University of Pretoria

EFFECTIVE SCHEDULING OF HYBRID BATCH AND CONTINUOUS PROCESSES

by

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Thesis submitted in fulfillment of the requirements for the degree

Master of Science (Applied Sciences)

in the

Faculty of Engineering, Built Environmental and Information Technology

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University of Pretoria

Pretoria

January 2009

SYNOPSIS

An integrated Mixed-Integer Linear Programming (MILP) formulation for short-term scheduling of hybrid batch and continuous processes is presented in this thesis. This work is hatched from a realization that very few of the optimization formulations on production planning and scheduling address hybrid batch and continuous processes. In addition, the few of the proposed formulations are user friendly for practical applications. One of the better formulations is the continuous-time State Sequence Network (SSN) formulation proposed by Majozi & Zhu (2001) which explains scheduling of multipurpose batch processes. This thesis explores the applications of this formulation as a robust optimization tool to make scheduling decisions in large-scale manufacturing facilities and extends it to address hybrid batch and continuous processes.

DEDICATION

To my parents and grandparents

DECLARATION

I, Benard Monte Ongwae, declare that unless indicated otherwise, this dissertation is my own work.

This work has not been submitted in whole, or in part, for an award of a degree at another University.

Benard Monte Ongwae
January 2009

ACKNOWLEDGEMENTS

I would like to thank the following for their support in realizing this work;

My supervisor, Prof. Thokozani Majozi of the University of Pretoria. I came to the university with somewhat shaky projects from a pharmaceutical plant three years go and he welcomed me into his elite group of students. Since then, he has given me guidance and freedom when needed.

Mr. Leon Kruger, Director of Operations at Johnson & Johnson South Africa for seeing the potential in me. Leon appointed me as Process Engineer, Johnson & Johnson and gave me numerous opportunities including academic support at the University of Pretoria. I wouldn't be where I am today without his support.

The Technical support group, Sasol Polymers Sasolburg.

My family, Onty and son Shawn for the space and time during the duration of this thesis.

ABSTRACT

A large number of optimization formulations on production planning and scheduling for different manufacturing processes have been published over the past two decades. Very few of the proposed formulations are user friendly for practical applications. One of these formulations is the continuous-time State Sequence Network (SSN) formulation proposed by Majozi & Zhu (2001).

This thesis explores the application of this formulation as a robust optimization tool to make scheduling decisions in large scale manufacturing facilities. The formulation is extended to accommodate hybrid batch and continuous processes common in most Fast Moving Consumer Goods (FMCG) industries.

GLOSSARY

State	This is an input into or output from a unit
Unit	This is a processing or storage vessel
Time point	Is an instant/point in time within a time horizon
Capacity	The maximum or optimum amount that can be produced, stored by a unit
Variable	Quantity capable of assuming any set of values
Binary variables	Binary variables are integer variables that can only have values of 1 or 0
Formulation	A systematic expression of concept terms
Batch process	This is where materials required for a process are fed into the processing unit at the start of the process. Products are only taken out at the end of the processing period.
Continuous process	This is where materials required are continuously fed and/or products from the process are continuously produced during the course of the process.
Parameter	Is a constant in an equation or in a mathematical formulation
Approach	The method used in dealing with a problem or accomplishing a solution
Model	A schematic description of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics
Objective function	Something worked toward or striven for; a goal

Optimization The procedure or procedures used to make a system or design as effective or functional as possible, especially the mathematical techniques involved.

Algorithm A step-by-step computational procedure for solving a problem in a finite number of steps

Uncertainty Improbability or a condition of being in doubt

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CHAPTER 1

INTRODUCTION- NATURE AND PURPOSE OF THE INVESTIGATION

1.1. Background

Short-term scheduling of batch processes has received a lot of research attention with significant publications made in the past two decades. However, scheduling of hybrid batch and continuous processes that exist in most industrial plants has received little attention. For example, in the pharmaceutical, food, paint and dairy industries, batch processes are used to manufacture products in bulk quantities while sophisticated machines continuously package the bulk quantities into smaller units that are sold to consumers. In large-scale polyethylene and polypropylene manufacturing plants, polymerization support catalysts are produced using batch processes before continuously being injected into reactors.

Scheduling decisions entail the determination of optimal sequence of events and allocation of tasks to available resources in order to achieve a given performance criterion. In general, the performance criterion is to maximize production over a fixed time horizon or to minimize the makespan for a given output. However, initial formulations on scheduling were confined to small size problems and did not guarantee global optimality (Majozi & Zhu, 2001). The shortcomings from these initial formulations have triggered increased research in the past two decades resulting in the development of numerous approaches aimed at addressing these problems.

Most of the methods developed are too complex or inadequate in solving industrial problems. On the other hand commercially available software packages for production scheduling or high level planning decisions are expensive and uneconomical to implement for small manufacturing facilities. It is therefore important to develop alternative but effective approaches that can be beneficial to such facilities.

The purpose of this investigation was to explore the benefits of applying a robust mathematical optimization technique to make scheduling decisions in hybrid batch and continuous processes. In most cases, these decisions have significant impact on overall company profitability by defining capital utilization and operating costs.

1.2. Thesis structure

The remaining chapters are organized as follows: Chapter 2 presents an overview of various methods used to address scheduling problems. Chapter 3 reviews State Sequence Network (SSN) representation of Majozi and Zhu (2001). A literature example (Ierapetritou and Floudas, 1998) is used to demonstrate the modeling capabilities of this representation. The chapter is also used to demonstrate the capabilities of SSN representation to model hybrid batch and continuous processes.

Chapter 4 presents a case study from a multinational pharmaceutical manufacturing facility based in South Africa to illustrate the capability of the proposed approach to solve large-scale hybrid batch and continuous problems. The implications of the results from the industrial case study are discussed in Chapter 5. Finally, Chapter 6 gives conclusions drawn from this work and gives directions for further research opportunities.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

Batch processing is predominately used to produce multiple, low-volume, high-value-added products in a single plant. These products are made based on their own recipes by sharing available resources. This allows for a higher degree of flexibility in scheduling but adds to the complexity of operations. Scheduling involves the determination of the order in which different tasks are carried out on different equipments and the detailed timing of the execution of all tasks to optimize plant operation.

It is important to note that some of the techniques proposed to solve batch-scheduling problems are as complex and diverse as the problem itself. This chapter reviews some of the literature on batch processes published over the past three decades.

2.2. Characteristics of scheduling problems

A general scheduling problem can be defined as follows.

Given the following:

i) a set of N products and their required quantities

ii) a set of M machines or equipment which are available for the production of these products

iii) production recipe for each product, i.e. the processing times for each task at suitable units, and the amount of the materials required

iv) a performance index to be optimized and

v) a set of rules which govern the production processes, such as order of operations, intermediate storage policies, allowances made for subdividing product runs etc.

Determine:

i) the optimal schedule for all tasks involved within a required time period

ii) the amount of materials processed by each task at any time point within the required time period and

iii) best performance index

Scheduling problems may vary in several aspects to optimize an appropriate performance criterion. These aspects will determine the level of difficulty when modeling these processes and include the following:

i) *Sequential / non-sequential:* In sequential processes, different products follow in the same processing sequence. Processing stages can either be single stage or multistage. A stage can consist of only one unit or several parallel units per stage. In non-sequential processes, products do not necessarily follow same processing sequence (Floudas *et al.* 2004).

ii) *Open shop vs. closed shops:* In an open shop policy, all production orders are by customer request and no inventory is stocked. In a closed shop policy, all customer requests are serviced from inventory and production tasks are for inventory replenishment decisions (Rippin, 1983, Musier *et al.* 1986).

iii) *Preemptive vs. nonpreemptive production runs:* Under preemptive production, a batch may be split up or interrupted during its run. In nonpreemptive production, a task cannot be interrupted until it is completed. (Behzad *et al.* 2007)

iv) *Multiproduct vs. multipurpose plants:* In a multiproduct plant, products are produced successively in a sequence of single product campaigns, one product at a time. One route is followed through the plant for each product. An alternative to this is a multi-plant structure with two or more independent multiproduct plants operating in parallel. In a multipurpose plant, more than one product will be present in a plant at the same time. The same product will follow different processing routes through the plant at different times. These alternative routes may or may not be predefined. (Rippin, 1983).

v) *Demand patterns:* Demand for product can be specified either at the end of the time horizon of interest or at specified points within the time horizon, (Floudas *et al.* 2004).

vi) *Changeovers:* There are three main types of changeovers in batch plants (Floudas *et al.* 2004), as follows.

a) *Sequence dependent changeovers:* where a unit is cleaned or set up to handle a different type of product. This is done to avoid cross contamination between different types of products or for safety reasons.

b) *Time or frequency dependent changeovers*: where a unit is cleaned or set up after processing a specified number of tasks.

c) *None: w*here a unit is utilized without cleaning or setting up between different tasks.

vii) *Operational policies:* Operational policies govern material transfer between different processing units and can be classified into eight different categories (Romero *et al.* 2004);

a) *Unlimited intermediate storage (UIS)*: Within this policy, the processing equipment is available immediately after completing a task. This is achieved by transferring processed material into readily available intermediate storage. On practical terms, the available intermediate storage between processing units cannot be exhausted.

b) *No-intermediate storage (NIS)*: Within this policy, a unit is not available until it has completed a task and its contents transferred to the next stage. This policy does not permit the use of intermediate storage.

c) *Finite intermediate storage (FIS)*: Within this policy, the available storage capacity between two processing units is fixed and can be exhausted. This constraint is a common feature in most manufacturing industries.

d) *Zero wait (ZW)*: Within this policy, intermediate products must be consumed immediately after being produced. A good example is where unstable intermediate products must be consumed immediately after production.

e) *Unlimited Wait (UW):* Within this policy, intermediate materials can wait in storage or in the current processing unit as long as it is necessary.

f) *Finite Wait (FW)*: Within this policy, intermediate materials can wait in storage or in the current processing unit for a limited set time period. *FW* is necessary to limit the wait time of unstable intermediate material in storage or in a processing unit as traditionally encountered in the paint industry.

g) *Mixed intermediate storage (MIS)*: In most plants, there is a combination of *FIS*, *NIS*, *FW* etc. This is termed mixed intermediate storage.

h) *Common intermediate storage (CIS)*: Within this operational policy, storage is shared amongst all processing units. A storage vessel can be allowed to receive the same product from different processing units. Alternatively, different processing units can receive same input material from a common storage, (Huang and Chen, 2007).

If restrictions are placed on the availability of intermediate storage, machine idle times will increase and this will in turn increase the makespan. Having intermediate storage decouples upstream and downstream operations allowing independence between the two subprocesses in terms of capacity constraints as well as cycle times, (Floudas *et al.* 2004).

One of the key mathematical items in scheduling problems is time representation. Mathematical scheduling formulations can be classified into two main categories, i.e. discrete and continuous time models (Floudas *et al.* 2004). In discrete-time approaches, the time horizon is divided into a number of time intervals of equal durations as shown in Figure 2.1 (a). The beginning or ending of a task occurs at the boundaries of these time intervals.

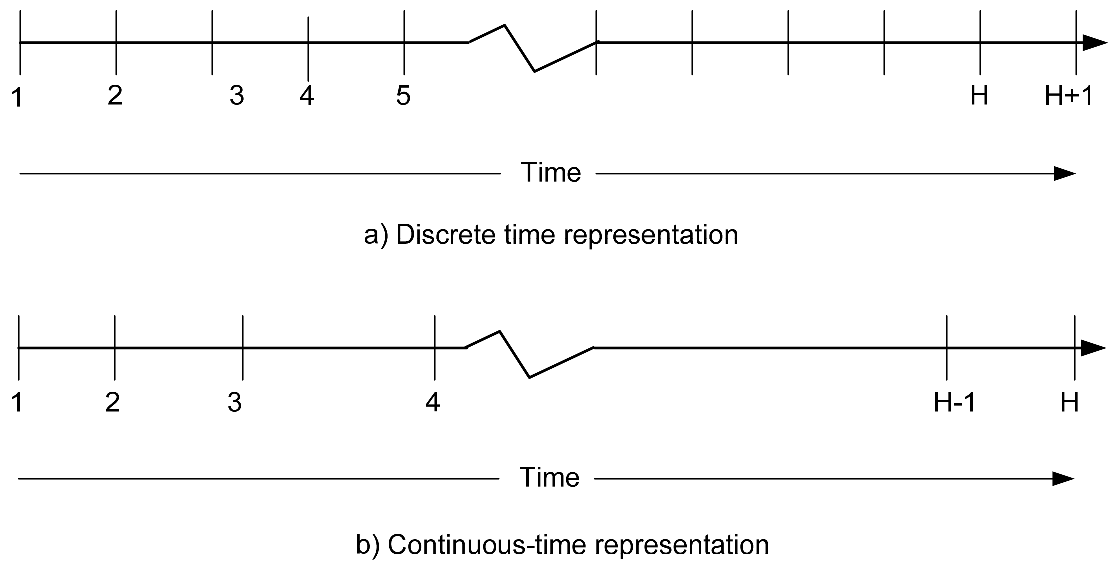


Figure 2.1: (a) Discrete and (b) Continuous time representations

The selection of the number of uniform time intervals in discrete time representation will present a compromise between the quality and accuracy of the solution and the required computational effort. To achieve a suitable approximation of the original problem, it is usually needed to use a time interval that is sufficiently small. The advantage of discrete-time representation is that it provides a reference grid of time for all operations in a scheduling problem. This representation has two limitations. Firstly, since time is continuous, discretization of time is only an approximation. Secondly, large size mixed integer linear programming (MILP) problems arise from a large number of discrete time intervals. This leads to combinatorial problems that are either difficult or impossible to solve for practical applications.

In continuous-time representations, events are allowed to take place at any point in the continuous time domain as shown in Figure 2.1 (b). Modeling this flexibility is achieved by introducing variable event times, which can be common to each unit or unique for each unit. The possibility of eliminating inactive event-time intervals with the continuous-time approaches results in smaller mathematical problems. However, because of the variable timing of events, modeling the scheduling processes become more challenging with complicated mathematical models.

2.3. Mixed integer linear programming (MILP)

A MILP program is a problem that can be expressed as follows:

$$\begin{aligned}
&\min f(x, y) = Z \\
&subject\ \ to: \\
&h(x, y) \le 0 \\
&g(x, y) = 0 \\
&x \in R^n, y \in \{0,1\}
\end{aligned}$$

where h and g are all linear functions.

In the MILP approach, a scheduling problem is formulated as a mixed integer linear program and solved by available optimization packages. The scheduling performance criterion is formulated as a linear objective function and scheduling constraints as linear equalities and / or inequalities (Huang and Chen, 2007).

2.4. MILP algorithms

Recipe networks similar to flowsheet representation in continuous processes applied to complex batch plants create ambiguities especially in determining exact material sequences and process description. To overcome this problem, Kondili *et al.* (1993) presented a general discrete-time State Task Network (STN) MILP framework for scheduling a wide range of multipurpose and multiproduct batch plants.

STN representation allows for a detailed description of operations in a batch plant. This representation contains two types of nodes namely state nodes and task nodes. State nodes represent feeds, intermediaries, and products and are denoted by circles. Task nodes represent processing tasks and are denoted by rectangles. Interconnecting arcs represent state and tasks precedence. This formulation uses discrete-time approach, where the time horizon is divided into a number of time intervals of equal duration. Events such as the beginning or end of task are forced to coincide at the intervals of these time slots. For allocation of tasks to units with capacity constraints, binary variables W_{ijt} are used to assign the start of task i in unit j at the beginning of a time interval t. The main drawbacks of this formulation are twofold. Firstly, time is continuous and the descritization of time compromises the accuracy of the solution of the original problem. In addition, this formulation leads to a large number of binary variables even when dealing with small problems. In general, the more binary variables a problem has, the more computational effort is required to find a solution. This is also true for small but complex problems with few binary variables. For both cases, the large computational effort required to find a solution might prove impractical when dealing with large scale industrial applications. It is worth mentioning that STN representation is revolutionary and forms the basis of most work done in this field.

Zhang and Sargent (1994) proposed a continuous time formulation based on Resource Task Network (RTN). In RTN, a production facility is treated as a collection of tasks and units where resources are used and some formed. Resources can be feeds, production equipment, utilities, personnel, intermediate products etc. This formulation considers cleaning and transportation as tasks in addition to processing tasks. The drawback to this formulation is that it leads to a large number of binary variables rendering it difficult to handle using available MILP solvers.

Schilling and Pantelides (1996) first introduced the concept of time points and time slots as shown in Figure 2.2. Each time point represents the beginning and/or end of a particular task, and each time slot is defined by the boundaries of two consecutive time points. A time slot is duration of a task. Using the same concept, Zhang and Sargent (1996), allowed the variation of batch sizes for the same task starting at the same point in different units. They also allowed this interaction to occur discretely at distinct time points.

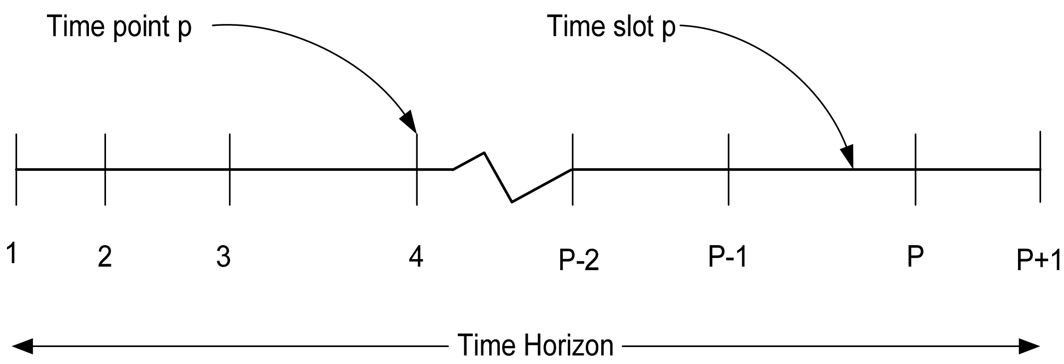


Figure 2.2: The concept of time points- Schilling and Pantelides (1996)

In most scheduling procedures, demands must be met at the end of the time horizon without due consideration of intermediate orders of products. To this effect, Pinto and Grossmann (1994) proposed a continuous time formulation based on parallel time coordinates for units and orders to address intermediate order problems. This approach was limited to fixed batch sizes and did not take into account resource constraints.

The common feature of all the formulations discussed above is the use of a binary variable to describe the assignment of a unit j to a task i at any point in time, n. This leads to a large number of binary variables of a dimension $|I|\times|J|\times|N|$, where, $|I|$, $|J|$ and $|N|$ denote the number of tasks, units and event points respectively. Ierapetritou and Floudas (1998) proposed a continuous-time formulation aimed at reducing the number of these binary variables. Their procedure used the STN representation and was based on the concept of event points, similar to time points proposed by Schilling and Pantelides (1996) as shown in Figure 2.2. In this formulation, an event point represents the beginning or an end of a task at a particular point in the time horizon. The main contribution of this work is the decoupling of unit events from task events by assigning two separate binary variables: $yv(j,n)$ to units, and $wv(i,n)$ to tasks. This leads to $|N|\times(|I|+|J|)$ number of binary variables. Compared to previous formulations, this particular formulation results in a smaller number of binary variables; $|N| \times (|I| + |J|)$ compared to $|I| \times |J| \times |N|$. However, it also has drawbacks. Firstly, in situations where stages involve several units, it predicts a relatively larger number of binary variables, which are later reduced by exploiting the one-to-one correspondence between units and tasks. Moreover for large problems, the reduction of binary variables might not be straightforward. The second drawback is the modeling of duration constraints as the function of batch size where the processing times consists of fixed and variable processing terms. This implies that the larger the batch, the longer the processing time in the same unit. This is not entirely true and imposes restrictions on time leading to suboptimal results.

Ierapetritou *et al.* (1999) presented a continuous time formulation based on the work by Ierapetritou and Floudas (1998) to handle intermediate due dates. This formulation has similar drawbacks discussed above.

Majozi and Zhu (2001) presented a continuous-time formulation motivated by the drawbacks of Ierapetritou and Floudas (1998). They introduced a new kind of representation, the State Sequence Network (SSN) representation consisting of only states. The main difference between SSN representation and STN representation is that in SSN only states are considered while tasks are implicitly incorporated in the subsequent formulation. This formulation uses time points to denote the use or production of a state. A single binary variable $y(s, p)$ associated with the use of a state at a time point is used throughout the formulation leading to small MILP problems. The paper presents two options of modeling time constraints. One option is to model the duration constraints as a function of batch size. The other option is to incorporate the degrees of freedom inherent in batch manufacturing processes. The optimal number of time points is determined using an iterative procedure proposed by Ierapetritou and Floudas (1998). The formulation depends on predefining an *effective state* where more than one input states are utilized in a unit at the same time. The same authors extended this formulation in integrating planning and scheduling, Zhu and Majozi (2001).

Mendez and Cerdá (2004) presented a continuous time MILP mathematical framework for reactive scheduling of resource-constrained multistage batch facilities. This formulation takes into account the schedule currently in progress, updated information on old production batches still to be processed, new order arrivals, present plant status and actual availability of renewable discrete resources like processing units and manpower. This formulation is able to update a schedule in progress when unforeseen events like deviations in processing times, equipment breakdown or batch reprocessing occur. To avoid full-scale rescheduling, the approach allows partial modifications to a schedule in progress consisting of starting time shifting, limited resource reallocation, and local batch reordering at any utilized resource item. This formulation has two major drawbacks. First it does not allow batch splitting or merging. Secondly, a large number of binary variables of the dimension $|I|\times|J|\times|S|$ is encountered, where $|I|$, $|J|$ and $|S|$ denote the number of tasks, units and stages respectively. Other works on reactive scheduling have been presented by Hasebe *et al.* (2000).

Majozi and Zhu (2004) presented a planning model based on SSN that determines the optimal allocation of operators on normal and penalized days. A normal day is a day that is not a holiday or a weekend. Rates of pay for penalized days differ from rates of pay for normal days. For example in South Africa, the basic conditions of employment Act (1997) dictate that 1.5 $\times$normal day pay be paid out for work done on a Saturday and 2$\times$normal day pay for work done on a Sunday and/or any holiday. There are also special allowances for work done during night shifts. The legislation also sets limits to the number of work hours that a person is allowed per day. It is important to note that each country has similar labor laws. This procedure takes into account operator quality and the corresponding impact they have on plant performance. The drawback of this model is the assumption that operator performance is directly related to operator rank. This is not necessarily true because there are many variables other than competencies that can affect operator performance on hourly or daily basis. Moreover, rank does not always translate to performance or productivity. The authors have acknowledged the qualitative nature of their work.

Floudas *et al.* (2004) presented an enhanced continuous-time formulation that considers intermediate due dates with resource constraints. This work is based on the work of Ierapetritou and Floudas (1998) and incorporates several features such as various storage policies (*UIS, FIS, NIS, ZW* and *MIS*), resource constraints, batch mixing and splitting and sequence-dependent changeovers. The key feature of this formulation is the continuous-time representation where tasks are allowed to continue over several event points. This enables resource quantities to be determined at the beginning of each utilization point. This formulation has the same drawbacks as its predecessor (Ierapetritou and Floudas, 1998).

Friedler *et al*. (1998) and Sanmartí *et al.* (2002) proposed S-graph framework based approaches for production scheduling different from STN, RTN and SSN representations. In this graph theoretic framework, nodes represent production tasks with arcs denoting the precedence between them. Two types of arcs namely recipe and schedule arcs are specified. An additional node called a product node is assigned for each product. A non-negative value $c(i, j)$ denotes the weight of an arc between nodes i and j. The task corresponding to node j cannot start before $c(i, j)$ time after the start of the task corresponding to node i. For example, in Figure 2.3 below, Si denotes the set of those equipment that are assigned task node i. The duration of task 1 is 7 time units and node 7 is the product node. This representation could handle batch splitting, *NIS* and *UIS* operational policies. However, *ZW* and *FIS* policies were not clearly addressed.

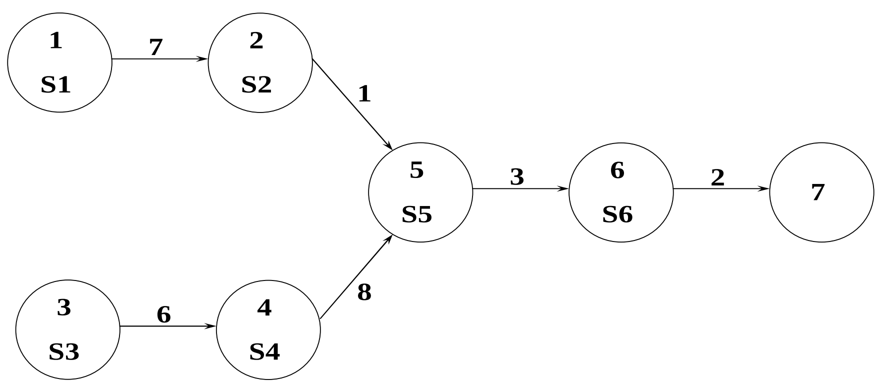


Figure 2.3: Example of an S-Graph representation

Friedler and Majozi (2006) extended this approach with the objective of maximizing economic performance indexes over a fixed time horizon. The approach was based on the S-graph detailed in the work of Sanmartí *et al.* (2002). This technique does not require any presupposition of the number of time points. For optimization, the procedure depends on a guided search algorithm that is guaranteed to terminate at a global optimum. This approach only addresses *NIS* operational policy.

2.5. Summary and conclusions

It is evident from the above review that short term scheduling has received a lot of research attention in the past two decades. The roots of the majority of most recent mathematical formulations can be traced to STN representation originally presented by Kondili *et al.* (1993).

Most works in literature address short-term scheduling of batch processes, while scheduling of combined batch and continuous processes has received less attention. However, most industrial plants have linked batch and continuous processes. For example, in Fast Moving Consumer Goods (FMCG) industries, production follows the following standard manufacturing route: Bulk product manufacturing → Storage→ Packaging → Warehousing. Packaging processes are continuous while all the other processes are either batch or semi-batch in nature. Products from these plants dominate the market, a motivation for further research in scheduling combined batch and continuous processes.

CHAPTER 3

METHODOLOGY DEVELOPMENT

3.1. Introduction

As discussed in Chapter 2, most production networks involve a combination of both batch and continuous processes. This prompted Ierapetritou and Floudas (1998) to extend their earlier work on scheduling of multipurpose batch plants to describe semi-continuous and continuous processes. In particular, the formulation follows a continuous-time STN representation and uses different binary variables for both task and unit events. The formulation also allows for variable processing times with respect to the amount of material being processed by specific tasks.

This thesis mirrors the work of Ierapetritou and Floudas (1998) in SSN based formulation of Majozi and Zhu (2001). The advantages of SSN representation over STN representation and their subsequent formulations are detailed in Chapter 2.

The notation used in this thesis is similar to that used by Majozi and Zhu (2001) and is given below. Some elements used by Ierapetritou and Floudas (1998) have been incorporated to describe continuous processes.

Nomenclature

Sets

I	$= \{\, i \mid i$ is a processing task $\}$
N	$= \{\, n \mid n$ is an event point $\}$
I_{st}	$= \{\, i_{st} \mid i_{st}$ is a storage task $\}$
S_{out}, j	$= \{\, s_{in}, j \mid s_{in}, j$ is an input state into unit $j \}$
S_{out}, j	$= \{\, s_{out}, j \mid s_{out}, j$ is an output state from unit $j \}$
S	$= \{\, s \mid s$ is any state $\} = s_{in}, j \cup s_{out}, j$

S^*_{in}, j	$= \{ s^*_{in}, j \mid s^*_{in}, j$ is an effective state $\}$ $s^*_{in}, j \subseteq S_{in}, j$
P	$= \{ p \mid p$ is a time point$\}$
J	$= \{ j \mid j$ is a processing unit $\}$
J_{st}	$= \{ j_{st} \mid j_{st}$ is a storage unit $\}$

Variables

$B(i, j, n)$	= amount of material undertaking task i in unit j at point n
$T^f(i, j, n)$	= time that task i starts in unit j at point n
$T^s(i, j, n)$	= time that task i finishes in unit j at point n
$yv(j, n)$	= binary variables that assign the utilization of unit j at event point n
$wv(i, n)$	= binary variables that assign the beginning of task i at event point n
T^f	= time at which a task is terminated
T^s	= time at which a task is initiated
$t_p(s, p)$	= time at which state s is produced at a time point p , $s \in S_{out,j}$
$t_u(s, p)$	= time at which state s is utilized at a time point p, $s \in S_{in,j}$
$q_s(s, p)$	= the amount of state s stored at a time point p
$m_u(s, p)$	= the amount of state s utilized at a time point p
$m_p(s, p)$	= the amount of state s produced at a time point p
$y(s, p)$	= decision binary variable associated with the usage of state s at a time point p
$t_a(s)$	= is the actual processing time of state s , $s \in S^*_{in}, j$
$\rho^+(s)$	= positive slack variable for variation in processing time for state s due to added degrees of freedom, $s \in S^*_{in}, j$
$\rho^-(s)$	= negative slack variable for variation in processing time for state s due to added degrees of freedom, $s \in S^*_{in}, j$

Parameters

$R_{i,j}^{\max}$ = maximum of material processed by task i required to start operating unit j

$R_{i,j}^{\min}$ = minimum rate of material processed by task i required to start operating unit j

$R_{i,j}$ = the average rate of material processed by task i required to start operating unit j

V_j = the capacity of a particular unit j

H = the time horizon of interest

$\tau(s_{in}, _j, p)$ = the mean processing time of a state

$Q_s^O(s)$ = the initial stored amount of state s

H = the time horizon of interest

$\upsilon(s)$ = the allowable percentage in processing time for state s

$rate_p(i_{st})$ = rates of production of material into storage

$rate_p(i_{st})$ = rates of consumption of material from storage

$rate(s_j)$ = the rate of utilization of a state s by unit j (kg/hour)

The key constraints from the published works of Ierapetritou and Floudas (1998) are explained as follows.

Allocation constraints

$$\sum_{i \in I_j} wv(i,n) = yv(j,n), \qquad \forall j \in J, n \in N \tag{1}$$

These constraints state that if a task i starts at an event point n it should take place in one of the suitable processing units. $wv(i,n)$ are binary variables denoting the start of task i in an event point n while $yv(j,n)$ are binary variables denoting the utilization of unit j at an event point n.

$$R_{i,j}^{\min}[T^f(i,j,n) - T^s(i,j,n)] \le B(i,j,n) \le R_{i,j}^{\max}[T^f(i,j,n) - T^s(i,j,n)]; \quad (2)$$

$$\forall i \in I, \forall j \in J_i, n \in N$$

These constraints give the limitation of the minimum and maximum processing rates of unit j when processing task i and state that the amount of material processed $B(i,j,n)$ at an event point n should be between $R_{i,j}^{\min}[T^f(i,j,n) - T^s(i,j,n)]$ and $R_{i,j}^{\max}[T^f(i,j,n) - T^s(i,j,n)]$, where $[T^f(i,j,n) - T^s(i,j,n)]$ is the duration of task i in unit j at an event point n. If the production rate is constant, then the following equality is true;

$$B(i,j,n) = R_{i,j}[T^f(i,j,n) - T^s(i,j,n)] \quad (3)$$

where $R_{i,j}$ is the constant rate of task i in unit j at an event point n. Note that for this formulation, an even point denoted the entire duration of a task.

Storage Constraints

The duration of a storage task is based on the difference between the rate of production of material into storage and the rate of consumption of the same material from storage. For a batch-continuous process, the rate of production will be zero, while for a continuous-continuous processes; the net rate will be the difference between the rates of production and consumption of material in storage. Based on these principles, the following constraints will hold;

$$[T^f(i_{st}, j_{st}, n) - T^s(i_{st}, j_{st}, n)] \ge \frac{B(i_{st}, j_{st}, n)}{rate_s(i_{st})}, \forall i_{st} \in I_{st}, \forall j_{st} \in J_{st}, n \in N \quad (4)$$

$$rate_s(i_{st}) = rate_p(i_{st}) - rate_c(i_{st}), \text{ and } rate_p(i_{st}) \ne rate_c(i_{st})$$

Where $rate_p(i_{st})$ and $rate_c(i_{st})$ are the rates of production and consumption of material in storage respectively. If $rate_p(i_{st}) = rate_c(i_{st})$, the quantity of material in storage will remain unchanged indefinitely making constraint (4) above infeasible.

Based on the above principles, we formulate a SSN based model for scheduling hybrid batch and continuous processes based on the novel concepts presented by Majozi and Zhu (2001). In this section, a brief review of the basic ideas of this formulation is presented. More specifically, the proposed approach has the following features.

i) follows a continuous-time representation in which the time points denoting the start or ending of a task are unknown and constitute variables to the optimization problem.

ii) a single binary variable $y(s,p)$ associated with the use of a state at a time point is used throughout the formulation leading to small MILP problems compared to formulations based on STN or RTN representations.

iii) the capacity of a unit in which a particular task takes place sets an upper limit for the amount of states used or produced by the corresponding task. The upper limit of the capacity of a unit is not necessarily its volume, rather the maximum amount that the unit can handle. For a mixing vessel this amount will be the amount that can be stirred without causing any spillage.

iv) the presence of a state in a unit will coincide with a corresponding processing task.

v) the usage of an input state in a unit will result in the production of an output state from the same unit, albeit at different time points. The input state could be interpreted as a raw material whilst the output state as the product.

In this thesis, additional parameters and variables are included to accommodate semicontinuous and continuous processes.

Figure 3.1 shows SSN building blocks. Figure 3.1 (a) highlights the transition from state S to state S'. This implies that there is a unit operation between the two states. Figure 3.1 (b) shows a mixing process with several input states yielding a new state. These could be materials entering a reactor to yield a product. Figure 3.1 (c) shows a separation unit where state s as the input is split into output states S' and S''.

The use of an input state s in unit j at time point p will result in the production of an output state s' from the same j unit at a time point $p+1$. If several states are used simultaneously in a unit only one state is assigned the binary variable. The state assigned the binary variable is called the "effective state". This will result to $|E| \times |P|$ number of binary variables where $|E|$ is the number of effective states and $|P|$ is the total number of time points in the formulation.

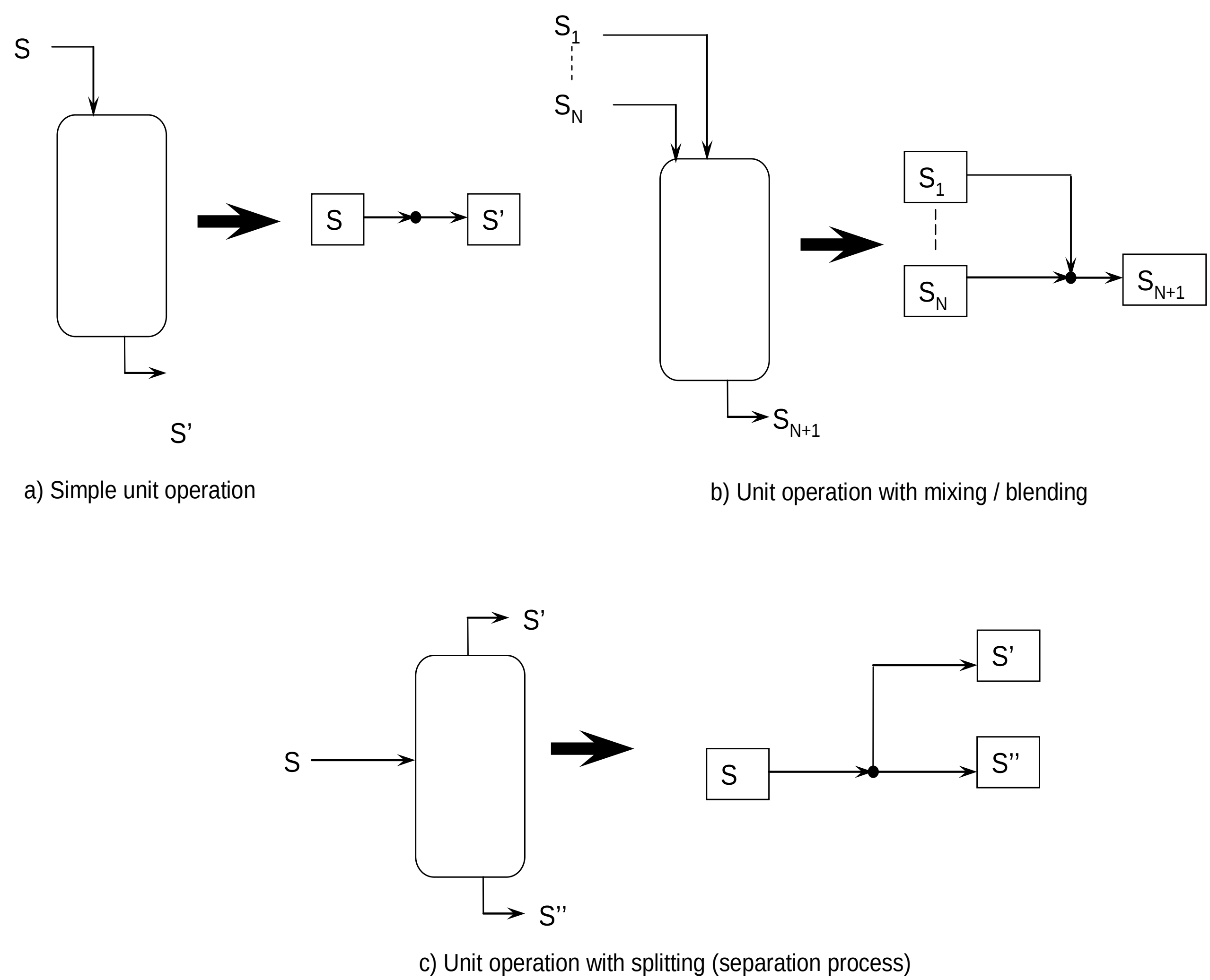


Figure 3.1: Building blocks for SSN

3.2. Mathematical model for SSN

The mathematical model presented below is extracted from Majozi and Zhu (2001).

Capacity constraints

$$V_j^L y(s_j^*, p) \le \sum_{s \in S_{in}} m_u(s,p) \le V_j^U y(s_j^*, p), \quad \forall \mathrm{j} \in \mathrm{J},\ \mathrm{p} \in \mathrm{P} \tag{5}$$

$$\begin{aligned} rate^{\min}(s_j)[t_p(s_j^{'}, p) - t_u(s_j, p-1)] \le m_u(s_j, p-1) y(s_j^*, p-1) \le \\ rate^{\max}(s_j)[t_p(s_j^{'}, p) - t_u(s_j, p-1)], \quad \forall \mathrm{j} \in \mathrm{J},\ \mathrm{p} \in \mathrm{P} \end{aligned} \tag{6}$$

Constraint (5) limits the total amount of all materials consumed in a unit at a time point p to the minimum and maximum capacities of that particular unit on condition that the corresponding effective state s^* is used. V_j^L and V_j^U denote the minimum and maximum capacities of unit *j*. Equation (6) is similar to equation (5)and is applicable to continuous processes. This equation mirrors STN constraint (2) of Ierapetritou and Floudas (1998) that limits the processing rates of a continuous process to be within the minimum and maximum processing rates ($rate^{\min}(s,j)$ and $rate^{\max}(s,j)$). If the processing rate is constant, then the following equation is true:

$$rate(j)[t_p(s,j,p) - t_u(s,j,p-1)] = m_p(s,j,p), \qquad j \in J, p \in P \tag{7}$$

Material balances

$$\sum_{s \in s_{in},j} m_u(s, p-1) = \sum_{s' \in s_{out,j}} m_p(s', p)\ ,\ j \in J, p \in P \tag{8}$$

$$q_s(s,p_1) = Q_s^O(s) - m_u(s,p_1)\ ,\ s \ne \text{product},\ p_1 = \text{starting point} \tag{9}$$

$$q_s(s,p) = q_s(s,p-1) - m_u(s,p)\ ,\ s = \text{feed}, p \in P\ ,\ p > p_1 \tag{10}$$

For a continuous process this is true;

$$q_s(s,p) = q_s(s,p-1) - \sum_j rate(s,j)[t_p(s_j^*,p) - t_u(s_j,p-1)];$$
$$p \in P,\ p > p_1,\ s = feed \quad (10a)$$

$$q_s(s,p) = q_s(s,p-1) - m_u(s,p) + m_p(s,p)\ ,\ s \neq \text{product, feed},\ p \in P, p > p_1 \quad (11)$$

$$q_s(s,p_1) = Q_s^O(s) - d(s,p_1)\ ,\ s = \text{product},\ p_1 = \text{starting point} \quad (12)$$

$$q_s(s,p) = q_s(s,p-1) + m_p(s,p) - d(s,p)\ ,\ s = \text{product, byproduct},\ p \in P, p > p_1 \quad (13)$$

Constraint (8) gives the material balance around a particular unit j, and states that the total amounts of all material produced from a unit at a time point p will be the same as the total amounts of all material consumed by the unit at a previous time point $p-1$. Constraint (9) states that the amount of state s stored at the first time point is the amount of state s originally stored at the beginning of the process adjusted accordingly with the amount used at the first time point. This constraint does not apply to a product or byproduct of a process as it is assumed that these are not used in any unit after production. Constraint (10) applies only to the feed states and implies that the amount of state s stored a time point p is the difference between the amounts stored at the time point p and the amount stored at the previous time point $p-1$. This constraint applies to the feed states only because the feed states are the ones that are only consumed in the process. Constraint (11) applies to intermediate states and implies that the amount of state s stored at a time point p is the amount stored at the previous time point $p-1$ adjusted accordingly with the amount that has been produced and used. This is because the intermediate states are the only ones consumed and produced in a process. Constraint (12) states that the amounts of product or byproduct stored at the first time point p_1 is the difference between the initial stored amounts adjusted with the amount delivered to customers at the first time point. Constraint (13) states that the amount of state s stored at time point p is that amount stored at the previous time point $p-1$ adjusted with the amount delivered to customers and the amount produced at time point p. The term $d(s,p)$ refers to the amount delivered to customers at a time point p and applies only to products and byproducts.

Duration constraints

This method considers two approaches for duration constraints. The first approach models the duration constraints as a function of batch size. In the second approach, the duration constraints do not depend on batch sizes but are modeled with considerations of practicalities in batch plants. This is done by using mean processing times collected from historical data. This way, elements such as operator response times, equipment deviations, quality of raw materials etc. are taken into account.

Duration constraints as a function of batch size

$$t_p(s_{out,j}, p) = t_u(s_{in,j}, p-1) + a(s^*_{in,j})y(s^*_{in,j}, p-1) + b(s_{\sin,j})\sum_{s_{in},j} m_u(s, p-1), \qquad \forall j \in J,\ \forall p \in P \tag{14}$$

$$b(s_{\sin,j}) = \frac{t^{\max}(s^*_{in,j}) - t^{\min}(s^*_{in,j})}{V_j^{\max} - V_j^{\min}}, \quad \forall j \in J \tag{15}$$

$$a(s^*_{in,j}) = \tau(s^*_{in,j})[1 - \upsilon(s^*_{in,j})] = t^{\min}(s^*_{in,j}), \quad \forall j \in J \tag{16}$$

$$t^{\max}(s^*_{in,j}) = \tau(s^*_{in,j})[1 + \upsilon(s^*_{in,j})], \quad \forall j \in J \tag{17}$$

Equation (14) gives the total time required to process a batch of material as a function of the batch size. The term $b(s_{\sin,j})$ in equation (15) gives the time required to process a unit amount of material of the batch corresponding to effective state s^*. Equations (16) and (17) present the minimum and maximum time required to process the effective state in a batch.

To accommodate factors that contribute to deviations in batch processing times, it is important to model the duration constraints on dependable mean processing times. This is best done by analyzing historical data of the process under different conditions and experiences.

$$t_p(s_{out,j}, p) = t_u(s^*_{in,j}, p-1) + \tau(s^*_{in,j})y(s^*_{in,j}, p-1), \quad \forall j \in J,\ \forall p \in P,\ \forall s_{sout} \in S_{out} \tag{18}$$

Constraint (18) defines the processing time of a batch corresponding to an input effective state s^*, and states that the time elapsed between the usage of input states and the production of output states from a unit corresponds to the processing time of a batch and this is subject to the usage of an effective state s^*.

Storage considerations

The duration of a storage task where the stored material is continuously used and/or produced depends on the rates of the production and consumption of the stored material, Floudas *at al.,* (1998).

$$t_p(s'_{j_{st}}, p) - t_u(s_{j_{st}}, p, p-1) = \frac{m_u(s_{j_{st}}, p-1)}{rate(s_{j_{st}})}, \qquad (19)$$

$$rate(s_{j_{st}}) = rate_p(s_{j_{st}}) - rate_c(s_{j_{st}}) \qquad (20)$$

where $rate_p(s, j_{st})$ and $rate_c(s, j_{st})$ are the corresponding constant production and consumption rates of the stored material. For a continuous–continuous storage conditions (where the stored material is produced and used continuously at the same time), $rate^p(s, j_{st})$ and $rate^c(s, j_{st})$ are > 0, while in batch-continuous storage conditions (where the stored material is not produced but used continuously, $rate^p(s, j_{st})$=0, while $rate^c(s, j_{st})$> 0. Constraints (19) and (20) are necessary to properly accommodate storage linking/supplying a continuous process.

Sequencing constraints

$$t_u(s^*_{in,j}, p) \geq \sum_{s_{in,j}} \sum_{s_{out,j}} \sum_{p' \leq p} [t_p(s_{out,j}, p') - t_u(s_{out,j}, p'-1)], \qquad (21)$$

$$\forall j \in J,\ s_{out,j} \in S_{out,j},\ s_{in,j} \in S_{in,j},\ p \in P,$$

$$t_u(s^*_{in,j}, p) \geq t_p(s^*_{out,j}, p),\ \forall j \in J, \quad p \in P \qquad (22)$$

$$t_u(s_{in,j}, p) \geq t_p(s_{out,j'}, p), \ \forall j, j' \in J, \quad p \in P, \quad s_{out,j'} = s_{in,j} \tag{23}$$

Equations (21) and (22) imply that an effective state can only be utilized in a unit if and only if all other previous tasks in that unit have been completed. Equation (23) is applicable where a unit is assigned to more than one task and implies that a state s can only be utilized in a unit j after it has been produced from another unit j'. This is true only if state s is an input state into unit j and is an output state from unit j'. Equations (22) and (23) are only applicable to intermediates as they are the only states that are both produced and used.

Time horizon constraints

$$t_u(s_{in,j}, p) \leq H, \qquad \forall j \in J, \ s_{in,j} \in S_{in,j}, \ p \in P \tag{24}$$

$$t_p(s_{out,j}, p) \leq H, \qquad \forall j \in J, \ s_{out,j} \in S_{out,j}, p \in P \tag{24}$$

Equations (24) and (25) imply that the usage or production of a state from any unit should be within the time horizon of interest *H*.

Storage constraints

$$q_s(s, p) \leq Q^{\max}(s), \qquad \forall s \in S, \quad p \in P \tag{26}$$

According to Equation (26), the amount of a state stored cannot exceed the maximum allowed for storage.

Objective function

The objective function can take several forms depending on the information provided. In most cases the objective function can be to maximize product throughput or minimize the makespan. In maximizing product throughput, the time horizon is fixed, while in minimizing makespan, the product throughput is fixed, hence the makespan minimization problem versus the converge of the throughput maximization problem.

$$Maximize \sum_s \sum_p d(s,p) \quad , \qquad s = product, \;\; p \in P, \quad \text{or} \qquad (27)$$

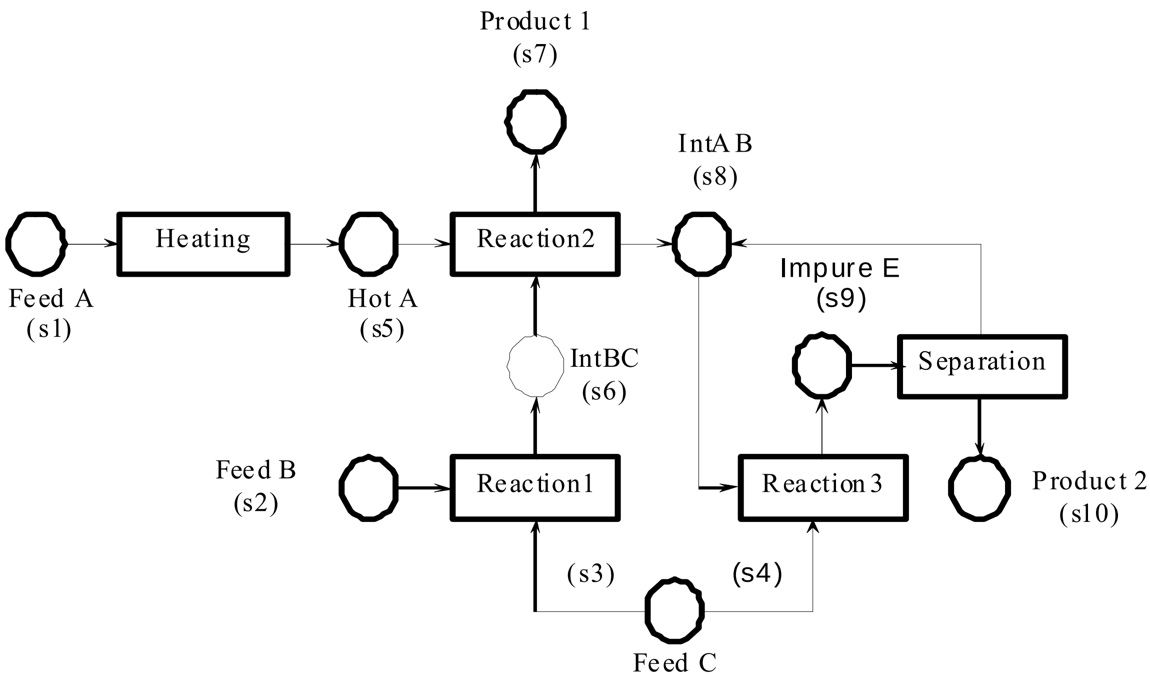
$$Minimize \;\; (H)\,. \qquad (28)$$

3.3. Illustrative example

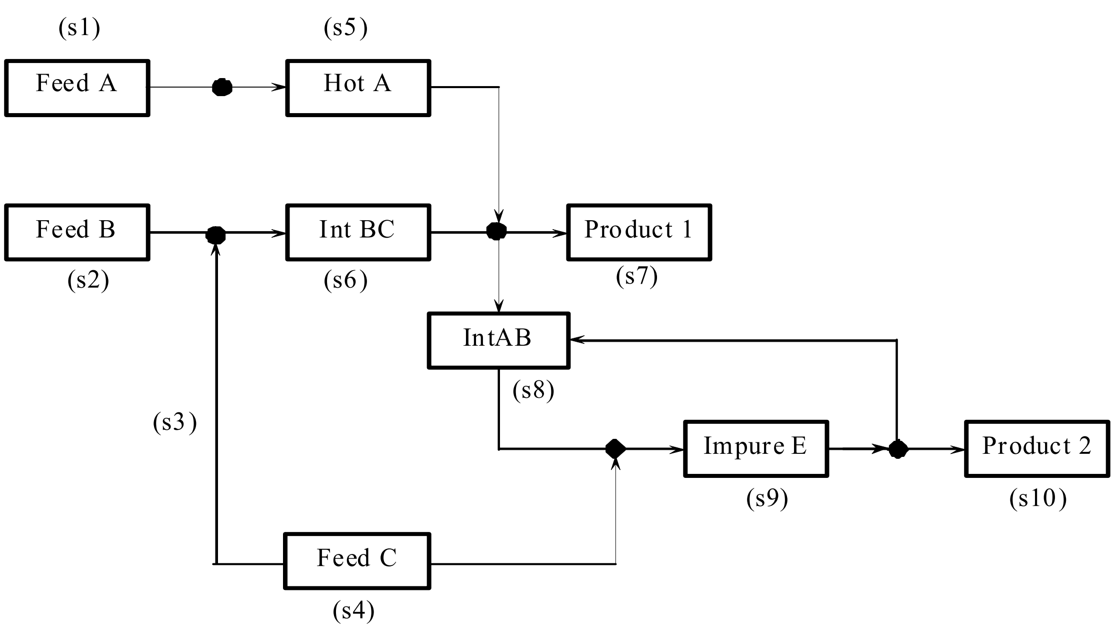
The illistrative example given below has been studied extensively in the literature: Kondili *et al.* (1993), Ierapetritou & Floudas (1998) and Majozi & Zhu (2001). Two products are produced from five processing stages; heating, three reactions and separation of an impure product E. The five processing stages occur in four units , i.e., a heater, two reactors and a separation unit. Feed C is used in two different reactions. The streams of feed C into the two reactions are assigned different states, i.e., s_3 to reaction 1 and s_4 to reaction 3. Table 3.1 represents the process data, while Figures 3.2 (a) and 3.2 (b) present the STN and SSN representation of this example.

Table 3.1: Data for the literature example (Majozi & Zhu, 2001)

Unit ID	Unit	Capacity	Suitability	Mean Processing time (h)
1	heater	100	heating	1
2	Reactor 1	50	reaction 1-3	2.0,2.0, 1
3	Reactor 2	80	reaction 1-3	2.0,2.0, 1
4	still	200	separation	1 for product 2, 2 for intAB
State		**Storage capacity**	**Initial amount**	**price**
feed A		unlimited	unlimited	0.0
feed B		unlimited	unlimited	0.0
feed C		unlimited	unlimited	0.0
hot A		100	0.0	0.0
intAB		200	0.0	0.0
intBC		150	0.0	0.0
impure E		200	0.0	0.0
product 1		unlimited	0.0	10.0
product 2		unlimited	0.0	10.0



(a) STN Representation for the illustrative example



(b) SSN representation of the illustrative example

Figure 3.2: (a) STN and (b) SSN networks for the illustrative example

Selection of effective states

More than one state is used in the reactors at the same time. It is therefore important to choose the effective states for each reaction to effectively model this problem.

$$s^*_{in,j} = \{s_1\}, \quad j = 1$$

$$s^*_{in,j} = \{s_9\}, \quad j = 4$$

$$s^*_{in,j} = \{s_2, s_6, s_8\}, \quad j = 2,3$$

Three reactions take place in units 2 and 3 at different stages of the process. State s_2 is the effective state for reaction 1, state s_6 the effective state for reaction 2 and s_8 is the effective state for reaction 3 in both units.

At any time point there are 8 active effective states leading to $8 \times P$ binary variables.

Material balances: capacity constraints

Heater

$$m_u(s_1, j_1, p) \leq 100 y(s_1, j_1, p), \forall p \in P$$

Reaction 1, Unit 2

$$m_u(s_2, j_2, p) + m_u(s_3, j_2, p) \leq 50 y(s_2, j_2, p), \forall p \in P$$

Reaction 1, Unit 3

$$m_u(s_2, j_3, p) + m_u(s_3, j_3, p) \leq 80 y(s_2, j_3, p), \forall p \in P$$

Reaction 2, unit 2

$$m_u(s_5, j_2, p) + m_u(s_6, j_2, p) \leq 50y(s_6, j_2, p), \forall p \in P$$

Reaction 2, unit 3

$$m_u(s_5, j_3, p) + m_u(s_6, j_3, p) \leq 80y(s_6, j_3, p), \forall p \in P$$

Reaction 3, unit 2

$$m_u(s_4, j_2, p) + m_u(s_8, j_2, p) \leq 50y(s_8, j_2, p), \forall p \in P$$

Reaction 3, unit 3

$$m_u(s_4, j_3, p) + m_u(s_8, j_3, p) \leq 80y(s_8, j_3, p), \forall p \in P$$

Material balances: unit mass balances

Heater

$$m_u(s_1, j_1, p-1) = m_p(s_5, j_1, p), \forall p \in P$$

Reaction 1, unit 2 (reactor 1)

$$m_u(s_2, j_2, p-1) + m_u(s_3, j_2, p-1) = m_p(s_6, j_2, p), \ \forall p \in P$$

Reaction 1, unit 3 (reactor2)

$$m_u(s_2, j_3, p-1) + m_u(s_3, j_3, p-1) = m_p(s_6, j_3, p), \ \forall p \in P$$

Reaction 2, unit 2 (reactor 1)

$$m_u(s_5, j_2, p-1) + m_u(s_6, j_2, p-1) = m_p(s_7, j_2, p) + m_p(s_8, j_2, p), \ \forall p \in P$$

Reaction 2, unit 3 (reactor 2)

$$m_u(s_5, j_3, p-1) + m_u(s_6, j_3, p-1) = m_p(s_7, j_3, p) + m_p(s_8, j_3, p), \ \forall p \in P$$

Reaction 3, unit 2 (reactor 1)

$$m_u(s_4, j_2, p-1) + m_u(s_8, j_2, p-1) = m_p(s_9, j_2, p), \ \forall p \in P$$

Reaction 3, unit 2 (reactor2)

$$m_u(s_4, j_3, p-1) + m_u(s_8, j_3, p-1) = m_p(s_9, j_3, p), \ \forall p \in P$$

Still

$$m_u(s_9, j_4, p-1) = m_p(s_8, j_4, p) + m_p(s_{10}, j_4, p), \ \forall p \in P$$

Material balances: storage mass balance

State 1

$$q_s(s_1, p_1) = Q_s^0(s_1) - m_u(s_1, j_1, p_1)$$

$$q_s(s_1, p) = q_s(s_1, p-1) - m_u(s_1, j_1, p) \ \ \forall p \in P, p > p_1$$

States 2,3 and 4

$$q_s(s, p_1) = Q_s^0(s) - \sum_j m_u(s, j, p_1), \quad j = 2,3, \mathrm{s} = 2,3,4$$

$$q_s(s_2, p) = q_s(s_2, p-1) - \sum_j m_u(s_2, j, p), \quad j = 2,3, \ \forall p \in P, p > p_1$$

State 5

$$q_s(s_5, p_1) = Q_s^0(s_5) - \sum_j m_u(s_5, j, p_1)$$

$$q_s(s_5, p) = q_s(s_5, p-1) + m_p(s_5, j_1, p) - \sum_j m_u(s_5, j, p), \; j = 2,3, \; \forall p \in P, p > p_1$$

State 6

$$q_s(s_6, p_1) = Q_s^0(s_6) - \sum_j m_u(s_6, j^*, p_1)$$

$$q_s(s_6, p) = q_s(s_6, p-1) + \sum_j m_p(s_6, j, p) - \sum_j m_u(s_6, j, p), \; j = 2,3, \; \forall p \in P, p > p_1$$

State 7

$$q_s(s_7, p_1) = Q_s^0(s_7) - d(s_7, p_1)$$

$$q_s(s_7, p) = q_s(s_7, p-1) + \sum_j m_p(s_7, j, p) - d(s_7, p_1), \; j = 2,3, \; \forall p \in P, p > p_1$$

State 8

$$q_s(s_8, p_1) = Q_s^0(s_8) - \sum_j m_u(s_8, j, p_1)$$

$$q_s(s_8, p) = q_s(s_8, p-1) + \sum_j m_p(s_8, j, p) - \sum_j m_u(s_8, j, p), \; j = 2,3, \; \forall p \in P, p > p_1$$

State 9

$$q_s(s_9, p_1) = Q_s^0(s_9) - m_u(s_9, j_4, p_1)$$

$$q_s(s_9, p) = q_s(s_9, p-1) + \sum_j m_p(s_9, j, p) - m_u(s_9, j_9, p)\,,\ j = 2,3,\ \ \forall p \in P\,, p > p_1$$

State 10

$$q_s(s_{10}, p_1) = Q_s^0(s_{10}) - d(s_{10}, p_1)$$

$$q_s(s_{10}, p) = q_s(s_{10}, p-1) + \sum_j m_p(s_{10}, j_4, p) - d(s_{10}, p)\,,\ \ \forall p \in P\,, p > p_1$$

Duration constraints: batch processing time modeled as a function of batch size

The minimum, maximum and the variable processing times for each of the tasks is given in Table 3.2 below

Table 3.2: Processing times for literature example

	Processing times (h)			
Unit	**Min**	**Mean**	**Max**	**Variable**
1	0.667	1	1.33	0.0067
2	1.33,1.33,0.67	2.0,2.0, 1	2.67,2.67,1.33	0.0267, 0.0267,0.0133
3	1.33,1.33,0.67	2.0,2.0, 1	2.67,2.67,1.33	0.0167,0.0167,0.0083
4	0.67 for product 2	1 for product 2	1.33 for product	0.0033 for product 1,
4	1.33 for intAB	2 for intAB	2, 2.67 for intAB	0.0067 for intAB

A 33% variation from the mean processing time is allowed in this literature example.

$$t_p(s_5, j_1, p) = t_u(s_1, j_1, p-1) + 0.667\,y(s_1, j_1, p-1) + 0.0067\,m_u(s_{1,} j_1, p-1) \quad \forall p \in P$$

$$t_p(s_6, j_2, p) = t_u(s_2, j_2, p-1) + 1.33(s_2, j_2, p-1) + 0.0267\big(m_u(s_{2,} j_2, p-1) + (s_{3,} j_2, p-1)\big), \quad \forall p \in P$$

$$t_p(s_6, j_3, p) = t_u(s_2, j_3, p-1) + 1.33y(s_2, j_3, p-1) + 0.0167\left(m_u(s_{2,} j_3, p-1) + (s_{3,} j_3, p-1)\right), \quad \forall p \in P$$

$$t_p(s_8, j_2, p) = t_u(s_6, j_2, p-1) + 1.33(s_6, j_2, p-1) + 0.0267\left(m_u(s_{5,} j_2, p-1) + (s_{6,} j_2, p-1)\right), \quad \forall p \in P$$

$$t_p(s_8, j_3, p) = t_u(s_6, j_3, p-1) + 1.33y(s_6, j_3, p-1) + 0.0167\left(m_u(s_{5,} j_3, p-1) + (s_{6,} j_3, p-1)\right), \quad \forall p \in P$$

$$t_p(s_9, j_2, p) = t_u(s_8, j_2, p-1) + 0.67(s_8, j_2, p-1) + 0.0133\left(m_u(s_{4,} j_2, p-1) + (s_{8,} j_2, p-1)\right), \quad \forall p \in P$$

$$t_p(s_9, j_3, p) = t_u(s_8, j_3, p-1) + 0.67y(s_8, j_3, p-1) + 0.0083\left(m_u(s_{4,} j_3, p-1) + (s_{8,} j_3, p-1)\right), \quad \forall p \in P$$

$$t_p(s_{10}, j_4, p) = t_u(s_9, j_4, p-1) + 0.67y(s_9, j_4, p-1) + 0.0033\left(m_u(s_{3,} j_3, p-1) + m_u(s_{3,} j_3, p-1)\right), \forall p \in P$$

Duration constraints: batch processing time modeled as a function of mean processing times

The duration constraints below are modeled with an assumption of zero deviation from the mean processing times. In other words, it is assumed that duration is fixed regardless of capacity in a unit

$$t_p(s_5, j_1, p) = t_u(s_1, j_1, p-1) + 1y(s_1, j_1, p-1), \ \forall p \in P$$

$$t_p(s_6, j_2, p) = t_u(s_2, j_2, p-1) + 2y(s_2, j_2, p-1), \quad \forall p \in P$$

$$t_p(s_6, j_3, p) = t_u(s_2, j_3, p-1) + 2y(s_2, j_3, p-1), \quad \forall p \in P$$

$$t_p(s_8, j_2, p) = t_u(s_6, j_2, p-1) + 2y(s_6, j_2, p-1), \ \forall p \in P$$

$$t_p(s_8, j_3, p) = t_u(s_6, j_3, p-1) + 2y(s_6, j_3, p-1), \quad \forall p \in P$$

$$t_p(s_9, j_2, p) = t_u(s_8, j_2, p-1) + y(s_8, j_2, p-1), \ \forall p \in P$$

$$t_p(s_9, j_3, p) = t_u(s_8, j_3, p-1) + y(s_8, j_3, p-1), \quad \forall p \in P$$

$$t_p(s_{10}, j_4, p) = t_u(s_9, j_4, p-1) + y(s_9, j_4, p-1), \ \forall p \in P$$

$$t_p(s_8, j_4, p) = t_u(s_9, j_4, p-1) + 2y(s_9, j_4, p-1), \quad \forall p \in P$$

Sequencing constraints

In this example, Equation (1) is not applicable to units 1 and 4 because these two units are assigned to only one task.

States s_1 *and* s_5

$$t_u(s_1, j_1, p) \geq t_p(s_5, j_1, p), \ \forall p \in P$$

States s_2, s_3 *and* s_6

$$t_u(s_2, j, p) \geq t_p(s_6, j, p), \ \forall p \in P, j = 2,3$$

$$t_u(s_2, j, p) = t_u(s_3, j, p), \ \forall p \in P, j = 2,3$$

$$t_u(s_6, j, p) \geq t_p(s_6, j, p), \ \forall p \in P, j = 2,3$$

States s_5, s_6 *and* s_8

$$t_u(s_6, j, p) \geq t_p(s_8, j, p), \ \forall p \in P, j = 2,3$$

$$t_u(s_5, j, p) = t_u(s_5, j, p), \ \forall p \in P, j = 2,3$$

$$t_u(s_8, j, p) \geq t_p(s_8, j, p), \ \forall p \in P, j = 2,3$$

States s_4, s_8 *and* s_9

$$t_u(s_8, j, p) \geq t_p(s_9, j, p), \ \forall p \in P, j = 2,3$$

$$t_u(s_8, j, p) = t_u(s_4, j, p), \ \forall p \in P, j = 2,3$$

$$t_u(s_9, j_4, p) \geq t_p(s_9, j, p), \ \forall p \in P, j = 2,3$$

States s_9 *and* s_{10}

$$t_u(s_9, j_4, p) \geq t_p(s_{10}, j_4, p), \ \forall p \in P$$

$$t_u(s_3, j_3, p) \geq t_p(s_4, j_3, p), \ \forall p \in P$$

The above correspond to equations (20) and (21)

$$t_u(s^*, j, p) \geq t_p(s_6, j, p') - t_u(s_2, j, p'-1) + t_p(s_8, j, p') - t_u(s_6, j, p'-1) - t_p(s_9, j, p') - t_u(s_8, j, p'-1)$$
$$j = 2,3, \quad \text{p} \geq \text{p'}, \ s^* = s_2, s_6, s_8$$

Time horizon constraints

The time horizon of interest in this example is 8 hours and therefore for this particular example, the following will apply.

States 1, 5

$$t_u(s_1, j_1, p) \leq 8, \ \forall p \in P$$

$$t_p(s_5, j_1, p) \leq 8, \ \forall p \in P$$

States 2, 3, 4, 5, 6, 7, 8, 9

$$t_u(s, j, p) \leq 8, \ \mathrm{s} = s_2, s_3, s_6, \mathrm{s}_8, \quad \mathrm{j} = 2,3, \forall p \in P$$

$$t_p(s, j, p) \leq 8, \ \mathrm{s} = s_6, \mathrm{s}_7, \mathrm{s}_8, \mathrm{s}_9, \mathrm{j} = 2,3, \forall p \in P$$

States 8, 9, 10

$$t_u(s_9, j_4, p) \leq 8, \ \forall p \in P$$

$$t_p(s, j_4, p) \leq 8, \ \mathrm{s} = \mathrm{s}_8, \mathrm{s}_{10}, \quad \forall p \in P$$

Objective function

For this example, the objective function is to maximize the product throughput (States s_7 and s_{10}) as follows;

$$\text{Maximize } R = 10 \times \sum_p \left(d(s_7, p) + d(s_{10}, p) \right), \forall p \in P$$

3.4. Results and discussions

Five time points and an 8-hour time horizon were used for approach 1 (batch processing times modeled as a function of batch size). Six time points and an 8-hour time horizon were used for the second approach (modeling batch-processing times to equal mean processing times). The results from the two approaches are shown in Table 3.3.

Both approaches gave a zero integrality gap implying best equipment utilization. Both results were obtained using GAMS, CPLEX solver version 9.1.2 in a 1.8 GHz Mobile Intel Pentium 4-M CPU with 261,560 KB RAM.

The results in the 2^{nd} column in Table 3.3 were obtained by modeling duration constraints independent from batch size giving an objective value of 2255, while the results on the 3^{rd} column were obtained by modeling duration constraints as a function of batch size giving an objective value of 1499.89. The first approach gives 50.34% more output compared to modeling the batch time as a function of batch size.

Tables 3.4 and 3.5 show the values of the binary variables at different time points for the two approaches. The value of the binary variables at the last time points for the two approaches is zero because no state can be utilized at the last time point. Figures 3.3 and 3.4 respectively present the Gantt chart for the two approaches.

Table 3.3: Results from the first literature example showing results of the two approaches

	Approach 1	Approach 2
Time points	5	6
Number of constraints	702	1128
Total number of variables	450	742
Total number of binary variables	40	48
MIP solution	1499.89	2255
CPU time (s)	0.06	0.06

Table 3.4:Values for the binary variables at the six time points: (Approach 1)

y(s,j,p)	p1	p2	p3	p4	p5
s_1,j_1	1	1	1	0	0
s_2,j_2	1	0	1	0	0
s_2,j_3	1	0	0	0	0
s_6,j_2	0	1	0	0	0
s_6,j_3	0	1	0	1	0
s_8,j_2	0	0	0	0	0
s_8,j_3	0	0	1	0	0
s_9,j_4	0	0	0	1	0

Table 3.5: Values for the binary variables at the five time points: (Approach 2)

y(s,j,p)	p1	p2	p3	p4	p5	p6
s_1,j_1	1	0	0	1	0	0
s_2,j_2	1	0	0	1	0	0
s_2,j_3	1	0	0	0	0	0
s_6,j_2	0	1	1	0	0	0
s_6,j_3	0	1	0	0	1	0
s_8,j_2	0	0	0	0	0	0
s_8,j_3	0	0	1	1	1	0
s_9,j_4	0	0	1	1	1	0

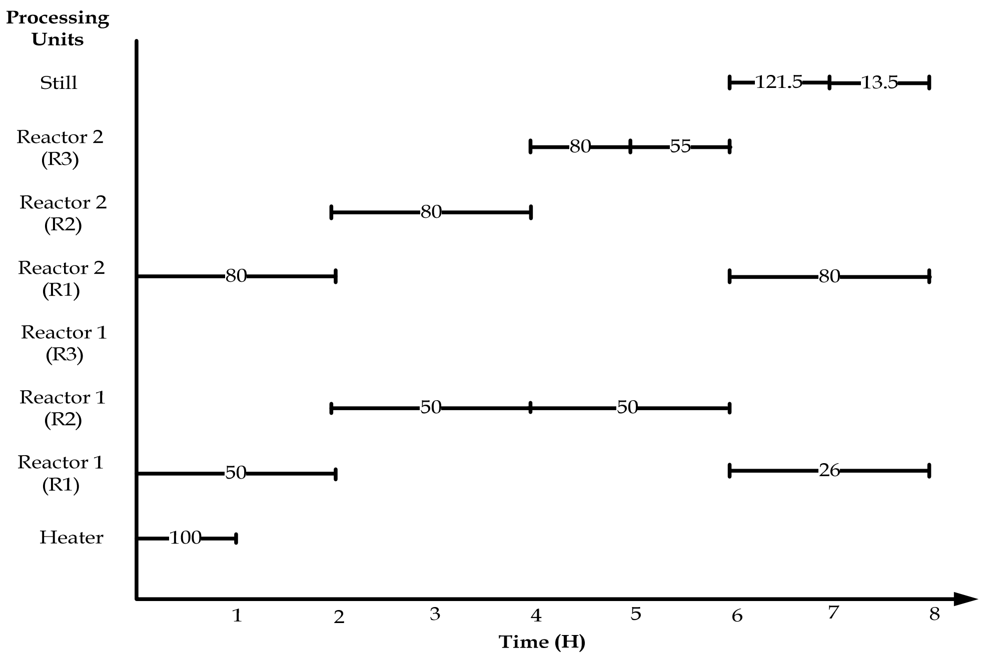


Figure 3.1: Gantt chart for the literature example: batch time modeled to mean processing time

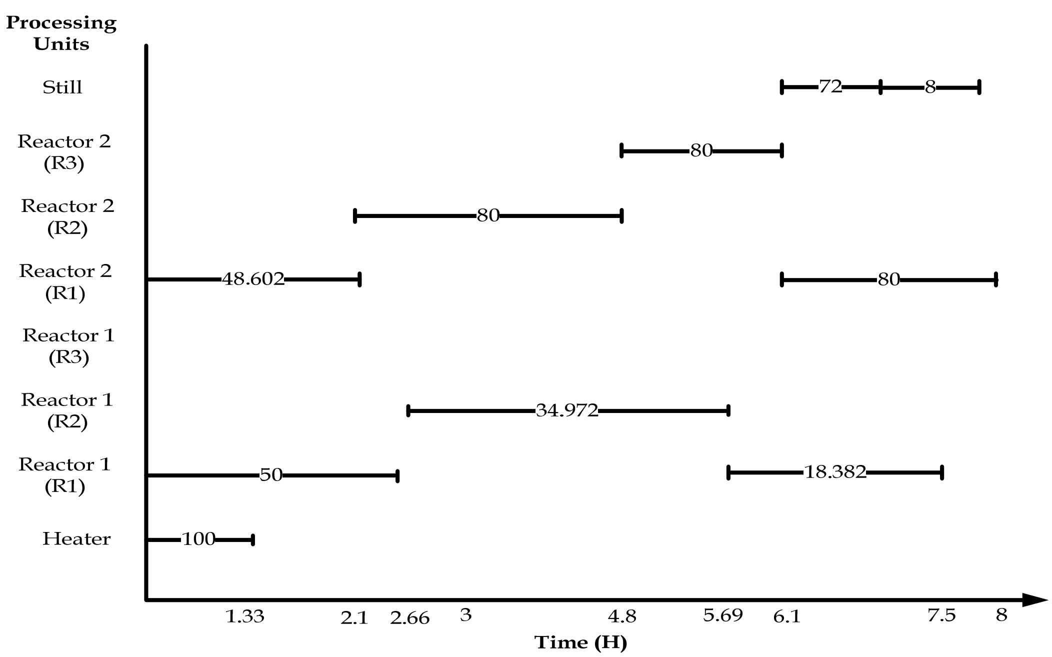


Figure 3.2: Gantt chart for the illustrative example; batch time modeled as a function of batch size

3.5. Conclusions

Continuous-time SSN based scheduling formulation has been detailed in Chapter 3 complete with an illustrative example. Following this, the formulation was extended to an existing manufacturing establishment. This is detailed in Chapter 4.

CHAPTER 4

INDUSTRIAL CASE STUDY

4.1. Introduction

This chapter supports the capability of continuous-time SSN based formulation to solve large scale industrial scheduling problems. Due to economies of scale, rewards emanating from coordinated operations are significant in large industrial problems. This was the case in the multinational pharmaceutical goods manufacturing company based in South Africa where this case study was done.

Section 4.2 outlines the production process under consideration. For reasons of confidentiality, full details of the systems and the final schedule obtained cannot be disclosed. It is important to note that although the actual data of the process has been disguised, this work has the same essential features as for the plant under consideration. In addition, this problem is sufficiently large for the detailed scheduling formulation to be tractable. Section 4.3 details the main features of the solution to the presented case study.

4.2. Problem outline

Overview

The case study considered in this thesis comprises of two stages. In the first stage, a common base used to manufacture pharmaceutical emulsions is produced in bulk quantities. In the second stage, the base is split to feed three machines where it is blended with additives to produce different products. These products are then packed and labeled. The base preparation process is batch in nature and lasts for 2 hours. The blending and packing processes are continuous. The case study is presented by the schematic Figure 4.1 and the processing data is detailed by Tables 4.1 and 4.2. Figure 4.2 is the SSN representation of the process.

Process description

Three steps are involved in the bulk base preparation process in this particular plant. The first step is the preparation of organic components where mineral oil, petroleum jelly, waxes and preservatives are blended together at between 68-72^{O}C for 1 hour. This process is represented by input states $s_1 - s_3$ and output state s_9 in SSN Figure 4.2.

The second step is the preparation of inorganic components in separate vessels where purified water and emulsifying agents are blended together at 68^{O}C -72^{O}C also for 1 hour. This process is represented by input states s_4 and s_5 with output state s_{10}. Subject to the availability of manpower, the two processes can be performed simultaneously since they use separate vessels.

The third processing stage is the mixing the organic and inorganic components presented by states s_9 and s_{10} respectively. This is done in the vessels containing the inorganic components. Both the organic and the inorganic components of the emulsion must be between 68^{O}C-72^{O}C at the time when the two are mixed together. Usually the inorganic components mixing vessels are larger in volume compared to their organic mixing counterparts. The mixed contents are then cooled 45^{O}C resulting to a thick bulk base presented by state s_{11}.

Blending of the bulk base with different perfumes, color pigments and Active Pharmaceutical Ingredients (API) produces different products. This is done continuously by the blending and packing machines. Note that perfumes, color pigments, API and packing materials (bottles, jars, labels, caps and boxes) are unique for each product. These are collectively denoted by states s_6, s_7 and s_8 for the three blending and packaging machines. The blended base is then put into individual bottles or jars. The filled bottles or jars are then labeled and packed into boxes for dispatch.

The final products from the three blending and packing machines are denoted by states s_{12}, s_{13} and s_{14}. It is important to note that the packing process can be terminated at any point within the time horizon on satisfaction of customer orders.

For flexibility, the plant has a combined closed and open shop policies. In closed shop policy, packaged products from the factory are taken to the main warehouse to replenish stock levels. The plant also supplies directly to the customers bypassing warehouse storage, a clear reflection of an open shop policy.

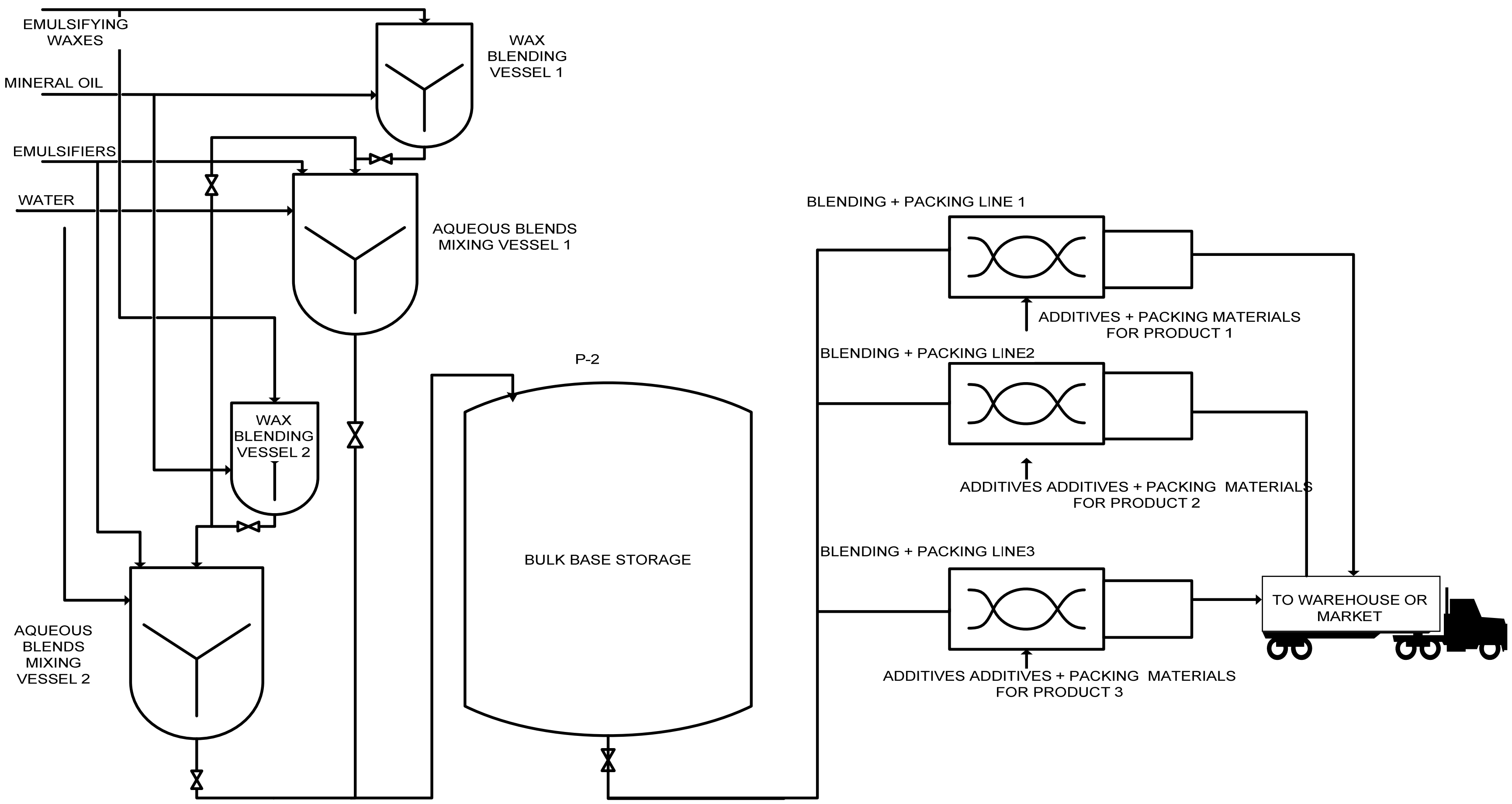


Figure 4.1: Flow diagram for Industrial case study

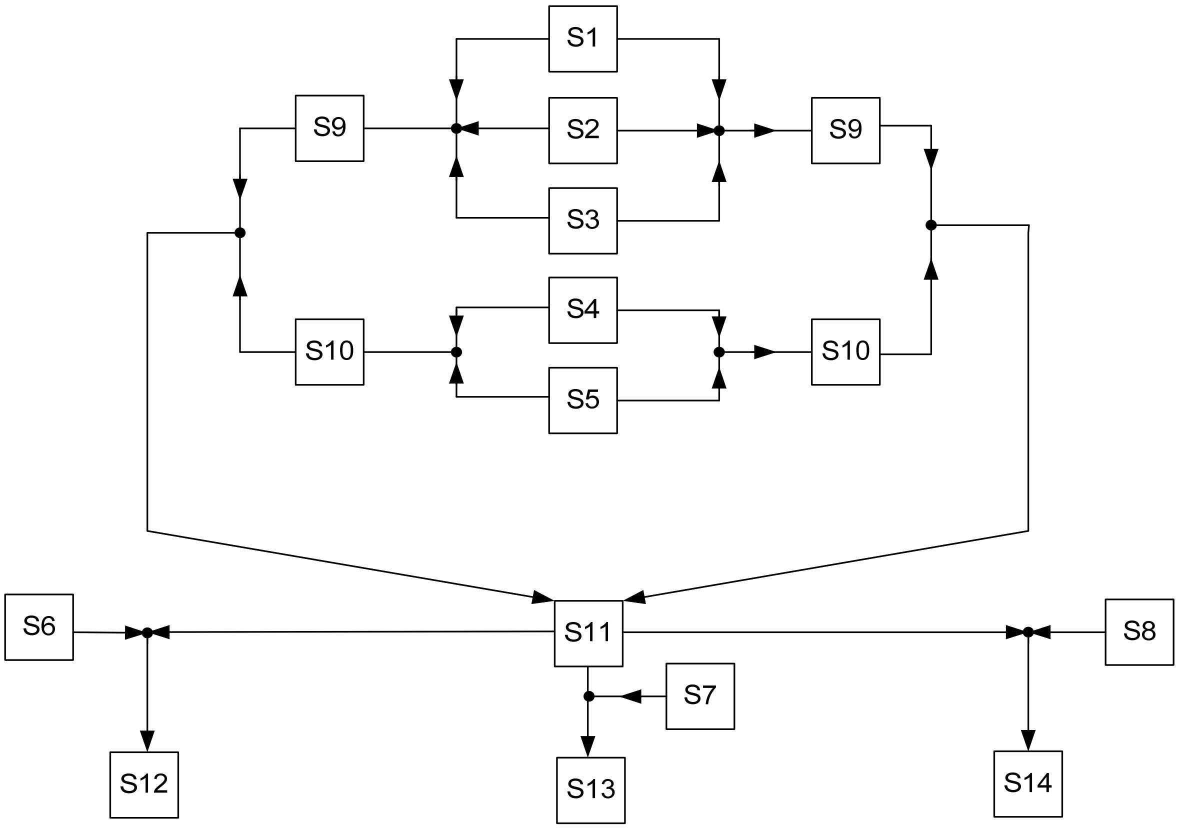


Figure 4.2: State Sequence Network for Industrial case study

Table 4.1: Industrial Case Study Data

Unit	*Capacity	Suitability	Mean Processing time
1, 2	1000, 1000	un-aq mixing	1
3, 4	4000, 6000	aq mixing1,2	1, 1
5, 6, 7	Continuous	Packaging	2000,2200,2400 kg/hr (constant continuous processing rate)

Table 4.2: Storage and Capacity data

State	Storage *Capacity	Initial amount	Price (R/kg)
1	unlimited	unlimited	10.0
2	unlimited	unlimited	10.0
3	unlimited	unlimited	10.0
4	unlimited	unlimited	10.0
5	unlimited	unlimited	10.0
6	unlimited	unlimited	10.0
7	unlimited	unlimited	10.0
8	unlimited	unlimited	10.0
9	0	unlimited	10.0
10	0	unlimited	10.0
11	20000	0.0	0.0
12	unlimited	0.0	1000
13	unlimited	0.0	1000
14	unlimited	0.0	1000

Objective functions

Production targets are fixed for each product coming from this plant with strict due dates. Special conditions, however, do arise for monopoly and for newly launched products. These two categories of products either dominate the market or have unprecedented market share growth. It is important to note that for these products, predictive or forecasted data is used for campaign runs. This case study addresses two objective functions for the two production scenarios.

The first objective function is to minimize the makespan to accomplish given fixed production targets. The second objective function is to maximize throughput in campaign runs aimed at replenishing stock levels for monopoly products or to satisfy large market orders for newly launched products.

4.3. Modeling the industrial case study

Selection of effective states

It is easy to relate the network of the industrial case study to the full SSN model described in Chapter (3). However, the key concepts and equations elaborating the batch-continuous and continuous sections of the plant are given as below.

Units 3 and 4 are used for two different tasks. These tasks are stages 2 and 3 of the process. State s_5 is the effective state for stage 2, while s_{10} is the effective state for stage 3. The choice of effective states for the case study is given as follows.

$$s^*_{in,j} = \{ s_1 \}, \quad j = 1,2$$

$$s^*_{in,j} = \{ s_5, s_{10} \}, \quad j = 3,4$$

$$s^*_{in,j} = \{ s_{11} \}, \quad j = 5,6,7$$

Material balances: capacity constraints

Unit 1 & 2

$$m_u(s_1, j, p) + m_u(s_2, j, p) + m_u(s_3, j, p) \le V_j^U y(s_1, j, p), \ \forall p \in P, j = 1, 2$$

Units 3 & 4, Mixing Process 1

$$m_u(s_4, j, p) + m_u(s_5, j, p) \le V_j^U y(s_5, j, p), \ \forall p \in P, j = 3, 4$$

Units 3 & 4, Mixing Process 2

$$m_u(s_9, j, p) + m_u(s_{10}, j, p) \le V_j^U y(s_{10}, j, p), \ \forall p \in P, j = 3, 4$$

Units 5, 6, and 7 are continuous processing units. Constant processing rates have been assumed for these units.

Units 5

$$[t_p(s_{12}, j_5, p) - t_u(s_{11}, j_5, p-1)] = \frac{m_p(s_{12}, j_5, p)}{rate(j_5)}, \quad \forall p \in P;$$

Units 6

$$[t_p(s_{13}, j_6, p) - t_u(s_{11}, j_6, p-1)] = \frac{m_p(s_{13}, j_6, p)}{rate(j_6)}, \quad \forall p \in P;$$

Units 7

$$[t_p(s_{14}, j_7, p) - t_u(s_{11}, j_7, p-1)] = \frac{m_p(s_{14}, j_7, p)}{rate(j_7)}, \quad \forall p \in P;$$

These constraints echo Equation (7) and give the relationship between the processing rates, material processed and the timing of continuous processes.

Material balances: batch-continuous processes

This feature is evident in the batch production and continuous consumption of state s_{11}. The following relationship holds;

$$\begin{aligned} q_s(s_{11}, p) = q_s(s_{11}, p-1) + \sum_{j^b} m_p(s_{11}, j^b, p) - \\ \big(rate(j_{s11,5})[t_p(s_{12}, j_5, p) - t_u(s_{11}, j_5, p-1)\,] + \\ rate(j_{s11,6})[t_p(s_{13}, j_6, p) - t_u(s_{11}, j_6, p-1)\,] + \\ rate(j_{s11,7})[t_p(s_{14}, j_7, p) - t_u(s_{11}, j_7, p-1)\,]\big), \\ j^b = 3,4, \quad p > 1. \end{aligned}$$

where $rate(j_{s11,5})$, $rate(j_{s11,6})$ and $rate(j_{s11,7})$ are the processing rates for state s_{11} by units j_5, j_6 and j_7 respectively. This equation echoes Equation (10b).

4.4. Results and discussions

Maximization of throughput

10 and 14 time points were used for 8 and 12-hour time horizons respectively. Both results were obtained using GAMS, CPLEX solver version 9.1.2 in a 3.0 GHz Intel Pentium 4-M CPU with 502MB RAM. 8 and 12-hour time horizons were chosen to reflect actual shift durations allowed by law.

Objective functions of 43920 kg (17280 kg for product 1, 14640 kg for product 2 and 12000 kg for product 3) were obtained for the 8 hour time horizon. The 12 hour time horizon gave an objective value of 71760 kg (25920 kg for product 1, 21840 kg for product 2 and 24000 kg for product 3).

Table 4.3 shows the results arising from the two time horizons while Tables 4.4 and 4.5 show the values of the binary variables at different time points for the two time horizons. Note that the values of the binary variables at the last time points for the two approaches are zero because no state can be utilized at the last time point.

Table 4.3: Results from the Industrial Case Study showing results 8 and 12-hour time horizons

	Time Horizon (Hours)	
	8	12
Time points	10	14
Number of constraints	3392	4752
Total number of variables	3153	4533
Total number of binary variables	980	1372
MIP solution	43920	71760
CPU time (s)	25.38	998

Table 4.4: Values for the binary variables for 8 hour time horizon

y(s,j,p)	p1	p2	p3	p4	p5	p6	p7	p8	p9
s_1,j_1	1	0	1	0	1	1	1	0	0
s_1,j_2	0	1	1	1	1	1	0	0	0
s_4,j_3	1	1	0	1	1	1	0	0	0
s_4,j_4	1	0	1	0	0	0	1	0	0
s_{10},j_3	0	0	1	0	0	0	1	1	0
s_{10},j_4	0	1	0	1	1	1	0	1	0
s_{11},j_5	1	1	1	1	1	1	1	1	1
s_{11},j_6	1	1	1	1	1	1	1	1	1
s_{11},j_7	1	1	1	1	1	1	1	1	1

Table 4.5: Values for the binary variables for 12-hour time horizon

y(s,j,p)	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13
s_1,j_1	1	0	1	0	0	0	0	1	1	1	1	0	0
s_1,j_2	0	1	1	1	1	1	1	1	1	0	1	0	0
s_4,j_3	1	1	0	0	0	1	1	1	0	1	1	0	0
s_4,j_4	1	0	1	0	1	0	0	0	0	1	1	0	0
s_{10},j_3	0	0	1	1	1	0	0	0	0	1	1	1	0
s_{10},j_4	0	1	0	1	0	1	1	0	0	0	0	0	0
s_{11},j_5	1	1	1	1	1	1	1	1	1	1	1	1	1
s_{11},j_6	1	1	1	1	1	1	1	1	1	1	1	1	1
s_{11},j_7	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 4.3 and Figure 4.4 respectively present the Gantt chart for the results obtained from the 8 and 12-hour time horizons respectively.

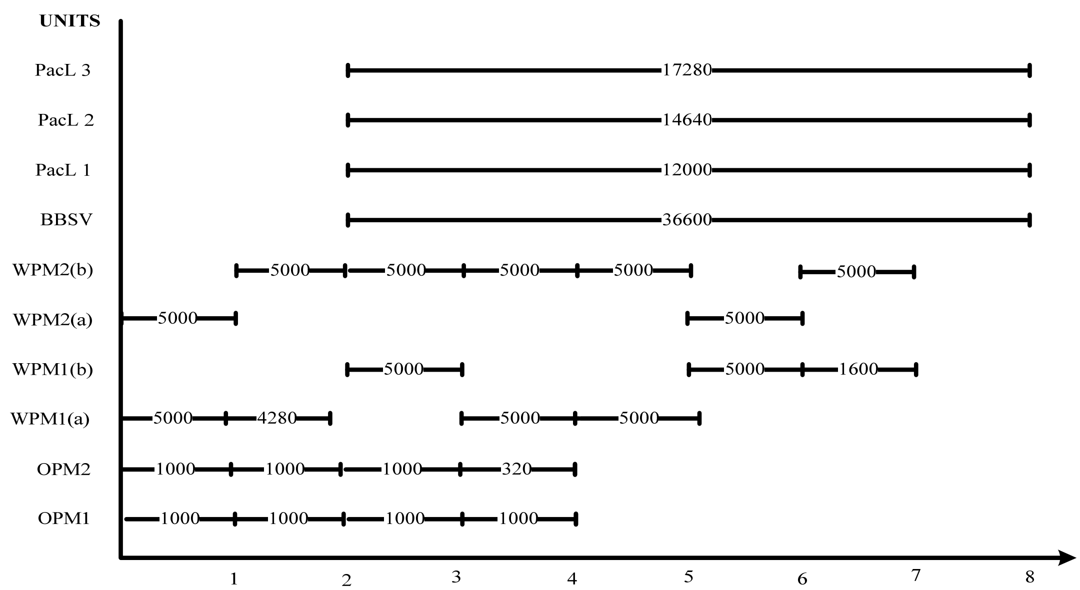


Figure 4.3: Gantt chart for the Industrial Case Study- 8 Hours Time Horizon

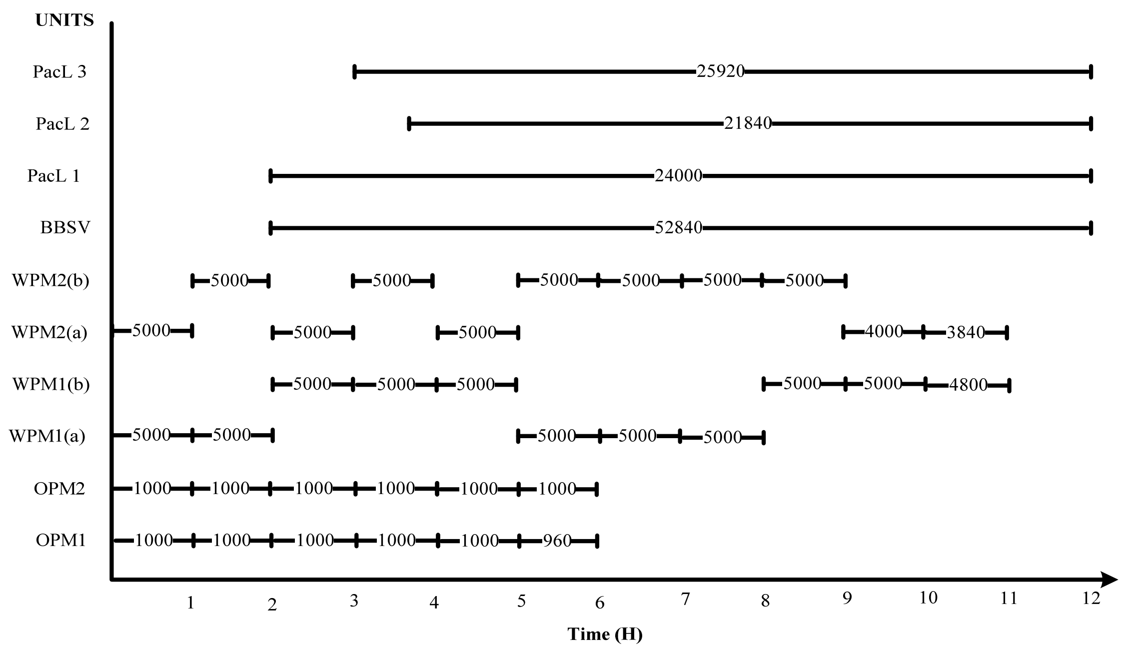


Figure 4.4: Gantt chart for the Industrial Case Study - 12 Hours Time Horizon

Minimization of the makespan

To minimize the makespan, time required to accomplish a fixed production target is unknown. A target of 60000 kg (20000 kg for each of the three products) was used to demonstrate the robustness of the model. An objective function of 10.512 hours was obtained using 11 time points.

Table 4.6 shows the results arising from the makespan minimization problem while Table 4.7 shows the values of the binary variables at different time points. Note that the values of the binary variables at the last time points are zero because no state can be utilized at the last time point. Figure 4.5 presents the Gantt chart for these results

Table 4.6: Results from the Industrial Case Study showing results for 60000 kg: Fixed output distributed evenly between three products

	Production target: 60,000 kg: 20,000 kg for product 1 20,000 kg for product 2 and 20,000 kg for product 3
Time points	11
Number of constraints	3732
Total number of variables	3494
Total number of binary variables	1078
MIP solution (H)	10.5152
CPU time (s)	0.422

Table 4.7: Values for the binary variables for the 60000 kg: Fixed output distributed evenly between three products

y(s,j,p)	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10
s_1,j_1	1	0	1	1	1	1	1	1	0	0
s_1,j_2	1	1	0	1	1	1	1	0	0	0
s_4,j_3	1	0	0	1	0	0	1	0	0	0
s_4,j_4	1	0	0	1	0	0	1	0	0	0
s_{10},j_3	1	1	0	1	0	1	1	1	0	0
s_{10},j_4	0	1	1	0	1	1	0	1	1	0
s_{11},j_5	1	1	1	1	1	1	1	1	1	1
s_{11},j_6	1	1	1	1	1	1	1	1	1	1
s_{11},j_7	1	1	1	1	1	1	1	1	1	1

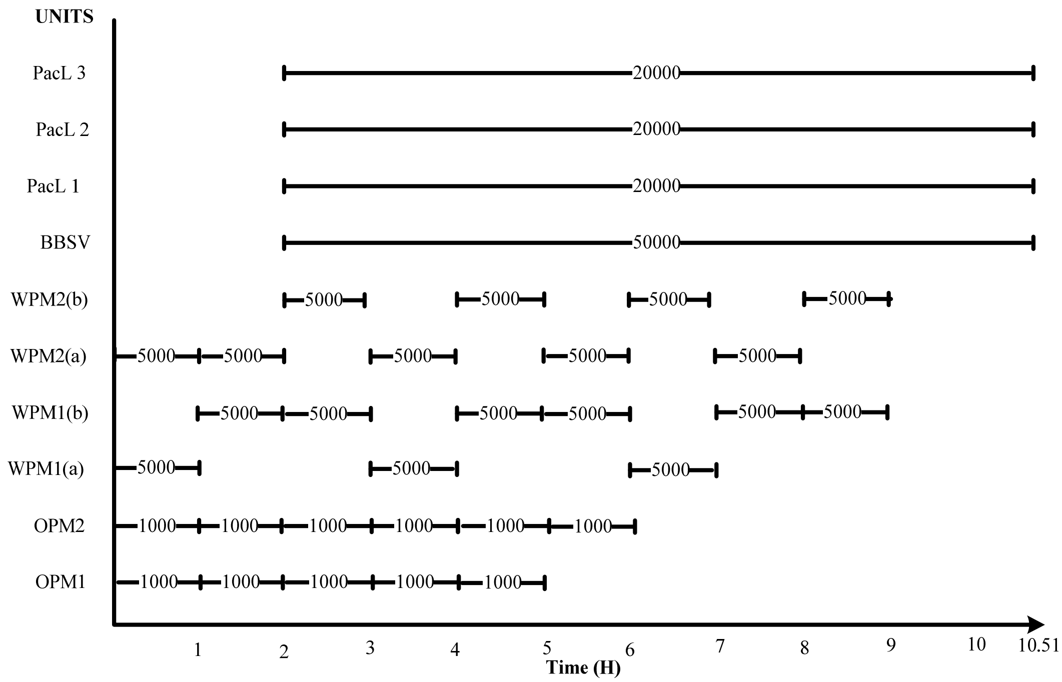


Figure 4.5: Gantt chart for the Industrial Case Study - 60000 kg: Fixed output distributed evenly between three products

4.5. Conclusions

This chapter has detailed the application of Continuous-time SSN based formulation to an existing hybrid batch and continuous industrial problem. The two time horizons used in the throughput maximization problem present the two common industrial shift durations in South Africa. These durations are in compliance with the labor law which restricts working hours to a maximum of 12. The second part of the industrial problem looked at minimizing the makespan to produce a fixed target.

The business and operational implications around the findings of this industrial case study are discussed in Chapter 5.

CHAPTER 5

DISCUSSIONS

5.1. Introduction

This chapter explores the consequences arising from the results obtained from the research case study. Section 5.2 elaborates the rationale behind the two common operating durations encountered in the industry and their business and operational implications emanating from the choice of the time horizons. Section 5.3 analyses and compares results of the two time horizons. A comprehensive discussion on the balance between productivity and safety issues emanating working longer hours is also detailed in this section. Section 5.4 gives conclusions drawn from the discussions in the Chapter.

5.2. Time horizon decisions

To comply with legislation, most facilities in South Africa operate 3 × 8 hours or 2 × 12 hours shifts per day. In most cases, it is not economical to operate more than 3 shifts of less than 8 hours.

5.3. Discussions

Table 5.1 illustrates economical implications arising from the two modes of operation. A constant hourly capital cost financially equivalent to 10 product units is assumed across the shifts. 2 × 12-hour shifts result in 9.44% more output per day compared to 3 × 8-hour shifts. In addition, there are costly additional hidden costs emanating from frequent shift changeovers. These include loss of vital information between operators and resultant downtime breaking production continuity. For this particular case, it will be beneficial to operate 2 × 12-hour shifts.

Table 5.1: Implications: Choice between 3 × 8 hours or 2× 12 hours shift operations

Product	Units	Total	Capital Cost		Units	Total	Daily Capital Cost	Daily Nett Profit
s12	17280	43920	800	24 hrs (3 X 8 Hr Shifts)	51840		2400	
s13	14640		800		43920	131760	2400	124560
s14	12000		800		36000		2400	
s12	25920	71760	1200	24 hrs (2 X 12 Hr Shifts)	51840		2400	136320
s13	21840		1200		43680	143520	2400	
s14	24000		1200		48000		2400	

The results above justify organization and structural changes to incorporate longer working hours. However, there are concerns raised about the impact of these changes. In most cases, workers become unproductive due to fatigue and sleepiness as a result of working 12-hour shifts. Emanating from fatigue is compromised safety due to lowered alert levels.

A comprehensive study on the relationship between accumulated fatigue, productivity, accidents and injuries due to long working hours in the process industry is detailed by Lilley, Feyer, Kirk and Gander (2001). Fatigue was found to be experienced by 78% of 12 hour shift workers. The study also found that certain groups of workers related long working hours to reduced sleep, compromised recovery time, and reduction of work pace.

Research has proved that specific tasks and numbers of breaks taken during a workday are independently associated with high fatigue levels at work. Near-miss injury events are common among those reporting a high level of fatigue at work. Accidents and lost-time injury are associated with length of time at work. Altogether, these indicate that fatigue and aspects of work organization may be associated with compromised productivity and safety in the workplace. Through monitoring and analysis of historical safety v/s working hours' data, manufacturing facilities should be able to get the best balance between working hours, productivity and safety.

In addition, the presented formulation can easily be extended to cover other aspects of a plant for unparallel benefits. The predicted benefits include but are not limited to the 5 broad categories given below.

i) **Cost savings on equipment, energy, labor, and environmental benefits**

a) This will be through planning and scheduling towards the most profitable plant for higher efficiencies in production, costs, taxes, transportation, etc. given the realities of the loading on the entire plant.

b) Selecting the best equipment examining expense, unneeded components, excess capacity, availability, and keeping longer gaps for better contingency capability

c) Reducing setup and cleaning costs both by campaign ordering and equipment selections

d) Reducing energy costs, peak energy usage, and steam spikes

e) Scheduling to reduce labor overtime, material, and other costs

f) Environmental scheduling for solid waste and wastewater reduction.

ii) **Material inventory Savings**

a) Determining and eliminating excess long-term inventory, leaving only what is needed for production targets, plus sufficient margin for unforeseen changes

b) Shortening the time feed, product, and work-in-progress (intermediate material) must be in inventory

c) Better for mixed feeds and product blending

d) Less frequent buying from competitors to meet customer commitments

e) Stochastic modeling for uncertainty in demand and unforeseen events

iii) **Time Savings for Orders**

a) Cutting order lead times through more intelligent and dynamic allocation of resources

b) Improving due date performance improved prediction of what is predictable, to adapt better to what is not

c) Improved throughput of high demand products. This is important for short-term plant profitability

iv) **Agility in changing conditions/ uncertainty**

This has been recommended as a possible arena of future direction of research in Chapter 6.

a) Unexpected equipment outages, or even equipment available sooner than expected

b) Responsive to situations of atypical feeds and off-grade material

c) Rework or disposal of bad/contaminated material

d) Immediate response to rush orders, quotations, order changes, feed availability, etc.

e) Improved maintenance planning - Coordination between maintenance and operations of the tradeoffs in changing the time equipment or an entire plant is shutdown and started up

f) Quick decisions due to real-time data

v) **Predictability**

a) Fewer scheduling mistakes due to both carelessness / operator errors and being unaware of situations in other parts of a plant. This can be eliminated through integrated plant scheduling

b) Shift view - a log sheet of the future for operators and shift production supervisors. By working towards a set target, there is always more focus on overall plant productivity.

c) Actual versus planned at any point in time as a measure of Overall Equipment Effectiveness (OEE), which has widely been accepted as a universal productivity measurement tool

$$OEE = \frac{Actual\ producton}{Planned\ producton} \times Quality\ factor$$

d) Prediction of bottlenecks of energy and other resources

e) Process analysis tool for both plant engineers and process designers

f) Reduce variability of both batch quality and predicted completion times

g) Compare different equipment and site, plant retrofitting, upgrade, and training. The formulation can be a powerful tool in investigating changes in capacity, different feeds, inventory levels, and peak energy usage

h) Wiser capital expenditure.

i) Training tool for planning people, and a link between scheduling and planning. This has been discussed briefly by Zhu and Majozi (2001).

j) Pre-planned changes under unpredictable events to reduce regulatory problems

5.4. Conclusions

The results from the industrial case study presented in Chapter 4 have been discussed in this chapter. It is clear that there are more benefits from running longer hours; however, these benefits can be overruled by other practical factors as detailed.

This chapter has also given a comprehensive list of other benefits that can emanate from a reliable scheduling framework and should be a motivation for any plant to implement such. However, it is important to keep in mind the impossibility of buying an off-the-shelf customized package that can meet any plant's requirement. Even most standard packages have to be customized to address specific needs. Note that the cost of customizing some of these packages to integrate specific needs of a plant can equate to the cost of the packages themselves. It is important therefore for small plants to utilize tools similar to the work presented in this thesis for the same benefits.

CHAPTER 6

CONCLUSIONS

6.1. Introduction

This thesis has presented a continuous-time SSN based formulation for scheduling large-scale hybrid batch and continuous processes. These processes are found in most chemical and FCMG companies where production of bulk products is done in batch modes of operations while packaging of the bulk products into smaller units are done through continuous processes.

The key contributions of this work are discussed in section 6.2. Finally, potential opportunities arising from this work are presented to motivate for further research.

6.2. Contributions

It is evident that until recently, most scheduling models have ignored hybrid batch and continuous processes. The work presented in this thes allows a large number of the implicating features associated with this hybridization to be taken into account in a unified manner.

This thesis has exploited the drawbacks of the STN based formulation of Ierapetritou and Floudas (1998) with respect to model size, number of binary variables etc. in scheduling continuous processes. The advantages of SSN based formulation of Majozi and Zhu (2001) over STN based formulations are discussed in Chapter 2.

It is important to note that this approach can be easily extended to incorporate sequence-dependent equipment changeovers, different operational policies and intermediate storage linking the hybrid processes. It has been proven through the industrial case study in Chapter 4 that the formulation will give closest approximations to the true production capabilities of the hybrid batch and continuous processes.

The formulation is useful in itself as a decision support tool for production planning and other scheduling requirements in large-scale manufacturing facilities.

6.3. Potential directions for future work

The work presented in this thesis points to numerous research opportunities. These include the following.

i) Incorporation of uncertainty elements into hybrid batch and continuous scheduling problems. Most process-scheduling problems addressed in the literature, including what has been covered in this thesis have been deterministic in nature. All industrial operations encounter unforeseen disruptions on daily basis. These include elements such as product and raw material price fluctuations, equipment breakdowns, operator absenteeism etc. These contribute to varying degrees of uncertainty in these operations. It is important for scheduling methodologies to fully incorporate these uncertainty elements to truly represent industrial realities.

ii) Extension of this framework to address supply chain and logistic activities. These include problems involving warehousing to accommodate mixed and dedicated storage, inventory control and material incompatibilities. Several transportation options involving different lead-times, unit costs and allowable quantities can be investigated using this framework to allow optimal choices to be made.

ABBREVIATIONS

MILP	Mixed Integer Linear Programming
STN	State Task Network
SSN	State Sequence Network
RTN	Resource Task Network
MINLP	Mixed Integer Nonlinear Programming
GAMS	General Algebraic Modeling System

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