FORECASTING SOUTH AFRICAN MACROECONOMIC VARIABLES WITH A MARKOV-SWITCHING SMALL OPEN-ECONOMY DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODEL*

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July 2016

Abstract

This paper seeks to identify evidence of regime-switching behaviour in the monetary policy response function and the variance of the shocks. It makes use of various specifications of a small open-economy Markov-switching dynamic stochastic general equilibrium (DSGE) model that is applied to South African data from 1989 to 2014. While the in-sample statistics suggest that some of the regime-switching models may provide superior results, the out-of-sample statistics suggest that the inclusion of various forms of regime-switching does not significantly improve upon the forecasting performance of the model. The results also suggest that the central bank response function has been consistently applied over the sample period.

JEL Classifications: E32, E52, F41.

Keywords: Monetary policy, inflation targeting, Markov-switching, dynamic stochastic general equilibrium model, Bayesian estimation, small open-economy.

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*The authors would like to thank the anonymous reviewer who provided generous and insightful comments. The remaining errors are those of the authors.

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1 Introduction

The South African economy has been subject to important political transitions, changes in monetary policy frameworks and economic crises. Recent events include the end of white minority rule and the formation of a democratic government in 1994, the adoption of a new inflation-targeting monetary policy framework in 2000, changes in central bank leadership, and a domestic currency crisis towards the end of 2001 (that lead to the formation of a formal commission of enquiry).[1] In addition, the economic and financial crises in other global markets also affected the South Africa economy, particularly through the relative risk-premium.[2]

This paper focuses on the degree to which monetary policy may have changed in light of these events, in the sense that we are able to identify regime-switching in the reaction function of a central bank, over recent periods of time. In addition, we also seek to identify any regime-switching behaviour in the variances of the structural disturbances. To investigate this phenomena, we consider the in-sample and out-of-sample statistics of an estimated small open-economy dynamic stochastic general equilibrium (DSGE) model that allows for various forms of regime-switching.

Early contributions to the literature that consider the use of Markov-switching in vector autoregressive (VAR) models include Sims and Zha (2004), Sims and Zha (2006), and Sims *et al.* (2008). These papers suggest that there is evidence of regime-switching in the monetary policy rule of the United States, when the models are applied to data over relatively long sample periods.[3] This work also suggests that regime-switching models should be used to describe monetary policy in those cases where the framework has changed (i.e. from one that considers monetary aggregates to one that is primarily concerned with prices).

More recently, Liu *et al.* (2009), Farmer *et al.* (2009), Farmer *et al.* (2011), Liu and Mumtaz (2011), Liu *et al.* (2011) and Alstadheim *et al.* (2013) provide examples of DSGE models that incorporate Markov-switching behaviour. These models allow for the analysis of samples with multiple regime changes, where they are largely focused on the way in which the central bank reacts to various factors that influence the policy rule. In addition, Alstadheim *et al.* (2013) consider how changes in the volatility of the respective shocks may influence the behaviour of the central bank. Most of these studies suggest that the assumption of a time-invariant central bank reaction function (as well as constant volatility) may provide an inaccurate description of monetary policy in these countries, during the respective time periods that were considered.

The use of regime-switching models that seek to describe changes in the monetary policy reaction function in South Africa are considered in quite a large number of recent studies (Naraidoo and Gupta (2010); Naraidoo and Raputsoane (2010, 2011, 2015); Kasai and Naraidoo (2011, 2012, 2013); Naraidoo and Paya (2012)). These papers model various kinds of asymmetric behaviour in the preferences of the central bank (South African Reserve Bank, SARB), leading to nonlinear reduced-form Taylor-type rules. Nonlinearities are not

[1]Over the sample period the respective leaders of central bank include: Stals [1989], Mboweni [1999], Marcus [2009] and Kganyago [2014]; where the date of appointment is included in brackets.

[2]These crises would include the emerging market crises that originated in Mexico [1994], Asia [1997], Russia [1998] and Argentina [1999]. As South Africa has a relatively liquid financial market, the dot.com and other asset pricing bubbles may have also influenced the variance of economic shocks at particular points in time. Then lastly, the recent global financial crisis and the period of quantitative easing in developed world economies may have affected both the monetary policy response function and the variance of the shocks.

[3]These studies extend the work of Clarida *et al.* (2000) and Lubik and Schorfheide (2004), by considering the application of Markov-switching models for the identification different monetary policy regimes. Computational details that describe a robust method for the calculation of the posterior density for the complex likelihood function are contained in Sims and Zha (2004) and Sims *et al.* (2008).

only considered in the output-gap and inflation, but also in a financial conditions index to capture changes in the financial state of the South African economy. In general, all these studies suggest that the fit of the regime-switching models is superior to that of linear models in both in-sample and out-of-sample evaluations. In addition, these papers also suggest that the SARB does respond to financial conditions, especially during episodes of crises.[4]

The results of this paper suggest that when South African data is taken to a DSGE model that allows for regime-switching in the central bank reaction function and the variances of the structural shocks, the behaviour of the central bank would appear to be reasonably consistent.[5] However, there may have been a few isolated incidents where the central bank responded relatively strongly to inflationary pressure. This finding is supported by the parameter estimates of the models that have a superior in-sample fit, the smoothed transition probabilities of the models that allow for regime-switching in the monetary policy rule, and the out-of-sample forecasting results.

We also note that allowing for changes in the variances of the risk-premium shock would appear to improve the in-sample fit of the model (which is the only variance of all the shocks that experiences a relatively large change). This finding is largely consistent with the work of Sims and Zha (2006), who find that allowing for changes in the variances of shocks improves the in-sample fit of a Markov-switching structural VAR that is applied to U.S. data.[6] However, the superior in-sample statistics do not translate into significant improvements in the out-of-sample forecasting results in this case.

The rest of the paper is organized as follows. Section 2 describes the methodology, while section 3 provides details of the data. The in-sample results are discussed in section 4 and out-of-sample results are discussed in section 5. The conclusion is contained in section 6.

2 Methodology

2.1 Theoretical model

The structure of the model follows that of Alpanda *et al.* (2011), which incorporates several small open-economy features of the South African economy.[7] This model structure is largely based on the frameworks of Galí and Monacelli (2005) and Justiniano and Preston (2010). After all variables are log-linearised around their steady-state, the equations that characterise the equilibrium conditions of the *non* Markov-switching version of model may be expressed as follows.

The domestic household's Euler condition yields a partially forward-looking IS curve in consumption:

$$c_t = \frac{1}{1+\zeta}\mathbb{E}_t\left[c_{t+1}\right] + \frac{\zeta}{1+\zeta}c_{t-1} - \frac{1-\zeta}{\sigma\left(1+\zeta\right)}\left(i_t - \mathbb{E}_t\left[\pi^c_{t+1}\right] - \Theta_t\right) \tag{1}$$

where σ is the inverse intertemporal-elasticity of substitution and habits in consumption are represented by ζ. The exogenous demand shock, is represented by Θ, whose natu-

[4]While these findings are of significant interest, the use of reduced-form models for monetary policy investigations have been criticized by Lucas (1976) for not incorporating forward-looking behaviour, while Galí (2008) and Christiano *et al.* (2011) note that reduced-form models have been largely unable to describe some of the essential features of monetary policy.

[5]To the best of our knowledge, this is the first paper that considers the use of a Markov-switching DSGE model that has been applied to South African data.

[6]Stock and Watson (2003), Primiceri and Justiniano (2008) and Fernández-Villaverde *et al.* (2010) also make note of importance of allowing for changes in the variances of shocks.

[7]See, Alpanda *et al.* (2010*a*) and Alpanda *et al.* (2010*b*) for further details of the derivation of the model.

ral logarithm follows an AR(1) process, with persistence parameter ρ_c, and error, $\varepsilon_{c,t} \sim$ i.i.d.$N[0, \sigma_c^2]$. The rate of consumer price inflation is expressed as π_t^c.

The relation between consumption and domestic output can be derived from the goods market clearing condition as:

$$y_t = (1-\alpha)c_t + [(1-\alpha)\eta\alpha + \eta\alpha]\,\tau_t + \alpha y_t^\star + \eta\alpha\psi_{f,t} \tag{2}$$

where α is the share of imports in consumption, η is the elasticity of substitution between domestic and foreign goods, y_t and $y_t^\star$ are domestic and foreign output, respectively, whilst $\tau_t = p_{f,t} - p_{h,t}$ is the terms of trade, and $\psi_{f,t}$ is the deviation of imported goods prices from the law-of-one-price.

Time differencing the terms-of-trade yields $\tau_t = \tau_{t-1} + p_{f,t} - p_{h,t}$, where $p_{h,t}$ and $p_{f,t}$ are inflation rates associated with the domestic and foreign goods prices, respectively. The domestic producer's problem yields a partially forward-looking New Keynesian Phillips curve for domestic price inflation:

$$\pi_{h,t} = \frac{\delta}{1+\delta\beta}\pi_{h,t-1} + \frac{\beta}{1+\delta\beta}\mathbb{E}_t[\pi_{h,t+1}] + \frac{(1-\theta_h)(1-\theta_h\beta)}{\theta_h(1+\delta\beta)}mc_t \tag{3}$$

where β is the time-discount parameter, δ determines the degree with which prices are indexed to past domestic price inflation, and θ_h is the probability that the firms cannot adjust their prices in any given period. The above Phillips curve ties current domestic inflation rate to past and expected future inflation as well as the marginal costs of the firm. Marginal cost is given as, $mc_t = \varpi_t - a_t + \gamma\tau_t + \eta_t^p$, where ϖ_t is the real wage rate, a_t is the level of productivity in the production function that follows an exogenous AR(1) process, and η_t^p is a domestic cost-push shock that also follows an AR(1) process.

Similarly, foreign goods price inflation follows a forward-looking Phillips curve:

$$\pi_{f,t} = \beta\mathbb{E}_t[\pi_{f,t+1}] + \frac{(1-\theta_f)(1-\theta_f\beta)}{\theta_f}\psi_{f,t} \tag{4}$$

where θ_f is the probability that the importers cannot adjust their prices in any given period. Overall consumer price inflation in the domestic country is given by $\pi_t = (1-\alpha)\pi_{h,t} + \alpha\pi_{f,t}$.

Staggered wage setting by households yields the following wage inflation Phillips curve:

$$\pi_{w,t} - \varphi_w\pi_{t-1} = \beta\mathbb{E}_t[\pi_{w,t+1}] - \varphi_w\beta\pi_t + \frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w(1+\xi_w\gamma)}\mu_t^w \tag{5}$$

where $\pi_{w,t}$ is the nominal wage inflation, φ_w is a parameter determining the degree of inflation indexation of nominal wage inflation, γ is the inverse of the elasticity of labour supply, and ϵ_w is the elasticity of substitution between differentiated labour services of households in the labour aggregator function. The wedge between the real wage and the marginal rate of substitution between consumption and labour in the household's utility function is μ_w, which may be expressed as,

$$\mu_t^w = \frac{\sigma}{1-\zeta}(c_t - \zeta c_{t-1}) + \gamma(y_t - a_t) - \varpi_t + \eta_t^w \tag{6}$$

where η_t^w is a wage cost-push shock that follows an AR(1) process. The relationship between nominal wage inflation and real wages can be expressed as $\pi_{w,t} = \varpi_t - \varpi_{t-1} + \pi_t$.

The uncovered interest parity (UIP) condition is then given by,

$$\mathbb{E}_t[q_{t+1}] - q_t = (r_t - \mathbb{E}_t[\pi_{t+1}]) - (r_t^\star - \mathbb{E}_t[\pi_{t+1}^\star])) + \phi_t \tag{7}$$

where $q_t = e_t + p_t^\star - p_t$ is the real exchange rate. This is related to the terms-of-trade and the gap from the law-of-one-price, which is expressed as, $q_t = (1-\alpha)\tau_t + y_{f,t}$. Time differencing the real exchange rate yields the relationship between real and nominal depreciation rates, where $q_t - q_{t-1} = \Delta e_t + \pi_t^\star - \pi_t$. The variable $\phi_t = \mu_t^\phi + \chi \cdot nfa_t$ captures the time-varying country risk-premia. It is determined by the sum of an exogenous component, μ_t^ϕ, which follows an AR(1) process, and the net foreign asset position of the country, nfa_t, where χ is an elasticity parameter. The net asset position of the country evolves over time according to

$$nfa_t - \frac{1}{\beta} nfa_{t-1} = y_t - c_t - \alpha(\tau_t - \phi_{f,t}). \tag{8}$$

The central bank then makes use of the nominal interest rate as its policy instrument in an open-economy Taylor rule that allows for the inclusion of the exchange rate in its reaction function. In addition, we assume that the central bank targets the expected future value of inflation, and as such we make use of an expectational operator for this critical variable. Hence,

$$i_t = \rho\, i_{t-1} + (1-\rho)\left[\varrho_\pi \mathbb{E}_t\left(\pi_{t+1}^c\right) + \varrho_y \tilde{y}_t + \varrho_d d_t\right] + \varepsilon_{i,t} \tag{9}$$

The rest of the world is modelled as a closed-economy version of the domestic economy, which can be represented by the representative IS curve (where the use of the * denotes foreign versions of the domestic counterparts):

$$y_t^\star = \frac{1}{1+\zeta}\mathbb{E}_t[y_{t+1}^\star] + \frac{\zeta}{1+\zeta} y_{t-1}^\star - \frac{1-\zeta}{\sigma^\star(1+\zeta)}\left(r_t^\star - \mathbb{E}_t[\pi_{t+1}^\star] + \mu_t^{d\star}\right) \tag{10}$$

a New Keynesian Phillips curve,

$$\pi_t^\star = \frac{\delta^\star}{1+\delta^\star\beta}\pi_{h,t-1} + \frac{\beta}{1+\delta^\star\beta}\mathbb{E}_t[\pi_{h,t+1}^\star] + \frac{(1-\theta^\star)(1-\theta^\star\beta)}{\theta^\star(1+\delta^\star\beta)} mc_t^\star \tag{11}$$

where the foreign marginal cost is given by,

$$mc_t^\star = \left(\frac{\sigma^\star}{1-\zeta} + \gamma^\star\right) y_t^\star - \left(\frac{\sigma^\star\zeta}{1-\zeta}\right) y_{t-1}^\star - (1+\gamma^\star) a_t^\star + \mu_t^{w,\star} \tag{12}$$

and a foreign Taylor rule that is specified as,

$$i_t^\star = \rho^\star i_{t-1}^\star + (1-\rho^\star)\left[\varrho_\pi^\star \pi_t^\star + \varrho_y^\star \tilde{y}_t^\star\right] + \varepsilon_t^{i\star} \tag{13}$$

2.2 Markov-switching

In the version of the model that incorporates Markov-switching in the domestic monetary policy reaction function, the Taylor rule in (9) is expressed as,

$$i_t = \rho_\kappa\, i_{t-1} + (1-\rho_\kappa)\left[\varrho_{\kappa,\pi}\mathbb{E}_t\left(\pi_{t+1}^c\right) + \varrho_{\kappa,y}\tilde{y}_t + \varrho_{\kappa,d} d_t\right] + \varepsilon_{i,t} \tag{14}$$

where κ is used to denote a two-state discrete Markov process taking values $\kappa \in \{1,2\}$ with transition probabilities ν^{1-2} and ν^{2-1}, that influence the current state of the two regime model. In this case, we denote the regime that has a smaller response to output deviations as $\kappa = 1$, while the regime for a larger response to output deviations is denoted $\kappa = 2$.[8]

In addition to the above specification, we also seek to identify different regimes in the variance of the structural shocks. In the more general specification of the model, this results in the inclusion of an additional ten parameters, where the notation ς_ϑ^i would refer to the

[8]While the distinction between the two regimes is controlled by the size of $\varrho_{\kappa,y}$, we see that the more significant difference in the central bank response function (when comparing the two regimes) relates to the reaction to a change in the rate of inflation.

variance in the corresponding monetary policy shock, $\varepsilon_{i,t} \sim \text{i.i.d.}N[0, \varsigma^i_\vartheta]$. In this case, $\vartheta \in \{1, 2\}$, is a two-state discrete Markov process, where the regime with the smaller variance in the shocks is denoted $\vartheta = 1$.[9]

In addition to these two models, we also consider the results of a specification that allows for Markov-switching in both the policy reaction function and the variance of the shocks, where each of these phenomena are controlled by separate (independent) chains. The set of models that we consider is then further augmented with a model that limits the regime-switching in the variance of structural disturbances to only the risk-premium shock, as this is of particular importance in an emerging market economy such as South Africa.[10] Then lastly, in a separate model we incorporate regime-switching in the monetary policy rule and the risk-premium shock, where a separate chain is used for each of these elements.[11]

2.3 Solution and estimation

As the solution in each state, is a function of the solution in the other states, traditional solution methods for constant-parameter linear rational expectations models may not be used.[12] Therefore, we make use of the methods developed in Maih (2015) that are relatively similar to those of Svensson (2007), Farmer *et al.* (2011) and Foerster *et al.* (2014). In essence, this method seeks to identify the minimum state variable solutions after applying the concept of mean square stability. This characterisation allows us to specify the general form of the Markov-switching rational expectations model as,

$$\mathbb{E}_t \left[A^+_{s_{t+1}} x_{t+1} (\bullet, s_t) \ ; \ A^0_{s_t} x_t (s_t, s_{t-1}) \ ; \ A^-_{s_t} x_{t-1} (s_{t-1}, s_{t-2}) \ ; \ B_{s_t} \varepsilon_t \right] = 0 \tag{15}$$

where x_t is a $n \times 1$ vector of endogenous variables and $\varepsilon_t \sim N(0, \varsigma_\vartheta)$ is the vector of exogenous shocks. The stochastic regime index s_t switches between two possibilities, such that $s_t = 1, 2$. These probabilities are assumed to be constant in this model, where s_t denotes the state of the system today and s_{t-1} denotes the state in the previous period.[13]

The parameters in the model are estimated with Bayesian techniques, where all the unobserved variables, states of the Markov chains, and parameter values are treated as random variables. The Kalman filter that is used to compute values for the unobserved processes takes the weighted average of unobserved processes for each state of the Markov chains, where the weights are provided by the probability values. These techniques are applied in Kim and Nelson (1999) and Sims *et al.* (2008), where the probabilities are calculated according to the Hamilton (1989) procedure.

After computing an approximation of the likelihood function we make use of the prior distributions to find the mode of the posterior distribution. Thereafter, we are able to initialize the Markov Chain Monte Carlo (MCMC) procedure that is used to construct the full posterior distribution and marginal data density. Details of the prior parameter values that are used in the calculation of the posterior estimates are similar to those that were used in Alpanda *et al.* (2011).[14]

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[9] Similar notation is used for the variance in the other stochastic shocks and the regime with the larger variance in the shocks is denoted $\vartheta = 1$.

[10] Alpanda *et al.* (2010*a*) provide a motivation for the inclusion of a risk-premium shock, when modelling South African macroeconomic data.

[11] Additional results from each of these models may be found in the online technical appendix.

[12] The online technical appendix includes further details regarding the solution and estimation techniques.

[13] This solution algorithm is implemented with the aid of the RISE toolbox that has been developed by Maih (2014).

[14] Additional results are also included in the online appendix.

3 Data

The dataset extends over the period 1989q1 to 2014q4. The start date of the sample is motivated by the findings of Du Plessis & Kotzé (2010; 2012), who suggest that there is a significant structural change in most macroeconomic variables that would impact on the measure of the business cycle during the mid-1980s.[15]

Essentially, we estimate the model with ten observed variables for measures of: domestic output growth, $\tilde{y}$, GDP-deflator inflation, π, consumer inflation, π^c, nominal interest rate, i, nominal wage inflation, π^w, nominal productivity, z, nominal currency depreciation, d, foreign output growth, y^*, foreign GDP-deflater inflation, π^*, and foreign nominal interest rate, i^*.

All of the data for the South African economy was obtained from the South African Reserve Bank, with the exception of consumer prices, which were obtained from Statistics South Africa.[16] The data for the United States economy was obtained from the Federal Reserve System. Measures of output, inflation, productivity and currency depreciation are transformed to growth rates, while interest rates are expressed as annualised rates.

4 Results

4.1 In-sample statistics

Table 1 contains the in-sample statistics for the six models that have been estimated on the full set of in-sample data. These models include a specification that does not include Markov-switching, along with the model that allows for switching in the monetary policy rule only. Thereafter, the third specification allows for Markov-switching in the variance of all the structural shocks (only), while the fourth specification allows for switching in both the monetary policy rule and the variance of all the structural shocks. The fifth specification considers the inclusion of Markov-switching in the variance of the risk-premium shock (only), while the final specification allows for switching in the monetary policy rule and the variance of the risk-premium shock.

	log-post:	log-lik:	log-prior:	log-MDD
No-switching	3329	3395	-66	3195
Switching: monetary policy only	3345	3415	-70	3171
Switching: variance of shocks	3371	3426	-55	3191
Switching: monetary policy & shocks	**3102**	**3168**	-66	2849
Switching: risk-premium shock	3345	3417	**-72**	**3201**
Switching: mon. policy & risk-prem.	3353	3425	-71	3200

Table 1: In-sample estimation statistics

The results include details of the log-posterior, log-likelihood, log-prior and log-marginal data density (MDD) that is estimated with the use of a Laplace approximation.[17] The

[15] Hence, if the sample period started prior to this structural break the Markov-switching model would possibly only pick up on this behaviour and leave the remaining sample as one that is characterised as a single regime.

[16] To create a single measure of consumer price inflation we combine the respective measures that existed prior to 2008 with that which was established under the current methodology, using the monthly weighting procedure that is discussed in Du Plessis *et al.* (2015).

[17] Christiano *et al.* (2011) provide details regarding the computation of this statistic.

only models that provide a log-MDD that is above the variant that does not make use of any regime-switching are the two that allow for switching in the variance of the risk-premium shocks. It should also be noted that the difference in the in-sample statistics of the three best performing models is not all that large and may not be significant (as they are approximations).

In addition, we also note that the model that provides the best log-posterior and log-likelihood is the heavily parametrised model that allows for regime-switching in the monetary policy rule and the variance of all the shocks (where separate chains control the two forms of regime-switching). Unfortunately, this specification has a relatively poor log-MDD and when we look at all the parameter estimates that are provided by this model, we note that a relatively large number of parameters would appear to be poorly identified.[18]

Furthermore, when we consider the parameter estimates of the model that allows for regime-switching in the variance of all the shocks, we note that with the exception of risk-premium shock, the difference in the mode values for the variance of other shocks hardly differs between each regime. Hence, allowing for a change in regime for these other shocks would not appear to be all that important.

4.2 Parameter estimates

After considering the in-sample statistics that were reported above, we summarise the parameter estimates for the case where there is no regime-switching, where there is regime-switching in the variance of the risk-premium shocks (only), and where there is regime-switching in the monetary policy rule and the variance of the risk-premium shocks.

Table 2 provides details of the prior and posterior parameter estimates for the monetary policy rule in the three models, where the results for the models that do not include regime-switching are reported under regime one (although these results would obviously apply to both regimes). These results report on the posterior mode and standard deviation for each of the parameters, where the transition probabilities, ν^{1-2}, ν^{2-1}, ω^{1-2} and ω^{2-1}, make use of flat priors.

The monetary policy rule parameter estimates of the two models that do not allow for regime-switching are similar. For example, the degree of interest rate smoothing differs by a small amount and the central bank response to a change in output takes values, $(1-\rho)\varrho_y = 0.101$ and 0.099, for no regime-switching and regime-switching in the risk-premium (only) models. In addition the response to changes in the exchange rate are both extremely close to zero, at $(1-\rho)\varrho_d = 0.010$ and 0.009, for these respective models. The only slight difference between these models pertains to the response to inflation, where the model that does not allow for regime-switching has a slightly larger response, where $(1-\rho)\varrho_\pi = 0.287$, against a value of 0.279 for the model that allows for regime-switching in the risk-premium (only).

In the case of the model that allows for regime-switching in the monetary policy rule, the posterior estimate for the interest rate smoothing coefficients are $\rho(\kappa = 1) = 0.7$ and $\rho(\kappa = 2) = 0.884$, which would suggest that there is much less interest rate smoothing when in the first regime. In addition, the interest rate smoothing in the second regime is greater than what is provided by the models that do not allow for regime-switching in the monetary policy rule. When calculating the response to inflation in the two regimes, we note that $(1-\rho)\varrho_\pi(\kappa = 1) = 0.426$ and $(1-\rho)\varrho_\pi(\kappa = 2) = 0.202$, which would suggest that the response to inflation is relatively large when in the first regime (i.e. almost 50% greater than the response that is provided by the no regime-switching alternative). When in the second regime, the response is less than half that of the first regime and approximately 30%

[18] All of the parameter mode and standard deviations for each of the models have been included in the online appendix, where we also include figures for distributions of parameters (and their means) in the three models that provide superior in-sample statistics.

				No-switching		Risk-premium (only)		Mon. pol & risk-prem.	
Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.	Post Mode	Post Std.	Post Mode	Post Std.
$\rho\ (\kappa=1)$	beta	0.75	0.10	0.823	0.025	0.846	0.019	0.700	0.055
$\rho\ (\kappa=2)$	beta	0.75	0.10					0.884	0.017
$\varrho_\pi\ (\kappa=1)$	gamma	1.50	0.25	1.624	0.209	1.803	0.203	1.419	0.252
$\varrho_\pi\ (\kappa=2)$	gamma	1.50	0.25					1.740	0.208
$\varrho_y\ (\kappa=1)$	gamma	0.25	0.12	0.570	0.201	0.643	0.223	0.198	0.106
$\varrho_y\ (\kappa=2)$	gamma	0.25	0.12					0.607	0.223
$\varrho_d\ (\kappa=1)$	gamma	0.12	0.05	0.057	0.021	0.061	0.022	0.115	0.038
$\varrho_d\ (\kappa=2)$	gamma	0.12	0.05					0.046	0.019
ν^{1-2}	uniform	0.00	1.00					0.893	0.322
ν^{2-1}	uniform	0.00	1.00					0.053	0.043

Table 2: Prior and posterior parameter estimates - Monetary policy rule

				No-switching		Risk-premium (only)		Mon. pol & risk-prem.	
Parameter	Distribution	Prior Mean	Prior Std.	Post Mode	Post Std.	Post Mode	Post Std.	Post Mode	Post Std.
$\varsigma^d\ (\vartheta=1)$	weibull	0.23	0.29	0.0036	0.0008	0.0019	0.0005	0.0020	0.0006
$\varsigma^d\ (\vartheta=2)$	weibull	0.23	0.29			0.0112	0.0042	0.0091	0.0038
ω^{1-2}	uniform	0.00	1.00			0.0019	0.0005	0.0489	0.0392
ω^{2-1}	uniform	0.00	1.00			0.0112	0.0042	0.6836	0.2709

Table 3: Prior and posterior parameter estimates - Variance of shocks

lower than the model that does not allow for regime-switching.

The parameter estimates for the central bank response to changes in output would suggest that when in regime-one, the central bank does not respond too aggressively, where $(1-\rho)\varrho_y(\kappa=1)=0.059$. In addition, when in regime-two the central bank response to changes in output is $(1-\rho)\varrho_y(\kappa=2)=0.070$. Both of these responses are relatively low, when compared to the models that do not allow for regime-switching (i.e. between 40% and 30% lower for the two regimes, respectively). Similarly, the response of the central bank to changes in the exchange rate suggest that in while the response is slightly larger in regime one, $(1-\rho)\varrho_d(\kappa=1)=0.034$, it is still relatively close to zero. The other measures for the response to changes in the exchange rate are all much closer to zero.

To summarise these results, we firstly note that the coefficients for the model that does not include regime-switching are similar to those of Alpanda *et al.* (2010*a*,*b*, 2011), Steinbach *et al.* (2009) and Ortiz and Sturzenegger (2007). From the results of the regime-switching model, we note that the central bank employs a relatively small amount of interest rate smoothing when in regime-one. In addition, its response to inflation is relatively aggressive and it does not react all that strongly to changes in output, when in this regime. In contrast with these results, when in regime-two, the parameter estimates would suggest that the central bank exhibits a large degree of interest rate smoothing and responds in a more moderate manner to changes in inflation.

Table 3 contains the parameter estimates for the variance of the risk-premium shocks, where we note that the difference in the size of the ς^d parameter between the two regimes is relatively large. For example, the coefficient estimate in the model that allows for regime-switching in the variance of the risk-premium shock (only), is relatively small in regime-one; however, during regime-two it is almost six times that of the estimate in regime-one.

Similar results are evident for the model that allows for regime-switching in the variance of the risk-premium shock and monetary policy rule, where the coefficient in regime-two is more than four times that of the estimate in regime-one. In addition, we also note that the estimates for the two regime-switching models during regime-one are almost half the size of those that are provided by the model that does not allow for regime-switching. Hence, these results would suggest that there is a large distinction in the size of the variance of the risk-premium shocks, when the model allows for regime-switching behaviour.

4.3 Transition probabilities

The smoothed transition probabilities for the model that incorporates regime-switching features in both the monetary policy rule and the variance in the risk-premium shock are displayed in Figures 1 and 2. These probabilities have been plotted against the respective variables that are included in the central bank reaction function. Figure 1 shows the smoothed transition probability for the chain that is applied to the monetary policy rule, where a probability value of one (on the right-hand side axis) corresponds to the regime where the central bank reacts aggressively to change in inflation (i.e. where $\kappa=1$).[19]

When considering these transition probabilities for a change in regime in the monetary policy rule, it is worth noting that there is no level shift. Hence, it would suggest that over the sample, monetary policy has been applied relatively consistently, despite the change to an inflation-targeting regime, the introduction of new central bank leaders, etc. The only period where the transition probability rose to a unitary value, occurred in 1999, during the time of the emerging market crisis. In addition, this also corresponds to the date of the appointment of Governor Mboweni to leadership of central bank. This result is also supported by the transition probabilities of the other specifications of these models, which

[19] The smoothed transition probabilities for all the other models are included in the online appendix.

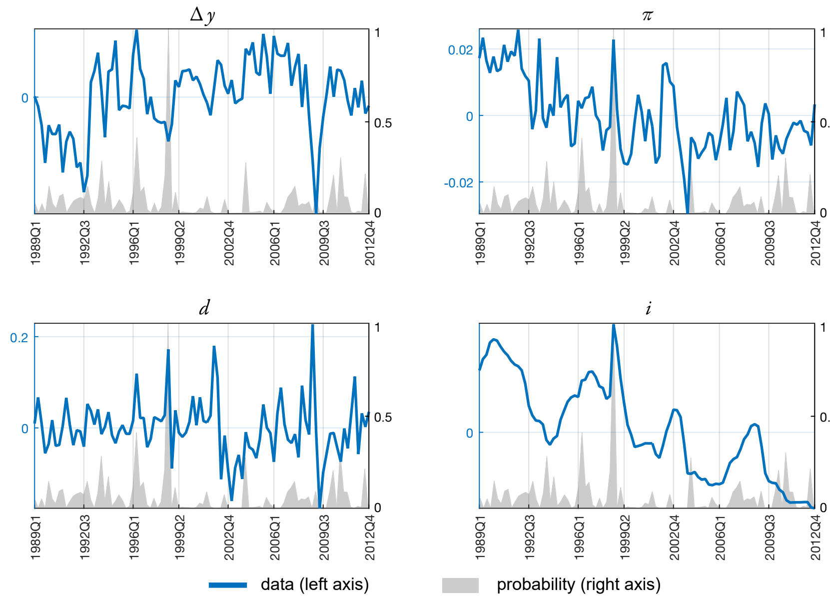

Figure 1: Smoothed transition probability ($\kappa = 1$) - Switching in monetary policy rule and variance of risk-premium shock

are provided in the online appendix.

When we turn our attention to the smoothed transition probabilities in the variance of the risk-premium shock, which are presented in Figure 2, we note that the periods where the variance of this shock is large corresponds to the time of the Argentinian emerging market and global financial crises (i.e. when $\vartheta = 2$). Note also that large depreciations in the exchange rate (positive values in d) are associated with movements into state, $\vartheta = 2$. Hence, these results would suggest that the effect of the risk-premium could be larger than would be the case when we only allow for a single state of the economy.

In addition, we also note that these smoothed transition probabilities are highly similar to those that are provided by the model specification that allows for regime-switching in the variances of all the shocks (with no regime-switching in the monetary policy rule) and the model that allows for regime-switching in the variance of the risk-premium shock (only). This would suggest that most of the regime-switching activity in the variance of the shocks is constrained to the risk-premium shock (which is also supported by the parameter estimates).

5 Forecasting

The results of the out-of-sample forecasting exercise are contained in Table 4. To generate the first of these forecasts, we estimate the model using an in-sample period that ends in 2001q4. We then generate a one- to eight-step ahead forecast, before we update the in-sample data to 2002q1 for the subsequent re-estimation that is used to generate the second forecast. The evaluation of the forecasts is conducted after calculating the root-mean squared-error (RMSE) for the one- to eight-step ahead forecasts over the entire out-of-sample period, which extends over ten years. In addition, we also employ the statistic of Diebold and Mariano (1995), which may be used to describe the significance of the differences in the respective RMSEs. This statistic is used to compare the forecasting performance of the regime-switching model that is responsible for the lowest RMSE against the non regime-

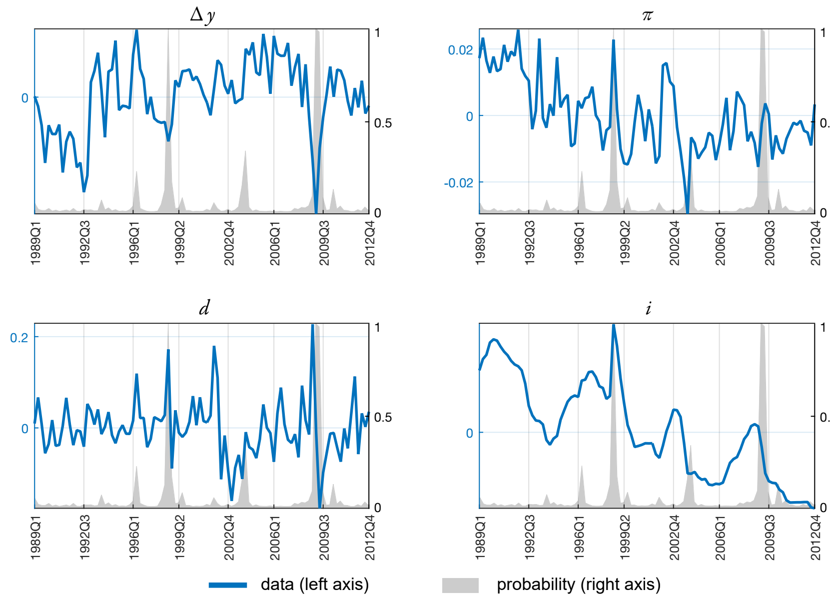

Figure 2: Smoothed transition probability ($\vartheta = 2$) - Switching in monetary policy rule and variance of risk-premium shock

switching alternative, for each particular step and variable. For example, we calculate the Diebold-Mariano statistic to compare the model that employs regime-switching in the variance of the risk-premium shock against the non regime-switching alternative at the five-step ahead horizon for output growth. In each of these tables, bold entries indicate the minimum RMSEs, and where the Diebold-Mariano statistic exceeds the ± 1.96 confidence interval, we attach a [*] to those values.

After taking the average over time for the one- to eight-step ahead RMSEs, the forecasts of output suggest that the model that does not include any regime-switching behaviour may provide slightly better out-of-sample results over the short-term. However, with an increase in horizon the differences in the RMSEs become very small, where some of the regime-switching models provide a slightly better RMSEs when the horizon is greater than a year. In addition, the Diebold-Mariano statistics for each step-ahead forecast suggest that the forecasting errors are not significantly different from one another (i.e. when comparing the best regime-switching model against the non regime-switching alternative).

The results for the short-term inflation forecasts are similar to those of output. However, in this case the RMSEs of the regime-switching models are also inferior over longer horizons. In addition, the Diebold-Mariano statistics once again suggest that there is no significant difference in the forecasting performance of the selected models. Then lastly, the out-of-sample results for interest rates are particularly poor for the regime-switching model, where the RMSEs at each step are relatively high and the Diebold-Mariano statistics suggest that the difference between these results at the short-term horizons are significant.

This would suggest that while some of the regime-switching models provide superior in-sample statistics (i.e. the model with regime-switching in the variance of the risk-premium shock and the model with regime-switching in the monetary policy rule and variance of the risk-premium shock), these models are unable to provide out-sample statistics that significantly improve upon the model that does not employ regime-switching, over this particular sample period.

	Forecast Horizons							
	1 step	2 step	3 step	4 step	5 step	6 step	7 step	8 step
Output								
No-switching	**0.03**	0.037	**0.044**	**0.047**	0.047	0.045	0.043	0.043
Policy switch	0.032	0.041	0.047	0.048	0.046	**0.044**	0.043	**0.042**
Volatility switch	0.034	**0.037**	0.045	0.048	0.047	0.045	**0.043**	0.043
Policy & Volatility	0.051	0.047	0.051	0.049	0.049	0.046	0.044	0.047
Risk-premium	0.034	0.040	0.046	0.047	**0.046**	0.044	0.043	0.043
Policy & Risk-prem	0.144	0.046	0.077	0.066	0.055	0.047	0.044	0.044
DM-statistic	-0.682	0.261	-0.422	-0.166	1.296	0.640	0.223	0.013
Inflation								
No-switching	0.057	0.062	0.059	0.051	0.048	0.046	**0.043**	**0.043**
Policy switch	0.063	0.067	0.061	0.051	0.047	**0.045**	0.045	0.044
Volatility switch	0.063	0.069	0.069	0.059	0.051	0.046	0.043	0.043
Policy & Volatility	0.070	0.069	0.062	0.054	0.049	0.048	0.046	0.048
Risk-premium	**0.056**	**0.058**	**0.055**	**0.047**	**0.045**	0.045	0.043	0.044
Policy & Risk-prem	0.121	0.133	0.094	0.065	0.050	0.047	0.045	0.046
DM-statistic	1.149	1.300	1.009	0.946	0.665	0.235	-0.051	-0.114
Interest Rate								
No-switching	**0.015**	**0.023**	**0.029**	**0.033**	**0.037**	**0.041**	**0.045**	**0.048**
Policy switch	0.019	0.027	0.032	0.035	0.039	0.043	0.047	0.050
Volatility switch	0.017	0.027	0.034	0.038	0.042	0.045	0.048	0.051
Policy & Volatility	0.025	0.036	0.042	0.047	0.050	0.053	0.056	0.058
Risk-premium	0.019	0.027	0.033	0.037	0.041	0.045	0.049	0.052
Policy & Risk-prem	0.027	0.035	0.038	0.041	0.044	0.048	0.051	0.054
DM-statistic	-2.335*	-2.319*	-1.525	-0.912	-0.781	-0.789	-0.750	-0.769

Table 4: Root-Mean Squared-Errors and Diebold-Mariano statistics (2002q1-2012q4)

6 Conclusion

This paper considers the use of various specifications of Markov-switching DSGE models for the South African economy. The initial results suggest that the models with regime-switching in the variance of the risk-premium shock, and the model that incorporates regime-switching in both the monetary policy rule and the variance of the risk-premium shock, could provide superior in-sample statistics.

In addition, these results suggest that there is little evidence of a level shift in the transition probabilities in the central bank reaction function. This would imply that the central bank has been fairly consistent with the application of policy over this sample period. However, there may have been a few isolated events that resulted in a strong reaction of the central bank to inflationary pressure, during the the late 1990s.

The models are also used to identify changes in the variance of the shocks, where we note that the only parameter value that experiences a large change (between the two regimes) pertains to that of the risk-premium. When considering the smoothed transition probabilities, we note that there was a large increase in the size of the variance of this shock during the emerging market crisis in 1999, and the start of the global financial crisis in 2007/8.

The out-of-sample results suggest that in many instances, the model that is responsible for the lowest RMSE does not employ regime-switching. This is evident over short-term forecasts for output, long-term forecasts for inflation and over all horizons for interest rates.

When comparing the differences between these forecasting errors, we note that in most cases the difference is largely insignificant, except for short-term forecasts of the interest rate (where the model that does not employ regime-switching is superior).

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