

**THE DEVELOPMENT AND OPTIMISATION OF AN
INVENTORY MODEL FOR UNSCHEDULED
MAINTENANCE ON AIRCRAFT**

by

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List of Acronyms

EOQ	-	Economic Order Quantity
ACMI	-	Aircraft, Crew, Maintenance and Insurance
UN	-	United Nations
WFP	-	World Food Program
ACC	-	Aircraft Characteristic Chapter

Executive Summary

Naturelink Aviation is involved in Private Air Charters, Aircraft Leasing, Aircraft Sales, In-house Training and Maintenance. The company has a fleet of eleven Embraer EMB120 and six PAC P750XL aircraft contracted in services across Africa. Naturelink Aviation seeks improvement regarding the management of inventory for unscheduled maintenance at outstations in Africa.

It was established that the company does not have an automated inventory systems for the outstations, mainly because outstations are dependent on contracts. Currently the engineer at the outstation has to manage the unscheduled maintenance inventory and determine when top-up stock needs to be ordered.

The decision was made to develop a multiproduct Economic Order Quantity model to determine optimal order quantities, number of cycles and order times. The main objective is to determine when items of different frequency usage can be ordered together instead of sending orders only days apart.

After this project it was clear that reorder points often coincide or happen close together in addition to the crew rotation schedule. This information enabled the development of a sample reorder schedule, it requires constant updated and verifications, because reorder points and EOQs may change. Naturelink Aviation will now test the schedule and if needed rectifications and adjustments will be made before the final distribution and implementation among the fleet.

1 Background and Introduction

1.1 Background

Naturelink Aviation was established in 1997 at Wonderboom Airport in Pretoria. Today the company offers Private Air Charters, Aircraft Leasing, Aircraft Sales, In-house Training and Maintenance to the aviation market and operates throughout Africa, in Asia and the Middle East. Specialised in ACMI regarding aircraft leasing the company is also accredited by the UN and WFP. The strength and potential for success is vested with the extensive knowledge, connections and experience of flying in Africa and the ability in handling off-base operations. Naturelink Aviation pursues a truly global placement strategy.

The maintenance and refurbishment division is the primary backbone of the company. Naturelink is an Embraer service centre and is licensed to service a large range of aircraft types and engines, serving Africa and beyond. It is of utmost importance that all aircraft be kept in peak operating condition, thus reliable and timely maintenance is essential.

Naturelink Aviation is involved in the WFP and other charters. The company has many outstations throughout Africa. Maintaining these aircraft in good condition is vital, as it costs up to \$6000 per day if a plane is grounded. Unscheduled and Check A maintenance can be done at the outstations. The planes are only required to return to South Africa for the Check C maintenance. Unscheduled maintenance involves rectifying unforeseen breakdowns during operations at outstations. Essential, yet cost effective, inventory levels for maintenance spare parts has to be kept at each outstation to ensure that unnecessary transportation costs and grounded costs are not spend. Optimum transportation of these parts is also important.

Naturelink Aviation wants to improve their inventory management for unscheduled maintenance at outstation in Africa. The project was approached from three aspects:

1. Data gathering, capturing, updating and analysis.
2. Development of the inventory model for the replaceable rotatables and consumables.
3. Testing, rectification and optimisation of the inventory model.

Figure 1: Project Structure



1.2 Project Definition

Naturelink Aviation's current unscheduled maintenance inventory is managed manually by the engineer at the outstation. This procedure is very time-consuming, ineffective and unreliable as human error is inevitable. Naturelink Aviation wants to improve the management of this inventory with regards to time, costs and reliability.

1.3 Project Aim

The main objective of this project is to develop and optimise an inventory model to assist Naturelink Aviation in improving their inventory management of unscheduled maintenance of each aircraft in contracted service.

1.4 Project Scope

Naturelink Aviation has a fleet of eleven Embraer EMB120 and six PAC P750XL aircraft. There are approximately 160 replaceable rotables and consumables available to the Embraer aircraft, contrasting to roughly 130 for the PAC aircraft. This project will be conducted on an Embraer aircraft as the pilot study, in order to test, rectify and optimise the inventory model before it is implemented on the remaining fleet. Scheduled maintenance will be excluded, because it is administrated separately and runs independently.

All data gathering and capturing, development and eventually optimisation of the inventory model will be done by the student engineer. Testing and rectification of the inventory model will also be handled by the student engineer with the assistance of Naturelink's maintenance and logistics departments.

2 Literature Study and Method Identification

2.1 General Inventory Model Study

2.1.1 Definition of Inventory

Inventory can be defined as any goods sold or used by an organisation to create the goods and services provided by the organisation while being stored. These goods include anything from raw material and work in progress to finished goods and supplies.

2.1.2 Definition of Inventory System

An inventory system monitors inventory levels through inventory policies and controls identified by the organisation. Through the inventory system the levels of inventory that has to be maintained, when inventory replenishments should occur and the size of the orders is established. In an inventory system there are three types of costs that are subject to control, at least two has to be controlled at a particular time:

1. Replenishment cost of inventory (setup or order cost).
2. Carrying cost of inventory (holding cost).
3. Shortage cost.

2.1.3 Definition of Inventory Model

An inventory model is a mathematical equation that enables organisations to determine the optimal quantity and the frequency of ordering and keeping goods and services available to the customer without interruption or delay.

2.1.4 Definition of Inventory Policies

Inventory policies are statements of an organisation's goals and approach to inventory management. These policies answer the questions of "When?" and "How much?" of the replenishments of inventory.

2.1.5 Purpose of Inventory

2.1.5.1 Predictability

Controlling the quantity of raw material, parts and consumables used in a process at a given time is essential for capacity planning and production scheduling. Inventory can act as a buffer for processes involved.

2.1.5.2 Fluctuation in Demand

Customer satisfaction and/or meeting production demand are vital to success in business, but demand is never completely constant, therefore maintaining inventory on hand is essential to prevent loss of sales and be prepared for unexpected fluctuations. This on hand inventory can be reduced to a minimum when customer and market patterns are studied, recorded and analysed.

2.1.5.3 Unreliability of Supply

Inventory provides a buffer for unreliable suppliers or products that are scares to obtain. To decrease on hand inventory these unreliable suppliers should be rehabilitated or replaced.

2.1.5.4 Price Protection

Planning orders will enable taking advantage of good deals and prevent hasty buying at peak market times, for example all prices tend to decrease after Christmas and New Year, before or after this period of time suppliers take advantage of the marketing season.

2.1.5.5 Quantity Discounts

Discounts or price breaks are often available when bulk quantities of product are bought.

2.1.5.6 Lower Ordering Costs

Ordering higher quantities of product at less frequent rates will result in lower ordering cost. This is a good option if holding costs are relatively low and/or ordering cost is high.

2.1.6 Purpose of Inventory Theory

Inventory theory is used by manager is establish rules that can be used to maintain inventory levels at the lowest cost possible while meeting customer demand.

2.1.7 Types of Inventory

2.1.7.1 Raw Material Inventory

Raw material is used to produce partial or completed products.

2.1.7.2 Work-In-Process Inventory

Inventory is called work-in-process inventory in the stages where raw material is being converted into partial products, finished goods and subassemblies; inventory in the production system. This inventory should be kept at the lowest possible level, but usually increases due to work delays, long movement times between operations and queuing bottlenecks.

2.1.7.3 Finished Goods Inventory (Cycle Stock)

Finished goods are ready for customer sales. Buffer inventory can be build up for unpredictable demand or predictable seasonal demand.

2.1.7.4 Consumables

Consumables are all inventory that will be used up some time along an organisation's production/service runs. These are often treated as raw material.

2.1.7.5 Service, Repair, Replacement and Spare Items (S & R items)

These items are used after sales. As long as a product is used in the market and services and repairs are needed, it will never be obsolete.

2.1.7.6 Safety Stock (Buffer Inventory)

Safety stock can serve several purposes internal and external to an organisation's processes. It enables decoupling of processes and operations to run independently, while also compensating for uncertainty in demand and supply.

2.1.7.7 Seasonal Stock

This type of stock is not different from any other finished inventory it is merely linked to seasonal changes, for example there is a greater demand for bathing suits during summer time/season.

2.1.7.8 Anticipatory Stock

Inventory produced in anticipation of a specific event, for example Valentine's Day. This inventory is not linked to seasons.

2.1.8 Inventory Cost

2.1.8.1 Setup Cost (Production Change Cost)

Setup cost is associated with the changing of materials, equipment and paperwork and the removal of previous inventory for the production of a new product. This cost is often related to ordering cost.

2.1.8.2 Ordering Cost

Ordering cost includes all costs related to the placement and retrieval of an order. All managerial, clerical and tracking costs associated to an order are included in the ordering cost.

2.1.8.3 Purchasing Cost (per unit)

This is the variable cost per unit purchased and includes raw material cost, variable labour and overhead costs involved with the purchasing of a unit of product. Shipping cost is also included in purchasing cost when shipping does occur.

2.1.8.4 Holding Cost (Inventory Carrying Cost)

Holding cost is the cost linked to a single unit being carried by an organisation for a single unit of time. This time varies from organisation to organisation. Holding cost includes all of the following aspects: storage cost, handling cost, insurance cost, taxes on inventory, depreciation and any costs due to breakage, theft and obsolescence.

2.1.8.5 Shortage Cost

Whenever a customer order can not be met there is cost involved called shortage cost. This can be cost associated to correcting the back order or the lost income due to sales being lost.

2.1.9 Types of Demand

Demand is divided into two categories namely deterministic demand and probabilistic or stochastic demand.

The dictionary definition for deterministic, probabilistic and stochastic is:

1. Deterministic – constant, known.
2. Probabilistic – likelihood, chance.
3. Stochastic – random.

Deterministic demand is the most ideal type of demand as it is constant and known. Probabilistic demand uses historical data to enable the determination of demand with a high probability rate and standard deviation. Stochastic demand is demand that shows not pattern and is random all the time.

2.1.10 Three Simple Inventory Systems

2.1.10.1 One-Bin System

In this system periodic replenishments are done at fixed periods of time, filling up inventory to a maximum level no matter the size of the order. This is a Fixed-Time Period Model (P-model).

2.1.10.2 Two-Bin System

The focus of this system is not time but inventory level. Two bins of inventory are used, whenever a bin is empty the other bin is being used and an order with a fixed quantity is ordered. This is a Fixed-Order Quantity Model (Q-model).

2.1.10.3 Optimal Replenishment System

This system is also a Fixed-Time Period Model (P-model) like the One-Bin System, except that inventory levels are reviewed and calculated at fixed periods of time before a decision of replenishment is made. The maximum inventory level (M), influenced by demand, ordering cost and shortage costs, has to be determined as well as the minimum order size (Q). To determine whether or not replenishment is needed at the time of the review the current inventory level (I) is deducted from the maximum inventory level (M) and only when this number (q) is greater than the minimum order size (Q) an order placed for replenishment. If the q is smaller than Q an inventory review will be conducted again at the next scheduled time to determine if replenishment is needed then. $M - I = q$, $q > Q$ place order, else wait.

2.1.11 Inventory Policies

As stated earlier, Inventory Policies contribute to the decisions of “When?” and “How much?” inventory replenishment should take place. According to Naddor (1966), the first question is usually answered in one of two ways:

1. Inventory should be replenishment when the amount in the inventory is equal to or below s_0 quantity units.

2. Inventory should be replenished every t_0 time units.

The second question is also usually answered in one of two ways:

1. The quantity to be ordered is q_0 quantity units.
2. A quantity should be ordered so that the amount in inventory is brought to a level of S_0 quantity units.

(r,q) Policy – when inventory reaches the level r , q units of inventory is ordered. It is often also referred to as a two-bin policy.

(t,q) Policy – after time t has lapsed, q units of inventory is ordered.

(s,S) Policy – when inventory levels equals or drops below s , an order is send to fill up the inventory level to S .

(t,S) Policy – after time t has lapsed, an order is placed to fill up inventory to level S .

These policies are reviewed periodically or continuously. Periodic reviews only occur at predetermined, specific intervals, while continuous reviews are done constantly at any time.

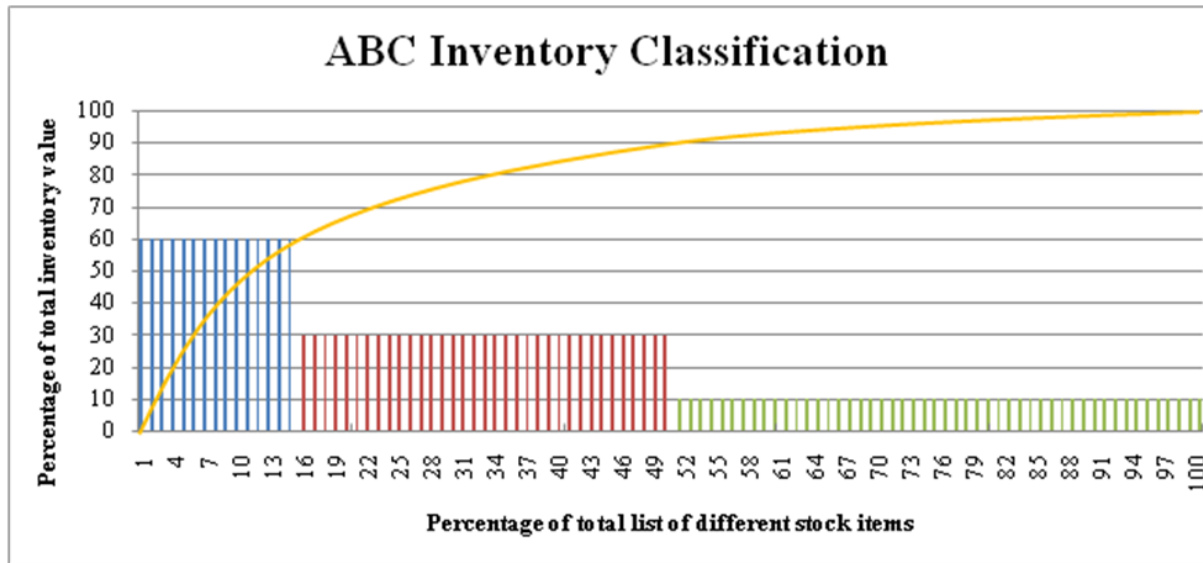
2.1.12 ABC Analysis

Many organisations have a large variety of inventory and the management of every single type of inventory is rather costly. In the nineteenth century Vilfredo Pareto conducted a study and realised that a mere 20% of people controls 80% of all wealth and that this theory is also plausible in other aspects of life. This is probable especially in inventory systems and therefore the ABC Inventory Classification System was developed.

The system is divided into three categories, namely:

1. High Rand Volume (Type A items).
Roughly the top 15% of items most commonly used.
5 % to 20% of inventory items contribute 55% to 65% of annual turnover.
2. Moderate Rand Volume (Type B items).
Approximately 35% of items used fairly often.
20% to 30% of inventory items contribute 20% to 40% of annual turnover.
3. Low Rand Volume (Type C items).
The last 50% of inventory items rarely used.
50% to 70% of inventory items contribute 5% to 25% of annual turnover.

Figure 2: Pareto Graph of ABC Analysis



2.2 Specified Inventory Model Study

It was determined from the data analysis that was done that an inventory model has to be developed for the smooth running of the unscheduled maintenance at outstations. Literature studies were done on similar scenarios and different inventory model possibilities were identified.

Queyranne (2000) approached his investigation of a multiproduct production and inventory model using the net present value. During his study comparisons to the EOQ model were regularly made. It was found that as long as deterministic demand is used the two models vary very little, but with probabilistic or stochastic demand the differences are major. The EOQ model uses long-run average cost and is fairly simple compared to the complexity of using net present value the results are straightforward and robust.

DeCroix and Arreola-Risa (1998) used the base-stock policy in addressing the problem of determining the optimal production and inventory policy for a multiproduct production-inventory system. The problem concerns homogeneous products with an average determined demand for each time period and random demand during the period over an infinite horizon. In order to see the effect of optimal production and inventory policy when the products are

not homogeneous a heuristic algorithm had to be generated. This developed into a very complex algorithm that had to be implemented on each product.

In Gelders and Van Looy's (1978) case study it was realised that an EOQ model is not always sufficient when slow and fast moving items are involved in one system. If an $EOQ < 1$ is obtained the item is mismanaged with the wrong inventory model, a knap-sack approach will be better for these slow movers. In general an EOQ model is fully adequate for medium and slow moving items.

In the utility fuel inventory study done by Morris et al. (1986) the optimal order policy was determined by a Markov model, the states represented different inventory levels and order periods.

The Economic Order Quantity Model:

The EOQ model is a fairly simple model to use to manage inventory levels and ordering. The model can be used in many different scenarios, but there are assumptions and conditions that also have to be met. For example, according to Winston (2004), an EOQ model is acceptable when the standard deviation of the annual demand is smaller than the actual EOQ value.

Fixed-Order Economic Order Quantity Model:

In a fixed-order system the order quantity stays constant. The inventory levels are monitored and when the stock level reaches a specific point an order is placed. The only danger regarding stock depleting to zero is during lead time. The demand range can be determined by the analysis of past demand data. Safety stock is dependent of the service level desired, the safety stock must not be too high as this will incur excess inventory cost.

Multiproduct Economic Order Quantity Model:

In the multiproduct EOQ system products of different frequency usage are set alongside each other and comparisons regarding each item's reorder frequency is made. Reordering cycles of the slower items are lined up to that of the fast moving items and corresponding order cycles can be determined.

The combination of a fixed-order EOQ model and multiproduct EOQ model will most probability be sufficient with the improving the unscheduled maintenance inventory management, required by Naturelink Aviation. Graph illustrations can be seen below.

Figure 3: Basic Economic Order Quantity Model

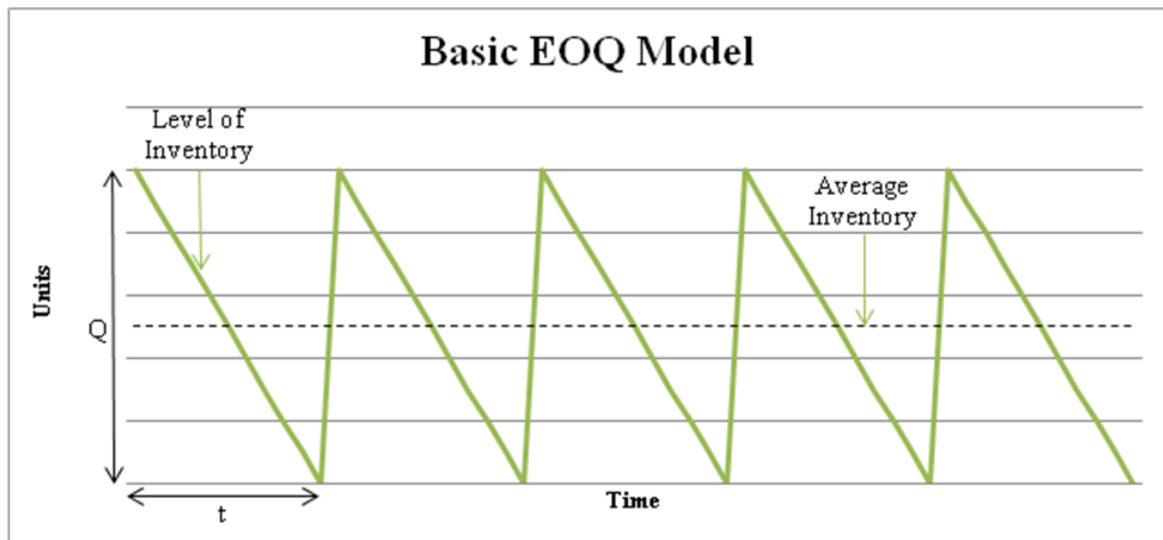


Figure 4: Fixed-Order Economic Order Quantity Model

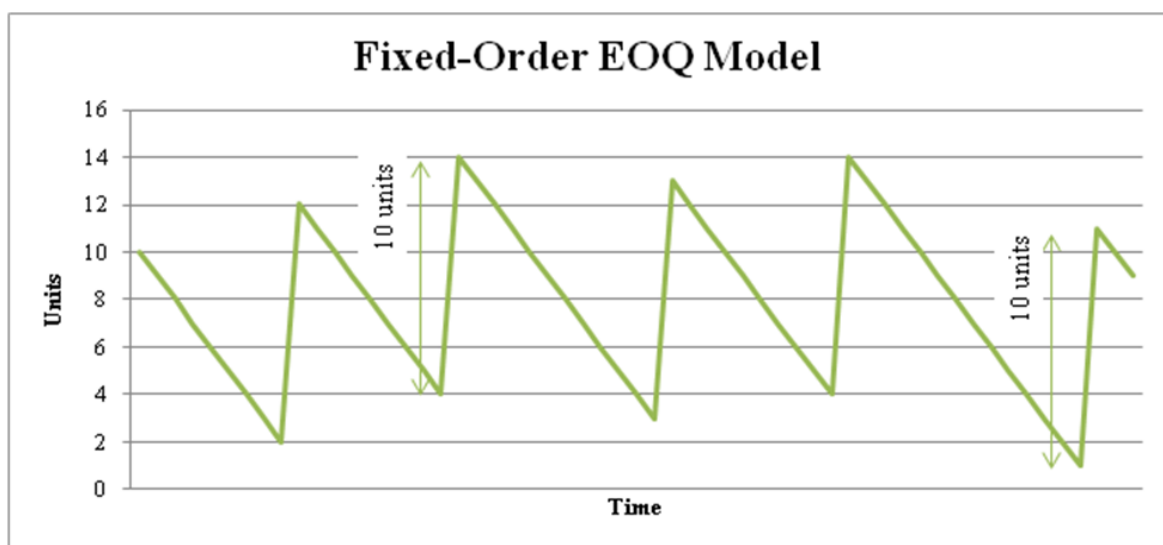
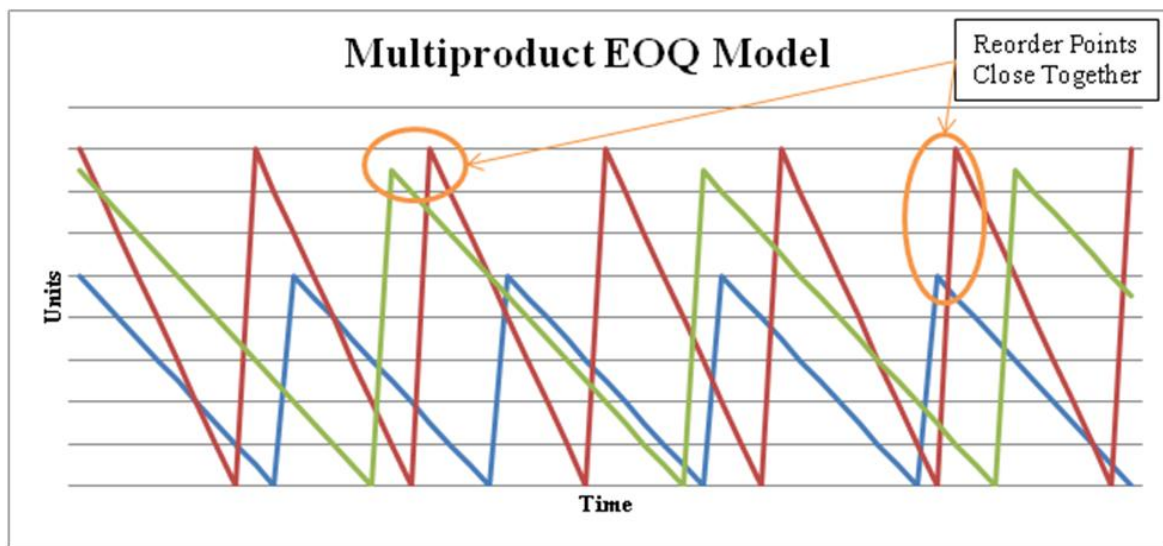


Figure 5: Multiproduct Economic Order Quantity Model



3 Data Gathering and Analysis

3.1 Data Gathering

Initial meetings and discussions were held with Naturelink Aviation's management and possible problem areas in which Industrial Engineering techniques can be applied were identified. The improvement of inventory management concerning unscheduled maintenance was selected and data gathering regarding these operations and maintenance were collected. The company supplied long term data from their database and weekly reports on latest maintenance activities (July 2009 to present) were collected on a monthly basis to continually update the data as it was reported. Every two weeks Naturelink Aviation was visited to discuss findings and problems with the data gathered and occasionally information about stock levels or crew rotations etc. were discussed and received through email.

The associated people working with the weekly reports identified some problems and setbacks that they experience with these reports. Numerous errors and omissions are made on the weekly reports especially concerning serialised and non-serialised rotables. The information is either incomplete or incorrect; this can cause major problems if something goes wrong, especially with the serialised rotables, because traceability is crucial in the aviation industry. In addition to this unreliable data the weekly reports can arrive up to two weeks late, with effect that the errors and omissions are only taken note of and corrected weeks after maintenance was carried out. Poor internet availability in some of the very rural countries is sometimes responsible as it complicates the sending of large weekly report data files.

Maintenance data (from the database) and weekly reports containing daily data of unscheduled maintenance were gathered and analysed. On the weekly report the following important information regarding maintenance are specified:

1. Problems on the aircraft.
2. Actions taken to rectify the problems.
3. The problem report date and rectification date.
4. Usage of rotables, serialised or non-serialised, and consumables.
5. Part and serial numbers where required.

6. Quantity usage of spare parts.
7. ACC allocation – ACC chapter are used for the sub-grouping of replaceable items, for example, any rotatable or consumable associated with landing gear is assigned to ACC chapter 32 – LANDING GEAR.
8. Remaining stock available.

It was identified that there are no automated inventory systems at the outstations, this is because the outstations changes from time to time, as they are dependent on contracts. Currently the outstation engineer has to manage the spare inventory by himself, but crew rotate on a six week cycle, so every six weeks a new engineer takes over the management of inventory and this can result in errors regarding the quantities of stock available. It is unreasonable to expect an engineer to always check the inventory levels of all 160 replaceable rotatables and consumables at the beginning of each rotation cycle. Furthermore, the engineer has to manually request an order for top-up inventory. The judgement of the re-order point therefore has to be determined by the engineer and this can result in inventory running out before the top-up goods arrive either due to a too low safety stock or custom clearances that can linger for up to two weeks. Therefore, because there is no proper inventory system for the outstations the lack of the development of a good and accurate inventory model is to be expected.

3.2 Data Analysis

Since there is currently no automated inventory system in place, data from the weekly reports was captured manually in an Excel spreadsheet for easy sorting into ACC chapter, identification of items and interpretation. This was only a quick temporary step in order to see what conclusion can be drawn from the available data.

A graph was drawn up for each item used in unscheduled maintenance to see and determine when inventory levels are depleted and top-up will be needed. From these graphs the pattern of an EOQ graph for a model with varying time intervals could be seen. It was also, again, established that there fast, medium and slow item flows. By merely studying the graphs possible improvements regarding inventory levels was identified.

The conclusion drawn from the data and graphs was that a multiproduct EOQ model with time intervals that vary has to be developed. All the requirements to develop an EOQ model was met when Naturelink Aviation provided the years of historical data of the demand for each replaceable rotatable and consumable.

Below some of the EOQ graphs that was developed is illustrated. The first graph is an idealistic representation of an EOQ model without any shortages and varying time intervals, this is followed by sample graphs of rotatables and consumables.

Figure 6: Idealistic Economic Order Quantity Model Graph (with no shortages and varying time intervals)

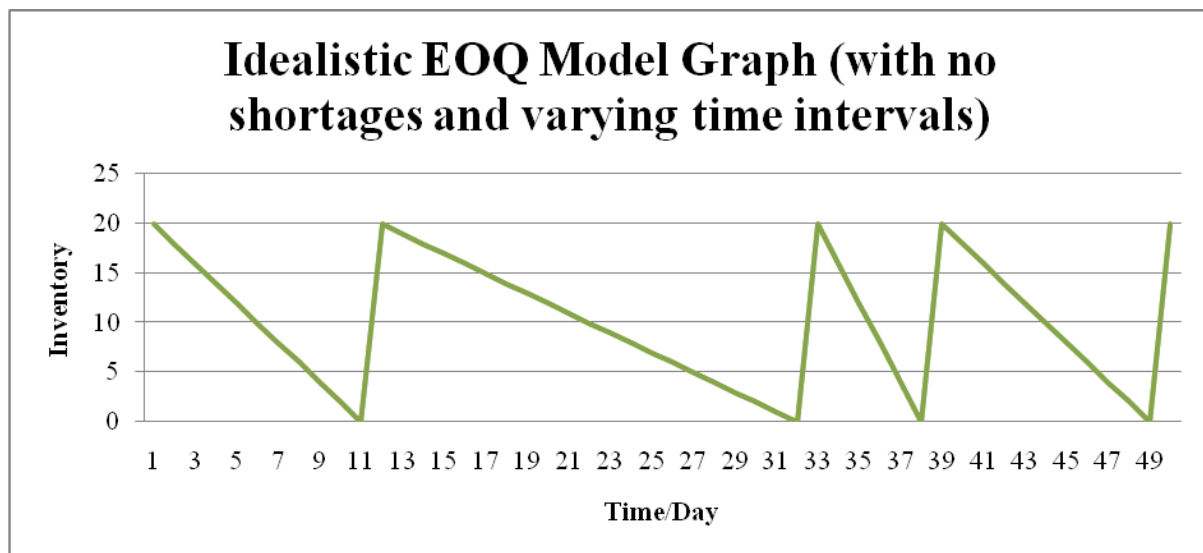


Figure 7: Economic Order Quantity Graph: Engine Oil

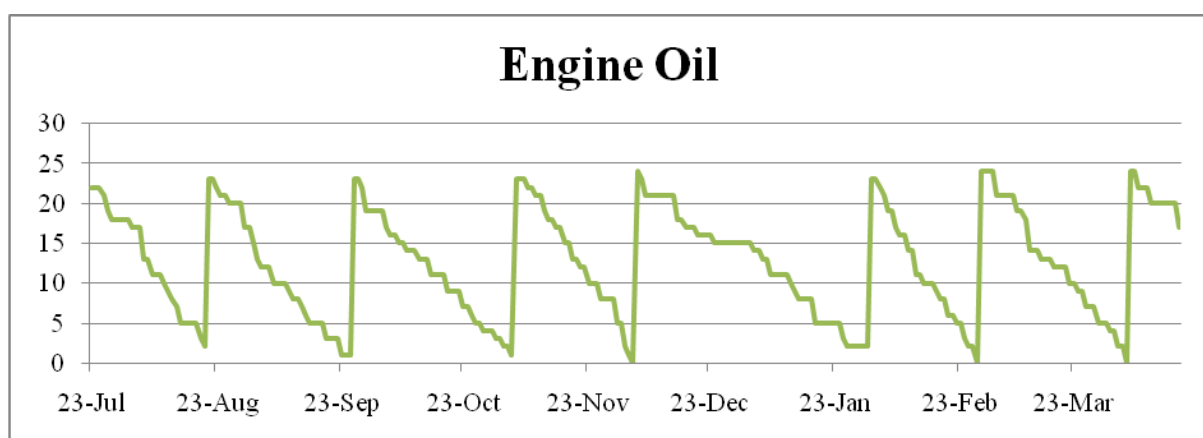


Figure 8: Economic Order Quantity Graph: Hydraulic Fluid

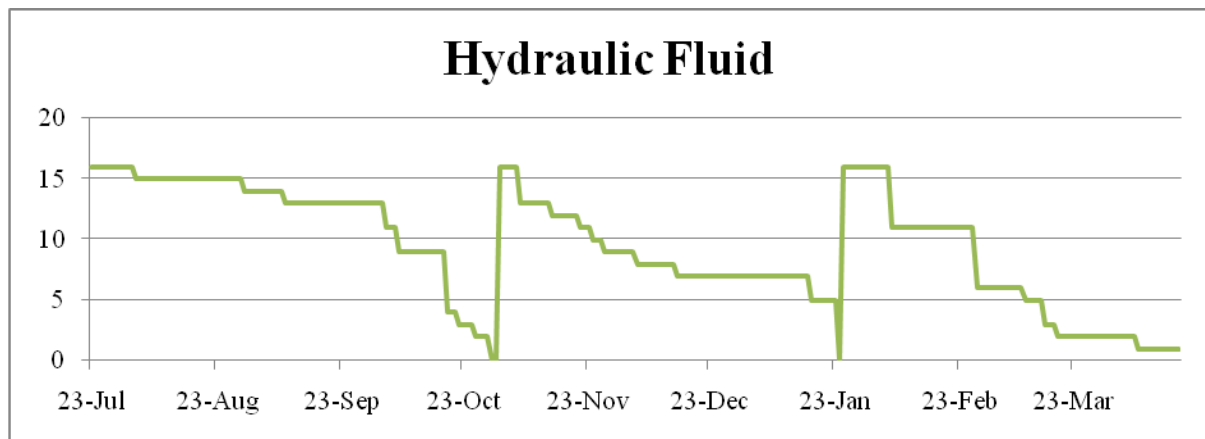
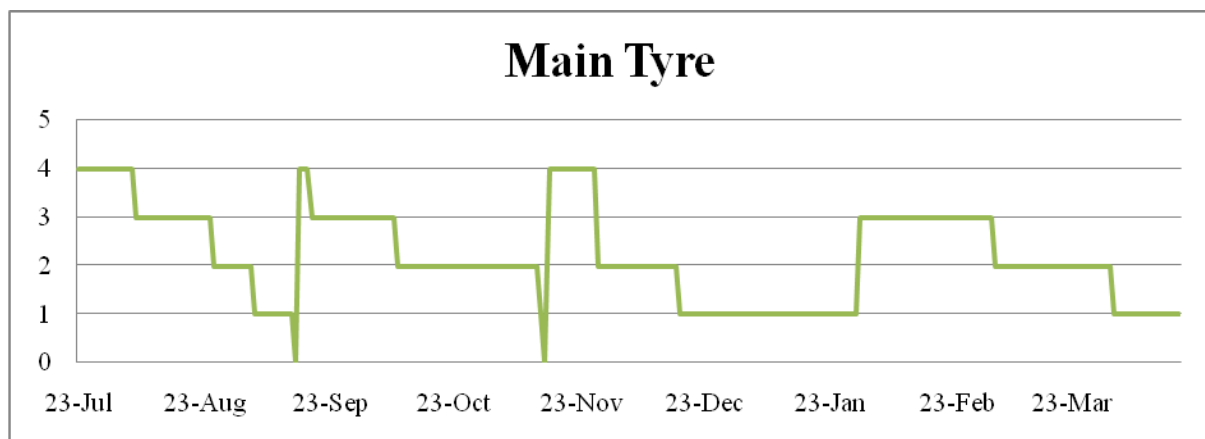


Figure 9: Economic Order Quantity Graph: Main Tyre



4 Conceptual Solution

4.1 *The Economic Order Quantity Model*

The EOQ model is a relatively basic model that enables organisations to determine the optimal order quantity and order point. There are assumptions and conditions that have to be met to be able to implement the EOQ model.

4.1.1 Assumptions of a Basic Economic Order Quantity Model

1. Demand has to be continuous, constant and known (deterministic)*.
2. Replenishment is constant and known.
3. Lead time has to be zero or at least constant and known.
4. For each order ordering, setup and holding costs is incurred.
5. All customer demand has to be satisfied, therefore no shortages are allowed.
6. Purchasing prices must be constant and independent from order quantity and time.
7. No inventory in transit is allowed, the delivery cost has to be included in the purchase price.
8. All ordered inventory items has to be independent.
9. Unlimited capital must be available.
10. Planning period has to be infinite.

* Take note that under specific conditions it is acceptable for demand to be uncertain; this is discussed in section 4.1.6.

4.1.2 Variables of an Economic Order Quantity Model

D	= annual demand of units	TC°	= optimal total cost
K	= ordering or setup cost	L	= lead time in days
H	= holding cost/unit/year	ROP _u	= reorder point (units)
q°	= optimal order quantity	ROP _t	= reorder point (time)
n°	= optimal order cycles	Q	= quantity of units in inventory cycle
t°	= optimal order time	SS	= safety stock

4.1.3 Equations of an Economic Order Quantity Model

Optimal Order Quantity

$$q^{\circ} = \sqrt{\frac{2DK}{H}}$$

Optimal Order Cycles

$$n^{\circ} = \frac{D}{q^{\circ}}$$

Optimal Order Time

$$t^{\circ} = \frac{1}{n^{\circ}}$$

Optimal Total Cost

$$TC^{\circ} = 0.5q^{\circ}H + \frac{DK}{q^{\circ 2}}$$

Total Annual Ordering Cost

$$HC = \frac{1}{2}q^{\circ}H$$

Total Annual Holding Cost

$$OC = \frac{DK}{q^{\circ}}$$

Reorder Point in Units

$$ROP \text{ (units)} = (SS + L) \times \text{Annual Demand}$$

Reorder Point in Time

$$ROP \text{ (time)} = (\text{Annual demand} \div 365 \text{ days}) \times \text{Lead time}$$

Total Annual Cost

$$TC = OC + HC$$

4.1.4 Illustrations of an Economic Order Quantity Model

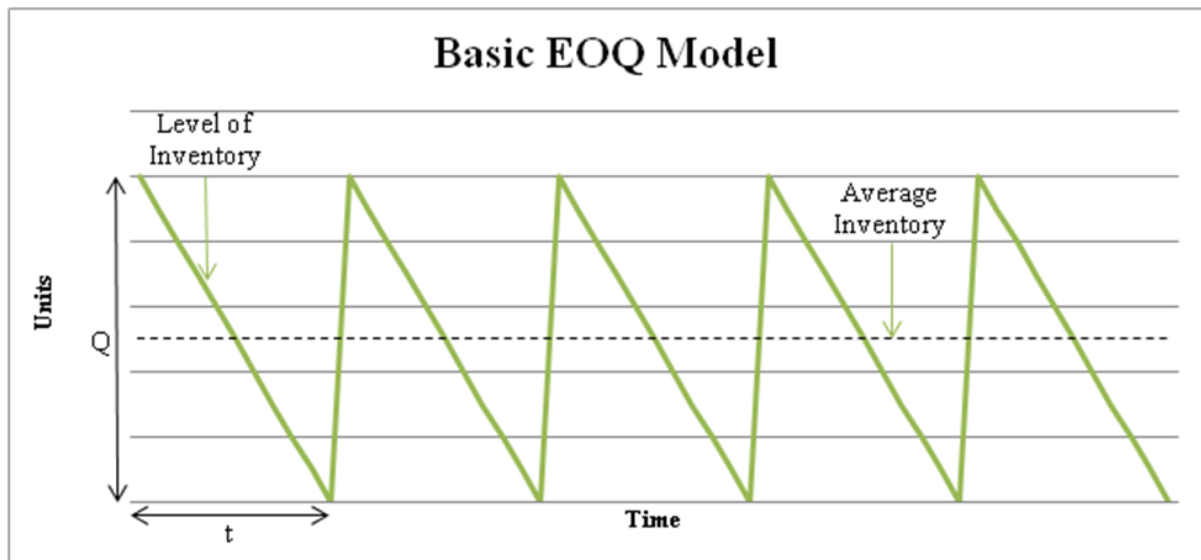
The diagram below is an example of a simple EOQ model. The variables K, h, D, and L has to be entered by the user. The EOQ, orders per year, reorder point (units), total holding cost, total ordering cost and total overall cost will be calculated.

Figure 10: Simple Economic Order Quantity Model

	A	B	C	D	E	F
1						
2		Simple EOQ Model				
3		Setup/Order Cost (K)	Holding Cost (h)	Annual Demand (D)	Lead Time (L)	
4		25.00	1.50	500	6.5	
5						
6		EOQ	129	Total Holding Cost	96.82	
7		Orders per Year	4	Total Ordering Cost	96.82	
8		Reorder Point	23	TOTAL COST	193.65	
9						

The diagram below is an idealistic behaviour representation of an EOQ model. The demand is constant, there is no lead time and shortages and replenishment occurs at a constant rate.

Figure 11: Behaviour Graph of a Basic Economic Order Quantity Model



4.1.5 The Economic Order Quantity Model with Multiple Products

The objective of this model will be to establish the optimal order quantity at the optimal time, taking into consideration the inventory levels of all the other items and the lead time, to optimise the frequency of spending parcels.

Figure 12: Multiproduct Economic Order Quantity Model

	A	B	C	D	E	F	G
1							
2		Multiproduct EOQ Model					
3			Item 1	Item 2	Item 3	Item n	
4		Annual demand	10000	1000	100	...	
5		Holding cost	75	75	75	...	
6		Order cost	630	630	630	...	
7		q	410	130	41	...	
8		n	24.40	7.72	2.44	...	
9		t (days)	14.96	47.31	149.61	...	
10							
11			Item 1	Item 2	Item 3	Item n	
12		Order quantity	410	130	41	...	
13		Order cycle in which item n has to be send	1	3	10	...	
14							
15		Total cost	R 15 407.93	R 4 898.06	R 1 574.54	...	
16							

4.1.6 The Economic Order Quantity Model with Uncertain Demand

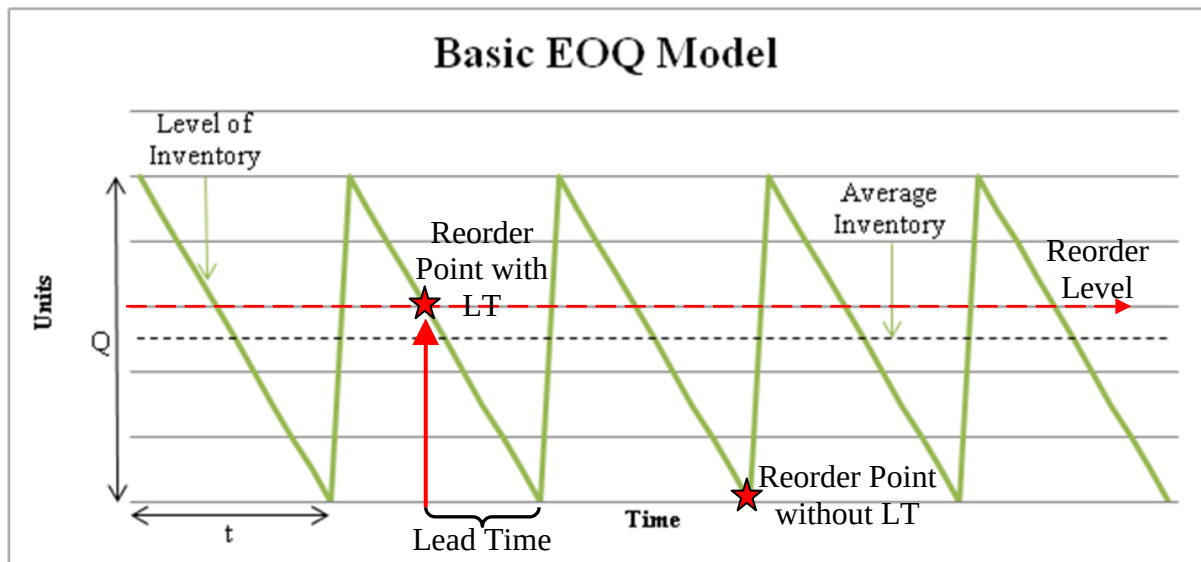
According to Winston (2004), an EOQ model may be sufficient as an inventory model when demand of uncertain to adequately approximate the EOQ. There are two conditions that have to be satisfied. Firstly, good and proper historical data has to be available to establish the average demand that will be required for the model period. Secondly, the EOQ value has to be larger than the standard deviation of the historical demand, otherwise this may lead to under stocking and incur shortage costs.

4.1.7 The Effect of Non-Zero Lead Time

The determination of whether or not there will be lead time involved in the replenishment of inventory and the size of this is absolutely crucial to consider when determining the optimal inventory model, else shortages may occur. The ordering/setup cost and holding cost will be unaffected by lead time and the EOQ model will still optimise and minimise the total cost. The reorder point or level has to be calculated accurately to ensure smooth running of the EOQ model. Whenever this level is reached an order has to be placed.

The lead time can be shorter or longer than the cycle time. Therefore the reorder point can either be in the current cycle time or in a previous cycle time. The diagram below shows a lead time that is shorter than the cycle time and indicates the reorder point and level.

Figure 13: The Effect of Lead Time on an Economic Order Quantity Model



EOQ models with regard to discounts and back orders will not be discussed because it will not form part of the inventory model for Naturelink Aviation. Shortages are not allowed and there is no possibility for price breaks.

5 Proposed Solution

Due to the vast number of different rotables and consumables and the size of Naturelink Aviation's fleet the company requested that the inventory model will be fairly simple and easy to operate. It was therefore decided that an EOQ model will be developed in an Excel database for ease of understanding the model and the program used.

5.1 *ABC Analysis*

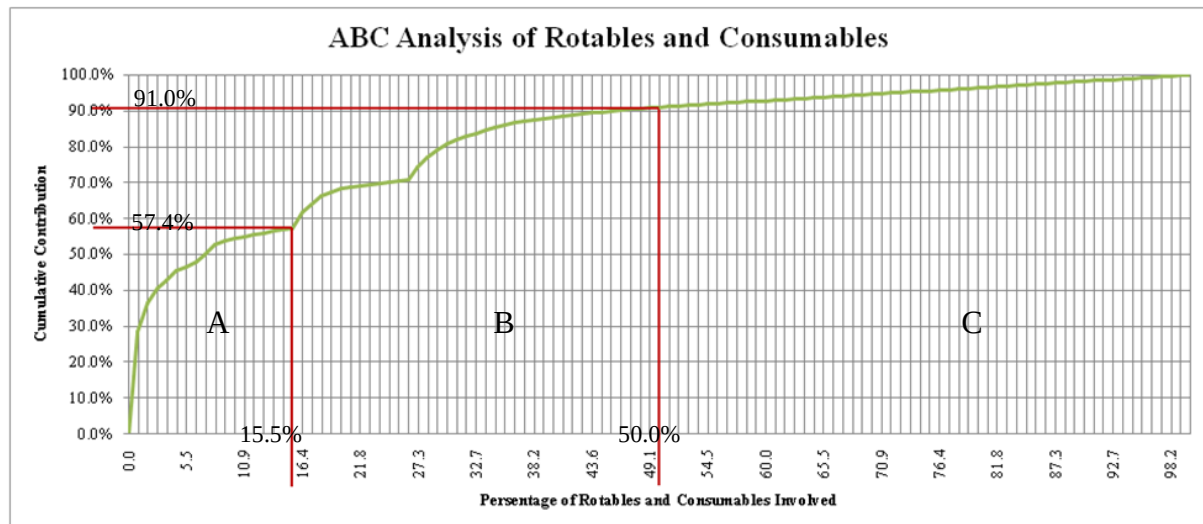
An ABC Analysis was conducted of all the rotables and consumables involved in any unscheduled maintenance on the Embraer aircraft. The ABC Analysis was not conducted in terms of Rand Volume contribution of the selling of each part because Naturelink is not a sales agent, it was rather conducted in terms of frequency usage. The aircraft can be grounded due to necessary replacement of cheap rotables or consumables and not just expensive spare parts. The analysis was conducted by comparing quantities used and the frequency of each kind of repair. The results of the ABC analysis was tabulate and as in theory it can be seen that just over 15% of all the rotables and consumables contribute to nearly 60% of usage and frequency.

Table 1: ABC Analysis

ABC Category	Number of Units	% of Product Rotable and Consumable Range	% of Overall Usage and Frequency
A	17	15.5%	57.4%
B	38	34.5%	33.6%
C	55	50.0%	9.0%
Total	110	100%	100%

The following graph was drawn in Excel to further illustrate the findings of the ABC analysis.

Figure 14: Pareto Graph of ABC Analysis for Unscheduled Maintenance Spares



For the purpose of this project it was decided that the focus will only be on the top 5% of all rotatables and consumables used, because eventually the same model equations will be incorporated for all the rotatables and consumables involved in unscheduled maintenance. This 5% consists of 6 items, they are tabulated below with their individual contribution to the overall usage and frequency.

Table 2: Top 5% of Rotables and Consumables

Spare Rotable/ Consumable Name	Item Code	Unit Price	Overall Usage %
Engine Oil	BP 3280	R36.38	28.6%
Hydraulic Fluid	Fluid 41	R232.43	7.7%
Compressor Wash Fluid	Turco 6783-50	R442.50	4.3%
Navigation Light	1308	R9.15	2.6%
Tyre Main Wheel	247R28-1	R4300.80	2.1%
Tyre Nose Wheel	185F03-5	R3091.20	1.4%

5.2 Inventory Cost and Variable Calculations

5.2.1 Demand and Standard Deviation

The annual demand for each item was determined by calculating the average demand of an item per day, because all data is in terms of days. The historical data spanned over 8 years. This average daily demand is then converted into annual demand by multiplying it with 365 days.

$$\text{Average Annual Demand} = \text{Average Daily Demand} \times 365 \text{ Days}$$

The daily standard deviation was determined by using Arena's Input Analyser. The standard deviation is converted to annual standard deviation by multiplying it with the square root of 365 days.

$$\text{Annual Standard Deviation} = \text{Daily Standard Deviation} \times \sqrt{365} \text{ Days}$$

Table 3: Average Annual Demand and Standard Deviation

Spare Rotable / Consumable Name	Item Code	Average Annual Demand	Standard Deviation
Engine Oil	BP 3280	331	17.577
Hydraulic Fluid	Fluid 41	105	12.113
Compressor Wash Fluid	Turco	70	8.196
Navigation Light	1308	52	7.088
Tyre Main Wheel	247R28-1	30	4.967
Tyre Nose Wheel	185F03-5	19	4.623

5.2.2 Ordering Cost

The ordering cost includes shipping cost and documentation cost. Naturelink Aviation always uses the same size container, it measures 156cm x 163cm x 156cm. Air freight is charged per kilogram, but this is not necessarily the weight of the parcel. The chargeable weight is either the actual weight of the parcel or the volumetric weight depending which is higher.

Volumetric weight is calculated by:

$$\frac{\text{Length (cm)} \times \text{Breadth (cm)} \times \text{Height (cm)}}{\text{Volumetric Factor}}$$

The volumetric factor used for International air freight is 6000 and the current rate at which Naturelink Aviation is charged at is R16.89. The documentation cost per shipment is R50.00. Naturelink Aviation determined that one container can carry 3.064 items of each rotatable and consumable.

The ordering cost per shipment:

$$\frac{\text{Volumetric Weight} \times \frac{R}{kg} + \text{Documentation Fee}}{\text{Total Number of Item Types} \times 3.064} = R22.88$$

5.2.3 Holding Cost

Holding cost is made up of storage cost, a percentage of engineering labour cost and insurance. The average number of units stored at one time at the current outstation of the sample aircraft has been established by Naturelink Aviation to be 1833.3.

Storage cost:

$$\frac{\text{Storage Area} \times \text{Monthly Rate} \times 12 \text{ Months}}{1833.3}$$

$$\frac{4.3 \times 5.2 \times 42.97 \times 12}{1833.3} = R6.29$$

Labour percentage:

$$\frac{\text{Annual Salary} \times 2.5\%}{1833.3}$$

$$\frac{40000 \times 12 \times 2.5\%}{1833.3} = R6.55$$

Insurance:

$$\frac{\text{Storage Area} \times \text{Monthly Rate} \times 12 \text{ Months} \times 16\%}{1833.3}$$

$$\frac{4.3 \times 5.2 \times 42.97 \times 12 \times 16\%}{1833.3} = R1.01$$

Holding cost per unit per year = R13.84

5.2.4 Lead Time and Standard Deviation

From the historical data that was obtained from Naturelink Aviation an average lead time and standard deviation was calculated. The average lead time is 12.1 days (0.03315 years) with a standard deviation of 1.92 days (0.00526 years).

5.2.5 Safety Stock

After much discussion it was decided to keep a safety stock level equivalent to that of 7 days' unit usage. This will ensure that even with unexpected fluctuations there will be sufficient stock of rotables and consumables.

Table 4: Safety Stock Levels

Spare Rotable / Consumable Name	Item Code	Safety Stock
Engine Oil	BP 3280	7
Hydraulic Fluid	Fluid 41	3
Compressor Wash Fluid	Turco	2
Navigation Light	1308	1
Tyre Main Wheel	247R28-1	1
Tyre Nose Wheel	185F03-5	1

5.3 The Economic Order Quantity Model

The EOQ model in Excel is fairly basic. Only the orange and red areas has to be changed or updated, while the areas coloured in blue and green contains equations and formulas and does not need to be altered. The exchange rate, lead time, annual demand and unit cost have to be reviewed at appropriate times and changed or updated if needed. The blue areas contains values that are predetermined, such as ordering cost, holding cost and converting lead time from days to years. The green areas illustrate the results of the model, these are the EOQ, orders per year, order time intervals and reorder point.

Figure 15: Economic Order Quantity Model in Excel

ECONOMIC ORDER QUANTITY MODEL FOR UNSCHEDULED MAINTENANCE							
Current R/\$ Exchange Rate	R 7.50		Lead Time in Days	12.1	Lead Time in Years	0.03315	
	Engine Oil (BP 3280)	Hydraulic Fluid (Fluid 41)	Compressor Wash Fluid (Turco 6783-50)	Navigation Light (1308)	Tyre Main Wheel (247R28-1)	Tyre Nose Wheel (185F03-5)	
Annual Demand	331	105	70	52	30	19	
Unit Cost	R 36.38	R 232.43	R 442.50	R 9.15	R 4 300.80	R 3 091.20	
Holding Cost	R 13.84	R 13.84	R 13.84	R 13.84	R 13.84	R 13.84	
Ordering Cost	R 22.88	R 22.88	R 22.88	R 22.88	R 22.88	R 22.88	
EOQ	33	19	15	13	10	8	
Orders per Year	10.01	5.65	4.61	3.96	3.03	2.43	
Order Time Intervals	36.47	64.62	79.15	92.08	120.29	150.49	
Reorder Point	11	4	2	2	1	1	

An approximate exchange rate has to be entered

Average lead time over the historical data was determined, but a review every 6 months or year is necessary

Lead time is converted from days to year

The EOQ value and Reorder Point are the two most important variables to be calculated

5.4 Results from the Economic Order Quantity Model

Table 5: Important Results from the Economic Order Quantity Model

Spare Rotable / Consumable Name	Item Code	Average Annual Demand	EOQ	Reorder Point	Orders per Year	Order Time Intervals (days)
Engine Oil	BP 3280	331	33	11	10.01	36.47
Hydraulic Fluid	Fluid 41	105	19	4	5.65	64.62
Compressor Wash Fluid	Turco 6783-50	70	15	2	4.61	79.15
Navigation Light	1308	52	13	2	3.96	92.08
Tyre Main Wheel	247R28-1	30	10	1	3.03	120.29
Tyre Nose Wheel	185F03-5	19	8	1	2.43	150.49

The following table and six graphs illustrates the variation that occurs within the standard deviation range of each rotatable or consumable. The reorder point calculated here is the original reorder point plus the safety stock.

Table 6: Range of Variables over the Standard Deviation of Engine Oil Demand

Spare Rotable / Consumable Name	Average Demand	Stock Level	EOQ	Reorder Point	Safety Stock
Engine Oil	313-349	37-42	31-35	17-18	6-7
Hydraulic Fluid	93-117	18-23	16-21	5-6	2-3
Compressor Wash Fluid	62-78	15-18	13-17	3-4	1-2
Navigation Light	45-59	11-14	11-15	2-3	1
Tyre Main Wheel	25-35	9-12	8-12	1-2	1
Tyre Nose Wheel	14-24	6-10	6-10	1	1

Figure 16: Range of Variables over the Standard Deviation of Engine Oil Demand

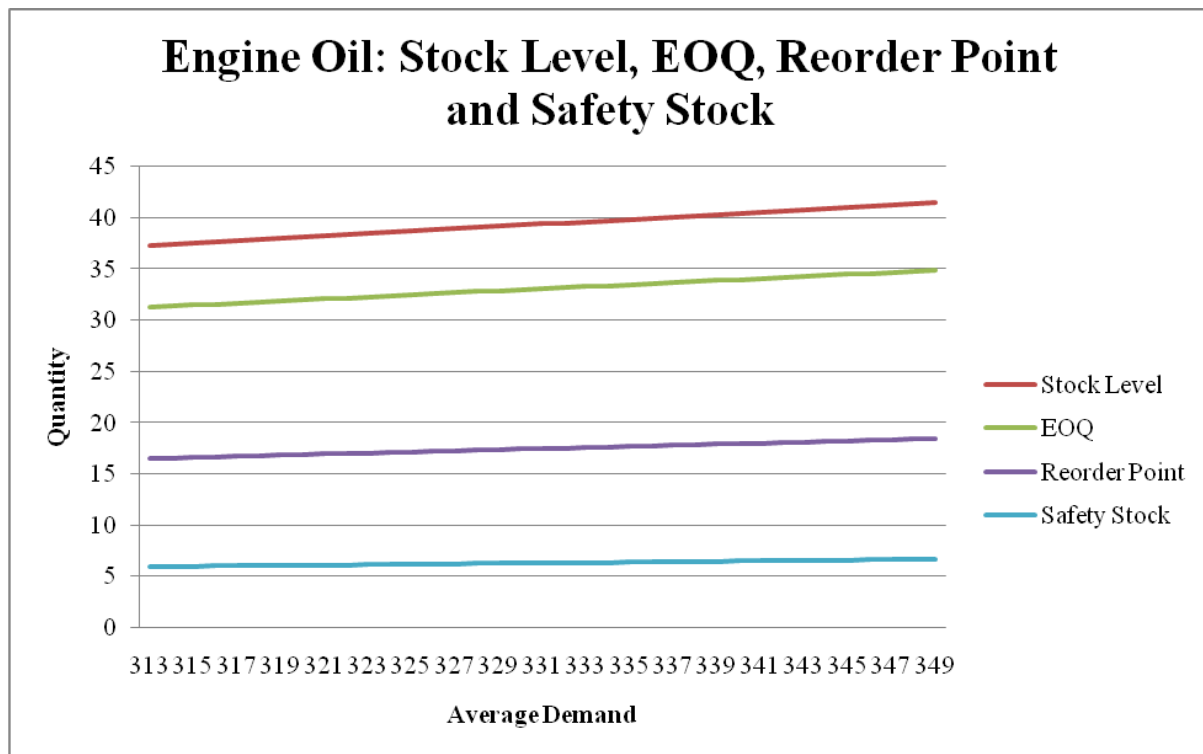


Figure 17: Range of Variables over the Standard Deviation of Hydraulic Fluid Demand

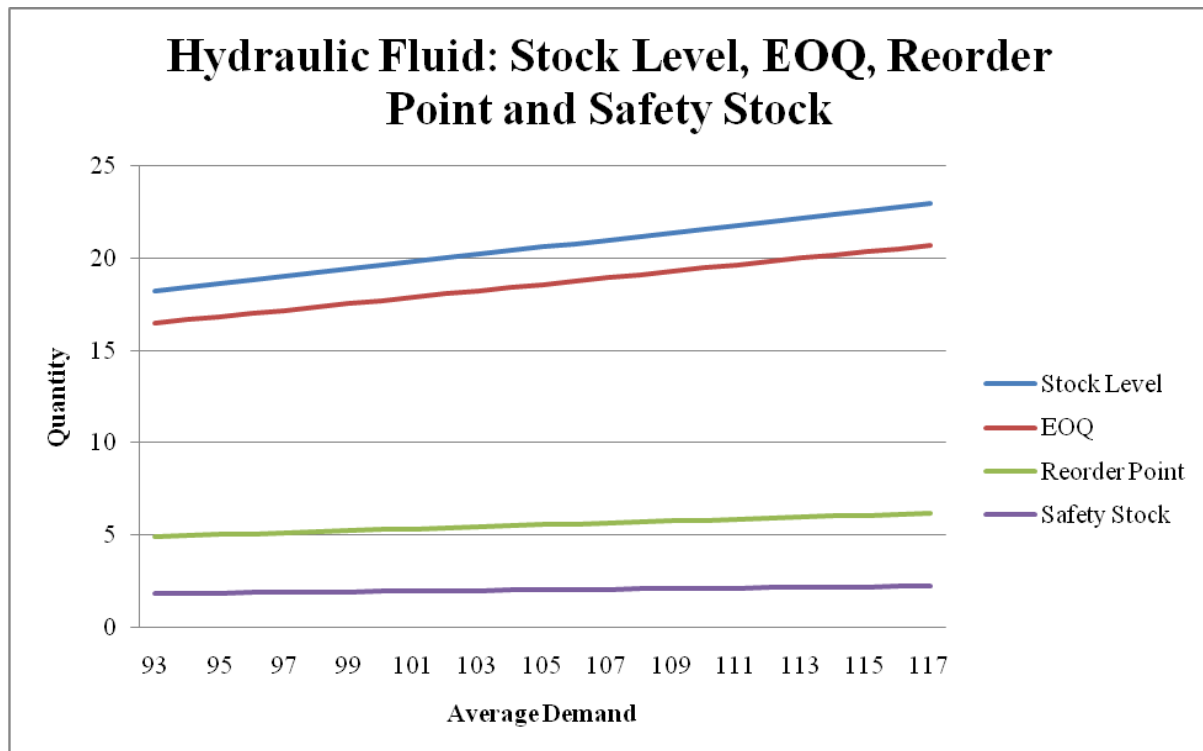


Figure 18: Range of Variables over the Standard Deviation of Compressor Wash Soap Demand

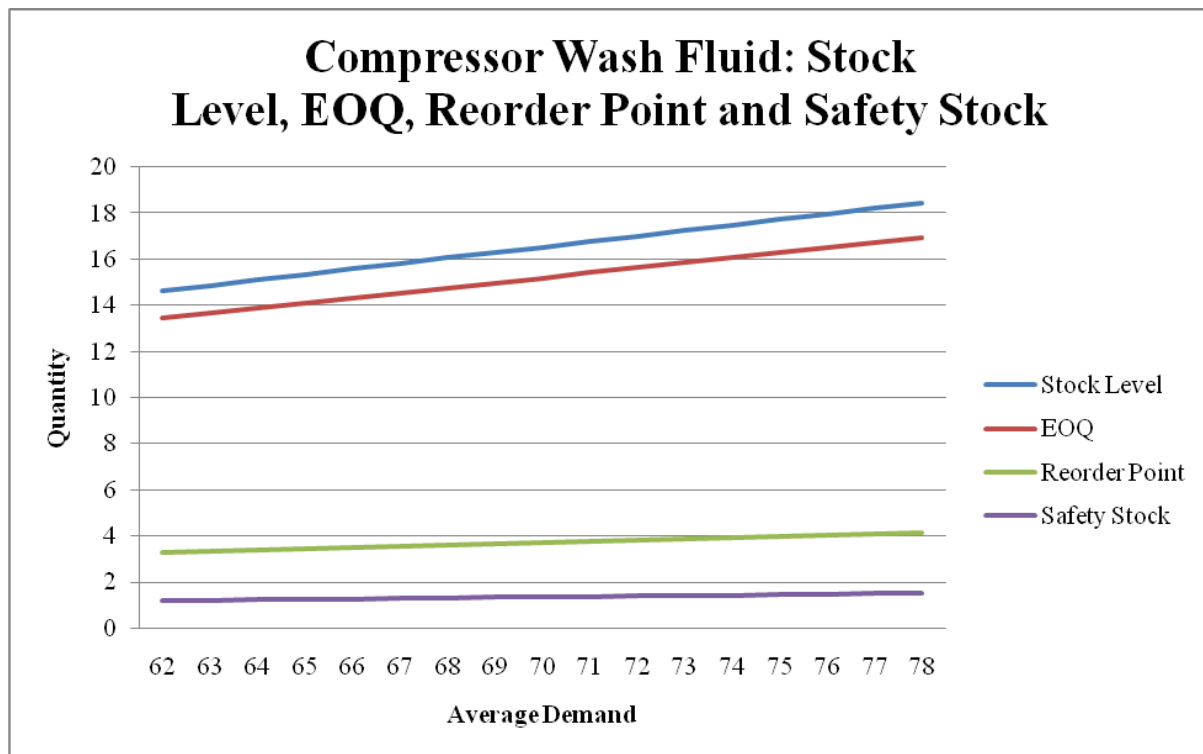


Figure 19: Range of Variables over the Standard Deviation of Navigation Light Demand

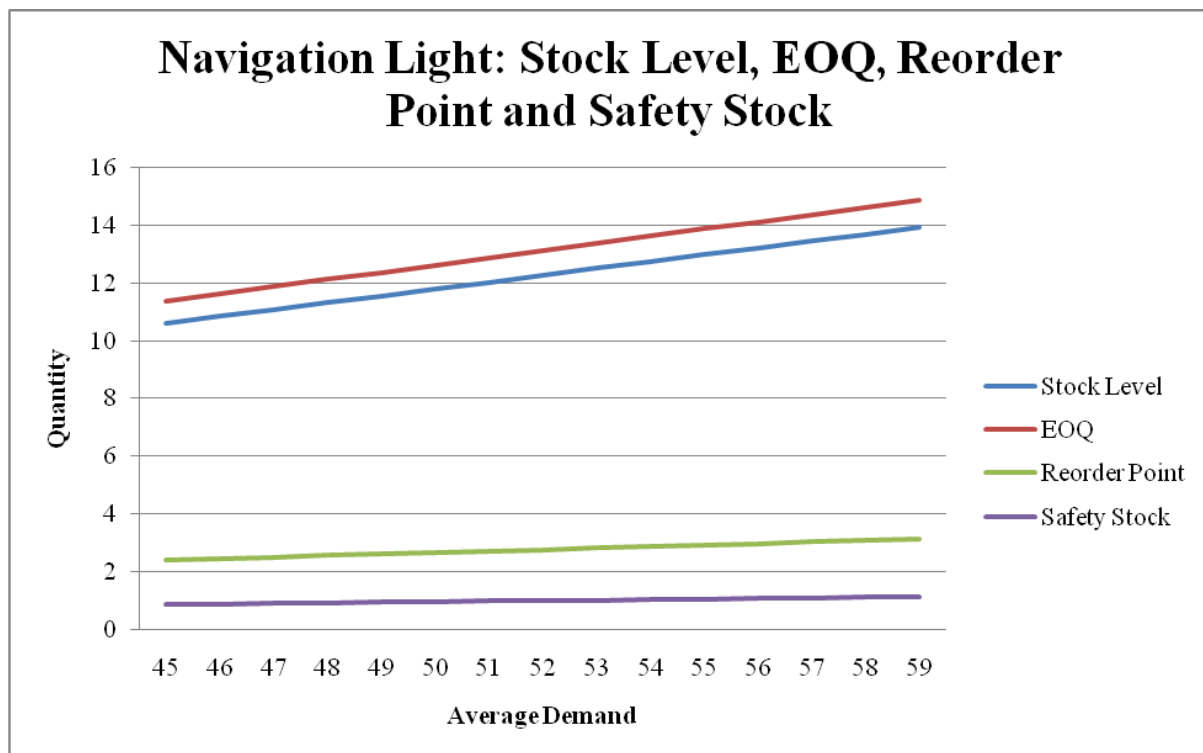


Figure 20: Range of Variables over the Standard Deviation of Tyre Main Wheel Demand

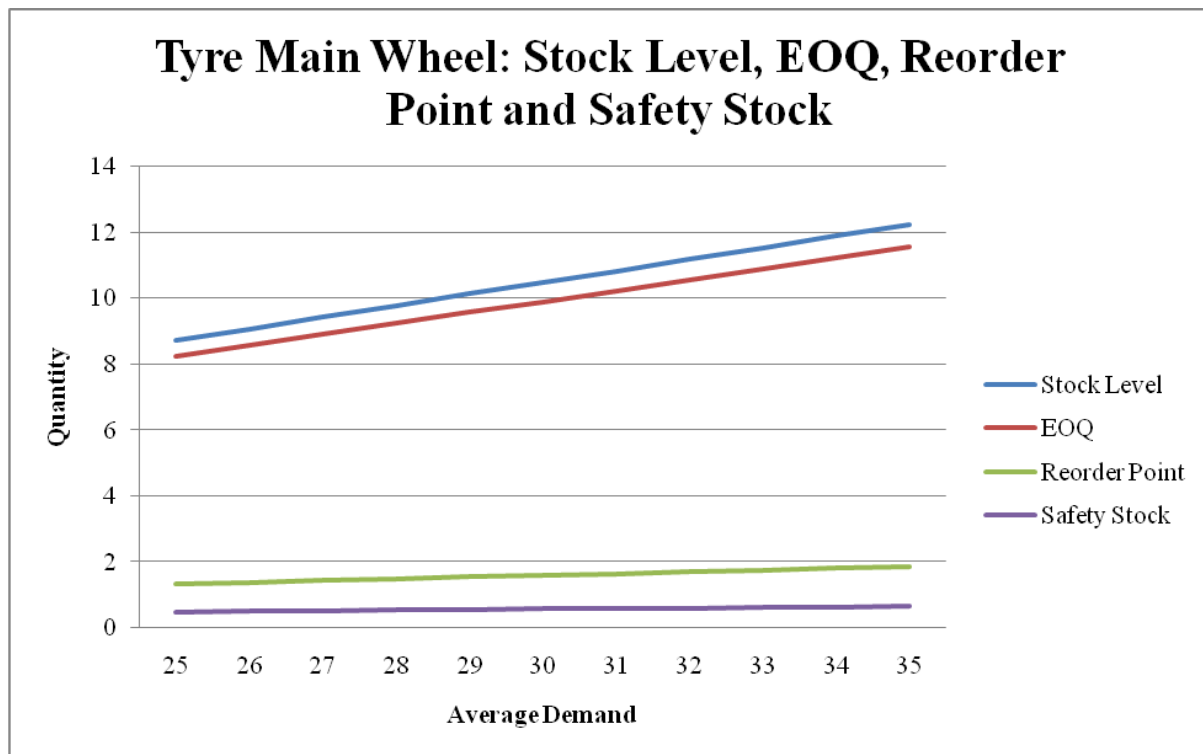
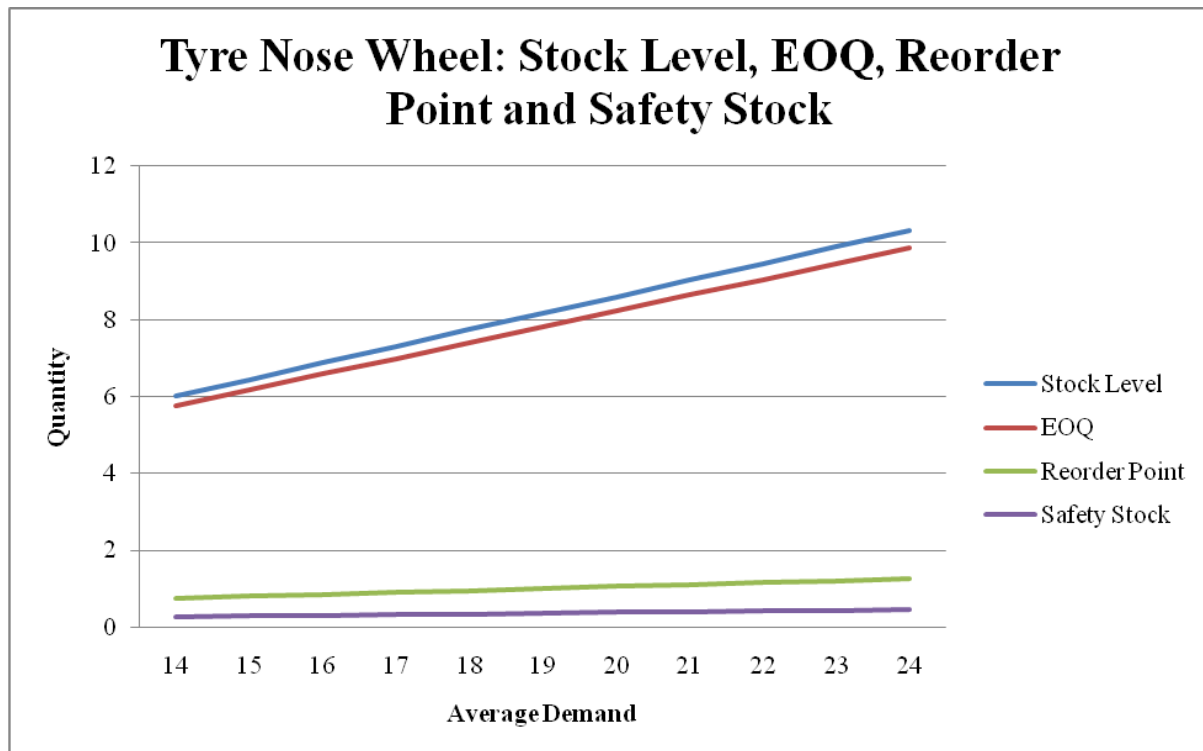
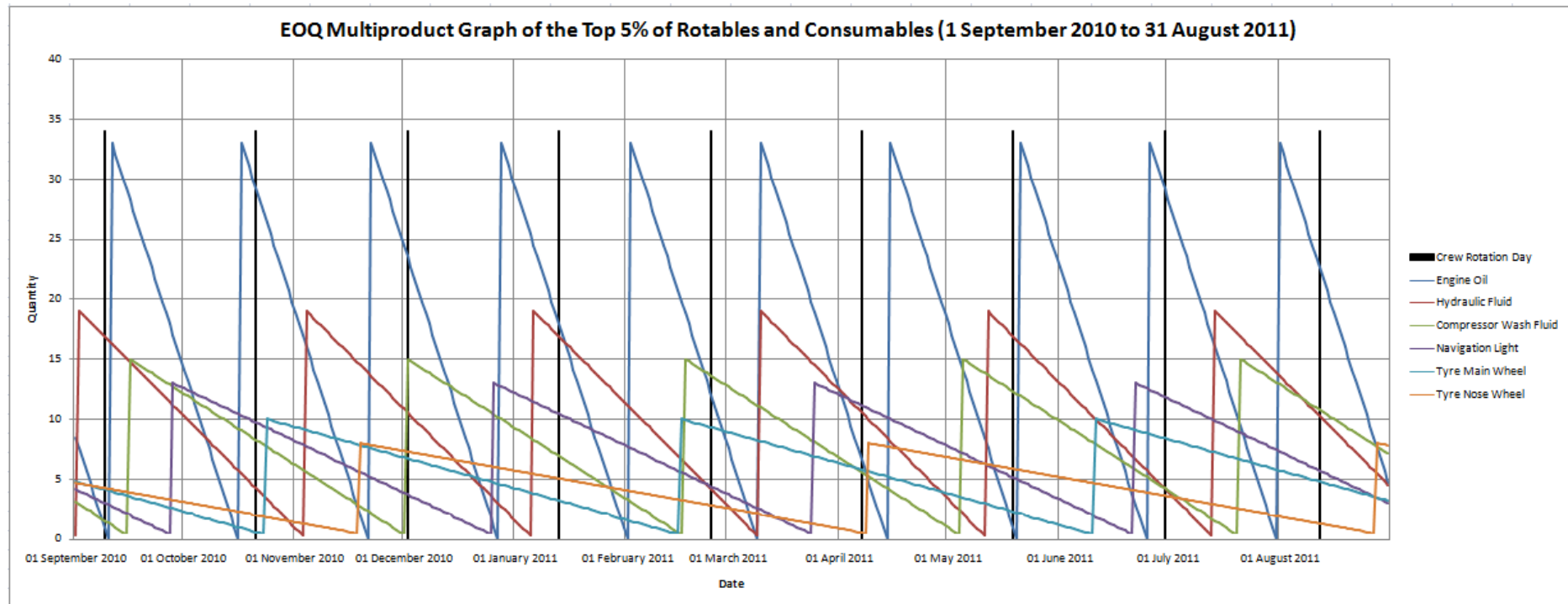


Figure 21: Range of Variables over the Standard Deviation of Tyre Nose Wheel Demand



From this EOQ graph it can easily be seen that the reorder points of the different rotables and consumables either overlap or occurs in short succession of each other. In addition the crew rotation schedule also shows that when needed they are able to take small / light parts along to the outstations.

Figure 22: Economic Order Quantity Graph of the Top 5% of Rotables and Consumables



6 Conclusion

With the assistance of Naturelink Aviation all of the dependent and independent variables was determined and calculated. Since air freight is rather expensive the combination of orders at different time intervals has to be carefully calculated and constantly updated and verified, because reorder points and EOQs may change. From the EOQ graphs that was developed through out the project it can clearly be seen that reorder point sometimes coincide or occur close together. A fairly simple schedule will be tested by Naturelink and if modifications are required these will be made before distributing and implementing through out the entire fleet.

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