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A NOVEL MATHEMATICAL MODEL AND NUMERICAL SIMULATION OF MASS AND HEAT TRANSFER IN BIOLEACHING PROCESS OF COPPER AGGLOMERATE.

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ABSTRACT

The bioleaching unlike the leaching is a process catalyzed by bacteria to obtain copper. Generally, the material containing copper is previously agglomerated to build piles. A three dimensional mathematical model to describe the fluid mechanics, mass and heat transport that consider source terms for oxidation reaction, biological kinetic, and oxygen depletion due to methanogenic bacteria (15-45 ºC), is presented. The results of computational simulations made with original programs using the proposed mathematical model, the finite volume method and RECT computational package were validated comparing it with experimental results obtained from a leaching pile of tailing agglomerated. These shown good concordances between them. When the effect of the catalyzing bacteria was included, is possible to predict their effect over the temperature, oxygen concentration and acid consume for a pile with bioleaching process.

INTRODUCTION

The bioleaching is a leaching process catalyzed by microorganisms and applied to copper sulfate minerals to improve the dissolution kinetic. The bacteria involved in the process are generally autotrophic, aerobic and chemosynthetic and resistant to extreme acid and high metal concentration media. The copper extraction from minerals such as chalcocite (Cu_2S) and covellite (CuS) by bioleaching is practiced commonly in mines around the world. The thermophilic bacteria are the most studied due to the advantages in biohydrometallurgy processes. In order to reach maximum copper recuperation from the pile 80-90%, is required 250-350 days of process [1]. The oxidation reactions are exothermic and allow self-heating the pile which improves the copper recuperation.

A mathematical model for the self-heating in piled material has been introduced by Sidhu et al. [2] and using by same authors [3-5] to predict with finite volume numerical simulations the self-heating in compost pile. This model has the particularity of group the effect of the growth of microorganisms colony in an parametrized equation, comparatively simple in front of classical equations of biologic kinetic. The oxidation reaction of the organic matter may be describe by and Arrhenius type equation [2]. In order to predict the solute flow in soil is used an equation that considers the advection (Darcy law), diffusion (Fick law) and hydrodynamic dispersion. This equation consider that solute is diluted in water thus is flowing with it [6].

In this paper is proposed a mathematical model for the oxygen flow, energy and acid solution inside of copper bioleaching pile located in field. The three dimensional mathematical model is solved with the finite volume method [7]. Numerical results with analytical solution and Numerical results with experimental data are compared in order to validate the algorithm accuracy. The results show that is possible to predict the solute flow and the water transport with errors less than 5%. The three dimensional simulation describe the internal distribution of the variables; water temperature increases, oxygen and acid consume.

NOMENCLATURE

A_1	[1/s]	Pre-exponential factor for the oxidation
A_2	[-]	Pre-exponential factor for the inhibition of biomass growth
C	[kg/m^3]	Concentration
$C_{p,T}$	[J/kgK]	Coefficient dependent on geometric, thermal and material property values
D	[m^2/s]	Diffusion coefficient.
E	[J/mol]	Activation energy
k	[W/mK]	Thermal conductivity
h	[m]	Head pressure
K	[cell/l]	Cell crowding
K_d	[m^3/kg]	Distribution coefficient
G'''	[W/m^3]	Volumetric heat generation density
R	[J/K mol]	Ideal gas constant
R_f	[-]	Retardation factor
t	[d]	time
T	[K]	Temperature
S	[-]	Source term
x	[m]	Cartesian axis direction
y	[m]	Cartesian axis direction
z	[m]	Cartesian axis direction

Special characters

ρ	[kg/m^3]	Density
$\rho_{s,b}$	[kg/m^3]	Bulk density
θ	[-]	Volumetric water content
f	[-]	Function
μ	[1/s]	Reaction velocity
ξ	[m]	Tortuosity
ε	[-]	Porosity
λ	[kg/m^{3}d]	Dispersivity
v	[m/d]	Porous mean velocity
ϕ	[-]	Variable
δ	[-]	First order degradation constant

Subscripts

b	biomass
T	Temperature
eff	Volumetric effective expression
irr	Irrigation
mgr	Mass growth
fl	Fluid phase
ox	Oxygen
max	Maximum
m	Mass
s	Sulfur
Fe	ferrous

MATHEMATICAL MODEL

For the temperature and oxygen concentration was assumed that the flow velocity inside the porous media is low, the convective terms values are very small compared with the diffusive terms values and hence in these cases the convective terms may be despised. The mathematical model considers the diffusion of temperature and oxygen and convection-diffusion of water and solute in porous media with chemical and biological reactions [4]. This model assumes that the microorganisms growth is possible due to ability to consume on iron and sulfur. The equation (1) shown the model for the temperature and oxygen

$$\left(\rho C_{p,T}\right)_{eff}\frac{\partial T}{\partial t} = k_{eff}\nabla^2 T + G'''_{T,ox} + G'''_b\,;$$

$$\varepsilon_{fl}\frac{\partial C_{ox}}{\partial t} = D_{eff}\nabla^2 C_{ox} - G'''_{ox} \qquad (1)$$

Heat and mass transfer properties in the porous media are defined in terms of the pile porosity ε_{fl},

$$k_{eff} = \varepsilon_{fl}k_{air} + (1-\varepsilon_{fl})k_m\,;$$

$$\left(\rho C_{p,T}\right)_{eff} = \varepsilon_{fl}\rho_{air}C_{p,air} + (1-\varepsilon_{fl})\rho_m C_{p,m}\,;\quad D_{eff} = \varepsilon_{fl}D_{air,m} \quad (2)$$

In these equations G_b''',$G_{e,ox}'''$, are the generations caused by biomass growth and the chemical reactions, and -G_{ox}''', is the oxygen consume due to chemical reaction and temperature increases. The generation and consume terms are characterized by a reaction velocity (μ).

$$G'''_{T,ox} = Q_{T,ox}(1-\varepsilon_{fl})\rho_m C_{ox}\mu_{mox}\,;G'''_b = Q_b(1-\varepsilon_{fl})\rho_b F_{ox}\mu_{mgr}\,;$$

$$\mu_{mox} = A_{ox}e^{\left(\frac{-E_{ox}}{RT}\right)} \qquad (3)$$

Here, μ_{mox}, is and Arrhenius type equation used to describe the oxidation velocity due to chemical reaction and μ_{mgr} is the velocity of the reactions catalyzed by microorganisms. In the present model was assumed that bacteria that consume iron and sulfur are the main cause of the self-heating. The microbial growth rate is expressed as a maximum microbial growth modified by limiting factors. The growth rate of iron oxidizing microbes is given by

$$\mu_{Fe} = \mu_{Fe,\max}f(T)[1-e^{\left(-\frac{C_{H^+}}{C_{H^+\max}}\right)}]\left[\frac{C_{Fe^{2+}}}{K_{Fe^{2+}}+C_{Fe^{2+}}}\right]. \quad (4)$$

$$\left[\frac{C_{ox}}{K_{ox}+C_{ox}}\right]\left[\frac{K_I}{K_I+N_{Fe^{2+}}}\right]$$

This equation shows a dependence of the growth rate of iron oxidizing microbes on temperature $f(T)$, pH (C_{H^+}), ferrous ion concentration ($C_{Fe^{2+}}$), oxygen concentration (C_{ox}), and cell crowding (K_I). A similar expression is written for the growth rate of sulfur oxidizing microbes. Sulfur oxidizing microbes grow by oxidizing elemental sulfur at the surface of the ore particles. Therefore, it will be assumed that planktonic sulfur oxidizers do not grow. The growth rate of adsorbed sulfur oxidizers is given by

$$\mu_S = \mu_{S,\max}f(T)\left[\frac{g_S}{K_S+g_S}\right]\left[\frac{C_{ox}}{K_{ox}+C_{ox}}\right]\left[\frac{K_I}{K_I+N_S}\right] \quad (5)$$

In this article we propose a parametrized equation to describe the metabolic activity of biomass.

$$f(T) = \frac{A_1e^{\left(\frac{-E_1}{RT}\right)}}{1+A_2e^{\left(\frac{-E_2}{RT}\right)}} \qquad (6)$$

The formulation of equation (6) encapsulates that activation and inactivation processes occur over different temperature ranges. At low temperatures the metabolic activity of the biomass grows with increasing temperature. These processes are governed by the parameters A_1 and E_1. Equation (6) can be derived theoretically by assuming that the biomass growth rate is determined by a rate limiting step in which there is an equilibrium between "activated" and "unactivated" forms of the biomass [8]. From this, thermodynamic, perspective the term $A_2 \exp(-E_2/RT)$ represents the temperature dependence of the equilibrium constant whilst the expression $A_1 \exp(-E_1/RT)$ is the maximum forward rate of reaction in the rate limiting step. Roels [8], notes that "Although this model is based on a highly simplified image of the complexity of the growth process, it can be considered an efficient tool to model the temperature dependence of the maximum rate of growth". Eq. (6) has been used to model the maximum specific biomass growth rate in the aerobic biodegradation of the organic fraction of municipal solid waste [9]. It has also been used in a number of models for solid-state fermentation processes [10-13]. Finally, the total microbial growth rate for the mesophilic microorganisms in the bioleaching pile is obtained

$$\mu_{mgr} = \mu_{Fe} + \mu_s \tag{7}$$

This expression was incorporated in equation (3) for the heat generation by biomass growth.

The solute transfer in soil is calculated with an equation that considers advection, diffusion and hydrodynamic dispersion in porous media with chemical reactions.

$$\begin{aligned} \frac{\partial(\theta C_s + \rho_{s,b} s)}{\partial t} &= -\frac{\partial}{\partial y}\left[\theta D_{e,y}\frac{\partial C_s}{\partial y}\right] \\ &-\frac{\partial(J_w C_s)}{\partial y} - \frac{\partial}{\partial x}\left[\theta D_{e,x}\frac{\partial C_s}{\partial x}\right] \\ &-\frac{\partial}{\partial z}\left[\theta D_{e,z}\frac{\partial C_s}{\partial z}\right] - S \end{aligned} \tag{8}$$

$$\begin{aligned} &D_e = D_l^s + D_{lh}; D_l^s = \xi_1(\theta) D_l^w; \xi_1(\theta) = \frac{\theta^{\frac{7}{3}}}{\theta_s^2}; \\ &D_{lh} = \lambda \nu; \ \nu = \frac{J_w}{\theta}; \quad J_w = K_s \frac{\partial H}{\partial y} \end{aligned} \tag{9}$$

Here, C_s is the soil solute concentration, $\rho_{s,b}$ the bulk density, s soil solute absorption, $D_{e,x}, D_{e,y}, D_{e,z}$, are the solute effective dispersion coefficient of soil in x,y,z coordinate respectively, D_l^s the solute effective diffusion coefficient in soil, $\xi_l(\theta)$ is the tortuosity, D_l^w solute diffusion coefficient in free water, D_{lh} is the hydrodynamic dispersion coefficient, ν water mean velocity in the porous, λ is the dispersivity, J_w is the Darcy flux, H is the water head potential ($H = h + y$) in the soil, K_s is the saturated hydraulic conductivity and S is the source term. If an equilibrium lineal adsorption is considered:

$$s = K_d C_s , \tag{10}$$

where K_d is the distribution coefficient corresponding to the slope of adsorption isotherm. When the assumption of linear absorption and water flow in steady state in soil are considered, the equation (8) turns to [6]:

$$\begin{aligned} &R_f \frac{\partial C_s}{\partial t} = D_{e,y}\frac{\partial^2 C_s}{\partial y^2} - \nu\frac{\partial C_s}{\partial y} + D_{e,x}\frac{\partial^2 C_s}{\partial x^2} \\ &+ D_{e,z}\frac{\partial^2 C_s}{\partial z^2} - \delta R_f C_s \end{aligned} \tag{11}$$

$$R_f = 1 + \frac{\rho_{s,b} K_d}{\theta} , \tag{12}$$

where R_f is the unit-less retardation factor and δ is a first-order degradation constant. The dependence of the solute parameter D_l^w and K_d on the temperature is calculated through the follows equation [14].

$$f(T) = exp^{\frac{E_p(T - T_{ref})}{RTT_{ref}}} , \tag{13}$$

where E_p , is the activation energy of the process being modeled, R is the universal gas constant, and T_{ref} (293.15 K or 20 ºC) is the reference temperature for optimal production.

The Richards equation is commonly used to describe the water transfer in soils.

$$\begin{aligned} &\frac{\partial \theta(h)}{\partial t} = \frac{\partial}{\partial y}\left[K(h)_y \frac{\partial h}{\partial y} - K(h)\right] \\ &+ \frac{\partial}{\partial x}\left[K(h)_x \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial z}\left[K(h)_z \frac{\partial h}{\partial z}\right] \end{aligned} \tag{14}$$

Here, θ is the water content, h the hydraulic pressure, $K(h)_x$, $K(h)_y$ and $K(h)_z$, are the hydraulic conductivity in x,y,z coordinate respectively. When the water content in the soil is either steady state or saturated with water, the gradients in equation (14) are zero and the water content value in the domain is known. The van Genuchten model [15] for the variation of water content with the hydraulic pressure was used in all cases.

NUMERICAL METHOD

The system of equations that governs this problem (1, 12 and 14) was solved numerically using the finite volume method, Patankar [7]. Each one of the governing equations was written in the general form of the transport equation, with unsteady, diffusion and linearized source terms:

$$\frac{\partial \phi}{\partial t} = div(\Gamma \cdot grad\phi) + Sc + Sp \cdot \phi \tag{15}$$

A first-order accuracy in time was used in the numerical scheme to account for the unsteady heat and mass terms

$$\frac{\partial \phi}{\partial t} = \frac{\phi^{t+\Delta t} - \phi^t}{\Delta t} \tag{16}$$

The diffusion coefficient (Γ) and source terms (Sc, Sp) for each dependent variable ϕ are given in Table 1.

An original computational program, written in Fortran language, with a combination of the TDMA and Gauss-Seidel method an iterative solver [7] was used to predict temperature and oxygen concentration inside the compost pile.

The pile geometry was discretized using uniform grid of: 40x26x40 nodes in the x,*y*,z directions, respectively, to verify that the results obtained were not influenced by the mesh size. Numerical simulations were carried out for pile: 50x22x5.5 m of width, large and height.

Temperature increases initially smoothly while oxygen decreased slowly with time until sudden changes, caused by the material heat generation. Therefore, a strategy based on the use of dynamic time steps was implemented, with lower time steps when the unsteady terms were higher. The different values of the time steps used in the unsteady calculations were in the interval.

$$600s \le \Delta t \le 86400s \tag{17}$$

The iterative procedure ends at each time step when the maximum difference between iteration for $\phi = T, C_{ox}, C_s$, satisfied in each control volume the convergence criteria

$$\phi_{i,j}^{k+1} - \phi_{i,j}^{k} \le 10^{-4} \tag{18}$$

Table 1. Diffusion coefficients and source terms for the waste pile mathematical model.

ϕ	Γ	*Sc*	*Sp*
$\theta(h)$	$K(h)$	$\dfrac{\Delta\theta(h)}{\Delta t}$	$\dfrac{\Delta K(h)}{\Delta y}$
T	k_{eff}	$\dfrac{T^{p}}{\Delta t} + G_{e,ox}''' + G_{b}'''$	$-\dfrac{1}{\Delta t}$
C_{ox}	D_{eff}	$\dfrac{C_{ox}{}^{p}}{\Delta t}$	$-G_{ox}''' - \dfrac{1}{\Delta t}$
C_s	D_e	$-\nu\dfrac{\Delta C_s}{\Delta y} + \dfrac{R_f C_s{}^{p}}{\Delta t}$	$-\delta R_f - \dfrac{R_f}{\Delta t}$

VALIDATION AND NUMERICAL CASES

Water flow in experimental leaching pile

In this section the experimental results obtained from agglomerated tailing material are compared with numerical results.

The experimental procedure is explained below. The tailing material with humidity of 13.2% (p/p), was agglomerated using 22.62 kg of sulfuric acid and 11.7 l of refining solution by ton of dry material. The product is an agglomerated material with final humidity of 14.4% (p/p). This material was disposal in a field pile of size 50mx22mx5.5m large, wide and high, respectively with lateral slopes of 45º. An impermeable carpet with drainage tubes were placed in the pile base. The bulk density of the piled material was 1550 kg/m^3.

The pile was drip irrigated on a surface of 308 m^2 with configuration of 1m between tubes and 0.9 m between droppers. The water average flow was 3.41 L/h m^2. In figure 1 is shown the daily volume on the surface.

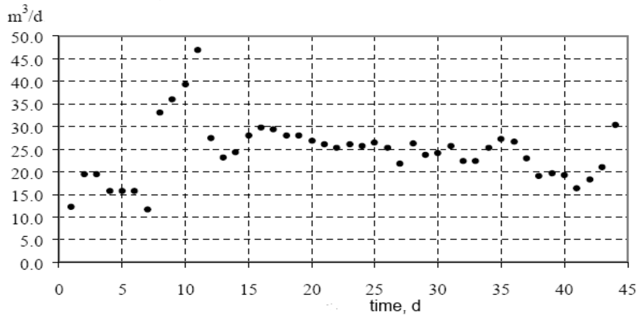

Figure 1. Water daily volume on the surface of leaching pile

In the final steps of the pile construction, samples of the piled material each 1m of high from different point along the pile were taken. These samples were used to observe the granulometry spatial variation of the material. The results indicate the formation of two material types. The first is similar to the sand and the second to pelletized material that was not agglomerated in the process and seem spheres from 0.01m to 0.08m of diameter with high humidity. These pellets are distributed mainly in the lower zone of the pile. For this reason the pile was divided in three horizontal layers. The first is located in the upper zone and has 1m of thickness. It is composed by homogeneous mixture of agglomerated material and pellets. The second layer is located in the intermediate zone and is composed by 3.5m of agglomerated tailing. The third is a lower layer of 1m thickness of pellets material. The hydrodynamics properties were determinate for each layer. In order to find the experimental suction curve were used pressure cells with range between 0 to 1 bar. The saturated hydraulic conductivity was determinate by using of constant head permeameter. Experimental values of the suction curve and saturated conductivity are incorporated to the RECT program [15,16] in order to obtain the van Genuchten parameters for the water content and unsaturated hydraulic conductivity for each layer defined in the granulometry study detailed in previous paragraph. In figure 2 are observed the experimental results and adjusted curves obtained using the RECT program and the van Genuchten model [15].

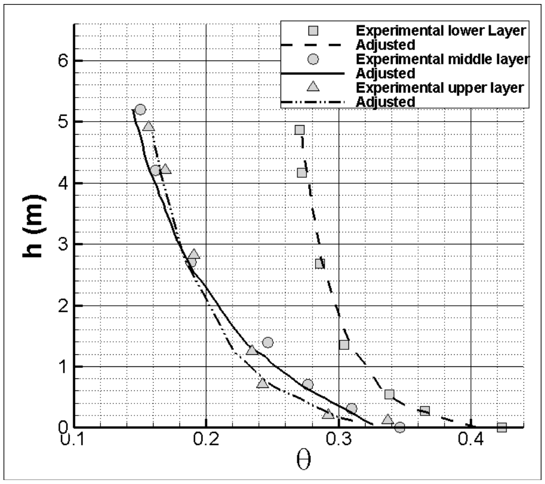

Figure 2 Experimental point and suction curves calculated by RECT for each layer.

The van Genuchten parameters for each layer are shown in table 2.

Table 2. Van Genuchten parameters obtained with RECT program for each layer.

Parameters	upper layer	middle layer	lower layer
θ_r (m^3/m^3)	0	0	0.140
θ_s (m^3/m^3)	0.325	0.330	0.407
α (1/d)	3.023	1.368	7.060
N	1.265	1.411	1.200
M	0.209	0.292	0.167
L	0.500	0.5	0.5
K$_s$ (m/d)	1.776	1.680	2.141

The time evolution of the water flux in the pile base from experimental data and numerical simulation are compared after 43d in figure 3. Numerical simulations results are obtained solving the Richards equation (14). The numerical values are close to the experimental. Higher difference of q is after 43d and reaches 0.002 m/d.

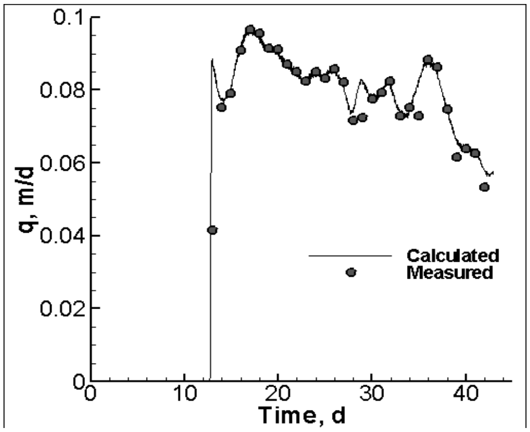

Figure 3 Measured and calculated transient values of the flux at bottom of the pile.

Comparison for the solute transport model

In this section are presented comparisons between analytical and numerical results for solute transport in soil. Considering the transport in homogeneous and isotropic porous media, steady and unidirectional water flow and lineal solute adsorption, the transport equation (8) becomes:

$$R_f \frac{\partial C_s}{\partial t} = D_L \frac{\partial^2 C_s}{\partial y^2} - v \frac{\partial C_s}{\partial y} + D_T \frac{\partial^2 C_s}{\partial x^2} + D_T \frac{\partial^2 C_s}{\partial z^2} - \delta R_f C_s + \lambda; \tag{19}$$

$$R_f = 1 + \frac{\rho_b K_d}{\theta}; \quad K_d = \frac{s}{C_s} \tag{20}$$

where D_T y D_L are soil dispersion coefficients in y,x,z coordinates, R_f is the soil retardation factor, K_d is the distribution coefficient and λ, δ are the degradation constants to chemical reaction of zero and first order, respectively. The analytical solution of equation (19) is [6].

$$\begin{aligned} C_s(x,y,z,t) = {} & \frac{C_{s,i}}{4} \int_{P(t)}^{t} \left[\frac{R_f z^2}{4\pi D_L \tau^3} \right] \exp\left[-\frac{\delta\tau}{R_f} - \frac{(R_f z - v\tau)^2}{4R_f D_L \tau} \right] \\ & \left[erfc\left[\frac{x-a}{2(D_T\tau / R_f)^{1/2}} \right] - erfc\left[\frac{x+a}{2(D_T\tau / R_f)^{1/2}} \right] \right] \\ & \left[erfc\left[\frac{y-b}{2(D_T\tau / R_f)^{1/2}} \right] - erfc\left[\frac{y+b}{2(D_T\tau / R_f)^{1/2}} \right] \right] d\tau \\ & + \frac{\lambda}{2R_f} \int_0^t \exp\left[-\frac{\delta\tau}{R_f} \right] \left[\begin{array}{l} erfc\left[\dfrac{v\tau - R_f z}{2(R_f D_L \tau)^{1/2}} \right] - \\ \exp\left[\dfrac{vz}{D_L} \right] erfc\left[\dfrac{R_f z + v\tau}{2(R_f D_L \tau)^{1/2}} \right] \end{array} \right] d\tau \end{aligned} \tag{21}$$

The calculate domain have dimensions of 120m x 200m x 120m and mesh of 88x98x88 nodes in x,y,z direction, respectively. The time step is 1d. Initially, solute-free medium is subjected to a source of $C_{s,i}$=1kg/m^3. The rectangular surface source has dimensions x=0-50m and z=0-50m at y=200m. Then, the boundary conditions may be written as: $C_s(x,y,200,t)=C_{s,i}$=1kg/m^3 if $0<x<50$m or $0<z<50$m; $C_s(x,y,200,t)=0$ if $50<x<120$m or $50<z<120$m. The input transport parameters for simulations are listed in Table 6.

Tabla 6 Input parameters for comparison with analytical solution

v m/d	δ $1/d$	λ kg/m^3d	D_T m^2/d	D_L m^2/d	R_f	$C_{s,i}$ kg/m^3	a m	b m
0.1	0.01	0	0.5	1	3	1	52	52

The three dimensional results for solute concentration obtained to 0.1 kg/m^3 after 10, 50, 100 and 365 days, are shown in figure 5.

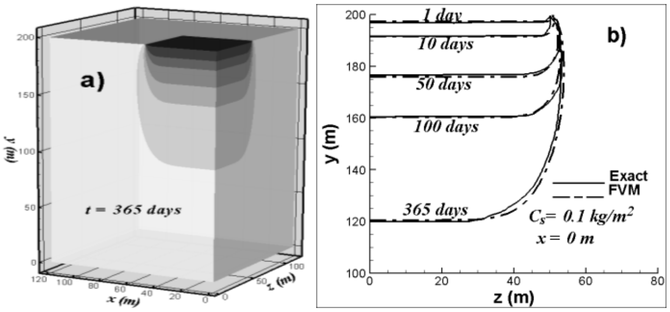

Figure 5 **Concentration profile**: a) three dimensional numerical results and b) comparison with analytical data.

The numerical results show good concordance with the analytical results. The highest errors are less than 2m and may be observed after 365d of infiltration. The finite volume method is able to solve adequately the solute transport equation in soil.

STUDY CASE

In this section the results for temperature, reactive flux of acid solution and oxygen obtained using the mathematical model and computational simulation for a bioleaching pile, are presented. The size of the pile corresponds to the detailed in the water flow validation. The initial conditions are: $T(x,y,z,0) = 293$K; $C_{ox}(x,y,z,0)$=0.08 kg/m^3; $C_s(x,y,z,0)$=1 kg/m^3 for the temperature, oxygen and acid concentration. The temperature and oxygen concentration boundary conditions on surface are 283K and 0.08 kg/m^3, and in the base of the pile adiabatic and impermeable, respectively. In the first internal node of the upper zone intermittent irrigation of the acid solution each 45 days was applied. The acid solution concentration is $C_s(x,5.4m,z,t_{irr})$=8 kg/m^3. Over lateral walls zero concentration was imposed. The pile base has free flow. In Figure 6 the 3D model with uniform mesh used in this simulation is shown.

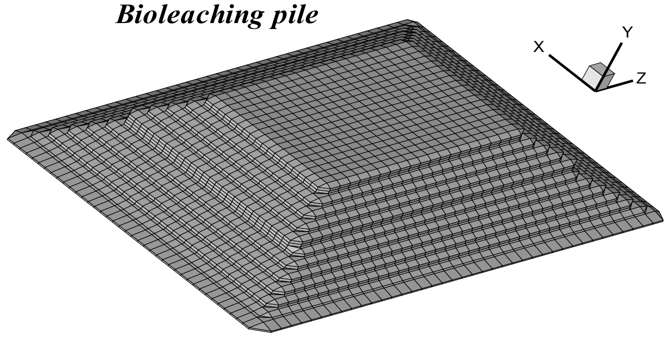

Figure 6 Bioleaching pile, digital 3D model and mesh.

The values of the hydro dispersive parameters are shown in table 7. Other values for the equations (1)-(3) are detailed in table 8.

Table 7 Hydro dispersive parameters for the study case.

ρ kg/m^3	δ 1/d	D_e m^2/d	k_d m^3/kg
1550	0.01	4.1e-4	3.5e-5

Table 8 Properties used in the mathematical model to energy and oxygen [3].

A_c, m^3/kg s	Q_b J/kg	Q_c J/kg	$C_{p,air}$ J/kg K	$C_{p,m}$ J/kg K	$D_{air,m}$ m^2/s	k_{air}, W/m K
5.7e8	7.66e6	5.5e9	1005	3320	2.4e-7	0.026
K_m, W/m K	ε_{fl}	ρ_{air}, kg/m^3	ρ_b, kg/m^3	ρ_m, kg/m^3	C_{ox}, kg/m^3	
0.18	0.3	1.17	575	1550	0.008	

In figure 7, the bi dimensional temperature distribution inside the bio lixiviation pile at three different times in middle (z=25m) and corner (z=42.4m) zones are presented. The maximum temperatures are observed in the middle-base zone. The temperature decrease approaching to the surface. In the surface of the pile the temperature is around of 10K over the ambient. The temperature inside the pile increases with passed time contrary to what happens on the surface where decrease 6K between 122 d and 365 d. Temperatures decreases to corner of pile (z=42.4m) keeping the higher temperatures in the middle-base zone.

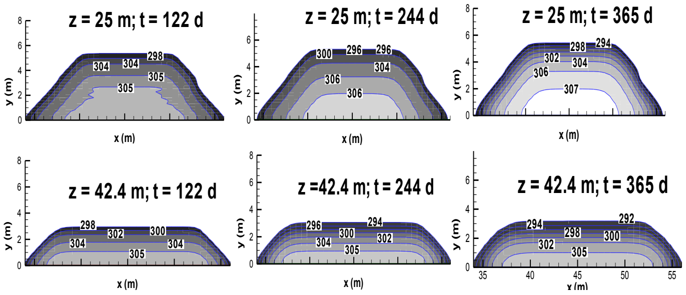

Figure 7 Temperature distributions inside of bio leaching pile at three different times and two positions.

Figure 8 show the bi dimensional distributions of oxygen concentration in the same positions and times of figure 7. The oxygen concentration decrease from 8e-3 kg/m^3 to 6.6e-3 kg/m^3, after 122 d in the middle zone (z=25m). Different values of oxygen concentration inside the pile are not observed at this time. In z=42.4m, the concentration to the surface have variations between 6.6e-3 kg/m^3 to 7.2e-3 kg/m^3. After 233 d, oxygen concentration decrease in the middle zone to 4.42e-3 kg/m^3. Similar tendency is observed after 365 d with minimum values of 2.81e-3 kg/m^3 in the middle zone (z=25m) and 3.01e-3kg/m^3 at the corner (z=42.4m). The values of oxygen concentration are higher near to the lateral walls. The oxygen consume is higher in the middle-base zone.

In Figure 9 are presented the solution concentration profile (a) and temperature (b) along of y coordinate in the middle of the pile, x=11m and z=25m.

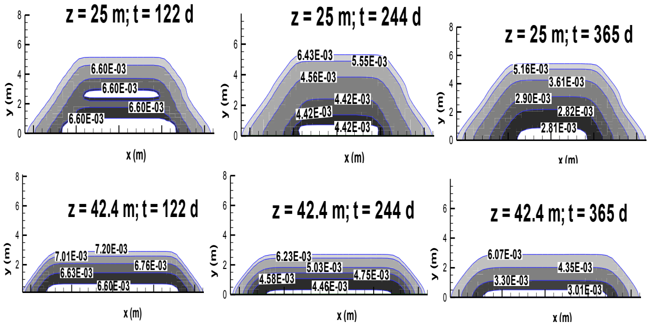

Figure 8. Oxygen concentration distribution inside of the bioleaching pile at three different times

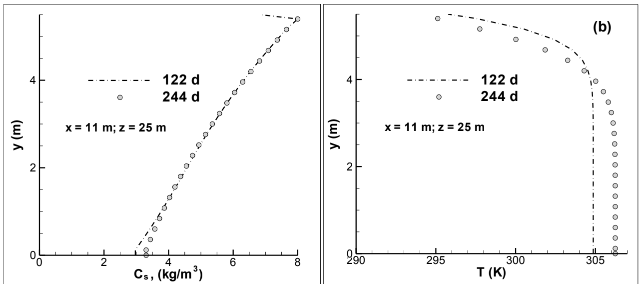

Figure 9. Acid concentration (a) and temperature (b) profile in the middle of the bio leaching pile.

Maximum concentrations are observed in the solution irrigation zone in the surface of the pile. This is agree with the figure 9 in which the maximum concentration is 8kg/m^3 at y=5.4m decreasing faster to surface (C_s=7 kg/m^3) and slower to inside the pile. The profiles after 122d and 244d are similar. Maximum differences in the solution concentration of 0.5 kg/m^3 are verified in the pile base. The highest concentration occurs after 244d product of acid accumulation due to hydro dispersive properties of material particularly of adsorption. In figure 9(c) are shown the temperature profile in the pile center for two different times. The maximum temperature are observed in the pile base y=0m and reaches 304.9K and 306.1m after 122d and 244d. The higher temperatures are verified at 4m and 3m of high after 122d and 244d, respectively. From here, temperatures values decreases exponentially to values near to 295K due to the proximity of the ambient temperature.

CONCLUSION

The mathematical model and numerical simulation allow to describe the heat generation by microorganism growth and oxidation reactions, the oxygen consumption and the solution reactive transport inside of bioleaching pile. The accuracy of the mathematical model and numerical method has been validated with experimental data and analytical equation. The numerical simulation allows to describe the internal distribution of temperature, oxygen and acid according to general tendencies of a bioleaching pile in field.

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