4 SYMMETRY MODELS FOR SQUARE CONTINGENCY TABLES WITH ORDERED CATEGORIES

Contingency tables are considered where the same variable with ordered categories is measured for both members of a matched pair. Responses are summarized in a two-way table in which both classifications have the same categories. One of the matters of interest in the analysis of square tables is the pattern of symmetry that may be exhibited by the cell probabilities in terms of their location relative to the main diagonal of the table. These models are discussed in more detail by Agresti (1984), Agresti (1990), Matthews (1995) and Tomizawa (1990).

4.1 SYMMETRY MODEL (S)

Consider an $I \times I$ contingency table with categorical variable $C = \{1, 2, \ldots, I\}$. A Poisson sampling procedure is assumed. Let Y_{ij} be the count in cell (i, j), y_{ij} the observed value of Y_{ij} and $n = \sum\sum y_{ij}$ the total counts. The counts can be arranged in a vector $\mathbf{Y}' = (Y_{11}, Y_{12}, \ldots, Y_{II})$ with $E(\mathbf{Y}) = \boldsymbol{\mu}$, the vector of expected counts. Let π_{ij} denote the probability that an observation falls in cell (i, j).
There is symmetry if

$$\pi_{ij} = \pi_{ji} \quad \text{for} \quad i \neq j.$$

Thus, if

$$\log\left(\mu_{ij}/\mu_{ji}\right) = \log \mu_{ij} - \log \mu_{ji} = 0 \quad \text{for} \quad i < j.$$

This can also be written as the constraint

$$\mathbf{g}(\boldsymbol{\mu}) = \mathbf{C} \log \boldsymbol{\mu} = \mathbf{0}$$

where, in the case of a 4×4 table the matrix $\mathbf{C}$ is given by

$$\begin{array}{cccccccccccccccc}
\mu_{11} & \mu_{12} & \mu_{13} & \mu_{14} & \mu_{21} & \mu_{22} & \mu_{23} & \mu_{24} & \mu_{31} & \mu_{32} & \mu_{33} & \mu_{34} & \mu_{41} & \mu_{42} & \mu_{43} & \mu_{44} \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0
\end{array} \tag{55}$$

Furthermore

$$\mathbf{G}_\mu = \frac{\partial}{\partial \boldsymbol{\mu}} \mathbf{g}(\boldsymbol{\mu}) = \mathbf{C}\mathbf{D}_\mu^{-1}.$$

The ML estimate for the vector with expected frequencies is given by

$$\begin{aligned}
\widehat{\boldsymbol{\mu}}_c &= \mathbf{y} - (\mathbf{G}_\mu \mathbf{V}_\mu)' \left(\mathbf{G}_\mathbf{y} \mathbf{V}_\mu \mathbf{G}_\mu'\right)^{-1} \mathbf{g}(\mathbf{y}) + o(\|\mathbf{y} - \boldsymbol{\mu}\|) \\
&= \mathbf{y} - \mathbf{C}' \left(\mathbf{C}\mathbf{D}_\mathbf{y}^{-1}\mathbf{C}'\right) \mathbf{C} \log(\mathbf{y}) + o(\|\mathbf{y} - \boldsymbol{\mu}\|).
\end{aligned}$$

The degrees of freedom for the likelihood ratio statistic is $I(I-1)/2$.

4.2 CONDITIONAL SYMMETRY (CS)

The conditional symmetry model is defined as

$$\pi_{ij} = \begin{cases} \tau\psi_{ij} & \text{when} \quad i < j \\ \psi_{ij} & \text{when} \quad i \geq j, \end{cases}$$

where $\psi_{ij} = \psi_{ji}$.
This is similar to

$$\log\left(\mu_{ij}/\mu_{ji}\right) = \log\tau \quad \text{for} \quad i < j$$

or

$$\log\mu_{ij} - \log\mu_{ji} = \log\tau \quad \text{for} \quad i < j.$$

This model can be formulated as $\mathbf{g}(\boldsymbol{\mu}) = \mathbf{0}$. Consider a 4×4 table with

$$\boldsymbol{\mu}' = \left(\mu_{11}, \mu_{12}, \mu_{13}, \mu_{14}, \mu_{21}, \mu_{22}, \mu_{23}, \mu_{24}, \mu_{31}, \mu_{32}, \mu_{33}, \mu_{34}, \mu_{41}, \mu_{42}, \mu_{43}, \mu_{44}\right).$$

Then

$$\mathbf{C}\log(\boldsymbol{\mu}) = \mathbf{X}\log\tau$$

where $\mathbf{C}$ is the matrix given in (55) and $\mathbf{X}' = (1,1,1,1,1,1) = \mathbf{1}_6'$.
Let $\mathbf{P} = \mathbf{I} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$. The constraint for the model is

$$\mathbf{g}(\boldsymbol{\mu}) = \mathbf{PC}\log(\boldsymbol{\mu}) = \mathbf{K}\log(\boldsymbol{\mu}) = \mathbf{0}$$

where $\mathbf{K} = \mathbf{PC}$.
Furthermore

$$\mathbf{G}_\mu = \frac{\partial}{\partial\boldsymbol{\mu}}\mathbf{g}(\boldsymbol{\mu}) = \mathbf{KD}_\mu^{-1}.$$

The ML estimate for the vector with expected frequencies is obtained iteratively with

$$\begin{aligned} \widehat{\boldsymbol{\mu}}_c &= \mathbf{y} - \left(\mathbf{G}_\mu\mathbf{V}_\mu\right)'\left(\mathbf{G}_\mathbf{y}\mathbf{V}_\mu\mathbf{G}_\mu'\right)^{-1}\mathbf{g}(\mathbf{y}) + o\left(\|\mathbf{y} - \boldsymbol{\mu}\|\right) \\ &= \mathbf{y} - \mathbf{K}'\left(\mathbf{KD}_\mathbf{y}^{-1}\mathbf{K}'\right)\mathbf{K}\log(\mathbf{y}) + o\left(\|\mathbf{y} - \boldsymbol{\mu}\|\right). \end{aligned}$$

The ML estimate for τ is obtained by

$$\widehat{\tau} = \exp\left[\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{C}\log\left(\widehat{\boldsymbol{\mu}}_c\right)\right].$$

The degrees of freedom for the likelihood ratio statistic is $(I+1)(I-2)/2$.

4.3 DIAGONALS-PARAMETER SYMMETRY (DPS)

Goodman (1979) defines the diagonals-parameter symmetry model as

$$\pi_{ij} = \begin{cases} \delta_{j-i}\psi_{ij} & \text{when} \quad i < j, \\ \psi_{ij} & \text{when} \quad i \geq j, \end{cases}$$

where $\psi_{ij} = \psi_{ji}$.
Consider a 4×4 table. The model can be written as

$$\mathbf{C}\log(\boldsymbol{\mu}) = \mathbf{X}\log\boldsymbol{\delta}$$

where $\mathbf{C}$ is the matrix given in (55),

$$\mathbf{X} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

and $\boldsymbol{\delta}' = (\delta_1, \delta_2, \delta_3)$.
Let $\mathbf{P} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. The constraint for the model is

$$\mathbf{g}(\boldsymbol{\mu}) = \mathbf{PC}\log(\boldsymbol{\mu}) = \mathbf{K}\log(\boldsymbol{\mu}) = \mathbf{0}$$

where $\mathbf{K} = \mathbf{PC}$.
The ML estimates for the expected frequencies are obtained iteratively by

$$\widehat{\boldsymbol{\mu}}_c = \mathbf{y} - \mathbf{K}'\left(\mathbf{K}\mathbf{D}_{\mathbf{y}}^{-1}\mathbf{K}'\right)\mathbf{K}\log(\mathbf{y}) + o\left(\|\mathbf{y} - \boldsymbol{\mu}\|\right).$$

The ML estimate for $\boldsymbol{\delta}$ is obtained by

$$\widehat{\boldsymbol{\delta}} = \exp\left[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{C}\log(\widehat{\boldsymbol{\mu}}_c)\right].$$

The degrees of freedom for the likelihood ratio statistic is $(I-1)(I-2)/2$.

4.4 LINEAR DIAGONALS-PARAMETER SYMMETRY (LDPS)

The linear diagonals-parameter symmetry model is defined as

$$\pi_{ij} = \begin{cases} \rho^{j-i}\psi_{ij} & \text{when} \quad i < j, \\ \psi_{ij} & \text{when} \quad i \geq j, \end{cases}$$

where $\psi_{ij} = \psi_{ji}$.
Consider a 4×4 table. The model can be written as

$$\mathbf{C}\log(\boldsymbol{\mu}) = \mathbf{X}\log\rho$$

where $\mathbf{C}$ is the matrix given in (55) and $\mathbf{X}' = (1, 2, 3, 1, 2, 1)$.
Let $\mathbf{P} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. The constraint for the model is

$$\mathbf{g}(\boldsymbol{\mu}) = \mathbf{PC}\log(\boldsymbol{\mu}) = \mathbf{K}\log(\boldsymbol{\mu}) = \mathbf{0}$$

where $\mathbf{K} = \mathbf{PC}$.
The ML estimates for the expected frequencies are obtained iteratively by

$$\widehat{\boldsymbol{\mu}}_c = \mathbf{y} - \mathbf{K}'\left(\mathbf{K}\mathbf{D}_{\mathbf{y}}^{-1}\mathbf{K}'\right)\mathbf{K}\log(\mathbf{y}) + o\left(\|\mathbf{y} - \boldsymbol{\mu}\|\right).$$

The ML estimate for ρ is obtained by

$$\widehat{\rho} = \exp\left[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{C}\log(\widehat{\boldsymbol{\mu}}_c)\right].$$

The degrees of freedom for the likelihood ratio statistic is $(I+1)(I-2)/2$.

4.5 ANOTHER LINEAR DIAGONALS-PARAMETER SYMMETRY MODEL (ALDPS)

Another linear diagonals-parameter symmetry model (ALDPS) is defined by Tomizawa (1990) as

$$\pi_{ij} = \begin{cases} \rho^{I-(j-i)}\psi_{ij} & \text{when} \quad i < j, \\ \psi_{ij} & \text{when} \quad i \geq j, \end{cases}$$

where $\psi_{ij} = \psi_{ji}$.
Consider a 4×4 table. The model can be written as

$$\mathbf{C}\log(\boldsymbol{\mu}) = \mathbf{X}\log\rho$$

where $\mathbf{C}$ is the matrix given in (55) and $\mathbf{X}' = (3, 2, 1, 3, 2, 3)$.

Let $\mathbf{P} = \mathbf{I} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$. The const

$$\mathbf{g}(\boldsymbol{\mu}) = \mathbf{PC}\log(\boldsymbol{\mu}) = \mathbf{K}\log(\boldsymbol{\mu}) = \mathbf{0}$$

where $\mathbf{K} = \mathbf{PC}$.
The ML estimates for the expected frequencies are obtained iteratively by

$$\widehat{\boldsymbol{\mu}}_c = \mathbf{y} - \mathbf{K}'\left(\mathbf{K}\mathbf{D}_{\mathbf{y}}^{-1}\mathbf{K}'\right)\mathbf{K}\log(\mathbf{y}) + o\left(\|\mathbf{y} - \boldsymbol{\mu}\|\right).$$

The ML estimate for ρ is obtained by

$$\widehat{\rho} = \exp\left[\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{C}\log\left(\widehat{\boldsymbol{\mu}}_c\right)\right].$$

The degrees of freedom for the likelihood ratio statistic is $(I+1)(I-2)/2$.

4.6 2-RATIOS-PARAMETER SYMMETRY (2RPS)

The 2-ratios-parameter symmetry model is defined by Tomizawa (1990) as

$$\pi_{ij} = \begin{cases} \phi\theta^{j-i-1}\psi_{ij} & \text{when} \quad i < j, \\ \psi_{ij} & \text{when} \quad i \geq j, \end{cases}$$

where $\psi_{ij} = \psi_{ji}$.
Consider a 4×4 table. The model can be written as

$$\mathbf{C}\log(\boldsymbol{\mu}) = \mathbf{X}\log\boldsymbol{\zeta}$$

where $\mathbf{C}$ is the matrix given in (55),

$$\mathbf{X} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$$

and $\boldsymbol{\zeta}' = (\phi, \theta)$.
Let $\mathbf{P} = \mathbf{I} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$. The constraint for the model is

$$\mathbf{g}(\boldsymbol{\mu}) = \mathbf{PC}\log(\boldsymbol{\mu}) = \mathbf{K}\log(\boldsymbol{\mu}) = \mathbf{0}$$

where $\mathbf{K} = \mathbf{PC}$.
The ML estimates for the expected frequencies are obtained iteratively by

$$\widehat{\boldsymbol{\mu}}_c = \mathbf{y} - \mathbf{K}'\left(\mathbf{K}\mathbf{D}_{\mathbf{y}}^{-1}\mathbf{K}'\right)\mathbf{K}\log(\mathbf{y}) + o\left(\|\mathbf{y} - \boldsymbol{\mu}\|\right).$$

The ML estimate for $\boldsymbol{\zeta}$ is

$$\widehat{\boldsymbol{\zeta}} = \exp\left[\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{C}\log\left(\widehat{\boldsymbol{\mu}}_c\right)\right].$$

The degrees of freedom for the likelihood ratio statistic is $\left(I^2 - I - 4\right)/2$.

4.7 QUASI SYMMETRY (QS)

Quasi symmetry is defined as

$$\pi_{ij} = \alpha_i \beta_j \psi_{ij} \qquad \text{for all} \quad i, j,$$

where $\psi_{ij} = \psi_{ji}$.
Consider a 4×4 table. The model can be written as

$$\mathbf{C} \log(\boldsymbol{\mu}) = \mathbf{X} \log \boldsymbol{\theta}$$

where $\mathbf{C}$ is the matrix given in (55),

$$\mathbf{X} = \begin{pmatrix} 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 \end{pmatrix}$$

and $\boldsymbol{\theta}' = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$.
Let $\mathbf{P} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. The constraint for the model is

$$\mathbf{g}(\boldsymbol{\mu}) = \mathbf{PC} \log(\boldsymbol{\mu}) = \mathbf{K} \log(\boldsymbol{\mu}) = \mathbf{0}$$

where $\mathbf{K} = \mathbf{PC}$.
The ML estimates for the expected frequencies are obtained iteratively by

$$\widehat{\boldsymbol{\mu}}_c = \mathbf{y} - \mathbf{K}'\left(\mathbf{K}\mathbf{D}_{\mathbf{y}}^{-1}\mathbf{K}'\right)\mathbf{K}\log(\mathbf{y}) + o\left(\|\mathbf{y} - \boldsymbol{\mu}\|\right).$$

The ML estimate for $\boldsymbol{\theta}$ is obtained by

$$\widehat{\boldsymbol{\theta}} = \exp\left[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{C}\log(\widehat{\boldsymbol{\mu}}_c)\right].$$

The degrees of freedom for the likelihood ratio statistic is $(I-1)(I-2)/2$.

4.8 EXAMPLE

Table 4.1, taken from Agresti (1984) and also discussed by Tomizawa (1990) is the father's and son's occupational mobility data in Britain. The table relates father's and son's occupational status category. The symmetry models discussed in this chapter were fitted to the data. Table 4.2 gives the expected cell frequencies for each model, Table 4.3 gives the goodness of fit statistics, and Table 4.4 gives the ML estimates for the model parameters.

TABLE 4.1: Occupational Status for British Father-Son Pairs.

	Son's Status					
Father's Status	1	2	3	4	5	Total
1	50	45	8	18	8	129
2	28	174	84	154	55	495
3	11	78	110	223	96	518
4	14	150	185	714	447	1510
5	3	42	72	320	411	848
Total	106	489	459	1429	1017	

TABLE 4.2: Occupational Status for British Father-Son Pairs.

	Son's Status					
Father's Status	1	2	3	4	5	Total
1	50	45	8	18	8	129
	–	(36.5)[a]	(9.5)	(16.0)	(5.5)	
	–	(40.7)[b]	(10.6)	(17.8)	(6.1)	
	–	(41.4)[c]	(10.0)	(18.1)	(8.0)	
	–	(38.9)[d]	(10.7)	(19.1)	(6.9)	
	–	(41.1)[e]	(10.4)	(17.0)	(5.7)	
	–	(41.0)[f]	(10.5)	(17.3)	(5.8)	
	–	(42.2)[g]	(10.7)	(18.8)	(7.3)	
2	28	174	84	154	55	495
	(36.5)	–	(81.0)	(152.0)	(48.5)	
	(32.3)	–	(90.3)	(169.5)	(54.1)	
	(31.6)	–	(91.8)	(159.7)	(54.9)	
	(34.1)	–	(86.3)	(172.0)	(58.0)	
	(31.9)	–	(91.2)	(166.4)	(51.6)	
	(32.0)	–	(90.9)	(167.6)	(52.5)	
	(30.8)	–	(78.3)	(155.2)	(56.8)	
3	11	78	110	223	96	518
	(9.5)	(81.0)	–	(204.0)	(84.0)	
	(8.4)	(71.7)	–	(227.5)	(93.7)	
	(9.0)	(70.2)	–	(213.2)	(88.3)	
	(8.3)	(76.7)	–	(217.5)	(95.0)	
	(8.6)	(70.8)	–	(229.7)	(91.9)	
	(8.5)	(71.1)	–	(229.0)	(92.6)	
	(8.3)	(83.7)	–	(215.0)	(101.0)	
4	14	150	185	714	447	1510
	(16.0)	(152.0)	(204.0)	–	(383.5)	
	(14.2)	(134.5)	(180.5)	–	(427.7)	
	(13.9)	(144.3)	(176.8)	–	(434.6)	
	(12.9)	(132.0)	(190.5)	–	(408.9)	
	(15.0)	(137.6)	(178.3)	–	(431.8)	
	(14.7)	(136.4)	(179.0)	–	(430.6)	
	(13.2)	(148.8)	(193.0)	–	(441.0)	
5	3	42	72	320	411	848
	(5.5)	(48.5)	(84.0)	(383.5)	–	
	(4.9)	(42.9)	(74.3)	(339.3)	–	
	(3.0)	(42.1)	(79.7)	(332.4)	–	
	(4.1)	(39.0)	(73.0)	(358.2)	–	
	(5.3)	(45.4)	(76.1)	(335.2)	–	
	(5.2)	(44.5)	(75.4)	(336.4)	–	
	(3.7)	(40.2)	(67.0)	(326.0)	–	
Total	106	489	459	1429	1017	

[a]Estimated expected frequencies for symmetry model (S).
[b]Estimated expected frequencies for conditional symmetry model (CS).
[c]Estimated expected frequencies for diagonals–parameter symmetry model (DPS).
[d]Estimated expected frequencies for linear diagonals–parameter symmetry model (LDPS).
[e]Estimated expected frequencies for another linear diagonals–parameter symmetry model (ALDPS).
[f]Estimated expected frequencies for 2-ratio–parameter symmetry model (2RPS).
[g]Estimated expected frequencies for quasi symmetry model (QS).

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TABLE 4.3: Goodness of Fit statistics.

Model	df	χ^2	G^2	AIC^+
S	10	37.22	37.46	17.46
CS	9	10.30	10.35	−7.65
DPS	6	6.44	6.44	−5.56
LDPS	9	17.09	17.13	−0.87
ALDPS	9	10.05	10.13	−7.87
2RPS	8	9.96	10.02	−5.98
QS	6	4.67	4.66	−7.34

TABLE 4.4: ML Estimates of model parameters.

Model	Parameters	ML Estimates
S	–	–
CS	τ	$\widehat{\tau} = 1.26$
DPS	$\boldsymbol{\delta}' = (\delta_1, \delta_2, \delta_3, \delta_4)$	$\widehat{\delta}_1 = 1.31$, $\widehat{\delta}_2 = 1.11$, $\widehat{\delta}_3 = 1.30$, $\widehat{\delta}_4 = 2.67$
LDPS	ρ	$\widehat{\rho} = 1.14$
ALDPS	ρ	$\widehat{\rho} = 1.07$
2RPS	$\boldsymbol{\zeta}' = (\phi, \theta)$	$\widehat{\phi} = 1.28$, $\widehat{\theta} = 0.96$
QS	$\boldsymbol{\theta}' = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$	$\widehat{\alpha}_1 = 1.17$, $\widehat{\alpha}_2 = 1.00$ $\widehat{\alpha}_3 = 1.03$ $\widehat{\alpha}_4 = 0.98$ $\widehat{\alpha}_5 = 0.84$ $\widehat{\beta}_1 = 0.85$ $\widehat{\beta}_2 = 1.00$ $\widehat{\beta}_3 = 0.97$ $\widehat{\beta}_4 = 1.02$ $\widehat{\beta}_5 = 1.19$

Discussion of results

Tomizawa (1990) selected the best model by using the modified AIC defined as

$$AIC^+ = G^2 - 2\,(df)\,.$$

The best fitting model is the one with the smallest AIC^+, which in this example is the ALDPS and CS models.

For the ALDPS model $\widehat{\rho} = 1.065$. Thus, the proportion of father-son pairs for which the son had a k grades higher status category than the father, for $k = 1, 2, 3, 4$, is estimated to be $(1.065)^{5-k}$ times higher than the proportion in which the father had the k grades higher status category.

For the CS model $\widehat{\tau} = 1.26$ which means that for each pair of categories, (i, j) and (j, i), the proportion of father-son pairs for which the son had the higher status is estimated to be 1.26 times higher than the proportion in which the father had the higher status.

The program for this example is given in the appendix and can be used for any square contingency table with ordered categories.