ASSESSING CATENARY ARCHES FOR BRIDGE DESIGN IN SOUTH AFRICA: A PRELIMINARY STUDY

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ABSTRACT

Bridges form a vital component of South Africa’s transport infrastructure, yet many are susceptible to failure due to environmental stressors and suboptimal design. Recent disasters, including the 2024 KwaZulu-Natal floods, have underscored the fragility of current bridge systems and the need for resilient structural solutions. While circular arches are prevalent due to their historical effectiveness, this preliminary study investigates the viability of catenary arches as an alternative. Through comparative finite element analysis, the structural performance of catenary and circular arches was evaluated, focusing on moment and stress distribution, deformation, and material efficiency. Results reveal that catenary arches offer more balanced internal force distribution, reduced material consumption, and enhanced resistance to lateral forces. These characteristics suggest that catenary forms may serve as a more resilient and cost-effective bridge solution in regions prone to extreme conditions. This study lays the groundwork for further optimisation and supports the integration of catenary geometries into future South African bridge designs.

Keywords: Catenary Arc, Funicular, Circular Arc, Bridges, Sustainability.

1. INTRODUCTION

1.1 Background

Bridges are critical components of South Africa’s transportation network, facilitating the smooth movement of people and goods. However, South Africa has experienced a series of bridge failures in recent years, leading to significant disruptions and safety concerns. Factors such as poor design, financial constraints, seismic activity, and inadequate maintenance contribute to these failures. The repercussions of neglecting maintenance cannot be overstated, as severe deterioration of bridge structures often occurs before their expected service life is reached. One prevalent mode of deterioration in bridges constructed from concrete reinforced with steel, is steel corrosion, leading to delamination. This issue is a leading cause of bridge failure and collapse globally, including in South Africa (Community, 2018). Despite efforts to maintain existing reinforced concrete (RC) bridges and repair those beyond the maintenance threshold, solutions such as cathodic protection, sacrificial anodes, and galvanising steel are often costly and require specialised expertise (Stephens et al., 2019).

The vulnerability of South Africa's bridge infrastructure was further exposed during the 2024 KwaZulu-Natal floods, which caused extensive damage to infrastructure, including bridges. According to the KwaZulu-Natal Department of Transport, hundreds of bridges were either damaged or destroyed, cutting off access to essential services such as schools, workplaces, and shops (Mhlongo, 2024). The floods led to the collapse or severe damage of numerous bridges, disrupting connectivity in affected areas. The financial burden of repairing the damaged infrastructure, including bridges, is estimated at approximately R3 billion, underscoring the magnitude of the disaster and the extensive efforts required for recovery (Mhlongo, 2024). This event highlights the urgent need for more resilient and sustainable bridge designs that can withstand extreme weather conditions and maintain connectivity in disaster-prone regions.

Among the various solutions to address these challenges associated with traditional supports, catenary shell structures emerge as a promising alternative. Unlike traditional designs, catenary shell structures offer several advantages. They are exceptionally strong, capable of spanning large distances without additional support, and surprisingly lightweight. These qualities not only enhance their performance but also make them environmentally friendly. Additionally, Catenary arcs are good at resisting lateral loads, making them suitable in flood and windy conditions. Moreover, shell structures allow for innovative and dynamic shapes, offering architects and engineers greater flexibility in design (Gohnert, Bulovic, & Bradley, 2018; Gohnert & Bradley, 2022; WTN, 2023).

1.2 Problem Statement

Semi-circular arch bridges have long been recognized as optimal structures due to their ability to efficiently distribute compressive forces, minimize stress concentrations, and provide stability under varying load conditions. These bridges, widely used for centuries, offer durability and structural efficiency, making them a reliable choice for bridge construction. However, despite their advantages, semi-circular arches exhibit uneven stress and moment distributions, particularly at the ends, leading to higher localized forces and increased reinforcement requirements. This limitation, coupled with increasing demands for more cost-effective and resilient designs, necessitates exploring innovative alternatives. South Africa's recent infrastructure challenges, including extensive damage during the 2024 KwaZulu-Natal floods, underscore the urgency of identifying even more optimal solutions to address the country's diverse terrain, financial constraints, and vulnerability to environmental extremes.

1.3 Aim of Paper

The aim of this study is to assess the feasibility of employing catenary-shaped arches as an advanced alternative to semi-circular arch bridges for South Africa's bridge infrastructure. By leveraging the natural compression efficiency of the catenary form, this study seeks to demonstrate improved structural performance, material efficiency, and cost-effectiveness, offering a more resilient and sustainable solution to modern bridge design challenges.

1.4 Scope of Paper

This study focuses on evaluating the feasibility of adopting catenary-shaped arches as an optimal alternative to semi-circular arch bridges for South Africa’s bridge infrastructure. The scope of this research encompasses the key aspects as shown in Table 1.

By addressing these aspects, the research seeks to provide a comprehensive understanding of the potential advantages of catenary-shaped arches, positioning them as a superior alternative to semi-circular arch bridges for modern bridge construction in South Africa.

Table 1: Key aspects of the research scope

Aspect	Description
Historical and global context	Review of traditional and contemporary applications of arched bridges globally, highlighting lessons and innovations applicable to South African infrastructure.
Environmental and load response	Assessment of structural performance under standard and extreme loading scenarios (e.g., dead loads, live loads, wind), using TMH and SANS standards.
Numerical modelling	Development and validation of finite element models in MIDAS software, using plate elements to simulate real-world bridge conditions and assess sensitivity to design changes.
Structural performance	Comparative analysis of the structural behaviour of catenary and semi-circular arches, focusing on stress distribution, moment balance, and deformation patterns under identical loading conditions.
Material efficiency and cost	Evaluation of concrete volume and weight, with emphasis on potential material savings and cost reductions through the adoption of catenary arch profiles.
Future research recommendations	Identification of gaps in current findings and suggestions for further studies, including parametric variation of the scaling factor "a", material optimisation, and behaviour under seismic and flood conditions.

2. HISTORICAL CONTEXT AND GLOBAL INSIGHTS ON ARCHED BRIDGES

2.1 Arching Action

Arching action is a well-established phenomenon that occurs in walls, beams, and slabs. Masonry arches, many of which have stood for over a thousand years, remain stable and functional in many cases. The secret to their strength lies in the way their shape aligns with the stress flow, which takes the form of an arch (Figure 1). Arching action is essentially the natural path of stress, or how stress would naturally move if there were no barriers (Gohnert, 2022). Despite early recognition of arching in structures, the exact geometry of the compression arch in beams remained unidentified. Gohnert and Bradley (2023:753-754) used a segmental numerical method, referred to as the funicular arch theory, to analyse compression flow in beams and demonstrated that the shape of the thrust line corresponds precisely with the catenary curve. This discovery confirmed that the natural path of compressive forces in a beam under uniform load is indeed catenary.

Historically, low-rise segmental arches have been preferred in bridge construction due to their favourable structural behaviour. As the rise of a circular arch decreases, the geometry increasingly approximates that of a catenary curve, which is considered the optimal form for structures under gravitational loading, as it facilitates the transfer of forces primarily through axial compression. This approximation allows low-rise arches to efficiently channel compressive forces along their curvature, thereby reducing bending moments and limiting the need for substantial abutments. However, this efficiency comes with increased horizontal thrust at the supports, rendering the structure more sensitive to foundation movement or settlement. Consequently, the stability and performance of such arches are heavily dependent on the robustness of their abutments (Stone Arch Bridges, 2024).

Figure 1: Farmington River Railroad Bridge, USA (Phelan, 2013)

2.2 Case Studies of Circular and Catenary Arc Bridges

Arches have been utilized for centuries as a structurally efficient form, evolving from intuitive designs to scientifically optimized structures. Historical examples as detailed below demonstrate the enduring relevance and advantages of catenary principles in load distribution and material efficiency.

2.2.1 Basento Bridge in Italy

The Basento Bridge, also known as the Musmeci Bridge depicted in Figure 2, is an iconic example of modern engineering employing catenary principles. Completed in 1976 by engineer Sergio Musmeci, the bridge features a single thin membrane of reinforced concrete moulded into continuous catenary-shaped arches (Marmo et al., 2019). Its design exemplifies material efficiency and structural stability, with the concrete shell functioning as both the load-bearing structure and a pedestrian walkway. The bridge's resilience during the 1980 Irpinia earthquake, which measured 6.89 Mw, underscores the effectiveness of catenary shapes in mitigating seismic forces (Domus, 2007). This case highlights how catenary forms can distribute loads evenly across the structure, reducing stress concentrations and enhancing durability.

Figure 2: Basento river bridge (Luongo, 2016)

2.2.2 The Devil's Bridge in Italy

The Devil's bridge depicted in Figure 3 was built in the 11th century and later modified in the 14th century, the Bridge is loacted in Borgo a Mozzano. This bridge showcases an early application of catenary-like principles in medieval bridge design. Its distinctive humpback shape features asymmetrical arches, enabling effective load distribution and structural stability (Allison, 2013). Although the arch approximates a catenary, it is not a true catenary in the mathematical sense. True catenary curves equation (1) *(see in section 3)* and are formed naturally by a freely hanging chain under uniform gravity. In contrast, the Devil's Bridge arch was shaped more intuitively and does not follow the precise geometry of a mathematically derived catenary (Nesi, 2017). This bridge demonstrates how the principles underlying catenary curves, efficient handling of compressive forces and reduction of tensile stresses, have informed bridge construction long before formal mathematical derivation of the catenary curve.

Figure 3: The Devil's bridge (Orlandini, 2018)

2.2.3 The Roman Bridge of Alcántara, Spain

Constructed between 103 and 106 AD, the Roman Bridge of Alcántara (Figure 4) illustrates the Roman mastery of arched structures. Although primarily semi-circular in design, the bridge's reliance on compression to transfer loads is a precursor to the principles embodied in catenary arches (Pérez et al., 2018). The use of durable materials, such as opus caementicium (Roman concrete), combined with the efficient force distribution of the arches, has allowed the structure to remain intact for nearly two millennia (Karolak and Jasieńko, 2020). This case study underscores the longevity and efficiency of arch-based designs, providing a foundation for the subsequent refinement of catenary forms.

Figure 4: The Roman bridge of Alcántara, Spain (Germany, 2018)

2.2.4 Summary of Insights

The historical applications of arches, as illustrated in these case studies, demonstrate a progression from semi-circular and asymmetrical forms to true catenary shapes. Each example highlights the structural advantages of arch-based designs, including their ability to withstand significant loads and environmental challenges while minimizing material usage. By understanding these historical developments, modern engineers can leverage the natural efficiency of catenary arches to address contemporary challenges in bridge construction, such as cost constraints, environmental resilience, and durability. The transition from intuitive, semi-circular arches to mathematically optimized catenary forms representation of a critical evolution in structural engineering. These case studies provide a foundation for adopting catenary arches as an advanced solution for bridge design, particularly in regions like South Africa, where infrastructure must balance sustainability, resilience, and cost-efficiency.

3. THE CATENARY ARC THEORY

The term “catenary” is derived from the Latin word “catēna,” which means "chain." In physics and geometry, a catenary is the curve that an idealized hanging chain or cable assumes under its own weight when supported only at its ends in a uniform gravitational field (Figure 5). The curve has a U-like shape, superficially similar in appearance to a parabola, but it is not a parabola (Gohnert, 2022).

Mathematically, the catenary curve is the graph of the hyperbolic cosine function. The equation of the catenary curve can be expressed as:

$$y = a\cosh\left(\frac{x}{a}\right) \qquad (1)$$

Where “a” is the scaling factor determined from defining the length and the height of the arc replacing the y with L and x with H until the value of “a” converges. The lower the “a” the flatter the arc and the higher the “a” the sharper the arc (Gohnert, 2022). The catenary theory was first proposed by Robert Hooke in 1675. The mathematical properties of the catenary curve were studied by Hooke in the 1670s, and its equation was derived by Leibniz, Huygens, and Johann Bernoulli in 1691 (Gohnert and Bradley, 2023).

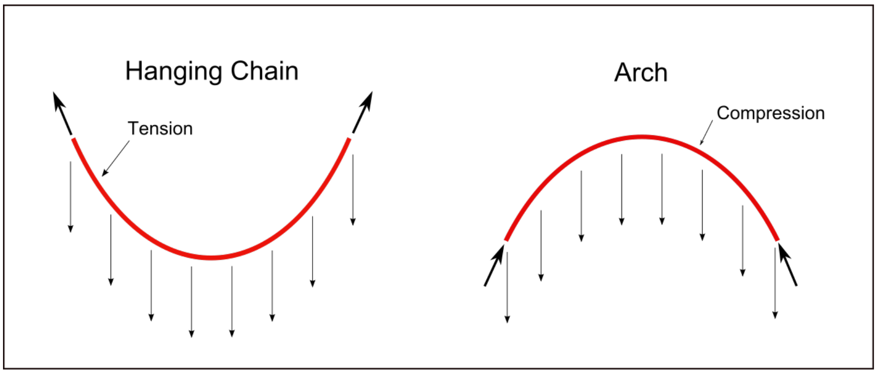

Figure 5: Catenary Shape (Lwphillips, 2013)

4. THE CIRCULAR ARC THEORY

The circular arch as depicted in Figure 6 is a classical architectural and engineering design based on the geometry of a circle. It has been widely used in bridges, vaults, and other structures for its aesthetic appeal and structural efficiency (Karnovskiĭ, 2012). Unlike the catenary arch, which is optimal for uniformly distributed loads, the circular arch's performance depends heavily on its half-angle and the load distribution.

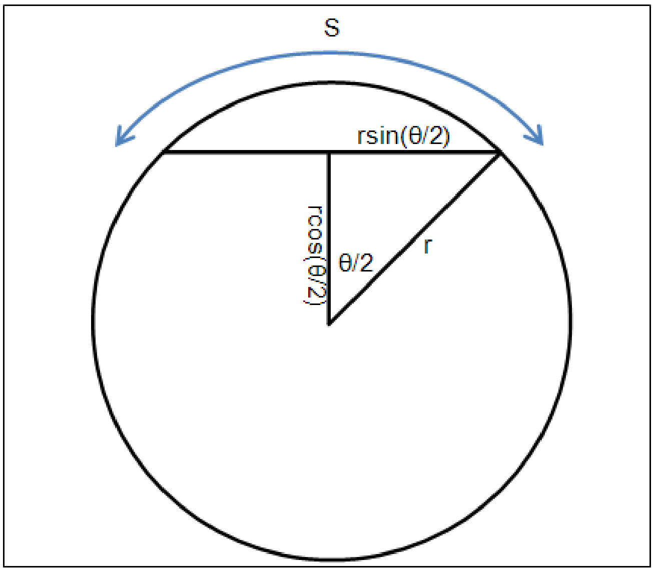

Figure 6: Circular arch

A circular arch is defined as a segment of a circle, characterized by its radius r, central angle θ and subtended arc length s. The relationship between these parameters is given by:

$$s = r \times \theta \tag{2}$$

The performance and stability of the arch are influenced by the half-angle α, which is half of the total subtended angle ($\alpha = \frac{\theta}{2}$). The half-angle determines the extent of the arch and plays a critical role in its structural behaviour under load. A half-angle near 45 degrees ensures that forces above this point remain purely compressive, with minimal bending effects. When the arch geometry and half-angle are optimized, the circular arch can achieve behaviour similar to a catenary arch in the compression region. However, deviations from uniform load or an improper half-angle introduce bending moments, making the circular arch less efficient under such conditions.

5. DESIGN PROCEDURE AND MODEL FORMULATION

The design and analysis of two bridge structures were performed using MIDAS software, ensuring structural integrity and performance under various loading conditions. The approach emphasized consistency in key parameters and adhered to relevant standards for validation. The sketches of the models are depicted in Figure 7 and Figure 8 below and the models under analysis are depicted in Figure 9 and 10:

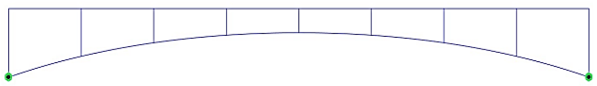

Figure 7: Catenary arc model sketch

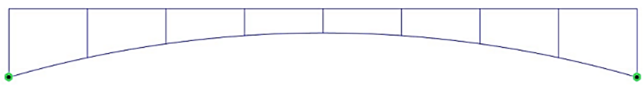

Figure 8: Circular arc model

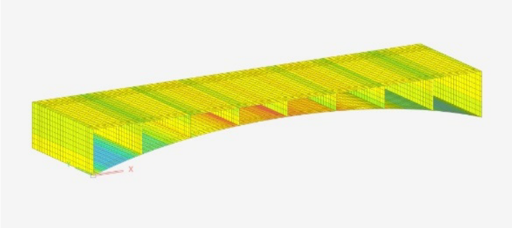

Figure 9: Catenary arc model

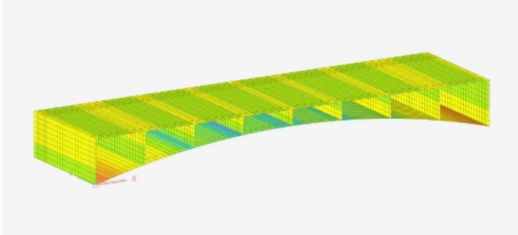

Figure 10: Circular arc model

5.1 Modelling Parameters

Both bridges share common parameters: an 80,492 mm span, nine columns (300 mm x 22,000 mm), and a bridge width of 22,000 mm. The deck includes four lanes (each 3,700 mm wide), outer shoulders (3,000 mm), inner shoulders (1,200 mm), and walkways (1,500 mm on each side). The bridges have a height of 5,500 mm, a deck depth of 500 mm, and an arc thickness of 1,000 mm. Both models are modelled using pinned supports at their boundaries. The concrete used is Grade 60, ensuring the necessary strength and durability (it is important to note that no reinforcement steel was used during the design on both the bridges).

5.2 Unique Characteristics

The Circular arc bridge model features a circular arch design with a diameter of 150,000 mm and a half-angle of 31.13 degrees, optimizing compressive forces and minimizing bending. The catenary arc bridge model uses a catenary arch profile with a scaling factor “a” of 2 chosen as the preliminary value for this design.

5.3 Structural Analysis

Finite element analysis (FEA) was conducted using plate elements, meshing the structure into approximately 7,000 segments for accurate modelling.

5.4 Design Codes and Loading Conditions

The design followed SANS 10160 for static loading, including self-weight (dead load), dead load of wearing surfaces (SIDL), pedestrian loads, and wind loads. Traffic loading adhered to TMH7 standards, with lane loads (36 kN/m), abnormal loads (16 kN), and concentrated loads (300,000 kN/mm²), plus a combined load (NC + 0.67 NA).

Several static load combinations were used to assess the structural response:

LC1: 1.5 DL

LC2: 1.2 DL + 1.6 LL

LC3: 1.2 DL + 0.5 LL + 1.3 WL

LC4: 0.9 DL + 1.3 WL

5.5 Moving Load Applications

Moving load applications were modelled per TMH7 standards, considering both symmetrical and asymmetrical scenarios to replicate real-world traffic conditions.

5.6 Global Axes Layout

The following Figure presents the layout of the global axes that was used when modelling the bridges.

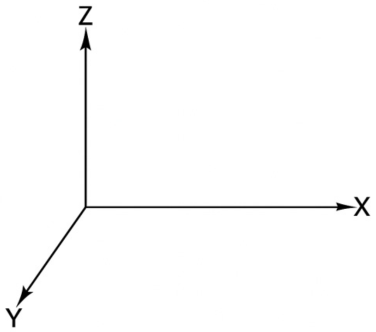

Figure 11: Global axes layout

5.7 Units' Definition

Since the analysis is based on plate elements rather than solid elements, the associated stresses, moments, and forces are defined as follows:

- Moments (kN·mm/mm): These represent the bending moment generated by a force in kilonewtons (kN) applied over a distance in millimetres (mm), normalised per unit width (mm) of the plate element.
- Stresses (kN/mm²): These indicate the internal stress resulting from a force in kN acting over the area of the face of the plate element, measured in square millimetres (mm²).
- Deformations (mm): These refer to the displacement or deflection of the structure under load, expressed in millimetres.
- Forces (kN): These are the linear internal forces acting within the plate elements, measured in kilonewtons

6. RESULTS AND DISCUSSION

6.1 Distribution of Moments on the Arcs Under the Worst Load Case Scenario (LC2)

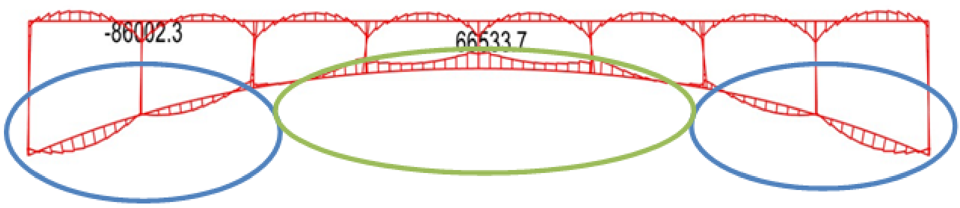

Figure 12: Distribution of moments over the catenary arc

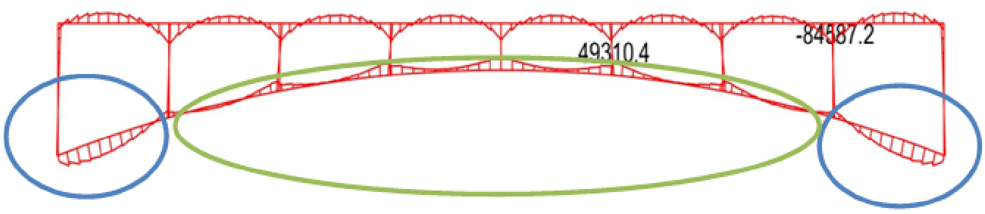

Figure 13: Distribution of moments over the circular arc

6.1.1 Discussions

The moment distribution in the catenary and circular arches (Figures 12 and 13, respectively) reveals significant differences in structural response under similar loading conditions. In the catenary arch, bending moments are concentrated: positive at the crown and negative near the supports. This results in a relatively symmetrical internal force distribution. Although the catenary profile is theoretically optimal for resisting vertical loads through axial compression, real-world conditions such as fixed support restraints and non-uniform loading introduce bending moments. In this case, peak values reach approximately +66,337 kNmm/mm at midspan and −86,002.3 kNmm/mm at the supports.

In contrast, the circular arch demonstrates a broader and less concentrated moment distribution. Its peak positive moment at midspan is lower (+49,310.4 kNmm/mm), but it extends over a wider central region. The negative moments near the supports, while similar in magnitude to those observed in the catenary arch (-84,587.2 kNmm/mm), persist across a larger portion of the structure. This wider spread of high bending demand can complicate reinforcement detailing and reduce material efficiency, particularly when considered in light of design requirements for anchorage, crack width control, and permissible redistribution (SANS, 2000; CEN, 2004a).

From a design perspective, the catenary arch benefits from a more localized concentration of bending, which may allow reinforcement to be used more efficiently. On the other hand, the extended zones of bending in the circular arch introduce additional complexity in both detailing and construction. Design codes such as Eurocode 2 allow up to 30 percent moment redistribution provided that ductility, anchorage, and rotational capacity are verified (CEN, 2004a; CEN, 2004b). These provisions are critical in addressing potential internal force redistributions that may arise during service, particularly under cracked or time-dependent conditions.

In summary, while both arches develop bending due to practical boundary and loading conditions, the catenary arch exhibits a more concentrated and predictable moment pattern. Adherence to structural design codes ensures that such differences are addressed appropriately in terms of safety, efficiency, and durability.

6.2 Distribution of Stresses on the Arcs

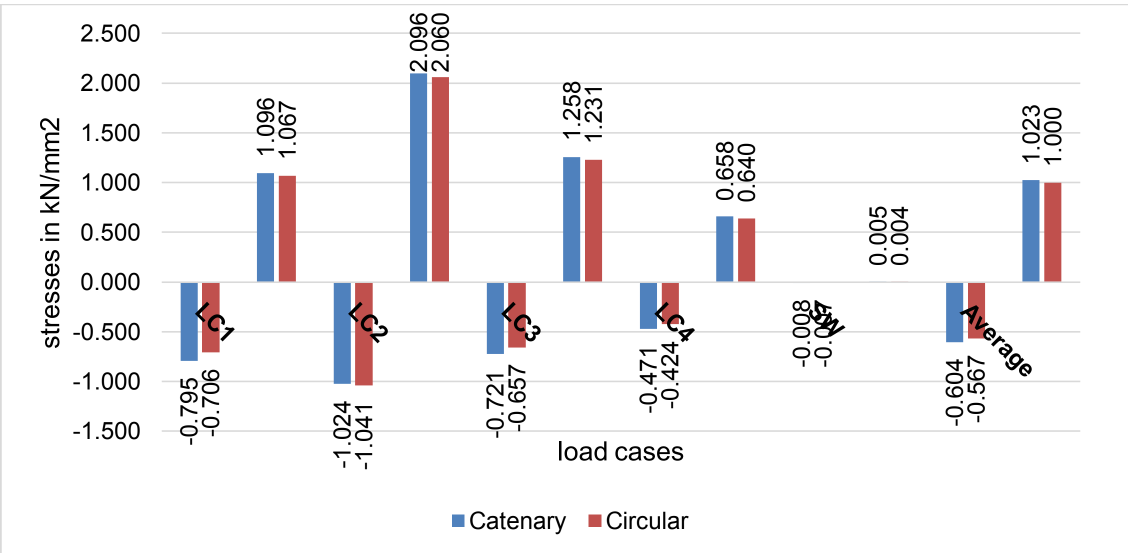

Figure 14: The maximum stresses in global X direction in both arcs

6.2.1 *Discussions*

An analysis of stress distribution revealed that both the catenary and circular arches demonstrate similar overall behaviour across all loading scenarios with minimal values (almost zero) of stress when looking at behaviour under self-weight. However, under the most critical condition, Load Case 2 (illustrated in Figure 14), subtle differences emerge. The circular arc appears to perform slightly better in terms of tensile stress distribution (positive values), whereas the catenary arc shows a marginal advantage in handling compressive stresses (negative values). In this scenario, the recorded stresses for the circular arc are -1.04 kN/mm² and 2.06 kN/mm², while the catenary arc registers -1.024 kN/mm² and 2.096 kN/mm². The differences between the two are minimal, suggesting that both structural forms are equally effective in managing stress even under severe loading. Across all evaluated load cases, both arches consistently display comparable stress responses, indicating their reliability and structural efficiency. This overall similarity supports the conclusion that either design is a viable option for practical application.

6.3 Deformations

Table 2: Average deformations for both arcs in the z and x direction under load case 2 in mm

	Catenary	Circular
DZ upwards (mm)	0	0
DZ downwards (mm)	-2103,27	-2046,28
Dx to the left (mm)	-50,01	-65,49
Dx to the right(mm)	56,93	69,46

6.3.1 *Discussions*

The deformations in the global z-direction follow a similar pattern in both arcs, with both deforming downward and no upward deformations observed. The average vertical deformation for the catenary arc is approximately **2.1** meters, while for the circular arc, it is slightly lower at about **2.05** meters, as shown in Figure 11 (deformations are excessive since the structures are not reinforced). However, a notable difference is observed in the horizontal deformations in the x-direction. The deformations to the left are **0.05** meters for the catenary arc and **0.065** meters for the circular arc, while the deformations to the right are **0.056** meters and **0.07** meters, respectively. This indicates that the catenary arc is more effective at resisting pull-out forces compared to the circular arc, and that the circular arc is effective in resisting vertical forces.

6.4 Overall Structural Performance

In this analysis, only the worst-case loading scenario, referred to as Loading Case 2, was evaluated. As presented in Table 3, the forces and moments in the X and Z directions (Fx, Fz, Mx, and Mz) are distributed almost equally. Both the catenary and circular arches demonstrate highly similar behaviour, with nearly identical values observed for all plate forces and stresses. This similarity can be explained by the fact that, as a circular arch becomes shallower, its behaviour begins to resemble that of a shallow catenary arch, which is inherently suited to carrying compressive forces, as discussed in Section 4. This is further supported by the concept of the arching effect, which highlights that a reduction in the rise of a circular arch causes its geometry to increasingly approximate that of a catenary curve. These findings suggest that both structural forms respond similarly to applied loads, efficiently distributing forces and stresses, and thus offering comparable structural performance.

Table 3: The average force and stress distribution in both bridges

Force/Stress	Average	Catenary	Circular
fx (kN)	positive	76633	76102
	negative	-76645	-76063
fz (kN)	positive	3266	3025
	negative	-3334	-3087
Mx(kNmm/mm)	hogging	1201158	1080253
	sagging	-1197515	-1081412
Mz (kNmm/mm)	hogging	10451363	10384601
	sagging	-10464296	-10397463

6.5 Bill of Materials

The overall bill of materials was extracted from MIDAS software, which calculates weights based on material density and volume. The results indicate that, in terms of area and volume, the circular arc design exhibits slightly higher values for thinner sections, such as columns, due to their increased length. In contrast, the catenary arc shows marginally higher values for thicker sections, particularly the arches, as its shallower profile results in a greater surface area. In terms of total weight, the circular arc is slightly heavier overall, especially in the thinner components. However, when considering the combined area, volume, and weight, the catenary arc demonstrates greater material efficiency (see Table 4). The difference in Volume is 3000L, which reduces the costs associated with concrete. This suggests that further optimisation of the catenary arc could lead to even greater reductions in material usage, particularly since the circular arc used in this study is already optimised.

Table 4: Overall area, volume and weight of both the arcs

Attribute	Circular Arc	Catenary Arc	Difference
Total Area (mm^2)	4.609×10^9	4.596×10^9	$\mathbf{1.3 \times 10^7}$
Total Volume (mm^3)	2.992×10^{12}	2.989×10^{12}	$\mathbf{3.0 \times 10^9}$
Total Weight (kN)	68,810	68,740	**70**

7. DISCUSSION OF RESULTS

The comparative analysis of the catenary and circular arcs highlights key differences in their structural performance, material efficiency, and deformation behaviour. While both arcs perform similarly under standard conditions, the catenary arc consistently proves to be a more efficient and optimal design choice, particularly when considering extreme loading scenarios and material utilization.

7.1 Stress and Moment Distribution

The catenary arc exhibits a more balanced moment distribution, with approximately half of its length experiencing positive moments and the other half negative moments. This even distribution reduces localized stress concentrations, minimizing the risk of material fatigue or failure. In contrast, the circular arc is dominated by positive moments along most of its length, with high negative moments concentrated at its ends. These concentrated stresses necessitate extensive reinforcement in the circular arc, increasing both material usage and construction complexity. Under worst-case loading conditions (Load Case 2), the catenary arc maintains lower negative moments (-86,004 vs. -84,631kNmm/mm) and higher positive moments (66,771 vs. 41,390kNmm/mm), demonstrating superior force distribution and structural efficiency.

7.2 Stress Behaviour

Both arcs exhibit primarily compressive stresses and perform comparably in managing applied forces across all load cases. Under the worst-case scenario, the circular arc shows slightly lower compressive and tensile stresses (-1.04 and 2.06kN/mm^2) compared to the catenary arc (-1.024 and 2.096 kN/mm^2). However, these differences are negligible and unlikely to affect long-term performance. The catenary arc's consistent stress flow offers better predictability, enhancing its structural efficiency and reducing the risk of stress-related failures.

7.3 Deformation Patterns

In the global z-direction, both arcs deform downward, with the catenary arc showing a slightly larger average vertical deformation (2.1 m) compared to the circular arc (2.05 m). However, in the horizontal x-direction, significant differences are observed. The catenary arc demonstrates smaller horizontal deformations (0.05 m left, 0.056 m right) compared to the circular arc (0.065 m left, 0.07 m right). This indicates that the catenary arc offers superior resistance to pull-out forces and lateral loads, making it particularly advantageous for regions prone to wind or seismic activity. In contrast, the circular arc's larger horizontal deformations may require additional bracing, increasing construction complexity and costs. It is acknowledged that the vertical deformations reported here are not representative of practical bridge designs, as the models were intentionally constructed using unreinforced concrete to enable a controlled comparison of arch geometries under consistent conditions.

7.4 Material Utilization

From a material perspective, the catenary arc is more efficient, with an overall lower total volume, area, and weight. For instance, the total area of the catenary arc is $4.596 * 10^9$ mm² compared to $4.609 * 10^9 mm^2$ for the circular arc. Similarly, the catenary arc's total volume is $2.989 * 10^{12}\ mm^3$, slightly lower than the circular arc's $2.992 * 10^{12} mm^3$. In terms of weight, the catenary arc totals 68,740 kN, which is marginally lighter than the circular arc at 68,810 kN. These differences, while small, reflect the catenary arc's ability to optimize material usage. This efficiency aligns with sustainability goals, as less material results in reduced environmental impact and cost savings, particularly in large-scale infrastructure projects.

7.5 Structural Implications and Design Recommendations

The catenary arc's superior moment distribution, reduced horizontal deformation, and efficient material usage make it the more resilient and cost-effective design. Its reliance on compressive forces minimizes the need for reinforcement, reducing susceptibility to issues like corrosion and long-term maintenance. The circular arc, while robust, is less efficient due to its higher horizontal deformations and concentrated negative moments, necessitating more complex reinforcement strategies.

8. FUTURE RESEARCH

As this is a preliminary study, there remains considerable scope for further research into the effective application of catenary arches within the South African context. A more detailed parametric analysis is required to explore the influence of variables such as the scaling factor "a", introduced in Section 3. In this study, a scaling factor of 2 was used, which resulted in a relatively shallow catenary profile that closely mimicked the behaviour of the optimised circular arch. However, further optimisation of this parameter may yield improved structural performance.

Future research will focus on examining how variations in the scaling factor affect stress distribution, deformation, and overall structural efficiency. Additionally, the potential of catenary forms to improve performance under wind loads, seismic forces, and flood conditions warrants in-depth investigation. These factors are particularly relevant in the South African environment, where climatic and geological risks pose serious challenges to infrastructure.

Subsequent studies should also assess the material requirements for optimised catenary arches, with a focus on quantifying steel savings and cost reductions in construction. Moreover, the influence of column placement on stress distribution and load transfer mechanisms should be evaluated to inform best practices in bridge support configuration.

9. CONCLUSIONS

This study assessed the comparative performance of catenary and circular arch bridges under South African design and environmental conditions. While both arch forms exhibit similar overall structural responses, the catenary arch consistently outperformed the circular alternative in several key areas. It demonstrated more uniform moment distribution, superior lateral stability, and reduced horizontal deformation, all of which contribute to a simpler and potentially more efficient reinforcement strategy. Moreover, the catenary arch showed marginally improved material efficiency, with a lighter total structure and lower concrete volume.

Given the demands of cost-effectiveness, environmental resilience, and long-term durability in South African infrastructure, the catenary arch emerges as a promising structural solution. Its performance under extreme loading conditions and capacity for material optimisation make it well suited to the country's engineering and economic landscape. Future research should further explore parameter variation, seismic and hydrodynamic responses, and reinforcement optimisation to fully leverage the advantages of the catenary form in modern bridge design.

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