Time Series Analysis of Persistence in Crude Oil Price Volatility across Bull and Bear Regimes*

Luis A. Gil-Alana[1], Rangan Gupta[2], Olusanya E. Olubusoye[3], and OlaOluwa S. Yaya[3]
[1]Department of Economics & ICS, University of Navarra, Pamplona, Spain
[2]Department of Economics, University of Pretoria, Pretoria, South Africa
[3]Department of Statistics, University of Ibadan, Ibadan, Nigeria

Highlights

- We analyze persistence in price and volatility of WTI oil price;
- We identify bull and bear market phases;
- We also conduct recursive estimation;
- Oil price is non-stationary and volatility exhibits long-memory;
- Degree of persistence increases when market phases are identified.

Abstract

This paper deals with the analysis of crude oil prices in the context of fractional integration and using bull and bear phases over monthly periods between September, 1859 to July, 2015. We examine both the log prices series as well as volatility, approximated by means of the absolute and the squared returns. The results for the whole sample indicate that the log-prices are nonstationary, with an order of integration close to 1 or even higher than 1, while the squared and absolute returns show evidence of long memory behavior. Upon separating the sample according to bull and bear periods, we observe an increase in the order of integration in both the log-prices and the two measures of volatility. Our results have important policy implications.

Keywords: Bull and bear regimes; Oil price; Persistence; Volatility; West Texas Intermediate market

1. Introduction

Following the seminal work of Hamilton (1983), a considerable body of literature has been published connecting movements in oil returns and its price volatility with recessions and inflationary episodes in the US economy (see for example, Balke et al., (2002, 2010), Brown and Yücel (2002), Barsky and Kilian (2004), Jones et al. (2004), Kilian (2008a,b, 2009a,b), Elder and

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Serletis (2010), Nakov and Pescatori (2010), Baumeister and Peersman (2013a,b), Kang and Ratti (2013a,b), Antonakakis et al., (2014a,b), and Bjørnland and Larsen (2015), references cited therein). According to Hamilton (2008), nine out of ten recessions in the US since World War II have been preceded by an increase in oil prices.[1] In fact, Hamilton (2009) even goes so far as to argue that a large proportion of the recent downturn in the US economy during the "Great Recession" can also be attributed to the oil price shock of 2007-2008.

So overall, the importance of the role played by the oil market for the US economy is, in general, undeniable. Hence, from the perspective of a policy maker, it is of paramount importance to determine whether the shocks to oil prices and its volatility are temporary or permanent in nature in order to help in better policy design. Against this backdrop, in this paper, we use a long-memory approach, which in turn, provides a direct measure of persistence, unlike unit root tests, to determine the persistence property of oil prices and its volatility.

There are three oil markets: the European Brent, the West Texas Intermediate (WTI) and the Organization of Petroleum Exporting Countries (OPEC), and the prices in the three markets are related to one another, so one market can explain by itself what happens at the international oil markets. In this study, we concentrate on the persistence properties of WTI oil price volatility covering the monthly periods of September, 1859 to July, 2015. In other words, we analyse persistence in volatility of the WTI oil returns spanning more than 150 years of the history of this market, with the starting date corresponding to the beginning of the modern era of the petroleum industry with the drilling of the first oil well in the U.S. at Titusville, Pennsylvania in August,

[1] Hamilton (2013) documents historical oil price shocks; identifying Drake's Oil Discovery of 1859, US Civil war, post-World-War-II, the Suez Crisis of 1956-57, the OPEC oil embargo of 1973-1974, the Iranian revolution of 1978-1979, the Iran-Iraq War initiated in 1980, the first Persian Gulf War in 1990-91, and the oil price spike of 2007-2008 and many other minor economic disturbances that followed each of the post war oil shocks.

1859.[2] Note that, even though our focus is on oil price volatility or uncertainty, for the sake of completeness, we also analyse the persistence property of oil prices in itself. For a detailed discussion of the literature on the persistence of oil prices, the reader is referred to Gil-Alana and Gupta (2014).

The persistence of volatility in oil prices are considered in Dvir and Rogoff (2014), Ewing and Malik (2010), Erten and Ocampo (2012), Zellou and Cuddington (2012), Bildirici and Ersin (2013, 2014), Charles and Darné (2014) and Mu and Ye (2015). Dvir and Rogoff (2014). The papers found that the real price of oil has undergone three phases: first, from 1861 to 1878 with high prices and high volatility persistence. The second phase, between 1878 and 1973 was characterized by low prices and less volatility persistence, while in the third identified phase, from 1973 till 2009, high prices with high volatility persistence is identified. Ewing and Malik (2010) examined the volatility persistence in oil prices by incorporating endogenously determined structural changes into a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models and found that oil price shocks dissipated sharply but did have a strong initial impact. Erten and Ocampo (2012) applied the asymmetric band-pass filter to decompose the time series of crude oil prices from 1875 to 2010 into a trend component and super-cycles lasting from 20 to 70 years and found an upward trend in oil prices before the 1920s, a downward trend between the 1920s and the 1960s, and an upward trend after 1960. Zellou and Cuddington (2012) also found evidence of cycles and trend in the prices of oil from 1861 to 2010, with significant cycles after the First World War period as the after match effect of global financial crises, with the significant one occurring in 2008. The weak evidence of cycles also occurred before the

[2] The beginning of new era in Oil production was marked in 1959 when Edwin Drake successful produced usable quentities of crude oil for commercial purposes from 69-foot well Pennsylvania. Drake discovery broke the oil market and his prices for oil rose and the prices of WTI oil quickly fell averaging $9.60 per barrel in 1860 which further reduced to10 cents per barrel by the end of 1861.

World war period. They further stated that the upward long term trend after the World war is considerably weaker when compared to the significant cycles. Bildirici and Ersin (2013; 2014) considered using a regime dynamic conditional variance modelling approach in predicting oil price dynamics. Bildirici and Ersin (2014) actually extended their work to include modelling persistence of volatility in the volatility series of WTI oil prices and found that the fractionally integrated power GARCH model performed best among other model variants. Charles and Darné (2014) considered volatility persistence in the three crude oil markets (WTI, Brent and OPEC) between 1985 and 2010 and identified significant structural changes where the volatility persistence were found to reduce when the variance changes were introduced into the volatility modelling process. Mu and Ye (2015) employed a methodology that allows one to disentangle the long term trend in oil prices from cyclical movements. Using data from 1861 to 2010, the authors identified a deterministic quadratic trend and two types of cycles with the short one having a 6 year-period along with long cycles of 29 year-periods. Their results further indicated smaller growth rate for the long term trend.

The prices in each of the oil markets can be viewed with respect to the level of persistence of volatility in the returns. Also, there may be non-linear dynamics in the time series of oil and this is synonymous to the financial theory of bull and bear phases. Given the history of oil prices, it is possible to examine the dynamics of oil price over time, identify possible market phases and investigate volatility persistence at each market phase. That is the focus of this paper. Salisu (2014) evaluated the comparative performance of symmetric and asymmetric volatility models for Brent and WTI oil prices using data before, during and after the global crisis. The results showed evidence of leverage effects in both oil prices. The results further showed that bad news in the oil market would increase volatility in crude oil prices more than good news, and in terms of persistence of volatility, the WTI oil price series was found to be more persistent than the Brent oil price series. Salisu (2014) only considered one event that has provoked a crisis in oil prices, but it is of importance to study first the dynamics of breaks in oil prices and relate this with the volatility persistence.

Few studies have examined modelling crude oil market volatility using the classical GARCH model and its variants. Narayan and Narayan (2007) examined the volatility series of oil at different sub-samples and obtained inconsistent evidence of asymmetry and persistence of shocks, and on the overall series, the shocks tended to be permanent, and exerting asymmetric effects on the volatility. Alom, Ward and Hu (2012) examined the asymmetry and persistency of volatility in the prices of crude oil and its products using the GARCH-type modelling framework, and found different persistence levels of volatility. Salisu and Fasanya (2012) analyzed WTI and Brent crude oil prices using the structural break tests of Narayan and Popp (2010) and Liu and Narayan (2010), and obtained results that support identifying structural breaks in oil price time series before empirical analysis of price volatility. Salisu and Mobolaji (2013) investigated returns and volatility transmission between oil prices and the US-Nigeria exchange rate using an approach which allowed for the operation of two structural breaks and found the timing of the structural breaks to coincide with the period of global financial crisis.

The above mentioned review of the univariate works shows that the analysis of crude oil prices has not been extended to cover prices: (i) based on bull and bear markets-based sub-samples; (ii) using fractional integration techniques to investigate persistence of volatility based on the absolute and squared returns of the oil prices, and; (iii) covering over 150 years of data, which in turn, includes the entire history of the modern oil industry in the US. The identification of bull and bear financial phases of crude oil is achieved by applying an algorithm by Pagan and

Soussonouv (2003). To the best of our knowledge, Narayan and Narayan (2007), Dvir and Rogoff (2014) and Salisu (2014) are the only papers that have attempted to identify sub-samples for oil prices, and which have modeled volatility. In Narayan and Narayan (2007), there is no clear-cut justification for choosing the series subsamples applied in the paper. Dvir and Rogoff (2014) obtained their sub-samples based on low and high oil prices which corresponded to periods of low and high volatilities, while Salisu (2014) based his data sub-samples on periods of global financial crisis. In the present study, we are motivated by the recent bull and bear behaviour in the oil markets (see Tokic, 2010; 2015).[3] This approach of identifying market phases over 150 years of data also helps us to control for the sensitivity of the estimates of persistence due to structural breaks in the oil market.

The rest of the paper is then structured as follows: Section 2 describes the methodology used in the paper which is based on fractional integration and Pagan and Sossounov' (2003) algorithm for the detection of bull and bear phases. Section 3 presents the data and the main empirical results, while Section 4 concludes the manuscript.

2. Methodology

2.1 Fractional dependence approach

The methodology used in the paper to model the WTI series is based on fractional integration. The idea behind this concept is that the number of differences required in the series to render it stationary I(0) may be a fractional number. In other words, we say a time series $\{x_t, t = 0, \pm 1, ...\}$ is integrated of order d, and denoted by $x_t \approx I(d)$, if it can be represented as

[3]Tokic (2015) found that the 2014 oil price collapse was triggered by the fall in the dollar-euro exchange rate as a result of the sudden economic growth outlook divergence between the United States and the European communities. Before this period, oil price had experienced bubble tagged as 2008 oil bubble (see Charles and Darné, 2009; Tokic, 2010; Wang and Liu, 2010; Ortiz-Cruz et al., 2012 and Stevens and de Lamirande, 2014).

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, ..., \tag{1}$$

where L is the lag-operator ($Lx_t = x_{t-1}$); d can be any real value, and u_t is I(0) defined as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. Note that, for any real value d, the polynomial in the left hand side in (1) can be expressed in terms of its Binomial expansion such that:

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - d L + \frac{d(d-1)}{2} L^2 - \ldots , \tag{2}$$

implying that the higher the value of d is, the higher the degree of association between observations that are widely separated in time. Thus, the parameter *d* plays a crucial role in determining the degree of persistence of the series. If $d = 0$ in (1), clearly $x_t = u_t$, the process is short memory, it is covariance stationary, and it may be weakly Autoregressive Moving Average (ARMA) autocorrelated, with the values in the autocorrelation function decaying exponentially fast. If *d* belongs to the interval (0, 0.5), x_t is still covariance stationary though the autocorrelations will take longer to disappear than in the previous case of I(0) behaviour; if d belongs to [0.5, 1), the process is no longer covariance stationary though it is still mean reverting in the sense that shocks will tend to disappear in the long run.[2] Finally, if $d \geq 1$, x_t is nonstationary and not mean reverting. Processes of the form given by (1) with positive non-integer d are called fractionally integrated, and when u_t is ARMA(p, q) x_t is known as a Fractionally Integrated ARMA (or ARFIMA) model.

The methodology used in the paper to estimate the fractional differencing parameter is based on the Whittle function in the frequency domain, using both parametric (Dahlhaus, 1989) and semiparametric (Robinson, 1995; Abadir et al., 2007) methods. The use of other approaches,

[2]In the case of d in the interval [0.5, 1) some authors argue that "mean reversion" is a misnomer given the nonstationarity nature of the process (Phillips and Xiao, 1999).

such as the Maximum Likelihood (ML) method in the time domain (Sowell, 1992; Beran, 1995) produced essentially the same results.

Given that our paper covers over 150 years of data, it is not illogical to assume that there are likely to be regime changes in the oil market. Understandably, parameter estimates across these regimes cannot be expected to remain the same. Hence, we supplement our long memory estimations for the full-sample by repeating the econometric analysis across the bull and bear phases of the oil market. The following sub-section discusses how we identify these two regimes.

2.2 Algorithm for identifying the Market Phases

Following Pagan and Sossounov (2003), steps of an algorithm based on their Fortran programme is written to split the WTI oil price time series into bull and bear phases. The bull phase is the period between an immediate trough and the next peak, while the bear phase is the period between an immediate peak and the next trough of the time series.

The steps of the algorithm are presented as follow:

(1a). Determine the initial turning points in raw data by choosing local peaks (troughs) as occurring when they are the highest (lowest) values in a window of eight months on either side of the date.

(1b). Enforce alternation of turns by selecting the highest of the multiple peaks (or the lowest of the multiple troughs).

(2a). Eliminate turns within six months of beginning and end of the series.

(2b). Eliminate peaks (or troughs) at both ends of the series which are lower or higher.

(2c). Eliminate cycles whose duration is less than 16 months.

(2d). Eliminate phases whose duration is less than four months (unless fall or rise exceeds 20%).

3. Data, Results and Discussion

The data considered in this work are the monthly crude oil prices of the West Texas Intermediate (WTI) market, which spans the monthly periods of September 1859 and July 2015 (i.e., 1872 data points). The data is sourced from the Global Financial Database. Note that, we are using nominal oil prices here, i.e., oil prices in current US dollars. There are three reasons behind such a choice: (1) Reliable data on seasonally-unadjusted Consumer Price Index (CPI) at monthly frequency from FRED database of the St. Louis Federal Reserve (primary source Bureau of Labor Statistics) is only available from 1913, hence we would lose over fifty years of data if real oil prices were used. Some seasonally unadjusted CPI data starting in 1871 is available from Professor Robert J. Shiller's website.[4] Again, this would imply losing 12 years or so of data and also important events in the oil industry during the 1860s (see Table 1). But since standard approaches of seasonal adjustments, like the X-13, cannot handle more than 50 years of data, these could not be used either; (ii) More importantly, as indicated by Hamilton (2011), what is more important in terms of explaining US business cycle movements is the nominal and not real oil price. Hamilton (2011) suggests that using real values induces measurement errors. More specifically, he writes: "…deflating by a particular number, such as the CPI, introduces a new source of measurement error, which could lead to deterioration in the forecasting performance. In any case, it is again quite possible that there are differences in the functional form of forecasts based on nominal prices instead of real prices" (page 370). Hence, Hamilton (2011) recommends modeling and forecasting of nominal oil prices, returns, and volatility and not real values of the same, and; (iii) The concept of volatility is based on squared or absolute nominal returns, and not real returns. This is understandable, since using real prices would mean that the volatility is driven by two components, the nominal oil price and the CPI, so one would not

[4] http://www.econ.yale.edu/~shiller/data.htm.

know the true data generating structure and persistence behaviour of the returns. These are the three serious reasons, we stick to nominal WTI oil prices rather than real values of the same.

Plots of the original time series of (oil price), log-transformed returns, absolute and squared returns are given in Figures 1(i) – 1(iv) below. From the time plot of prices in Figure 1(i), we observed the oil price at $20 in 1859 and this dropped sharply to $1 around 1860s as a result of Drake's oil that broke the WTI oil market, and later rose to $12 around 1864 as a result of scarcity caused by US civil war (Ripy, 1999). This maintained the stable prices of $0.50-$3 until 1967 when it started experiencing astronomical rises again. Series of events that caused increase and decrease in the prices of oil are stated in Table 1 below, of which the outbreaks of war between the oil producing economies had significant effects in the pricing of WTI oil at the markets.

Figures 1(ii) – 1(iv) show the returns (as well as the volatility, approximated by the squared and absolute returns) display higher volatility, especially at the beginning and at the end of the sample.

We first applied Pagan and Soussonouv (2003) algorithm to identify the market phases of oil prices. The identification is summarized in Table 1, where the first bear phase commenced at December 1859 with oil price of $20 and ended at December 1861 with $0.10 price. Table 1 further indicated the price of oil at the end of each market phase as well as the event that occurred during phase.

Figure 1: Time series plots

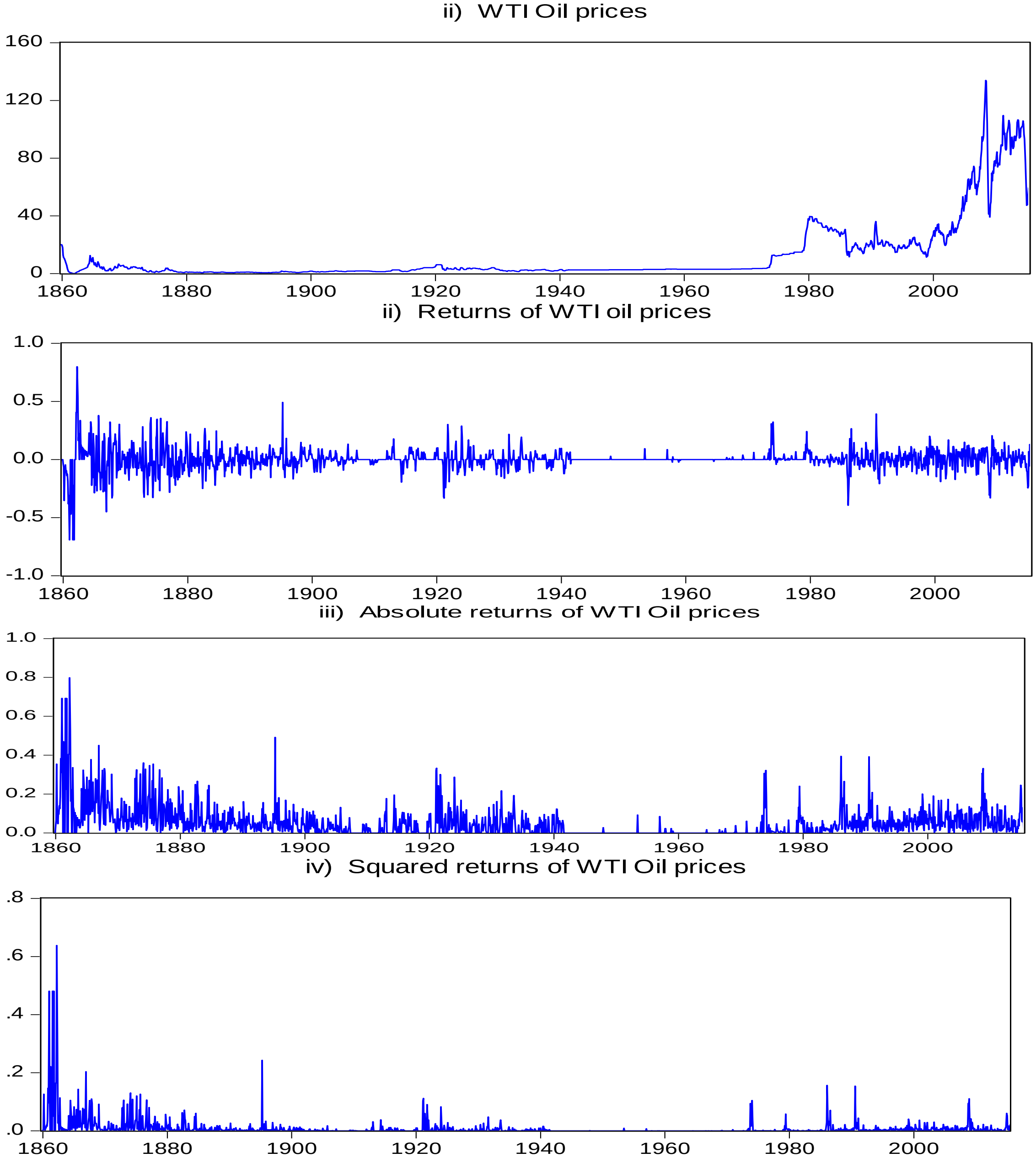


Table 1: Bull and Bear Market Phases of WTI Crude Oil

Phases	Market	Period	Phase ending value $	Events that Occurred
1st	1st Bear	1859M12-1862M01	0.10	Drake's Oil Discovery
2nd	1st Bull	1862M01-1864M07	12.50	U.S. Civil War: First Oil Shock
3rd	2nd Bear	1864M07-1892M10	0.51	Evolution of many Oil production industries
4th	2nd Bull	1892M10-1920M12	6.10	High demand for power and transportation
5th	3rd Bear	1920M12-1933M05	1.30	The West Coast Gasoline Famine of 1920 and the Great Depression
6th	3rd Bull	1933M05-1980M06	39.50	The early post-war era and boom in US oil production, OPEC embargo and Iranian Revolution
7th	4th Bear	1980M06-1986M03	12.61	The Great Price collapse period
8th	4th Bull	1986M03-2008M06	133.88	First Persian gulf war; More global industries depending on oil; East Asian crisis; Venezuelan unrest and the second Persian Gulf War.
9th	5th Bear	2008M06-2009M02	39.09	Global financial crisis
10th	5th Bull	2009M02-2013M09	106.29	Recovery from global financial crisis
11th	6th Bear	2013M09-2015M01	47.22	The birth of the shale revolution; oversupply of oil in the market;

We follow the algorithm proposed by Pagan and Sossounov (2003) in identifying the market phases.

We next focus on the estimation of the fractional differencing parameter, first with a parametric approach (in Table 2) and then using a "local" Whittle semi-parametric method (in Table 3). In the parametric context, we consider both the cases of uncorrelated (white noise) and autocorrelated errors. In the latter case, we use Bloomfield's (1973) method, which is a

Table 2: Estimates of d and their corresponding confidence bands with a parametric method

		No det. terms	An intercept	A linear trend
Log WTI	Wh. Noise	1.17 (1.13, 1.21)	1.30 (1.26, 1.35)	1.30 (1.26, 1.35)
	Autoc.	1.01 (0.93, 1.09)	1.07 (1.00, 1.12)	1.07 (1.00, 1.12)
Abs. Rtns.	Wh. Noise	0.33 (0.30, 0.36)	0.30 (0.28, 0.33)	0.29 (0.27, 0.32)
	Autoc.	0.37 (0.32, 0.40)	0.32 (0.29, 0.35)	0.31 (0.28, 0.34)
Sq. Rtns.	Wh. Noise	0.25 (0.23, 0.28)	0.24 (0.22, 0.27)	0.24 (0.21, 0.26)
	Autoc.	0.32 (0.28, 0.36)	0.30 (0.27, 0.35)	0.29 (0.26, 0.34)

The values in parenthesis refer to the 95% confidence intervals for the estimation of d.

Table 3: Estimates of d using a local Whittle semiparametric method

M	40	41	42	43	44	45	46	47	48	49	50
Log WTI	0.708	0.717	0.727	0.735	0.743	0.755	0.755	0.761	0.767	0.771	0.781
Abs. Rtns.	0.477	0.474	0.475	0.485	0.496	0.499	0.500	0.500	0.500	0.500	0.495
Sq. Rtns.	0.291	0.292	0.295	0.304	0.316	0.320	0.323	0.332	0.337	0.328	0.328

m refers to the bandwidth number.

nonparametric approach of modelling I(0) errors that produce autocorrelation decaying exponentially to zero as in the AR case. Starting with the parametric method, we consider the three standard cases of no regressors, an intercept, and with an intercept and a linear time trend, and using the log WTI oil prices series, we observe that the unit root null hypothesis is rejected in favour of higher order of integration if the errors are uncorrelated; however, allowing for autocorrelation, though the estimated value of d is slightly above 1, the unit root null hypothesis (i.e., $d = 1$) cannot be rejected. Focusing now on the volatility, measured in terms of the squared and absolute returns, the estimated value of d is found to be significantly positive in all cases, providing evidence in favour of long memory behaviour often experienced in the volatility series of assets prices.

Using the "local" Whittle semi-parametric methods (Table 3), the results for a selected group of bandwidths $m = 40, 41, ..., 50$ are displayed in Table 3. We observe that the estimates of d for the log WTI oil prices are around 0.75 for all bandwidth numbers, implying now some evidence of mean reverting behaviour. For the squared and absolute returns, the same evidence of long memory (i.e., $d > 0$) as with the parametric method is now obtained with the semi-parametric one.

Table 4 focuses on the different subsamples according to the bull and bear phases previously identified. In general we observed an increase in the degree of integration in the bull periods compared with the bear ones. If we focus on the absolute and squared returns (in Tables 5 and 6) the results are a slightly more ambiguous in the sense that we do not observe any systematic increase or decrease in the estimated values of d when moving from a bull period to a bear one.[5]

[5] Since our algorithm identified the end of the last bear phase in January, 2015, while oil prices were still falling, based on the suggestions of an anonymous referee, we re-conducted the estimation over the period of September, 2013 to July, 2015. Not surprisingly, the additional six data points did not affect our results qualitatively and only in a minor way quantitatively. Complete details of these results are available upon request from the authors.

Table 4: Estimates of d and their corresponding confidence bands with a parametric method

Log WTI	u_t	No det. terms	An intercept	A linear trend
Bear 1	Wh. Noise	1.01 (0.78, 1.38)	1.01 (0.88, 1.36)	0.93 (0.54, 1.39)
	Autoc.	0.76 (-0.08, 1.44)	0.90 (-0.08, 1.33)	0.26 (-0.56, 1.57)
Bull 2	Wh. Noise	0.78 (0.46, 1.29)	1.69 (1.40, 2.14)	1.50 (1.29, 1.84)
	Autoc.	0.30 (-0.61, 0.92)	xxx	xxx
Bear 2	Wh. Noise	0.97 (0.89, 1.08)	1.12 (1.00, 1.26)	1.11 (1.00, 1.26)
	Autoc.	0.79 (0.69, 0.95)	0.69 (0.58, 0.85)	0.73 (0.62, 0.86)
Bull 2	Wh. Noise	1.17 (1.09, 1.28)	1.26 (1.17, 1.37)	1.26 (1.17, 1.37)
	Autoc.	1.01 (0.85, 1.20)	1.07 (0.92, 1.24)	1.07 (0.93, 1.24)
Bear 3	Wh. Noise	0.87 (0.75, 1.07)	1.34 (1.14, 1.63)	1.33 (1.13, 1.61)
	Autoc.	0.60 (0.47, 0.79)	0.50 (0.38, 0.76)	0.54 (0.35, 0.79)
Bull 3	Wh. Noise	1.43 (1.34, 1.50)	1.46 (1.39, 1.54)	1.46 (1.39, 1.54)
	Autoc.	1.36 (1.24, 1.52)	1.31 (1.19, 1.48)	1.31 (1.19, 1.48)
Bear 4	Wh. Noise	0.86 (0.72, 1.07)	1.67 (1.48, 1.89)	1.66 (1.46, 1.89)
	Autoc.	0.71 (0.41, 1.06)	0.11 (-0.11, 0.40)	xxx
Bull 4	Wh. Noise	1.05 (0.98, 1.15)	1.09 (0.99, 1.23)	1.09 (0.99, 1.23)
	Autoc.	1.05 (0.92, 1.22)	0.84 (0.73, 1.02)	0.86 (0.73, 1.02)
Bear 5	Wh. Noise	0.38 (-0.09, 1.39)	xxx	xxx
	Autoc.	xxx	xxx	xxx
Bull 5	Wh. Noise	1.00 (0.83, 1.23)	1.04 (0.76, 1.34)	1.04 (0.85, 1.28)
	Autoc.	0.91 (0.54, 1.30)	xxx	0.94 (0.29, 1.29)
Bear 6	Wh. Noise	0.64 (0.17, 1.18)	1.64 (1.36, 1.91)	1.66 (1.38, 1.93)
	Autoc.	xxx	xxx	xxx

The values in parenthesis refer to the 95% confidence intervals for the estimation of d. XXX means that convergence was not achieved.

Table 5: Estimates of d and their corresponding confidence bands with a parametric method

Abs. returns	u_t	No det. terms	An intercept	A linear trend
Bear 1	Wh. Noise	-0.01 (-0.18, 0.34)	-0.02 (-0.33, 0.40)	-0.12 (-0.58, 0.37)
	Autoc.	-0.06 (-0.73, 0.54)	-0.11 (-0.71, 0.67)	xxx
Bull 1	Wh. Noise	0.65 (0.33, 1.09)	0.47 (0.25, 0.85)	0.36 (0.06, 0.84)
	Autoc.	xxx	-0.16 (-0.61, 0.34)	
Bear 2	Wh. Noise	0.26 (0.20, 0.32)	0.20 (0.16, 0.25)	0.11 (0.05, 0.19)
	Autoc.	0.27 (0.19, 0.35)	0.22 (0.16, 0.29)	0.10 (0.02, 0.21)
Bull 2	Wh. Noise	0.25 (0.19, 0.33)	0.22 (0.16, 0.30)	0.19 (0.13, 0.28)
	Autoc.	0.10 (-0.01, 0.22)	0.08 (-0.01, 0.21)	0.03 (-0.06, 0.17)
Bear 3	Wh. Noise	0.20 (0.16, 0.46)	0.21 (0.11, 0.36)	0.19 (0.08, 0.34)
	Autoc.	0.00 (-0.16, 0.18)	0.00 (-0.12, 0.15)	0.00 (-0.12, 0.18)
Bull 3	Wh. Noise	0.43 (0.37, 0.50)	0.42 (0.36, 0.49)	0.43 (0.37, 0.50)
	Autoc.	0.28 (0.16, 0.43)	0.29 (0.19, 0.49)	0.30 (0.18, 0.46)
Bear 4	Wh. Noise	0.64 (0.49, 0.81)	0.63 (0.47, 0.81)	0.63 (0.49, 0.81)
	Autoc.	xxx	xxx	xxx
Bull 4	Wh. Noise	0.17 (0.07, 0.28)	0.14 (0.06, 0.23)	0.14 (0.06, 0.23)
	Autoc.	0.05 (-0.09, 0.25)	0.06 (-0.08, 0.20)	0.05 (-0.09, 0.23)
Bear 5	Wh. Noise	0.13 (-0.81, 1.13)	0.17 (-0.79, 1.16)	0.18 (-0.77, 1.17)
	Autoc.	xxx	xxx	xxx
Bull 5	Wh. Noise	0.03 (-0.41, 0.30)	0.01 (-0.14, 0.21)	-0.01 (-0.16, 0.21)
	Autoc.	0.27 (-0.53, 0.93)	0.18 (-0.23, 1.22)	0.82 (-0.07, 1.17)
Bear 6	Wh. Noise	0.76 (0.48, 1.06)	0.74 (0.39, 1.04)	0.72 (0.42, 1.04)
	Autoc.	xxx	xxx	xxx

The values in parenthesis refer to the 95% confidence intervals for the estimation of d. XXX means that convergence was not achieved.

Table 6: Estimates of d and their corresponding confidence bands with a parametric method

Sq. returns	u_t	No det. terms	An intercept	A linear trend
Bear 1	Wh. Noise	-0.05 (-0.18, 0.20)	-0.09 (-0.32, 0.25)	-0.51 (-0.88, 0.15)
	Autoc.	-0.06 (-0.62, 0.54)	-0.11 (-0.71, 0.66)	xxx
Bull 1	Wh. Noise	0.05 (-0.01, 0.12)	0.05 (-0.01, 0.11)	0.22 (-0.10, 0.76)
	Autoc.	0.10 (-0.02, 0.22)	0.08 (-0.01, 0.21)	xxx
Bear 2	Wh. Noise	0.18 (0.13, 0.24)	0.15 (0.11, 0.21)	0.07 (0.02, 0.13)
	Autoc.	0.27 (0.19, 0.36)	0.22 (0.16, 0.29)	0.10 (0.02, 0.20)
Bull 2	Wh. Noise	0.05 (-0.01, 0.12)	0.05 (-0.01, 0.11)	0.02 (-0.04, 0.09)
	Autoc.	0.10 (-0.02, 0.22)	0.08 (-0.01, 0.21)	0.03 (-0.06, 0.18)
Bear 3	Wh. Noise	0.27 (0.15, 0.42)	0.22 (0.12, 0.35)	0.20 (0.10, 0.34)
	Autoc.	0.01 (-0.16, 0.18)	0.00 (-0.11, 0.15)	0.00 (-0.12, 0.18)
Bull 3	Wh. Noise	0.31 (0.25, 0.37)	0.30 (0.25, 0.37)	0.31 (0.24, 0.38)
	Autoc.	0.28 (0.16, 0.42)	0.29 (0.18, 0.44)	0.30 (0.18, 0.46)
Bear 4	Wh. Noise	0.36 (0.17, 0.56)	0.35 (0.16, 0.56)	0.36 (0.20, 0.55)
	Autoc.	xxx	xxx	xxx
Bull 4	Wh. Noise	0.16 (0.07, 0.27)	0.15 (0.06, 0.25)	0.15 (0.06, 0.25)
	Autoc.	0.05 (-0.09, 0.27)	0.06 (-0.08, 0.23)	0.05 (-0.09, 0.25)
Bear 5	Wh. Noise	0.13 (-0.46, 1.15)	0.15 (-0.44, 1.15)	0.13 (-0.47, 1.16)
	Autoc.	xxx	xxx	xxx
Bull 5	Wh. Noise	0.07 (-0.14, 0.28)	0.05 (-0.09, 0.23)	0.07 (-0.08, 0.31)
	Autoc.	0.27 (-0.12, 0.93)	0.17 (-0.23, 1.24)	0.82 (-0.08, 1.18)
Bear 6	Wh. Noise	0.71 (-0.04, 1.03)	0.69 (-0.06, 1.07)	0.69 (0.30, 1.06)
	Autoc.	xxx	xxx	xxx

The values in parenthesis refer to the 95% confidence intervals for the estimation of d. XXX means that convergence was not achieved.

We then conducted some recursive estimates of d, starting with the first bear period and adding successively the observations corresponding to each bull and bear sub-periods. The results for the log WTI oil prices, absolute and square returns are reported, respectively in Tables

7, 8 and 9. Figure 2 displays the plots for each of the cases under both white noise and autocorrelated (Bloomfield) disturbances.

Table 7: Estimates of d and their corresponding confidence bands with a parametric method

Log WTI	u_t	No det. terms	An intercept	A linear trend
Bear 1	Wh. Noise	1.01 (0.78, 1.38)	1.01 (0.88, 1.36)	0.93 (0.54, 1.39)
	Autoc.	0.76 (-0.08, 1.44)	0.90 (-0.08, 1.33)	0.26 (-0.56, 1.57)
Cycle 1	Wh. Noise	1.20 (1.05, 1.41)	1.48 (1.34, 1.68)	1.47 (1.34, 1.68)
	Autoc.	1.18 (0.89, 1.57)	1.56 (1.22, 1.98)	1.52 (1.22, 1.99)
Bear 2	Wh. Noise	1.14 (1.05, 1.23)	1.31 (1.23, 1.42)	1.31 (1.23, 1.41)
	Autoc.	0.99 (0.84, 1.14)	1.09 (0.98, 1.24)	1.09 (0.98, 1.24)
Cycle 2	Wh. Noise	1.15 (1.09, 1.22)	1.31 (1.24, 1.38)	1.31 (1.24, 1.38)
	Autoc.	1.00 (0.89, 1.11)	1.11 (1.00, 1.20)	1.11 (1.00, 1.20)
Bear 3	Wh. Noise	1.15 (1.10, 1.22)	1.31 (1.25, 1.38)	1.31 (1.25, 1.38)
	Autoc.	0.97 (0.86, 1.08)	1.06 (0.98, 1.17)	1.06 (0.98, 1.17)
Cycle 3	Wh. Noise	1.17 (1.12, 1.22)	1.32 (1.27, 1.37)	1.32 (1.27, 1.37)
	Autoc.	1.01 (0.94, 1.08)	1.07 (0.99, 1.13)	1.07 (0.99, 1.13)
Bear 4	Wh. Noise	1.14 (1.09, 1.19)	1.33 (1.29, 1.39)	1.33 (1.29, 1.39)
	Autoc.	0.94 (0.87, 1.03)	1.10 (1.02, 1.18)	1.10 (1.02, 1.18)
Cycle 4	Wh. Noise	1.17 (1.13, 1.22)	1.30 (1.25, 1.34)	1.30 (1.25, 1.34)
	Autoc.	1.01 (0.94, 1.07)	1.03 (0.97, 1.10)	1.03 (0.97, 1.10)
Bear 5	Wh. Noise	1.16 (1.11, 1.21)	1.31 (1.27, 1.36)	1.31 (1.27, 1.36)
	Autoc.	0.93 (0.86, 1.00)	1.07 (1.01, 1.13)	1.07 (1.01, 1.13)
Cycle 5	Wh. Noise	1.17 (1.13, 1.21)	1.30 (1.26, 1.35)	1.30 (1.26, 1.35)
	Autoc.	0.99 (0.92, 1.07)	1.06 (0.97, 1.10)	1.06 (0.97, 1.10)
Whole sample	Wh. Noise	1.17 (1.13, 1.21)	1.30 (1.26, 1.35)	1.30 (1.26, 1.35)
	Autoc.	1.01 (0.93, 1.09)	1.07 (1.00, 1.12)	1.07 (1.00, 1.12)

The values in parenthesis refer to the 95% confidence intervals for the estimation of d. XXX means that convergence was not achieved.

Table 8: Estimates of d and their corresponding confidence bands with a parametric method

Abs. returns	u_t	No det. terms	An intercept	A linear trend
Bear 1	Wh. Noise	-0.02 (-0.18, 0.34)	-0.03 (-0.37, 0.40)	-0.12 (-0.63, 0.39)
	Autoc.	-0.15 (-0.91, 0.25)	-0.40 (-0.93, 0.46)	xxx
Cycle 1	Wh. Noise	0.26 (0.08, 0.52)	0.21 (0.06, 0.47)	0.17 (-0.02, 0.47)
	Autoc.	0.17 (-0.19, 0.62)	0.11 (-0.12, 0.47)	-0.06 (-0.35, 0.50)
Bear 2	Wh. Noise	0.29 (0.23, 0.35)	0.24 (0.18, 0.30)	0.16 (0.09, 0.24)
	Autoc.	0.32 (0.25, 0.43)	0.24 (0.18, 0.32)	0.10 (0.00, 0.23)
Cycle 2	Wh. Noise	0.31 (0.27, 0.35)	0.26 (0.23, 0.30)	0.19 (0.15, 0.25)
	Autoc.	0.36 (0.31, 0.42)	0.29 (0.25, 0.34)	0.18 (0.11, 0.25)
Bear 3	Wh. Noise	0.31 (0.27, 0.35)	0.27 (0.24, 0.34)	0.23 (0.19, 0.28)
	Autoc.	0.34 (0.29, 0.39)	0.27 (0.24, 0.33)	0.22 (0.17, 0.28)
Cycle 3	Wh. Noise	0.33 (0.30, 0.36)	0.29 (0.27, 0.32)	0.26 (0.23, 0.29)
	Autoc.	0.36 (0.32, 0.40)	0.30 (0.27, 0.34)	0.26 (0.22, 0.31)
Bear 4	Wh. Noise	0.34 (0.32, 0.37)	0.31 (0.28, 0.34)	0.28 (0.25, 0.31)
	Autoc.	0.39 (0.35, 0.43)	0.32 (0.29, 0.36)	0.29 (0.25, 0.32)
Cycle 4	Wh. Noise	0.33 (0.30, 0.36)	0.30 (0.27, 0.32)	0.29 (0.26, 0.31)
	Autoc.	0.36 (0.33, 0.39)	0.31 (0.28, 0.34)	0.30 (0.27, 0.34)
Bear 5	Wh. Noise	0.33 (0.31, 0.36)	0.30 (0.28, 0.33)	0.30 (0.27, 0.32)
	Autoc.	0.37 (0.33, 0.40)	0.32 (0.30, 0.35)	0.31 (0.28, 0.35)
Cycle 5	Wh. Noise	0.32 (0.30, 0.35)	0.29 (0.27, 0.32)	0.29 (0.27, 0.31)
	Autoc.	0.35 (0.32, 0.39)	0.31 (0.28, 0.34)	0.31 (0.27, 0.35)
Whole sample	Wh. Noise	0.33 (0.30, 0.36)	0.30 (0.28, 0.33)	0.29 (0.27, 0.32)
	Autoc.	0.37 (0.32, 0.40)	0.32 (0.29, 0.35)	0.31 (0.28, 0.34)

The values in parenthesis refer to the 95% confidence intervals for the estimation of d. XXX means that convergence was not achieved.

It is observed that, with the exception of one single case (log WTI oil price series with autocorrelation), there is, in every case, an increase in the estimated value of d as we increase the number of observations in the sample, especially for the initial periods of the sample, and this happens not only for the log-prices but also for the two measures of the volatility.

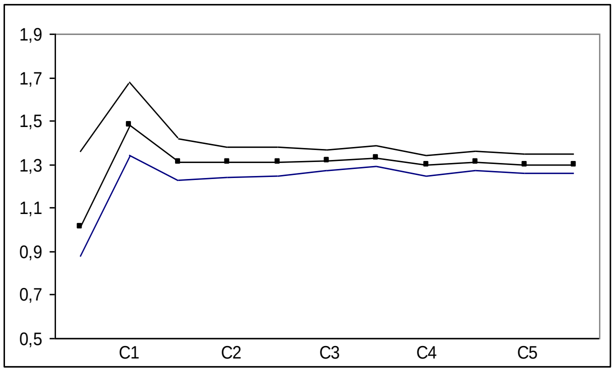
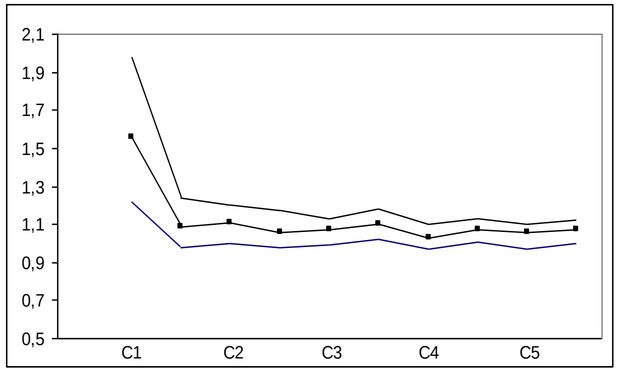
Table 9: Estimates of d and their corresponding confidence bands with a parametric method

Sq. returns	u_t	No det. terms	An intercept	A linear trend
Bear 1	Wh. Noise	-0.05 (-0.20, 0.20)	-0.08 (-0.30, 0.25)	-0.53 (-1.31, 0.14)
	Autoc.	-0.06 (-0.72, 0.53)	-0.11 (-0.71, 0.71)	xxx
Cycle 1	Wh. Noise	0.18 (0.04, 0.39)	0.17 (0.03, 0.37)	0.13 (-0.02, 0.37)
	Autoc.	0.20 (-0.06, 0.57)	0.18 (-0.05, 0.54)	0.10 (-0.17, 0.53)
Bear 2	Wh. Noise	0.24 (0.19, 0.30)	0.22 (0.17, 0.28)	0.17 (0.12, 0.24)
	Autoc.	0.31 (0.23, 0.41)	0.28 (0.20, 0.37)	0.21 (0.12, 0.32)
Cycle 2	Wh. Noise	0.24 (0.21, 0.29)	0.23 (0.19, 0.27)	0.20 (0.16, 0.24)
	Autoc.	0.33 (0.27, 0.39)	0.29 (0.24, 0.35)	0.25 (0.19, 0.32)
Bear 3	Wh. Noise	0.25 (0.21, 0.28)	0.23 (0.20, 0.27)	0.21 (0.17, 0.25)
	Autoc.	0.32 (0.26, 0.39)	0.28 (0.24, 0.34)	0.26 (0.21, 0.33)
Cycle 3	Wh. Noise	0.25 (0.23, 0.28)	0.24 (0.21, 0.26)	0.22 (0.20, 0.25)
	Autoc.	0.33 (0.28, 0.38)	0.30 (0.26, 0.35)	0.28 (0.24, 0.33)
Bear 4	Wh. Noise	0.26 (0.23, 0.29)	0.24 (0.22, 0.27)	0.23 (0.20, 0.26)
	Autoc.	0.34 (0.30, 0.39)	0.30 (0.27, 0.35)	0.29 (0.25, 0.35)
Cycle 4	Wh. Noise	0.25 (0.23, 0.28)	0.24 (0.22, 0.26)	0.23 (0.21, 0.26)
	Autoc.	0.33 (0.29, 0.36)	0.29 (0.26, 0.33)	0.29 (0.26, 0.34)
Bear 5	Wh. Noise	0.26 (0.23, 0.28)	0.24 (0.22, 0.27)	0.24 (0.21, 0.26)
	Autoc.	0.33 (0.29, 0.38)	0.31 (0.27, 0.35)	0.30 (0.26, 0.34)
Cycle 5	Wh. Noise	0.25 (0.23, 0.28)	0.24 (0.22, 0.26)	0.23 (0.21, 0.26)
	Autoc.	0.32 (0.28, 0.37)	0.29 (0.26, 0.34)	0.29 (0.25, 0.33)
Whole sample	Wh. Noise	0.25 (0.23, 0.28)	0.24 (0.22, 0.27)	0.24 (0.21, 0.26)
	Autoc.	0.32 (0.28, 0.36)	0.30 (0.27, 0.35)	0.29 (0.26, 0.34)

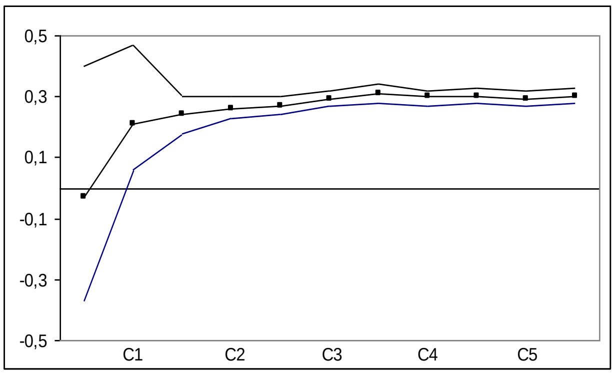
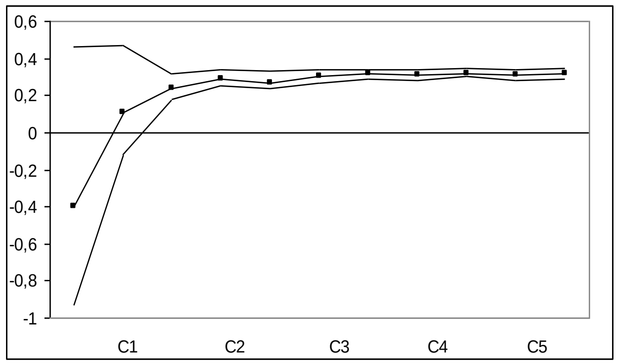
The values in parenthesis refer to the 95% confidence intervals for the estimation of d. XXX means that convergence was not achieved.

Figure 2: Recursive estimates across the cycles

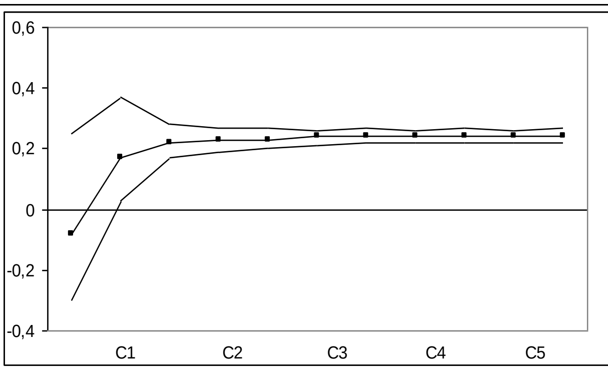
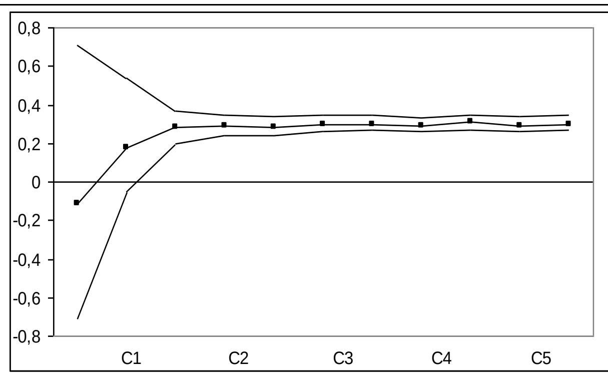
i) Log of WTI

White noise errors	Autocorrelation
	

ii) Absolute returns of WTI

White noise errors	Autocorrelation
	

iii) Squared returns of WTI

White noise errors	Autocorrelation
	

4. Concluding Remarks

This paper has focused on the analysis of the log-WTI prices along with its volatility approximated by means of the absolute and squared return series. We looked at the degree of persistence of the series corresponding to the whole sample as well as at the different subsamples according to the bull and bear periods identified by the algorithm developed by Pagan and Sossounov (2003). Our results show that the log WTI series is nonstationary with an order of integration close to 1 or even higher than 1, while the absolute and squared returns display long memory behaviour. This means that the series is highly persistent with shocks having permanent effects and thus, in the event of shocks, requires strong policy measures to recover the original trends. Once we separate the sample in terms of the bull and bear periods, we observe an increase in the degree of persistence as we increase the number of periods in the sample. Our results thus indicate that identifying bull and bear phases of the oil market is important, since this implies that the effect of the shocks of oil price and its volatility on the macroeconomy will have a more persistent effect than what would be estimated if we did not identify such market phases. Understandably, more persistence might imply government intervention in terms of policy to neutralise the impact of such effects, especially if it is negative, and perceived to be long-lasting.

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