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TURBULENT MIXING IN 19-ROD ASSEMBLY WITH NONEQUIVALENT ELEMENTARY CELLS

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ABSTRACT

This paper present experimental results of turbulent mixing in 19 smooth and rough rods assembly with thermohydraulic nonequivalece of elementary cells and a pitch-to-diameter ratio s/d = 1.35 in a circular shell.

INTRODUCTION

The efforts of researchers, dealing with the problem of determining the coefficient of turbulent mixing in rod bundles, were devoted to searching for the relations that would render the cellular methods more universal.

As of now, hydrodynamics of flow in the channels of complicated shape has been investigated inadequately. The available studies demonstrate that the rate of the turbulent interchannel transfer in actual assemblies depends in a complex manner on secondary flows and directional convective flows. However, convective flows are determined mainly by various structural, technological and operational deviations in the bundle geometry, which are sometimes not known exactly. This explains the fact that, even in what might appear to be identical conditions, various researchers obtain markedly dissimilar values of the coefficient of interchannel mixing in the bundles of circular rods.

The anisotropy of the coefficient of turbulent diffusion and secondary flows depend on the flow regime and geometry of cells. This presents certain difficulties in calculating the velocity and temperature fields even in the system of connected cells, which are channels of ideal shape. There are also other mechanisms of mass and heat transfer in actual assemblies [1]. The deviation of geometric dimensions of the rod lattice from ideal average values results in that the convective flow at the coolant, which is stable in time, becomes randomly distributed in space.

The next factor influential on mixing in actual rod assemblies is spacer elements. The authors of Ref. [2] cite a number of the reports of foreign firms that demonstrate that an increment in turbulent mixing increases in proportion to the reduction of the flow area of the cell due to the presence of the spacer grid. With a distance from the spacer grid, the coefficient of turbulent diffusion decreases asymptomatically until flow is stabilized.

Noticeable secondary flows, the anisotropy of turbulent transfer and a complex influence of spacer elements complicate the theoretical development of the model of turbulent mixing even in the simplest cases of two neighboring cells. Therefore, the only reliable method of determining the coefficient of turbulent diffusion in rod assemblies is an experiment.

The rate of exchange between the coolant flows in neighboring cells of the rod assemblies can be characterized by the mixing coefficient that represents a portion of the flow, exchanging momentum and energy on unit length, relative to the total flow in the channel [3]

$$\mu_q = G_{k,n} / G_k \,, \tag{1}$$

where $G_{k,n}$ is the coolant flow in the lateral direction from cell k to cell n, and G_k is the axial coolant flow in cell k.

In calculating the temperature fields in this case, a system of equations for cells of the rod assembly is solved, which is closed using experimental coefficients of interchannel mixing μ_q [3].

A great many studies are concerned with turbulent diffusion in rod assemblies. Reviews of these studies can be found, for example, in Refs. [1, 2].

The terms, defining turbulent diffusion between the elementary cells k and n, enter into the balance equation as the Reynolds stresses $\tau_{k,n}$, and in the energy equation they appear as the expression of the heat flux $q_{k,n}$:

$$\tau_{k,n} = \frac{\rho \nu_\mu}{l_{k,n}} \left(u_n - u_k \right), \tag{2}$$

$$q_{k,n} = \frac{\rho \mu_q}{y_{k,n}} \left(i_n - i_k \right). \tag{3}$$

The quantities ν_μ and μ_q are related by the turbulent Prandtl number Pr_t:

$$\frac{\nu_\mu}{\nu} = \frac{\mu_q}{\mu}\frac{Pr_t}{Pr}. \quad (4)$$

In most studies of flow mixing, it is assumed that $Pr_t = 1$ and the rate of heat and mass transfer is the same ($Le_t = 1$). Then, it remains to experimentally determine the magnitudes of ν_μ or μ_q.

The applicability and accuracy of the cellular methods were examined in Refs. [4, 5]. Consideration was given to the energy equation alone. Convective heat transfer was disregarded. The solution, obtained using the cellular method, was compared with more "exact" solutions that give a detailed picture of the temperature distribution in the cells themselves. The object of comparison was either the difference of the average temperatures $t_{av,k}$ and $t_{av,n}$ in cells k and n, or the thermal mixing length $y_{k,n}$ (see Eq. (3)), which was calculated from the relation

$$y_{k,n} = \frac{t_{av,k} - t_{av,n}}{\frac{1}{w_{k,n}}\int \left.\frac{\partial t}{\partial \varphi}\right|_{k,n} dw_{k,n}}, \quad (5)$$

where the temperature gradient $\left.\frac{\partial t}{\partial \varphi}\right|_{k,n}$ was determined along the boundary $w_{k,n}$ between the neighboring cells.

The thermal mixing length $y_{k,n}$ is a function of the temperature gradient $\left.\frac{\partial t}{\partial \varphi}\right|_{k,n}$, which, in turn, can depend on the assembly geometry, the flow distribution among cells, the heat release in the rods, the coolant temperature at the entrance to the assembly, etc.

Quantity of μ_q (eqs. 3 and 4) is not universal characteristic for describing turbulent diffusion. This is attested by different forms of the relations and generalizing experimental data and are presented in review [2].

Figure 1 presents the coefficients of turbulent thermal conductivity obtained by various authors. Attention should be given to the fact that the ratio μ_q/μ, calculated using the local approach that was implemented in the HERA program using the integral approach, are not identical at s/d = 1.3. In comparison with the local approach, the integral approach gives about 14% smaller values of μ_q at $Re_{k,n} = 10^5$. In turn, the values of μ_q, calculated using the local approach, are half as large as the experimental values determined by Rowe and Angle [6]:

$$\frac{\mu_q}{\mu} = 0.0062\,Re_{k,n}^{0.857}. \quad (6)$$

The authors of Ref. [2] explain this by an insufficient length for stabilization of turbulent diffusion in experiments. However, there can also be other reasons. Kjellström [14] detected a significant anisotropy of the coefficient of turbulent viscosity: the ratio of the coefficients of turbulent viscosity in the circumferential and radial directions, averaged over the thickness $y_{k,n}$ and the circumference, is 4.79. If this value is used in the HERA model, the dependence for μ_q/μ will be in good agreement with the experimental dependence obtained by Rowe and Angle.

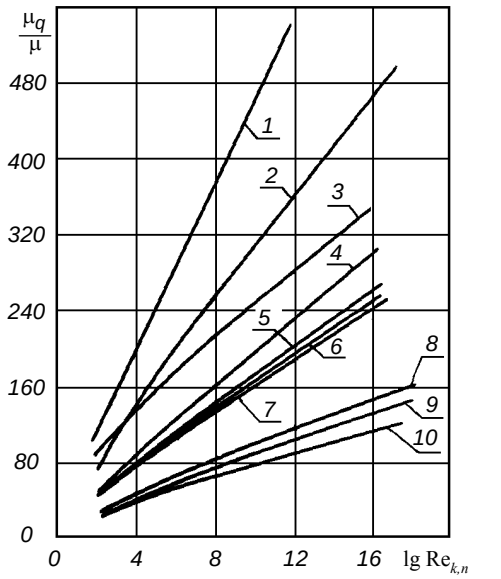

Figure 1 Turbulent thermal conductivity as a function of $Re_{k,n}$. 1, according to [6]; 2, according to [7]; 3, according to [8]; 4, according to [9]; 5, according to [10]; 6, according to [6]; 7, according to [11]; 8, calculation using the HERA program (a local approach) [2]; 9, calculation using the HERA program (an integral approach); and 10, according to Ref. [12]

Nomenclature

A	[-]	Coefficient in Eq.(7) – turbulent mixing coeficient
B	[-]	Coefficient in Eq.(7)
d	[m]	Rod diameter
DARS		Lithuanian abbreviation of computer program for calculation of gas-cooled reactor
G	[kg/s]	Flow rate of the coolant
i	[J/kg]	Enthalpy
l	[m]	Distinctive length between centres of the cell
N	[-]	Number of experimental points
u	[m/s]	Flow velocity
q	[W/m^2]	Heat flux
rem		Root-mean-squere
s	[m]	Rod spacing
t	[^{o}C]	Temperature
w	[m/s]	Velocity from cell *k* to cell *n*
y	[m]	Thermal mixing length
	[kg/s]	Flow rate through cell
Pr	[-]	Prandtl number
Re	[-]	Reynolds number
Le	[-]	Lewis number

Special characters

φ	[deg]	Angle
ν	[m^2/s]	Kinematics viscosity
ρ	[kg/m^3]	Density
τ	[N/m]	Shear stresses
ξ	[-]	Resistance coefficient

Subscripts

av	Average
c	Calculated
ex	Experiment
h	Hydraulic
k,n	Between the cells *k* and *n*
max	Maximum
min	Minimum
q	Heat
r	Rough
ran	Random
sm	Smooth
syst	Systematic
t	Turbulent

INVESTIGATIONS METHODOLOGY

In the present study, the coefficient of turbulent viscosity was determined from the equation

$$\mu_q / \mu = A \mathrm{Re}_{k,n}^{B} \sqrt{\xi_r / \xi_{sm}} \,. \qquad (7)$$

where the correction term ξ_r/ξ_{sm} was introduced for rough assemblies following recommendations of Ref. [13].

As shown above, the coefficient B in bundles depends on the friction law and is equal to 0.875–0.9 in the range $10^5 \le \mathrm{Re}_{k,n} \le 10^6$. Guided by the increased values of $\mathrm{Re}_{k,n}$, which can reach 10^6 in the considered models, we assume $B = 0.9$. This value of B is taken by many researchers. The parameter A for a specific assembly can be determined from the experimental flow temperatures $t_{ex,k}$ in the subcell of the measured cross section. Having the preset parameters of experimental conditions and varying the coefficient A, with the aid of the DARS program [15] it is possible to calculate the average coolant temperatures in cells $t_{c,k}$ in the same measured cross section. The rms deviation (Δt_{rem}) of the calculated temperatures $t_{c,k}$ from the experimental temperatures $t_{ex,k}$ is determined from the equation

$$\Delta t_{rem} = \sqrt{\sum_{i=1}^{N} \frac{1}{N} \left(\Delta t_{c,i} - \Delta t_{ex,i} \right)^2} \,. \qquad (8)$$

On the basis of the extremum of the function $\Delta t_{rem} = f(A)$, the minimum value of the parameter A ($A_{\min}$) is found at each experimental point characterized by specified values of G (the coolant flow through the assembly) and q (the mean heat flux). Having determined $A_{\min}$ at all experimental points, using the least-square method we find the final value of A for the entire range of operating conditions in the experiment and determine the rms deviation σ_A

$$\sigma_A = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(A_{\min,i} - A_i \right)^2} \,. \qquad (9)$$

where n is the number of experimental points and i is the running number of the experimental point. In order to determine the coefficient of turbulent mixing, experimental studies were conducted for a 19-rod bundle with spacer grids.

RESULTS

In order to determine the coefficient of turbulent mixing in a 19-rod assembly with nonequivalent cells, four experimental runs were performed with the bundles of smooth and rough rods. In these experiments, the average values of the coolant flow rates and temperatures in the cells were determined from the measured velocity and temperature fields around the rods in a 30° segment (Figure 2). Measurements around each rod were performed at 6° intervals in the azimuthal direction and at 1 mm intervals in the radial direction. In the first two and in the fourth experimental runs, the distance between spacer grids was 720 mm, and in the third run it was increased to 1440 mm. Analysis of the experimental data manifested that, in most experiments, the total coolant flow through the assembly, measured by a diaphragm, differs by no more than 4% in a smooth bundle and by 9% in a rough bundle from that obtained in the case of integration of the velocity field in the measured cross section.

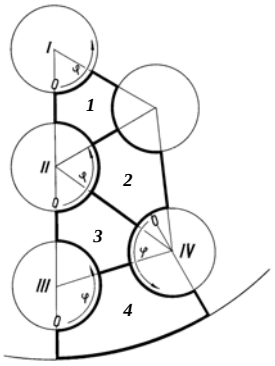

Figure 2 Splitting of the cross section in which the velocity and temperature fields were measured. *1-4* numbers of elementary cells, *I-IV* - numbers of rods

Figure 3 presents differences in the relative coolant flows through four cells, calculated using the DARS program ($y_{c,k}$) and determined experimentally ($y_{ex,k}$). The indicated differences are related to the relevant experimental values of the relative flow ($y_{ex,k}$). In the calculations, yhe shape factors of the cells were borrowed from [16], and the coefficient of turbulent viscosity was predicted using Eq. (7) at $A = 0.067$ and $B = 0.9$. As seen from Figure 3, in the two bundles, there are both a random deviation of $y_{c,k}$ from $y_{ex,k}$, caused by the error in the measurement of $y_{ex,k}$, and a systematic deviation, resulting from the error in the calculational model of the flow hydraulics in cells.

Table 1 presents the systematic deviations Δ_{syst} and the maximum random deviations $2\sigma_{ran}$, estimated with a 95% confidence coefficient, from the values, shown by a dash line in Figure 3.

For a smooth bundle, the values of $y_{c,k}$ in cells Nos. *1* and *4* are higher than the experimental values by 1.6% and 3.2%, respectively, with maximum random deviations of 1.9% and 2.8%. For a rough bundle, the systematic overestimate of $y_{c,k}$ in comparison with the experimental value is observed in cells

Nos. *1* and *3* (2.2% and 1.7%, respectively) with maximum random deviations of 3.4% and 4.5%.

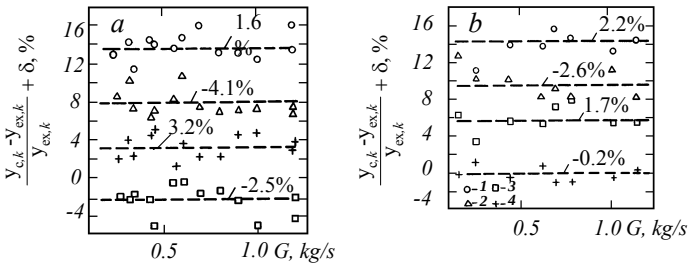

Figure 3 Relative deviation of the calculated coolant flow rates from the experimental values in cells in various experiments at A = 0.0067 in smooth (*a*) and rough (*b*) assemblies. Dots: *1–4*, in respective elementary cells. In elementary cells Nos. *1* and *2*, δ = 12; and Nos. *3* and *4*, δ = 0

Table 1 Systematic and random deviations in determining flow rates in cells

Deviation, %	Number of the elementary cell			
	1	2	3	4
	a. assembly of smooth rods			
Δ_{syst}	1.6	-4.1	-2.6	3.2
$2\sigma_{ran}$	1.9	2.4	2.4	2.8
	b. assembly of rough rods			
Δ_{syst}	2.2	-2.6	1.7	-0.2
$2\sigma_{ran}$	3.4	2.4	4.5	1.7

The systematic deviation of the calculated coolant flow rates from the experimental values can be attributed to the systematic deviation of the hydraulic diameters of the cells from the values specified in the calculation. It is not difficult to prove that, in the Blausius friction law, the shape parameter of the cell is proportional to $d_{h,k}^{1.25}$ Hence, it follows that the deviation of the calculated coolant flow rates from the experimental values is 1.6–4.1%.

The systematic deviation of the calculated values of $y_{c,k}$ from the experimental values can be eliminated through a formal selection of the shape factor for each cell separately. However, this is a very laborious task, since a multiple solution of the system of differential equations using the DARS program is required.

It is impossible to eliminate random deviations of $y_{c,k}$ from $y_{ex,k}$. The obtained random deviations of $y_{c,k}$ from $y_{ex,k}$ (1.9–2.8% for a smooth bundle and 1.7–3.4% for a rough bundle) are somewhat smaller than the difference in the coolant flows measured by the diaphragm and obtained by integrating the velocity profiles (~ 4%). They result mainly from the fact that the velocity profiles were measured, not over the total cross section, but only in a 30° segment.

If the correction of the shape factors of individual cells is rejected, then, according to Table 1, the maximum relative error in the determination of the coolant flow in the cells $y_{ex,k}$ is 6.5%, and the error in the measurement of the air heating in cells is 2.7% (this is the value obtained after estimating the experimental errors in the determination of separate quantities). When the effect of thermal heating is disregarded, the error of measuring $y_{ex,k}$ increases up to 9.2%.

In the overwhelming majority of experiments, the temperature of the coolant heating, calculated from the heat release in the rods of a smooth bundle, is higher by no more than 7% than that obtained by integration of the temperature field. In a rough bundle, this difference is about ±4%. In all experimental runs, there is the same picture of the distribution of average temperatures of the coolant among the cells. The maximum flow temperature in the assembly cross section is observed in cell No. *1*, and the minimum in cell No. *4*. The coolant temperatures in cells Nos. *2* and *3* is approximately the same. The maximum nonuniformity of the average temperature over the cells in the cross sections is close to the nonuniformity of the average temperature of the coolant heating in it.

Coefficient of turbulent mixing

As mentioned above, the coefficient of turbulent mixing was determined from Eq. (7). For all experimental conditions, the minimum value of the parameter A (A_{min}) was found that was characterized by the preset values of the coolant flow through the assembly and by the mean heat flux. The final value of the parameter A for the entire region of operating conditions in the experiment was determined using the least-squares method.

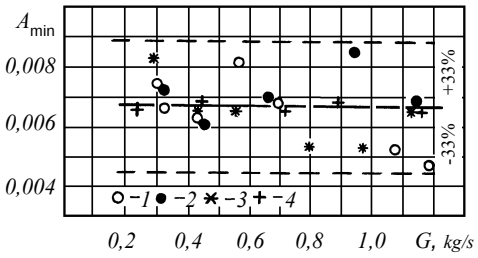

Figure 4 Minimum coefficient of turbulent mixing as a function of the coolant flow rate. Dots: *1–4*, experimental runs. The solid line represents the expectation values. The dash line shows the maximum deviation

A separate processing of the data of all experimental runs demonstrated that, for a 95% confidence coefficient, we should restrict ouserselves to a zeroeth-degree polynomial (Figure 4), i.e., that the approximating function is a constant independent of the coolant flow and calculated as the arithmetic mean of A (the expectation value)

$$\overline{A} = \frac{1}{N}\sum_{i=1}^{N} A_{\min,i} \cdot \qquad (10)$$

As seen from Figure 4, in experiments with smooth assemblies with spacer grids placed at intervals of 720 mm, A = 0.0067, which is only 8% higher than in Eq. (6) recommended in Ref. [9].

The relative rms deviation of experimental points $\overline{\sigma}_A = \sigma_A / \overline{A}$ is 16.4%, and the estimate of the maximum deviation with a 95% confidence coefficient is $2\sigma \approx 33\%$.

The expected value of A with an increased interval between spacer grids (1440 mm) is 0.0064 at the same $\overline{\sigma}_A$.

Figure 5 shows the A dependences of the calculated differences of average temperatures in smooth and rough bundles with nonequivalent cells in four cells in the measured cross section at arbitrary A and A = 0.0067. The curves are calculated using the DARS program. The values of the coolant heating in the cells (Δt_k) are referred to the average value of heating in the measured cross section (Δt_{av}). The dependences in Figure 5 make it possible to evaluate the error of the calculational model and compare it with the experimental error. As follows from Figure 5, with a 33% deviation of the coefficient A from a nominal value of 0.0067, the maximum relative error in determining the flow temperature, observed in the first (central) cell, accounts for 7% of the average air heating in the measured cross section of a smooth bundle and 3% in a rough bundle. After comparing this value with the estimate of the experimental error (9.2%), it can be concluded that the latter is larger than the computational error. Thus, the adequacy of the calculational model of the DARS program is proved in which the coefficient of turbulent mixing is expressed by Eq. (7), where A = 0.0067.

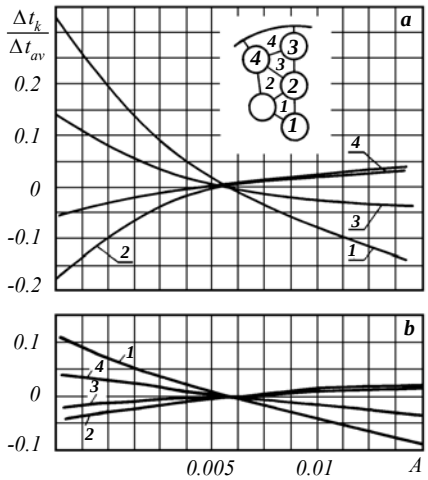

Figure 5 Relative error of calculating the air temperature in cells as a function of the mixing coefficient. *a*, smooth bundle; *b*, rough bundle. *1–4*, numbers of cells and rods

CONCLUSION

The turbulent mixing coefficients, obtained as a result of data processing, are of integral character along the assembly with account for the inlet effects. To a certain extent, provision is also made here of the convective transfer effects, caused by various secondary flows, which can be brought about by spacer elements, and also of those resulting from the difference of the calculational model for convective transfer among cells according to the DARS program from the actual picture in the experiment. Therefore, strictly speaking, μ_q represents a certain effective coefficient of turbulent thermal conductivity.

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