Gordon Institute of Business Science
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Improved performance prediction and the implications for the efficient frontier

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Abstract

The research project was conducted on the Johannesburg Stock Exchange (JSE) and was aimed at evaluating the performance of asset pricing models and the role they play in modern portfolio theory’s efficient frontier. Furthermore, style investing was evaluated as an alternative approach to portfolio construction. The research performed placed emphasis on the factors used by Gene Fama and Kenneth French’s 3- and 5- factor models and Mark Carhart’s 4-factor model, but Edward Altman’s Z-Score and its factors were also evaluated for their possible application as part of an asset pricing model or portfolio selection tool.

The results indicate that asset pricing models explain a significant proportion of historic returns on the JSE but performed much worse at explaining future returns. As a result, the theoretical efficient frontier does not materialise in actual future returns. Lastly, Style Investing that considers the factors used in APMs, was found to be a superior portfolio selection approach to the asset pricing models themselves.

Keywords

CAPM, Fama French, Carhart, Altman, Efficient frontier, Style Investing

Declaration

I declare that this research project is my own work. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the Gordon Institute of Business Science, University of Pretoria. It has not been submitted before for any degree or examination in any other University. I further declare that I have obtained the necessary authorisation and consent to carry out this research.

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Date:

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"All models are wrong, but some are useful"

- George Edward Pelham Box

1.1 Purpose

The purpose of the research conducted was to critically assess the ability of different asset pricing models (APMs) to predict the future performance of stocks on the Johannesburg Stock Exchange (JSE). Additionally, the research aimed to understand the linkage between expected performance and realised performance and the subsequent implications on the practical validity of the theoretical efficient frontier. Lastly, the research evaluated the performance of Style Investing in relation to APMs in the context of portfolio selection.

1.2 Context of the Study

1.2.1 Pricing Models

Any investment carries risk. It is the objective of each investor to maximise the amount of return achieved whilst minimising the amount of risk taken on with an investment. Logically it follows that an investor will require additional compensation for investing in assets perceived to be higher risk. It thus follows that there exists a positive relationship between risk and return and it is this relationship that an investor ultimately wants to manage and optimise. This is no simple feat as future returns and risk events are not known at the point of making the investment decision. Capital asset pricing models aim to provide insight into the "return" part of this dilemma.

Since the early 1960's researchers such as Treynor (1962; 1961), Lintner (1965a, 1965b) and Mossin (1966) have been working on statistical models that aim to explain returns achieved by individual listed companies. Together, their work gave rise to the fist, and probably the most prominent, pricing model, the *Capital Asset Pricing Model (CAPM)*. CAPM makes use of a single independent variable, historic price fluctuation relative to that of the market, to explain or estimate the return on either a single share or a portfolio of shares. Over the years, other prominent researchers, such as Fama and French (1992, 1993, 2015) and Carhart (1997) have expanded on the CAPM by incorporating additional information on the individual companies being assessed. The ultimate objective of improving upon the original CAPM was to reduce the remaining unexplained variance between the expected and realised returns, thereby increasing the level of reliance that an investor can place upon the models utilised.

1.2.2 Portfolio Optimisation

The second part of the investor's dilemma is one of portfolio construction and consequently, optimisation. In his seminal work, Harry Markovitz (1952), put forward the notion that diversification of risk is required in order to reduce the variability of both the magnitude- and timing of a portfolio's returns. Markovitz further proposed that given certain expectations around individual stock returns and the covariance of returns between stocks in a portfolio, individual stocks in a portfolio could be weighted in a way that would maximize returns for a given level of portfolio volatility. This relationship between the optimised expected portfolio returns and the acceptable level of volatility gave rise to what is today known as the *Efficient Frontier*. In part due to this seminal work, Markovitz won both the John von Neumann Theory Prize as well as a Nobel memorial prize in Economic Sciences.

Fundamental to the practical application of Markovitz's portfolio selection technique is the ability to reliably and accurately produce return expectations for individual stocks. Arguably, what is also important, but what this research did not investigate, is the expectation of future covariance between individual shares.

1.2.3 Style Investing

Style investing is the practice of creating portfolios by selecting assets based upon their characteristics as opposed to selecting individual shares based upon their individual risk and return expectations. The approach relies on the premise that "unrelated securities will move together simply because they have been grouped into the same asset class" (Barberis and Shleifer, 2003, p. 2). This is in contradiction to the work in the field of pricing models, which states that the price of a stock is only a reflection of the discounted value of all future dividends (Ohlson, 1995) and that changes in expectations are driven by individual company fundamentals. Barberis, Shleifer and Wurgler (2005) suggested that comovements in stock returns are, at least in part, due to individual investors classifying stocks into groups with similar characteristics and that, as these investors adjust their demand for these characteristics, the demand for the underlying securities also shifts, causing a correlated change in share price. Wahal and Yavuz (2013) found this herd-like behaviour amplifies the movement of stock returns and therefore, it is imperative that a knowledgeable investor take note of Style Investing.

1.3 Problem Statement

Benjamin Graham, the renowned investment sage, once gave the sound advice to "Buy not on optimism, but on arithmetic." As the quote implies, the logical and informed investor seeks to reduce risk and increase returns by making calculated and informed decisions. The reality is however quite different as investors tend to irrationally push market prices away from the price that a logical, informed investor would pay (Ohlson, 1995). This behaviour essentially creates a mismatch between the value- and price of stocks, where value can be seen as the net present value of all future dividends (Ohlson, 1995). Informed investors can thus achieve excess returns by investing in stocks where the price of the stock is below the value.

As the timing and size of all future dividends are not known in advance, the true value of a stock cannot be known in advance, and at best can only be estimated. Investors thus develop views on the future returns of stocks, based upon conceptual expert judgment models or analytical predictive models based on historical performance. All models, conceptual- or analytical-, rely on historical experience and the premise that the past experience will extend into the future. All models are an oversimplification of a complex world and thus subject to error. Regardless of the approach followed, a certain level of uncertainty (or "risk") remains inherent. The value of these models, therefore, lies in their ability to guide an informed investor towards higher returns under lower levels of risk. As the aphorism coined by the statistician, George Box, goes: *“All models are wrong, but some are useful”*.

With the array of asset pricing models, optimisation techniques and style investing approaches available to investors, insight into the benefits and/or risks that these models offer could prove valuable to both investors and researchers alike. Additionally, investors and researchers interested in the JSE might have concerns with the applicability of these investment tools are that they were developed on very large stock exchanges and as such, might not be appropriate for use on the JSE.

This research thus set out to investigate the following areas, specifically in the context of the JSE:

1. Do APMs explain historic returns on the JSE?
2. Can APMs be used to reliably estimate future returns?
3. Do MPT and the Efficient Frontier work in practice, when making use of APMs?
4. Can Style Investing, focused on popular APMs drivers, achieve returns comparable to the APMs themselves?

1.4 Significance of the Study

The research conducted aimed to add value in the academic- and business spheres. Specifically, attempting to answer academic questions that have not been well explored in the South African context, whilst providing insight and guidance to investors on practically implementable investment strategies. Discussed in more detail below, are the contributions made to the academic- and business spheres:

1.4.1 Value to Academia

In terms of asset pricing models on the JSE, Ward and Muller (2012) showed that CAPM does not add value when used on the Johannesburg Stock Exchange, whilst Auret and Sinclaire (2006), showed that the book to market value of equity ratio employed by Fama and French in their 3 factor model (1992), is indicative of stock returns on the JSE. Both of the aforementioned findings were echoed by Strugnell, Gilbert, and Kruger (2011). There has however been limited research conducted on Fama and French's 5 factor model (2015) or Carhart's model (1997) on the JSE. Ward and Muller (2013) again made significant contributions to the body of knowledge in the field of Style Investing on the JSE. However, apart from their "Combination" model (composition not described), their analysis is primarily univariate.

The body of knowledge in the field of asset pricing models on the JSE is limited, and apart from Ward and Muller, very little has been done in the field of Style Investments. The research conducted has therefore added to the body of knowledge by making contributions in the fields of asset pricing and style investment, whilst, together with Du Pisanie (2018) being one of the pioneers in combining the two fields.

1.4.2 Value to Business

South Africa has one of the biggest and most advanced equity exchanges on the African continent, as a report by Wills Tower Watson (2018) indicated when it listed eight of South Africa's major institutional investment firms under the 500 biggest in investment firms in the world. Investment firms earn income in two main ways: management- and performance fees (Chen, 2019). Whenever an investment firm outperforms the benchmark set, it is entitled to earn performance fees. Therefore, any firm that can utilise the research performed to outperform their benchmark, will be able to extract value. Additionally, the style-based investment approach outlined in the research is simple enough for any private individual or small business to apply and extract above market returns over extended periods of time. Lastly, since the research illustrates the performance differentials between different styles and/or statistical model-based approaches, hedge funds would be able to leverage the research to inform both their long- and short positions.

1.5 Definition of Terms

Abbreviation	Definition
AFS	Annual Financial Statements
ANOVA	ANalysis Of VAriance
APMs	Asset Pricing Models
B/M	Book to Market Value of Equity
CAPM	Capital Asset Pricing Model
CH4	Carhart 4 Factor Model
CMA	Conservative minus Aggressive
EBIT	Earnings Before Interest and Taxes
EF	Efficient Frontier
EMI	Efficient minus Inefficient
FF3	Fama-French 3 Factor Model
FF5	Fama-French 5 Factor Model
HML	High minus Low
J203	JSE All Share Index
JSE	Johannesburg Stock Exchange
MPT	Modern Portfolio Theory
NPN	Naspers Ltd
RMSE	Root Mean Squared Error
RMW	Robust minus Weak
SMB	Small minus Big
SR	Sharpe Ratio
WML	Winners minus Losers

Chapter 2: Literature Review

"As each generation comes into the world devoid of knowledge, its first duty is to obtain possession of the stores already amassed"

- Horace Mann

As early as the 1960s, researchers have developed and tested numerous theoretical and mathematical models aimed at predicting the pseudo random nature of financial markets. One of these pioneers, Edward I. Altman (1968) made use of the financial ratios of individual companies to predict the future performance of said companies, by combining the available information with the assistance of statistical techniques. Through the use of discriminant analysis, Altman created his "Z-score" that made use of profitability, leverage, liquidity and other financial ratios, to predict the likelihood of a company becoming insolvent. The use of financial ratios in the estimation of future stock performance later became commonly known as Fundamental Analysis. Although Altman's work was seminal in its own right, it was preceded by others that paved the way for the integration of the fields of statistics, fundamental financial analysis, capital asset pricing, and portfolio theory.

2.1 Asset Pricing

In his seminal work, "Portfolio Selection", Harry Markowitz (1952) pointed out that building a portfolio first starts with estimates of the performances of individual stocks and then moves on the process of selecting an appropriate portfolio of stocks. The following literature review sections are broken up in this way and will critically evaluate core papers whereupon the proposed research will fundamentally be based:

One of the pioneers in the field of stock return prediction, William F. Sharpe (1964), was among the first researchers to formalise the notion that a distinct relationship between risk and reward exists. As Sharpe pointed out, an investor who is faced with two distinct investment options, wherein he expects the same level of return from both, but is more certain of the returns that would be achieved from one of the two investments, will naturally opt for the more reliable and predictable option. This is because the unpredictability of the returns poses a risk to the investor. It then follows that the more unstable an investment is perceived to be, the riskier it would be perceived to be, and as such, it would attract less demand and consequently, its price should, in fact, be lower. Simply put, an investor would only purchase the more volatile stock if it had a lower price, regardless of the fact that the expected returns between the two investment opportunities are equal. Sharpe (1964) made use of the standard deviation in historical returns of a stock as a measure of the risk of the stock. A few years later, Sharpe expanded on his initial work by proposing that the return an investor expects can be represented on a straight line (Sharpe, 1966), that is a function of the risk-free rate of return, plus compensation for risk (see Equation (1)):

$$E_i = p + b\sigma_i \tag{1}$$

Together with the work performed by other pioneers in the field such as Treynor (1962), Lintner (1965a, 1965b) and Merton (1972), Sharpe's work would eventually give birth to the well-known Capital Asset Pricing Model (CAPM), set out in Equation (2). Today, CAPM is a widely used model that provides investors with a view of what returns to expect (R_i) for a given level of systematic risk (β_i) and the prevailing difference between returns on risky assets (R_m), typically stocks, and returns on risk-free assets (R_f), typically government bonds.

$$R_i = R_f + \beta_i(R_m - R_f) \tag{2}$$

where

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{3}$$

In recent years, research has shown that the single factor (systematic risk) model of CAPM is not reliable (Frazzini and Pedersen, 2014; Ward and Muller, 2012).

The failure of CAPM can, at least in part, be attributed to its simplicity. With only a single factor to capture all the nuances between different types of stocks, the model is constrained and therefore struggles to maintain accuracy as other factors affect the level of returns achieved. Other researchers noticed this inadequacy and constructed multifactor models.

Fama and French (1992) proposed a 3-factor model (FF3) which has also become seminal in its own right. Later, they expanded the FF3 to a 5-factor (FF5) model (2015). In their initial three-factor model, Fama and French expanded on CAPM by adding factors that would account for the market capitalisation or “Size” of a firm, as well as the book value of equity relative to the market value of the equity or “Book to Market”. Thereafter, the model was again expanded to include factors that would gauge the level of “Profitability” and the level of “Investment” in the firm being assessed. By including more factors, Fama and French were able to improve upon the CAPM model.

Prior to the inclusions of profitability and the investment level to their model, Fama and French also evaluated another factor that was not directly related to the company’s fundamentals. This factor evaluated the effect that the share price “Momentum” has on future returns. They found that historical good performing stocks outperformed historically bad performing stocks, even when controlling for the “Size” and “Book o Market” factors (Fama and French, 2012). These results supported earlier findings by Carhart (1997) who found that stock performance over the preceding year assisted his 4-factor model (CH4) to outperform Fama and French’s 3-factor model. The practice of evaluating the performance of a stock does not fall into the realm of fundamental analysis, but rather belongs to the field of Technical Analysis. Locally, researchers such as Cox and Britten (2019), Basiewicz and Auret (2010) and Du Pisanie (2018) have evaluated APMs on the JSE and generally found the models as being able to capture a large part of historic variances in observed share returns.

Technical analysis is the practice of predicting market prices and returns based solely on the price, quantity, and timing of trades made, together with the pattern these indicators form over time. Although not as popular with academics as it is with practitioners, some researchers have shown that Technical analysis possesses the ability to predict stock performance. Blume, Easley, and O'Hara (1994) showed that the volume being traded provides additional information to investors, whilst Zhu and Zhao (2009) showed that investors can make use of the current trading price of stock and compare that to the moving average of the stock price over time to better time their trading decisions. And as mentioned earlier, Fama and French (2012) and Carhart (1997) found that in most cases, the recent change in stock price (momentum) was indicative of the future performance of stocks.

With all the factors available to predict the future performance of stocks, it is only natural to question how this information can be combined and how will weight or importance be assigned to each factor. Linear regression, as employed by Fama and French (1992, 1993, 2015), provides a suitable solution to this problem, but the field of statistics offers numerous solutions:

Lam (2004) proposed a Neural Network that integrated fundamental and technical analysis to predict stock performance. Ward and Muller (2013) proposed a pseudo-stepwise-regression model that has combined the realms of technical-, fundamental- as wells as market analysis factors. Ward and Muller illustrated that their "Combined Style Strategy" outperformed the market average in the long run, whilst simply selecting the portfolio-based quintile rankings and equally distributing funds to each stock in the portfolio.

2.2 Portfolio Optimisation

Harry Markowitz's seminal work (1952), focused on achieving optimal portfolio level performance for a given level of risk appetite. Markowitz set the scene for his research by pointing out that the maxim stating that investors simply want the highest possible return, was in fact only partially true. He pointed out that investors typically diversified their portfolios since they not only valued high returns, but also certainty in their returns.

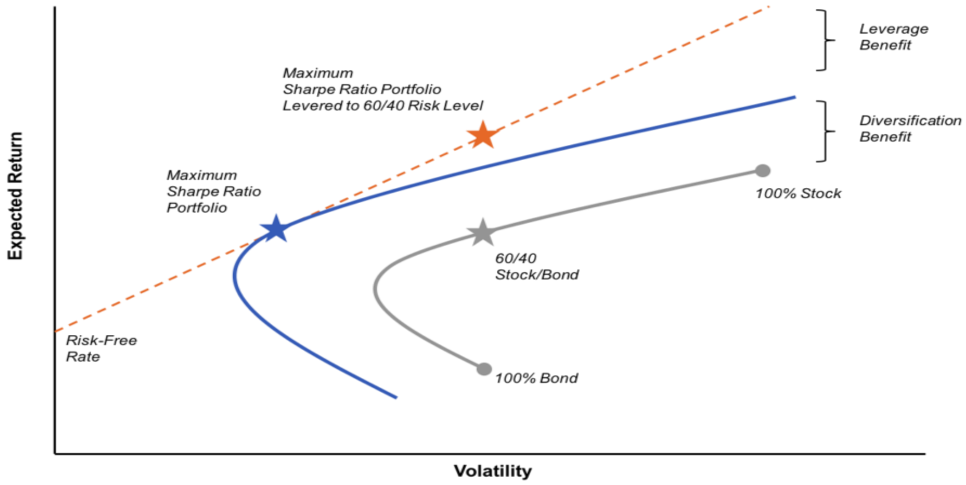

Figure 1: Efficient Frontier & the Sharpe Ratio

(Source:https://blog.thinknewfound.com/2017/12/portable-beta-making-returns-youre-already-getting/mpt-efficient-frontier/)

Markowitz mathematically illustrated that, given a set of expected returns, the variances of said expected returns (a proxy for risk) and the covariances between the returns of different stocks, a combination of stocks can be selected that would theoretically maximise the return for an investor, given the investor's acceptable level of risk. Furthermore, Markowitz then illustrated that an efficiency frontier is created as the optimal level of return is found for different levels of acceptable risk (see Figure 1). Markowitz's work gave rise to what is today known as Modern Portfolio Theory (MPT).

Markowitz's solution to portfolio optimisation revolved around the idea that expected variation in returns would need to be compensated for, through increased returns. Since Markowitz's solution is dependent upon a non-linear input (variance), the optimisation becomes complex and computationally taxing to solve. For this reason, other researchers have proposed similar, but linear, optimisation techniques. For instance, Konno and Yamazaki (1991), poposed a mean-absolute deviation approach wherein the average error between historic return expectation and actual historic returns are set by the investor and thereafter the portfolio is optimised for maximal returns. As it was with Markowitz, Konno and Yamazaki also illustrated that an efficiency frontier can be created by adjusting the level of acceptable error optimizing each point.

As few years after Markowitz's similar work, Treynor (1962) developed an index to rank the risk-reward relationship (see Equation (4)). In turn, Sharpe (1966) then expanded on this Treynor work and created what is today known as the Sharpe Ratio (SR – see Equation (5)). The SR makes use of a portfolio's historic returns and the variance in those returns, relative to the risk-free rate of return to gauge the risk-reward relationship. Sharpe then went one step further and linked the SR to Markowitz's (1952) portfolio optimisation. The intersect between the SR and the Efficient Frontier curve represents the most theoretically optimal portfolio combination and any point on the SR line can be then be achieved through leveraging (see Figure 1).

$$TI_i = \frac{R_p - R_f}{\beta_p} \qquad (4)$$

$$SR_i = \frac{R_p - R_f}{\sigma_p} \qquad (5)$$

One of the core assumptions underpinning MPT is that investors have access to funds at the risk-free rate of return. Theoretically, investors can borrow money at the risk-free rate of return, invest these funds in the theoretically optimal portfolio and in doing so, achieve returns in excess of the efficient frontier. This practice is known as leveraging.

2.3 Style Investing

Under the classical finance paradigm, prices are primarily driven by a company's fundamental data which informs investor expectations of future cashflows. And therefore, stock prices and movements should largely be agnostic to arbitrary information such as the location where the stock is traded or the relative size of a company. Furthermore, the performance should also be uncorrelated to the performance of stocks in other unrelated sectors. Froot and Dabora (1999), however, found that the stock performance of large corporate "Siamese twins"[1] did not adhere to the classical finance paradigm. They found that the stock price movements of these companies were strongly related to the performance of the stock exchange where the shares were traded. Similarly, the work done by Fama and French (1992, 2015) showed that companies with similar traits, such as similar market capitalisation, book to market value of equity ratios, levels of profitability or levels of investment, had similar stock performance. Pindyck and Rotenberg (1993) found that correlations between stock prices of unrelated shares, that could not be explained by macro-economic variables, which led them to develop the hypothesis that this correlation or "co-movement" of stock prices was primarily driven by market segmentation. Noting the aforementioned research and their findings, Barberis, Shleifer and Wurgler (2005) set out to test if the co-movement of stock performance could be, at least in part, be explained by the trading activities of institutional investors or large groups of like-minded individual investors that have classified stocks into groups based on some criterion. They found that stocks start to comove with other stocks on the S&P 500 Index, soon after being included in the index.

Investing in unrelated stocks based upon some criterion, such as being part of an index, and reducing emphasis or even ignoring the fundamental information of the individual stocks, is known as "style investing". For instance, unit trusts will construct their portfolios based on some "style", such as "Value Funds"[2] or "Property Funds".

In South Africa, Ward and Muller (2013) made significant contributions to the field of Style Investment research. Amongst other styles, they evaluated (directly or indirectly[3]) in the "Size", "Book to Market", "Profitability", "Investment" and "Momentum" factors identified by Fama and French (1992, 2015) and Carhart (1997),

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[1] Froot and Dabora (1999) referred to pairs of large companies that shared cashflows and traded in multiple locations as "Siamese Twins". More specifically, they evaluated, three pairs of Siamese Twins: 1) Royal Dutch Petroleum and Shell Transport and Trading, 2) Unilever NV and Unilever PLC and 3) SmithKline Beecham.

[2] Value funds look for companies with low price to earnings ratios or high book to market ratios.

[3] "Profitability" was indirectly measured as "Return on Equity" and "Return on Capital". "Investment" was indirectly measured as "Net Asset Growth".

respectively. With the exception of the “Size” factor, their analysis largely supported the assertion that these factors are related to future performance.

2.4 Altman’s Z-Score and its factors

In 1968, Edward Altman published a paper wherein he, through the use of statistical modelling and fundamental company data, was able to predict company failure with 95% accuracy (Altman, 1968). Altman made use of a statistical technique known as “Discriminant Analysis” to develop his world renowned “Z-Score” (see Equation (6)) which combines five financial ratios into a single value that can be used to predict the likelihood of a company filing for bankruptcy. Intuitively, these ratios should also be able to predict a company’s likelihood to succeed or even produce above market returns. The following sub-sections are dedicated to each of the factors in Altman’s Z-Score:

$$Z - Score = 1.2x_1 + 1.4x_2 + 3.3x_3 + 0.6x_4 + 0.99x_5 \qquad (6)$$

where $x_1 = Working\ Capital/Total\ Assets$

$x_2 = Retained\ Earnings/Total\ Assets$

$x_3 = EBIT/Total\ Assets$

$x_4 = Working\ Capital/Total\ Liabilities$

$x_5 = Sales/Total\ Assets$

2.4.1 Working Capital to Total Assets Ratio (X1)

Working capital is the difference between a company’s short term assets and short term debt and is indicative of a company’s liquidity level (Firer, Ross, Westerfield, and Jordan, 2012). Altman stated that firms that continually generated operational losses would have declining levels of working capital over time. Altman found this ratio to have more predictive power than the other two well-known and commonly used liquidity ratios, the quick- and current ratios (Altman, Iwanicz-Drozdowska, Laitinen, and Suvas, 2014).

2.4.2 Retained Earnings to Total Assets Ratio (X2)

Altman included this measure as, in his view, it provided insight into both the company's and long-run cumulative profitability and its age (Altman, 1968). Altman points out that this measure could be discriminating against young companies, as these companies would not have had time to accumulate retained earnings. He, however, defends the measure by pointing out that, all else being equal, young companies do indeed have a higher risk of failure, making the "discrimination" appropriate (Altman, 2013).

2.4.3 Earnings before Interest and Taxes to Total Assets Ratio (X3)

A measure of a company's ability to generate profit from its assets, not unlike the profitability measures assessed by Ward and Muller (2013). Altman preferred this measure over other measures of profitability as it removed taxation and leverage effects (Altman, 1968).

2.4.4 Market Value of Equity to Book Value of Total Liabilities Ratio (X4)

Not unlike Fama and French's "Book to Market" equity measure (Fama and French, 1992), Altman considered market pricing and perception through this measure. However, where Fama and French were trying to identify exaggerated market prices, Altman used his measure to gauge how much financial loss a company would be able to absorb before it became insolvent. Altman found that the market value of equity was more predictive than the book value (Altman, 1968).

2.4.5 Sales to Total Assets (X5)

Although this measure was the least significant on a univariate basis, Altman included it as his discriminant analysis pointed to the measure's interactions with other variables as making it important and significant. Furthermore, in Altman's view, this measure was indicative of a company's management's ability to deal with competitive forces.

Chapter 3: Research Questions and Hypotheses

"An experiment is a question which science poses to Nature, and a measurement is the recording of Nature's answer"

- Max Planck

3.1 Literature Review Summary

Capital asset pricing and portfolio theory is a well-established research field. Numerous approaches have been developed to estimate the future returns of stocks. Researchers have proposed a number of factors that drive- or are indicative of future performance, whilst others such as Altman (1968) made use of similar factors to predict company failure.

Once the expected performance of each potential stock is determined through APMs, well-established research around portfolio optimisation provides practitioners and researchers alike, with a theoretical framework that aims to improve upon the performance of equally weighted portfolios.

Capital asset pricing models and modern portfolio theory are not the only options available to investors; The simple and eloquent approach of Style analysis has become popular with institutional investors and empirical research has shown it to be an approach for achieving long-term above market returns (Muller and Ward, 2013).

3.2 Gaps in the Literature

On APM and MPT, literature is somewhat lacking on the link between improved estimates, such as those developed by Fama and French (1992, 1993, 2015) and Carhart (1997), and the effect on the improved estimates have on the location of the realised efficient frontier. This is one of the gaps that the research performed aimed to address.

Furthermore, there seems to be very little literature, that brings the worlds of APM and

Style investment together. This might be since the main idea behind Style analysis and -investment is that groups of individuals and/or companies can create momentum by following a particular style and APMs do not subscribe to this notion. Regardless, the research tested if APMs can be used as a "Style".

Lastly, Altman (1968) developed his seminal "Z-Score" to predict company failure. When developing this score, he made use of fundamental analysis and statistical modelling. Intuitively, these same fundamental factors, should be able to predict company success and therefore stock performance. There exists very little research on the topic, especially in the South African context. The research preformed also set out to, at least in part, fill this gap.

3.3 Objectives, Questions, Hypotheses

The aforementioned literature gaps leave numerous questions unanswered. The following research questions were aimed at providing answers to these gaps. In formulating the research questions, the null hypotheses were constructed to be a contrarian view to the commonly accepted literature reviewed.

Table 1: Asset Pricing Model Equations

$$R_{CAPM} = \alpha + B_1 CAPM \tag{7}$$

$$R_{FF3} = \alpha + B_1 CAPM + B_2 SMB + B_3 HML \tag{8}$$

$$R_{FF5} = \alpha + B_1 CAPM + B_2 SMB + B_3 HML + B_4 RMW + B_5 CMA \tag{9}$$

$$R_{CH4} = \alpha + B_1 CAPM + B_2 SMB + B_3 HML + B_6 WML \tag{10}$$

Table 2: Research Objective 1

Research Objective #1 Understand the ability of Asset Pricing Models to explain **Historic Returns** on the JSE					
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	*H_0 Rejection Criteria*	*Method Employed*	*See Equation*
1.1	How well does CAPM explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by CAPM (H_0: $\beta_1 = 0$)	$\beta_1 \neq 0$, implying that CAPM reduces the unexplained variance in historic returns. Prob(F-Statistic) < 0.05	Simple Linear Regression	(7)
1.2	How well does FF3 explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by FF3 (H_0: $\beta_1 = \beta_2 = \beta_3 = 0$)	β_1 or β_2, or $\beta_3 \neq 0$, implying that any of the factors in FF3 reduces the unexplained variance in historic returns Prob(F-Statistic) < 0.05	Multiple Linear Regression	(8)
1.3	How well does FF5 explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by FF5 (H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$)	β_1 or β_2, or β_3, or β_4, or $\beta_5 \neq 0$, implying that any of the factors in FF5 reduces the unexplained variance in historic returns Prob(F-Statistic) < 0.05	Multiple Linear Regression	(9)
1.4	How well does CH4 explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by CH4 (H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_6 = 0$)	β_1 or β_2, or β_3, or $\beta_6 \neq 0$, implying that any of the factors in CH4 reduces the unexplained variance in historic returns Prob(F-Statistic) < 0.05	Multiple Linear Regression	(10)
1.5	Which model explains the most variance in historic returns?	All models are equally good (H_0: $RMSE_{CAPM} = RMSE_{FF3} = RMSE_{FF5} = RMSE_{CH4}$)	The RMSEs of all models are equal Prob(F-Statistic) < 0.05	ANOVA + Tukey	N/A

Asset Pricing Models (APMs) where not intended to be used as predictive tools, but rather sets out to explain return variations (Du Pisanie, 2018). However, their use in Modern Portfolio Theory requires this feature and therefore, understanding their ability to predict future returns is imperative.

Table 3: Research Objective 2

Research Objective #2 Understand the ability of Asset Pricing Models to explain **Future Returns** on the JSE					
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	H_0 Rejection Criteria	*Method Employed*	*See Equation*
2.1	How well does CAPM explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by CAPM ($H0: \beta 1 = 0$)	$\beta 1 \neq 0$, implying that CAPM reduces the unexplained variance in future returns Prob(F-Statistic) < 0.05	Simple Linear Regression	(7)
2.2	How well does FF3 explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by FF3 ($H0: \beta 1 = \beta 2 = \beta 3 = 0$)	$\beta 1$ or $\beta 2$, or $\beta 3 \neq 0$, implying that any of the factors in FF3 reduces the unexplained variance in future returns Prob(F-Statistic) < 0.05	Multiple Linear Regression	(8)
2.3	How well does FF5 explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by FF5 ($H0: \beta 1 = \beta 2 = \beta 3 = \beta 4 = \beta 5 = 0$)	$\beta 1$ or $\beta 2$, or $\beta 3$, or $\beta 4$, or $\beta 5 \neq 0$, implying that any of the factors in FF5 reduces the unexplained variance in future returns Prob(F-Statistic) < 0.05	Multiple Linear Regression	(9)
2.4	How well does CH4 explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by CH4 ($H0: \beta 1 = \beta 2 = \beta 3 = \beta 6 = 0$)	$\beta 1$ or $\beta 2$, or $\beta 3$, or $\beta 6 \neq 0$, implying that any of the factors in CH4 reduces the unexplained variance in future returns Prob(F-Statistic) < 0.05	Multiple Linear Regression	(10)
2.5	Which model explains the most variance in future returns?	All models are equally good (H_0: $RMSE_{CAPM} = RMSE_{FF3} = RMSE_{FF5} = RMSE_{CH4}$)	The RMSEs of all models are equal Prob(F-Statistic) < 0.05	ANOVA + Tukey	N/A

The practical application of MPT optimisation requires reliable estimates of risk (return volatility) and return. Therefore, a thorough understanding of the realised returns, relative to the expected returns and historic volatility is required:

$$R_{Actual} = \alpha + B_1 R_{Expected} \tag{11}$$

Table 4: Research Objective 3

Research Objective #3 Understand the link between Asset Pricing Models, Portfolio Optimisation and Realised Returns					
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	H_0 Rejection Criteria	*Method Employed*	*See Equation*
3.1	Does the Efficient Frontier materialise when making use of CAPM as the expected return component of MPT?	CAPM expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns	A breakdown of the monotonic positive relationship between the standard deviation of the portfolio and realised returns	Simulated Portfolios, Graphical Inspection	(11)
3.2	Does the Efficient Frontier materialise when making use of FF3 as the expected return component of MPT?	FF3 expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns	A breakdown of the monotonic positive relationship between the standard deviation of the portfolio and realised returns	Simulated Portfolios, Graphical Inspection	(11)
3.3	Does the Efficient Frontier materialise when making use of FF5 as the expected return component of MPT?	FF5 expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns	A breakdown of the monotonic positive relationship between the standard deviation of the portfolio and realised returns	Simulated Portfolios, Graphical Inspection	(11)
3.4	Does the Efficient Frontier materialise when making use of CH4 as the expected return component of MPT?	CH4 expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns	A breakdown of the monotonic positive relationship between the standard deviation of the portfolio and realised returns	Simulated Portfolios, Graphical Inspection	(11)

Table 5: Research Objective 4

Research Objective #4 Understand the behaviour of Asset Pricing Models in a Portfolio Selection Setting					
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	H_0 Rejection Criteria	*Method Employed*	*See Equation*
4.1	Can CAPM return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of CAPM, are not ordered according to model expectations	Over the period observed, the cumulative returns on the portfolio with the highest expected return is materially greater than the portfolio with the lowest expected return	Graphical time-series[4]	N/A
4.2	Can FF3 return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of FF3, are not ordered according to model expectations	Over the period observed, the cumulative returns on the portfolio with the highest expected return is materially greater than the portfolio with the lowest expected return		N/A
4.3	Can FF5 return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of FF5, are not ordered according to model expectations	Over the period observed, the cumulative returns on the portfolio with the highest expected return is materially greater than the portfolio with the lowest expected return		N/A
4.4	Can CH4 return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of CH4, are not ordered according to model expectations	Over the period observed, the cumulative returns on the portfolio with the highest expected return is materially greater than the portfolio with the lowest expected return		N/A
4.5	Can Altman's Z-Score return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of Altman's Z-Score expectations, are not ordered according to Z-Score ranking	Over the period observed, the cumulative returns on the portfolio with the highest expected return is materially greater than the portfolio with the lowest expected return		N/A

The final research objective was to propose a long-short investment strategy that would make leverage the learnings from the

[4] The approach followed will mimic that of Ward and Muller (2013), see Figure 45 (page 85)

other research objectives. Ultimately, the proposed strategy would either make use of APMs, factor-based style investment and/or a combination of these to achieve long-term growth on a long-portfolio whilst identifying portfolios that could be shorted to enable leveraging to take place. To establish whether the proposed strategy should make use of APMs or Style-based investing, the long-term performance of these approaches needed to be evaluated. This led to the following research questions:

Table 6: Research Objective 5

Research Objective #5 Understand the Performance of APM Portfolio selection vs Style Investing					
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	*H_0* Rejection Criteria	*Method Employed*	*See Equation*
5.1	Making use of the FF3 factors only, does Style Investing or APM portfolio select perform best?	The cumulative difference in performance between the long and short portfolios are equal, regardless of the approach followed when employing FF3 factors	Over the period observed, the cumulative return on a long/short strategy is different between the two approaches when employing FF3 factors	Graphical time-series	N/A
5.2	Making use of the FF5 factors only, does Style Investing or APM portfolio select perform best?	The cumulative difference in performance between the long and short portfolios are equal, regardless of the approach followed when employing FF5 factors	Over the period observed, the cumulative return on a long/short strategy is different between the two approaches when employing FF5 factors		N/A
5.3	Making use of the CH4 factors only, does Style Investing or APM portfolio select perform best?	The cumulative difference in performance between the long and short portfolios are equal, regardless of the approach followed when employing CH4 factors	Over the period observed, the cumulative return on a long/short strategy is different between the two approaches when employing CH4 factors		N/A

**** CAPM is omitted from the last research objective as the single factor does not allow for granular portfolio selection.*

Chapter 4: Research Methodology

"When you can measure what you are speaking about, and express it in numbers, you know something about it"

- Lord Kelvin

4.1 Choice of Methodology

4.1.1 Philosophy

The research was conducted in the positivist paradigm, following a highly structured approach to enable the replication of results (Saunders and Lewis, 2017). Positivism was chosen as the research philosophy as the stated research problems together with the availability of secondary data lend itself an approach focused on facts that are observable and quantifiable. In line with the positivist research philosophy, the proposed research aimed to interpret observed relationships with the view that these are possible cause-effect relationships (Creswell, 2014), without making any inferences based upon the unobserved phenomenon. Lastly, given the literature review conducted, positivism is the most commonly applied research philosophy in this field of research.

4.1.2 Approach

In line with the Positivist philosophy chosen, the research was deductive in nature. The research attempted to understand possible causal relationships between dependent and independent variables. Hypotheses around these relationships were formed, based upon theory and observation. Furthermore, these hypotheses were (statistically) tested for their validity (Saunders and Lewis, 2017).

Other considerations that informed the decision to make use of a deductive approach, include:

- Time constraints: Given the short timeframe wherein the research needed to be conducted, a deductive approach is preferred as it is usually less time consuming.

- Internal- and External validity: With the deductive approach, research validity is improved as results and conclusions are repeatable and result from hypothesis tests that are not open to interpretation by the researcher.

4.1.3 Methodological Choices

The research conducted only made use of secondary quantitative data. However, the data essentially consisted of two major components namely, stock financial ratios (fundamental) as well as stock market information such as price and quantity traded. According to Saunders & Lewis (2017), the use of multiple sources of quantitative data implies that the chosen methodology is thus multi-quantitative.

4.1.4 Purpose of the Research Design

The research conducted was descripto-explanatory in nature as it sought to describe and provide an understanding of the possible causal relationships that exist between dependent and independent variables by re-examining secondary data (Saunders and Lewis, 2017). More specifically, the research aimed to describe the relationship between certain individual company- and/or market factors and their relationship with individual stock performance. Furthermore, the research intended to explain the interaction between different stocks, their individual expected performances and portfolio level performance.

4.1.5 Strategy

As the research "intervened" in the observed relationships by controlling for certain characteristics, whilst none of the independent variables were manipulated, the research strategy followed was Quasi-Experimental design, also known as Causal-Comparative design (Creswell, 2014). The data was not be altered except where errors were identified and addressed. The research took its direction from the data and did not attempt to influence the observed relationships. This approach was followed to reduce the possibility of observational bias affecting results.

4.1.6 Time Horizon

By its very nature, the stock market is unpredictable, and the complex interaction and levels of interdependencies change over time. As such, simply making use of cross-sectional data to assess the relationships between dependent- and independent variables, would not have provided a true reflection of the actual relationships being evaluated. Further to this, evaluating only one firm over a period of time would also not have provided the research with a full view of the complex interactions. For instance, if a single mining company had to be chosen as the subject of the research, the relationship observed between the firm's Return on Equity (RoE) would be distorted by the performance of the mining industry over the period being assessed and would therefore not be representative of the true relationship between RoE of a firm and future stock performance. The research made use of panel- or "longitudinal" data, consisting of numerous individual stocks tracked over several years. By doing so, it will avoid the pitfalls posed by cross-sectional data, as well as single subject timeseries data.

4.1.7 Techniques and Procedures

The research only made use of publicly listed secondary data. This was done as the use of secondary data had several distinct advantages:

- Cost-effectiveness: From both a time and monetary point of view, it is much more cost-effective to make use of data that has already been collected by a third-party.
- Data Quality: Since the research made use of only publicly listed firms, the data required is readily available in electronic format from the IRESS research domain provided by the Gordon Institute of Business Science or indirectly from the Johannesburg Stock Exchange via Yahoo! Finance.
- Repeatability and Reliability: As the data is publicly available, any future researcher that wanted to validate or test the findings of the research, can access the same data with relative ease.

4.2 Population

The research made use of companies listed on the Johannesburg Stock Exchange (JSE) between 2000 and 2019. Like Ward and Muller (2013), the research focused on the analysis of the largest companies (by market capitalisation) that represent 99% of the value on the JSE (see Figure 2). In doing so, almost 50% of the available data was not used. As such, the research contains an element of selection bias, however, this approach was followed as smaller companies on the JSE are typically thinly traded and have volatile returns that are not driven by the factors that were evaluated during the course of the research.

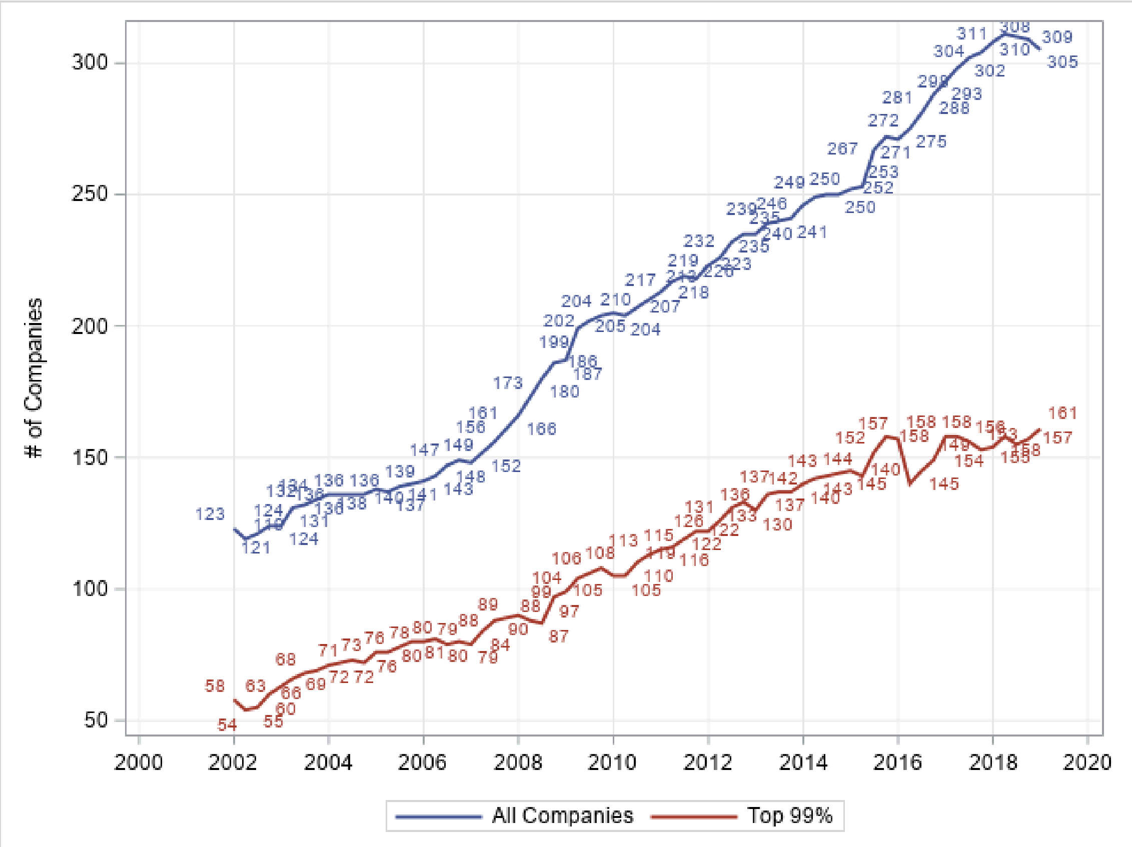

Figure 2: Number of companies making up 99% of JSE Market Cap

4.3 Unit of Analysis

The proposed research had two distinct units of analysis:

i. Individual Company Level

Companies listed on the JSE were analysed to understand the relationship of their financial and market-related data (independent variables) with their future performance. Amongst others, metrics of company data, such as Asset Turnover, Return on Equity, Book to Market value of Equity were collected and/or calculated over the time horizon of the research. The individual company level unit of analysis was then used to construct the expected return forecasting models.

ii. Portfolio level

A portfolio of company stocks was used as another unit of analysis when the performance of selected portfolios is compared amongst each other and the market average. Ultimately, it is at this unit of analysis level that the research aimed to evaluate the efficient frontier discussed in earlier sections.

4.4 Sampling Size and Method

Not all data collected was used. Below is an explanation of the process followed and the reasoning behind it.

4.4.1 Sample Size

As explained in Section 4.2, only the data from the top 99% of companies on the JSE, by market capitalisation, was used. This is similar to the approaches followed by Ward and Muller (2013) and Du Pisanie (2018). However, unlike their studies, the approach followed was to always use the top 99%, where their studies made use of the 160 companies, or all companies if fewer were available at any particular point in time. Figure 2 illustrates the number of companies included in every point in time cohort.

The limited number of companies on the JSE does not lend itself to making use of cross-validation samples. For instance, splitting the early 2000’s data into 50/50 “Build” and “Validation” samples, through simple random sampling, stratified by cohort, would have yielded cohorts with around 25 records each. Therefore, introducing sampling would have increased the risk invalid results being obtained due to low volumes and the subsequent effect on reliability.

4.4.2 Sample Method

The panel data consisted of monthly information for each company on the JSE, including the number of shares in issue and the share price at the end of each month. The combination of information on the share price and the number of shares in issue enabled the calculation of each company's market capitalisation at each month's end.

Each month was treated as a cohort. Then for each cohort, the total market capitalisation on the JSE was calculated. Thereafter, each company's contribution to the total market capitalisation was calculated, per cohort. Companies were sorted by their contribution, per cohort, and in descending order. The cumulative contribution was then calculated per cohort, and companies were included in the research data as long as the cumulative contribution was below 99%.

4.5 Measurement Instrument

As stated earlier, the research made use of secondary financial data sourced from directly- or indirectly from the JSE. Therefore, the measurement instrument in question relates to the process and procedures related to creation, audit, and reporting of financial data, as well as the mechanisms within the JSE that relate to ensuring market data such as price and volume are reliable and accurate. Below, the merits of the reliability and validity of the information used are discussed:

4.5.1 Validity

The validity of research "is concerned with whether the findings are really about what they appear to be about" (Saunders and Lewis, 2017, p. 134). The validity of the findings of this research are subject to the following factors identified by Saunders and Lewis (2017):

4.5.1.1 Quality of Data

The research conducted and the subsequent findings place heavy reliance on the data used. The data is direct- or indirectly sourced from the Johannesburg Stock Exchange or publicly available, published, company annual financial statements (AFS). The 2018 Global Competitiveness Report (World Economic Forum, 2018) ranked the South Africa 55th and 56th (out of 140 countries) on the strength of auditing and reporting standards and shareholder governance respectively. The same report ranked its equity market, which is managed by the JSE, 2nd in the world.

4.5.1.2 Subject Selection

As mentioned in Section 4.4, the author knowingly and willingly excluded the smallest companies on the JSE. This imposes a risk to the validity of the research findings. The author would, however, argue that the 1% of the market capitalisation excluded could not materially alter the findings of the research.

4.5.1.3 Mortality and Censoring

If one compares the number of companies in the data used in this research (see Figure 2) to that used by Ward and Muller (2012) (see Figure 46), it becomes clear that the data used in this study is not complete. Upon further investigation, the author found that companies that delisted from the JSE, willingly or due to failure, prior to 2005, did not form part of the research data. This in effect is data censoring and brings the validity of the of the research findings into question.

Upon discovering the gap in the data, the author set out to test the impact on the validity of the data by reconstructing the Johannesburg All Share Index (J203). The cumulative and monthly returns of the official index versus that of the reconstructed index can be seen in Figure 3.

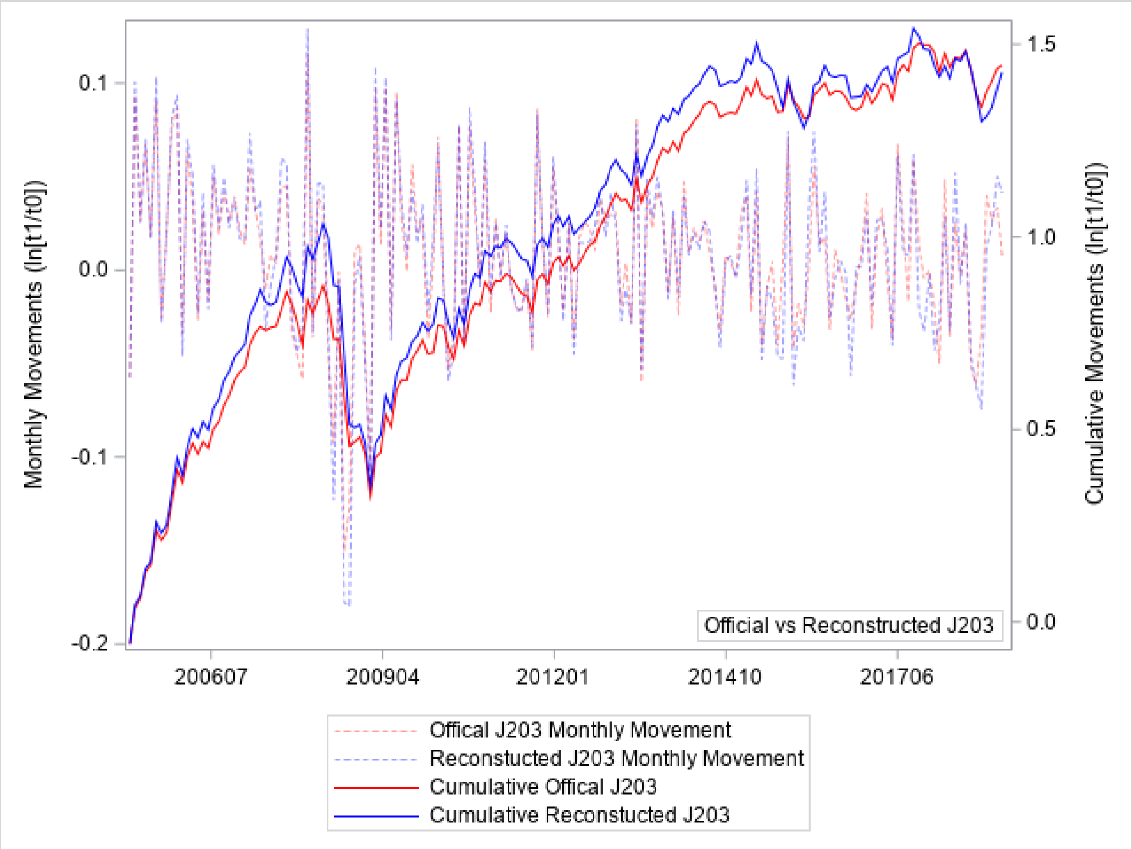

Figure 3: Reconstructed JSE All Share Index (J203)

The results obtained from the exercise provided comfort around the validity of the results obtained.

4.5.1.4 The ambiguity of Causal Direction

The last risk factor that the validity of the findings face, is a misinterpretation of the causal direction between the research's dependent- and independent variables. As there is no statistical test for causal direction, the author essentially needed to impose his views when setting up the research model.

Since the research model followed is rooted in around 50 years' worth of literate that supports the notion that fundamental company information is indicative of future performance, the author does not deem this to be a risk to the validity of the research findings.

4.5.2 Reliability

For research to be deemed reliable, “it must employ data collection methods and analysis procedures which produce consistent findings” (Saunders and Lewis, 2017, p. 135). Saunders and Lewis (2017) identify a number of factors that could negatively influence the reliability of research. From the factors listed, the author identified the following as posing a risk to the validity of the research performed:

4.5.2.1 Data Errors

Given the sheer number of records and datasets used in the research, data anomalies are bound to play a part and should, therefore, be controlled and minimised. Section 4.5.3, elaborates on how data errors were treated in during the research process. However, from a reliability point of view, it is important to note that data errors were considered and catered for as much as possible. Again, the results obtained from the reconstruction of the J203 index provides some assurance on the handling of data errors (see Figure 3).

4.5.2.2 Observer Error and -Bias

The research performed required an extensive amount of statistical programming and data manipulation. Although exceptional care was taken and quality controls were implemented, the situation allows for human error to affect the findings reached. To minimise the risk of human error negatively affecting the reliability of the research findings, the author included numerous checks on the distribution of the input- and output data, and also multiple analyses, making use of different sets of code and checked the results for consistency.

In addition to observer error, observer bias on the part of the author could skew results. For instance, if the author wanted the small market cap portfolios to outperform the large cap portfolios, the author could have selected optimal points to split the portfolio to obtain the desired results. To reduce the risk of bias affecting results, the author followed the positivist philosophy (discussed in Section 4.1.1) and maintained a consistent approach, which is outlined in Section 4.7, throughout the analysis performed.

4.5.3 Data Quality Control

To promote the reliability of the results obtained, extensive data checks were performed. Where data issues were identified, updated, corrected data was used instead. Where no corrected data was available, erroneous records were excluded from the research project.

A second problem encountered was that some companies delisted, which can lead to the research being tainted with survivorship bias. Although this was not explicitly addressed from a data point of view, the analytical process employed limited the impact of such delisting occurrences significantly. To test the integrity and reliability of the data used, the author reconstructed the JSE All Share Index (J203). Figure 3 shows the monthly- and cumulative returns on the official- and reconstructed indices.

4.6 Data Gathering Process

Secondary data was obtained from Ward and Muller (2013), which they gathered from IRESS and Yahoo! Finance over many years. The data was provided in the form of Microsoft Access® databases which was not conducive to the author's analytical requirements. The data was therefore converted to SAS® datasets.

4.7 Analysis Approach

The research performed included several distinct phases which will be discussed in more detail below:

4.7.1 Asset Pricing Models

The mathematical formulation of CAPM was described in Section 2.1 but is repeated here for ease of reference. As can be seen from Equations (12) and (13), CAPM requires three inputs, the risk-free rate of return (R_f), the market rate of return (R_m), and the rate of return of the company or portfolio being assessed (R_i).

$$R_i = R_f + \beta_i(R_m - R_f) \quad (12)$$

where

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \quad (13)$$

The research made use of the historic South African 10-Year Bond Yield as the R_f component[5]. For the R_m component, the J203 index was used. Together with monthly returns from individual companies R_i, the β_i parameter could be calculated.

4.7.1.1 Calculating Beta (β_i) and CAPM

Individual company Betas were calculated for every month in the panel data. Beta was calculated by tracking the natural logarithm of monthly returns on both the J203 and the company being assessed (see Figure 4). It is important to note that dividends paid were included in the calculation as additional returns.

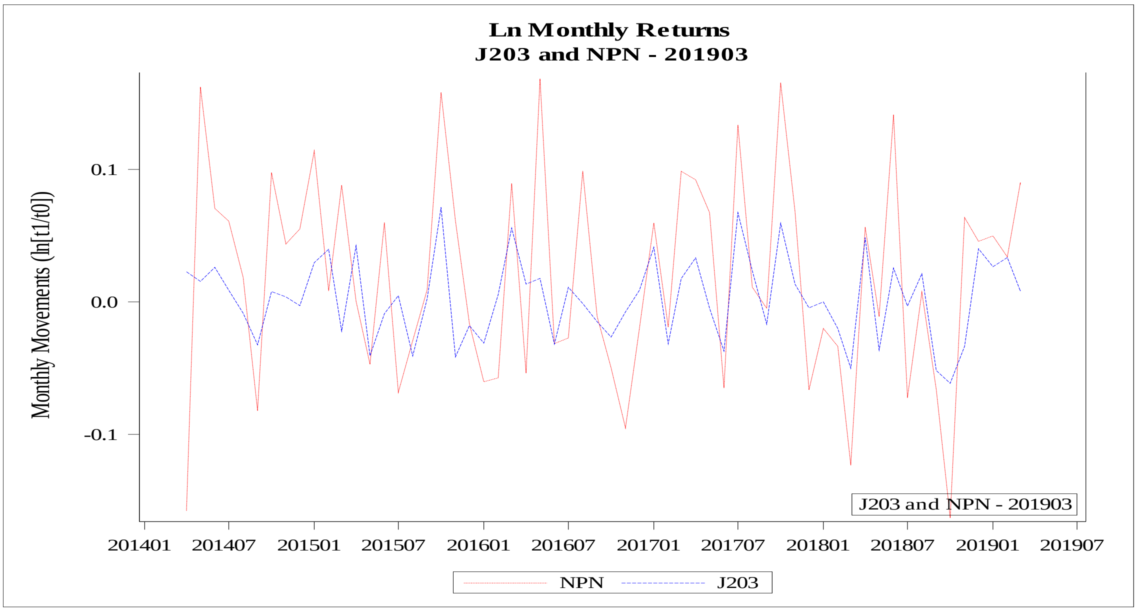

Figure 4: Ln Monthly Returns of the J203 and Naspers Ltd (NPN)

[5] Source: https://za.investing.com/rates-bonds/south-africa-10-year-bond-yield-historical-data

By making use of the most recent 60 months of returns available at each point in time, the author was able to fit linear regression models that produced (see Figure 5) the Beta parameters over time.

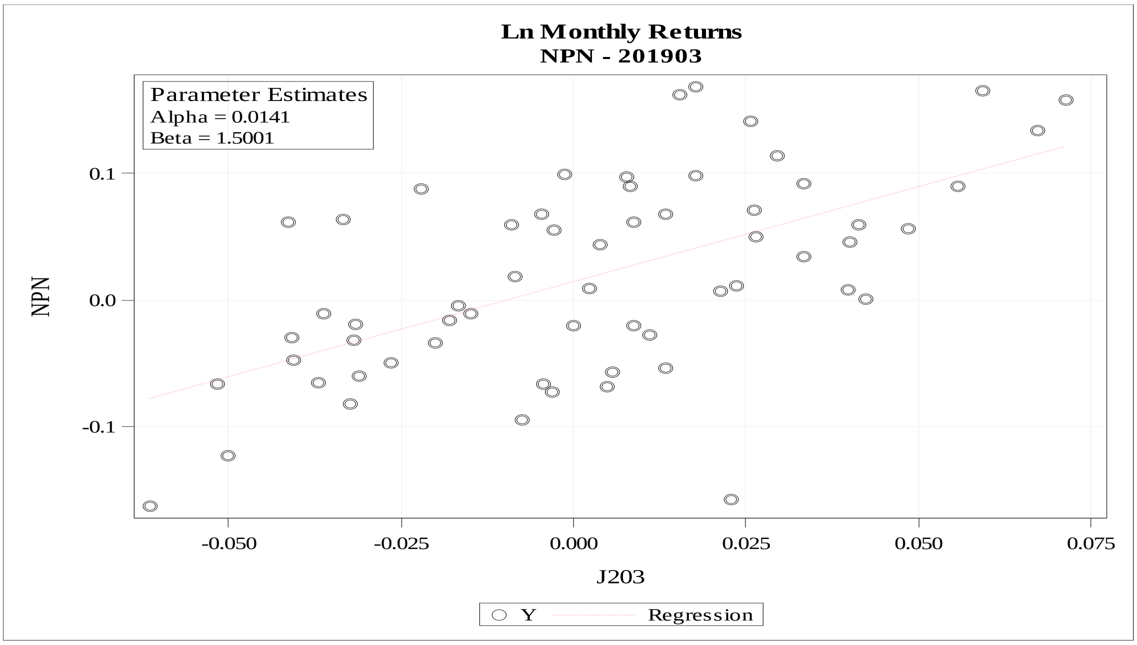

Figure 5: Beta Estimation by Linear Regression

This approach followed allowed for the continual updating of the Beta parameter, as the example in Figure 6 illustrates.

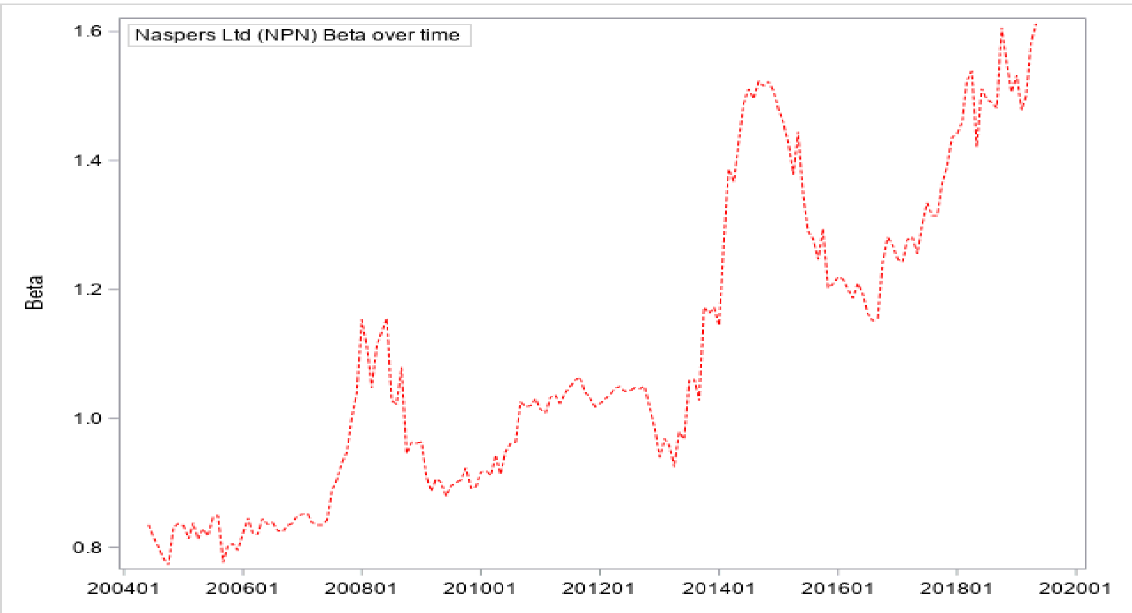

Figure 6: Estimated Beta for NPN over time

Once a monthly Beta for each share was calculated as above, calculating the CAPM estimated returns was a simple process of solving Equation (12) for each company and at each point in time.

4.7.1.2 *Calculating other Asset Pricing Model Factors*

The Fama-French 3-factor (FF3), Fama-French 5-factor (FF5) and Carhart 4-factor (CH4) models all require the effects of certain "factors", such as the observed difference in performance between portfolios consisting of only "small" companies and portfolios only consisting of "large" companies, to be calculated prior to the running of linear regression models. This section outlines the steps followed in calculating these factors:

4.7.1.3 Calculate Factors per APM

Following a similar approach to Fama and French (2015), portfolios were created according to factor classification. The author made use of the same labels as Fama and French to maintain consistency with the literature[6].

$$Number\ of\ Distinct\ Populations = 2^{Number\ of\ Decision\ Parameters} \qquad (14)$$

At each point in time, the companies that make up research population (see Section 4.2) were split into sub-populations, following the tree diagrams below (Figure 7, Figure 8 and Figure 9). At each decision point, the median value of the variable determining the outcome of the decision point was used to split the population into two further sub-populations. This approach leads the number of distinct sub-populations growing exponentially as the number of parameters considered increases (see Equation (14)). The benefit of splitting the population at the decision parameter's median value independently for each decision point is that it results in the similar numbers of companies ending up in each portfolio, which somewhat

[6] Size: Small (S) or Big (B); Book to Market: High (H) or Low (L); Profitability: Robust (R) or Weak (W); Investment: Aggressive (A) or Conservative (C); Momentum: Winners (W) or Losers (L)

circumvents the problem that Du Pisanie (2018) encountered (see Figure 47).

For every factor, the performance difference between the portfolios exhibiting one characteristic was compared to the performance of the portfolios exhibiting the other characteristic. For example, to calculate the SMB_{FF3} factor, the average performance of the "Big, High B/M" (BH) and the "Big, Low B/M" (BL) performance was subtracted from the average performance of the "Small, High B/M" (SH) and the "Small, Low B/M" (SL) portfolios. This is similar to the approach followed by Fama and French (2015) as portfolios were equally weighted when calculating the factor value.

4.7.1.4 Fama-French 3 Factor

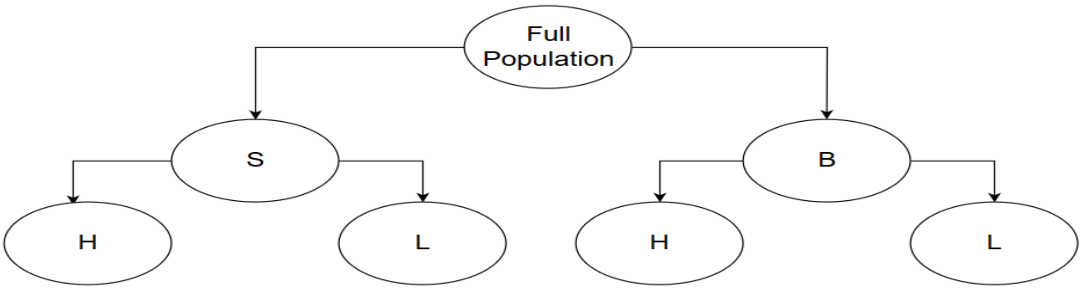

Figure 7: Population Splits (Fama-French 3-Factor Model)

The two factors, other than the CAPM, for the FF3 model, were calculated on a monthly basis thought equations (15) and (16).

$$SMB_{FF3} = (SH + SL)/2 - (BH + BL)/2 \tag{15}$$

$$HML_{FF3} = (SH + BH)/2 - (SL + BL)/2 \tag{16}$$

4.7.1.5 Fama-French 5 Factor

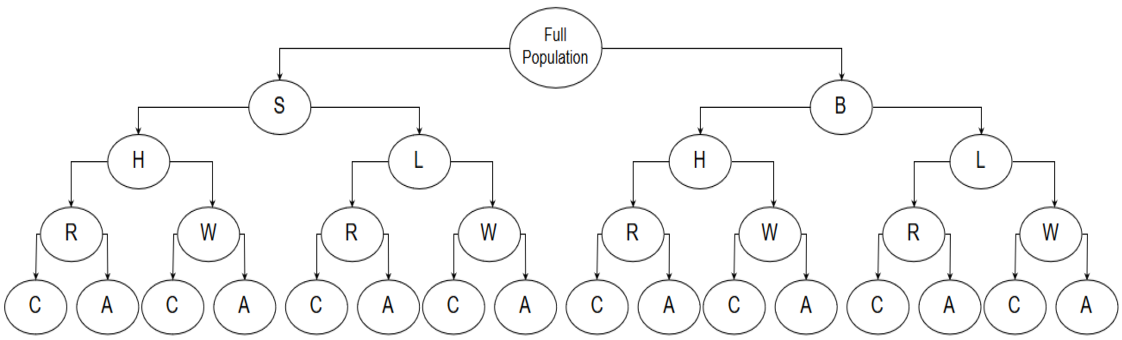

Figure 8: Population Splits (Fama-French 5-Factor Model)

In a similar fashion to the approach followed for the FF3 factors, the FF5 factors were also calculated, however, the increase in the number of factors complicates matter slightly since the number of portfolios increases rapidly.

$$SMB_{FF5} = (SHRC + SHRA + SHWC + SHWA + SLRC + SLRA + SLWC + SLWA)/8 - (BHRC + BHRA + BHWC + BHWA + BLRC + BLRA + BLWC + BLWA)/8 \quad (17)$$

$$HML_{FF5} = (SHRC + SHRA + SHWC + SHWA + BHRC + BHRA + BHWC + BHWA)/8 - (SLRC + SLRA + SLWC + SLWA + BLRC + BLRA + BLWC + BLWA)/8 \quad (18)$$

$$RMW_{FF5} = (SHRC + SHRA + BHRC + BHRA + SLRC + SLRA + BLRC + BLRA)/8 - (SHWC + SHWA + BHWC + BHWA + SLWC + SLWA + BLWC + BLWA)/8 \quad (19)$$

$$CMA_{FF5} = (SHRC + SHWC + BHRC + BHWC + SLRC + SLWC + BLRC + BLWC)/8 - (SHRA + SHWA + BHRA + BHWA + SLRA + SLWA + BLRA + BLWA)/8 \quad (20)$$

4.7.1.6 Carhart's Model

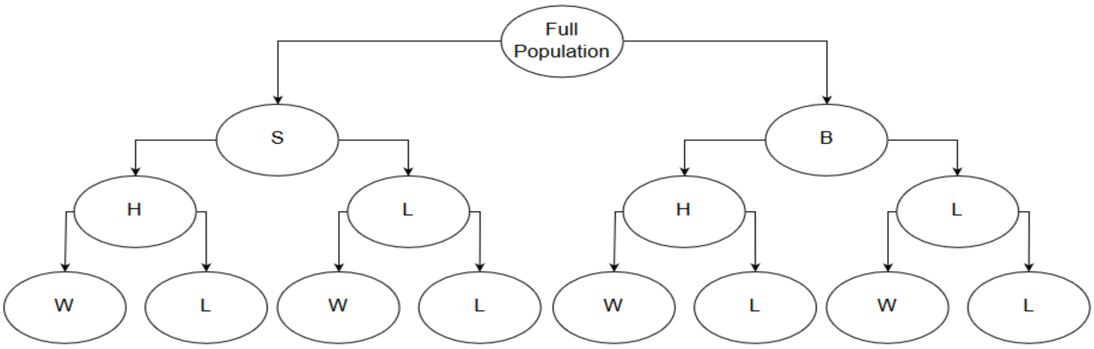

Figure 9: Population Splits (Carhart Model)

Carhart (1997) expanded on Fama and French's (1993) original 3-Factor model by including share price momentum as a new factor, splitting "Winners" from "Losers". In the research performed, total investor return, which included dividends, was used to rank companies and select the "Winner" and "Loser" portfolios.

$$SMB_{CH4} = (SHW + SHL + SLW + SLL)/4 - (BHW + BHL + BLW + BLL)/4 \quad (21)$$

$$HML_{CH4} = (SHW + SHL + BHW + BHL)/4 - (SLW + SLL + BLW + BLL)/4 \quad (22)$$

$$WML_{CH4} = (SHW + SLW + BHW + BLW)/4 - (SHL + SLL + BHL + BLL)/4 \quad (23)$$

The Momentum factor that Carhart (1997) introduced was calculated as in Equation (23).

4.7.1.7 Research Objective #1: Evaluating APMs on Historic Performance

With CAPM and other required factors calculated, linear regression models could be run to solve the parameters in Equations (7), (8), (9) and (10). Like Du Pisanie (2018), regressions were run on the preceding 36 months' returns, however, unlike Du Pisanie, the linear regressions were run for every month, as opposed to every 3 months. The regressions were run for all four models, every month, between January 2005 and March 2019, with model parameters calibrated for each portfolio[7]. Calibrating the models on each portfolio allows the parameters to better fit each individual portfolio. Thus, a total of 684 models were calibrated (171 months x 4 models). Each individual model's fit-statistics[8] together with the root-mean-squared-errors (RMSE) obtained, were stored so that it could be used to evaluate the research questions as described in Table 2.

4.7.1.8 Evaluation of Model Performance: F-Statistic

To assess the ability of each model to explain the variance in observed returns, the F-Static was used. The F-Statistic by itself is only an indication of the model's ability to explain the variance, with a higher value being better, however, it does not indicate if the results are statistically significant. Thus, the probability of randomly obtaining the calculated F-Statistic was also calculated and it was then used to reject the null hypothesis, at a significance level of 1%.

4.7.1.9 Determining which model performs best

The compare model performance, one could look at the F-Statistics, with higher F-

---

[7] A portfolio would constitute all companies that fall into the same leaf of either Figure 7, Figure 8 or Figure 9, depending on the model being calibrated.

[8] F-value and Prob(F)

values being indicative of a superior model. However, in the event that F-values are close to one another, it becomes hard to make the determination with a level of confidence. For this reason, the author opted to calculate the RMSE that each model generated and compare the RMSEs obtained over the evaluation period between the models, with the assistance of an analysis of variance (ANOVA). Additionally, a Tukey analysis was run to determine the appropriate (potential) grouping of models.

4.7.1.10 Research Objective #2: Evaluating APMs on Future Performance

The approach followed to evaluate the performance of the AMPs in predicting future returns, was largely similar to the approach followed to determine explanatory performance on the historic returns; however, a change in approach was needed.

Traditionally, APMs are simply used to explain the variances in historic return performance. However, this is of little value in practice if the expected performances produced by these models do not correlate with future returns.

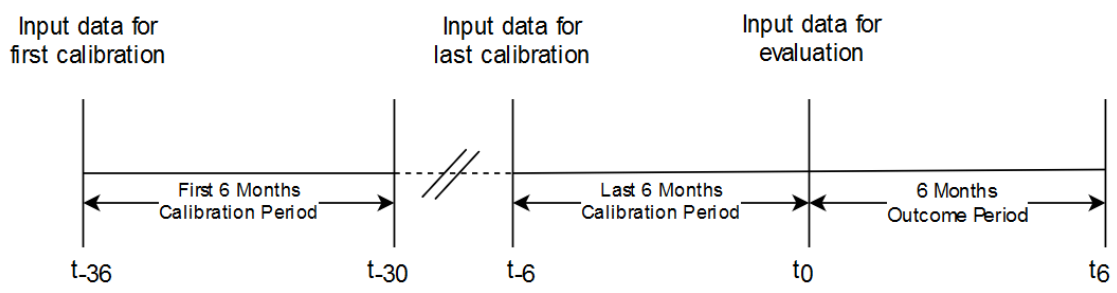

Figure 10: Calibration and Evaluation Timeline

It was thus necessary to simulate this situation by calibrating and evaluating the models over different periods of time. Figure 10 illustrates the approach followed. Data at the beginning of each calibration period was used as the input parameters to the linear regressions (Equations (7), (8), (9) and (10)). The cumulative returns over the next six months were then used as the dependent variable that the models would need to estimate. This was done for 30 (36-6) periods and the combined data from all 30 periods was then used to calibrate the models.

Lastly, to evaluate the performance of the now calibrated model, the data as at the start of the evaluation period was used in conjunction with the model and calibrated its *β-values*. By following this approach, the calibration and evaluation periods do not overlap.

4.7.2 Research Objective #3: Linking APM performance to the Realised Efficient Frontier

Research Objective #3 is aimed at understanding the real-life application of the APMs in the creation of an efficient frontier (EF) and furthermore, to better understand the effect that APM accuracy (or lack thereof) can have on the EF. The author made use of the following steps to test the aforementioned relationships:

The author, through simulation, created 1,000 random portfolios, each consisting of 16 companies, over the timespan of the data. For each portfolio, its standard deviation and actual average monthly returns were calculated.

Next, for each model, the portfolios that constituted the Efficient Frontier, by virtue of having the highest expected return for a given level of standard deviation, were marked in the data (the red dots in Figure 11).

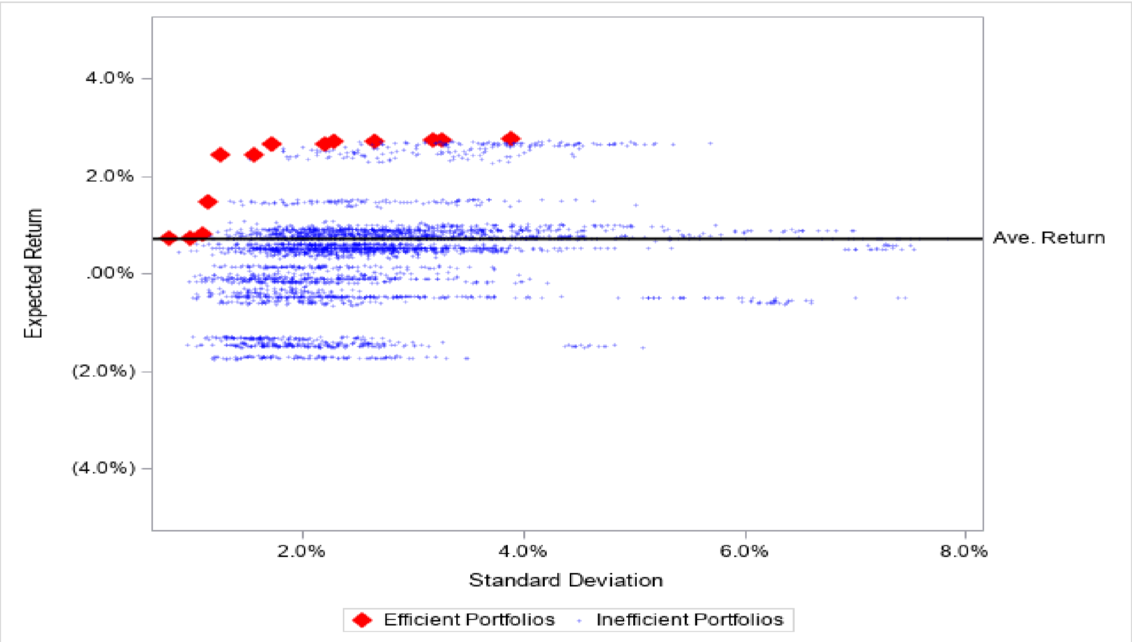

Figure 11: CAPM Simulated Efficient Frontier

Now, if Modern Portfolio Theory and the Efficient Frontier could be applied in practice by making use of an APM as the expectation of risk, the marked portfolios should remain close to the north-western edge of the distribution, when plotted against actual returns. Furthermore, the return on these portfolios should ideally monotonically increase in return, in line with the increased standard deviation.

4.7.3 Research Objective #4: APMs and Altman's Z-Score for Portfolio Selection

The author opted to make use of the graphical time-series approach employed by Ward and Muller (2013) to evaluate the cumulative returns that could be achieved under different portfolio selection strategies. This approach was selected as it, in the author's view, provides the truest reflection of the long-term real-world implications of correctly, or incorrectly, selecting companies to include in a portfolio.

In line with the approach followed by Ward and Muller (2013), companies were allocated to portfolios and the portfolio allocation was refreshed every quarter. Companies could, therefore, move between portfolios because either their circumstances or those of their counterparts have changed, every 3 months. Figure 12 illustrates the cumulative returns achieve following this approach, where portfolios are selected at random.

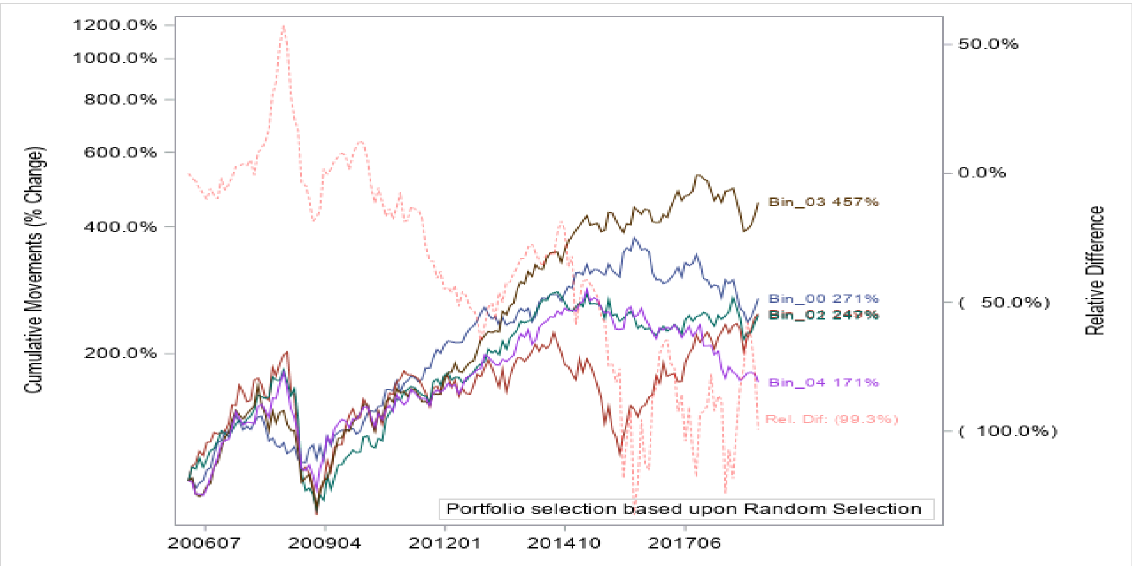

Figure 12: Random Portfolio Selection

The fourth research objective was to evaluate the feasibility of using APM prediction or Altman's Z-Score (1968) as portfolio selection tools. Effectively, this would mean that an investor could go long on a portfolio which the model deemed to have high expected returns, whilst simultaneously going short on a portfolio which the model deems to have low, or negative, expected returns.

The author decided to include the Z-Score as, in theory, Altman's score should be able to predict which companies will do badly going forward as it was developed to predict failure. This feature could be valuable to investors looking to short companies. Section 2.4 unpacks the Altman's Z-Score (1968) which consists of five fundamental ratios that Altman proved to be indicative of companies being in distress. Some of these ratios closely resemble the factors that Fama and French (1993, 2015) and Carhart (1997) introduced with their models. For instance, Altman's fourth variable "market value of equity to book value of total liabilities" is very close to Fama and French's "market value of equity to book value of equity". Furthermore, one could say that, intuitively, the Z-Score variables and the asset pricing model factors provide similar information on the companies being assessed. It thus follows that Altman's Z-Score should be able to act as a ranking tool that could be used to select portfolios.

The models' expectations (derived in Section 4.7.1.10) and Altman's Z-Score, were used to split the top 99% of companies into five mutually exclusive portfolios. These portfolios refreshed with new sets of companies every quarter, according to the updated model estimates. It is important to note that the companies in these portfolios were equally weighted as was done by Ward and Muller (2013).

The returns generated by each portfolio were tracked over time on a cumulative basis. The ability of the models to act as portfolio selection tools were then evaluated based upon the difference in actual cumulative returns, between the portfolio with the highest expectation of return and the portfolio with the lowest expectation of return.

4.7.4 Research Objective #5: APMs vs Style Investing for Portfolio Selection

The last research objective essentially was to establish which portfolio selection approach is best at identifying long- and short portfolios, with the intention of maximising the mean difference in returns between these portfolios.

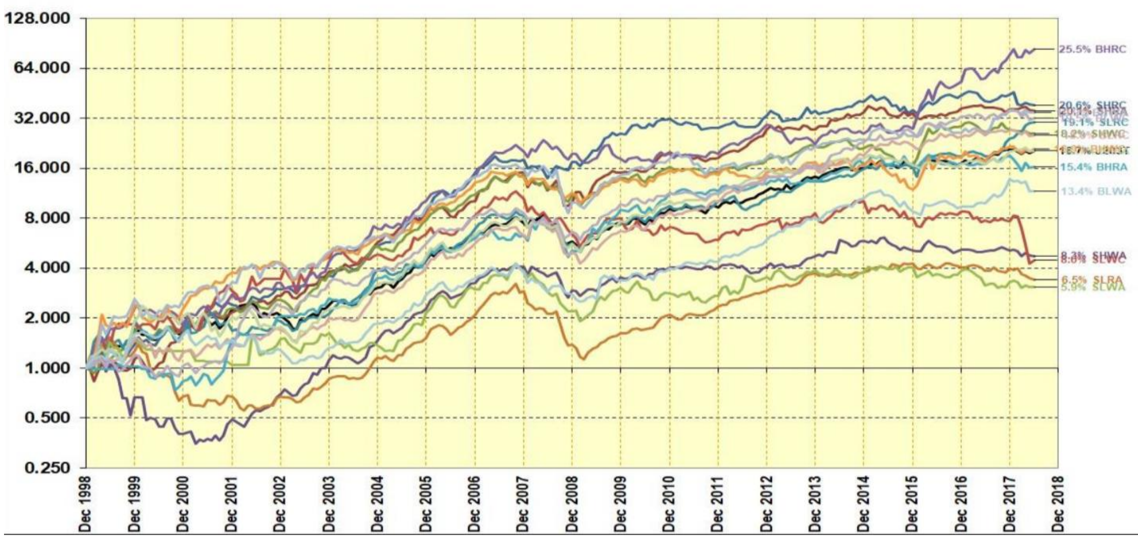

Figure 13: Style Investing Based Upon APM Factors (Du Pisanie)

(Source: (Du Pisanie, 2018))

If one does not apply MTP portfolio weight optimisation, but simply equally weight companies in a portfolio, APM portfolio selection becomes a simple exercise once the model expectations are calculated; One would simple go long on companies with high expected returns and short those that have low or negative expected returns. The author opted to make use of the top 20% of companies, according to expected returns, to act as the long portfolio, with the bottom 20% making up the short portfolio.

In a Style Investment setting, this methodology is less cumbersome as no regression models need to run. Du Pisanie (2018) tracked the cumulative performance of the sixteen unique style portfolios that could be created by segmenting the available companies by the factors in Fama and French's 5-factor model (Fama and French, 2015). The result of this analysis (see Figure 13) showed that high B/M companies with robust profitability and that invests conservatively, outperformed low B/M companies with weak profitability that invested aggressively. This aligned well with the body of knowledge on factor premiums. The author thus opted to follow the theory on APM factors with the style investment-based approach to selecting long- and short portfolios. For instance, with the Fama-French 3-Factor model (1992), the long portfolio would consist of the "Small", "High B/M" companies, whilst the short portfolio would consist of the "Big", "Low B/M" companies.

Du Pisanie's approach was to assign factor flags based upon a company's ranking on the entire array of companies available (Du Pisanie, 2018). This led to very large and very small portfolios being created (see Figure 47). Therefore, the author decided to segment the population in the same fashion as was done for the factor calculation (described in Section 4.7.1) with the intension of creating equally sized portfolios.

4.8 Limitations of Analysis

Like all analytical projects, the analyses performed in this research project is subject to limitations which need to be considered when evaluating the results obtained. Below is a list of analytical limitations that the author encountered during the analytical process, or considers to be factors that could influence the results obtained:

4.8.1 Statistical test assumptions may have been violated

The statistical procedures employed such as the ANOVA and Linear Regression have assumptions around the distribution and interdependence of the information that it makes use of. The author did not test whether these assumptions hold and therefore, the procedures may have produced statistically inaccurate results.

4.8.2 Altman's Z-Score

Altman's Z-Score (1968) was developed to predict company failure, based on a very different period in time and did not make use of data from the Johannesburg Stock Exchange. Therefore, making use of the Z-Score without adapting it according to the needs of this research project is a shortcoming. The author noted that this as an area of concern and therefor focused on the underlying ratios of the score.

4.8.3 Evaluation of the Efficient Frontier

The technique followed to evaluate the efficient frontier did not involve optimising the selection- and weights of the shares assigned to a portfolio, as the efficient frontier calculation process requires. Instead, random portfolios were created and the most optimal of them were selected as the efficient frontier. This implies that the mathematically perfect efficient frontier was not selected. However, the author does not believe this to have had a material impact on the results obtained.

4.8.4 Human Error and Short Timeframe

Albert Einstein said: “A person who never made a mistake never tried anything new”. With that in mind, the author notes that the analyses could be subject to human error despite the numerous steps taken to minimise the likelihood and/or magnitude of the error. In addition to the risk of human error, the author notes that the timeframe wherein the analysis needed to take place limited the quality and depth thereof.

4.9 Main Assumptions

To perform the analysis described in the sections above, a few assumptions needed to be made. Although assumptions are always required in any quantitative analysis project, it is important to take note of these as flawed assumptions could jeopardise the quality and reliability of the results obtained. Below are the two main assumptions made during the analysis performed:

4.9.1 Data reliability

The data source provides an accurate and reliable history of the past. This would include the companies’ financial statement information as well as the share- and bond price data used in the analyses.

4.9.2 Accounting Standardisation

The International Financial Reporting Standards (IFRS) govern the way in which financial statements should be put together. In theory, this ensures that all companies listed on the Johannesburg Stock Exchange would have comparable information in similarly named line items on the balance sheets and income statements. The analyses performed assumes that the IFRS standard was followed and implemented correctly.

Chapter 5: Results

“The goal is to transform data into information, and information into insight”

- Carly Fiorina

5.1 Asset Pricing Model Factors

5.1.1 Fama-French 3 Factor

Figure 14 shows a 12-month moving average of the factor values for FF3, and it is the first indication that the factors have lost predictive power over recent years as the strong positive skewness observed in the early 2000s has diminished or even fallen away in recent years.

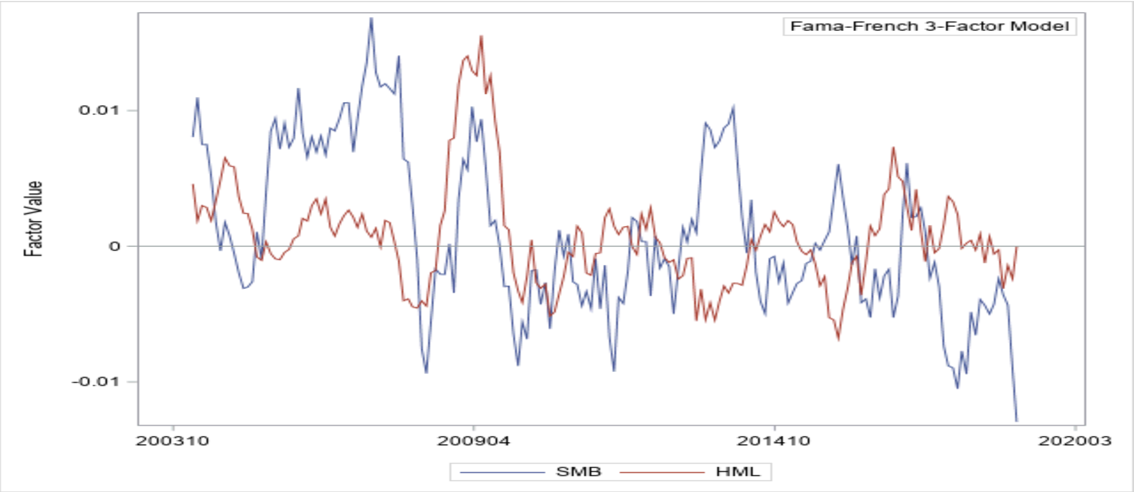

Figure 14: Calculated Factor Premiums (FF3)

Figure 15 illustrates the same information, but this time, the cumulative returns the factors generated over the period of observation is shown. It echoes the sentiment that the early margins offered by these factors have not been maintained in recent years. The “HML” factor has not provided any additional growth since 2009 whilst the “SMB” factor has performed negatively over that same period.

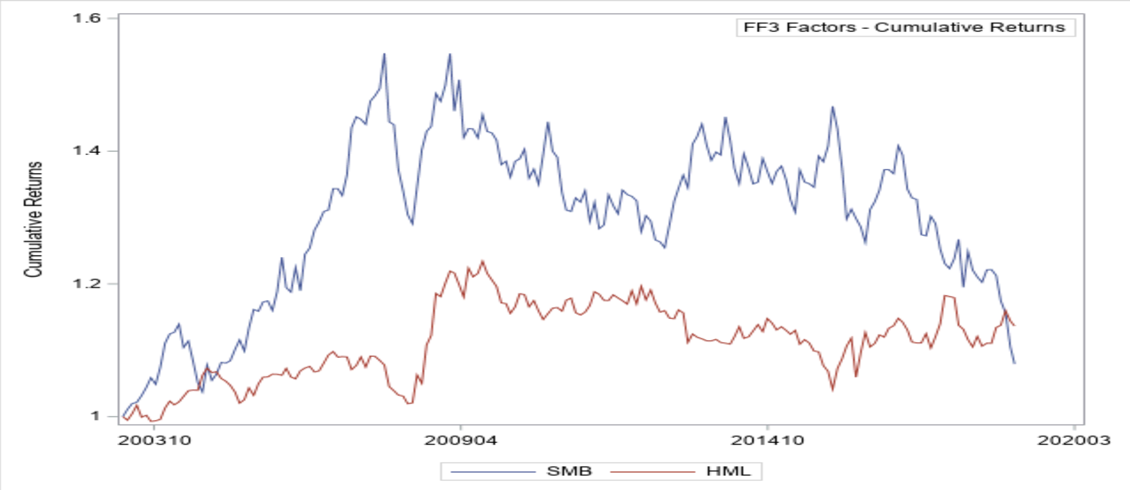

Figure 15: FF3 Cumulative Factor Returns

5.1.2 Fama-French 5 Factor

Figure 16 shows the moving average of the additional factors[9] that Fama and French added to their 3-Factor model to arrive at their 5-Factor model (Fama and French, 2015). As was seen with SMB and HML factors in the FF3 Model above, the positive skewness of the RMW and CMA factors also diminishes in recent periods, pointing to a deterioration of the explanatory power of the factors.

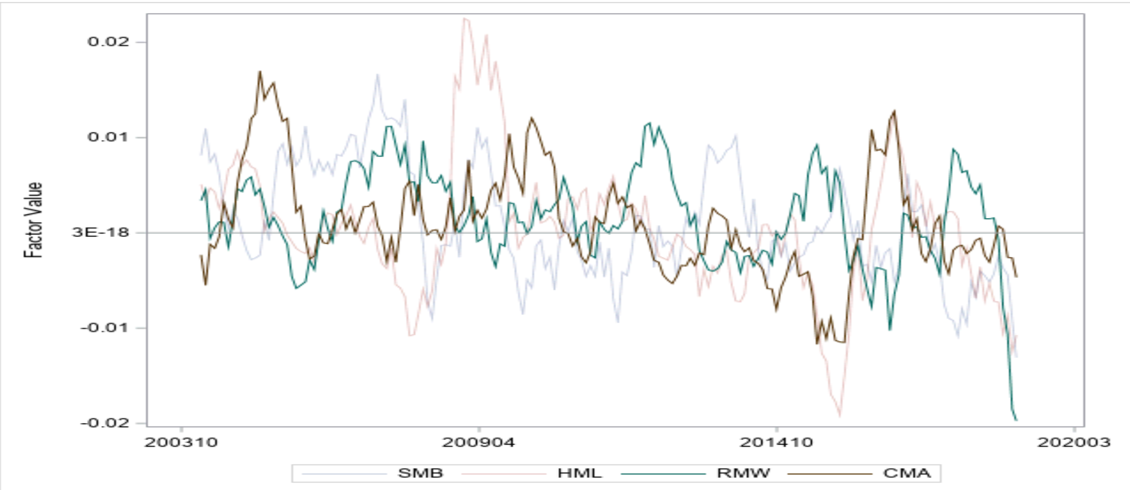

Figure 16: Calculated Factor Premiums (FF5)

[9] The moving averages for SMB_{FF5} and HML_{FF5} can be seen more clearly in Figure 48 (page 88)

On a cumulative basis is can however be seen these two factors provided a positive return up until the end of 2015, substantially longer than the SMB and HML factors. However, since 2016, these two factors have also yielded negative returns.

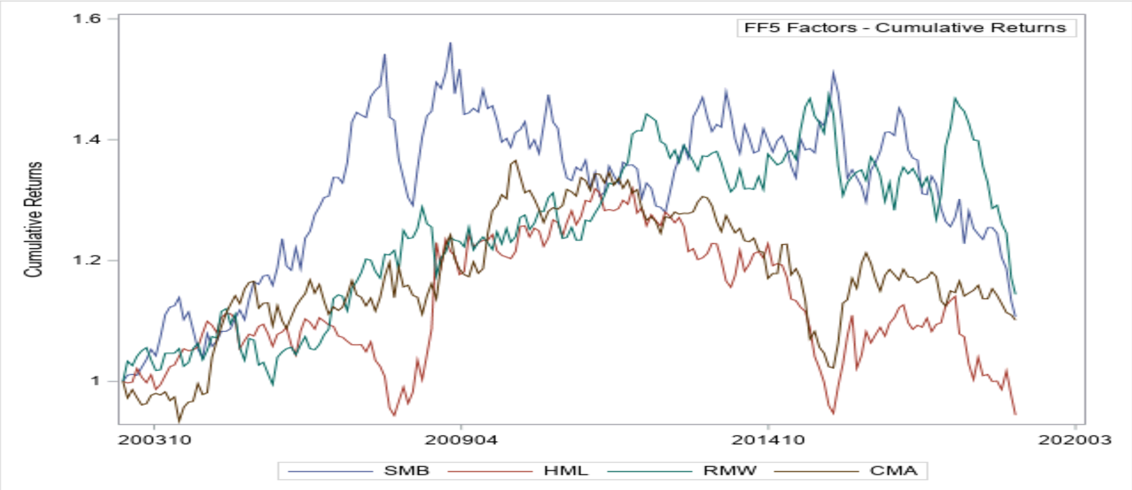

Figure 17: FF5 Cumulative Factor Returns

5.1.3 Carhart Model

Carhart (1997) introduced the WLM factor, building on Fama and French's 3-factor model (Fama and French, 1992). The 12-month moving average of this factor is shown in Figure 18 and it indicates that generally, the factor has positive skewness. This initial view supports Carhart's findings that "Winner" portfolios tend to outperform "Loser" portfolios (Carhart, 1997).

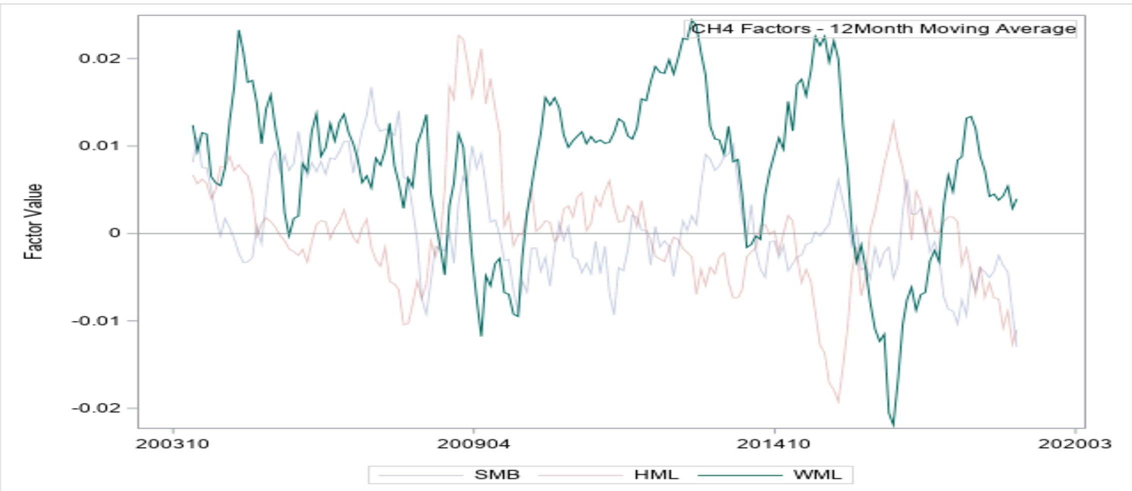

Figure 18: Calculated Factor Premiums (CH4)

On a cumulative basis, the WML factor significantly outperforms the other factors, SMB and HML, in his model which has initially been proposed by Fama and French (1992). Figure 19 shows the cumulative effect of the WML factor's strong positive skewness over a prolonged period of time.

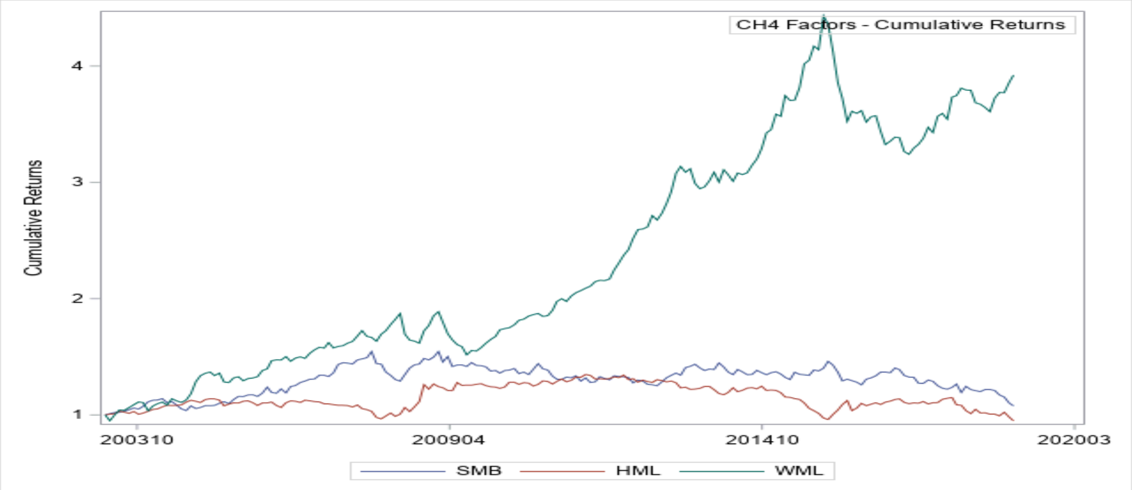

Figure 19: CH4 Cumulative Factor Returns

5.2 Asset Pricing Model Performance

5.2.1 *Research Objective #1: Historic Returns*

Research Objective #1 centred around the evaluation of APMs on the JSE. Figure 20 shows the resultant F-values from the monthly regressions performed, which were described in Section 4.7.1.7. Higher F-values are indicative of a model's superior ability to explain the variance observed in the dependent variable.

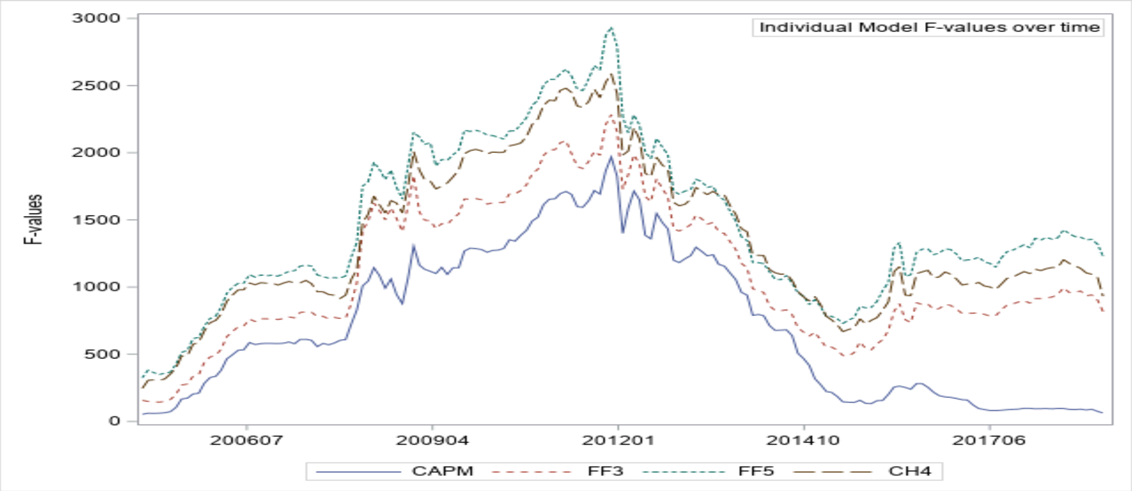

Figure 20: APM F-Values on Historic Returns

Although informative, F-Values need to be checked for statistical significance, and on historic returns, all models were found to be statistically significant over all periods (detailed information available in Appendix 3 – APM Fit Statistics).

Table 7: ANOVA - Research Question 1.5

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.002561	0.000854	15.5	<.0001
Error	680	0.037447	5.51E-05		
Corrected Total	683	0.040008			

The final research question under the first research objective was to understand which model was best at explaining the variances observed in historic returns. An ANOVA and Tukey analysis was performed to address this question. The results are shown in Table 7 and Table 8, respectively.

Table 8: Tukey's Studentised Range Test for RMSE - Research Question 1.5

Means with the same letter				
are not significantly different.				
Tukey Grouping		Mean	N	APM
	A	0.063497	171	CAPM
	B	0.06131	171	FF3
	B			
C	B	0.059542	171	CH4
C				
C		0.058368	171	FF5

5.2.2 *Research Objective #2: Future Returns*

Research Objective #2 was concerned with whether APMs would maintain their explanatory power when used as predictor models. Essentially the same analysis as above was repeated, but changes were made to the timing of information provided to the models. This was discussed in more detail in Section 4.7.1.10. The F-Values generated by the linear regression models calibrated are shown in Figure 21. Again, the F-Values need to be read in conjunction with the probabilities of randomly achieving those values – detailed information is available in Appendix 3 – APM Fit Statistics.

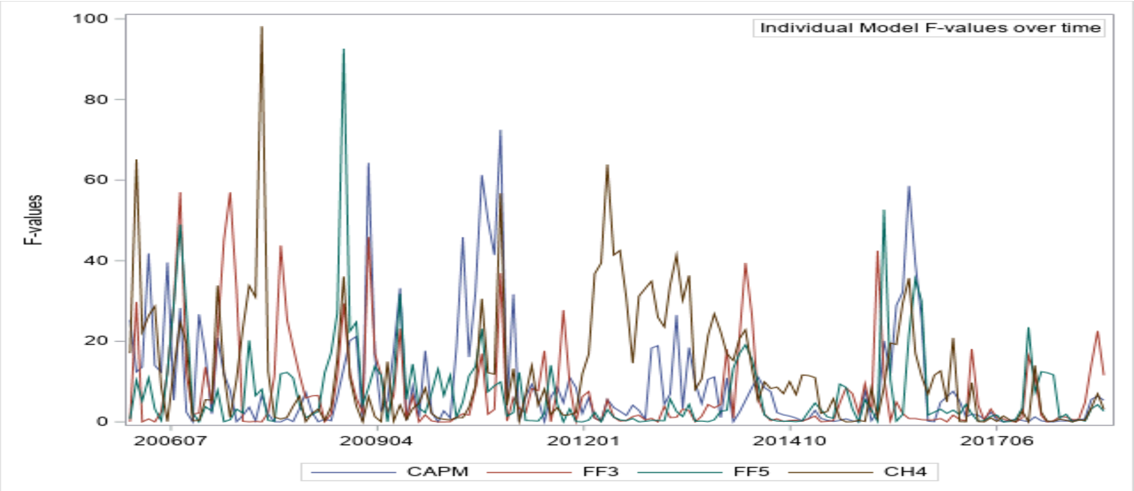

Figure 21: APM F-Values on Future Returns

To determine which model was superior in explaining the variance in future returns, again, and ANOVA analyses were performed (see Table 9). However, since the results from the ANOVA indicated that no means were different, the Tukey analysis was not performed.

Table 9: ANOVA - Research Question 2.5

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.025884	0.008628	1.41	0.2386
Error	620	3.792145	0.006116		
Corrected Total	623	3.818029			

Some periods yielded statistically significant F-values whilst other did not (see Appendix 3 – APM Fit Statistics). The inconsistency of the results obtained between periods forced additional analysis to be performed. To obtain a single view of the APM performances the following additional steps where followed:

- Append data from all periods into a single dataset, so that one model could be fit to the data
- Randomly split the entire dataset into two distinct and mutually exclusive populations: "Build" & "Validation" (see Figure 22)
- Fit each as of the APMs as described before, making use of the "Build" dataset only
- Apply newly fitted models to the "Validation" data set
- Make use of linear regression to determine the model predictions' ability to explain the variance of future returns.

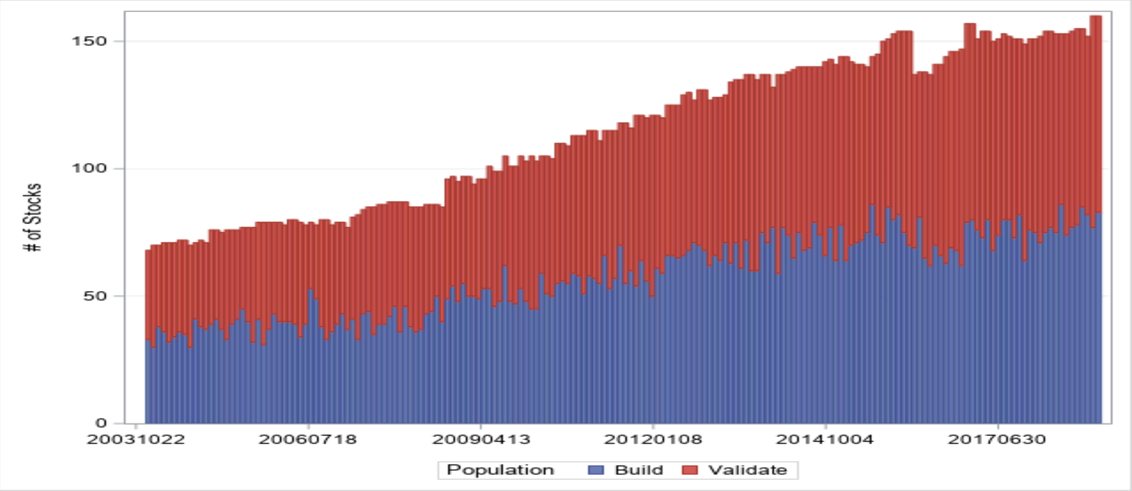

Figure 22: Build and Validation Populations for APM Evaluation on Future Returns

The results of evaluating the model fitted on the "Build" data, on the "Validation" population are shown in Table 10 below:

Table 10: APM Estimate Performance on Future Data (Full Panel)

CAPM					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	0.015784	0.015784	16.33	<.0001
Error	10441	10.09283	0.000967		
Corrected Total	10442	10.10862			
Fama-French 3-Factor					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	0.064819	0.064819	67.38	<.0001

Error	10441	10.0438	0.000962		
Corrected Total	10442	10.10862			
Fama-French 5-Factor					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	0.077646	0.077646	80.82	<.0001
Error	10441	10.03097	0.000961		
Corrected Total	10442	10.10862			
Carhart 4-Factor					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	0.161184	0.161184	169.18	<.0001
Error	10441	9.947435	0.000953		
Corrected Total	10442	10.10862			

5.3 Research Objective #3: Efficient Frontier

The third research objective was to evaluate the appropriateness of the use of APMs as input into the determination of an efficient frontier. The research approach followed was to randomly generate thousands of portfolios, determine the portfolios that would have constituted the theoretical efficient frontier, and then to evaluate the realised return/risk relationship for consistency with the theory.

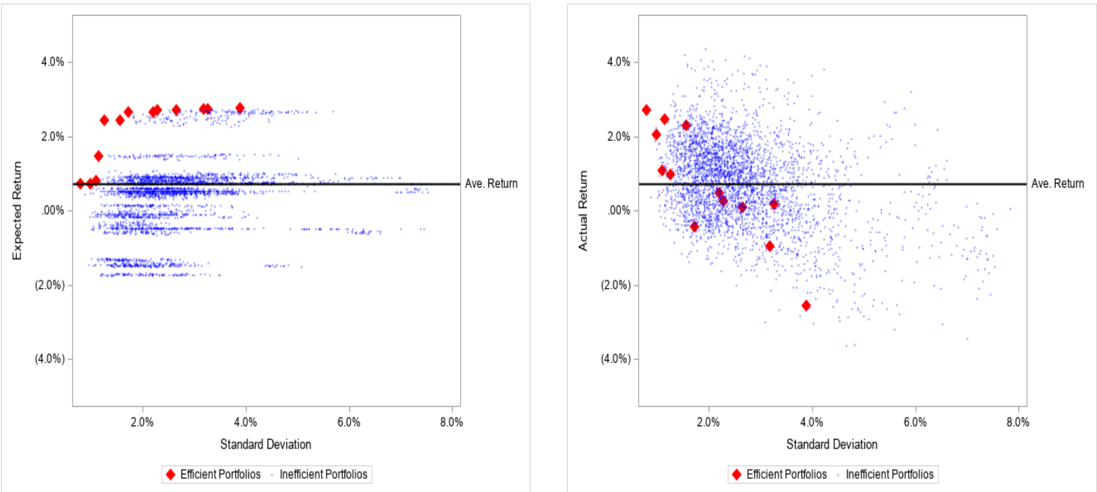

Figure 23: Theoretical and Realised Efficient Frontier (CAPM)

The lines observed on the left-hand graph in Figure 23 are formed by high concentrations of portfolios around similar levels of expected returns. This happens as CAPM has only the one β_i-parameter that drives the expected return, through the MRP. Portfolios that have similar average β_i-parameters will also have similar expectations when the MRPs are comparable. Thus, the limited number of drivers of expected risk in the CAPM creates a lumped distribution of expected returns.

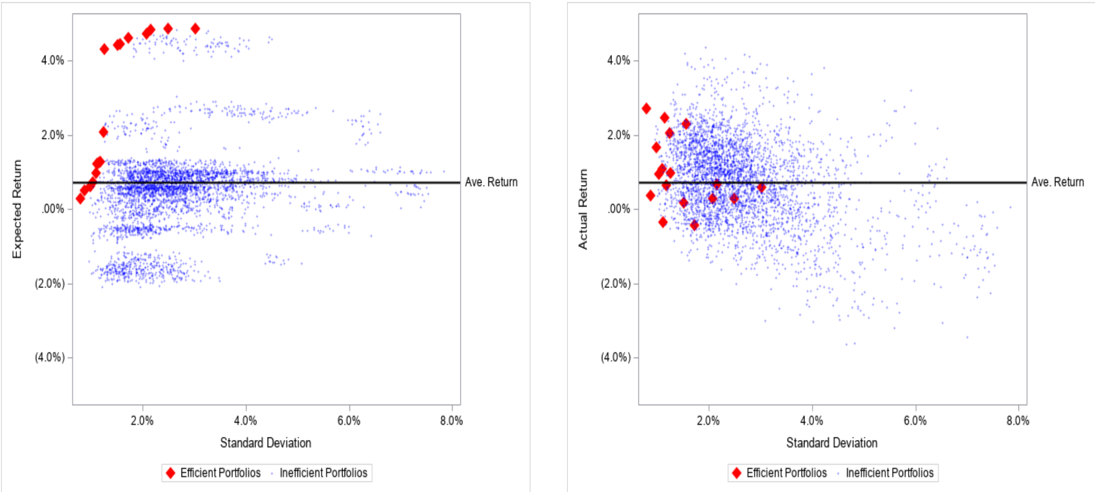

Figure 24: Theoretical and Realised Efficient Frontier (FF3)

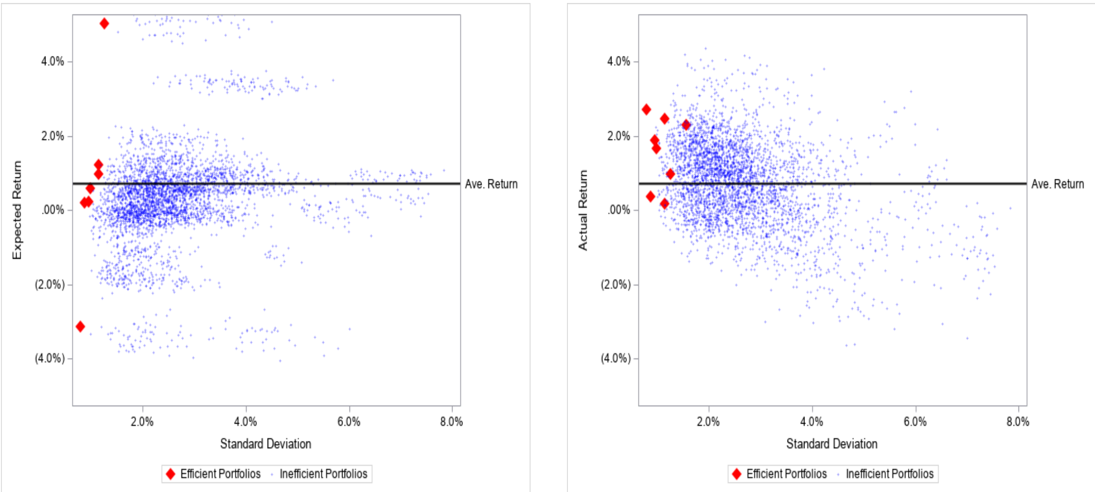

Figure 25: Theoretical and Realised Efficient Frontier (FF5)

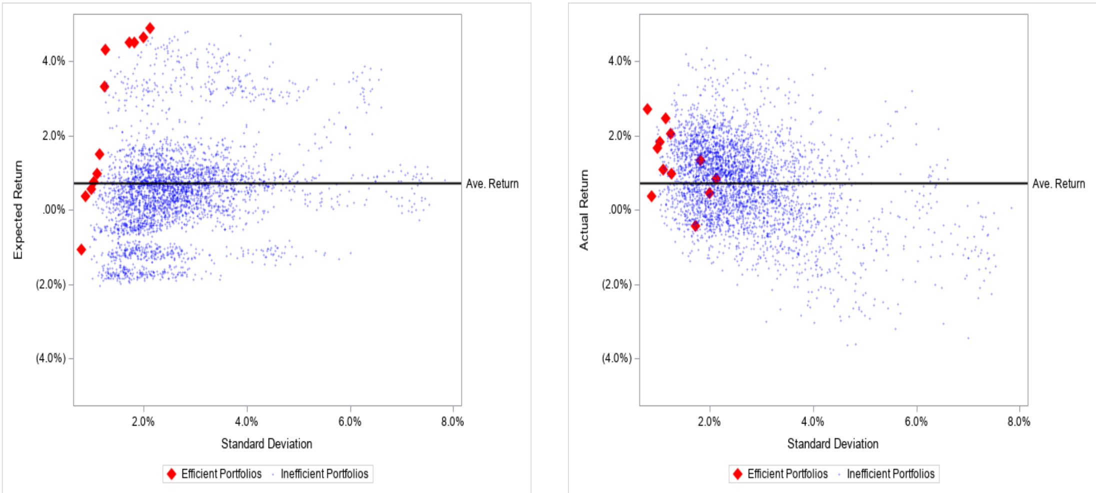

Figure 26: Theoretical and Realised Efficient Frontier (CH4)

5.4 Research Objective #4: Asset Pricing Models as Portfolio Selection Tools

The fourth research objective was to understand the role that APMs could play in the selection of companies for either long or short portfolios. Principally, the "portfolio selection criteria" was based upon the model's expectation of future returns (as calculated in Section 5.2.2). The figures below show the results of applying the strategy discussed in Section 4.7.3, between January 2016 and December 2018. The red-dotted line shows the cumulative returns between the portfolio with the highest expected return (Bin 4) and the portfolio with the lowest expected return (Bin 0). Ideally, the red-dotted line should have a high positive value that increases over time.

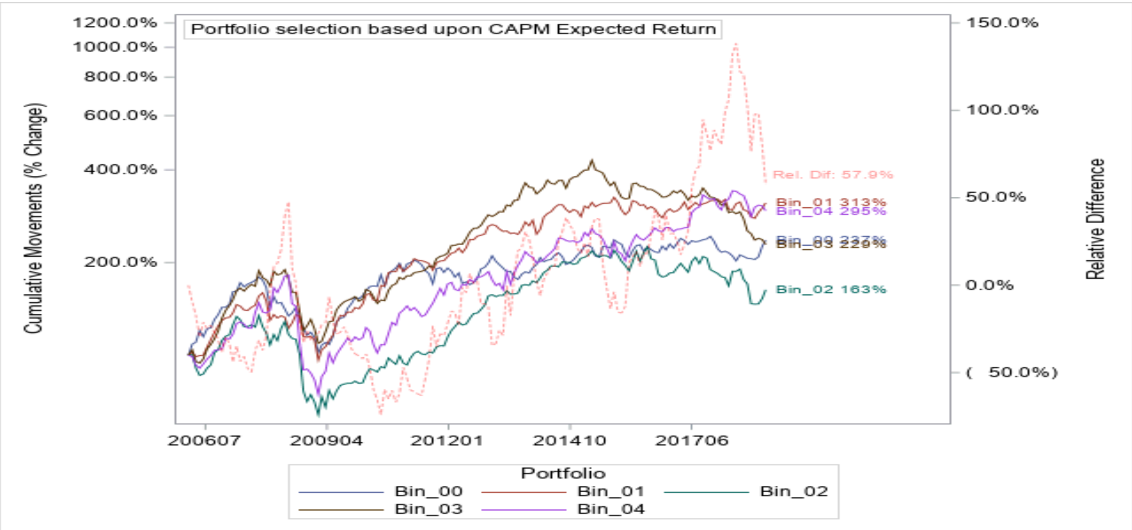

Figure 27: Portfolio Returns on CAPM Expected Returns

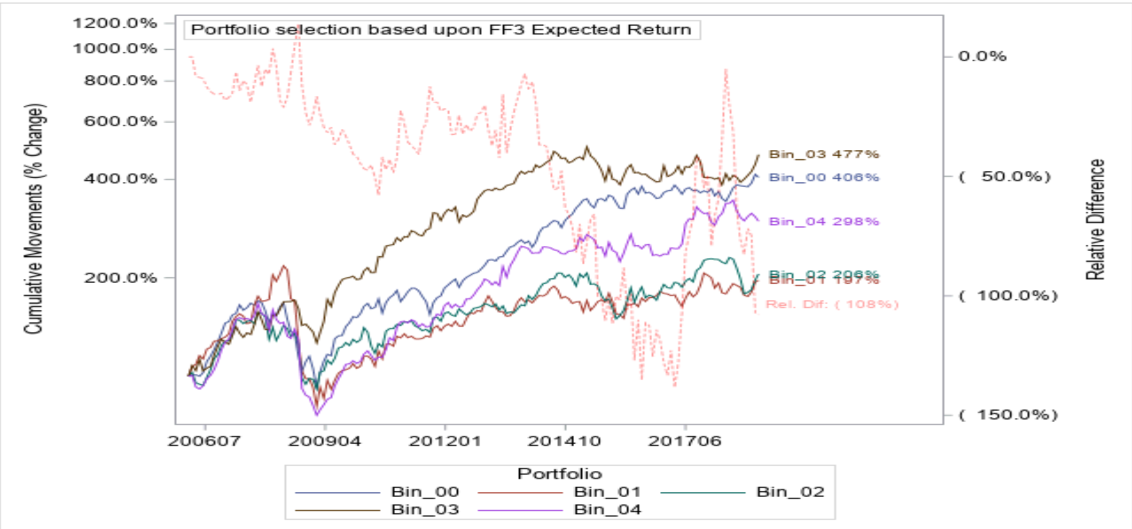

Figure 28: Portfolio Returns on FF3 Expected Returns

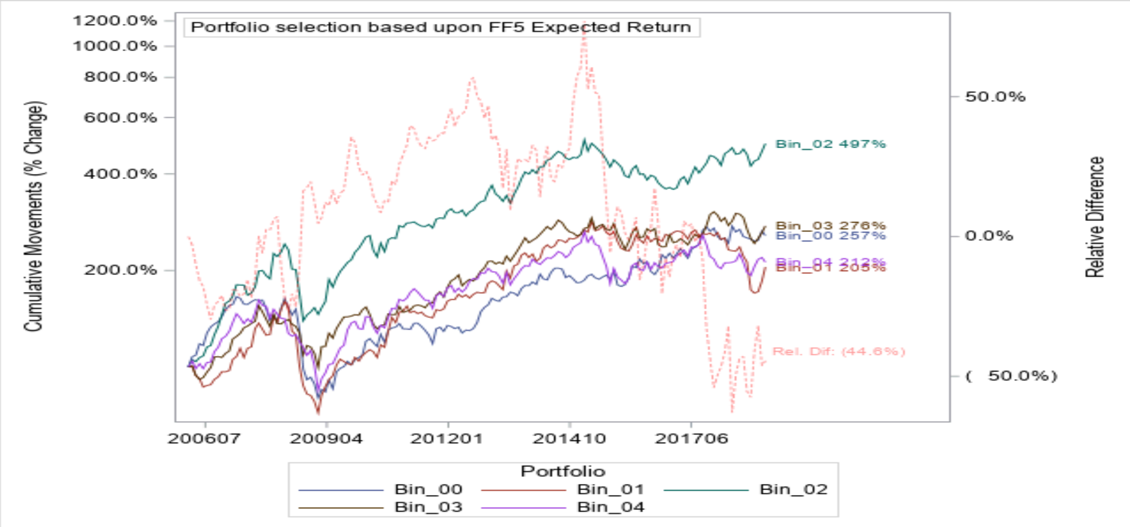

Figure 29: Portfolio Returns on FF5 Expected Returns

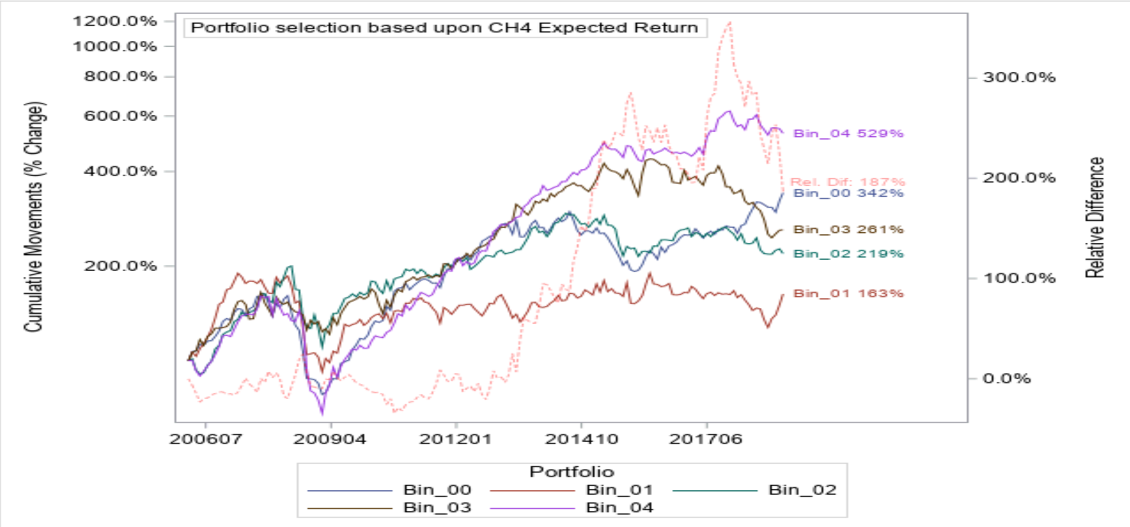

Figure 30: Portfolio Returns on CH4 Expected Returns

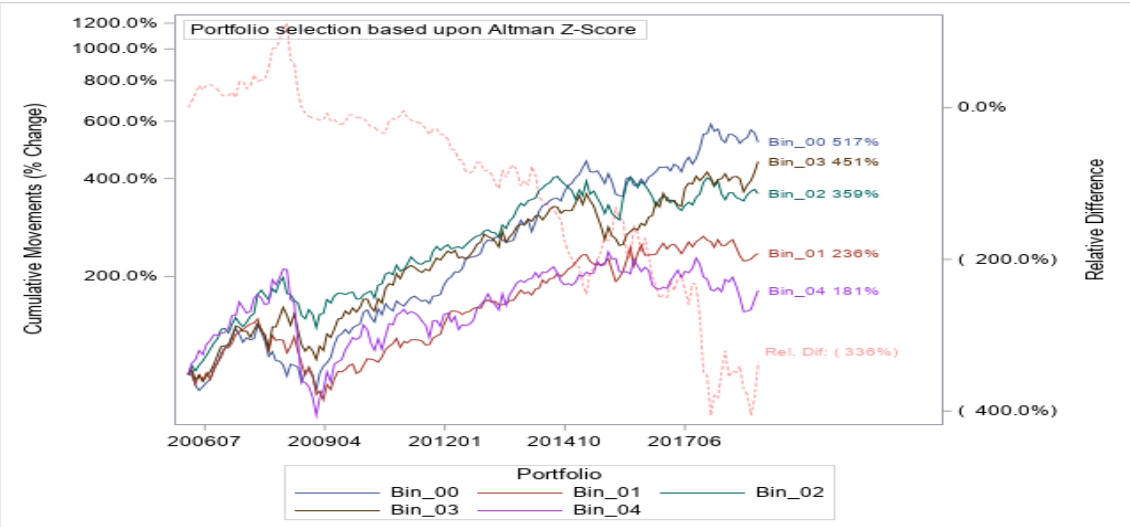

Figure 31: Portfolio Returns on Altman's Z-Score

5.5 Research Objective #5: APMs vs Style Investing for Portfolio Selection

The approach outlined in Section 4.7.4 involved the creation of long and short portfolios, under an APM- and Style Investing approach to company selection. The factors of each of the APM models were used to drive both the regression model and the style-base portfolio selection. The figures below show the long-term cumulative returns that each strategy achieved:

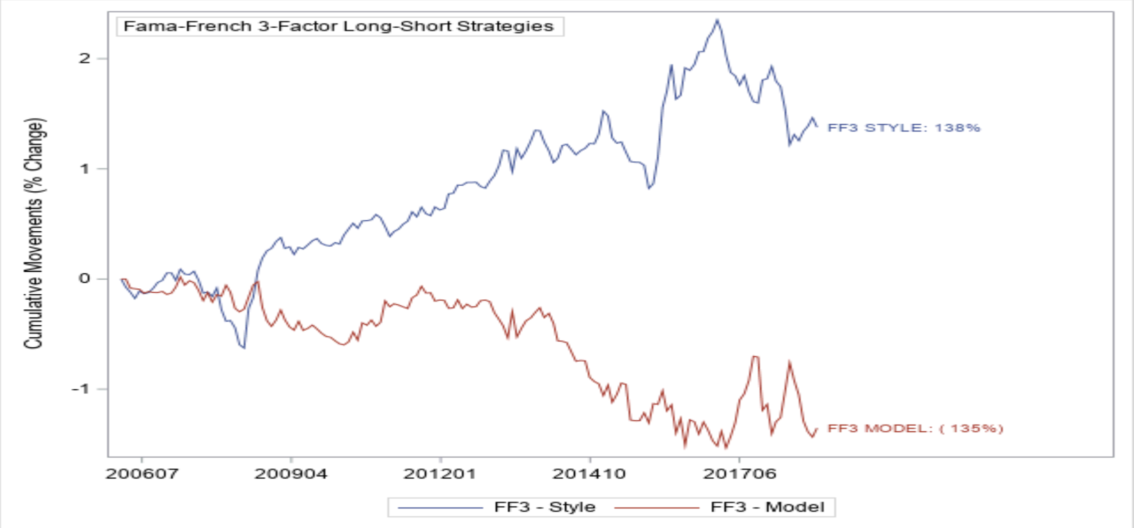

Figure 32: APM vs Style Investing (FF3)

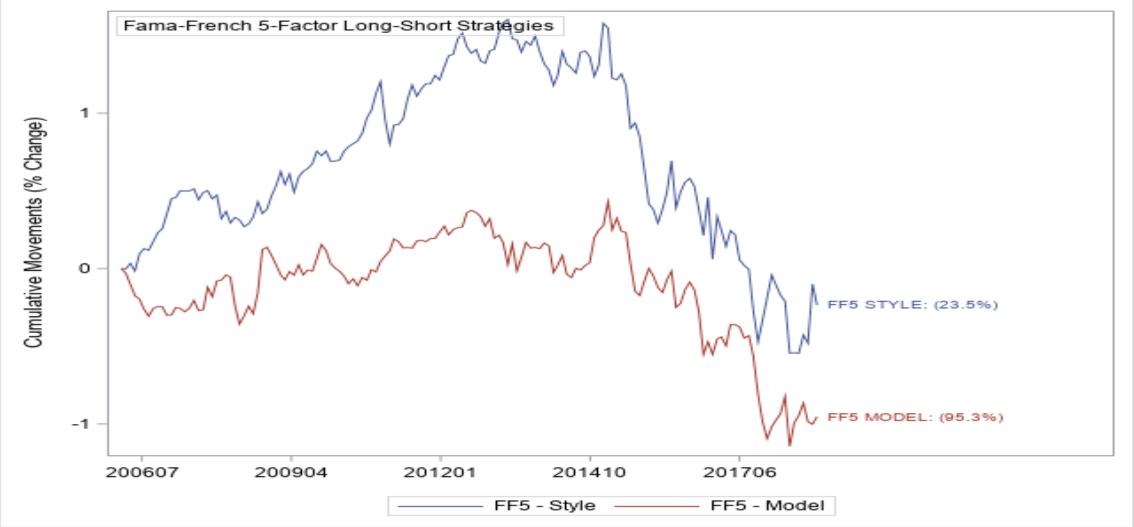

Figure 33: APM vs Style Investing (FF5)

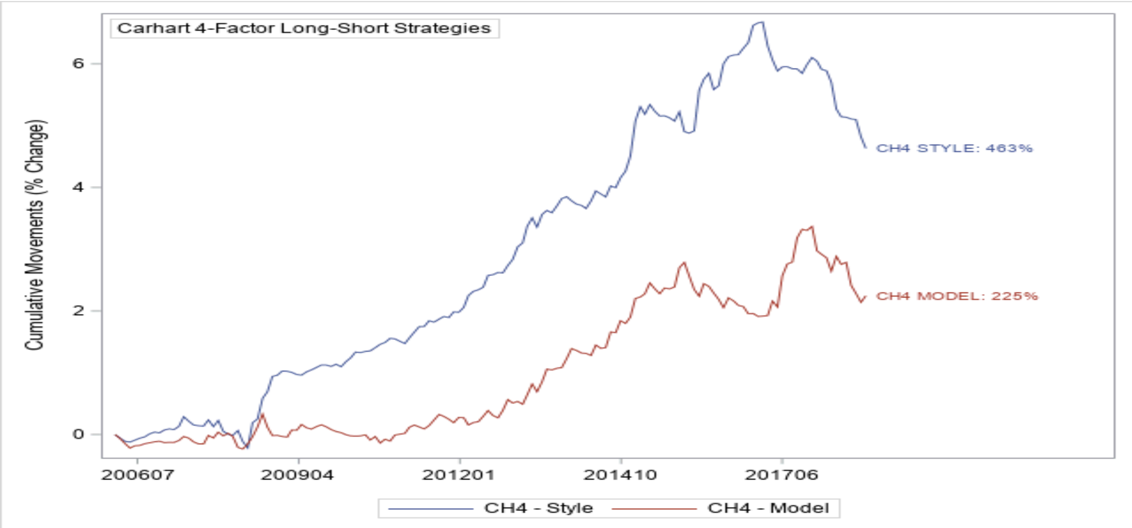

Figure 34: APM vs Style Investing (CH4)

Chapter 6: Discussion of Results

"In theory, there is no difference between theory and practice. In practice there is"

- Yogi Berra

6.1 Asset Pricing Model Factors

6.1.1 Small-minus-Big

The SMB factor is used in both Fama and French's 3-factor and 5-factor models as well as Carhart's 4-factor model (Carhart, 1997; Fama and French, 1992, 2015). The literature on the model suggests that there is a "size effect" that needs to be considered when the variance of returns on the market is being decomposed. The literature dictates that generally, small companies earn more returns than their larger counterparts, which leads to SMB having a positive skewness.

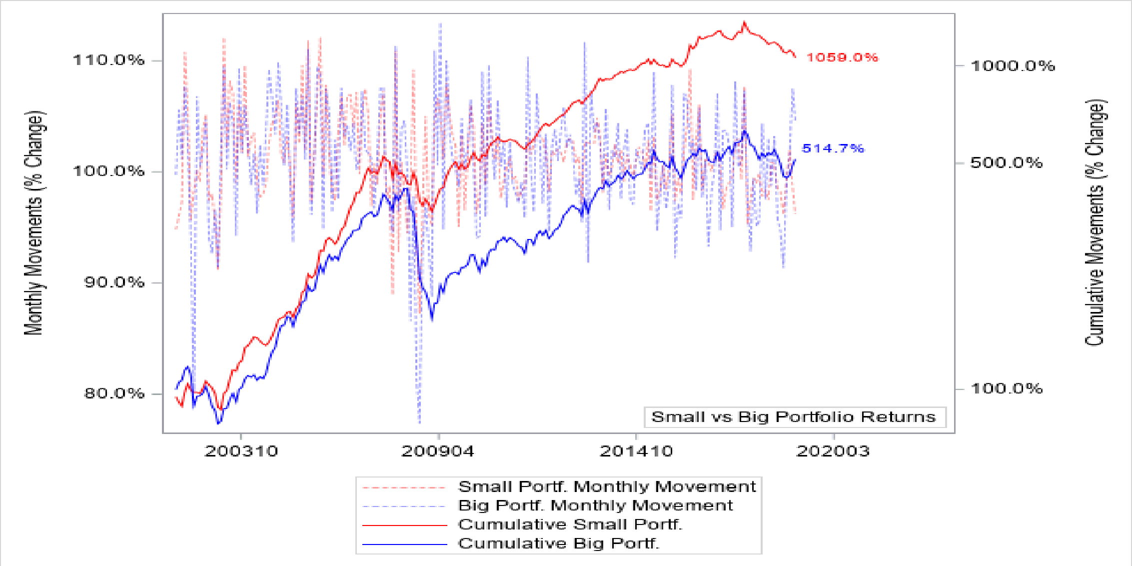

Figure 35: Cumulative Returns on SMB as an Investment Style

The results obtained during this study supports the findings of the pioneering researchers such as Fama and French (1992). In addition to that, it also supports the findings of South African researchers, such as Muller and Ward (2013), Basiewicz and Auret (2010) that have found the factor to be present on the JSE. It was however interesting to note that the strength of the factor diminished over time, in all models calculated, and in recent years turned negative. This might be attributed to a change in investor sentiment; As the perceived riskiness of the JSE increased post the 2008 world financial crises, investors might have sought the safety and security that larger firms offer. This would support Barberis, Shleifer and Wurgler's theory that co-movements are partly driven by large institutional investors that group companies by some characteristic and trade the group of shares (Barberis et al., 2005).

6.1.2 High-minus-Low

The HML factor is the second factor in Fama and French's 3-factor model and it too appears in their 5-factor model as well as Carhart's 4-factor model (Carhart, 1997; Fama and French, 1992, 2015). The factor assesses a company's balance sheet valuation of its equity against its market valuation and with this information, ultimately splits the market by "value stocks" and "growth stocks". The literature states that value stocks (High B/M) have historically outperformed growth stocks. The research conducted showed that this was also the case on the JSE. The factor is however significantly less informative/predictive of returns than its counterpart SMB (see Figure 35 and Figure 36).

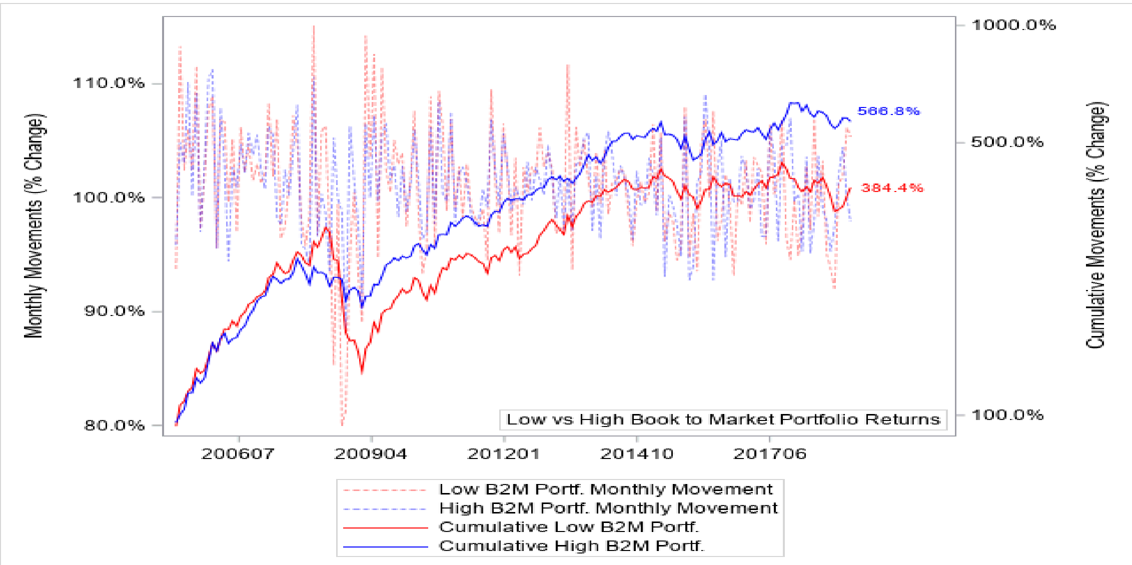

Figure 36: Cumulative Returns on HML as an Investment Style

6.1.3 Robust-minus-Weak

The profitability of a company is a natural place for any investor to start when assessing the future prospects of the company. Fama and French expanded on their 3-factor model by including the RMW factor in their 5-factor model (Fama and French, 1992, 2015). Locally, Cox and Britten (2019) found that on the JSE, a 3-factor model comprising of the SMB and RMW factors were among the strongest 3-factor model that they could produce.

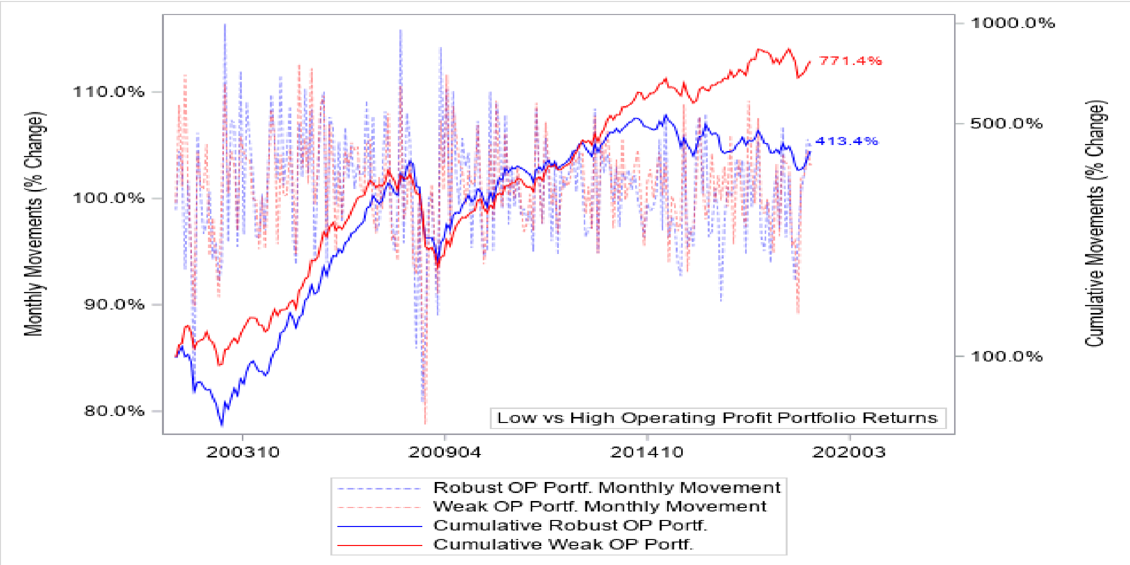

Figure 37: Cumulative Returns on RMW as an Investment Style

The results produced by this research is quite interesting when it comes to the RMW factor. Over the initial periods, there is some support for the findings in the earlier research literature which states that the robust profitability portfolios should outperform the weak profitability portfolios (Fama and French, 2015). However, since the world financial crises of 2008, the factor has not performed well and from 2015 onwards, companies with “weak” profitability actually outperformed their “robust” counterparts (see Figure 37). This phenomenon has could be correlated with the SMB deterioration discussed earlier and together, the deterioration of these factors could spell disaster for a regression model that bases its return expectations on historical performance. In the FF3 and FF5 however, the SMB and HML factors are controlled for which help the factor maintain its, weak, but positive cumulative return profile (see Figure 17).

6.1.4 Conservative-minus-Aggressive

The CMA factor in Fama and French's (2015) 5-factor model refers to the recent level of investment of a company. Fama and French found that a high level of investment was indicative of lower expected return. The results of this study are however contradictory to the views of Fama and French. Rather, it finds that the factor is at best, a weak indication of future performance on the JSE, which supports the findings of Cox and Britten (2019).

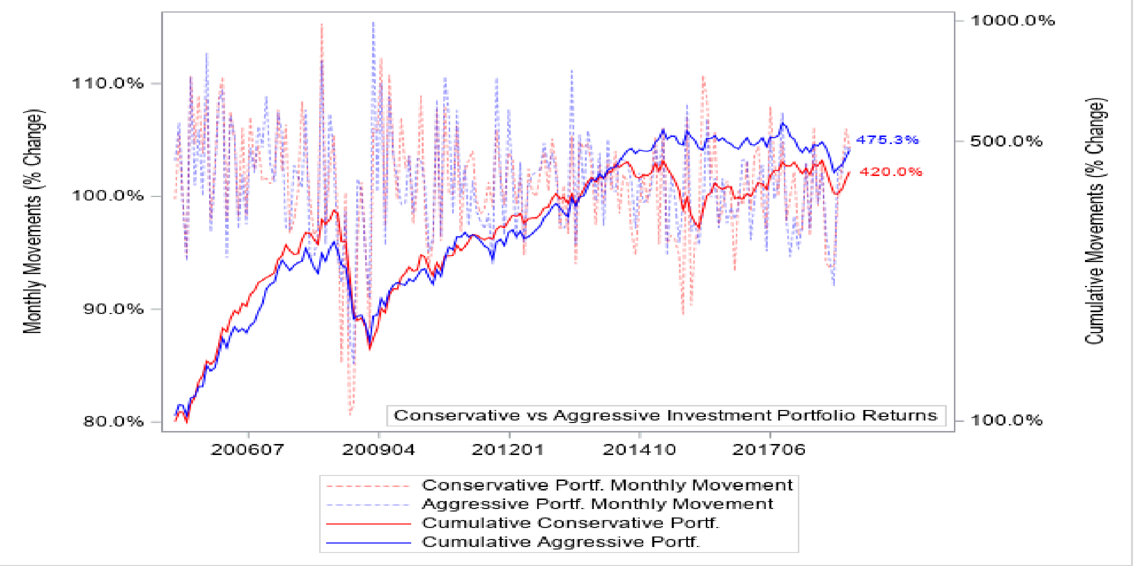

Figure 38: Cumulative Returns on CMA as an Investment Style

6.1.5 Winners-minus-Losers

Carhart (1997) introduced the concept of performance momentum into his 4-factor model with the WML factor. Fama and French (2012) also found that momentum was a strong indicator of future performance (see Table 17).

This study found that momentum was the strongest factor of all evaluated. It was especially powerful when controlling for the SMB and HML factors, as is done in Carhart's model.

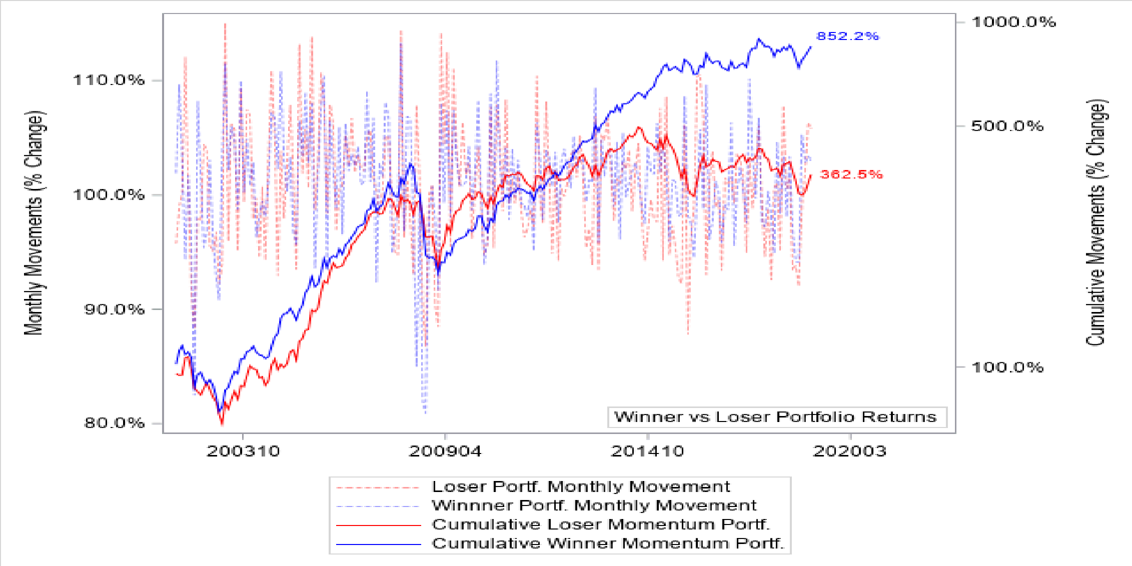

Figure 39: Cumulative Returns on WML as an Investment Style

6.1.6 Altman's Z-Score Factors

One of the research objectives of this study was to evaluate Altman's Z-Score's ability to rank companies in terms of their future performance and returns (Altman, 1968).

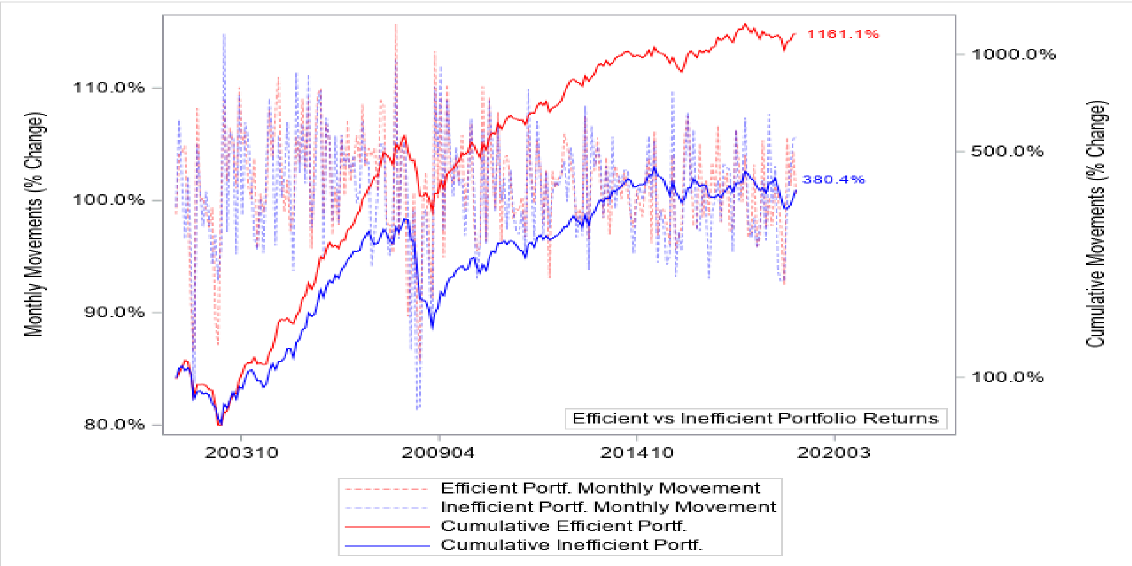

Figure 40: Cumulative Returns on EMI as an Investment Style

During this process, the author also evaluated the performance of the score's individual factors and the Sales to Total Assets (X5 variable) ratio proved to be very indicative of performance on the JSE. The ratio essentially measures a company's ability to utilise its assets and find ways to be efficient and effective. This leads to improvements to its sales revenue levels, without substantially growing its asset base which would incur costs.

As discussed in Section 2.4.5, Altman believed that the factor was also indicative of a company's ability to excel in a competitive environment. Given the lack of economic growth over the last 10 years in South Africa, any company looking to grow had to take market share from its competitors. It should thus be expected that Altman's X5 variable could act as an investment style on the JSE.

The author named this factor *"Efficient-minus-Inefficient" (EMI)* and utilised it in a suggested investment approach that is discussed in Section 6.3 later in this chapter.

6.2 Asset Pricing Model Performance

6.2.1 Research Objective #1: Historic Returns

The first object of the research was to evaluate the ability of the different APMs to explain the observed variance of returns on the Johannesburg Stock Exchange. Additionally, the research sought to understand which model provides the most explanatory power and is ultimately the strongest of the models tested.

Figure 20 indicates the performance of the models though the F-values generated during the linear regressions ran. Somewhat unsurprisingly, the F-values increase as the number of factors included in the models increase, from the CAPM's single factor through to Fama and French's 5-factor model (FF5) (Fama and French, 2015). These results thus correlate with those of Cox and Britten (2019). More interestingly, Carhart's 4 F-factor model (CH4) performs similarly to FF5 whilst it does not contain the information on levels of investment or profitability. Conversely, FF5 still outperforms CH4, without the explicit momentum information.

The analysis of variance (ANOVA) performed confirms that there is a statistically significant difference between the average root-mean-squared-errors that the models generated. Additionally, the Tukey analysis points out that there are differences between the models, with only CH4 not being statistically significantly different form either FF3 or FF5. Therefore, statistically speaking, there is little difference between the ability FF3 and CH4, and CH4 and FF5 to reduce the error term.

It is interesting to note that the F-values for all models peaked towards the end of 2012 and deteriorated from there. This can be attributed to the deterioration of the information provided by the underlying factors, discussed in Section 6.1.

Lastly, we can conclude that all the APMs evaluated have some ability to explain the variance in returns on the JSE. The FF5 model is the most predictive among the models evaluated.

Table 11: Research Objective #1 - Conclusions

Research Objective #1 Understand the ability of Asset Pricing Models to explain **Historic Returns** on the JSE			
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	*Conclusion*
1.1	How well does CAPM explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by CAPM ($H_0: \beta_1 = 0$)	The null hypothesis is rejected, and we conclude that CAPM can explain historic returns on the JSE.
1.2	How well does FF3 explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by FF3 ($H_0: B_1 = B_2 = \beta_3 = 0$)	The null hypothesis is rejected, and we conclude that FF3 can explain historic returns on the JSE.
1.3	How well does FF5 explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by FF5 ($H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$)	The null hypothesis is rejected, and we conclude that FF5 can explain historic returns on the JSE.
1.4	How well does CH4 explain historic returns on the JSE?	There is no relationship between the actual historic returns and expected returns generated by CH4 ($H_0: \beta_1 = \beta_2 = \beta_3 = \beta_6 = 0$)	The null hypothesis is rejected, and we conclude that CH4 can explain historic returns on the JSE.
1.5	Which model explains the most variance in historic returns?	All models are equally good ($H_0: RMSE_{CAPM} = RMSE_{FF3} = RMSE_{FF5} = RMSE_{CH4}$)	The null hypothesis is rejected, and we conclude there is a significant difference in the explanatory power of the models. The Tukey analysis indicates that only CH4 not being statistically significantly different form either FF3 or FF5. The analysis indicates that FF5 explains most of the variance in historic returns.

6.2.2 Research Objectives #2 & #3: Future Returns & Efficient Frontier

Figure 21 shows the F-values for the regression performed on future returns. When this is contrasted with the F-values achieved on historic returns (Figure 20) it becomes clear that APMs perform substantially worse when being applied to out-of-sample problems. The ANOVA performed indicates that there is no statistical difference between the models when it comes to their ability to predict future returns. For this reason, no Tukey analysis was performed. These results were both surprising and disappointing to the author. It was expected that the underlying relationships between the dependent and independent variables would hold over time and this would enable robust and reliable predictions.

Table 12: Future Return Regressions - p-Value Information

	Level	CAPM	FF3	FF5	CH4
Avg. p-Value		0.228	0.273	0.238	0.175
% of Statistically Significant p-Values	alpha: 5%	50.6%	41.0%	42.3%	61.5%
% of Statistically Significant p-Values	alpha: 10%	55.8%	45.5%	51.3%	66.0%

Despite the less than encouraging results obtained from the regressions, the author believed that Carhart's 4-factor model could provide some level of confidence after further analysis of the p-values obtained (see Table 12).

To confirm the findings shown in Table 12, additional regression analysis was performed wherein cohorts across the timespan of the data were combined and then split into build- and validation populations. The APM models were then calibrated and evaluated on these distinct populations. The findings of this exercise confirmed that Carhart's 4-factor model performed best at predicting future returns.

Table 13: Research Objective #2 - Conclusions

Research Objective #2 Understand the ability of Asset Pricing Models to explain **Future Returns** on the JSE			
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	*Conclusion*
2.1	How well does CAPM explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by CAPM (H0: β1 = 0)	The null hypothesis is rejected, and we conclude that CAPM can explain future returns on the JSE.
2.2	How well does FF3 explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by FF3 (H0: β1 = β2 = β3 = 0)	The null hypothesis is rejected, and we conclude that FF3 can explain future returns on the JSE.

2.3	How well does FF5 explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by FF5 (H0: β1 = β2 = β3 = β4 = β5 = 0)	The null hypothesis is rejected, and we conclude that FF5 can explain future returns on the JSE.
2.4	How well does CH4 explain future returns on the JSE?	There is no relationship between the actual future returns and expected returns generated by CH4 (H0: β1 = β2 = β3 = β6 = 0)	The null hypothesis is rejected, and we conclude that CH4 can explain future returns on the JSE.
2.5	Which model explains the most variance in historic returns?	All models are equally good (H_0: $RMSE_{CAPM} = RMSE_{FF3} = RMSE_{FF5} = RMSE_{CH4}$)	The null hypothesis is rejected, and we conclude there is a significant difference in the explanatory power of the models. The FF3 and FF5 models performed the worst, followed by CAPM, with CH4 performing the best.

The third objective of the research was to link the APM expectations to the Efficient Frontier, with the aim of determining whether the theoretical efficient frontier would at all be a feasible solution in a practical environment.

Given the less than ideal results obtained from the future return prediction models fitted, it was not surprising to find that the theoretical efficient frontier does not hold perfectly in practise – at least not given the results obtained during this study. On a positive note, however, it should be noted that for the most part, the APMs could identify portfolios with above average returns.

The failure of the APMs to predict future returns to an adequate level to maintain the efficient frontier can be due to a number of reasons. Firstly, the JSE is a small stock exchange which is exposed to numerous external forces that affect the returns generated. These factors, such as the global financial sector climate, local politics, and currency volatility will wreak havoc on any model unless the model is able to consider and forecast these factors. Secondly, given the small number of companies on the JSE, an investor that wishes to outperform the market needs to select relatively small portfolios to do so. This again leads to volatility in the results obtained. Lastly, the number of factors that are considered in the models might be sufficient to explain historic price volatility, but are, arguably, insufficient to capture the complex relationship that exists between the current status quo and the future returns that will materialise. Since the Modern Portfolio Theory and the Efficient Frontier place heavy reliance on these models, the results of this research would suggest that the theories cannot be reliably applied to the JSE.

Table 14: Research Objective #3 - Conclusions

<table>
<tr><th colspan="4">Research Objective #3
Understand the link between Asset Pricing Models, Portfolio Optimisation and Realised Returns</th></tr>
<tr><th>#</th><th>Research Question Specifics</th><th>Null Hypothesis (H₀)</th><th>Conclusion</th></tr>
<tr><td>3.1</td><td>Does the Efficient Frontier materialise when making use of CAPM as the expected return component of MPT?</td><td>CAPM expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns</td><td rowspan="4">None of the APMs were able to maintain the Efficient Frontier. Furthermore, the models did not maintain the positive relationship between risk and return.

The null hypothesis could therefore not be rejected.</td></tr>
<tr><td>3.2</td><td>Does the Efficient Frontier materialise when making use of FF3 as the expected return component of MPT?</td><td>FF3 expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns</td></tr>
<tr><td>3.3</td><td>Does the Efficient Frontier materialise when making use of FF5 as the expected return component of MPT?</td><td>FF5 expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns</td></tr>
<tr><td>3.4</td><td>Does the Efficient Frontier materialise when making use of CH4 as the expected return component of MPT?</td><td>CH4 expected returns are insufficiently reliable to accurately identify portfolios that will ultimately constitute the EF, whilst maintaining the positive relationship between historic volatility risk and realised returns</td></tr>
</table>

6.2.3 Research Objective #4: Asset Pricing Models as Portfolio Selection Tools

As noted in the previous section, the APMs were able to select above average return producing portfolios, which leads to the next research objective that is concerned with the ability of APMs, and Altman's Z-Score to select portfolios based upon their expected returns.

Both the FF3 and FF5 models performed poorly when used to select portfolios. This was mainly driven by the historically observed premiums the factors that drive the model, not being reflective of future performance. Over the period observed, the CH4 model, on the other hand, performed quite well. It achieved a 244% difference in the cumulative returns, between the portfolio with the highest expected returns and the portfolio with the lowest expected returns.

Table 15: Research Objective #4 - Conclusions

Research Objective #4 Understand the behaviour of Asset Pricing Models in a Portfolio Selection Setting			
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	*Conclusion*
4.1	Can CAPM return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of CAPM, are not ordered according to model expectations	The null hypothesis cannot be rejected. Although the long portfolio outperformed the short portfolio, it did not do so convincingly and additionally, the model was not able to rank portfolio performance.
4.2	Can FF3 return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of FF3, are not ordered according to model expectations	The null hypothesis cannot be rejected. Although the long portfolio outperformed the short portfolio, it did not do so convincingly and additionally, the model was not able to rank portfolio performance.
4.3	Can FF5 return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of FF5, are not ordered according to model expectations	The null hypothesis cannot be rejected. Although the long portfolio outperformed the short portfolio, it did not do so convincingly and additionally, the model was not able to rank portfolio performance.
4.4	Can CH4 return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of CH4, are not ordered according to model expectations	**The null hypothesis was rejected**. The long portfolio outperformed the short portfolio convincingly and with the exception of the short portfolio, the model was able to maintain portfolio performance ranking.
4.5	Can Altman's Z-Score return expectations be used to create Style portfolios that generate long-term positive (or negative) returns?	The long-term returns of the different portfolios selected, by making use of Altman's Z-Score expectations, are not ordered according to Z-Score ranking	The null hypothesis cannot be rejected. Although the long portfolio outperformed the short portfolio, it did not do so convincingly and additionally, the model was not able to rank portfolio performance.

Altman's Z-Score is quite interesting (Altman, 1968); At first glance, the result would suggest that the Z-score is terrible at predicting winner and loser portfolios, as the results seem to be the opposite of what is expected. However, if one considers that a high Z-score implies that a company is deemed safe from financial distress, the further implications become quite important: Altman's Z-Score calculation was discussed in Section 2.4 but is repeated here for convenience:

$$Z - Score = 1.2x_1 + 1.4x_2 + 3.3x_3 + 0.6x_4 + 0.99x_5 \qquad (24)$$

where $x_1 = Working\ Capital/Total\ Assets$

$x_2 = Retained\ Earnings/Total\ Assets$

$x_3 = EBIT/Total\ Assets$

$x_4 = Working\ Capital/Total\ Liabilities$

$x_5 = Sales/Total\ Assets$

As Equation (24) would suggest, higher levels of working capital (x_1 and x_4) drive the score higher. However, from a future returns point of view, an investor might rather view this as an inefficiency in the company. Similarly, higher levels of retained earnings (x_2) would imply that the return on equity is less than optimal. On the other hand, higher levels of sales (x_5) and profitability (x_3) would be attractive to an investor. Therefore, from the point of view of an investor that is interested in future returns, the Z-score has factors that work against each other.

All of the factors that make up the Z-score where individually assessed for their ability to act as portfolio selection factors or "styles" (see Appendix 4 – Altman's Z-Score Factor Style Returns). Among the factors, the sales to total assets ratio (x_5) was identified as being the strongest indicator of future performance. For this reason, the author created an APM factor called Efficient-minus-Inefficient (EMI), referring to the efficient use of assets to generate revenues. Figure 41 illustrates the cumulative return generated by splitting the available companies into efficient and inefficient portfolios monthly.

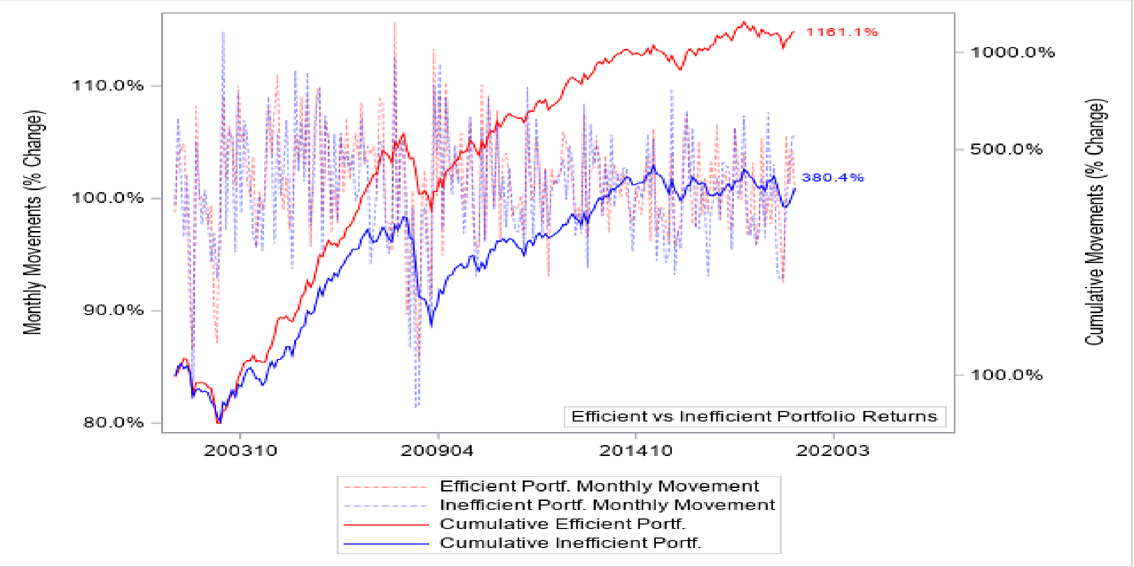

Figure 41: Efficient-minus-Inefficient

As was done in Section 6.1, the 12-month moving average as well as the cumulative return premium earned by the efficient portfolio were calculated and are illustrated in Figure 42 and Figure 43 respectively.

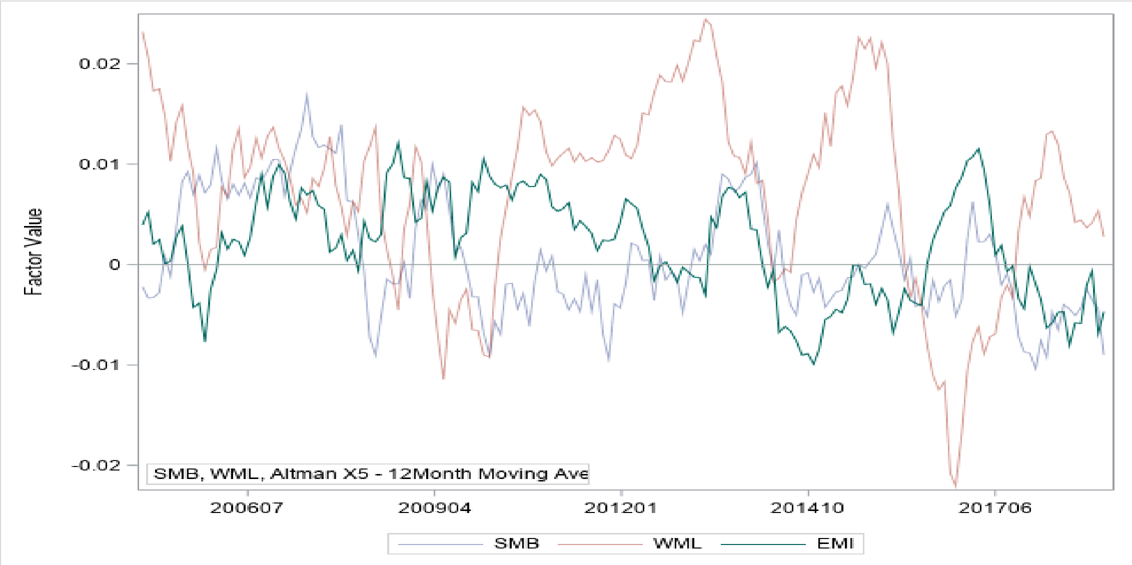

Figure 42: Calculated Factor Premium – EMI

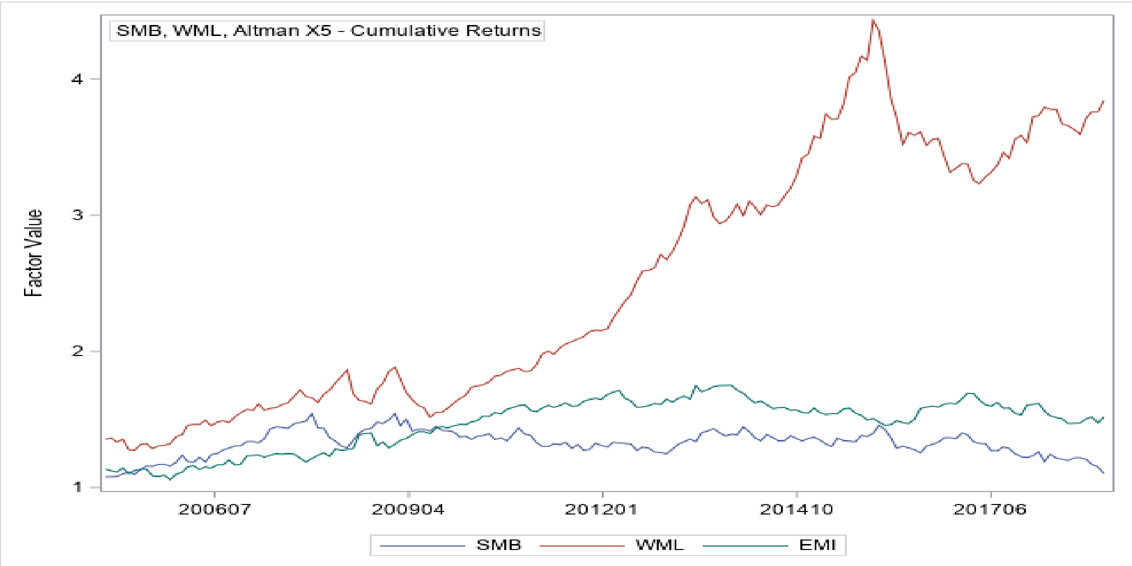

Figure 43: Cumulative Factor Returns (EMI)

Although the EMI factor is not as informative as the WML factor, it does seem to carry sufficient information to outperform the SMB factor over the period of evaluation.

6.3 Research Objective #5: APMs vs Style Investing for Portfolio Selection

The final research objective was aimed at understanding whether an APM-based or a style-based approach to portfolio selection would yield the best long-term results under conditions where long and short portfolios would need to be selected.

Given the disappointing results obtained when evaluating the Fama-French 3- and 5-factor models' ability to predict future returns, it is not surprising that the style-based approach to portfolio selection significantly outperformed the APM-based approach for these models (Fama and French, 1992, 2015). However, the Carhart's 4-factor model (Carhart, 1997), performed relatively well at predicting future returns and it was therefore expected that it would perform well at portfolio selection. Again, it was surprising to find that despite the 225% differential in returns between the APM's long- and short portfolios, it was still convincingly outperformed by the style-based approach, which achieved a 463% differential achieved after slipping from its high of more than 600%.

Table 16: Research Objective #5 - Conclusions

Research Objective #5			
Understand the Performance of APM Portfolio selection vs Style Investing			
#	*Research Question Specifics*	*Null Hypothesis (H_0)*	*Conclusion*
5.1	Making use of the FF3 factors only, does Style Investing or APM portfolio select perform best?	The cumulative difference in performance between the long and short portfolios are equal, regardless of the approach followed when employing FF3 factors	The null hypothesis is rejected. The Style-based investment approach significantly outperformed the APM portfolios selection methodology when making use of the FF3 factors.
5.2	Making use of the FF5 factors only, does Style Investing or APM portfolio select perform best?	The cumulative difference in performance between the long and short portfolios are equal, regardless of the approach followed when employing FF5 factors	The null hypothesis is rejected. The Style-based investment approach significantly outperformed the APM portfolios selection methodology when making use of the FF5 factors.
5.3	Making use of the CH4 factors only, does Style Investing or APM portfolio select perform best?	The cumulative difference in performance between the long and short portfolios are equal, regardless of the approach followed when employing CH4 factors	The null hypothesis is rejected. The Style-based investment approach significantly outperformed the APM portfolios selection methodology when making use of the CH factors.

It should be noted that the significant losses suffered on the style-based strategy, since the high of 600% was achieved, is concerning; If this strategy was employed by an intuitional investor, losses such these would have spelt disaster as explaining a 25% loss in value to the client would be problematic at best.

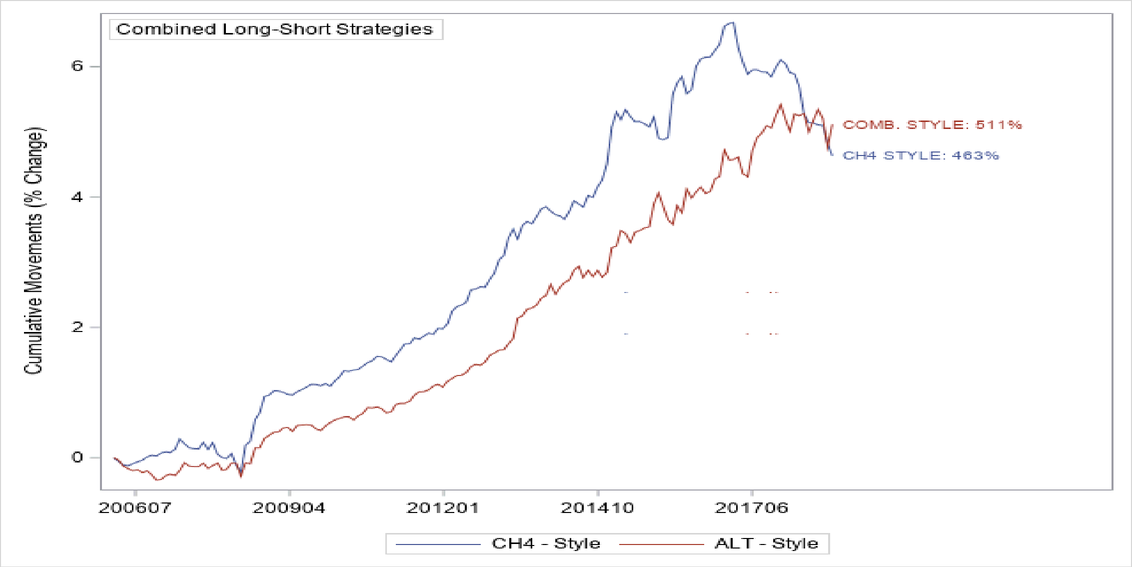

Figure 44: CH4 vs Altman X5 Style Strategies

Noting this, the author set out to combine the style factors that performed best over time. This included SMB, WML, and EMI. Figure 44 illustrates the cumulative returns that were achieved by following the combined strategy.

For most of the period under assessment, the CH4 strategy outperformed the combined strategy. However, the combined strategy was relatively unaffected by the crash that the CH4 strategy experienced. Additionally, the growth trajectory of the combined strategy was more consistent over time, albeit at a subdued level.

Looking back, one might be forgiven for thinking that the combined strategy is the best choice, given the recent losses incurred on the CH4 strategy. However, doing so would be with the benefit of hindsight and up until the losses started occurring, the CH4 would have been the best strategy by some margin. That said, the author is of the view that by basing the decision of long and short portfolios on multiple strategies, potential returns and -risks are inherently being exchanged. This could be seen in the relative stability of the cumulative returns observed on the combined strategy.

Chapter 7: Conclusions

"Statisticians, like artists, have the bad habit of falling in love with their models"

- George Edward Pelham Box

7.1 Principal Findings

The research aimed to evaluate a multitude of investment models and techniques on the Johannesburg Stock Exchange (JSE). These included the seminal capital asset pricing model (CAPM) which stemmed from the work of the pioneers in the field such, Treynor (1962), Lintner (1965a, 1965b), Merton (1972) and Sharpe (1964). And extended further to include the multifactor models introduced by modern day experts in the field like Fama and French (1992, 1993, 2012, 2015) and Carhart (1997). The research found that these models were able to explain significant amounts of historic price variations on the JSE, which is in supports prior research in the field (Basiewicz and Auret, 2010; Cox and Britten, 2019).

Next, the research focused on the ability of these asset pricing models (APMs) to predict future returns, as reliable expected returns are required in portfolio optimisation and the functioning of the efficient frontier (Markowitz, 1952). The research found that Fama and French's 3- and 5-factor models did not perform well when tasked with predicting future returns (Fama and French, 1992, 2015). The research mainly attributed the poor performance to the deterioration in the predictive power of the underlying model factors (SMB, HML, RMW, CMA). Carhart's 4-factor model, on the other hand, performed relatively well at predicting future returns primarily attributable to its incorporation of the momentum (WML) factor, which proved to have superior predictive power (Carhart, 1997). However, Carhart's model, like all the other APMs evaluated, also failed to select portfolios that maintained the efficient frontier into the future.

The research found that the APMs evaluated did not provide sufficiently accurate predictions of future returns to allow for the practical implementation of portfolio optimisation and/or the efficient frontier. It was however noted that the relatively small size of the JSE and South Africa's foreign exposure and political uncertainty makes accurate predictions of return on the JSE, at best, problematic. Despite the disappointing findings produced, when evaluating APMs on the realised efficient frontier, it was noted that the APMs, generally, selected portfolios with above average returns. This implied that APMs could be used to select portfolios that would outperform the market on a consistent basis. The research then sought to understand if the APMs could be used as portfolio selection tools.

In line with the findings around the APMs ability to predict future returns, Fama and French's 3- and 5 factor models performed poorly, whilst Carhart's 4 factor model performed relatively well when it came to selecting of portfolios (Carhart, 1997; Fama and French, 1992, 2015).

As the process of asset pricing model building and calibration is quite cumbersome, whilst Style Investment is a relatively simple process, the research sought to understand the improved prediction benefit, if any, derived from making use of APMs over a Style Investing approach. Following a graphical time series approach that mimicked that employed by Ward and Muller (2013), the research compared the long term effects of selecting both long- and short portfolios under the two competing portfolio selection frameworks. It was found that Style Investing outperformed the APM approach on a consistent basis.

Lastly, the research sought to understand Altman's Z-Score and its factors' behaviour in an investing setting (Altman, 1968). The Z-Score was developed to estimate the likelihood of company failure and not the future returns that an investor would be interested in. However, the factors it utilises are similar to those employed by Fama and French in their factor models (Fama and French, 1992, 2015) and since "distress" can be seen as being on the opposite end of the performance scale to high levels of returns, it made sense that it could be used to rank companies in terms of their future returns potential. The research, however, found that the score did not perform well, as the factors it used worked against each other when it came to prediction of high returns and that the score would need to be calibrated to perform optimally in the investment setting. Furthermore, the research found that Sales to Total Assets ratio employed by Altman's Z-Score performed very well as an investment style, outperforming the Size (SMB) factor employed by Fama and French.

7.2 Implications for Management

The research performed provides a cautionary note to individual and corporate investors alike. The asset pricing models assessed during the research performed well at understanding historic risks, but their reliability in terms of return forecasting is, at best, questionable. When the inaccuracy leads to unreliable portfolio estimates, the concept of modern portfolio theory becomes meaningless and provides investors with a false sense of comfort that optimal returns will be achieved. That said, the APMs have a powerful ability to identify drivers of returns. These drivers can then be used to follow a Style Investing approach.

Tracking the return premiums of each individual factor illustrated that the explanatory power of factors changes over time. It is thus imperative that fund managers asses and track factor performance over time and adjust their portfolio selection criteria as and when needed.

7.3 Limitations of the Research

As with any quantitative or qualitative research endeavour, this project was subject to factors that limited the quality of the research outcomes. Below is an outline of the limiting factors encountered during the research project:

7.3.1 Endogeneity

As noted by Basiewicz and Auret (2010), the problem of endogeneity is a criticism that is often raised against asset pricing models. Endogeneity is a problem that arises when both the dependent and independent variables are correlated by another factor which is not present in the model. This could lead to incorrect assumptions being drawn from the statistical procedures employed during the research project.

7.3.2 Sample size and Data timeframe

The research project made use of data from the Johannesburg Stock Exchange (JSE) between 2000 and 2019, roughly 170 months' worth of data. Considering that Fama and French (2015) made use of around 600 months' worth of data in their research, the sample size used in this research project may be considered small and therefore less significant conclusions can be drawn from it. Additionally, the number of companies at any point in time in this research peaked at around 160, whilst Fama and French conducted their research on the New York Stock Exchange which hosts more than 2,400 companies (New York Stock Exchange, 2019). Therefore, comparatively speaking, the JSE is tiny and thus, the credibility of any results obtained from research conducted inherently suffers.

7.3.3 Formulation of Research Objectives

The author struggled to narrow down the research objectives and as a result, the research project has broad objectives. This takes away from the depth of analysis that could be reached. That said, the research's broad approach followed enabled it to find the lack of linkage between future expected performance and portfolio construction under the modern portfolio paradigm.

7.3.4 Depth of Discussion

The author acknowledges his lack of experience in the field of research and the negative effects that this has had on the quality and depth of discussion on important topics covered during the research project.

7.4 Further Work

The research conducted left numerous unanswered questions which leaves the opportunity for further research in the field. Some of these are discussed below:

7.4.1 A new multi-factor model

As discussed earlier, Altman's Z-Score made use of the Sales to Asset ratio, which this research project found to be highly indicative of future returns achieved. This research project, however, stopped short of building a factor model similar to those of Fama and French (1992, 2015) and Carhart (1997). The research found that the combination of the SMB, WML, and EMI (Sales to Assets) factors was able to form a strong style-based investment strategy. Therefore, the author expects that these factors would also form a powerful multi-factor capital asset pricing model.

7.4.2 Improving the split point

The research required the creation of sub-populations (see Figure 7, Figure 8 and Figure 9) which ultimately were used to calibrate the APM factors and drive the style-investment portfolios. With the intention of keeping sub-portfolio sizes equal, the author opted to make each split equal to the median value of the variable under consideration. Intuitively, this is not the optimal point to split a continuous variable. Further work into where the optimal splitting point is would add to the field of research and provide guidance on how to improve the cumulative returns generated by an investment strategy.

7.4.3 Non-monotonic Effects

When evaluating Altman's Z-Score and its factors (Altman, 1968), the author noted that some of the factors did not exhibit a monotonic relationship with the dependent variable (excess returns). For instance, very high B/M values could indicate that, as the body of knowledge suggests, a company is undervalued and that a high probability of excess returns exists. However, this does not imply that higher B/M values will always lead to higher returns. Extremely high values might indicate that the company suffers from other exogenous factors, that the market views as threatening to the continued existence of the company. It, therefore, follows that the true relationship between factors used in APMs and/or style investing might need to be investigated further.

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Appendix 1 – Additional Information – Other Research

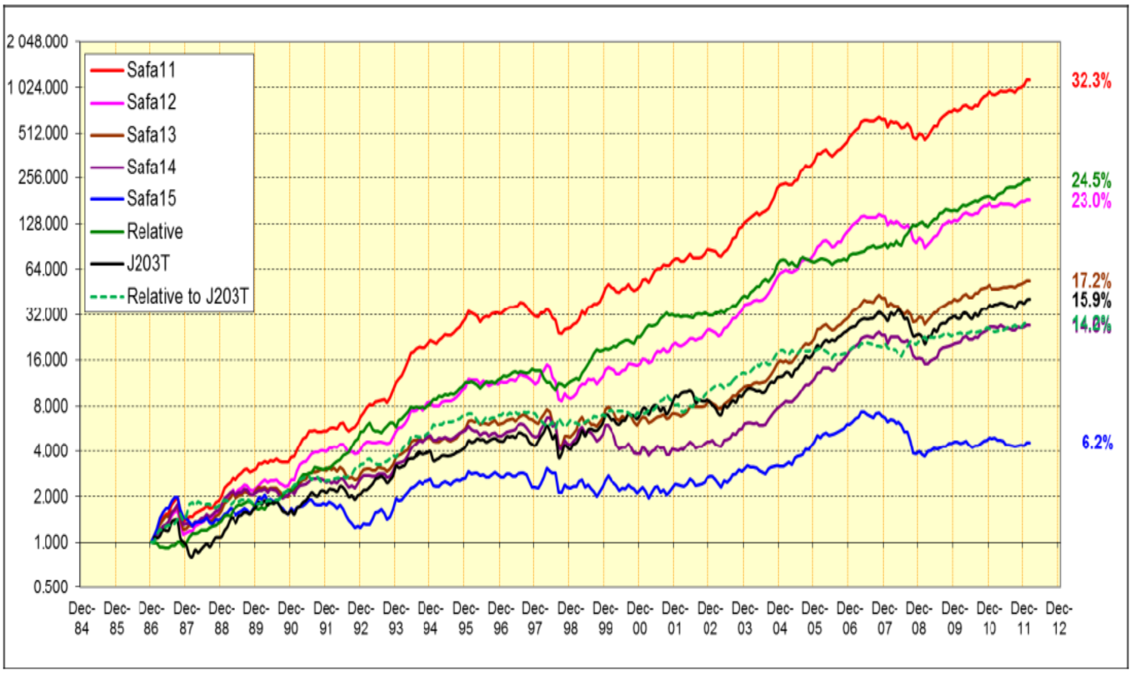


Figure 45: Cumulative Returns Combination Style Returns (Ward and Muller)

(Source: (Muller and Ward, 2013))

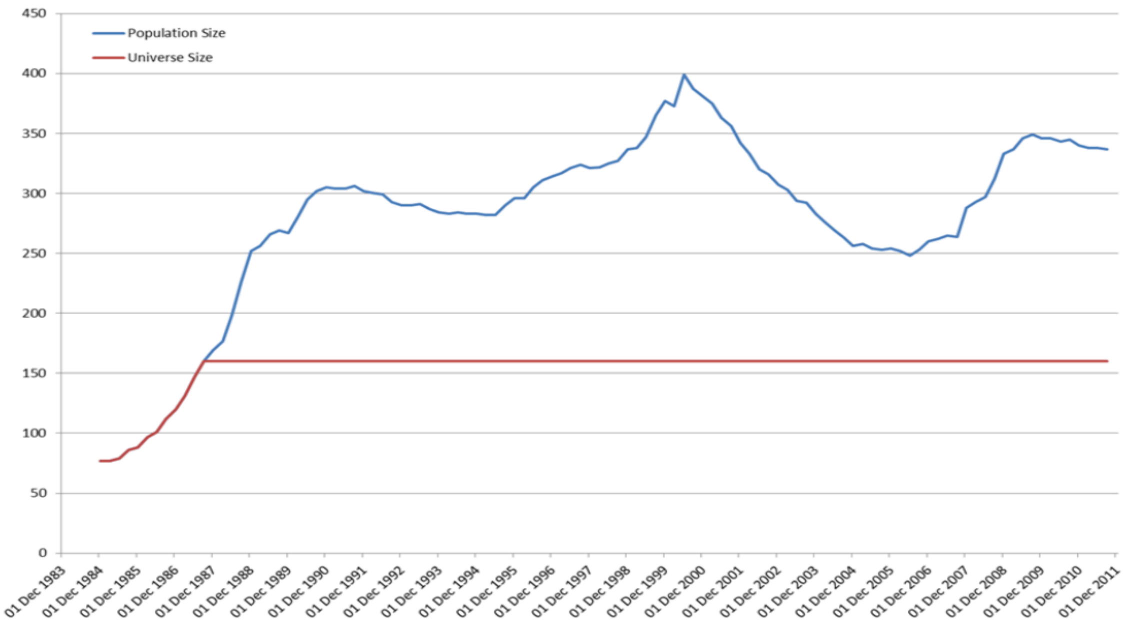


Figure 46: Number of Shares on the JSE over time (Ward and Muller)

(Source: (Muller and Ward, 2013))

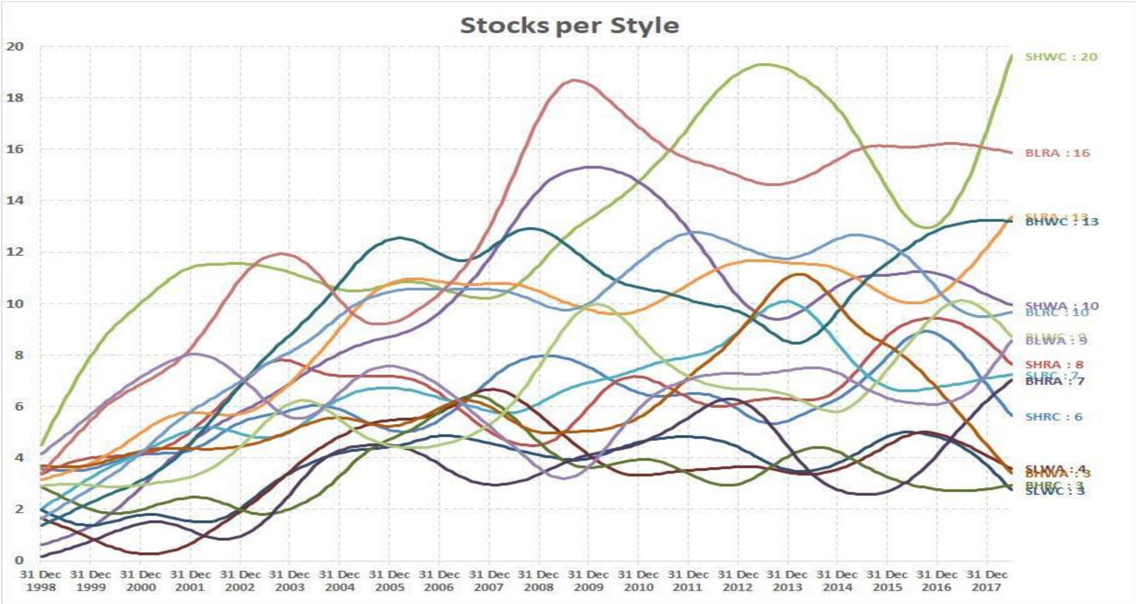


Figure 47: Stocks per Style (Du Pisanie)

(Source: (Du Pisanie, 2018))

Table 17: Monthly Excess Returns Size x Momentum (Fama and French)

Panel B: Monthly excess returns for 25 portfolios formed on size and momentum

	Mean				
	Low	2	3	4	High
Global					
Small	0.20	0.66	0.80	1.15	1.57
2	0.17	0.52	0.54	0.78	1.12
3	0.28	0.46	0.55	0.57	0.85
4	0.26	0.46	0.52	0.57	0.86
Big	0.12	0.32	0.38	0.55	0.61
North America					
Small	0.54	0.96	1.19	1.51	1.96
2	0.52	0.95	0.95	1.00	1.50
3	0.57	0.75	0.90	1.07	1.27
4	0.54	0.79	0.84	0.82	1.29
Big	0.36	0.52	0.44	0.74	0.97
Europe					
Small	−0.28	0.38	0.61	1.03	1.75
2	−0.16	0.46	0.66	0.88	1.45
3	0.19	0.43	0.63	0.77	1.11
4	0.27	0.52	0.65	0.77	1.11
Big	0.22	0.47	0.69	0.65	0.77
Japan					
Small	0.17	0.26	0.15	0.24	−0.05
2	−0.10	−0.03	−0.06	0.03	−0.09
3	−0.14	−0.22	−0.12	−0.02	−0.05
4	−0.11	−0.11	−0.18	−0.16	−0.05
Big	−0.10	−0.28	−0.31	−0.12	−0.06
Asia Pacific					
Small	0.60	1.04	1.31	1.95	1.73
2	−0.14	0.83	0.85	1.08	1.18
3	0.18	0.60	0.71	1.19	1.24
4	0.48	0.96	0.85	0.99	1.23
Big	1.11	0.72	1.07	1.06	1.12

(Source: (Fama and French, 2012))

Appendix 2 – Additional Information – This Research

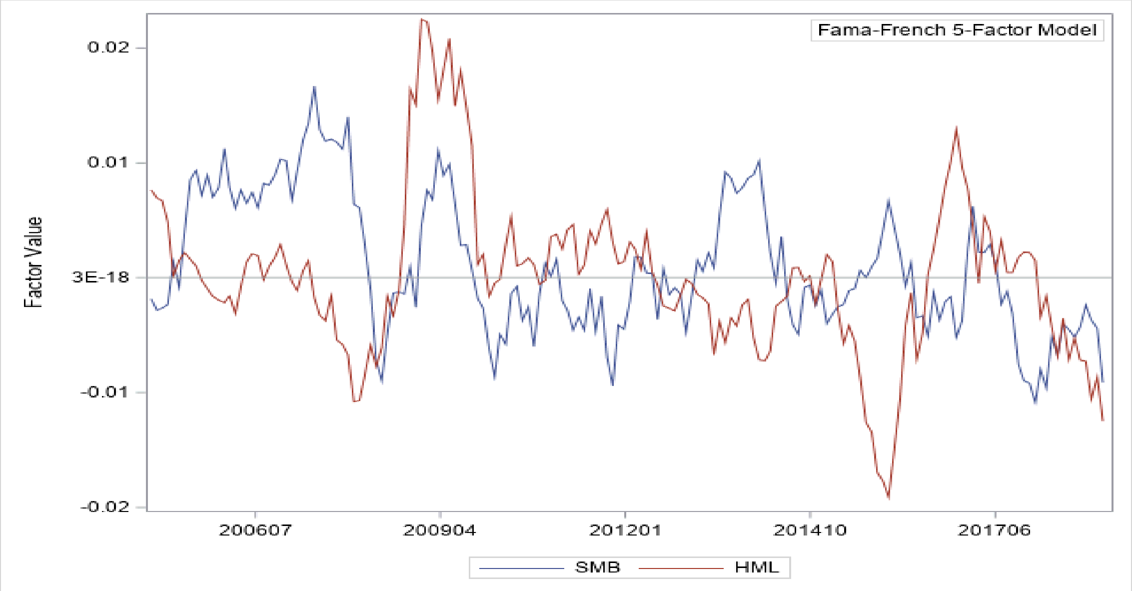


Figure 48: Calculated Factors (FF5) – SMB & HML

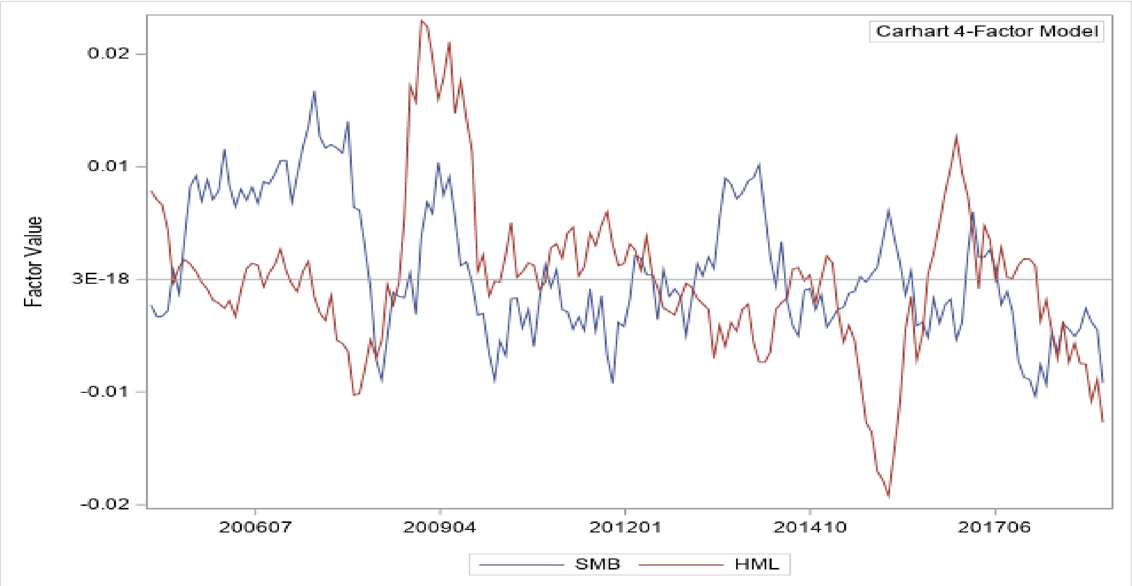


Figure 49: Calculated Factors (CH4) – SMB & HML

Appendix 3 – APM Fit Statistics

Historic Return Models

Calibrati on Point	CAPM F-value	CAPM Prob(F)	FF3 F-value	FF3 Prob(F)	FF5 F-value	FF5 Prob(F)	CH4 F-value	CH4 Prob(F)
200501	53.18	<.001	156.98	<.001	324.5	<.001	245.38	<.001
200502	60.89	<.001	148.52	<.001	382.51	<.001	306.24	<.001
200503	59.55	<.001	146.38	<.001	364.84	<.001	308.44	<.001
200504	60.98	<.001	143.49	<.001	348.59	<.001	306.46	<.001
200505	64.1	<.001	147.83	<.001	358.33	<.001	315.23	<.001
200506	72.17	<.001	162.25	<.001	366.76	<.001	352.2	<.001
200507	103.82	<.001	200.8	<.001	424.89	<.001	396.27	<.001
200508	167.05	<.001	271.17	<.001	513.67	<.001	486.83	<.001
200509	172.72	<.001	277.66	<.001	529.56	<.001	488.09	<.001
200510	204.32	<.001	336.15	<.001	616.42	<.001	573.58	<.001
200511	209.72	<.001	347.37	<.001	625.52	<.001	587.92	<.001
200512	284.35	<.001	439.52	<.001	708.76	<.001	672.45	<.001
200601	326.6	<.001	479.78	<.001	766.51	<.001	734.42	<.001
200602	336.31	<.001	492	<.001	784.57	<.001	752.98	<.001
200603	380.3	<.001	536.64	<.001	850.38	<.001	798.2	<.001
200604	465.25	<.001	630.6	<.001	953.08	<.001	893.62	<.001
200605	495.53	<.001	667.87	<.001	998.51	<.001	940.03	<.001
200606	529.59	<.001	704.77	<.001	1027.49	<.001	978.9	<.001
200607	532.81	<.001	709.43	<.001	1028.64	<.001	979.86	<.001
200608	586.93	<.001	760.9	<.001	1092.31	<.001	1030.56	<.001
200609	571.91	<.001	743.65	<.001	1073.61	<.001	1017.31	<.001
200610	579.1	<.001	764.46	<.001	1090.07	<.001	1032.71	<.001
200611	581.39	<.001	763.54	<.001	1084.07	<.001	1028.85	<.001
200612	580.55	<.001	764.09	<.001	1088.4	<.001	1030.09	<.001
200701	579.5	<.001	758.54	<.001	1081.53	<.001	1014.82	<.001
200702	581.92	<.001	765.79	<.001	1098.38	<.001	1030.34	<.001
200703	591.3	<.001	777.65	<.001	1117.24	<.001	1041.54	<.001
200704	578.09	<.001	770.7	<.001	1122.37	<.001	1032.45	<.001
200705	610.53	<.001	810.77	<.001	1156.17	<.001	1031.57	<.001
200706	610.98	<.001	815.67	<.001	1162.74	<.001	1049.85	<.001
200707	603.76	<.001	813.8	<.001	1156.19	<.001	1025.19	<.001
200708	556.73	<.001	779.05	<.001	1090.49	<.001	965.05	<.001
200709	580.58	<.001	782.15	<.001	1077.56	<.001	964.78	<.001
200710	570.28	<.001	763.86	<.001	1066.96	<.001	945.2	<.001

200711	583.27	<.001	774.24	<.001	1069.99	<.001	938.73	<.001
200712	603.35	<.001	766.98	<.001	1067.35	<.001	914.34	<.001
200801	609.59	<.001	766.34	<.001	1082.56	<.001	940.47	<.001
200802	728.48	<.001	882.97	<.001	1228.19	<.001	1059.65	<.001
200803	829.88	<.001	1028.79	<.001	1331.75	<.001	1129.32	<.001
200804	1008.41	<.001	1420.65	<.001	1751.13	<.001	1478.33	<.001
200805	1045.43	<.001	1490.23	<.001	1784.25	<.001	1542.66	<.001
200806	1144.48	<.001	1625.57	<.001	1928.57	<.001	1677.52	<.001
200807	1084.88	<.001	1569.12	<.001	1875.62	<.001	1612.24	<.001
200808	991.76	<.001	1500.86	<.001	1801.37	<.001	1550.69	<.001
200809	1060.7	<.001	1589.65	<.001	1870.65	<.001	1644.3	<.001
200810	944.57	<.001	1511.25	<.001	1742.31	<.001	1625.32	<.001
200811	876.54	<.001	1413.58	<.001	1656	<.001	1551.84	<.001
200812	1062.86	<.001	1600.6	<.001	1893.87	<.001	1741.74	<.001
200901	1304.85	<.001	1829.68	<.001	2152.63	<.001	2008.73	<.001
200902	1163.35	<.001	1558.55	<.001	2116.61	<.001	1870.91	<.001
200903	1132.86	<.001	1498.25	<.001	2062.32	<.001	1799.76	<.001
200904	1116.97	<.001	1493.59	<.001	2074.28	<.001	1788.16	<.001
200905	1100.92	<.001	1435.1	<.001	1902.27	<.001	1729.88	<.001
200906	1146.94	<.001	1472.7	<.001	1949.76	<.001	1752.23	<.001
200907	1094.43	<.001	1464.86	<.001	1943.13	<.001	1775.02	<.001
200908	1144.17	<.001	1512.57	<.001	1984.7	<.001	1820.45	<.001
200909	1143.41	<.001	1545.56	<.001	2020.29	<.001	1872.01	<.001
200910	1267.89	<.001	1656.26	<.001	2167.45	<.001	1992.67	<.001
200911	1289.72	<.001	1653.65	<.001	2158.81	<.001	2015.63	<.001
200912	1286.34	<.001	1658.88	<.001	2167.49	<.001	2025.24	<.001
201001	1279.67	<.001	1650.47	<.001	2155.82	<.001	2014.14	<.001
201002	1259.06	<.001	1617.42	<.001	2131.65	<.001	1991.09	<.001
201003	1272.74	<.001	1627.05	<.001	2128.24	<.001	2005.47	<.001
201004	1275.38	<.001	1629.8	<.001	2115.63	<.001	2002.47	<.001
201005	1287.43	<.001	1628.66	<.001	2101.48	<.001	2002.74	<.001
201006	1352.51	<.001	1694.06	<.001	2162.44	<.001	2051.8	<.001
201007	1341.84	<.001	1693.39	<.001	2163.02	<.001	2059.7	<.001
201008	1386.02	<.001	1718.55	<.001	2209.81	<.001	2072.07	<.001
201009	1423.04	<.001	1761.9	<.001	2261.69	<.001	2121.08	<.001
201010	1491.34	<.001	1843.15	<.001	2351.53	<.001	2210.74	<.001
201011	1520.04	<.001	1877.55	<.001	2391.59	<.001	2251.33	<.001
201012	1613.1	<.001	1978.89	<.001	2502.17	<.001	2351.77	<.001
201101	1653.94	<.001	2021.32	<.001	2545.82	<.001	2391.53	<.001
201102	1657.35	<.001	2022.53	<.001	2547.5	<.001	2387.39	<.001

201103	1695.16	<.001	2073.79	<.001	2587.77	<.001	2463.79	<.001
201104	1712.07	<.001	2090.84	<.001	2622.76	<.001	2480.82	<.001
201105	1689.67	<.001	1993.22	<.001	2570.83	<.001	2449.34	<.001
201106	1598.21	<.001	1896.55	<.001	2473.19	<.001	2347.11	<.001
201107	1594.33	<.001	1882.4	<.001	2462.21	<.001	2338.77	<.001
201108	1642.55	<.001	1934.01	<.001	2549.05	<.001	2379.98	<.001
201109	1718.97	<.001	2011.28	<.001	2648.83	<.001	2473.78	<.001
201110	1691.99	<.001	1980.92	<.001	2614.44	<.001	2409.73	<.001
201111	1865.4	<.001	2208.49	<.001	2871.94	<.001	2524.13	<.001
201112	1968.16	<.001	2283.55	<.001	2934.3	<.001	2588.84	<.001
201201	1825.64	<.001	2159.27	<.001	2795.55	<.001	2442.58	<.001
201202	1399.93	<.001	1728.23	<.001	2267.61	<.001	1983.18	<.001
201203	1584.93	<.001	1836.55	<.001	2148.3	<.001	2012.34	<.001
201204	1714.27	<.001	1984.84	<.001	2281.07	<.001	2186.63	<.001
201205	1648.81	<.001	1917.61	<.001	2207.5	<.001	2101.93	<.001
201206	1385.39	<.001	1674.71	<.001	2004.64	<.001	1847.38	<.001
201207	1358.69	<.001	1635.92	<.001	1954.97	<.001	1821.15	<.001
201208	1548.29	<.001	1806.4	<.001	2106.7	<.001	1966.71	<.001
201209	1480.35	<.001	1732.68	<.001	2042.25	<.001	1907.97	<.001
201210	1432.82	<.001	1670.74	<.001	1984.36	<.001	1877.05	<.001
201211	1199.33	<.001	1430.56	<.001	1712.94	<.001	1629.71	<.001
201212	1182.22	<.001	1417.69	<.001	1691.09	<.001	1605.09	<.001
201301	1210.65	<.001	1436.14	<.001	1714.94	<.001	1616.72	<.001
201302	1234.61	<.001	1465.35	<.001	1726.03	<.001	1654.01	<.001
201303	1296.98	<.001	1537.11	<.001	1803.08	<.001	1742.09	<.001
201304	1265.58	<.001	1508.81	<.001	1785.09	<.001	1714.36	<.001
201305	1232.24	<.001	1467.53	<.001	1742.12	<.001	1688.3	<.001
201306	1241.35	<.001	1481.43	<.001	1751.04	<.001	1713.1	<.001
201307	1169.57	<.001	1410.44	<.001	1677.11	<.001	1686.01	<.001
201308	1155.78	<.001	1399.28	<.001	1646.74	<.001	1686.68	<.001
201309	1104.2	<.001	1337.39	<.001	1561.22	<.001	1605.82	<.001
201310	1056.34	<.001	1280.77	<.001	1493.29	<.001	1541.6	<.001
201311	960.14	<.001	1176.12	<.001	1373.13	<.001	1434.52	<.001
201312	940.64	<.001	1143.9	<.001	1346.94	<.001	1411.86	<.001
201401	789.71	<.001	986.71	<.001	1181.38	<.001	1236.7	<.001
201402	796.19	<.001	975.45	<.001	1181.68	<.001	1235.92	<.001
201403	786.55	<.001	959.51	<.001	1174.37	<.001	1235.21	<.001
201404	713.45	<.001	860.01	<.001	1114.09	<.001	1130.55	<.001
201405	677.75	<.001	826.08	<.001	1057.75	<.001	1106.72	<.001
201406	679.29	<.001	823.74	<.001	1054.86	<.001	1095.2	<.001

201407	681.92	<.001	830.79	<.001	1086.02	<.001	1099.43	<.001
201408	638.15	<.001	795.71	<.001	1028.83	<.001	1048.65	<.001
201409	506.53	<.001	696.39	<.001	956.04	<.001	967.32	<.001
201410	468.36	<.001	665.02	<.001	932.27	<.001	927.45	<.001
201411	417.04	<.001	634.59	<.001	869.81	<.001	896.79	<.001
201412	318.61	<.001	664.28	<.001	906.96	<.001	929.61	<.001
201501	273.97	<.001	606.59	<.001	855.24	<.001	869.74	<.001
201502	222.6	<.001	550.74	<.001	779.45	<.001	782.2	<.001
201503	216.25	<.001	553.38	<.001	782.43	<.001	756.2	<.001
201504	184.91	<.001	530.66	<.001	764.24	<.001	718.8	<.001
201505	145.48	<.001	490.4	<.001	728.74	<.001	668.02	<.001
201506	143.05	<.001	487.3	<.001	754.64	<.001	684.54	<.001
201507	141.06	<.001	512.26	<.001	772.88	<.001	694.81	<.001
201508	157.34	<.001	591.41	<.001	856.12	<.001	764.03	<.001
201509	133.97	<.001	542.23	<.001	845.42	<.001	725.49	<.001
201510	132.17	<.001	532.31	<.001	848.49	<.001	752.72	<.001
201511	154.22	<.001	576.89	<.001	893.66	<.001	772.85	<.001
201512	155.59	<.001	608.41	<.001	974.3	<.001	826.49	<.001
201601	201.11	<.001	675.69	<.001	1034.87	<.001	890.12	<.001
201602	256.22	<.001	821.4	<.001	1290.45	<.001	1115.64	<.001
201603	262.59	<.001	876.07	<.001	1331.74	<.001	1152.38	<.001
201604	252.76	<.001	757.14	<.001	1079.71	<.001	939.42	<.001
201605	239.33	<.001	735.42	<.001	1091.98	<.001	936.54	<.001
201606	281.84	<.001	883.75	<.001	1258.29	<.001	1100.8	<.001
201607	281.01	<.001	869.71	<.001	1282.09	<.001	1113.48	<.001
201608	253.52	<.001	874.5	<.001	1284.04	<.001	1125.29	<.001
201609	216.54	<.001	832.22	<.001	1241.79	<.001	1068.31	<.001
201610	191.25	<.001	840.19	<.001	1253.13	<.001	1076	<.001
201611	182.44	<.001	870.47	<.001	1284.2	<.001	1112.89	<.001
201612	178.71	<.001	863.85	<.001	1279.83	<.001	1095.76	<.001
201701	170.86	<.001	836.95	<.001	1246.58	<.001	1060.8	<.001
201702	161.14	<.001	803.6	<.001	1196.1	<.001	1015.37	<.001
201703	159.32	<.001	805.42	<.001	1201.69	<.001	1022.31	<.001
201704	122.28	<.001	805.5	<.001	1205.82	<.001	1021.08	<.001
201705	96.12	<.001	808.87	<.001	1222.72	<.001	1034.73	<.001
201706	88.09	<.001	797.14	<.001	1187.64	<.001	1008.14	<.001
201707	81.39	<.001	783.74	<.001	1175.85	<.001	1000.46	<.001
201708	80.78	<.001	786.61	<.001	1147.68	<.001	981.15	<.001
201709	83.03	<.001	831.21	<.001	1217.71	<.001	1023.11	<.001
201710	86.99	<.001	866.47	<.001	1263.73	<.001	1065.31	<.001

201711	89.58	<.001	875.25	<.001	1275.61	<.001	1073.72	<.001
201712	91.31	<.001	888.31	<.001	1293.73	<.001	1095.25	<.001
201801	97.62	<.001	899.59	<.001	1315.76	<.001	1115.43	<.001
201802	97.24	<.001	879.88	<.001	1291.74	<.001	1100.35	<.001
201803	93.8	<.001	915.09	<.001	1364.06	<.001	1132.96	<.001
201804	94.73	<.001	914.83	<.001	1350.24	<.001	1122.8	<.001
201805	95.36	<.001	918.54	<.001	1369.66	<.001	1137.06	<.001
201806	93.7	<.001	926.2	<.001	1356.14	<.001	1145.8	<.001
201807	96.2	<.001	943.99	<.001	1366.33	<.001	1145.54	<.001
201808	95.99	<.001	995.12	<.001	1426.05	<.001	1204.34	<.001
201809	89.53	<.001	959.12	<.001	1398.24	<.001	1176.83	<.001
201810	88.14	<.001	961.34	<.001	1375.74	<.001	1159.2	<.001
201811	90.82	<.001	971.08	<.001	1371.62	<.001	1139.13	<.001
201812	85.11	<.001	935.53	<.001	1351.91	<.001	1105.74	<.001
201901	90.29	<.001	941.73	<.001	1354.85	<.001	1095.05	<.001
201902	74.36	<.001	902.05	<.001	1318.18	<.001	1069.55	<.001
201903	62.83	<.001	819.55	<.001	1231.15	<.001	934.75	<.001

Future Return Models

Calibration Point	CAPM F-value	CAPM Prob(F)	FF3 F-value	FF3 Prob(F)	FF5 F-value	FF5 Prob(F)	CH4 F-value	CH4 Prob(F)
200601	25.31	<.001	0.04	0.845	0.74	0.391	17.06	<.001
200602	12.41	<.001	29.76	<.001	10.35	0.001	65.14	<.001
200603	13.61	<.001	0	0.991	5.17	0.023	22.28	<.001
200604	41.85	<.001	0.75	0.388	10.91	0.001	26.36	<.001
200605	14	<.001	0.01	0.942	3.21	0.074	28.7	<.001
200606	12.36	<.001	2.48	0.116	0.32	0.569	8.74	0.003
200607	39.58	<.001	12.26	<.001	12.24	<.001	0.09	0.768
200608	5.35	0.021	29.3	<.001	29.22	<.001	13.01	<.001
200609	28.21	<.001	56.98	<.001	48.96	<.001	24.74	<.001
200610	2.43	0.12	12.11	<.001	27.43	<.001	19.48	<.001
200611	0	0.959	1.91	0.167	2.24	0.135	0.28	0.595
200612	26.7	<.001	2.2	0.139	0.02	0.896	0.38	0.54
200701	16.62	<.001	13.63	<.001	3.78	0.052	5.51	0.019
200702	1.99	0.159	4.35	0.037	2.69	0.102	5.39	0.021
200703	20.89	<.001	21.85	<.001	7.85	0.005	33.87	<.001
200704	11.88	<.001	45.19	<.001	0.04	0.835	11.91	<.001

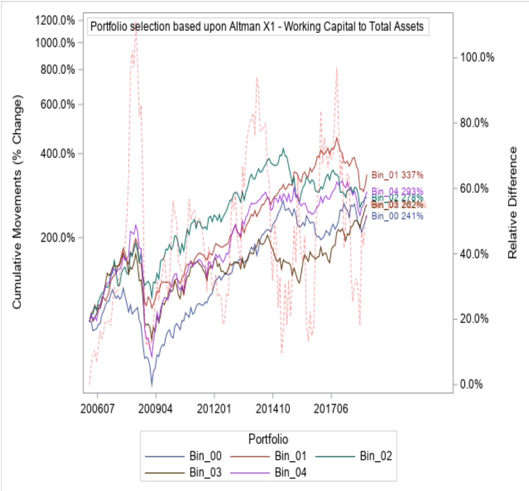
200705	7.74	0.006	56.96	<.001	0.55	0.46	1.12	0.291
200706	0.04	0.833	32.71	<.001	3.18	0.075	10.6	0.001
200707	1.57	0.211	0.16	0.689	2.2	0.139	23.02	<.001
200708	3.67	0.056	0.02	0.891	20.23	<.001	33.81	<.001
200709	0.3	0.581	0.12	0.725	6.5	0.011	30.96	<.001
200710	6.68	0.01	0	0.953	8.1	0.005	98.17	<.001
200711	1.94	0.164	2.25	0.134	0.37	0.545	12.53	<.001
200712	0.04	0.849	11.35	<.001	0.16	0.689	1.19	0.276
200801	0.01	0.91	43.82	<.001	11.94	<.001	0.71	0.398
200802	0.79	0.376	25.31	<.001	12.29	<.001	1.16	0.282
200803	0.08	0.777	18.55	<.001	10.92	0.001	3.87	0.05
200804	3.3	0.07	11.71	<.001	4.21	0.041	6.37	0.012
200805	7.31	0.007	5.92	0.015	1.05	0.306	0.06	0.813
200806	2.34	0.127	6.46	0.011	1.85	0.174	2.16	0.143
200807	0.02	0.899	6.56	0.011	2.61	0.107	3.19	0.075
200808	0.7	0.401	0.03	0.853	12.21	<.001	0.37	0.541
200809	0.27	0.602	1.93	0.165	17.09	<.001	3.75	0.053
200810	6.23	0.013	11.36	<.001	27.03	<.001	15.66	<.001
200811	.	.	29.39	<.001	92.6	<.001	36.07	<.001
200812	20.18	<.001	12.06	<.001	22.5	<.001	11.01	<.001
200901	21.19	<.001	5.93	0.015	24.75	<.001	3.98	0.047
200902	0.13	0.719	2.32	0.128	4.26	0.039	0	0.958
200903	64.3	<.001	45.9	<.001	8.52	0.004	6.22	0.013
200904	16.29	<.001	16.93	<.001	13.8	<.001	1.31	0.253
200905	2.77	0.096	11.63	<.001	11.55	<.001	0.04	0.844
200906	5.2	0.023	2.16	0.142	0	0.946	14.95	<.001
200907	17.49	<.001	6.57	0.011	9.99	0.002	0.18	0.676
200908	33.19	<.001	23.21	<.001	31.8	<.001	4.22	0.04
200909	0.37	0.546	0.62	0.429	5.57	0.019	0.54	0.463
200910	8.91	0.003	6.55	0.011	14.37	<.001	3.33	0.068
200911	1.14	0.286	0	0.986	3.46	0.063	4.63	0.032
200912	17.66	<.001	1.76	0.185	2.26	0.133	8.41	0.004
201001	2.33	0.128	0.25	0.62	7.38	0.007	2.06	0.151
201002	0.03	0.857	0	0.96	13.09	<.001	0.92	0.338
201003	2.74	0.098	0	0.949	6.51	0.011	0.67	0.413
201004	0.96	0.328	0.04	0.84	11.58	<.001	0.51	0.477
201005	18.43	<.001	1.15	0.284	1.1	0.294	1.08	0.3
201006	45.85	<.001	2.01	0.157	4.96	0.026	1.05	0.306
201007	16.09	<.001	1.66	0.198	11.42	<.001	3.64	0.057

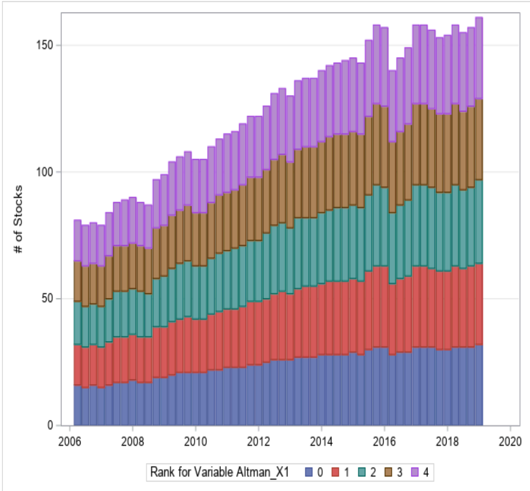
201008	31.98	<.001	8.24	0.004	13.72	<.001	8.89	0.003
201009	61.28	<.001	16.96	<.001	23.13	<.001	30.6	<.001
201010	50.08	<.001	1.9	0.168	7.47	0.006	12.19	<.001
201011	41.4	<.001	3.09	0.079	8.99	0.003	11.84	<.001
201012	72.49	<.001	36.94	<.001	9.94	0.002	56.73	<.001
201101	4.23	0.04	0.22	0.638	1.4	0.237	4.14	0.042
201102	31.66	<.001	6.03	0.014	2.32	0.128	13.23	<.001
201103	0.03	0.872	3.37	0.067	12.31	<.001	0.26	0.613
201104	6.23	0.013	2.24	0.134	0.35	0.553	6.66	0.01
201105	9.34	0.002	8.25	0.004	0.32	0.571	13.96	<.001
201106	5.62	0.018	7.76	0.005	0.22	0.642	4.14	0.042
201107	0.05	0.815	17.65	<.001	2.04	0.153	8.15	0.004
201108	6.16	0.013	0.01	0.93	14.07	<.001	1.79	0.181
201109	8.68	0.003	7.71	0.006	4.49	0.034	3.54	0.06
201110	4.74	0.03	27.75	<.001	0	0.966	1.63	0.202
201111	11.03	<.001	9.31	0.002	3.25	0.072	1.8	0.18
201112	8.53	0.004	0.16	0.693	0.11	0.743	2.27	0.132
201201	2.08	0.15	6.27	0.012	0	0.993	11.93	<.001
201202	6.02	0.014	7.54	0.006	0.45	0.504	16.86	<.001
201203	0.96	0.327	0.91	0.341	2.38	0.123	36.78	<.001
201204	0.21	0.646	0.13	0.713	0.05	0.829	39.41	<.001
201205	5.56	0.019	5.47	0.02	3.05	0.081	63.88	<.001
201206	3.58	0.059	1.16	0.281	1.18	0.277	41.4	<.001
201207	2.64	0.104	0.2	0.653	0.5	0.478	42.55	<.001
201208	1.59	0.207	0.35	0.552	0.29	0.59	30.67	<.001
201209	4.14	0.042	1.33	0.25	0.85	0.356	14.57	<.001
201210	2.84	0.092	1.67	0.196	0	0.971	31.2	<.001
201211	0.63	0.427	0.5	0.48	0.15	0.697	33.13	<.001
201212	18.33	<.001	0.9	0.344	0.37	0.542	34.92	<.001
201301	18.9	<.001	0	0.968	0.34	0.561	25.96	<.001
201302	4.38	0.037	3.85	0.05	0.28	0.595	23.57	<.001
201303	6.91	0.009	1.08	0.3	5.67	0.017	33.94	<.001
201304	26.55	<.001	1.15	0.284	2.6	0.107	41.39	<.001
201305	3.38	0.066	3.19	0.074	1.33	0.249	30.2	<.001
201306	18.46	<.001	2.8	0.094	4.35	0.037	36.35	<.001
201307	9.63	0.002	0.02	0.895	0.15	0.696	8.07	0.005
201308	4.54	0.033	1.43	0.232	0.23	0.635	10.66	0.001
201309	10.57	0.001	4.33	0.038	0.05	0.826	21.49	<.001
201310	11.17	<.001	3.49	0.062	0.53	0.466	26.8	<.001
201311	1.07	0.3	4.34	0.037	2.64	0.104	22.36	<.001

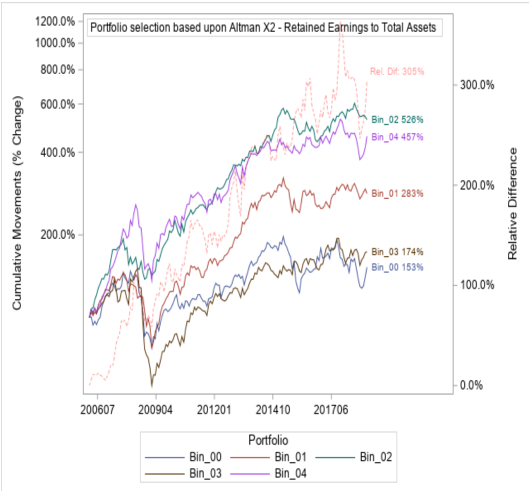
201312	10.89	0.001	18.03	<.001	2.87	0.09	16.92	<.001
201401	0.03	0.853	0.96	0.327	13.3	<.001	15.21	<.001
201402	2.5	0.114	21.58	<.001	17.07	<.001	20.72	<.001
201403	.	.	39.46	<.001	19.13	<.001	22.86	<.001
201404	8.44	0.004	26.59	<.001	15.59	<.001	13.45	<.001
201405	11.11	<.001	5.62	0.018	9.64	0.002	5.42	0.02
201406	8.5	0.004	1.95	0.163	1.69	0.194	9.93	0.002
201407	7.53	0.006	0.3	0.582	0.65	0.422	8.27	0.004
201408	2.3	0.13	0.78	0.379	0.23	0.634	8.64	0.003
201409	1.75	0.186	0.11	0.741	0.18	0.675	7.02	0.008
201410	1.45	0.229	0.33	0.563	0.17	0.678	10.01	0.002
201411	0.95	0.33	0.42	0.517	0.07	0.792	6.78	0.009
201412	0.31	0.578	0.17	0.676	0.27	0.601	11.65	<.001
201501	0.78	0.377	0.78	0.376	2.55	0.11	11.5	<.001
201502	2.86	0.091	1.46	0.227	4.69	0.031	11	<.001
201503	0.89	0.345	0.03	0.861	3.08	0.08	2.13	0.144
201504	0.41	0.521	0.14	0.711	1.25	0.264	2.59	0.108
201505	0.12	0.728	0.34	0.558	0.91	0.341	5.76	0.017
201506	0.37	0.546	4.7	0.03	9.4	0.002	0.77	0.381
201507	0.77	0.38	8.44	0.004	8.57	0.003	0.01	0.94
201508	0.27	0.601	7.01	0.008	3.25	0.072	0.05	0.819
201509	0.06	0.813	2.08	0.15	0.02	0.9	0.41	0.522
201510	7.83	0.005	9.53	0.002	5.49	0.019	0.11	0.744
201511	0.28	0.595	2.32	0.128	2.98	0.085	8.38	0.004
201512	2.4	0.121	42.48	<.001	0.15	0.698	1.14	0.285
201601	20.11	<.001	11.15	<.001	52.67	<.001	8.59	0.003
201602	11.93	<.001	0.3	0.586	10.55	0.001	19.58	<.001
201603	28.89	<.001	4.94	0.026	0.17	0.68	19.2	<.001
201604	32.24	<.001	2.05	0.152	1.89	0.169	29.53	<.001
201605	58.58	<.001	0.81	0.367	21.56	<.001	35.68	<.001
201606	39.32	<.001	0.84	0.361	35.52	<.001	17.09	<.001
201607	25.45	<.001	0.56	0.453	29.97	<.001	11.4	<.001
201608	0.18	0.676	0.47	0.493	1.68	0.195	6.87	0.009
201609	0.06	0.809	0.51	0.475	2.43	0.119	11.58	<.001
201610	4.85	0.028	0.84	0.36	3.15	0.076	12.67	<.001
201611	.	.	0.01	0.916	2.11	0.146	4.95	0.026
201612	7.61	0.006	1.69	0.194	2.93	0.087	20.85	<.001
201701	5.18	0.023	0.62	0.43	1.84	0.175	0.09	0.769
201702	1.27	0.26	0.27	0.603	4.36	0.037	0.03	0.859
201703	1.93	0.165	18.09	<.001	1.98	0.159	9.75	0.002

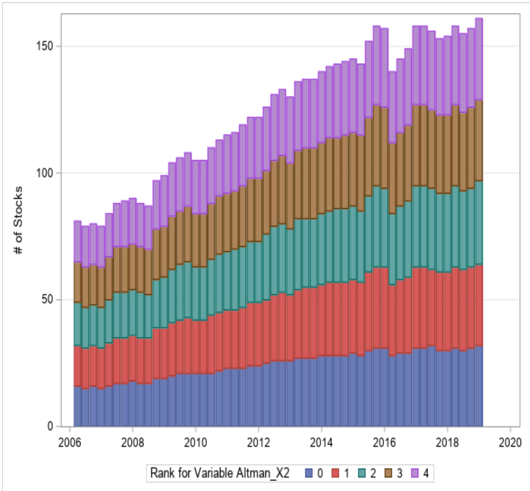
201704	1.66	0.197	4.19	0.041	0.19	0.665	0.08	0.784
201705	0.73	0.392	0.41	0.523	0.35	0.553	0.01	0.931
201706	2.44	0.119	3.2	0.074	1.4	0.236	1.02	0.314
201707	0.51	0.474	1.07	0.3	1.71	0.191	0.12	0.726
201708	0.01	0.938	0.74	0.389	0	0.97	1.48	0.224
201709	0.4	0.526	0.15	0.7	0.09	0.759	0.52	0.469
201710	0.62	0.432	0	0.973	0.69	0.405	0.01	0.912
201711	0.36	0.55	1.03	0.31	3.44	0.064	2.67	0.102
201712	0.05	0.823	16.38	<.001	23.48	<.001	0.01	0.917
201801	1.25	0.263	11.48	<.001	7.37	0.007	14.04	<.001
201802	0.29	0.593	1.62	0.204	12.46	<.001	2.39	0.123
201803	0.06	0.813	0	0.967	12.21	<.001	0.22	0.642
201804	0	0.973	0.25	0.615	11.68	<.001	0.18	0.669
201805	0.26	0.611	1.3	0.255	0.99	0.321	0.71	0.401
201806	0.23	0.635	1.21	0.271	1.89	0.169	0.42	0.516
201807	0.4	0.525	0.56	0.453	0.01	0.927	0	0.99
201808	0.37	0.543	0.6	0.438	0.59	0.443	0.48	0.49
201809	0.87	0.351	4.82	0.028	0.21	0.648	0.39	0.535
201810	5.57	0.018	14.1	<.001	3.23	0.072	3.52	0.061
201811	6.38	0.012	22.56	<.001	4.28	0.039	6.98	0.008
201812	5.36	0.021	11.56	<.001	2.65	0.104	3.13	0.077

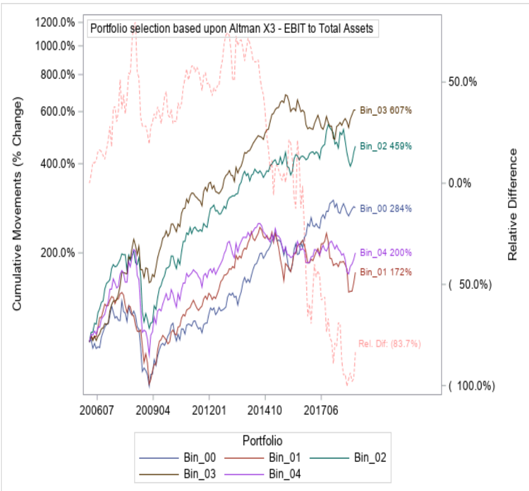
Appendix 4 – Altman's Z-Score Factor Style Returns

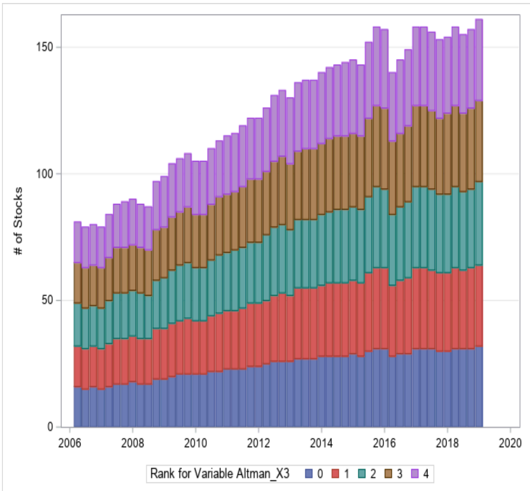


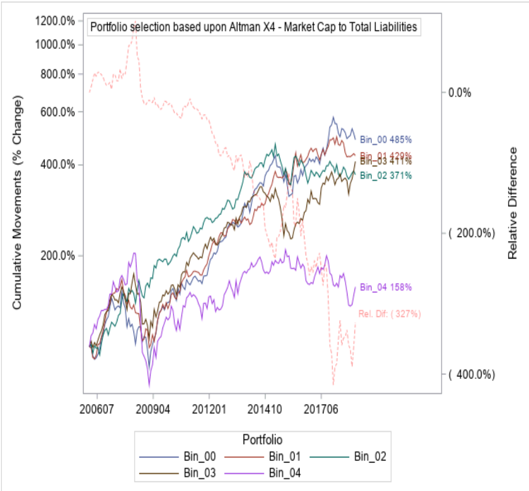


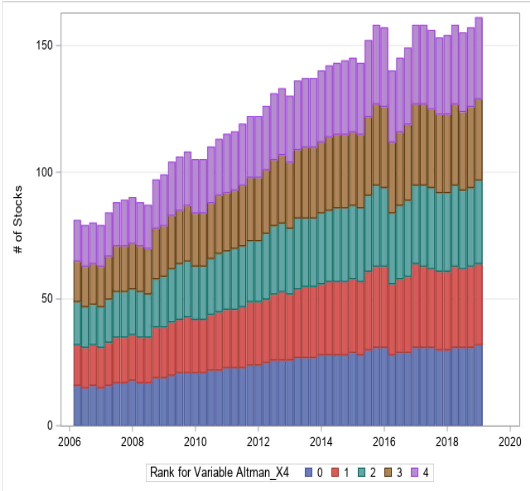


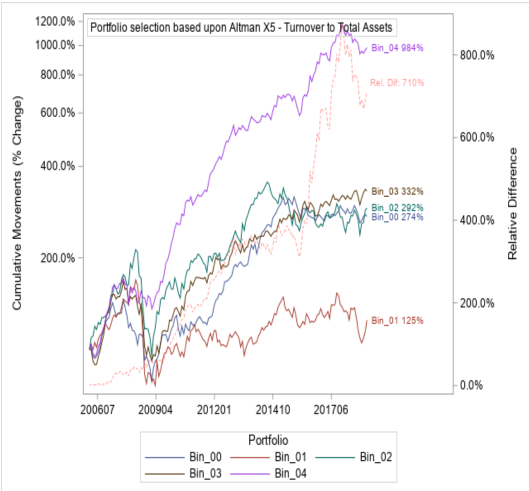


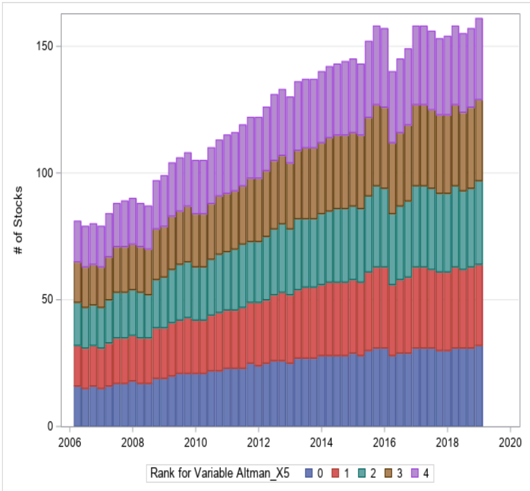





Portfolio selection based upon Altman X4 - Market Cap to Total Liabilities
Cumulative Movements (% Change)
Relative Difference
1200.0%
1000.0%
800.0%
600.0%
400.0%
200.0%
0.0%
(200.0%)
(400.0%)
Bin_00 485%
Bin_02 371%
Bin_04 158%
Rel. Dif. (327%)
200607
200904
201201
201410
201706
Portfolio
Bin_00
Bin_01
Bin_02
Bin_03
Bin_04


# of Stocks
150
100
50
0
2006
2008
2010
2012
2014
2016
2018
2020
Rank for Variable Altman_X4
0
1
2
3
4


Portfolio selection based upon Altman X5 - Turnover to Total Assets
Cumulative Movements (% Change)
Relative Difference
1200.0%
1000.0%
800.0%
600.0%
400.0%
200.0%
0.0%
Bin_04 984%
Rel. Dif. 710%
Bin_03 332%
Bin_02 292%
Bin_00 274%
Bin_01 125%
200607
200904
201201
201410
201706
Portfolio
Bin_00
Bin_01
Bin_02
Bin_03
Bin_04


# of Stocks
150
100
50
0
2006
2008
2010
2012
2014
2016
2018
2020
Rank for Variable Altman_X5
0
1
2
3
4

Appendix 5 – Ethical Clearance

Gordon Institute of Business Science
University of Pretoria

25 July 2019

Riaan du Plooy

Dear Riaan

Please be advised that your application for Ethical Clearance has been approved.

You are therefore allowed to continue collecting your data.

Please note that approval is granted based on the methodology and research instruments provided in the application. If there is any deviation change or addition to the research method or tools, a supplementary application for approval must be obtained

We wish you everything of the best for the rest of the project.

Kind Regards

GIBS MBA Research Ethical Clearance Committee