

NATURAL-CONVECTIVE HEAT TRANSFER IN SQUARE CAVITY WITH VISCOUS DISSIPATION

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ABSTRACT

The natural-convective heat transfer in a square cavity is considered. The left and right boundaries are maintained at constant temperature T_0 . Top and bottom horizontal walls are assumed adiabatic. The problem is considered in Boussinesq approximation. It is supposed that fluid flow is laminar and all its thermophysical properties are constant. There are terms with a viscous dissipation in energy equation. The initial temperature of fluid is more than temperature of boundaries. It is found values of Grashof and Eckert numbers when stationary total heat flow through side walls is more than zero in the wide range of initial temperatures. In the works [1], [2] it was obtained source of energy due to natural convection with time-varying boundary conditions. In this paper it is also obtained source of energy but with constant boundary conditions. The problem is solved numerically by the control volume method and algorithm SIMPLER [3].

INTRODUCTION

The laminar fluid flow with a gravity force in the Boussinesq approximation is studied by many authors for a long time. It is a classic example of the natural-convection problem. However, it is interesting to add terms with viscous dissipation to the energy equation, so we can find new solutions of this classic problem. And the stationary solution will depend on the initial temperature distribution for fixed boundary conditions. It is clear that if initial temperature is equal to the boundary temperatures then there is no fluid flow and the total integral heat flow through the side walls is zero. We've found that there is a nonzero heat flow through the side walls in some range of initial temperatures, Grashof and Eckert numbers.

There is range of natural-convective problems where we have to take into consideration viscous dissipation, for example, hydrodynamics in strong gravitational fields and geological processes, processes wherein large decelerations or high rotating speeds take place [4]. As a result there are some physical models of medium such as boundary-layer flows with

free convection, porous medium and thermoviscous fluids that require contribution of viscous dissipation.

One of the first works dedicating to influence of viscous dissipation on natural convective flow is article [4] of Gebhart B. He investigated the influence of viscous dissipation on a two-dimensional laminar boundary-layer flow around a semi-infinite plate (parallel to the body force). A perturbation method was used. Gebhart B. continued this work with and Mollendorf J. [5] where were presented results of numerical calculations. A lot of authors use these works as classic ones. In the works [6], [7] the laminar free convection along a vertical plate is also studied taking into account the viscous dissipation of the fluid. The article [6] considers a vertical plate placed in a thermally stratified environment. The plate is stationary heated in the work [7]. The papers [8-10] have deal with effects of viscous dissipation on magnetohydrodynamics natural convection flow along vertical surfaces with different geometric forms. Free convection and mass transfer in boundary layer near vertical permeable porous plate is presented by Malga B. S. et al. [11]. The effect of viscous dissipation on the Darcy free convection boundary-layer flow over a vertical plate also is discussed by Magyaria E. et al. [12]. Kairi R.R. et al. [13] investigate the influence of the effect of viscous dissipation and radiation on natural convection heat transfer from vertical flat plate in a non-Darcy porous media saturated with non-Newtonian fluid of variable viscosity. RamReddy Ch. Et al. [14] have analysed the effects of viscous dissipation and magnetic field on free convection heat and mass transfer along a vertical plate embedded in a nanofluid saturated non-Darcy porous medium have been studied. Roja P. et al. [15] have studied the unsteady two-dimensional laminar flow of a viscous dissipative micropolar fluid past a semi-infinite, vertical porous plate moving steadily and subjected to a thermal radiation and mass transfer. The free convection boundary-layer flow of an incompressible fluid along a uniformly heated sphere with heat generation in presence of viscous dissipation has been investigated by Aktar S. et al. [16].

With respect to flows of fluids with viscosity depending on temperature Leslie G. A. et al. [17] have investigated the influence of thermoviscosity effect on the two-dimensional flow of a thin film of viscous fluid on either the outer side or the inner side of a uniformly rotating horizontal cylinder. Rigatos A.P. et al [18] have studied the two-dimensional adiabatic unsteady flow of an incompressible Newtonian liquid with temperature-dependent viscosity between two horizontal parallel plates. If the top plate has shear then the flow looks like the uniform-shearing flow as we approach to the boundaries. The steady free convection flow with variable viscosity in the boundary layer near a vertical plate has been described by Elbashbeshy E.M.A. et al. [19], [20]. The paper [20] also takes into account presence of the magnetic field.

There are not so many investigations that consider internal hydrodynamic problems with viscous dissipation and constant thermophysical properties. For example such work as [21] where Jha B.K. et al. have analysed fully developed laminar natural-convective flow of incompressible fluid with viscous dissipation between two parallel vertical plates with steady-periodic temperature regime on the boundaries.

NOMENCLATURE

c	[J/kgK]	Specific heat capacity
g	[m/s ²]	Gravitational acceleration
L	[m]	Length of cavity side
P	[Pa]	Pressure
P^*	[Pa]	Reference pressure [$\mu^2/\rho_0 L^2$]
T	[K]	Temperature
T_0	[K]	Initial temperature in the cavity for $\alpha=1$, temperature of the left and the right boundaries
u	[m/s]	Component of velocity in the direction of axis Ox
v	[m/s]	Component of velocity in the direction of axis Oy
v^*	[m/s]	Reference velocity [$\mu/\rho_0 L$]
t	[s]	Time
x, y	[m]	Cartesian coordinates
β	[1/K]	Coefficient of thermal expansion
λ	[W/mK]	Thermal conductivity
μ	[P]	Dynamic viscosity
ρ_0	[kg/m ³]	Density
τ^*	[s]	Reference time [$\rho_0 L^2/\mu$]

Dimensionless characteristics

Ec	[-]	Eckert number [$\mu^2/L^2\rho_0^2cT_0$]
Ec^*	[-]	Critical value of Eckert number for which natural-convective flow decays for fixed $Gr=2 \times 10^4$
Gr	[-]	Grashof number [$g\beta T_0 L^3\rho_0^2/\mu^2$]
Gr^*	[-]	Critical value of Grashof number for which natural-convective flow decays for fixed $Ec=10^{-3}$
Nu_r	[-]	Total heat flux on the right wall
Nu_l	[-]	Total heat flux on the left wall
Pr	[-]	Prandtl number [$c\mu/\lambda$]
\tilde{P}	[-]	Pressure [$(P+\rho_0 g y)/P^*$]
q_r	[-]	Local density of heat flux [$-\partial\theta/\partial X$ for $X=1$]
U	[-]	Component of velocity in the direction of axis Ox [u/v^*]
V	[-]	Component of velocity in the direction of axis Oy [v/v^*]
X, Y	[-]	Cartesian coordinates [x/L and y/L]
ε	[-]	Initial temperature [$\alpha-1$]
θ	[-]	Temperature [$(T-T_0)/T_0$]
τ	[-]	Time [t/τ^*]

Special characters

α	[-]	Coefficient in temperature initial condition
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PROBLEM FORMULATION

The heat transfer in a square enclosure filled with fluid is considered. The top and bottom horizontal walls are adiabatic, and the right and the left side walls are maintained at a constant temperature T_0 .

The flow is assumed to be laminar and two-dimensional. The fluid is incompressible and its thermophysical properties are treated as constant everywhere, except for the density in the body force term in the momentum equation for velocity v .

The system of equations in Boussinesq approximation can be written as follows:

$$\begin{aligned} \rho_0 \frac{\partial u}{\partial t} + \rho_0 u \frac{\partial u}{\partial x} + \rho_0 v \frac{\partial u}{\partial y} &= -\frac{\partial P}{\partial x} + \mu \Delta u; \\ \rho_0 \frac{\partial v}{\partial t} + \rho_0 u \frac{\partial v}{\partial x} + \rho_0 v \frac{\partial v}{\partial y} &= -\frac{\partial P}{\partial y} + \mu \Delta v - \rho_0 g [1 - \beta(T - T_0)]; \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0; \\ \rho_0 c \frac{\partial T}{\partial t} + \rho_0 c u \frac{\partial T}{\partial x} + \rho_0 c v \frac{\partial T}{\partial y} &= \lambda \Delta T + \\ &+ \mu \left[2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right]. \end{aligned}$$

The boundary conditions are

$$x=0: \quad u=v=0, \quad T=T_0;$$

$$x=L: \quad u=v=0, \quad T=T_0;$$

$$y=0: \quad u=v=0, \quad \frac{\partial T}{\partial y}=0;$$

$$y=L: \quad u=v=0, \quad \frac{\partial T}{\partial y}=0.$$

The initial conditions are:

$$t=0: \quad u=v=0, \quad T=\alpha T_0.$$

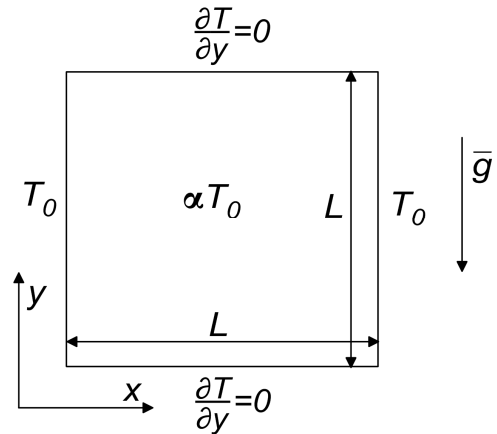


Figure 1 Enclosure geometry and temperature boundary and initial conditions.

The system of equation with initial and boundary conditions has been considered in dimensionless form for convenience of numerical analysis. The characteristic parameters for this problem are

$$v^* = \frac{\mu}{\rho_0 L}, \quad \tau^* = \frac{\rho_0 L^2}{\mu}, \quad P^* = \frac{\mu^2}{\rho_0 L^2}$$

For the dimensionless variables taken as

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \tau = \frac{t}{\tau^*}, \quad \theta = \frac{T - T_0}{T_0},$$

$$U = \frac{u}{v^*}, \quad V = \frac{v}{v^*}, \quad \tilde{P} = \frac{P + \rho_0 g y}{P^*}$$

the above mentioned system of equations transforms to

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial \tilde{P}}{\partial X} + \Delta U;$$

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial \tilde{P}}{\partial Y} + \Delta V + \text{Gr} \cdot \theta;$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0;$$

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\text{Pr}} \Delta \theta +$$

$$+ \text{Ec} \left[2 \left(\frac{\partial U}{\partial X} \right)^2 + 2 \left(\frac{\partial V}{\partial Y} \right)^2 + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y} \right)^2 \right]$$

and the dimensionless boundary conditions are

$$X = 0: \quad U = V = 0, \quad \theta = 0;$$

$$X = 1: \quad U = V = 0, \quad \theta = 0;$$

$$Y = 0: \quad U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0;$$

$$Y = 1: \quad U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0.$$

The dimensionless initial conditions are as follows:

$$\tau = 0: \quad U = V = 0, \quad \theta = \varepsilon.$$

Thus, the problem is defined with four dimensionless parameters: $\varepsilon = \alpha - 1$, Grashof number $\text{Gr} = g\beta T_0 L^3 \rho_0^2 / \mu^2$, Prandtl number $\text{Pr} = c\mu/\lambda$, Eckert number $\text{Ec} = \mu^2 / L^2 \rho_0^2 c T_0$.

NUMERICAL METHOD

The present problem is solved by control volume method with SIMPLER algorithm [3] using power law scheme and the second order approximation for boundary conditions. The grid 202×202 has been found to be satisfactory for analyzing this problem. Computations are carried out for fixed Prandtl number $\text{Pr} = 1$. Grashof number has been varied from 10^4 to 2×10^4 , Eckert number has been varied from 0.5×10^{-3} to 10^{-3} .

Dimensionless total heat fluxes on the left and right walls were used for describing of heat transfer through the domain:

$$Nu_l = -\int_0^1 \frac{\partial \theta}{\partial X} \Big|_{X=0} dY, \quad Nu_r = -\int_0^1 \frac{\partial \theta}{\partial X} \Big|_{X=1} dY.$$

ANALYSIS OF OBTAINED RESULTS

Let us consider the case with $\text{Gr} = 2 \times 10^4$ and $\text{Ec} = 10^{-3}$ in details. The stationary state of the system is achieved for the time $\tau > 1$. The initial temperature in the domain is assumed to be bigger than on the boundaries. Therefore heat flux on the right boundary will be in positive direction of Ox axis according to Fourier law and heat flux on the left boundary will be in negative direction of Ox . Absolute values of Nu_r and Nu_l are equal in any moment of time. Consider Nu_r for different values of parameter ε . On the fig. 2, 3 we see that total heat flux on the right wall falls down to zero for time $\tau \approx 1$ for $\varepsilon \leq 0.19$. For $\varepsilon \leq 0.19$ viscous dissipation doesn't have influence on stationary solution of the problem, natural-convective flow decays during the time. However for $\varepsilon = 0.2$ Nu_r has already started to increase at $\tau = 0.05$ (fig.3). Value of Nu_r in stationary state is equal 46.063 for all $\varepsilon \geq 0.2$. For all $\varepsilon \geq 0.2$ natural-convective flow is forming during the time in the domain (fig.7). The temperature field is nonzero and the same for all $\varepsilon \geq 0.2$ at $\tau > 3$ (fig.6). At that, the both maximal values of temperature and maximal values of local density of heat flux $-\partial\theta/\partial X$ for $X = 1$ are achieved for $Y > 0.7$ in stationary state (fig.8).

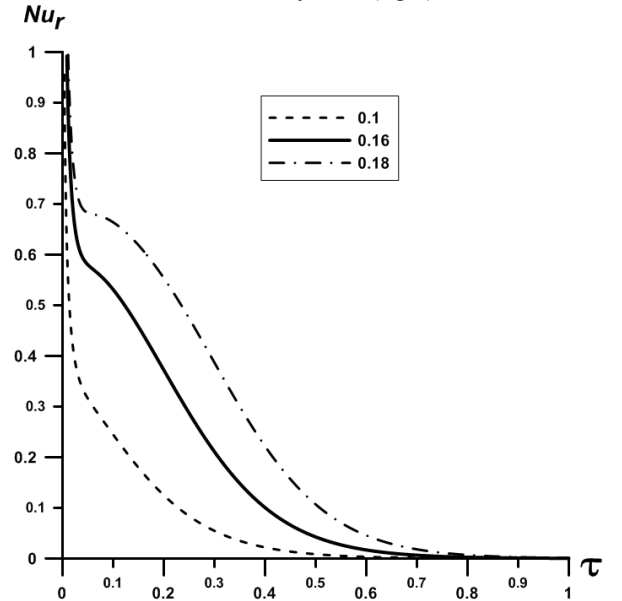


Figure 2 Heat flux Nu_r dependence on time at $\text{Gr} = 2 \times 10^4$, $\text{Ec} = 10^{-3}$, $\varepsilon = 0.1, 0.16, 0.18$.

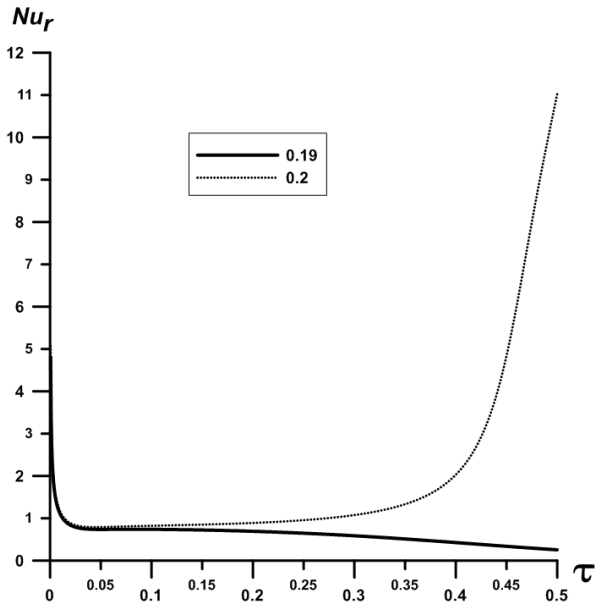


Figure 3 Heat flux Nu_r dependence on time at $Gr=2 \times 10^4$, $Ec=10^{-3}$, $\epsilon=0.19, 0.2$.

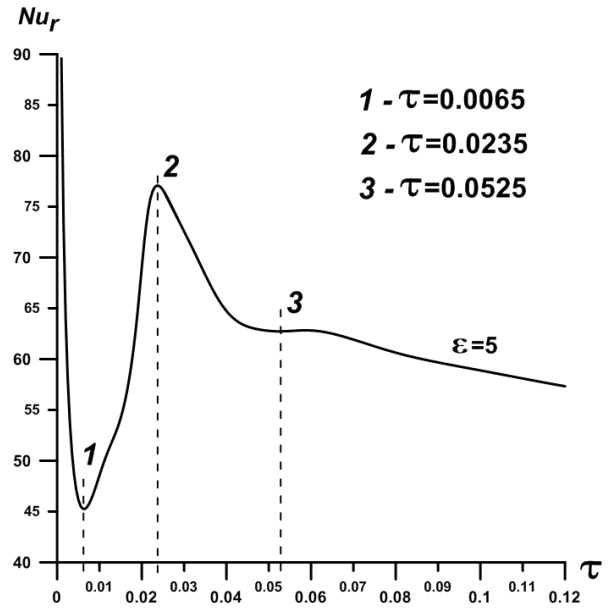


Figure 5 Heat flux Nu_r dependence on time at $Gr=2 \times 10^4$, $Ec=10^{-3}$, $\epsilon=5$.

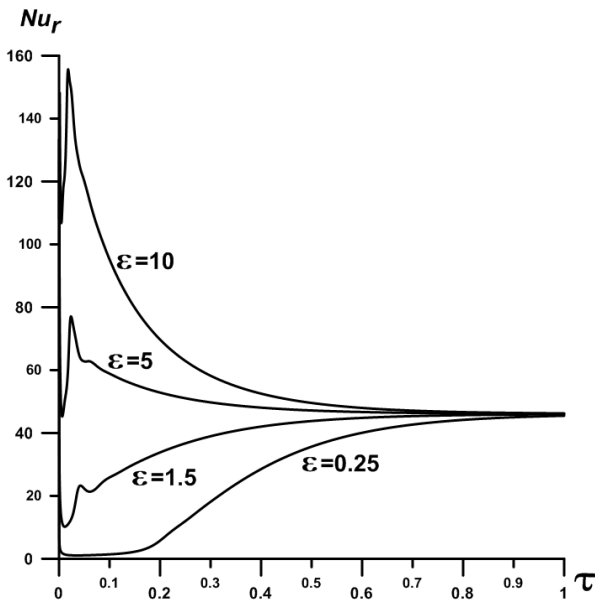


Figure 4 Heat flux Nu_r dependence on time at $Gr=2 \times 10^4$, $Ec=10^{-3}$, $\epsilon=0.25, 1.5, 5, 10$.

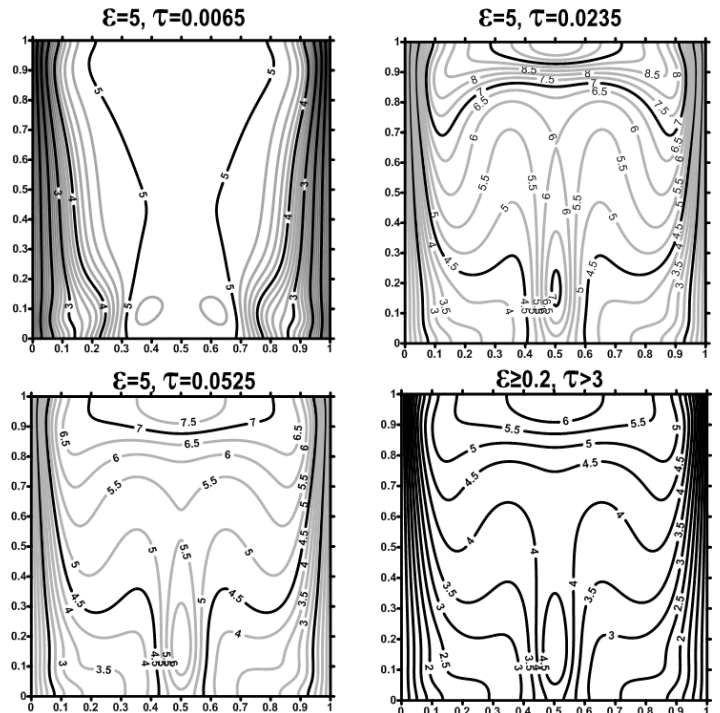


Figure 6 Representative plot of isotherms: $Gr=2 \times 10^4$, $Ec=10^{-3}$, $\epsilon=5$, $\tau=0.0065, 0.0235, 0.0525$ and $\epsilon \geq 0.2$, $\tau > 3$.

Fig. 6 and 7 demonstrates temperature fields and flow fields respectively for $\epsilon=5$ on time moments when there are extreme values of total heat flux on the boundary (Fig.5). Let us consider temperature fields for $\epsilon=5$ on fig.6. After initial time moment cavity cools down due to influence of boundary conditions. But at the moment $\tau=0.0065$ there has already been places where temperature is more than initial temperature of the domain: $0.35 < X < 0.45$, $0.55 < X < 0.65$ and $0.05 < Y < 0.15$. During the time temperature is becoming more than initial temperature in the main part of the domain.

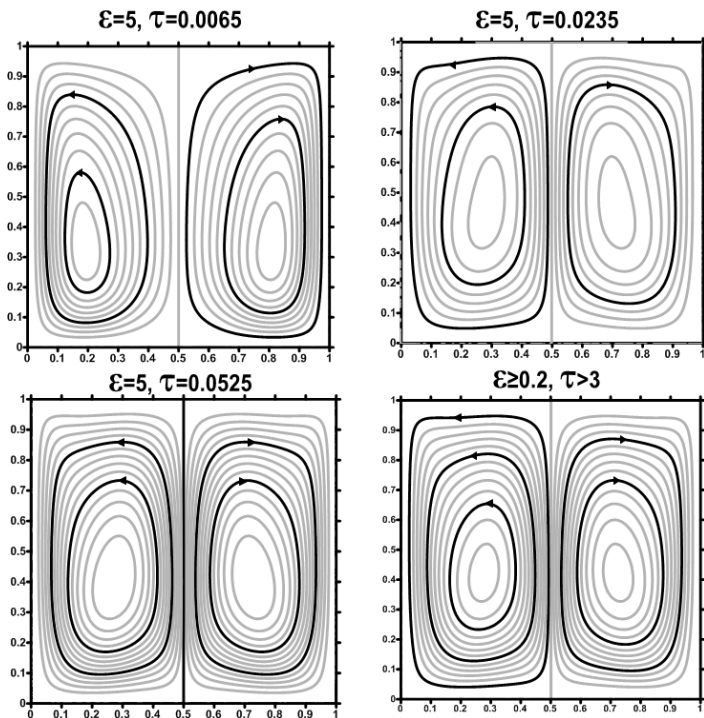


Figure 7 Representative plot of stream function: $Gr=2 \times 10^4$, $Ec=10^{-3}$, $\varepsilon=5$, $\tau=0.0065$, 0.0235 , 0.0525 and $\varepsilon \geq 0.2$, $\tau > 3$.

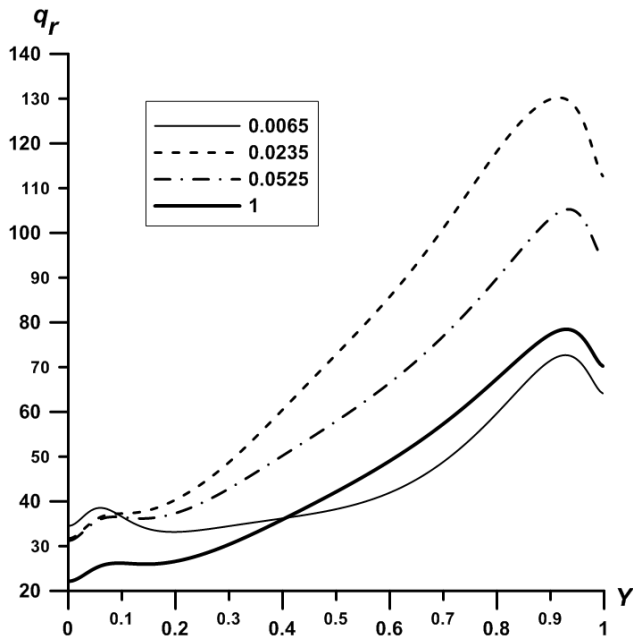


Figure 8 Local heat flux q_r dependence on space coordinate Y at $Gr=2 \times 10^4$, $Ec=10^{-3}$, $\varepsilon=5$ for different time moments - $\tau=0.0065$, 0.0235 , 0.0525 , 1 .

Fig.9 shows dependence of Nu_r on Eckert number for fixed Grashof number $Gr=2 \times 10^4$. And there is dependence of Nu_r on Grashof number for fixed Eckert number $Ec=10^{-3}$ on the fig.10. The plots mean that total heat flux on the wall falls down with decreasing Eckert and Grashof number. Maximal value of local density of heat flux q_r also falls down with decreasing of Eckert or Grashof number (fig.11). Because energy inflow falls down due to viscous dissipation with decreasing of Eckert and Grashof number. Values of Eckert number on the fig.9 and Grashof number on the fig.10 for which natural-convective flow decays are $Ec < 0.737 \times 10^{-3}$ and $Gr < 1.471 \times 10^4$ respectively. Minimum value of total heat flux on the boundary for which we observe stationary natural-convective flow is the same for dependences on fig.9 и 10 and is equal 3.98. It's impossible to get $0 < Nu_r < 3.98$ in this problem.

Fig. 12, 13 present evolution of temperature and flow fields with decreasing of Eckert number.

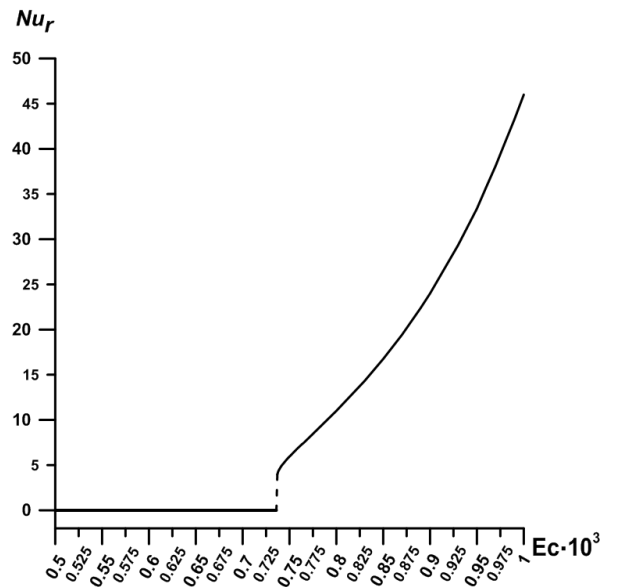


Figure 9 Heat flux Nu_r dependence on Eckert number at $Gr=2 \times 10^4$.

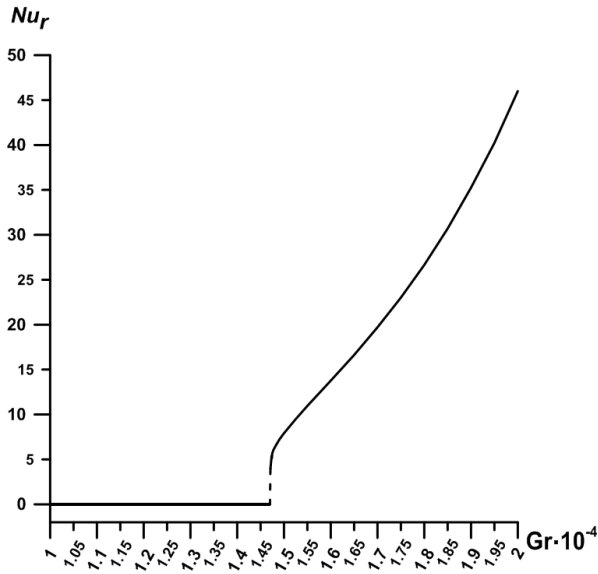


Figure 10 Heat flux Nu_r dependence on Grashof number at $Ec=10^{-3}$

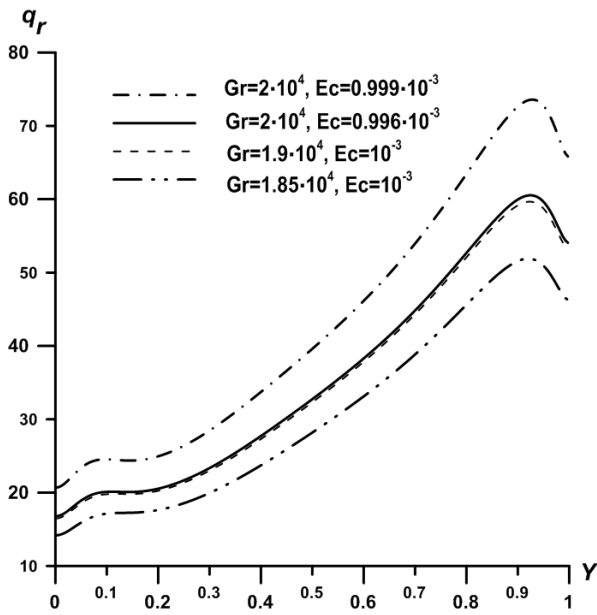


Figure 11 Local heat flux q_r dependence on space coordinate Y for different Gr and Ec , $\varepsilon \geq 0.2$ for stationary state.

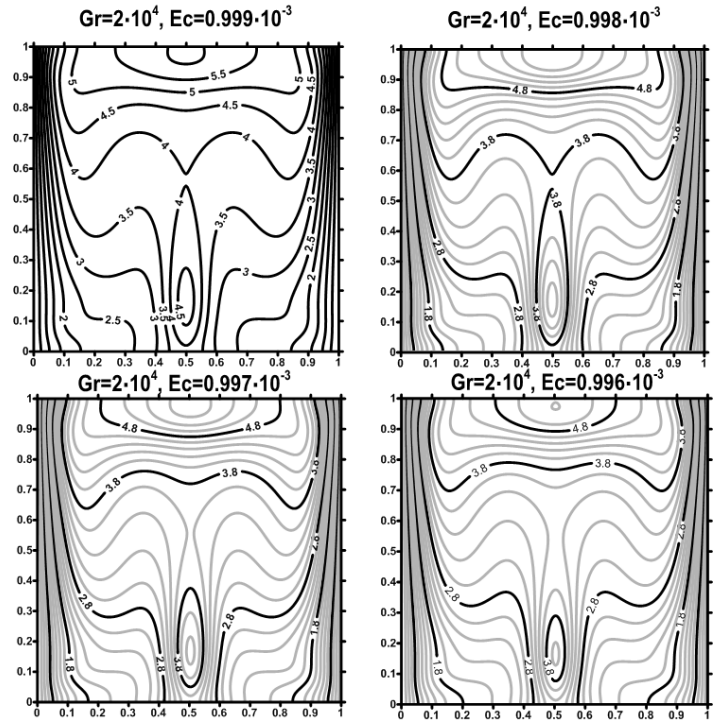


Figure 12 Representative plots of isotherms: $Gr=2 \times 10^4$, $Ec \times 10^3 = 0.999, 0.998, 0.997, 0.996$, $\varepsilon \geq 0.2$ for stationary state.

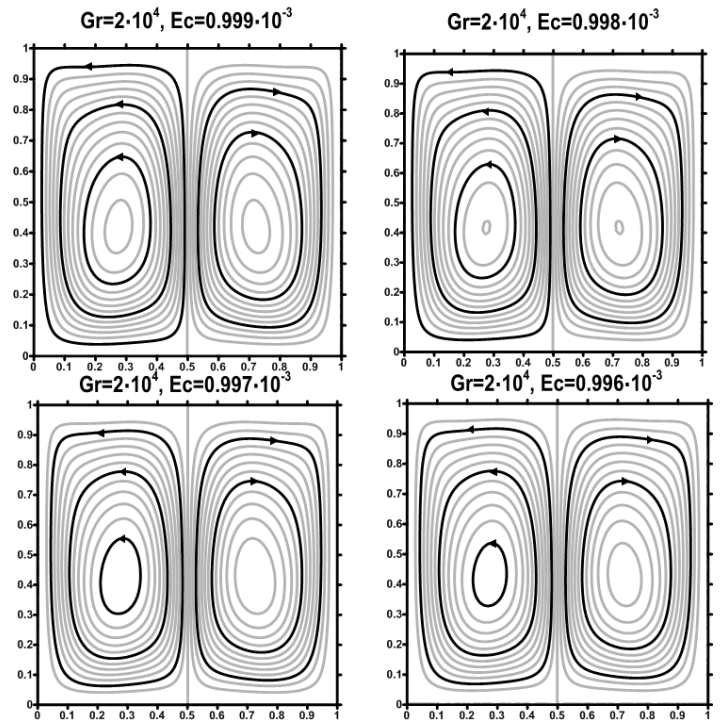


Figure 13 Representative plots of stream function: $Gr=2 \times 10^4$, $Ec \times 10^3 = 0.999, 0.998, 0.997, 0.996$, $\varepsilon \geq 0.2$ for stationary state.

CONCLUSION

It's found that there is a nonzero heat flow through the side walls for $\varepsilon \geq 0.2$. Value of Nu_r in stationary state is equal 46.063 for all $\varepsilon \geq 0.2$. For $0 < \varepsilon \leq 0.19$ viscous dissipation doesn't have influence on stationary solution of the problem, natural-convective flow decays during the time.

Absolute value of total heat flux on the wall falls down with decreasing Eckert and Grashof numbers. Value of Eckert number for which natural-convective flow decays for fixed $Gr = 2 \times 10^4$ is $Ec^* = 0.736 \times 10^{-3}$. Value of Grashof number for which natural-convective flow decays for fixed $Ec = 10^{-3}$ is $Gr^* = 1.47 \times 10^4$. Minimum value of total heat flux on the right boundary for which we observe stationary natural-convective flow is the same for dependences of Nu_r on Ec and Nu_r on Gr and is equal 3.98. There is not solution of the problem with $0 < Nu_r < 3.98$.

REFERENCES

- [1] Kalabin, E.V., Kanashina, M.V. and Zubkov, P.T., Natural-convective heat transfer in a square cavity with time-varying side-wall temperature, *Numerical Heat Transfer, Part A*, 47, 2005, pp. 621-631.
- [2] Kalabin, E.V., Kanashina, M.V. and Zubkov, P.T., Heat transfer from the cold wall of a square cavity to the hot one by oscillatory natural convection, *Numerical Heat Transfer, Part A*, 47, 2005, pp. 609-619.
- [3] Patankar S.V., *Numerical Heat Transfer and Fluid Flow*: Hemisphere Publ. Corp., 1980, 200 p.
- [4] Gebhart, B. Effect of viscous dissipation in natural convection, *Journal of Fluid Mechanics*, 14, 1962, pp. 225-232.
- [5] Gebhart, B., Mollendorf, J., Viscous dissipation in external natural convection flows, *Journal of Fluid Mechanics*, 38 (1), 1969, pp.97-107.
- [6] Hossain, M. A., Saha, Suvash C., Gorla, R. S. R. Viscous dissipation effects on natural convection from a vertical plate with uniform surface heat flux placed in a thermally stratified media, *International Journal of Fluid Mechanics Research*, 32(3), 2005, pp. 269-280.
- [7] Pantokratoras, A. Effect of viscous dissipation in natural convection along a heated vertical plate, *Applied Mathematical modelling*, 29(6), 2005, pp. 553-564.
- [8] Kabir, K.H., Alim, Md. A., Andallah, L.S. Effects of viscous dissipation on MHD natural convection flow along a vertical wavy surface, *Journal of Theoretical and Applied Physics*, 7 (31), 2013.
- [9] Abdullah-Al-Mamun, Ariful Azim, N.H.Md., Maleque, Md. A. Combined effect of conduction and viscous dissipation on magnetohydrodynamic free convection flow along a vertical plate, *Journal of Naval Architecture and Marine Engineering*, 4, 2007, pp. 87-98.
- [10] Kishore, P.M., Rajesh, V., Vijayakumar Verma, S. Effects of heat transfer and viscous dissipation on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature, *Journal of Naval Architecture and Marine Engineering*, 7, 2010, pp. 101-110.
- [11] Malga, B.S., Kishan, N. Viscous dissipation effects on unsteady free convection and mass transfer flow past an accelerated vertical porous plate with suction, *Advances in Applied Science Research*, 2 (6), 2011, pp. 460-469.
- [12] Magyaria, E., Reesb, D.A.S. Effect of viscous dissipation on the Darcy free convection boundary-layer flow over a vertical plate with exponential temperature distribution in a porous medium, *Fluid Dynamics Research*, 38, 2006, pp. 405-429.
- [13] Kairi, R.R., Murthy, P.V.S.N., Ng, C.O. Effect of viscous dissipation on natural convection in a non-Darcy porous medium saturated with non-Newtonian fluid of variable viscosity, *The Open Transport Phenomena Journal*, 3, 2011, pp. 1-8.
- [14] RamReddy, Ch., Murthy, P.V.S.N., Chamkha, A.J., Rashad, A.M. Influence of viscous dissipation on free convection in a non-Darcy porous medium saturated with nanofluid in the presence of magnetic field, *The Open Transport Phenomena Journal*, 5, 2013, pp. 20-29.
- [15] Roja, P., Sankar Reddy, T., Bhaskar Reddy, N. Radiation and mass transfer effects on MHD free convective flow of a micropolar fluid past an infinite vertical porous moving plate embedded in a porous medium with viscous dissipation, *International Journal of Scientific and Research Publications*, 3 (6), 2013.
- [16] Aktar, S., Ruma M.B.M, Parveen, N. Viscous dissipation effects on natural convection flow along a sphere with heat generation, *Global Journal of Science Frontier Research*, 10 (1), 2010.
- [17] Leslie, G.A., Wilson, S.K., Duffy, B.R. Thermoviscous coating and rimming flow, *Q. Jl. Mech. Appl. Math.* XX (XX), 2012, pp. 1-41.
- [18] Rigatos, A.P., Charalambakis, N.C. Two-dimensional adiabatic Newtonian flow with temperature-dependent viscosity, *International Journal of Engineering Science*, 39, 2001, pp. 1143-1165.
- [19] Elbashareshy, E.M.A., Ibrahim, F.N. Steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate, *Journal of Physics D: Applied Physics*, 26 (12), 1993, pp. 2137-2143.
- [20] Elbashareshy, E.M.A. Free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of the magnetic field, *International Journal of Engineering Science*, 38, 2000, pp. 207-213.
- [21] Jha, B.K., Abjibade, A.O. Effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions, *Thermophysics and Aeromechanics*, 18 (4), 2011, pp. 561-571.