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Investigating learners' misconceptions on division of whole numbers to identify a goal of research lesson – a case of lesson study

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ABSTRACT

Identifying and addressing learners' misconceptions is critical for effective mathematical instruction, as unresolved conceptual gaps hinder long-term mathematical development. Diagnostic assessment provides a valuable means of uncovering these misconceptions and informing targeted teaching strategies. Despite this, classroom practices often prioritise procedural fluency over deep conceptual understanding, allowing learners to progress without fully addressing underlying difficulties. This study investigated Grade 6 learners' misconceptions about the division of whole numbers using a two-tier multiple-choice diagnositc test and unstructured interviews. Guided by Revised Bloom's Taxonomy and Newman's Error Analysis Theory within an interpretivist qualitative case study design, the study employed thematic analysis to categorise learners' errors. The findings revealed three dominant types of errors: transformation errors, comprehension errors, and processing skill errors. These results underscore the importance of strengthening learners' grasp of foundational concepts such as place value and mathematical language. The study recommends that teachers explicitly integrate diagnostic assessments into classroom practice and design instructional interventions that directly target specific error types, thereby supporting both conceptual understanding and procedural fluency in division.

Keywords: division, two-tier diagnostic assessment, misconceptions, conceptual understanding, lesson study

INTRODUCTION

Mathematics has been a core component of school curricula globally since the 1890s, and its development has been significantly influenced by technological innovations (Alfageh et al., 2024). As education continues to evolve in response to such advancements, the necessity for ongoing research and curricular adaptation becomes increasingly evident. Despite this progress, mathematics education continues to face global challenges, with persistent concerns about learners' achievement levels (Mullis et al., 2020). In the South African context, the performance of learners in mathematics remains notably low when compared to their international counterparts. According to Aunio et al. (2016), South African learners demonstrate limited proficiency in basic mathematical skills. This is further substantiated by the Trends in International Mathematics and Science Study (TIMSS) 2019 results, which ranked South Africa among the lowest-performing countries out of 64 participating countries at both Grade 4 and Grade 8 levels (Reddy et al., 2019). A significant issue highlighted by these results is learners' weak grasp of foundational numeracy concepts.

Understanding basic mathematical operations is critical for learners' overall progress in mathematics. Foundational skills, such as number sense and arithmetic operations, form the basis upon which more complex mathematical reasoning and problem-solving abilities are built (Alfageh et al., 2024; Aunio et al., 2016). However, research indicates that classroom instruction often emphasizes rote counting and obtaining correct answers over conceptual understanding (Graven & Venkatakrishnan, 2018). This practice limits learners' mathematical development and contributes to widespread misconceptions. In particular, conceptual clarity in basic operations such as divisions is essential. When learners understand these operations deeply, they are better equipped to engage with more advanced mathematical content (Alanazi & Threlfall, 2008; Aunio et al., 2016). Teachers play a pivotal role in identifying and addressing such learning difficulties. As Sekao (2023) notes, accurately diagnosing learners' challenges is fundamental to effective teaching. A deeper understanding of learners' misconceptions enables teachers to adjust their instructional strategies to better meet individual learning needs, thereby enhancing both teaching effectiveness and learning outcomes (Alfageh et al., 2024).

While several studies have explored learners' misconceptions about division, much of the existing literature has been conducted in contexts outside South Africa (Alanazi, 2008; Aunio et al., 2016). There is comparatively limited research that investigates these misconceptions in underperforming systems such as South Africa, where issues of language, prior knowledge, and resource constraints may shape learners' errors in unique ways. Moreover, earlier studies have typically catalogued common misconceptions but have provided less focus on their instructional implications—specifically, how teachers can use knowledge of these misconceptions to design responsive lesson plans. This study, therefore, makes two distinct contributions: First, by offering an updated and context-specific account of South African learners' misconceptions about whole number division; and second, by extending prior work to highlight how these misconceptions can directly inform instructional practices and lesson planning. This study, therefore, aimed to investigate learners' misconceptions about the division of whole numbers within the South African context, where persistent challenges in mathematics performance create distinctive learning difficulties. Beyond simply identifying common misconceptions, the goal was to generate context-specific insights that extend existing research and to examine how these misconceptions can be used as a basis for improving lesson planning and instructional practices. Accordingly, the study was guided by the following research questions:

RQ1 What are the common misconceptions learners have about the division of whole numbers?

RQ2 How can the learners' misconceptions about the division of whole numbers inform lesson planning?

Mathematical Thinking Lesson Study in South Africa

Lesson Study is a collaborative professional development approach that aims to improve teaching practices through systematic and reflective inquiry into classroom instruction (Seino & Foster, 2021; Sekao, 2023; Sekao & Engelbrecht, 2022; Takahashi, 2006; Takahashi & McDougal, 2018). In the South African context, Lesson Study has been adopted to deepen learners' understanding of mathematical concepts and to enhance the quality of teaching, thereby contributing to educational improvement and the professional growth of teachers (Sekao & Engelbrecht, 2022). The South African Lesson Study cycle typically comprises five stages:

(1) Diagnostic assessment and analysis,

(2) Collaborative lesson planning,

(3) Lesson presentation and observation,

(4) Post-lesson reflection, and

(5) Lesson improvement (Sekao, 2023; Sekao & Engelbrecht, 2022).

This cycle encourages teachers to work collectively to design, deliver, observe, and refine lessons with the aim of improving both teaching and learning. The process begins with diagnostic assessment, a critical tool for identifying learners' strengths and weaknesses. It provides valuable insights into learners' misconceptions and informs the development of targeted instructional strategies (Alfageh et al., 2024; Department of Basic Education (DBE), 2011). During this phase, teachers evaluate learners' understanding to design lesson plans that respond to the specific learning needs uncovered during the assessment.

In the second stage—collaborative lesson planning—teachers work in teams to develop lesson plans grounded in the findings from the diagnostic assessment. This collaborative process integrates the diverse pedagogical knowledge and experiences of participating teachers to design a structured and responsive instructional framework (Takahashi & McDougal, 2018). The third stage involves the presentation of the research lesson by one teacher, while the rest of the Lesson Study team observes learners' engagement and thinking throughout the lesson (Sekao, 2023). The fourth stage, post-lesson reflection, involves collective analysis and discussion of the observed lesson. During this phase, teachers critically reflect on teaching strategies and learner responses, often with input from external experts such as experienced educators or academic researchers (Seino & Foster, 2021). In the final stage, teachers propose improvements to the lesson based on their observations, reflections, and the insights gained throughout the cycle (Sekao, 2023; Takahashi & McDougal, 2018).

A defining feature of Lesson Study is its emphasis on teacher collaboration. This collaborative practice fosters deeper understanding, shared problem-solving, and pedagogical innovation (Sekao, 2023; Sekao & Engelbrecht, 2022). It enables teachers to critically evaluate their instructional approaches and learner outcomes, promoting continuous professional learning, the development of teaching expertise, and the cultivation of mathematical thinking (Takahashi, 2006). Furthermore, Lesson Study nurtures a culture of continuous improvement that can enhance learner engagement and achievement, ultimately contributing to improved educational outcomes (Alfageh et al., 2024). In this study, we focus specifically on the first stage of the Lesson Study cycle: diagnostic assessment. By implementing diagnostic assessment, we aim to analyse learners' misconceptions and use these insights to inform more effective and targeted mathematics instruction.

The Role of Diagnostics Assessment in Mathematics

Studies Diagnostic assessment plays a central role in the Lesson Study cycle and is widely recognised as one of its most critical stages (Vetter, 2022). It enables teachers to gain a clear understanding of learners' current levels of understanding and identify specific learning gaps. Based on this analysis, teachers can set targeted instructional goals and conduct Lesson Study aligned with these objectives (Alfageh et al., 2024). The process of setting clears and achievable learning goals serves as a guiding framework for effective instructional planning within Lesson Study. By leveraging diagnostic assessment, educators not only enhance their professional knowledge but also support the development of learners' mathematical thinking (Alfageh et al., 2024; Chufama & Sithole, 2021). Importantly, diagnostic assessment goes beyond merely identifying errors—it provides insights into the underlying causes of learners' misconceptions. Understanding these root causes enables more strategic and effective pedagogical interventions (Liu et al., 2020; Vetter, 2022).

Adapting teaching strategies to reflect learners' actual understanding is expected to significantly improve the overall quality of instruction. Teachers should only proceed with formal instruction once they have a comprehensive understanding of their learners' conceptual readiness (Boster, 2024). When foundational mathematical concepts are not well established, learners risk falling behind, and continued instruction without addressing these gaps may widen the disconnect between the teacher's expectations and learners' actual abilities (Liu et al., 2020). This instructional gap can have serious consequences, including declining performance in mathematics and the development of negative attitudes toward the subject. Therefore, by conducting diagnostic assessments at the outset, teachers are better equipped to design lesson objectives that respond to learners' needs. Within the Lesson Study framework, this enables the delivery of more targeted, effective instruction that supports conceptual consolidation and ultimately enhances the quality of mathematics education.

Learners' Difficulty When Dealing with the Division of Whole Numbers

Division is often perceived as the inverse of multiplication and conceptually linked to repeated subtraction, just as multiplication is linked to repeated addition (Alanazi, 2008). This traditional viewpoint positions multiplication and division as straightforward extensions of addition and subtraction (De Castro et al., 2002). However, Siegler and Lortie-Forgues (2015) argue that these operations are distinct mathematical constructs that require learners to adopt different perspectives on numbers and operations. Furthermore, Alanazi (2008) emphasises that the relationship among the four basic operations—addition, subtraction, multiplication, and division—is not isolated or linear but rather interconnected. A deep conceptual understanding of multiplication and division depends on prior mastery of addition and subtraction, and thus mathematical thinking cannot progress effectively without first solidifying these foundational concepts. This developmental process forms part of what Alanazi (2008) refers to as conceptual structure—a framework comprising key concepts and their interrelationships.

Effective mathematical learning requires both conceptual knowledge and procedural knowledge (Rittle-Johnson et al., 2001; Tan Sisman & Aksu, 2016). Conceptual knowledge refers to an understanding of the principles and relationships that underlie mathematical operations, providing a meaningful context for why procedures work (Tan Sisman & Aksu, 2016). Procedural knowledge, on the other hand, relates to the execution of specific rules and algorithms for performing operations (Rittle-Johnson et al., 2001). A balanced integration of these two types of knowledge fosters deeper learning. In the context of division, learners must understand how it connects to and builds upon prior conceptual knowledge of addition, subtraction, and multiplication. A critical element underpinning this conceptual foundation is the understanding of place value, which is vital for interpreting multi-digit numbers and performing long division accurately. Themane and Luneta (2021, p. 3) stress that "an understanding of place value is the most crucial aspect of mathematics that underpins much of the number work undertaken by children." Strengthening learners' grasp of place value enhances their ability to make sense of numerical operations and supports the development of number sense.

To foster conceptual understanding, De Castro et al. (2002) advocate for instructional strategies that promote the internalisation of number concepts. These include the use of visual representations such as area models and pie charts, which help learners visualise the effects of division and multiplication. Additionally, Ndlovu et al. (2019) recommend a blended approach that combines the use of concrete manipulatives and language-based instruction. Concrete objects—like counters or base-ten blocks—enable learners to physically engage with abstract concepts, making them more accessible, particularly at the primary school level (De Castro et al., 2002). Complementing this, mathematical language instruction encourages learners to verbalise their reasoning, thus deepening their conceptual understanding (Götze, 2018; Themane & Luneta, 2021). Facilitating peer discussions around division tasks, for instance, can sharpen both learners' mathematical thinking and their communication skills. Götze (2018) and Ndlovu et al. (2019) highlight that this dual strategy of manipulatives and verbal engagement significantly enhances learners' mathematical understanding and problem-solving competence.

In summary, promoting procedural fluency must be grounded in conceptual understanding. By constructing a solid conceptual base in operations such as addition and subtraction, and building upon this foundation to explore multiplication and division, learners are better equipped to grasp complex calculations and develop flexible mathematical thinking. This integrated instructional approach not only addresses learners' common difficulties with whole number division but also contributes to a more coherent and connected mathematical knowledge system (Alanazi & Threlfall, 2008; Götze, 2018; Ndlovu et al., 2019).

THEORETICAL FRAMEWORK OF THE RESEARCH

This study was guided by two complementary theoretical lenses: Revised Bloom's Taxonomy (Anderson et al., 2001) and Newman's error analysis (Newman, 1977). These frameworks were employed to design the diagnostic test and to interpret learners' errors in whole-numbers division, respectively.

Revised Bloom's Taxonomy provided the foundation for developing test items varying cognitive levels. Specifically, the taxonomy was applied to ensure that the diagnostic test contained a balanced distribution of items targeting lower-order levels such as "Remember" and "Understand", middle-order levels such as "Apply", and higher-order levels such as "Analyse". For instance, a recall-level question might ask learners to calculate $24 \div 6$, whereas an analysis-level item could require them to justify why $15 \div 4$ results in both a quotient and a remainder. By mapping each test item explicitly to a cognitive level, the diagnostic tool moved beyond assessing rote recall and enabled us to probe the depth of learners' conceptual and procedural understanding. Newman's error analysis framework was then systematically applied to interpret learners' responses. Following Newman's five stages:

1) Reading,
2) Comprehension,
3) Transformation,

Table 1. Alignment of revised bloom's taxonomy levels with diagnostic test items and Newman's error types

Bloom's taxonomy Level	Diagnostic test item	Possible Newman error type(s)	Illustrative example of error
Remember (Recall facts/procedures)	Compute (54÷ 2).	Process skills error	Learner sets up long division correctly but subtracts incorrectly, giving 26 instead of 27.
Understand (Interpret meaning)	You have 48 pencils. You distribute the pencils to 3 people equally. How many pencils does each person receive?	Comprehension error	Learner interprets the problem as multiplication: ($48\times 3 = 144$).
Apply (Use a procedure in context)	Solve: "A teacher divides 45 books into groups of 9. How many groups are there?"	Transformation error	Learner understands context but translates it as (45×9) instead of ($45 \div 9$).
Analyze (Explain/justify reasoning)	Explain why ($15 \div 4$) gives a quotient and a remainder.	Comprehension and transformation errors	Learner states "15 cannot be divided by 4" (fails to recognize remainder concept).

4) Process skills, and
5) Encoding (Rahman & Effendy, 2019).

This article focused specifically on three types of errors most relevant to division misconceptions: comprehension, transformation, and process skill errors. These were chosen because of their strong alignment with common conceptual and procedural misconceptions in mathematics education literature. Comprehension errors, which resemble conceptual errors, involve a misunderstanding of the fundamental concepts, rules, or relationships that underpin mathematical reasoning (Alanazi, 2008; Rahman & Effendy, 2019). At the primary school level, where conceptual foundations are being built, these errors are especially significant. Transformation errors refer to difficulties in converting a verbal or symbolic problem into a mathematical expression. Learners exhibiting this type of error may misinterpret a problem statement or apply incorrect formulas (Anugrah & Kusmayadi, 2019; Rahman & Effendy, 2019). Process skill errors relate to procedural execution. These occur when learners know the correct procedure or formula but make errors in calculation, algorithmic steps, or logical sequencing during problem-solving (Rahman & Effendy, 2019). Learners may apply familiar strategies incorrectly due to misunderstandings or lack of fluency in procedures. Each learner's written response was coded according to the stage where the error first appeared. For example, a learner who misinterpreted "divide 20 sweets among 4 children" as multiplication committed a comprehension error, while one who understood the problem but incorrectly represented it as 20×4 displayed a transformation error. Similarly, a learner who correctly set up 20÷4 but made a subtraction slip in the long division algorithm exhibited a process skill error. This coding process enabled us to identify not only whether an answer was incorrect but also why it was incorrect and at which cognitive stage the difficulty emerged.

To make this operationalisation explicit, the diagnostic test items were aligned with Bloom's cognitive levels, and the learners' responses were analysed through Newman's error categories. This dual alignment ensured that the test captured both the level of cognitive demand and the specific nature of the errors. **Table 1** illustrates this alignment with sample items and typical learner errors.

Previous studies have applied either Bloom's Taxonomy or Newman's error analysis in mathematics education, but rarely together, and even less so in relation to whole-number division misconceptions. For example, Radmehr and Drake (2018) used Bloom's framework to analyse the cognitive demand of mathematics tasks, demonstrating how task design influences the kinds of thinking students engage in. Similarly, Rahman and Effendy (2019) and Anugrah and Kusmayadi (2019) operationalised Newman's framework to categorise common learner errors in problem-solving, highlighting the prevalence of comprehension and transformation difficulties. These studies confirm the individual usefulness of each framework but typically treat them in isolation.

A number of studies have deliberately combined Revised Bloom's Taxonomy with Newman's error analysis in examining whole-number division. This study, therefore, contributes to the field by integrating the two frameworks in a complementary manner: Bloom's taxonomy structured the design of diagnostic items across cognitive levels, while Newman's framework provided a systematic method for interpreting the nature of errors. This dual application is theoretically significant because it connects what kind of thinking a task demands with where learners struggle in executing it. Methodologically, the integration strengthens diagnostic power by allowing errors to be traced both to the cognitive level of the item and to the specific stage of problem-solving where misconceptions occur. Together, these frameworks provide a comprehensive lens for diagnosing and understanding mathematical misconceptions. When applied in tandem, they enable teachers to identify the nature and source of learners' errors, design targeted interventions, and adapt curriculum and assessment strategies accordingly (Prismana et al., 2018; Radmehr & Drake, 2018).

METHODOLOGY

This section outlines the research design, sampling procedures, data collection methods, and data analysis techniques employed in this study. A qualitative case study approach was adopted to investigate Grade 6 learners' misconceptions in whole-number division through a two-tier diagnostic test and follow-up interviews.

Research Design and Sampling

This study adopted a qualitative case study design. Case studies allow for in-depth exploration of a specific phenomenon within its real-life context, offering nuanced and comprehensive insights into the research problem (Baškarada, 2013; Tetnowski, 2015). The case study approach was particularly suited to this research, as it facilitated a detailed examination of learners' mathematical misconceptions in a natural classroom setting and enabled the development of informed and practical recommendations for instruction. Lesson study principles were incorporated as a structured collaborative research support strategy rather than as the primary research method. The Lesson Study team, consisting of the first author (researcher) and two practicing teachers from the school, collaboratively contributed to:

(a) The design and moderation of the two-tier diagnostic test,

(b) Development of interview questions responsive to identified misconceptions, and

(c) Analysis and validation of learners' responses.

The teachers did not participate in classroom observations, shared lesson planning, or joint teaching, so the study does not represent a full traditional lesson study cycle. Instead, lesson study principles—collaboration, reflective dialogue, and iterative development of assessment instruments—guided the team's work to ensure pedagogical relevance and methodological rigor. This approach allowed teachers to provide insights grounded in classroom practice while supporting research validity.

Participants

The primary participants were 18 Grade 6 learners enrolled in a single class at an independent primary school in South Africa. Grade 6 was deliberately chosen as it represents a critical transition point in mathematical development, where foundational arithmetic skills begin to be applied to problem-solving and higher-order thinking tasks (Ross et al., 2006). The teachers in the Lesson Study team were secondary participants, contributing expertise and guidance for the design, interpretation, and validation of learner data. No classroom teaching or observations were conducted by the Lesson Study team, and the teachers were not treated as subjects for data collection; their involvement focused solely on supporting the research process.

To this end, a diagnostic test focused on division was administered to surface learners' understanding and misconceptions. The study was conducted during the second period of the school day and lasted approximately 30 minutes. The diagnostic test was administered under the supervision of the class teacher, who also serves as the mathematics department head. This arrangement ensured a consistent testing environment and minimised external influence on learners' responses. The purpose of the diagnostic test was to identify misconceptions in learners' understanding of division and to examine how these misconceptions influenced their mathematical reasoning and performance. The case study design enabled in-depth analysis of how such misconceptions are constructed, how they manifest in learners' written responses, and how they may be addressed through more effective, conceptually focused instructional strategies (Rai & Thapa, 2015; Tetnowski, 2015)

Data Collection

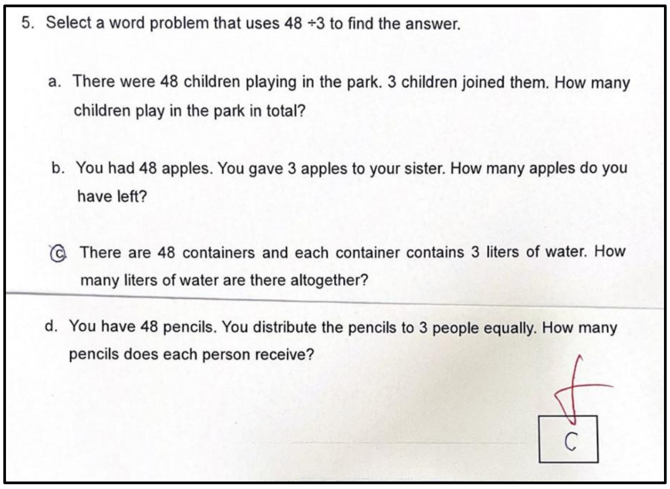
Data were collected primarily from leaners via a two-tier diagnostic test (2TDT) and unstructured interviews. The 2TDT, developed collaboratively with the Lesson Study team, consisted of five division-focused questions mapped to Revised Bloom's taxonomy to capture different cognitive levels. Each question was designed to probe learners' reasoning, not just their answers, allowing identification of conceptual and procedural misconceptions (Gurel et al., 2015). Following the administration of the test, learners' responses were analysed to identify misconceptions related to the division of whole numbers–this formed the basis of the document analysis. To ensure methodological rigor, teachers contributed data in the form of structured feedback and reflective notes on:

1) The alignment and clarity of the diagnostic test items,
2) Appropriateness of distractors reflecting anticipated misconceptions, and
3) Suggestions for follow-up interview questions that could probe learners' reasoning effectively.

These teacher contributions were documented in reflective memos and meeting notes, which were analysed qualitatively to identify points of consensus and divergence. Notes were coded thematically according to the focus of the feedback (e.g., item clarity, misconception alignment, instructional relevance), and these codes informed revisions to the test and interview protocol. This procedure ensured that teacher expertise was systematically integrated without treating them as research participants. Following the 2TDT administration, selected learners participated in unstructured interviews. Interview questions were iteratively refined based on the analysis of learners' test responses and the teachers' feedback, ensuring that discussions targeted specific misconceptions. Interviews also allowed exploration of learners' reasoning behind correct answers, revealing flawed or incomplete understanding that the 2TDT alone could not capture (Adhabi & Anozie, 2017; Gurel et al., 2015; Maree, 2020).

Data Analysis

Learner data were analysed using a deductive-inductive thematic analysis approach (Braun & Clarke, 2019; Terry et al., 2017), guided by Newman's classification of misconceptions (conceptual and procedural). Test items were designed with distractors reflecting anticipated misconceptions, supporting the deductive aspect, while inductive coding allowed emergent patterns to be captured. Each learner response was coded to identify comprehension, transformation, and process skill errors. Interview data were analysed using the same coding scheme, enabling triangulation of misconceptions. Teacher data (reflective memos and meeting notes) were analysed thematically to identify feedback categories, inform refinements to the diagnostic tools, and validate interpretations of learner misconceptions. This collaborative analytic process ensured that findings were grounded both in theoretical constructs and in classroom pedagogical knowledge, strengthening the study's credibility and rigor.


5. Select a word problem that uses 48 ÷3 to find the answer.

a. There were 48 children playing in the park. 3 children joined them. How many children play in the park in total?

b. You had 48 apples. You gave 3 apples to your sister. How many apples do you have left?

c. There are 48 containers and each container contains 3 liters of water. How many liters of water are there altogether?

d. You have 48 pencils. You distribute the pencils to 3 people equally. How many pencils does each person receive?

C

Figure 1. Question 5 in the diagnostic test (Source: Field study, Learner 1 script)

Validity and Reliability of Data Collection Tools

To ensure rigor, this study established credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985). Credibility was strengthened through methodological triangulation, combining diagnostic tests, interviews, and document analysis to cross-verify findings, while member checking with learners and the lesson study team, as well as peer review from an honours study group, further minimized researcher bias. Transferability was addressed by providing detailed descriptions of the research context, participants, and procedures, enabling readers to judge the applicability of the findings to similar contexts. Dependability was ensured through consistent documentation of the research design, data collection, and analysis procedures, supported by an audit trail that allows for replication of the study and demonstrates the stability of findings over time. Confirmability was promoted by acknowledging potential researcher bias, guaranteeing participant anonymity during interviews, and fostering an environment in which learners could freely express their views; cultural diversity was also considered to reduce interpretive bias. Additionally, triangulation across multiple data sources reinforced that the interpretations were firmly grounded in the data rather than researcher assumptions, thereby enhancing the overall validity and reliability of the data collection tools.

Limitations

This study had two main limitations. First, the relatively small sample size of 18 learners from a single class limited the diversity of responses and the generalizability of findings. As Obilor (2023) notes, larger samples enhance the reliability and accuracy of results; therefore, future research with broader and more diverse populations is necessary to strengthen the applicability of the findings. Second, the inclusion of a calculation diagram in the diagnostic test (e.g., for 54 ÷ 2) may have unintentionally influenced learners' reasoning processes. In some cases, interview responses suggested that learners relied on the diagram to inform their answers, which raises concerns about whether the test fully captured their independent understanding of division concepts. This design feature could have induced misinterpretations or masked deeper misconceptions, potentially affecting the accuracy of the data. Consequently, the results of this study should be interpreted with caution. Addressing these limitations in future research through larger samples and refined test designs would help generate more robust insights into learners' misconceptions about whole-number division.

FINDINGS

The study aimed to investigate how learners approach the division of whole numbers by analysing their responses to a diagnostic test. To identify the nature and origin of the errors made, Newman's error analysis framework was employed. This analytical approach enabled the identification of the specific stages in the problem-solving process where learners encountered difficulties. In particular, this study focused on three primary types of errors: transformation errors, comprehension errors, and processing skills errors. The results are presented according to these categories, beginning with transformation errors, which involve learners' difficulties in converting verbal statements into appropriate mathematical equations. This is followed by an analysis of comprehension errors and processing skill errors to provide a more comprehensive understanding of the nature of learners' misconceptions, followed by instructional implications derived directly from the empirical findings.

Transformation Errors

Figure 1 presents Question 5 of the diagnostic test, which assesses learners' ability to translate verbal problem statements into appropriate mathematical equations. Specifically, the question requires learners to determine which of the four given sentences correctly corresponds to the question 48 ÷ 3. While learners at earlier grade level may be introduced to two-digit ÷ single-digit calculations, the primary purpose of this item for Grade 6 learners is not assess arithmetic computation, but to evaluate their conceptual understanding of division in the context of word problems. This aligns with the South African Grade 6 curriculum, which emphasizes problem-solving and the application of foundational operations in real-life contexts.

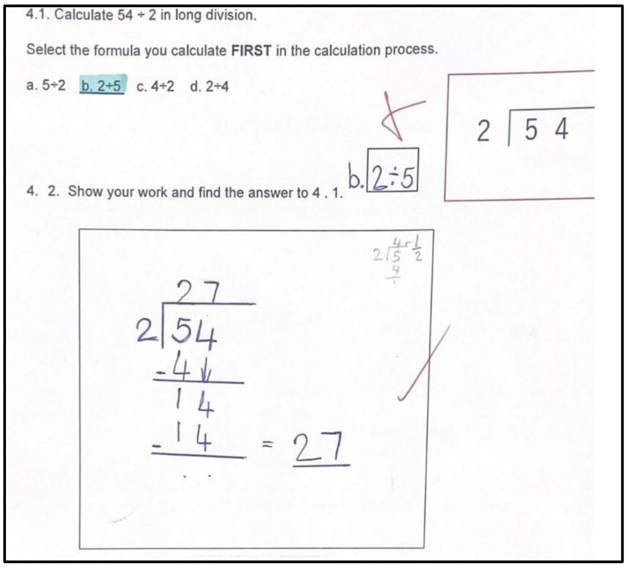


Figure 2. Result of error (Source: Field study, Learner 2 script)

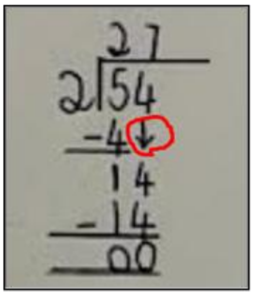


Figure 3. Working process (Source: Field study, Learner 3 script)

The item assesses the transformation phase, in which learners must interpret the verbal statement and convert it into a symbolic mathematical expression. Keywords such as 'distribute' and 'each' signal a division context, whereas terms like 'in total' are typically associated with multiplication. Several learners selected incorrect options, demonstrating a misinterpretation of these keywords and highlighting challenges in distinguishing between language cues for different operations. For example, some learners chose option C, implying multiplication, whereas the correct answer was option D, as shown in **Figure 1**. These findings suggest that learners' difficulties are conceptual and linguistic rather than computational, underscoring the importance of assessing transformation skills at this grade level to diagnose misconceptions in word-problem interpretation. Instructionally, this highlights the importance of explicitly teaching the meaning of mathematical keywords and providing learners with opportunities to collaboratively translate word problems into symbolic form, thereby correcting misinterpretations through guided discussion.

Comprehension Errors

Comprehension errors occur when learners rely on superficial cues or memorised procedures rather than a conceptual understanding of division. For example, in Question 4.1 '54 ÷ 2', one learner selected '2 ÷ 5', indicating a misunderstanding of the relationship between dividend and divisor (**Figure 2**).

During the interview, when the first author asked, "Why did you choose this option?", learner 1 responded, "I followed the order of the figures," referring to the numbers highlighted in red circles. This response suggests that the learners' choice was based not on a conceptual understanding of division process, but rather on superficial visual cues provided in the task. Such reasoning reflects a comprehension error, where learners rely on surface features rather than an understanding of the underlying mathematical relationships. Similarly, another learner (2) stated "division is because it divides a small number by a large number", reflecting an attempt to recall a simplified rule incorrectly. These responses illustrate a lack of conceptual understanding, consistent with the prior research showing that learners may answer correctly procedurally without grasping underlying mathematical relationships. Interestingly, all six learners who answered question 4.1 incorrectly, managed to answer question 4.2 correctly. This indicates that their correct responses were based on familiarity with the long division procedure rather than conceptual understanding. Evidently, when the first author asked learners during interviews to explain how they solved question 4.2 using the long division method, none was able to provide a correct explanation. This highlights a reliance on procedural knowledge and shows a limited conceptual understanding of division.

Furthermore, as shown in **Figure 3**, Question 4.2 shows the learners' working process during long division process. In this figure, an arrow circled in red can be seen. During the interview, the first author asked the learners "what the arrow meant." Learner 3 explained that it showed "bringing down 4 from the top".

This explanation reflects a misunderstanding of subtracting step involved in long division, specifically, the belief that digit 4 comes down from above rather than being the result of the subtraction operation (i.e., 54 – 40). Such a misconception indicates a lack of conceptual understanding and suggest that learners are relying on memorised procedures rather than meaningful reasoning. This procedural reliance may stem from classroom practices that overemphasise rote memorisation without sufficiently addressing the underlying concepts. When learners are taught to follow steps mathematically, they may struggle to apply these procedures in unfamiliar or complex contexts. Therefore, this type of comprehension error reinforces the importance of integrating conceptual explanations into instruction to support learners' deeper understanding of mathematical processes. Such responses align with prior studies (e.g., Rahman & Effendy, 2019) show that learners often produce correct answers procedurally while lacking deeper comprehension. To address these errors, teachers should incorporate visual models and manipulatives such as counters, number lines, or base-ten blocks to illustrate the relationship between dividend, divisor, and quotient. Equally important are guided discussions in which learners explain their reasoning, enabling them to move beyond rote memorisation and toward meaningful engagement with mathematical structures.

Processing Skill Error

In the comprehension error section, it was noted that learners who selected '2 ÷ 5' demonstrated a comprehension error stemming from a lack of basic conceptual understanding. However, from the perspective of Newman' s error analysis, this error could also be classified as a processing skill error. The incorrect selection of '2 ÷ 5' suggests that learners recalled and applied an inappropriate division calculation procedure. This indicates that while the learner may possess a general understanding of how to perform division, they struggle to select and execute the correct procedure in context. In other words, learners may grasp the conceptual meaning of division yet misapply it when attempting to solve problems. Even when they arrive at a correct answer, their use of incorrect procedure reflects a disconnect between conceptual understanding and procedural application.

Secondly, another example of a processing skill error emerged in the learner's expression during the interview, is where they stated 'bring down 4 from the top'. This response reflects a misunderstanding of the division algorithm, likely stemming from a lack of conceptual understanding of the calculation procedure. Particularly, the operation '54 – 40' should be understood as decomposing 54 into 50 and 4, subtracting 40 from 50 to yield 10, and then adding the remaining 4 to obtain 14. Thus, the digit 4 is not 'brought down' from above but rather remains as part of the original number. This type of error suggests that the learner has misremembered and misapplied a procedural step, indicating a processing skill error that is closely tied to inadequate understanding of the underlying structure of the algorithm. Even when correct answers were reached, these explanations revealed a disconnect between procedural steps and the conceptual logic behind them. Such errors, consistent with Newman's (1977) definition, underscore the fragility of procedural knowledge when divorced from conceptual understanding. Teachers can address this by encouraging learners to verbalise each step of the algorithm, linking procedural actions to their conceptual rationale. Stepwise modelling, guided practice, and reflective dialogue can help ensure that learners' execution of procedures is anchored in an understanding of place value and numerical relationships.

Integration of Errors

The findings showed that the classification of an error can vary depending on the perspective from which it is analysed. A single error may be viewed as a processing skill error, a conceptual error, or a transformation error. This indicates that such errors are difficult to isolate, as they are often complex and intertwined. Even when an error appears to be procedural in nature, it may stem from a lack of conceptual understanding or be closely linked to other error types. For instance, the incorrect selection of '2 ÷ 5' revealed both conceptual and procedural difficulties. Similarly, the misconception regarding the matching of an equation to the problem statement in Question 5 was not only the result of a transformation error but also a reflection of limited conceptual understanding. Rahman and Effendy's (2019) study emphasised that learners often make multiple errors within a single problem. This suggests that teachers must be able to identify and address multiple overlapping error types rather than focusing on a single category. A multidimensional analysis of learner errors is therefore essential for accurately diagnosing misconceptions and informing appropriate instructional interventions.

Taking together, the evidence demonstrates a direct connection between the diagnostic findings and instructional strategies that can be implemented in the classroom. Transformation errors point to the need for explicit teaching of mathematical language and guided practice in translating word problems into symbolic expressions. Comprehension errors highlight the value of manipulatives, visual models, and structured reasoning discussions that deepen learners' grasp of mathematical concepts. Processing skill errors underscore the importance of stepwise modelling, learner verbalisation, and reflective practice to link procedural fluency with conceptual understanding. By drawing on these targeted strategies, teachers can address misconceptions more effectively and create a learning environment that promotes both procedural accuracy and deeper mathematical understanding. To ensure the examples provided are representative, all six learners who answered Question 4.1 incorrectly exhibited comprehension errors, and multiple learners demonstrated transformation errors in Question 5. Interview data triangulated learners' reasoning, confirming that the patterns observed in selected excerpts reflected broader trends across the class. Coding was conducted systematically: each response was reviewed, classified according to the definitions above, and verified collaboratively with the Lesson Study team to ensure reliability. This approach ensures that the categorisation is transparent, theoretically grounded, and reflective of common patterns rather than isolated cases.

DISCUSSION

The findings showed that learners made a range of errors, including comprehension errors, process skill errors, and transforming errors. Notably both the analysis of learners' written responses and interviews data indicated a predominance of comprehension errors. This aligns with Clarkson (1991), which identified comprehension errors as the most common type of error among learners. Comprehension, as defined by Alanazi (2008), involves understanding of the basic rules, concepts, and ideas underlying mathematics. Furthermore, Rahman and Effendy (2019) emphasise that learners must demonstrate sufficient competence at each cognitive stage before progressing to the next, particularly reaching the encoding stage. Without mastering earlier stages, learners' progression is hindered. Their study also reveals that learners often make multiple overlapping errors within a single problem, rather than isolated mistakes. Our findings confirm this pattern: A single error frequently involved a combination of a comprehension, procedural, and transformation errors, highlighting the complex and interrelated nature of learners' misconceptions. Central to these errors is a lack of conceptual understanding, suggesting that strengthening learners' foundational mathematical concepts is critical to reducing errors.

In this study, the most common conceptual errors included the incorrect selection of '2 ÷ 5' and the misexpression of the long division step as 'bring down 4 from the top'. These errors reflect a fundamental misunderstanding of place value, particularly learners' inaccurate grasp of the role and value of each digit within a number. As Themane and Luneta (2021) note, without a solid understanding of place values, learners tend to rely on rote memorisation rather than meaningful engagement with numerical relationships. This misexpression 'bring down 4 from the top' exemplifies how learners can execute procedures without understanding the underlying mathematics: The number 4 does not literally 'come down,' but results from the subtraction step '54 - 40 = 14,' representing the tens and one's positions in the original number. This misunderstanding underscores that many learners are taught to mimic steps in the long division algorithm without grasping the logic behind them, reinforcing the need for instruction that balance conceptual understanding with procedural fluency.

To address these misconceptions, instructional strategies must deepen learners' conceptual understanding of place value. This article aligns with the approaches proposed by Ndlovu et al. (2019), which integrate concrete objects and language-based instruction. Concrete objects, such as base-ten blocks, play money, decimal blocks, or hundred charts, allow learners to visually and physically represent abstract concepts, facilitating meaningful engagement with numerical structures (De Castro et al., 2002; Themane & Luneta, 2021). Language-based instruction complements these tools by enabling learners to articulate and reflect on their mathematical reasoning, fostering metacognitive awareness (Götze, 2018; Themane & Luneta, 2021). For example, learners can verbalise each step in long division, discuss strategies with peers, or present solutions in class followed by guided reflection. By combining manipulative-based activities with explicit mathematical language, teachers can promote both procedural fluency and deeper conceptual understanding, supporting learners to move beyond rote computation.

The diagnostic assessment also provides direct guidance for lesson planning. Transformation errors, where learners misinterpret word problems or reverse the operations, inndicating the need for lessons incorparating context-rich tasks and guided interpretation strategies. Comprehension errors highlight the important of explicit instruction on mathematical language, symbols, and structural understanding, while processing skill errors reveal gaps in learners' ability to accurately apply procedures within a conceptual understanding. These findings can inform the selection of teaching strategies, manipulatives, and representations, enabling teachers to design lessons that directly target learners' identified misconceptions and strengthen overall mathematical understanding.

CONCLUSION

This study revealed that Grade 6 learners' misconceptions about whole-number division were primarily manifested as transformation, comprehension, and processing skill errors, often overlapping within a single problem. The predominance of these errors can be attributed to a weak conceptual foundation, particularly regarding place value, which caused learners to rely on superficial cues, memorised procedures, or misinterpretations of mathematical language rather than meaningful reasoning. For instance, misexpressions such as "bringing down 4 from the top" illustrate how rote instruction of algorithms without conceptual grounding can reinforce misconceptions. Similarly, learners' misinterpretation of keywords in word problems highlights the linguistic challenges inherent in mathematical problem-solving. These findings suggest that learners' difficulties are not random mistakes but stem from instructional practices that overemphasise procedural fluency while underemphasising conceptual understanding. As a response, the study recommends instructional strategies that deliberately integrate concrete manipulatives and language-based approaches. Manipulatives help learners visualise and make sense of abstract concepts like place value, while structured opportunities for verbal explanation, peer discussion, and reflection enable them to articulate reasoning and correct misconceptions. By combining these approaches, teachers can more effectively address learners' misconceptions, strengthen both conceptual and procedural understanding, and promote more positive engagement with mathematics.

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