The Effect of an Augmented Input Intervention on Subtraction Word-Problem Solving for Children with Intellectual Disabilities: A Preliminary Study

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Abstract

Children with intellectual disabilities experience well documented challenges in the development of problem solving, particularly related to mathematical word-problem solving. Such challenges are related to both linguistic and conceptual skills. While interventions to assist children in developing conceptual skills are well reported, linguistic interventions within the mathematical field are limited. This study aimed to determine the effect of an augmented input intervention, on subtraction word-problem solving for children with intellectual disabilities. A multiple baseline across behaviours design was used and replicated across the seven participants. Intervention was provided in three tiers (one week each) for change, combine, and compare subtraction word-problems. Intervention was withdrawn for a four week period and followed by a maintenance phase. Results suggested that for older participants with higher receptive language and numerical operation scores, the augmented input intervention provided significant gains across all three types of word-problems. Overall, four of the seven participants showed improvement in subtraction word- problem solving. Augmented input may provide language supports for subtraction word-problem solving for some children with mild intellectual disability.

Keywords*:* aided language stimulation, augmented input, intellectual disabilities, mathematics, subtraction, word-problem solving

Mathematics is an important part of the school curriculum and plays an important role in facilitating the development of problem solving. Word-problems in particular allow for the development of independent reasoning, organisation, and problem-solving abilities (Browder et al., 2018; Krawec, 2014). However, word-problems require children to understand the language of the word-problem (semantics), represent the mathematical components accurately, plan and solve the solution strategy, and retrieve answers from memory (Jitendra, Petersen-Brown, et al., 2013; Jitendra, Rodriguez, et al., 2013; Root et al., 2017).

For children with intellectual disabilities, difficulties with memory, encoding, retrieval, and strategy employment, are well documented (Zheng et al., 2011. These difficulties are often cited as reasons for excluding word-problem solving from the mathematics curriculum (Kroesbergen & Van Luit, 2005; Root et al., 2017) opting to rather focus on elementary mathematical skills such as numbers, measurement, and computation (Browder et al., 2008; Butler et al., 2001; Taber-Doughty et al., 2011) despite research evidence of numerous replications showing that these children met mastery of word-problem solving (Root et al., 2020). The difficulties that children with intellectual disabilities experience in learning word-problem solving and consequently problem solving, have largely been neglected (Browder et al., 2008, 2018; Root et al., 2017; Spooner et al., 2019; Zheng et al., 2011).

In order to identify effective mechanisms for teaching children with intellectual disabilities more advanced mathematics, a meta-analysis conducted in 2008 and followed up in 2019 provided sufficient evidence of the use of systematic and explicit instruction, technology-aided instruction, graphic organizers, and manipulatives, to be confirmed as evidence based practices (Browder et al., 2008; Spooner et al., 2019). Many of these strategies are highlighted in Schema-Based Intervention (SBI), a validated mathematics intervention (Jitendra et al., 2015), which focuses on teaching children conceptual foundations by matching the problem to a known schema which is then used to assist them in the organisation of the information and the solving of the problem (Jitendra, Petersen-Brown, et al., 2013). In spite of the benefits demonstrated when SBI is used to teach mathematics to children with intellectual disabilities, this approach focuses on the conceptual components of mathematical problems (Brosh et al., 2018; Browder et al., 2018; Jitendra et al., 2015), without a specific focus on language, for example. Although specific vocabulary related to problems are taught, does a child identify differences between the functionality of possessive pronouns such as 'his' and 'hers', or the use of 'each' as a pronoun or an adjective – which can change the meaning of a sentence. This, in spite of the evidence that language impairments are a confounding but, separate factor, within mathematics skills (Rhodes et al., 2015).

Word-problems in particular present challenges as they are “linguistically presented problems requiring arithmetic solutions” (Zheng, et al. 2013, p. 97). The implication of this is that to accurately solve them, children need to first comprehend the word-problem (Riley & Greeno, 1988; Root et al., 2017; Schumacher & Fuchs, 2012; Spooner et al., 2019) before a representation of the mathematical components can be developed and a solution strategy operationalised (Jitendra, Petersen-Brown, et al., 2013; Root & Browder, 2019). The lack of facilitation of the language associated with word-problems is particularly relevant when word-problems are seen as the foundations for the development of independent reasoning, organisation, and problem-solving (Browder et al., 2018; Krawec, 2014). For although the solving of word-problems without full linguistic comprehension is possible within a limited frame of reference, if these skills are to be generalised across life-skills, the linguistic component of the word-problems is critical (Browder et al., 2018; Riley & Greeno, 1988; Root & Browder, 2019).

Augmentative and alternative communication (AAC) is a scientific field of research and clinical practice that uses a variety of compensatory strategies, techniques and devices (American Speech-Language-Hearing Association, 2019). Augmented input is a specific AAC strategy which has been reported to be an effective mechanism in supporting the development of both receptive and expressive language in children with disabilities (Allen et al., 2017; Sennott et al., 2016). Augmented input has been defined as the “incoming communication or language from a communicative partner that included speech and is supplemented by components of an augmentative and alternative communication system” (Romski & Sevcik, 1988, p.283). The supplementation of communication or language may be conducted using no technology techniques for example manual signs, low technology techniques, for example communication boards with symbols which represent words or high technology devices, for example electronic mobile devices with pictures and possibly voice output (American Speech-Language-Hearing Association, 2019).

Although augmented input may facilitate language development for children who have intellectual disabilities (Allen et al., 2017; Sennott et al., 2016), the impact of augmented input on the specific language learning required for learning mathematics is currently not known. As children with intellectual disabilities, commonly face language impairments in addition to memory, and encoding challenges (Rondal, 2001; Van der Schuit et al., 2011), programmes to address challenges in learning mathematics, need to also include the development of language (in particular receptive) skills.

Accordingly, the purpose of this study was to describe the effect of an augmented input intervention in facilitating change, combine, and compare subtraction word-problems in children with intellectual disability.

Method

Research design.

A multiple baseline experimental design across behaviours replicated across seven participants was used. In this type of design, each participant serves as their own control for an intervention presented sequentially across independent tasks (one does not influence the other) – termed tiers (Coon & Rapp, 2018; Ledford & Gast, 2018). The effect of the intervention is considered both through visual and statistical analysis, and the replication of results across tiers can infer a causal relationship between the dependent and independent variables, while replication of results across participants strengthens the inference (Kratochwill et al., 2012). A multiple baseline across behaviours design is applicable when the skills being learned are not know by the participants at the start of the intervention, cannot be reversed and can be learnt for one behaviour without influencing the performance in any other behaviours (Wolery et al., 2010).

The independent variable was a scripted augmented input programme provided in a group setting to teach three types of subtraction word-problems, namely change, combine, and compare (Riley et al., 1983). The dependent variable was the number of accurate answers provided by each participant for each type of subtraction word-problem, measured daily.

Participants

The ethics committee of the relevant higher education institution approved the study. Permission was also obtained from the relevant authority of an independent school for learners with special educational needs. All parents of participants provided consent, and assent was obtained daily from the participants themselves prior to the implementation of the study.

Participants for this study were selected using purposive sampling. Thirty-four information packs and consent letters were sent to the parents of potential participants; 20 of which were returned with consent confirmed. The educators of the participants whom had consented to be included in the study then completed a questionnaire on the participants, following which a pre-intervention assessment was conducted. Following the pre-intervention assessment, 13 children were excluded as they did not meet the selection criteria of (i) being within the 8.00 to 12.11 (years. months) age range; (ii) having an IQ score between 40 and 69 as measured by the KBIT-2 (Kaufman & Kaufman, 2004); (iii) attending a school with English as the language of learning and teaching for two years; (v) being able to correctly identify 90% of the PCS symbols in response to the spoken label from a 36 PCS symbol overlay; (vi) displaying receptive language skills scores between 4.00 and 8.00 years on the PPVT-4 (Dunn & Dunn, 2007); (vii) obtaining mathematical knowledge scores between 4.00 and 8.00 years on the WIAT-II Mathematical Reasoning subtest (Wechsler, 2005); (viii)

Note. [a] KBIT-2 (Kaufman & Kaufman, 2004) [b] Peabody Picture Vocabulary Test-4 (Dunn & Dunn, 2007) [c] WIAT-II (Wechsler, 2005)

Table 1

Participant Descriptions (N=7)

	Participant 1	Participant 2	Participant 3	Participant 4	Participant 5	Participant 6	Participant 7
Gender	Female	Male	Male	Male	Female	Female	Male
Chronological age (yr.month)	9.10	11.05	9.07	11.06	9.10	9.01	10.08
Home language	English	English	SiSwati	isiZulu	English	English	isiZulu
Primary diagnosis	Developmental Delay	Low-Functioning Academic Ability	Autism Spectrum Disorder (ASD)	Learning Disability	Down syndrome	Autism Spectrum Disorder (ASD)	Low-Functioning Academic Ability
Current therapy	None	None	Speech (weekly)	None	None	None	None
Intelligence Quotient [a]	56	61	60	49	48	59	40
Receptive language age equivalent score [b] (yr.month)	5.00	7.05	4.02	5.11	4.02	4.02	4.02
Numerical operations age equivalent score [c] (yr.month)	5.08	7.00	5.08	7.00	5.08	6.00	6.08
Mathematical reasoning age equivalent score [c] (yr.month)	5.00	5.08	4.04	6.00	5.04	5.04	4.00
Informal Counting Test	100%	100%	100%	100%	100%	100%	100%
PCS Identification Test	100%	100%	93.6%	100%	93.6%	93.6%	93.6%

counting 10 counters with 100% accuracy; and (ix) having no previous exposure to AAC. Table 1 provides a description of the participants.

Materials

The materials developed for, and used in this study, are described below.

Development. The augmented input intervention was developed following the outline provided by Riley et al. (1983) and Riley and Greeno (1988) who hypothesised and accepted a three step semantic model for solving problems (Riley et al., 1983; Riley & Greeno, 1988). The model is described in terms of its three distinct steps. First, the word-problem is transformed from an external to an internal representation of the words. To do this, sets based on an appropriate schema and representing the word-problem are created. The sets highlight memory and the linguistic ability to map the word-problem in a way that allows the solution to be determined (Pellegrino & Goldman, 1987; Riley & Greeno, 1988). Second, cognitive representation of the word-problem based on the sets is realised. In this step quantitative procedures that best represent the word-problem are decided upon. Manipulatives are set out to provide a concrete representation of the spoken and visually presented word-problem. Third, the manipulatives are physically moved to solve the word-problem, hence the word-problem is not only presented in an auditory format, but also visually.

For this study three types of subtraction word-problems (change, compare, and contrast) were identified and sets for their solution developed. Six problems for each type of word-problem, identical except for numeric values, were created. The problems for each script were presented randomly within each script (change, compare, and contrast). Examples of scripts are provided in Table 2.

Table 2

Presentation of Intervention word-problems by the researcher

	Simultaneous Techniques	Procedure	Word problem[a, b] (example)
Aim To transform the word-problem from an external to an internal representation. Theory: The intervention highlights memory and the linguistic ability to map the word-problem in a way that allows the solution to be determined (Pellegrino & Goldman, 1987; Riley & Greeno, 1988).	Augmented input (speaking while pointing to symbols representing key words: statement:question ratio of 80:20 and > 80% of words are augmented)	The researcher explains the mathematical problem verbally, while also pointing to each symbol, e.g. says "*Zinzi has 7 sweets*" while pointing to the symbols "girl" (representing Zinzi), "7" and "sweets". The teacher then says "*She gives some to Joe*", while pointing to the symbols for "girl' (representing she/Zinzi); "give" and "boy" (representing "Joe"	*Change: Change is unknown* (ZINZI HAS 7 SWEETS. She GIVES SOME to JOE. Now ZINZI HAS 5 SWEETS. HOW MANY SWEETS does ZINZI GIVE to JOE?) *Combine: Subset it unknown* (ZINZI and JOE HAVE 9 SWEETS TOGETHER. ZINZI HAS 4 SWEETS. HOW MANY SWEETS does JOE HAVE?) *Compare: Difference is unknown* (ZINZI HAS 6 SWEETS. JOE HAS 2 SWEETS. HOW MANY SWEETS does JOE HAVE LESS than ZINZI?)
	Cognitive representation of the word-problem based on the sets is realised.	Concrete manipulatives (glass counters) are used to represent the problem.	
	Cognitive representation of the solution process is realised.	The concrete manipulatives (glass counters) are physically moved to solve the word-problem.	

[a] Words in capital letters were augmented with graphic symbols , meaning that the researcher pointed to the symbols while saying the words.
[b] The linguistic structure of the word problems was the same each time they were presented within a tier. Only numerical values changed with numbers from 1-9 being used, and ensuring that the answer also fell into this range.

Augmented input. Augmented input involves the researcher pointing with a finger to key concepts, displayed as graphic symbols on a communication board while speaking, thereby providing the children with simultaneous visual and auditory input. A script was used to ensure that the input provided remained consistent. This script also encorporated two of the principles of augmented input, namely that a 80:20 (statement: question) ratio is used, and that a minimum of 80% of the symbols on the communication board is used during the session (Goossens, 1989). During the session, the use of the 20 glass counters was also demonstrated. Each intervention session was 40 minutes in duration.

Communication board. The communication boards were developed following the recommendations of Dada, Huguet, and Bornman (2013). To begin with the scripts for each type of word-problem analysed, and the key words within each explanation were identified. This process resulted in 36 key words being identified. A peer panel of fifteen children with typical development (aged 5.00 – 5.11) then selected which graphic symbols (PCS; Mayer-Johnson, 1987) best represented the key words. The 36 symbols were arranged onto communication boards; a large board (1000mm x 900mm) for the researcher, and identical smaller boards (420mm x 297mm) for each of the participants. The symbols were arranged in vertical syntactic groups from left to right: miscellaneous words, verbs, descriptors, prepositions, and nouns, according to the Fitzgerald key (Fitzgerald et al., 1984). In addition, each board also included 11 numerals (0-10) and two calculation mats in keeping with Riley et al.'s (1983) semantic model. Each graphic symbol (75mm x 60mm) was printed in colour with the corresponding label typed in bold, Grade One Font, size 35, above the symbol. The background colours of the symbols were in accordance with the convention of (Goossens, 1994). A black and white image of the board is included in Figure 1.

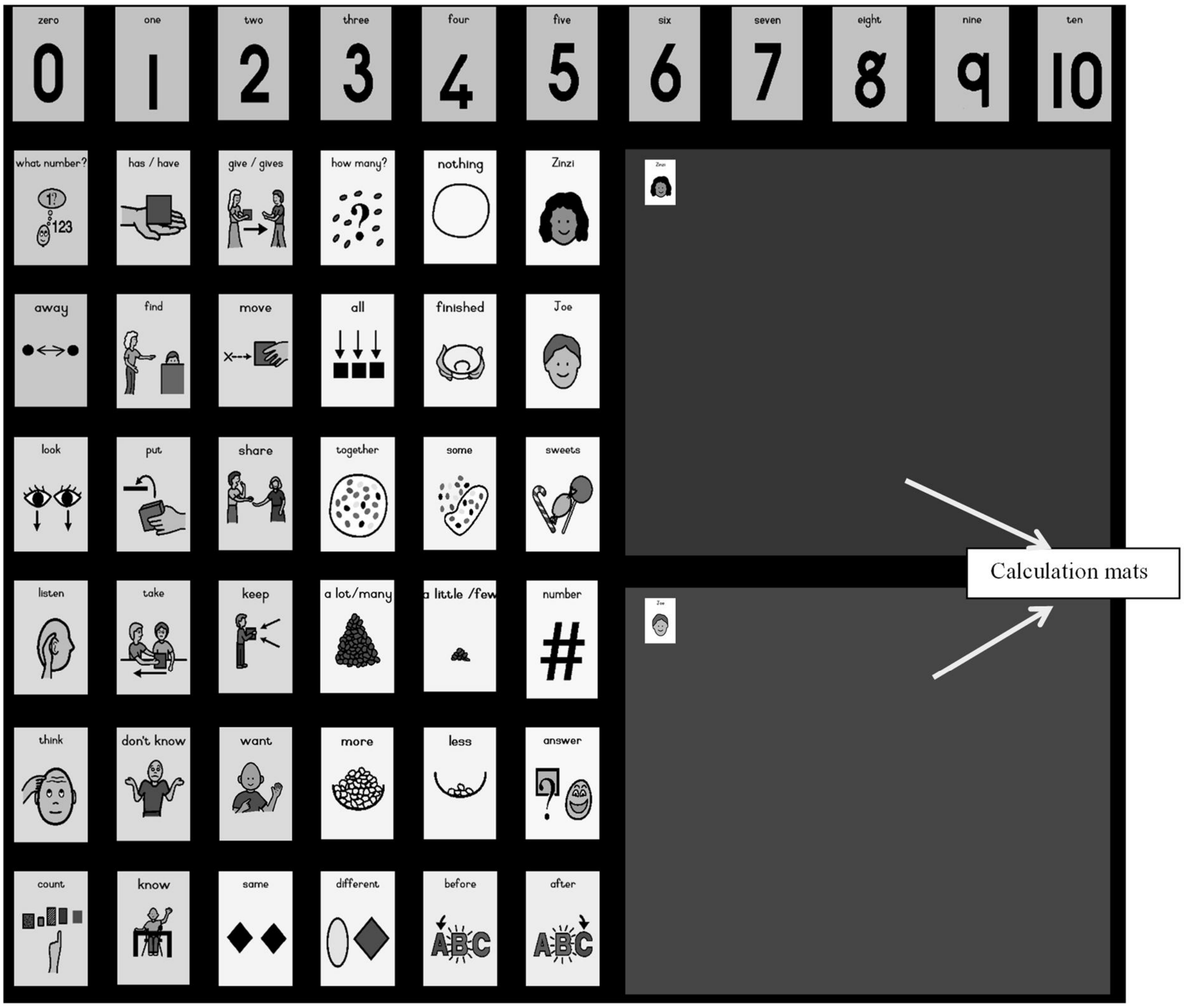

Figure 1: Facilitator board for aided language stimulation

Probe. For the probe, each participant was provided with a communication board (smaller boards), 20 glass counters, a pencil, and a probe answer sheet. The probe consisted of eighteen scripted word-problems (see Table 2 for examples of each word problem). Each word-problem was read to the participants twice. Probes were 20 minutes in duration. The participants were required to calculate and record the solution for each word-problem independently. Solutions were recorded on the probe answer sheet by circling the answer (0-10). During probes the communication board, and glass counters were visible and available to the participants but no augmented input was provided. Participants were allowed to use a method of their choice to solve the word-problems. Non-contingent reinforcement such as "*Good job*!" was given.

Procedures

Setting. The augmented input intervention was implemented in a classroom at the school that the participants attended. The intervention and probes were provided to all seven participants simultaneously in group format. Participants were seated in individual desks in a semi-circle around the researcher who sat next to the large communication board described earlier. The study was conducted in the morning during the time allocated to mathematics in the school time table. The participants received no additional mathematics input during the whole course of the research study – including baseline and withdrawal.

Research Phases. Within a multiple baseline design across behaviours and participants four phases of a study were are conducted for each participant, baseline, intervention, withdrawal, and maintenance (Ledford & Gast, 2018; Wolery et al., 2010).

Baseline phase. Prior to the commencement of the intervention, the baseline phase was completed. The baseline comprised three consecutive probes. The probes were also conducted in a group format. The probes were 20 minutes in duration and were video recorded. Each participant had their own smaller replica (297mm x 420mm) of the researchers larger communication board (1000mm x 900mm), 20 glass counters, a pencil and a probe answer sheet which comprised of a numbers table showing the numbers 0 to 10 repeated in 18 rows. While the researcher's communication board was still visible in the front of the classroom, no augmented input was provided during the probes. Eighteen scripted word-problems that were similar to those used in the intervention but with different numeric values were randomly read to the participants in the group. Each word-problem was read twice. The participants were allowed to use their individual communication boards, counters or a method of their choice to solve the subtraction word-problems. Participants were required to calculate the solution for each word-problem independently, then find and circle the correct number representing their answer (0-10), in the row allocated to the specific word-problem on the probe answer sheet. As participants worked non-contingent reinforcement such as "*Good* *job!"* were given. On completion of the probes, the probe answer sheets were collected and participants received stickers as a reward.

Intervention phase. The implementation of the augmented input intervention commenced after a stable baseline phase for all the participants was noted. The intervention phase was conducted over three weeks. A week was allocated to each type of subtraction word-problem. During Week 1, six change word-problems for subtraction were taught in accordance with the augmented input scripts, the researcher's communication board and the counters. This was followed by teaching six combine (Week 2) and six compare (Week 3) word-problems. The seven participants received the intervention simultaneously in a group format, each session lasting 40 minutes, conducted in the morning during the time allocated to mathematics in the school time table.

For each word problem the researcher read the problem while pointing to the symbols for the identified words on the communication board, for example, "ZINZI (point) HAS (point) 7 (point) SWEETS (point). Then she GIVES (point) SOME (point) to JOE(point). Now ZINZI (point) HAS (point) 5 (point) SWEETS(point). HOW (point) MANY (point) SWEETS (point) does ZINZI (point) GIVE (point) to JOE (point)?" An example of the three different types of subtraction word-problems used are included in Table 2. Simultaneously with the augmented input (speaking while pointing to symbols) the researcher demonstrated each subtraction word-problem and the solution process using the counters. After the problem solving had been demonstrated, participants had to find the correct solution from the numerals between 0 and 10 on the answer sheet and circle it. The other five word-problems were then explained in the same manner. This process was repeated over the five days with the word-problems presented in a random sequence each day. After each intervention session, the participants were given a break and then returned to the setting for the probes. Probes were conducted daily on 18 word-problems using the same procedures outlined in the baseline phase. The probe word-problems were similar (except for the numerals used) to those taught and randomly presented.

In the second week of intervention the combine type word-problems were taught using the procedure described for the change type word-problems while the change type word-problems were no longer taught but were still probed daily. The compare type word-problems were taught using the same procedure as for the change and combine type word-problems in the third week. At the end of the third week, intervention was stopped which meant that the compare type word-problems did not move into the post-intervention phase.

Withdrawal phase. A withdrawal period of four weeks followed the intervention. During the withdrawal phase the participants remained in their with no word problem solving instruction.

Maintenance probe. A maintenance probe was conducted on three consecutive days following the withdrawal period to determine whether participants had maintained their ability to solve subtraction word-problems independently.

Data analysis

Comparisons of performance were made between tiers for individuals as well as between participants using visual analysis (Jitendra et al., 2015), supplemented by the statistical analysis of the improvement rate difference (IRD; Maggin et al., 2011).

Visual Analysis of data. The results from the probes were analysed for individual participants across each phase and tier of the study in terms of level, immediacy of effect, trend, variability, and overlap between phases. The visual analysis of data was used to determine if there was an effect which could infer a causal relationship between the dependent and independent variables (Kratochwill et al., 2012).

Level. The median of the dependent variable probes within a phase (Horner & Kratochwill, 2012; Ledford & Gast, 2018). A change in level between phases, suggests an effect of the independent variable. For this study a change in level of +/-2 was considered evidence of an effect (where results are percentages, change in level of 20% is suggested. For this study 2 represents 33%; Ledford & Gast, 2018).

Immediacy of effect. An indicator of how quickly a change in levels occurred on the introduction of the independent variable. Immediacy is calculated by determining difference between the means of the last three data points in a phase and the first three data points in the next phase. For this study, a difference of +/-2 was considered sufficient to indicate immediacy (Kratochwill et al., 2012).

Trend. The slope of the best-fitting line for the outcome measures (Kratochwill et al., 2012). The trend for each phase of the study was determined by drawing a line between medians for the first half and the second half of the phase (White & Haring, cited: Ledford & Gast, 2018). A change in trend between phases, provides evidence of the effect of the independent variable (Gast & Spriggs, 2010). For this study a change in trend of +/-2 was considered evidence of an effect (Ledford & Gast, 2018).

Baseline stability. A determination of the percentage of baseline probe points which fall within a predetermined range of the trend line (stability envelope). Where ≤20% of data falls outside of the stability envelope, a stable trend is suggested. For this study a stability envelope of 2 and a flat trend (trend <2) were required (Horner & Kratochwill, 2012; Ledford & Gast, 2018).

Overlap. The proportion of overlapping data from one phase to the next. Percentage of Non-overlapping Data (PND) was selected as an overlap measure this study due to its widespread use in single-subject research (in addition to the calculation of IRD; Kratochwill et al., 2012). PND is calculated as follows:

$$\text{PND} = \frac{\text{number of data points in treatment phase higher than the highest in baseline phase}}{\text{total number of data points for treatment phase}}$$

Statistical analysis of data. Improvement Rate Difference (IRD) was used as a statistical measure of effect size. IRD provides an indication of the improvement in performance between baseline and intervention phases. IRD is considered a robust improvement rate, equal to Cohen's Kappa and Cramer's V. It has strong inter-scorer reliability, correlates with para-metric and non-parametric effect sizes, and meets APA publication standards of providing the Confidence Interval (CI), (Parker et al., 2009). IRD is calculated in two steps: First the improvement rate within each baseline and treatment phase is calculated. In baseline phase, improved data points are equal to or greater than any data points in the treatment phase. In treatment phase, improved data points are greater than all data points in baseline phase (Parker et al., 2009). Secondly the IRD is the difference between the IR of treatment and baseline phases is determined:

$$\text{IR} = \frac{\text{number of improved data points}}{\text{total number of data points in that phase}}$$

$$\text{IR}_T - \text{IR}_B = \text{IRD}$$

An omnibus IRD in which treatment and baseline probes from the full study are considered was also conducted. IRD Values 0.7 and greater show large effects, values between 0.5 and 0.7 show moderate effects, and values of 0.5 and below show small or questionable effects. (Parker et al., 2009). Confidence intervals were calculated for the IRD at 90% using the StatsDirect3 software (Buchan, 2000).

Results

The treatment fidelity and data reliability are presented first, followed by the results of the augmented input intervention for each participant.

Integrity and reliability of the study

Treatment integrity. The content of the augmented input intervention exceeded the minimum levels of 80:20 for the statement: question ratio (Dada & Alant, 2009; Goossens, 2000): combine tier ratio 84:16; change and compare ratio 92:8). The augmented input frequency for all three scripts exceeded the required 80% level (Dada & Alant, 2009; Goossens, 2000): combine tier mean of 98.10%; compare tier mean 97.08%; change tier mean 94.26%.

Procedural integrity. Procedural integrity for intervention sessions was 99.7 %, and probe procedures was 99.6%.

Reliability. Inter-rater reliability for the scoring of probes was 100%.

Visual analysis of data for word-problems

The visual analysis of probe data for this study provided evidence of stability in baseline trend for all participants. See Figures 2 through 8 for participant data.

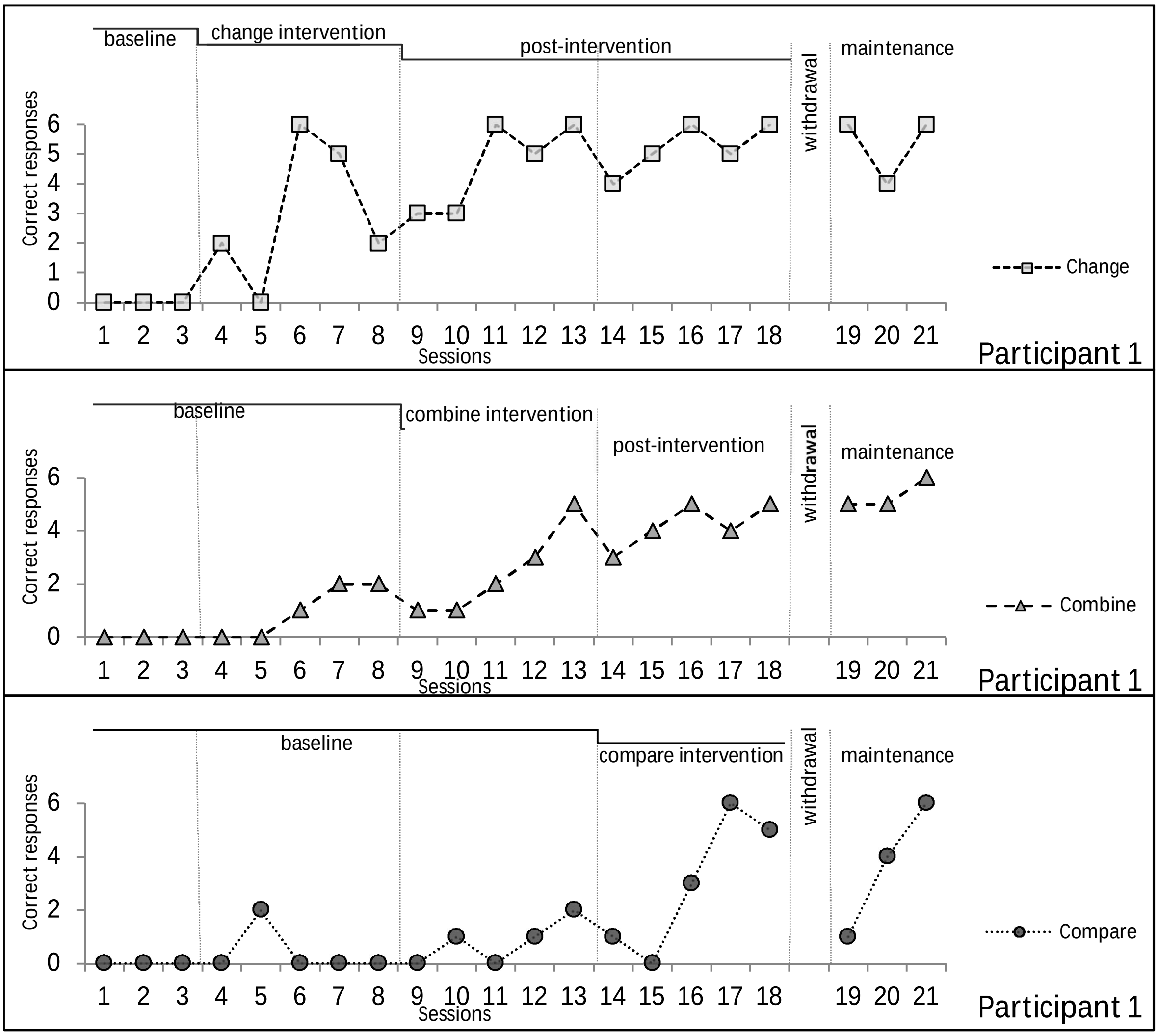

Figure 2. Number of correct responses for three types of subtraction word-problems participant 1.

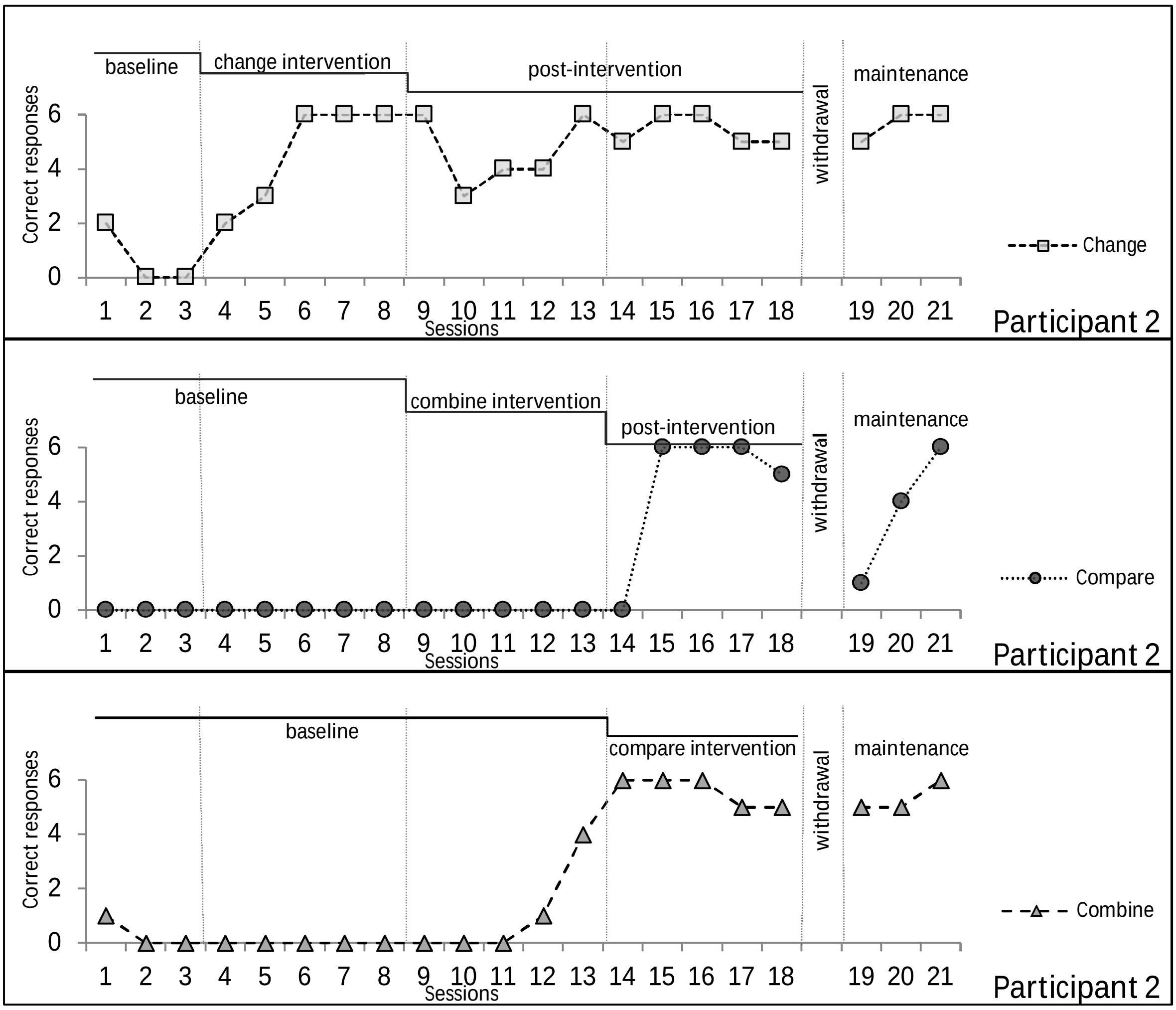

Figure 3. Number of correct responses for three types of subtraction word-problems participant 2.

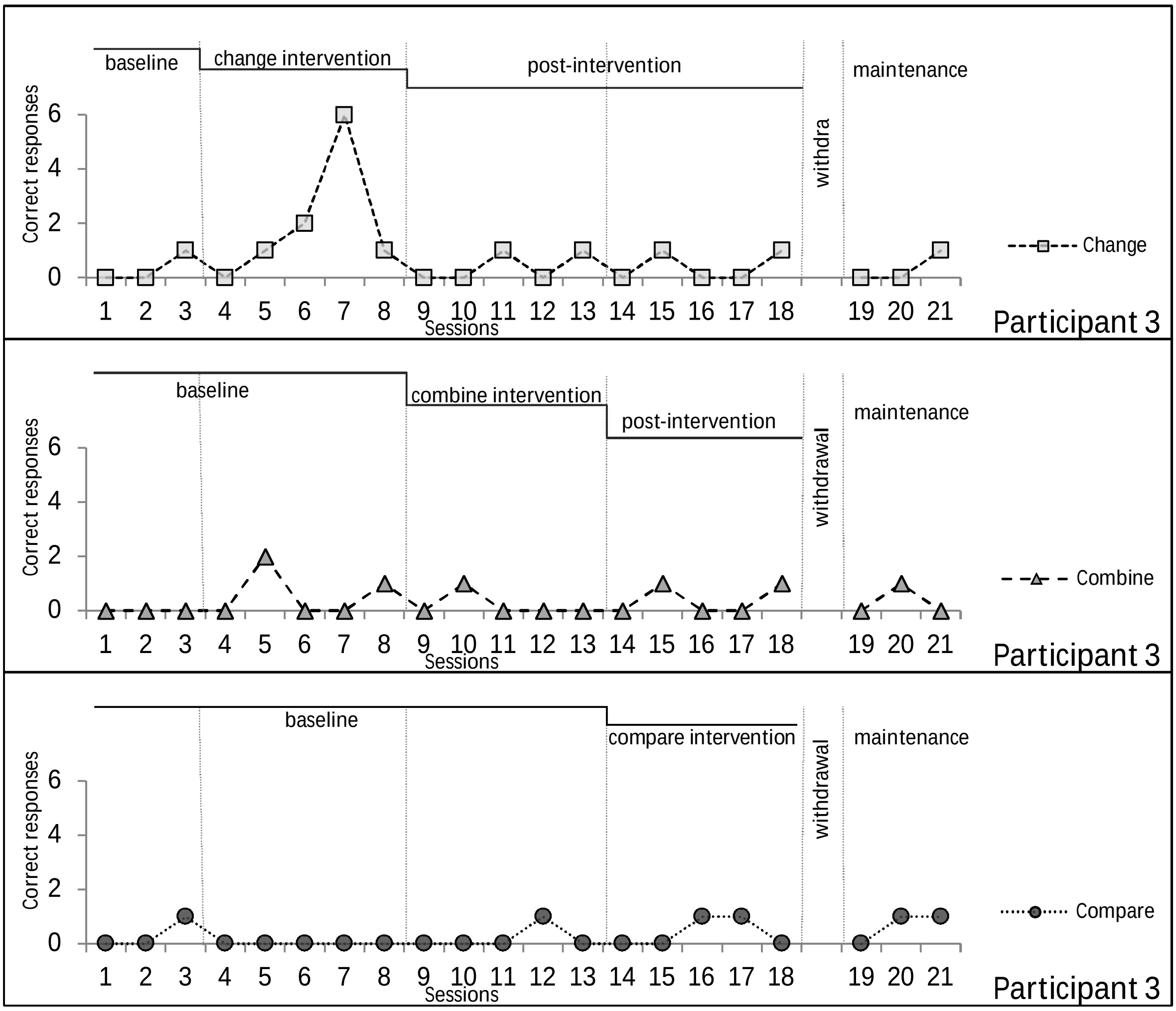

Figure 4. Number of correct responses for three types of subtraction word-problems participant 3.

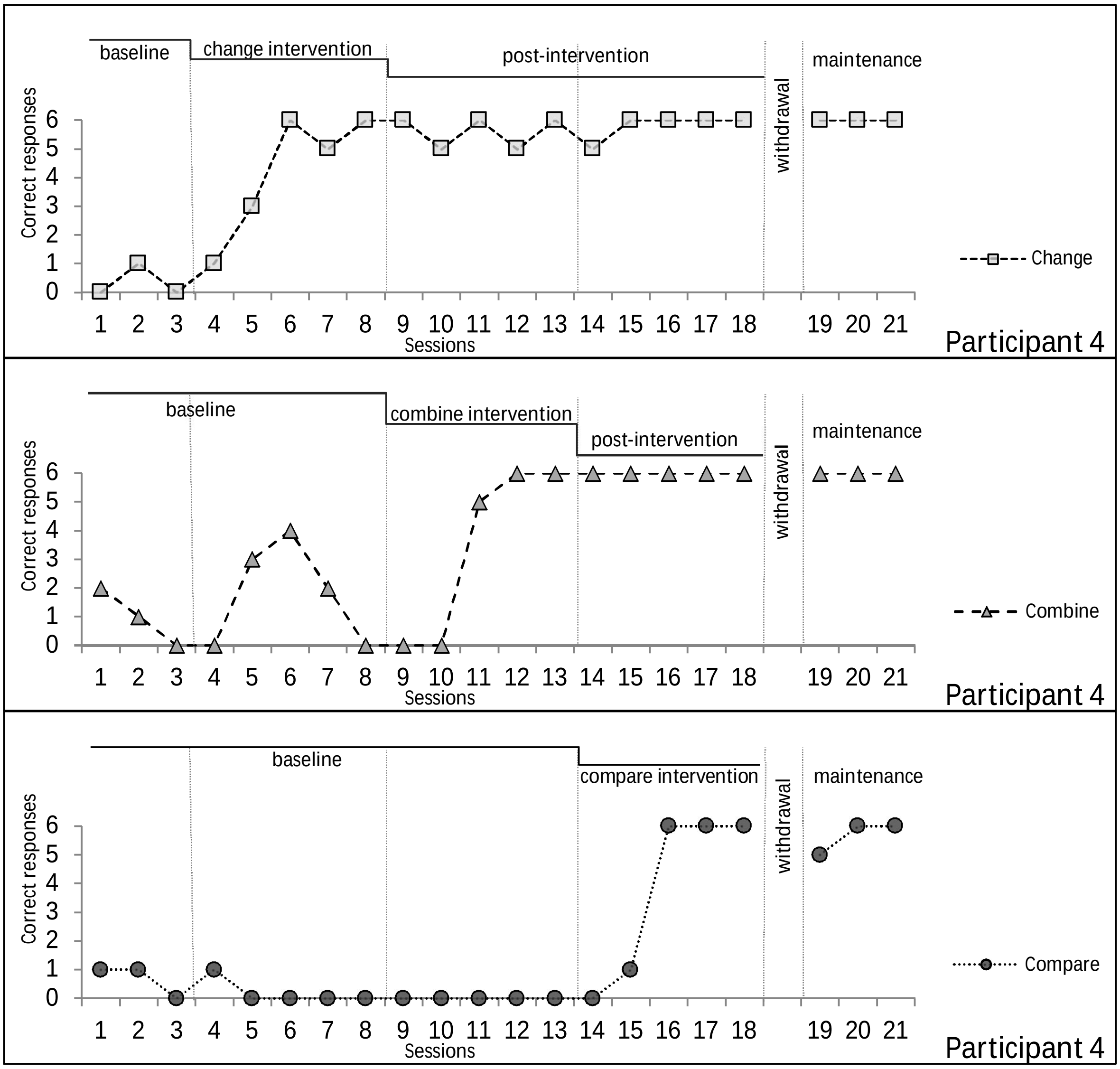

Figure 5. Number of correct responses for three types of subtraction word-problems participant 4.

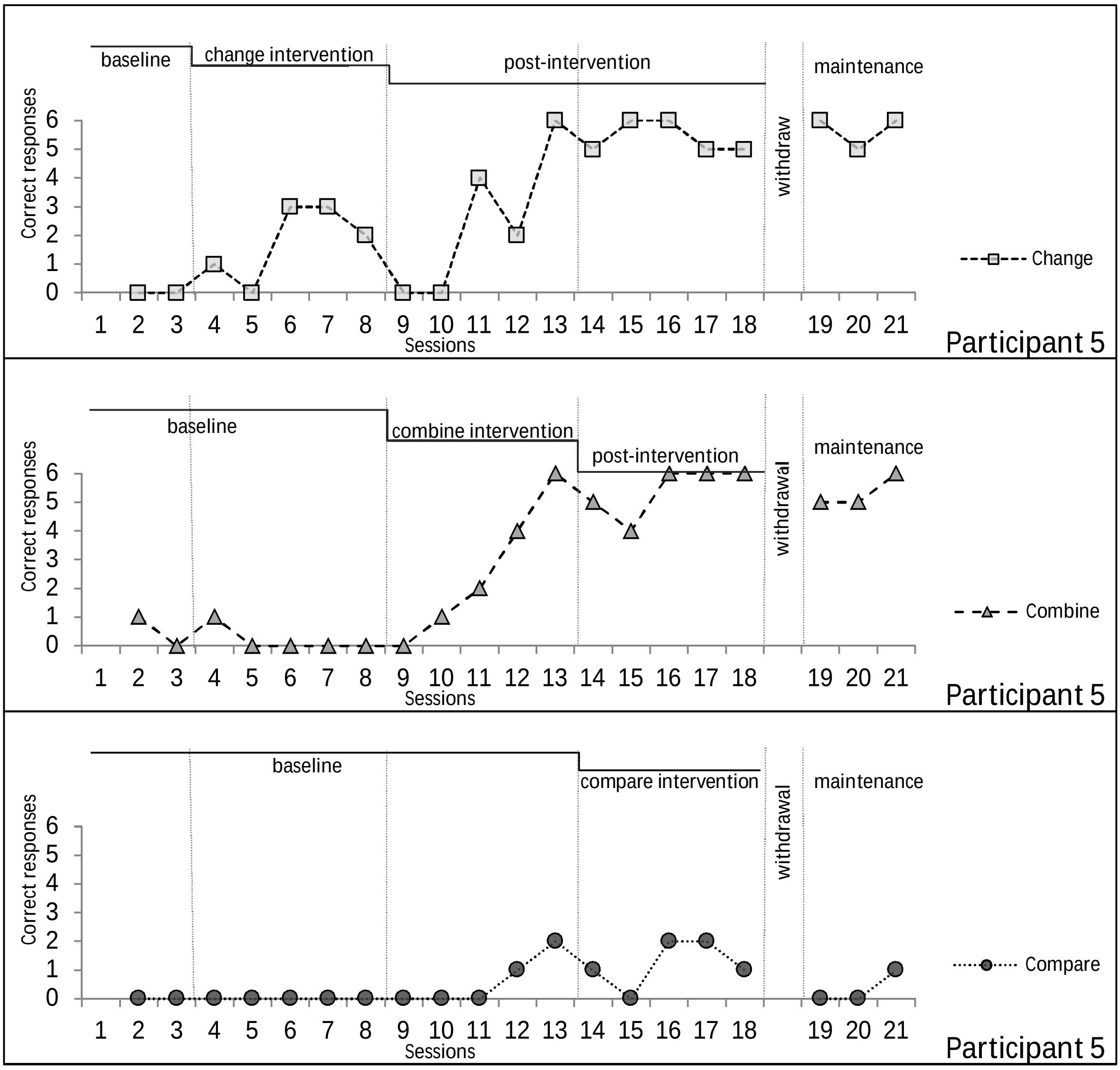

Figure 6. Number of correct responses for three types of subtraction word-problems participant 5.

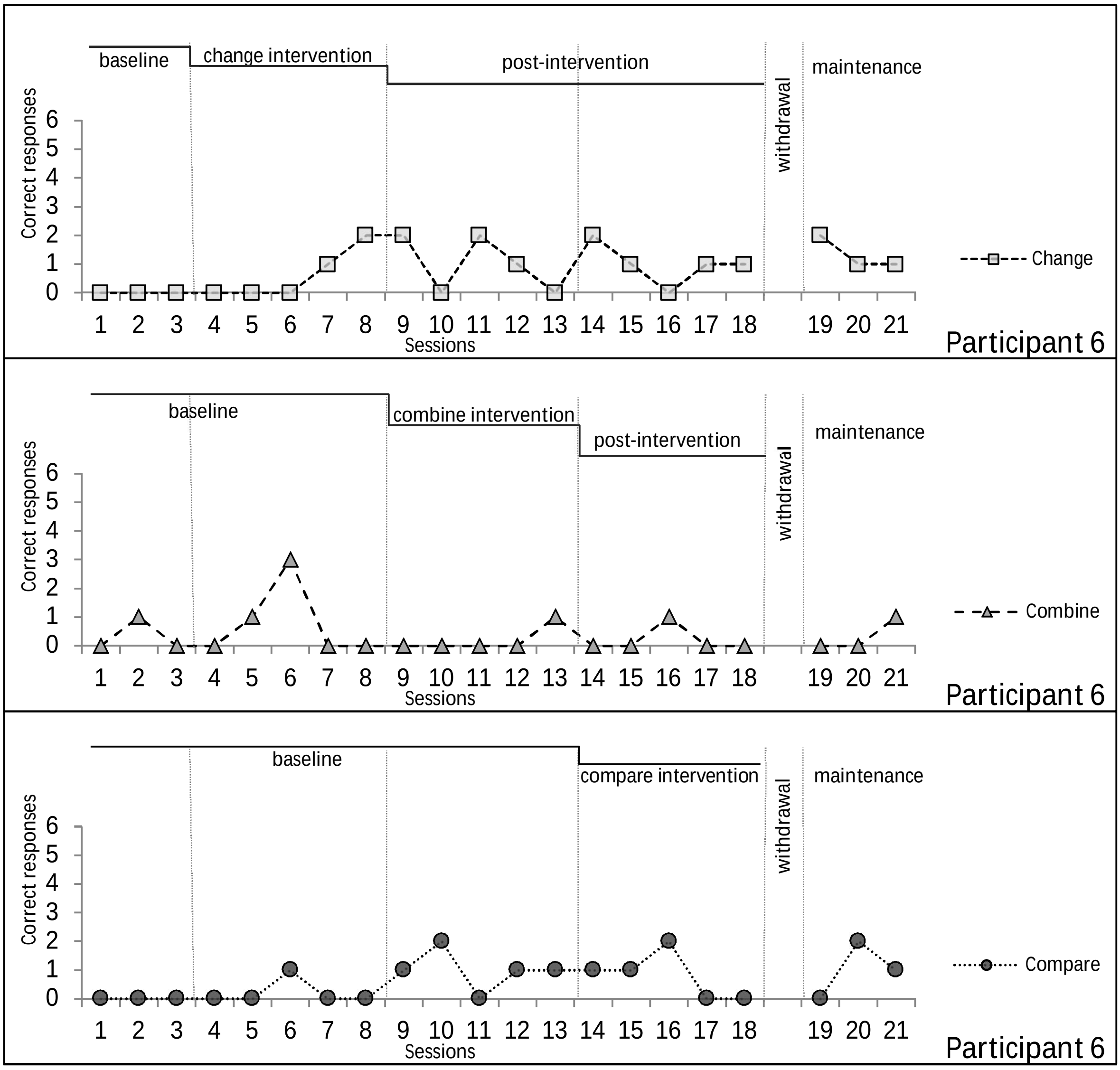

Figure 7. Number of correct responses for three types of subtraction word-problems participant 6.

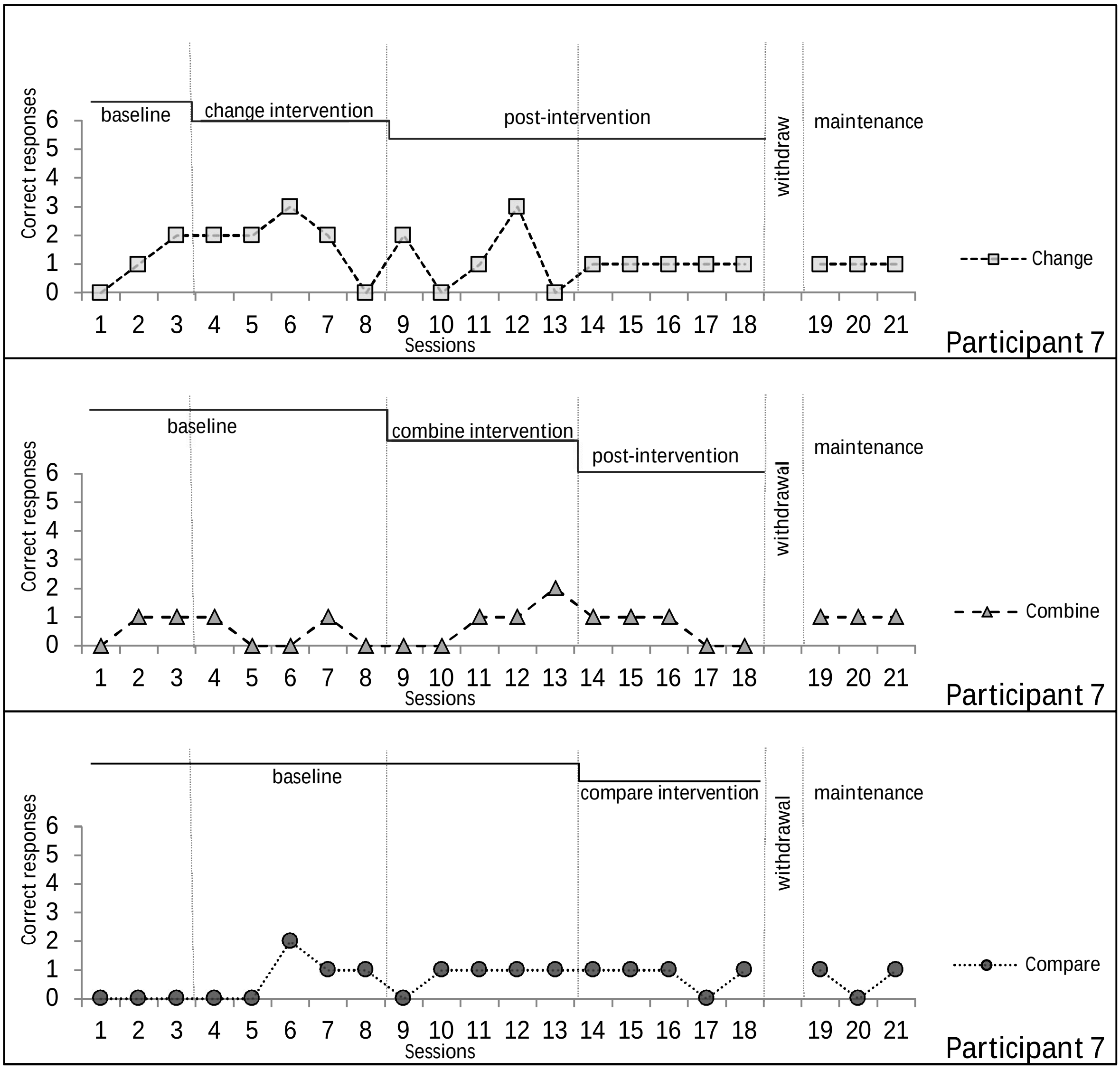

Figure 8. Number of correct responses for three types of subtraction word-problems participant 7.

Changes in level were evident for participants one and four across all tiers. Changes in trend were evident for participants two and four across all tiers.Immediacy of effect was recorded for three participants in the change tier (P1, P2, and P4), and one participant in the compare tier (P2). Participants one, four, and six showed a high percentage of non-overlapping data between the baseline and change tier.

The change tier recorded changes in effect across all visual analysis measures for two participants (P2 and P4), for three out of four visual analysis measures for one participant

Table 3
Effect of intervention on accuracy scores for subtraction word-problems for participants

	Word-problem tier	Baseline		Intervention	Post-intervention†	Maintenance	PND	IRD (CI at 90%)	Level RC	Trend RC	Immediacy of effect
		Stability[β]	Probes								
Participant 1	Change	✓	0/18	15/30	49/60	16/18	80%	**80% (17%, 96%)****	+2*	+1.5	+2*
	Combine	✓	5/48	12/30	21/30	16/18	40%	40% (6%, 73%)	+2*	+1.5	-1
	Compare	✓	6/78	15/30	-	11/18	60%	60% (27%, 86%)**	+3*	+4.5*	-1
	Total		11/144	40/90	70/90	43/54	-	60% Omnibus IRD**	-	-	-
Participant 2	Change	✓	2/18	23/30	50/60	17/18	80%	**80% (17%, 96%)****	+6*	+5.5*	+3*
	Combine	✓	1/48	5/30	28/30	16/18	20%	**80% (42%, 96%)****	0	+2.5*	0
	Compare	✓	0/78	23/30	-	11/18	100%	**80% (43%, 96%)****	+6*	+2.5*	+5*
	Total		3/144	51/90	78/90	44/54	-	**80% Omnibus IRD****	-	-	-
Participant 3	Change	✓	1/18	9/30	4/60	1/18	40%	46% (14%, 83%)	+1	+2*	+1
	Combine	✓	3/48	1/30	2/30	1/18	0%	0% (27%, 37%)	0	-1	-1
	Compare	✓	2/78	2/30	-	2/18	0%	0% (18%, 36%)	0	+0.5	-1
	Total		6/144	12/90	6/90	4/54	-	15.3% Omnibus IRD	-	-	-
Participant 4	Change	✓	1/18	27/30	57/60	18/18	80%	**80% (17%, 96%)****	+5*	+3.5*	+3*
	Combine	×	12/48	23/30	30/30	18/18	60%	60% (24%, 86%)**	+4*	+4*	-2
	Compare	✓	3/78	19/30	-	17/18	60%	60% (27%, 86%)**	+6*	+5.5*	+1
	Total		16/144	69/90	87/90	53/54	-	66.7% Omnibus IRD**	-	-	-
Participant 5	Change	✓	0/15	12/30	27/60	17/18	60%	**80% (6%, 96%)****	+2.5*	+3*	+1
	Combine	✓	2/42	13/30	27/30	16/18	60%	60% (20%, 87%)**	+2*	+5.5*	+1
	Compare	✓	3/72	6/30	-	1/18	0%	63% (20%, 86%)	+1	+1	0
	Total		5/129	30/90	54/90	34/54	-	67.7% Omnibus IRD**	-	-	-
Participant 6	Change	✓	0/18	3/30	8/60	4/18	70%	40% (-20%, 74%)	0	+1.5	0
	Combine	✓	5/48	1/30	1/30	1/18	0%	0% (-27%, 37%)	0	0	0
	Compare	✓	6/78	4/30	-	3/18	0%	0% (-18%, 36%)	+1	-2	0
	Total		11/144	8/90	9/90	8/54	-	13.3% Omnibus	-	-	-
Participant 7	Change	×	3/18	9/30	11/60	3/18	20%	0% (-38%, 51%)	+1	-3	+1
	Combine	✓	4/48	4/30	3/30	3/18	20%	20% (-10%, 58%)	+0.5	+2.5*	0
	Compare	✓	8/78	4/30	-	2/18	0%	0% (-18%, 36%)	0	-2	0
	Total		15/144	17/90	14/90	8/54	-	6.7% Omnibus IRD	-	-	-

Note.[β] A stable baseline was determined by trend stability and a flat trend.
† The period following intervention, but prior to withdrawal, for the change and combine tiers, was termed post-intervention, (change: 10 days; combine: 5 days).
Evidence of effect as determined statistically **IRD 0.5- 0.7 (moderate effect) ****IRD ≥0.7 (large effect).**

(P1). The compare tier recorded changes in effect across all visual analysis measures for one participant (P2).

Statistical analysis data for word-problems

IRD reported a large effect of intervention for participant two, overall (omnibus IRD 0.6) and for each tier (change 0.8, CI, 0.17:0.96; combine 0.8, CI, 0.42:0.96, compare 0.8, CI, 0.43:0.96). Participants 1 and 4 showed a large effect of 0.8 in the change tier (CI, 0.17:0.96). Participants one, four, and five provided evidence of a moderate effect of intervention overall (omnibus IRD of 0.6, 0.66, and 0.67 respectively). Participants one, four, and five showed evidence of moderate effects in the compare tier (IRD 0.6, CI, 0.27:0.86; IRD 0.6, CI, 0.24:0.86; and IRD 0.63, CI, 0.20:0.86 respectively), while participants four and five showed a moderate effect for the combine tier (IRD, 0.6 and CI, 0.24:0.86; and 0.20:0.87 respectively). See Table 3 for the statistical analysis results.

Discussion

The results of this study, on the effect of an augmented input intervention on the ability of school-age children with intellectual disabilities to solve change, combine, and compare subtraction word-problems will be discussed below. The results do not provide strong evidence overall that the use of an augmented input intervention is sufficient for improving the abilities of children with intellectual disabilities to perform subtraction word-problems, however it may be useful to discuss the variability in performance between participants by referring to the literature.

The first factor to be considered is that of language. Language comprehension has long been identified as a contributory factor to word-problem solving (Browder et al., 2018; Root et al., 2017; Root et al., 2020), for example participants may confuse the meaning of the words “altogether” and “each” in combine-type word-problems. This study showed effects in both the change and compare tiers of intervention for four participants, hence for these participants the use of augmented input may have aided their comprehension. The participants who did not benefit from the intervention presented with lower language scores on the PPVT-4 (4.02 age equivalent), while the one participant who benefitted the most from the augmented-input intervention had the highest language score (7.05 age equivalent), and two participants with language scores of 5.00 and 5.11 (age equivalent) also reported omnibus IRD gains across all tiers. These results support the literature which proposes that existing receptive language skills may enhance the benefit provided by augmented input intervention (Dada & Alant, 2009).

A second factor for consideration is the participants' initial accuracy of graphic symbol (PCS) identification. Although the differences in PCS identification during screening were small (6.4%), the participants who achieved 100% accuracy in the PCS identification also scored highest in the probes, while the participants who scored slightly lower did not show benefit from the intervention. Hence the ability to recognise symbols may be a factor for consideration when determining if augmented input is an appropriate tool for children with intellectual disabilities.

A third factor for consideration was the numerical operations and mathematical reasoning age equivalent scores of participants. It is particularly interesting to note that participants two and four, who scored 7.00 for numerical operations, both showed effects of the intervention across all three tiers, which were not seen for participants who scored below 7.00 for numerical operations. In contrast, no pattern of gains in relation to scores could be identified for mathematical reasoning. Hence numerical operational skills appear to positively benefit word-problem solving, but for the subtraction word-problems used in this study mathematical reasoning did not.

A fourth factor for consideration is that of chronological age which is purported to affect word-problem solving. Older children are expected to achieve higher results than their younger counterparts for word-problems due to older children having had more opportunity to develop schematised knowledge and employ it spontaneously (Bernardo, 1999). This pattern was observed for participants two and four in this study, both of whom were eleven years old. In contrast, participants three and six were younger (nine years old), and did not show effects of the intervention.

A fifth factor for consideration is the use of concrete apparatus for learning (a pedagogical factor). As advocated for use with children with intellectual disabilities, clear instruction and concrete apparatus (counters) to demonstrate, was used to facilitate word-problem solving in this study (Browder et al., 2008; Gersten et al., 2009; Spooner et al., 2019). Counters were used to represent the solution process of the verbally presented word-problems (Carbonneau et al., 2013). During probe testing, participants 1, 2, 4, and 5 who scored highest in the probes were observed to use the counters to solve the probe word-problems, while the other three participants who scored lowest in the word-problems probes, chose not use the counters. Although the participants' reasons for using the counters/or not are unknown, this study further supports those who propose that the use of concrete materials can facilitate word-problem solving (Browder et al., 2008; Carbonneau et al., 2013; Gersten et al., 2009; Spooner et al., 2019).

Finally, regardless of factors which may influence results, certain children will always defy the odds. In this study, participant five was the exception to language scores, PCS recognition, numerical operation scores, and age. For all of these factors she fell below the level at which effect was expected based on other participants, and yet, showed effects across all three tiers of intervention.

Clinical Implications

Although not a comprehensive approach for all children with intellectual disabilities, the inclusion of augmented input with other established mathematical problem solving strategies (e.g. SBI) for subtraction word-problems could benefit specific children. In addition the intervention was provided in a group setting, hence it may provide a viable alternative to one-on-one intervention for some children, and the materials developed for this study (communication board, scripts, and answer sheets) are relatively inexpensive and may be easily reproduced in low-resource settings.

Limitations and Future Directions

Limitations of this study include that the intervention was informed by a teaching rather than a learning criterion. In addition, the study targeted a range of missing medial and final amounts which have varied levels of difficulty. Perhaps, teaching the problem types to mastery with missing final quantities first would have decreased the difficulty level experienced by participants when working with missing medial quantities.

It is possible that the participants who did not benefit may have done so with extended teaching periods. Secondly, the effect of the augmented input intervention has limited generalizability to other populations or operations.

Future research could include (i) comparative studies of comparing the augmented input and alternative interventions. (ii) the effect of an augmented input intervention using more complex word-problems that require higher levels of problem-solving, and (iii) the effect of an augmented input intervention with another operation (e.g. addition).

Conclusion

In conclusion, subtraction word-problem solving is a complex interplay between child and pedagogical factors. For participants with higher receptive language scores, numerical operation scores, and ages, strong effects of augmented input intervention were identified. While the participants with lower receptive language scores, numerical operation scores, and ages did not benefit from the intervention.

Declaration of interest:

The authors report no conflicts of interest. The authors alone take responsibility for the content and writing up of this paper.

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