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Portfolio optimisation using alternative risk measures ☆

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ABSTRACT

We use a numerical methods algorithm based on gradient descent to optimise investment portfolios of global indices using raw and forecasted risk measures at differing frequencies. The results permit a comparison of how the characteristics of risk measures other than the variance and standard deviation impact portfolio performance. Asymmetric risk measures result in superior portfolio returns, while risk measures incorporating unsquared deviations outperform those incorporating squared deviations. Risk measures forecasted using the exponentially weighted moving average (EWMA) methodology do not yield significant increases in portfolio returns. Semi-absolute deviation, mean absolute deviation and downside semi-deviation perform favourably in producing higher returns.

1. Introduction

The use of variance as a risk measure in the risk-return trade-off proposed by Modern Portfolio Theory (MPT) is subject to criticism. One criticism addressed by optimising portfolio risk versus returns using alternative risk measures, notably downside semi-variance (DSV) and downside semi-deviation (DSSD), is that it minimises upside risk (Markowitz, 1952; Byrne & Lee, 2004; Boasson et al., 2011; Cardoso et al., 2019). Portfolios formulated using these risk measures have both outperformed (Rigamonti & Lučivjanská, 2022) and underperformed (Jacobsen, 2005; Stanković et al., 2020) portfolios optimised using variance. Studies comparing portfolios optimised using mean absolute deviation (MAD) to variance-optimised portfolios have typically reported higher total returns, as have studies using semi-absolute deviation (SAD) (Stanković et al., 2020; Chen et al., 2023). Studies using value-at-risk (VaR) reported similar and lower returns relative to variance-optimised portfolios (Jacobsen, 2005; Stanković et al., 2020), whereas conditional VaR (CVaR) yielded mixed results (Lohre et al., 2010; Hafsa, 2015; Nguyen et al., 2018).

Our study can be framed within the context of literature that compares the performance of alternative risk measures in portfolio optimisation. Righi and Borenstein (2018) compare the performance of 11 risk measures. Using a broad sample of U.S. stocks (over 2000), they find that no risk measure clearly dominates, although loss-deviation measures consistently exhibit favourable performance. This is attributed to the unique characteristics of each measure, making each suitable within a different context. Cardoso et al. (2019) use six risk measures for portfolio optimisation with cardinality constraints and rebalancing for a sample of Brazilian assets.

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CVaR and VaR yield higher accumulated returns and higher monthly returns and reduce drawdown risk without limiting potential upside gains. Ramos et al. (2023) assess the performance of seven risk measures for portfolio optimisation for risk minimisation using S&P100 constituents. Depending on the objective, different risk measures perform optimally, such as yielding the highest absolute returns, superior diversification, risk reduction, and superior risk-adjusted performance. The authors recognise that risk measures differ mathematically and conceptually. Stanković et al. (2020) evaluate the performance of five alternative risk measures, which they claim are commonly used in research, using a subset of Serbian stocks. The best-performing portfolios, in terms of highest returns and Sharpe ratios, are those optimised using MAD and SAD. Sant'Anna et al. (2022) analyse the behaviour of tracking portfolios relative to a benchmark using seven risk measures as constraints for S&P500 stocks. As in Ramos et al. (2023), the results suggest that different risk measures excel in different contexts. Some measures yield high returns (expected loss measures) but a high tracking error. Other measures, such as deviation measures and expected shortfall, perform well in reducing tracking error and risk, making them useful as constraints. Hunjra et al. (2020) compare the performance of four risk measures in portfolio construction for three South Asian markets (Pakistan, Bangladesh & India) under varying economic conditions. Portfolios optimised using the CVaR95% measure yield the highest returns across all scenarios. This measure's favourable performance is attributed to its ability to control downside risk. Gaivoronski et al. (2005) compare six risk measures within the context of portfolio optimisation aimed at tracking a benchmark using a sample of the largest companies in the industrial sector on the Oslo Stock Exchange. They develop algorithms for both static and dynamic portfolio management, and performance is measured using tracking error and downside tracking error. Although they do not arrive at a unanimous answer as to what the best measure is, they argue that consideration should be given to the requirements of different types of investors. Lam et al. (2010) optimise portfolios comprising stocks that are part of the Kuala Lumpur Composite Index (KLCI) using four risk measures. The minimax risk measure outperforms the other measures considered (variance, mean absolute deviation, and semi-variance), providing the highest mean return and risk-adjusted performance, particularly for investors with a strong aversion to downside risk. However, MAD yields the lowest risk.

Our study contributes to literature on the comparative performance of alternative risk measures in maximising returns and risk-adjusted returns. It differentiates itself from similar studies which are limited to national markets. Our sample is broader; we optimise portfolios comprising *global* sector, industry group, and industry indices (or their analogous exchange traded funds (ETFs)) using ten raw and forecasted risk measures (twenty in total) at daily, weekly, and monthly recalculation frequencies. We consider a more recent period which is longer than that of the comparative studies discussed, from February 2012 to March 2022, encompassing the COVID-19 outbreak (2020) and its aftermath. While the number of alternative risk measures considered is not as extensive as that in Righi et al. (2018), Sant'Anna et al. (2022), and Ramos et al. (2023), it investigates measures not considered by these studies. It is more extensive than the studies of Gaivoronski et al. (2005), Lam et al. (2010), Cardoso et al. (2019), Stanković et al. (2020), and Hunjra et al. (2020). Instead of relying on linear programming or historical simulation, we make use of a gradient descent optimisation algorithm that we develop and thereby do away with restrictive distributional assumptions. The risk measures that we compare – variance (V), standard deviation (SD), DSV, DSSD, MAD, SAD, VaR, and CvaR – are popular and widely used but have not been compared in a single study (see Grootveld & Hallerbach, 1999; Gaivoronski & Pflug, 2005; Mansini et al., 2007; Andreu & Torra, 2009; Lwin et al., 2017; Sehgal & Jagadesh, 2023; Chen et al., 2023). Our study also seeks to address another criticism of MPT; that of stable return distributions and persistent risk-return ratios (Boasson et al., 2011; Iyiola et al., 2012, Stanković et al., 2020, Rigamonti & Lučivjanská, 2022), which is contradictory to the time-varying nature of risk (Rachev et al., 2008). Consequently, we investigate the use of an exponentially weighted moving average (EWMA) process to model risk measure forecasts. To the best of our knowledge, there are no studies on portfolio optimisation using EWMA risk forecasts in a comparative context using these measures. Our study differs from Cardoso et al. (2019), who restrict their application of EWMA and GARCH to modelling variance and not for forecasting a broader set of risk measures. Finally, we discuss the characteristics of the risk measures considered, aiming to explain their performance with reference to the empirical results.

Our results indicate that three alternative risk measures, namely DSSD, MAD, and SAD, consistently outperform all other risk measures. Performance is measured using annualised returns and Sharpe ratios (SRs) and compared to that of portfolios optimised using variance and standard deviation. Favourable risk measure characteristics include asymmetry and unsquared deviations, with measures possessing these traits outperforming counterparts that lack them. Using forecasted risk measures to optimise portfolios does not yield statistically significant improvements in portfolio performance. This is potentially attributable to the methodology applied to forecast risk, EWMA, or the result of market cycles of different durations. These results continue to hold when the sample is subdivided into pre-COVID-19 and COVID-19 periods. Implications for portfolio managers and researchers, together with limitations and avenues for further research, are outlined in the conclusion.

2. Data and methodology

The sample comprises daily data spanning the period 29 February 2012 to 25 March 2022 (N=2602 trading days) and consists of three tiers, namely ten S&P Global 1200 sector indices, 24 MSCI global industry group, and 69 MSCI industry indices (each index comprises an asset in the portfolio, see Table A1 in Appendix A for descriptive statistics). The sample length is comparable to that of Ramos et al. (2023) and Stanković et al. (2020), and is longer than that of Righi and Borenstein (2018). It reflects market performance during both calmer and more turbulent periods, capturing bear and bull markets, the COVID-19 outbreak, and its aftermath which includes a global energy crisis and the outbreak of the Russia-Ukraine war. It recognises the role of persistent influences and significant

Table 1
Descriptive statistics for sector indices.

Sector	Energy	Fin	Cons. Staples	Cons. Discret	Health Care	Indus-trials	Info. Tech	Materials	Utilities	Comm.
Mean	0.0002	0.0004	0.0004	0.0005	0.0005	0.0004	0.0007	0.0003	0.0004	0.0003
Median	0.0002	0.0007	0.0005	0.0009	0.0007	0.0006	0.0011	0.0004	0.0008	0.0005
Std Deviation	0.0151	0.0109	0.0072	0.0095	0.0086	0.0094	0.0115	0.0105	0.0091	0.0089
Variance	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Kurtosis	25.1026	18.8054	13.9923	12.6219	9.3263	17.1441	12.9207	10.5556	24.3023	10.8187
Skewness	-0.7006	-0.7877	-0.6956	-0.7324	-0.3732	-0.5546	-0.4020	-0.4474	-0.4734	-0.6229
SW	0.8525*	0.8623*	0.8990*	0.9048*	0.9185*	0.8796*	0.8960*	0.9292*	0.8450*	0.9120*

Notes: This table reports descriptive statistics for sector indices. The total number of daily observations is 2606 ($N = 2602$ trading days). The Shapiro-Wilk (SW) test is applied to test for normality. The null hypothesis that daily returns are normally distributed is rejected in all instances (all *p*-values less than 10e-100). * indicates statistical significance at the 1% level. Descriptive statistics for industry groups and industries are reported in Appendix A.

but transient factors, in line with the nature of the return-generating process (see Korosteleva, 2022; Duggan, 2023; Szczygielski et al., 2020; 2024). Descriptive statistics for the S&P Global 1200 sector indices are reported in Table 1.

Returns are first estimated for each index, followed by each risk measure in Table 2, as well as their EWMA forecasts using a 3-month rolling period of 66 daily return observations.[1] The risk-free rate used is the US three-month T-bill rate. The EWMA forecast, $\widehat{m}_t$, of each risk measure m at time step t, is calculated recursively as follows:

$$\widehat{m}_t = \lambda_t x_t + (1 - \lambda)\widehat{m}_{t-1} \tag{1}$$

where x_t is an observation for risk measure x at time t and λ_t is a smoothing parameter between 0 and 1, recalculated for each period by minimising the heteroskedasticity-adjusted mean absolute error between the EWMA and the 'true' risk measure calculated using daily returns over the most recent month (Bollen, 2015). Mean returns $\overline{(r)}$ are assumed to be zero while determining the deviations from the mean which are aggregated to calculate each risk measure (Alexander, 2008). Consequently, for V, SD, DSV, DSSD, VaR, and CvaR, squared returns, r_t^2, are substituted for x_t in period t, while for MAD and SAD, we substitute absolute returns, $|r_t|$ for x_t. We use the EWMA to forecast risk measures. Although there are alternative forecasting methods, such as ARCH(*p,q*)/GARCH(*p,q*) modelling, these models have limitations and may not be universally applicable. For example, they are parametric, requiring the specification of an explicit model and the identification of an underlying error distribution, making them complex, restrictive, and prone to misspecification (Golosnoy et al., 2015; Szczygielski et al., 2023). In contrast, EWMA requires only a single parameter to be estimated from the training period data, and the model is well-known, widely used by practitioners, and readily implementable (Hartkopf & Reh, 2023).

In Table 2, mean returns are denoted by $\bar{r}$, n is the number of observations per estimation/calculation period, α is the significance level for VaR and CvaR, and Q denotes the quantile function. Each is calculated using a 3-month sample period using portfolio returns, where r_t denotes returns at time t. Returns, risk measures, and EWMAs are used to calculate asset weights[3] for a long-only portfolio on the Capital Market Line (CML) tangential to the efficient frontier (Sharpe, 1964). Portfolios are rebalanced at daily, weekly, and monthly frequencies.[4] Rebalancing at these intervals is undertaken to test the impact of different rebalancing periods on strategy returns while remaining cognisant that the risk and returns in each out-of-sample period are unlikely to remain substantially similar to that of the period used to optimise the portfolio (see Rachev et al., 2008).

[1] Where $P_{i,t}$ is the index value for index i at time t and $w_{i,t}$ is the index i's asset weighting at time t, the return for index i at time t, $r_{i,t} = P_{i,t} / P_{i,t\text{-}1}$ and the portfolio return, $r_t = \sum_{i=1}^{10} w_{i,t} r_{i,t}$. Monthly, weekly, and daily portfolios are formed using 66 daily observations, corresponding to the number of business days in a three-month period.

[2] Following Rockafellar & Uryasev (2000) and Krokhmal et al. (2002), we elect to set α at the 5% and 10% level for VaR_α (eq.(8)) and $CVaR_\alpha$ (eq. (9)) while acknowledging that an α of 5% is commonly used in the literature (see Ramos et al., 2023). By setting α at both 5% and 10% we follow convention while taking into account that 10% has also been used. Because VaR_α and $CVaR_\alpha$ are sensitive to the tail end of the loss distribution, they focus on extreme losses. While this can be beneficial for managing tail risk, it may also lead to overly conservative estimates, particularly if the actual distribution of returns is not as extreme as anticipated. This could result in underestimating potential performance in favour of avoiding unlikely extreme losses. Portfolios optimised for VaR or CVaR at 99% (α of 1%) may end up being overly concentrated in assets perceived as safer. This could limit diversification benefits and potentially reduce the overall efficiency of the portfolio. Hunjra et al.'s (2020) results support this to some extent; they find that while CVaR0.05 produces the most favourable results, CVaR0.01 produces the worst results across economic scenarios.

[3] For the purposes of the study, a portfolio is synonymous with its asset weights as the portfolio is defined by the weights (in percentage of total asset value) of its ten constituent assets.

[4] The term rebalancing is used here to describe a complete recalculation of the portfolio at each time step, rather than simply resetting the asset weights to the originally-calculated weights periodically. Daily industry group and industry runs were extremely computationally intensive due to the optimisation process requiring the calculation of more than 100 partial derivatives for each asset and each time period. Consequently, we limit the daily recalculation of strategies to sector-level data.

Table 2
Risk measure and sample statistic derivations.

Risk Measure	Formula	
Variance	$V = \frac{\sum_{t=1}^{n}(r_t - \bar{r})^2}{(n-1)}$	(2)
Standard deviation	$SD = \sqrt{\frac{\sum_{t=1}^{n}(r_t - \bar{r})^2}{(n-1)}}$	(3)
Downside semi-variance	$DSV = \frac{\sum_{t=1}^{n}(r_t - \bar{r})^2}{(n-1)}$ *where* $r_t < 0$	(4)
Downside semi-deviation	$DSSD = \sqrt{\frac{\sum_{t=1}^{n}(r_t - \bar{r})^2}{(n-1)}}$ *where* $r_t < 0$	(5)
Mean absolute deviation	$MAD = \frac{1}{n}\sum_{t=1}^{n}\left\lvert r_t - \frac{\sum_{t=1}^{n}(r_t)^2}{n}\right\rvert$	(6)
Semi-absolute deviation	$SAD = \sqrt{\frac{1}{n}}\sum_{t=1}^{n}\left\lvert r_t - \frac{\sum_{t=1}^{n}(r_t)^2}{n}\right\rvert$	(7)
5% and 10% Value at Risk[2]	$VaR_\alpha = -Q_{1-\alpha}(r_1, r_2, \cdots, r_n)$	(8)
5% and 10% Conditional Value at Risk	$CVaR_\alpha = \; = \frac{\sum_{t=1}^{n}(r_t - \bar{r})}{n}$ *where* $r_t < VaR_\alpha$	(9)

Notes: This table shows the derivation of the risk measure sample statistics used in the study.

We develop our own algorithm based on non-parametric gradient descent-like numerical methods to determine the optimal portfolio for period *t*. This permits us to implement the same solution across different risk measures, thereby controlling for differences in parameterisation that linear programming-related optimisation solutions might incur. It avoids opaque ('black box') elements in optimisation which commercial solver software could impart, and permits the controlled inclusion of constraints.[5] To optimise a portfolio, we calculate the partial derivative, $\frac{\partial S_t}{\partial w_i}$, of the slope ($S_t$) of the CML with respect to the weighting for each asset in the portfolio, w_i, by calculating the risk and return of portfolios with incrementally different asset weights for each asset (see Fig. 1, steps 3–6). Asset weights are then adjusted in the direction which increases the slope of the CML (step 9). This procedure is repeated in successively smaller increments until the change in the slope becomes negligible (steps 7–8), indicating a local maximum in the slope of the CML and an optimal portfolio (steps 7 & 11).

Once the portfolio for period *t* is optimised, its return is calculated out-of-sample for the next single period, $t+1$ (step 16)[6] and the rolling window is moved forward by a single time step (steps 12–14). Iterating across all periods results in a return for a specific risk measure's investment strategy, with the strategy consisting of a portfolio for each time step. We repeat the above procedure for each risk measure. Sample statistics are used for expected portfolio returns and risk measures to avoid making distributional assumptions, with the exception of the EWMA for VaR and CVaR, for which a non-parametric EWMA is not feasible.[7]

As sector indices can be traded using tracking ETFs (unlike industry group and industry indices which do not have equivalent ETF trackers), the effect of ETF total expense ratios, NASDAQ trading fees, and commissions is reflected by reducing each period's returns by the expected fees associated with holding or adjusting the portfolio. We assume the same fee structure for every index (tradeable or not) based on average fees on the tradeable ETFs which averaged 0.11% across strategies, ranging between 0.1% and 0.13%. The optimisation process is undertaken for all possible combinations of the ten risk measures, at the sector, industry group, and industry levels, recalibration frequencies, EWMA forecasts, and traditional backward-looking risk measures. This resulted in 140 investment strategies that were compared and used to draw inferences.

3. Results

3.1. Risk measure performance

Annualised returns for 140 investment strategies are reported in Table 3, with corresponding SRs in Table 4. The performance of each risk measure across raw and forecasted measures, tiers, and rebalancing periods is evaluated using the Wilcoxon Signed-Rank test.

[5] Algorithmic outputs were extensively tested for robustness using Monte Carlo simulations and we undertook extensive tests to confirm that the numerical methods used resulted in well-optimised portfolios for each risk measure. For example, 50,000 portfolios were simulated with random asset weights. Comparisons of the resultant risk/return ratios to those generated by our algorithm showed that the latter were higher, indicating better optimisation (more efficient portfolios). Our tests also show that the algorithm is well-suited for handling discontinuities in VaR, having a higher probability of arriving at a solution compared to linear programming when risk measures are known to be intractable to closed-form solutions. Results of the robustness tests are available upon request from the authors.

[6] i.e. if weekly, then the following week, or if monthly, then the following month.

[7] An EWMA is an aggregation, while VaR is a percentile. By aggregating, we condense returns into a single measure and lose information necessary to rank returns and select the percentile.

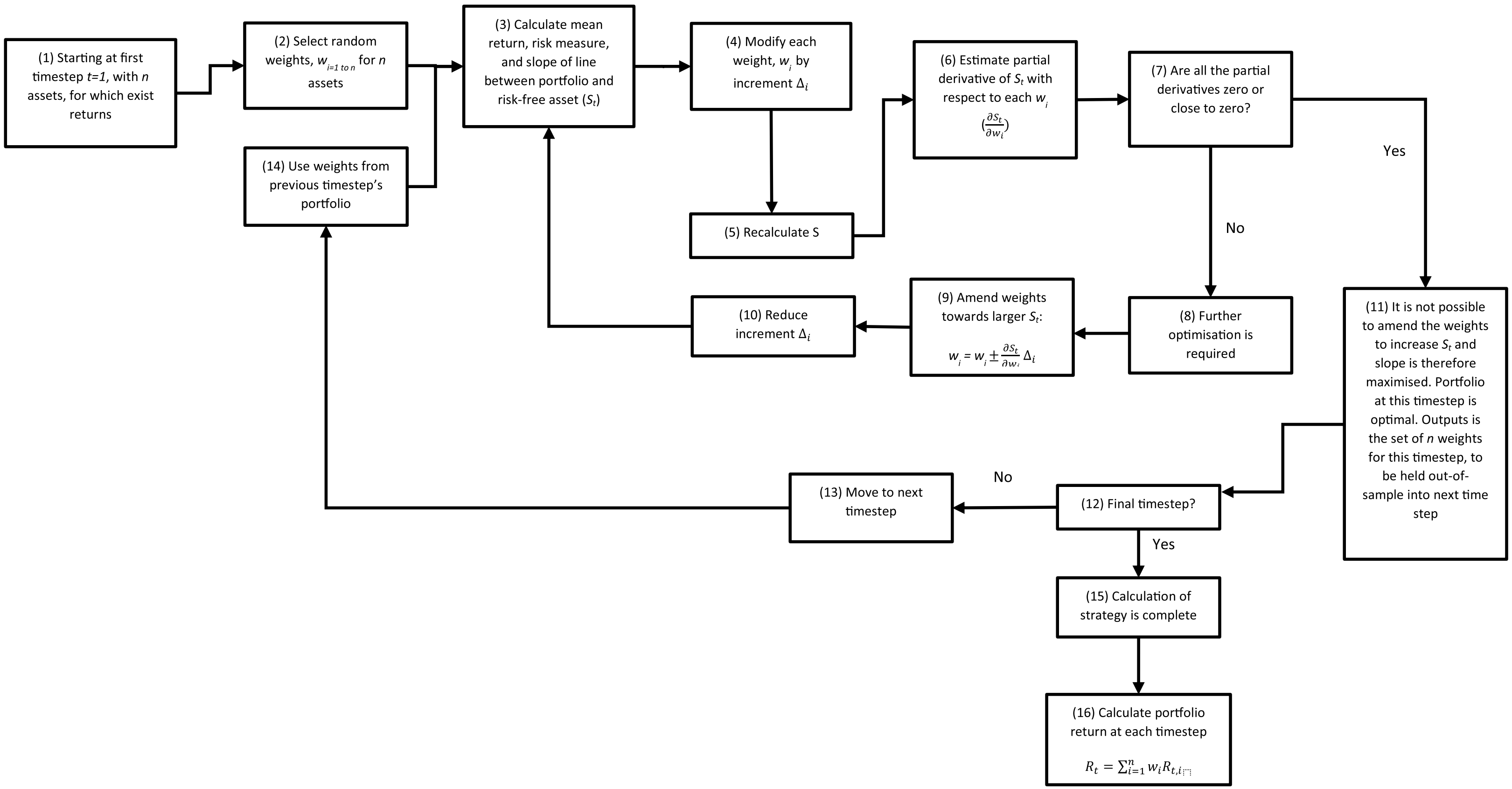

Fig. 1. Optimisation process.
Notes: Fig. 1 sets out the steps followed to calculate each successive period's optimised portfolio in an investment strategy. Up to several hundred iterations of steps 3 to 10 are required to maximise the slope of the CML and thereby to find an optimal portfolio for the period.

Table 3
Annualised strategy returns.

Tier	Calculation period	Raw/ EWMA	V	SD	DSV	DSSD	MAD	SAD	VaR0.1	VaR0.05	CVaR0.1	CVaR0.05
						Panel A						
Sector	**Daily**	**Raw**	9.60%	10.97%	10.88%	10.91%	10.94%	12.59%	9.31%	8.84%	9.91%	10.52%
		EWMA	11.26%	8.7%	7.09%	7.71%	8.94%	8.71%	8.81%	9.01%	9.11%	9.63%
	Weekly	**Raw**	10.66%	11.71%	9.73%	13.93%	12.19%	13.49%	9.56%	8.05%	11.6%	11.86%
		EWMA	10.71%	12.34%	12.7%	11.45%	16.18%	15.73%	12.62%	12.59%	5.10%	7.49%
	Monthly	**Raw**	11.33%	10.98%	10.49%	13.07%	12.6%	15.18%	9.26%	11.13%	10.9%	11.8%
		EWMA	7.61%	11.42%	7.69%	10.63%	11.5%	11.61%	10.89%	11.25%	8.88%	11.94%
Industry Group	**Weekly**	**Raw**	9.22%	14.48%	9.98%	13.71%	12.64%	11.59%	10.94%	14.56%	15.42%	16.26%
		EWMA	10.53%	18.87%	14.92%	19.18%	20.00%	19.47%	18.69%	18.71%	14.89%	15.76%
	Monthly	**Raw**	11.77%	14.51%	12.41%	13.13%	13.74%	14.16%	12.63%	12.15%	15.51%	15.31%
		EWMA	6.55%	10.52%	11.1%	13.02%	11.04%	12.29%	10.16%	10.16%	11.93%	10.28%
Industry	**Weekly**	**Raw**	10.66%	11.71%	9.73%	13.93%	12.19%	13.49%	9.56%	8.05%	11.6%	11.86%
		EWMA	10.71%	12.34%	12.7%	11.45%	16.18%	15.73%	12.62%	12.59%	5.10%	7.49%
	Monthly	**Raw**	11.33%	10.98%	10.49%	13.07%	12.6%	15.18%	9.26%	11.13%	10.9%	11.8%
		EWMA	7.61%	11.42%	7.69%	10.63%	11.5%	11.61%	10.89%	11.25%	8.88%	11.94%
						Panel B						
Raw			10.66%	12.20%	10.53%	13.11%	12.42%	13.67%	10.08%	10.57%	12.27%	12.78%
EWMA			9.29%	12.24%	10.56%	12.02%	13.62%	13.60%	12.10%	12.23%	9.14%	10.65%
						Panel C						
Sector			10.20%	11.03%	9.77%	11.29%	12.06%	12.89%	10.08%	10.15%	9.26%	10.54%
Industry Group			9.52%	14.60%	12.11%	14.76%	14.36%	14.38%	13.11%	13.90%	14.44%	14.41%
Industry			10.08%	11.62%	10.16%	12.28%	13.12%	14.01%	10.59%	10.76%	9.13%	10.78%
						Panel D						
Overall			9.97%†	12.22%*	10.55%†	12.56%*	13.02%*††	13.64%*††	11.09%†	11.40%	10.70%	11.71%**
Delta (V)			0.00%	2.24%	0.58%	2.59%	3.05%	3.66%	1.12%	1.43%	0.73%	1.74%
Delta (SD)			-2.24%	0.00%	-1.67%	0.35%	0.81%	1.42%	-1.13%	-0.82%	-1.52%	-0.50%

Notes: This table reports the annualised returns for each of the 140 strategies as well as average returns aggregated by Raw/EWMA categorisation and by tier, and overall. V = variance, SD = standard deviation, DSV = downside semi-variance, DSSD = downside semi-deviation, MAD = mean absolute deviation, SAD = semi-absolute deviation, VaR0.1 = 10% value at risk, VaR0.05 = 5% value at risk, CvaR0.1 = 10% conditional value at risk, CvaR0.05 = 5% conditional value at risk. Overall is the average annualised return across all strategies optimised using a given risk measure. Delta (V) and Delta (SD) are returns for strategies optimised using a given risk measure less returns for strategies optimised using variance and SD, respectively, with the latter being risk measures that are traditionally used in portfolio optimisation.

Statistical significance based on Wilcoxon Signed-Rank tests:

Relative to variance-optimised benchmark: * $p \leq 0.01$ ** $p \leq 0.05$ *** $p \leq 0.1$

Relative to standard deviation-optimised benchmark: † $p \leq 0.01$ †† $p \leq 0.05$ ††† $p \leq 0.1$

Table 4
Sharpe ratios.

Tier	Calculation period	Raw/ EWMA	V	SD	DSV	DSSD	MAD	SAD	VaR0.1	VaR0.05	CvaR0.1	CvaR0.05
						Panel A						
Sector	**Daily**	**Raw**	0.65	0.70	0.68	0.69	0.69	0.82	0.58	0.54	0.62	0.65
		EWMA	0.77	0.55	0.45	0.48	0.57	0.56	0.55	0.57	0.58	0.62
	Weekly	**Raw**	0.71	0.73	0.62	0.95	0.76	0.85	0.60	0.49	0.73	0.73
		EWMA	0.71	0.77	0.81	0.71	1.03	1.00	0.79	0.79	0.30	0.44
	Monthly	**Raw**	0.91	0.79	0.82	1.07	0.92	1.11	0.70	0.82	0.78	0.85
		EWMA	0.59	0.86	0.56	0.76	0.87	0.89	0.82	0.85	0.71	0.84
Industry Group	**Weekly**	**Raw**	0.53	0.81	0.57	0.75	0.74	0.70	0.62	0.83	0.87	0.91
		EWMA	0.66	1.08	0.89	1.16	1.15	1.12	1.07	1.07	0.78	0.86
	Monthly	**Raw**	0.79	0.88	0.87	0.82	0.84	0.88	0.77	0.76	0.96	0.96
		EWMA	0.43	0.64	0.68	0.76	0.70	0.76	0.62	0.62	0.59	0.52
Industry	**Weekly**	**Raw**	0.71	0.73	0.62	0.95	0.76	0.85	0.60	0.49	0.73	0.73
		EWMA	0.71	0.77	0.81	0.71	1.03	1.00	0.79	0.79	0.30	0.44
	Monthly	**Raw**	0.91	0.79	0.82	1.07	0.92	1.11	0.70	0.82	0.78	0.85
		EWMA	0.59	0.86	0.56	0.76	0.87	0.89	0.82	0.85	0.71	0.84
						Panel B						
Raw			0.75	0.78	0.71	0.90	0.80	0.90	0.65	0.68	0.78	0.81
EWMA			0.64	0.79	0.68	0.76	0.89	0.89	0.78	0.79	0.57	0.65
						Panel C						
Sector			0.72	0.73	0.66	0.78	0.81	0.87	0.67	0.68	0.62	0.69
Industry Group			0.60	0.85	0.75	0.87	0.86	0.87	0.77	0.82	0.80	0.81
Industry			0.73	0.79	0.70	0.87	0.90	0.96	0.73	0.74	0.63	0.72
						Panel D						
Overall			0.69†††	0.78***	0.70†††	0.83**	0.85*††	0.90*†	0.72†	0.74	0.67†††	0.73
Delta (V)			0.00	0.09	0.01	0.14	0.16	0.21	0.02	0.04	-0.02	0.04
Delta (SD)			-0.09	0.00	-0.09	0.05	0.06	0.11	-0.07	-0.05	-0.11	-0.05

Notes: This table shows SRs for each of the 140 strategies as well as average returns aggregated by Raw/EWMA categorisation and by tier, and overall. V = variance, SD = standard deviation, DSV = downside semi-variance, DSSD = downside semi-deviation, MAD = mean absolute deviation, SAD = semi-absolute deviation, VaR0.1 = 10% value at risk, VaR0.05 = 5% value at risk, CvaR0.1 = 10% conditional value at risk, CvaR0.05 = 5% conditional value at risk. Overall is the average SR across all strategies optimised using a given risk measure. Delta (V) and Delta (SD) are returns for strategies optimised using a given risk measure less returns for strategies optimised using variance and SD, respectively, with the latter being risk measures that are traditionally used in portfolio optimisation.
Statistical significance based on Wilcoxon Signed-Rank tests:
Relative to variance-optimised benchmark: * $p \leq 0.01$ ** $p \leq 0.05$ *** $p \leq 0.1$
Relative to standard deviation-optimised benchmark: † $p \leq 0.01$ †† $p \leq 0.05$ ††† $p \leq 0.1$

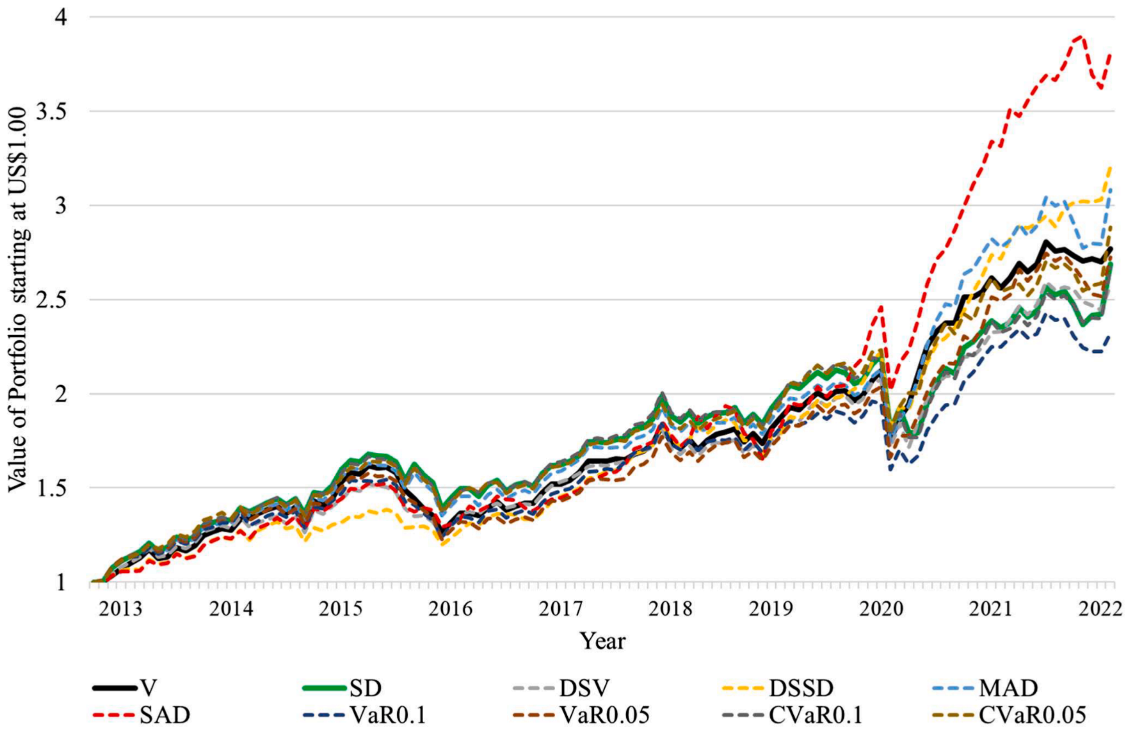

Fig. 2. Performance of monthly raw risk measure-optimised strategies (sector level).
Notes: This figure plots the value of a hypothetical $1.00 portfolio invested on 2 October 2012 for strategies based on sector indices optimised using raw risk measures and recalculated on a monthly basis. The performance of the SAD strategies is denoted by the solid red line. The solid black line denotes variance-optimised strategies and the solid green line denotes SD-optimised strategies.

We seek to determine whether there is a statistically significant difference between annualised returns and the SRs of strategies optimised using each risk measure and those optimised using variance and SD.[8]

Panel D of Table 3 reveals that overall, SAD-optimised strategies yield the highest average annualised returns of 13.64%, with a corresponding SR of 0.9 (Panel D of Table 4). The next best performers are MAD-optimised strategies (13.02% return, 0.85 SR), followed by DSSD (12.85%, 0.83). Furthermore, the performance of SAD-optimised strategies is consistent, with Panel A of Tables 3 and 4 showing that this strategy yields the highest or closest-to-highest annualised returns and SRs for almost every combination of sector, industry group, industry, calculation period, and backward-looking versus EWMA forecast. There are some (limited) exceptions to the consistent outperformance of SAD-optimised strategies. For daily EWMA-optimised strategies, V (11.26%, 0.7) outperforms SAD (8.72%, 0.56), and for weekly raw risk measure-optimised strategies, CVaR0.05 (16.27%, 0.91) outperforms SAD (11.6%, 0.7). Strong SAD performance is consistent with the finings of Jacobsen (2005) and Stanković et al. (2020). Comparing the performance of SAD strategies to other risk measure-optimised strategies, including variance- and SD-optimised strategies in Fig. 2, SAD strategies appear to perform particularly well during rebounding markets (2020–2022) and commensurately with other strategies during other phases of the market (2013–2019) prior to the COVID-19 outbreak. In Section 3.4, we consider the performance of risk measures prior to the outbreak of COVID-19, that is, prior to 1 January 2020 and then during the COVID-19 period from 1 January 2020.

Overall, the results suggest that optimising strategies using SAD generally results in superior annualised returns and risk-adjusted

[8] We are interested in whether portfolio optimisation using each alternative risk measure improves portfolio returns relative to the benchmark. We test the statistical significance of the difference between the annualised returns for variance- and standard deviation-optimised portfolios and those optimised using each alternative risk measure across the 14 different scenarios modelled for each risk measure (that is, each of three recalculation periods, three tiers and raw vs EWMA). Testing for the statistical significance of the differences in returns between variance- and standard deviation-optimised returns and the returns of a strategy optimised using an alternative risk measure for a single one of the 14 scenarios would not establish whether the alternative risk measure results in better performance. The reasons for the outcome of such a single test could be attributed to coincidence or to interaction effects. Consequently, significance tests are conducted at a risk measure level rather than strategy level. We also applied the *t*-test as a secondary confirmatory test. The conclusions remain unchanged.

Table 5
Differences between EWMA- and raw risk measure-optimised strategies.

Calculation period	Tier	V	SD	DSV	DSSD	MAD	SAD	VaR0.1	VaR0.05	CVaR0.1	CVaR0.05	Average
						Panel A						
Daily[10]	**Sector**	1.66%	-2.27%	-3.78%	-3.19%	-2.00%	-3.88%	-0.49%	0.17%	-0.80%	-0.89%	-1.55%**
						Panel B						
Weekly	**Sector**	0.05%	0.62%	2.97%	-2.49%	3.98%	2.24%	3.06%	4.54%	-6.50%	-4.37%	0.41%
	Industry Group	1.30%	4.39%	4.95%	5.47%	7.37%	7.88%	7.75%	4.15%	-0.53%	-0.50%	4.22%**
	Industry	0.05%	0.62%	2.97%	-2.49%	3.98%	2.24%	3.06%	4.54%	-6.50%	-4.37%	0.41%
	Average Delta	0.47%	1.88%	3.63%	0.16%	5.11%	4.12%	4.62%	4.41%	-4.51%	-3.08%	1.68%
						Panel C						
Monthly	**Sector**	-5.22%	-3.98%	-1.31%	-0.11%	-2.70%	-1.88%	-2.47%	-1.99%	-3.57%	-5.03%	-2.83%***
	Industry Group	-3.72%	0.44%	-0.80%	-2.43%	-1.11%	-3.56%	1.63%	0.12%	-2.01%	0.14%	-1.13%
	Industry	-3.72%	0.44%	-2.80%	-2.43%	-1.11%	-3.56%	1.63%	0.12%	-2.01%	0.14%	-1.33%*
	Average Delta	-4.22%	-1.03%	-1.64%	-1.66%	-1.64%	-3.00%	0.26%	-0.58%	-2.53%	-1.58%	-1.76%***
						Panel D						
Average of Panel A, B, C		-1.37%	0.04%	0.03%	-1.10%	1.20%	-0.08%	2.02%	1.66%	-3.13%	-2.13%	-0.29%

Notes: This table reports differences between EWMA strategy annualised return and the raw strategy annualised return. V = variance, SD = standard deviation, DSV = downside semi-variance, DSSD = downside semi-deviation, MAD = mean absolute deviation, SAD = semi-absolute deviation, VaR0.1 = 10% value at risk, VaR0.05 = 5% value at risk, CVaR0.1 = 10% conditional value at risk, CVaR0.05 = 5% conditional value at risk. Positive values indicate EWMA-optimised portfolios outperform raw risk measure-optimised strategies.
Statistical significance (Wilcoxon Signed-Rank test):
Difference between EWMA and Raw annualised returns: * $p \leq 0.10$** $p \leq 0.05$ *** $p \leq 0.01$.

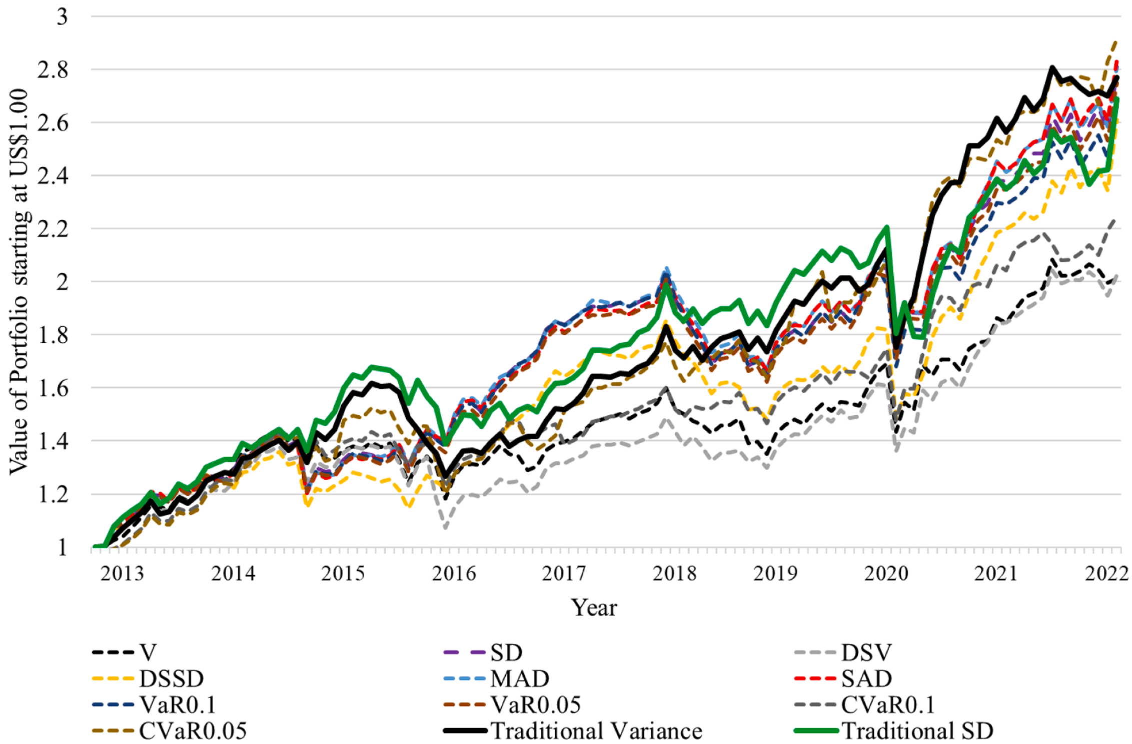

Fig. 3. Performance of monthly EWMA risk measure-optimised strategies (sector level).
Notes: This figure plots the value of a hypothetical $1.00 portfolio invested on 2 October 2012 for strategies based on sector indices optimised using EWMA risk measures and recalculated monthly. The performance of the traditional raw variance-optimised strategy is denoted by the solid blue line and the performance of the traditional raw SD-optimised strategy is denoted by the solid green line.

returns relative to the other risk measures investigated, notable amongst these being variance.[9] While our study focuses on performance, as measured by annualised returns and risk-adjusted returns, we also consider portfolio risk. A brief overview of these results followed by a discussion is reported in Appendix B.

3.2. EWMA performance

Next, we investigate whether portfolios constructed using risk forecasts result in better-performing portfolios than those constructed using the most recent raw backward-looking risk measure. Table 5 reports the differences in annualised returns between strategies optimised using EWMA and those optimised using the traditional backward-looking raw risk measures (hereafter, the 'return differential'). The return differential ranges from -6.5% (Raw CVaR0.1 outperforms EWMA CVaR significantly in Panel B of Table 5) to 7.88% (EWMA SAD outperforms raw SAD significantly). Thus, while certain risk measures exhibit enhanced performance when forecasted using EWMA, others do not. Overall, the average differential across all categories is -0.29% and is statistically insignificant (Panel D). The application of an EWMA does not universally improve returns. However, it is noteworthy that strategies recalibrated on a weekly basis for industry groups demonstrated significant benefits, producing a significant average return differential of 4.22% (Panel B) and an average positive return albeit an insignificant return differential of 1.68% overall, compared to that of daily strategies (-1.55%, Panel A) and monthly strategies (-1.76% in Panel C) which yielded negative return differentials.

Whether the use of EWMA forecasts is effective may be explained by several factors. First, smoothing a risk measure using an EWMA removes discontinuities in risk measures calculated for similar but slightly different portfolios. When optimising, this permits simpler and more accurate portfolio optimisation because the numerical methods optimisation algorithm is dependent upon gradient

[9] The Information Technology (IT) sector performed particularly well in recent years. To gain insight into whether the performance of the SAD-optimised strategy is attributable mainly to the relative outperformance of this sector, we compared asset allocation for the SAD-optimised strategies to that of the variance-optimised strategies. Using monthly strategies as an example, the SAD-optimised strategy allocated on average 78.6% vs 53.4% (variance) to the top five performing sectors. Variance allocated 46.6% vs 21.4% (SAD) to the bottom five performing sectors. Specifically, the SAD-optimised strategy resulted in 37.5% of the portfolio being allocated to the IT sector on average whereas the variance-optimised strategy allocated 14.6% on average to this sector. The conclusion is that despite this sector's favourable performance, SAD's inherent favouring of top performing portfolios is responsible for this strategy's outperformance.

[10] See footnote 4.

descent which does not respond well to discontinuities. Second, we postulate that smoothing certain risk measures, such as conditional value-at-risk, which is dependent upon parameterisation assumptions, may result in over-smoothing or excessive information loss. Third, and most importantly, an EWMA may simply be a poor tool for estimating future risk. An unadjusted EWMA is a trailing estimator suitable for application to monotonic data, performing poorly with cyclical time series (Yu et al., 2020). Consequently, if the market follows cycles of different durations (Lee et al., 2021), the presence of short-duration cycles may result in poor returns when trades are based on smoothed, longer-duration signals provided by the EWMA. The strong performance of weekly EWMA strategies but poor performance of daily and monthly EWMA strategies (Panels A and C) may therefore be attributable to underlying risk following cycles of different periods.

Fig. 3 plots monthly sector-level EWMA-optimised strategies, analogous to those in Fig. 2 which were optimised using raw risk measures. While almost all strategies optimised using the EWMA of the alternative risk measures (dotted lines) outperform the strategy optimised using the EWMA of variance (dotted black line), they generally do not outperform the traditional raw variance-optimised strategy (black solid line). For EWMA SD-optimised portfolios (dotted purple line), the performance is similar to that of the raw SD-optimised strategy. This suggests that using risk measure forecasts to optimise portfolios does not result in improved portfolio performance.

3.3. A comparison of risk measure characteristics

Besides general statements as to the undesirability of symmetrical risk measures for portfolio optimisation (see Byrne & Lee, 2004; Boasson et al., 2011; Cardoso et al., 2019), we find no corroboration drawn from empirical analysis of how the characteristics of risk measures impact portfolio performance in the literature. To address this gap, we compare the performance of each risk measure with the aim of inferring how their underlying characteristics impact portfolio performance.

The criticism that the use of symmetrical risk measures minimises upside risk and downside risk (see Section 1) appears to be well founded. Strategies optimised using DSV, DSSD, and SAD outperform their symmetrical counterparts. In Panel D of Tables 3 and 4, where DSV outperforms V by 0.58% annualised return and 0.01 SR, DSSD exceeds SD by 0.35% return and 0.05 SR, and SAD exceeds MAD by 0.62% return and 0.05 SR. While such differences may appear to be marginal on an annualised basis, they are substantial over the sample period, with DSV outperforming V by 15.18%, DSSD outperforming SD by 10.67%, and SAD outperforming MAD by 21.01%.[11]

The higher annualised returns for MAD- and SAD-optimised strategies relative to the other strategies may be attributable to these risk measures conferring a penalty upon the inclusion of an asset in a portfolio proportional to the difference between each period's return and the mean return rather than the square thereof. This is suggested by comparing the returns for MAD (13.02%) and SAD (13.64%) (Panel D of Table 3), which comprise unsquared return observations, to those for V (9.97%) and DSV (10.55%), which comprise squared return observations. Similarly, Panel D of Table 4 demonstrates higher SRs for portfolios optimised using risk measures based on unsquared returns, where MAD (0.85) and SAD (0.90) are higher than V (0.69) and DSV (0.7).

The use of raw deviations rather than squared deviations in calculating measures makes them less sensitive to outliers relative to variance, SD, DSV, and DSSD (Byrne & Lee, 2004). Greater sensitivity to outliers penalises the inclusion of assets with outliers, measured by deviations in returns from the mean. This may be particularly problematic for symmetrical risk measures, such as variance and SD, which reduce the weighting of assets with large upside return deviations. However, assuming at least some return distribution symmetry, penalising portfolio weightings for assets with large downside return deviations would also negatively impact portfolio returns, as large upside return deviations associated with those same assets will be excluded. This effect will be exacerbated in bull markets, as more large upside deviations would be excluded than large downside deviations.

A comparison of variance- and DSV-optimised strategies ('squared strategies') to SD- and DSSD-optimised strategies ('unsquared strategies') indicates that unsquared strategies result in higher annualised returns. This is evident from the overall returns in Panel D of Table 3, where SD (12.22%) exceeds V (9.97%) by 2.25% and DSSD (12.56%) exceeds DSV (10.55%) by 2.01%. This is because variance is the square of the SD. Where variance is the denominator of the function being optimised, the optimisation algorithm attributes more importance to risk than where SD is the denominator, such that assets with lower risk and lower returns will be more heavily weighted in squared strategies, while assets with proportionately higher risk and returns will be more heavily weighted in unsquared strategies. Any risk-reduction benefit of avoiding volatile indices is outweighed by a concomitant reduction in returns, as indicated by higher SRs for unsquared strategies relative to squared strategies, demonstrated by the SRs in Panel D of Table 4, where SD (0.78) exceeds V (0.69) and DSSD (0.83) exceeds DSV (0.7). In summary, portfolios optimised using asymmetrical risk measures outperform portfolios optimised using symmetrical risk measures. Furthermore, portfolios optimised using risk measures based on unsquared returns outperform those optimised using risk measures based on squared returns. Optimising portfolios with risk measures which attribute more importance to risk than to returns tends to result in lower portfolio returns.

3.4. Performance pre-COVID-19 and during COVID-19

Because our sample encompasses the COVID-19 period which constituted a highly disruptive and unprecedented event that resulted in global stock markets declining rapidly (March 2020), followed by a rapid recovery (by October 2020), we investigate

[11] To arrive at this comparison, we subtract compounded annualised returns for V, DSV and MAD from DSV, DSSD and SAD, respectively, assuming that are 250 trading days in a year.

whether this event impacts the relative portfolio performance.

While there is more variation in performance during the COVID-19 period (prior to 1 January 2020) across tiers and raw and forecasted measures, SAD-optimised portfolios yield the highest returns pre-COVID-19 overall and the third-highest overall returns (of 11.12% & 17.20%, respectively; Panel D of Table A2 & A3 in Appendix A) after DSSD (21.03%) and MAD (18.43%) during the COVID-19 period (after 1 January 2020). DSSD, MAD, and SAD outperform both V- and SD-optimised portfolios across both periods, and all alternative risk measures outperform V-optimised portfolios across both periods (albeit not always significantly).

For the full period, asymmetrical measures generally outperformed their symmetrical counterparts in magnitude. The same is true in the pre-COVID-19 period; examples are DSV (9.77%) outperforming V (8.96%), DSSD (10.05%) outperforming SD (9.93%), and SAD (11.12%) outperforming MAD (10.13%). During COVID-19, DSV (14.03%) outperforms V (8.63%) and DSSD (21.03%) outperforms SD (16.59%). The exception is MAD (18.43%) which outperforms SAD (17.20%). As with the full-period results, unsquared strategies generally outperform their squared counterparts. MAD (10.13%), SAD (11.12%), DSSD (10.05%), and SD (9.93%) outperform V (8.96%) and DSV (9.77%) prior to COVID-19. During the COVID-19 period, DSSD (21.03%), MAD (18.43%), SAD (17.20%), and SD (16.59%) outperform DSV (14.03%) and V (8.63%). During the COVID-19 period, the degree of outperformance is greater relative to the period prior to the COVID-19 outbreak. Pre-COVID-19 and COVID-19 SRs (see Tables A4 and A5 in Appendix A) exhibit similar patterns. The alternative risk measures generally yield higher risk-adjusted returns than either or both benchmarks. Prior to COVID-19, SAD (0.89) is the best performing measure, whereas following the COVID-19 outbreak, DSSD-optimised strategies yield the highest overall SR (0.87), followed by MAD (0.75), and SAD (0.71). Unsquared strategies outperform their squared counterparts, with SAD (0.89) yielding the highest SR prior to COVID-19 and DSSD (0.87) performing best during COVID-19. Asymmetrical measures outperform their symmetrical counterparts, with DSSD and SAD (both 1.01) yielding the highest SR prior to COVID-19 and DSSD (0.87) yielding the highest SR during COVID-19 and SAD (0.71) the third highest.

Sub-period analysis again suggests that the characteristics of the risk measures used to optimise portfolios have an impact on performance. Alternative risk measures tend to outperform either V- or SD-optimised portfolios or both across both periods, this being especially true for DSSD, MAD and SAD. SAD-optimised portfolios perform consistently well in both periods. As in the full sample period, unsquared and asymmetrical measures perform relatively well compared to their counterparts.

4. Conclusion

Our results show that the best performing measure is the semi-absolute deviation. Portfolios optimised using this measure yield the highest annualised returns and SRs. Other measures that perform favourably are DSSD and MAD, outperforming variance and SD-optimised portfolios. When the sample is divided into pre-COVID-19 and COVID-19 periods, these measures continue to perform well, although their relative performance rankings change. Portfolio managers wishing to maximise performance should use these measures for optimisation rather than using variance or SD. This is especially the case during volatile periods, as suggested by an increased return differential. The measure that performs worst is variance, severely underperforming SD, suggesting that using this measure should be avoided. Certain measures, specifically VaR and CVaR, underperform relative to SD, although they outperform variance. Overall, these findings suggest that while different is not always better, sometimes, it can be.

Replacing raw risk measures with EWMA-based risk forecasts results in similar portfolio performance relative to raw measures, although strategies recalibrated weekly benefit. The conclusion is that EWMA is not an ideal tool for estimating future risk, particularly if risk is cyclical. Portfolio managers and researchers studying portfolio optimisation who wish to account for the time-varying nature of risk and to estimate future risk within a portfolio optimisation context may need to use alternative methods. This could be a productive area for future research. Our study leads to some generalisable implications which should be taken into account when considering the *type* of risk measures that perform favourably. These are related to the characteristics of risk measures. Desirable characteristics appear to be asymmetry (DSV, DSSD and also VaR and CVaR with the latter two outperforming V but not SD), the use of unsquared returns (MAD, SAD, VaR & CVaR), the de-emphasis of outliers (SD, DSSD, MAD and SAD) and generally not diminishing the impact of expected returns during optimisation (e.g. SD as opposed to V). We hope this information is useful for portfolio managers faced with the decision of selecting risk measures for portfolio optimisation.

Our study is subject to several limitations which also serve as a motivation for further research. The sample comprises global sectors, industry groups, and industry equity-based indices, representing an abstraction. Results may differ if the sample comprises such groupings for a single national market or the sample composition is more granular, that is, comprising individual stocks or sub-industries, or if a different sample period is considered. An avenue for further research lies in replicating these results at the national market level using similar groupings. Relatedly, the focus is on equity-based indices. This calls into question whether these results – and the risk measures considered – are applicable to different asset classes, such as real estate investment trusts (REITs), commodities, or currency-based portfolios. Although our choice of alternative risk measures is motivated by literature which views them as popular and widely used, the subjectivity of this approach is a potential limitation. Further research could be undertaken to objectively and exhaustively identify the risk measures used in portfolio optimisation in practice. Such research could better inform further studies that seek to compare the performance of risk measures in portfolio optimisation.

CRediT authorship contribution statement

Douglas Austen Lorimer: Conceptualization, Methodology, Software, Resources, Data curation, Writing – original draft, Visualization, Investigation. **Cornelis Hendrik van Schalkwyk:** Supervision, Writing – review & editing, Project administration. **Jan Jakub Szczygielski:** Supervision, Writing – review & editing.

Data availability

Available upon request.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.frl.2024.105758.

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