic creep range, and range C is the incremental collapse stage. A diagram illustrating the different categories of permanent deformation is shown in Fig. 6.

Many studies over past decades has described the PD under repeated loads using the shakedown theory, the details and evolution of this theory described in many of previous studies [36, 102]. This theory has been widely applied on UGMs by adopting a specific visual criterion and boundaries as proposed by Werkmeister et al. [102, 104]. These criteria were also adopted by BS EN 13286–7 guideline in selection and ranking of the UGMs [30]. It is important to recognize that existing shakedown criteria, such as those developed by Werkmeister et al.[102] and those presented on Table 2, are not universally applicable across all environmental and climatic conditions. These criteria were derived under specific testing protocols and material properties, which may not accurately reflect the diverse range of UGMs used globally. For instance, a study conducted by Chen et al.[104] highlighted the limitations of applying Werkmeister's boundaries to reinforced UGMs. Their research, which involved UGMs reinforced with geogrids, demonstrated that the original shakedown limits needed to be adjusted to account for the enhanced mechanical behavior introduced by geosynthetic reinforcement. This finding underscores the necessity of updating existing criteria to suit specific applications and material configurations. Similarly, research by Gu et al. [58] raised critical concerns regarding the applicability of Werkmeister's criteria to UGMs in the state of Texas. Their study concluded that the shakedown boundaries, developed for specific materials and environmental conditions, failed to accurately characterise the behaviour of the Texas UGMs tested. These findings support the view that shakedown classifications derived from one region or material type may not be suitable for others. Furthermore, recent studies have proposed redefining shakedown boundaries based on alternative parameters, such as the effective cyclic stress ratio, to better capture material response under repeated loading. These evolving approaches indicate a shift toward more adaptable and locally relevant frameworks. Given these concerns, there is a significant risk of mischaracterising local materials if existing shakedown boundaries are applied without modification. This issue arises for two main reasons: The RLT test protocols used to develop current shakedown boundaries were conducted under fixed conditions that do not account for variations in local environments or pavement loading scenarios. The UGMs used in the development of these criteria may possess physical and mechanical properties that differ substantially from those of materials available in other regions.

4 Area for the Further Research

4.1 PD Behaviour of Recycled Materials

The availability of natural UGMs sourced from borrow pits and quarries is steadily declining, primarily because these materials are non-renewable resources [77]. Continuous extraction to meet the demands of large-scale infrastructure projects such as roads, railways, and embankments has resulted in the depletion of high-quality natural aggregates in many regions. The overexploitation of these borrow pits not only leads to the loss of natural landscapes and habitats, but also contributes to soil erosion, groundwater contamination, and air pollution through dust emissions. Moreover, aggregate extraction and processing are energy-intensive activities that produce significant greenhouse gas emissions, thereby exacerbating climate change. Given these environmental and sustainability challenges, there is an increasing global awareness of the need to reduce dependence on natural aggregates and to develop more sustainable alternatives. To address this scarcity, researchers and engineers have begun exploring the use of recycled materials as substitutes for natural UGMs in pavement and road foundation layers [105, 106]. These recycled aggregates (RAs) are commonly derived from construction and demolition waste, reclaimed asphalt pavement (RAP), recycled concrete aggregate (RCA), and various industrial by-products such as steel slag, crushed brick, and fly ash [107]. The utilisation of such materials offers substantial environmental benefits, including reduced landfill waste, lower demand for virgin aggregate extraction, decreased carbon footprint, and enhanced resource circularity in the construction sector.

Over the last two decades, numerous studies have been conducted to evaluate the mechanical, physical, and durability properties of recycled aggregates for use in road construction. Research findings generally indicate that recycled materials can achieve comparable stiffness, strength, and bearing capacity to those of natural aggregates when properly processed, graded, and stabilized with binders or additives. Nevertheless, the mechanical performance of recycled UGMs often varies widely due to their heterogeneous composition, higher porosity, and presence of adhered mortar or asphalt residues. These factors contribute to reduced particle strength, increased susceptibility to crushing, and greater water absorption, which may affect their long-term performance under cyclic loading. Despite the growing body of research on recycled aggregates, there remains a significant knowledge gap regarding their permanent deformation behavior when subjected to repeated or cyclic loading conditions. Permanent deformation, or rutting, is one of the critical failure modes in pavement structures, influencing both structural integrity and service life. The complex microstructure of recycled aggregates characterized by residual cementitious phases, variable particle morphologies, and weaker interparticle bonding can lead to nonlinear and time-dependent deformation responses that differ substantially from those of natural UGMs. Understanding these behaviors is essential to ensure that recycled materials perform reliably under realistic traffic and environmental conditions. To promote the sustainable and safe use of recycled materials in pavement engineering, further research is necessary to address several key aspects. These include a detailed evaluation of the elastoplastic and resilient behavior of recycled UGMs under varying stress states, loading frequencies, and moisture conditions; the development of constitutive models capable of predicting their deformation and failure characteristics; and the assessment of their long-term field performance through large-scale experimental and numerical studies. In addition, optimizing processing and stabilization techniques such as mechanical crushing, blending, and chemical stabilization can enhance the mechanical integrity and deformation resistance of recycled UGMs. In conclusion, the decline in natural UGM resources and the increasing emphasis on environmental sustainability have accelerated the global shift toward the use of recycled and alternative granular materials in road construction. However, the permanent deformation behavior of these materials remains insufficiently understood, necessitating intensive experimental, analytical, and modeling studies to ensure their durability, reliability, and long-term performance. Achieving this will support the development of sustainable, resource-efficient, and climate-resilient pavement infrastructures for the future.

4.2 Artificial Intelligence

The application of Artificial Intelligence (AI) has become increasingly widespread across various branches of engineering due to its powerful ability to analyse complex, nonlinear, and multidimensional data [108, 109]. In fields such as civil, geotechnical, and pavement engineering, AI techniques have shown remarkable potential for predictive modeling, optimization, pattern recognition, and decision support [110–113]. Among the various AI-based methods, Artificial Neural Networks (ANNs) have emerged as one of the most effective computational tools for capturing complex relationships between input parameters and system responses, particularly in materials and structural behavior analysis [79, 114, 115]. In the context of pavement and geotechnical engineering, ANNs have been successfully applied to model diverse phenomena, including resilient modulus prediction, soil classification, compaction characteristics, settlement behavior, and bearing capacity estimation [21, 116, 117]. These models are capable of learning from experimental or field datasets and generalizing the underlying patterns without requiring explicit mathematical formulations of the physical processes involved [18]. This makes ANNs particularly suitable for modeling materials such as UGMs, whose mechanical behavior is highly nonlinear and influenced by multiple interacting factors such as stress level, moisture content, loading frequency, particle size distribution, and mineralogical composition. However, when it comes to PD modeling of UGMs under cyclic loading, the application of AI and ANN-based methods remains limited. Only a few studies have addressed this specific area. For example, Ghorbani et al. [68, 108] developed an ANN model to predict the permanent deformation of granular materials using laboratory test data. While their findings demonstrated the potential of ANN in capturing the complex deformation behavior, the available models were based on relatively small datasets and older network architectures, which limited their predictive accuracy and generalization capability. Moreover, most previous studies relied on static network configurations and did not incorporate modern deep learning architectures such as convolutional neural networks (CNNs), recurrent neural networks (RNNs), or hybrid models that could better represent time-dependent deformation patterns in cyclic loading conditions. The current gap in the literature highlights the need for comprehensive and systematic research on the application of AI particularly ANN and deep learning models to simulate and predict permanent deformation behavior in both natural and recycled UGMs. Future studies should focus on developing data-driven predictive frameworks that can integrate large experimental datasets, sensor-based monitoring data, and numerical simulation results. By training these models on extensive, high-quality datasets, researchers can significantly improve their reliability and robustness. Furthermore, integrating ANNs with mechanistic-empirical approaches could enhance the interpretability of AI predictions, allowing for a better understanding of the physical mechanisms controlling deformation. Such hybrid modeling approaches combining AI algorithms with constitutive models or finite element simulations represent a promising direction for next-generation pavement analysis and design. The use of explainable AI (XAI) methods can also provide insights into the relative importance of key input variables (e.g., stress ratio, confining pressure, fines content, or moisture variation), helping engineers identify dominant factors that control permanent deformation behavior.

5 Conclusion

Over the years, research on granular materials has predominantly focused on their resilient behavior, largely due to the practical challenges associated with investigating permanent deformation. Resilient testing is relatively quick and allows for the evaluation of a wide range of stress conditions using a single specimen. In contrast, permanent deformation testing is time-intensive and requires separate specimens for each stress scenario. As a result, our understanding of the resilient response of granular materials has advanced more significantly than our knowledge of their long-term deformation behavior. This literature review examines the rutting mechanisms of UGMs and evaluates existing empirical and semi-empirical models for predicting permanent deformation. Most of these models have been developed based on laboratory data, particularly from RLT testing. The current predictive framework used in the mechanistic-empirical pavement design system Pavement ME Design is also discussed. However, in its current implementation, Pavement ME Design does not account for the shear strength of aggregate materials or the specific stress states induced by applied wheel loads when estimating rutting in unbound base and subbase layers. To more accurately characterise and predict the performance of unbound aggregate layers under repeated traffic loading, it is crucial that mechanistic-empirical pavement design methods incorporate the influence of applied stress levels on rutting behaviour. The accumulation of permanent strain in granular materials is influenced by several interrelated factors, including stress level, principal stress rotation, number of load applications, moisture content, stress history, compaction density, fines content, gradation, and aggregate type. Despite this, a comprehensive model capable of capturing the combined effects of these variables on permanent deformation behavior remains lacking. There is a pressing need to develop more generalised, theoretically sound computational models for predicting permanent strain in UGMs, models that fully integrate the key influencing factors. The adoption of advanced techniques such as artificial intelligence and machine learning is also encouraged, as these approaches offer significant potential to enhance model accuracy and capture complex material behaviors. This underscores the importance of continued and intensive research in this area moving forward.

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Declarations

Conflict of Interest The authors declared that this is paper is original work and not published anywhere.

Ethics Approval and Consent to Participate All research participants are part of a study.

Consent for Publication We, the undersigned, *give our consent for the publication of identifiable details.*

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Joseph Anochie Boateng is a professional engineer with the Engineering Council of South Africa (ECSA) certification and holds a PhD in civil engineering from University of Illinois at Urbana-Champaign, USA. He is currently working as Associate Professor of Civil Engineering at the University of Pretoria in South Africa. Joseph has more than 22 years combined experience in consulting and research work in road engineering and has provided consultancy services to local and international clients including road agencies, government departments and local governments in South Africa, Tanzania, Uganda, Ghana, Rwanda, South Sudan, Malawi, and the UK Aid/DFID. At the same time, Joseph has been involved in strategic R&D and innovation in bituminous and concrete materials technology, design and construction of high and low-volume roads, and forensic investigation of road failures in several countries in Africa. He has extensive hands-on experience in field investigation and laboratory testing of road materials and has led the development of several policy documents including guidelines, manuals, and specifications to support the road and asphalt industry in South Africa, Tanzania, and Ghana. Joseph has provided leadership in projects involving pavement design, asphalt design and testing using software, standards, and specifications (e.g., winDCP, mePADS, AASHTO Design Guide 1993; AASHTO Ware Pavement ME Design, COTO, TRH4, Sabita Manuals 35 & 39 – main author of both). Joseph is working closely with the South Africa Road Federation (SARF) and South African universities on asphalt and road engineering courses that are aimed at training and building capacity, as well as accelerating development of the necessary skills to address the shortage of civil engineers and technicians in Southern Africa. Since 2014, Joseph has developed and presented courses in asphalt mix design, and quality control and quality assurance of road construction to regional managers, engineers, and technicians from Tanzania National Roads Agency. Joseph is a member of local and international road engineering steering committees (e.g., South Africa BitMat, SAICE, Transportation Research Board (TRB) AFP70, ASCE Geo-Institute Pavements), and has chaired sessions at annual TRB and southern African transportation conferences. He is a regular reviewer of accredited journals (e.g., SAICE, ASCE Transportation Engineering, Construction and Building Materials). From 2015 to 2020. Tshwane University of Technology appointed him as Industrial Professor for his contribution to the Department of Civil Engineering's teaching, research, and postgraduate student supervision. Joseph is currently rated by the South African National Roads Agency Ltd (SANRAL) on a scale of 96/100 for his achievements in road engineering.

James Maina is a Professor and Deputy Dean at University of Pretoria. Prior to joining the university, he was working at the CSIR where he joined as senior researcher in 2005 and became the first black Chief Researcher in 2009. This he achieved through his outstanding leadership in the advancement of science, engineering and technology specifically in his field of expertise, namely, transport infrastructure engineering. Maina hails from Tanzania where he obtained his initial training as a civil engineer and graduated as the best civil engineering student. He then joined Tanzanian National Construction Council (NCC) as assistant consultant. In 1994, and as a result of his academic achievements, he received the prestigious Monbusho scholarship for post-graduate studies in civil engineering in Japan. He studied at the Miyazaki University where he obtained both his MEng in 1997 and PhD in 2000. Upon his graduation, Maina joined the Research Institute of NIPPO Corporation in Tokyo. NIPPO is the biggest road construction company in Japan. In 2003, through the fellowship of Japan Society for the Promotion of Science (JSPS), Maina moved to Tokyo Denki University as a visiting scholar until 2005 when CSIR offered him a position as a senior researcher at their Pretoria campus. Between May 2013 and Sept 2014, Maina took a Sabbatical leave from the CSIR and went to work in Doha, Qatar as a senior pavement expert on a Quality Assurance/Quality Control project funded by the Qatar's Public Works Authority (PWA) and managed by Italian national roads agency - ANAS S.q.A. Maina returned to South Africa and joined the University of Pretoria as a professor of civil engineering in October 2014.

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