

A METAHEURISTIC APPROACH TO THE ASSIGNMENT,  
SCHEDULING AND ROUTING OF CARE WORKERS IN THE  
HOME AND COMMUNITY-BASED SCENARIO IN SOUTH  
AFRICA

by

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# Executive Summary

The HIV/AIDS pandemic has reached an unprecedented scale in South Africa. The burden this has placed on the country's already restricted health care resources has led to a greater emphasis on the value of home-based care. Home-based care is a service model appropriate to a number of contexts from palliative and elderly care to support of orphaned and vulnerable children.

Home-based care not only relieves the burden on hospitals and community clinics, but is an affordable alternative to institutional care. In order to recognise home-based care as an equal, rather than an inferior alternative to institutional care, good service delivery is imperative.

The numerous restrictions encapsulated by the home-based care problem, such as adhering to time windows and efficient skill and need matching contributes to the complexity of the problem. The wide geographical dispersion of households in South Africa exacerbates this problem. As far as possible the assignment of beneficiaries to care workers should be done in such a way that the travelling time of the care workers will be minimised.

Through an extensive literature study, the home-based care problem has been identified as a combination of the Nurse Rostering Problem and the Vehicle Routing Problem with Time Windows. Both of these are optimisation problems that determines the optimal assignment of beneficiaries to care workers and minimise the time spent travelling, while adhering to constraints.

A hybrid approach is proposed to address this problem. A conceptual solution algorithm that combines the Variable Neighbourhood Search and Tabu Search meta-heuristics is considered in this document. The algorithm will be enhanced by adding a heuristic ordering algorithm which will construct an initial solution.

The complete hybrid algorithm was evaluated by applying it to test data from home care visits to orphaned and vulnerable children in Hearbeat's Nellmapius Community Project.

The solution algorithm provided sufficient results with care worker schedules not exceeding four hours a day and an equal workload between care workers.

The results indicate that the algorithm does have the ability to improve service delivery of home-based care by improved assignment, routing and scheduling of care workers. However, to ensure feasibility of this algorithm, careful implementation and monitoring is imperative.

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# List of Acronyms

|                |   |
|----------------|---|
| <b>AIDS</b>    | Acquired Immune Deficiency Syndrome                   |
| <b>AMP</b>     | Adaptive Memory Procedure                             |
| <b>CP</b>      | Constraint Programming                                |
| <b>CVI</b>     | Child Vulnerability Index                             |
| <b>GA</b>      | Genetic Algorithm                                     |
| <b>GUI</b>     | Graphical User Interface                              |
| <b>HIV</b>     | Human Immunodeficiency Virus                          |
| <b>IP</b>      | Integer Programming                                   |
| <b>ISP</b>     | Iterated Swap Procedure                               |
| <b>LIP</b>     | Linear Integer Programming                            |
| <b>LP</b>      | Linear Programming                                    |
| <b>LS</b>      | Local Search  |
| <b>MDVRP</b>   | Multi-depot Vehicle Routing Problem                   |
| <b>MDVRPTW</b> | Multi-depot Vehicle Routing Problem with Time Windows |
| <b>MIP</b>     | Mixed Integer Programming                             |
| <b>MP</b>      | Mathematical Programming                              |
| <b>MTSP</b>    | Multiple Travelling Salesman Problem                  |

|              |  |
|--------------|--|
| <b>NP</b>    | Non-Deterministic Polynomial-Time                |
| <b>NRP</b>   | Nurse Rostering Problem                          |
| <b>OVC</b>   | Orphaned and Vulnerable Children                 |
| <b>OVRP</b>  | Open Vehicle Routing Problem                     |
| <b>PSO</b>   | Particle Swarm Optimisation                      |
| <b>PVRP</b>  | Periodic Vehicle Routing Problem                 |
| <b>SA</b>    | Simulated Annealing                              |
| <b>TS</b>    | Tabu Search                                      |
| <b>TSP</b>   | Travelling Salesman Problem                      |
| <b>VNS</b>   | Variable Neighbourhood Search                    |
| <b>VRP</b>   | Vehicle Routing Problem                          |
| <b>VRPHE</b> | Vehicle Routing Problem with Heterogeneous Fleet |
| <b>VRPTW</b> | Vehicle Routing Problem with Time Windows        |

# Chapter 1

## Introduction and Background

### 1.1 Rationale for home-based care in South Africa

With a total of 5.7 million people living with HIV, South Africa is home to the largest population of HIV/AIDS affected people in the world (UNAIDS, 2009). The HIV/AIDS pandemic, other communicable diseases such as tuberculosis, infectious diseases, non-communicable diseases and various other health problems have reached an unprecedented scale, having an impact on all South Africans (Department Of Health, 2001).

South Africa is a country with limited health care resources, an increasing demand for health care and an unequal distribution of income. As a result it is the impoverished communities which are most affected by HIV/AIDS due to their restricted access to health care (Department Of Health, 2001).

The effects of AIDS are not only reflected by the country's health statistics, but also by the number of orphans in the country. Nearly 3.8 million children, 21% of South Africa's 18 million children, have lost one or both their parents. This number is expected to increase to 5.7 million over the next five years due to AIDS (Pendlebury et al., 2009).

According to Peu et al. (2008), the value of home and community-based care<sup>1</sup> services in South Africa became apparent as a result of the rapid growth of the HIV/AIDS pandemic.

Home-based care can be aimed at any of the following:

- Assisting frail, elderly people with everyday activities.
- Assisting people during convalescence .
- Providing support to orphans or other healthy people in need.
- Supporting persons living with HIV/AIDS or any other debilitating disease.
- Providing palliative care for terminally ill patients.

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<sup>1</sup>For the remainder of the document, the term home and community-based care will only be referred to as home-based care

- Supporting people with mental illnesses and other mental and physical disabilities.

Home-based care not only relieves the burden on hospitals and community clinics, but is also an affordable alternative to institutional care. Home-based care is also addressing the problem of the increase in children left orphaned and vulnerable due to HIV, by assigning some of the care responsibilities to trained care workers in the community.

## 1.2 Examination of the problem

Home-based care is an increasing trend in the South African health sector. To justify home-based care as an alternative to institutional care, good service delivery is of the utmost importance. Restrictions that hamper good service delivery such as unreliable public transport, legal occupational restrictions and care worker incompetence should be addressed. An overview of typical constraints relevant to home-based care can be viewed in Figure 1.1.

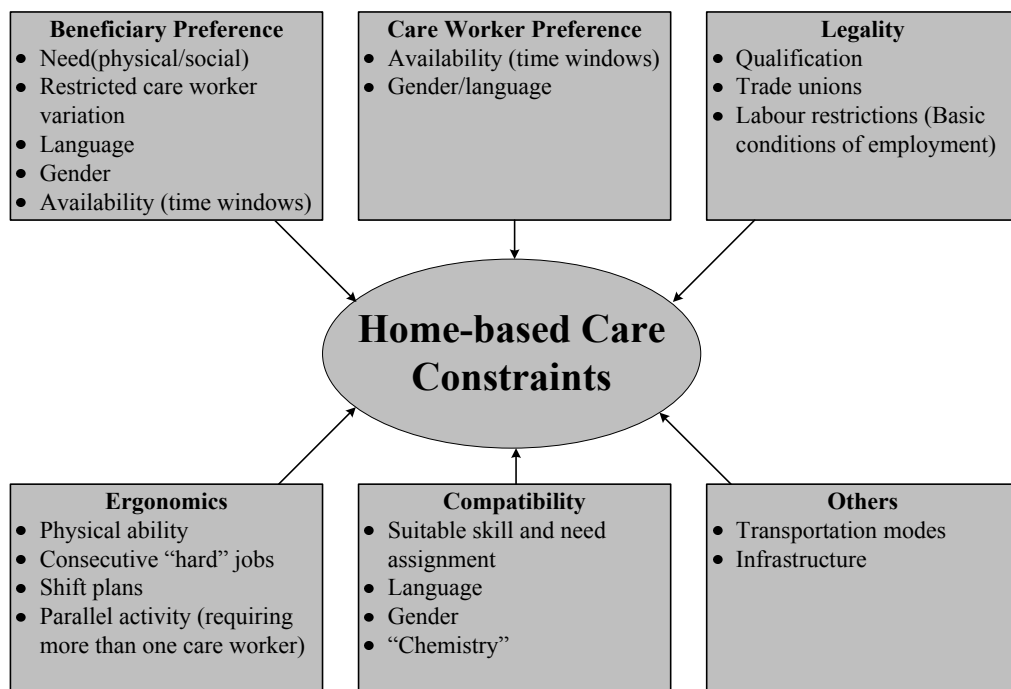


Figure 1.1: Overview of constraints relevant to the home-based care problem

In the typical home-based care scenario, the needs of the beneficiary are determined by an assessor. The care duties are then assigned to care workers, according to the care worker’s skill, ability, knowledge and expertise (Peu et al., 2008). When planning home visits, it should also be taken into consideration that the beneficiaries have preferential time windows in which they would like to be visited, and as far as possible the beneficiaries should be visited by the same care worker. The rationale

behind this is to provide a sense of safety and continuity for the beneficiaries. The beneficiaries should not be exposed to a new face with each visit, enabling them to build a trusting relationship with the care worker (Eveborn et al., 2006; Peu et al., 2008). It becomes clear that even just assigning care workers to beneficiaries according to time window preferences requires a lot of coordination to ensure good service delivery.

The service delivery of care workers is further constrained by transport availability and travel time. Since most care workers make use of public transport or travel by foot, the distance to be travelled in a day is often too far and some visits cannot be made. It would be ideal if the daily travel time of each care worker can be minimised, while at the same time dividing the care duties equally between the workers and meeting the other home care requirements.

After analysing this problem the question is:

*“How can the assignment, routing and scheduling of care workers be improved in order to advance service delivery in the Home Based Care sector in South Africa?”*

### **1.3 Heartbeat—A real world case**

Heartbeat is a non-profit and public benefit organisation, founded as a response to the orphan challenges in South Africa. Heartbeat endeavours to empower orphaned and vulnerable children (OVC) to reach their full potential by addressing the needs of the children through basic service provision. Key programmes provided in Heartbeat communities include care and academic support at after school centres, food provision and community food garden development, educational and recreational extracurricular activities and home care.

Of the 3.1 million orphaned children a few are fortunate enough to be cared for by a grandparent or other relative. Numerous children are, however, forced to take on the role of caregiver and provider. These children, who live without adequate care and protection, are more susceptible to exploitation and child labour.

Through regular home visits to OVC families, Heartbeat attempts to provide a sense of stability, protection and care by assuming the role of caregiver or parent. Trained home care workers provide counselling, homework and chore assistance and mediation of disputes, to name a few. The frequency of the home visits vary depending on the type of households with child-headed household visited most frequently and relative-headed households visited the least.

Limited staff capacity and an overload of destitute OVC are two major challenges faced by Heartbeat. This problem is amplified by the geographical spread of households, resulting in long travelling distances required to reach OVC. Consequently, the quality of the home care services suffers greatly.

Minimising the travel time of each care worker while simultaneously matching beneficiary needs to care worker skills will drastically improve the quality of home-care services. Not only will this improved assignment, scheduling and routing of home care workers reduce the hassles a home-care worker experience, but it will also enhance service delivery.

Heartbeat’s Nellmapius community project, one of 14 community projects, will be used as a research subject during the development of a home-based care assignment, scheduling and routing algorithm. Nellmapius is a semi-urban settlement east of Pretoria with an estimated population size of 65 000. Despite the availability of basic amenities, government services such as health care are limited. Unemployment, crime and drug abuse are prevalent in this area. Health statistics also indicates a high HIV prevalence which is reflected in the large number of OVC in Nellmapius. Nellmapius is a typical example of a community with a great need for home-based care, but very little resources. Therefore, it is an ideal setting for the development of such an algorithm.

## 1.4 Research Design and Methodology

The main focus of this project will be the development of a generic computerised solution algorithm. This algorithm will assist in solving the home-based care problem by developing daily assignment, routing and scheduling solutions. The solution will be based on the following user inputs:

- Care worker profile: Skill, availability time window (working hours), geographical location and transportation mode.
- Beneficiary profile: Requirements and needs, availability time windows, geographical location and care worker preference.

The development of a *generic* algorithm is paramount in order to provide flexibility for algorithm application. This will ensure that the algorithm can be applied to all the different variations of home based care in South Africa.

The first step towards the development of this algorithm is to review cases similar to the home-based care problem. This will aid in the identification of solution approaches for this type of problem. Comparing these solution approaches, the most applicable algorithms should be adapted and integrated and then refined to fit the home-based care problem. The main objective of this new algorithm will be to minimise the travel time of care workers while still meeting beneficiary requirements. For implementation purposes the algorithm will then be coded in the *GNU Octave*® development environment, an open source programming language.

The robustness and flexibility of the algorithm will be tested using data from Heartbeat’s Nellmapius project. Results will enable verification and validation of the algorithm which will pinpoint necessary model refinements and customisation.

To ensure the maximum utility of this algorithm, a user-friendly and accessible graphical user interface (GUI) should be developed to accompany the algorithm. The purpose of this interface will be to allow the organisations, who wish to use the algorithm for routing and scheduling, to input the required data in a familiar environment (not using a specific programming language). The development and coding of the GUI falls outside the scope of the project. However, information system design specifications will be developed to assist with this work.

## 1.5 Document Structure

Chapter 2 presents a critical review of the home-based care problem as well as the two sub problems - Nurse Rostering Problem (NRP) and the Vehicle Routing Problem (VRP). Applicable variants of the VRP are also introduced. A study of the exact and approximation solution methods for the problem is also presented in this chapter. Chapter 3 continues the discussion of the chosen solution approaches as well as their adaptation for the problem at hand. Chapter 4 presents the computational results obtained from applying the algorithm to the Heartbeat Nellmapius project. The analysis of the results as well as algorithm performance will also be discussed in Chapter 4. Chapter 5 will conclude the report with further research suggestions and specification requirements for the development of a user interface for the algorithm.

# Chapter 2

## Literature review

### 2.1 Problem Identification

The home-based care problem encapsulates two distinct sub-problems: The assignment of beneficiaries to suitable care workers, identified as the Nurse Rostering Problem (NRP), and the optimal scheduling and routing of care workers, identified as an extension of the Vehicle Routing Problem (VRP).

#### 2.1.1 The Nurse Rostering Problem

The NRP is described by Burke et al. (1999) as: “The assignment of duties to a set of people with different qualifications, work regulations and preferences.” The similarities between the NRP and the assignment of care workers are clear. Solving this problem will ensure that the skills of care workers and requirements of beneficiaries are matched and that the time availability of care workers will be taken into consideration.

The complexity of the NRP is mainly as a result of the vast number of hard and soft constraints encountered in this type of problem. Hard constraints have to be met under any circumstances in order for the schedule to be considered feasible (Burke et al., 2010; Akjiratikarl et al., 2007). Typical examples are:

- Qualification requirements.
- Working hours.
- Worker requirements for shifts.

Alternately, soft constraints allow unavoidable constraint violations without rendering a solution infeasible. Soft constraints include:

- Leave applications.
- Shift type preferences (i.e. night or weekend shifts).
- Avoiding unfavourable shift successions (i.e. a night shift followed by an early day shift).

## 2.1.2 The Vehicle Routing Problem

In the classical Vehicle Routing Problem (VRP) a sequence of deliveries are generated for a set of  $K$  identical vehicles in a fleet with a carrying capacity  $Q$ . This fleet, which is based at a single depot, must deliver order quantities  $q_i$  with  $i \in \{1, \dots, n\}$  to  $n$  customers without exceeding the capacity of a vehicle,  $Q$ . This should be done in such a way that all customer demands are met, without exceeding the maximum route length  $D$  (Taillard, 1999; Li et al., 2007). The mathematical formulation of the VRP, as adapted from Joubert (2006), is presented as:

$$x_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ travels from node } i \text{ to } j, \text{ where} \\ & i, j \in \{1, 2, \dots, N\} | i \neq j, \text{ and } k \in \{1, 2, \dots, K\} \\ 0 & \text{otherwise} \end{cases} \quad (2.1)$$

$$\min z = \sum_{i=0}^N \sum_{\substack{j=0 \\ j \neq i}}^N \sum_{k=1}^K c_{ij} x_{ij}^k \quad (2.2)$$

subject to

$$\sum_{i=0}^N \sum_{k=1}^K x_{ij}^k = 1 \quad \forall j \in \{1, 2, \dots, N\} \quad (2.3)$$

$$\sum_{j=0}^N \sum_{k=1}^K x_{ij}^k = 1 \quad \forall i \in \{1, 2, \dots, N\} \quad (2.4)$$

$$\sum_{i=0}^N x_{ip}^k - \sum_{j=0}^N x_{pj}^k = 0 \quad \forall p \in \{1, 2, \dots, N\}, k \in \{1, 2, \dots, K\} \quad (2.5)$$

$$\sum_{j=0}^N q_j \left( \sum_{i=0}^N x_{ij}^k \right) \leq Q \quad \forall k \in \{1, 2, \dots, K\} \quad (2.6)$$

$$\sum_{i=0}^N \sum_{j=0}^N t_{ij} x_{ij}^k \leq D \quad \forall k \in \{1, 2, \dots, K\} \quad (2.7)$$

$$\sum_{j=1}^N x_{0j}^k \leq 1 \quad \forall k \in \{1, 2, \dots, K\} \quad (2.8)$$

$$\sum_{i=1}^N x_{i0}^k \leq 1 \quad \forall k \in \{1, 2, \dots, K\} \quad (2.9)$$

$$x_{ij}^k \in \{0, 1\} \quad \forall i, j \in \{1, 2, \dots, N\}, k \in \{1, 2, \dots, K\} \quad (2.10)$$

The term  $c_{ij}$  in (2.2) can be used interchangeably for travel cost, travel distance or travel time, depending on the objective of the specific problem. In this specific problem the objective is to minimise the total travelling cost while constraints (2.3)

and (2.4) ensure that only one vehicle handles deliveries to a given customer. Constraint (2.5) ensures that a vehicle will leave a node it has arrived at. The total capacity that a single vehicle can deliver cannot exceed  $Q$  in (2.6) and (2.7) limits the maximum travel distance,  $D$  ( $t_{ij}$  may also be defined as the travel time in which case (2.7) will limit the maximum route duration of each vehicle). Constraints (2.8) and (2.9) ensure that a vehicle is not scheduled on more than one route.

## 2.2 Variants of the Vehicle Routing Problem

The VRP is a very practical and challenging problem often encountered in the logistics sector. However, the rigid assumptions that the classic VRP makes often restricts its applicability to real-life problems (Ho et al., 2008). These assumptions include utilisation of a homogenous fleet, servicing from a single depot and that each vehicle can only serve one route (Joubert, 2006). By adding additional constraints to this problem, these assumptions can be corrected, but this will increase the complexity of a problem formulation that is already difficult to solve in itself. Several variants of the VRP exist which is also applicable to the home-based care problem.

### 2.2.1 The Vehicle Routing Problem with Time Windows

Dondo and Cerdá (2007) describes a time window as a time period during which a service can be done. This time is usually defined by an earliest time and latest time at which service may commence at a specific node (Joubert, 2006). Time windows can be further characterised as *hard* or *soft*. In the case of a *hard* time window, the arrival time at a node must fall within the specified time window, else the arrival will not be allowed. A time window may be considered as *soft* when a delivery is permitted outside the specified time window, but usually a penalty cost is incurred (Akjiratikarl et al., 2007; Joubert, 2006).

The routing of home care workers has been identified as an extension of the Vehicle Routing Problem with Time Windows (VRPTW) by Akjiratikarl et al. (2007), Eveborn et al. (2006), Bertels and Fahle (2006) and Cheng and Rich (1998). In the home-based care problem it is the availability of beneficiaries and care workers that typically give rise to time windows.

### 2.2.2 The Multi-Depot Vehicle Routing Problem

As a variant of the VRP, the multi-depot VRP (MDVRP) is applied to cases where a company has more than one depot to consider. Hence, the single homogeneous fleet will be dispersed between the multiple depots. This results in an additional problem of assigning depots to customers prior to scheduling the fleet (Ho et al., 2008).

Another variant which relates to the MDVRP is the open vehicle routing problem (OVRP). In the OVRP a vehicle does not return to a central depot after servicing the last customer on the route. This problem is most often encountered when contracted delivery men make use of their own transport.

Cheng and Rich (1998) identified the home-based care problem as a VRPTW with many depots (MDVRPTW). In this approach each care worker's home serves as a depot housing a single vehicle (i.e. the care worker). This method eliminates the travelling of the care worker to and from a central depot, since the routes start and end at the care worker's home.

### 2.2.3 The Vehicle Routing Problem with Heterogeneous Fleet

The VRP with a heterogeneous fleet (VRPHE) is applied to problems where the fleet of vehicles vary with regard to capacity or cost. Bertels and Fahle (2006) states the importance of incorporating this variant of the VRP into the home-based care problem. In many home care scenarios the care workers make use of different transportation modes resulting in varying travelling velocities and time. The home-based care problem will however not be treated as VRPHE, since the travelling speed between nodes will be predetermined.

## 2.3 Solution Approaches

There are very limited publications which examine the solution approaches of the home-based care problem defined in Chapter 1. That is, the solution which combines the NRP and the VRP. Separately, these problems have attracted much more research. The solution approaches for the two separate problems as well as the limited solution approaches that have been applied to the home-based care problem will be discussed.

Steeg and Schröder (2007) identified both the NRP and the VRPTW as two well known non-deterministic polynomial-time hard (NP-hard) problems. As a result, the home-based care problem can also be classified as NP-hard, since it combines the NP-hard NRP and VRPTW.

NP-hard problems are most often difficult to solve optimally (Steeg and Schröder, 2007) or unpractical to solve optimally due to the vast amount of computational time required for all but the smallest problem instances. Such problems are rather solved by applying approximation or heuristic algorithms (Winston and Venkataramanan, 2003). Due to the extensive application of heuristics and metaheuristics found in the literature, a brief overview of these approaches will be given:

Heuristics are approximation solution algorithms which construct initial solutions and then apply search methods to improve this feasible initial solution (Winston and Venkataramanan, 2003). Although optimality is not guaranteed, this usually results in good, feasible solutions. Heuristics do, however, have the tendency to get trapped in the first local optimum found. This is a result of the initial solution upon which the improvement search was based (Winston and Venkataramanan, 2003; Joubert, 2006).

Metaheuristics are solution techniques which orchestrate decision making techniques to search for an improved solution in the solution space (Glover and Kochenberger, 2003; Joubert, 2006). This method differs from heuristic techniques since the robust searches also allow non-improving feasible moves. According to Rardin

(1998), this characteristic of metaheuristics reduces the probability of getting stuck at local optima.

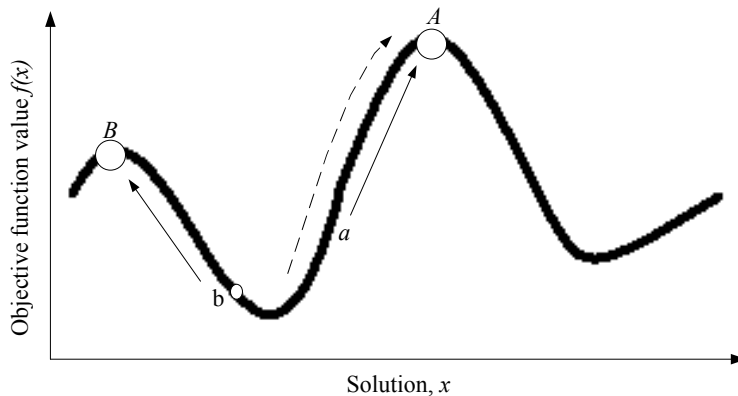


Figure 2.1: Comparison of global and local optima (Joubert, 2006)

Figure 2.1 illustrates the difference between the solution approaches of heuristics and metaheuristics. If a heuristic starts with initial solution  $a$ , it can only improve until its local optimum  $A$  is reached. Although the local optimum  $A$  also happens to be the global optimum, the same approach follows for initial solution  $b$ , which can only improve until  $B$  is reached. This is a result of the restriction of heuristics which only allows improving moves. The ability of metaheuristics, which prevent getting stuck at local optima ( $B$ ) by allowing non-improving moves, is illustrated by the dotted line. The solution starts at  $b$ , but first worsens while moving in the direction of  $a$ , before it improves towards  $A$  (Joubert, 2006).

### 2.3.1 Nurse Rostering Problem

Bertels and Fahle (2006) comment on the importance of solving the NRP in the home-based care problem as efficiently as possible, in order to obtain feasible and applicable solutions. This is necessary due to the high interdependencies between the rostering constraints and optimised routes.

In their paper *Nurse Rostering Problems - A bibliographic survey*, Cheang et al. (2003) state that the two methodologies commonly used to solve the NRP are mathematical programming (MP) and heuristics. Although exact algorithms exist for the NRP, little research has been done on applications because in the field of nurse rostering, problems are usually very large and the application of metaheuristic approaches seems to dominate.

Very recently, Burke et al. (2010) proposed a hybrid model for highly-constrained NRP's. An integer programme (IP) is used to solve the sub-problem which contains only the hard constraints. The hard constraints are the constraints that have to be met in order to consider the solution as acceptable and feasible. A variable neighbourhood search (VNS) is then applied which focuses on enforcing the excluded soft constraints while attempting to improve the initial results of the IP. Curtois et al. (2008) follow a very similar approach to Burke et al. (2010), by also developing a

hybrid VNS algorithm. The algorithm uses a repeating and iterative VNS which is followed by a disruption and repair process and is further improved through backtracking.

Hansen and Mladenović (2001) describe the VNS as a metaheuristic which exploits the idea of a systematic neighbourhood change within a search algorithm. In contrast with general metaheuristics which only apply a local search in the neighbouring search space, the VNS explore increasingly distant neighbourhoods and moves from one neighbourhood to the next only if an improvement will be made by the move. This allows the search to avoid a descent to a local minimum and to escape the valley which contains them (Hansen and Mladenović, 1997).

Bai et al. (2000) examine a stochastic ranking method for solving constrained optimisation problems such as the NRP. This method is combined with a genetic algorithm (GA) and simulated annealing (SA) metaheuristic.

GA's are search techniques inspired by the biological evolution of species, which is associated with the aim of continuous improvement to always search for the better solution (Dréo et al., 2003). The core focus of this algorithm is based on the development of new solutions through the mutation or random selection and change of old solutions (Cheang et al., 2003). According to Rardin (1998), GA's search for globally optimal solutions through the intellectual exploitation of random search methods, emulating biological evolution, will ensure that the current best optimum is always in the solution.

Despite the interest observed in applying GA to rostering problems, Aickelin and Dowsland (2000) comment that the classic GA metaheuristic is not well equipped to handle the conflicting objectives and constraints which are often encountered in the NRP. They approach the problem by decomposing it into two stages which are formulated as IP's. Stage one is a standard knapsack problem which assures that there are sufficient personnel to meet the constraints and stage two is concerned with the allocation of shifts. The adapted GA is implemented in the second stage.

Another acclaimed approach that has been used to solve the NRP is the MP-based near-optimal method. Cheang et al. (2003) mention the work of Valouxis and Housos who used this specific method. An initial solution is obtained by solving an IP which is further improved using a Tabu Search (TS).

### **2.3.2 Vehicle Routing Problem**

Ho et al. (2008) solved the MDVRP by developing a hybrid GA which consists of a grouping, routing, scheduling and improvement component. The Clark and Wright saving method, VNS and iterated swap procedure (ISP) are hybridised in this solution algorithm. The first two are concerned with the generation of the initial solution while the ISP aims to improve the solution.

Very little research has been done on the development of solution algorithms for the OVRP. Li et al. (2007) found that all the existing algorithms for the OVRP are based on heuristics with good performance by the VNS, TS, and record-to-record travel which is a variant of SA. In their work, Li et al. (2007) specifically refer to the work of Brandão who uses a VNS to generate an initial feasible solution and an improving TS with embedded insertion and swap methods.

Dondo and Cerdá (2007) present a discrete 3-phase cluster-based heuristic approach to solve the multi-depot VRPTW and a heterogeneous fleet. Phase 1 uses a heuristic clustering algorithm which clusters the depots in order to reduce the computational burden of the problem. Phase 2 and 3 are concerned with the scheduling and routing and is done by solving a mixed integer linear problem (MILP) and travelling salesman problem (TSP) respectively.

Joubert (2006) developed an integrated and intelligent metaheuristic for the constrained VRP. This metaheuristic integrates an Artificial Neural Network (ANN), TS, GA and fuzzy c-clustering. The goal of this metaheuristic is to find the best possible solution for any real-life vehicle routing and scheduling problem.

Semet and Taillard (1993) developed a TS based method for solving a VRPHE. Despite using this standard method very satisfactory results were obtained. Semet and Taillard (1993) comment that introducing an iterated constructive process into this TS method may result in further improvement of the solution.

Taillard (1999) approached the VRPHE by developing a heuristic column generation method. For each vehicle type in the fleet the classic VRP (with a homogeneous fleet) are solved with an adaptive memory procedure (AMP). The AMP is based on the TS method of Semet and Taillard (1993). Once all the homogeneous VRP are solved, they are combined to produce a solution to the VRPHE.

The Tabu Search (TS) method, which allows local search (LS) methods to overcome local optima, is based on the properties of human memory. When a local optimum is encountered, LS techniques are employed to search for improved solutions in the neighbouring search space. The LS techniques allow non-improving moves (Winston and Venkataramanan, 2003). The TS algorithm contains a memory-based property, preventing future mistakes based on mistakes made in the past (Dréo et al., 2003). By using this memory, previously visited solutions are stored in a tabu list which prevents this solution being revisited within a predetermined number of iterations. The TS has the ability to start with a single initial solution as well as a population of initial solutions. The latter is obtained through the implementation of a beam search (Winston and Venkataramanan, 2003).

### 2.3.3 Home-based care

Eveborn et al. (2006) approached the assignment of visits to care workers, using a set partitioning model combined with a repeated matching heuristic to determine the best solution. This heuristic uses the branch-and-bound algorithm in combination with a successive shortest path algorithm (Engquist, 1982; Eveborn et al., 2006). The merit of the solutions obtained was judged by evaluating the travel time of the care workers and the amount of patients not assigned care workers. Both of these had to be minimised. Bertels and Fahle (2006) criticise this approach, as excessive computational time is required for column generation approaches, such as the set partitioning model used by Eveborn et al. (2006). Despite this criticism, this system was successfully implemented in Sweden, showing a reduction in both the planning and travel time for home care visits (Eveborn et al., 2006).

Stegg and Schröder (2007) follows a hybrid approach to solve the periodic home-

based care problem. Constraint programming and a guided branch-and-bound search was applied to find an initial feasible solution for the combinatorial NRP and periodic vehicle routing problem with time windows (PVRPTW). The VNS metaheuristic, which is guided by a SA metaheuristic, attempts to improve this initial solution while an underlying constraint programme assures feasibility of the NRP.

The VNS technique follows a different approach to local search techniques by changing the solution significantly. The SA metaheuristic is a non-randomised stochastic search method which simulates the physical process of combining particles in a cooling system (Winston and Venkataramanan, 2003). According to Glover and Kochenberger (2003) the key factor of SA is its ability to escape local optima by allowing moves which worsen the objective function value in the hopes of finding the global optimum. This memory less approach allows any feasible improvement in the objective function value, while the acceptance of inferior solutions are based on the Boltzman function evaluation criterion (Winston and Venkataramanan, 2003).

Another hybrid approach for solving the home-based care problem was applied by Bertels and Fahle (2006) in the development of their PARPAP<sup>®</sup> software which serves as a planning tool for staff rostering and routing in the home-based care industry. An initial feasible solution was obtained using a combination of constraint programming (CP) and linear programming (LP). The improvement algorithm which was then applied combines CP and the TS metaheuristic. This combinatorial application delivered near optimal results and also performed superior to single paradigm applications of CP, TS, GA or SA.

In the works of Akjiratikarl et al. (2007), the home health care problem was identified as an extension of the multiple depot VRPTW (MDVRPTW). The relatively novel Particle Swarm Optimisation (PSO) metaheuristic was adapted to solve this problem. An initial solution was developed by assigning the home care tasks with the earliest start time priority and minimum distances first. The local improvement procedures (LIP) search algorithm, which is embedded in the PSO, was then applied to obtain the global best solution through insertion and swap procedures. By applying this approach the total distance travelled by the care worker is minimised, while capacity and time window constraints are satisfied.

PSO is a population based search technique which emulates the collective behaviour of animals i.e. birds flocking and fish schooling (Akjiratikarl et al., 2007). The development of this algorithm was inspired by the theory of socio-psychology, which posits that an individual's movement is influenced by their own and their neighbour's last behaviour (Dréo et al., 2003). In PSO, the decision of a particle's adjustment in position is dependent on their current speed and direction of movement as well as the best preceding position of the particle itself and its neighbours (Akjiratikarl et al., 2007; Dréo et al., 2003). This is an indication of memory-based decision making. Hence, the PSO attempts to balance the exploration and exploitation by combining a local search (particle's best position retrieved from memory) with a global search (neighbouring experience retrieved from memory) (Akjiratikarl et al., 2007).

Cheng and Rich (1998) presents the home health care routing and scheduling problem as a mixed integer problem (MIP) based on the multiple travelling salesman

problem (MTSP). A two phased algorithm is implemented as solution. The first phase is concerned with a parallel tour building procedure which develops routes simultaneously, while the second phase improves on these developed routes by trying to find the best schedule, based on a set of constraints. This approach yielded good results and had a short computational time, but was only tested with a very small dataset, consisting of four nurses and 10 patients.

## 2.4 Selection of Solution Techniques

Based on the literature it is clear that there are several solution approaches that have proved successful in solving the home care problem as well as the NRP and VRP. The use of hybrid approaches, combining the strengths of heuristics and metaheuristics, seems to be the norm. In the attempt to develop a robust generic solution algorithm for solving the home-based care assignment, routing and scheduling problem in South Africa, an appropriate combination of metaheuristics should be chosen with the intent of hybridisation.

The solution algorithms for the home-based care problem discussed in the literature have been developed for very problem specific situations. Since the South African home-based care problem also contains problem specific restrictions it is decided to approach the problem as a combinatorial NRP and VRPTW.

From the conducted literature-study the dominant use of two metaheuristics for the NRP became apparent — The GA and the VNS. As stated by Aickelin and Dowsland (2000), the conflicting constraints often encountered in the NRP inhibit the ability of GA's to consistently produce good solutions. Based on this, the VNS will be applied to the nurse rostering component of the problem. The study also indicates that the VNS, SA, TS and GA metaheuristics, or combinations thereof are suited to solve the vehicle routing component of the problem. This part of the problem requires determining a good sequencing of the set of jobs determined by the VNS for a specific care worker. TS presents the advantage of requiring less computational time than SA and GA (mainly as a result of algorithm complexity) while still obtaining good solutions. It is also suitable to be used in conjunction with other metaheuristics. Since a hybrid approach will be used and Li et al. (2007) mention the successful use of a VNS with TS, the TS is chosen to solve the VRP.

Both the VNS and TS are metaheuristics that improve an initial solution received as input to the algorithms. A construction heuristic needs to be developed in order to generate an initial solution. Burke et al. (2010) introduce a heuristic ordering method as construction heuristic to solve the NRP. An adaptation of this algorithm will be developed to serve as construction algorithm.

The solution of the home-based care problem thus entails a hybrid approach which combines the strengths of the VNS and TS metaheuristics as well as a construction heuristic. The VNS will be applied to the NRP and will investigate possible improving shift and care worker switches. Simultaneously, a TS will be used to determine the best sequencing of this proposed change and the resulting travel time.

## 2.5 Conclusion

This chapter presented a look into the problem of the optimal assignment and routing of care workers in a home-based care scenario. The home-based care problem has been identified as a combination of two NP-hard problems—The Nurse Rostering Problem and the Vehicle Routing Problem. Both these problems are difficult to solve optimally and metaheuristics are most often applied to get a good solution. Several solution approaches exist to solve this problem - each with its own merits. A hybrid approach, combining the Variable Neighbourhood Search and Tabu Search metaheuristics is deemed suitable to solve the home-based care problem. In the following chapter the discussion of the chosen solution approaches will be extended and the adapted conceptual solution algorithm for the home-based care problem presented.

# Chapter 3

## Solution Algorithm

The home-based care problem is a combination of two well-known NP-hard problems - The Nurse Rostering Problem and the Vehicle Routing Problem with time windows. Due to the complexity associated with the problems, and the vast computational time required to solve them optimally, it is decided to employ a hybridised metaheuristic approach. Although this approach will not be able to guarantee optimality, both the Variable Neighbourhood Search (VNS) and the Tabu Search (TS) have demonstrated their ability to obtain near optimal solutions.

The remainder of this chapter describes the proposed solution algorithm. Firstly, the individual algorithms will be discussed after which the conceptual hybrid algorithm will be presented. Table 3.1 presents a list of the terminology that will be used frequently throughout this chapter:

Table 3.1: Solution Algorithm Terminology

| Term        | Description   |
|-------------|---|
| $J_i$       | Care activities that need to be performed, $i \in I$                          |
| $N_n$       | Set of care workers, $n \in N$  |
| $R^n$       | Roster for care worker $n$ consisting of all jobs assigned to the care worker |
| $M_n^T$     | Maximum allowed daily schedule duration for care worker $n$                   |
| $p_{jn}$    | Penalty value associated with assigning activity $j$ to care worker $n$       |
| $T^n$       | Visiting schedule for care worker $n$   |
| $S'$        | Set containing individual visiting schedules for all care workers             |
| $t_k^{(n)}$ | Starting time of job $k$ , for care worker $n$ , $k \in J$                    |
| $k_{max}$   | Stopping criteria for VNS   |
| $t_{max}$   | Stopping criteria for TS  |
| $T'_{opt}$  | Incumbent TS duration   |
| $S'_{opt}$  | Incumbent schedule solution   |
| $N_k$       | Neighbourhood structure of VNS  |

## 3.1 Solution Algorithm

Due to the complexity of solving the home-based care problem optimally, a hybrid solution algorithm is proposed. The algorithm will combine the VNS and TS metaheuristics. The sequential development of this algorithm will be based on the sequence used by Bertels and Fahle (2006) in their solution approach for the home-based care problem. The steps, discussed in more detail subsequently, include: Pre-processing of data, development of an initial solution, and improvement of the care activity assignment and the sequencing of the individual rosters.

### 3.1.1 Pre-processing of data

The aim of this step is to reduce the data complexity of a problem. A 0-1 matrix is developed which indicates care worker/care activity compatibility with regard to skill and time windows. A one will be an indication of a care activity with a time window that coincides with the time window of a care worker who possesses the skills necessary to perform the activity. A zero is an indication of incompatibility. In the case where there exist only one care worker who is compatible with a care activity, the care activity,  $J_i$ , is fixed in the care worker's roster.

This concept will be explained through the matrix in Table 3.1. As the matrix indicates the only critical compatibility combination exist for care activity  $J_3$ , which only care worker  $N_2$  can perform. All the other care activities have more than one care worker that can perform the specific activities, and are therefor not regarded as critical.

Table 3.2: Compatibility Matrix

|       | $N_1$ | $N_2$ | $N_3$ | $N_4$ | $N_5$ |
|-------|-------|-------|-------|-------|-------|
| $J_1$ | 1     | 0     | 1     | 1     | 0     |
| $J_2$ | 1     | 1     | 0     | 0     | 1     |
| $J_3$ | 0     | 1     | 0     | 0     | 0     |
| $J_4$ | 1     | 1     | 1     | 1     | 0     |
| $J_5$ | 0     | 1     | 1     | 0     | 1     |

### 3.1.2 Construction of an initial solution

Both the VNS and TS are metaheuristics which find good solutions through the iterative improvement of a feasible solution. It is thus necessary to construct an initial feasible solution as input to the metaheuristics. As stated before, the quality of the final solution is highly dependent on the initial solution of the problem since this will direct the search techniques employed by the metaheuristics. A heuristic ordering method, similar to the one applied to the NRP by Burke et al. (2010), is proposed for the development of an initial feasible solution. This feasible solution  $S = \{R^{(1)}, \dots, R^{(N)}\}$  consists of  $N$  rosters, where  $R^{(n)} = \{J_1^{(n)}, \dots, J_k^{(n)}\}$  represents

the roster for each care-worker. The pseudo-code for the construction heuristic of the initial solution can be viewed in Algorithm 1.

Evaluation criteria for determining the difficulty of a care activity is predetermined and a weight is assigned to each criterion. The weighted sum of the difficulty of assignment for each care activity (except those already assigned during the pre-processing phase) is determined and the care activities are sorted from most to least difficult. The care activities are then, in turn, assigned to the care worker that incurs the least penalty,  $p_{jn}$ , in receiving the activity. Maximum allowable working hours for care workers, a constraint which is not considered by Burke et al. (2010), will also be incorporated into the construction heuristic. During the assignment of care activities the maximum allowed work time,  $M_n^T$ , of each care worker may not be exceeded. The assignment should also endeavour to equally distribute the workload of each care worker as far as possible.

Since the main objective of the solution is to determine the optimal routing which will result in the shortest schedule duration for each care worker, the duration of the care worker schedule should now be determined. For the initial solution, the current ordering of jobs, as determined through the preceding assignments will be used. This will result in a solution  $S' = \{T^1, \dots, T^N\}$  where  $T^{(n)} = \{(j_0^{(n)}, t_0^{(n)}), \dots, (j_k^{(n)}, t_k^{(n)}), (j_{k+1}^{(n)}, t_{k+1}^{(n)})\}$ , and  $t$  is the starting time for each corresponding job. The starting time  $t$  for each job is calculated by adding the duration of the previous job and the travelling time,  $d_{ij}$ , between two nodes to the starting time of the previous job. The nodes  $j_0$  and  $j_{k+1}$  are also included in this schedule to incorporate the travelling time to and from the care worker's home or the central depot as well.

### 3.1.3 Variable Neighbourhood Search

The Variable Neighbourhood Search (VNS) is a simple and effective metaheuristic for combinatorial problems which applies the systematic change of neighbourhoods within a local search algorithm with the aim to find an improved solution (Hansen and Mladenović, 2001). This is a relatively novel approach and differs from other metaheuristics with regard to search space (Burke et al., 2010). Although the larger search space is mostly the attractive feature of this metaheuristic, constructing the correct neighbourhood structures is of vital importance for the success of the metaheuristic (Hansen and Mladenović, 2001). The basic structure of the VNS, as adapted from Curtois et al. (2008), is presented in Algorithm 2.

The algorithm starts by determining an initial feasible solution within the defined neighbourhood structures (Step 0). A local search technique then allows the search for possible improved solutions within increasingly distant neighbourhoods (Step 2). An incumbent solution tracks the best solution found and is updated only when an improved solution is found (Step 3). Once all the neighbourhoods have been searched and an improved solution found the algorithm terminates (Step 1).

The VNS will be applied to the nurse rostering component of the home-based care problem. The proposed VNS algorithm is developed combining aspects identified in the work of Curtois et al. (2008) and Burke et al. (2010).

The initial solution  $S = \{R^{(1)}, \dots, R^{(N)}\}$ , as obtained from the heuristic ordering

---

**Algorithm 1:** Pseudo-code of the construction heuristic

**Input** : Penalty value  $p_{nj}$  for assigning care worker  $n$  to job  $j$ .

Criteria for difficulty of job assignment.

Weight factor for each criterion.

Distance matrix.

**Output:** Roster set  $S = \{R^{(1)}, \dots, R^{(N)}\}$ .

Schedule set  $S' = \{T^{(1)}, \dots, T^{(N)}\}$ .

**Step 1:** *Compute difficulty of shift assignment.* Determine the weighted sum of the difficulty of assignment for each care activity  $j \in J$ .

**Step 2:** *Determine order<sub>j</sub>.* Sort the care activities in a descending order according to difficulty of assignment.

**Step 3:** *Assign.* According to  $order_j$ , assign the care activity to the care worker who will incur the smallest penalty,  $p_{jn}$ , without violating the  $max\_time_n$  constraint of the care worker. Once a care activity has been assigned remove it from the list. Repeat this step until all care activities have been assigned.

**Step 4:** *Compute total schedule duration.* For each roster  $R^{(n)}$ , determine the corresponding schedule  $T^{(n)}$  and total schedule duration for each schedule.

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**Algorithm 2:** High Level Variable Neighbourhood Search Algorithm

**Step 0: Initialisation.** Define a set of neighbourhood structures  $N_k$ , where  $k = 1, \dots, k_{max}$  and determine a feasible initial solution  $x$ . Set the best solution to the current solution  $x_{opt} \leftarrow x$  and set  $k \leftarrow 1$ .

**Step 1: Stopping.** If  $k = k_{max}$  then stop. The algorithm only terminates after all neighbourhoods have been explored.

**Step 2: Explore.** Explore the neighbourhood  $N_k$  of  $x$  until the best solution  $x'$  in  $N_k$  is found.

**Step 3: Incumbent Solution.** If  $x'$  is better than  $x_{opt}$ , replace  $x_{opt} \leftarrow x'$ .

**Step 4: Increment.** If  $x_{opt} \leftarrow x'$  set  $k \leftarrow 1$  and return to step 1, else set  $k \leftarrow k + 1$  and return to step 1.

---

method described in Section 3.1.2, will be used as the initial solution. A VNS is then applied to the solution. The following adjustments to neighbourhood structure define the neighbourhoods of the VNS:

1. Swapping a care activity assigned to a care worker, with an assigned care

activity of another care worker.

2. Swapping two care activities of the same care worker between different days in the schedule.

The TS, which will be discussed in the next section, is embedded in the VNS. The purpose of the TS is to determine the best routing for each proposed move of the VNS in order to determine the total time of the proposed roster. The total schedule time will be an indication of the quality of the proposed move.

From the initial solution, the search within the first neighbourhood commences. This search induces a single swap of a care-activity from a pair of care-workers. The quality of the proposed swap is then evaluated by calculating the total schedule time of the new proposed roster with the TS. If the best solution (shortest schedule) obtained in the current neighbourhood is not an improvement of the solution in the previous neighbourhood, the algorithm extends its search to search for another move that will result in an improvement. Otherwise, the solution is updated as the incumbent solution and the algorithm returns to the first neighbourhood of the new solution. When there are no more improving moves left within the first neighbourhood, or the stopping criteria is met, the search extends to the second neighbourhood.

The search within the second neighbourhood proposes an exchange between care activities in a specific care worker's schedule. This search is conducted for each of the care workers. If the proposed move results in an improved solution, the incumbent solution is updated to the current result. This process is repeated until the stopping criteria is met.

The process is illustrated by the pseudo-code in Algorithm 3.

### 3.1.4 Tabu Search

The TS is an established metaheuristic and has proved successful in numerous applications to the VRPTW. It prevents the tendency of getting stuck at local optima by also allowing non-improving moves in the improving search (Rardin, 1998).

Algorithm 4 illustrates the high-level structure of the TS as adapted from Rardin (1998). This algorithm will be adapted and applied to the scheduling component of the home-based care problem. As can be seen from the high level algorithm, the first step of the TS is to determine an initial feasible solution. Although any feasible solution will be sufficient, the solution quality of the TS is notorious for its sensitivity to the initial solution. Since the TS is embedded within both neighbourhoods of the VNS, the proposed swap of the VNS will supply the initial solution to the TS. The search space of the TS is confined to only the jobs within the specific roster under examination in the VNS. The main purpose of this algorithm is to generate solutions with improved schedule ordering and resultantly shorter duration. Every allowable, non-tabu move should be evaluated. If the resulting schedule time from the proposed move is shorter than the current best solution, the incumbent solution has to be updated. For each swap within the roster, the activity with which another activity is replaced has to be moved to the tabu list for a specific duration. This will prevent subsequent searches from proposing the same move. This is an iterative

---

**Algorithm 3:** Pseudo-code for Variable Neighbourhood Search Algorithm

**Input** : Initial Solution Roster set  $S = \{R^{(1)}, \dots, R^{(N)}\}$ .  
Initial Solution Schedule set  $S' = \{T^{(1)}, \dots, T^{(N)}\}$ .  
**Output:** Improved Roster set  $S_{opt} = \{R^{(1)}, \dots, R^{(N)}\}$ .  
Improved Schedule set  $S'_{opt} = \{T^{(1)}, \dots, T^{(N)}\}$ .

**Neighbourhood 1: Assigning a care activity to a different care worker**

Set  $k \leftarrow 1$

**while**  $k \leq k_{max}$  **do**  
Explore the neighbourhood  $N_k$ . For each proposed move  
calculate duration of  $S'$ .  
**if**  $S' \leq S'_{opt}$  **then**  
     $S'_{opt} \leftarrow S'$   
    set  $k \leftarrow 1$   
**else**  
    set  $k \leftarrow k + 1$

**Neighbourhood 2: Swapping care workers of roster sets**

**for** *care workers*=1:n **do**

Set  $k \leftarrow 1$

**while**  $k \leq k_{max}$  **do**  
Explore the neighbourhood  $N_k$ . For each proposed move  
calculate duration of  $S'$ .  
**if**  $S' \leq S'_{opt}$  **then**  
     $S'_{opt} \leftarrow S'$   
**else**  
    set  $k \leftarrow k + 1$

---

---

**Algorithm 4: High Level Tabu Search Algorithm**

**Step 0: Initialisation.** Determine a feasible initial solution  $\mathbf{T}^{(0)}$  as well as an iteration limit  $t_{max}$ . Set the current best solution  $\mathbf{T}_{opt} \leftarrow \mathbf{T}^{(0)}$  and the iteration count  $t \leftarrow 0$ . Currently the tabu list is empty.

**Step 1: Stopping.** If  $t = t_{max}$ , or if no move can be made to a feasible neighbour of  $\mathbf{T}^{(t)}$ , then stop.  $\mathbf{T}_{opt}$  is then used as the approximate optimum.

**Step 2: Move.** Pick an allowable feasible non-tabu move  $\Delta\mathbf{T}^{(t+1)}$ . Update  $\mathbf{T}^{(t+1)} \leftarrow \mathbf{T}^{(t)} + \Delta\mathbf{T}^{(t+1)}$ .

**Step 3: Step.** Update  $\mathbf{T}^{(t+1)} \leftarrow \mathbf{T}^{(t)} + \Delta\mathbf{T}^{(t+1)}$ .

**Step 4: Incumbent Solution.** If  $z(\mathbf{T}^{(t+1)})$  is better than  $z(\mathbf{T}_{opt})$ , replace  $\mathbf{T}_{opt} \leftarrow \mathbf{T}^{(t+1)}$ .

**Step 5: Tabu List.** Remove any tabu moves that have been on the tabu list long enough. Also, add new moves including any that would immediately return from  $\mathbf{T}^{(t+1)}$  to  $\mathbf{T}^{(t)}$ .

**Step 6: Increment.** Increase  $t \leftarrow t + 1$  and go to step 1.

---

process and should be repeated until the stopping criterion,  $t_{max}$ , is met. The stopping criteria and the tenure duration of a value in the tabu list will determine solution quality. The incumbent solution,  $\mathbf{T}'_{opt}$ , recorded upon termination of the algorithm, is regarded as the approximate optimum solution.

## 3.2 Hybrid algorithm

The solution algorithm presented in this paper represents an iterative process in which a TS is embedded within a VNS. This is preceded by a heuristic ordering algorithm which constructs an initial feasible solution as input to the VNS. This overall process is illustrated by the Pseudo-code in Algorithm 5.

## 3.3 Conclusion

This chapter presented the conceptual hybrid solution algorithm that is proposed to solve the home-based care problem in South Africa. This proposed algorithm combines the strengths of the VNS and TS metaheuristics. It also includes a heuristic ordering algorithm which is employed to construct an initial feasible solution.

The remainder of the report will consist of the computational results obtained by applying this solution algorithm to a real-world case. This will allow further

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**Algorithm 5:** Pseudo-code for overall hybrid algorithm

**Create initial solution with heuristic ordering method**

**REPEAT : For Neighbourhood 1**

Variable Neighbourhood Search

For each proposed move apply **Tabu Search**

**if** *current duration*  $\leq$  *best duration* **then**

    set best schedule  $\leftarrow$  current schedule

    set best duration  $\leftarrow$  current duration

**else**

    set current schedule  $\leftarrow$  best schedule

**UNTIL : Search Terminated - Proceed to Neighbourhood 2**

**REPEAT : For Neighbourhood 2**

Variable Neighbourhood Search

For each proposed move apply **Tabu Search**

**if** *current duration*  $\leq$  *best duration* **then**

    set best schedule  $\leftarrow$  current schedule

    set best duration  $\leftarrow$  current duration

**else**

    set current schedule  $\leftarrow$  best schedule

**UNTIL : Search Terminated**

---

analysis and improvement of the algorithm. The concluding chapter will discuss further improvements and relating research that can still be done.

# Chapter 4

## Model Execution

In this chapter the proposed solution algorithm is verified and tested. This is done by applying data from Heartbeat’s Nellmapius project site, and comparing the results.

### 4.1 Heartbeat

The Nellmapius project currently serves 132 households. The home visits are conducted by five care workers with the purpose of assessing the need of the children, addressing problems within the household, and providing assistance with everyday activities.

The recommended frequency of visits per target group is illustrated in the table below:

Table 4.1: Recommended frequency of home-visits (UNAIDS, 2009)

| <b>Target Group</b>                       | <b>Frequency</b>     |
|---|----------------------|
| Child-Headed Household                    | Three times per week |
| Potential Child-Headed Household          | Three times per week |
| Relative-Headed Household (without grant) | Once a week          |
| Relative-Headed Household (with grant)    | Once a month         |

The purpose of the high frequency contact between care workers and child-headed households as well as potential child-headed households (children living with terminally ill parents), is to provide stability to these homes by means of assigning the responsibilities of parents to the care workers. For this reason it is highly recommended that a specific care worker should be assigned to each of these households, to ensure stability. Due to the large need in Nellmapius, Heartbeat’s ideal ratio of 1 care worker for every 10 households is exceeded by far (1:26), and this is severely impairing the quality of the care. This also often leads to variation of care workers with the critical visits.

A complete map of Nellmapius, with the locations of the care workers and the various households, is presented in Figure 4.1.

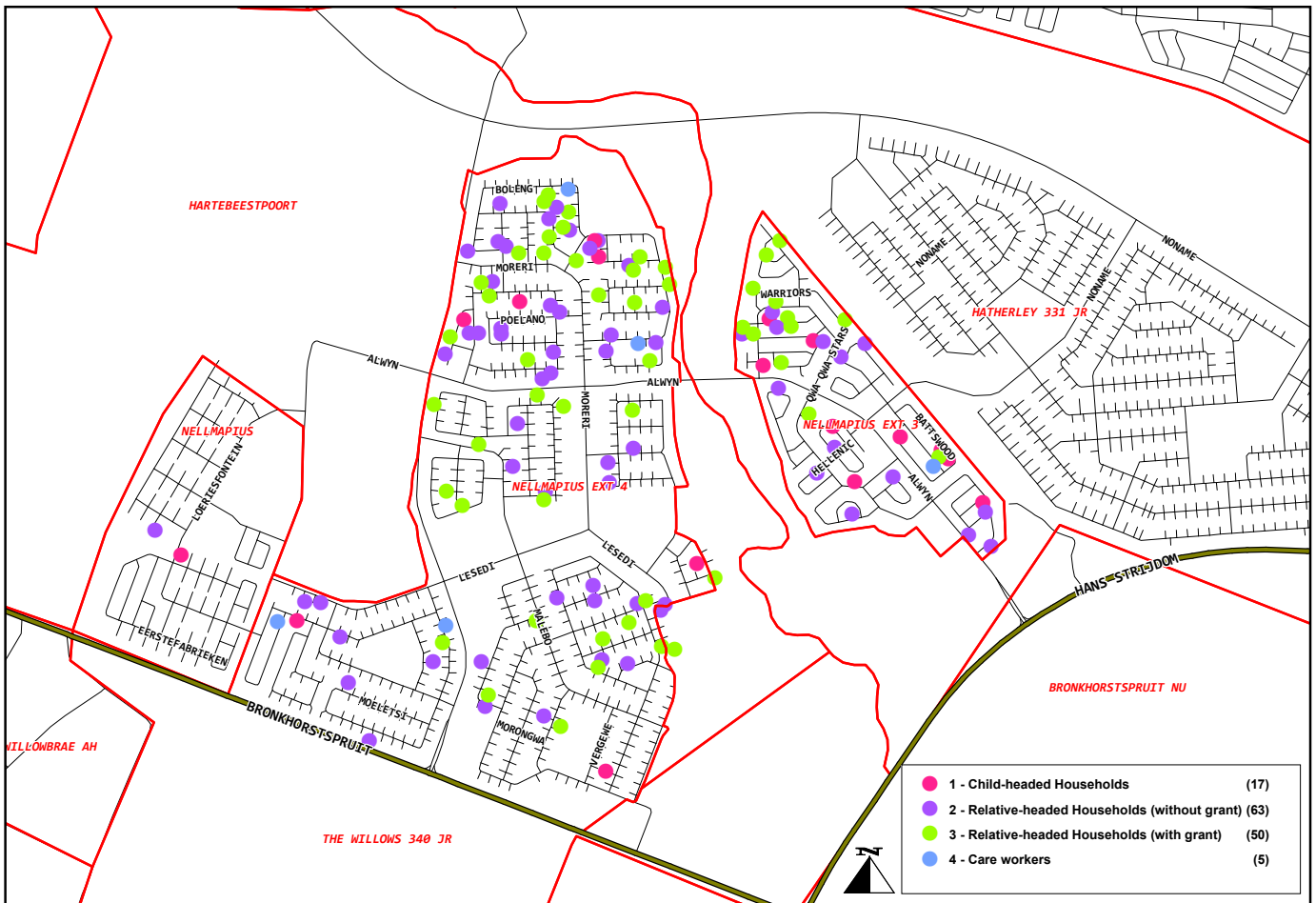


Figure 4.1: Geographical representation of Heartbeat care worker and household location in Nellmapius, Gauteng

Currently no structured approach is being followed for the assignment of visits to care workers—resulting in unfulfilled demand of visits. This is mainly a result from OVC not being at home when a care worker wants to conduct a home visit, because no prior arrangements were made. These visits are seldom followed up, due to a lack of time and poor monitoring of performed visits. The geographical dispersal of the households also contribute to this problem. The households located on the outskirts of the settlement, furthest from the care workers, are often neglected because of the extra effort required to perform visits at these households.

The following sections will examine the results of the solution algorithm for solving the home-based care problem of OVC for Heartbeat’s Nellmapius project.

## 4.2 Route and Schedule construction

### 4.2.1 Data gathering and Preparation

Applicable data, to be used as input for the algorithms described in Chapter 3, is a critical component of the solution algorithm. For this reason it was necessary to follow an approach that would transform the raw data received from Heartbeat, into processable input data for the solution algorithm.

Heartbeat is currently in the process of conducting a census to collect information to update their data basis. A list of the households in Nellmapius served by Heartbeat, containing addresses, type of household and the number of beneficiaries, collected during this census, was provided by Heartbeat.

The data was cleaned and standardised, and through the process of geocoding the geographic coordinates of the addresses, expressed as latitude and longitude, were obtained. The geographic coordinates allow the data to be mapped and entered into geographical information systems (GIS). The data pertaining to the household information is contained in Appendix A.

The most critical data set for the solution algorithm is the internodal distances. The GIS programme, MapInfo, was used to determine the road distances between all the households and care workers, and an internodal distance matrix was constructed. The distance matrix contains 18,084 combinations, and therefore requires ample space. A partial distance matrix can be viewed in Appendix B, and the complete distance matrix is included on the compact disc accompanying the document.

Of the 132 households served by Heartbeat in Nellmapius, 17 are child-headed or potential child-headed households, 64 are relative-headed households not receiving grants, and 51 are relative-headed households receiving grants. In order to meet the frequencies of visits prescribed in Table 4.1, for the 132 households, a total of 511 visits need to be conducted during a four-week period.

As discussed in Chapter 3, the development of care worker schedules to perform these 511 visits was done through the following stages:

- Pre-processing of data.
- Construction of an initial solution.
- Improvement of initial solution.

## 4.2.2 Pre-processing of data

The care workers employed by Heartbeat are all community members who received the same informal training from Heartbeat. Since no skill variation exist between the care workers, it is proposed that the critical visits—the child-headed households and potential child-headed households—are permanently assigned to a specific care worker. These assignments should preferably be made to a care worker within a close proximity. This will ensure that a trusted adult is nearby in case of an emergency, prevent care worker variation and as a result add to the sense of stability that is provided.

The following table depicts the assignment of the critical visits to care workers, where Household ID refers to the unique identification Heartbeat uses for the specific household.

These visits are fixed to the schedule of the applicable care worker and won't be adjusted in the following steps.

Table 4.2: Pre-processing: Assignment of critical visits to care workers

|                     | <b>Worker 1</b>  | <b>Worker 2</b> | <b>Worker 3</b> | <b>Worker 4</b> | <b>Worker 5</b> |
|---------------------|------------------|-----------------|-----------------|-----------------|-----------------|
| <b>Household ID</b> | 28, 46, 116, 124 | 48, 107         | 19, 53, 90      | 25, 32          | 2, 8, 9, 16, 55 |

### 4.2.3 Initial Solution

The construction heuristic, elucidated in Algorithm 1, was used for the construction of the initial solution.

The remaining visits, not assigned during the pre-processing phase, were ordered in a descending order from the most difficult to least difficult to assign. The criteria used was the type of visit, with relative-headed households not receiving grants regarded more critical than relative-headed households receiving grants.

The visits were then assigned to the care worker who will incur the smallest penalty for conducting the visit. The penalty is calculated taking into consideration the distance between the care worker and the household, as well as the number of visits already performed by the care worker.

This assigned visits were then ordered into a schedule, to serve as an initial solution.

The results obtained from the construction heuristic are presented in Appendix C. For each worker a five day schedule, over a four week period is constructed. The numbers indicate the specific Heartbeat identification for the household, as defined by Visit ID in Appendix A.

Although the solution algorithm focuses on the effective assignment of visits to care workers, and the improved routing and scheduling of the assigned visits, it was necessary to include visit duration for computation purposes. The total schedule duration includes the travelling time between the nodes as well as the visit duration, and highlights the trade-off between the travelling distance of the care worker and the number of visits conducted by the care worker.

Because the visits conducted by care workers usually takes approximately 30 minutes, it was decided use a normal distribution for visit durations , with a mean of 30 minutes and standard deviation of 2 minutes.

### 4.2.4 Improved Solution

The hybrid solution algorithm was applied to the results obtained from the initial solution with the intent of improving this solution.

The first variable neighbourhood search (VNS) of the solution algorithm was performed in the extended neighbourhood of the schedules of all the care workers. Visits, assigned to the care workers through the construction heuristic, were exchanged between the care workers in order to reduce the total schedule duration and to achieve more equal schedule durations between the care workers. This resulted in a fair assignment of care activities where a trade-off was made between travelling distance and duration, and the number of visits, so that the care worker of whom it is required to travel further distances, will perform less visits.

Once this search was completed, the algorithm advanced to the second neighbourhood structure. During this search the effect of exchanging visits in an existing schedule of a care worker was evaluated for each care worker.

For each of these searches a VNS was executed for 200 iterations, with an embedded tabu search (TS) performing 200 iterations within each iteration of the VNS to determine the optimal routing for each proposed swap of the VNS.

### 4.3 Results

The improved care worker schedules, as obtained from the solution algorithm are included in Appendix D.

For illustration purposes Figure 4.2 provides a typical representation of an approximated optimal daily route for care workers in Nellmapius as obtained from the solution algorithm.

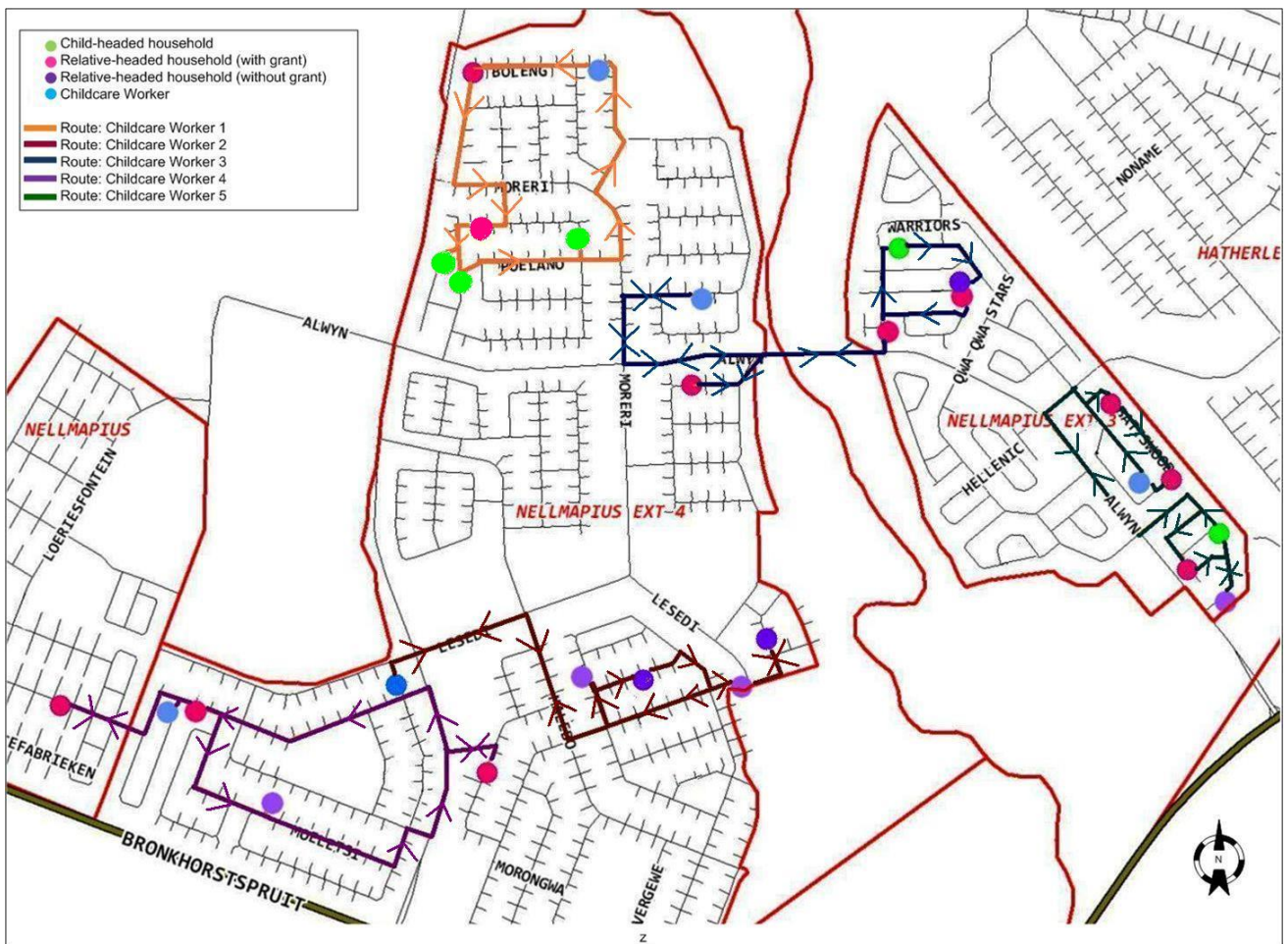


Figure 4.2: Geographical representation of an approximated optimal route for care workers (Week 4, Day1)

Figure 4.3 presents a detailed representation of the approximated daily route for care worker 1 presented in Figure 4.2. Care workers depart from their own

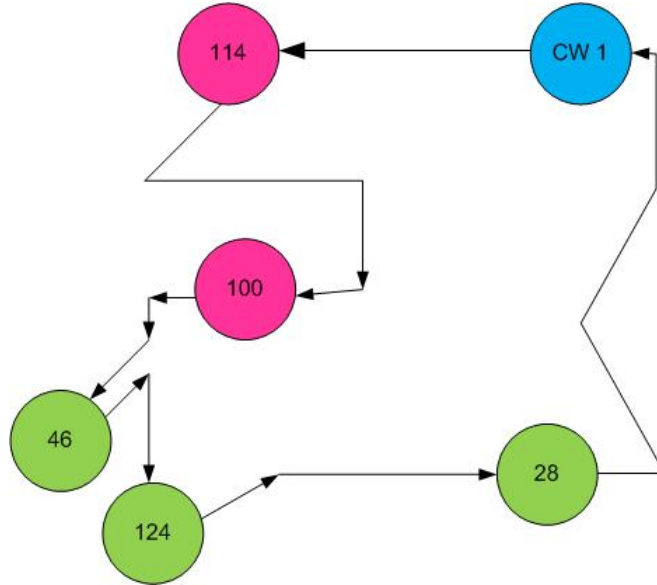


Figure 4.3: Detailed route: Care Worker 1 (Week 4, day1)

residences (light blue nodes) and then perform home visits in the order proposed by the improved schedule. When travelling between nodes, it is assumed that the care worker will follow the shortest route, which is predetermined by the GIS programme. The estimated travelling durations, service times, arrival and departure times for the route presented in Figure 4.3 are presented in Table 4.3:

Table 4.3: Arrival and departure times for an approximated optimal route for care worker 1

| Location      | Arrival Time | Service Time (minutes) | Departure Time |
|---------------|--------------|------------------------|----------------|
| Care Worker 1 | N/A          | N/A                    | 14:00          |
| Household 114 | 14:05        | 31                     | 14:36          |
| Household 100 | 14:40        | 30                     | 15:10          |
| Household 46  | 15:12        | 29                     | 15:41          |
| Household 124 | 15:42        | 30                     | 16:12          |
| Household 28  | 16:17        | 28                     | 16:45          |
| Care Worker 1 | 16:49        | N/A                    | N/A            |

A summary of the results of the final improved worker schedules, included in Appendix D, which resulted from the solution algorithm is presented in Table 4.4. From this results it becomes evident that the solutions obtained from the algorithm address the problems currently experienced by Heartbeat, in the following ways:

**Fair work distribution** The number of visits assigned to each care worker varies between four and six per day. Taking into consideration both the travelling duration of the care worker and the number of visits performed the total schedule duration of each care worker compares favourably the other care workers’.

Table 4.4: Summary of algorithm results

| <b>Objective Function</b>     | <b>Worker 1</b> | <b>Worker 2</b> | <b>Worker 3</b> | <b>Worker 4</b> | <b>Worker 5</b> |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Total solution duration (min) | 3671.45         | 3594.39         | 3651.22         | 3602.66         | 3670.38         |
| Duration Week 1               | 941.23          | 902.18          | 909.06          | 879.71          | 913.50          |
| Duration Week 2               | 912.52          | 900.81          | 918.78          | 900.29          | 919.38          |
| Duration Week 3               | 908.94          | 894.47          | 916.14          | 908.80          | 919.64          |
| Duration Week 4               | 908.76          | 896.93          | 907.24          | 913.86          | 917.86          |
| Total Visits                  | 109             | 92              | 107             | 91              | 111             |
| Max Daily visits              | 6               | 5               | 6               | 5               | 6               |
| Min Daily visits              | 5               | 4               | 5               | 4               | 5               |

**Effective routing and scheduling** The improved routing and scheduling of the initial solution resulted in care worker schedules not exceeding 3.5 hours on any day. This allows care workers to perform the visits in the afternoon, after school, between 2pm and 6pm.

**Limited care worker variation** By fixing the visits of child-headed households to specific care workers' schedules, it is ensured that these visits will always be conducted by the same care worker. This will limit the variation of care workers for these critical visits, and enable the children to develop a trusting relationship with the specific care worker. By assigning these visits to the care worker living closest to the family will also add to the security of the child-headed households by ensuring that an adult with whom the children have developed a trusting relationship is close by in case of an emergency.

**Visit planning** A big advantage of the assignment, scheduling and routing algorithm is that the entire monthly schedule can be determined in a short duration and enable the care workers to make appointments with the children in advance, which allows the children to plan to be at home when the visit is to be conducted.

## 4.4 Algorithm Performance

Because Heartbeat's current home visit process is extremely ineffective, there are no results that can be used as benchmark for evaluating the performance of the solution algorithm.

As discussed in chapter 2 and chapter 3, using an exact algorithm to solve a problem with this complexity and magnitude optimally, is usually either impossible, or very ineffective due to the vast amount of computation time required.

For the purpose of evaluating the algorithm performance it was decided to replace the final routing component of the algorithm with an exact algorithm. An integer linear programme was designed and solved using LINGO®, an optimisation

modelling software. The LINGO® model that was used can be viewed in Appendix E.

The solution was also compared to the preferred constraint of limiting the daily schedule duration to four hours, in order to perform visits during day time, between 2pm and 6 pm.

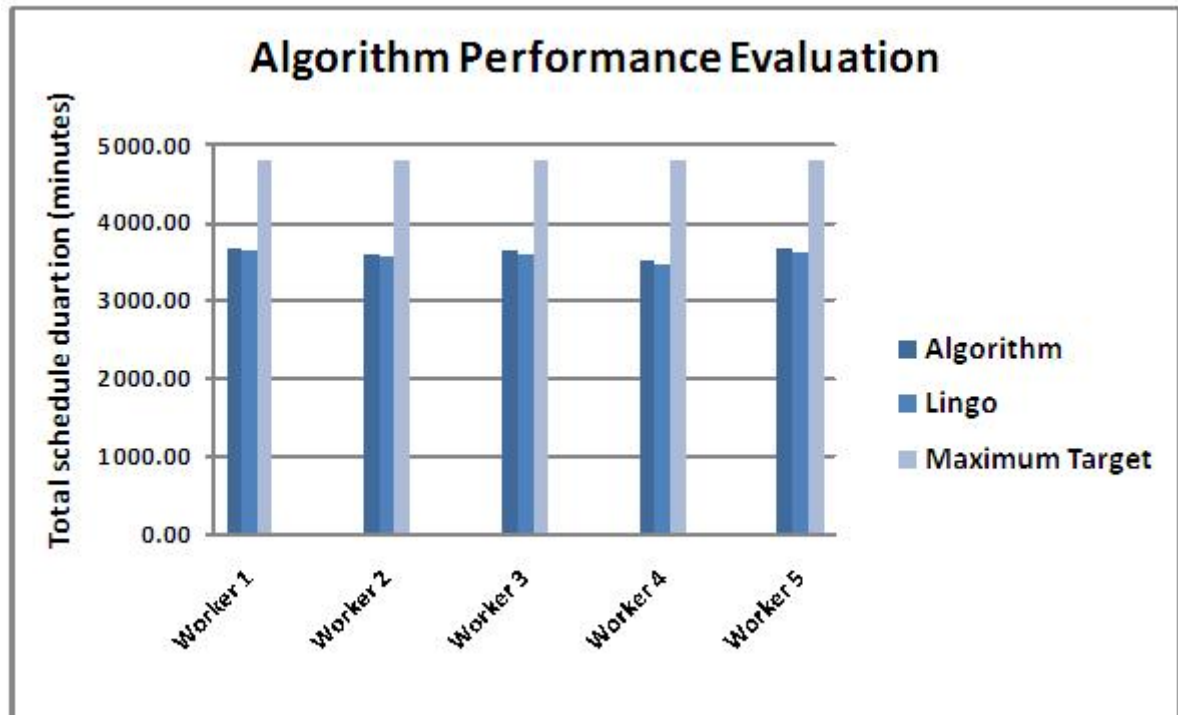


Figure 4.4: Algorithm Performance

Figure 4.4 depicts the results of the performance of the solution algorithm compared to the solution obtained from using LINGO® and the maximum schedule duration constraint, for each worker for the entire four week period. As indicated by the graph, the solution algorithm performs favourably by exceeding the required algorithm performance by an average of 23.51%. The exact solution component included for evaluation purposes also presents an almost insignificant improvement of only 1.81% on average.

Appendix F contains the complete algorithm evaluation results per week, for each of the care workers.

## 4.5 Conclusion

This chapter presented the process of applying the solution algorithm developed in this project to a real world case, as well as the results obtained from this process.

The improvement of home visits at Heartbeat’s Nellmapius project site through the solution algorithm resulted in all the required visits being performed during a four week period, no care worker’s daily schedule exceeding 3.5 hours, and all care workers working approximately the same duration over this period.

The evaluation of the algorithm's performance revealed very satisfactory results, with the algorithm performing 23.51% better than the maximum schedule duration constraint. Replacing the final routing component of the approximation algorithm with an exact routing algorithm also only resulted in a mere 1.81% improvement.

The following chapter will discuss the work that can still be done to further improve the results of the algorithm and ensure effective implementation of the algorithm.

# Chapter 5

## Conclusion

### 5.1 Future Work

Despite the good results obtained from applying the solution algorithm, there still exist ample opportunities for further improvements. The most prominent and critical of these improvements is the development and implementation of a user interface. This will obviate the need for the user to master the programming language. To mitigate the possibility of human errors, the skill of the user need to be taken into consideration and the minimum amount of data should be entered manually.

The solution algorithm requires an inter-nodal distance matrix as input. The data used for testing, with regard to the inter-nodal distances of Nellmapius, was created with the help of a geographical informations system (GIS) specialist. Applying the solution algorithm to other problems will also require the input data regarding the distances between the applicable nodes. Without the aid of a GIS specialist, one way of obtaining these values is the physical measurement of the distances, but this will be extremely time consuming for larger networks. It is proposed that the implementation of a GIS is investigated. Not only will this produce an accurate inter-nodal distance matrix within a short period of time, but can also provide useful information such as speed limits, obstructions in the road and the infrastructure of the environment.

For implementation purposes the solution algorithm was programmed in *GNU Octave*®, an open source programming language.

### 5.2 Recommendations for Implementation

The aim of this project states the need to improve the service delivery of home-based care in South Africa. Despite the improvements made by the application of the solution algorithm, it alone will not result in the improvement of home-based care. Implementation and thorough evaluation of this algorithm is critical to the success of this proposed solution.

Strebel (2004) lists several key issues that should be considered and addressed when implementing a programme for OVC, such as the proposed solution algorithm. These include:

- Close monitoring and evaluation, and early intervention and correction is essential to ensure programme viability.
- Necessity of beneficiary involvement in implementation of OVC models to highlight possible pitfalls.
- Due to the large and continually increasing number of OVC, involvement and training of community members as care workers is a good solution to the resource problem.
- For optimal OVC home care the focus should be expanded from physical and developmental needs to include psychosocial needs of OVC as well, making counselling training essential for care workers.
- Model flexibility is crucial in order to adapt to varying community needs.
- Continuous improvements of all applied models is imperative to assure model relevance.

### 5.3 Conclusion

Home-based care is becoming more prevalent in South Africa, mainly due to the effects of the HIV/AIDS pandemic. The novelty of home-based care as well as inadequate service delivery by care workers has caused reluctance to recognise home-based care as an equal alternative of institutional care.

The aim of this project was to improve the assignment, routing and scheduling of care workers, in order to advance service delivery in the home-based care sector in South Africa.

This problem presented itself as a classical optimisation problem where the travelling duration or distance need to be minimised while adhering to constraints. It combines two well known operations research problems, the Vehicle Routing Problem with Time Windows and the Nurse Rostering Problem.

Both the Vehicle Routing Problem with Time Windows and the the Nurse Rostering Problem are non-deterministic polynomial-time hard problems. Due to the complexity of optimally solving these problems, an approximation solution algorithm was developed which combines a Variable Neighbourhood Search and Tabu Search.

The solution algorithm and a construction heuristic algorithm for initial solution were successfully constructed in the *GNU Octave*® programming environment. Algorithm performance was evaluated by examining the algorithm results from data of Heartbeat's Nellmapius project.

The algorithm can be implemented in its current state, but future projects can be undertaken for further refinement of the algorithm and to develop a user interface to ease utilisation of the algorithm.

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# Appendix A

## Household and Care Worker Information

Table A.1: Household and Care Worker Information

| ID | Address                     | X Coordinate | Y Coordinate | Visit Type |
|----|-----------------------------|--------------|--------------|------------|
| 1  | 1284 Aces Street            | 28.367462    | -25.730877   | 3          |
| 2  | 2127 African Wanders Street | 28.374857    | -25.738496   | 1          |
| 3  | 2057 African Wanders Street | 28.375145    | -25.739865   | 2          |
| 4  | 2020 African Wanders Street | 28.37376     | -25.737794   | 3          |
| 5  | 2129 African Wonders Street | 28.374933    | -25.7388     | 2          |
| 6  | 1339 Arsenal's Street       | 28.370004    | -25.734004   | 2          |
| 7  | 1833 Battswood Street       | 28.371778    | -25.7377     | 2          |
| 8  | 1958 Battswood Street       | 28.373441    | -25.736896   | 1          |
| 9  | 1961 Battswood Street       | 28.37368     | -25.737156   | 1          |
| 10 | 1294 Black Leopards Street  | 28.367928    | -25.730447   | 3          |
| 11 | 2397 Boleng Street          | 28.357844    | -25.729185   | 2          |
| 12 | 2410 Boleng Street          | 28.360712    | -25.729563   | 3          |
| 13 | 2292 Boleng Street          | 28.357255    | -25.73078    | 2          |
| 14 | 2417 Boleng Street          | 28.36073     | -25.730126   | 2          |
| 15 | 1527 Cape Town Spurs Street | 28.367875    | -25.734986   | 2          |
| 16 | 1807 Celtic Street          | 28.36972     | -25.736143   | 1          |
| 17 | 1814 Celtic Street          | 28.368919    | -25.735781   | 3          |
| 18 | 1625 Colts Street           | 28.370383    | -25.738866   | 2          |
| 19 | 1237 Costa Do Sol Street    | 28.367553    | -25.732832   | 1          |
| 20 | 1048 Costa Do Sol Street    | 28.366646    | -25.733107   | 3          |
| 21 | 1189 Costa Do Sol Street    | 28.368307    | -25.733089   | 3          |
| 22 | 1232 Costa Do Sol Street    | 28.368176    | -25.732817   | 3          |
| 23 | 1185 Costa Do Sol Street    | 28.367809    | -25.733102   | 2          |
| 24 | 2080 Crusaders Street       | 28.37437     | -25.739485   | 2          |
| 25 | 602 Delagoa Bay Avenue      | 28.347463    | -25.740112   | 1          |
| 26 | 3994 Dimakatso Street       | 28.362885    | -25.73566    | 3          |
| 27 | 3260 Dipuo Street           | 28.356478    | -25.733934   | 2          |

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Table A.1 – Continued

| ID | Address                  | X Coordinate | Y Coordinate | Visit Type |
|----|--------------------------|--------------|--------------|------------|
| 28 | 3301 Dipuo Street        | 28.357123    | -25.732859   | 1          |
| 29 | 3255 Dipuo Street        | 28.356654    | -25.733393   | 3          |
| 30 | 3373 Direlago Street     | 28.36053     | -25.73554    | 3          |
| 31 | 5540 Ditumediso Steet    | 28.352899    | -25.742648   | 2          |
| 32 | 5762 Ditumediso Street   | 28.351415    | -25.742142   | 1          |
| 33 | 3771 Emela Street        | 28.356529    | -25.738141   | 3          |
| 34 | 3737 Emela Street        | 28.357072    | -25.738586   | 3          |
| 35 | 5586 Gaugela Street      | 28.356407    | -25.742798   | 3          |
| 36 | 5622 Getrouheid Street   | 28.35608     | -25.743403   | 2          |
| 37 | 3538 Gontse Street       | 28.358964    | -25.736085   | 2          |
| 38 | 3386 Gontse Street       | 28.359627    | -25.735189   | 3          |
| 39 | 2547 Hallelujah Crescant | 28.361431    | -25.730676   | 2          |
| 40 | 2552 Hallelujah Crescant | 28.361716    | -25.73043    | 2          |
| 41 | 2823 Hallelujah Crescant | 28.363686    | -25.733587   | 2          |
| 42 | 2689 Hallelujah Crescent | 28.362155    | -25.733344   | 2          |
| 43 | 2214 Hallelujah Crescent | 28.364146    | -25.731806   | 3          |
| 44 | 2542 Hallelujah Crescent | 28.36096     | -25.73106    | 3          |
| 45 | 2218 Hallelujah Crescent | 28.364012    | -25.731257   | 3          |
| 46 | 2551 Hallelujah Crescent | 28.361585    | -25.730433   | 1          |
| 47 | 1739 Hellenic Street     | 28.36979     | -25.736801   | 2          |
| 48 | 4964 Hlagisa Street      | 28.361971    | -25.746759   | 1          |
| 49 | 5046 Hlompho Street      | 28.36044     | -25.745396   | 3          |
| 50 | 5093 Hlompho Street      | 28.35985     | -25.745065   | 2          |
| 51 | 4238 Hulp Street         | 28.363855    | -25.74183    | 2          |
| 52 | 2515 Itekela Street      | 28.358998    | -25.730817   | 3          |
| 53 | 1165 Kabylie Street      | 28.369065    | -25.733533   | 1          |
| 54 | 1055 Kabylie Street      | 28.366631    | -25.73333    | 2          |
| 55 | 1771 Kaizer Chiefs Road  | 28.370471    | -25.73785    | 1          |
| 56 | 3610 Kamogelo Street     | 28.359915    | -25.738158   | 2          |
| 57 | 3328 Kamogelo Street     | 28.359856    | -25.738409   | 3          |
| 58 | 3100 Karabo Street       | 28.360176    | -25.733872   | 2          |
| 59 | 3032 Karabo Street       | 28.360088    | -25.732425   | 2          |
| 60 | 2013 Leeds Street        | 28.372034    | -25.736488   | 1          |
| 61 | 6149 Lesedi Street       | 28.35169     | -25.741562   | 2          |
| 62 | 6156 Lesedi Street       | 28.352235    | -25.741566   | 2          |
| 63 | 5315 Lokile Street       | 28.357966    | -25.744418   | 3          |
| 64 | 5269 Malebo Road         | 28.359596    | -25.742143   | 3          |
| 65 | 3691 Malebo Road         | 28.357629    | -25.736714   | 3          |
| 66 | 3674 Malebo Street       | 28.3588      | -25.737388   | 2          |
| 67 | 5953 Moeletsi Street     | 28.353184    | -25.744042   | 2          |
| 68 | 5996 Moeletsi Street     | 28.353893    | -25.745834   | 2          |
| 69 | 4665 Mogwera             | 28.361849    | -25.743327   | 2          |

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Table A.1 – Continued

| ID  | Address                     | X Coordinate | Y Coordinate | Visit Type |
|-----|-----------------------------|--------------|--------------|------------|
| 70  | 4219 Mokwadi Street         | 28.363878    | -25.742926   | 3          |
| 71  | 4215 Mokwadi Street         | 28.364316    | -25.743014   | 3          |
| 72  | 2475 Moreri Street          | 28.35986     | -25.730829   | 3          |
| 73  | 2388 Moreri Street          | 28.358367    | -25.729295   | 2          |
| 74  | 4545 Motlotlegi Street      | 28.361588    | -25.741503   | 2          |
| 75  | 4518 Motlotlegi Street      | 28.361535    | -25.741059   | 2          |
| 76  | 1326 Ntokozo Street         | 28.370142    | -25.732859   | 3          |
| 77  | 4619 Odirile Avenue         | 28.362767    | -25.74221    | 3          |
| 78  | 4648 Odirile Avenue         | 28.361867    | -25.742695   | 3          |
| 79  | 4368 Odirile Avenue         | 28.363334    | -25.741522   | 3          |
| 80  | 4244 Odirile Street         | 28.363997    | -25.741625   | 2          |
| 81  | 4564 Odirile Street         | 28.363053    | -25.741616   | 2          |
| 82  | 3114 Ofentse Street         | 28.359298    | -25.734097   | 3          |
| 83  | 2875 Ofentse Street         | 28.360104    | -25.734512   | 2          |
| 84  | 2872 Ofentse Street         | 28.3598      | -25.734698   | 2          |
| 85  | 1966 Orlando Pirates Street | 28.373351    | -25.737111   | 3          |
| 86  | 4704 Phedisana Street       | 28.362731    | -25.743445   | 2          |
| 87  | 3148 Poelano Street         | 28.358395    | -25.733131   | 2          |
| 88  | 2963 Poelano Street         | 28.360398    | -25.732647   | 2          |
| 89  | 3197 Poelano Street         | 28.358401    | -25.733299   | 2          |
| 90  | 1080 Qwa Qwa Stars Road     | 28.367358    | -25.734275   | 1          |
| 91  | 1255 Qwa Qwa Stars Road     | 28.367783    | -25.732326   | 3          |
| 92  | 1216 Qwa Qwa Stars Road     | 28.367672    | -25.73264    | 2          |
| 93  | 1052 Qwa Qwa Stars Road     | 28.36702     | -25.733298   | 3          |
| 94  | 374 Qwa Qwa Stars Road      | 28.346576    | -25.739347   | 2          |
| 95  | 2438 Raadpleeg Street       | 28.360521    | -25.730019   | 3          |
| 96  | 2364 Raadpleeg Street       | 28.360008    | -25.72904    | 3          |
| 97  | 2467 Raadpleeg Street       | 28.360051    | -25.73032    | 3          |
| 98  | 2446 Raadpleeg Street       | 28.360291    | -25.729423   | 2          |
| 99  | 2366 Raadpleeg Street       | 28.359864    | -25.729243   | 3          |
| 100 | 2458 Raadpleeg Street       | 28.360033    | -25.729777   | 2          |
| 101 | 1349 Ravens Street          | 28.370833    | -25.733612   | 2          |
| 102 | 4717 Saamroep Street        | 28.36171     | -25.743572   | 3          |
| 103 | 4418 Saamroep Street        | 28.360306    | -25.741422   | 2          |
| 104 | 1149 Santos Street          | 28.367966    | -25.734181   | 3          |
| 105 | 1114 Santos Street          | 28.369402    | -25.733547   | 2          |
| 106 | 3450 Sebaka Street          | 28.356106    | -25.735497   | 3          |
| 107 | 4186 Serumola Street        | 28.365082    | -25.740393   | 1          |
| 108 | 3874 Serumola Street        | 28.365708    | -25.740809   | 3          |
| 109 | 2798 Setso Street           | 28.36295     | -25.732337   | 3          |
| 110 | 2787 Setso Street           | 28.363916    | -25.732492   | 2          |
| 111 | 3230 Somandla Street        | 28.357716    | -25.731728   | 3          |

Continued on Next Page...

Table A.1 – Continued

| <b>ID</b> | <b>Address</b>            | <b>X Coordinate</b> | <b>Y Coordinate</b> | <b>Visit Type</b> |
|-----------|---------------------------|---------------------|---------------------|-------------------|
| 112       | 3003 Somandla Street      | 28.357991           | -25.732156          | 3                 |
| 113       | 4058 Somandla Street      | 28.362912           | -25.736832          | 2                 |
| 114       | 3225 Somandla Street      | 28.35809            | -25.731718          | 2                 |
| 115       | 4330 Somandla Street      | 28.357716           | -25.743398          | 2                 |
| 116       | 5349 Sterkte Street       | 28.359034           | -25.732319          | 1                 |
| 117       | 2829 Sterkte Street       | 28.363487           | -25.734144          | 3                 |
| 118       | 2160 Sterkte Street       | 28.361982           | -25.733851          | 2                 |
| 119       | 1673 Strikers Street      | 28.369185           | -25.737612          | 2                 |
| 120       | 2585 Thabo Street         | 28.36291            | -25.731357          | 3                 |
| 121       | 2750 Thabo Street         | 28.363144           | -25.730934          | 3                 |
| 122       | 2583 Thabo Street         | 28.362761           | -25.731188          | 2                 |
| 123       | 2527 Thabo Street         | 28.358575           | -25.730613          | 2                 |
| 124       | 2560 Thabo Street         | 28.361733           | -25.730951          | 1                 |
| 125       | 2321 Thereso Street       | 28.358301           | -25.730487          | 2                 |
| 126       | 4118 Tokelo Street        | 28.362087           | -25.737872          | 2                 |
| 127       | 4109 Tokelo Street        | 28.362049           | -25.737281          | 2                 |
| 128       | 4302 Tsebiso Street       | 28.357848           | -25.744776          | 2                 |
| 129       | 3134 Tshepo Street        | 28.357623           | -25.733287          | 2                 |
| 130       | 3289 Tshepo Street        | 28.357313           | -25.733292          | 2                 |
| 131       | 2627 Tumelo Street        | 28.361731           | -25.732102          | 3                 |
| 132       | 1038 Warriors Street      | 28.367              | -25.731894          | 3                 |
| CW1       | 2399 Boleng Street        | 28.360688           | -25.728847          | –                 |
| CW2       | 5425 Gaugela Street       | 28.356504           | -25.742288          | –                 |
| CW3       | 2817 Hallelujah Crescent  | 28.363074           | -25.733602          | –                 |
| CW4       | 6100 Mapule               | 28.350754           | -25.742166          | –                 |
| CW5       | 1996 Orlando Pirates Road | 28.37316            | -25.737401          | –                 |

# Appendix B

## Internodal Distance Matrix

Table B.1: Partial internodal distance matrix

|    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1  | 0    | 1169 | 1311 | 1034 | 1201 | 448  | 907  | 933  | 972  | 69   | 1020 | 719  | 1062 | 706  | 477  | 652  | 586  | 972  | 226  | 271  | 270  | 236  | 260  | 1227 | 2339 |
| 2  | 0    | 1169 | 0    | 161  | 36   | 724  | 333  | 236  | 197  | 1177 | 2072 | 1798 | 2037 | 1760 | 832  | 600  | 693  | 468  | 1003 | 1057 | 925  | 956  | 962  | 1227 | 2339 |
| 3  | 1311 | 0    | 170  | 0    | 125  | 863  | 430  | 386  | 348  | 1322 | 2183 | 1916 | 2137 | 1875 | 943  | 710  | 801  | 509  | 1133 | 1057 | 1058 | 1092 | 92   | 2881 | 2857 |
| 4  | 1034 | 140  | 279  | 0    | 169  | 587  | 207  | 109  | 74   | 1043 | 1932 | 1658 | 1899 | 1620 | 693  | 462  | 555  | 373  | 864  | 917  | 786  | 823  | 205  | 2750 | 2750 |
| 5  | 1201 | 36   | 125  | 169  | 0    | 755  | 352  | 269  | 230  | 1209 | 2097 | 1825 | 2060 | 1786 | 857  | 623  | 716  | 474  | 1032 | 1085 | 954  | 986  | 991  | 98   | 2863 |
| 6  | 448  | 724  | 863  | 587  | 755  | 0    | 465  | 489  | 528  | 464  | 1383 | 1095 | 1378 | 1064 | 249  | 234  | 563  | 289  | 289  | 365  | 206  | 235  | 251  | 779  | 2450 |
| 7  | 907  | 333  | 430  | 207  | 352  | 465  | 0    | 196  | 208  | 929  | 1752 | 1487 | 1710 | 1445 | 513  | 280  | 371  | 198  | 714  | 753  | 644  | 677  | 673  | 340  | 2546 |
| 8  | 933  | 236  | 386  | 109  | 269  | 489  | 196  | 0    | 39   | 940  | 1852 | 1572 | 1827 | 1537 | 397  | 397  | 478  | 391  | 772  | 832  | 692  | 723  | 732  | 314  | 2729 |
| 9  | 972  | 197  | 348  | 74   | 230  | 528  | 208  | 39   | 0    | 979  | 1888 | 1610 | 1861 | 1573 | 654  | 428  | 520  | 396  | 810  | 869  | 730  | 761  | 770  | 278  | 2750 |
| 10 | 69   | 1177 | 1322 | 1043 | 1209 | 464  | 929  | 940  | 979  | 0    | 1060 | 758  | 1111 | 750  | 524  | 684  | 625  | 1006 | 278  | 335  | 308  | 275  | 307  | 1241 | 2405 |
| 11 | 1020 | 2072 | 2183 | 1932 | 2097 | 1383 | 1752 | 1852 | 1888 | 1060 | 0    | 312  | 201  | 330  | 1281 | 1523 | 1427 | 1775 | 1131 | 1055 | 1217 | 1192 | 1169 | 2160 | 1716 |
| 12 | 719  | 1062 | 2037 | 1899 | 2060 | 1378 | 1710 | 1827 | 1861 | 1111 | 201  | 399  | 0    | 382  | 1247 | 1485 | 1388 | 1709 | 1134 | 1047 | 1219 | 1199 | 1886 | 1900 | 1000 |
| 13 | 1062 | 2037 | 1899 | 1899 | 2060 | 1378 | 1710 | 1827 | 1861 | 1111 | 201  | 399  | 0    | 382  | 1247 | 1485 | 1388 | 1709 | 1134 | 1047 | 1219 | 1199 | 1886 | 1900 | 1000 |
| 14 | 706  | 477  | 1062 | 1062 | 1786 | 1064 | 1445 | 1537 | 1573 | 750  | 330  | 67   | 382  | 0    | 962  | 1204 | 1109 | 1471 | 801  | 729  | 888  | 862  | 840  | 1843 | 1858 |
| 15 | 477  | 652  | 2037 | 2037 | 1786 | 1064 | 1445 | 1537 | 1573 | 750  | 330  | 67   | 382  | 0    | 962  | 1204 | 1109 | 1471 | 801  | 729  | 888  | 862  | 840  | 1843 | 1858 |
| 16 | 652  | 586  | 1062 | 1062 | 1786 | 1064 | 1445 | 1537 | 1573 | 750  | 330  | 67   | 382  | 0    | 962  | 1204 | 1109 | 1471 | 801  | 729  | 888  | 862  | 840  | 1843 | 1858 |
| 17 | 586  | 972  | 2037 | 2037 | 1786 | 1064 | 1445 | 1537 | 1573 | 750  | 330  | 67   | 382  | 0    | 962  | 1204 | 1109 | 1471 | 801  | 729  | 888  | 862  | 840  | 1843 | 1858 |
| 18 | 972  | 226  | 1062 | 1062 | 1786 | 1064 | 1445 | 1537 | 1573 | 750  | 330  | 67   | 382  | 0    | 962  | 1204 | 1109 | 1471 | 801  | 729  | 888  | 862  | 840  | 1843 | 1858 |
| 19 | 271  | 270  | 236  | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  |
| 20 | 270  | 236  | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 |
| 21 | 270  | 236  | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 |
| 22 | 236  | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 |
| 23 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  |
| 24 | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 |
| 25 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 | 260  | 1227 | 2339 |
| 26 | 730  | 1288 | 1365 | 1158 | 1305 | 765  | 955  | 1108 | 1137 | 799  | 944  | 764  | 840  | 700  | 525  | 713  | 628  | 864  | 585  | 490  | 638  | 641  | 592  | 1274 | 1916 |
| 27 | 1196 | 1984 | 2060 | 1853 | 2001 | 1408 | 1651 | 1798 | 1828 | 1258 | 585  | 692  | 385  | 644  | 1192 | 1401 | 1312 | 1555 | 1160 | 1062 | 1235 | 1224 | 1183 | 1969 | 1340 |
| 28 | 1100 | 1957 | 2043 | 1823 | 1976 | 1347 | 1624 | 1793 | 1793 | 1158 | 445  | 551  | 248  | 507  | 1146 | 1365 | 1273 | 1545 | 1085 | 991  | 1164 | 1150 | 1112 | 1951 | 1488 |
| 29 | 1162 | 1984 | 2064 | 1851 | 2002 | 1391 | 1651 | 1793 | 1824 | 1222 | 518  | 632  | 318  | 586  | 1182 | 1396 | 1306 | 1562 | 1136 | 1040 | 1213 | 1201 | 1161 | 1973 | 1401 |
| 30 | 900  | 1530 | 1601 | 1401 | 1545 | 1002 | 1197 | 1353 | 1381 | 969  | 811  | 713  | 668  | 646  | 767  | 959  | 873  | 1095 | 795  | 696  | 857  | 856  | 808  | 1511 | 1659 |
| 31 | 2036 | 2335 | 2337 | 2242 | 2336 | 2041 | 2046 | 2239 | 2254 | 2105 | 1692 | 1773 | 1491 | 1715 | 1792 | 1905 | 1846 | 1871 | 1900 | 1806 | 1947 | 1954 | 1904 | 2264 | 724  |
| 32 | 2117 | 2476 | 2483 | 2379 | 2478 | 2151 | 2180 | 2371 | 2387 | 2186 | 1693 | 1803 | 1494 | 1749 | 1902 | 2027 | 1964 | 2010 | 1994 | 1898 | 2045 | 2050 | 2000 | 2408 | 538  |
| 33 | 1414 | 1908 | 1947 | 1794 | 1917 | 1481 | 1588 | 1766 | 1788 | 1482 | 1078 | 1118 | 882  | 1057 | 1236 | 1392 | 1318 | 1444 | 1301 | 1203 | 1358 | 1359 | 1310 | 1863 | 1103 |
| 34 | 1401 | 1851 | 1887 | 1739 | 1859 | 1446 | 1534 | 1715 | 1736 | 1470 | 1125 | 1145 | 932  | 1083 | 1199 | 1346 | 1275 | 1386 | 1277 | 1180 | 1331 | 1334 | 1284 | 1803 | 1154 |
| 35 | 1794 | 1983 | 1979 | 1896 | 1982 | 1742 | 1705 | 1899 | 1912 | 1864 | 1631 | 1645 | 1437 | 1582 | 1496 | 1584 | 1534 | 1524 | 1634 | 1546 | 1671 | 1682 | 1632 | 1908 | 1116 |
| 36 | 1870 | 2035 | 2026 | 1951 | 2033 | 1811 | 1761 | 1957 | 1969 | 1939 | 1707 | 1725 | 1511 | 1661 | 1566 | 1649 | 1600 | 1578 | 1708 | 1620 | 1743 | 1755 | 1705 | 1956 | 1107 |
| 37 | 1070 | 1677 | 1740 | 1552 | 1691 | 1174 | 1346 | 1509 | 1536 | 1138 | 832  | 800  | 659  | 736  | 936  | 1119 | 1037 | 1231 | 970  | 870  | 1032 | 1030 | 983  | 1651 | 1460 |
| 38 | 956  | 1630 | 1703 | 1501 | 1647 | 1089 | 1297 | 1451 | 1480 | 1023 | 742  | 681  | 585  | 610  | 859  | 1056 | 969  | 1197 | 869  | 769  | 935  | 931  | 885  | 1613 | 1578 |
| 39 | 628  | 1664 | 1779 | 1524 | 1690 | 971  | 1348 | 1442 | 1478 | 677  | 425  | 154  | 449  | 116  | 835  | 1069 | 977  | 1328 | 684  | 611  | 768  | 744  | 720  | 1688 | 2066 |
| 40 | 596  | 1652 | 1770 | 1512 | 1679 | 953  | 1340 | 1428 | 1465 | 643  | 445  | 152  | 485  | 116  | 827  | 1060 | 969  | 1326 | 665  | 596  | 748  | 723  | 702  | 1679 | 2116 |
| 41 | 502  | 1293 | 1396 | 1156 | 1316 | 659  | 967  | 1085 | 1119 | 571  | 819  | 577  | 768  | 521  | 465  | 694  | 601  | 926  | 665  | 596  | 748  | 723  | 702  | 1679 | 2116 |
| 42 | 621  | 1450 | 1548 | 1313 | 1472 | 820  | 1121 | 1244 | 1278 | 688  | 679  | 477  | 609  | 413  | 625  | 851  | 758  | 1068 | 565  | 468  | 641  | 630  | 433  | 1304 | 2102 |
| 43 | 361  | 1356 | 1475 | 1216 | 1383 | 660  | 1046 | 1132 | 1169 | 424  | 746  | 456  | 751  | 118  | 534  | 766  | 676  | 1042 | 374  | 300  | 458  | 435  | 410  | 1385 | 2295 |
| 44 | 677  | 1682 | 1793 | 1543 | 1707 | 1001 | 1362 | 1463 | 1499 | 729  | 403  | 181  | 400  | 114  | 851  | 1084 | 992  | 1332 | 716  | 630  | 800  | 778  | 751  | 1701 | 1991 |
| 45 | 362  | 1405 | 1527 | 1265 | 1432 | 700  | 1099 | 1178 | 1215 | 418  | 708  | 408  | 728  | 378  | 589  | 819  | 731  | 1101 | 411  | 348  | 495  | 469  | 449  | 1437 | 2278 |
| 46 | 617  | 1669 | 1786 | 1529 | 1695 | 972  | 1356 | 1445 | 1482 | 664  | 425  | 137  | 463  | 95   | 843  | 1076 | 985  | 1340 | 683  | 614  | 767  | 743  | 721  | 1695 | 2096 |
| 47 | 726  | 562  | 660  | 429  | 583  | 324  | 231  | 380  | 407  | 759  | 1573 | 1303 | 1527 | 1258 | 289  | 76   | 149  | 246  | 514  | 538  | 456  | 490  | 474  | 569  | 2677 |
| 48 | 1922 | 1646 | 1585 | 1605 | 1632 | 1694 | 1462 | 1650 | 1648 | 1984 | 2143 | 2056 | 1973 | 1989 | 1492 | 1468 | 1460 | 1264 | 1711 | 1650 | 1711 | 1735 | 1691 | 1540 | 1926 |
| 49 | 1830 | 1699 | 1658 | 1641 | 1690 | 1650 | 1477 | 1672 | 1675 | 1895 | 1954 | 1889 | 1777 | 1822 | 1430 | 1441 | 1419 | 1280 | 1629 | 1560 | 1641 | 1661 | 1614 | 1602 | 1685 |
| 50 | 1820 | 1736 | 1701 | 1674 | 1728 | 1658 | 1505 | 1700 | 1705 | 1886 | 1907 | 1852 | 1727 | 1785 | 1433 | 1455 | 1429 | 1309 | 1625 | 1552 | 1640 | 1659 | 1611 | 1643 | 1604 |

<sup>a</sup>The complete internodal distance matrix is included on the compact disc accompanying this document

# Appendix C

## Initial solution

Table C.1: Schedule: Initial Solution

| Worker1 | Week 1               | Week 2               | Week 3               | Week 4               |
|---------|----------------------|----------------------|----------------------|----------------------|
| Day 1   | 11 28 46 98 116 124  | 11 28 46 98 116 124  | 11 28 46 98 116 124  | 11 28 46 99 116 124  |
| Day 2   | 12 28 46 100 116 124 | 13 39 52 100 122 125 | 13 39 59 100 122 125 | 13 39 59 100 122 125 |
| Day 3   | 13 39 59 111 122 125 | 14 40 59 114 123     | 14 40 73 114 123     | 14 40 73 114 123     |
| Day 4   | 14 40 73 114 123     | 28 46 73 116 124     | 28 46 88 116 124     | 28 46 88 116 124     |
| Day 5   | 28 46 88 116 124     | 28 46 88 116 124     | 28 46 96 116 124     | 28 46 98 116 124     |
| Worker2 | Week 1               | Week 2               | Week 3               | Week 4               |
| Day 1   | 34 50 74 86 113      | 35 50 74 86 113      | 36 50 74 86 113      | 36 51 74 102 113     |
| Day 2   | 36 51 75 103 126     | 36 51 75 103 126     | 48 51 75 103 126     | 48 56 75 103 126     |
| Day 3   | 48 56 77 107 127     | 48 56 78 107 127     | 48 56 79 107 127     | 48 57 80 107 127     |
| Day 4   | 48 63 80 107         | 48 64 80 107         | 48 69 80 107         | 48 69 81 107         |
| Day 5   | 48 69 81 107         | 48 69 81 107         | 49 70 81 107         | 50 71 86 107         |
| Worker3 | Week 1               | Week 2               | Week 3               | Week 4               |
| Day 1   | 1 19 53 82 92 118    | 6 23 53 83 95 118    | 6 23 53 83 97 118    | 6 23 53 83 101 120   |
| Day 2   | 6 23 53 83 101 121   | 15 26 53 90 101 131  | 15 30 53 90 101 132  | 15 38 53 90 105      |
| Day 3   | 15 41 53 90 105      | 19 41 53 90 105      | 19 41 53 90 105      | 19 41 54 90 108      |
| Day 4   | 19 42 54 90 109      | 19 42 54 90 110      | 19 42 54 90 110      | 19 42 58 90 110      |
| Day 5   | 19 43 58 90 110      | 19 44 58 92 112      | 19 45 58 92 117      | 19 53 72 92 118      |
| Worker4 | Week 1               | Week 2               | Week 3               | Week 4               |
| Day 1   | 25 32 62 87 129      | 25 32 61 84 128      | 25 32 62 84 128      | 25 32 62 87 128      |
| Day 2   | 25 32 66 89 130      | 25 32 62 87 129      | 25 32 65 87 129      | 25 32 66 89 129      |
| Day 3   | 25 32 67 94          | 25 32 66 89 130      | 25 32 66 89 130      | 25 32 67 94 130      |
| Day 4   | 27 37 68 115         | 27 33 67 94          | 27 37 67 94          | 27 37 68 106         |
| Day 5   | 31 61 84 128         | 31 37 68 115         | 31 61 68 115         | 31 61 84 115         |
| Worker5 | Week 1               | Week 2               | Week 3               | Week 4               |
| Day 1   | 2 5 9 16 55 60       | 2 7 9 16 55 60       | 2 7 9 17 55 76       | 2 7 9 18 55 85       |
| Day 2   | 2 7 9 18 55 91       | 2 8 9 18 55 93       | 2 8 9 18 55 104      | 2 8 9 20 55 119      |
| Day 3   | 2 8 9 21 55 119      | 2 8 10 22 55 119     | 2 8 16 24 60 119     | 2 8 16 24 60         |
| Day 4   | 3 8 16 24 60         | 3 8 16 24 60         | 3 8 16 47 60         | 3 8 16 47 60         |
| Day 5   | 4 8 16 47 60         | 5 9 16 47 60         | 5 9 16 55 60         | 5 9 16 55 60         |

# Appendix D

## Improved Solution

Table D.1: Improved Solution: Worker 1

| <b>Week 1 Schedule</b> | <b>Duration</b> | <b>Week 2 Schedule</b> | <b>Duration</b> |
|------------------------|-----------------|------------------------|-----------------|
| 73-11-111-116-114-28   | 200.2           | 129-28-114-88-11       | 172.41          |
| 28-88-116-123-46-100   | 203.52          | 14-98-13-28-116-46     | 199.67          |
| 12-13-59-124-125       | 171.5           | 46-100-116-59-124      | 166.22          |
| 98-28-14-124-46        | 168.5           | 116-122-125-73-124-40  | 206.37          |
| 39-46-122-124-116-40   | 197.51          | 39-46-124-28-52        | 167.85          |
| <b>Total Duration</b>  | <b>941.23</b>   | <b>Total Duration</b>  | <b>912.52</b>   |
| <b>Week 3 Schedule</b> | <b>Duration</b> | <b>Week 4 Schedule</b> | <b>Duration</b> |
| 125-46-114-73-13-11    | 205.05          | 114-100-46-124-28      | 169.4           |
| 96-116-46-39-122-124   | 196.92          | 99-11-125-122-116-28   | 206.62          |
| 123-28-116-40-124      | 168.51          | 124-40-46-123-14       | 162.86          |
| 88-59-28-116-98        | 167.84          | 116-88-59-39-46        | 163.69          |
| 46-100-28-124-14       | 170.62          | 98-28-13-116-73-124    | 206.19          |
| <b>Total Duration</b>  | <b>908.94</b>   | <b>Total Duration</b>  | <b>908.76</b>   |

Table D.2: Improved Solution: Worker 2

| <b>Week 1 Schedule</b> | <b>Duration</b> | <b>Week 2 Schedule</b> | <b>Duration</b> |
|------------------------|-----------------|------------------------|-----------------|
| 103-74-51-48-50        | 185.26          | 75-107-48-78           | 162.7           |
| 36-107-113-126         | 159.73          | 36-127-113-107-81      | 191.61          |
| 48-63-77-75-56         | 196.78          | 51-50-48-69-80         | 198.68          |
| 48-127-107-34          | 174.2           | 48-103-74-86           | 158.71          |
| 86-69-81-107-80        | 186.22          | 35-107-126-56-64       | 189.12          |
| <b>Total Duration</b>  | <b>902.18</b>   | <b>Total Duration</b>  | <b>900.81</b>   |
| <b>Week 3 Schedule</b> | <b>Duration</b> | <b>Week 4 Schedule</b> | <b>Duration</b> |
| 48-70-49-50-36         | 185.26          | 81-107-74-103          | 150.37          |
| 113-126-127-86-48      | 197.35          | 57-113-127-75          | 152.27          |
| 48-107-80-51           | 160.00          | 48-80-71-51-107        | 195.04          |
| 75-79-74-81-107        | 186.75          | 56-50-86-48-36         | 197.24          |
| 69-103-56-107          | 162.11          | 69-48-102-126-107      | 202.01          |
| <b>Total Duration</b>  | <b>894.47</b>   | <b>Total Duration</b>  | <b>896.93</b>   |

Table D.3: Improved Solution: Worker 3

| <b>Week 1 Schedule</b> | <b>Duration</b> | <b>Week 2 Schedule</b> | <b>Duration</b> |
|------------------------|-----------------|------------------------|-----------------|
| 41-110-1-92-105        | 165.47          | 26-90-105-53-92        | 165.45          |
| 118-42-58-83-43-19     | 196.26          | 19-95-44-112-131       | 173.62          |
| 23-53-90-121-19        | 173.81          | 58-90-23-19-83         | 173.89          |
| 109-101-6-15-90-53     | 201.25          | 15-53-54-6-101-90      | 201.38          |
| 82-54-53-90-19         | 172.27          | 41-53-110-42-118-19    | 204.45          |
| <b>Total Duration</b>  | <b>909.06</b>   | <b>Total Duration</b>  | <b>918.78</b>   |
| <b>Week 3 Schedule</b> | <b>Duration</b> | <b>Week 4 Schedule</b> | <b>Duration</b> |
| 83-53-101-90-54        | 173.16          | 15-90-105-53-19        | 163.46          |
| 30-42-97-23-19         | 174.8           | 101-92-23-72-42        | 176.16          |
| 45-19-92-15-90         | 163.85          | 38-110-118-6-90-19     | 209.96          |
| 53-132-19-6-90-118     | 201.62          | 41-120-90-19-53        | 167.87          |
| 53-105-41-58-117-110   | 202.71          | 83-53-54-108-58        | 189.78          |
| <b>Total Duration</b>  | <b>916.14</b>   | <b>Total Duration</b>  | <b>907.24</b>   |

Table D.4: Improved Solution: Worker 4

| <b>Week 1 Schedule</b> | <b>Duration</b> | <b>Week 2 Schedule</b> | <b>Duration</b> |
|------------------------|-----------------|------------------------|-----------------|
| 27-25-94-32            | 163.89          | 130-33-128-62          | 167.88          |
| 32-25-31-68-115        | 190.32          | 66-129-27-37-84        | 197.39          |
| 32-61-37-129           | 157.82          | 25-32-67-61            | 143.59          |
| 25-87-84-89-130        | 201.96          | 25-94-68-32-31         | 186.02          |
| 66-67-128-62           | 165.75          | 115-87-89-25-32        | 205.41          |
| <b>Total Duration</b>  | <b>879.71</b>   | <b>Total Duration</b>  | <b>900.29</b>   |
| <b>Week 3 Schedule</b> | <b>Duration</b> | <b>Week 4 Schedule</b> | <b>Duration</b> |
| 115-32-27-87           | 176.45          | 32-84-66-94            | 167.94          |
| 84-89-130-37-25        | 202.62          | 67-115-32-25           | 154.28          |
| 94-25-31-32-128        | 193.25          | 31-25-32-68-128        | 194.29          |
| 25-32-61-62            | 137.41          | 61-62-130-106-87       | 195.63          |
| 68-67-65-129-66        | 199.06          | 25-37-27-89-129        | 201.72          |
| <b>Total Duration</b>  | <b>908.80</b>   | <b>Total Duration</b>  | <b>913.86</b>   |

Table D.5: Improved Solution: Worker 5

| <b>Week 1 Schedule</b> | <b>Duration</b> | <b>Week 2 Schedule</b> | <b>Duration</b> |
|------------------------|-----------------|------------------------|-----------------|
| 3-2-18-16-55-24        | 200.8           | 55-10-93-16-2          | 175.76          |
| 2-7-9-60-55-4          | 193.1           | 16-9-5-3-60-8          | 193.75          |
| 2-8-9-60-55-119        | 191.49          | 9-55-119-16-2-60       | 193.22          |
| 5-8-16-21-91           | 171.49          | 2-54-8-47-22           | 169.71          |
| 9-8-16-47-60           | 156.61          | 60-7-9-8-55-18         | 186.93          |
| <b>Total Duration</b>  | <b>913.5</b>    | <b>Total Duration</b>  | <b>919.38</b>   |
| <b>Week 3 Schedule</b> | <b>Duration</b> | <b>Week 4 Schedule</b> | <b>Duration</b> |
| 16-47-60-8-55          | 162.59          | 2-8-24-3-9             | 166.5           |
| 9-2-8-55-18-7          | 189.44          | 9-60-8-119-55-2        | 201.71          |
| 9-76-55-16-2-60        | 205.21          | 9-20-16-55-7           | 174.72          |
| 119-24-2-9-5-3         | 198.3           | 2-18-16-60-55          | 172.9           |
| 60-17-16-104-8         | 164.1           | 16-47-5-60-85-8        | 202.04          |
| <b>Total Duration</b>  | <b>919.64</b>   | <b>Total Duration</b>  | <b>917.86</b>   |

# Appendix E

## Exact routing algorithm: LINGO®code

Model:

Sets:

Node/1..5/:U;

TimeWindow/2..N/:T;

Link(Node,Node):Dist,X;

Endsets

Data:

Dist=@ole('Distance.xls','distance');

TimeWindow=@ole('Time.xls','timew');

enddata

N=@SIZE(Node);

MIN=@sum(Link:Dist\*X);

@For(Node(K):@sum(Node(I):X(I,K))=1;);

@For(Node(K):@sum(Node(J):X(K,J))=1;);

@For(Node(K):@For(Node(J)—J#GT#1#AND#K#GT#1:

U(J)-U(K)+N\*X(J,K);N-1;));

@For(Link:@Bin(X););

end

# Appendix F

## Algorithm evaluation results

Table F.1: Summary of algorithm evaluation results

| <b>Worker 1</b> | <b>Solution Algorithm<br/>Results (min)</b> | <b>LINGO<br/>Results (min)</b> | <b>Maximum Schedule<br/>Duration (min)</b> |
|-----------------|---|--------------------------------|--|
| Week 1          | 941.23                                      | 925.00                         | 1200                                       |
| Week 2          | 912.52                                      | 895.26                         | 1200                                       |
| Week 3          | 908.94                                      | 892.08                         | 1200                                       |
| Week 4          | 908.76                                      | 892.65                         | 1200                                       |
| Total           | 3671.45                                     | 3604.99                        | 4800.00                                    |
| <b>Worker 2</b> | <b>Solution Algorithm<br/>Results (min)</b> | <b>LINGO<br/>Results (min)</b> | <b>Maximum Schedule<br/>Duration (min)</b> |
| Week 1          | 879.18                                      | 869.23                         | 1200                                       |
| Week 2          | 877.81                                      | 862.11                         | 1200                                       |
| Week 3          | 871.47                                      | 858.85                         | 1200                                       |
| Week 4          | 873.93                                      | 865.46                         | 1200                                       |
| Total           | 3502.39                                     | 3455.65                        | 4800.00                                    |
| <b>Worker 3</b> | <b>Solution Algorithm<br/>Results (min)</b> | <b>LINGO<br/>Results (min)</b> | <b>Maximum Schedule<br/>Duration (min)</b> |
| Week 1          | 909.06                                      | 892.56                         | 1200                                       |
| Week 2          | 918.78                                      | 905.18                         | 1200                                       |
| Week 3          | 916.14                                      | 904.96                         | 1200                                       |
| Week 4          | 907.24                                      | 895.42                         | 1200                                       |
| Total           | 3651.22                                     | 3598.12                        | 4800.00                                    |
| <b>Worker 4</b> | <b>Solution Algorithm<br/>Results (min)</b> | <b>LINGO<br/>Results (min)</b> | <b>Maximum Schedule<br/>Duration (min)</b> |
| Week 1          | 879.71                                      | 866.31                         | 1200                                       |
| Week 2          | 900.29                                      | 889.33                         | 1200                                       |
| Week 3          | 908.80                                      | 899.34                         | 1200                                       |
| Week 4          | 913.86                                      | 906.91                         | 1200                                       |
| Total           | 3602.66                                     | 3561.89                        | 4800.00                                    |
| <b>Worker 5</b> | <b>Solution Algorithm<br/>Results (min)</b> | <b>LINGO<br/>Results (min)</b> | <b>Maximum Schedule<br/>Duration (min)</b> |
| Week 1          | 913.50                                      | 909.92                         | 1200                                       |
| Week 2          | 919.38                                      | 914.40                         | 1200                                       |
| Week 3          | 919.64                                      | 912.84                         | 1200                                       |
| Week 4          | 917.86                                      | 909.62                         | 1200                                       |
| Total           | 3670.38                                     | 3646.78                        | 4800.00                                    |