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PRIVATE CONSUMPTION EXPENDITURE IN SOUTH AFRICA: THE ROLE OF PRICE EXPECTATIONS AND LEARNING

by

RENEÉ KOEKEMOER

Submitted in fulfilment of the requirements
for the degree

DOCTOR COMMERCII

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And finally, though not the least, towards our Heavenly Father, for countless blessings.

Reneé Koekemoer.

I am still not confident that I understand the difference between supply and demand.

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SUMMARY

PRIVATE CONSUMPTION EXPENDITURE IN SOUTH AFRICA: THE ROLE OF PRICE EXPECTATIONS AND LEARNING

by

RENEÉ KOEKEMOER

PROMOTOR: **PROF. J. H. VAN HEERDEN**

DEPARTMENT: **ECONOMICS**

DEGREE: **DOCTOR COMMERCII (ECONOMETRICS)**

Private consumption expenditure is a major component of aggregate demand, accounting for roughly 60 per cent of gross domestic product in South Africa. Any attempt to explain the dynamics of the South African economy by means of an econometric model must therefore capture aggregate consumption as accurately as possible. The principal objective of the study is to derive a model for consumer behaviour in South Africa, in order to test the hypothesis that consumers are forward-looking with respect to prices when considering consumption expenditure decisions.

Modelling the expectations formation process of the consumer is therefore central in this study. It is assumed that consumers learn through a Kalman filter-based (boundedly rational learning) process for updating their expectations, conditional on prior errors made when forecasting the future price level. The first stage of implementing the boundedly rational learning approach involves the estimation of the time-varying mechanism, which represents economic agents using incomplete historical information to form expectations. The expectations rule is formulated in an attempt to capture the psychological learning process of intelligent economic agents who, despite having incomplete information, learn about their environment as time progresses. In the next stage, the expectations formation mechanism is incorporated into the behavioural equations. The theoretical specification of the behavioural equations is based on the forward-looking theories of consumption, in particular the life-cycle model of Modigliani and Brumberg, and Ando and Modigliani, and the permanent-income hypothesis of Friedman.

Consumption expenditure, for purposes of this study, is disaggregated into three categories, namely durable consumption, non-durable consumption and services. These categories, as well as total private consumption expenditure, are considered separately for unique determinants to be included in the information set. The Johansen approach, a multivariate cointegration technique, is applied in the estimation of the behavioural equations. Empirical findings prove that consumption, non-human (financial) wealth and current disposable income constitute a long-run equilibrium relationship in the case of total consumption expenditure. The same holds for expenditure on durables. In the case of non-durable consumption, a long-run cointegration equation includes only current disposable income as explanatory variable. Variables that contribute towards explaining the short-run dynamics of the system include wealth stock, the return on wealth, current disposable income, interest rates, relative prices and a variable reflecting labour market conditions, namely the employment rate in the non-agricultural sector. Interest rates prove to be significant in the explanation of durable consumption only, while the employment rate variable is only significant in the non-durable consumption function. Apart from the above, the one-period-ahead price expectations variable (the result from the Kalman filter estimation) is included in the behavioural functions to test for the role of forward-looking inflation in consumption expenditure decisions. This variable only proves to be significant in the case of durable and total private consumption expenditure.

Two sets of empirical results are thus presented: first, the time-varying coefficients of the price expectations rule and associated Kalman filter result of the estimated one-period-ahead consumer price level and second, the set of behavioural equations containing the price expectations variable.

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SAMEVATTING

PRIVATE VERBRUIKSBESTEDING IN SUID-AFRIKA: DIE ROL VAN PRYSVERWAGTINGS EN LEER

deur

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DEPARTEMENT: **EKONOMIE**

GRAAD: **DOCTOR COMMERCII (EKONOMETRIE)**

Private verbruiksbesteding is 'n belangrike vraagkantkomponent wat verantwoordelik is vir ongeveer 60 persent van Suid-Afrika se bruto binnelandse produk. Enige poging om die dinamiek van die Suid-Afrikaanse ekonomie met behulp van 'n ekonometriese model te verklaar, moet dus totale verbruiksbesteding so akkuraat moontlik vaslê. Die hoofdoel van hierdie studie is om 'n model vir Suid-Afrikaanse verbruikersgedrag af te lei en om die hipotese te toets dat verbruikersverwagtings met betrekking tot toekomstige prysvlakke bestedingsbesluite beïnvloed.

Die modellering van die proses waarvolgens verbruikers verwagtings vorm is sentraal tot hierdie studie. Daar word aanvaar dat verbruikers hul verwagtings opdateer volgens 'n Kalman filter-gebaseerde (beperkte rasionale) leerproses, voorwaardelik tot vorige foute wat begaan is tydens die vooruitskatting van die prysvlak. Die eerste stap in die implimentering van hierdie benadering behels die beraming van die tydsveranderlike meganisme wat verteenwoordigend is van die ekonomiese agent se gebruik van onvolledige historiese inligting in die vorming van prysverwagtings. Die verwagtingsreël word geformuleer in 'n poging om die psigologiese leerproses vas te lê van intelligente ekonomiese agente wat, ten spyte van onvolledige inligting, mettertyd vanuit hulle omgewing leer. In die volgende stap word die meganisme van verwagtingsvorming in die gedragsvergelykings geïnkorporeer. Vooruitskouende verbruiksteorieë, spesifiek die lewensiklusmodel van Modigliani en Brumberg, en Ando en Modigiani, en die permanente-inkome hipotese van Friedman, vorm die basis van die teoretiese spesifikasie van die gedragsvergelykings.

Vir doeleindes van hierdie studie is verbruiksbesteding opgedeel in drie kategorieë, naamlik duursame verbruik, nie-duursame verbruik en dienste. Hierdie kategorieë, sowel as totale verbruiksbesteding, word afsonderlik beskou ten einde unieke determinante vir die inligtingstel te bepaal. Empiriese bevindings bewys dat daar in die geval van totale verbruiksbesteding 'n langtermyn ewewigsverwantskap tussen verbruik, nie-menslike (finansiële) welvaart en huidige besteebare inkome bestaan. Dieselfde geld vir die besteding op duursame goedere. In die geval van nie-duursame verbruik, sluit die langtermyn ko-integrasie vergelyking slegs huidige besteebare inkome as verklarende veranderlike in. Die welvaartsvlak, opbrengs op welvaart, huidige besteebare inkome, rentekoerse, relatiewe pryse en 'n veranderlike wat toestande in die arbeidsmark reflekteer, naamlik indiensname in die nie-landbousektor, dra by tot die verklaring van die korttermyn dinamika van die stelsel. Rentekoerse is slegs betekenisvol in die verklaring van duursame verbruik, terwyl die indiensnamekoers slegs in die nie-duursame verbruiksfunksie betekenisvol is. Behalwe vir die bogenoemde, is die prysverwagting ten opsigte van een periode in die toekoms (die resultaat van die Kalman filter beraming) in die gedragsvergelykings ingesluit om die rol van inflasieverwagtings in die besluite rakende verbruiksbesteding te toets. Hierdie veranderlike is slegs betekenisvol in die geval van duursame- en totale verbruiksbesteding.

Die Johansen-benadering, 'n meerveranderlike ko-integrasie tegniek, is gebruik in die beraming van die gedragsvergelykings. Twee stelle empiriese resultate is ingesluit: eerstens die tydsveranderlike koëffisiënte van die prysverwagtingsreël en die ooreenstemmende Kalman filter resultaat van die beraamde vlak van die verbruikersprys een-periode-vooruit, en tweedens, die stel gedragsvergelykings wat die prysverwagtings bevat.

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LIST OF ABBREVIATIONS

ADF	Augmented Dickey Fuller
AIC	Akaike Information Criterion
AIH	Absolute Income Hypothesis
APC	Average propensity to consume
BMR	Bureau for Market Research
ECM	Error correction model
EG	Engle-Granger
DBSA	Development Bank of Southern Africa
DF	Dickey Fuller
GEM	Global Econometric Model
LCH	Life Cycle Hypothesis
MPC	Marginal propensity to consume
NIESR	National Institute for Economic and Social Research
PIH	Permanent Income Hypothesis
REF	Rational Expectations Hypothesis
SARB	South African Reserve Bank
SBC	Schwarz Baysian Criterion
STATS SA	Statistics South Africa
VAR model	Vector autoregressive model

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CHAPTER 1

INTRODUCTION

The study of consumer behaviour – what, how much, and when individuals consume – has been a lifetime occupation of thousands of economists.
(Hall and Taylor 1993:267)

1.1 INTRODUCTION AND BACKGROUND

Private consumption expenditure accounts for approximately 60 per cent of gross domestic product in South Africa. Any attempt to explain the dynamics of the South African economy by means of an econometric model must therefore capture aggregate consumption expenditure as accurately as possible. In order to understand consumer behaviour and price expectations formation by the average South African consumer, it is essential to be well informed on the composition of the South African population as well as socio-economic and demographic aspects such as income distribution, literacy rate and age distribution. For this purpose, a condensed socio-economic profile of the South-African consumer is included in section 1.4. The objective and layout of the study is presented in sections 1.2 and 1.3 respectively.

1.2 OBJECTIVE OF THE STUDY

In this study, the issues of concern are mainly: what determines consumption expenditure decisions and how do price expectations affect these decisions. The principal objective is to test the hypothesis that South African consumers are forward-looking with respect to prices when considering consumption expenditure decisions. To test this hypothesis, the relevant economic theory has to be considered in order to derive a theoretical model. A model for price expectations formation must be selected and the price expectations effect must subsequently be incorporated into a model of consumption expenditure.

Consumption expenditure, for purposes of this study, is disaggregated into three categories, namely durable consumption, non-durable consumption and services. These categories, as well as total private consumption expenditure, must be considered separately for unique determinants to be included in the information set. Starting with the theoretical model, a set of behavioural equations has to be estimated and subjected to thorough diagnostic checking in order to validate the model. If the price expectations variable is found to be statistically significant in explaining private consumption expenditure, one may conclude that South African consumers do consider the expected future price level when making consumption expenditure decisions.

1.3 LAYOUT OF THE STUDY

This first chapter is intended to provide a background and motivation for the study as well as a synopsis of documented results. The theoretical research and empirical findings are contained in Chapters 2 to 5 while Chapter 6 is devoted to final conclusions.

Chapter 2 commences with an account of the relevant economic theory of private consumption expenditure. A number of different theories of consumption developed in response to the simple Keynesian consumption function are discussed. These include the *life-cycle model* of Modigliani and Brumberg and also Ando and Modigliani and the *permanent-income model* of Friedman. The aforementioned theories are jointly referred to as the *forward-looking theory of consumption*. A survey of empirical results of international studies on consumption expenditure is conducted, paying particular attention to wealth and income effects, the current and expected price level and liquidity constraints on the level of consumption expenditure. The specifications of consumption functions in some well-known international macro-models are compared to conclude the chapter.

Expectations, that is anticipations or views of the future formed by economic agents, are considered in Chapter 3. This chapter surveys the theoretical developments on expectations formation, starting with the adaptive expectations hypothesis, followed by rational expectations (or model-consistent expectations) and finally the process of learning (or boundedly rational learning). The Kalman filter, a variable parameter estimation technique,

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which will be utilised in the estimation of consumer price expectations in South Africa, is discussed. This chapter also deals with the implementation of expectations in macro-economic models, including reference to international studies concerning the learning approach.

Chapter 4 provides an overview of the relevant econometrics literature for the estimation of dynamic models that are applied in the empirical estimation of a learning model for private consumption expenditure in South Africa. This chapter highlights the problem of spurious regression when modelling non-stationary data series, and reports econometrics literature addressing this issue. Single equation estimation techniques are discussed namely two-step Engle-Granger estimation and three-step Engle-Yoo estimation, primarily to point out the limitations thereof as opposed to a multivariate estimation technique like the Johansen approach. The importance of diagnostic checking in the modelling process is also stressed.

In Chapter 5 a learning model for private consumption expenditure in South Africa is derived. The hypothesis that South African consumers are forward-looking with respect to prices when making consumption expenditure decisions is tested. In this study, it is assumed that consumers learn through a Kalman filter-based (boundedly rational learning) process for updating their expectations conditional on prior forecasting. The first stage of implementing the boundedly rational learning approach would involve the estimation of the time-varying mechanism, which represents economic agents using incomplete historical information to form expectations. In the next stage, the expectations formation mechanism is incorporated into the behavioural equations. Two sets of empirical results are presented in this chapter: first, the time-varying coefficients of the price expectations rule and associated Kalman filter result of the estimated one-period-ahead consumer price level and second, the set of behavioural equations containing the price expectations variable.

Consumption expenditure is disaggregated into the following categories: durable consumption, non-durable consumption and services. Empirical estimation results for total private consumption expenditure are reported, followed by durable consumption expenditure and non-durable consumption expenditure. Since stochastic estimation of total private consumption expenditure is believed to be more reliable than that of consumption expenditure on services, the latter is deterministically determined as the residual of the total and the other two categories.

Chapter 6 summarises the empirical results and reports final conclusions with respect to consumption expenditure behaviour of the South African consumer.

1.4 SOCIO-ECONOMIC PROFILE OF THE AVERAGE SOUTH AFRICAN CONSUMER

An assessment of the socio-economic profile of South Africa can provide important insights into average consumer behaviour and can be instrumental in the *a priori* theorisation on the wealth and income effects on consumption, price expectations formation and the effect of price expectations on consumption expenditure decisions.

The socio-economic profile of South Africa is characterised by an unequal income distribution, very high unemployment rates and concomitant inescapable poverty.

The sample period under consideration in this study spans the 1970s up to 1998. Over this period, real economic growth as measured by changes in the real gross domestic product displays an overall downward trend, with average economic growth equal to 3.2 per cent for the 1970s, 2.2 per cent for the 1980s and 1.3 per cent for the 1990s. Average population growth declined slightly from 2.4 per cent for the 1970s to about 2.1 per cent during the 1990s. Disposable income per capita in real terms increased only by 0.4 per cent over the period under consideration, while a slight decrease of 0.25 per cent for the 1990s was realised (South African Reserve Bank, Quarterly Bulletin, Various issues).

Despite being classified as an upper middle-income country by the World Bank, South Africa has a high level of absolute poverty. A household is deemed to live in poverty if its total income is less than the poverty income level. The minimum living level as defined by the Bureau of Market Research (BMR) is R950 per month for a household of four persons (Whiteford and van Seventer 1999:32). This translates to US$1.80 per person per day at an exchange rate of R6.00/US$ (which is substantially higher than the US$1.00 per person per day poverty line frequently used in international studies). Based on the standard set by the BMR, 57 per cent of the South African population are living in poverty, amounting to nearly 23 million people (*op. cit.*:32). Poor living conditions are further highlighted by the lack of access to basic services, considering the fact that 55 per cent of households do not have access to tap water in their dwelling, while 12 per cent of households are without any sanitation facilities (Stats SA 1998). The aggregate poverty gap in 1996 was estimated to be R41.5 billion, which amounts to just over 10 per cent of total income earned by households (Whiteford and van Seventer 1999:34). Viewed simplistically, this means that a perfectly targeted annual transfer of 10 per cent of total household income from non-poor households to poor households could eliminate poverty in South Africa.

The South African income distribution rates among the most unequal in the world, and there has been little change for all population groups together over the sample period. The Gini-coefficient increased from 0.68 in 1975 to 0.69 in 1996. Changes occurred within population groups, with the most notable change within the African population group from 0.47 in 1975 to 0.66 in 1996 and within the White population group from 0.36 in 1975 to 0.55 in 1996 (*op. cit.*:17). Despite the significant income redistribution towards previously disadvantaged population groups, racial inequalities still persist and white per capita income was almost nine times higher than African per capita income in 1996, and 4.5 and 2.3 times higher than coloured and Asian per capita incomes respectively.

Income inequality among households is also very high and has even risen slightly between 1991 and 1996 with the richest 10 per cent of households' share of total income rising from 52.3 per cent to 53 per cent. This was accompanied by a slight decline in the poorest 40 per cent of households' income from 3.8 per cent in 1991 to just 3.4 per cent in 1996. Unemployment levels have risen dramatically over the period and reached a level of 36.3 per cent or 6.7 million people in 1998. The economy has lost about 272 000 jobs over the period 1991 to 1996 while the labour force grew in size by more than 300 000 people each year (Viljoen 1998).

High unemployment levels, particularly among less-skilled and unskilled persons, cannot be separated from the relative low literacy rate of the population. The literacy rate (completion of primary school education) of the adult population (20 years and older) was

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64 per cent in 1996. On the other hand, 19.3 per cent of the adult population has received absolutely no schooling (Stats SA 1998). Population Development indicators (DBSA 1995:25) suggest that in 1991, nearly one in every ten children of school-going age was not attending school.

What can be concluded from the above, is that a large portion of South African consumers is subjected to very low living standards, earns very little income and possesses virtually no wealth. Furthermore, low income levels lead to severe liquidity constraints on the consumer through limited access to credit and thus influence the intertemporal distribution of consumption expenditure over their life span. Lastly, seeing that living standards of 57 per cent of the population fail to comply with minimum standards, it can be expected that whatever little income these consumers earn, will be spent on non-durable goods, leaving little for durable consumption and even less for wealth accumulation.

CHAPTER 2

CONSUMPTION THEORY

Much of the most insightful empirical work in macroeconomics over the past twenty years has been concerned with consumption...
(Romer 1996:309)

2.1 INTRODUCTION

This chapter commences with an account of the relevant economic theory of consumption expenditure, to support the theoretical derivation for a model of private consumption. Reference to a number of studies on private consumption expenditure is made, paying particular attention to the effect of aspects like wealth, prices, liquidity constraints and expectations on consumption. The specification of consumption functions in some well-known international macro-models are compared to conclude the chapter. In Chapter 5, the South African situation is evaluated against the backdrop of the above analysis when an empirical estimation of private consumption expenditure functions is presented.

2.2 THE KEYNESIAN VS THE NEW CLASSICAL APPROACH

A Keynesian economist thinks about consumption theory in terms of private domestic behavioural relations underlying the IS schedule. The effects of income and interest rates on consumption would be stressed and adding the LM schedule would complete the model. A new classical economist on the other hand, would specify a production function and then would allow prices and interest rates to adjust to clear all markets (Abel 1990:726).

A number of different theories of consumption have been developed in response to the deficiencies in the simple Keynesian consumption function. Since the 1950s, economic models of consumption behaviour have explicitly recognised that in making consumption decisions, consumers consider their lifetime resources rather than simply their current income. Both the *life-cycle model* of Modigliani and Brumberg (1954) and Ando and

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Modigliani (1963) and the *permanent-income model* of Friedman (1957) are based on the notion that consumers prefer smooth streams of consumption over time. Hall and Taylor (1993:278) refer to these theories jointly as the *forward-looking theory of consumption*.

2.3 THE UNDERLYING CHOICE-THEORETIC FRAMEWORK

The life-cycle and permanent-income hypotheses, which are the major theories of consumption behaviour, both relate consumption to lifetime income.

The underlying choice-theoretic framework emphasises that a consumer has an intertemporal utility function that depends on consumption in every period of life. A first principle of microeconomics is that consumers structure their consumption plans to maximise their satisfaction or utility. According to Abel (1990:729), the consumer maximises utility subject to a single lifetime budget constraint. There is no static or period-by-period, budget constraint that requires consumption in a period to equal the income in that period. It is access to capital markets that allows consumers to choose a sequence of consumption expenditures over time, which will be smoother than the sequence of income. Access to perfect capital markets would allow consumers to borrow or lend as much as they would like, given an exogenous interest rate.

Varian (1993:179-92) examines consumer behaviour by considering the choices involved in saving and consumption over time - the consumer's intertemporal choices. The shape of the consumer's indifference curves would indicate his tastes for consumption at different times. Well-behaved preferences, where the consumer is willing to substitute some present consumption for future consumption, would be the most reasonable. How much he is willing to substitute, depends on his subjective pattern of consumption. Convexity of preferences is very natural in this context, since it means that the consumer would rather have an 'average' amount of consumption each period than a lot today and nothing tomorrow or *vice versa*. The consumer's optimal combination of consumption in any two periods, say, is where the budget line is tangent to an indifference curve.

Thomas (1993:253) makes it clear that "there is no way in which we can derive the absolute income hypothesis (the Keynesian consumption function) from a traditional microeconomic analysis". The feasible approach towards an analysis of consumer behaviour would therefore have to be conducted within a new classical framework.

2.4 THE BASIC FORWARD-LOOKING THEORY OF CONSUMPTION

The basic notion that individual consumers are forward-looking decision-makers is embodied in the life-cycle and permanent income theories. The life-cycle theory derives its name from its emphasis on a family looking ahead over its entire lifetime. The permanent income theory is named for its distinction between permanent income, which a household expects to be long-lasting, and transitory income, which is expected to quickly disappear. In practice the theories differ primarily in the types of equations used to express the basic idea of forward-looking consumers and how they are implemented empirically.

2.4.1 The life-cycle hypothesis

The analysis of section 2.3 corresponds with the work of Fisher (1907). It was later adopted and generalised by Modigliani and Brumberg (1954) in their life-cycle hypothesis. They assumed that a household plans its lifetime consumption pattern so as to maximise the total utility it obtains from consumption during its lifetime.

Assuming that households do not plan to leave assets to their heirs, algebraically a household of age T maximises a utility function of the form

$$U = U(C_T, C_{T+1}, C_{T+2}, \ldots, C_L) \tag{2.1}$$

where C_i $(i=T,T+1,T+2,\ldots L)$ is planned consumption at age i and L is the household's expected age at 'death'. Since the household plans to exactly exhaust its resources during its lifetime, (2.1) is maximised subject to the lifetime budget constraint

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$$A_{T-1} + Y_T + \sum_{i=T+1}^{N} \frac{Y_i^e}{(1+r)^{i-T}} = \sum_{i=T}^{L} \frac{C_i}{(1+r)^{i-T}} \,. \tag{2.2}$$

where

A_{T-1}	=	non-human wealth (i.e. physical and financial assets) carried over from the household's (T-1)th year
Y_T	=	household's earned or non-property income at age T
Y_i^e	=	household's expected non-property income at age i
r	=	the interest rate and
N	=	household's age at retirement.

Modigliani and Brumberg adopt the simplifying assumption that the utility function (2.1) is homothetic[1]. This implies that the planned current consumption is given by

$$C_T = \gamma_T W_T \tag{2.3}$$

where W_T is the household's total expected lifetime resources at age T. That is, it is the sum of all the terms on the left-hand side of the budget constraint (2.2)

$$W_T = A_{T-1} + Y_T + \sum_{i=T+1}^{N} \frac{Y_i^e}{(1+r)^{i-T}} \,. \tag{2.4}$$

Similarly, planned consumption in future years is given by

$$C_i = \gamma_i W_T \qquad i = T+1, T+2, \ldots, L \,. \tag{2.5}$$

The γ_is in equations (2.3) and (2.5) will depend on the rate of interest and the household's tastes and preferences. However, they will also depend on the age of the household. Because resources are to be exhausted during its lifetime, the nearer the household is to

[1] Graphically, this would mean that the slopes of the indifference curves are the same along any straight line drawn through the origin. Thus for a given rate of interest, as W_T increases and the budget line shifted outward parallel to itself, the optimal ratio C_T/C_{T+1} remains unchanged regardless of the magnitude of W_T. The ratio C_T/C_{T+1} will however depend on the tastes of the consumer, as represented by the precise form of his indifference map, and on the rate of interest.

'death', the larger the proportion of its resources it plans to expend during any given year. The important aspect of (2.3) and (2.5) is that the γ_is are independent of the magnitude of W_T. Thus the household keeps the ratios of its planned consumption expenditures on any two future years unchanged no matter what the size of its lifetime resources.

Ando and Modigliani (1963) adopted equation (2.3) for empirical estimation from aggregate time series data. Problems in estimating the expected non-property income for consumers meant that their final equation simply involved regressing aggregate consumption C_t, on aggregate current non-property income Y_t, and the aggregate wealth of consumers A_{t-1}. Their most important finding was that, for annual US data for 1929-59, A_{t-1} was a significant determinant of C_t. The marginal propensity to consume (MPC) out of net worth was estimated to be in the region 0.07 to 0.10. Their aggregate consumption function was therefore of the form $C_t = \alpha A_{t-1} + \beta Y_t$.

2.4.2 The permanent-income hypothesis

Although the permanent-income hypothesis shares many similarities with the life-cycle hypothesis, the former was developed independently and found its first definite form in the work of Friedman.

Friedman generalizes the two period case to an 'indefinitely long horizon' rather than to a remaining life-span. Friedman also introduces the concepts of present period planned of *permanent consumption*, C^p, and *permanent income*, Y^p.

According to Friedman, permanent consumption would be a function of present period total wealth W, and the rate of interest:

$$C^p = q(W, r). \tag{2.6}$$

Total wealth in the Friedman formulation is the discounted sum of all future receipts, including income from non-human assets. Wealth in period t would be

$$W_t = Y_t + \frac{Y_{t+1}}{1+r} + \frac{Y_{t+2}}{(1+r)^2} + \frac{Y_{t+3}}{(1+r)^3} + \ldots \tag{2.7}$$

where Y_t is total expected receipts in period t.

Friedman also makes use of the simplifying assumption that the consumer's utility function is homothetic, and equation (2.6) then becomes

$$C^P = qW \tag{2.8}$$

where the factor of proportionality q, is dependent on the consumers' tastes and on the rate of interest.

Theoretically, permanent income is defined as the maximum amount a consumer could consume while maintaining his wealth intact. It is in fact the return on wealth, i.e. $Y^p = rW$. Equation (2.8) can thus be rewritten as

$$C^p = q(\frac{Y^p}{r}) = kY^P \tag{2.9}$$

where q = rk.

The quantity k in equation (2.9) depends on the tastes of the household and on the rate of interest. However, under conditions of uncertainty, Friedman also introduces an additional motive for saving - the need to accumulate a reserve of wealth for contingencies. Since human wealth constitutes a less satisfactory reserve than non-human wealth, the proportion of permanent income consumed, k, is made to depend, also, on the proportion of total wealth which is held as non-human wealth. For a given rate of interest, this ratio is directly proportional to the ratio of non-human wealth to permanent income for which Friedman uses the symbol w. Thus we have

$$C^P = k(r, w, u)Y^P \tag{2.10}$$

where u is a *portmanteau* variable reflecting consumers' tastes.

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When attempts are made to relate the permanent income hypothesis to actual data, obvious problems are faced. Current or 'measured' income is clearly different from the theoretical concept of permanent income and even if adequate 'flow of services' data on current or 'measured' consumption were available, this would still differ from planned or permanent consumption. According to Friedman, measured income, Y, consists of two components – a permanent component Y^P, and a transitory component Y^t. Measured consumption C, is similarly divided into permanent consumption, C^P, and transitory consumption C^t. Thus we have

$$Y = Y^P + Y^t \qquad \text{and} \qquad C = C^P + C^t. \tag{2.11}$$

The empirical definition of Y^P is that it is the normal or expected unfortuitous income of the consumer. This roughly corresponds to the theoretical definition but is purposely left vague by Friedman since "the precise line to be drawn between permanent and transitory components is best left to the data themselves, to be whatever seems to correspond to consumer behaviour" (Friedman 1957:23). In practice, permanent income would be whatever quantity the consumer regarded as determining his planned consumption. The transitory component of income is to be regarded that which arises from accidental or chance occurrences, while permanent and transitory consumption may be interpreted as planned and 'unplanned' consumption respectively. Based on Friedman's assumption that Y^t is uncorrelated with C^t, any unforeseen increment in income does not result in unplanned consumption. This is obviously open to debate. Friedman however justifies this premise by pointing out that even if income is other than expected, the consumer would tend to stick to his consumption plan, but adjust his asset holdings.

2.4.3 Comparison of the life-cycle and permanent-income hypotheses

From the above analysis, it is clear that there are basic similarities between the life-cycle and permanent-income hypotheses. According to the life-cycle hypothesis, a change in current income Y_T, will influence current consumption C_T, only to the extent that it changes W_T, the household's expected lifetime resources. Normally, changes in Y_T, unless they lead to revisions in expectations concerning future income, i.e. to changes in the Y_i^es, can be expected to have little influence on current consumption unless the household is near 'death'. Similarly, in Friedman's model an increase in current income influences current

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consumption only to the extent that it changes W and, hence, permanent income. Furthermore, in both cases, the 'proportionality postulate' is not vital. In the case of the life-cycle hypothesis, current consumption would remain a function of total lifetime resources, although the relationship would no longer be one of strict proportionality. In the permanent income hypothesis, C^P remains a function of W and hence, of permanent income rather than current income.

There are also relatively minor, but clear differences between the models. The annuitisation of total wealth means that the stock of non-human assets does not appear explicitly in Friedman's consumption function. However, Friedman does distinguish between the influences of human and non-human wealth on consumption. The factor of proportionality, k, i.e. average propensity to consume (APC) out of permanent income, is dependent on the ratio of the two. Non-human wealth - physical and financial assets - appears explicitly in the formulation of wealth in the life-cycle model, equation (2.4). Finally, in the life-cycle model, the household merely looks ahead towards the end of its life. Friedman's annuitisation of total wealth suggests that his household has an infinite life or at any rate attaches as much importance to the consumption of its heirs as its own. One of the major implications of the life-cycle hypothesis is that saving is done by consumers when they are working to provide for consumption when they are retired. The implication will not be captured in a model in which the consumer lives and earns income forever.

2.5 A REVIEW OF RESEARCH ON AGGREGATE CONSUMPTION EXPENDITURE

This section highlights the development in the empirical application of the basic forward-looking theories.

2.5.1 Early time series estimations under the permanent-income hypothesis

Friedman (1957) adopted a distributed lag formulation with geometrically declining weights, when suggesting that, for aggregate time series data, permanent income in period t may be estimated by

$$Y_t^P = \lambda Y_t + \lambda(1-\lambda)Y_{t-1} + \lambda(1-\lambda)^2 Y_{t-2} + \ldots \qquad 0 < \lambda < 1. \tag{2.12}$$

The argument is that consumers will assign the largest weight to their current income in assessing their permanent income and successively declining weights to past income. According to Thomas (1993:263) this formulation stresses the 'expected' nature of permanent income. Equation (2.12) implies that permanent income is determined by an *adaptive expectations hypothesis*. In this case, it is

$$Y_t^P - Y_{t-1}^P = \lambda(Y_t - Y_{t-1}^P) \qquad 0 < \lambda < 1. \tag{2.13}$$

Thus differences between permanent and measured income lead to an adjustment in the perceived level of permanent income. The extent of the adjustment depends on the size of λ.

Friedman's time series work was, however, based on versions of equation (2.12). He computed various time series for Y^P_t using a different value for λ in each case, truncating after 16 terms. Using annual real per capita US data for 1905-51, he ran regressions of the form $\hat{C}_t = \hat{\alpha} + \hat{\beta} Y_t^P$ for each Y^P_t series and chose the value of λ which provided the closest fit. The highest R^2 was obtained for λ=0.33. For this equation, the intercept term was insignificant with a very low t-ratio and the estimate of k was $\hat{\beta}$ =0.88. This supported the hypothesis that the relationship between C^P and Y^P was one of proportionality. The value obtained for $\hat{\beta}$ was also close to the observed APC for the period.

It is commonly known that Friedman's estimating procedure can be simplified by application of the Koyck transformation. Evans (1969) estimated this version, using US annual data for 1929-62. He obtained the following result:

$$\underset{}{\hat{C}_t = \underset{(0.041)}{0.280Y_t} + \underset{(0.052)}{0.676C_{t-1}}}. \tag{2.14}$$

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Interpreting (2.14) in terms of Friedman's hypothesis yields $\lambda=1\text{-}0.676=0.324$ and a value for $k=0.280/\lambda=0.86$. Both these estimates were very close to those obtained by Friedman.

Since the data series are non-stationary, the disturbance term in (2.14) is almost certain to be autocorrelated, the combination of autocorrelated disturbance terms and a lagged dependent variable means the OLS estimators will be biased and inconsistent. This problem is addressed in section 4.2.

2.5.2 The effect of prices and inflation on consumption – the influential study by Davidson, Hendry, Srba and Yeo (1978): an error correction approach

The mid-1970s saw a breakdown of the form of consumption function discussed in the previous section. These functions implied a constant savings ratio and could not explain its rise during the 1970s. Davidson, Hendry, Sarb and Yeo (1978) found econometric support for the presence of an inflation term in the equation and also concluded that the rise in inflation in the 1970s was the factor behind the rising savings ratio. Their study was also highly influential in introducing the error correction approach to econometrics - the first study that attempted to deal with non-stationarity in the data and the spurious correlation problem. The study of Davidson *et al.* also represented the first explicit use in this area of the Hendry type general to specific methodology.

Davidson *et al.* concentrated on three previous studies of non-durable consumption and personal disposable income for UK data - that of Hendry (1974), Ball *et al.* (1975) and Wall *et al.* (1975) - and tried to explain why these previous investigators came to such widely different conclusions.

Their analysis proceeded by noting seven potential explanations for the main differences between the three studies, namely the choice of (i) data series, (ii) methods of seasonal adjustment, (iii) other data transformations, (iv) functional forms, (v) lag structures, (vi) diagnostic statistics and (vii) estimation methods. Even after 'standardising' the models on a common basis for (i)-(iv), the models still seemed to lead to different conclusions. Remaining possible reasons could only be (v)-(vii).

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The above standardisation by sample period, etc., enabled Davidson *et al.* to 'nest' the three competing hypotheses as special cases of a general hypothesis or estimation equation. This enabled them to test, on purely statistical grounds, which provided the best description of the relationship between income and consumption. The best model appeared to be that of Wall *et al.*, which was of the form

$$\Delta C_t = \alpha_0 + \alpha_1 \Delta Y_t + \alpha_2 \Delta Y_{t-1} \qquad \alpha_0 > 0 \tag{2.15}$$

with ΔC_t and ΔY_t the quarterly changes in consumption and income.

Davidson *et al.* pointed out that this statistically preferred equation (2.15) had some unacceptable economic properties: first, the equation had no static equilibrium solution. Also, the equation implied that the adjustment of consumption to any change in income is complete after just two quarters, and moreover, was apparently independent of any disequilibrium in the previous levels of the variables C_t and Y_t. The model thus accounted only for short-run behaviour. Davidson *et al.* (*op. cit.*:686) resolved the last of the above problems by adopting an error correction approach and presented the following error correction model in log-linear form (Δ_4 denotes the four period or annual difference for quarterly data):

$$\begin{aligned} \Delta_4 \hat{C}_t = &+0.47_1 \Delta_4 Y_t - 0.21\Delta_1 \Delta_4 Y_t - 0.10(C_{t-4} - Y_{t-4}) \\ &\quad (0.04) \qquad\quad (0.05) \qquad\qquad (0.02) \\ &+ 0.01\Delta_4 D_t^0 - 0.13\Delta_4 P_t - 0.28\Delta_1\Delta_4 P_t \\ &\quad (0.003) \qquad (0.07) \qquad (0.15) \end{aligned} \tag{2.16}$$

Davidson *et al.* invoked the Deaton hypothesis[2] by adding price variables to the equation in order to rectify the consistent overprediction of consumption during the period 1971-75, when there was a steady increase in the UK propensity to save.

[2] Deaton (1978) argued that it is *accelerating* inflation that reduces consumer expenditure. Davidson *et al.* followed him by including first and second differenced forms of the price variable in their specifications of UK consumption.

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Hendry and Ungern-Sternberg (1980), indicated that the importance of the variables $\Delta_4 P_t$ and $(C_{t-4}-Y_{t-4})$ can be explained in terms of a wealth effect. They began by noting that equations such as (2.16) have a major flaw as a complete account of the dynamic behaviour of flow variables. Since C_t and Y_t are rarely equal, this means that some latent asset stock must be changing and changes in this stock may itself affect the change in C_t. This, of course, is merely another way of saying that wealth effects may influence consumption.

Wealth, or 'cumulative saving' effects, were introduced into their model by assuming that consumers seek to maintain constant ratios not only between consumption and income, but also between the latent asset stock and income. They proposed an 'equilibrium relationship', $A_t = BY_t$, where A_t is the latent asset stock or wealth variable.

The disequilibrium 'costs' or 'losses' are incurred if C_t or A_t differs from their equilibrium values. The consumer is assumed to minimise a quadratic function of these losses subject to a budget constraint. This eventually leads to an equation that is a generalisation of (2.16), since it will contain two equilibrium errors $(C_{t-1}-Y_{t-1})$ and $(A_{t-1}-Y_{t-1})$. The latter reflects the extent of the previous period disequilibrium between asset stock and income.

Drobny and Hall (1989) established that the conventional variables used in the Davidson *et al.* specification do in fact not constitute a cointegrating vector. It also failed to perform well over the first five years of the 1980s. They contributed the failure partly to relative income effects within the overall distribution of incomes. If, as empirical results suggest, the propensity to consume of higher-rate income tax payers is different from that of standard-rate taxpayers, large changes in tax differentials (such as those of 1979 in the United Kingdom) may have large effects on aggregate expenditure on non-durable goods. They therefore introduced a tax rate differential variable and, together with disposable income and the wealth-to-income ratio, they found a cointegrating vector to exist. They further added a dummy variable for announced VAT changes to the long-run equilibrium equation.

Their preferred specification for the error correction model, based on the two-step Engle-Granger estimation procedure was (*op. cit.*:459):

$$\Delta\hat{C}_t = \underset{(0.002)}{0.008} + \underset{(0.07)}{0.33}\Delta Y_t - \underset{(0.05)}{0.12}\Delta\Delta Y_t + \underset{(0.002)}{0.007}D_t - \underset{(0.05)}{0.20}\Delta P_t - \underset{(0.092)}{0.14}\Delta C_{t-1} - \underset{(0.073)}{0.19}Z_{t-1} \quad (2.17)$$

with Z_{t-1} the lagged residual from the cointegrating regression.

Prior to the study by Davidson *et al.*, Branson and Klevorick (1969) added a price variable to the simple life-cycle hypothesis to test for the effect of 'money illusion' in US consumption. Economic theory suggests that the price coefficient in the consumption function should be equal to zero. A rise in the price level, with real income and real wealth remaining constant, must imply an equiproportionate rise in money income and money wealth and hence should not change consumption expenditure. If the price coefficient is positive, then this would imply that consumers are exhibiting the phenomenon commonly known as 'money illusion'. A positive coefficient means that a rise in P_t, with Y_t and W_t constant, results in a rise in consumption. Consumers must therefore be treating the equiproportionate rise in money income and money wealth as if it were a rise in real income and real wealth and 'not noticing' the rise in prices. A negative coefficient, however, implies some sort of reverse illusion. In the face of equiproportionate changes in the price level, money income and money wealth, consumers reduce consumption. This suggests that they believe that their real income and wealth have fallen when in fact they have not. In some way they are 'noticing' the rise in prices, but not the equiproportionate rises in money income and money wealth. Branson and Kleverick found their equivalent of the price coefficient to be significantly greater than zero and concluded that a significant degree of money illusion existed in the US consumption function.

The rapid inflation first experienced by many Western economies during the 1970s led to a number of attempts to establish links between the inflation rate, rather than the price level, and consumption. The economic rationale for the inclusion of inflation was as a proxy for the inflation loss on liquid assets, but Deaton (1987) suggested an alternative explanation in that variable inflation created uncertainty, and hence a decision to postpone consumption. Greater uncertainty regarding future real income during times of high inflation will lead to greater precautionary savings. Subsequent work by Hendry and Ungern-Sternberg (1981)

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specifically included a liquid assets term and argued that the inflation loss on this variable should be deducted from the income variable.

2.5.3 Wealth effects

The second main challenge to the consumption function came in the mid-1980s, when this time models of the type discussed in section 2.5.2 failed to explain the sharp fall in the savings-ratio in the United Kingdom. According to Bai and Whitley (1997:70), modellers reacted to the impact of forecast failures in the late 1980s by assuming that financial deregulation had increased liquidity of physical assets held by the personal sector. Deregulation resulted in the personal sector increasing its debt-to-income ratio. At the same time this boost to demand stimulated a rise in asset prices (especially house prices) but the ratio of debt to assets rose in spite of asset appreciation. Physical wealth began to appear in consumption equations. The equations then began to appear more like the life-cycle models of consumption rather than the original Keynesian form.

Although researchers agreed all along that wealth is an important determinant of consumption, the reason why earlier studies did not include explicit wealth variables was basically a lack of data on total non-human wealth. In instances where researchers attempted to include some measure of non-human wealth, they had to rely either on the liquid asset component of such wealth, or construct their own series from past data on saving. Examples include Zellner *et al.* (1965), and Townsend (1976) who found liquid asset variables to be important determinants of consumption, and Stone (1973) who constructed wealth data for the United Kingdom using the relationship $W_t = W_0 + \sum_{i=1}^{t} S_i$, where W_0 refers to wealth in some 'bench-mark' year for which data is available and S_i to saving in year i.

The study of Ball and Drake (1964) was the first study to explicitly pay attention to the precise type of consumer behaviour that may cause wealth variables to be important. In the Ball-Drake model, individuals are assumed to be short-sighted in the face of uncertainty, and their basic motive for saving is a broad precautionary one. The arguments in the consumer's utility function are current real consumption and real non-human wealth. That

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is $U_t = U(C_t, W_t)$, where W_t is wealth at the end of the (short planning) period over which utility is maximised and C_t is consumption during that period. That is, the consumer does not ignore the future but safeguards against its uncertainties by accumulating wealth. The more wealth he accumulates, the more secure he feels and, given his rate of consumption, the more utility he derives. The future is therefore allowed for without making the possibly unrealistic assumption of intertemporal utility maximisation required by the life-cycle and the permanent-income hypotheses.

However, these models of consumption behaviour in turn failed to explain the rise in the savings-ratio that occurred in the United Kingdom in 1990-91 and research began to focus on the forward-looking behaviour of consumers, going back to the original life-cycle model adjusted for forward-looking behaviour by Hall (1978) and subsequently adjusted by Hayashi (1982) to deal with liquidity constraints.

2.5.4 Liquidity constraints and credit constraints

The assumption in the basic forward-looking theory is that consumers have access to perfect capital markets and can borrow or lend at an exogenous rate of interest, while in reality, a substantial fraction of consumers is unable to consume as much as predicted by the forward-looking theory because they are unable to finance their desired level of consumption. These consumers are said to be liquidity constrained if they are unable to maintain expenditure by liquidating financial assets, or credit constrained if they are unable to borrow as much as they would like to at the prevailing interest rate.

The importance of liquidity constraints from the viewpoint of macroeconomics is that the relation between consumption and contemporaneous income is generally different for liquidity constrained consumers than it is for consumers who do not face binding liquidity constraints. The literature on liquidity constraints on consumption suggests that aggregate consumption responds to changes in both permanent and current income. This is equivalent to distinguishing between forward-looking consumers (the wealth constrained), who smooth their consumption according to the life-cycle hypothesis and backward-looking or credit-constrained consumers (liquidity constrained), whose consumption is restricted by their current incomes (Bai and Whitley 1997:73).

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Liquidity constraints were first introduced by Hall and Mishkin (1982) and tested by Hayashi (1982) and Campbell and Mankiw (1991). The basic notion is that many households have small initial values of assets and are unwilling or unable to borrow. Hence their current consumption is constrained by current income. Aggregate consumption is then given by the behaviour of both unconstrained and constrained households:

$$C_t = (1-\pi)C^U + \pi C^C . \tag{2.18}$$

The importance of liquidity constraints in consumption behaviour has been widely recognised; important empirical implementations include Campbell and Mankiw (1989), Abel (1990), Darby and Ireland (1993) and Campbell and Deaton (1989). They assume that some consumers can borrow or lend at an exogenous rate of interest while others, who would like to increase their current borrowing in order to increase current consumption, are unable to do so because of liquidity constraints.

Bai and Whitley (1997) utilises this insight as foundation for a model that behaves quite differently in the two groups of consumers.

The consumption of forward-looking consumers in their model is given by

$$C_{1t} = \beta(A_{ft} + H_{ft}) \tag{2.19}$$

where C_{1t} is the consumption of forward-looking consumers, A_{ft} is net financial and physical wealth (real non-human wealth) at time t, and H_{ft} is human wealth at time t for forward-looking consumers.

The credit-constrained consumers' consumption is equal to their non-property income:

$$C_{2t} = YD_{2t}, \tag{2.20}$$

so that aggregate consumption is a linear function of total wealth of the two types of consumers:

$$C_t = \beta(A_{ft} + H_{ft}) + YD_{2t} + u_t . \tag{2.21}$$

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Flavin (1985) and Muelbauer and Bover (1986), also concluded that the addition of a liquidity constraint does improve the explanation of the data. The liquidity constraint contains a shadow price which operates as an interest rate, so that a consumer faced with a liquidity constraint will behave as if faced with a higher interest rate. Current consumption therefore becomes more expensive and consumers substitute future consumption.

2.5.5 The role of expectations in consumption

During the twenty years after the Second World War, the adaptive expectations hypothesis enjoyed considerable popularity as a simple and apparently sensible model of how economic agents form expectations. Under the permanent income hypothesis, permanent income can be regarded as determined by an adaptive expectations process, i.e. through estimation of equations such as (2.13). The deficiencies of the adaptive expectations hypothesis, however, gradually became more apparent. It was pointed out that to model expectations of a variable adaptively, implies irrational behaviour on the part of economic agents if that variable grows at a constant rate over time. The agents' forecast error will consistently be positive. If the agent continues forming expectations adaptively under these circumstances, he will soon realise he is consistently underpredicting the variable under consideratioin and a rational individual will start taking this into account when he makes his predictions.

The 1980s saw the introduction of rational expectations (also termed model-consistent expectations) into a number of forecasting models. In simple terms, rational expectations implies that agents have access to all relevant information and make the best possible use of it when forming expectations regarding any variable. Relevant information, of course, includes knowledge of government policy aims. Hall and Garratt (1992b) point out that the economic evidence for the importance of expectations is almost uniformly based on the weak form of rational expectations (i.e. that agents do not make systematic mistakes), rather than the strong form (that they use a particular model to form their expectations). It is clearly a significant step to go from the statement that agents are 'on average' correct in their expectations to the much stronger one that they use a particular model which they believe completely. Hall and Garratt learnt from practical implementation of rational expectations within a forecasting context, that the presence of rational expectations in

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models tends to cause jumps in the initial period value of the variable. This occurs because agents anticipate the future and therefore make all the required adjustments in the current period.

Learning is a natural assumption, which overcomes the complications of the rational expectations hypothesis, and also avoids the need for the unrealistic information assumption of the strong rational expectations hypothesis. It is based on the notion that expectations are formed by intelligent agents who are not fully informed but are able to learn from their environment as time progresses. A specific expectations rule is therefore set up and the parameters of this rule are allowed to change in a way which represents the process of the agents learning about the economy.

With regard to modelling consumption expenditure, price expectations may hypothetically influence current consumption. Agents have a rule for forming expectations about the next period's consumer prices, which might involve such factors as past values of, say, interest rates, the exchange rate as well as lagged values of the consumer price index. They use this rule with their best estimate of the parameters attached to the rule to form an expectation of consumer prices in the next period and this affects consumption in the current period. During the next period, agents are able to observe the actual outcome for consumer prices and to compare it with their original expectation, which then gives them a direct measure of their expectation error. Given this error, the parameters of the expectations rule may then be revised to produce a better forecast for the following period. By repeating this procedure over a number of periods, the agents come to learn a set of parameters which will give them the best forecast of consumer prices; thus they learn about the structure of the model.

In practice, a set of 'rolling' regressions may be performed, using OLS, each period adding the latest expectation error to the data set, or alternatively, a more sophisticated mechanism based on the Kalman filter may be used for the variable parameter estimation of the expectations rule. Price expectations formation and the implementation of the price expectations variable in the consumption function are addressed in Chapter 3 and Chapter 5 respectively.

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2.5.6 Comparing specifications in different macro-models

A brief overview of the specification of consumption functions for some of the main UK macroeconometric models (Whitley 1994:34) can be found in Table 2.1. In addition, the approach towards addressing the notion of expectations is also reported. Inevitably, focussing on models at one point in time runs the risk that the description might be outdated at present. However, it may be believed that while details may change, the underlying features remain very much the same.

According to Whitley, many accounts of consumption equations written by the modellers themselves describe the consumption equation as based on the life-cycle hypothesis. This is a rather vague description in practice, since the empirical models include explanations of consumption by income alone, a combination of real incomes and wealth, and wealth alone. From Table 2.1 it is evident that there seems to be consensus on the inclusion of a combination of wealth and income, with wealth usually defined as the sum of financial and physical assets, in the determination of consumption spending. In addition, either real or nominal interest rates are included. This suggests a life-cycle approach modified by liquidity constraints.

2.6 CONCLUSION

This chapter reviewed the literature on consumption theory, with specific reference to the development of the forward-looking theory of consumption.

The most common approach in the 1970s was to treat consumers as constrained in their purchase decisions by current income. The mid-1970s saw the breakdown of this form of equation in the face of rising inflation world-wide and a rise in the savings ratio, specifically in the United Kingdom. This led to the introduction of price or inflation variables to consumption functions. Researchers also became aware of the problem of non-

Table 2.1 A comparison of specifications of consumption functions

Model	**Consumption**	**Expectations**
London Business School (LBS)	Real income, housing and financial wealth, nominal interest rates, demographic factors	Exchange rate either forward looking or learning approach
National Institute for Economic and Social Research (NIESR)	Non-credit-financed spending a function of financial wealth, real income and real interest rates	Rational in exchange rate, prices, wages, stock building
Liverpool University (LPL)	Private sector demand determined by wealth and long-term interest rates	Rational
HM Treasury (HMT)	Total wealth, inflation-adjusted income, real interest rates	Adaptive or implicit
Bank of England (BE)	Financial wealth, housing wealth, real income, mortgage equity withdrawal	Adaptive or implicit
Oxford Economic Forecast (OEF)	Income, total wealth, nominal interest rates, relative price durables/non-durables	Adaptive or implicit

Source: Whitley (1994:235).

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stationary data and spurious regressions and the error correction approach was adopted in the late 1970s. The second main challenge to consumption specifications came in the mid-1980s when models failed to explain the sharp fall in the savings ratio in the United Kingdom. This gave rise to the explicit introduction of wealth variables in consumption equations. When models of consumption behaviour again failed to explain the rise in the savings ratio that occurred in 1990-91 in the United Kingdom, the forward-looking theory of consumption was adapted to deal with liquidity constraints. The most recent development in consumption specification has been to introduce a price expectations variable into the behavioural equation. This has been done in this study and is reported in Chapter 5.

CHAPTER 3

EXPECTATIONS, LEARNING AND THE KALMAN FILTER

Expectations are formed by intelligent agents who are not fully informed and who, as a consequence, must learn about their environment.
(Hall and Garratt 1992a:52)

3.1 INTRODUCTION

Individuals frequently form expectations about the future level of prices e.g. when making consumption expenditure decisions and during wage bargaining. Expectations are formed conditional on economic agents' perceptions of the current economic environment or regime as well as possible time-related changes in that environment.

Expectations, i.e. anticipations or views of the future, have featured prominently in macroeconomic literature from the inception of the concept. Since 1930, when Irving Fisher introduced the 'anticipated rate of inflation' as the difference between the nominal and real interest rates, expectations have played an important role in economic theory. Formal analytical treatment of expectations formation, however, only emerged over the last quarter of this century. As a result, important advances in this area have occurred.

The development of macroeconomic models over the past thirty years has forced economists to recognise that expectations are not to be treated as exogenous - or to be ignored at will - but instead are central to our understanding of the functioning of the economy (Holden *et al.* 1985:1).

Historically, there have been two distinct methods of dealing with expectations in economic analysis - one is the direct measuring of expectations by conducting surveys to determine expectations, the other is to provide a simple model of expectations formation.

Much early empirical work on expectations centred around attempts to provide direct measures of agents' expectations (e.g. Katona 1951, 1958; Tobin 1959 and Eisner 1965).

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Much of this research was directed towards understanding the psychological underpinnings of individual expectations formation.

Direct measures are, for obvious reasons, not a very plausible method of obtaining expected values for future outcomes of economic variables. Although undoubtedly useful in preparing economic forecasts, gathering direct measures of expectations is costly and time consuming; in addition the data become outdated rather quickly. Furthermore, and perhaps more importantly, direct measures of agents' expectations provide little insight into how expectations would change as policy changes. The breakthrough that led to a more general approach to expectations modelling came with the realisation that expectations could be treated as an unobservable component. In this study, the latter method is adopted: a simple model of expectations formation is specified and the coefficient vector of the expectations rule is treated as an unobservable component.

This chapter surveys the theoretical developments on expectations formation by economic agents, starting with the *adaptive expectations hypothesis*. This is followed by *rational expectations* (or *model-consistent expectations*), and finally the process of *learning* (or *boundedly rational expectations*). A variable parameter estimation technique, namely the Kalman filter, which will be utilised in the estimation of consumer price expectations in South Africa, is discussed in section 3.4. Section 3.5 deals with the implementation of expectations in macro-models and section 3.6 reports on a number of international studies concerning the learning approach. The empirical result of application of the variable parameter estimation technique to the price expectations rule, is reported in Chapter 5. Price expectations are then implemented in a set of 'forward-looking' consumption functions, the result of which is also documented in Chapter 5.

3.2 EXPECTATIONS IN MACRO MODELS

Any macro model containing expectations can, in general terms, be defined as

$$y_{it} = f_i(Y_t, X, {}_tY^e_{t+1}) \qquad i = 1 \ldots n \qquad t = 1 \ldots T \qquad (3.1)$$

where

Y	=	the vector of current and lagged values of the n endogenous variables ($y_{it}...y_{it-k}$, where k is the lag depth of the model)
X	=	the vector of exogenous variables over all time periods ($x_0...x_T$) and
${}_tY^e_{t+1}$	=	expectation of Y for period t+1, based upon information available at period t.

This expectation may, in general, be viewed as being derived from another set of relationships, namely

$$ {}_ty^e_{t+1} = g_k(Y_t, X) \qquad k = 1...m. \tag{3.2}$$

The next section provides an exposition of different approaches in theory and practice to find expressions for equation (3.2).

3.3 MODELS OF EXPECTATIONS FORMATION

This section briefly discusses the theoretical development of expectations analysis and also refers to its implementation in empirical models.

3.3.1 Adaptive expectations mechanisms

During the twenty years after the Second World War, the adaptive expectations hypothesis (AEH), first proposed by Cagan (1956), enjoyed considerable popularity amongst econometricians as a simple and apparently sensible model of how economic agents form expectations.

In modelling expectations of, say, future price levels, the hypothesis of adaptive expectations states that

$$({}_{t-1}p^e_t - {}_{t-2}\,p^e_{t-1}) = \Phi(p_{t-1} - {}_{t-2}p^e_{t-1}) \qquad 0 < \Phi < 1 \tag{3.3}$$

where ${}_{t-1}p_t^e$ represents the expected value of the price level in period t formed in period t−1.

The individual therefore harbours a series of expectations for future outcomes of the price level and, in each period, the expectation for the future is revised in a way proportional with the most recently observed error.

Rearranging (3.3) yields

$${}_{t-1}p_t^e = \Phi p_{t-1} + (1-\Phi)\,{}_{t-2}p_{t-1}^e \tag{3.4}$$

and by successively substituting for the lagged expectations we get

$${}_{t-1}p_t^e = \Phi p_{t-1} + \Phi(1-\Phi)p_{t-2} + \Phi(1-\Phi)^2 p_{t-3} + \dots \tag{3.5}$$

Unobservable expectations with respect to the price level may thus be measured purely in terms of past observations of the actual price level.

The above seemed intuitively appealing, as it states that expectations of the future are a simple extrapolation of the past. The deficiencies of this approach, however, gradually became apparent. The adaptive expectations rule would, for example, cause consistent and growing mistakes in periods when prices grow at a constant rate, of say 10 per cent. The adaptive expectations model would then consistently under-estimate the level of p, and it would do so at an increasing absolute amount over time, implying that an economic agent would make entirely predictable errors, even in the very long run. It is hard to believe that such a feature could be true of an intelligent economic agent.

One way of addressing this problem is to generalise the rule to an extrapolative one, which can cope with growing variables. Fixed parameter rules in general are, however, liable to perform poorly in one circumstance or the other. An example of this is a change in the regime generating the underlying variable. The Lucas critique (Lucas 1976) pointed out

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that when expectations are modelled by functions of lagged variables, the parameters of these functions may vary as the regime for determining the expectations variable changes. So if we assume that agents are intelligent and that they avoid consistent expectations errors, then any fixed parameter extrapolative expectations model will be unable to accommodate policy or other regime changes (Currie and Hall 1994:93).

3.3.2 The concept of rational expectations

The predominant paradigm for modelling expectations is the rational expectations hypothesis. The concept of rational expectations (RE) was originally formulated by Muth (1961). He suggested that agents form their expectations in the same way that they undertake other activities – that is, they use economic theory to predict the value of the variable and this is their 'rational' expectation. Rational expectations are thus simply predictions based on economic theory, using the information available at the time the predictions are made (Holden *et al.* 1985:18).

Walters (1971) preferred the term 'consistent' to 'rational' since the expectations are consistent with the relevant economic theory – it assumes that agents use information efficiently in such a way that the aggregate of all agents' expectations will not be systematically wrong. The seventies witnessed an explosion in theoretical work incorporating rational expectations, which then became closely linked with the neo-classical approach to macroeconomics, to the extent that the two approaches were widely regarded as synonymous.

In the full or strong form of the rational expectations hypothesis, it is assumed that economic agents have a complete knowledge of the economic system about which they need to form expectations. This knowledge includes the functional form, the parameters of the system, as well as any exogenous process governing the system.

Formally, the rational expectations hypothesis (REH) may be stated as follows:

$$ {}_t p^e_{t+s} = E_t(p_{t+s}) \tag{3.6} $$

where

p	=	the variable being forecasted (i.e. the price level)
${}_t p^e_{t+s}$	=	the prediction of the price level for time t+s, formed at time t and
E_t	=	the statistical expectation conditional on information available at time t, when the forecast is made.

The rational expectations hypothesis requires that the prediction made by the forecaster be consistent with the prediction generated by the model, conditional on information available at that time. Setting s=1, equation (3.6) implies:

$$ p_{t+1} = ({}_t p^e_{t+1}) + \varepsilon_{t+1}. \tag{3.7} $$

This equation asserts that the price fluctuates about its forecast level with a purely random error ε_{t+1} that has a zero mean. This relationship between the price and its prediction, according to Turnovsky (1996:59), characterises an efficient market. It means that prices fully reflect available information, thus eliminating any systematic opportunities for making supernormal profits. As an empirical description, this assumption is an appealing one for asset and financial markets in which information is generally readily available. But it is probably less appealing for forecasting such quantities as the expectations of the consumer price index, which are likely to be based on far inferior information.

Several arguments for and against the rational expectations hypothesis are found in the literature. Some authors still maintain that one of the most compelling arguments in favour of the rational expectations hypothesis, is the relative weakness of the alternatives. As already noted, traditional expectations schemes, such as the adaptive expectations hypothesis, involve systematic forecasting errors. This is not particularly desirable, since one would expect individuals to learn this eventually and to abandon such rules or to modify them in some way. By contrast, the rational expectations hypothesis generates

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expectations that are self-fulfilling to within a random error, which cannot be predicted on the basis of information available at the date when the expectations are formed.

The most important objections to the rational expectations hypothesis concern the fact that the application thereof requires not only knowledge of the underlying structure of the model, but also of the relevant parameters. Specialists are unable to agree on a model, but even if they should find agreement, they usually obtain varying estimates for relevant coefficients. Much less will consumers, who are presumably less sophisticated in economic theory, but whose expectations we are attempting to model, have such information. This informational argument clearly has merit.

The first estimated macroeconomic models incorporating rational expectations were those of Anderson (1979) and Fair (1979). The latter was a fairly large model with 84 equations, including expectations of future prices of the stock and bond markets. The solution technique in these applications (Fair and Taylor 1983) consists of a two part iteration scheme: first, values for the expectations variable are considered given and conventional Gauss-Seidel solution methods are used to solve the model conditional on these given values for expectations; second, the expectations variables are set equal to the solution values from the model derived in the first stage. The whole process is then repeated until the expectations variables used in the first stage are consistent with the updated values of the second stage.

The rational expectations hypothesis has failed to become general practice in a forecasting context, due to a number of practical problems associated with this approach. Hall (1995:979) points out that the presence of rational expectations tends to cause jumps in the initial period value of a range of variables, for example the exchange rate, which is considered implausible by forecasters. However, it is in the large empirical models where the weakness of the rational expectations assumption has become most apparent. As a tool for analysing long-run behaviour, an assertion of rational expectations is both powerful and useful, but it is not a good representation of the short-run dynamic behaviour of many markets. Implementation in large models has in fact emphasised this weakness (Currie and Hall 1994:109). Another practical problem related to the above, is that many policy options may be impossible to analyse under rational expectations. Hall and Garratt

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(1998:257), e.g. illustrated how a permanent rise in interest rates would lead to an infinitely large jump in the exchange rate under the open arbitrage condition and the model then fails to yield a solution.

Another fact to consider is the possibility that the information set may contain errors. In such instances, rational expectations assumptions will, by exploiting the information set to the full, make larger overall errors than the adaptive expectations assumption which makes less efficient use of the information.

One conclusion to be drawn from the above, may be that agents do not have full information and full model-consistent expectations but that they learn about regime changes over time and assimilate new information. It would, therefore, be natural to progress from *strong* rational expectations to a *weaker* model, which allows for the possibility of making errors in the short run, while ruling out long-run systematic or predictable errors. This gives rise to learning models. Learning models represent a shift from the assumption of full rationality on the part of economic agents in forming expectations, towards the idea of bounded rationality which is more consistent with the psychology literature on learning processes.

3.3.3 Learning

Learning models of expectations have increasingly received attention in the theoretical literature over the past decade or more. In response to the limitations of the rational expectations hypothesis imposed by the stringent assumption of full knowledge of the true structural model, a number of economists have introduced processes describing the way in which economic agents learn about the underlying economic structure over time.

Cuthbertson (1988:224) reports on an early attempt by Friedman (1979) to provide an alternative optimising framework to rational expectations. Friedman advocates that given the true model $y_t = x_t\beta + u_t$ (where u_t is a white noise error term), agents may sequentially update their estimate of the fixed true parameter vector β as more information on (y_t, x_t) becomes available – a process of recursive least squares. Cuthbertson extends Friedman's

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framework to include the case where (i) agents have some prior information about β (at time $t=0$) and (ii) β is allowed to vary stochastically. Friedman (1979:33-34) alludes to the latter outcome when he discusses the possibility that agents may perceive a simple linear model with time varying parameters as a good approximation to the complex 'true' model. Such a model would then represent a boundedly rational learning model of expectations.

The concept of boundedly rational learning is the result of a slightly weaker informational assumption. It implies that agents use some 'reasonable' rule to form expectations and that the form of this rule remains constant over time while agents 'learn' the parameters of this rule. Agents are therefore not assumed to instantaneously know the 'true' model but they do use information optimally (or efficiently).

Implementing boundedly rational learning in a macro model entails the specification of a relatively simple expectations rule and then allowing the parameters of this rule to vary, thus correcting previous errors made by that rule. Over time, the expectations rule would be expected to adjust to the particular regime under which the model is operating and a rational expectations solution will be established. However, in the short term, the learning rule allows for errors and hopefully generates a more plausible path to equilibrium.

A serious issue with regard to the learning approach, and to a certain extent unresolved, is the selection of an appropriate expectations rule. In many instances, the reduced form of the structural equation containing the expectations variable, is the best vantage point for formulating the expectations rule (e.g. DeCanio 1979; Radner 1982; Bray and Slavin 1986; Hall and Garratt 1992a, 1992b).

The specification of an expectations rule, such as equation (3.2), is based on the assumption that some element of the rule – usually the parameter vector – is not known with certainty. The basic idea is that over time, the economic agent will methodically increase his knowledge about the true values of these parameters. Equation (3.2) may therefore be restated to explicitly include the parameters of the rule:

$$ {}_t y^e_{t+1} = g_k(Y_t, X, \xi_t) \qquad k = 1 \ldots m \tag{3.8} $$

where ξ_t is the vector of agents' estimates of the parameters at period t. Having specified the learning rule or the expectations rule, a mechanism that governs the evolution of these parameters through time needs to be specified.

The way in which the parameters change can be determined in a number of ways. At the simplest level, agents could simply perform a set of 'rolling' regressions using ordinary least squares, each period adding the latest expectation error to the data set. According to Hall and Garratt (1992a:52), this simple form of learning does not respond well to structural changes. The more sophisticated mechanism based on the Kalman filter, is therefore preferable, although the steps are conceptually similar.

The Kalman filter, proposed as an optimal method of implementing the learning process, will be discussed in the next section. 'Optimal' in this context would mean that expectations are correct on average and have minimum mean square prediction errors.

To standardise on notation, note that ${}_{t-1}p_t^e$ and ${}_tp_{t+1}^e$ in equations (3.5), (3.7) and (3.8) may also be written as $p_{t|t-1}^e$ and $p_{t+1|t}^e$. The latter corresponds to the notation used in the next section where a technical exposition of the Kalman filter algorithm is presented – the Kalman filter may be viewed as a form of adaptive expectations where the adjustment parameter, and thus the price expectation, is updated each period based on new information.

3.4 A TECHNICAL EXPOSITION OF STATE-SPACE MODELS AND THE KALMAN FILTER

State-space models were originally developed by control engineers (Wiener 1949, Kalman 1960, 1963). Kalman filtering, named after the contributions of R.E. Kalman, found applications in, for example, the technology of radars, aircraft stabilisation, the determination of coordinates in nautical or aerospace applications and chemical processes. It was not until the 1980s that state-space models received attention in economics literature

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(amongst others, Lawson 1980; Harvey *et al.* 1986; Cuthbertson 1988; Barrel *et al.* 1994; Currie and Hall 1994).

There are two major benefits to representing a dynamic system in state-space form. First, the state-space allows unobserved variables (known as the state variables) to be incorporated into, and estimated along with, the observable model. Second, state-space models can be estimated using the Kalman filter, a powerful recursive algorithm. Apart from estimating vector autoregressions with coefficients that change over time, this algorithm also provides a way to calculate exact finite-sample forecasts and the exact likelihood function for Gaussian ARMA processes. Also, factor matrix autocovariance-generating functions or spectral densities may be determined by means of the Kalman filter (Hamilton 1994:372).

State-space models have been applied in the economics and econometrics literature to model unobserved variables such as permanent income, expectations, measurement errors, missing observations, unobserved components (cycles and trends) and the natural rate of unemployment. Extensive surveys of applications of state-space models in econometrics can be found in Hamilton (1994:372-408) and Cuthbertson *et al.* (1992:191-225). Cuthbertson *et al.* (*op cit.*:191) distinguish between two types of models especially amenable to representation via the Kalman filter, namely *unobservable components* models and *time-varying parameter* models. In this study, the state-space model with stochastically time-varying parameters has been applied to a linear regression, i.e. the price expectations rule, in which the coefficient vector changes over time.

The next section describes how a dynamic system can be written in state-space form, which form is suitable for the application of the Kalman filter.

3.4.1 The state-space representation of a dynamic system

The state-space representation of the dynamics of an $(n\times1)$ vector, y_t, is given by the following system of equations:

$$y_t = A'x_t + H'\xi_t + w_t \tag{3.9}$$

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{3.10}$$

where A', H' and F are matrices of parameters of dimension (n×k), (n×r) and (r×r), respectively, and x_t is a (k×1) vector of exogenous or predetermined variables. ξ_t is a (r×1) vector of possibly unobserved state variables, known as the *state vector*. The first equation is known as the *observation* (or measurement) equation and the second is known as the *state* (or transition) equation. The (n×1) and (r×1) disturbance vectors w_t and v_t are assumed to be independent white noise with

$$E(v_t v'_\tau) = \begin{cases} Q & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{3.11}$$

$$E(w_t w'_\tau) = \begin{cases} R & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{3.12}$$

where Q and R are (r×r) and (n×n) matrices, respectively. The disturbances v_t and w_t are assumed to be uncorrelated at all lags:

$$E(v_t w'_\tau) = 0 \qquad \text{for all } t \text{ and } \tau. \tag{3.13}$$

The statement that x_t is predetermined or exogenous means that x_t provides no information about ξ_{t+s} or w_{t+s} for s=0,1,2,... beyond what is contained in $y_{t-1}, y_{t-2},\ldots,y_1$. Thus, x_t could include lagged values of y or variables which are uncorrelated with ξ_τ and w_τ for all τ.

The system of equations (3.9) through (3.13) is typically used to describe a finite series of observations $\{y_1, y_2,\ldots,y_T\}$ for which assumptions about the initial value of the state vector ξ_t are needed.

The various parameter matrices (F, Q, A, H or R) could be functions of time, as will be discussed in section 3.4.3.

Substituting (3.17) into (3.18),

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1}) . \quad (3.21)$$

The coefficient matrix in (3.21) is known as the *gain matrix* and is denoted K_t:

$$K_t \equiv FP_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1} . \quad (3.22)$$

For a derivation of these equations, see Hamilton (1994:379-380). Equations (3.17) and (3.18) may be called the updating equations and the value of $\hat{\xi}_{t+1|t}$ denotes the best forecast of ξ_{t+1} based on a constant and a linear trend function of $(y_t, y_{t-1}, \ldots, y_1, x_t, x_{t-2}, \ldots x_1)$. The matrix $P_{t|t+1}$ gives the MSE of this forecast.

The forecast of y_{t+1} is then given by

$$\hat{y}_{t+1|t} \equiv \hat{E}(y_{t+1|t} \mid x_{t+1|t}, \vartheta_t) = A'x_{t+1} + H'\hat{\xi}_{t+1|t} \quad (3.25)$$

with the associated MSE

$$E[(y_{t+1} - \hat{y}_{t+1|t})(y_{t+1} - \hat{y}_{t+1|t})'] = H'P_{t+1|t}H + R. \quad (3.26)$$

What still remains outstanding in the exposition above, is the estimation of the unknown parameter matrices F, Q, A, H and R, which are determined through maximum likelihood estimation of these parameters. The Kalman filter was motivated in the above discussion in terms of linear projections. The forecasts $\hat{\xi}_{t|t-1}$ and $\hat{y}_{t|t-1}$ are thus optimal within the set of forecasts that are linear in (x_t, ϑ_{t-1}), where $\vartheta_{t-1} \equiv (y'_{t-1}, y'_{t-2}, \ldots, y'_1, x'_{t-1}, x'_{t-2}, \ldots, x'_1)'$.

If the initial state ξ_1 and the innovations $\{w_t, v_t\}_{t=1}^{T}$ are multivariate Gaussian[3], then the stronger claim can be made that the forecasts $\hat{\xi}_{t|t-1}$ and $\hat{y}_{t|t-1}$ calculated by the Kalman filter are optimal among any functions of (x_t, ϑ_{t-1}). Moreover, if ξ_1 and $\{w_t, v_t\}_{t=1}^{T}$ are

---

[3] The vector $Y = (Y_1, Y_2, \ldots, Y_n)'$ has a multivariate Gaussian or multivariate normal distribution if $f_Y(y) = (2\pi)^{-n/2}|\Omega|^{-1/2}\exp[(-\frac{1}{2})(y-\mu)'\Omega^{-1}(y-\mu)]$ and is written as $Y_t \sim N(\mu, \Omega)$ with the mean μ an $(n\times 1)$ vector and the variance-covariance a matrix Ω of dimension $(n\times n)$ which is symmetric and positive semidefinite.

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Gaussian, then the *distribution* of y_t conditional on (x_t, ϑ_{t-1}) is Gaussian with mean given by (3.23) and variance given by (3.24):

$$y_t \mid x_t, \vartheta_{t-1} \sim N((A'x_t + H'\hat{\xi}_{t|t-1}),(H'P_{t|t-1}H + R));$$

that is

$$\begin{aligned} & f_{y_t|x_t,\vartheta_{t-1}}(y_t \mid x_t, \vartheta_{t-1}) \\ & \quad = (2\pi)^{-n/2} \mid H'P_{t|t-1}H + R \mid^{-1/2} \\ & \quad\quad \times \exp\{-\tfrac{1}{2}(y_t - A'x_t - H'\xi_{t|t-1})'(H'P_{t|t-1}H + R)^{-1} \\ & \quad\quad \times (y_t - A'x_t - H'\xi_{t|t-1})\} \qquad \text{for } t = 1,2,\ldots,T. \end{aligned} \tag{3.25}$$

From the above equation, the sample log likelihood can be constructed, namely

$$\sum_{t=1}^{T} \log f_{y_t|x_t,\vartheta_{t-1}}(y_t \mid x_t, \vartheta_{t-1}). \tag{3.26}$$

Expression (3.26) can then be maximised numerically with respect to the unknown parameters in the matrices F, Q, A, H and R. Burmeister and Wall (1982) illustrate this application.

To summarise, the state vector ξ_t and its mean squared error P_t are recursively estimated by the set of equations (3.17) through (3.20), which also represents the key equations for the Kalman filter. $\xi_{s|v}$ is the forecast of the state vector at time period s, given information available at time v. Note that the recursion for P does not depend on the forecasted state vector, or on the observed data (y_t,x_t).

To implement the Kalman filter, the starting values must be specified and the unknown matrices must be replaced by their estimates. By default, a software package like EViews obtains starting values by treating these matrices as fixed coefficients and estimating them using OLS.

Once starting values are obtained, the parameters A, H, F, R and Q are estimated by maximising the log likelihood function under the assumption that the distribution of y_t, conditional on x_t and past values of (y_t,x_t), is multivariate normal (Gaussian).

3.4.3 State-space model with stochastically varying coefficients

In the above discussion, it was assumed that the matrices F, Q, A, H and R were all constant. The Kalman filter can also be adapted for more general state-space models in which the values of these matrices depend on the exogenous or lagged dependent variables included in the vector x_t.

Equations (3.9) and (3.10), i.e. the state-space representation may thus be altered to:

$$y_t = a(x_t) + [H(x_t)]'\xi_t + w_t \tag{3.27}$$

$$\xi_{t+1} = F(x_t)\xi_t + v_{t+1}. \tag{3.28}$$

Here $F(x_t)$ denotes an $(r \times r)$ matrix whose elements are functions of x_t; $a(x_t)$ similarly describes an $(n \times 1)$ vector-valued function and $H(x_t)$ an $(r \times n)$ matrix-valued function.

It is assumed that conditional on x_t and on the data observed through date t−1, denoted $\vartheta_{t-1} \equiv (y'_{t-1}, y'_{t-2}, \ldots, y'_1, x'_{t-1}, x'_{t-2}, \ldots, x'_1)'$, the vector $(v'_{t+1}, w'_t)'$ has the Gaussian distribution

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} \mid x_t, \vartheta_{t-1} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q(x_t) & 0 \\ 0 & R(x_t) \end{bmatrix}\right). \tag{3.29}$$

Equations (3.27) to (3.29) generalise the earlier framework by allowing for stochastically varying parameters, but it is more restrictive in the sense that a Gaussian distribution is assumed in (3.29). Hamilton (1994:399-400) explains the role of the Gaussian requirement, as set out below.

Suppose it is taken as given that $\xi_t \mid \vartheta_{t-1} \sim N(\hat{\xi}_{t|t-1}, P_{t|t-1})$. Assuming as before that x_t contains only strictly exogenous variables or lagged values of y, this also describes the distribution of $\xi_t \mid x_t, \vartheta_{t-1}$. It follows from the assumptions in (3.27) to (3.29) that

$$\begin{bmatrix} \xi_t \\ y_t \end{bmatrix} \mid x_t, \vartheta_{t-1} \sim N\left(\begin{bmatrix} \hat{\xi}_{t|t-1} \\ a(x_t) + [H(x_t)]'\hat{\xi}_{t|t-1} \end{bmatrix}, \begin{bmatrix} P_{t|t-1} & P_{t|t-1}H(x_t) \\ H'(x_t)P_{t|t-1} & [H(x_t)]'P_{t|t-1}H(x_t) + R(x_t) \end{bmatrix}\right). \tag{3.30}$$

Conditional on x_t, the terms $a(x_t)$, $H(x_t)$ and $R(x_t)$ can all be treated as deterministic. Thus, the formula for the conditional distribution of Gaussian vectors[4] can be used to deduce that

$$\xi_t \mid y_t, x_t, \vartheta_{t-1} \equiv \xi_t \mid \vartheta_t \sim N(\hat{\xi}_{t|t}, P_{t|t}). \tag{3.31}$$

where

$$\hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} + \left\{P_{t|t-1}H(x_t)\left[[H(x_t)]'P_{t|t-1}H(x_t) + R(x_t)\right]^{-1} \times \left[y_t - a(x_t) - [H(x_t)]'\xi_{t|t-1}\right]\right\} \tag{3.32}$$

and

$$P_{t|t} = P_{t|t-1} - \left\{P_{t|t-1}H(x_t) \times \left[[H(x_t)]'P_{t|t-1}H(x_t) + R(x_t)\right]^{-1}[H(x_t)]'P_{t|t-1}\right\}. \tag{3.33}$$

It then follows from (3.27) and (3.29) that $\xi_{t+1} \mid \vartheta_t \sim N(\xi_{t+1|t}, P_{t+1|t})$, where

$$\hat{\xi}_{t+1|t} = F(x_t)\hat{\xi}_{t|t} \tag{3.34}$$

and

$$P_{t+1|t} = F(x_t)P_{t|t}[F(x_t)]' + Q(x_t). \tag{3.35}$$

Equations (3.32) through (3.35) are the Kalman filter equations (3.17) to (3.20), with the parameter matrices replaced with their time-varying analogs. Thus, as long as the initial state ξ_1 is treated as $N(\hat{\xi}_{1|0}, P_{1|0})$, the Kalman filter iterations proceed as before. The obvious generalisation of (3.25) may still be used to calculate the likelihood function.

The only difference from the constant-parameter case is that the inference (3.32) is a non-linear function of x_t. This means that although (3.32) gives the optimal inference if the disturbances and initial state are Gaussian, it cannot be interpreted as the linear projection of ξ_t on ϑ_t, with non-Gaussian disturbances.

3.4.4 The Kalman filter in terms of conventional least squares procedures

The exposition above is mainly in applied statistical terms. Cuthbertson (1988:234-241) presents the econometrics of the Kalman filter using conventional least squares procedures. He begins by reinterpreting the familiar problem of estimation subject to linear restrictions

[4] The conditional density of ξ_t given y_t, is found by dividing the joint density by the marginal density (Hamilton 1994:101-102, 399).

in terms of a Kalman filter updating equation, utilising the Kalman gain. This is followed by a reinterpretation of the Theil-Goldberger pure and mixed estimator as a 'one-shot' Kalman filter updating equation combining *a priori* and sample information. In both of the above cases, Cuthbertson reinterprets the standard formulae for the covariance matrix of the parameters as Kalman filter variance updating equations. He also concludes by presenting the relatively complex case of the state-space form with variable parameters.

3.5 IMPLEMENTATION OF THE LEARNING PROCESS IN A MACROECONOMIC MODEL

The learning process is generally implemented in model context as follows. The model essentially contains three blocks of equations, namely:

$$y_{it} = f_i(Y_t, X, {}_tY^e_{t+1}) \qquad i = 1\ldots n, \qquad t = 1\ldots T \tag{3.36}$$

$$ {}_ty^e_{t+1} = g_k(Y_t, X, \xi_t) \qquad k = 1\ldots m \tag{3.37}$$

$$\xi_t = \xi_{t-1} + v_t \qquad v_t \sim N(0,Q). \tag{3.38}$$

Equation (3.37) represents the measurement (or observation) equation(s) of the state-space model in Kalman filter terms, while equation (3.38) represents the state (or transition) equation(s).

Assuming that the value of ξ_{t-1} is known, the last block of equations for the expected value of ξ_t can be solved, which is simply the Kalman filter prediction equations for ξ, called the state vector. Given ξ, the second block of equations for the expected value of ${}_tY^e_{t+1}$ can be solved, and given this, the first block of equations for Y_t can finally be solved.

Q, the covariance matrix of the errors in the equations governing the evolution of the parameters (or in Kalman filter terms, the state equation error terms) is given by the original estimation and an estimation for P_{t-1} can be obtained (the uncertainty of the parameters or state variables). The Kalman filter prediction equations for P can then be used to derive an estimate of ${}_{t-1}P_t$. Having solved the complete model for Y_t, we can

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determine the one-step-ahead prediction error, that is the error that occurs between the expectation of the vector Y_t derived from the learning model and the model's final solution for Y_t. The one-step-ahead prediction error is a combination of stochastic error terms of the measurement and state equations. Given this and the estimate of $P_{t|t-1}$, the Kalman filter updating equations can be used to derive revised estimates of P_t and ξ_t. The updating is done on the basis of the observed errors between the whole model solution and the original expectations model forecast.

The process is then repeated for the next period, starting from the new updated estimates of ξ_t to predict ξ_{t+1}, and so on. In this way, the learning model will adjust its own parameters to cope with any change in structure or regime of the whole model. In the forecasting period, the final values of the state vector are used, in addition to other exogenous input, to solve the model for Y_t.

The underlying assumptions of this process are still quite strong, as agents are still assumed to process all available information in an optimal fashion and a substantial degree of sophistication on the part of the economic agent is still assumed. The learning model may, however, fulfil the criteria for weak rational expectations, since agents are not assumed to have full information. They will most likely make mistakes in the short run, but systematic errors over an extended period of time are ruled out.

Considering the model's response to any regime change reveals that the parameters of the learning rule respond over time to that change. The behaviour of the parameters of the rule, provides an important insight into the form of equilibria which may emerge from the system. Marcet and Sargent (1988) summarise the main theoretical results. The concepts of learning is characterised as the process of changing the parameters of the rule. If these parameters settle down to some fixed level, learning may be regarded as having ceased and this is sometimes called an expectational equilibrium (or E-equilibrium). Marcet and Sargent demonstrate that this stable condition is also a full rational expectations equilibrium.

3.6 EMPIRICAL LITERATURE ON THE LEARNING APPROACH

Although the theoretical literature on learning has grown from the early work of Friedman (1975 and 1979), the empirical literature has remained relatively sparse. Two empirical implementations of the learning approach in large-scale macro models will be presented as representative examples of the application of the Kalman filter to an expectations rule.

The learning approach to the treatment of expectations was first adopted in the exchange rate sector of the London Business School (LBS) model of the UK economy. Hall and Garratt (1992b) present an empirical model for the Sterling/Deutchmark real exchange rate assigning an important role to exchange rate expectations, which are assumed to be formed through a Kalman filter-based learning process.

The structural form of the exchange rate equation, derived from a capital stock model with government intervention is:

$$E_t = \alpha_0 + \alpha_1 E^e_{t+1} + \alpha_2 E_{t-1} + \alpha_3 r_t + \alpha_4 r_{t-1} + \alpha_5 T_t + \alpha_6 T_{t-1} \qquad (3.39)$$

with

E_t	=	log of the real Sterling/Deutchmark exchange rate
r_t	=	real interest rate differential between UK and German short-term rates and
T_t	=	log of the ratio of exports to imports which is a measure of the real trade balance.

The end result of a search is reported as a restricted form of the above equation, estimated by three stage least squares, namely

$$E_t = 0.0329 + 0.675 E^e_{t+1} + 0.299 E_{t-1} + 0.352 r_t \qquad (3.40)$$

which, in the long-run, exactly equals uncovered interest rate parity (UIP).

The time-varying rule for exchange rate expectations is derived from the structural equation (*op. cit.*:10) by rearranging equation (3.39) to give:

$$E^e_{t+1} = \beta_0 + \beta_1 E_t + \beta_2 E_{t-1} + \beta_3 r_t + \beta_4 r_{t-1} + \beta_5 T_t + \beta_6 T_{t-1}. \qquad (3.41)$$

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Lagging this equation by one period and using it to substitute out the term in E_t, after collecting terms, gives:

$$E^e_{t+1} = \beta_0 + (\beta_1^{\,2} + \beta_2)E_{t-1} + \beta_1\beta_2 E_{t-2} + (\beta_1\beta_3 + \beta_4)r_{t-1} \\ + \beta_1\beta_4 r_{t-2} + \beta_3 r_t + (\beta_1\beta_5 + \beta_6)T_{t-1} + \beta_1\beta_6 T_{t-2} + \beta_5 T_t. \tag{3.42}$$

The β coefficients are combinations of the α coefficients, and the contemporaneous terms in the above equation are replaced by assuming partial reduced form equations for r_t and T_t, respectively, as follows:

$$r_t = C_1(L)r_{t-1} + C_2(L)GDP_{t-1} + C_3(L)INF_{t-1} \tag{3.43}$$

$$T_t = D_1(L)T_{t-1} + D_2(L)GDP_{t-1} + D_3(L)PO_{t-1} + D_4(L)E_{t-1} \tag{3.44}$$

where PO is the log of the oil price and INF is the rate of inflation which could include both UK and German inflation or the differential and GDP is the log of real GDP. C_i and D_i are polynomial lag operators.

The terms r_t and T_t my thus be eliminated and collection of terms yields:

$$E^e_{t+1} = \beta_0 + (\beta_1^{\,2} + \beta_2 + \beta_5 D_4(L))E_{t-1} + \beta_1\beta_2 E_{t-2} + (\beta_1\beta_2 + \beta_4 + \beta_3 C_1(L))r_{t-1} \\ + \beta_1\beta_4 r_{t-2} + (\beta_1\beta_5 + \beta_6 + \beta_5 D_1(L))T_{t-1} + \beta_1\beta_6 T_{t-2} + \beta_3 C_2(L)GDP_{t-1} \\ + C_3(L)INF_{t-1} + \beta_5(D_2(L)GDP_{T-1} + D_3(L)PO_{t-1}). \tag{3.45}$$

Equation (3.45) is regarded as the basic partial reduced form rule which agents use to form their expectations. The above was simplified by dropping any lagged terms greater than t-3 by introducing a stochastic constant (*op. cit.*:11).

The basic structure was then used in a specification search to produce the following equation for expectations formation:

$$E^e_{t+1} = \gamma_{0t} + \gamma_{1t}E_{t-1} + \gamma_{2t}r_{t-1} + \gamma_{3t}INF^{UK}_{t-1} + \gamma_{4t}T^{UK}_{t-1} + \gamma_{5t}PO_{t-1}. \tag{3.46}$$

Note that all lagged information is dated t–1 when the equation is used to forecast E_{t+1}, i.e. the information set does not include current information. The parameters γ are restricted forms of the β coefficients. Hall and Garratt (*op. cit.*:11) note that their derivation of the expectations equation is by no means exclusive. In principle, formation of expectations could be the result of information from anywhere in the model, provided that it is relevant.

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The time-varying parameters are then assumed to be generated by the following process:

$$\gamma_{it} = \gamma_{it-1} + \varepsilon_{it} \ . \tag{3.47}$$

Equation (3.46) may be regarded as the standard measurement equation and the set of equations (3.47) as the state equations. The model has thus been formulated in state-space form and the Kalman filter can be applied to estimate the time-varying parameters conditional on the variance of the error terms of (3.46) and the covariance matrix of (3.47), which is assumed to be diagonal. The estimation reported (*op. cit.*:14-18) suggests that the most important determinants of the forward exchange rate are real interest rate differentials and the previous period's exchange rate. All parameter values displayed a reasonable degree of time variance, with the main movements in the coefficients on the real interest rate differential and the ratio of exports to imports for the UK, reflecting a faster rate of learning on these variables.

Another example where the Kalman filter was employed to estimate the time-varying parameter rule of expectations is a study conducted by Barrel *et al.* (1994:173). In this study, the boundedly rational learning approach was applied to wage behaviour in three countries, namely the UK, France and Italy. A time-varying parameter model for forecasting prices was first estimated. This defined the information set of the economic agents. The expectations rule together with the structural equations in which the expectations are embedded were then incorporated into the global econometric model (GEM), developed by the National Institute of Economic and Social Research (NIESR) in the UK and jointly maintained by the Institute and the London Business School.

Barrel *et al.* (*op. cit.*:174) in deriving the time-varying rule for price expectations, noted that by definition, it did not follow from a tightly formulated theory. The selection was however also not completely *ad hoc* and variables were selected to capture important endogenous linkages operating in the model.

The dependent variable for the price expectations equation in each of the three countries is the change in the log of consumer price inflation one period ahead. The information set includes the change of the log of the home country inflation, lagged by one or two periods,

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a short-term interest rate lagged by one period, the change of the log of capacity utilisation, lagged by one period, and the change of the log of the relevant spot nominal exchange rate, with respect to the Deutschmark. The only exception is France, where German inflation was also included.

Barrel *et al.* (*op. cit.*:175) first report an OLS estimation of the expectations rule, proving that in all three cases price expectations are autoregressive with lagged home country inflation the most important variable. The other variables are for the most part insignificant but are retained for the reason that they might play an important role in capturing endogenous linkages in the model. The estimates of time-varying parameters reported demonstrate a reasonable degree of variation over the period. The hyperparameters associated with the autoregressive terms imply that learning will occur rapidly with respect to changes in these variables and, by contrast, learning with respect to the other variables will be slow.

Finally, Barrel *et al.* (*op. cit.*:183) report on the outcomes of a set of simulations where the learning mechanism is in operation. These are then compared with those under rational expectations and a fixed parameter adaptive expectations mechanism. The simulations are the realignment of the franc, lira and pound within the ERM, and an oil price shock. The authors conclude from the results of these simulations that filter-based learning models caused prices to rise more sharply than they do under model-consistent or strongly rational expectations. This suggests that policy analysis within models relying only on model-consistent expectations, can be seriously misleading.

3.7 CONCLUSION

In this chapter, the theoretical development on the formation of expectations by economic agents was surveyed, starting with the adaptive expectations hypothesis, rational expectations (also called model-consistent expectations), and, most recently, boundedly rational learning. Practical problems with the implementation of the adaptive expectations hypothesis and the rational expectations hypothesis in large-scale macro models have been highlighted. The process of learning has been proposed as an alternative where the

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possibility that economic agents can make consistent prediction errors, even in the very long run, is ruled out. Boundedly rational learning seems to be an appealing alternative; full information on the part of the economic agent is not required, but it is accepted that the intelligent agent, knowing the structure of the expectations model, will assimilate new information as time progresses and adjust the parameters of the model accordingly.

An exposition of the state-space representation of a system and the Kalman filter as time-varying parameter estimation technique were presented. A brief account of a number of international studies in this regard concludes this chapter. Chapter 5 describes the application of the Kalman filter to a price expectations rule for expectations formation by South African consumers, as well as the incorporation of the price expectations variable into a set of behavioural equations.

3.4.2 Estimation by the Kalman filter

Given observations (y_t,x_t) for $t = 1,2,\ldots,T$, one of the ultimate objectives is to estimate the unknown parameters in the system based on these observations. The parameters to estimate would include the matrices A, H, F, R, and Q, and the state vector ξ_t of which inferences need to be made. The Kalman filter is a recursive algorithm for sequentially updating the state vector given past information. More technically, it is an algorithm for calculating linear least squares forecasts of the state vector on the basis of data observed up to date t,

$$\hat{\xi}_{t+1|t} \equiv \hat{E}(\xi_{t+1} \mid \vartheta_t),$$

where

$$\vartheta_t \equiv (y'_t, y'_{t-1}, \ldots, y'_1, x'_t, x'_{t-1}, \ldots, x'_1)' \qquad (3.14)$$

and $\hat{E}(\xi_{t+1} \mid \vartheta_t)$ denotes the linear projections of ξ_{t+1} on ϑ_t and a constant. The Kalman filter calculates these forecasts recursively, generating $\xi_{1|0}, \xi_{2|1},\ldots,\xi_{T|T-1}$ in succession. Associated with each of these forecasts is a mean squared error (MSE) matrix, represented by the following (r×r) matrix:

$$P_{t+1|t} = E[(\xi_{t+1} - \hat{\xi}_{t+1|t})(\xi_{t+1} - \hat{\xi}_{t+1|t})']. \qquad (3.15)$$

The recursion would begin with $\hat{\xi}_{1|0}$, which denotes a forecast of ξ_1 based on no observations of y or x. This is just the unconditional mean of ξ_1,

$$\hat{\xi}_{1|0} = E(\xi_1),$$

with associated MSE (variance of ξ_1),

$$P_{1|0} = E\{[\xi_1 - E(\xi_1)][\xi_1 - E(\xi_1)]'\}. \qquad (3.16)$$

Then, iteration on the following set of equations takes place:

$$\hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1}) \qquad (3.17)$$

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t} \qquad (3.18)$$

and

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1} \qquad (3.19)$$

$$P_{t+1|t} = FP_{t|t}F' + Q \qquad \text{for } t=1,2,\ldots,T. \qquad (3.20)$$

Substituting (3.17) into (3.18),

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1}) . \tag{3.21}$$

The coefficient matrix in (3.21) is known as the *gain matrix* and is denoted K_t:

$$K_t \equiv FP_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1} . \tag{3.22}$$

For a derivation of these equations, see Hamilton (1994:379-380). Equations (3.17) and (3.18) may be called the updating equations and the value of $\hat{\xi}_{t+1|t}$ denotes the best forecast of ξ_{t+1} based on a constant and a linear trend function of $(y_t, y_{t-1}, \ldots, y_1, x_t, x_{t-2}, \ldots x_1)$. The matrix $P_{t|t+1}$ gives the MSE of this forecast.

The forecast of y_{t+1} is then given by

$$\hat{y}_{t+1|t} \equiv \hat{E}(y_{t+1|t} \mid x_{t+1|t}, \vartheta_t) = A'x_{t+1} + H'\hat{\xi}_{t+1|t} \tag{3.25}$$

with the associated MSE

$$E[(y_{t+1} - \hat{y}_{t+1|t})(y_{t+1} - \hat{y}_{t+1|t})'] = H'P_{t+1|t}H + R. \tag{3.26}$$

What still remains outstanding in the exposition above, is the estimation of the unknown parameter matrices F, Q, A, H and R, which are determined through maximum likelihood estimation of these parameters. The Kalman filter was motivated in the above discussion in terms of linear projections. The forecasts $\hat{\xi}_{t|t-1}$ and $\hat{y}_{t|t-1}$ are thus optimal within the set of forecasts that are linear in (x_t, ϑ_{t-1}), where $\vartheta_{t-1} \equiv (y'_{t-1}, y'_{t-2}, \ldots, y'_1, x'_{t-1}, x'_{t-2}, \ldots, x'_1)'$.

If the initial state ξ_1 and the innovations $\{w_t, v_t\}_{t-1}^{T}$ are multivariate Gaussian[3], then the stronger claim can be made that the forecasts $\hat{\xi}_{t|t-1}$ and $\hat{y}_{t|t-1}$ calculated by the Kalman filter are optimal among any functions of (x_t, ϑ_{t-1}). Moreover, if ξ_1 and $\{w_t, v_t\}_{t-1}^{T}$ are

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[3] The vector $Y = (Y_1, Y_2, \ldots, Y_n)'$ has a multivariate Gaussian or multivariate normal distribution if $f_Y(y) = (2\pi)^{-n/2}|\Omega|^{-1/2}\exp[(-\frac{1}{2})(y-\mu)'\Omega^{-1}(y-\mu)]$ and is written as $Y_t \sim N(\mu, \Omega)$ with the mean μ an $(n \times 1)$ vector and the variance-covariance a matrix Ω of dimension $(n \times n)$ which is symmetric and positive semidefinite.

Gaussian, then the *distribution* of y_t conditional on (x_t, ϑ_{t-1}) is Gaussian with mean given by (3.23) and variance given by (3.24):

$$y_t \mid x_t, \vartheta_{t-1} \sim N((A'x_t + H'\hat{\xi}_{t|t-1}), (H'P_{t|t-1}H + R));$$

that is

$$\begin{aligned} & f_{y_t|x_t,\vartheta_{t-1}}(y_t \mid x_t, \vartheta_{t-1}) \\ & \quad = (2\pi)^{-n/2} \mid H'P_{t|t-1}H + R \mid^{-1/2} \\ & \quad \times \exp\{-\tfrac{1}{2}(y_t - A'x_t - H'\xi_{t|t-1})'(H'P_{t|t-1}H + R)^{-1} \\ & \quad \times (y_t - A'x_t - H'\xi_{t|t-1})\} \qquad \text{for } t = 1,2,\ldots,T. \end{aligned} \tag{3.25}$$

From the above equation, the sample log likelihood can be constructed, namely

$$\sum_{t=1}^{T} \log f_{y_t|x_t,\vartheta_{t-1}}(y_t \mid x_t, \vartheta_{t-1}). \tag{3.26}$$

Expression (3.26) can then be maximised numerically with respect to the unknown parameters in the matrices F, Q, A, H and R. Burmeister and Wall (1982) illustrate this application.

To summarise, the state vector ξ_t and its mean squared error P_t are recursively estimated by the set of equations (3.17) through (3.20), which also represents the key equations for the Kalman filter. $\xi_{s|v}$ is the forecast of the state vector at time period s, given information available at time v. Note that the recursion for P does not depend on the forecasted state vector, or on the observed data (y_t, x_t).

To implement the Kalman filter, the starting values must be specified and the unknown matrices must be replaced by their estimates. By default, a software package like EViews obtains starting values by treating these matrices as fixed coefficients and estimating them using OLS.

Once starting values are obtained, the parameters A, H, F, R and Q are estimated by maximising the log likelihood function under the assumption that the distribution of y_t, conditional on x_t and past values of (y_t, x_t), is multivariate normal (Gaussian).

3.4.3 State-space model with stochastically varying coefficients

In the above discussion, it was assumed that the matrices F, Q, A, H and R were all constant. The Kalman filter can also be adapted for more general state-space models in which the values of these matrices depend on the exogenous or lagged dependent variables included in the vector x_t.

Equations (3.9) and (3.10), i.e. the state-space representation may thus be altered to:

$$y_t = a(x_t) + [H(x_t)]'\xi_t + w_t \tag{3.27}$$

$$\xi_{t+1} = F(x_t)\xi_t + v_{t+1}. \tag{3.28}$$

Here $F(x_t)$ denotes an $(r\times r)$ matrix whose elements are functions of x_t; $a(x_t)$ similarly describes an $(n\times 1)$ vector-valued function and $H(x_t)$ an $(r\times n)$ matrix-valued function.

It is assumed that conditional on x_t and on the data observed through date t−1, denoted $\vartheta_{t-1} \equiv (y'_{t-1}, y'_{t-2}, \ldots, y'_1, x'_{t-1}, x'_{t-2}, \ldots, x'_1)'$, the vector $(v'_{t+1}, w'_t)'$ has the Gaussian distribution

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} | \, x_t, \vartheta_{t-1} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q(x_t) & 0 \\ 0 & R(x_t) \end{bmatrix}\right). \tag{3.29}$$

Equations (3.27) to (3.29) generalise the earlier framework by allowing for stochastically varying parameters, but it is more restrictive in the sense that a Gaussian distribution is assumed in (3.29). Hamilton (1994:399-400) explains the role of the Gaussian requirement, as set out below.

Suppose it is taken as given that $\xi_t \,|\, \vartheta_{t-1} \sim N(\hat{\xi}_{t|t-1}, P_{t|t-1})$. Assuming as before that x_t contains only strictly exogenous variables or lagged values of y, this also describes the distribution of $\xi_t | x_t, \vartheta_{t-1}$. It follows from the assumptions in (3.27) to (3.29) that

$$\begin{bmatrix} \xi_t \\ y_t \end{bmatrix} | \, x_t, \vartheta_{t-1} \sim N\left(\begin{bmatrix} \hat{\xi}_{t|t-1} \\ a(x_t) + [H(x_t)]'\hat{\xi}_{t|t-1} \end{bmatrix}, \begin{bmatrix} P_{t|t-1} & P_{t|t-1}H(x_t) \\ H'(x_t)P_{t|t-1} & [H(x_t)]'P_{t|t-1}H(x_t) + R(x_t) \end{bmatrix}\right). \tag{3.30}$$

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Conditional on x_t, the terms $a(x_t)$, $H(x_t)$ and $R(x_t)$ can all be treated as deterministic. Thus, the formula for the conditional distribution of Gaussian vectors[4] can be used to deduce that

$$\xi_t \mid y_t, x_t, \vartheta_{t-1} \equiv \xi_t \mid \vartheta_t \sim N(\hat{\xi}_{t|t}, P_{t|t}). \tag{3.31}$$

where

$$\hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} + \{P_{t|t-1}H(x_t)[[H(x_t)]'P_{t|t-1}H(x_t) + R(x_t)]^{-1} \times [y_t - a(x_t) - [H(x_t)]'\xi_{t|t-1}]\} \tag{3.32}$$

and

$$P_{t|t} = P_{t|t-1} - \{P_{t|t-1}H(x_t) \times [[H(x_t)]'P_{t|t-1}H(x_t) + R(x_t)]^{-1}[H(x_t)]'P_{t|t-1}\}. \tag{3.33}$$

It then follows from (3.27) and (3.29) that $\xi_{t+1} \mid \vartheta_t \sim N(\xi_{t+1|t}, P_{t+1|t})$, where

$$\hat{\xi}_{t+1|t} = F(x_t)\hat{\xi}_{t|t} \tag{3.34}$$

and

$$P_{t+1|t} = F(x_t)P_{t|t}[F(x_t)]' + Q(x_t). \tag{3.35}$$

Equations (3.32) through (3.35) are the Kalman filter equations (3.17) to (3.20), with the parameter matrices replaced with their time-varying analogs. Thus, as long as the initial state ξ_1 is treated as $N(\hat{\xi}_{1|0}, P_{1|0})$, the Kalman filter iterations proceed as before. The obvious generalisation of (3.25) may still be used to calculate the likelihood function.

The only difference from the constant-parameter case is that the inference (3.32) is a non-linear function of x_t. This means that although (3.32) gives the optimal inference if the disturbances and initial state are Gaussian, it cannot be interpreted as the linear projection of ξ_t on ϑ_t, with non-Gaussian disturbances.

3.4.4 The Kalman filter in terms of conventional least squares procedures

The exposition above is mainly in applied statistical terms. Cuthbertson (1988:234-241) presents the econometrics of the Kalman filter using conventional least squares procedures. He begins by reinterpreting the familiar problem of estimation subject to linear restrictions

[4] The conditional density of ξ_t given y_t, is found by dividing the joint density by the marginal density (Hamilton 1994:101-102, 399).

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in terms of a Kalman filter updating equation, utilising the Kalman gain. This is followed by a reinterpretation of the Theil-Goldberger pure and mixed estimator as a 'one-shot' Kalman filter updating equation combining *a priori* and sample information. In both of the above cases, Cuthbertson reinterprets the standard formulae for the covariance matrix of the parameters as Kalman filter variance updating equations. He also concludes by presenting the relatively complex case of the state-space form with variable parameters.

3.5 IMPLEMENTATION OF THE LEARNING PROCESS IN A MACROECONOMIC MODEL

The learning process is generally implemented in model context as follows. The model essentially contains three blocks of equations, namely:

$$y_{it} = f_i(Y_t, X, {}_tY^e_{t+1}) \qquad i = 1\ldots n, \qquad t = 1\ldots T \tag{3.36}$$

$$ {}_ty^e_{t+1} = g_k(Y_t, X, \xi_t) \qquad k = 1\ldots m \tag{3.37}$$

$$\xi_t = \xi_{t-1} + v_t \qquad v_t \sim N(0,Q). \tag{3.38}$$

Equation (3.37) represents the measurement (or observation) equation(s) of the state-space model in Kalman filter terms, while equation (3.38) represents the state (or transition) equation(s).

Assuming that the value of ξ_{t-1} is known, the last block of equations for the expected value of ξ_t can be solved, which is simply the Kalman filter prediction equations for ξ, called the state vector. Given ξ, the second block of equations for the expected value of ${}_tY^e_{t+1}$ can be solved, and given this, the first block of equations for Y_t can finally be solved.

Q, the covariance matrix of the errors in the equations governing the evolution of the parameters (or in Kalman filter terms, the state equation error terms) is given by the original estimation and an estimation for P_{t-1} can be obtained (the uncertainty of the parameters or state variables). The Kalman filter prediction equations for P can then be used to derive an estimate of ${}_{t-1}P_t$. Having solved the complete model for Y_t, we can

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determine the one-step-ahead prediction error, that is the error that occurs between the expectation of the vector Y_t derived from the learning model and the model's final solution for Y_t. The one-step-ahead prediction error is a combination of stochastic error terms of the measurement and state equations. Given this and the estimate of $P_{t|t-1}$, the Kalman filter updating equations can be used to derive revised estimates of P_t and ξ_t. The updating is done on the basis of the observed errors between the whole model solution and the original expectations model forecast.

The process is then repeated for the next period, starting from the new updated estimates of ξ_t to predict ξ_{t+1}, and so on. In this way, the learning model will adjust its own parameters to cope with any change in structure or regime of the whole model. In the forecasting period, the final values of the state vector are used, in addition to other exogenous input, to solve the model for Y_t.

The underlying assumptions of this process are still quite strong, as agents are still assumed to process all available information in an optimal fashion and a substantial degree of sophistication on the part of the economic agent is still assumed. The learning model may, however, fulfil the criteria for weak rational expectations, since agents are not assumed to have full information. They will most likely make mistakes in the short run, but systematic errors over an extended period of time are ruled out.

Considering the model's response to any regime change reveals that the parameters of the learning rule respond over time to that change. The behaviour of the parameters of the rule, provides an important insight into the form of equilibria which may emerge from the system. Marcet and Sargent (1988) summarise the main theoretical results. The concepts of learning is characterised as the process of changing the parameters of the rule. If these parameters settle down to some fixed level, learning may be regarded as having ceased and this is sometimes called an expectational equilibrium (or E-equilibrium). Marcet and Sargent demonstrate that this stable condition is also a full rational expectations equilibrium.

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3.6 EMPIRICAL LITERATURE ON THE LEARNING APPROACH

Although the theoretical literature on learning has grown from the early work of Friedman (1975 and 1979), the empirical literature has remained relatively sparse. Two empirical implementations of the learning approach in large-scale macro models will be presented as representative examples of the application of the Kalman filter to an expectations rule.

The learning approach to the treatment of expectations was first adopted in the exchange rate sector of the London Business School (LBS) model of the UK economy. Hall and Garratt (1992b) present an empirical model for the Sterling/Deutchmark real exchange rate assigning an important role to exchange rate expectations, which are assumed to be formed through a Kalman filter-based learning process.

The structural form of the exchange rate equation, derived from a capital stock model with government intervention is:

$$E_t = \alpha_0 + \alpha_1 E^e_{t+1} + \alpha_2 E_{t-1} + \alpha_3 r_t + \alpha_4 r_{t-1} + \alpha_5 T_t + \alpha_6 T_{t-1} \tag{3.39}$$

with

E_t = log of the real Sterling/Deutchmark exchange rate

r_t = real interest rate differential between UK and German short-term rates and

T_t = log of the ratio of exports to imports which is a measure of the real trade balance.

The end result of a search is reported as a restricted form of the above equation, estimated by three stage least squares, namely

$$E_t = 0.0329 + 0.675 E^e_{t+1} + 0.299 E_{t-1} + 0.352 r_t \tag{3.40}$$

which, in the long-run, exactly equals uncovered interest rate parity (UIP).

The time-varying rule for exchange rate expectations is derived from the structural equation (*op. cit.*:10) by rearranging equation (3.39) to give:

$$E^e_{t+1} = \beta_0 + \beta_1 E_t + \beta_2 E_{t-1} + \beta_3 r_t + \beta_4 r_{t-1} + \beta_5 T_t + \beta_6 T_{t-1}. \tag{3.41}$$

Lagging this equation by one period and using it to substitute out the term in E_t, after collecting terms, gives:

$$\begin{aligned} E^e_{t+1} &= \beta_0 + (\beta_1^{\,2} + \beta_2)E_{t-1} + \beta_1\beta_2 E_{t-2} + (\beta_1\beta_3 + \beta_4)r_{t-1} \\ &\quad + \beta_1\beta_4 r_{t-2} + \beta_3 r_t + (\beta_1\beta_5 + \beta_6)T_{t-1} + \beta_1\beta_6 T_{t-2} + \beta_5 T_t. \end{aligned} \tag{3.42}$$

The β coefficients are combinations of the α coefficients, and the contemporaneous terms in the above equation are replaced by assuming partial reduced form equations for r_t and T_t, respectively, as follows:

$$r_t = C_1(L)r_{t-1} + C_2(L)GDP_{t-1} + C_3(L)INF_{t-1} \tag{3.43}$$

$$T_t = D_1(L)T_{t-1} + D_2(L)GDP_{t-1} + D_3(L)PO_{t-1} + D_4(L)E_{t-1} \tag{3.44}$$

where PO is the log of the oil price and INF is the rate of inflation which could include both UK and German inflation or the differential and GDP is the log of real GDP. C_i and D_i are polynomial lag operators.

The terms r_t and T_t my thus be eliminated and collection of terms yields:

$$\begin{aligned} E^e_{t+1} &= \beta_0 + (\beta_1^{\,2} + \beta_2 + \beta_5 D_4(L))E_{t-1} + \beta_1\beta_2 E_{t-2} + (\beta_1\beta_2 + \beta_4 + \beta_3 C_1(L))r_{t-1} \\ &\quad + \beta_1\beta_4 r_{t-2} + (\beta_1\beta_5 + \beta_6 + \beta_5 D_1(L))T_{t-1} + \beta_1\beta_6 T_{t-2} + \beta_3 C_2(L)GDP_{t-1} \\ &\quad + C_3(L)INF_{t-1} + \beta_5(D_2(L)GDP_{T-1} + D_3(L)PO_{t-1}). \end{aligned} \tag{3.45}$$

Equation (3.45) is regarded as the basic partial reduced form rule which agents use to form their expectations. The above was simplified by dropping any lagged terms greater than t-3 by introducing a stochastic constant (*op. cit.*:11).

The basic structure was then used in a specification search to produce the following equation for expectations formation:

$$E^e_{t+1} = \gamma_{0t} + \gamma_{1t}E_{t-1} + \gamma_{2t}r_{t-1} + \gamma_{3t}INF^{UK}_{t-1} + \gamma_{4t}T^{UK}_{t-1} + \gamma_{5t}PO_{t-1}. \tag{3.46}$$

Note that all lagged information is dated t−1 when the equation is used to forecast E_{t+1}, i.e. the information set does not include current information. The parameters γ are restricted forms of the β coefficients. Hall and Garratt (*op. cit.*:11) note that their derivation of the expectations equation is by no means exclusive. In principle, formation of expectations could be the result of information from anywhere in the model, provided that it is relevant.

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The time-varying parameters are then assumed to be generated by the following process:

$$\gamma_{it} = \gamma_{it-1} + \varepsilon_{it} \ . \tag{3.47}$$

Equation (3.46) may be regarded as the standard measurement equation and the set of equations (3.47) as the state equations. The model has thus been formulated in state-space form and the Kalman filter can be applied to estimate the time-varying parameters conditional on the variance of the error terms of (3.46) and the covariance matrix of (3.47), which is assumed to be diagonal. The estimation reported (*op. cit.*:14-18) suggests that the most important determinants of the forward exchange rate are real interest rate differentials and the previous period's exchange rate. All parameter values displayed a reasonable degree of time variance, with the main movements in the coefficients on the real interest rate differential and the ratio of exports to imports for the UK, reflecting a faster rate of learning on these variables.

Another example where the Kalman filter was employed to estimate the time-varying parameter rule of expectations is a study conducted by Barrel *et al.* (1994:173). In this study, the boundedly rational learning approach was applied to wage behaviour in three countries, namely the UK, France and Italy. A time-varying parameter model for forecasting prices was first estimated. This defined the information set of the economic agents. The expectations rule together with the structural equations in which the expectations are embedded were then incorporated into the global econometric model (GEM), developed by the National Institute of Economic and Social Research (NIESR) in the UK and jointly maintained by the Institute and the London Business School.

Barrel *et al.* (*op. cit.*:174) in deriving the time-varying rule for price expectations, noted that by definition, it did not follow from a tightly formulated theory. The selection was however also not completely *ad hoc* and variables were selected to capture important endogenous linkages operating in the model.

The dependent variable for the price expectations equation in each of the three countries is the change in the log of consumer price inflation one period ahead. The information set includes the change of the log of the home country inflation, lagged by one or two periods,

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a short-term interest rate lagged by one period, the change of the log of capacity utilisation, lagged by one period, and the change of the log of the relevant spot nominal exchange rate, with respect to the Deutschmark. The only exception is France, where German inflation was also included.

Barrel *et al.* (*op. cit.*:175) first report an OLS estimation of the expectations rule, proving that in all three cases price expectations are autoregressive with lagged home country inflation the most important variable. The other variables are for the most part insignificant but are retained for the reason that they might play an important role in capturing endogenous linkages in the model. The estimates of time-varying parameters reported demonstrate a reasonable degree of variation over the period. The hyperparameters associated with the autoregressive terms imply that learning will occur rapidly with respect to changes in these variables and, by contrast, learning with respect to the other variables will be slow.

Finally, Barrel *et al.* (*op. cit.*:183) report on the outcomes of a set of simulations where the learning mechanism is in operation. These are then compared with those under rational expectations and a fixed parameter adaptive expectations mechanism. The simulations are the realignment of the franc, lira and pound within the ERM, and an oil price shock. The authors conclude from the results of these simulations that filter-based learning models caused prices to rise more sharply than they do under model-consistent or strongly rational expectations. This suggests that policy analysis within models relying only on model-consistent expectations, can be seriously misleading.

3.7 CONCLUSION

In this chapter, the theoretical development on the formation of expectations by economic agents was surveyed, starting with the adaptive expectations hypothesis, rational expectations (also called model-consistent expectations), and, most recently, boundedly rational learning. Practical problems with the implementation of the adaptive expectations hypothesis and the rational expectations hypothesis in large-scale macro models have been highlighted. The process of learning has been proposed as an alternative where the

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possibility that economic agents can make consistent prediction errors, even in the very long run, is ruled out. Boundedly rational learning seems to be an appealing alternative; full information on the part of the economic agent is not required, but it is accepted that the intelligent agent, knowing the structure of the expectations model, will assimilate new information as time progresses and adjust the parameters of the model accordingly.

An exposition of the state-space representation of a system and the Kalman filter as time-varying parameter estimation technique were presented. A brief account of a number of international studies in this regard concludes this chapter. Chapter 5 describes the application of the Kalman filter to a price expectations rule for expectations formation by South African consumers, as well as the incorporation of the price expectations variable into a set of behavioural equations.

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CHAPTER 4

ESTIMATION OF BEHAVIOURAL EQUATIONS: COINTEGRATION ANALYSIS IN ECONOMETRIC MODELLING

Applied econometrics cannot be done mechanically: it needs understanding, intuition and skill. (Cuthbertson et al. 1992: v)

4.1 INTRODUCTION

This chapter provides a brief overview of the relevant econometrics literature for the estimation of dynamic models. Aspects like non-stationarity and the problem of spurious regressions, integrating and cointegrating properties of the data, as well as the estimation of single equations and multivariate systems using cointegration techniques will be covered. The Engle-Granger framework will be taken as point of departure, followed by the Engle-Yoo extension of the basic model, and finally the Johansen approach. Links between the short and the long run will be explored, the concept of an error correction model (ECM) will be discussed as well as the role of diagnostic testing and dynamic simulation in econometric modelling.

The Johansen technique has been employed in this study for the empirical estimation of the long-run equilibrium relationships in the behavioural equations that are reported in Chapter 5, namely a learning model for private consumption expenditure in South Africa. The single equation techniques are reported merely to emphasise the limitations of employing these techniques as opposed to a multivariate estimation technique like the Johansen approach.

4.2 THE PROBLEM OF NON-STATIONARITY AND SPURIOUS REGRESSIONS

Early reference to the problem of spurious correlations is made by Yule in a 1926 paper, noting that in analysing time-series data, it is possible to observe spurious correlations.

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Darnell (1994:378) defines a spurious correlation as an observed sample correlation between series which, though appearing to be statistically significant, is a reflection of a common trend rather than a reflection of any underlying association. In the context of regressions using time-series data, it is possible to regress a variable y_t on another variable x_t, obtain a high R^2 statistic, large computed t-values and a very low Durbin-Watson statistic. This combination of statistics is a classic symptom of a spurious regression: one which has the superficial appearance of a good fit, especially when the model has been re-estimated using some adjustment for autocorrelation, such as Cochrane-Orcutt.

It is generally the presence of trends in the underlying data generation process that leads to the spurious identification of relationships between variables, simply because the trends in the data series generate statistical association, rather than meaningful causal relationships. The fundamental problem with regressing non-stationary series is that t- and F-tests no longer have the standard distributions associated with stationary series. With non-stationary series, there is a tendency to reject the null hypothesis of no association between individual variables, as well as for all regressors jointly. Moreover, this tendency increases with sample size. Ordinary least squares (OLS) estimation therefore presents problematic inferences when the data set contains non-stationary data. Detrending the data series would solve the problem only for trend stationary data and simply differencing the data to remove the non-stationary stochastic trend would not be appropriate, since the use of differenced variables, although avoiding the spurious regression problem, will also remove any long-run information (Banarjee *et al.* 1993, 82-84).

When considering long-run relationships, it becomes necessary to consider the underlying properties of the process that generates time series variables, i.e. a distinction must be made between stationary and non-stationary variables since, as suggested above, failure to do so can lead to spurious regression. The next section reports proposed techniques to establish integrating and cointegrating properties of the data.

4.3 INTEGRATING AND COINTEGRATING PROPERTIES OF THE DATA

Stationarity is a key concept in cointegration analysis. A definition for stationarity will be given in this section, followed by an exposition of a testing procedure to establish the order of integration of an individual time series. This will be followed by an introduction to techniques to apply when data series are non-stationary, involving the concept of cointegration.

4.3.1 Stationarity

A (weakly) stationary variable may be defined as a series with a constant mean and constant, finite variance[5]. Thus, a time series (x_t) is stationary if its mean, $E(x_t)$, is independent of t, and its variance, $E[x_t-E(x_t)]^2$, is bounded by some finite number and does not vary systematically with time. A non-stationary series on the other hand, will have a time-varying mean, or variance, so that any reference to the mean or variance should include reference to the particular time period under consideration.

Whether a variable is stationary depends on whether is has a unit root. Comparing stationary and non-stationary variables is also related to the different types of time trends that can be found in variables. Non-stationary variables contain stochastic, or random, trends, while stationary variables contain deterministic, or fixed, trends. It is data with random trends which often leads to spurious correlations. Next, a testing procedure for the presence of unit roots in data series will be presented. When data series are found to contain one or more unit roots, i.e. series are non-stationary, it is important to establish cointegration between variables, in order to infer a non-spurious causal long-run relationship between the series under consideration, as is discussed in section 4.3.3.

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[5] Strict stationarity demands not only that the mean and variance of the series are independent of time, but also that all other higher moments are independent of time. In the cointegration framework, the requirement of weak stationarity generally suffices.

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4.3.2 Unit roots and order of integration

As already noted above, non-stationarity implies the presence of a unit root in the time series under consideration. Testing for a unit root can be used to establish the order of integration.

If a series must be differenced d times before it becomes stationary, then it is said to be integrated of order d, denoted I(d). Thus a series x_t is I(d) if x_t is non-stationary but $\Delta^d x_t$ is stationary. That means that the series has d unit roots (solutions) associated with its ARIMA(p,d,q) representation, $(1-L)^d \phi(L)x_t = \theta(L)e_t$, for some p and q with $\phi(L)$ and $\theta(L)$ polynomials in the lag operator (L) and e_t a stationary process (Cuthbertson *et al.* 1992:130).

In order to test for the presence of unit roots, and hence for the degree of integration of individual series, a number of statistical tests may be used. The most popular of these are based on the class of tests developed by Dickey and Fuller (1979, 1981). Surveys by Diebold and Nerlove (1988); Pagan and Wickens (1989); Dolado *et al.* (1990); and Muscatelli and Hurn (1995)), amongst others, provide accounts of these tests.

The testing strategy followed in this study to determine the order of integration of individual time series is the one suggested by Dolado *et al.* (1990), employing the augmented Dickey-Fuller (ADF) test. A practical application of the Dolado testing strategy is provided by Sturm and De Haan (1995:69).

Dickey and Fuller (1981) test the null hypothesis of non-stationarity versus stationarity, suggesting ordinary least squares estimation of

$$\Delta Y_t = \eta_0 + \eta_1 \text{Trend} + \eta_2 Y_{t-1} + \sum_{i=1}^{m} \eta_{2+i} \Delta Y_{t-i} + \varepsilon_t \qquad (4.1)$$

where Y_t is the series being tested, m is the number of lags in the testing equation and ε_t is the residual. Lagged values of the dependent variable are included to take account of any serial correlation, and m is chosen so as to ensure that the residuals are white noise.

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Dolado *et al.* suggest commencing the test with the specification of (4.1). The test is implemented through the usual t-statistic of $\hat{\eta}_2$, denoted as τ_τ. Under the null hypothesis, τ_τ will not follow the standard t-distribution; adjusted values as computed by MacKinnon (1990) have to be used for evaluation. If τ_τ is significant, the null of non-stationarity is rejected, and the series is stationary. This then concludes the test.

If τ_τ is insignificant however, the joint null hypothesis that $\eta_1=\eta_2=0$ using the F-statistic, denoted as Φ_3, is tested. The relevant critical values from Dickey and Fuller (1981) are used. If Φ_3 is significant, the test for a unit root must be conducted again, in this instance using the critical values of the standard t-distribution.

If the trend is not significant in the maintained model, the next step would be to estimate equation (4.1) without a trend ($\eta_1\equiv0$). Once again the unit root test must be conducted, now denoting the t-statistic of $\hat{\eta}_2$ as τ_μ and using the relevant critical values from MacKinnon. If the null hypothesis is rejected, there is again no need to continue.

If the null is not rejected, the joint null hypothesis $\eta_0=\eta_2=0$ with use of the F-statistic, denoted as Φ_1, is tested, employing the critical values reported by Dickey and Fuller. Again, if it is significant, the unit root test must be conducted, using the standardised normal distribution.

If not, the constant must be removed from the testing equation as well ($\eta_0\equiv\eta_1\equiv0$). The new statistic is called τ. MacKinnon also reports relevant critical values for this t-statistic. The last step is to examine whether the null hypothesis is rejected or not, i.e. whether the series is stationary or not.

The number of lags used in the estimated equations may be determined as suggested by Perron (1989). Perron suggested starting with eight lags. If the last lag is insignificant at a 10 per cent level (using the standard normal distribution), it is omitted. Next, seven lags are included. Again it is tested whether the last lag is significant (or there are no lags left, in which case the test is called the Dickey-Fuller (DF) test). This large significance level is taken because, as Perron (1989:1384) pointed out, 'including too many regressors of lagged first-differences does not affect the size of the test but only decreases its power. Including too few lags may have a substantial effect on the size of the test'. Furthermore, Molinas (1986) noticed that 'a rather large number of lags has to be taken in the ADF test in order to capture the essential dynamics of the residuals'. Alternatively, the lag truncation parameter can be selected in order to minimise Akaike's information criterion (AIC) and to obtain stationarity of the residuals.

The result of applying the above test procedure to data series employed in this study is reported in Chapter 5, section 5.3.2. Tables 5.2, 5.3 and 5.4 contain values of test statistics for data in levels and first and second differenced form where relevant.

In cases where the ADF test proves to be inconclusive, other tools may be relied upon, for example graphical representations of the data in levels and first and second differenced form. It is also common to investigate whether a series is stationary by visual inspection of the graph of the sample autocorrelations against time, known as the *correlogram*, calculated by dividing the sample autocovariances by the sample variance. Alternative tests may also be considered. Phillips and Perron (1988) and Perron (1988) for example have suggested a non-parametric procedure in order to take account of the serial correlation in the model. Their procedure yields a number of 'modified' DF-type statistics, also known as Z-statistics. The advantage of these modified Z-statistics is that asymptotically, they eliminate the nuisance parameters that are present in the DF-statistics when the errors are not independently and identically distributed (IID). However, the main drawback in computing these Z-statistics is that the researcher has to decide *a priori* on the number of residual autocovariances which are to be used in implementing the corrections suggested by Phillips and Perron (Muscatelli and Hurn 1995:175).

4.3.3 The concept of cointegration

According to Harris (1995:6), the economic interpretation of cointegration states that if two (or more) series are linked to form an equilibrium relationship spanning the long run, then even though the series themselves may contain stochastic trends and thus be non-stationary, they will nevertheless move closely together over time and the difference between them will be stable (i.e. stationary).

The formal definition of cointegration of two variables, developed by Engle and Granger (1987:253) is as follows: time series x_t and y_t are said to be *cointegrated of order d,b* where $d \geq b \geq 0$, written as

$$x_t, y_t \sim CI(d,b),$$

if

(i) Both series are integrated of order d,

(ii) There exists a linear combination of these variables, say $\alpha_1 x_t + \alpha_2 y_t$, which is integrated of order d–b.

The vector $[\alpha_1, \alpha_2]$ is called the *cointegrating vector.*

A straightforward generalisation of the above definition for the case of n variables is as follows: if x_t denotes an $(n\times1)$ vector of series $x_{1t}, x_{2t}, \ldots, x_{nt}$ and

(i) each x_{it} is I(d),

(ii) there exists an $(n\times1)$ vector α such that $x'_t \alpha \sim I(d-b)$, then

$x'_t \alpha \sim CI(d, b)$.

Condition (i) in the above definition can be relaxed, as follows: if a linear combination of any two time series y_t and x_t is formed and each is integrated of a different order, then the resulting series will be integrated at the higher of the two orders of integration. Thus if $y_t \sim I(1)$ and $x_t \sim I(0)$ (or $y_t \sim I(0)$ and $x_t \sim I(1)$), these two series cannot possibly be cointegrated as the I(0) series has a constant mean while the I(1) series tends to drift over time. Consequently the error $(y_t - \alpha x_t \sim I(1))$ between them would not be stable over time. It is however possible to obtain cointegration between 3 or more series even if all series are not integrated of the same order. Pagan and Wickens (1989) pointed out that, in this instance, a subset of the higher-order series must cointegrate to the order of the lower-order series. If $y_t \sim I(1)$, $x_t \sim I(2)$ and $z_t \sim I(2)$, and a cointegration relationship between x_t and z_t is found, such that $v_t\ (=x_t - \lambda z_t) \sim I(1)$, then this result can potentially cointegrate with y_t, to obtain $w_t\ (=y_t - \gamma v_t) \sim I(0)$.

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Furthermore, if there are $n>2$ variables in the model, there may be more than one cointegrating vector. It is possible for up to $n-1$ linearly independent cointegration vectors to exist, which has implications for testing and estimating cointegration relationships.

4.4 MODELLING COINTEGRATED SERIES THROUGH ERROR CORRECTION MODELS

This section will describe how cointegrated non-stationary variables can be used to formulate and estimate a model with an error correction mechanism. The fact that variables are cointegrated implies that there is some adjustment process which prevents the errors in the long-run relationship from becoming increasingly larger. Engle and Granger (1987:255-258) have shown that for any set of variables that are cointegrated of order 1,1, that is CI(1,1), there exists a valid error correction representation of the data. The *Granger Representation Theorem* formalises this theoretical connection between cointegration and error correction.

Different approaches towards establishing cointegration between variables and estimating the long-run relationship, and the subsequent specification of an error correction model representing the short-run adjustment towards equilibrium, will be discussed below. The discussion will commence with single equation cointegration techniques, namely the Engle-Granger (1987) approach and the Engle-Yoo (1989) extension of this procedure, followed by a multivariate cointegration technique known as the Johansen (1988, 1989) approach.

4.4.1 Engle-Granger estimation

The first approach was originally proposed by Engle and Granger (1987). Consider the long-run relation for the bivariate case (the extension to the multivariate case is direct) with the form:

$$y_t = \beta x_t + u_t, \tag{4.2}$$

where both y_t and x_t are I(1), and with β an unknown coefficient. The Engle-Granger two-step procedure would entail the following steps: first, estimate equation (4.2) by ordinary

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least squares and test for stationarity of the residuals. This entails testing whether $u_t \sim I(1)$ against the alternative that $u_t \sim I(0)$. Second, if the null hypothesis of no cointegration can be rejected, equation (4.3) below can be estimated, replacing β by its previously computed OLS estimate, $\hat{\beta}$:

$$\Delta y_t = \phi(L)\Delta y_{t-1} + \Theta(L)\Delta x_t + \alpha(y_{t-1} - \hat{\beta}x_{t-1}) + \varepsilon_t, \tag{4.3}$$

where ε_t is an error term and α is negative. Δy_t, Δx_t and $(y_{t-1} - \hat{\beta}x_{t-1})$ are all I(0) and consequently, provided that the model is properly specified, ε_t is also I(0). Equation (4.3) represents the *error correction model*, containing the long-run cointegration relationship in the form of the lagged residual obtained from the estimated long-run cointegration equation (called the *error correction mechanism*) as well as the short-run dynamic structure allowing for adjustment towards equilibrium. Engle and Granger (*op. cit.*:254) motivate the inclusion of the error correction mechanism as follows: "The idea is simply that a proportion of the disequilibrium from one period is corrected in the next period". The slope coefficient α in equation (4.3) indicates the speed of adjustment towards equilibrium.

Testing for cointegration in the first step of the procedure entails the following. As with univariate unit root tests, the unit root in the residual (implying no cointegration between the variables) is based on a t-test with a non-normal distribution. However, unless the value of β is already known (not estimated by equation (4.2)), it is also not possible to use the standard Dickey-Fuller tables of critical values. There are two major reasons for this. First, because of the way it is constructed, the least square estimator chooses the parameter vector which minimises residual variance, even if the variables are not cointegrated, causing the error, u_t, to appear as stationary as possible. Thus the standard DF distribution would tend to over-reject the null. Second, the distribution of the test statistic under the null is affected by the number of regressors included in equation (4.2). MacKinnon (1991:273-75) uses response surface analysis to obtain approximate finite sample critical values for the conventional ADF-test on residuals of the long-run relationship, and must be used in this instance (Harris 1995:54).

Engle and Granger (1987) also suggest the use of a second test for cointegration, namely the cointegrating regression Durbin-Watson (CRDW) test, proposed by Sargan and

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Bhargava (1983). This test is computed from the cointegrating regression. This statistic should be compared with the critical values from either Table II or III (*op. cit.*:269-70); under the null of non-cointegration, CRDW should be close to zero and so the null is rejected if the statistic exceeds the critical value. According to Engle and Granger, this statistic should however be used only for a quick approximate result.

The Engle-Granger two-step approach has the advantage of relative simplicity. Where there is a unique cointegrating vector, it allows for the use of the superconsistency property of OLS to obtain consistent estimates of the cointegrating vector. However, there are a number of critical limitations to the technique, which severely limits its usefulness and applicability. One important limitation pertains to the fact that inferences cannot be drawn using standard t-statistics about the significance of the parameters of the static long-run model, since the distribution of the estimators of the cointegrating vector is generally non-normal. Furthermore, while the static regression gives consistent estimates of the cointegrating vector, these estimates are not fully efficient.

Engle and Yoo (1989) proposed a third step to the Engle and Granger two-step estimation to overcome the above-mentioned problems.

4.4.2 Engle-Yoo estimation

The Engle-Yoo estimation procedure only provides an extension (third step) to the two-step Engle-Granger approach. The third step provides a correction of the parameter estimates of the first stage static regression, which provides a set of standard errors allowing the valid calculation of standard t-tests.

The third stage simply consists of a further regression of the conditioning variables from the static regression multiplied by minus the error correction parameter, regressed on the errors from the second-stage error correction model. The coefficients from this model are the corrections to the parameter estimates while their standard errors are the relevant standard errors for the inference (Cuthbertson *et al.*:141).

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The three steps are then: first, estimate the standard cointegrating regression of the form:

$$y_t = \beta x_t + u_t, \tag{4.4}$$

where u_t is the OLS residual to give first-stage estimates for β, namely β^1. The second-stage dynamic model has to be estimated next, using the residuals from the cointegrating regression to impose the long-run constraint:

$$\Delta y_t = \phi(L)\Delta y_{t-1} + \Theta(L)\Delta x_t + \alpha u_{t-1} + \varepsilon_t. \tag{4.5}$$

The third stage consists of the regression

$$\varepsilon_t = \delta(-\alpha)x_t + v_t. \tag{4.6}$$

The correction of the first stage estimates then is

$$\beta^3 = \beta^1 + \delta \tag{4.7}$$

and the correct standard errors for β^3 are given by the standard errors for δ in the third-stage regression.

4.4.3 Problems associated with the single equation approach

A number of problems related to a single equation approach towards modelling the static long-run equilibrium relationship and the dynamic short-run properties of the underlying data generating process have been pointed out in the preceding sections. More limitations will subsequently be highlighted, followed by a strategy known as the Johansen maximum likelihood estimation procedure to address these deficiencies.

One important limitation that has been addressed pertains to the fact that standard t-statistics cannot be used to draw inferences about the significance of the parameters of the static long-run model, due to the non-normal distribution of the estimators of the cointegrating vector.

Perhaps the most severe limitation of the single equation model follows from the possibility of more than one cointegrating vector present in the data when the cointegration regression contains more than two variables. Including n variables in the equation may yield (n–1) equilibrium relationships governing the joint evolution of the variables, and hence there may exist (n–1) cointegrating relationships in the data. Single equation techniques assume that only one cointegrating vector exists in the data. Should this not be the case, the consequence will be inefficiency in the estimation, since the cointegrating vector will be a linear combination of all cointegrating relationships present in the data.

Furthermore, the Engle-Granger approach effectively ignores the short-run dynamics when estimating the long-run equilibrium relationship, or the cointegrating vector. Estimating the model by means of only including the long-run equilibrium relationship (the data in levels) effectively shifts the short-run dynamics to the error term, subjecting the residuals to serial correlation.

Yet another problem with residual based tests worth noting, pertains to the endo-exogenous division of variables. Charemza and Deadman (1997:151) point out that the distinction between the types of variables appearing in a multiple equation system is in stark contrast with single equation structural modelling. Usually, what is on the left-hand side of a single equation structural model is simply treated as endogenous and what is on the right-hand side as exogenous. Charemza and Deadman (*op. cit.*:150) refer to a model of aggregate income, Y_t, and aggregate consumption, C_t, to demonstrate that a rise in income will lead to a rise in consumption. However due to the income identity, it is impossible to change the value of C_t, without influencing Y_t. Both variables would thus be regarded as endogenous variables and be described as jointly dependent variables. This type of observed simultaneity between variables is disregarded in a single equation approach.

Multivariate estimation techniques are able to address the above problems by detecting all possible long-run cointegration relationships present in the data, accommodating short-run properties when estimating the long-run relationships and by taking simultaneity between variables into account. Harris (1995:61) demonstrates that information is lost unless endogenous variables appear on the left-hand side of the estimated equations in a multivariate model, except for the case where all the variables in the cointegration vector are weakly exogenous. The Johansen approach, a multivariate estimation technique, namely, is addressed in the next section.

4.4.4 The Johansen approach

What has come to be termed the Johansen approach emerged from the work of Johansen (1988, 1989) and Johansen and Juselius (1990). One means of understanding the impulse of the approach comes from a contribution of Sims (1980), who noted that in macroeconomic systems many variables are likely to be interdependent, rendering exogeneity of variables rare. Sims's suggestion for dealing with this problem was to use vector autoregressive (VAR) estimation.

An exposition of the Johansen approach will be presented next. The method begins by expressing the data generation process of a vector z_t of n potentially endogenous variables, as an unrestricted vector autoregression (VAR) in the levels of the variables, involving up to k lags of z_t:

$$z_t = A_1 z_{t-1} + \ldots + A_k z_{t-k} + u_t \tag{4.8}$$

where z_t is (n×1) and each of the A_i is an (n×n) matrix of parameters. The system of equations (4.8) can be reparameterised in ECM form:

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + u_t \tag{4.9}$$

where $\Gamma_i = -(I-A_1-\ldots-A_i)$, $i=1,\ldots,k-1$ and $\Pi = -(I-A_1-\ldots-A_k)$. This way of specifying the system contains information on both the short and long-run adjustment to changes in z_t, via the estimation of $\hat{\Gamma}_i$ and $\hat{\Pi}$. Thus, Π defines the long-run levels solution to equation (4.8). It can be shown that $\Pi = \alpha\beta'$, where α represents the speed of adjustment to equilibrium, while β is a matrix of long-run coefficients such that the term $\beta' z_{t-k}$ embedded in (4.9) represents up to (n−1) cointegration relationships in the multivariate model which ensures that the z_t converge to their steady-state solutions. Assuming that z_t is a vector of non-stationary I(1) variables, then all the terms in (4.9) which involve Δz_{t-i} are I(0), while Πz_{t-k} must also be stationary for $u_t \sim I(0)$ to be white noise. Stationarity of the term Πz_{t-k},

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i.e. a set of I(1) variables which form linear combinations that are I(0), implies that it contains the long-run *cointegrating relationships between the variables in levels*.

The Johansen technique therefore reveals the exact number of cointegrating relationships present between variables in the model, as well as the nature of these relationships. The way in which the information is extracted involves a method known as *reduced rank regression*. Rewriting equation (4.9) yields:

$$\Delta z_t + \alpha\beta' z_{t-k} = \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1}\Delta z_{t-k+1} + u_t \,. \tag{4.10}$$

It is possible to correct for short-run dynamics (i.e. eliminate their effect) by regressing Δz_t and z_{t-k} separately on the right-hand side of (4.10). That is, the vectors R_{0t} and R_{kt} are obtained from:

$$\Delta z_t = P_1 \Delta z_{t-1} + \ldots + P_{k-1}\Delta z_{t-k+1} + R_{0t} \tag{4.11}$$

$$z_{t-k} = T_1 \Delta z_{t-1} + \ldots + T_{k-1}\Delta z_{t-k+1} + R_{kt} \,. \tag{4.12}$$

The above can then be used to form residual (product moment) matrices:

$$S_{ij} = T^{-1}\sum_{t=1}^{T} R_{it} R'_{jt} \qquad i,j=0,k. \tag{4.13}$$

The maximum likelihood estimates of β is obtained as the eigenvectors corresponding to the r highest eigenvalues from solving the equation

$$|\, \lambda S_{kk} - S_{k0} S_{00}^{-1} S_{0k} \,| = 0 \tag{4.14}$$

which gives the n eigenvalues $\hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_n$ and the corresponding eigenvectors $\hat{V} = (\hat{v}_1, \ldots, \hat{v}_n)$. The r elements in $\hat{V}$ which determine the linear combinations of stationary relationships can be denoted $\hat{\beta} = (\hat{v}_1, \ldots, \hat{v}_r)$, that is, these are the cointegration vectors. Johansen (1992b) points out that a test for the rank of Π equal to 1 (r=1), for example, is essentially a test that $\hat{\lambda}_2 = \hat{\lambda}_3 = \ldots = \hat{\lambda}_n = 0$, whereas $\hat{\lambda}_1 > 0$.

Thus, testing for cointegration amounts to a consideration of the rank of Π, that is, finding the number of r linearly independent columns in Π. If Π has full rank (there are r=n

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linearly independent columns) then the variables z_t are I(0), while if the rank of Π is zero, there are no cointegration relationships. The interesting case is when Π has reduced rank, i.e. when there are $r \leq (n-1)$ cointegration vectors present.

To test the null hypothesis that there are at most r cointegration vectors amounts to:

$$H_0: \lambda_i = 0 \qquad i = r+1, \ldots, n \tag{4.15}$$

versus H_A: $\lambda_i \neq 0$, where only the first r eigenvalues are non-zero. The restriction can be imposed for different values of r. The log of the maximised likelihood function for the restricted model is then compared to the log of the maximised likelihood function of the unrestricted model and a standard likelihood ratio test is computed (although with a non-standard distribution). That is, it is possible to test the null hypothesis using what has become known as the *trace* statistic:

$$\lambda_{trace} = -2\log(Q) = -T\sum_{i=r+1}^{n}\log(1-\hat{\lambda}_i) \qquad r = 0,1,2,\ldots,n\text{-}2,n\text{-}1. \tag{4.16}$$

where Q=(restricted maximum likelihood ÷ unrestricted maximum likelihood). Asymptotic critical values are provided in Osterwald-Lenum (1992). Harris (1995:88) points out that when only a small sample of observations on z_t is available, there are likely to be problems with the power and size properties of the above test when using asymptotic critical values.

Another test of the significance of the largest λ_r, is the so-called maximal-eigenvalue or λ_{max} statistic:

$$\lambda_{max} = -T\log(1-\hat{\lambda}_{r+1}) \qquad r = 0,1,2,\ldots,n\text{-}2,n\text{-}1.$$

This tests that there are r cointegration vectors present against the alternative that r+1 such vectors exist.

This section is concluded by presenting a number of distinct steps in the practical estimation process as set out by Harris (1995:76):

(i) Test the order of integration of each variable entering the multivariate model.

(ii) Select the correct lag length, k, of the vector autoregressive (VAR) model, to ensure that the (vector) error correction model has Gaussian errors. Model selection criteria as the Akaike information criterion (AIC) or the Schwarz Bayesian criterion may be used, as well as F tests for the hypothesis that the i-period lag is zero.

(iii) Determine whether the system needs to be conditioned on any I(0) variables, including dummy variables.

(iv) Test for the reduced rank of the system. The Johansen cointegration test may be used to test whether $\Pi(=\alpha\beta')$ has reduced rank; and to determine the value of r, with $r \leq (n-1)$ the number of cointegration vectors present in β.

(v) Determine whether the system is to be estimated with deterministic variables (constant and trend) or not. There are three possible options: (1) the VAR may be specified without any constant term; (2) the VAR may have a restricted constant term which appears only as a part of the cointegrating vectors so that the ECM from (4.9) contains any constants within the term Πz_{t-k} only; (3) the VAR may have an unrestricted constant (Cuthbertson *et al.* 1995:148).

(vi) Test for weak exogeneity. Weakly exogenous variables may be removed from the left-hand side of the equation, while remaining in the long-run model.

(vii) Test the linear hypothesis on cointegrating relationships as well as for unique cointegrating vectors. This entails imposing restrictions motivated by economic arguments (e.g. that some of the β_{ij}s are zero, or that homogeneity restrictions are needed such as $\beta_{1j}=-\beta_{2j}$) and then testing whether the columns of β are identified.

(viii) Impose joint tests of restrictions on the α loading matrix, (i.e. the speed-of-adjustment parameters) and the β cointegrating vector.

4.5 THE ROLE OF DIAGNOSTIC TESTING AND DYNAMIC SIMULATION IN ECONOMETRIC MODELLING

According to Hendry (1980:403), the three golden rules of econometrics are test, test, test. Diagnostic checking is therefore a very important part of the whole process of model selection – "Rigorously tested models, which adequately describe the available data, encompass previous findings and were derived from well based theories would greatly

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enhance any claim to be scientific." (*op. cit.*:403). Thus, whether the error correction model has been derived using single equation estimation techniques, or a more sophisticated multivariate estimation technique, in order to assess the validity of the model, it must be subjected to a battery of diagnostic tests.

Diagnostic tests, or mis-specification tests, are designed to test the adequacy of the specification of a regression equation. Darnell (1994:93) summarises the possible ways in which an equation might be mis-specified: (i) the set of regressors may be incomplete – some variables may have been omitted; (ii) the parameter vector may not be constant; (iii) the functional form may be incorrect; (iv) one or more of the regressors may not be exogenous; (v) the error term may be autocorrelated; (vi) the error term may be heteroscedastic; and (vii) the error term may be non-normally distributed.

Tests developed to test for mis-specification include the following: tests of omitted variables may be carried out as a linear hypothesis test – additional variables are included in the model and their exclusion is tested as an F-test; constancy of the parameter vector may be examined using a Chow test or a test of predictive failure or may be examined using recursive residuals; the functional form may be examined using Ramsey's RESET test or a Box-Cox test; exogeneity may be examined by Hausman's test; autocorrelation and heteroscedasticity may be examined by a number of tests, including the Lagrange multiplier test and the Box-Pierce and Lung-Box tests for serial correlation, the Breuch-Pagan test for heteroscedasticity and Engle's test for autoregressive conditional heteroscedasticity; and the normality assumption may be tested by the Bera-Jarque statistic and by testing for outliers. The detail of these tests will not be discussed here, but can been found in the literature (amongst others Godfrey 1988; Darnell 1993; Cuthbertson *et al.* 1995; Greene 1997).

Cuthbertson *et al.* (1995:106) point out that often an ECM is constructed so that it passes a set of diagnostic tests. Tests for parameter constancy and encompassing tests then become of increasing importance in testing competing models.

Dynamic, in-sample simulation and deterministic analysis of the response characteristics of the model, testing whether short and long-run response characteristics correspond to

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theoretical priors and long-run equilibrium properties of the data often prove helpful in assessing the validity of the model. The process would consist of conducting a dynamic baseline forecast for each stochastic equation. An exogenous shock is applied to the system and the adjustment path towards a new equilibrium is then determined. Dynamic out-of-sample simulation can be useful in establishing the forecasting performance of the model.

4.6 CONCLUSION

This chapter highlighted the problem of spurious regression when modelling non-stationary data series and reported econometrics literature addressing this issue. Single equation estimation techniques were discussed to point out the limitations thereof as opposed to a multivariate estimation technique like the Johansen approach. The Johansen technique however poses it own demands, and the discussion included the presentation of a number of distinct steps to consider when applying this technique. Finally, the importance of diagnostic testing was re-emphasised.

CHAPTER 5

A LEARNING MODEL OF PRIVATE CONSUMPTION EXPENDITURE IN SOUTH AFRICA

Most economists accept that beliefs about the future are an important determinant of behaviour today. (Begg et al. 1991:568)

5.1 INTRODUCTION

In this chapter, the hypothesis that South African consumers are forward-looking with respect to prices when making consumption expenditure decisions, is tested. It is assumed that consumers learn using a Kalman filter-based (boundedly rational learning) process for updating their expectations conditional on prior errors made when forecasting the price level.

The first stage of implementing the boundedly rational learning approach would involve the estimation of the time-varying mechanism, which represents economic agents using incomplete historical information to form expectations. In the second stage, the expectations formation mechanism is incorporated into the behavioural equations. The theoretical specification of the private consumption expenditure function (or categories of consumption expenditure) would therefore include a price expectations variable, namely the expected one-period-ahead consumer price level.

Two sets of empirical results will be presented in this chapter: first, the time-varying coefficients of the price expectations rule – that is the state equations of the state-space form of the price expectations rule – and second, the set of behavioural equations containing the price expectations variable. Consumption expenditure is disaggregated into the following categories: durable consumption (including durables and semi-durables), non-durable consumption and services. Empirical estimation results for total private consumption expenditure are presented, followed by durable consumption expenditure and non-durable consumption expenditure. Since stochastic estimation of total consumption expenditure is believed to be more reliable than that of the consumption expenditure on services, expenditure on services is deterministically determined as the residual of the total and the other two categories.

Behavioural equations are subjected to extensive diagnostic testing to ensure that the model is statistically well specified. Deterministic analysis of the response characteristics of the model is also conducted to ensure that short and long-run response characteristics correspond to theoretical priors and long-run equilibrium properties of the data.

5.2 THE THEORETICAL MODEL

The theoretical model for private consumption expenditure is developed in this section. Its empirical estimation results are presented in section 5.5. *A priori* expectations of elasticities of variables included in the long-run equilibrium relationships of the different categories of consumption expenditure will be pointed out, as well as expected short-run dynamic properties of the equations.

Since price expectations feature as a variable contributing towards the short-run dynamic structure of the behavioural equations, a theoretical model for the formation of price expectations by the South African consumer will also be proposed.

5.2.1 The consumption function

The theoretical specification of the behavioural equations for each of the categories of consumption, based on the forward-looking theories of consumption, will include an income variable, some wealth variable, and possibly long-term or short-term interest rates representing monetary conditions. These are according to theory (as discussed in Chapter 2), the variables to consider for the long-run equilibrium or steady-state relationship:

$$c_t = Ay_t^{\alpha}w_t^{\beta}r_t^{\gamma}e^{\varepsilon_t} \tag{5.1}$$

$$0<\alpha<1;\ 0<\beta<1;\ \alpha+\beta=1;\ \gamma<0$$

with

c_t	=	private consumption expenditure in period t
y_t	=	personal disposable income in period t
w_t	=	financial wealth stock in period t
r_t	=	a representative interest rate in period t and
ε_t	=	the stochastic disturbance term.

The explicit inclusion of both the income and the wealth variables is justified by the forward-looking theories of consumption, and further motivated by the Ball-Drake hypothesis regarding the derivation of utility from the accumulation of wealth (section 2.5.3). Both α and β are expected to be positive with $\alpha+\beta=1$.

Within an intertemporal consumption optimisation framework[6], the interest rate constitutes a trade-off between current and future consumption. A rise in the rate of return on accumulated savings increases the opportunity cost associated with current consumption and should raise the savings rate, thus lowering current consumption. On the other hand, the future income stream expected from the higher rate of return on savings may encourage current consumption (equations (2.8) and (2.9) of the permanent income hypothesis). Interest rates may therefore have a negative or a positive effect on consumption expenditure. The substitution effect is however expected to be larger than the income effect; hence a negative expected sign on interest rates.

5.2.1.1 *A priori* expectations of income elasticity and wealth elasticity

As discussed in section 1.4, South Africa is characterised by absolute poverty and an unequal distribution of both income and wealth. A substantial portion of the population possesses virtually no wealth and earns a small income, if any. More specifically, 90 per cent of the population earns less than 50 per cent of total income (Whiteford and van Seventer 1999:ii), while the unemployment rate ranges in the region of 36 per cent (Stats SA 1998). The large portion of the population constrained by very low income levels,

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[6] The first order condition characterising optimal consumption behaviour would be $u'(c_t)/E[u'(c_{t+1})] = (1+\rho)/(1+r_t)$ with ρ the rate of time preference.

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spends virtually all of its income on consumption, with very little left to be utilised for wealth accumulation. Wealth therefore plays an insignificant role in their consumption expenditure decisions. For South Africa as a whole, one would expect an income elasticity close to unity for *total* consumption expenditure, and a low wealth elasticity, while for *durable* consumption, a relatively larger wealth elasticity would be expected. For *non-durable* consumption, wealth would not be expected to be a driving factor in the long run. Wealth may however contribute towards the short-run dynamic structure of non-durable consumption expenditure.

5.2.1.2 Short-run dynamics

Other variables that were considered as explanatory of consumer behaviour include lagged consumption expenditure, i.e. expenditure patterns of the past, variables reflecting labour market sentiment, for example the employment rate, relative prices, lagged personal savings, credit leasing finance, instalment sale credit, stock market prices, etc. The expected price level, that is the one-period-ahead consumer price level forecasted by the Kalman filter, can be used to test for the effect of price expectations on consumption expenditure behaviour within a boundedly rational learning framework. Most of the above variables proved to contribute towards explaining the short-run dynamics of the system.

A priori, one would expect interest rates only to be significant in explaining durable consumption expenditure and, perhaps, total consumption expenditure. Conversely, interest rates are not expected to have a significant influence on non-durable expenditure decisions, given that the majority of South African consumers are subjected to liquidity constraints and have limited access to bank or any other form of credit as a consequence of their extremely low income levels. In addition, these consumers spend virtually their entire income on non-durables for immediate consumption at a subsistence level, ruling out a substitution effect following interest rate changes. These consumers have no savings; an interest rate increase therefore fails to affect their future income, and thus an income effect is equally improbable.

Likewise, price expectations are expected to play a significant role in explaining durable and aggregate consumption expenditure. As motivated previously, the majority of South

African consumers are trapped in a relatively rigid pattern of consumption. Liquidity constraints disqualify them from increasing current consumption, to for example hedge against expected price increases. Absolute poverty rules out further reductions in current consumption; these consumers are frequently consuming at a subsistence level. As before, substitution and income effects are effectively disabled. Price expectations are, therefore, unlikely to have a significant effect on these consumers who allocate virtually all income towards non-durable expenditure.

Variables reflecting labour market conditions, like the employment or unemployment rate, may also play a role in explaining consumption, particularly non-durable consumption. Adverse developments in the labour market often affect the unskilled workforce first. Their wages are likely to be low and mainly directed towards non-durable consumption. Fluctuations in labour market conditions will therefore be reflected by changes in non-durable expenditure patterns.

5.2.2 A model for price expectations formation

A boundedly rational learning approach towards the formation of price expectations is intuitively attractive. It is also consistent with psychology literature on learning processes, unlike the rational expectations hypothesis, which demands full information from economic agents regarding the model as well as its parameters.

The boundedly rational learning approach is based on the assumption that expectations are formed by intelligent agents who are not fully informed, but learn about their environment as time progresses. The learning model requires the specification of a time-varying parameter rule of expectations formation. In many applications, the expectations rule is derived from the reduced form of the structural equation. An application to the exchange rate sector of the London Business School model as discussed in section 3.4, serves as an example in this regard. Citing Hall and Garratt (1992b:11), "How one derives an expectations equation is by no means exclusive. In principle, formation of expectations could be the result of information anywhere in the model, if it is thought to be relevant".

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In setting up the expectations rule in this case, the application of price expectations to wage behaviour of countries in the global econometric model (GEM) (Barrel *et al.* 1994:174) is followed. The criteria used to decide on the inclusion of variables in the time-varying expectations rule, according to Barrel, do not follow from a tightly formulated theory, but inclusion of variables are also not completely *ad hoc*. Rather, variables are selected to capture important endogenous linkages in the model and also enable the price expectations equation to adjust the coefficients on existing variables in accordance with movements in variables not included in the behavioural equation.

The expectations rule may therefore be represented by the following theoretical specification:

$$\Delta cpi_{t+1} = e^{\xi_{1t}} \Delta cpi_{t-1}^{\xi_{2t}} \Delta rl_{t-1}^{\xi_{3t}} \Delta exch_{t-1}^{\xi_{4t}} e^{w_t} . \qquad (5.2)$$

The dependent variable for the price expectations equation is the change in the consumer price index one period ahead (Δcpi_{t+1}). The information set or the independent variables include the change in the representative long-term interest rate (Δrl_{t-1}) and the change in the rand/US dollar spot nominal exchange rate ($\Delta exch_{t-1}$). The above specification implies that information of period t−1 is utilised to form price expectations in period t with respect to period t+1.

Alternative specifications that deserve consideration and empirical testing include the 3-month bankers' acceptance rate, or the prime interest rate or even the M3 money supply, instead of the representative long-term interest rate. The spot rand/US dollar exchange rate may possibly be replaced in the specification by e.g. import prices, the terms of trade, or the effective exchange rate. In addition, the wage rate and capacity utilisation in the economy may also be considered as possible explanations of the expected price level. The GEM specification (*op. cit.*:174) mentioned earlier, does include a capacity utilisation variable in the information set, in addition to prices, interest rates and the relevant exchange rate.

The above theoretical specification in a South African context may be motivated as follows. The specification, including lagged prices, interest rates and the exchange rate is an attempt to model the psychological expectations formation process of the (often unsophisticated) consumer. Given that 19 per cent of the adult South African population is illiterate (has not completed primary school) (Stats SA 1998), the adjustment of parameters of an expectations rule based on variables like the terms of trade, capacity utilisation or the money supply probably implies an unrealistically sophisticated consumer. Information about changes in prices, interest rate levels and the exchange rate is perhaps more accessible to the average consumer than any other economic variables influencing price changes. Price expectations are expected *a priori* to be autoregressive for the most part, with a less significant contribution by other variables included in the information set.

The first step in implementing the learning model of price expectations would be to formulate the expectations rule in state-space form.

Again, consider the general form of the state-space representation for a model with stochastically varying coefficients:

$$y_t = a(x_t) + [H(x_t)]'\xi_t + w_t \tag{5.3}$$

$$\xi_{t+1} = F(x_t)\xi_t + v_{t+1} \tag{5.4}$$

The price expectations rule is a regression in which the coefficient vector changes over time:

$$y_t = x'_t \beta_t + w_t \tag{5.5}$$

$$(\beta_{t+1} - \bar{\beta}) = F(\beta_t - \bar{\beta}) + v_{t+1} \tag{5.6}$$

where x_t is a $(k\times 1)$ vector that includes lagged values of y or variables that are independent of the regression disturbance w_τ for all τ.

If the eigenvalues of the $(k\times k)$ matrix F are all inside the unit circle, then $\bar{\beta}$ is interpreted as the average or steady state value of the coefficient vector. If it is further assumed that

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} | \, x_t, \vartheta_{t-1} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & \sigma^2 \end{bmatrix}\right), \tag{5.7}$$

then (5.5) to (5.7) will be recognised as a state-space model of the general form of (5.3) to (5.4) with state vector $\xi_t = (\beta_t - \bar{\beta})$. The regression in (5.5) can be written as

$$y_t = x'_t \bar{\beta} + x'_t \xi_t + w_t \tag{5.8}$$

which is an observation equation of the form (5.3) with $a(x_t) = x'_t \bar{\beta}$, $H(x_t)=x_t$ and $R(x_t)=\sigma^2$.

In terms of the specification of equation (5.2), y_t and x_t in equations (5.3) will be given by:

$$\begin{aligned} y_t &= \Delta cpi_{t+1} \\ x'_t &= [1, \Delta cpi_{t-1}, \Delta rl_{t-1}, \Delta exch_{t-1}]' . \end{aligned} \tag{5.9}$$

The unknown parameters of the system will be estimated along with the (4×1) state vector, ξ_t. The state vector will be assumed to evolve through time according to a random walk with drift process, that is $\xi_{t+1}=\xi_t+v_{t+1}$.

Since all variables are integrated of order 1 (see section 5.3.2), first differences are taken and all variables are utilised in natural logarithm form. The empirical result of the expectations rule will be reported in section 5.4.

5.3 THE DATA

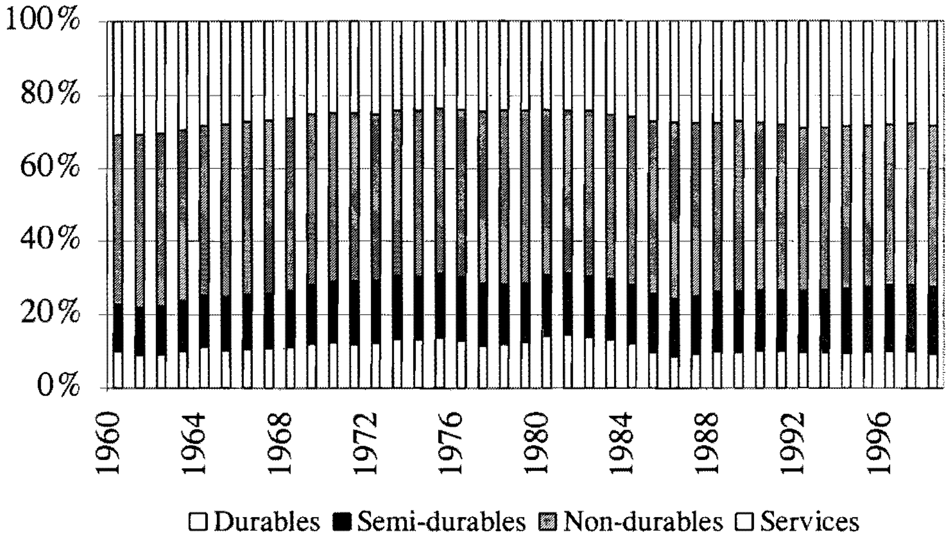
The sources and construction of the data series used to empirically estimate the theoretical models above are discussed in this section, as well as the univariate characteristics of the data.

5.3.1 Sources of data and calculations

Total private consumption expenditure is disaggregated into 4 categories, namely durable consumption, semi-durable consumption, non-durable consumption and services. Expenditure on durables on average accounts for 11 per cent of total consumption expenditure, while semi-durable expenditure accounts for 16 per cent of the total. This average percentage for non-durables is 46 per cent and for services 27 per cent. All data is published in real terms. Figure 5.1 gives an indication of this distribution over time.

The income variable is represented by real disposable income. The consumer price index (CPI) is used to deflate this variable. The 3-month bankers' acceptance rate and the eskom rate are considered as the representative short-term and the long-term interest rates respectively.

Figure 5.1 Private consumption expenditure



Source: South African Reserve Bank, Quarterly Bulletin, Various issues.

Non-human wealth would ideally include net financial wealth of households, housing wealth and possibly an index for stock market prices. The magnitude of the *stock* of wealth in South Africa is however not available in time series format. An indication of the *flow* of financial wealth for the household sector is available from the National Financial Account in the form of the financing balance (i.e. financial assets - financial liabilities)[7].

The (financial) wealth stock variable was then constructed by accumulating financial flows for the household sector from 1970, assuming a base value in that year. The return on wealth variable was constructed by means of the representative long-term interest rate. A

[7] Financial assets and liabilities as reported in the National Financial Account consist of 24 items including gold and other foreign reserves; cash and demand monetary deposits; short, medium and long-term monetary deposits; deposits with other institutions and other financial institutions; treasury and other bills; bank loans and advances; trade credit and short-term loans; short and long-term government stock; non-marketable government bonds; securities of local authorities and public enterprises; other loan stock and preference shares; ordinary shares; foreign branch/head office balances; long-term and mortgage loans; interest in retirement and life funds; amounts receivable/payable; other assets/liabilities and a balancing item.

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constraining factor is that data on financial wealth is only available from 1970 onwards, whereas the other variables under consideration date back to 1960 and some to 1946. Wealth variables were also deflated using the CPI.

The source of all the data used is the *Quarterly Bulletin* of the South African Reserve Bank. For a list of variables refer to Table 5.1. All variables are used in natural logarithmic form.

Table 5.1 List of variables

Series	**Description**
$ctot_t$	Real total private consumption expenditure
$cdur_t$	Real private consumption expenditure on durables and semi-durables
$cndur_t$	Real private consumption expenditure on non-durables
$cserv_t$	Real private consumption expenditure on services
yd_t	Real personal disposable income
w_t	Real financial wealth
rw_t	Real return on financial wealth
$empl\_na_t$	Employment in the non-agricultural sectors
$relp_t$	Prices of durables relative to prices of non-durables, $(Pdur/Pndur)_t$
rs_t	Nominal short-term interest rate (3-month bankers' acceptance rate)
rl_t	Nominal long-term interest rate (eskom rate)
$exch_t$	Rand/US dollar spot nominal exchange rate
cpi_t	Price level (consumer price index)
cpi_e_t	Expected one period ahead price level ($cpi\_e_t = cpi_{t+1}$)
$cpi\_e\_f_t$	Kalman filter prediction of expected one period ahead price level

5.3.2 Order of integration

In analysing the univariate characteristics of the data, the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests were employed to establish the order of integration

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of the data series. The testing strategy discussed in section 4.3.2 was used; namely as suggested by Dolado *et al.* (1990) and as applied by Sturm and De Haan (1995:69).

The number of lags used in the estimated equations was determined in a similar way as suggested by Perron (1989:1384), namely starting with eight lags and testing downwards, until the last lag is significant or there are no lags left.

In addition, graphing the data series in levels as well as their first and second differences, looking at autocorrelation functions (correlograms) and spectrum analysis, proved to be helpful when ADF-test results were inconclusive.

Tables 5.2, 5.3 and 5.4 report the outcomes of the ADF-tests for all relevant data series employed in estimations. The series tested are listed in the first column. The second column reports whether a trend and a constant (Trend), only a constant (Constant), or neither one (None) is included. In the third column, the number of lags used is reported. The next column shows the ADF t-statistic, called τ_τ when a trend and a constant are included, τ_μ when only a constant is included, and τ when neither is included. The last column reports the F-statistic, Φ_3 (Φ_1), testing whether the trend (constant) is significant under the null hypothesis of no unit root.

According to Table 5.2, ADF-tests rendered two of the variables stationary in levels, namely the log of non-durable consumption and the log of the consumer price index. However, by simply looking at graphical representations of these series it becomes obvious that these series cannot be stationary in levels. Table 5.3 indicates that all variables are indeed integrated of order 1. From experience and evidence from other tools, prices are known to be integrated of either order 1 or 2. The second differenced form of the consumer price index was therefore also subjected to a unit root test. In Table 5.4, it is once again clear that the ADF test is inconclusive in establishing the order of integration of the consumer price index. The increasing rate of the change in the general price level between 1970 and 1986 and the slowdown in the rate of change from then onwards, could be the reason for the ADF-test's failure to be conclusive in this instance, see Figure 5.7. In this study, the consumer price index is regarded as I(1) and used in first differenced form. All variables that were employed in empirical estimation are presented in Figure 5.2.

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Figure 5.2 Graphical representation of data series employed in estimations, all data in natural logarithmic form

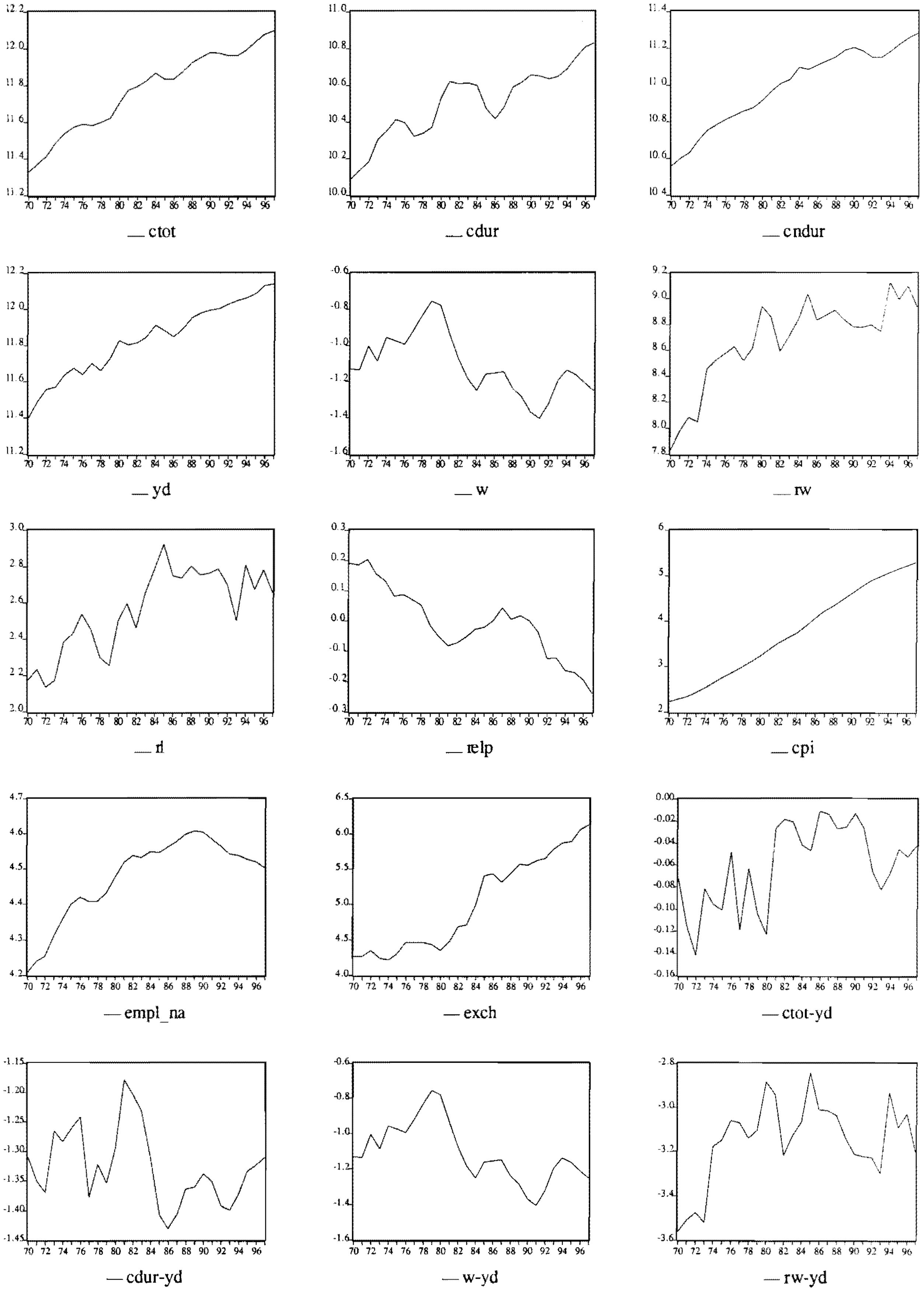

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Table 5.2 Augmented Dickey-Fuller tests for non-stationarity, levels, 1970-1997 (All data series in natural logarithmic form)

Series	Model	Lags	$\tau_\tau, \tau_\mu, \tau$[a]	Φ_3, Φ_1[b]
$ctot_t$	Trend	4	-1.94	4.17
	Constant	2	-2.19	4.99
	None	6	1.00	
dur_t	Trend	1	-3.39	7.23
	Constant	4	-2.45	4.07
	None	2	1.74	
$cndur_t$	Trend	0	-1.62	4.64
	Constant	0	-2.90**	8.46**
	None	1	2.48	
yd_t	Trend	2	-3.59	5.01
	Constant	2	-2.71	2.83
	None	0	3.88	
w_t	Trend	8	-3.23	5.19
	Constant	8	-2.87	4.23
	None	5	0.38	
rw_t	Trend	0	-2.83	5.02
	Constant	8	-2.45	1.74
	None	0	1.93	
rl_t	Trend	8	0.14	1.59
	Constant	8	-2.15	1.78
	None	8	1.34	
$relp_t$ $(Pdur/Pndur)_t$	Trend	6	-3.35	1.96
	Constant	8	-0.42	1.05
	None	2	-1.16	
$empl\_na_t$	Trend	3	0.81	10.93**
	Constant	8	0.07	4.22
	None	5	-0.77	
$exch_t$	Trend	3	-2.34	3.02
	Constant	2	0.73	1.43
	None	0	3.13	
cpi_t	Trend	1	-1.52	17.26**
	Constant	4	-2.46**	12.09**
	None	6	1.02	

*(**) Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), -3.00(-3.77) when only a constant is included (τ_μ) and -1.96(-2.68) when neither is included (τ). The standard normal critical value is -1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

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Table 5.3 Augmented Dickey-Fuller tests for non-stationarity, first differenced, 1970-1997 (All data series in natural logarithmic form)

Series	Model	Lags	τ_τ,τ_μ,τ[a]	Φ_3,Φ_1[b]
$\Delta ctot_t$	Trend Constant None	3	-4.21*	4.68
Δdur_t	Trend Constant None	1	-3.93*	5.26
$\Delta cndur_t$	Trend Constant None	3	-3.88*	4.80
Δyd_t	Trend Constant None	0	-5.83*	16.98**
Δw_t	Trend Constant None	3	-3.95*	4.09
Δrw_t	Trend Constant None	0	-6.20**	19.27**
Δrl_t	Trend Constant None	2	5.03**	13.02**
$\Delta relp_t$ $\Delta(Pdur/Pndur)_t$	Trend Constant None	0	-4.31**	9.31*
$\Delta empl\_na_t$	Trend Constant None	2	5.02**	6.49
$\Delta exch_t$	Trend Constant None	2	-4.61**	7.81*
Δcpi_t	Trend Constant None	0 0	-2.25 -2.89**	4.76 8.38**

*(**) Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), -3.00(-3.77) when only a constant is included (τ_μ) and -1.96(-2.68) when neither is included (τ). The standard normal critical value is −1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

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Table 5.4 Augmented Dickey-Fuller tests for non-stationarity, second differenced, 1970-1997 (Data series in natural logarithmic form)

Series	**Model**	**Lags**	$\tau_\tau,\tau_\mu,\tau^a$	$\Phi_3,\Phi_1{}^b$
$\Delta\Delta cpi_t$	Trend	2	-0.59	2.18
	Constant	0	-2.09	4.36
	None	0	-0.35	

*(**) Significant at a 5(1)% level.
a At a 5(1)% significance level the MacKinnon critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), -3.00(-3.77) when only a constant is included (τ_μ) and -1.96(-2.68) when neither is included (τ). The standard normal critical value is –1.703 (-2.473).
b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

5.4 ESTIMATION TECHNIQUE USED

The estimation technique employed for estimation of behavioural equations is the Johansen maximum likelihood estimation methodology. The main advantage of the Johansen technique over the Engle and Yoo three-step procedure has been pointed out in Chapter 4 and is, in essence, the inability of the latter to determine the number of cointegrating relationships present in the data. (If there are n variables included in the model, as many as (n–1) linearly independent cointegration vectors may possibly exist.) Assuming that there is one cointegrating vector present, when there are in fact more, leads to inefficiency in the sense that only a linear combination of these vectors may be obtained when estimating a single equation model. Other advantages of the Johansen procedure have also been discussed in Chapter 4.

Although the Johansen technique as a means of establishing the number of cointegrating relationships is readily available in econometrics software packages, the technique imposes several demands. The distinct steps in the estimation process have also been highlighted in Chapter 4, and these will be followed in the practical estimation. The process starts by confirming the order of integration of all variables. Also, the correct lag length of the vector autoregressive (VAR) model is determined so as to ensure that the vector error correction model has Gaussian errors. The Johansen cointegration test is then used to test for the reduced rank of the system, i.e. to determine the number of cointegrating relationships present in the data. Next, the inclusion of deterministic variables (constant and trend) must be established by considering whether the data contain trends. This is followed by testing for weak exogeneity and by testing the linear hypotheses on the cointegrating relationships. The final steps in the process normally entail testing for unique cointegrating vectors and imposing joint tests of restrictions on the α loading matrix and the β cointegrating vector.

In this study, the residual obtained from the cointegrating vector, – i.e. the equilibrium error estimated from the long-run equilibrium relationship, is implemented in an unrestricted single equation error correction model (ECM) and the residual diagnostics are used to test whether error terms are white noise. The coefficient of the equilibrium residual in the ECM also provides information about the speed of adjustment towards equilibrium.

The estimation results obtained for the three estimated private consumption expenditure categories (i.e. total, durables and non-durables) are presented in section 5.5.2.

5.5 ESTIMATION RESULTS

The application of the Kalman filter estimation to the state-space representation of the time-varying price expectations rule is presented in this section, including the subsequent implementation of the Kalman filter price expectations forecast in the behavioural equations.

5.5.1 Time-varying parameter estimation of the expectations rule

The first step in the estimation process would be to estimate the structural equation $\Delta cpi_t = f(\Delta cpi_{t-2}, \Delta rl_{t-2}, \Delta exch_{t-2})$ with fixed parameters by means of ordinary least squares (OLS), including all variables in natural logarithmic form. Table 5.5 reports the expectations equation estimated by OLS. This is intended to give an indication of the approximate form of price expectations when estimated with time varying parameters. It

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appears from Table 5.5 that price expectations are mainly autoregressive. The most significant term is the lagged consumer price index while the other variables are, for the most part, insignificant. They are retained, however, because it is believed that they may serve to capture some endogenous interaction in the broad macro model.

Table 5.5 OLS regression of price expectations equation

Dependent variable: Δcpi_t

Variables	**Coefficient**	**Std. Error**	**t-Statistic**
constant	0.023696	0.010126	2.340120
Δcpi_{t-2}	0.767885	0.101243	7.584574
Δrl_{t-2}	0.018168	0.036861	0.492869
$\Delta exch_{t-2}$	-0.000037	0.000238	-0.154866
sample period (adjusted): 1963 to 1998 $\overline{R}^2 = 0.6216$ s.e. = 0.0274			

Table 5.6 reports the hyperparameters and residual diagnostics for the price expectations equation when estimated using the Kalman filter. No convergence was found over the sample period 1960 to 1998 or 1965 to 1998, and therefore the estimation was conducted over the sample period 1970 to 1998. Convergence was in this instance achieved after 70 iterations with a convergence factor of 0.001. Finding no convergence is not uncommon when conducting estimation with the Kalman filter (Hamilton 1994:387-388). The EViews user's guide (1997:544) suggests that different starting values for ξ_0 and P_0 or a different solution algorithm[8] be used when problems of this nature are encountered. By default,

[8] In this instance, the default algorithm used by EViews for models which may be estimated using first derivative methods was selected, namely the Marquardt algorithm. This algorithm modifies the Gauss-Newton algorithm by adding a correction matrix (or ridge factor) to the Hessian approximation. The ridge

Eviews estimates initial values by running OLS on the observations equation, treating ξ_0 as a fixed parameter.

The variances of the error term of the observation equation and the covariance matrix of the state equation error terms, which is assumed to be diagonal, are reported in the top part of the table. $R(1,1)=\sigma^2$ represents the variance of the error term of the observation equation and Q represent the covariance matrix of the error terms or adjustment factors of the state equations. The latter, which are often called hyperparameters, determine the speed of learning and reflect the signal to noise ratio for each variable; hence they also reflect the rate of convergence of the model. The hyperparameters only make sense when compared to the variance of the error term of the observation equation, which in this case is for all practical purposes equal to zero. In this comparison, the magnitude of the hyperparameters is indicative of a fairly rapid learning process with respect to lagged prices, and to a lesser extent with respect to interest rates and the exchange rate.

Put differently, the covariance matrix of the state equation error terms represents the 'forgetting factor' (Hall and Garratt 1992b:7). When the matrix contains only zeros, the Kalman filter will generate OLS estimations of the state vector, ξ. As the diagonal elements of the covariance matrix increase, the parameters are allowed to change more rapidly, and in effect 'forget' the past.

The final values of the state vector, ξ, with associated standard errors, are also reported in the top part of the table. ξ_1, ξ_2, and ξ_3 represent the time-varying coefficients of the independent variables in the expectations rule, Δcpi_{t-2}, Δrl_{t-2} and $\Delta exch_{t-2}$ respectively. The intercept coefficient of the price expectations rule is defined as time-independent (constant).

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correction handles numerical problems when the outer product is near singular and may improve the convergence rate.

As an alternative, the BHHH-algorithm (Berndt, Hall, Hall and Hausman) may be selected. This algorithm is referred to as Gauss-Newton for general nonlinear least squares problems, and Berndt, Hall, Hall and Hausman (BHHH) for maximum likelihood problems. It follows Newton-Raphson, but replaces the negative of the Hessian by an approximation formed from the sum of the outer product of the gradient vectors for each observation's contribution to the objective function. For least squares and log-likelihood functions, this approximation is asymptotically equivalent to the actual Hessian when evaluated at the parameter values which maximise the function. When evaluated away from the maximum, this approximation may be quite poor (EViews 1998:621).

Table 5.6 Hyperparameters, final value of the state vector and equation diagnostics from the time varying Kalman filter estimates

Dependent variable: Δcpi_t

Variables	Coefficient	Std.Error	t-Statistic
R(1,1)	6.90E-65	0.004206	2.19E-125
Q(1,1)	0.015835	0.026962	0.587311
Q(2,2)	0.000659	0.691802	0.000953
Q(3,3)	0.000440	0.757571	0.000581
constant	0.066396	0.004206	15.78748
Final ξ_1	0.111234	0.162426	0.684828
Final ξ_2	-0.029184	0.042777	-0.682235
Final ξ_3	-0.028425	0.052550	-0.540908
sample period (adjusted): 1970 to 1998 $\bar{R}^2 = 1.0000$ s.e. = 7.62E-15 BJ (normality) = 0.018052 [0.991] LB(1)=0.0496 [0.824] LB(2)=5.4780 [0.065] LB(3)=6.1115 [0.106] LB(4)=6.1125 [0.191]			

Note: ξ_1, ξ_2, and ξ_3 are the time-varying coefficients of Δcpi_{t-2}, Δrl_{t-2} and $\Delta exch_{t-2}$, respectively (all variables in natural logarithmic form).

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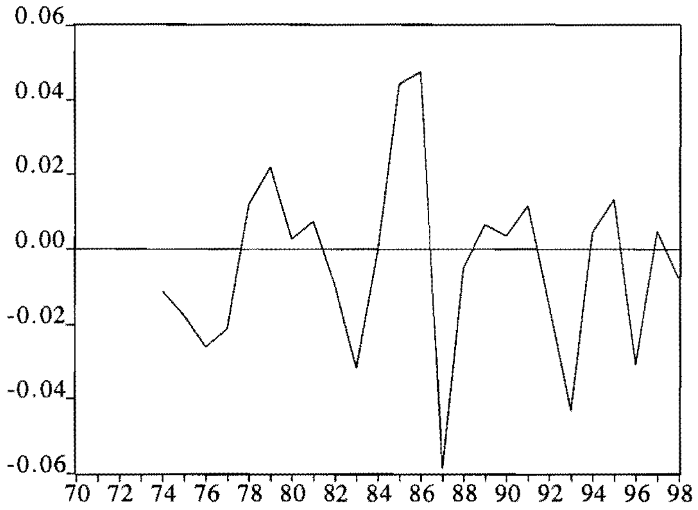
Figure 5.3 is a representation of the one-step-ahead forecast error, that is the residual of the observation equation. Residual diagnostics reported in Table 5.6 indicate adherence to the normality assumption and there is no serial correlation present. The regression statistics (e.g. the R-squared) for time-varying coefficient models are computed by EViews, using the smoothed residuals $y_t - A'x_t - H'\xi_{t|T}$ from the observations equation. The R^2 and adjusted R^2 are equal to unity in this instance.

Figure 5.3 Residual: one-step ahead forecast error graph



Each of the estimated time-varying slope coefficients, assumed to evolve as a random walk with drift, displays a reasonable degree of variation over the period. The intercept coefficient of the expectations rule was not allowed to vary. Figures 5.4 to 5.6 illustrate the evolution of the time-varying coefficients.

The time-varying coefficient of lagged prices clearly displays the largest degree of variation. This coefficient varies roughly between 1.0 and 0.0, with an upward trend between 1974 and 1986, coinciding with a period of rapid increase in the general domestic price level. Double-digit inflation figures were first recorded in 1974 and ascribed mainly to the oil price shocks experienced in 1973. This was repeated in 1979. The high inflation figures during this period reflected the world-wide upward trend in general price levels. Although the main trading partners of South Africa managed to curb their domestic inflation during the 1980s, the upward trend in South African inflation continued. Inflation peaked in 1986 at 18.4 per cent, a period in South African history dominated by international diplomatic and economic isolation. Since 1986 onwards, there has been a general downward tendency in inflation in South Africa, slowing down to a single digit from 1993 onwards. This process was aided by consistently strict monetary policy.

It is interesting to observe that the evolution of the time-varying coefficient of the lagged price level in the price expectations rule to a certain extent mimicked the trend of the actual change in price levels. First, an overall increasing trend emerged from 1974 to 1986 (after an initial downward adjustment of the coefficient), followed by a descent from a high of 0.8 in 1986 to a final value of 0.11 in 1998 when the lowest inflation rate in 28 years was recorded in South Africa, namely 6.8 per cent. What can be gathered from the above may be that consumers are indeed learning from new information as time progresses. In periods of high and rapidly increasing price levels, consumers continuously adjust the parameters of the rule upwards and, as soon as they realise that price levels are declining, they start adjusting their parameters downward, leading to lower expected price levels.

The coefficient of the second variable in the information set of the price expectations rule displays a considerably smaller degree of variation, especially when taking into account that the scale in Figure 5.5 is only 60 per cent of that of Figure 5.4. The coefficient evolves around 0.04 within the band (-0.02, 0.12). The general trend is slightly upward for the period 1974 to 1985, when interest rates peaked at 18.62 per cent, followed by a movement sideways in line with interest rate movements.

The variation in the coefficient of the lagged spot nominal rand/US dollar exchange rate is also significantly less than that of lagged prices. The coefficient varies around –0.016, between -0.089 and 0.017. The coefficient evolves rather smoothly, with only one significant jump in 1985, a year marked by the largest depreciation in the rand/US dollar exchange rate in history, namely a depreciation of 50 per cent from R1,48/US dollar to R2,23/US dollar. This pronounced depreciation may clearly be attributed to political and economic isolation, the debt standstill and disruptive balance of payments adjustments.

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Figure 5.4 Coefficient on lagged consumer price inflation

Figure 5.5 Coefficient on lagged long-term interest rate

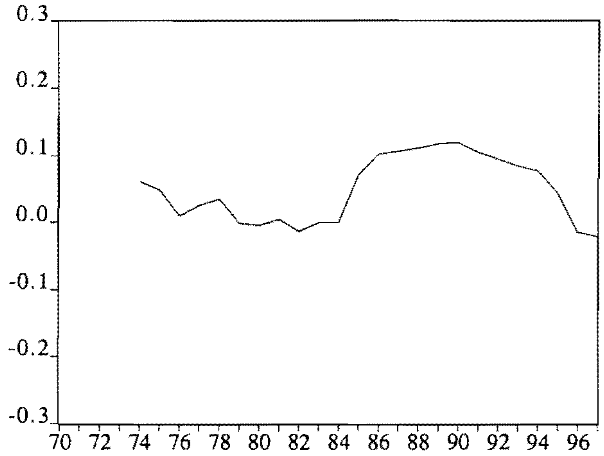

Figure 5.6 Coefficient on lagged rand/US dollar exchange rate

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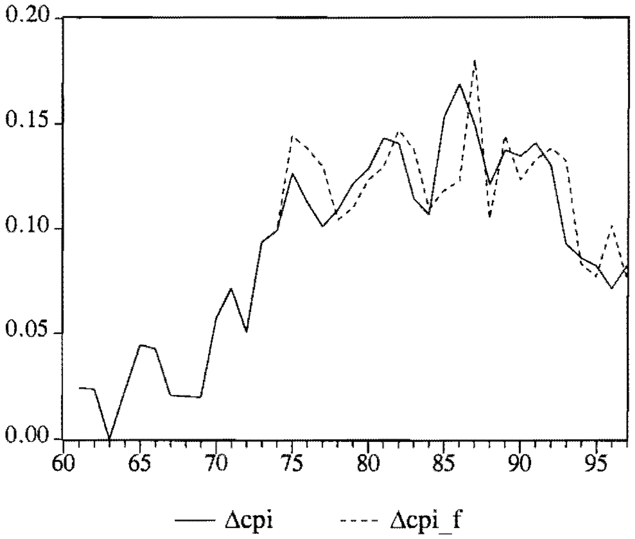
It is obvious from the above that the most important variable in the information set of the price expectations rule is the lagged price level. The other two variables display a lesser degree of variation and are also statistically less significant in explaining price expectations. Furthermore, the larger variance of the one-period-ahead forecast of the first element of the state vector, namely that of the lagged price level, is also indicative of a faster rate of learning with respect to observed changes in the actual price level than that of the other two independent variables.

The actual price level and its Kalman filter prediction for the specification $\Delta cpi_t = f(\Delta cpi_{t-2}, \Delta rl_{t-2}, \Delta exch_{t-2})$ are shown in Figure 5.7.

Figure 5.7 Kalman filter estimation of expected price level: Δcpi and Δcpi_f



In order to implement the price expectations variable in the consumption functions, the above was repeated for the specification

$$\Delta cpi_{t+1} = f(\Delta cpi_{t-1}, \Delta rl_{t-1}, \Delta exch_{t-1}) \quad (5.10)$$

or put differently,

$$\Delta cpi\_e_t = f(\Delta cpi\_e_{t-2}, \Delta rl_{t-1}, \Delta exch_{t-1}) \quad (5.11)$$

with $\Delta cpi\_e_t = \Delta cpi_{t+1}$.

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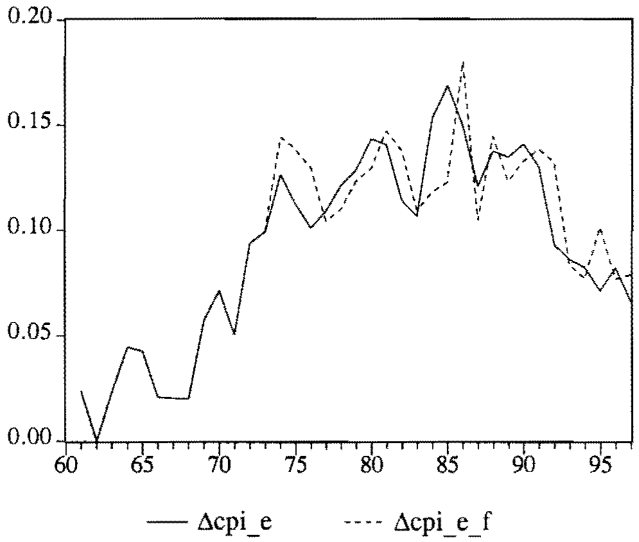
The reason for implementing the price expectations rule in the form of (5.11), is that the software package used to dynamically solve the consumption expenditure model is unable to determine a solution for the dependent variable with Δcpi_{t+1} included in the information set.

The estimation result of model (5.11) corresponds exactly to that reported in Table 5.6, with the exception that the adjusted sample period is from 1969 to 1997 instead of from 1970 to 1998. The graphical representation of the actual and fitted values for Δcpi_e_t in Figure 5.8 confirms this fact. The fitted value, $\Delta cpi\_e\_f_t$, is therefore the variable to be implemented in the consumption functions as the variable reflecting the price expectations of the consumer. Citing Cuthbertson (1988:226) in this regard: "It is worth noting at the outset that an expectations series generated using the Kalman filter with time-varying parameters may be directly used in behavioural equations where the structural parameters are assumed constant."

Figure 5.8 Kalman filter estimation of expected price level: Δcpi_e and Δcpi_e_f



5.5.2 Forward-looking consumption functions

Finally, the behavioural equations can be estimated, containing the one-period-ahead forecast of the expectations variable as part of the short-run dynamics of the system; that is the actual result obtained through the Kalman filter estimation.

The long-run relationship between private consumption expenditure, wealth and personal disposable income, in the case of total private consumption expenditure and durable consumption expenditure, was estimated in the form c-yd = f(w-yd), thus constraining the sum of wealth and income elasticities to unity (all variables in natural logarithm form), for the specification

$$c_t = Ayd_t^{\alpha} w_t^{\beta} e^{\varepsilon_t} \qquad \text{with } 0<\beta<1;\ 0<\alpha<1;\ \alpha+\beta=1. \tag{5.12}$$

In the case of non-durable consumption, only current disposable consumption is included in the long-run equilibrium equation, with an expected income elasticity close to unity. Interest rates are believed to play a significant role in the explanation of the durable consumption expenditure category and, since durable consumption (including durables and non-durables) constitutes 27 per cent of total consumption, possibly also in the total private consumption expenditure function. Interest rates are, however, not included in the long-run equilibrium equation, but rather considered as contributing towards the short-run dynamic structure of the system.

Since expenditure on durables accounts only for 11 per cent and semi-durables for 16 per cent of total private consumption expenditure, the durable consumption function was estimated on the aggregate of the two components.

In all instances, a vector autoregressive (VAR) model with a lag length of 1 was used to test for the number of cointegrating relationships. Although the Akaike information criterion (AIC) and the Schwarz Bayesian criterion (SBC) suggested that a model with lag length 2 should be selected, the values of the information criteria in all cases were very close for the two models. Granger (1991:561) also notes in this regard that the AIC criterion is known to 'overfit' in the sense that if the true value of p for VAR(p) is p_0 (finite), then the criterion is inclined to choose a value for p that is somewhat greater than p_0. The validity of the Johansen estimation technique depends on white noise error terms – the minimum lag length that renders residuals of the ECM white noise would thus be acceptable. In all instances, the error correction models passed all the standard tests of serial correlation, normality, etc. where long-run coefficients were obtained with a VAR of order 1. The limited sample of the data from 1970 to 1997 (constrained by the unavailability of the financial wealth variable) and the fact that the Johansen cointegration

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test is rather data-hungry, contributed towards the decision to use a VAR with lag length 1 throughout. The resulting long-run coefficients, in most instances, turned out to be very similar for a VAR(1) and a VAR(2). For all behavioural equations, values for the AIC and SBC as well as the long-run coefficients obtained with a specified lag length of 1 and 2, respectively, are reported.

The results of individual stochastic functions are subsequently reported in sections 5.5.2.1 to 5.5.2.3. Three possible error correction specifications for durable consumption expenditure are presented. The selection of the best model, based on model selection criteria and economic theory, is motivated.

5.5.2.1 Total private consumption expenditure

The long-run relationship between total private consumption expenditure, wealth and personal disposable income was estimated in the form c-yd = f(w-yd), thus constraining the sum of wealth and income elasticities to unity (all variables in natural logarithmic form). Both wealth variables were tested, namely the stock of wealth, w, and the return on wealth, rw. In this instance, total private consumption expenditure, the return on wealth and personal disposable income constituted a cointegrating relationship.

(i) The cointegration equation

In order to test for cointegration between (c-yd) and (rw-yd), the Johansen test was employed. The correct lag length for the VAR must be selected to ensure that the error correction model has Gaussian errors. Both AIC and SBC suggest that a VAR model with a lag length of 2 should be used. The result of these measures is reported in Table 5.7.

Table 5.7 Selection of the order of the VAR: Total private consumption expenditure

Order of VAR	AIC	SBC
1	57.2617	55.3180
2	53.7687	50.6234

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The values of the test statistics proved to be close for the two models. Since the sample contains only 27 observations and seeing that the Johansen procedure is rather data-hungry, the cointegration test was conducted for a VAR of both length 1 and 2.

In the case of a VAR with lag length 2, cointegration could only be established at a 10 per cent significance level. For a VAR with lag length equal to 1, the Johansen likelihood ratio test for the number of cointegrating relationships (denoted by r), based on the maximum eigenvalue and the trace of the stochastic matrix, suggests one cointegrating relationship at a 5 per cent significance level between the variables in the long-run relationship (the cointegration equation). This result is presented in Table 5.8.

Table 5.8 Johansen test for the number of cointegrating relationships: total private consumption expenditure

Cointegration LR test based on maximal eigenvalue of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
$r=0$	$r=1$	16.8664	15.8700	13.8100
$r \leq 1$	$r=2$	7.0921	9.1600	7.5300
Cointegration LR test based on trace of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
$r=0$	$r \geq 1$	23.9585	20.1800	17.8800
$r \leq 1$	$r=2$	7.0921	9.1600	7.5300

Note: VAR order = 1

The long-run relationship was estimated with an unrestricted constant and the long-run cointegration relationship, for rank equal to 1 ($r=1$), is reported in Table 5.9 (figures in brackets denote coefficients normalised on the dependent variable).

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Table 5.9 Estimate of cointegration equation

Dependent variable: $(ctot - yd)_t$

Variables	Cointegrating vector
$(ctot\text{-}yd)_t$	4.4839 (-1.0)
$(rw\text{-}yd)_t$	-0.88128 (0.19655)
constant	-2.5077 (0.55926)
Sample Period: 1971-1997 Order of VAR = 1; r=1	

The above result suggests the presence of a long-run relationship of the form

$$ctot_t = (1\text{-}0.19655)yd_t + (0.19655)rw_t + 0.55926. \qquad (5.13)$$

The income elasticity therefore amounts to 0.80345 and the wealth elasticity to 0.19655 for the period under consideration. (For a VAR(2), the wealth elasticity amounts to 0.17302 and the intercept coefficient equals 0.48837.) Coefficients are of the correct sign and the magnitudes correspond with *a priori* expectations expressed in the hypothesised long-run relationship.

The residual derived from the above, allows for the specification of the error correction model, representing the short-run dynamic adjustment process. This result is reported in Table 5.10.

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(ii) The error correction model

Both measures of non-human wealth were introduced in the specification of total private consumption expenditure. While the return to wealth is utilised in the long-run cointegration relationship, the actual real stock of wealth seems to contribute to the short-run dynamics of the system, so do lagged values of wealth stock and current real disposable income, lagged total consumption expenditure and the future expected price level. Lagged values of the dependent variable, total private consumption expenditure, would account for habit persistence or historic expenditure patterns, tastes of the consumer, etc. The expected price level, that is the one-period-ahead Kalman filter estimation, is introduced to test the hypothesis that consumers are forward-looking with respect to prices. Al variables are used in natural logarithmic form.

Diagnostic statistics suggest that the equation is statistically well-specified, with no violations of the Gaussian assumption. The use of a VAR of order 1 for the derivation of the long-run relationship, is therefore justified. Figures in square brackets denote probability values (the probability of falsely rejecting the null hypothesis of a zero restriction on the coefficient or diagnostic). BJ denotes the Bera-Jarque test for the normality assumption; LB and LM denote tests for serial correlation; Ramsey's RESET is a test for misspecification and ARCH and White denote tests for the homoskedasticity assumption. All test statistics are distributed χ^2, with figures in round brackets denoting degrees of freedom. Recursive estimates also confirmed stability in the parameters.

The result of *ex post* simulation of the private consumption function is presented graphically in Figure 5.9. The fitted values also constitute the base-line forecast against which the response characteristics of the system to exogenous income and wealth shocks are analysed.

Table 5.10 An error correction model for total private consumption expenditure

Dependent variable: $\Delta ctot_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	0.040825	0.011592	3.521846
$resid_{t-1}$	-0.205334	0.070992	-2.892357
$\Delta ctot_{t-1}$	-0.235168	0.105383	-2.231562
Δyd_t	0.670364	0.067966	9.863180
Δw_t	-0.316304	0.040438	-7.821941
Δw_{t-1}	0.120829	0.032498	3.717989
Δcpi^*_{t+1}	-0.216683	0.088190	-2.456994

sample period: 1975 to 1997
R^2 = 0.909710
$\bar{R}^2$ = 0.875851
s.e. = 0.009458

Normality:	BJ(2)=0.7026	[0.7026]
Serial correlation:	LB(8)=3.4096	[0.9096]
	LM(2)=2.9052	[0.2340]
Heteroscedasticity:	ARCH(1)=0.0415	[0.8385]
	White(1)=6.6778	[0.8781]
Stability:	RESET(2)=0.7714	[0.6799]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter prediction of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

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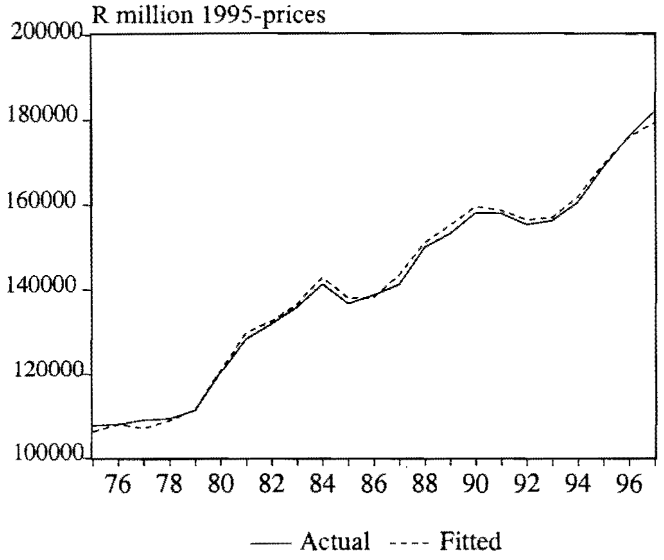
Figure 5.9 The overall dynamic fit of the total private consumption expenditure function



5.5.2.2 Durable private consumption expenditure

The long-run relationship between durable private consumption expenditure, wealth and personal disposable income was estimated in the form cdur-yd = f(w-yd), thus constraining the sum of wealth and income elasticities to unity (all variables in natural logarithmic form). Both wealth variables were tested, namely the stock of wealth, w, and the return to wealth variable, rw. In the case of durable private consumption expenditure, durable consumption, the stock of financial wealth and personal disposable income constituted a cointegrating relationship.

(i) The cointegration equation

In order to test for cointegration between (cdur-yd) and (w-yd), the Johansen test was employed. As in the case of total private consumption expenditure, both AIC and SBC suggest that a model with a lag length of 2 should be used. The information criteria are reported in Table 5.11.

Table 5.11 Selection of the order of the VAR: Durable private consumption expenditure

Order of VAR	AIC	SBC
1	45.2640	43.3203
2	42.4419	39.2966

The values of the information criteria prove to be very close for the two models. Since the sample contains only 27 observations, the cointegration test was conducted for both a VAR of length 1 and 2.

As with previous results, in the case of a lag length of 2, cointegration could not be established at either a 5 or a 10 per cent significance level. For a VAR with lag length equal to 1, the Johansen likelihood ratio test for the number of cointegrating relationships, based on the maximum eigenvalue and the trace of the stochastic matrix, suggests the presence of one cointegrating relationship at a 5 per cent significance level. These results are presented in Table 5.12.

Table 5.12 Johansen test for the number of cointegrating relationships: durable private consumption expenditure

Cointegration LR test based on maximal eigenvalue of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
$r=0$	$r=1$	24.2811	15.8700	13.8100
$r \leq 1$	$r=2$	2.1750	9.1600	7.5300
Cointegration LR test based on trace of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
$r=0$	$r \geq 1$	26.4561	20.1800	17.8800
$r \leq 1$	$r=2$	2.1750	9.1600	7.5300

Note: VAR order = 1

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The long-run cointegration relationship is presented in Table 5.13. With respect to the inclusion of deterministic variables in the long-run equation, including a constant, but no trend, yielded economically viable results. Figures in brackets denote coefficients normalised on the dependent variable.

Table 5.13 Estimate of cointegration equation

Dependent variable: $(cdur - yd)_t$

Variables	Cointegrating vector
(cdur-yd)t	-3.0301 (-1.0)
$(w\text{-}yd)_t$	0.72817 (0.24031)
constant	-3.2266 (-1.0648)
Sample Period: 1971-1997 Order of VAR = 1; r=1	

The above result suggests the presence of a long-run relationship of the form

$$cdur_t = (1\text{-}0.24031)yd_t + (0.24031)w_t - 1.0648. \tag{5.14}$$

The income elasticity therefore amounts to 0.75969 and the wealth elasticity to 0.24031 for the period under consideration. The signs and magnitudes of coefficients correspond to *a priori* expectations. (Although cointegration could not be proved for a VAR model with lag length 2, the slope coefficient of the relationship for r=1 in this instance turned out to be equal to 0.25208 and the intercept coefficient to be equal to –1.0484).

The residual derived from the above, allows for the specification of the error correction model, representing the short-run adjustment to equilibrium reported in Table 5.14. Two alternatives are also presented, reported in Table 5.15 and Table 5.16.

(ii) The error correction model

Only the stock of wealth variable as a measure of non-human wealth was introduced in the specification of durable private consumption expenditure. Variables contributing to the short-run dynamics of the system include current real disposable income, real financial wealth stock as well as lagged values of this variable, a representative long-term interest rate lagged by one period, relative prices (that is the ratio between prices of durables relative to non-durables) and the future expected price level. The expected price level, that is the one-period-ahead Kalman filter prediction, is introduced to test the hypothesis that consumers are forward-looking with respect to prices when making durable consumption expenditure decisions. Al variables are used in natural logarithmic form. Since the intercept coefficient had a low t-statistic (t=1.33) in the original estimation, signalling statistical insignificance, it was omitted from the equation, with the following result.

Diagnostic statistics suggest that the equation is statistically well-specified, and that the use of a VAR with lower length than was selected by the information criteria, was justified in this case, since the errors of the ECM adhere to the Gaussian assumption. Figures in square brackets denote probability values (the probability of falsely rejecting the null hypothesis of a zero restriction on the coefficient or diagnostic). BJ denotes the Bera-Jarque test for the normality assumption; LB and LM denote tests for serial correlation; Ramsey's RESET is a test for misspecification and ARCH and White denote tests for the homoskedasticity assumption. All test statistics are distributed χ^2, with figures in round brackets denoting degrees of freedom. Recursive estimates also confirmed stability in the parameters.

The result of *ex post* simulation of the durable private consumption function is presented graphically in Figure 5.10.

Table 5.14 An error correction model for durable private consumption expenditure

Dependent variable: $\Delta cdur_t$

Variable	Coefficient	Std. Error	t-Statistic
$resid_{t-1}$	-0.339951	0.089959	-3.778934
Δyd_{t-1}	1.206395	0.111066	10.86197
Δw_t	-0.547395	0.057194	-9.570830
Δw_{t-1}	0.252309	0.070234	3.592415
Δrl_{t-1}	-0.096311	0.031806	-3.028070
$\Delta relp_{t-1}$	-0.458177	0.126838	-3.612293
Δcpi^*_{t+1}	-0.082389	0.039528	-2.084328

sample period (adjusted): 1975 to 1997

$R^2 = 0.938790$

$\bar{R}^2 = 0.915837$

s.e. = 0.017725

Normality:	BJ(2)=0.7912	[0.6733]
Serial correlation:	LB(8)=6.6335	[0.5570]
	LM(2)=2.4212	[0.2980]
Heteroscedasticity:	ARCH(1)=0.0182	[0.8927]
	White(1)=19.1092	[0.1608]
Stability:	RESET(2)=2.3362	[0.3110]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter estimation of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

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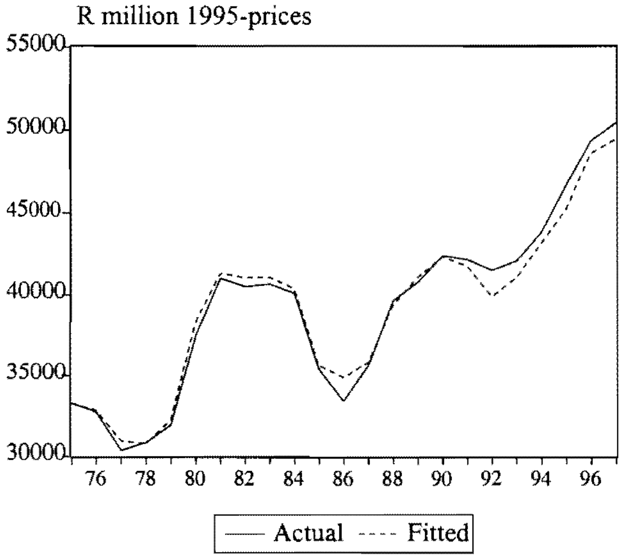
Figure 5.10 The overall dynamic fit of the durable private consumption expenditure function



The intercept term was suppressed in the above case. Although it is generally preferable to include unrestricted constants in the ECM, even if not statistically significant, it is also considered acceptable to restrict the constant in the ECM in order to obtain a more reasonable model. Davidson, Hendry, Srba and Yeo (1978:687) did this in their 1978 paper on consumption in order for the error correction model to work (see equation (2.16)). However, the dynamic simulation of the equation seems to perform poorly over the last number of observations, constantly underestimating the level of durable consumption expenditure.

It must be pointed out that even if the unrestricted constant is retained, the results are acceptable. This result is reported in Table 5.15, with the overall fit of this specification represented in Figure 5.11. The *ex post* simulation, in this instance, yields a satisfactory in-sample fit.

One may argue that the p-value of 0.2 associated with the t-value of 1.339 of the constant in Table 5.15, although not significant, can still be in an ambiguous range and that bias may be introduced by restricting the constant to zero.

Table 5.15 An error correction model for durable private consumption expenditure, containing an unrestricted constant

Dependent variable: $\Delta cdur_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	0.029138	0.021748	1.339805
$resid_{t-1}$	-0.280156	0.092791	-4.096925
Δyd_{t-1}	1.179692	0.110222	10.70286
Δw_t	-0.559272	0.056523	-9.89457
Δw_{t-1}	0.232467	0.070133	3.314681
Δrl_{t-1}	-0.080064	0.033328	-2.402260
$\Delta relp_{t-1}$	-0.44224	0.126367	-3.357082
Δcpi^*_{t+1}	-0.308901	0.173410	-1.781335

sample period (adjusted): 1975 to 1997

$R^2 = 0.945332$

$\bar{R}^2 = 0.919821$

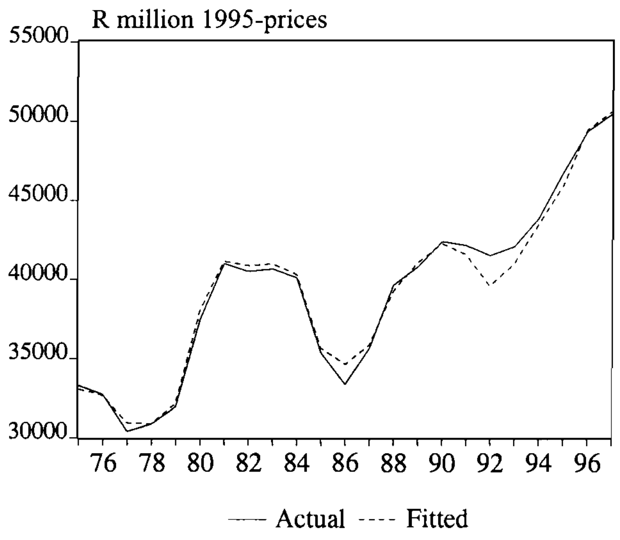
s.e. = 0.017300

Normality:	BJ(2)=1.7408	[0.4188]
Serial correlation:	LB(8)=5.9891	[0.6480]
	LM(2)=2.5668	[0.2771]
Heteroscedasticity:	ARCH(1)=0.0982	[0.7540]
	White(1)=15.3248	[0.3563]
Stability:	RESET(2)=0.9783	[0.6131]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter estimation of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

The motivation for restricting the constant of the ECM in the first place, was because a t-value of the price expectations variable of 1.78 in absolute terms (with an associated p-value of 0.0951), may not be significant enough to make a strong case in favour of the effect of price expectations on durable consumption expenditure decisions. However, a p-value of less than 0.10 may often be considered significant 'enough'. When performing a two-tailed test on the coefficient of Δcpi^*_{t+1}, the cut-off value of 10 per cent significance with 15 degrees of freedom would be -1.753. This perhaps provides more evidence on the significance of the relationship between price expectations and durable consumption. Since the signs (and magnitudes) of the coefficients in the ECM are generally not interpreted, a one-tailed test (that yields a critical value of 1.753 on a 5 per cent significance) would not be appropriate in this instance.

Figure 5.11 The overall dynamic fit of the durable private consumption expenditure function, containing an unrestricted constant



As a last option, if one is prepared to accept that the South African consumer is not interest rate-sensitive in making durable consumption expenditure decisions, the ECM reported in Table 5.16, may also be regarded as an acceptable structural explanation of durable consumption expenditure behaviour in South Africa.

Table 5.16 An error correction model for durable private consumption expenditure, excluding the interest rate variable from the information set

Dependent variable: $\Delta cdur_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	0.048148	0.023081	2.086034
$resid_{t-1}$	-0.482087	0.094022	-5.127377
Δyd_{t-1}	1.176291	0.125574	9.367303
Δw_t	-0.523292	0.062099	-8.426698
Δw_{t-1}	0.168625	0.070948	2.280311
$\Delta relp_{t-1}$	-0.387292	0.142910	-2.710045
Δcpi^*_{t+1}	-0.470578	0.182092	-2.584289

sample period (adjusted): 1975 to 1997

$R^2 = 0.924301$

$\bar{R}^2 = 0.895913$

s.e. = 0.019711

Normality:	BJ(2)=1.0547	[0.9730]
Serial correlation:	LB(8)=6.4233	[0.6000]
	LM(2)=0.0349	[0.9828]
Heteroscedasticity:	ARCH(1)=0.0524	[0.8189]
	White(1)=14.895	[0.2474]
Stability:	RESET(2)=0.4219	[0.8097]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter estimation of the expected price level, that is the variable $cpi\_e\_f_t$ displayed in Figure 5.7.

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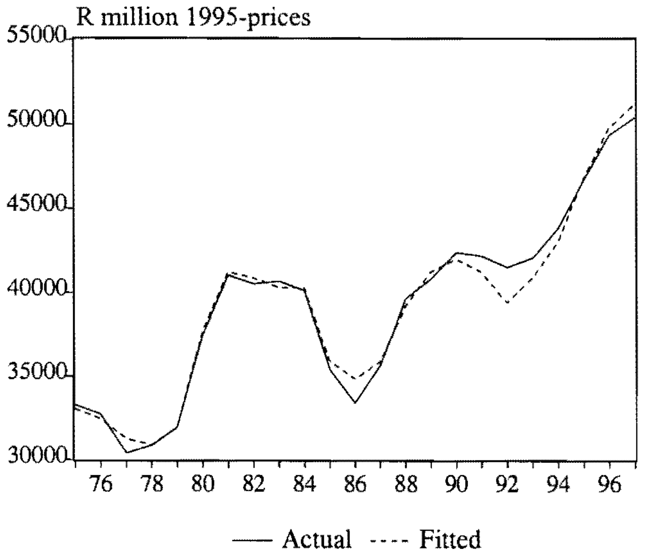
Figure 5.12 The overall dynamic fit of the durable private consumption expenditure function, excluding the interest rate variable from the specification



Based on diagnostic statistics, all three models appear to be statistically well specified. The dynamic fit of the first option, where the intercept coefficient was restricted to zero, was less than satisfactory. The third option, suggesting interest rate insensitivity on the part of the consumer, does not correspond with economic theory which states that, within the intertemporal optimisation framework, interest rates constitute a trade-off between current and future consumption. On this basis, interest rates certainly should play a role, especially in explaining durable consumption. On these grounds, the second option may be considered optimal. Model selection criteria, namely the adjusted R-squared statistic and the Akaike information criterion support this decision while the Schwarz Bayesian criterion favours the model with restricted intercept. This result is reported in Table 5.17.

The model of which the overall dynamic fit is depicted in Figure 5.11 will therefore be regarded as the baseline forecast for durable consumption against which the response characteristics of the system to exogenous shocks will be analysed. This result is reported in section 5.8.

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Table 5.17 Model selection criteria for the durable consumption expenditure function

Model	Adjusted R^2	AIC	SBC
Option 1: Restricted intercept	0.915837	-4.981926	-4.636341
Option 2: Unrestricted intercept	0.919821	-5.008005	-4.613051
Option 3: Exclusion of interest rates	0.895913	-4.769461	-4.423876

5.5.2.3 Non-durable private consumption expenditure

A long-run relationship was established between non-durable consumption expenditure and personal disposable income in the form c = f(yd), with variables in natural logarithmic form. No restrictions were placed on the income elasticity, but it is expected to be close to unity. The series for non-durable consumption expenditure and personal disposable income also constituted a cointegrating relationship.

(i) The cointegration equation

In order to test for cointegration between cndur and yd, the Johansen cointegration test was employed. First, the AIC and SBC were used to establish the appropriate lag length of the VAR model, i.e. the minimum lag length, which would ensure Gaussian errors for the error correction model. Both AIC and SBC suggest that a model with a lag length of 2 should be used. The result of these measures is reported in Table 5.18.

The values of the test statistics prove to be close for the two models. For reasons argued before, the cointegration test was conducted for both a VAR(1) and a VAR(2) model.

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Table 5.18 Selection of the order of the VAR: Non-durable private consumption expenditure.

Order of VAR	AIC	SBC
1	67.3497	66.0175
2	67.0948	64.4304

For a VAR with lag length equal to 1, the Johansen likelihood ratio test for the number of cointegrating relationships, based on the maximum eigenvalue and the trace of the stochastic matrix, suggests one cointegrating relationship at a 5 per cent significance level between the variables in the long-run relationship (the cointegration equation). This result is presented in Table 5.19 and the long-run cointegration relationship in Table 5.20. The specification, with constant and trend restricted to zero (no deterministic variables included in long-run relationship) yielded results in line with those hypothesised.

Table 5.19 Johansen test for the number of cointegrating relationships: non-durable private consumption expenditure

Cointegration LR test based on maximal eigenvalue of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	r=1	32.1893	11.0300	9.2800
r ≤ 1	r=2	3.5239	4.1600	3.0400
Cointegration LR test based on trace of the stochastic matrix				
Null	**Alternative**	**Statistic**	**95% Critical value**	**90% Critical value**
r=0	r ≥ 1	35.7132	12.3600	10.2500
r ≤ 1	r=2	3.5239	4.1600	3.0400

Note: VAR order = 1

Table 5.20 Estimate of cointegration equation

Dependent variable: $cndur_t$

Variables	**Cointegrating vector**
$cndur_t$	-1.3686 (-1.0)
yd_t	1.2861 (0.93969)
Sample Period: 1970-1997 Order of VAR = 1; r=1	

The above result suggests the presence of a long-run relationship of the form

$$cndur_t = 0.93969yd_t \ . \qquad (5.15)$$

The income elasticity therefore amounts to 0.93969, which is relatively close to unity, for the period under consideration. (For a VAR model with lag length of 2, a cointegrating relationship with income elasticity equal to 0.93969 was established, a result almost identical to the reported result of a VAR with a lag length of 1.)

The residual derived from the above, allows for the specification of the error correction model, reported in Table 5.21.

(ii) The error correction model

Variables contributing to the short-run dynamics of the system include current real disposable income, real financial wealth stock as well as relative prices (that is the ratio between prices of durables and non-durables) and a variable reflecting labour market conditions, namely the employment rate in the non-agricultural sector. The future expected price level was not significant in the explanation on non-durable consumption expenditure, nor did interest rates prove to be statistically significant. All variables were used in natural logarithmic form. The constant had a low t-score (t=-0.81), signalling statistical insignificance, but could not be dropped form the equation for the following reason. The specification should always include a constant in one form or another; just for scaling of the data, a constant should be included for the cointegrating vector. If the data has non-zero growth, this may be coming from an exogenous variable or from a drift term (constant) in an unrestricted ECM equation. The constant was therefore retained in this instance.

Diagnostic statistics suggest that the equation is statistically well-specified, and that a VAR with lag length of 1 was sufficient to render a white noise error term for the error correction model. Figures in square brackets denote probability values (the probability of falsely rejecting the null hypothesis of a zero restriction on the coefficient or diagnostic). BJ denotes the Bera-Jarque test for the normality assumption; LB and LM denote tests for serial correlation; Ramsey's RESET is a test for misspecification and ARCH and White denote tests for the homoskedasticity assumption. All test statistics are distributed χ^2, with figures in round brackets denoting degrees of freedom. Recursive estimates confirmed stability in the parameters.

The result of *ex post* simulation of the private non-durable consumption function is presented graphically in Figure 5.13. The fitted values once again constitute the base-line forecast against which the response characteristics of the system to exogenous income and wealth shocks will be analysed.

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Table 5.21 An error correction model for non-durable private consumption expenditure

Dependent variable: $\Delta cndur_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	-0.006294	0.007789	-0.808036
$resid_{t-1}$	-0.223873	0.059900	-3.737458
Δyd_t	0.272376	0.086796	3.138119
Δw_t	-0.102052	0.033484	-3.047824
$\Delta relp_t$	0.260865	0.085547	3.049375
$\Delta emplna_t$	0.373183	0.136430	2.735335

sample period (adjusted): 1971 to 1997

$R^2 = 0.753823$

$\bar{R}^2 = 0.695209$

s.e. = 0.013030

Normality:	BJ(2)=2.2545	[0.3239]
Serial correlation:	LB(8)=5.0631	[0.7510]
	LM(2)=1.3352	[0.5129]
Heteroscedasticity:	ARCH(1)=0.6945	[0.4046]
	White(1)=9.8576	[0.4531]
Stability:	RESET(2)=6.9559	[0.0309]

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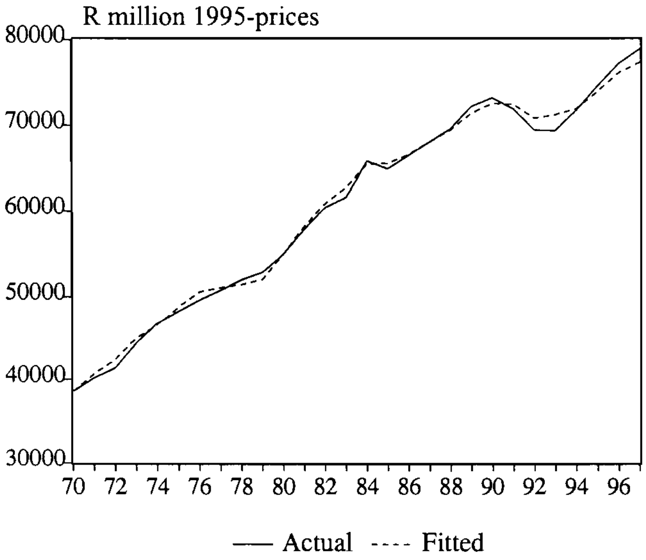
Figure 5.13 The overall dynamic fit of the non-durable private consumption expenditure function



5.6 LONG-RUN RELATIONSHIPS: COMPARISON OF RESULTS OBTAINED FROM JOHANSEN AND ENGLE-GRANGER PROCEDURES

In order to prevent cluttering of the reported results obtained for long-run relationships between consumption, income and wealth variables, results obtained with the Engle-Granger estimation technique are compared separately in this section with results obtained with the Johansen approach. These results are documented in Table 5.22. The Engle-Granger equilibrium relationships were not incorporated into error correction models, thus the third-step correction as suggested by Engle and Yoo (1989) was not conducted. EG-cointegration denotes the ADF unit root test statistic, which should be evaluated against the MacKinnon (1991:275) response surface critical values.

The results contained in Table 5.22 confirm the discussion in section 4.4.3, stating that even if a single cointegrating vector exist between variables in the long-run relationship, the results obtained with a single equation estimation technique would differ from the result obtained with the Johansen technique. The reason for this is, as stated before, that the Engle-Granger approach effectively ignores the short-run dynamics when estimating the

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long-run equilibrium relationship. Another reason for the difference in outcome pertains to the endo-exogenous division of variables. Any simultaneity between variables is disregarded in a single equation estimation approach. Charemza and Deadman (1997:150) noted in this regard that a rise in income (or wealth) would lead to a rise in consumption. Due to the income identity, however, it would be impossible to change the value of consumption without influencing income (or wealth). All three variables would thus be regarded as endogenous variables and be described as jointly dependent variables.

Table 5.22 Comparison of long-run relationships between the Johansen approach and the Engle-Granger procedure

Categories of consumption	**Johansen**	**Engle and Granger**
Total	$ctot_t = (1\text{-}0.19655)yd_t$ $+(0.19655)rw_t$ $+0.55926$	$ctot_t = (1\text{-}0.06699)yd_t$ $+(0.06699)rw_t$ $+0.15083$ EG-cointegration: -3.2710
Durable	$cdur_t = (1\text{-}0.24031)yd_t$ $+(0.24031)w_t$ -1.06480	$cdur_t = (1\text{-}0.47729)yd_t$ $+(0.47729)w_t$ -1.71432 EG-cointegration: -3.2013
Non-durable	$cndur_t = 0.93969yd_t$	$cndur_t = 0.928874yd_t$ EG-cointegration: -2.0592 *or* $cndur_t = 1.03218yd_t$ -1.22256 EG-cointegration: -2.4713

5.7 THE MODEL SOLUTION

Since stochastic estimation of total consumption is believed to be more reliable than that of consumption expenditure on services, expenditure on services was deterministically determined as the residual of the total and the other two categories. The quality of the in-sample fit is presented below as measured by the root-mean-squared-percentage-error (Klein *et al.* 1983:76):

$$\text{RMSPE} = \sqrt{\sum_{t=1}^{T}[(y_t - \hat{y}_t)/y_t]^2 / T} \times 100\,. \tag{5.16}$$

As expected, the root-mean-squared-error value for services is higher than for the categories stochastically estimated, but the measure is still within the 5 per cent acceptable range (*op. cit.*:76).

Table 5.23 RMSPE for behavioural equations

Private consumption expenditure categories			
Total	**Durable**	**Non-durable**	**Services**
0.81%	1.80%	1.15%	4.24%

5.8 RESPONSE CHARACTERISTICS OF ERROR CORRECTION MODELS

Deterministic analysis of the response characteristics of the model was conducted to test whether the short and long-run response characteristics correspond to theoretical priors and long-run equilibrium properties of the data. The process consists of conducting a dynamic baseline forecast for each behavioural equation. An exogenous shock is then applied to the income variable and subsequently to the wealth variable(s), and the adjustment path towards a new equilibrium outcome is determined.

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A shock of 10 per cent was considered to be significant and first, personal disposable income was allowed to rise 10 per cent above its actual level from 1977 onwards. This date was considered appropriate to commence with the exogenous shock, since the adjusted sample period over which the estimation and baseline forecast were conducted was from 1975 to 1997. The same procedure was repeated, applying an exogenous shock of 10 per cent to the financial wealth variable, with the associated effect on return to wealth, from 1977 onwards.

The expected response in total private consumption expenditure as a result of a sustained 10 per cent exogenous shock to personal disposable income would, for example, ultimately mean an adjustment towards a new equilibrium level, 8.03 per cent higher than would be expected without the exogenous shock. A 10 per cent increase in the wealth variable, on the other hand, would only result in a 1.97 per cent increase in total private consumption.

The expected multiplier effects as indicated by the respective elasticity values in the long-run cointegration equations and the actual outcome over the sample period are reported in Table 5.24 and Table 5.25.

Table 5.24 Expected and actual multiplier effect of a 10 per cent exogenous shock to personable disposable income from 1977 onwards

Categories of consumption	**Income elasticity**	**Expected result (% change in consumption)**	**Final value (% change in consumption)**
Total	0.80345	8.0345%	7.8748%
Durable	0.75969	7.5969%	7.5092%
Non-durable	0.93935	9.3935%	9.3223%

Table 5.25 Expected and actual multiplier effect of a 10 per cent exogenous shock to financial wealth from 1977 onwards

Categories of consumption	**Wealth elasticity**	**Expected result (% change in consumption)**	**Final value (% change in consumption)**
Total	0.19655	1.9655%	1.79748%
Durable	0.24031	2.4031%	2.3168%
Non-durable	–	0%	-0.0061%

Both durable and non-durable consumption expenditure categories reach their new equilibrium values (in accordance with the respective long-run coefficients of the cointegration equations) within the sample period. After an initial overshooting, adjustment proceeds along smooth paths towards the new equilibrium levels of expenditure, as illustrated by Figures 5.15 and 5.16. For total consumption, the new expected equilibrium value is not quite reached, but appears to be well within reach. The graphical representation of the adjustment path in Figure 5.14 indicates that the adjustment path is progressing well towards this value. The speed of adjustment is generally determined by the coefficient of the lagged residual of the long-run cointegration equation in the error correction model.

Vertical axes measures the difference in outcome of the baseline simulation and the simulation subjected to the exogenous shock, as percentage of the level of the dependent variable in the baseline outcome.

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Figure 5.14 Response characteristics of total private consumption expenditure

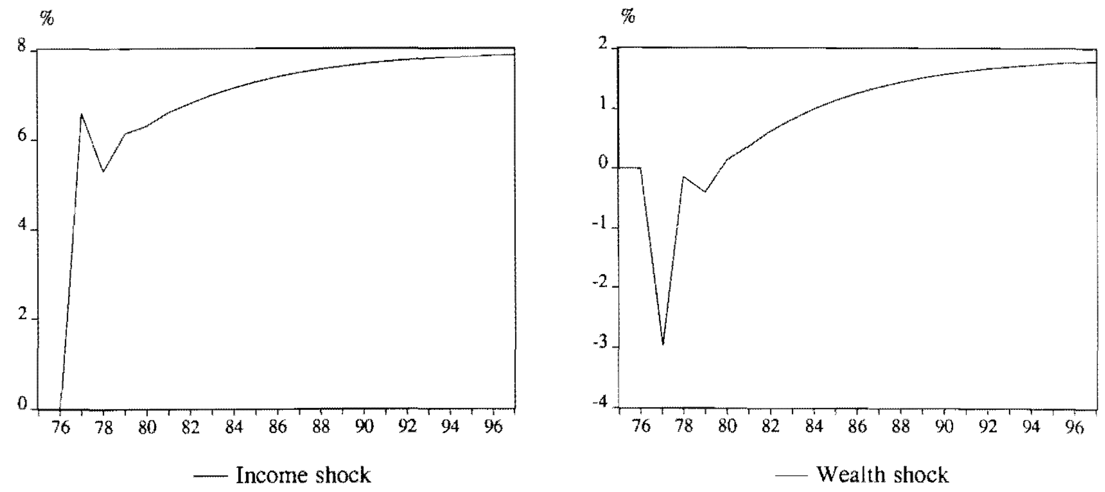


Figure 5.15 Response characteristics of durable private consumption expenditure

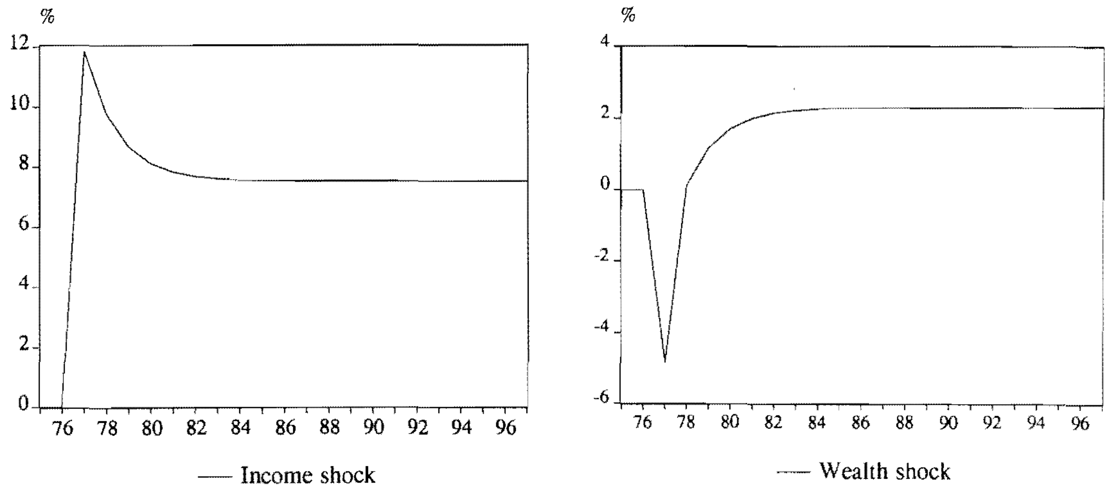
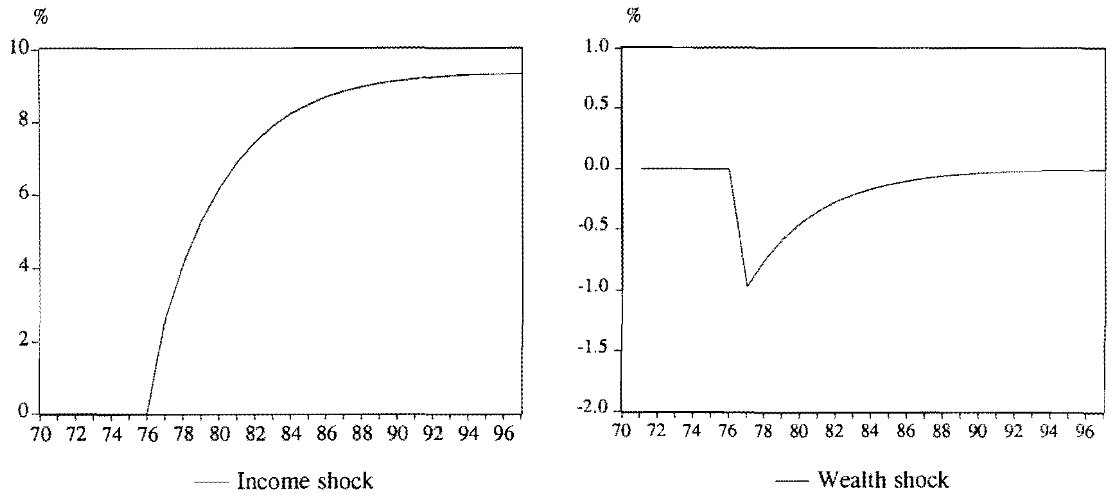


Figure 5.16 Response characteristics of non-durable private consumption expenditure



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5.9 CONCLUSION

This chapter tested the hypothesis that South African consumers are forward-looking with respect to prices when making consumption expenditure decisions. Furthermore, it was also tested whether the assumption that consumers learn through a boundedly rational learning process from expectations errors made in the past and that they find a way of increasing their knowledge about the true values of the parameters in the expectations rule, is correct.

The expectations rule was formulated in an attempt to capture the psychological learning process of intelligent economic agents. Variables influencing price expectations were taken to be the lagged observed price level, lagged interest rates and the lagged exchange rate. The most important variable in the information set proved to be the lagged price level. The time-varying coefficient of this variable also displayed the largest variation with an upward tendency in periods with high and increasing price levels and *visa versa*.

The price expectations variable was subsequently implemented in a set of behavioural equations, namely the private consumption expenditure functions. Consumption expenditure was disaggregated into durable consumption, non-durable consumption and services. Since it was believed that stochastic estimation of total consumption expenditure would be more reliable than that of services, expenditure on services was deterministically determined as the residual of the total and the other two categories.

Consumption, non-human (financial) wealth and current disposable income constituted a long-run equilibrium relationship in the case of total consumption expenditure and expenditure on durables. In the case of non-durable consumption, a long-run cointegration equation included only current disposable income as an explanatory variable. Variables that contributed towards explaining the short-run dynamics of the system included wealth stock, the return on wealth, current disposable income, interest rates, relative prices and a variable reflecting labour market conditions, namely the employment rate in the non-agricultural sector. Interest rates proved to be significant in the explanation of durable consumption only, while the employment rate variable was only included in the non-durable consumption function. Apart from the above, the one-period-ahead price expectations

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variable (the result from the Kalman filter estimation) was included in the behavioural functions to test for the role of forward-looking inflation in consumption expenditure decisions. This variable only proved significant in the durable and total private consumption expenditure functions.

Diagnostic testing proved that the behavioural equations were statistically well specified while deterministic analysis of the response characteristics of the models proved that short and long-run response characteristics corresponded to theoretical priors and long-run equilibrium properties of the data.

From the results reported in this chapter, it may be concluded that both the time-varying Kalman filter prediction for price expectations and the estimation of the error correction models for private consumption expenditure yielded satisfactory results.

CHAPTER 6

CONCLUSION

6.1 RESTATING THE OBJECTIVE OF THE STUDY

The principal objective of this study was to derive a model for private consumption expenditure in South Africa, in order to test a hypothesis comprising three components. First, it was tested whether consumers are forward-looking with respect to prices when considering consumption expenditure decisions. Modelling the price expectations formation process of consumers was therefore central in this study; citing Begg *et al.* (1991:568) in this regard: "Most economists accept that beliefs about the future are an important determinant of behaviour today".

Second, the hypothesis that consumers learn through a Kalman filter-based (boundedly rational learning) process when updating their expectations was tested. Learning as expectations formation mechanism implies intelligent economic agents who, although not fully informed, are able to assimilate information and learn about their environment as time progresses. It is therefore accepted that consumers have knowledge about the structure of the expectations rule, but do not know the parameters. The unobservable component in the model of price expectations formation is thus taken to be the coefficient vector of the expectations rule.

Third, the theoretical specification of the behavioural equations based on the forward-looking theories of consumption, in particular the life-cycle model of Modigliani and Brumberg, and Ando and Modigliani, and the permanent-income hypothesis of Friedman, was tested empirically. In practice, these theories express the notion that consumers are forward-looking and consider not only current disposable income when making consumption expenditure decisions – as dictated by Keynes's absolute-income hypothesis – but income over their entire life span. This implies that a wealth variable as well as an interest rate variable should enter the long-run equilibrium equation, in addition to the income variable.

6.2 THE STUDY

Consumption expenditure, for purposes of this study, was disaggregated into durable consumption, non-durable consumption and expenditure on services. The study commenced with an assessment of the socio-economic profile of the average South African consumer, whose profile bears important implications for *a priori* theorisation on consumer behaviour and price expectations formation. The most pronounced characteristic of the socio-economic profile of South Africa as depicted in Chapter 1, is an unequal income distribution, with the wealthiest 10 per cent of households' share in total income equal to 53 per cent. A large portion of the population is consequently living in poverty – 57 per cent of the population's income is less than an income level of US$1.80 per person per day. A contributing factor to this is the high unemployment level, partially due to the poor growth performance of the economy, but also to a relatively unproductive, unskilled labour force.

The above holds implications for consumer behaviour. *A priori* it was expected that total private consumption expenditure would, in the long run, be dependent upon personal disposable income as well as financial wealth. This would also be true for durable consumption expenditure, while non-durable consumption would in the long run be guided only by personal disposable income. This conviction was motivated by the assumption that a large portion of the population constrained by very low income levels, would spend virtually all their income on consumption and mostly on non-durables, with very little left to be utilised for wealth accumulation. Wealth was therefore *a priori* expected to play an insignificant role in their consumption expenditure decisions.

These consumers would also be subjected to liquidity constraints due to low income levels and limited or no access to credit. These consumers also normally have no savings; interest rates changes therefore were not expected to influence their consumption expenditure decisions. Liquidity constraints further disqualify these consumers from increasing current consumption to hedge against expected price increases. For this reason, interest rates and price expectations were not expected to be significant in explaining non-durable consumption, although they were considered important determinants for durable and total consumption expenditure. Lastly, variables reflecting labour market conditions were considered explanatory of consumption expenditure levels, particularly non-durable

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consumption, since adverse developments in the labour market often affect the unskilled workforce first. These workers' wages are likely to be low and mainly directed towards non-durable consumption.

The above, then, constituted the *a priori* theorisation on the information set of the behavioural equations. Price expectations were considered an important determinant of the short-run dynamic structure of durable private consumption expenditure as well as total private consumption expenditure.

In order to test the significance of price expectations in explaining consumption expenditure, and the hypothesis that consumers learn about price changes through a boundedly rational learning process, an expectations rule first had to be derived. The expectations rule was formulated in an attempt to capture the psychological learning process of intelligent economic agents as accurately as possible.

In setting up the expectations rule in this case, the application of price expectations to wage behaviour of countries in the global econometric model (GEM) (Barrel *et al.* 1994:174) was followed. The dependent variable in the expectations rule was taken to be the one-period-ahead consumer price level. The information set included consumer prices, lagged by one period, lagged interest rates and the exchange rate, lagged by one period. The only deviation from the GEM specification was the exclusion of the capacity utilisation variable. The motivation of this theoretical specification used in a South African context was the following: the specification, including lagged prices, interest rates and the exchange rate is an attempt to model the psychological expectations formation process of the (often unsophisticated) consumer. Given that 19 per cent of the population is illiterate (has not completed primary school) (Stats SA 1998), the adjustment of parameters of an expectations rule based on variables like capacity utilisation, the terms of trade, money supply and so forth, probably implies an unrealistically sophisticated consumer. Information about price changes and changes in interest rates and the exchange rate is perhaps more accessible to the average consumer than any other economic variables influencing price changes.

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The Kalman filter was applied to the expectations rule to obtain the time-varying parameters of the rule. Each of the coefficients, assumed to evolve according to a random walk with drift process, displayed a reasonable degree of variation over the period. The most important variable in the explanation of price expectations was the lagged price variable. This coefficient also displayed the largest degree of variation, an indication of a fairly rapid rate of learning with respect to this variable. An interesting observation regarding the evolution of the time-varying coefficient of lagged prices, was that it mimicked the actual price trend to a certain extent. The interpretation of this could be that in periods of high and rapidly increasing price levels, consumers continuously adjust this parameter of the rule upwards and, as soon as they realise that price levels are declining, they start adjusting the parameter downwards, leading to lower expected price levels.

The one-period-ahead price level obtained with the Kalman filter estimation process was the variable that was incorporated into the learning model of consumption expenditure. The Kalman filter result represented price expectations, which allowed the hypothesis that consumers consider price expectations when making consumption expenditure decisions to be tested empirically.

Total private consumption expenditure, durable consumption expenditure and non-durable consumption expenditure were determined stochastically while expenditure on services was determined as the residual of the total and the other two categories. Estimation was conducted by means of the Johansen technique, a multivariate cointegration technique. Empirical estimation of the behavioural equations proved that consumption, non-human (financial) wealth and current disposable income constitute a long-run equilibrium relationship in the case of total consumption expenditure. The same holds for expenditure on durables. In the case of non-durable consumption, a long-run cointegration equation included only current disposable income as an explanatory variable. Variables that contributed towards explaining the short-run dynamics of the system include wealth stock, the return on wealth, current disposable income, interest rates, relative prices and a variable reflecting labour market conditions, namely the employment rate in the non-agricultural sector. Interest rates proved to be significant in the explanation of durable consumption only, while the employment rate variable was only included in the non-durable consumption function. Apart from the above, the one-period-ahead price expectations

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variable (the result from the Kalman filter estimation) was included in the behavioural equations to test for the role of forward-looking inflation in consumption expenditure decisions. This variable proved significant in the case of durable and total private consumption expenditure.

6.3 CONCLUSION

The hypothesis that South African consumers consider price expectations when making consumption expenditure decisions has been validated. The fact that consumers may be regarded as intelligent agents who are able to assimilate information and learn from their environment as time progresses has been proven. The forward-looking theories of consumption further proved to be the appropriate model to use for modelling private consumption expenditure in South Africa, including personal disposable income and financial wealth as well as interest rates in the information set.

The large portion of the South African population that consumes at a subsistence level and the consumption of which is mainly directed towards non-durable consumption, accounts for the fact that non-durable consumption is not in the long run guided by financial wealth. Liquidity constraints on poor consumers also explain the insignificance of any interest rate or price expectations effect on non-durable consumption.

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