PREDICTING STOCK RETURNS AND VOLATILITY USING CONSUMPTION-AGGREGATE WEALTH RATIOS: A NONLINEAR APPROACH

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ABSTRACT

Recent empirical evidence based on a linear framework tends to suggest that a Markov-switching version of the consumption-aggregate wealth ratio (cay^{MS}), developed to account for structural breaks, is a better predictor of stock returns than the conventional measure (*cay*) – a finding we confirm as well. Using quarterly data over 1952:Q1-2013:Q3, we however provide statistical evidence that the relationship between stock returns and *cay* or cay^{MS} is in fact nonlinear. Then, given this evidence of nonlinearity, using a nonparametric Granger causality test, we show that it is in fact *cay* and not cay^{MS} which is a stronger predictor of not only stock returns, but also volatility.

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1. INTRODUCTION

In a seminal contribution, Lettau and Ludvigson (2001) showed that the consumption-aggregate wealth ratio (*cay*) is a strong predictor of quarterly real stock returns and the equity premium. The paper shows that a wide class of optimal models of consumer behaviour imply that the log-*cay* (human capital plus asset holdings) summarizes expected returns on aggregate wealth or the market portfolio, thus making it a strong predictor of quarterly stock returns. Ever since, a large number of studies have confirmed this finding (see Rapach and Zhou, 2013 for a detailed literature review). More recently, Bianchi *et al.* (2014) provide evidence of infrequent shifts, or breaks, in the mean of *cay*. Given this, Bianchi *et al.* (2014) introduce a Markov-switching

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version of this measure i.e., *cay*^{MS} and show that it has superior forecasting power for quarterly excess stock market returns compared to the conventional estimate.

Against this backdrop, the objective of our paper is to compare the predictive ability of *cay* and *cay*^{MS} not only for real stock returns, but also their volatility, using the nonparametric causality test of Nishiyama *et al.* (2011). This test is developed to study higher order causality and is inherently based on a nonlinear dependence structure between the variables. Our decision to use a nonparametric approach also emanates from the fact that the behaviour of stock returns and their relationship vis-à-vis *cay* and *cay*^{MS}, might be nonlinear. This framework is widely used in the linear predictive regression-based literature on forecasting stock markets, including that by Lettau and Ludvigson (2001) and Bianchi *et al.* (2014). Obviously, if the statistical tests reveal nonlinearity - the existence of which we show below – the results from linear models cannot be relied upon. To the best of our knowledge, this is the first paper that uses a nonparametric framework to compare the predictive ability of *cay* and *cay*^{MS} for the value-weighted Center for Research in Security Prices (CRSP) index-based returns used by Lettau and Ludvigson (2001) and Bianchi *et al.* (2014), as well as for the volatility of returns (measured by squared returns). The only related study is that of Ludvigson and Ng (2007), which analysed the predictive ability of *cay* for both excess-returns and its volatility using a linear predictive regression framework. The rest of the paper is organized as follows: Section 2 presents the empirical methodology, while Section 3 discusses the data and presents the results. Finally, Section 4 concludes.

2. METHODOLOGY

Hereafter we briefly describe the methodology proposed by Nishiyama *et al.* (2011), with the test restricted to the case when the examined series follow a stationary nonlinear autoregressive process of order one under the null. Nishiyama *et al.* (2011) motivated the high-order causality by using the following nonlinear dependence between series

$$x_t = g(x_{t-1}) + \sigma(y_{t-1})\epsilon_t \qquad (1)$$

where $\{x_t\}$ and $\{y_t\}$ are stationary time series and $g(.)$ and $\sigma(.)$ are unknown functions which satisfy certain conditions for stationary. In general, y_{t-1} has information in predicting x_t^K for a given integer K. Consequently, the null hypothesis of non-causality in the K^{th} moment is given by

$$H_0: E(x_t^K | x_{t-1}, \ldots, x_1, y_{t-1}, \ldots, y_1) = E(x_t^K | x_{t-1}, \ldots, x_1)\ w.p.1. \qquad (2)$$

where $w.p.1$ abbreviates to "with probability one". Formally, we say that y_t does not cause x_t up to the K^{th} moment if

$$H_0: E(x_t^K | x_{t-1}, \ldots, x_1, y_{t-1}, \ldots, y_1) = E(x_t^K | x_{t-1}, \ldots, x_1)\ w.p.1. \quad \text{for all } k = 1, \ldots, K \qquad (3)$$

For $K = 1$, this definition reduces to non-causality in mean. Nishiyama *et al.* (2011) note that, it is easy to construct the test statistic $\hat{S}_t^{(k)}$ for each $k = 1, \ldots, K$. We implement the test for $k = 1$ to test for causality in the 1st moment (non-causality in mean), and for $k = 2$ in the 2nd moment (non-causality in variance).

3. Empirical results

The value adjusted CRSP index (CRSP-VW), obtained from the Center for Research in Security Prices, is deflated by the personal consumption expenditure chain type price deflator (2009=100) to give us the real stock price.[1] The stock return is computed as the real log returns (*rcrspr*), and its volatility (*rcrspv*) as the squared values of the returns. The data on the two measures *cay* and cay^{MS} spans 1952:Q1-2013:Q3 and are obtained from Sydney Ludvigson's website. As we want to compare the predictive ability of *cay* and cay^{MS}, we standardize them by dividing with their respective standard deviations.

[1] As pointed out by Lettau and Ludvigson (2001), the CRSP Index (which includes the NYSE, AMEX, and Nasdaq) is believed to provide a better proxy for nonhuman components of total asset wealth because it is a much broader measure than S&P Index.

We start off with the standard linear Granger causality test. To ensure that our results are comparable with the nonparametric test, we use a lag-length of one in the vector autoregressive (VAR) model.[2] As can be seen from Table 1, the null hypothesis that *cay* or cay^{MS} does not Granger causes *rcrspr* is overwhelmingly rejected, with cay^{MS} showing greater predictability, thus confirming the results of Bianchi *et al.* (2014). We also conducted the Bai and Perron (2003) tests of multiple structural breaks on an AR(1) and a VAR(1) model of *rcrspr* including *cay* or cay^{MS} and we carried out the linear Granger causality tests. Interestingly, we could not detect any structural breaks.

TABLE 1: LINEAR GRANGER-CAUSALITY TEST

Dependent Variable: *rcrsp* (1952:Q1-2013:Q3)	
cay	7.78***
cay^{MS}	17.19***

Note: *** indicates rejection of the null hypothesis of absence of Granger causality at 1% level

Next, we used the BDS test (Brock *et al.*, 1996) on the residuals of an AR(1) and a VAR(1) model. As reported in Table 2, for *rcrspr* the test is found to overwhelmingly reject the null hypothesis of *i.i.d* structure for many possible dimensions, thus implying an omitted nonlinear structure. This might be evidence of a possible nonlinear relationship between *rcrspr* and *cay* or cay^{MS}. To further substantiate this indication and considering that the results from the linear Granger causality test cannot be relied upon, we rely on the utilization of the nonparametric causality test proposed by Nishiyama *et al.* (2011).

[2] The results are robust to the choice of various lag-lengths.

TABLE 2: BDS TEST

Dimension	AR(1)	*cay-based* VAR(1)	cay^{MS}*-based* VAR(1)
2	0.03	0.03	0.10
3	0.00	0.00	0.01
4	0.00	0.00	0.00
5	0.00	0.00	0.00
6	0.00	0.00	0.00

Note: Entries are *p*-values for the null of serial independence in the error structure of *rcrspr* after using an AR(1) filter or two VAR(1) model specifications i.e., [*rcrspr, cay*] and [*rcrspr*, cay^{MS}] respectively

The results are reported in Table 3. As it can be seen, *cay* and cay^{MS} are found to cause not only *rcrspr* but also *rcrspv*. Yet, unlike the linear Granger causality test, the results tend to suggest that *cay* is a stronger predictor, relative to cay^{MS}, of returns as well as volatility of returns. Based also on the BDS evidence of a nonlinear data generating process for the stock returns we consider the detected nonlinear causality results for *cay* and cay^{MS} obtained from the Nishiyama *et al.* (2011) test far more reliable.[3]

TABLE 3: NONLINEAR CAUSALITY TEST

Dependent Variable: *rcrsp* and *rcrspv* (1952:Q1-2013:Q3)

	Test statistics	
	$\hat{S}_T^{(1)}$	$\hat{S}_T^{(2)}$
cay	77.29**	47.54**
cay^{MS}	48.31**	22.93**

Note: ** indicates rejection of null hypothesis of non-causality at a 5% level

$\hat{S}_T^{(1)}$: Test statistic for causality in-mean; $\hat{S}_T^{(2)}$: Test statistic for causality in-variance

[3] We also conducted a robustness analysis on the stock returns of S&P500. Our results were qualitatively similar across the linear and nonparametric tests. The details of all test results are available upon request by the authors.

4. Conclusions

Ever since Lettau and Ludvigson (2001) and Ludvigson and Ng (2007) showed that the consumption-aggregate wealth ratio (*cay*) can predict stock returns and volatility under a linear regression framework, a large number of empirical studies confirmed their findings. Recently, Bianchi *et al.* (2014) introduced a Markov-switching version of this measure (cay^{MS}) to capture breaks in the mean of *cay*, and showed that this measure has a superior forecasting power compared to the conventional estimate. Initially our work confirms this finding using linear Granger causality tests. Nevertheless, we show that the evolution of stock returns as well as its relationship vs. the two *cay* measures is highly nonlinear. Using the nonparametric Granger causality test of Nishiyama *et al.* (2011) we demonstrate that, while it is true that both *cay* and cay^{MS} are strong predictors of stock returns, it is actually the conventional measure which predicts returns more robustly compared to cay^{MS}, as opposed to the findings of Bianchi *et al.* (2014). As the Nishiyama *et al.* (2011) allows for causality detection at higher moments, we further reveal that the results for return predictability carries over to stock volatility. To sum-up, when we account for inherent nonlinearities, *cay* is found to be a better predictor compared to cay^{MS} both for stock returns and volatility.

References

Bai, J. and Perron, P. 2003. Computation and analysis of multiple structural change models, *Journal of Applied Econometrics*, 18, 1-22.

Bianchi, F., Lettau, M. and Ludvigson, S. C. 2014. A Markov-Switching cay. Mimeo, New York University.

Brock, W., Dechert, D., Scheinkman, J. and LeBaron, B. 1996. A Test for Independence Based on the Correlation Dimension. *Econometric Reviews*, 15, 197–235.

Lettau, M. and Ludvigson, S. C. 2001. Consumption, Aggregate Wealth and Expected Stock Returns. *Journal of Finance*, 56, 815-849.

Ludvigson, S. C. and Ng, S. 2007. The empirical risk-return relation: a factor analysis approach. *Journal of Financial Economics*, 83, 171-222.

Nishiyama, Y., Hitomi, K., Kawasak, Y. and Jeong, K. 2011. A consistent nonparametric test for nonlinear causality - Specification in time series regression. *Journal of Econometrics*, 165, 112-127.

Rapach, D.E. and Guofu, Z. 2013. Forecasting Stock Returns in Handbook of Economic Forecasting, Volume 2A, Graham Elliott and Allan Timmermann (Eds.), Amsterdam: Elsevier, 328–383.