

Malt distribution network design for the  
South African Breweries

by

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# Executive summary

This is the formal document for a project to optimise the malt distribution network of the South African Breweries (SAB). SAB owns two malting plants that supply its seven breweries with malt. Both the supply of barley and the demand for malt are subject to seasonality. The variability of the supply and demand poses a logistical problem for SAB, as they must ensure that they are able to balance the supply and demand in the network. Malt and barley therefore have to be transported between the different facilities, requiring sufficient storage space in the network for the various commodities.

The aim of this project is to design a malt network for SAB in such a manner as to present them with a minimum cost solution for the network. The optimisation of their malt distribution network can hold great benefits for SAB, since an efficient distribution network is crucial to the functioning of a business.

To address SAB's malt network design problem, existing literature on similar problems were sought in order to gain insight into how to solve the problem. It was found that the problem can be best approached through the use of mathematical modelling. The problem SAB faces fits the description of a Capacitated Facility Location Problem, which can be modelled as a Mixed Integer Program. *GAMS* is used to model the mathematical model into an algorithm and the model is solved using *CPLEX* as a solver.

The solution to the model provides SAB with a decision support tool for capacity expansion and could point out problem areas in their current network.

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# List of Acronyms

<b>CFLP</b>	Capacitated Facility Location Problem
<b>DNDP</b>	Distribution Network Design Problem
<b>FAD</b>	Flavoured Alcoholic Drink
<b>FLP</b>	Facility Location Problem
<b>GA</b>	Genetic Algorithm
<b>IP</b>	Integer Programming
<b>LP</b>	Linear Programming
<b>MIP</b>	Mixed Integer Program
<b>ML</b>	Modelling Language
<b>SAA</b>	Sample Average Approximation
<b>SA</b>	Simulated Annealing
<b>SAB</b>	South African Breweries
<b>TS</b>	Tabu Search

# Chapter 1

## Introduction and problem background

Beer is one of the most popular and widely enjoyed alcoholic beverages in the world. There are many different types of beer, like lagers, ales, stouts and pilsners. Beer is made through the fermentation of sugars that are obtained from starch sources and is therefore distinguished from drinks, such as wine, where the sugars for fermentation is not acquired from starches.

The brewing industry is a huge global business and more than 133 billion litres of beer is sold annually. There are a number of dominant players in this industry. InBev is the largest brewing company in the world, with SABMiller in second place, followed by the third largest brewer Anheuser-Busch.

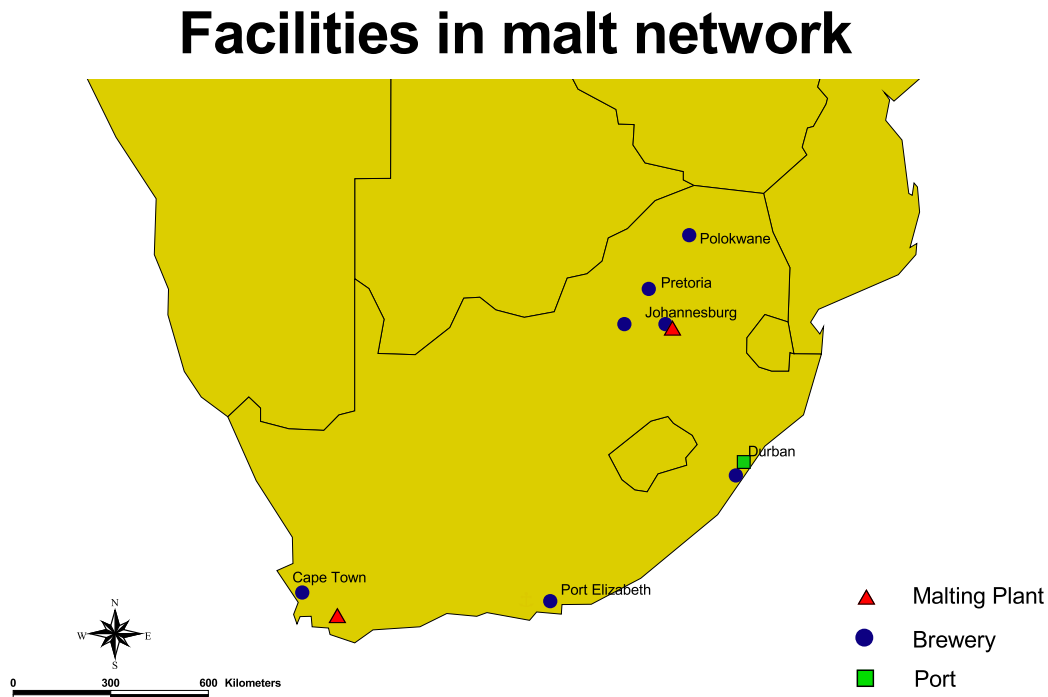
SAB was founded in 1895. In 2002 they bought Miller brewing Company and so SABMiller plc. was formed. SABMiller has activities that span across more than 60 countries and the group showed an annual turnover of more than US \$18,000 million in 2007.

SAB, which is a now subsidiary of SABMiller, have seven breweries located across South Africa, where ten beer and three Flavoured Alcoholic Drink (FAD) brands are brewed. In order for SAB to produce beer and the beer base from which FADs are made, they need three main ingredients namely water, yeast and malt.

Malt provides the starch based material from which the beer ferments. During the malting process, barley is converted into malt by allowing the barley to germinate and then kiln-drying the grain to stop the germination process. The selection of different malts is crucial to the brewing process, as the combination of malts contributes to the unique colour and flavour characteristics of each beer. It is clear to see that SAB would need a huge amount of malt in order to brew the 3.1 billion litres of beer they produce



Figure 1.1: Geographical layout of SAB's malt distribution network



on an annual basis.

It is very important that the network within which the malt and barley is distributed is efficient. Not only does malt transportation and storage costs amount to quite a large expense for SAB, but should a brewery suffer a severe malt shortage they might be unable to satisfy customer demand. The main aim of this project is to optimise the malt distribution network for the South African Breweries, in such a way as to present a minimum cost strategy to the supply chain.

## 1.1 Malt and barley availability and distribution

In order to put the malt distribution network in proper perspective, one must be aware of the geographical placement of the different production facilities. Therefore Figure 1.1 presents a map of South Africa on which the various facilities are indicated.

SAB owns two malting plants that are operated by the Southern Associated Maltsters. The one plant is located in Caledon in the Western Cape and the other one in Alberton, Gauteng.

The malting plants get their local supply of barley from crops that are cultivated in Western Cape and Northern Cape. Barley availability is seasonal and crop yields depend

heavily on factors such as rainfall and temperatures. In addition to the locally grown barley, the malting plants can also use imported barley to produce malt. Since the barley supply is variable, the volumes of the different types of malt that can be produced vary throughout the year.

The two malting plants have to supply each of the seven breweries, with assorted volumes of each type of malt. A brewery's malt usage will vary according to the volumes of the different brands of beer it brews, as each recipe for beer requires different proportions of the various types of malt. The demand for beer varies seasonally and thus the demand for malt will also fluctuate throughout the year. SAB can also import additional malt to augment the local supply.

The stochastic nature of the barley supply and the demand of malt, presents a malt and barley distribution and storage problem for SAB. They must ensure that each brewery can be supplied with the right volume and variety of malt at the right time.

In order to address the malt and barley network design problem, certain issues must be resolved. The research questions that we ask in this project are:

- Which malting facility must supply which of the breweries?
- Where should the surplus malt be stored?

Although network design problems are commonly found, they are definitely not trivial. A well planned distribution network is crucial to the functioning of any business and the infrastructure that is needed for such a network can be worth millions of Rands. Therefore, SAB will be able to benefit greatly from such a project as it will assist them in structuring their malt network more optimally.

## 1.2 Research design

The network design problem faced by SAB fits the broad definition of a type of problem that is known as the Capacitated Facility Location Problem (CFLP). The definition of a CFLP according to ArosteGUI et al. (2006) is:

“For CFLP, the problem is to determine which of  $N$  capacity-constrained facilities should be used to satisfy the demand for  $M$  customers at the lowest sum of fixed and variable costs.”

Therefore the distribution network design problem is modelled as a unique adaptation of the CFLP that is specifically tailored according to SAB's needs. The model includes adaptations that makes it suitable to solve the network design problem for all the relevant

malt varieties over a planning horizon defined by SAB. SAB decided that the problem should be solved for one year that is subdivided into 52 weekly periods.

The following deliverables can be expected on completion of the project:

- A mathematical model representing the optimisation of the network.
- A solution algorithm.
- The solution to a realisation of the model.
- A sensitivity analysis on the robustness of the optimal network solution.

### 1.3 Research Methodology

In order to accomplish the stated deliverables for the project, the work is structured into a number of phases.

The first phase is the analysis and understanding of the problem and the impact it has on the business. Network design problems are frequently encountered and literature has ample examples of how various authors have approached and solved distribution network design problems. Optimisation is identified as the appropriate approach to SAB's network design problem.

The network is then represented as a mathematical model. A CFLP is used to model the problem and the model is formulated as a Mixed Integer Program (MIP). Test data, obtained from SAB, is needed as inputs to the model. At this stage, the model is verified and validated. It is important to ascertain that all the constraints have been modelled correctly and to ensure that the model is a reasonable representation of SAB's malt network.

The next step is the solving of the model. The modelling language, *GAMS*, is used to code the solution algorithm and the solver, *CPLEX*, is used to solve the problem. After the model has been solved, the final phase is to perform a sensitivity analysis in order to determine the sensitivity of the solution to variation in various input parameters.

### 1.4 Document structure

The rest of the document is organised in the following manner. In the next chapter a literature review on the problem is presented to show how previous researchers have gone about solving similar network design problems. Chapter 3 presents a mathematical model of the malt and barley distribution network for SAB. In Chapter 4 the model is

solved and validated using optimisation software. The results are discussed and finally, sensitivity analysis is performed to test the robustness of the results and the impact that changes will have on the optimal distribution network. Chapter 5 concludes this document and discusses further research opportunities that stem from this project.

# Chapter 2

## Literature on distribution network design

### 2.1 Approaching a network design problem

SAB is faced with a problem where they need to decide how to optimally structure their malt and barley distribution network. It must be done in such a way as to ensure that the malt and barley is available when and where it is needed, while minimising the total cost of the network.

According to Coyle et al. (2003) there are three modelling approaches that can be used to approach a logistics network design problem, namely optimisation models, simulation models and heuristic models. They also state that whereas simulation and heuristic models can provide a good approximation for the optimum solution, an optimal solution cannot be guaranteed by these approaches. An optimisation model, on the other hand, has several distinct advantages over simulation and heuristic models. The most important advantages gained by using an optimisation model are :

- The best possible solution will be obtained for the given assumptions and data set.
- A complex model can be handled correctly.
- The cost savings between the exact and approximated solutions can be significant.

It would seem logical to opt for an optimisation modelling approach in order to obtain the best possible solution to the problem. However, sometimes it is not possible to solve a problem with optimisation, as some problems take enormous amounts of computational time to solve. If no algorithm that can solve the problem in polynomial time exists, the problem is classified as a nondeterministic (NP) class problem. Large instances of these

NP class problems usually need to be solved using some form of approximation, as the computational time increases non-polynomial with the size of the problem (Winston and Venkataramanan, 2003). Therefore, the size and complexity of the model will be the determining factor in whether or not the problem can be solved with an exact approach.

## 2.2 Approximation through heuristic and metaheuristic approaches

There are quite a number of heuristic approaches that can be used to approximate a network design problem and literature gives ample examples of authors who used heuristics techniques when solving large instances of these network design problems. One of the major drawbacks of heuristic techniques is that due to their greedy nature, they can easily get trapped in a local optimum and fail to find the global optimum solution.

For this reason metaheuristics were developed. Metaheuristics use intelligent search techniques and incorporates some procedure that prevents the algorithm from getting trapped in a local optimum. Three such metaheuristic approaches are the Tabu Search (TS), Simulated Annealing (SA) and Genetic Algorithm (GA) (Winston and Venkataramanan, 2003).

A study was performed by Arostegui et al. (2006) to test the performance of the TS, SA and GA heuristics on different variations of the Facility Location Problem (FLP). They specifically chose the FLP as a subject for the study due to its wide application in network design. They found that the superiority of a metaheuristic technique very much depends on the measure with which you evaluate the performance of the different techniques. However, whereas the performance of the GA and SA algorithms tended to be application specific, the TS showed very good performance on most of the problems. Therefore it was concluded that the TS gave the best overall performance and should be tried first, as it provides good results and is easy to implement.

The problems Arostegui et al. (2006) used for his analysis are very large, as there can be up to 100 facilities and more than 200 customers in the networks solved by them. Fortunately, the malt and barley network of SAB is much smaller. There are only two malting plants, seven breweries, one third party storage site and one site where imported malt can be offloaded. Sridharan (1995) stated that exact algorithms can solve medium sized problems, for example 50 plants and 50 customers, within reasonable time. Therefore, there is a reasonable chance that the SAB network design problem is small enough that it does not have to be approximated and that it can be solved by using an optimisation model.

## 2.3 Modelling the problem

The first step in modelling an optimisation problem is to determine the type of problem you are dealing with. SAB's problem best fits the profile of a problem called a Distribution Network Design Problem (DNDP). Miranda and Garrido (2008) classify a DNDP as a problem where a client (a brewery), who usually has a stochastic demand for a product (malt), needs to be serviced by some type of production facility (malting plant). The set of production facilities and their clients can be scattered throughout an entire geographic region.

A DNDP can be modelled in many ways, but an FLP, or one of its variants, is one of the most common approaches that are used in modelling DNDPs. A variant of the FLP is the Uncapacitated Facility Location Problem, where the assumption is made that all the facilities have an unlimited capacity. However, in SAB's case all the facilities have a maximum capacity, making the unlimited capacity assumption invalid. If all the facilities have a maximum amount of demand it can service, the problem becomes a Capacitated Facility Location Problem (Sridharan, 1995). In a case like SAB's, where the CFLP is extended over a number of periods, it is called a dynamic CFLP.

There are two stages to a CFLP. In the first stage the decision of where to locate the plants are made and in the second stage the customers are assigned to the different plants (Sridharan, 1995).

Quite a number of authors have used a CFLP to model a DNDP. Hinojosa et al. (2008) developed a model for a dynamic CFLP which allows for outsourcing as well as the building, or closing, of facilities. Melo et al. (2005) also developed a model to design a multi-commodity supply chain network, which allows for the creation of new facilities, as well as the expansion of existing facilities. Both these authors used a MIP to formulate their models.

Santoso et al. (2005) propose the use of a two-stage stochastic program to model a DNDP with uncertain parameters. The plant location decisions are made in the first stage of the model and in the second stage, the optimal distribution of the product is determined based on the realisations of the uncertain parameters. The deterministic model to their problem is also formulated as an MIP.

Sridharan (1995) states that an MIP can be used to formulate a CFLP, therefore an MIP would be the logical choice when modelling SAB's problem.

### 2.3.1 Deterministic versus stochastic modelling

Many typical facility location models use only deterministic values for demand and supply in the constraints (Miranda and Garrido, 2008). Unfortunately, the true demand often

contains uncertainty and in some cases there may be merit in including some random parameters in the model.

Santoso et al. (2005) developed a stochastic programming model and solution algorithm to solve a supply chain network design problem. The authors compared the solutions obtained from their stochastic programming model and algorithm to the answers from a deterministic model that uses only the mean values for the uncertain parameters. The three best candidate solutions that were found using the algorithm were compared to the answer of the deterministic, mean value model. They found that the average total cost of the minimisation problem was smaller for all three of the candidate solutions than for the mean value problem. It was also found that the variability of the solutions were smaller for the stochastic programming model solutions than for the mean value problem. Furthermore, the optimality gap, as well as the variability of the optimality gap was found to be smaller for the stochastic programming solutions.

However, including stochastic parameters in a model can add a lot of complexity to the model. In order to generate realistic distributions for the random parameters, realistic data pertaining to the parameters are needed. If the input distributions are not a reasonable representation of the true parameters, the whole effort will be futile. Therefore one must make a careful decision if the benefits that can be obtained from using stochastic programming to model SAB's network warrant the extra effort and complexity.

## 2.4 Solving the problem

After the model has been formulated, the next very important step is solving the problem. There are a number of methods available with which DNDPs can be solved. If the size and complexity of the model is small enough, it is possible to solve the problem within reasonable time using an exact method.

### 2.4.1 Branch-and-bound Algorithm

If the DNDP was formulated as an MIP, an enumerational tree can be used to obtain an exact solution for the CFLP. An example of such an enumerational tree approach is the branch-and-bound method, which is often used by researchers to solve Capacitated Facility Location Problems (Sridharan, 1995).

In order to use the branch-and-bound method, the constraints of the original MIP must be relaxed first. One of the methods that can be used to relax the integer constraints of a CFLP, is the Linear Programming (LP) relaxation (Sridharan, 1995). With the LP relaxation the integer constraints are relaxed, thereby enlarging the solution space from



feasible solution points to a feasible region which includes all real values between the feasible points.

The LP relaxation of the MIP is then solved and if all the integer variables assume values that are indeed integer, the optimal solution to the LP relaxation is also the solution to the original problem (Winston and Venkataramanan, 2003). In practice, it is not always the case that the solution to the relaxation is also the solution to the original MIP.

The branch-and-bound algorithm is then used to divide the solution space into sub regions and each region is explored in search of an optimal solution of the MIP (Atamturk and Savelsbergh, 2005). If the solution to the sub problem satisfies the integer constraints of the MIP, the solution becomes a candidate solution to the problem and if the solution violates the integer constraints, the problem is branched into two new sub problems.

The candidate solutions to the sub problems are used to create and update the bounds on the optimal solution. If the solution to a sub problem is infeasible or falls outside the current bounds on the problem, it is futile to further branch on that problem and the sub problem is fathomed. The algorithm terminates when all possible sub problems have either been explored or fathomed. The optimal solution to the MIP is the candidate solution with the best objective value at the time of termination.

Applying the branch-and-bound algorithm manually can become a tedious and time-consuming task. If the problem is large, it will be nearly impossible to find the solution by hand. Therefore optimisation software is an important tool in solving a real-life optimisation problem.

## 2.4.2 Optimisation software

There are quite a number of Modelling Languages (MLs) available with which DNDPs can be modelled and solved. Algebraic languages are a group of modelling languages in which the model can be presented to the program in a format that is similar to conventional mathematical notation (Kuip, 1993). These algebraic languages are declarative languages, which state what must be calculated, but not how it should be done (Steiger and Sharda, 1993). Declarative languages also allow the data to be separated from the model, which helps the modeller with maintaining the model (Atamturk and Savelsbergh, 2005).

To solve an Integer Programming (IP) or MIP problem, most software employs a LP based branch-and-bound algorithm (Atamturk and Savelsbergh, 2005). There are a wide variety of solvers available and *CPLEX*, *EXPRESS* and *XA* are just a few of the solvers that can be used to solve an MIP. Different solvers employ different strategies to accelerate the branch-and-bound algorithm, thus reducing the time it takes to solve the

problem.

*GAMS* and *LINGO* are examples of algebraic MLs, which can be used to solve SAB's DNDP. The main difference between *GAMS* and *LINGO* lies in the solvers. Where *GAMS* accesses solvers, like *CPLEX* or *EXPRESS*, that are sold separately to the modelling language, *LINGO* solvers are linked directly to the modelling language (LINDO Systems, 2001).

### 2.4.3 Developing a custom algorithm

Authors can also develop their own algorithms, based on a combination of other algorithms, to solve network design problems. Santoso et al. (2005) have developed an algorithm to solve a DNDP under uncertain conditions.

The objective function of the model by Santoso et al. (2005) can be split into a fixed investment cost and the expected value of future costs. The fixed costs mainly consist out of investment costs to open and run a facility, thus the random parameters have no direct effect on this cost. On the other hand, the expected costs point to the cost of transportation, production and shortages, which are directly influenced by the stochastic parameters.

Santoso et al. (2005) uses a method called the Sample Average Approximation (SAA) scheme, which allows them to evaluate the expected part of the objective function. In the SAA scheme, a random sample of  $N$  realisations of the stochastic parameters are generated and the expected costs are then approximated by the sample average. The problem becomes deterministic for each realisation of the random sample and can thus be solved using optimisation techniques.

The deterministic functions, that were obtained by the realisations of the random sample, can be optimised using a cutting plane algorithm like the Benders Decomposition. However, as the normal Benders Decomposition algorithm would require too many iterations, a few techniques to accelerate the algorithm were introduced.

Santoso et al. (2005) specifically developed this algorithm to solve large scale stochastic network design problems with a very large number of scenarios. They tested their accelerated Benders Decomposition algorithm on a problem which has 142 facilities, 238 customers and 13 product types and found that it delivers excellent solutions and is computationally effective.

## 2.5 Concluding the literature review

From the literature that was reviewed in this study, one may conclude that the malt and barley distribution network for SAB may be best approximated using a mixed integer programming model that is based on a CFLP. Given the relative small size of the distribution network, one should be able to solve the model to optimality using optimisation software. The model can then be solved using the modelling language *GAMS* in connection with the solver *CPLEX*.

# Chapter 3

## Mathematical Model

The main aim of the mathematical model is to determine the least cost malt distribution network for SAB. Whenever malt is transported from one facility to another, a transport cost is incurred and if malt is stored in a silo SAB incurs a storage cost. The production and purchase cost of the malt is another cost that is relevant to the problem. The optimal solution to the DNDP would be one that minimises the sum of these three costs. The model will also be able to answer the question as to whether or not SAB has enough malt storage space throughout the network.

For the purposes of the model the following assumptions were made:

1. Assume that the cheapest mode of transport is always used to transport malt between facilities.
2. All demand must be met on time.
3. Assume no significant malt losses occur during storage and shipment.
4. Assume third party storage facilities start without any opening stock, as to better determine the storage needs.

The formulation of the model posed a few interesting challenges due to the nature of the malting process. Each batch of malt, which is called a piece, has a unique chemical composition. To improve the overall properties of a batch of malt, different pieces of malt are blended together into a blend. Once the quality specifications of a blend have been approved, it may not be stored with any other blend, even if the two blends are of the same type of malt. This, of course, has huge storage implications for SAB, but it also places a crucial constraint on the model.

To incorporate the constraint that prevents different blends from mixing, each blend of malt has been treated as a unique entity. In the model, a *product* represents a blend

of malt and the destination of a shipment of malt is a specific silo, rather than a facility, like a brewery or malting plant.

### 3.1 Original formulation of the model

The following sets are used in the model:

- T** represents the set of periods (in weeks), such that  $\mathbf{T} = \{1, 2, \dots, T\}$
- S** represents the set of all the silos at all the facilities, such that  $\mathbf{S} = \{1, 2, \dots, S\}$
- P** represents the set of all the possible products (blends) that can be produced during the planning horizon, such that  $\mathbf{P} = \{1, 2, \dots, P\}$
- L** represents the set of breweries, such that  $\mathbf{L} = \{1, 2, \dots, L\}$
- K** represents the set of malting plants, such that  $\mathbf{K} = \{1, 2, \dots, K\}$
- $\mathbf{M}_k$  represents the sub set of silos that is located at malting plant or malt import facility  $k$ , where  $k \in \mathbf{K}$ , such that  $\mathbf{M}_k \subseteq \mathbf{S}$  and  $\mathbf{M}_k$  is pairwise null, such that  $\{M_i \cap M_j\}_{i,j=\{1,2,\dots,K\}|i<j} = \phi$
- F** represents the sub set of silos that are located at any malting plant or import facility, such that  $\mathbf{F} = \{\mathbf{M}_1 \cup \mathbf{M}_2 \cup \dots \mathbf{M}_K\}$
- $\mathbf{B}_l$  represents the sub set of silos that is located at brewery  $l$ , where  $l \in \mathbf{L}$ , such that  $\mathbf{B}_l \subseteq \mathbf{S}$  and  $\mathbf{B}_l$  is pairwise null, such that  $\{B_i \cap B_j\}_{i,j=\{1,2,\dots,L\}|i<j} = \phi$
- H** represents the sub set of silos that are located at any brewery, such that  $\mathbf{H} = \{\mathbf{B}_1 \cup \mathbf{B}_2 \cup \dots \mathbf{B}_L\}$
- W** represents the sub set of additional silos that can be build, where  $\mathbf{W} \subseteq \mathbf{S}$
- R** represents the set of different malt types, such that  $\mathbf{R} = \{1, 2, \dots, R\}$
- $\mathbf{Q}_r$  represents the sub set of products (blends) that are representative of the same malt type  $r$ , such that  $\mathbf{Q}_r \subseteq \mathbf{P}$  and  $r \in \mathbf{R}$
- $\mathbf{A}_k$  represents the sub set of products (blends)  $p$  that are produced at malting plant  $k$ , such that  $\mathbf{A}_k \subseteq \mathbf{P}$  and  $k \in \mathbf{K}$  and  $\mathbf{A}_k$  is pairwise null, such that  $\{A_i \cap A_j\}_{i,j=\{1,2,\dots,K\}|i<j} = \phi$
- $\tau$  represents the sub set of products (blends)  $p$  that are imported, such that  $\tau \subseteq \mathbf{P}$  and  $\{\tau \cap A_i\}_{i=\{1,2,\dots,K\}} = \phi$

The decision variables are:

- $x_{ijpt} \triangleq$  the amount (in tons) of product  $p$  that was moved from silo  $i$  to silo  $j$  in period  $t$ , such that  $i, j \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $v_{ipt} \triangleq$  the volume (in tons) of product  $p$  that is in silo  $i$  at the end of period  $t$  (after demand has been satisfied and dispatches made), such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $e_{ipt} \triangleq$  the volume (in tons) of product  $p$ , that has just been manufactured, that is put into silo  $i$  in period  $t$ , such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $f_{ipt} \triangleq$  the volume (in tons) of product  $p$ , that is taken from silo  $i$  in period  $t$  in order to satisfy demand at a brewery, such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $b_{lrt} \triangleq$  the inventory of malt type  $r$  at brewery  $l$  at the end of period  $t$ , where  $l \in \mathbf{L}, r \in \mathbf{R}, t \in \mathbf{T}$
- $u_{ipt} \triangleq \begin{cases} 1 & \text{if product } p \text{ is stored in silo } i \text{ in period } t, \text{ where } i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \\ 0 & \text{otherwise} \end{cases}$
- $y_i \triangleq \begin{cases} 1 & \text{if silo } i \text{ built, where } i \in \mathbf{W} \\ 0 & \text{otherwise} \end{cases}$
- $a_{pt} \triangleq \begin{cases} 1 & \text{if product } p \text{ is manufactured in period } t, \text{ where } p \in \mathbf{P}, t \in \mathbf{T} \\ 0 & \text{otherwise} \end{cases}$

The following parameters are necessary to solve the problem:

- $B_p \triangleq$  the given blend size (in tons) of product  $p$ , such that  $p \in \mathbf{P}$
- $C_s \triangleq$  the given capacity (in tons) of silo  $s$ , such that  $s \in \mathbf{S}$
- $D_{lrt} \triangleq$  the given demand (in tons) for malt type  $r$  at brewery  $l$  in period  $t$ , where  $r \in \mathbf{R}, l \in \mathbf{L}, t \in \mathbf{T}$
- $E_{ij} \triangleq$  the given transport cost (in Rand per ton) for malt that is transported from silo  $i$  to silo  $j$ , where  $i, j \in \mathbf{S}$
- $G_k \triangleq$  the given maximum malting capacity of malting plant  $k$ , where  $k \in \mathbf{K}$
- $N_i \triangleq$  the given variable storage cost (in Rand per ton) for malt that is stored in silo  $i$ , where  $i \in \mathbf{S}$
- $O_p \triangleq$  the given manufacturing or purchase cost (in Rand per ton) of product  $p$ , where  $p \in \mathbf{P}$
- $Z \triangleq$  the given cost (in Rands) to build one ton of additional storage
- $\alpha \triangleq$  the given number of periods for which the demand must be covered by the safety stock held at a brewery
- $\beta \triangleq$  number of periods for which the demand must be covered by the safety stock throughout the whole malt supply grid

The objective of the model is to find a solution that minimises the total sum of investment-, transportation-, storage- and purchase cost of the malt. Hence, the objective function of the model is formulated as follows in (3.1):

$$\min z = \sum_{i \in \mathbf{S}} \sum_{\substack{j \in \mathbf{S} \\ j \neq i}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} (E_{ij} + N_i) x_{ijpt} + \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} O_p B_p a_{pt} + \sum_{i \in \mathbf{W}} Z C_i y_i \quad (3.1)$$

subject to

$$v_{ipt-1} + \sum_{\substack{i' \in \mathbf{S} \\ i' \neq i}} x_{i'ipt} - \sum_{j \in \mathbf{S}} x_{ijpt} + e_{ipt} - f_{ipt} = v_{ipt} \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (3.2)$$

$$v_{ipt-1} + e_{ipt} + \sum_{\substack{i' \in \mathbf{S} \\ i' \neq i}} x_{i'ipt} \leq C_i u_{ipt} \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (3.3)$$

$$\sum_{p \in \mathbf{P}} u_{ipt} \leq 1 \quad \forall i \in \mathbf{S}, t \in \mathbf{T} \quad (3.4)$$

$$\sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} u_{ipt} \leq M y_i \quad \forall i \in \mathbf{W} \quad (3.5)$$

Equation (3.2) calculates the inventory in each silo at the end of a period. Constraint (3.3) ensures that the inventory in a silo does not exceed the silo capacity and it furthermore drives the binary variable that links the silo usage to the volume of malt stored in the silo. It may seem that the ending inventory  $v_{ipt}$ , as calculated in (3.2), could also be used to drive the binary silo utilisation variable  $u_{ipt}$ . However, if a silo is filled because it was emptied at the end of the previous period and again emptied during the current period, both  $v_{ipt-1}$  and  $v_{ipt}$  will be zero. If the variable  $v_{ipt}$  drives the binary variable  $u_{ipt}$ , the silo will register as unused for the current period even though it was in use. Therefore the left hand side of (3.3) cannot be replaced with  $v_{ipt}$ .

Constraint (3.4) ensures that only one product can be stored in a silo in any given period. Constraint (3.5) determines whether or not additional silos are built. From (3.4) we know that for each silo in each period the sum over  $p$  of  $u_{ipt}$  can never be more than one. Therefore in (3.5)  $M$  can assume a value that is equal to the total number of time periods.



$$\sum_{i \in \mathbf{M}_k} e_{ipt} = a_{pt} B_p \quad \forall k \in \mathbf{K}, p \in \mathbf{A}_k, t \in \mathbf{T} \quad (3.6)$$

$$\sum_{\substack{i \in \mathbf{S} \\ i \notin \mathbf{F}}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} e_{ipt} \leq 0 \quad (3.7)$$

$$\sum_{p \in \mathbf{A}_k} a_{pt} B_p \leq G_k \quad k \in \mathbf{K}, t \in \mathbf{T} \quad (3.8)$$

$$\sum_{t \in \mathbf{T}} a_{pt} \leq 1 \quad \forall p \in \mathbf{P} \quad (3.9)$$

Equation (3.6) ensures that all malt produced during a period is placed into silos. This equation is only repeated for products that are manufactured at malting plant  $k$ , thus ensuring that the malt can only be placed in silos at that specific malting plant. Constraint (3.7) ensures that  $e_{ipt}$  is zero for all silos that are not located at malting plants. Constraint (3.8) ensures that a malting plant does not produce more than its maximum capacity, while Equation (3.9) ensures that a product can only be manufactured once.

$$\sum_{i \in B_l} \sum_{p \in Q_r} f_{ipt} = D_{lrt} \quad \forall l \in \mathbf{L}, r \in \mathbf{R}, t \in \mathbf{T} \quad (3.10)$$

$$\sum_{\substack{i \in \mathbf{S} \\ i \notin \mathbf{H}}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} f_{ipt} \leq 0 \quad (3.11)$$

$$b_{lrt} = \sum_{i \in B_l} \sum_{p \in Q_r} v_{ipt} \quad \forall l \in \mathbf{L}, r \in \mathbf{R}, t \in \mathbf{T} \quad (3.12)$$

$$b_{lrt} \geq \sum_{t=t}^{t+\alpha} D_{lrt} \quad \forall l \in \mathbf{L}, r \in \mathbf{R}, t \in \mathbf{T} \quad (3.13)$$

$$\sum_{i \in \mathbf{S}} \sum_{p \in Q_r} v_{ipt} \geq \sum_{t=t}^{t+\beta} \sum_{l \in \mathbf{L}} D_{lrt} \quad \forall r \in \mathbf{R}, t \in \mathbf{T} \quad (3.14)$$

Constraint (3.10) ensures that all demand is met on time. The constraint is repeated separately for all breweries, thus ensuring that a brewery can only use malt that is already on the premises to satisfy the demand. Constraint (3.11) ensures that  $f_{ipt}$  is zero for all silos not located at a brewery. The total inventory at each brewery is calculated by Equation (3.12). Constraint (3.13) ensures that there are a certain number of weeks' worth of safety stock at each brewery. Although Constraint (3.14) is similar to (3.13), Constraint (3.14) ensures there is adequate safety stock spread throughout the whole malt supply grid, which can be a different amount than the sum of the safety stock held

at all the breweries.

$$x_{ijpt} \geq 0 \quad \forall i, j \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (3.15)$$

$$e_{ipt}, f_{ipt}, v_{ipt} \geq 0 \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (3.16)$$

$$b_{lrt} \geq 0 \quad \forall l \in \mathbf{L}, r \in \mathbf{R}, t \in \mathbf{T} \quad (3.17)$$

$$u_{ipt} \in \{0, 1\} \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (3.18)$$

$$a_{pt} \in \{0, 1\} \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (3.19)$$

$$y_i \in \{0, 1\} \quad \forall i \in \mathbf{W} \quad (3.20)$$

Constraints (3.15) to (3.17) ensure the variables are non negative, while Constraints (3.18) to (3.20) ensure that the relevant variables are binary.

**Table 3.1: Dimensions of the model**

Description	Number
Total number of silos	178
Total number of blends	289
Total number of malt types	2
Total number of malting plants	2
Total number of breweries	7
Total number of third party storage sites	1
Total number of malt import terminals	1
Total number of time periods	52

## 3.2 Model execution challenges

The mathematical model forms the basis from which the solution algorithm is developed. The overall dimensions of the sets of the mathematical model, as defined in Section 3.1, can be seen in Table 3.1. If these sets are applied to the model, there are a total of 419 476 356 possible variables. When the model was implemented in *GAMS*, it turned out that it is not possible to execute the model with the available computer hardware capacity. The sheer size of the model continuously caused *GAMS* to run out of memory during the execution of the model.

A considerable amount of time and effort was spent on attempts to reduce the memory usage of the model. Strategies included experimenting with the number of time periods used, as well as assigning all variables that can never assume a non-zero value in the model to zero. SAB's current network was examined to determine between which facilities no malt is shipped. The various attempts helped to reduce the number of variables that need to be solved from the initial 419 476 356 variables to 114 232 792 non-zero variables. Various sources of information, like user guides and *GAMS* customer support, were also consulted in an effort to resolve the memory problem. Unfortunately, it was to no avail and therefore it was decided to revisit some of the assumptions and constraints placed on the model.

Modelling each blend as a separate product adds much more realism to the model. However, the realism needs to be weighed against the complexity it adds to the model. Modelling all the separate blends in an effort to prevent the mixing of the blends causes the model to become huge. Moreover, when one models the separate blends, factors like the size of a blend in comparison to the size of the silos in which the blend is stored come into play. There is no doubt that that it is important for SAB to ensure that their silos sizes can match their blend sizes in some way. However, within the scope of this project, one has no control over the piece sizes and resulting blend sizes. Thus, for the model to

even stand a chance of finding a feasible solution, one would also need to model all silos and any other type of malt storage in the grid with the utmost accuracy.

In deciding on the amount of realism needed for the model to be able to fulfil its objective, one must look at the ultimate purpose of the model. This model is developed to determine which malting plant must supply which of the breweries with malt and to determine the storage requirements for the surplus malt. To answer the research questions posed in this project, it is not absolutely vital that the malt network is modelled with the utmost realism.

### 3.3 Reformulating the model

It was decided to relax the constraint viewing each blend of malt as a separate product. Furthermore, it was decided to develop two new model formulations by making a few adaptations to the original model.

The one model, as presented in Section A.1 of Appendix A, sees storage space as a bulk volume, rather than individual silos. To make provision for the fact that different malt types and blends are not allowed to mix, each facility has a working storage capacity which is equal to 85% of the facility's actual storage capacity. This model is used to present an overall view of the network and as the model is relatively small, it allows for easy experimentation with different scenarios. From here on, this model is referred to as Model 1. The model is used to solve the problem for the total period of one year. Time periods for the solution are first chosen as four quarters to present the reader with an uncomplicated solution and aid with model verification and validation. The time periods are changed to weeks and the model is solved for the original planning horizon, which is one year divided into 52 weeks.

The second model, presented in Section A.2 in Appendix A, incorporates more complexity into the formulation and storage space is modelled as individual silos. As the dimensions of the model are much bigger and the model thus takes longer to solve, the model is only solved for one quarter subdivided into thirteen weekly periods. The main aim of the second model is to provide a more detailed solution for one of the quarters. Henceforth, this model is referred to as Model 2.

In order to solve the models, a number of inputs are required from SAB. Chapter 4 discusses the input parameters to the model, as well as the execution of the model to obtain a solution to the DNDP.

# Chapter 4

## Execution of the Model

The two mathematical models that were presented in Sections A.1 and A.2 of Appendix A are formulated into computer algorithms. The algorithms are coded in *GAMS* and the source code of one of the algorithms can be seen in Appendix C. We will now turn our attention to the input parameters that are needed to solve the model.

### 4.1 Required data

The following sets of data, that were obtained from SAB, are the essential inputs to the algorithm:

- The weekly malt demand
- The storage capacities
- The transportation costs
- The storage costs
- The production or purchase cost of malt
- The different types of malt produced
- The amount of malt each malting plant produces during each period

The overall dimensions of the two mathematical models are presented in Tables 4.2 and 4.3. For the purposes of the model it is assumed that the penalty costs that can be seen in Table 4.4 are applicable to all facilities. SAB requires that there must be at least eight weeks of safety stock throughout the malt supply grid, with a minimum of three

**Table 4.1: Safety stock parameters**

Parameter	Value
$\alpha$	3 weeks
$\beta$	8 weeks

**Table 4.2: Dimensions of Model 1 which views storage as a bulk capacity**

Item	Total Number
Malt types	2
Malting plants	2
Breweries	7
Third party storage sites	1
Malt import terminals	1
Time periods	4 quarters or 52 weeks respectively

weeks' worth of safety stock at any given facility. These safety stock requirements leads to the safety stock parameter values that can be seen in Table 4.1.

The amount of malt in the opening stock of each facility at the start of the first period is not taken from the actual figures, but is calculated using the same constraints the model uses to determine the appropriate ending stock for each period.

#### 4.1.1 Data manipulation

Most of the input data first has to be reworked into a format that is compatible with the model formulation. Moreover, all the data has to be transformed into a file format that can be read by *GAMS*. In order to aid the data manipulation process and help prevent conversion errors, programmes that can convert the data sets into the necessary formats were written in *MATLAB*.

**Table 4.3: Dimensions of Model 2 which models storage as separate silos**

Item	Total Number
Silos	178
Malt types	2
Malting plants	2
Breweries	7
Third party storage sites	1
Malt import terminals	1
Time periods	13

**Table 4.4: Applicable Penalty costs**

Description	Cost
Imported malt	R1000 per Ton
Additional storage capacity	R2000 per Ton

After all the data has been processed into the correct format, it is entered into the model and the model is solved. The output that is obtained from *GAMS* has to be reworked so that the solutions can be presented in a more understandable format that is easy to interpret. Outputs of the model are exported to *Microsoft Excel*. Several calculations are performed on the data and summaries and graphs are made. The *Excel* workbook containing the solutions to the model will be presented to SAB. The results of the model are discussed in Section 4.3.

## 4.2 Model verification and validation

It necessary to ascertain that the model is functioning correctly and to determine if the model is a reasonable representation of the problem that was modelled. Verification and validation is performed throughout the modelling, coding and solving phases. If an error is discovered at any stage, the error is corrected and the model resolved. The model is also checked by inspecting the answers to see if they seem logical and possible. All models and algorithms are validated as best as possible. However, with a large model, like model 2, it is very difficult to be sure that the model is absolutely error free.

## 4.3 Model Results

The models were solved in *GAMS* using *CPLEX* as a solver. It was decided to experiment with a few scenarios on Model 1 and then select the scenario that appears to be the best and solve Model 2 for that scenario.

When solving Model 1, there was looked at the business rules that govern which facilities are allowed to dispatch malt. Currently SAB's network is structured in such a way that Alrode Maltings does not dispatch any malt and the malt is therefore stored at Alrode until Alrode brewery uses the malt. The breweries also do not dispatch malt in the current structure. Consequently the following scenarios were identified:

- Scenario 1: All facilities are allowed to dispatch malt to any other facility
- Scenario 2: Breweries are not allowed to any dispatch malt

**Table 4.5: Comparison of Scenarios 1, 2 and 3**

Criteria	Scenario 1	Scenario 2	Scenario 3
Solve time (seconds)	0.381	0.281	0.312
Iterations	311	276	269
Objective value	R 111 932 473	R 111 876 145	R 141 108 430
Extra storage required (Tons)	-	-	14342

**Table 4.6: Comparison of Scenarios 2 and 3 solved for one year in weekly periods**

Criteria	Scenario 2	Scenario 3
Solve time (seconds)	1.171	1.156
Iterations	4667	3745
Objective value	R127 192 697	R 144 853 075
Extra storage required (Tons)	-	11 154

- Scenario 3: Neither Alrode Malting Plant, nor the breweries are allowed to dispatch any malt

Model 1 was solved for all three scenarios for a period one year divided into quarters. A comparison of the results that were obtained by solving the three scenarios can be seen in Table 4.5. In Figures 4.1, 4.2 and 4.3 the total volume of malt shipped between different facilities are plotted as a percentage of the total volume of malt shipped in a year. The most imports take place during quarter 3. In quarter 3, both malting plants produce the same type of malt. This leading to a shortage in other types of malt, which then needs to be imported.

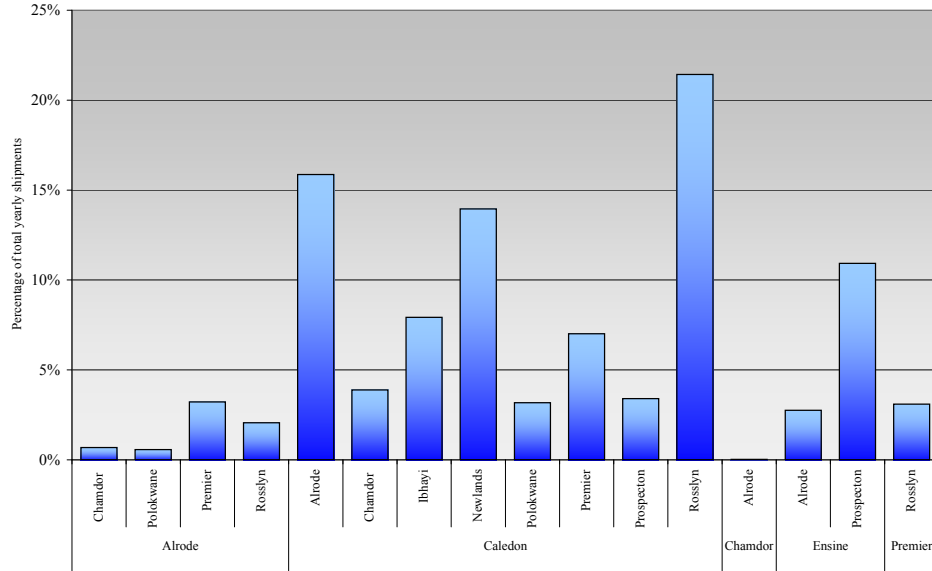
In Scenario 1, where all the breweries are allowed to dispatch malt, only one of the breweries does in actual fact dispatch malt to another facility. However, breweries are not currently equipped to handle dispatches and since the shipment of malt from the brewery is so small, it is not worthwhile further pursuing Scenario 1.

Scenarios 2 and 3 of Model 1 is then solved for the period of one year subdivided into weekly periods, as SAB specifically requested a weekly breakdown of the solution. Table 4.6 presents the solution statistics for the weekly models. The results obtained for the weekly periods is similar to the results obtained for the quarterly periods and will therefore not be discussed in detail here. More detailed results can be seen in Appendix B.

Scenario 2, where only the breweries are not allowed to dispatch malt, was chosen as the most appropriate Scenario to solve with Model 2. The model was solved for one quarter divided into thirteen weeks. As the third quarter (October - December) is SAB's



**Figure 4.1: Relative volumes of malt shipped between facilities in Scenario 1**



peak period, data for this quarter is used to solve Model 2. Due to the short time frame of this model, opening stock is not calculated, but the values that were obtained in Model 1 for the closing stock at the end of September were used in Model 2 as the opening stock for October. The results can be seen in Table 4.7.

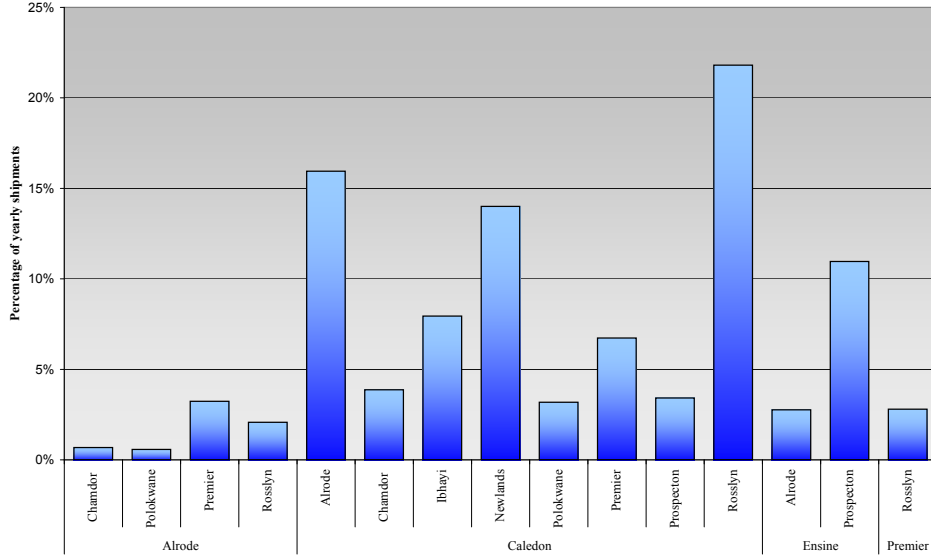
## 4.4 Analysis of the results

From the different scenarios that were experimented with, it appears that it is the best if both malting plants have the option to dispatch malt, but is not necessary for the

**Table 4.7: Results of Scenario 2 solved with Model 2 for 1 quarter**

Criteria	Value
Solve time (seconds)	1890
Iterations	174373
Non zero elements	1 747 807
Discrete variables	5 658
Objective value	R16 986 845

**Figure 4.2: Relative volumes of malt shipped between facilities in Scenario 2**



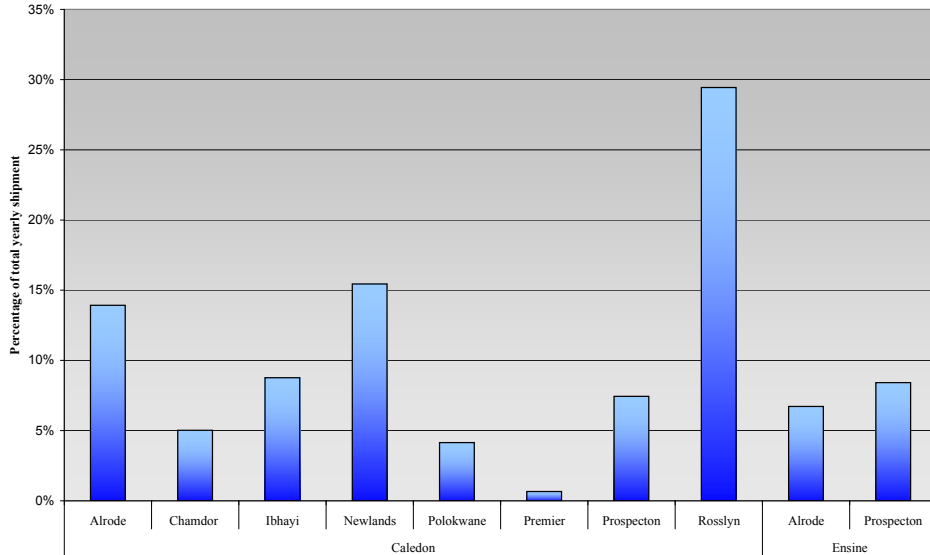
breweries to be able to dispatch malt. Figure 4.4 shows the average volume of malt that is shipped between facilities on a weekly basis given the scenario just described. Currently, SAB's network is not structured in this way and perhaps the current structure of the network is a contributory factor to their storage dilemma. The model outputs have shown that additional storage space is only required if Alrode Maltings is not allowed to dispatch malt.

## 4.5 Sensitivity Analysis

A sensitivity analysis forms a very important part of the analysis of the results, as it shows how sensitive the solution is to changes in the circumstances. It also gives an idea as to which parameters are the most critical to the problem.

There are two very important concepts in a sensitivity analysis, namely the marginal cost and the shadow price. The marginal cost shows by how much a coefficient in the objective function should improve, before the corresponding variable will assume a non-zero value. The shadow price, on the other hand, is the amount by which the optimal value improves if the right hand side of a constraint is improved by one unit. The

**Figure 4.3: Relative volumes of malt shipped between facilities in Scenario 3**

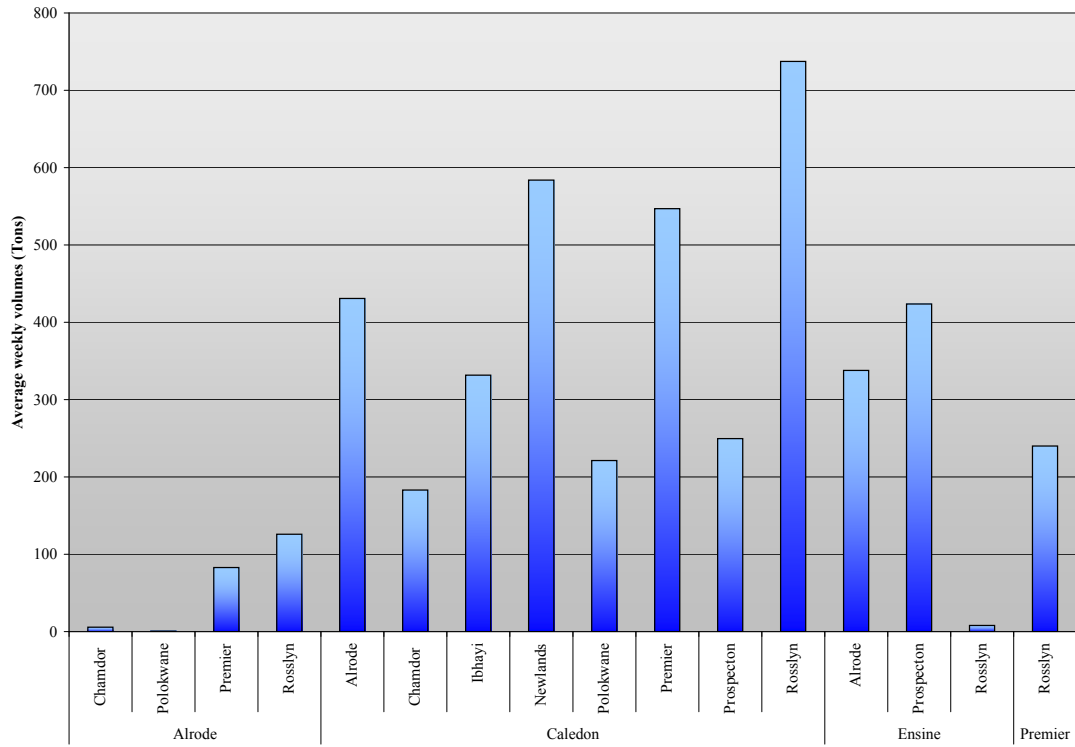


reduced cost and shadow price are only valid for the specific range in which the solution base remains unchanged.

The sensitivity analysis for SAB's model was performed automatically by *GAMS*. A sensitivity analysis consists out of a large amount of data as each variable has a marginal cost and each constraint has a dual price. Only the marginal cost of building additional storage space is discussed as it provides the most insight to the problem.

The marginal cost of extra storage space for Scenario 2, solved with Model 1 can be seen in Table 4.8. The range wherein the marginal costs are valid is form 0 to infinity for all the facilities. It is very interesting to see by how much the price of additional storage must improve before the model will consider building extra storage. The dual price of extra storage capacity is also zero in most instances, implying that one additional unit of storage space will not cause the objective value to improve.

**Figure 4.4: Average volumes of malt shipped between facilities**



**Table 4.8: Marginal cost of extra storage**

Facility	Marginal cost
Alrode	1757.64
Caledon	1328.86
Chamdor	1893.41
Ibhayi	1716.84
Newlands	1430.08
Polokwane	1899.37
Prospecton	2000
Rosslyn	1846.65
Ensign	1947.24
Premier	1911.76

# Chapter 5

## Conclusion and further research

The main aim of this project was to develop a decision support model for SAB that can help them to structure their malt distribution network more optimally. It was decided that a CFLP best describes SAB's problem and consequently the network was modelled as an MIP. The mathematical model was coded into *GAMS* and solved for a specific set of input data. The solution to the model presented suggestions as to where capacity expansions could possibly be done and also which malt movements present a minimum cost solution.

There is a lot of room for further research on this project. First of all, ways with which the original model, as formulated in Section 3.1, can be solved should be explored. Perhaps a heuristic method can be used to obtain answers to the model that treats blends as separate entities, as this version of the model is a much more realistic representation of the problem.

Incorporating randomness into the model is definitely an area which could be explored further. This should make the model more realistic, as there is a lot of uncertainty throughout the distribution network.

This project focussed on the malt side of the distribution network. However, it would be possible to expand this model to include the barley distribution network as well. By doing this, the whole malt and barley distribution network can be optimised from barley cultivation, right through the supply chain to the breweries. The model can also be implemented to determine the production schedules of the malting plants. Another suggestion is to expand the model to allow for the increase of malting capacity.

This project barely scratched the surface of the overall area of opportunity for optimisation in the malt and barley distribution network. However, it presented useful insights on the current malt network and gave some pointers as to which direction could be taken to structure the network more optimally.

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# Appendix A

## Model Reformulations

### A.1 Model 1: Simplified model with bulk storage

In Model 1, storage space is seen as a bulk volume, rather than individual silos.

The following sets are used in the model:

- T** represents the set of periods (in weeks), such that  $\mathbf{T} = \{1, 2, \dots, T\}$
- P** represents the set products (malt types) , such that  $\mathbf{P} = \{1, 2, \dots, P\}$
- S** represents the set of all facilities, such that  $\mathbf{S} = \{1, 2, \dots, S\}$
- L** represents the sub set of facilities that are breweries, such that  $\mathbf{L} \subseteq \mathbf{S}$
- K** represents the sub set of facilities that are malting plants, such that  $\mathbf{K} \subseteq \mathbf{S}$
- $\delta$  represents the sub set of facilities into which imported malt can be offloaded from the harbour, such that  $\delta \subseteq \mathbf{S}$

The decision variables are:

- $x_{ijpt} \triangleq$  the amount (in tons) of product  $p$  that was moved from facility  $i$  to facility  $j$  in period  $t$ , such that  $i, j \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $y_i \triangleq$  the amount (in tons) of extra storage that is built at facility  $i$  , such that  $i \in \mathbf{S}$
- $n_{pt} \triangleq$  the number of shipments of product  $p$  that is imported in period  $t$ , such that  $p \in \mathbf{P}, t \in \mathbf{T}$
- $\theta_{ipt} \triangleq$  the amount (in tons) of the imported product  $p$  that is offloaded into facility  $i$  , such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$



The following parameters are necessary to solve the problem:

- $\epsilon_{ipt} \triangleq$  the given amount (in tons) of product  $p$  that is manufactured at facility  $i$  in period  $t$ , such that  $i \in \mathbf{K}$
- $C_i \triangleq$  the given storage capacity (in tons) at facility  $i$ , such that  $i \in \mathbf{S}$
- $D_{ipt} \triangleq$  the given demand (in tons) for product  $p$  at facility  $i$  in period  $t$ , where  $i \in \mathbf{L}, p \in \mathbf{P}, t \in \mathbf{T}$
- $E_{ij} \triangleq$  the given transport cost (in Rand per ton) for malt that is transported from facility  $i$  to facility  $j$ , where  $i, j \in \mathbf{S}$
- $G_p \triangleq$  the given cost (in Rand per ton) to import malt type  $p$
- $N_i \triangleq$  the given variable storage cost (in R per ton) for malt that is stored at facility  $i$ , where  $i \in \mathbf{F}$
- $O_p \triangleq$  the given manufacturing cost (in Rand per ton) of product  $p$ , where  $p \in \mathbf{P}$
- $R \triangleq$  the given shipment size (in tons) of imported malt
- $Z \triangleq$  the given cost (in Rands) to build one ton of additional storage
- $\alpha \triangleq$  the given number of periods for which the demand must be covered by the safety stock held at a brewery
- $\beta \triangleq$  number of periods for which the demand must be covered by the safety stock throughout the whole malt supply grid

$$\min z = \sum_{i \in \mathbf{S}} \sum_{\substack{j \in \mathbf{S} \\ j \neq i}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} (E_{ij} + N_i) x_{ijpt} + \sum_{i \in \mathbf{S}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} O_p \epsilon_{ipt} + \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} G_p n_{p,t} + \sum_{i \in \mathbf{S}} Z y_i \quad (\text{A.1})$$

subject to

$$v_{ipt} = v_{ipt-1} + \sum_{\substack{i' \in \mathbf{S} \\ i' \neq i}} x_{i'ipt} - \sum_{j \in \mathbf{S}} x_{ijpt} + \epsilon_{ipt} - D_{ipt} + \theta_{ipt} \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.2})$$

$$\sum_{i \in \delta} \theta_{ipt} = n_{pt} R \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.3})$$

$$\sum_{\substack{i \in \mathbf{S} \\ i \notin \delta}} \theta_{ipt} \leq 0 \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.4})$$

$$v_{ipt} \geq \sum_{t=t}^{t+\alpha} D_{ipt} \quad \forall i \in \mathbf{L}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.5})$$

$$\sum_{i \in \mathbf{S}} v_{ipt} \geq \sum_{t=t}^{t+\beta} \sum_{i \in \mathbf{L}} D_{ipt} \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.6})$$

$$x_{ijpt} \geq 0 \quad \forall i, j \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.7})$$

$$\theta_{ipt} \geq 0 \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.8})$$

$$y_i \geq 0 \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.9})$$

$$n_{pt} \geq 0 \text{ and integer} \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.10})$$

The objective function of the model (A.1) is to find a solution that minimises the total sum of investment-, transportation-, storage- and purchase cost of the malt.

Equation (A.2) calculates the inventory at each facility at the end of a period, whereas (A.3) ensures that imported malt is offloaded at designated offloading facilities. Constraint (A.4) ensures that  $\theta_{ipt}$  assumes the value of zero if imported malt may not be offloaded at a specific facility. Constraint (A.5) ensures that there are a certain number of weeks' worth of safety stock at each brewery and (A.6) ensures that there is adequate safety stock spread throughout the whole malt supply grid.

Constraints (A.7) to (A.9) ensure that the relevant variables are non negative, while (A.10) ensures that the relevant variables are integer.

## A.2 Model 2: Model with silos as storage space

The model presented in this section is more complex than Model 1 in Section A.1, as silos are modelled as separate storage entities.

The following sets are used in the model:

- T** represents the set of periods (in weeks), such that  $\mathbf{T} = \{1, 2, \dots, T\}$
- S** represents the set of all the silos at all the facilities, such that  $\mathbf{S} = \{1, 2, \dots, S\}$
- P** represents the set products (malt types) , such that  $\mathbf{P} = \{1, 2, \dots, P\}$
- L** represents the set of breweries, such that  $\mathbf{L} = \{1, 2, \dots, L\}$
- K** represents the set of malting plants, such that  $\mathbf{K} = \{1, 2, \dots, K\}$
- M<sub>k</sub>** represents the sub set of silos that is located at malting facility  $k$ , where  $k \in \mathbf{K}$ , such that  $\mathbf{M}_k \subseteq \mathbf{S}$  and  $\mathbf{M}_k$  is pairwise null, such that  $\{M_i \cap M_j\}_{i,j=\{1,2,\dots,K\}|i<j} = \phi$
- F** represents the sub set of silos that are located at any malting plant, such that  $\mathbf{F} = \{\mathbf{M}_1 \cup \mathbf{M}_2 \cup \dots \mathbf{M}_K\}$
- B<sub>l</sub>** represents the sub set of silos that is located at brewery  $l$ , where  $l \in \mathbf{L}$ , such that  $\mathbf{B}_l \subseteq \mathbf{S}$  and  $\mathbf{B}_l$  is pairwise null, such that  $\{B_i \cap B_j\}_{i,j=\{1,2,\dots,L\}|i<j} = \phi$
- H** represents the sub set of silos that are located at any brewery, such that  $\mathbf{H} = \{\mathbf{B}_1 \cup \mathbf{B}_2 \cup \dots \mathbf{B}_L\}$
- W** represents the sub set of additional silos that can be build, where  $\mathbf{W} \subseteq \mathbf{S}$
- $\delta$  represents the sub set of silos into which imported malt can be offloaded from the harbour, such that  $\delta \subseteq \mathbf{S}$

The decision variables are:

- $x_{ijpt} \triangleq$  the amount (in tons) of product  $p$  that was moved from silo  $i$  to silo  $j$  in period  $t$ , such that  $i, j \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $v_{ipt} \triangleq$  the volume (in tons) of product  $p$  that is in silo  $i$  at the end of period  $t$  (after demand has been satisfied and dispatches made), such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $e_{ipt} \triangleq$  the volume (in tons) of product  $p$ , that has just been manufactured, that is put into silo  $i$  in period  $t$ , such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $f_{ipt} \triangleq$  the volume (in tons) of product  $p$ , that is taken from silo  $i$  in period  $t$  in order to satisfy demand at a brewery, such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $b_{lpt} \triangleq$  the inventory of malt type  $p$  at brewery  $l$  at the end of period  $t$ , where  $l \in \mathbf{L}, p \in \mathbf{P}, t \in \mathbf{T}$
- $n_{pt} \triangleq$  the number of shipments of product  $p$  that is imported in period  $t$ , such that  $p \in \mathbf{P}, t \in \mathbf{T}$
- $\theta_{ipt} \triangleq$  the amount (in tons) of the imported product  $p$  that is offloaded into silo  $i$ , such that  $i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T}$
- $u_{ipt} \triangleq \begin{cases} 1 & \text{if product } p \text{ is stored in silo } i \text{ in period } t, \text{ where } i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \\ 0 & \text{otherwise} \end{cases}$
- $y_i \triangleq \begin{cases} 1 & \text{if silo } i \text{ built, where } i \in \mathbf{W} \\ 0 & \text{otherwise} \end{cases}$

The following parameters are necessary to solve the problem:

- $\epsilon_{kpt} \triangleq$  the given supply (in tons) of product  $p$  at malting plant  $k$  in period  $t$
- $C_s \triangleq$  the given capacity (in tons) of silo  $s$ , such that  $s \in \mathbf{S}$
- $D_{lpt} \triangleq$  the given demand (in tons) for product  $p$  at brewery  $l$  in period  $t$ , where  $p \in \mathbf{P}, l \in \mathbf{L}, t \in \mathbf{T}$
- $E_{ij} \triangleq$  the given transport cost (in Rand per ton) for malt that is transported from silo  $i$  to silo  $j$ , where  $i, j \in \mathbf{S}$
- $G_p \triangleq$  the given cost (in Rand per ton) to import malt type  $p$
- $N_i \triangleq$  the given variable storage cost (in Rand per ton) for malt that is stored in silo  $i$ , where  $i \in \mathbf{S}$
- $O_p \triangleq$  the given manufacturing cost (in Rand per ton) of product  $p$ , where  $p \in \mathbf{P}$
- $R \triangleq$  the given shipment size (in tons) of imported malt
- $Z \triangleq$  the given cost (in Rands) to build one ton of additional storage
- $\alpha \triangleq$  the given number of periods for which the demand must be covered by the safety stock held at a brewery
- $\beta \triangleq$  number of periods for which the demand must be covered by the safety stock throughout the whole malt supply grid

The objective of the model is to find a solution that minimises the total sum of investment-, transportation-, storage- and purchase cost. Hence, the objective function of the model is formulated as follows in Equation (A.11):

$$\min z = \sum_{i \in \mathbf{S}} \sum_{\substack{j \in \mathbf{S} \\ j \neq i}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} (E_{ij} + N_i) x_{ijpt} + \sum_{k \in \mathbf{K}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} O_p B_{kpt} + \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} G_p n_{p,t} + \sum_{i \in \mathbf{W}} Z C_i y_i \quad (\text{A.11})$$

subject to

$$v_{ipt-1} + \sum_{\substack{i' \in \mathbf{S} \\ i' \neq i}} x_{i'ipt} - \sum_{j \in \mathbf{S}} x_{ijpt} + e_{ipt} + \theta_{ipt} - f_{ipt} = v_{ipt} \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.12})$$

$$v_{ipt-1} + e_{ipt} + \theta_{ipt} + \sum_{\substack{i' \in \mathbf{S} \\ i' \neq i}} x_{i'ipt} \leq C_i u_{ipt} \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.13})$$

$$\sum_{p \in \mathbf{P}} u_{ipt} \leq 1 \quad \forall i \in \mathbf{S}, t \in \mathbf{T} \quad (\text{A.14})$$

$$\sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} u_{ipt} \leq M y_i \quad \forall i \in \mathbf{W} \quad (\text{A.15})$$

Equation (A.12) calculates the inventory in each silo at the end of a period. Constraint (A.13) ensures that the inventory in a silo does not exceed the silo capacity and it furthermore drives the binary variable that links the silo usage to the volume of malt stored in the silo. Constraint (A.14) ensures that only one product can be stored in a silo in any given period. Constraint (A.15) determines whether or not additional silos are built. Following the same logic that was explained for Constraint (3.5) in Section 3.1,  $M$  can yet again assume a value that is equal to the total number of time periods.

$$\sum_{i \in \mathbf{M}_k} e_{ipt} = \epsilon_{kpt} \quad \forall k \in \mathbf{K}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.16})$$

$$\sum_{i \in \delta} e_{ipt} = n_{pt} R \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.17})$$

$$\sum_{\substack{i \in \mathbf{S} \\ i \notin \mathbf{F}}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} e_{ipt} \leq 0 \quad (\text{A.18})$$

$$\sum_{\substack{i \in \mathbf{S} \\ i \notin \delta}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} \theta_{ipt} \leq 0 \quad (\text{A.19})$$

Equation (A.16) ensures that all malt produced during a period is placed into silos. As (A.16) is repeated for each malting plant  $k$ , it also ensures that the malt can only be placed in silos at the malting plant where the malt was produced. Equation (A.17) ensures that imported malt is placed into silos. Constraint (A.18) ensures that  $e_{ipt}$  is zero for all silos that are not located at malting plants and Constraint (A.19) ensures that

silos into which the imported malt is placed are only located at the designated offloading facilities.

$$\sum_{i \in B_l} f_{ipt} = D_{lpt} \quad \forall l \in \mathbf{L}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.20})$$

$$\sum_{\substack{i \in \mathbf{S} \\ i \notin \mathbf{H}}} \sum_{p \in \mathbf{P}} \sum_{t \in \mathbf{T}} f_{ipt} \leq 0 \quad (\text{A.21})$$

$$b_{lpt} = \sum_{i \in B_l} \sum_{p \in \mathbf{P}} v_{ipt} \quad \forall l \in \mathbf{L}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.22})$$

$$b_{lpt} \geq \sum_{t=t}^{t+\alpha} D_{lpt} \quad \forall l \in \mathbf{L}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.23})$$

$$\sum_{i \in \mathbf{S}} v_{ipt} \geq \sum_{t=t}^{t+\beta} \sum_{l \in \mathbf{L}} D_{lpt} \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.24})$$

Equation (A.20) ensures that all demand is met on time. The constraint is repeated separately for all breweries, thus ensuring that a brewery can only use malt that is already on the premises to satisfy the demand. Constraint (A.21) ensures that  $f_{ipt}$  is zero for all silos not located at a brewery. The total inventory at each brewery is calculated by Equation (A.22). Constraint (A.23) ensures that there are a certain number of weeks' worth of safety stock at each brewery and (A.24) ensures there is adequate safety stock spread throughout the whole malt supply grid.

$$x_{ijpt} \geq 0 \quad \forall i, j \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.25})$$

$$e_{ipt}, f_{ipt}, v_{ipt}, \theta_{ipt} \geq 0 \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.26})$$

$$b_{lpt} \geq 0 \quad \forall l \in \mathbf{L}, r \in \mathbf{R}, t \in \mathbf{T} \quad (\text{A.27})$$

$$n_{pt} \geq 0 \text{ and integer} \quad \forall p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.28})$$

$$y_i \in \{0, 1\} \quad \forall i \in \mathbf{W} \quad (\text{A.29})$$

$$u_{ipt} \in \{0, 1\} \quad \forall i \in \mathbf{S}, p \in \mathbf{P}, t \in \mathbf{T} \quad (\text{A.30})$$

Constraints (A.25) to (A.27) ensure the variables are non negative, while (A.28) ensures that  $n_{pt}$  can only assume integer values. (A.29) and (A.30) ensure that the relevant variables are binary.

# Appendix B

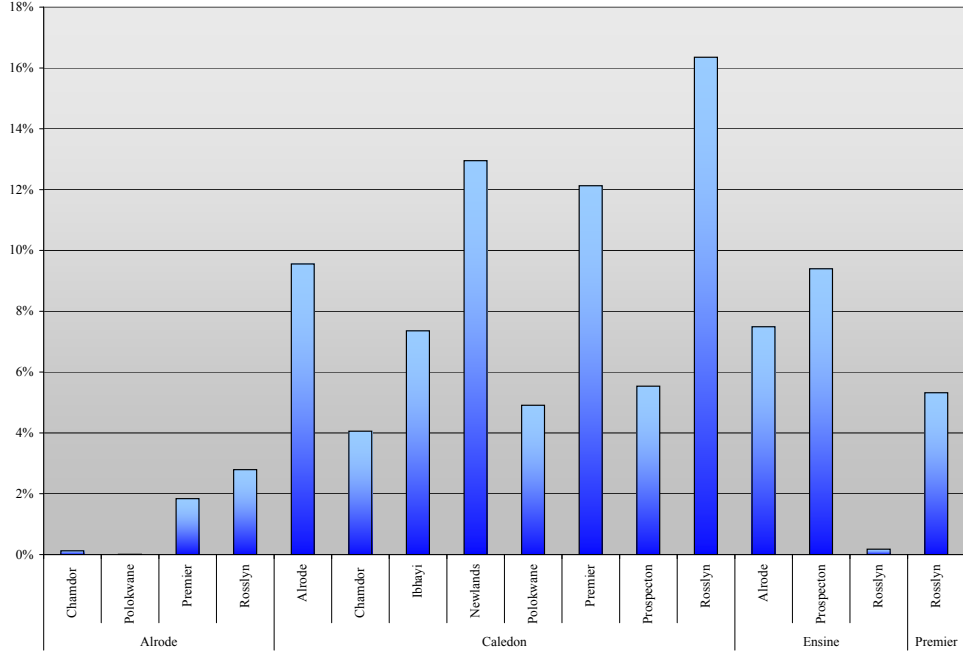
## Additional results of the model

### B.1 Results of Model 1 solved for weekly periods

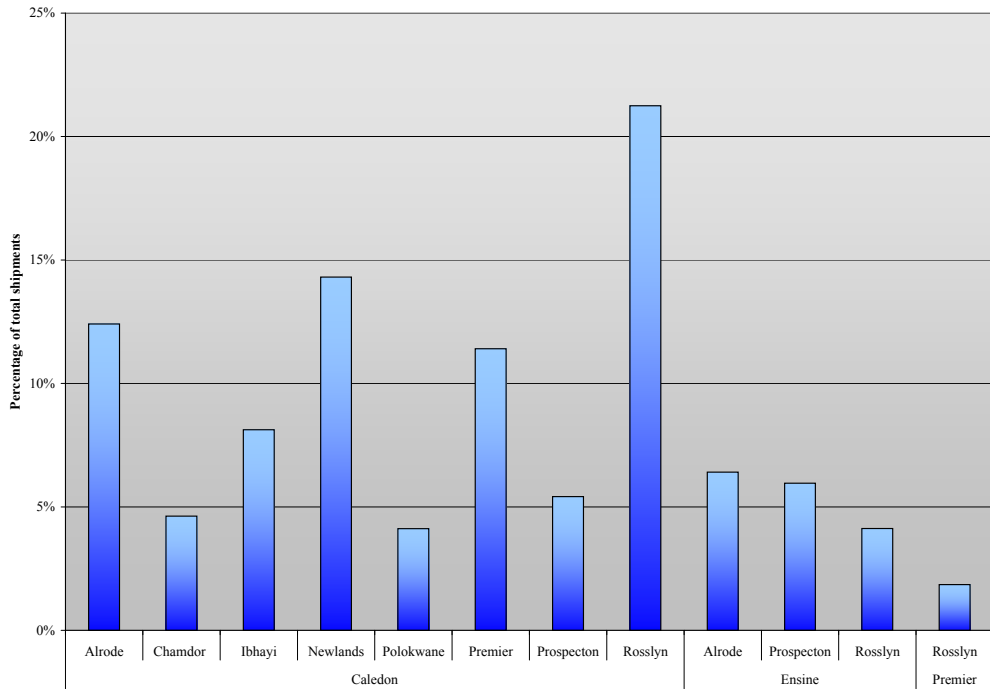
This appendix includes the more details on results that were obtained when solving Model 1 for one year subdivided into weekly periods. Figures B.1 and B.2 shows the total volume that was shipped between facilities as a percentage of the total yearly shipments. The degree to which storage capacity is utilised can be seen in Figures B.3 and B.4. Please note that the percentage utilisation refers to the percentage of the working capacity that is utilised.



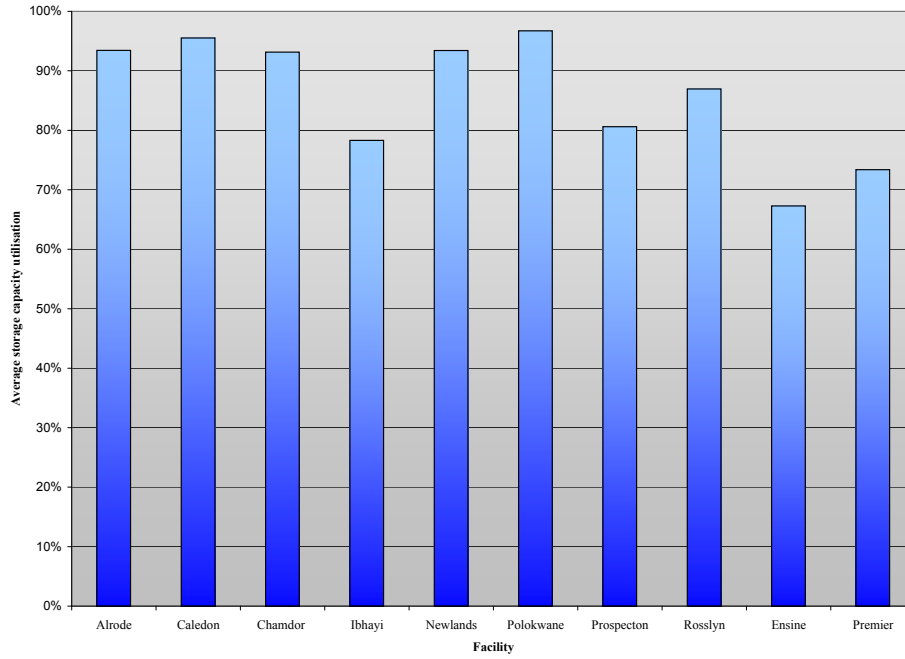
**Figure B.1: Scenario 2 in weeks: Volumes shipped between facilities as a percentage of the total yearly volume shipped**



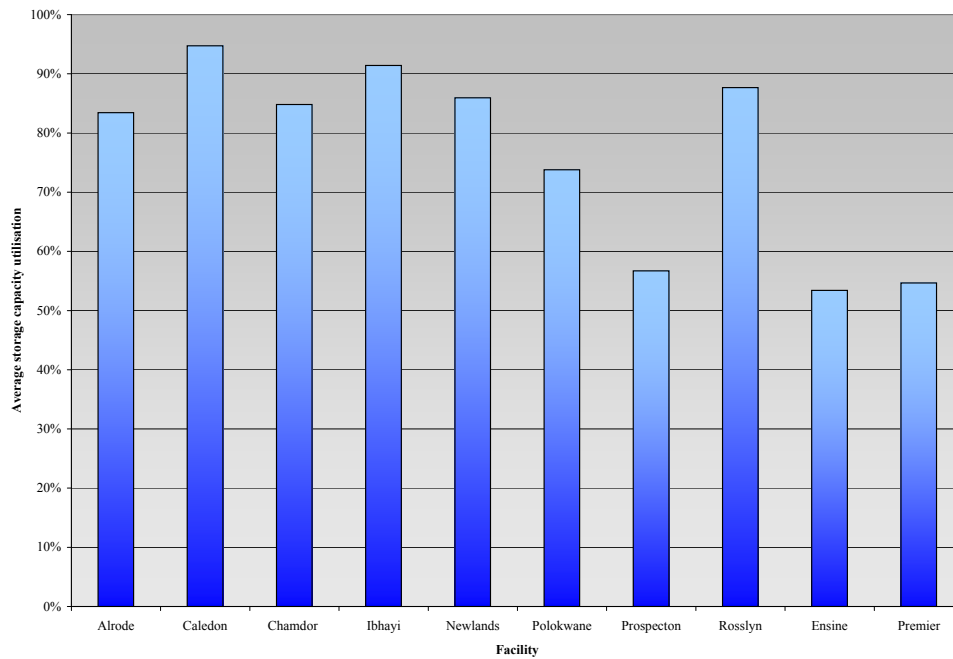
**Figure B.2: Scenario 3 in weeks: Volumes shipped between facilities as a percentage of the total yearly volume shipped**



**Figure B.3: Scenario 2 in weeks: Storage capacity utilisation**



**Figure B.4: Scenario 3 in weeks: Storage capacity utilisation**



# Appendix C

## *GAMS* algorithms for Model 1

The algorithm for Model 1 is presented in this appendix. The algorithm is written for the period of one year subdivided into weeks. To change this algorithm to one that can solve the problem for quarterly periods, the number of time periods need to be adapted to four and the appropriate data has to be included.

```
$Title Malt distribution network for SAB
```

```
$Ontext
```

```
This project finds the least cost malt distribution network for the South African Breweries. It is the final year project (BPJ 420) of Carin de Kock in 2008. This version of the model calculates the bulk weekly volumes to be shipped and allows blends to mix. The model solves the problem for weekly periods.
```

```
$Offtext
```

```
Sets
```

```
p    products (blends)           /1*2/
f    facilities                   /1*10/
t    time periods in weeks       /1*52/
```

```
alias(f,i,j)
alias(t,period);
```

```
Sets
```

k(i) the subset of facilities that are malting plants

/1 2/

l(i) the subset of facilities that are breweries

/1 3 4 5 6 7 8/

h(i) the subset of facilities into which imported malt can be offloaded

/9/;

#### Sets

AllLinks(i,j) all possible links between silos

/ 1\*10 . 1\*10/

AllowableLinks(i,j) silos between which malt movement is allowed

/

\$include AllowableLinks.inc

/

NotAllowedLinks(i,j) silos between which malt may not be moved;

NotAllowedLinks(i,j)= AllLinks(i,j)- AllowableLinks(i,j);

#### Parameters

Supply(i,p,t) The supply of product p at facility i (malting plants only)  
in period t

/

\$include SupplyINC.inc

/

O(p) purchase cost (in R per ton) of product p

/

1 0

2 0

/

ImportCost(p) the cost (in R per ton) to import product p

```
/
1 1000
2 1000
/
```

C(i) storage capacity at facility i (at 85% utilisation)

```
/
#include Capacity.inc
/
```

VarStorage(i) variable storage cost (in Rands) per ton of malt in silo s  
/10 15/

TransportCost(i,j) transport cost (in R per ton) between silo i and j  
/  
#include TransportCostINC.inc  
/

D(i,p,t) demand for malt type r at facility i (breweries only) in period t  
/  
#include DemandINC.inc  
/

OpeningGridStock(p) the safety stock of malt p in the grid at start of problem  
/  
#include GridOpen.inc  
/

BetaStart(i,p) opening stock of product p at facility i at start of problem  
/  
#include BreweryOpenStockINC.inc  
/

ShipmentSize(p) the given shipment size (in tons) of product p  
/  
1 5000  
2 5000  
/;

scalar

beta the number of weeks of safety stock required at a brewery  
/3/

gamma the number of weeks of safety stock required in the whole grid  
/8/

BuildCost the cost(in R) to build 1 ton of extra storage  
/ 2000/

MaxStorageBuild the maximum amount of extra storage space (in tons)  
that may be built  
/30000/

MaxCarriageVolume the max amount of malt that can be shipped from  
CAM per week  
/ 3960/

MaxImport max number of imported shipments per period  
/1/;

Variables

z total cost of transportation storage and purchase  
x(i,j,p,t) the amount(in tons) of product p moved from silo  
i to silo j in period t  
v(i,p,t) the volume (in tons) of product p in silo i at the

end of period t  
 $y(i,p,t)$  the volume (in tons) of product p offloaded into facility i in period t  
 $n(p,t)$  number of shipments of product p imported in period t  
 $\text{OpeningStock}(i,p)$  the opening stock of product p needed at facility i  
 $\text{ExtraStore}(i)$  the extra storage required at facility i;

Positive Variable x

v  
 y  
 OpeningStock  
 ExtraStore;

Integer variable n;

$x.fx(i,j,p,t)\$(\text{NotAllowedLinks}(i,j))=0;$   
 $\text{OpeningStock}.fx(i,p)\$(\text{ord}(i)=10)\text{or }(\text{ord}(i)=9))=0;$   
 $y.fx(i,p,t)\$(\text{not }h(i))=0;$

Equations

objective	objective function that minimises total transport inventory and purchase cost
$\text{period1inventory}(i,p,t)$	determines inventory in silo i in period 1
$\text{inventory}(i,p,t)$	determines inventory in silo i for all other periods except period 1
$\text{capacity\_constraint}(i,t)$	ensures a facility does not store more than its capacity
$\text{malt\_imported}(p,t)$	determines malt imported in a period into facility i
$\text{max\_imported}(t)$	the maximum nr of imported shipments allowed per period
$\text{Brewery\_safety\_stock}(i,p,t)$	ensures there is enough safety stock at each brewery
$\text{Grid\_safety\_stock}(p,t)$	ensures there is enough stock throughout the grid
$\text{caledon\_train}(i,t)$	constrains the amount of malt that can be shipped from caledon

OpenstockCalc(i,p)	the required amount of opening stock of product p at facility i
OpenstockCapacity(i)	the openingstock may not exceed capacity of facility i
GridOpenstockCalc(p)	calculates the openingstock in the grid
MaxExtraStore	the maximum extra capacity that may be built in the grid;

objective .. z =e= sum((i,j,p,t)\$AllowableLinks(i,j), (TransportCost(i,j) +VarStorage(i))\*x(i,j,p,t))  
+sum((p,t), n(p,t)\*ImportCost(p)\* ShipmentSize(p)  
+sum(i\$k(i),0(p)\*Supply(i,p,t)))  
+sum(i,BuildCost\*ExtraStore(i))

\*s.t.

period1inventory(i,p,t)\$(ord(t) eq 1)..  
v(i,p,t) =e= OpeningStock(i,p)  
+sum((j)\$AllowableLinks(j,i),x(j,i,p,t))  
-sum((j)\$AllowableLinks(i,j),x(i,j,p,t))  
+ Supply(i,p,t)-D(i,p,t)+y(i,p,t)\$h(i);

inventory(i,p,t)\$(ord(t)>=2)..  
v(i,p,t) =e= v(i,p,t-1)+sum((j)\$AllowableLinks(j,i),x(j,i,p,t))  
-sum((j)\$AllowableLinks(i,j),x(i,j,p,t))  
+ Supply(i,p,t)-D(i,p,t)+y(i,p,t)\$h(i);

capacity\_constraint(i,t).. sum(p,v(i,p,t))=l=C(i)+ExtraStore(i);

malt\_imported(p,t).. sum(i, y(i,p,t)) =e= n(p,t)\*ShipmentSize(p);

max\_imported(t).. sum(p,n(p,t))=l=MaxImport;

Brewery\_safety\_stock(i,p,t).. v(i,p,t)=g=  
sum( period\$((ord(period)>ord(t)) and  
(ord(period)<= (ord(t)+Beta))) ,D(i,p,t));



```

Grid_safety_stock(p,t)..      sum(i,v(i,p,t))=g=sum( i,sum( period$
                               ((ord(period)> ord(t)+1) and
                               ord(period) <= ord(t)+Gamma )),D(i,p,t));

OpenstockCalc(i,p)$ (ord(i) le 9) ..  OpeningStock(i,p)=g=BetaStart(i,p);

OpenstockCapacity(i)..      sum(p,OpeningStock(i,p))=l=C(i);

GridOpenstockCalc(p)..      sum(i,OpeningStock(i,p))=g=OpeningGridStock(p);

caledon_train(i,t)$ (ord(i) eq 2) ..
                               sum(j,sum(p,x(i,j,p,t))) =l= MaxCarriageVolume ;

MaxExtraStore..sum(i,ExtraStore(i))=l=  MaxStorageBuild;

Model SAB_maltmodel /all/ ;

SAB_maltmodel.holdfixed=yes;

option MIP=CPLEX;
option solslack=1;

SAB_maltmodel.iterlim = 1000000;
SAB_maltmodel.reslim  = 1000000;

Solve SAB_maltmodel using mip minimizing z;

$libinclude xldump x.l WeekModelXLdump.xls Sheet1!a1
$libinclude xldump v.l WeekModelXLdump.xls Sheet2!a1
$libinclude xldump n.l WeekModelXLdump.xls Sheet3!a1
$libinclude xldump OpeningStock.l WeekModelXLdump.xls Sheet4!a1
$libinclude xldump ExtraStore.l WeekModelXLdump.xls Sheet5!a1
$libinclude xldump x.m WeekModelXLdump.xls Sheet6!a1
$libinclude xldump capacity_constraint.m WeekModelXLdump.xls Sheet7!a1

```

Display x.l, v.l, ExtraStore.l, n.l, y.l, OpeningStock.l, OpeningStock.m ;