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Reconsideration**

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MEASURING THE WELFARE COST OF INFLATION IN SOUTH AFRICA: A RECONSIDERATION

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Abstract

In this paper, using the Fisher and Seater (1993) long-horizon approach, we estimate the long-run equilibrium relationship between money balance as a ratio of income and the Treasury bill rate for South Africa over the period of 1965:02 to 2007:01, and, in turn, use the obtained estimates of the interest elasticity and the semi-elasticity to derive the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach, as well, as Lucas's (2000) compensating variation approach. When, the results are compared to welfare cost estimates obtained recently by Gupta and Uwilingiye (2008), using the same data set, but based on Johansen's (1991, 1995) cointegration technique, the values are less by more than half in size than those obtained in the latter study, with the same being utmost ranging between 0.16 percent to 0.36 percent of GDP for the target-band of 3 percent to 6 percent of inflation. The paper, thus, highlights the fact that welfare cost estimates of inflation are sensitive to the methodology used to estimate the long-run equilibrium money demand relationships.

Keywords: Long-Horizon Regression; Money Demand; Welfare Cost of Inflation.

JEL Classification: E31; E41; E52.

1. INTRODUCTION

In a recent study, Gupta and Uwilingiye (2008) measured the welfare cost of inflation in South Africa, based on estimates of the interest elasticity and semi-elasticity of money demand functions, obtained using the Johansen (1991, 1995) methodology on quarterly data for M3, GDP and the Treasury bill rate. Given the estimates for the elasticities, the authors then calculated the welfare cost of inflation using Bailey's (1956) consumer surplus approach. Relying more on results obtained from the log-log specification of money demand, rather than the semi-log model for the same,¹ they indicated that the welfare cost in South Africa ranged between 0.34 percent and 0.67 percent of GDP, for a band of 3 to 6 percent of inflation, over the period of 1965:02 to 2007:01.

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¹ The authors' decision to place more confidence on the log-log model of money demand was due to two reasons: First, the R^2 and the *Adjusted R*² values of the inverse money demand relationship captured by the log-log specification was higher than the corresponding values of the semi-log model, and; Second, although there existed overwhelming evidence that suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen test with two lags. The authors, thus, had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship.

In this paper, we re-estimate the long-run relationship between money balance and interest rate for South Africa using the same data set and over the same period as used by Gupta and Uwilingiye (2008), but using an alternative approach, namely the long run horizon regression proposed by Fisher and Seater (1993). One of the advantages of using the long-horizon regression approach is that cointegration is neither necessary nor sufficient for tests on the interest rate elasticity of money demand. As in Gupta and Uwilingiye (2008), the coefficients obtained in regression for both alternative money demand specifications: double-log version originated by Meltzer (1963) with constant elasticity and a semi-log version originated by Cagan (1956) with constant semi-elasticity of money, are then used to calculate welfare cost of inflation. In addition, the welfare cost of inflation is then estimated not only by using Bailey's (1956) consumer's surplus approach, but as well as Lucas's (2000) compensating variation approach.

This need to compare the welfare cost estimates with the one obtained by Gupta and Uwilingiye, based on the Johansen (1991, 1995) methodology, emanates from the issue of sensitiveness of the estimates of the interest elasticity of alternative forms of money demand, based on alternative econometric techniques adopted to estimate the long-run relationship between money balance and the nominal interest rate. Given that, welfare cost estimates hinge critically on the estimate of the interest elasticity and semi-elasticity, it is important to check for the robustness of the results obtained using alternative econometric methodologies.

The claim, we make above, regarding the need to use alternative estimation techniques to obtain values for interest elasticity and semi-elasticity is not without empirical basis though.

Based on the long-horizon regression approach, proposed by Fisher and Seater (1993), Serletis and Yavari (2004), in their study dealing with the welfare cost of inflation for Canada and the United States, came up with much smaller figures compared to Lucas (2000), who had indicated that a reduction in the nominal rate from 0 percent to 3 percent would yield a benefit equivalent to 0.90 percent of real income. However, Serletis and Yavari (2005), while repeating the above study for Italy came up with very similar numbers for the welfare cost that they obtained earlier for Canada and the United States. The authors indicated that reducing the interest rate, in Italy, from 14 percent to 3 percent would yield a benefit equivalent to an increase in real income of 0.40 percent, which, in turn, was quite comparable to their estimates for Canada (0.35 percent) and the United States (0.45 percent) for the same percentage point reduction in the nominal interest rate. On the other hand, based on the Phillips-Ouliaris (1990) test for cointegration, Ireland (2007) found that a 10 percent rate of inflation when compared to price stability, in the United States, would imply a welfare cost of 0.21 percent of income. This figure, though lower than that of Lucas (1981, 2000) and Serletis and Yavari (2004), was in line with Fischer's (1981) findings of 0.30 percent. Clearly then, besides sample period and the country under investigation and alternative money demand specifications, welfare cost estimates are sensitive to alternative estimation methodologies, and, hence, our need to reconsider the welfare cost estimates obtained by Gupta and Uwilingiye (2008), cannot be overlooked.

Given that, inflation has an effect on economic activity, and ultimately on people's well-being as it reduces the purchasing power of money balances when inflation rises, a correct and fair evaluation of welfare cost of inflation is crucial, since inflation creates and amplifies distortions in many areas of economic activity and it has also an influence on all decisions of economic agents. Besides, in a country like South Africa, where the central bank targets inflation, the importance of investigating how substantial are the welfare costs of inflation under the current inflation target zone of 3 to 6 percent pursued by the South African Reserve Bank is of paramount importance. Since this would help us decide if there is a need to rethink the band of the target in terms of the welfare cost of inflation. To the best of our knowledge, this is the first attempt to measure the welfare cost of inflation for the South African economy, based on the long-run regression approach proposed by Fisher and Seater (1993).

The remainder of the paper is organized as follows: Section 2 provides a brief summary of the theoretical issues regarding the estimation of the welfare cost of inflation, while, Section 3 and 4, respectively, discusses the data and the long-horizon empirical methodology for the estimation of the log-log and the semi-log money demand specifications. Section 4 also presents the empirical estimates for the interest rate elasticity and the semi-elasticity, as well as the welfare cost estimates for the South African economy. Finally, Section 5 concludes.

2. THE THEORETICAL FOUNDATIONS

As indicated by Lucas (2000), money demand specification is vital in determining the appropriate size of the welfare cost of inflation. Lucas (2000) contrasts between two competing specifications for money demand. One, inspired by Meltzer (1963), relates the natural logarithm of m , a ratio of money balances to nominal income, and the natural logarithm of a short-term nominal interest rate r . Formally, this can be expressed as follows:

$$\ln(m) = \ln(A) - \eta \ln(r) \quad (1)$$

where $A > 0$ is a constant and $\eta > 0$ measures the absolute value of the interest elasticity of money demand. Another specification, adapted from Cagan (1956), links the log of m to the level of r via the following equation:

$$\ln(m) = \ln(B) - \xi r \quad (2)$$

where $B > 0$ is a constant and $\xi > 0$ measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the “*consumers’s surplus*”, that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if $m(r)$ is the estimated function, and $\psi(m)$ is the inverse function, then the welfare cost can be defined as:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x) dx = \int_0^r m(x) dx - rm(r) \quad (3)$$

As seen from Equation (3), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue rm to deduce the deadweight loss.

Since the function m has the dimensions of a ratio to income, so does the function w . The value of $w(r)$, represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at r or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by (1) or is $m(r) = Ar^{-\eta}$, the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A \left(\frac{\eta}{1-\eta} \right) r^{1-\eta} \quad (4)$$

While, for a semi-log money demand specification i.e., $m(r) = Be^{-\xi r}$, $w(r)$ is obtained by the following formula:

$$w(r) = \frac{B}{\xi} \left[1 - (1 + \xi r) e^{-\xi r} \right] \quad (5)$$

As can be seen from (4) and (5), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, and, hence, we first need to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

Besides providing the theoretical general equilibrium justifications for Bailey's consumer surplus approach, Lucas (2000), also takes a compensating variation approach in estimating the welfare cost of inflation. To start off, Lucas (2000) uses Brock's (1974) perfect foresight version of Sidrauski's (1967) Money-in-the-Utility (MIU) model, and defines the welfare cost of a nominal interest rate r , $w(r)$, to be the income compensation needed to leave the household indifferent between living in a steady-state with an interest rate constant at r and an otherwise identical steady-state with the interest rate of zero. With, $w(r)$ being obtained from the solution to the following equation:

$$u[1 + w(r)y, \phi(r)y] = u[y, \phi(0)y] \quad (6)$$

Assuming a homothetic current period utility function ($u(c,m) = \frac{1}{1-\sigma} \left[cf \left(\frac{m}{c} \right) \right]^{1-\sigma}$; $\sigma \neq 1$) and setting up the dynamic programming problem (see Lucas (2000) for details), Lucas obtains a differential equation in $w(r)$ of the following form:

$$w'(r) = \psi \left(\frac{\phi(r)}{1 + w(r)} \right) \phi'(r) \quad (7)$$

For any given money demand function, Equation (7) can be solved numerically for an exact welfare cost function $w(r)$. In fact, with equation (1), equation (7) can be written as:

$$w'(r) = \eta Ar^{(-\eta)} (1 + w(r))^{-\frac{1}{\eta}} \quad (8)$$

yielding a solution for log-log specification

$$w(r) = -1 + \left(1 - Ar^{1-\eta} \right)^{\frac{\eta}{\eta-1}} \quad (9)$$

While, for the semi-log model (7) yields

$$w'(r) = \left[\xi B e^{-\xi r} \left(r + \frac{1}{\xi} \log(1 + w(r)) \right) \right] \approx \left[\xi B e^{\xi r} \left(r + \frac{1}{\xi} w(r) \right) \right] \quad (10)$$

with a solution

$$w(r) = -e^{-\frac{B e^{-\xi r}}{\xi}} \left\{ e^{\frac{B e^{-\xi r}}{\xi}} - Ei \left[\frac{B}{\xi} \right] + Ei \left[\frac{B e^{-\xi r}}{\xi} \right] \right\} \quad (11)$$

and where $Ei(x) = -\int_x^\infty \frac{e^{-t}}{t} dt$, and one uses the principal value of the integral.

Note to calculate $w(r)$, in equations (9) and (11),² we use the estimates of η and ξ obtained from the long-horizon regression, discussed in Section 4, while, the values for A and B are obtained such that they match the geometric means of the data for the log-log and the semi-log specifications respectively, i.e., $A = \bar{m}/\left(\bar{r}\right)^{-\eta}$, $B = \bar{m}/\left(e^{-\xi\bar{r}}\right)$ with \bar{m} and \bar{r} being respectively the geometric means of m and r respectively.

3. DATA

In this study, we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The variables used in this study are the money balances ratio ($rm3$), generated by dividing the broad measure of money supply (M3)³ by the nominal income (nominal GDP), and short term interest rate, in our case, proxied by the 91 days Treasury bill rate (tbr).⁴ All series, except for the Treasury bill rate are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by $lrm3$ and $ltbr$, respectively.

4. EMPIRICAL METHODOLOGY AND RESULTS

Following Gupta and Uwilingiye (2008), and as it is standard in time series analysis, we start off by studying the univariate characteristics of the data. In this regard, we performed tests of stationarity on our variables ($lrm3$, $ltbr$ and tbr) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips- Perron (PP) test. As was observed by Gupta and Uwilingiye(2008), the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected. As the variables were found to be non-stationary, it paved the way for the long-horizon regression proposed by Fisher and Seater (1993) to avoid obtaining estimates for the interest rate elasticity and semi-elasticity based on spurious regressions. As stated at the onset, cointegration, is neither necessary nor sufficient for this approach, so we do not test specifically for cointegration.⁵

The basics of the long-horizon regression approach can be described as follows, by starting off with the following bivariate autoregressive representation:

$$\alpha_{mm}(L)\Delta^{(m)}m_t = \alpha_{mr}(L)\Delta^{(r)}r_t + \varepsilon_t^m \quad (12)$$

² The calculations were done using the DSolve routine in Mathematica, Version 5.

³ See Gupta and Uwilingiye (2008) for details regarding the reasons behind the choice of M3 as the appropriate monetary aggregate for South Africa, over narrower aggregates generally used in literature. Basically, the authors indicate that the ratio of M3 to GDP is less volatile when compared to the corresponding ratios of M1 and M2 to GDP, and also M3 was used to account for the financial innovations that have taken place in the South African economy over the sample period being used of our concern.

⁴ We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

⁵ The reader is referred to Gupta and Uwilingiye (2008) for the tests on stationality and cointegration on the variables of the model, reported in Tables 1 through 3.

$$\alpha_{rr}(L)\Delta^{(r)}r_t = \alpha_{mm}(L)\Delta^{(m)}m_t + \varepsilon_t^r \quad (13)$$

where $\alpha_{mm}^0 = \alpha_{rr}^0 = 1$, $\Delta = 1 - L$, where L is the lag operator, m is the money-income ratio, r is the nominal interest rate, and $\langle x \rangle$ represents the order of integration of x , so that if x is integrated of order γ , or $I(\gamma)$ in the terminology of Engle and Granger (1987), then $\langle x \rangle = \gamma$ and $\langle \Delta x \rangle = \langle x \rangle - 1$. The vector $(\varepsilon_t^m, \varepsilon_t^r)'$ is assumed to be independently and identically distributed normal with zero mean and covariance \sum_{ε} , the elements of which are $\text{var}(\varepsilon_t^m)$, $\text{var}(\varepsilon_t^r)$, $\text{cov}(\varepsilon_t^m, \varepsilon_t^r)$. A key result in Fisher and Seater (1993) applies to the case where $\langle m \rangle = \langle r \rangle = 1$, which is the case with our data as money balance as *lrm3*, *lbr* and *tbr* are all $I(1)$. In this case, the long-run derivative of m with respect to r , $\text{LRD}_{m,r}$, is given by:

$$\text{LRD}_{m,r} = \frac{\theta_{mr}(1)}{\theta_{rr}(1)} \quad (14)$$

with $\text{LRD}_{m,r}$ being interpreted as the long-run elasticity of m with respect to r . In fact, under the Fisher and Seater (1993) identification scheme, which assumes that r is exogenous in the long run, $\theta_{mr}(1)/\theta_{rr}(1)$ can be interpreted as $\lim_{k \rightarrow \infty} b_k$, where b_k is the coefficient from the regression:

$$\left[\sum_{j=0}^k \Delta^{(m)} m_{t-j} \right] = a_k + b_k \left[\sum_{j=0}^k \Delta^{(r)} r_{t-j} \right] + e_{kt} \quad (15)$$

and for $\langle m \rangle = \langle r \rangle = 1$, consistent estimate of b_k can be derived by applying ordinary least squares to the regression

$$m_t - m_{t-k-1} = a_k + b_k [r_t - r_{t-k-1}] + e_{kt}, \quad (16)$$

$$k = 1, \dots, K$$

Based on Equation (16) and for a value of $k=30$ as used by Serletis and Yavari (2004 and 2005), our estimate of the interest rate elasticity, η , is 0.1073, and interest semi-elasticity ξ is 1.0099, which, in turn, are much lower than the corresponding values of 0.2088 and 2.1991, obtained by Gupta and Uwilingiye (2008) based on the Johansen (1991 and 1995) methodology.

Once we obtain the estimated values for η and ξ , using long-horizon regression, we calculate the values of A and B such that the curves obtained pass through the geometric means of the data. This gives us values of $A = 0.4255$ and $B = 0.6035$. Note the values for A and B obtained by Gupta and Uwilingiye (2008), based on the cointegrating relationships were, respectively, 0.3323 and 0.6862.

Having obtained the estimates for η and ξ and the values for A and B , we are now in a position to obtain the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach

and Lucas' (2000) compensating variation method. The results have been reported in Table 1. Note for the sake of comparison, we also present the welfare cost estimates, based on the values of η, ξ, A and B , obtained by Gupta and Uwilingiye (2008), based on the Johansen (1991 and 1995) approach.

Table 1: Welfare cost estimates.

Inflation rate	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0034	0.0034	0.0018	0.0015	0.0037	0.0035	0.0019	0.0016
6	0.0067	0.0076	0.0035	0.0035	0.0072	0.0079	0.0037	0.0036
10	0.0108	0.0143	0.0057	0.0068	0.0117	0.0149	0.0062	0.0070
15	0.0156	0.0241	0.0084	0.0118	0.0172	0.0251	0.0092	0.0123

Based on the results reported in the Columns 2 and 3, and 4 and 5, the welfare cost estimates obtained under the consumer surplus approach, for 3 percent, 6 percent, 10 percent and 15 percent of inflation, using the Johansen (1991 and 1995) cointegration method and the long-horizon regression approach respectively, we see that welfare costs are substantially lower in the latter case. In fact they are nearly less by more than half, of the costs obtained using the cointegration approach for both the log-log and the semi-log specifications. When we compare Columns 6 and 7, and 8 and 9, we obtain a similar picture for the welfare cost estimates obtained using the compensating variation approach. Further, the welfare cost estimates within a specific estimation method, but across the consumer surplus approach and the compensating variation approach are quite similar, with the figures being slightly higher under the compensating variation method outlined by Lucas (2000). Specifically, for the log-log (semi-log) specification, estimated using the cointegration approach, under the consumer surplus approach [compensating variation approach], an increase in the inflation rate from 3 percent to 6 percent would increase the welfare cost from 0.67 percent of GDP to 1.08 percent of GDP [0.72 percent of GDP to 1.17 percent of GDP] (0.76 percent of GDP to 1.43 percent of GDP [0.79 percent of GDP to 1.4449 percent of GDP]). While, under the long-horizon approach the welfare cost estimates ranges between 0.18 percent of GDP to 0.35 percent of GDP and 0.19 percent of GDP to 0.37 percent of GDP with the log-log specification, obtained from the consumer surplus and the compensating variation approaches respectively, for an increase in the inflation rate from 3 percent to 6 percent, the corresponding values under the semi-log specification, for the same increase in the rate of inflation, are 0.15 percent of GDP to 0.35 percent of GDP and 0.16 percent of GDP to 0.36 percent of GDP. The bottom line is that, as in Serletis and Yavari (2004 and 2005), we find the welfare cost estimates based on the long-horizon approach tends to be much smaller when compared to other standard econometric method of arriving at the long-run equilibrium relationship between ratio of money balance to income and the nominal interest rate. And the reason is that, under the long-horizon approach estimates of interest rate elasticity and semi-elasticity tends to be comparatively lower, and this, in turn, given the fact that welfare cost estimates based on money demand estimations critically hinges on the size of interest rate elasticity and semi-elasticity, brings down the welfare cost of inflation when compared to estimates obtained via econometric methods, such as the Johansen (1991 and 1995) approach.

5. CONCLUSION

In this paper, using the Fisher and Seater (1993) long-horizon approach, we estimate the long-run equilibrium relationship between money balance as a ratio of income and the Treasury bill rate for South Africa over the period of 1965:02 to 2007:01, and, in turn, use the obtained estimates of the interest elasticity and the semi-elasticity to derive the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach, as well, as Lucas's (2000) compensating variation approach. When, the results are compared to welfare cost estimates obtained recently by Gupta and Uwilingiye (2008), using the same data set, but based on Johansen's (1991, 1995) cointegration

technique, the values are less by more than half in size than those obtained in the latter study. The paper, thus, highlights the fact that welfare cost estimates of inflation are sensitive to the methodology used to estimate the long-run equilibrium money demand relationships.

At this stage two aspects of the obtained results needs further emphasis: First, when compared to the literature, the welfare cost estimates obtained for South Africa, whether based on the long-horizon regression or the Johansen (1991 and 1995) cointegration approach, are relatively higher when compared to estimates available in the literature for other economies for similar levels of inflation rates; Second, it must be realised that whatever the estimation methodology used and whether one uses a consumer-surplus approach or a compensating variation method, based on our estimates, we can conclude that the SARB.s current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare, at least when compared to a Friedman-type deflationary rule of zero nominal rate of interest.

But, one cannot deny that a relevant question is: Given that welfare cost estimates are sensitive estimation methodologies and seem to vary quite a lot based on which econometric approach is undertaken, what is the true size of the welfare cost of inflation in South Africa? The answer to this question is difficult. But a fact that needs to be realised is that – econometric methodologies deriving welfare cost measures by estimating money demand relationships provide only the lower bounds to the welfare cost of inflation. Since, such welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Given these two additional sources of inflation costs, there is no denying the fact that one can achieve, possibly, larger gains by reducing the inflation target below 3 percent, the lower limit of the current inflation target band.

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