Inflation forecasting with rolling windows: An appraisal

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Abstract

We examine the performance of rolling windows procedures in forecasting inflation. We implement rolling windows augmented Dickey–Fuller (ADF) tests and then conduct a set of Monte Carlo experiments under stylized forms of structural breaks. We find that as long as the nature of inflation is either stationary or non-stationary, popular varying-length window techniques provide little advantage in forecasting over a conventional fixed-length window approach. However, we also find that varying-length window techniques tend to outperform the fixed-length window method under conditions involving a change in the inflation process from stationary to non-stationary, and vice versa. Finally, we investigate methods that can provide early warnings of structural breaks, a situation for which the available rolling windows procedures are not well suited.

KEYWORDS : Chow test, GARCH model, Markov switching model, Monte Carlo experiments, rolling windows

1 INTRODUCTION

The use of rolling windows has been shown to improve forecasting accuracy in the presence of structural breaks. [1] In this connection, a range of window selection techniques have been developed (see Inoue et al., 2017; Pesaran & Timmermann, 2007). Recently, however, Hall et al. (2023) were not able to find any improvement in forecasting accuracy using rolling window techniques, including those proposed by Pesaran and Timmermann (2007) and Inoue et al. (2017), compared with standard AR models with a fixed window size. Hall et al. (2023) conjectured that earlier studies finding that various window selection techniques had improved forecasting performance were based on the specific time series features of their data—for example, in the case of inflation forecasting, those studies were mainly focused on samples containing a combination of the non-stationary data generated from the 1970s and the 1980s and the stationary data generated from the 1990s, without taking account of the yet to be available stationary data generated from 2000 to 2020. In this paper, we assess that conjecture formally through a set of rolling window augmented Dickey–Fuller (ADF) tests and then a set of Monte Carlo experiments that assess the performance of these procedures under various forms of structural breaks. The Monte Carlo experiments indicate that a simple structural break in the constant is not sufficient to provide the varying-window techniques an advantage over a simple fixed window. However, when the nature of inflation changes from a non-stationary to a stationary process, and vice versa, there can be gains from using the varying-window techniques.

Apart from the issue of stationary versus non-stationary data, there remains a problem of being able to detect a break in a timely way so that the window selection procedure can be useful for policymakers. Varying-window techniques typically begin by using a standard structural break test to examine if a break has occurred. From a policy perspective, this procedure is not very useful because these tests only work well when the break is towards the middle of a sample period; they do not work well when there is a break at the very end of the period. Yet, the primary interest of a policymaker is to know if a break is occurring or has just occurred using the most recent data because the failure to react in a timely way can be costly. To give a recent example, the upsurge in inflation in 2021 and 2022 caught policy makers by surprise. As Giles, Romei, and Smith (2023, p. 3) put it: "many [policy makers] were late to spot just how big a problem this wave of inflation would prove." [2] In what follows, we assess three techniques that might have provided an early warning of a structural break in inflation during the 2021–2022 period.

The remaining structure of the paper is as follows. Section 2 provides a literature review on inflation forecasting using rolling window and window selection techniques. Section 3 considers the properties of inflation in the UK and the United States over the period from January 1960 to September 2022. Section 4 presents an extensive set of Monte Carlo experiments that investigate what type of structural change would give the varying-window techniques an advantage compared with other techniques. Section 5 investigates three methods of detecting a structural break at the very end of the sample. Section 6 concludes.

2 LITERATURE REVIEW

Many studies attribute forecasting failure to parameter instability, which can cause a lot of problems in macroeconomics forecasting and financial predictions if not properly dealt with (to name a few, Stock & Watson, 1996, 2007; Goyal & Welch, 2003; Koop & Potter, 2007; Clements & Hendry, 2008; Rossi, 2013). One effective way to tackle the issue, as suggested by Clements and Hendry (2006), is to resort to rolling window forecasting, a method which, in recent years, has been increasingly used (Clark & McCracken, 2009). Typically, the size of rolling window is either selected by an arbitrary number or by the past experience. For example, Stock and Watson (2007) used a rolling window of 40 quarters to forecast US quarter-on-quarter core inflation from 1970 to 2004. Koop and Korobilis (2011) used a rolling window of 72 months to forecast UK month-on-month and annual inflation and output growth from 1990 to 2008. Giraitis et al. (2013) adopted a rolling window of 20 or 30 quarters to forecast 97 US quarterly macroeconomic series from 1970 to 2008, including price level and money supply. Bańbura and Bobeica (2023) used a rolling window of 20 quarters to forecast the quarter-on-quarter inflation in the euro area from 1994 to 2018. Rossi and Inoue (2012), however, found that different window sizes may affect forecasting performances, suggesting that the window size could be important for forecasting accuracy.

Several approaches for selecting the optimal window size in a rolling regression have been proposed. A convenient choice is to simply use the observations after the last break (Clements & Hendry, 2006; Pesaran & Timmermann, 2007). But Pesaran and Timmermann (2005) found that including prebreak observations is more likely to improve forecasting performance as it reduces the small sample bias in the estimation, implying that the pre-break observations are informative. Pesaran and Timmermann (2007) argued that the optimal window length is a trade-off between the forecast bias from using pre-break observations and the forecast variance from shrinking the estimation sample. They developed a cross-validation method to select the number of pre-break observations to be included in the estimation sample. With such trade-off between the forecast bias and variance in mind, Inoue et al. (2017) have developed an alternative optimal window selection procedure. They found that their new technique works well when forecasting quarter-on-quarter US GDP growth and inflation with an initial rolling window of 100 quarters for the period between1984 and 2014. However, Hall et al. (2023) found that none of those methods above improves the forecasting accuracy against a fixed rolling window of 120 months in forecasting the monthly inflations in the United States, the euro area, and the UK during the period from 2010 to 2022. The authors speculated that the failure of window selection techniques is related with the time series property of the inflation series, with inflations in the United States and the UK being non-stationary in the 1970s and 1980s, but becoming stationary afterwards. The present paper provides a formal examination of the issue with a rolling ADF test and a large set of Monte Carlo experiments.

3 THE TIME SERIES PROPERTIES OF INFLATION

We begin by examining the time series properties of inflation in the United States and the UK. One important aspect of this analysis is the consequences of defining inflation as the month-on-month percentage change or the year-on-year percentage change—say, a particular month in 1 year over the corresponding month of the previous year. Typically, inflation is reported as an annual percentage change. Figures 1 and 2 show the annual percentage change in United States consumer prices and the UK consumer prices, respectively.

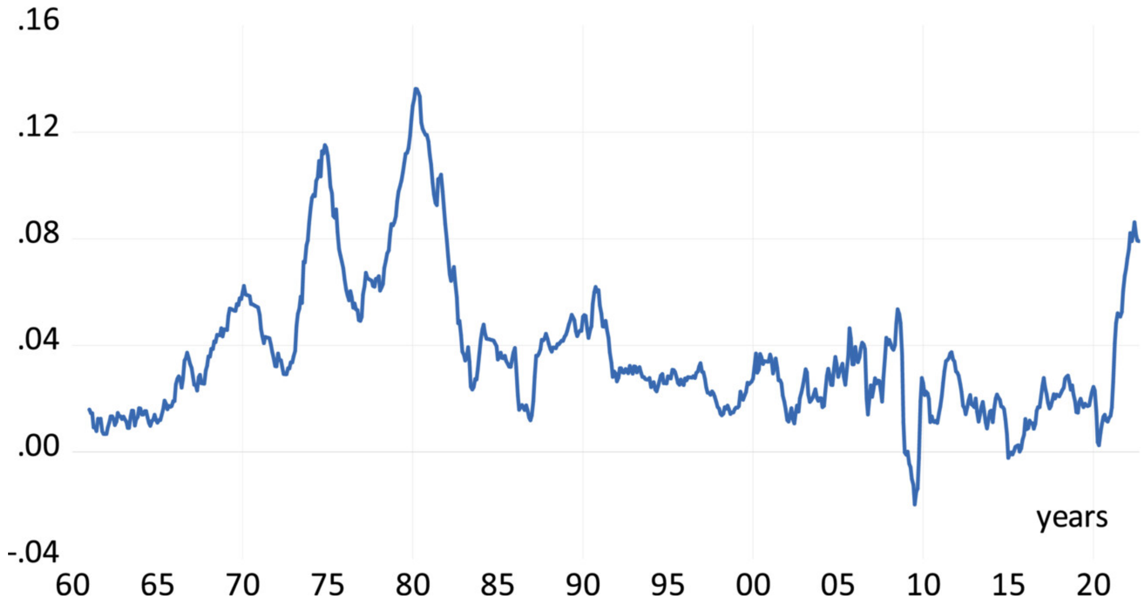

FIGURE 1. US CPI inflation; percentage annual change.

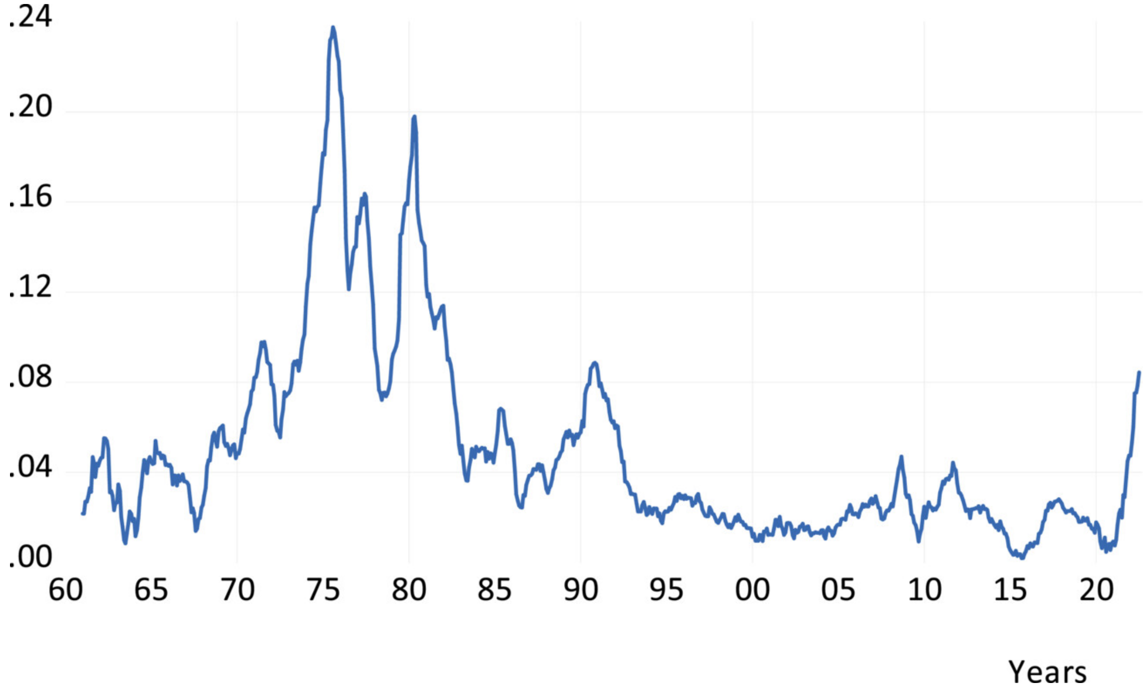

FIGURE 2. UK CPI inflation; percentage annual change.

While the data reported in Figures 1 and 2 are standard in terms of reporting inflation outcomes, they contain several characteristics which distinguish them.

1. If the data are monthly and if the year-on-year inflation rate is reported, then in a forecasting exercise, we will essentially already know the data embedded in the inflation rate in the prior 11 months. This circumstance makes the forecasting exercise easier, although in a spurious way.
2. If the error in a month-on-month inflation forecast is identically and independently distributed (IID), we would expect the error in an annual forecast to have an 11th order moving average error, which could be difficult to deal with. In other words, the annual rate of inflation will exhibit a high degree of serial correlation.
3. In testing the inflation rate for stationarity, the basic assumption of stationary tests is that the inflation rate should be measured as a first difference in the log of the price level. Creating an MA (11) error process would make testing unnecessarily complicated. In principle, the ADF and other stationarity tests can handle a high-order MA error process, but in practice, this high-order MA process likely to affect the results. This circumstance reflects the fact that the standard selection criteria will not select lags of sufficient length.

In what follows, we compare the consequence of using the standard measure of inflation with the month-on-month inflation rate. To illustrate the difference, Figures 3 and 4 show the month-on-month inflation rates for the United States and the UK, respectively.

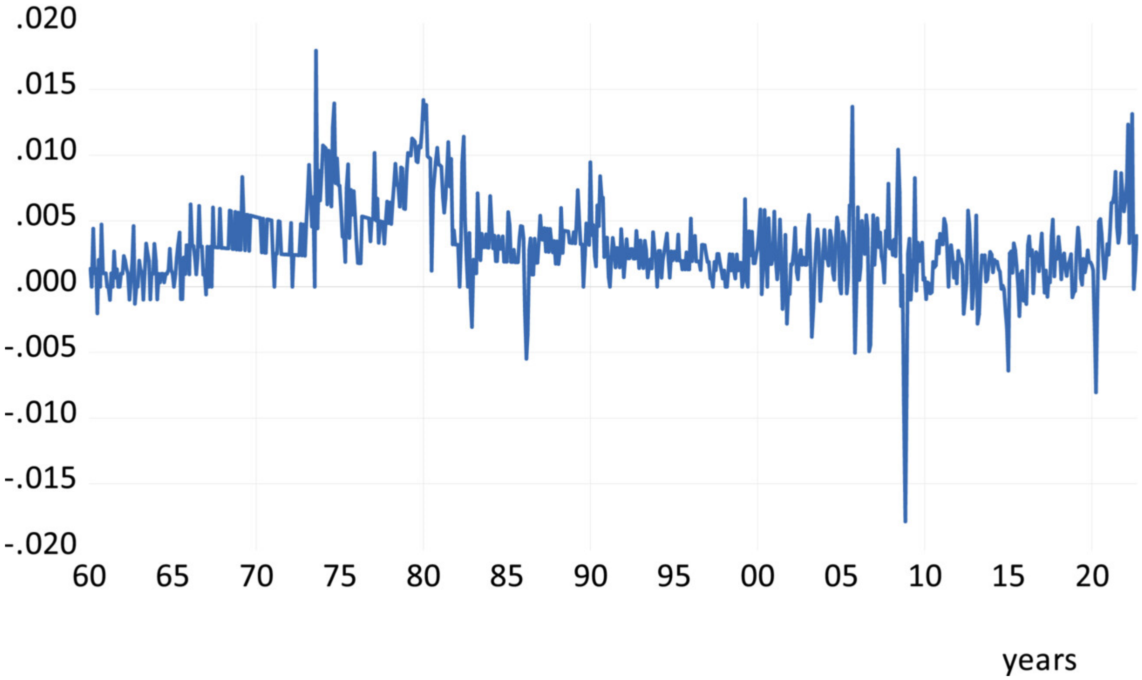

FIGURE 3. US CPI inflation; percentage month on month change.

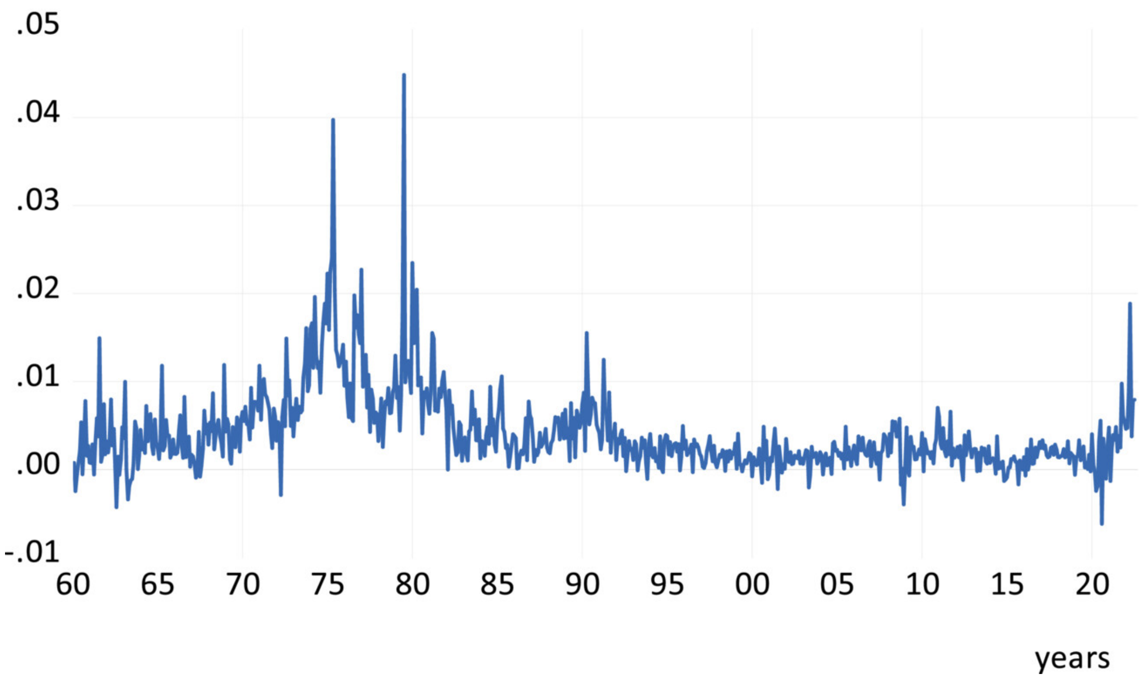

FIGURE 4. UK CPI inflation; percentage month on month change.

While in a sense the figures based on the year-on-year inflation rate and the month-on-month rate convey a similar story—for example, Figures 1 and 2 and Figures 3 and 4 each show the rise in inflation in the 1970s—they entail different time series properties. In what follows, we investigate whether this difference might be a factor underlying the results of the literature that employs rolling windows.

We should point out that there is little consensus as to how inflation should be defined. Inoue et al. (2017) explicitly state that they consider a month-on-month inflation rate, but other authors use annual inflation rates. The choice, however, can have important consequences for model specification. Consider the following example (We use a quarter-on-quarter example to simplify the notation, but an annual month-on-month exercise would be a simple extension). Suppose the true quarter-on-quarter inflation model is

$$p_t - p_{t-1} = \alpha_0 + \alpha_1 (p_{t-1} - p_{t-2}) + \varpi_t \quad (1)$$

where p_t is the log of the price level at period t on quarterly data, and $\overline{\omega}_t$ is a white noise error. If we estimate the model in an annual change form, then

$$p_t - p_{t-4} = (p_t - p_{t-1}) + (p_{t-1} - p_{t-2}) + (p_{t-2} - p_{t-3}) + (p_{t-3} - p_{t-4}) \quad (2)$$

Using (2), we can rewrite (1) as

$$p_t - p_{t-4} = \alpha_0 + a_1 (p_{t-1} - p_{t-2}) + (p_{t-1} - p_{t-2}) + (p_{t-2} - p_{t-3}) + (p_{t-3} - p_{t-4}) + \varpi_t \quad (3)$$

which shows that, as long as sufficient lags are included, the two forms should give equivalent error terms. However, if we were working with annual inflation, we would not normally include month-on-month changes in the lags but annual changes. Hence, we would be estimating a model of the form:

$$\begin{aligned} p_t - p_{t-4} = {} & \alpha_0 + \beta_1 (p_{t-1} - p_{t-5}) + \beta_2 (p_{t-2} - p_{t-6}) \\ & + \beta_3 (p_{t-3} - p_{t-7}) + \beta_4 (p_{t-4} - p_{t-8}) + \varpi *_t \quad (4) \end{aligned}$$

Here, we see that to get all the relevant price level terms in (3) into Equation (4) we would have to include unnecessary lags in the price level, which could lead to different terms that are insignificant. The annual inflation equation is then likely to be either (1) over-parameterized and, hence, inefficient, or (2) under-parameterized and, hence, biased.

We now turn to the suggestion of Hall et al. (2023) that previous studies have had in forecasting inflation may be due to the time series properties of the sample data. To test this conjecture, we developed a program to do a rolling window ADF test. We take 30-year rolling window samples starting from January 1970, with the last window ending in September 2022. This gives us 273 samples altogether. We then perform a sequence of ADF tests on these samples. For each test, the number of lags is selected using the Schwartz Bayesian criterion. We perform this procedure using both the monthly and annual changes. The dates in Figures 5-8 show the end date for each sample.

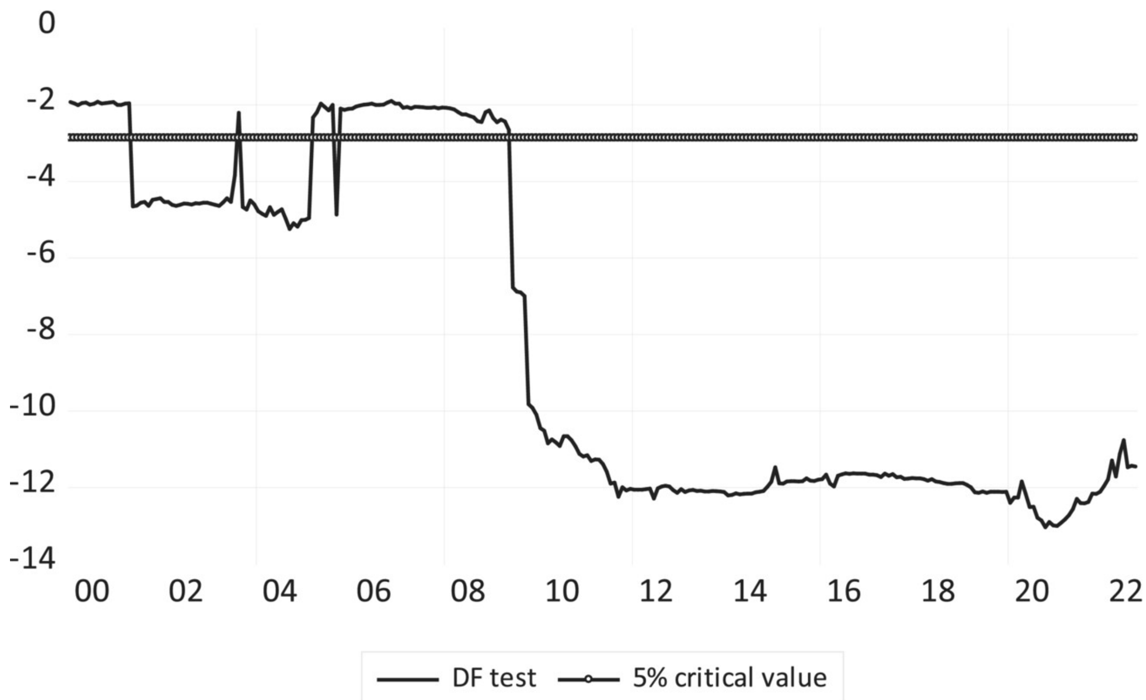

FIGURE 5. The ADF test for the US month on month inflation rate.

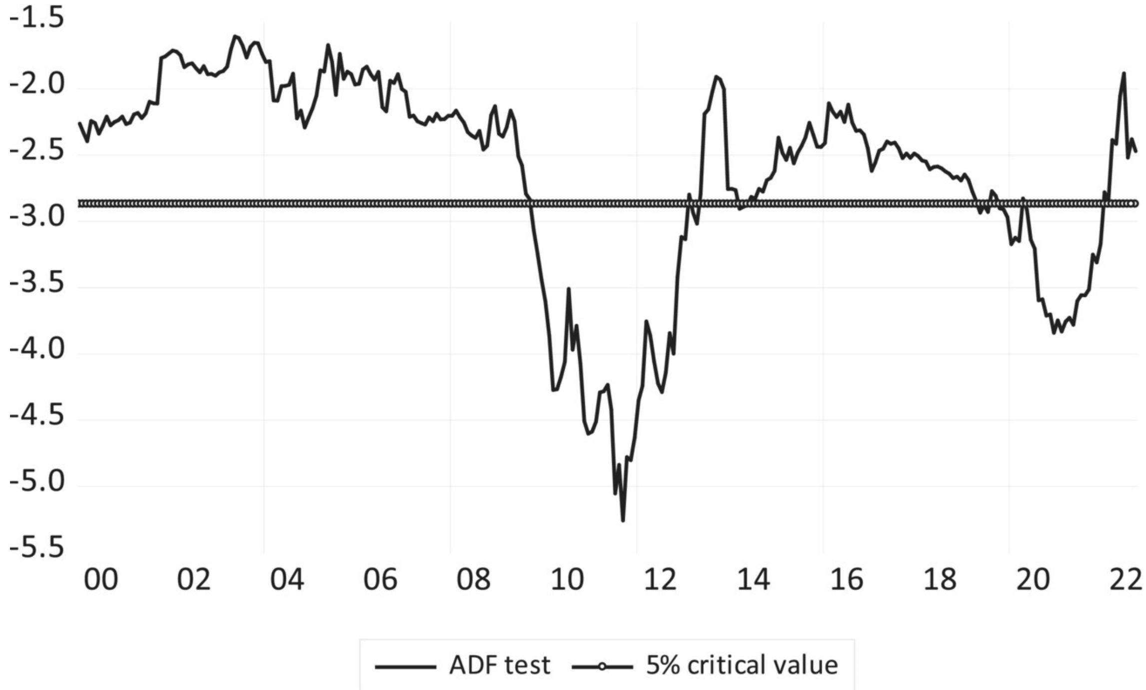

FIGURE 6. The ADF test for the US annual inflation rate.

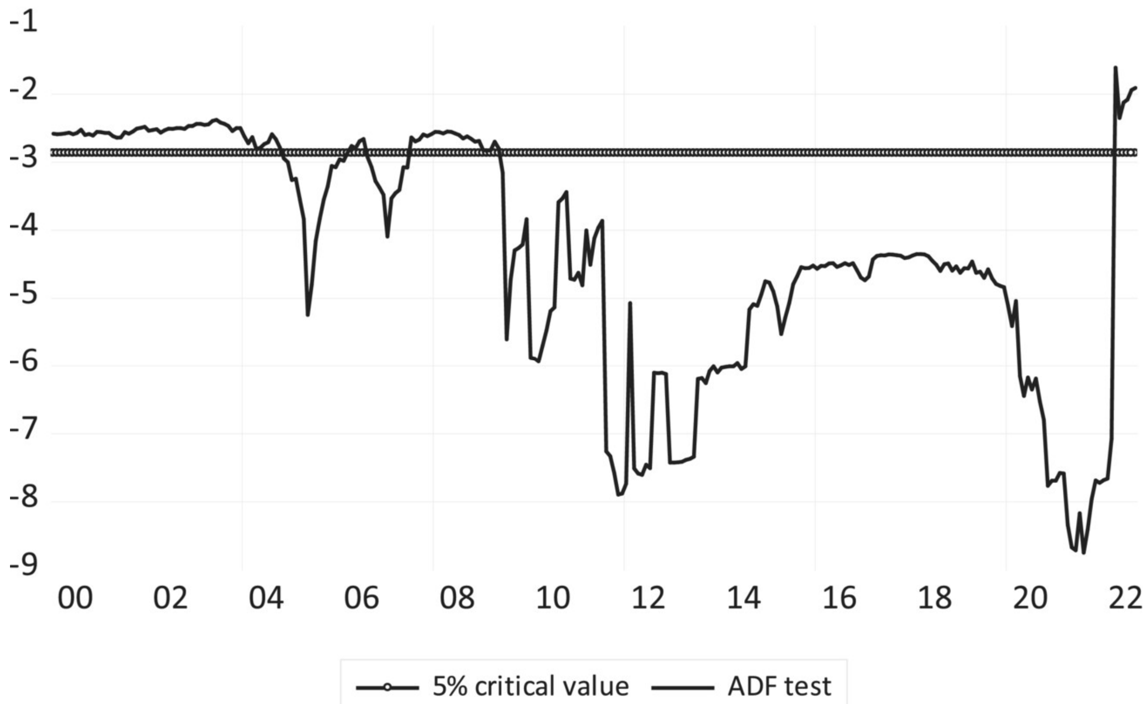

FIGURE 7. The ADF tests for the UK month on month inflation rate.

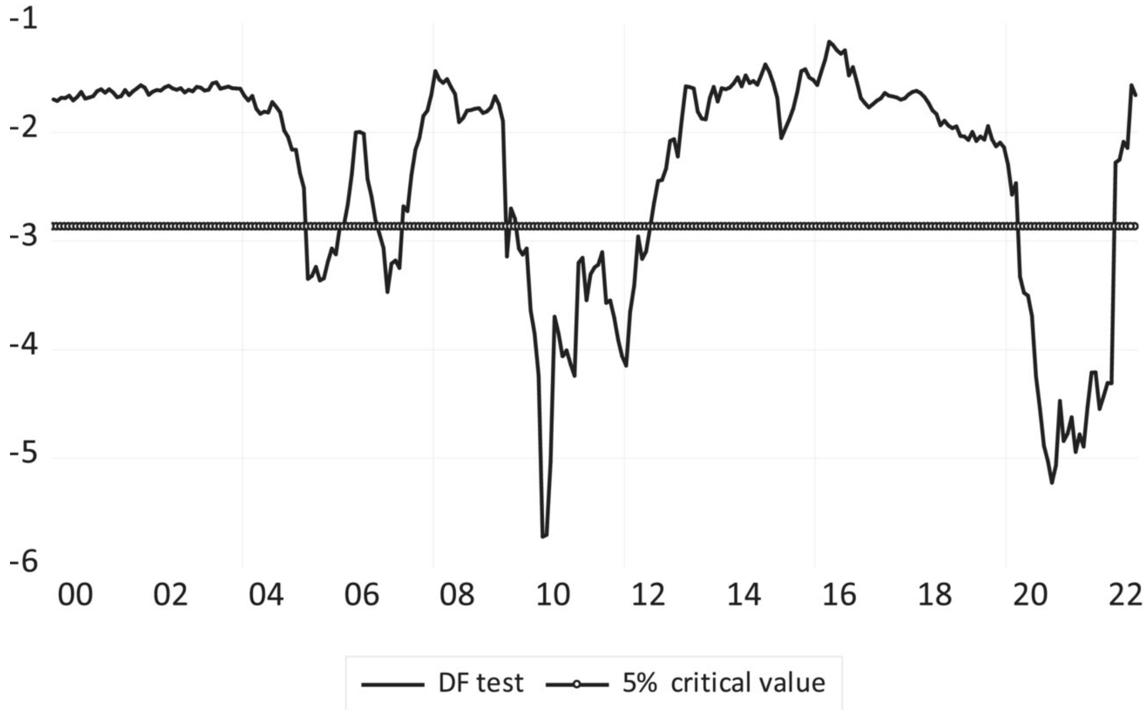

FIGURE 8. The ADF test for the annual UK inflation.

Figures 5 and 6 show the results for each sample using the month-on-month and the annual inflation rates, for the United States; Figures 7 and 8 show the comparable results for the UK.

Because at each stage of the moving window the number of lags in the ADF test is selected using the Schwartz criteria, there could potentially be different number of lags for each test. Comparing Figures 5 and 6, while there is some similarity between the figures, it is clear that the two sets of tests tell a different story. In the case of the month-on-month tests in Figure 5, inflation in the early part of the sample switches back and forth between stationarity and non-stationarity. After 2009, however, the test becomes strongly stationary and remains that way. Figure 6, however, suggests that the only period when US inflation was stationary was around 2009–2014. Apart from that period, it was mostly non-stationary.

Figures 7 and 8 provide a similar picture for the UK. Here, the annual change appears to be mostly non-stationary, with just a brief period of stationarity around 2009–2012, while the month on month series is mostly stationary after 2009, with a short return to non-stationarity, at the end of the period.

From Figures 5-8, it is evident that the time series properties of two 30-year inflation samples can be markedly different, especially between an earlier sample containing data from the 1970s and 1980s and a later sample that does not.

4 MONTE CARLO EXPERIMENTS

In what follows, we report the results of a set of Monte Carlo simulations. [3] Our objective is to identify the type of structural break or data definition (month-on-month or annual inflation) that leads to a superior performance for the varying window-selection approaches. We provide a range of possible scenarios including a stationary inflation rate, a non-stationary inflation rate, a break on the constant parameter of the underlying data generating process (DGP), and various switches between stationarity and non-stationarity in the DGP. We begin by defining the DGPs considered.

4.1 The DGPs

The DGP for the non-stationary period (modeled in second differences of the log CPI) is the following.

$$\Delta y_t = c_t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \epsilon_t \quad (5)$$

$$y_t = y_{t-1} + \Delta y_t \quad (6)$$

where c_t is a constant, which may undergo a structural break, and the β's are the coefficients; p is the AR lag length; ϵ_t is assumed to be identically, independently, and normally distributed with zero mean and unit variance; y_t represents inflation.

For the stationary period (modeled in first differences of the log CPI), we use the following equation to generate the series y_t, which mimics the data in the inflation level.

$$y_t = c_t + \sum_{i=1}^{p} \beta_i y_{t-i} + \epsilon_t \quad (7)$$

To cover a range of scenarios, we consider a total of eight DGPs; they are estimated on the basis of actual data for the United States and the UK. Table 1 [4] lists the eight DGPs with the parameters in the estimated equations rounded to three decimal places.

TABLE 1. DGPs based on the US and UK data.

DGP	Description	Parameters
1	Monthly change for non-stationary period in the United States.	$c = 0$ and β = {−0.574, −0.336, −0.336, −0.325, −0.206, −0.147}
2	Monthly change for stationary period in the United States.	c = 0.001 and β = {0.491, −0.171}
3	Annual change for non-stationary period in the United States.	c = 0 and β = {0.303, 0.234, −0.088, 0.010, 0.150, −0.012, 0.042, 0.032, 0.189, 0.057, 0.056, −0.403}
4	Annual change for stationary period in the United States.	c = 0.001 and β = {1.404, −0.620, 0.157}
5	Monthly change for non-stationary period in the UK.	c = 0 and β = {−0.590, −0.381, −0.283, −0.272, −0.252}
6	Monthly change for stationary period in the UK.	c = 0 and β = {0.181, 0.189, 0.167, 0.169}
7	Annual change for non-stationary period in the UK.	c = 0 and β = {0.353, 0.223, 0.037, −0.082, −0.030, 0.138}
8	Annual change for stationary period in the UK.	c = 0 and β = {1.050, 0.097, −0.109, 0.020, −0.104, 0.001, 0.132, −0.104, −0.004, −0.059, 0.182, −0.125}

Note: DGP, data generating process.

For each simulation, 240 + 200 + p observations are generated with zero initial values for y_1 to y_p. Then, the first 200 + p observations are discarded. Hence, each series has T = 240 observations, which is equivalent to 20 years of monthly data. When there is no break, $c_t = c$ for all t. In the case of a structural break at time $T_b < T$, $c_t = c_1$ for $t \leq T_b$, and $c_t = c_2$ for $t \geq T_b + 1$, where T_b = 120. The values of c_1 and c_2 vary in different cases, and are reported in Section 4.3.

4.2 Estimation and forecasting

For each DGP, we estimate AR(p) models in two specifications. First, we estimate Equation (5), which is the first difference specification (change of inflation), and we generate forecasts of Δy_t. Second, we estimate Equation (7), which is the level specification (inflation), and we generate forecasts of y_t. Note that when the series is non-stationary (in DGPs 1, 3, 5, 7), we are estimating an AR model with non-stationary variables. The lag length p is selected by either Akaike information criterion (AIC) or Bayesian information criterion (BIC); the maximum number of lags allowed is eight for all simulations.

Given the simulated series, we perform a one-step ahead rolling window forecasting exercise, which is commonly used in the literature. For example, given 240 observations (20 years of monthly data), we use the first 120 observations to estimate the model and forecast the 121st observation. Then we add the 121st observation and discard the first observation (so the estimation sample size remains at 120) to forecast the 122nd observation. As the window moves, we produce the forecasts for the 121st to 240th observations and the forecast errors associated with each observation. This is the standard rolling window forecasting procedure with a fixed-window size, typically used in the literature. We call this "Fixed" in what follows.

Apart from this standard rolling window, we also use various window selection techniques developed by Pesaran and Timmermann (2007) and Inoue et al. (2017). We include the post-break method of Pesaran and Timmermann (2007) (therefore, "Post-break"), two cross-validation methods of Pesaran and Timmermann (2007) (one with estimated break dates, "PTCV estimated break", and the other with unknown break dates, "PTCV unknown break"), and the optimal window selection method of Inoue et al. (2017) (therefore "IJR"). [5] For each sub-sample period, we first use the Bai and Perron's (1998) parameter constancy test to test for parameter breaks. If there is no break, then we use the whole sample of 120 observations. Otherwise, we apply the window selection technique accordingly.

Following the forecasting literature, we compare our forecasts with a standard random walk (RW) benchmark. For each simulation, we compute the ratios of the mean squared forecast errors (MSFEs) of AR forecasts based on window sizes selected by five different methods, respectively, over the MSFE of RW forecast. Then, we calculate the average for each ratio in the 5000 simulations. An average ratio that is smaller than one means the forecasting model beats the random walk. The lower the average ratio, the more accurate the forecast. To save space, we call the average ratio MSFE henceforth.

4.3 Simulation results in standard DGPs

We report the MSFEs ratios in the various simulation setups across the DGPs (noted above) in Tables 2–49. The ratio is rounded to three decimal places with the smallest ratio in each group highlighted in bold.

TABLE 2. DGP1, monthly change for non-stationary period in the United States, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.803**	0.885	0.814	0.813	0.839
No break BIC	0.836	0.867	0.834	0.833	0.844
Small break AIC	**0.859**	0.949	0.870	0.868	0.896
Small break BIC	0.899	0.948	0.901	0.899	0.912
Large break AIC	**0.883**	0.964	0.893	0.891	0.919
Large break BIC	0.907	0.944	0.908	0.906	0.915

Note: data generating process.

TABLE 3. DGP1, monthly change for non-stationary period in the United States, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.283**	0.300	0.285	0.285	0.289
No break BIC	0.290	0.296	0.291	0.291	0.292
Small break AIC	**0.302**	0.319	0.304	0.303	0.309
Small break BIC	0.305	0.311	0.306	0.306	0.307
Large break AIC	0.337	0.351	0.337	0.336	0.344
Large break BIC	0.333	0.337	0.333	**0.332**	0.334

Note: DGP, data generating process.

TABLE 4. DGP1.1, monthly change for non-stationary period in the United States, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.744**	0.824	0.752	0.751	0.767
No break BIC	0.764	0.804	0.769	0.768	0.775
Small break AIC	**0.807**	0.895	0.816	0.815	0.834
Small break BIC	0.825	0.876	0.832	0.831	0.840
Large break AIC	0.876	0.953	0.881	0.880	0.909
Large break BIC	0.877	0.913	0.877	**0.875**	0.887

TABLE 5. DGP1.1, monthly change for non-stationary period in the United States, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.283**	0.300	0.285	0.285	0.289
No break BIC	0.290	0.296	0.291	0.291	0.292
Small break AIC	**0.301**	0.320	0.304	0.303	0.308
Small break BIC	0.305	0.311	0.306	0.306	0.307
Large break AIC	0.337	0.351	0.336	0.336	0.343
Large break BIC	0.333	0.337	0.332	**0.332**	0.334

TABLE 6. DGP2, monthly change for stationary period in the United States, forecasting in level.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.738	0.778	0.743	0.742	0.752
No break BIC	**0.720**	0.731	0.722	0.722	0.724
Small break AIC	0.786	0.829	0.790	0.789	0.802
Small break BIC	0.768	0.781	0.769	**0.768**	0.771
Large break AIC	0.862	0.899	0.859	0.857	0.877
Large break BIC	0.863	0.828	0.827	**0.826**	0.827

TABLE 7. DGP2, monthly change for stationary period in the United States, forecasting in first difference.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.358**	0.380	0.360	0.360	0.365
No break BIC	0.370	0.379	0.371	0.371	0.373
Small break AIC	**0.360**	0.383	0.362	0.362	0.367
Small break BIC	0.371	0.381	0.373	0.372	0.374
Large break AIC	**0.366**	0.389	0.369	0.368	0.373
Large break BIC	0.377	0.385	0.378	0.378	0.380

TABLE 8. DGP2.1, monthly change for stationary period in the United States, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.738	0.778	0.743	0.742	0.752
No break BIC	**0.720**	0.731	0.722	0.722	0.724
Small break AIC	0.786	0.831	0.791	0.790	0.803
Small break BIC	0.768	0.781	0.768	**0.768**	0.771
Large break AIC	0.862	0.899	0.860	0.858	0.877
Large break BIC	0.864	0.829	0.828	**0.826**	0.828

TABLE 9. DGP2.1, monthly change for stationary period in the United States, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.358**	0.380	0.360	0.360	0.365
No break BIC	0.370	0.379	0.371	0.371	0.373
Small break AIC	**0.361**	0.383	0.362	0.362	0.367
Small break BIC	0.372	0.381	0.373	0.373	0.375
Large break AIC	**0.367**	0.389	0.369	0.369	0.373
Large break BIC	0.377	0.386	0.378	0.378	0.380

TABLE 10. DGP3, annual change for non-stationary period in the United States, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.830**	0.938	0.846	0.843	0.891
No break BIC	0.842	0.909	0.856	0.853	0.882
Small break AIC	**0.510**	0.583	0.520	0.518	0.548
Small break BIC	0.519	0.558	0.525	0.523	0.540
Large break AIC	**0.233**	0.267	0.238	0.237	0.252
Large break BIC	0.240	0.254	0.242	0.240	0.248

TABLE 11. DGP3, annual change for non-stationary period in the United States, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.696**	0.755	0.703	0.703	0.723
No break BIC	0.707	0.736	0.712	0.711	0.718
Small break AIC	**0.727**	0.789	0.733	0.732	0.756
Small break BIC	0.752	0.776	0.750	0.750	0.759
Large break AIC	**0.773**	0.840	0.780	0.779	0.808
Large break BIC	0.817	0.825	0.802	0.801	0.812

TABLE 12. DGP3.1, annual change for non-stationary period in the United States, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.204**	0.239	0.210	0.209	0.220
No break BIC	0.207	0.227	0.212	0.211	0.217
Small break AIC	**0.487**	0.561	0.499	0.497	0.525
Small break BIC	0.497	0.538	0.505	0.503	0.519
Large break AIC	**0.864**	0.959	0.875	0.871	0.924
Large break BIC	0.896	0.937	0.890	0.888	0.921

TABLE 13. DGP3.1, annual change for non-stationary period in the United States, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.696**	0.755	0.703	0.703	0.723
No break BIC	0.707	0.736	0.712	0.711	0.718
Small break AIC	**0.727**	0.790	0.734	0.733	0.756
Small break BIC	0.753	0.777	0.751	0.750	0.759
Large break AIC	**0.774**	0.842	0.781	0.779	0.809
Large break BIC	0.817	0.825	0.802	0.801	0.812

TABLE 14. DGP4, annual change for stationary period in the United States, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.857	0.950	0.868	0.866	0.892
No break BIC	**0.839**	0.887	0.849	0.847	0.860
Small break AIC	0.890	0.981	0.902	0.900	0.929
Small break BIC	**0.876**	0.919	0.883	0.881	0.895
Large break AIC	0.899	0.985	0.912	0.908	0.943
Large break BIC	**0.886**	0.916	0.891	0.887	0.903

TABLE 15. DGP4, annual change for stationary period in the United States, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.705	0.742	0.708	0.708	0.716
No break BIC	**0.690**	0.698	0.692	0.692	0.693
Small break AIC	0.715	0.752	0.718	0.718	0.726
Small break BIC	**0.699**	0.707	0.701	0.700	0.702
Large break AIC	0.743	0.787	0.748	0.748	0.757
Large break BIC	**0.723**	0.736	0.726	0.725	0.728

TABLE 16. DGP4.1, annual change for stationary period in the United States, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.857	0.950	0.868	0.866	0.892
No break BIC	**0.839**	0.887	0.849	0.847	0.860
Small break AIC	0.891	0.981	0.902	0.900	0.930
Small break BIC	**0.877**	0.920	0.884	0.882	0.896
Large break AIC	0.900	0.984	0.913	0.909	0.943
Large break BIC	**0.886**	0.916	0.891	0.887	0.903

TABLE 17. DGP4.1, annual change for stationary period in the United States, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.705	0.742	0.708	0.708	0.716
No break BIC	**0.690**	0.698	0.692	0.692	0.693
Small break AIC	0.715	0.753	0.719	0.718	0.727
Small break BIC	**0.699**	0.707	0.700	0.700	0.701
Large break AIC	0.743	0.786	0.748	0.747	0.757
Large break BIC	**0.722**	0.735	0.725	0.725	0.727

TABLE 18. DGP5, monthly change for non-stationary period in the UK, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.780**	0.859	0.791	0.790	0.816
No break BIC	0.809	0.848	0.812	0.811	0.824
Small break AIC	**0.833**	0.918	0.844	0.842	0.869
Small break BIC	0.863	0.914	0.868	0.866	0.880
Large break AIC	**0.853**	0.929	0.862	0.860	0.888
Large break BIC	0.876	0.914	0.879	0.877	0.887

TABLE 19. DGP5, monthly change for non-stationary period in the UK, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.274**	0.290	0.276	0.276	0.280
No break BIC	0.278	0.285	0.279	0.279	0.281
Small break AIC	**0.293**	0.310	0.295	0.295	0.300
Small break BIC	0.296	0.303	0.297	0.297	0.299
Large break AIC	0.327	0.342	0.327	**0.326**	0.334
Large break BIC	0.329	0.331	0.327	0.327	0.329

TABLE 20. DGP5.1, monthly change for non-stationary period in the UK, forecasting in level.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.719**	0.792	0.727	0.726	0.741
No break BIC	0.731	0.770	0.736	0.736	0.743
Small break AIC	**0.785**	0.867	0.794	0.793	0.812
Small break BIC	0.797	0.847	0.804	0.803	0.813
Large break AIC	**0.855**	0.932	0.860	0.858	0.888
Large break BIC	0.861	0.899	0.861	0.859	0.874

TABLE 21. DGP5.1, monthly change for non-stationary period in the UK, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.274**	0.290	0.276	0.276	0.280
No break BIC	0.278	0.285	0.279	0.279	0.281
Small break AIC	**0.293**	0.310	0.295	0.295	0.300
Small break BIC	0.296	0.303	0.297	0.297	0.298
Large break AIC	0.327	0.342	0.326	**0.326**	0.334
Large break BIC	0.329	0.331	0.327	0.326	0.328

TABLE 22. DGP6, monthly change for stationary period in the UK, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.643**	0.695	0.650	0.649	0.663
No break BIC	0.646	0.673	0.650	0.650	0.657
Small break AIC	**0.681**	0.734	0.688	0.687	0.704
Small break BIC	0.687	0.709	0.687	0.686	0.695
Large break AIC	**0.727**	0.781	0.735	0.733	0.756
Large break BIC	0.734	0.750	0.732	0.730	0.741

TABLE 23. DGP6, monthly change for stationary period in the UK, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.226	0.237	0.227	0.227	0.229
No break BIC	**0.224**	0.227	0.225	0.225	0.225
Small break AIC	0.229	0.239	0.230	0.230	0.232
Small break BIC	**0.226**	0.229	0.227	0.227	0.227
Large break AIC	0.236	0.247	0.237	0.237	0.239
Large break BIC	**0.232**	0.235	0.233	0.233	0.233

TABLE 24. DGP6.1, monthly change for stationary period in the UK, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	**0.643**	0.695	0.650	0.649	0.663
No break BIC	0.646	0.673	0.650	0.650	0.657
Small break AIC	**0.682**	0.735	0.689	0.688	0.705
Small break BIC	0.687	0.710	0.688	0.687	0.695
Large break AIC	**0.727**	0.782	0.736	0.734	0.756
Large break BIC	0.734	0.750	0.733	0.731	0.741

TABLE 25. DGP6.1, monthly change for stationary period in the UK, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.226	0.237	0.227	0.227	0.229
No break BIC	**0.224**	0.227	0.225	0.225	0.225
Small break AIC	0.229	0.240	0.230	0.230	0.232
Small break BIC	**0.226**	0.229	0.227	0.227	0.227
Large break AIC	0.236	0.247	0.237	0.237	0.240
Large break BIC	**0.232**	0.235	0.233	0.233	0.233

TABLE 26. DGP7, annual change for non-stationary period in the UK, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.819	0.936	0.834	0.832	0.874
No break BIC	**0.815**	0.878	0.829	0.826	0.849
Small break AIC	**0.393**	0.451	0.400	0.399	0.417
Small break BIC	0.394	0.420	0.397	0.396	0.404
Large break AIC	**0.154**	0.175	0.156	0.156	0.164
Large break BIC	0.156	0.162	0.155	0.155	0.158

TABLE 27. DGP7, annual change for non-stationary period in the UK, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.740	0.797	0.746	0.746	0.760
No break BIC	**0.732**	0.756	0.737	0.736	0.741
Small break AIC	0.787	0.848	0.793	0.792	0.810
Small break BIC	0.792	0.805	0.786	**0.786**	0.792
Large break AIC	0.844	0.907	0.850	0.848	0.873
Large break BIC	0.867	0.851	0.844	**0.842**	0.848

TABLE 28. DGP7.1, annual change for non-stationary period in the UK, forecasting in levels.

	Fixed	**Post-break**	**PTCV unknown break**	**PTCV estimated break**	**IJR**
No break AIC	0.125	0.145	0.128	0.127	0.132
No break BIC	**0.125**	0.136	0.127	0.127	0.129
Small break AIC	**0.348**	0.401	0.354	0.353	0.367
Small break BIC	0.349	0.375	0.353	0.352	0.359
Large break AIC	**0.849**	0.957	0.859	0.856	0.901
Large break BIC	0.865	0.897	0.854	0.852	0.881

TABLE 29. DGP7.1, annual change for non-stationary period in the UK, forecasting in first differences.

	Fixed	**Post-break**	**PTCV unknown break**	**PTCV estimated break**	**IJR**
No break AIC	0.740	0.797	0.746	0.746	0.760
No break BIC	**0.732**	0.756	0.737	0.736	0.741
Small break AIC	0.786	0.848	0.793	0.792	0.810
Small break BIC	0.792	0.805	0.786	**0.785**	0.791
Large break AIC	0.843	0.905	0.849	0.847	0.872
Large break BIC	0.866	0.851	0.844	**0.842**	0.848

TABLE 30. DGP8, annual change for stationary period in the UK, forecasting in levels.

	Fixed	**Post-break**	**PTCV unknown break**	**PTCV estimated break**	**IJR**
No break AIC	1.018	1.161	1.036	1.033	1.084
No break BIC	**1.016**	1.091	1.036	1.032	1.062
Small break AIC	**1.002**	1.134	1.021	1.018	1.068
Small break BIC	1.007	1.060	1.019	1.015	1.044
Large break AIC	**0.888**	1.000	0.905	0.901	0.952
Large break BIC	0.911	0.932	0.909	0.905	0.934

TABLE 31. DGP8, annual change for stationary period in the UK, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.562	0.602	0.566	0.566	0.576
No break BIC	**0.555**	0.572	0.558	0.558	0.561
Small break AIC	0.580	0.625	0.586	0.585	0.597
Small break BIC	**0.578**	0.599	0.582	0.581	0.586
Large break AIC	**0.620**	0.677	0.627	0.626	0.642
Large break BIC	0.630	0.657	0.631	0.631	0.638

TABLE 32. DGP8.1, annual change for stationary period in the UK, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	1.018	1.161	1.036	1.033	1.084
No break BIC	**1.016**	1.091	1.036	1.032	1.062
Small break AIC	**1.002**	1.133	1.021	1.017	1.068
Small break BIC	1.006	1.058	1.019	1.015	1.043
Large break AIC	**0.887**	0.998	0.903	0.899	0.949
Large break BIC	0.910	0.932	0.907	0.904	0.933

TABLE 33. DGP8.1, annual change for stationary period in the UK, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
No break AIC	0.562	0.602	0.566	0.566	0.576
No break BIC	**0.555**	0.572	0.558	0.558	0.561
Small break AIC	0.581	0.625	0.586	0.585	0.597
Small break BIC	**0.578**	0.599	0.582	0.582	0.586
Large break AIC	**0.621**	0.678	0.628	0.627	0.643
Large break BIC	0.631	0.658	0.632	0.632	0.639

TABLE 34. DGP1-DGP2, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.990	1.108	0.971	0.967	1.018
BIC	0.981	0.925	0.924	**0.921**	0.934

TABLE 35. DGP1-DGP2, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.461	0.532	0.466	0.466	0.479
BIC	**0.458**	0.475	0.461	0.461	0.465

TABLE 36. DGP2-DGP1, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.882	0.946	0.881	**0.879**	0.904
BIC	0.925	0.904	0.889	0.888	0.892

TABLE 37. DGP2-DGP1, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	**0.292**	0.311	0.294	0.294	0.298
BIC	0.296	0.305	0.297	0.297	0.299

TABLE 38. DGP3-DGP4, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.811	0.881	0.808	0.804	0.854
BIC	0.788	0.801	0.779	**0.776**	0.800

TABLE 39. DGP3-DGP4, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.865	0.961	0.870	0.870	0.895
BIC	0.834	0.872	**0.832**	0.833	0.842

TABLE 40. DGP4-DGP3, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	**0.892**	0.997	0.903	0.901	0.941
BIC	0.893	0.957	0.903	0.901	0.924

TABLE 41. DGP4-DGP3, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.730	0.786	0.735	0.734	0.750
BIC	**0.723**	0.749	0.727	0.726	0.731

TABLE 42. DGP5-DGP6, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.761	0.818	0.763	0.760	0.794
BIC	0.754	0.763	0.747	**0.745**	0.761

TABLE 43. DGP5-DGP6, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.260	0.279	0.262	0.261	0.266
BIC	**0.254**	0.260	0.255	0.255	0.256

TABLE 44. DGP6-DGP5, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	**0.821**	0.894	0.828	0.827	0.851
BIC	0.846	0.872	0.841	0.840	0.849

TABLE 45. DGP6-DGP5, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	**0.282**	0.299	0.284	0.284	0.288
BIC	0.287	0.293	0.288	0.288	0.289

TABLE 46. DGP7-DGP8, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.895	0.996	0.907	0.904	0.954
BIC	**0.893**	0.928	0.895	0.891	0.921

TABLE 47. DGP7-DGP8, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.624	0.680	0.631	0.631	0.647
BIC	**0.620**	0.646	0.622	0.622	0.629

TABLE 48. DGP8-DGP7, forecasting in levels.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.860	0.981	0.873	0.870	0.913
BIC	**0.859**	0.919	0.869	0.867	0.888

TABLE 49. DGP8-DGP7, forecasting in first differences.

	Fixed	Post-break	PTCV unknown break	PTCV estimated break	IJR
AIC	0.771	0.830	0.777	0.776	0.793
BIC	**0.760**	0.782	0.761	0.760	0.765

4.3.1 DGP1 monthly change for non-stationary period in the United States

The parameters in DGP 1 without break are $c = 0$ and β = {−0.574, −0.336, −0.336, −0.325, −0.206, −0.147}; we generate y_t using Equations (5) and (6). A break is applied on the constant term, where c_1 = 0, c_2 = 0.5 for a small break, and c_2 = 1 for a large break. Tables 2 and 3 report the results for the DGP1 based on US data in the cases of forecasting in levels and in first differences, respectively.

As expected, the AR models always beat the RW forecasts. In addition, the fixed-window techniques always produce the better forecast. The general conclusion here is that even in the presence of a large break in the constant, the window selection methods do not improve the forecast accuracy to any meaningful extent.

4.3.2 DGP1.1

To test the effect of a break in the opposite direction (that is, a decrease in the constant in the inflation equation), using US data, DGP1.1 keeps the AR parameters as DGP1 but applies c = 1 for the no-break case. Then a break is applied on the constant term, where c_1 = 1, c_2 = 0.5 for a small break, and c_2 = 0 for a large break. In the latter case, y_t follows DGP1 when $t > T_b$. The results are shown in Tables 4 and 5.

These results are similar to the DGP1. The varying window techniques gives very similar results to fixed and full windows. There is no significant gain here using the varying-window length techniques.

4.3.3 DGP2 monthly change for stationary period in the United States

We now examine the stationary US data. The parameters in DGP 2 without break are c = 0.001 and β = {0.491, −0.171}; we generate y_t using Equation (7). A break is applied on the constant term, where c_1 = 0.001, c_2 = 0.501 for a small break, and c_2 = 1.001 for a large break. Tables 6 and 7 give the results of this exercise for the DGP2.

In this case, the window selection methods provide some improvement when forecasting the DGP2 series with a large break in levels (as reported in Table 6).

4.3.4 DGP2.1

We next examine a negative shock to the constant term in the DGP using the US data. In this case, DGP2.1 uses the same AR parameters as DGP2 but applies c = 1.001 for the no-break case. Then, a break is applied on the constant term, where c_1 = 1.001, c_2 = 0.501 for a small break, and c_2 = 0.001 for a large break. In the latter case, y_t follows DGP2 when $t > T_b$. The results are given in Tables 8 and 9.

These results are similar to the DGP2 with some improvement from the varying-window selection techniques (as reported in Table 8).

4.3.5 DGP 3 annual change for non-stationary period in the United States

The parameters in DGP 3 without break are c = 0 and β = {0.303, 0.234, −0.088, 0.010, 0.150, −0.012, 0.042, 0.032, 0.189, 0.057, 0.056, −0.403}; we generate y_t using Equations (5) and (6). A break is applied on the constant term, where c_1 = 0, c_2 = 0.5 for a small break, and c_2 = 1 for a large break. Tables 10 and 11 give the results of this exercise for the DGP3 based on the non-stationary US data.

There is no gain from window selection methods. Fixed rolling window performs the best in both level and first difference specifications.

4.3.6 DGP3.1

We next assume that the trend growth in US inflation declines. With the same AR parameters as in DGP3, DGP3.1 applies c = 1 for the no-break case. Then a break is applied on the constant term, where c_1 = 1, c_2 = 0.5 for a small break, and c_2 = 0 for a large break. In the latter case, y_t follows DGP3 when $t > T_b$. The results are given in Tables 12 and 13.

We obtain similar results as under DGP3, with little improvements from the window selection techniques. In other words, the fixed window techniques outperforms all the variable-window techniques.

4.3.7 DGP 4 annual change for stationary period in the United States

Next, we examine the year-on-year US inflation rates with a positive break for the constant. The parameters in DGP 4 without break are c = 0.001 and β = {1.404, −0.620, 0.157}; we generate y_t using Equation (7). A break is applied on the constant term, where c_1 = 0.001, c_2 = 0.501 for a small break, and c_2 = 1.001 for a large break. Tables 14 and 15 give the results for the DGP4.

There is no gain from window selection methods. Fixed rolling window performs the best in both level and first difference specifications.

4.3.8 DGP4.1

We now assume a decline in the constant term. With the same AR parameters as DGP4, DGP4.1 applies c = 1.001 for the no-break case. Then, a break is applied on the constant term, where c_1 = 1.001, c_2 = 0.501 for a small break, and c_2 = 0 for a large break. In the latter case, y_t follows DGP4 when $t > T_b$. The results are reported in Tables 16 and 17.

We obtain similar results as DGP4; there is no gain from using the variable-window lengths.

4.3.9 DGP 5 monthly change for non-stationary period in the UK

We now repeat the above experiments for the DGPs based on the UK data. The parameters in DGP 5 without break are $c = 0$ and $\beta = \{-0.590, -0.381, -0.283, -0.272, -0.252\}$; we generate y_t using Equations (5) and (6). A break is applied on the constant term, where c_1 = 0, c_2 = 0.5 for a small break, and c_2 = 1 for a large break. Tables 18 and 19 report the results of this exercise for the DGP5.

In a similar way to the DGP based on US data, there is little gain in the forecasting accuracy from window selection methods.

4.3.10 DGP5.1

With the same AR parameters as DGP5, DGP5.1 applies c = 1 for the no-break case. Then, a break is applied on the constant term, where c_1 = 1, c_2 = 0.5 for a small break, and c_2 = 0 for a large break. In the latter case, y_t follows DGP5 when $t > T_b$. The results are shown in Tables 20 and 21.

We obtain similar results as under DGP5. While there is a small improvement from the window selection technique, it is marginal.

4.3.11 DGP 6 monthly change for stationary period in the UK

The parameters in DGP 6 without break are $c = 0$ and $\beta = \{0.181, 0.189, 0.167, 0.169\}$; we generate y_t in Equation (7). A break is applied on the constant term, where c_1 = 0, c_2 = 0.5 for a small break, and c_2 = 1 for a large break. Tables 22 and 23 report the results for the DGP6.

There is little difference between the fixed window and window selection methods in both specifications.

4.3.12 DGP6.1

With the same AR parameters as DGP6, DGP6.1 applies c = 1 for the no-break case. Then a break is applied on the constant term, where c_1 = 1, c_2 = 0.5 for a small break, and c_2 = 0 for a large break. In the latter case, y_t follows DGP6 when $t > T_b$. The results are shown in Tables 24 and 25.

We get similar results as DGP6. Window selection methods provide no improvement.

4.3.13 DGP 7 annual change for non-stationary period in the UK

The parameters in DGP 7 without break are $c = 0$ and $\beta = \{0.353, 0.223, 0.037, -0.082, -0.030, 0.138\}$; we generate y_t using Equations (5) and (6). A break is applied on the constant term, where $c_1 = 0$, $c_2 = 0.5$ for a small break, and $c_2 = 1$ for a large break. Tables 26 and 27 give the results of this exercise for DGP7.

There is some improvement from window selection methods when forecasting in first differences (as reported in Table 27).

4.3.14 DGP7.1

With the same AR parameters as DGP7, DGP7.1 applies $c = 1$ for the no-break case. Then a break is applied on the constant term, where $c_1 = 1$, $c_2 = 0.5$ for a small break, and $c_2 = 0$ for a large break. In the latter case, y_t follows DGP7 when $t > T_b$. The results are shown in Tables 28 and 29.

We get similar results as under DGP7. Window selection methods offer some improvement when forecasting in differences (as reported in Table 29).

4.3.15 DGP 8 annual change for stationary period in the UK

The parameters in DGP 8 without break are $c = 0$ and $\beta = \{1.050, 0.097, -0.109, 0.020, -0.104, 0.001, 0.132, -0.104, -0.004, -0.059, 0.182, -0.125\}$. The sum of the AR parameters is 0.981, very close to 1. We generate y_t in Equation (7). A break is applied on the constant term, where $c_1 = 0$, $c_2 = 0.5$ for a small break, and $c_2 = 1$ for a large break. Tables 30 and 31 report the results of this exercise for DGP8.

In this case, forecasts based on the AR models are worse than RW forecasts when there is no break or a small break in the DGP forecasts based on levels. AR forecasts are better when there is a large break. There is no gain from window selection methods. Fixed rolling window performs the best. The results show that when AR parameters are close to that of a unit root process, the random walk would outperform AR models, even when there is a small break. However, AR models are better when the break is large.

4.3.16 DGP8.1

With the same AR parameters as DGP8, DGP8.1 applies $c = 1$ for the no-break case. Then a break is applied on the constant term, where $c_1 = 1$, $c_2 = 0.5$ for a small break, and $c_2 = 0$ for a large break. In the latter case, y_t follows DGP8 when $t > T_b$. The results are reported in Tables 32 and 33.

We obtain similar results as under DGP8. If we use the levels model to forecast, the AR models only beat RW when there is a large break. Window selection methods provide little improvement.

Overall, window selection methods are not helpful in most DGPs, even when there is a break in the middle of the series. In cases in which they do give an improvement, it is usually very small. The conclusion from these simulations is that the presence of a simple break is not sufficient to warrant the use of variable-length window techniques. These results support the findings of Hall et al. (2023) that varying-length window techniques, even in the presence of a break in the constant, do not produce significant improvement in the presence of a single regime of stationary or non- stationary behavior. In the next section, we examine the case of involving a switch between a stationary and non-stationary regime.

4.4 Simulation results from DGPs with mixed regimes

We now combine the DGPs described above. For example, in the case of an existing structural break at time T_b = 120, when $t \leq T_b$, y_t follows DGP1 and when $t > T_b$, y_t follows DGP2. This is called DGP1-DGP2, corresponding to that under which the monthly change of US inflation moves from a non-stationary period to stationary period. Similarly, we have DGP2-DGP1, DGP3-DGP4, DGP4-DGP3, DGP5-DGP6, DGP6-DGP5, DGP7-DGP8, and DGP8-DGP7. The forecasting procedures are the same as described above. The results are reported in Tables 34–49.

The results in Tables 34–49 indicate that the window selection methods can provide better forecasts when the real DGPs contain mixed regimes, especially for the month-over-month inflation data. This finding is especially true when we switch from non-stationary to stationary inflation, which is exactly what happened in the actual data during the period from 1970 to 2000.

The results of the above Monte Carlo simulations can be summarized as follows. As long as the nature of inflation remains either stationary or non-stationary, varying-length window techniques offer little or no advantage over a fixed rolling window. This result holds whether inflation is defined as the year-on-year inflation rate (that is, the inflation rate from a particular month in 1 year relative to the same month in the prior year) or the month-on-month rate. Varying-length window techniques were found to provide an advantage when the nature of inflation changes from non-stationary to stationary, or vice versa. A change in the characteristics in inflation in the period comprising 1960 to 1999 compared to the period comprising 2000 to 2022 marked a change from one dataset containing around 20 years of non-stationary data followed by 10 years stationary data to another dataset containing around 20 years of stationary data followed by at most 2 years of non-stationary data. This helps explain the superior forecasting performance found in earlier studies using varying-length window techniques.

5 DETECTING A STRUCTURAL SHIFT

The various window selection techniques discussed in the previous section all rely on detecting the structural break using one of the standard structural break tests, such as the Bai and Perron's (1998) test. These techniques work well in a standard econometric setting, but they have an important drawback from a policy point of view. Specifically, these tests work well when the break is in the middle of the sample period. We can use these tests to analyze a historical period and ask if the period was subject to a structural break. In typical usage, it is accepted that these tests only work when the break date is after the first 20% of the observations and before the last 20%. However, in a policy setting, we are interested in knowing if there has been a structural break in the most recent data. Structural break tests are not designed for this purpose. In this section, we examine how quickly several techniques are able to detect a break during the period in which it takes place. We continue to use the inflation data for the United States and the UK. There remains the question whether annual inflation or month-on-month inflation provide a more timely way of detecting the break. The techniques we use are as follows:

1. *A one step ahead Chow test*. This test works as a part of recursive estimation; hence, it focuses on a series of forecasts for the last piece of data in each recursion.
2. *A GARCH model*. It is well known that the conditional variance in a standard GARCH model can change very rapidly. Examination of the conditional variance could potentially be a timely indicator of a regime change.
3. *A Markov chain switching regression model*. Here, when the parameters are allowed to switch between two distinct sets of parameters, experience tells us that the switch is typically very rapid such that the state probabilities can indicate a regime change or structural break in a timely fashion.

In each model, we use an AR(6) univariate model as the basis of the forecast analysis.

5.1 One-step-ahead Chow test

We present the one-step-ahead residual for period t and the standard error for the sample estimated from the beginning of the period up to period $t - 1$ for the period from January 2015 to September 2022. This procedure shows us when the one step ahead residual goes outside of the normal range of errors (i.e., one standard deviation). Note that we are not interested in a formal test here but rather an alert for possible structural change. The results from this procedure are presented in Figures 9 and 10 for the UK and the United States, respectively.

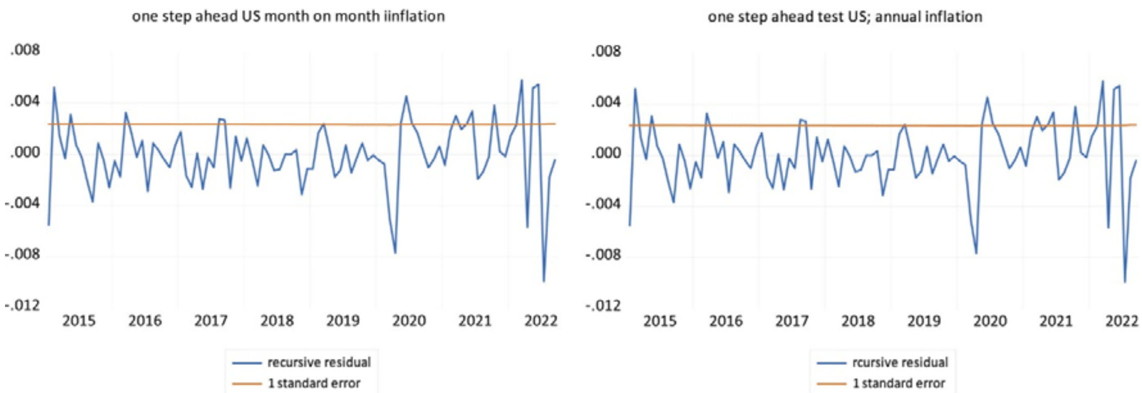

FIGURE 9. One-step-ahead tests for UK inflation.

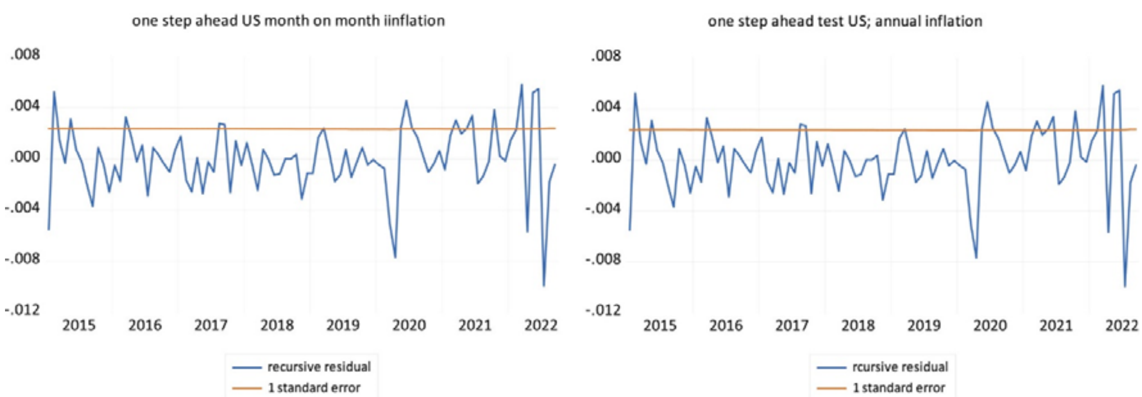

FIGURE 10. One-step-ahead tests for the US inflation.

For the UK, both procedures show a clear alert towards the second half of 2021, although there are signs of instability as early as mid-2020.

For the United States, the picture is very similar to that for the UK: there are signs of instability in 2020 but especially in 2021. There is little difference between the annual data and the month-on-month data, consistent with our findings in Section 3 because the test focuses on the residual. That is, this stability test should give similar results for both annual and month-on-month data.

5.2 GARCH model

We next consider the effectiveness of a GARCH model in identifying the change in the conditional variance as a structural break occurs. The conditional variance can be viewed as a measure of future uncertainty. Here, we again use an AR(6) model as the mean equation and a GARCH(1,1) specification for the variance process. The conditional variance for the UK and the United States are reported in Figures 11 and 12, respectively.

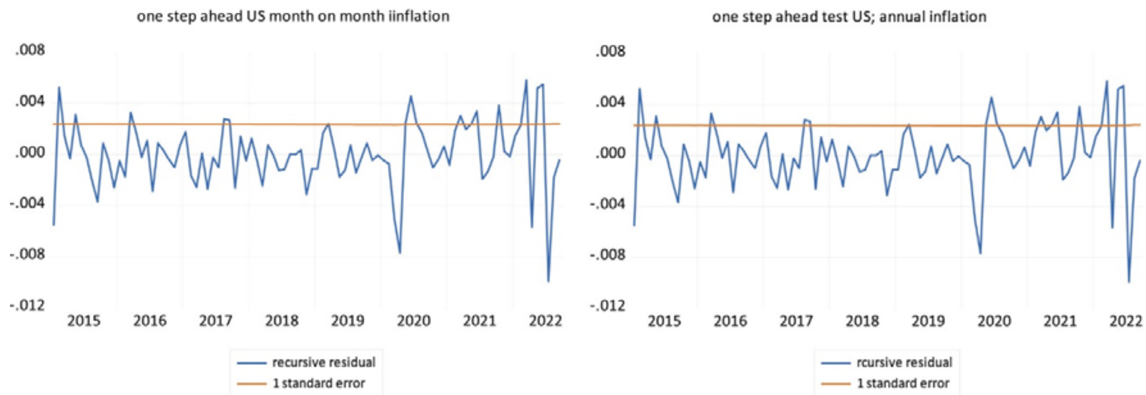

FIGURE 11. The conditional variance for the UK inflation.

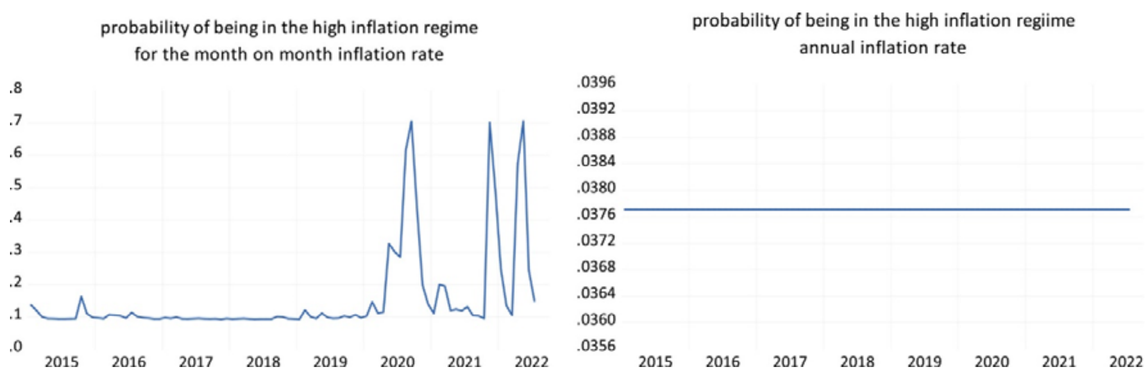

FIGURE 12. The conditional variance for US inflation.

As shown in Figure 11, there is clearly a strong rise in the conditional variance in the middle of 2020, followed by a larger rise through 2022. Both measures of inflation give similar results because this indicator focuses on the residual from the AR model. In other words, as long as there are sufficient lags in the AR model, the results from both equations should be essentially the same.

For the United States, Figure 12 shows a somewhat different picture for the two inflation measures. Both show a considerable rise in uncertainty in 2020. The annual inflation rate shows a similar rise in 2021, while the month-on-month measure does not show a rise until 2022. This result likely indicates that the AR(6) is not an adequate specification in the case of the United States. However, both measures of uncertainty would have alerted policymakers to a possibility of increased instability and uncertainty as early as mid-2020.

5.3 A switching regression model

A two state Markov chain switching regression model is perhaps the closest modeling technique to the properties of the inflation process (as indicated in Section 3). We showed that inflation seems to have periods when it is non-stationary and others when it is stationary. This is what a switching regression model would allow. We now specify an AR(6) model as the basic specification, but we allow the parameters of the AR(6) to endogenously switch between two sets of values following a Markov chain process. The interesting issue here is how quickly the model will switch into the non-stationary process. Figure 13 shows the probability of being in the high inflation state for both the annual and month-on-month inflation rates for the UK.

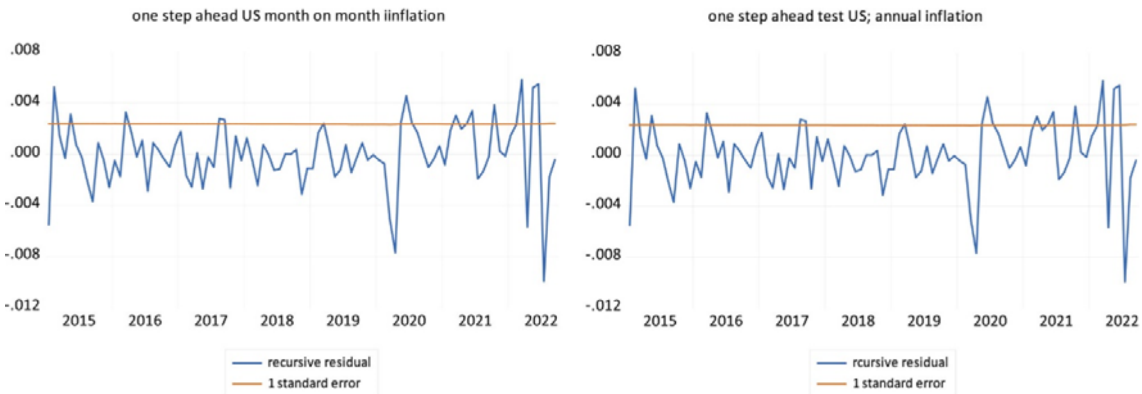

FIGURE 13. Probability of being in the high inflation regime for the UK.

Here the month-on-month and the annual inflation models perform very differently. The month-on-month model shows a clear move into a high-inflation regime from early 2020, while the annual inflation model shows no sign of moving into a high inflation regime.

Figure 14 shows the comparable results for the United States. Here, both series show a sharp rise in the probability of being in the high-inflation regime in 2020. The annual rate shows a larger probability in 2021, whereas both measures of inflation show a high probability in 2022. This model, therefore, provided a useful warning sign for policymakers.

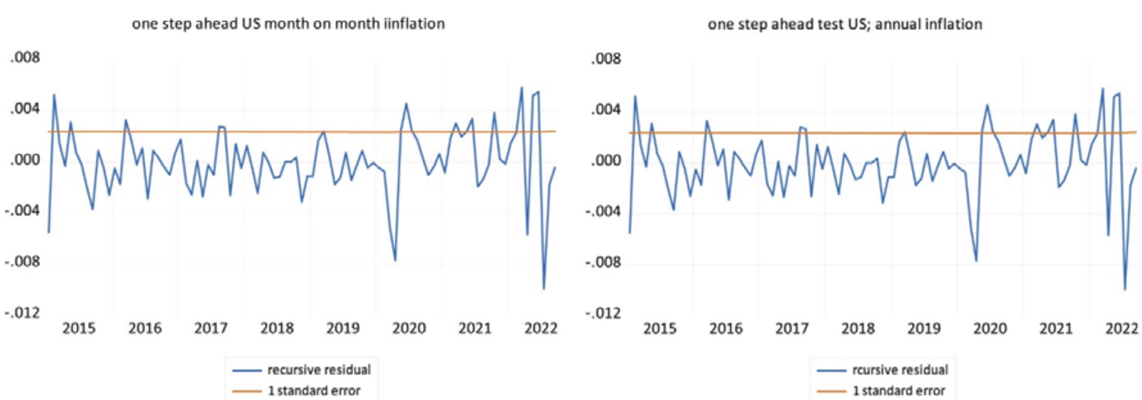

FIGURE 14. Probability of being in the high inflation regime for the United States.

6 CONCLUSIONS

Varying-window selection techniques are found to not help very much in forecasting inflation over the period 2000 onwards. We have argued that there have been substantial structural changes between the pre-2000 period and the post-2020 period. In particular, we have shown that the basic stationarity properties of inflation changed at that point from being non-stationary to being stationary. We then performed an extensive Monte Carlo study which showed that the various window selection techniques under structural breaks only help when the form of regime change takes exactly this form (that is, a change from non-stationary to stationary). We conclude that a change in the inflation process from non-stationary to stationary helped underpin the superior forecasting performance of varying-length window techniques in earlier studies. Our Monte Carlo experiments indicated that these techniques do not offer any improvement over fixed-window techniques in periods comprised of *only* stationary or non-stationary data.

We argued that, in a policy context, varying-length window techniques have limited applicability because they are only useful when the structural break occurs in the middle portion of the sample. From a policy perspective, we are interested in the latest data and the possibility that a structural break has, in fact, occurred. We investigated three possible ways to indicate that a break may be in the process of occurring; a one step ahead Chow test, a GARCH model, and a Markov chain switching regression model. All three models indicated that there was a strong possibility of a structural break in inflation beginning in 2020.

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DATA AVAILABILITY STATEMENT

All data are taken from publicly available data sources. The particular vintage of data used in this study is available upon request from the authors.

NOTES

[1] Rossi (2013) argued that the selection of the rolling window length is crucial in forecasting. Recent papers that have explored the moving window ideas include Medel et al. (2016), Inoue et al. (2017), Hong et al. (2017), and Tang et al. (2021).

[2] See, also, Banerjee et al. (2023) and Blanchard and Bernanke (2023).

[3] All the simulations reported here were carried out on the University of Leicester high performance computer ALICE

[4] For the DGP setup with a break, see the details described in Section 4.3.

[5] For a summary of these window selection techniques, see Hall, Tavlas and Wang (2023, Section 3).

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