










# Comparison of Polarized Radiative Transfer Codes Used by the EHT Collaboration

Ben S. Prather<sup>1</sup>, Jason Dexter<sup>2</sup>, Monika Moscibrodzka<sup>3</sup>, Hung-Yi Pu<sup>4,5,6</sup>, Thomas Bronzwaer<sup>3</sup>, Jordy Davelaar<sup>7,8,9</sup>, Ziri Younsi<sup>10,11</sup>, Charles F. Gammie<sup>12,13,14</sup>, Roman Gold<sup>15</sup>, George N. Wong<sup>16,17</sup>, Kazunori Akiyama<sup>18,19,20</sup>, Antxon Alberdi<sup>21</sup>, Walter Alef<sup>22</sup>, Juan Carlos Algaba<sup>23</sup>, Richard Anantua<sup>20,24,25</sup>, Keiichi Asada<sup>6</sup>, Rebecca Azulay<sup>22,26,27</sup>, Uwe Bach<sup>22</sup>, Anne-Kathrin Baczo<sup>22</sup>, David Ball<sup>28</sup>, Mislav Baloković<sup>29</sup>, John Barrett<sup>18</sup>, Michi Bauböck<sup>12</sup>, Bradford A. Benson<sup>30,31</sup>, Dan Bintley<sup>32,33</sup>, Lindy Blackburn<sup>20,24</sup>, Raymond Blundell<sup>24</sup>, Katherine L. Bouman<sup>34</sup>, Geoffrey C. Bower<sup>35,36</sup>, Hope Boyce<sup>37,38</sup>, Michael Bremer<sup>39</sup>, Christiaan D. Brinkerink<sup>9</sup>, Roger Brissenden<sup>20,24</sup>, Silke Britzen<sup>22</sup>, Avery E. Broderick<sup>40,41,42</sup>, Dominique Brogiere<sup>39</sup>, Sandra Bustamante<sup>43</sup>, Do-Young Byun<sup>44,45</sup>, John E. Carlstrom<sup>31,46,47,48</sup>, Chiara Ceccobello<sup>49</sup>, Andrew Chael<sup>50</sup>, Chi-kwan Chan<sup>50,51,52</sup>, Dominic O. Chang<sup>20,24</sup>, Koushik Chatterjee<sup>20,24</sup>, Shami Chatterjee<sup>53</sup>, Ming-Tang Chen<sup>53</sup>, Yongjun Chen (陈永军)<sup>54,55</sup>, Xiaopeng Cheng<sup>44</sup>, Ilje Cho<sup>21</sup>, Pierre Christian<sup>56</sup>, Nicholas S. Conroy<sup>13,24</sup>, John E. Conway<sup>49</sup>, James M. Cordes<sup>53</sup>, Thomas M. Crawford<sup>31,46</sup>, Geoffrey B. Crew<sup>18</sup>, Alejandro Cruz-Osorio<sup>11</sup>, Yuzhu Cui (崔玉竹)<sup>57,58</sup>, Mariafelicia De Laurentis<sup>11,59,60</sup>, Roger Deane<sup>61,62,63</sup>, Jessica Dempsey<sup>32,33,64</sup>, Gregory Desvignes<sup>22,65</sup>, Vedant Dhruv<sup>12</sup>, Sheperd S. Doeleman<sup>20,24</sup>, Sean Dougal<sup>28</sup>, Sergio A. Dzib<sup>22,39</sup>, Ralph P. Eatough<sup>22,66</sup>, Razieh Emami<sup>24</sup>, Heino Falcke<sup>9</sup>, Joseph Farah<sup>67,68</sup>, Vincent L. Fish<sup>18</sup>, Ed Fomalont<sup>69</sup>, H. Alyson Ford<sup>28</sup>, Raquel Fraga-Encinas<sup>9</sup>, William T. Freeman<sup>70,71</sup>, Per Friberg<sup>32,33</sup>, Christian M. Fromm<sup>11,22,72</sup>, Antonio Fuentes<sup>21</sup>, Peter Galison<sup>20,73,74</sup>, Roberto García<sup>39</sup>, Olivier Gentaz<sup>39</sup>, Boris Georgiev<sup>40,41,42</sup>, Ciriaco Goddi<sup>75,76,77,78</sup>, Arturo I. Gómez-Ruiz<sup>79,80</sup>, José L. Gómez<sup>21</sup>, Minfeng Gu (顾敏峰)<sup>54,81</sup>, Mark Gurwell<sup>24</sup>, Kazuhiro Hada<sup>82,83</sup>, Daryl Haggard<sup>37,38</sup>, Kari Haworth<sup>24</sup>, Michael H. Hecht<sup>18</sup>, Ronald Hesper<sup>84</sup>, Dirk Heumann<sup>28</sup>, Luis C. Ho (何子山)<sup>85,86</sup>, Paul Ho<sup>6,32,33</sup>, Mareki Honma<sup>82,83,87</sup>, Chih-Wei L. Huang<sup>6</sup>, Lei Huang (黄磊)<sup>54,81</sup>, David H. Hughes<sup>79</sup>, Shiro Ikeda<sup>19,88,89,90</sup>, C. M. Violette Impellizzeri<sup>69,91</sup>, Makoto Inoue<sup>6</sup>, Sara Issaoun<sup>24,151</sup>, David J. James<sup>93</sup>, Buell T. Jannuzi<sup>28</sup>, Michael Janssen<sup>22</sup>, Britton Jeter<sup>6</sup>, Wu Jiang (江悟)<sup>54</sup>, Alejandra Jiménez-Rosales<sup>9</sup>, Michael D. Johnson<sup>20,24</sup>, Svetlana Jorstad<sup>94</sup>, Abhishek V. Joshi<sup>12</sup>, Taehyun Jung<sup>44,45</sup>, Mansour Karami<sup>40,41</sup>, Ramesh Karuppusamy<sup>22</sup>, Tomohisa Kawashima<sup>95</sup>, Garrett K. Keating<sup>24</sup>, Mark Kettenis<sup>96</sup>, Dong-Jin Kim<sup>22</sup>, Jae-Young Kim<sup>22,97</sup>, Jongsoo Kim<sup>44</sup>, Junhan Kim<sup>34</sup>, Motoki Kino<sup>19,98</sup>, Jun Yi Koay<sup>6</sup>, Prashant Kocherlakota<sup>11</sup>, Yutaro Kofuji<sup>82,87</sup>, Shoko Koyama<sup>6,99</sup>, Carsten Kramer<sup>39</sup>, Michael Kramer<sup>22</sup>, Thomas P. Krichbaum<sup>22</sup>, Cheng-Yu Kuo<sup>6,100</sup>, Noemi La Bella<sup>9</sup>, Tod R. Lauer<sup>101</sup>, Daeyoung Lee<sup>12</sup>, Sang-Sung Lee<sup>44</sup>, Po Kin Leung<sup>102</sup>, Aviad Levis<sup>34</sup>, Zhiyuan Li (李志远)<sup>103,104</sup>, Rocco Lico<sup>21,105</sup>, Greg Lindahl<sup>24</sup>, Michael Lindqvist<sup>49</sup>, Mikhail Lisakov<sup>22</sup>, Jun Liu (刘俊)<sup>22</sup>, Kuo Liu<sup>22</sup>, Elisabetta Liuzzo<sup>106</sup>, Wen-Ping Lo<sup>6,107</sup>, Andrei P. Lobanov<sup>22</sup>, Laurent Loinard<sup>108,109</sup>, Colin J. Lonsdale<sup>18</sup>, Ru-Sen Lu (路如森)<sup>22,54,55</sup>, Nicholas R. MacDonald<sup>22</sup>, Jirong Mao (毛基荣)<sup>110,111,112</sup>, Nicola Marchili<sup>22,106</sup>, Sera Markoff<sup>113,114</sup>, Daniel P. Marrone<sup>28</sup>, Alan P. Marscher<sup>94</sup>, Iván Martí-Vidal<sup>26,27</sup>, Satoki Matsushita<sup>6</sup>, Lynn D. Matthews<sup>18</sup>, Lia Medeiros<sup>20,32,152</sup>, Karl M. Menten<sup>22</sup>, Daniel Michalik<sup>115</sup>, Izumi Mizuno<sup>32,33</sup>, Yosuke Mizuno<sup>11,58,116</sup>, James M. Moran<sup>20,24</sup>, Kotaro Moriyama<sup>11,18,82</sup>, Cornelia Müller<sup>9,22</sup>, Alejandro Mus<sup>26,27</sup>, Gibwa Musoke<sup>9,113</sup>, Ioannis Myserlis<sup>117</sup>, Andrew Nadolski<sup>13</sup>, Hiroshi Nagai<sup>19,83</sup>, Neil M. Nagar<sup>118</sup>, Masanori Nakamura<sup>6,119</sup>, Ramesh Narayan<sup>20,24</sup>, Gopal Narayanan<sup>43</sup>, Iniyan Natarajan<sup>20,24</sup>, Antonios Nathanail<sup>11,120</sup>, Santiago Navarro Fuentes<sup>117</sup>, Joey Neilsen<sup>121</sup>, Roberto Neri<sup>39</sup>, Chunchong Ni<sup>40,41,42</sup>, Aristeidis Noutsos<sup>22</sup>, Michael A. Nowak<sup>122</sup>, Junghwan Oh<sup>123</sup>, Hiroki Okino<sup>82,87</sup>, Héctor Olivares<sup>9</sup>, Gisela N. Ortiz-León<sup>22,109</sup>, Tomoaki Oyama<sup>82</sup>, Feryal Özel<sup>151</sup>, Daniel C. M. Palumbo<sup>20,24</sup>, Georgios Filippou Paraschos<sup>22</sup>, Jongho Park<sup>6,44</sup>, Harriet Parsons<sup>32,33</sup>, Nimesh Patel<sup>24</sup>, Ue-Li Pen<sup>6,40,124,125,126</sup>, Dominic W. Pesce<sup>20,24</sup>, Vincent Piétu<sup>39</sup>, Richard Plambeck<sup>127</sup>, Aleksandar PopStefanija<sup>43</sup>, Oliver Porth<sup>11,113</sup>, Felix M. Pötzel<sup>22,128</sup>, Jorge A. Preciado-López<sup>40</sup>, Dimitrios Psaltis<sup>151</sup>, Venkatesh Ramakrishnan<sup>118,129,130</sup>, Ramprasad Rao<sup>24</sup>, Mark G. Rawlings<sup>32,33,131</sup>, Alexander W. Raymond<sup>20,24</sup>, Luciano Rezzolla<sup>11,132,133</sup>, Angelo Ricarte<sup>20,24</sup>, Bart Ripperda<sup>8,16,134,151</sup>, Freek Roelofs<sup>9,20,24</sup>, Alan Rogers<sup>18</sup>, Eduardo Ros<sup>22</sup>, Cristina Romero-Cañizales<sup>6</sup>, Arash Roshaninshat<sup>28</sup>, Helge Rottmann<sup>22</sup>, Alan L. Roy<sup>22</sup>, Ignacio Ruiz<sup>117</sup>, Chet Ruzszyk<sup>18</sup>, Kazi L. J. Rygl<sup>106</sup>, Salvador Sánchez<sup>117</sup>, David Sánchez-Argüelles<sup>79,80</sup>, Miguel Sánchez-Portal<sup>117</sup>, Mahito Sasada<sup>82,135,136</sup>, Kaushik Satapathy<sup>28</sup>, Tuomas Savolainen<sup>22,130,137</sup>, F. Peter Schloerb<sup>43</sup>, Jonathan Schonfeld<sup>24</sup>, Karl-Friedrich Schuster<sup>24</sup>, Lijing Shao<sup>22,86</sup>, Zhiqiang Shen (沈志强)<sup>54,55</sup>, Des Small<sup>96</sup>, Bong Won Sohn<sup>44,45,138</sup>, Jason SooHoo<sup>18</sup>, Kamal Souccar<sup>43</sup>, He Sun (孙赫)<sup>34</sup>, Fumie Tazaki<sup>82</sup>, Alexandra J. Tetarenko<sup>139,151</sup>, Paul Tiede<sup>20,24</sup>, Remo P. J. Tilanus<sup>9,28,91,140</sup>, Michael Titus<sup>18</sup>, Pablo Torne<sup>22,117</sup>, Efthalia Traianou<sup>21,22</sup>, Tyler Trent<sup>28</sup>, Sascha Trippe<sup>141</sup>, Matthew Turk<sup>13</sup>, Ilse van Bemmel<sup>96</sup>, Huib Jan van Langevelde<sup>91,96,142</sup>, Daniel R. van Rossum<sup>9</sup>, Jesse Vos<sup>9</sup>, Jan Wagner<sup>22</sup>, Derek Ward-Thompson<sup>143</sup>, John Wardle<sup>144</sup>, Jonathan Weintraub<sup>20,24</sup>, Norbert Wex<sup>22</sup>, Robert Wharton<sup>22</sup>, Maciek Wielgus<sup>22</sup>, Kaj Wiik<sup>145</sup>, Gunther Witzel<sup>22</sup>, Michael F. Wondrak<sup>9,146</sup>, Qingwen Wu (吴庆文)<sup>147</sup>, Paul Yamaguchi<sup>24</sup>, Aristomenis Yfantis<sup>9</sup>, Doosoo Yoon<sup>113</sup>, André Young<sup>9</sup>, Ken Young<sup>24</sup>, Wei Yu

(于威)<sup>24</sup> , Feng Yuan (袁峰)<sup>54,81,148</sup> , Ye-Fei Yuan (袁业飞)<sup>149</sup> , J. Anton Zensus<sup>22</sup> , Shuo Zhang<sup>150</sup> ,  
Guang-Yao Zhao<sup>21</sup> , and Shan-Shan Zhao (赵杉杉)<sup>54</sup> 

The Event Horizon Telescope Collaboration

- <sup>1</sup> CCS-2, Los Alamos National Laboratory, P.O. Box 1663, Los Alamos, NM 87545, USA; [bprather@lanl.gov](mailto:bprather@lanl.gov)  
<sup>2</sup> JILA and Department of Astrophysical and Planetary Sciences, University of Colorado, Boulder, CO 80309, USA  
<sup>3</sup> Department of Astrophysics/IMAPP, Radboud University, P.O. Box 9010, 6500 GL Nijmegen, The Netherlands  
<sup>4</sup> Department of Physics, National Taiwan Normal University, No. 88, Sec.4, Tingzhou Road, Taipei 116, Taiwan, R.O.C.  
<sup>5</sup> Center of Astronomy and Gravitation, National Taiwan Normal University, No. 88, Sec. 4, Tingzhou Road, Taipei 116, Taiwan, R.O.C.  
<sup>6</sup> Institute of Astronomy and Astrophysics, Academia Sinica, 11F of Astronomy-Mathematics Building, AS/NTU No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan, R.O.C.  
<sup>7</sup> Department of Astronomy and Columbia Astrophysics Laboratory, Columbia University, 550 W. 120th Street, New York, NY 10027, USA  
<sup>8</sup> Center for Computational Astrophysics, Flatiron Institute, 162 Fifth Avenue, New York, NY 10010, USA  
<sup>9</sup> Department of Astrophysics, Institute for Mathematics, Astrophysics and Particle Physics (IMAPP), Radboud University, P.O. Box 9010, 6500 GL Nijmegen, The Netherlands  
<sup>10</sup> Mullard Space Science Laboratory, University College London, Holmbury St. Mary, Dorking, Surrey, RH5 6NT, UK  
<sup>11</sup> Institut für Theoretische Physik, Goethe-Universität Frankfurt, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany  
<sup>12</sup> Department of Physics, University of Illinois, 1110 West Green Street, Urbana, IL 61801, USA  
<sup>13</sup> Department of Astronomy, University of Illinois at Urbana-Champaign, 1002 West Green Street, Urbana, IL 61801, USA  
<sup>14</sup> NCSA, University of Illinois, 1205 W. Clark Street, Urbana, IL 61801, USA  
<sup>15</sup> CP3-Origins, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark  
<sup>16</sup> School of Natural Sciences, Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540, USA  
<sup>17</sup> Princeton Gravity Initiative, Princeton University, Princeton, NJ 08544, USA  
<sup>18</sup> Massachusetts Institute of Technology Haystack Observatory, 99 Millstone Road, Westford, MA 01886, USA  
<sup>19</sup> National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan  
<sup>20</sup> Black Hole Initiative at Harvard University, 20 Garden Street, Cambridge, MA 02138, USA  
<sup>21</sup> Instituto de Astrofísica de Andalucía-CSIC, Glorieta de la Astronomía s/n, E-18008 Granada, Spain  
<sup>22</sup> Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany  
<sup>23</sup> Department of Physics, Faculty of Science, Universiti Malaya, 50603 Kuala Lumpur, Malaysia  
<sup>24</sup> Center for Astrophysics | Harvard & Smithsonian, 60 Garden Street, Cambridge, MA 02138, USA  
<sup>25</sup> Department of Physics & Astronomy, The University of Texas at San Antonio, One UTSA Circle, San Antonio, TX 78249, USA  
<sup>26</sup> Departament d'Astronomia i Astrofísica, Universitat de València, C. Dr. Moliner 50, E-46100 Burjassot, València, Spain  
<sup>27</sup> Observatori Astronòmic, Universitat de València, C. Catedrático José Beltrán 2, E-46980 Paterna, València, Spain  
<sup>28</sup> Steward Observatory and Department of Astronomy, University of Arizona, 933 N. Cherry Avenue, Tucson, AZ 85721, USA  
<sup>29</sup> Yale Center for Astronomy & Astrophysics, Yale University, 52 Hillhouse Avenue, New Haven, CT 06511, USA  
<sup>30</sup> Fermi National Accelerator Laboratory, MS209, P.O. Box 500, Batavia, IL 60510, USA  
<sup>31</sup> Department of Astronomy and Astrophysics, University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637, USA  
<sup>32</sup> East Asian Observatory, 660 N. A'ohoku Place, Hilo, HI 96720, USA  
<sup>33</sup> James Clerk Maxwell Telescope (JCMT), 660 N. A'ohoku Place, Hilo, HI 96720, USA  
<sup>34</sup> California Institute of Technology, 1200 East California Boulevard, Pasadena, CA 91125, USA  
<sup>35</sup> Institute of Astronomy and Astrophysics, Academia Sinica, 645 N. A'ohoku Place, Hilo, HI 96720, USA  
<sup>36</sup> Department of Physics and Astronomy, University of Hawaii at Manoa, 2505 Correa Road, Honolulu, HI 96822, USA  
<sup>37</sup> Department of Physics, McGill University, 3600 rue University, Montréal, QC H3A 2T8, Canada  
<sup>38</sup> McGill Space Institute, McGill University, 3550 rue University, Montréal, QC H3A 2A7, Canada  
<sup>39</sup> Institut de Radioastronomie Millimétrique (IRAM), 300 rue de la Piscine, F-38406 Saint Martin d'Hères, France  
<sup>40</sup> Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada  
<sup>41</sup> Department of Physics and Astronomy, University of Waterloo, 200 University Avenue West, Waterloo, ON N2L 3G1, Canada  
<sup>42</sup> Waterloo Centre for Astrophysics, University of Waterloo, Waterloo, ON N2L 3G1, Canada  
<sup>43</sup> Department of Astronomy, University of Massachusetts, Amherst, MA 01003, USA  
<sup>44</sup> Korea Astronomy and Space Science Institute, Daedeok-daero 776, Yuseong-gu, Daejeon 34055, Republic of Korea  
<sup>45</sup> University of Science and Technology, Gajeong-ro 217, Yuseong-gu, Daejeon 34113, Republic of Korea  
<sup>46</sup> Kavli Institute for Cosmological Physics, University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637, USA  
<sup>47</sup> Department of Physics, University of Chicago, 5720 South Ellis Avenue, Chicago, IL 60637, USA  
<sup>48</sup> Enrico Fermi Institute, University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637, USA  
<sup>49</sup> Department of Space, Earth and Environment, Chalmers University of Technology, Onsala Space Observatory, SE-43992 Onsala, Sweden  
<sup>50</sup> Princeton Gravity Initiative, Jadwin Hall, Princeton University, Princeton, NJ 08544, USA  
<sup>51</sup> Data Science Institute, University of Arizona, 1230 N. Cherry Avenue, Tucson, AZ 85721, USA  
<sup>52</sup> Program in Applied Mathematics, University of Arizona, 617 N. Santa Rita, Tucson, AZ 85721, USA  
<sup>53</sup> Cornell Center for Astrophysics and Planetary Science, Cornell University, Ithaca, NY 14853, USA  
<sup>54</sup> Shanghai Astronomical Observatory, Chinese Academy of Sciences, 80 Nandan Road, Shanghai 200030, People's Republic of China  
<sup>55</sup> Key Laboratory of Radio Astronomy, Chinese Academy of Sciences, Nanjing 210008, People's Republic of China  
<sup>56</sup> Physics Department, Fairfield University, 1073 North Benson Road, Fairfield, CT 06824, USA  
<sup>57</sup> Research Center for Intelligent Computing Platforms, Zhejiang Laboratory, Hangzhou 311100, People's Republic of China  
<sup>58</sup> Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shengrong Road 520, Shanghai, 201210, People's Republic of China  
<sup>59</sup> Dipartimento di Fisica "E. Pancini," Università di Napoli "Federico II," Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126, Napoli, Italy  
<sup>60</sup> INFN Sez. di Napoli, Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126, Napoli, Italy  
<sup>61</sup> Wits Centre for Astrophysics, University of the Witwatersrand, 1 Jan Smuts Avenue, Braamfontein, Johannesburg 2050, South Africa  
<sup>62</sup> Department of Physics, University of Pretoria, Hatfield, Pretoria 0028, South Africa  
<sup>63</sup> Centre for Radio Astronomy Techniques and Technologies, Department of Physics and Electronics, Rhodes University, Makhanda 6140, South Africa  
<sup>64</sup> ASTRON, Oude Hoogeveensedijk 4, 7991 PD Dwingeloo, The Netherlands  
<sup>65</sup> LESIA, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, Université de Paris, 5 place Jules Janssen, F-92195 Meudon, France  
<sup>66</sup> National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Road, Chaoyang District, Beijing 100101, People's Republic of China  
<sup>67</sup> Las Cumbres Observatory, 6740 Cortona Drive, Suite 102, Goleta, CA 93117-5575, USA  
<sup>68</sup> Department of Physics, University of California, Santa Barbara, CA 93106-9530, USA  
<sup>69</sup> National Radio Astronomy Observatory, 520 Edgemont Road, Charlottesville, VA 22903, USA

- <sup>70</sup> Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, 32-D476, 77 Massachusetts Avenue, Cambridge, MA 02142, USA
- <sup>71</sup> Google Research, 355 Main Street, Cambridge, MA 02142, USA
- <sup>72</sup> Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Emil-Fischer-Str. 31, D-97074 Würzburg, Germany
- <sup>73</sup> Department of History of Science, Harvard University, Cambridge, MA 02138, USA
- <sup>74</sup> Department of Physics, Harvard University, Cambridge, MA 02138, USA
- <sup>75</sup> Universidade de Sao Paulo, Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Departamento de Astronomia, Sao Paulo, SP 05508-090, Brazil
- <sup>76</sup> Dipartimento di Fisica, Università degli Studi di Cagliari, SP Monserrato-Sestu km 0.7, I-09042 Monserrato, Italy
- <sup>77</sup> INAF—Osservatorio Astronomico di Cagliari, Via della Scienza 5, I-09047, Selargius, CA, Italy
- <sup>78</sup> INFN, Sezione di Cagliari, Cittadella Univ., I-09042 Monserrato (CA), Italy
- <sup>79</sup> Instituto Nacional de Astrofísica, Óptica y Electrónica. Apartado Postal 51 y 216, 72000. Puebla Pue., MÉxico
- <sup>80</sup> Consejo Nacional de Ciencia y Tecnología, Av. Insurgentes Sur 1582, 03940, Ciudad de México, MÉxico
- <sup>81</sup> Key Laboratory for Research in Galaxies and Cosmology, Chinese Academy of Sciences, Shanghai 200030, People's Republic of China
- <sup>82</sup> Mizusawa VLBI Observatory, National Astronomical Observatory of Japan, 2-12 Hoshigaoka, Mizusawa, Oshu, Iwate 023-0861, Japan
- <sup>83</sup> Department of Astronomical Science, The Graduate University for Advanced Studies (SOKENDAI), 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan
- <sup>84</sup> NOVA Sub-mm Instrumentation Group, Kapteyn Astronomical Institute, University of Groningen, Landleven 12, 9747 AD Groningen, The Netherlands
- <sup>85</sup> Department of Astronomy, School of Physics, Peking University, Beijing 100871, People's Republic of China
- <sup>86</sup> Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, People's Republic of China
- <sup>87</sup> Department of Astronomy, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
- <sup>88</sup> The Institute of Statistical Mathematics, 10-3 Midori-cho, Tachikawa, Tokyo, 190-8562, Japan
- <sup>89</sup> Department of Statistical Science, The Graduate University for Advanced Studies (SOKENDAI), 10-3 Midori-cho, Tachikawa, Tokyo 190-8562, Japan
- <sup>90</sup> Kavli Institute for the Physics and Mathematics of the Universe, The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, 277-8583, Japan
- <sup>91</sup> Leiden Observatory, Leiden University, Postbus 2300, 9513 RA Leiden, The Netherlands
- <sup>92</sup> School of Physics, Georgia Institute of Technology, 837 State St NW, Atlanta, GA 30332, USA
- <sup>93</sup> ASTRAVEO LLC, PO Box 1668, Gloucester, MA 01931, USA
- <sup>94</sup> Institute for Astrophysical Research, Boston University, 725 Commonwealth Avenue, Boston, MA 02215, USA
- <sup>95</sup> Institute for Cosmic Ray Research, The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8582, Japan
- <sup>96</sup> Joint Institute for VLBI ERIC (JIVE), Oude Hoogeveensedijk 4, 7991 PD Dwingeloo, The Netherlands
- <sup>97</sup> Department of Astronomy and Atmospheric Sciences, Kyungpook National University, Daegu 702-701, Republic of Korea
- <sup>98</sup> Kogakuin University of Technology & Engineering, Academic Support Center, 2665-1 Nakano, Hachioji, Tokyo 192-0015, Japan
- <sup>99</sup> Niigata University, 8050 Ikarashi-nino-cho, Nishi-ku, Niigata 950-2181, Japan
- <sup>100</sup> Physics Department, National Sun Yat-Sen University, No. 70, Lien-Hai Road, Kaosiung City 80424, Taiwan, R.O.C.
- <sup>101</sup> National Optical Astronomy Observatory, 950 N. Cherry Avenue, Tucson, AZ 85719, USA
- <sup>102</sup> Department of Physics, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong
- <sup>103</sup> School of Astronomy and Space Science, Nanjing University, Nanjing 210023, People's Republic of China
- <sup>104</sup> Key Laboratory of Modern Astronomy and Astrophysics, Nanjing University, Nanjing 210023, People's Republic of China
- <sup>105</sup> INAF-Istituto di Radioastronomia, Via P. Gobetti 101, I-40129 Bologna, Italy
- <sup>106</sup> INAF-Istituto di Radioastronomia & Italian ALMA Regional Centre, Via P. Gobetti 101, I-40129 Bologna, Italy
- <sup>107</sup> Department of Physics, National Taiwan University, No. 1, Sect. 4, Roosevelt Road, Taipei 10617, Taiwan, R.O.C
- <sup>108</sup> Instituto de Radioastronomía y Astrofísica, Universidad Nacional Autónoma de México, Morelia 58089, MÉxico
- <sup>109</sup> Instituto de Astronomía, Universidad Nacional Autónoma de México (UNAM), Apdo Postal 70-264, Ciudad de México, MÉxico
- <sup>110</sup> Yunnan Observatories, Chinese Academy of Sciences, 650011 Kunming, Yunnan Province, People's Republic of China
- <sup>111</sup> Center for Astronomical Mega-Science, Chinese Academy of Sciences, 20A Datun Road, Chaoyang District, Beijing, 100012, People's Republic of China
- <sup>112</sup> Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, 650011 Kunming, People's Republic of China
- <sup>113</sup> Anton Pannekoek Institute for Astronomy, University of Amsterdam, Science Park 904, 1098 XH, Amsterdam, The Netherlands
- <sup>114</sup> Gravitation and Astroparticle Physics Amsterdam (GRAPPA) Institute, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands
- <sup>115</sup> Science Support Office, Directorate of Science, European Space Research and Technology Centre (ESA/ESTEC), Keplerlaan 1, 2201 AZ Noordwijk, The Netherlands
- <sup>116</sup> School of Physics and Astronomy, Shanghai Jiao Tong University, 800 Dongchuan Road, Shanghai, 200240, People's Republic of China
- <sup>117</sup> Institut de Radioastronomie Millimétrique (IRAM), Avenida Divina Pastora 7, Local 20, E-18012, Granada, Spain
- <sup>118</sup> Astronomy Department, Universidad de Concepción, Casilla 160-C, Concepción, Chile
- <sup>119</sup> National Institute of Technology, Hachinohe College, 16-1 Uwanotai, Tamonoki, Hachinohe City, Aomori 039-1192, Japan
- <sup>120</sup> Department of Physics, National and Kapodistrian University of Athens, Panepistimiopolis, GR 15783 Zografos, Greece
- <sup>121</sup> Department of Physics, Villanova University, 800 Lancaster Avenue, Villanova, PA 19085, USA
- <sup>122</sup> Physics Department, Washington University, CB 1105, St. Louis, MO 63130, USA
- <sup>123</sup> Sejong University, 209 Neungdong-ro, Gwangjin-gu, Seoul, Republic of Korea
- <sup>124</sup> Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, ON M5S 3H8, Canada
- <sup>125</sup> Dunlap Institute for Astronomy and Astrophysics, University of Toronto, 50 St. George Street, Toronto, ON M5S 3H4, Canada
- <sup>126</sup> Canadian Institute for Advanced Research, 180 Dundas Street West, Toronto, ON M5G 1Z8, Canada
- <sup>127</sup> Radio Astronomy Laboratory, University of California, Berkeley, CA 94720, USA
- <sup>128</sup> Institute of Astrophysics, Foundation for Research and Technology—Hellas, Voutes, 7110 Heraklion, Greece
- <sup>129</sup> Finnish Centre for Astronomy with ESO, FI-20014 University of Turku, Finland
- <sup>130</sup> Aalto University Metsähovi Radio Observatory, Metsähovintie 114, FI-02540 Kylmälä, Finland
- <sup>131</sup> Gemini Observatory/NSF NOIRLab, 670 N. A'ohökü Place, Hilo, HI 96720, USA
- <sup>132</sup> Frankfurt Institute for Advanced Studies, Ruth-Moufang-Strasse 1, D-60438 Frankfurt, Germany
- <sup>133</sup> School of Mathematics, Trinity College, Dublin 2, Ireland
- <sup>134</sup> Department of Astrophysical Sciences, Peyton Hall, Princeton University, Princeton, NJ 08544, USA
- <sup>135</sup> Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan
- <sup>136</sup> Hiroshima Astrophysical Science Center, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8526, Japan
- <sup>137</sup> Aalto University Department of Electronics and Nanoengineering, PL 15500, FI-00076 Aalto, Finland
- <sup>138</sup> Department of Astronomy, Yonsei University, Yonsei-ro 50, Seodaemun-gu, 03722 Seoul, Republic of Korea
- <sup>139</sup> Department of Physics and Astronomy, Texas Tech University, Lubbock, TX 79409-1051, USA
- <sup>140</sup> Netherlands Organisation for Scientific Research (NWO), Postbus 93138, 2509 AC Den Haag, The Netherlands
- <sup>141</sup> Department of Physics and Astronomy, Seoul National University, Gwanak-gu, Seoul 08826, Republic of Korea
- <sup>142</sup> University of New Mexico, Department of Physics and Astronomy, Albuquerque, NM 87131, USA
- <sup>143</sup> Jeremiah Horrocks Institute, University of Central Lancashire, Preston PR1 2HE, UK

<sup>144</sup> Physics Department, Brandeis University, 415 South Street, Waltham, MA 02453, USA<sup>145</sup> Tuorla Observatory, Department of Physics and Astronomy, University of Turku, Finland<sup>146</sup> Radboud Excellence Fellow of Radboud University, Nijmegen, The Netherlands<sup>147</sup> School of Physics, Huazhong University of Science and Technology, Wuhan, Hubei, 430074, People's Republic of China<sup>148</sup> School of Astronomy and Space Sciences, University of Chinese Academy of Sciences, No. 19A Yuquan Road, Beijing 100049, People's Republic of China<sup>149</sup> Astronomy Department, University of Science and Technology of China, Hefei 230026, People's Republic of China<sup>150</sup> Bard College, 30 Campus Road, Annandale-on-Hudson, NY 12504, USA

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## Abstract

Interpretation of resolved polarized images of black holes by the Event Horizon Telescope (EHT) requires predictions of the polarized emission observable by an Earth-based instrument for a particular model of the black hole accretion system. Such predictions are generated by general relativistic radiative transfer (GRRT) codes, which integrate the equations of polarized radiative transfer in curved spacetime. A selection of ray-tracing GRRT codes used within the EHT Collaboration is evaluated for accuracy and consistency in producing a selection of test images, demonstrating that the various methods and implementations of radiative transfer calculations are highly consistent. When imaging an analytic accretion model, we find that all codes produce images similar within a pixel-wise normalized mean squared error (NMSE) of 0.012 in the worst case. When imaging a snapshot from a cell-based magnetohydrodynamic simulation, we find all test images to be similar within NMSEs of 0.02, 0.04, 0.04, and 0.12 in Stokes  $I$ ,  $Q$ ,  $U$ , and  $V$ , respectively. We additionally find the values of several image metrics relevant to published EHT results to be in agreement to much better precision than measurement uncertainties.

*Unified Astronomy Thesaurus concepts:* [Supermassive black holes \(1663\)](#); [Radiative transfer simulations \(1967\)](#); [Astronomy software \(1855\)](#)

## 1. Introduction

In 2019, the Event Horizon Telescope (EHT) Collaboration published images of the central black hole (BH) in the galaxy M87 (hereafter M87\*), which measured and interpreted the total intensity of radio emission in two bands near 230 GHz (Event Horizon Telescope Collaboration et al. 2019a, 2019b, 2019c, 2019d, 2019e, 2019f, hereafter [EHTC I](#), [EHTC II](#), [EHTC III](#), [EHTC IV](#), [EHTC V](#), [EHTC VI](#)). In 2021, additional results were released measuring the degree and distribution of linear polarization across the image of M87\*, measured via the Stokes parameters  $Q$  and  $U$  (Event Horizon Telescope Collaboration et al. 2021a, 2021b, hereafter [EHTC VII](#), [EHTC VIII](#)). Linear polarization results are also expected of the central Milky Way BH Sgr A\* accompanying total-intensity results published in 2022 (Event Horizon Telescope Collaboration et al. 2022a, 2022b, 2022c, 2022d, 2022e, 2022f).

In order to interpret polarized observations, the collaboration generated models of the accreting plasma around M87\*, usually with general relativistic magnetohydrodynamic (GRMHD) simulations. Simulated images were generated from the models via general relativistic radiative transfer (GRRT) calculations in order to predict the emission visible from Earth from the generated plasma state ([EHTC V](#); see also Wong et al. 2022). The total-intensity images produced by various GRRT codes were validated against analytically defined tests in Gold et al. (2020) and found to be in good agreement. That paper also compared the output of certain pairs of codes, but not all codes, when imaging GRMHD simulation data.

The interpretation of the linear-polarimetric EHT image performed in [EHTC VIII](#) also used synthetic images. Polarimetric

images are more complicated than total-intensity images, as they predict the linear and circular polarization parameters (Stokes  $Q$ ,  $U$ ,  $V$ ) in addition to the total intensity (Stokes  $I$ ). Predicting polarized emission involves solving the coupled polarized radiative transfer equations, which can introduce significant additional computational problems, such as the treatment of rapid Faraday rotation and the need to parallel transport the linear polarization direction through curved spacetime. The additional complexity merits a separate comparison of polarized radiative transfer schemes present in several of the codes compared in Gold et al. (2020). That comparison is presented in this paper.

This paper provides brief descriptions of the codes compared, specifications of the tests performed, and measurements of code error (where available) or similarity as a group, compared against parameter changes and estimated detector accuracy.

The paper is structured as follows. In Section 2 we briefly describe all codes participating in the comparison study. In Section 3 we define three test problems used to compare the codes. In Section 4 we define a metric to evaluate light curves and image similarities and present the results of the comparisons. The discussion of the results and limitations of the examined ray-tracing radiative transfer schemes is given in Section 5. We conclude our study in Section 6.

## 2. Participating Codes

### 2.1. BHOSS

The BHOSS code (Younsi et al. 2012, 2020) numerically integrates, for an arbitrary input spacetime metric tensor, the geodesic equations of motion coupled with the covariant polarized radiative transfer equations. The solution of the polarized radiative transfer equations is achieved via parallel propagation of a pair of mutually orthogonal basis 4-vectors that define the observer's frame. A fourth-order Runge–Kutta–Fehlberg method with fifth-order error estimate and adaptive step size control, hereafter RKF4(5), is typically used. In regions of higher Faraday depth, an RKF8(9) method is used,

<sup>151</sup> NASA Hubble Fellowship Program, Einstein Fellow.<sup>152</sup> NSF Astronomy and Astrophysics Postdoctoral Fellow.

and when the transfer equations are particularly stiff, a variable-order implicit RKF integrator is employed.

## 2.2. *ipole*

The *ipole* code<sup>153</sup> (Noble et al. 2007; Mościbrodzka & Gammie 2018) is a publicly available ray-tracing scheme for covariant polarized GRRT. *ipole* splits the radiative transfer problem into two steps. In the fluid frame it evolves the Stokes parameters taking into account synchrotron emission, absorption, and Faraday effects and using an analytic solution to the polarized transfer equations with constant coefficients. Currently two analytic solvers are implemented in the code (Landi Degl’Innocenti & Landi Degl’Innocenti 1985, and the one presented in Appendix A of Mościbrodzka & Gammie 2018). Analytic solvers make *ipole* solutions numerically stable even for plasma with large optical or Faraday depths. In the coordinate frame, the parallel transport of the Stokes parameters is accomplished by transport of coherency matrix rather than Stokes parameters themselves. Hence, *ipole* radiative transfer is coordinate and metric independent. *ipole* has been tested against another polarized ray-tracing code *grtrans* (see method paper Mościbrodzka & Gammie 2018 and the next subsection) and against a polarized Monte Carlo radiative transfer scheme (see Mościbrodzka 2020 and Appendix A of this work). *ipole* was used in EHTC VIII for calculating polarized images of models with accelerated electrons.

## 2.3. *ipole-IL*

*ipole-IL*,<sup>154</sup> usually also called *ipole* but suffixed in this comparison for clarity, is a fork of the original *ipole* code described above, with features designed for treating libraries of GRMHD snapshot files, particularly from *iharm3D* as a part of the PATOKA pipeline (Wong et al. 2022). It maintains the same transport scheme implemented in *ipole* but adds robustness features such as reorthogonalization of tetrad basis vectors and additional limiting cases for the analytic solutions and fits. It also adds compatibility with a number of different GRMHD codes and supports calculating emission from different electron energy distribution functions.

## 2.4. *grtrans*

The *grtrans* code<sup>155</sup> (Dexter & Agol 2009; Dexter 2016) solves the polarized radiative transfer equations along null geodesics in a Kerr spacetime. The radiative transfer equations are integrated either numerically (Hindmarsh 2019) or with quadrature methods (Landi Degl’Innocenti & Landi Degl’Innocenti 1985; Rees et al. 1989). The quadrature methods are the most accurate and efficient for calculations of polarized radiative transfer in Faraday thick problems and are used for the test problems discussed here.

## 2.5. *Odyssey*

The *Odyssey* code (Pu et al. 2016) is a public GPU-based code<sup>156</sup> that solves the unpolarized radiative transfer equation along null geodesics from the observer to the source (observer-to-source) in Kerr spacetime. In Pu & Broderick (2018), to

implement the polarization computations and fit the need for solving the Stokes parameters along the null geodesic from the source to the observer (source-to-observer), a two-stage scheme is proposed: (i) during the observer-to-source stage, unpolarized radiative transfer is computed backward in time; and (ii) inverse the time direction and trace the same geodesic during the source-to-observer stage, and simultaneously solve the four Stokes parameters. As a result, there are four additional ordinary differential equations (ODEs; related to the Stokes parameters) to be solved in the second stage compared to the first stage. By controlling the time direction directly in the code, there is no need to save the photon path during the observer-to-source stage for the use of the source-to-observer stage. However, the caveat is that solving the additional four Stokes parameters during the source-to-observer stage can be computationally costly, and the Runge–Kutta scheme may fail when complicated Faraday coefficients are introduced in a given problem.

In this work, to improve its speed and the stability, we improve the polarization scheme of *Odyssey* with the following: (i) a two-stage scheme is still adopted, without solving the four ODEs for Stokes parameters during the second (source-to-observer) stage; and (ii) instead, during the second stage, the Stokes parameters are solved along the geodesics with an implicit method (Pihajoki et al. 2018; Bronzwaer et al. 2020). In this new scheme, the accuracy of the polarization computation is automatically controlled by the accuracy of the geodesic computation. The modifications significantly improve the computational speed. For example, it takes about a second (including the time for reading GRMHD simulation data) for *Odyssey* to finish the computation for the GRMHD snapshot test problem (Section 3.3).

## 2.6. *RAPTOR*

The *RAPTOR* code (Bronzwaer et al. 2018, 2020)<sup>157</sup> is a public code that numerically integrates the equations of motion of light rays in arbitrary spacetimes and then performs polarized radiative transfer calculations along the rays. The code uses an adaptive Runge–Kutta–Fehlberg scheme to integrate the geodesic equation where the Christoffel symbols either can be provided analytically or are numerically computed on the fly by using a fourth-order centered finite-difference method. To integrate the polarized radiative transfer equation, *RAPTOR* uses a hybrid ImEx integration scheme that switches to an implicit integrator in case of stiffness, in order to solve the equation with optimal speed and accuracy for all possible values of the local optical/Faraday thickness of the plasma. The code uses an adaptive camera grid to optimize run time by adding resolution where needed (Davelaar & Haiman 2022) and can produce virtual reality visualizations (Davelaar et al. 2018). The code is fully interfaced with the nonuniform grid (adaptive mesh refinement) data format of the BHAC code (Davelaar et al. 2019). Radiative transfer coefficients are provided for the thermal electron distribution, but also the  $\kappa$  and power-law distributions.

## 3. Test Problems

Three test problems were used to evaluate the codes. The problems were chosen to reflect tests already present in the literature, highlighting specifically the aspects of code performance related to polarized transport. A previous

<sup>153</sup> Current version is available at <https://github.com/moscibrodzka/ipole>.

<sup>154</sup> <https://github.com/AFD-Illinois/ipole>

<sup>155</sup> <https://github.com/jadexter/grtrans>

<sup>156</sup> <https://github.com/hungyipu/Odyssey>

<sup>157</sup> <https://github.com/jordydavelaar/raptor>

**Table 1**  
Constant Coefficients for the Analytic Comparison Test Integrations

	$j_I$	$j_Q$	$j_U$	$j_V$	$\alpha_I$	$\alpha_Q$	$\alpha_U$	$\alpha_V$	$\rho_Q$	$\rho_U$	$\rho_V$
Emission/Absorption	2.0	1.0	0.0	0.0	1.0	1.2	0.0	0.0	0.0	0.0	0.0
Rotation	0.0	0.1	0.1	0.1	0.0	0.0	0.0	0.0	10.0	0.0	-4.0

**Note.** These mirror values in Mościbrodzka & Gammie (2018). See Section 3.1.

comparison (Gold et al. 2020) evaluated many of the same codes for similarity and accuracy in producing total-intensity images. An additional goal was to verify data product similarity when imaging the output of GRMHD simulations, a test evaluated only for certain pairs of codes considered in Gold et al. (2020). The tests are described here from least to most complex, with each testing a larger subset of code features.

### 3.1. Comparison to Analytic Result

The first test problem is a straightforward integration of the nonrelativistic polarized transfer equation using constant coefficients, chosen for the availability of an analytic solution from Landi Degl’Innocenti & Landi Degl’Innocenti (1985), allowing direct evaluation of code accuracy, in addition to code similarity. The test here is taken directly from Dexter (2016), with coefficients as listed in Mościbrodzka & Gammie (2018).

In the Stokes basis  $I$ ,  $Q$ ,  $U$ ,  $V$ , the nonrelativistic polarized radiative transfer equation is

$$\frac{d}{ds} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} j_I \\ j_Q \\ j_U \\ j_V \end{pmatrix} - \begin{pmatrix} \alpha_I & \alpha_Q & \alpha_U & \alpha_V \\ \alpha_Q & \alpha_I & \rho_V & -\rho_U \\ \alpha_U & -\rho_V & \alpha_I & \rho_Q \\ \alpha_V & \rho_U & -\rho_Q & \alpha_I \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}. \quad (1)$$

In the test, this equation is integrated twice with different subsets of nonzero coefficients. This minimizes the complexity of the analytic comparison functions, isolating any bugs in treating emission and absorption from those in Faraday rotation and conversion. The coefficients for each integration are given in Table 1.

### 3.2. Thin-disk Model

The second test problem consists of imaging emission from a thin opaque disk aligned to the midplane of a near-maximally spinning BH, as described in Novikov & Thorne (1973). This involves solving the geodesic equation in two contexts: first in tracing lines of sight from the camera through the Kerr metric, and second in parallel-transporting the direction of linearly polarized emission from the disk back to the camera. This test closely mirrors a figure from Schnittman & Krolik (2009), which is reproduced as a test in Dexter (2016).

The thin-disk test does not include any diffuse emission, that is, all transport coefficients  $j_S$ ,  $\alpha_S$ ,  $\rho_S = 0$  uniformly for all Stokes parameters  $S$ . Instead, the initial Stokes parameters are set as a boundary condition at the first midplane crossing of each geodesic when traced backward from the camera. As in Schnittman & Krolik (2009), the total flux  $F$  is taken from Page & Thorne (1974), and the intensity at the desired frequency  $I_\nu$

is obtained by calculating an effective temperature and assuming a blackbody distribution diluted by a hardening factor  $n = 1.8$ :

$$T_{\text{eff}} \equiv \left( \frac{F}{\sigma} \right)^{1/4}, \quad (2)$$

$$I_\nu = \frac{1}{n^4} B_\nu(n \cdot T_{\text{eff}}), \quad (3)$$

where  $F$  is the total emitted power per area of the thin disk and  $B_\nu(T)$  is the blackbody function of the temperature  $T$  and the emitted frequency  $\nu$  in the fluid frame.

The emitted intensity and horizontal polarization fraction in the outgoing direction are determined by assuming scattering from a semi-infinite atmosphere, as in Chandrasekhar (1960), Table 24, with the direction of linear polarization pointing along the plane of the disk. Emission is enabled only between  $r_{\text{ISCO}}$  and  $R_{\text{out}} = 100r_g$ , where  $r_g$  is the system gravitational radius  $GM_{\text{BH}}/c^2$ , with  $G$  the gravitational constant and  $c$  the speed of light. The fluid orbital angular velocity  $u^{\phi}$  is assumed to be Keplerian:

$$\frac{u^{\phi}}{u^t} = \frac{1}{r^{3/2} + a_*}, \quad (4)$$

where  $a_*$  is the dimensionless form of the BH angular momentum  $J$ ,  $a_* \equiv Jc/GM^2$  with  $-1 \leq a_* \leq 1$ . As the test will need to be implemented in many different codes, we simplify the original problem from Schnittman & Krolik (2009) by observing at only a single frequency rather than summing over a range. The full set of parameters used for this image is

$$a_* = 0.99 \quad (5a)$$

$$M_{\text{BH}} = 10 M_{\odot} = 1.477 \cdot 10^6 \text{cm} \frac{c^2}{G} \quad (5b)$$

$$D_{\text{source}} = 0.05 \text{ pc} \quad (5c)$$

$$\dot{M} = 0.1 \dot{M}_{\text{Edd}} \approx 2.218 \cdot 10^{-9} M_{\odot} \text{ yr}^{-1} \quad (5d)$$

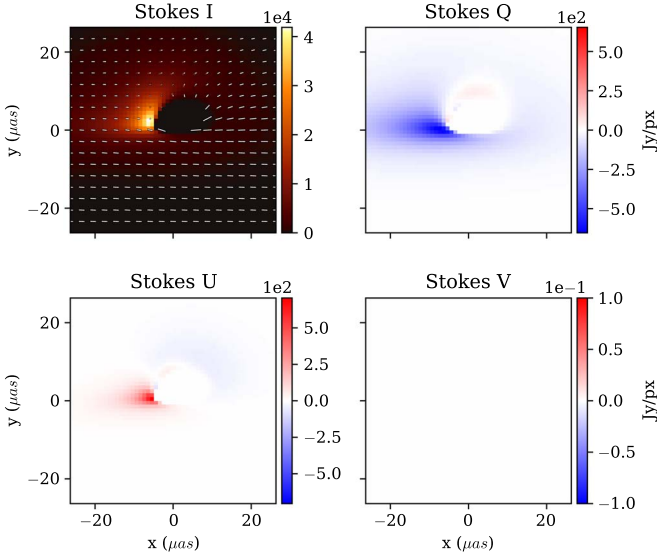
$$h\nu = 1 \text{ keV}, \quad (5e)$$

where these parameters define observation at a single frequency  $\nu$  of a BH of mass  $M_{\text{BH}}$  at distance  $D_{\text{source}}$ , characterized by a Kerr spacetime with BH spin parameter  $a_*$ , and accreting at rate  $\dot{M}$ . Following Dexter and Schnittman,  $\dot{M}_{\text{Edd}} \equiv \frac{L_{\text{Edd}}}{c^2}$ .

Example output from this test run with `ipole-IL` is shown in Figure 1. As no circularly polarized emission or Faraday conversion occurs in the problem, the Stokes  $V$  flux remains exactly zero.

#### 3.2.1. Camera

In this and the following test, the camera tetrad is constructed such that a geodesic at the center of the camera’s field of view (FOV) would have zero angular momentum  $k_{\phi}$ . The polar angle  $\theta_{\text{cam}}$  is defined relative to the BH angular



**Figure 1.** Example output of the thin-disk problem: the top left panel shows total flux with overplotted polarization direction vectors scaled by the linear polarization fraction, and the other panels show fluxes of Stokes  $Q$ ,  $U$ , and  $V$  at each pixel (the Stokes  $Q$ ,  $U$ , and  $V$  images). In this test Stokes  $V$  remains identically zero as expected.

momentum vector. This is identical to the camera definition from Gold et al. (2020).

The camera is placed at radius  $10^4 r_g$  in both tests, to reduce the discrepancy between pinhole and planar cameras. Note that  $R_{\text{cam}}$  is in this case and in practice much smaller than  $D_{\text{source}}$ —so long as  $R_{\text{cam}}$  is large enough to eliminate camera effects, the image intensity is invariant with distance.

The FOV of each test is given in two forms:  $DX$ , the in-plane distance from one edge of the imaged material to the other in gravitational radii  $r_g$ , and  $FOV$ , the angular size from Earth in microarcseconds ( $\mu\text{s}$ ).

Pixels on each image correspond to evenly spaced geodesics, starting from each pixel center. That is, an image with an FOV of  $80 \mu\text{s}$  to a side and a resolution  $NX$  of 80 pixels would be calculated using geodesics originating at  $-39.5$  to  $39.5 \mu\text{s}$  away from the FOV center, spaced  $1 \mu\text{s}$  apart in each cardinal direction. All images in these tests are square, with equal FOV from west to east and north to south.

For the thin-disk test, the camera parameters are

$$\theta_{\text{cam}} = 75^\circ \quad (6a)$$

$$NX = 80 \text{ pixels} \quad (6b)$$

$$DX = 40 r_g \quad (6c)$$

$$FOV = 78.99 \mu\text{s}. \quad (6d)$$

### 3.3. GRMHD Snapshot

The last test consists of imaging the relativistic thermal synchrotron emission at 230 GHz from one snapshot from a GRMHD simulation. This test exercises all aspects of the code, as well as code-specific choices, such as interpolation of fluid state recorded at discrete locations and calculation of the transport coefficients via fitting functions. The standard snapshot file used for this test is taken from a SANE simulation with spin  $a_* = 0.9375$ , performed using *iharm3D* (Prather et al. 2021) with a resolution of  $288 \times 128 \times 128$  cells in  $r$ ,  $\theta$ , and  $\phi$  respectively. Except for the coordinate system, this simulation exactly reflects the simulations performed with

*iharm3D* as a part of the library used in **EHTC V** and **EHTC VIII**, as described in Wong et al. (2022). Not all GRMHD codes can read all GRMHD output, as coordinate systems and fluid state descriptions can differ from code to code—thus, not every polarized radiative transfer code used in the EHTC can be directly compared with this test. The *iharm3D* format is chosen since it is readable by a majority of codes used in the EHTC, and in particular those codes relevant to studies in **EHTC VIII**.

The snapshot is taken at  $4500 r_g/c$  after simulation start, well into the run’s quiescent period. The file is available upon request for testing future codes. Since it involves creating an image from just one snapshot of the simulation, the test makes the assumption of “fast light,” i.e., that the fluid is static as light propagates from emission to observer.

The parameters of this test are chosen to reflect values for M87\*, specifically those used in creating the libraries of simulated images used in **EHTC V**, **EHTC VI** and **EHTC VIII**, hereafter the “EHT image libraries.” These are

$$M_{\text{BH}} = 6.2 \times 10^9 M_\odot \quad (7a)$$

$$D = 16.9 \text{ Mpc} \quad (7b)$$

$$\nu = 230 \text{ GHz}. \quad (7c)$$

The camera is defined as in Section 3.2.1, with parameters chosen to reflect the angle of the M87\* jet and the FOV observed by the EHT:

$$\theta_{\text{cam}} = 163^\circ \quad (8a)$$

$$\phi_{\text{cam}} = 0^\circ \quad (8b)$$

$$FOV = 160 \mu\text{s} \quad (8c)$$

$$DX = 44.17 r_g \quad (8d)$$

$$NX = 160 \text{ pixels}. \quad (8e)$$

In addition to the system parameters above, imaging a GRMHD simulation necessarily involves setting another scale factor that determines the density of accreting material and the strength of magnetic fields. It is expressed here as a mass unit,  $\mathcal{M}$ , which gives units to the unscaled density values from a simulation,  $\rho_{\text{code}}$ :

$$\rho_{\text{CGS}} = \frac{\mathcal{M}}{r_g^3} \rho_{\text{code}}. \quad (9)$$

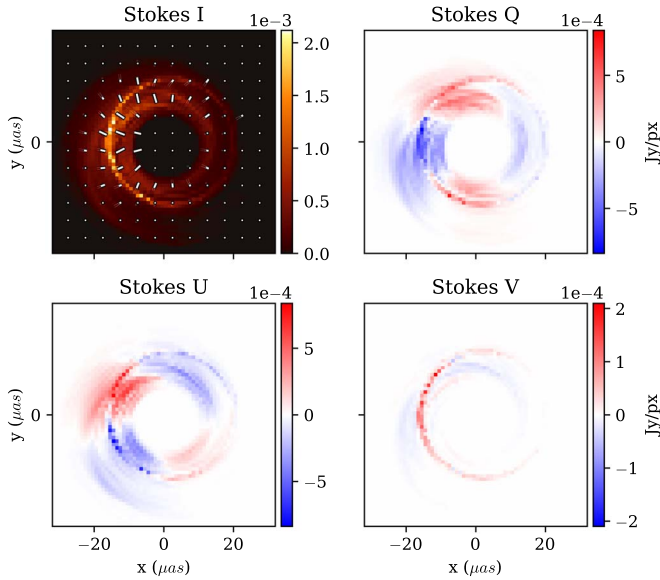
$\mathcal{M}$  is not known a priori, but it is highly correlated with the total image brightness. Thus, it is scaled so as to match the total image flux density to the observed compact flux density, usually by employing an iterative solver.

In the EHT image libraries,  $\mathcal{M}$  was fit such that images taken over the course of a full simulation would produce an average of 0.5 Jy of compact flux density (see Wong et al. 2022 for details). For this test, the sample image is fit alone using *ipole-IL* such that it produces 0.50 Jy when imaged with totally unpolarized transport, or about 0.47 Jy of Stokes  $I$  flux density when imaged using polarized transport. The value of  $\mathcal{M}$  and the corresponding accretion rate  $\dot{M}$  used for the test are listed below:

$$\mathcal{M} = 1.672 \cdot 10^{26} \text{ g} \quad (10a)$$

$$\dot{M} = 8.644 \cdot 10^{-5} M_\odot \text{ yr}^{-1} = 6.285 \cdot 10^{-7} \dot{M}_{\text{Edd}}, \quad (10b)$$

where  $\dot{M}_{\text{Edd}}$  is defined as earlier in this work.



**Figure 2.** Example output from *ipole-IL* running the GRMHD snapshot test. The image was produced using the parameters listed in Section 3.3, with the accretion rate parameter  $\mathcal{M}$  fit so as to produce about 0.5 Jy of total flux density at 230 GHz to match EHT observations.

Accretion flows around M87\* are strongly suspected to be two-temperature, with little thermal coupling between the ions and electrons (Mahadevan & Quataert 1997; Ryan et al. 2017; Sadowski et al. 2017). Since GRMHD simulations evolve only a single fluid with a single temperature, when simulating images the internal energy must be split between the ions and electrons based on a model. While this process is not well constrained, it is generally documented which electron distribution is being assumed, and the electron energy distribution model is standardized between codes when similar performance is expected.

Thus, for simplicity, in this test we set the electron temperature to a fixed ratio of 1/3 of the ion temperature, derived from the single-fluid GRMHD parameters by holding the total internal energy constant (see EHTC V):

$$T_e = \frac{2m_p u}{15k\rho}, \quad (11)$$

where  $u$  and  $\rho$  are the local fluid internal energy and rest-mass density per unit volume, respectively, and  $m_p$  and  $k$  are the proton mass and Boltzmann constant, respectively. Note that this is equivalent to the so-called “ $R_{\text{high}}$ ” model of EHTC V with  $R_{\text{low}} = R_{\text{high}} = 3$ . In splitting the total internal energy rather than setting the fluid and ion temperatures equal, it differs slightly from the original statement of the  $R_{\text{high}}$  model in Mościbrodzka et al. (2016).

The emission, absorption, and rotation coefficients are calculated based on the electron temperature (or more broadly, the electron energy distribution) using fitting functions approximating the full synchrotron emission calculations, which are expensive to compute. Codes in this comparison used a few different sets of fitting functions; further discussion is found in Section 5.3 and Appendix B.

Finally, as in EHTC V and commonly in the literature, emission is tracked only from regions of the simulation with  $\sigma \equiv B^2/\rho < 1$ . Regions with higher sigma (largely the polar “jet” regions) can overproduce emission if included, due to hot

material in the jet inserted by numerical floors to preserve stability of GRMHD algorithms.

Example output for the GRMHD snapshot test from *ipole-IL* is provided in Figure 2.

## 4. Results

### 4.1. Analytic Comparison Results

Results for the analytic integration tests from RAPTOR, Odyssey, *ipole*, and *ipole-IL* are shown in Figure 3. Raw output is plotted in the left panels, and differences from the analytic result are plotted in the right panels. This test verifies that the default accuracy parameters of each code allow them to match an analytic solution to within acceptable errors. Note that this is not a good measure of relative code accuracy or convergence—for convergence tests, see the accompanying code papers cited in Section 2.

Note that the results from two integrators are shown for RAPTOR as the “RK4” and “IE” variants. In normal integration, RAPTOR uses the “RK4” integrator, reserving the “IE” integrator for the few zones where Faraday rotation is too strong to take steps of an appropriate size with an explicit scheme (Bronzwaer et al. 2020).

In addition, the *ipole* scheme (also used in *ipole-IL*) is semianalytic: it uses the analytic solution for constant coefficients whenever it evolves the nonrelativistic polarized radiative transfer equations. Thus, *ipole* and *ipole-IL* will perform this test less accurately when taking more steps: a single step of any size would be exactly accurate, but multiple steps accrue round-off error.

### 4.2. Metrics

In the following two imaging tests, no exact result is available by which to evaluate code accuracy directly; rather, we evaluate consistency between all codes, both in overall image structure and in several metrics used to compare models with EHT results. In particular, we will use the definitions from EHTC VIII, computed over simulated images and used to compare models to the observed EHT result. These summary statistics include the total flux density  $F$ , image-integrated or “zero-baseline” linear and circular polarization fractions  $|m|_{\text{net}}$  and  $v_{\text{net}}$ , and the average linear polarization fraction over the resolved image  $\langle |m| \rangle$ . For an image represented as a vector of emitted flux per pixel in each Stokes parameter  $I_j$ ,  $Q_j$ ,  $U_j$ ,  $V_j$  over each pixel  $j$ , these values are defined as

$$F = \sum_j I_j \quad (12)$$

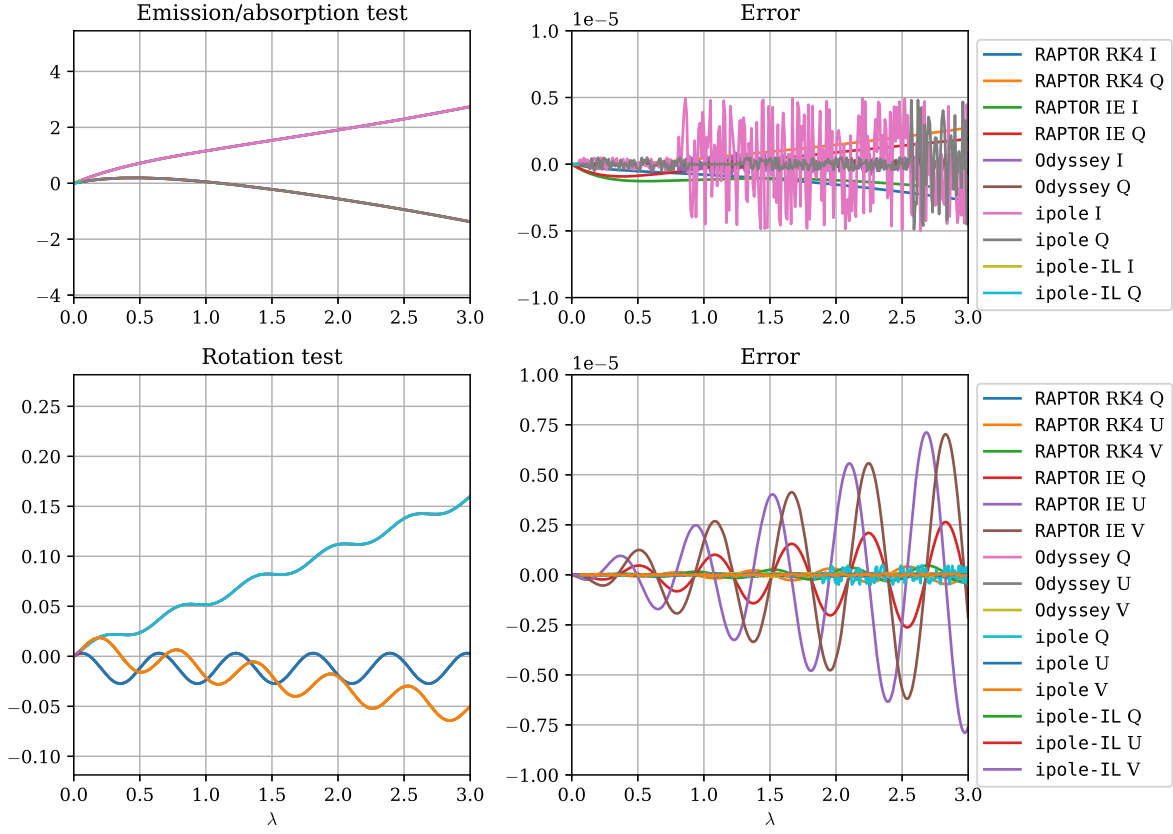
$$|m|_{\text{net}} = \frac{\sqrt{(\sum_j Q_j)^2 + (\sum_j U_j)^2}}{\sum_j I_j} \quad (13)$$

$$\langle |m| \rangle = \frac{\sum_j \sqrt{Q_j^2 + U_j^2}}{\sum_j I_j} \quad (14)$$

$$v_{\text{net}} = \frac{\sum_j V_j}{\sum_j I_j}. \quad (15)$$

Additionally, EHTC VIII used a complex coefficient reflecting the degree and angle of azimuthally symmetric linear polarization,  $\beta_2$  (see Palumbo et al. 2020). It is calculated by first centering the image, e.g., with *rex* (EHTC IV), and then





**Figure 3.** Comparison of integrator results for the analytic tests, plotting the relevant Stokes parameters against the dimensionless affine parameter  $\lambda$ , equivalent to length in this nonrelativistic test. Output from all codes overlaps to within line widths in the left panes.

taking the inner product of the complex linear polarization  $P = Q + iU$  with a rotationally symmetric function:

$$\beta_2 = \frac{1}{I_{\text{ann}}} \int_{\rho_{\min}}^{\rho_{\max}} \int_0^{2\pi} P(\rho, \phi) e^{-i2\phi} \rho d\phi d\rho \quad (16)$$

$$I_{\text{ann}} = \int_{\rho_{\min}}^{\rho_{\max}} \int_0^{2\pi} I(\rho, \phi) \rho d\phi d\rho, \quad (17)$$

where  $\rho/\phi$  are polar coordinates in the image plane, measured from/about the image center. This metric is expected to be useful only for images with a relatively low observer angle, as they will be more symmetric; thus, it is computed and compared only for the GRMHD snapshot test, which uses the low observer angle  $i = 17^\circ$  expected for M87\* and used for libraries of simulated images of that object.

The quantities  $\langle |m| \rangle$  and  $\beta_2$  are sensitive to image resolution. In order to mirror EHT measurements and reflect how simulated images were used in EHTC VIII, a circular Gaussian blur with an FWHM of  $20 \mu\text{as}$  was applied to all images before computing either resolution-dependent quantity.

In addition to the quantities used for direct comparison in EHTC VIII, we measure a point-source linear polarization direction or electric vector position angle (EVPA) east of due north on the sky, and thus in our Stokes convention defined as

$$\text{EVPA} = \frac{1}{2} \arg \left( \frac{\sum_j U_j}{\sum_j Q_j} \right). \quad (18)$$

As this metric is potentially volatile for images with low net linear polarization, and a corresponding measurement has not been made to which we might compare, we follow EHTC VIII in omitting this as a comparison metric—rather, we use it only in image summaries.

When evaluating image similarity, we use the normalized mean squared error (NMSE):

$$\text{NMSE}(A, B) = \frac{\sum_j |A_j - B_j|^2}{\sum_j |A_j|^2}, \quad (19)$$

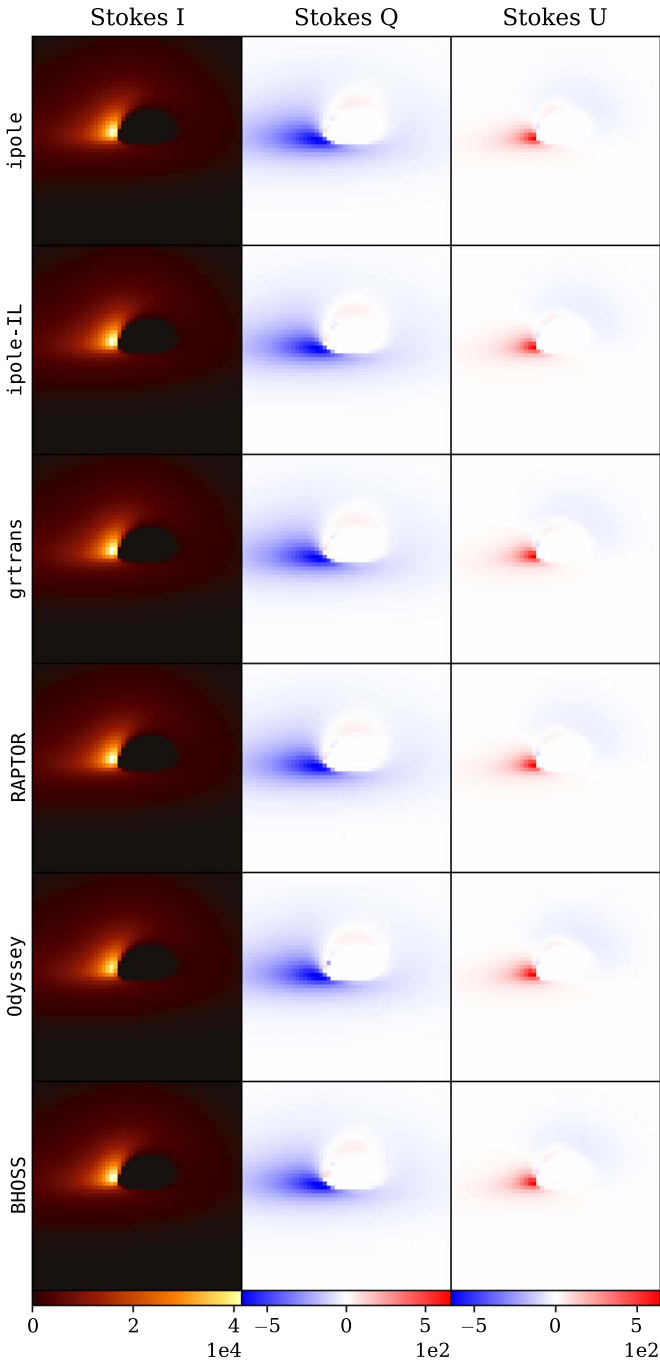
where  $A_j$  and  $B_j$  are the intensities of a particular Stokes parameter in two images at pixel  $j$ . Regardless of exact nomenclature, all NMSE values listed in this work and in Gold et al. (2020) use this normalization and are thus comparable.

Note that this definition of the NMSE is not symmetric under the ordering of  $A$  and  $B$ , and in particular as images get dimmer,  $\text{NMSE}(I, 0) = 1$ , whereas  $\text{NMSE}(0, I) = \infty$ . As it is normalized against the sum of squared pixel intensities, the NMSE becomes more volatile when evaluating dimmer images.

The NMSE was one of two metrics used to gauge similarity in Gold et al. (2020). We omit the other, the structural dissimilarity (DSSIM), since for the case of very similar images, values of the DSSIM are highly correlated with the NMSE (see Appendix C).

### 4.3. Thin-disk Test Results

Each code's output for the thin-disk test is plotted in Figure 4. The images are visually indistinguishable except in a



**Figure 4.** Full set of images produced for the thin-disk problem, in which codes produced an image from an analytic prescription for an opaque thin disk. The Stokes parameters  $I$ ,  $Q$ ,  $U$  are plotted separately, with Stokes  $V$  omitted, as it is uniformly zero.

few particular pixels, and this similarity is borne out in the comparison metrics. Stokes  $V$  is omitted from plots and comparisons for this test, as all codes produce exactly zero Stokes  $V$  across the entire image.

Recall that this test involves accurate evaluation of the geodesic equation and accurate parallel transport of the linear polarization vector from emission to camera. As the codes’ similarity in tracing geodesics was evaluated extensively in Gold et al. (2020), we focus here on demonstrating the latter through comparison of the resulting Stokes  $Q$ ,  $U$  images and the relevant image-integrated metrics. Table 2 lists total fluxes

**Table 2**  
Image-integrated Values for the Thin-disk Test

Code	Flux (Jy)	$ m _{\text{net}}$ (%)	$\langle  m  \rangle$ (%)	EVPA (deg)
ipole	6.841e+06	2.3341	2.378	88.123
ipole-IL	6.8699e+06	2.3224	2.3707	87.974
grtrans	6.8227e+06	2.3246	2.3709	88.01
RAPTOR	6.8689e+06	2.3265	2.3726	88.025
Odyssey	6.7338e+06	2.3527	2.3971	88.146
BHOSS	6.6949e+06	2.3439	2.3845	88.131

**Note.**  $v_{\text{net}}$  is omitted since it is uniformly zero, and  $\beta_2$  magnitude and angle are omitted since the image is not symmetric and thus the magnitude is very small.

and net polarization parameters for each image in the test. Figure 5 presents comparisons of each metric between each pair of images.

#### 4.4. GRMHD Snapshot Test Results

Results for the GRMHD snapshot test for ipole, ipole-IL, grtrans, Odyssey, and RAPTOR are listed in Table 3 and presented in Figures 6 and 7. Results are formatted similarly to the results of the thin-disk test, with the addition of a Stokes  $V$  component and circular polarization fraction in images and tables and the addition of the rotationally symmetric linear polarization coefficient  $\beta_2$  in tables of integrated values.

Except for the total flux density, the color bars in Figure 7 reflect the  $1\sigma$  values used to make cuts when evaluating models in EHTC VIII.

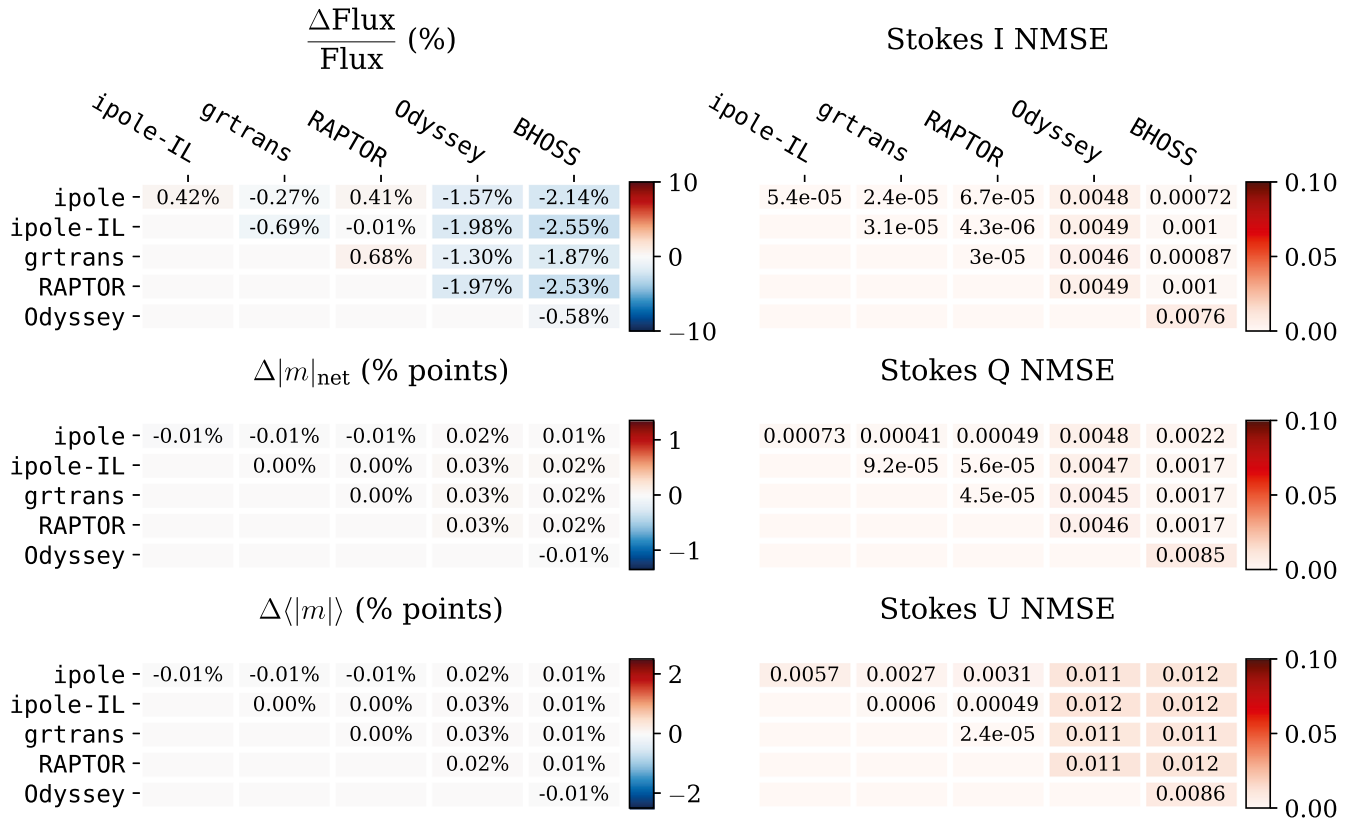
## 5. Discussion

### 5.1. Comparison to Observational Constraints

The values for each comparison metric used as cuts in EHTC VIII are listed in Table 4. The table values are based on  $1\sigma$  ranges for measurements of the same quantities in EHT data, described in EHTC VII. These ranges provide a comparison to evaluate code interchangeability—if the differences between codes are substantially less than the range of measurement uncertainties, the analysis is agnostic to the choice of code employed. The  $1\sigma$  ranges listed are also used as the color bar ranges in the colored table listings in Figure 7.

This comparison provides evidence that model evaluations as in EHTC VIII remain similar regardless of which of the included codes is employed. As recorded in Table 4, maximum code variation is universally less than 30% of the detector  $1\sigma$  range: in  $|m|_{\text{net}}$  (0.21% vs. 1.5%),  $v_{\text{net}}$  (0.08% vs. 0.4%),  $\langle |m| \rangle$  (0.70% vs. 2.5%),  $|\beta_2|$  (0.0026 vs. 0.015), and  $\angle\beta_2$  (3.4° vs. 17°). In any analysis based on cuts, code differences can shift a few particular images into or out of the final consideration. However, at these uncertainties no image from outside the  $1\sigma$  detector uncertainty would be consistent with the central observed value.

Broadening the comparison to different images and models shows promising similarities. Appendix D presents distributions of the image differences between ipole-IL and grtrans when run over thousands of snapshots of a very different model from the example: they show a wide variance but a smaller difference on average than in the example image, suggesting that image differences, at least between these codes,



**Figure 5.** Left column: tables comparing the absolute differences between the images produced by each pair of codes running the thin-disk test. The values themselves are provided in Table 2. Since the differences are symmetric, the table takes only the upper triangular portion of the comparison. Right column: tables comparing the NMSE between each pair of images, as defined in Section 4.2. To aid in comprehension, table cells are colored by value. Circular polarization is omitted; see test description in Section 3.2.

**Table 3**  
Image-integrated Values for the GRMHD Snapshot Test

Code	Flux (Jy)	$ m _{\text{net}}$ (%)	$v_{\text{net}}$ (%)	$\langle  m  \rangle$ (%)	EVPA (deg)	$ \beta_2 $	$\angle \beta_2$ (deg)
ipole-IL	0.47976	1.5232	0.64637	31.264	-78.241	0.28165	-20.307
ipole	0.47335	1.5904	0.62402	31.27	-77.243	0.28171	-20.622
RAPTOR	0.49396	1.6057	0.67381	31.893	-77.797	0.28346	-21.31
grtrans	0.45346	1.5135	0.69909	31.197	-77.647	0.28063	-17.903
Odyssey	0.46671	1.3928	0.70068	31.54	-71.432	0.28325	-20.713

**Note.** Definitions for all values are given in Section 4.2. Note that the EVPA is not used as a comparison metric in Figure 7.

are mostly stochastic, further suppressing any potential effect on a cuts-based analysis as in EHTC VIII.

### 5.2. Potential Measurement of System Parameters

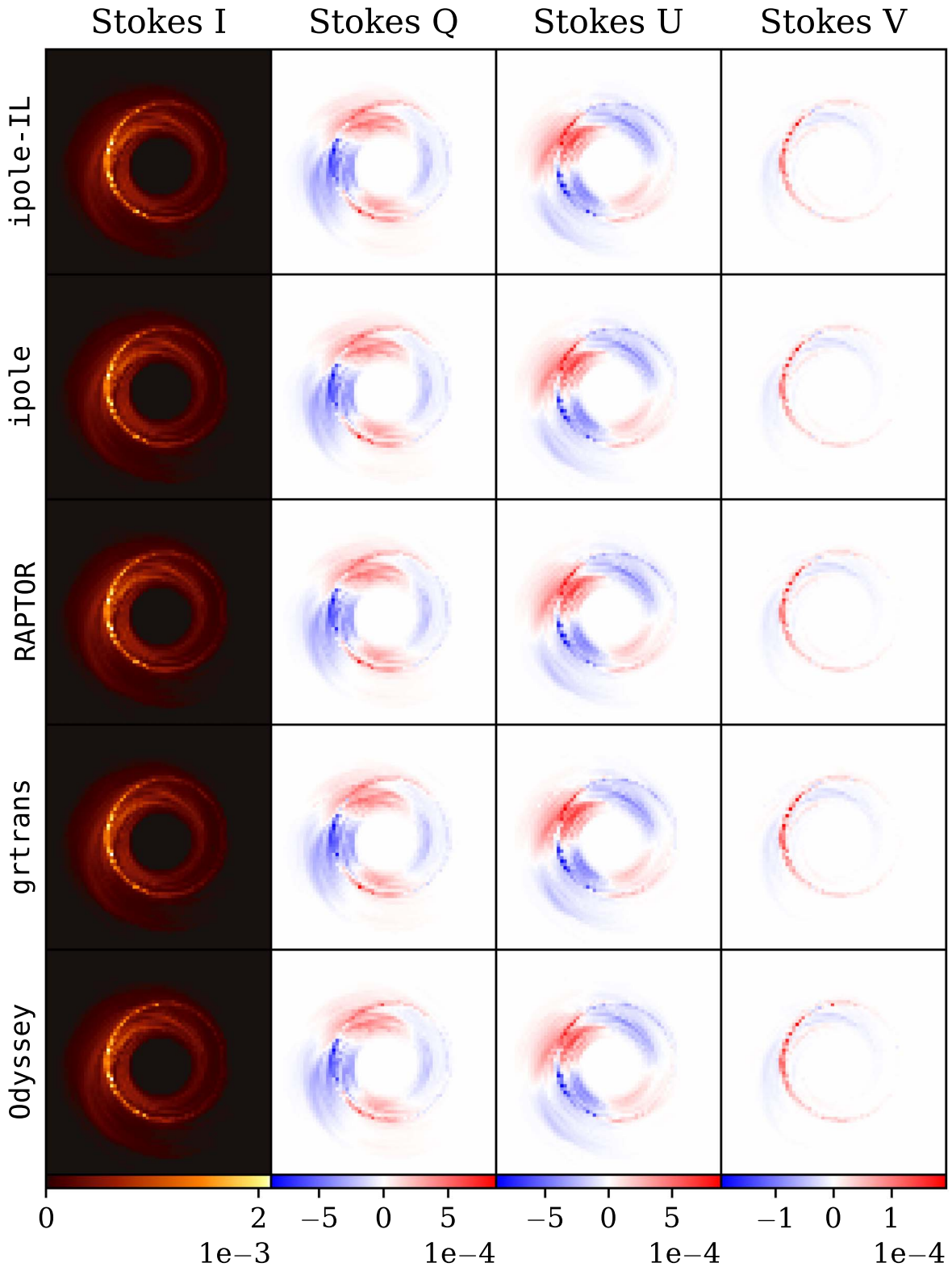
To translate the NMSE into a measure of code accuracy in testing model parameters, we define an “error budget” for each Stokes parameter, consisting of the largest NMSE between code results: 0.02 in  $I$ , 0.04 in  $Q$  and  $U$ , and 0.13 in  $V$ . Assuming perfect detector accuracy and modeling, this error characterizes which images are too similar to be effectively distinguished above code-to-code variations.

We then translate this error budget into constraints on the input parameters  $M_{\text{BH}}$ ,  $\mathcal{M}$ ,  $R_{\text{high}}$ , and the viewing angle by varying these parameters around the nominal values and calculating the resulting NMSE versus the nominal image, using ipole-IL. As illustrated in Figure 8, the required parameter changes are very modest; that is, the possible

constraints on system parameters are very precise. In imaging the example model, codes agree well enough to constrain the mass of M87\* to within 0.4% ( $2.5 \times 10^7 M_{\odot}$ ),  $\mathcal{M}$  to 9% ( $1.5 \times 10^{25}$ ), the observer angle to  $0.8^\circ$ , and the  $R_{\text{high}}$  parameter to within 0.18. These values are dependent on the base image—in particular, constraints on  $R_{\text{high}}$  will also depend on  $R_{\text{low}}$ , which was set differently in this case than for images in EHTC VIII.

### 5.3. Caveats and Limitations

There are a few caveats and limitations of the ray-tracing calculations presented in this work worth mentioning and improving in the future. Most glaringly, all ray-tracing codes use phenomenological postprocessing models of the electron energy distribution. In particular, this comparison adopts a fixed ratio of ion to electron temperature, which is not well motivated by EHT results. More accurate temperature

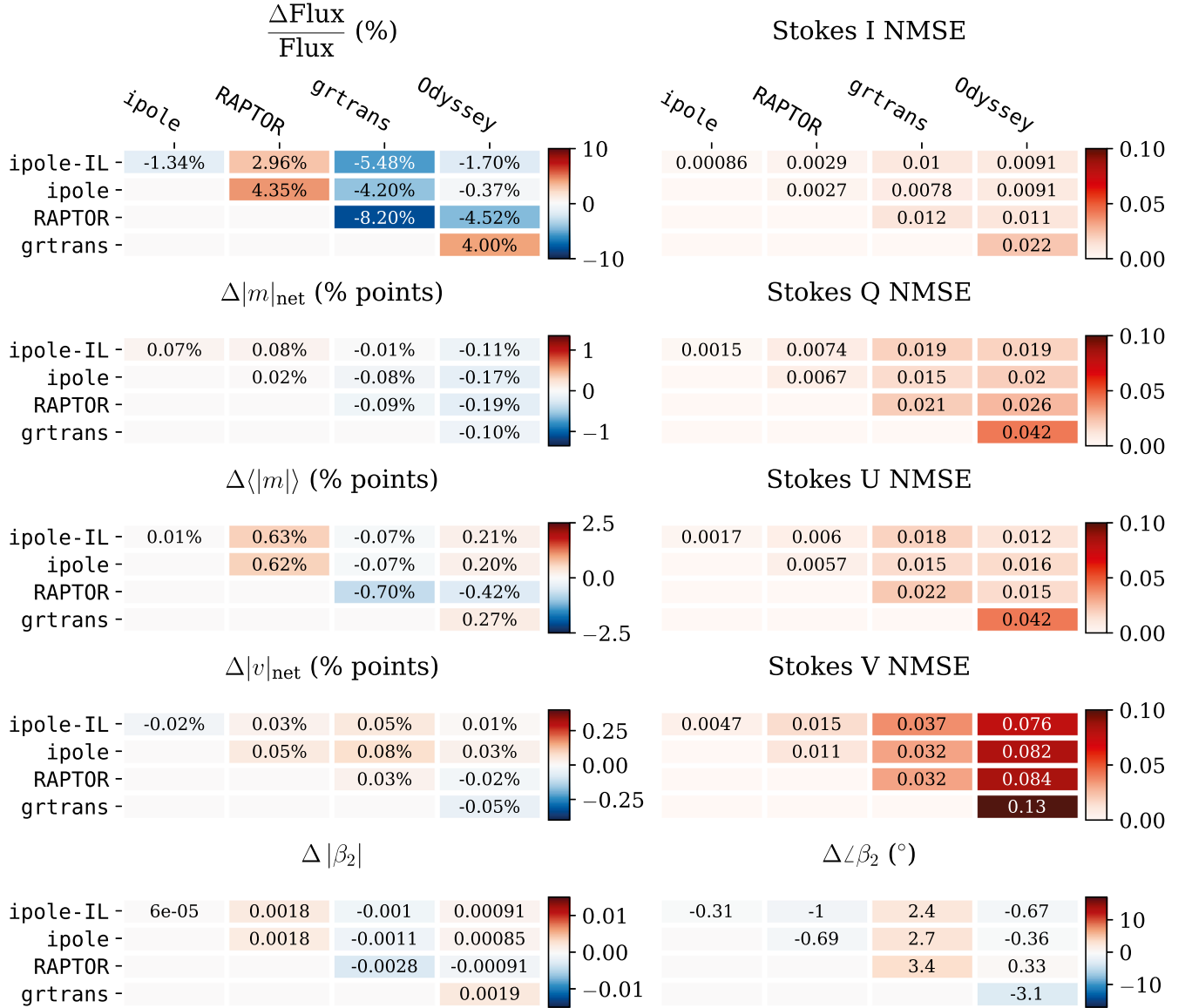


**Figure 6.** Comparison of images produced of the GRMHD test problem, in which codes produced an image given a simulated fluid snapshot, mirroring the analysis pipeline used in, e.g., EHTC V and EHTC VIII. The Stokes parameters  $I$ ,  $Q$ ,  $U$ ,  $V$  are plotted separately, with separate color bars.

prescriptions including cold electrons in the accretion disk dramatically increase Faraday rotation when viewed from the equator, scrambling the emission angle over regions of the image. Scrambled emission does not affect the total-intensity image, nor the measurable quantities in this comparison (except  $\angle\beta_2$ , which only makes sense to measure for face-on images

with low Faraday scrambling). Detailed study of code behavior in imaging Faraday-scrambled regions is left for future work.

All ray-tracing codes use synchrotron emissivities/absorptivities/rotativities in analytic forms that are fit formulae to synchrotron emissions integrated over (most often thermal) electron distribution function. The fit functions may differ from



**Figure 7.** Tables comparing each pair of images in the GRMHD snapshot test. Left column, bottom row: tables comparing all six image metrics used in EHTC VIII. Note that except for total flux density, all comparisons are absolute: that is, images with  $v_{\text{net}}$  of 0.4% and 0.8% would be listed with a difference of 0.4% points. Right column: tables listing the NMSE between each pair of images. See Section 4.2 for the definitions of all comparison metrics and Figure 5 for a description of table layout.

**Table 4**

Detector Uncertainty Compared Against Code Uncertainty in Observed Parameters

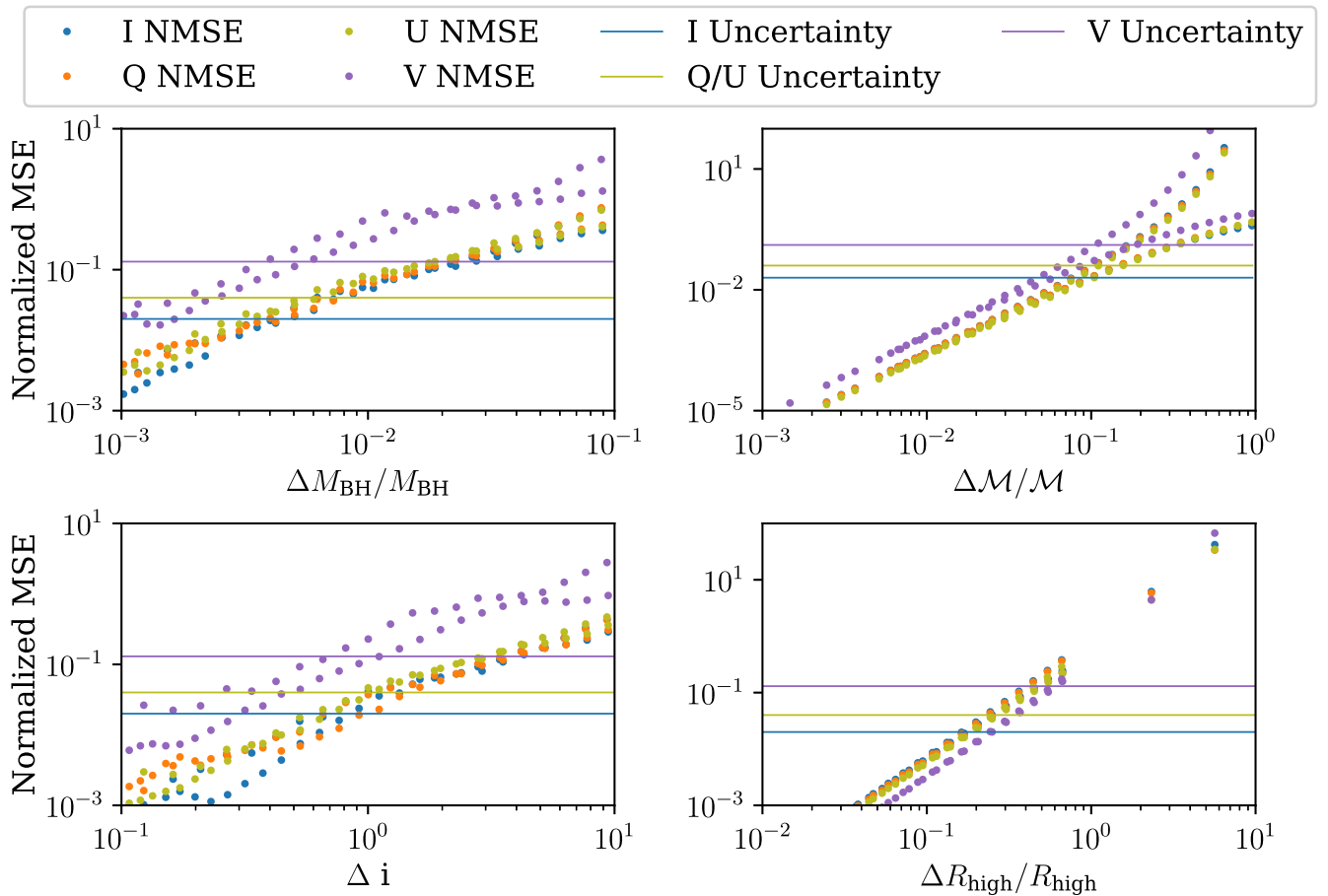
Parameter	$1\sigma$	Code Uncertainty	Uncertainty/ $\sigma$
$ m _{\text{net}}$	1.35 %	0.21 %	0.16
$v_{\text{net}}$	0.4 %	0.08 %	0.20
$\langle m \rangle$	2.5 %	0.70 %	0.28
$ \beta_2 $	0.015	0.0026	0.17
$\angle\beta_2$	17°	3.4°	0.20

**Note.** The “ $1\sigma$ ” column lists the  $1\sigma$  detector uncertainty of each parameter, as estimated in EHTC VII and used as an allowable range when comparing models in EHTC VIII. The measurement of  $\langle|m|\rangle$  was an upper bound; this upper bound was doubled to select the cut value. The “Code Uncertainty” column lists the greatest observed difference between codes when computing each parameter, and the final column lists this value as a proportion of the  $1\sigma$  value.

code to code and from the true emissivity and therefore introduce a small error to the integration. We discuss this issue in more detail in Appendix B. Other caveats concern the common assumption that the electrons are distributed isotropically, which may not be a good approximation for collisionless plasma surrounding Sgr A\* and M87\*.

Calculations presented in this work assume the infinite speed of light (so-called fast-light approximation), while in reality the light propagation timescale is comparable to the plasma dynamical timescale near the event horizon of the BH. Any future comparison of ray-tracing codes should include finite light propagation time effects. In such future comparison another source of error could be the time interpolations between GRMHD model time slices.

Finally, the linear polarization and the EVPA are sensitive to the external Faraday screen made of mildly or nonrelativistic electrons (which in practice could be located thousands of  $r_g$  away from the BH). Any inconsistencies in choosing the outer



**Figure 8.** Plots of image NMSE produced when varying several parameters (BH mass, angle,  $R_{\text{high}}$ , and accretion rate) around values used for the GRMHD snapshot test. For each difference, plotted along the  $x$ -axis, images were run with the parameter increased and decreased by that amount, resulting in two NMSE values, which are then overplotted. The largest NMSE between codes is plotted to provide a visual ceiling on “indistinguishable” images. Based on code agreement, current transport schemes can constrain all input parameters much more accurately than current models and detectors.

boundary of the ray-tracing integration may introduce discrepancies in the linear polarization maps. The latter is not a specific caveat of the ray-tracing itself, but it is a limitation when comparing models to observations.

## 6. Conclusions

In each of the tests conducted for this comparison, the several GRRT codes used within the EHT Collaboration have produced sufficiently similar results that they are functionally interchangeable for the collaboration’s uses. This is true both when measured in terms of image similarity (NMSE) and when measured directly in terms of the image metrics used to compare simulated polarized images to the EHT result in [EHTC VIII](#).

Using their default accuracy parameters, codes match the analytic result for the case of constant transport coefficients to better than 1 part in  $10^{-5}$ . They agree to within NMSE of 0.015 when imaging an analytically defined problem requiring parallel transport of the polarization vector.

In the more complex task of interpolating, translating, and imaging GRMHD output, codes agree to within an NMSE of 0.13 at worst, when measuring specifically the circular polarization map (NMSE of 0.045 in linear polarization, 0.02 in total intensity). Based on image similarity, the choice of imaging code will matter in model comparisons only when

trying to determine the BH mass to within 0.4%, accretion rate within 9%, or observer angle to within  $0.8^\circ$ . These values significantly outclass both the detection and modeling uncertainties available in the near future.

When measured with the image metrics used for model comparison in [EHTC VIII](#), all comparison images agree to much better precision than the detector uncertainty. Further, much of the difference that does appear is shown to be stochastic in nature. Thus, the choice of code is verified directly to have little effect on the analysis performed in that paper.

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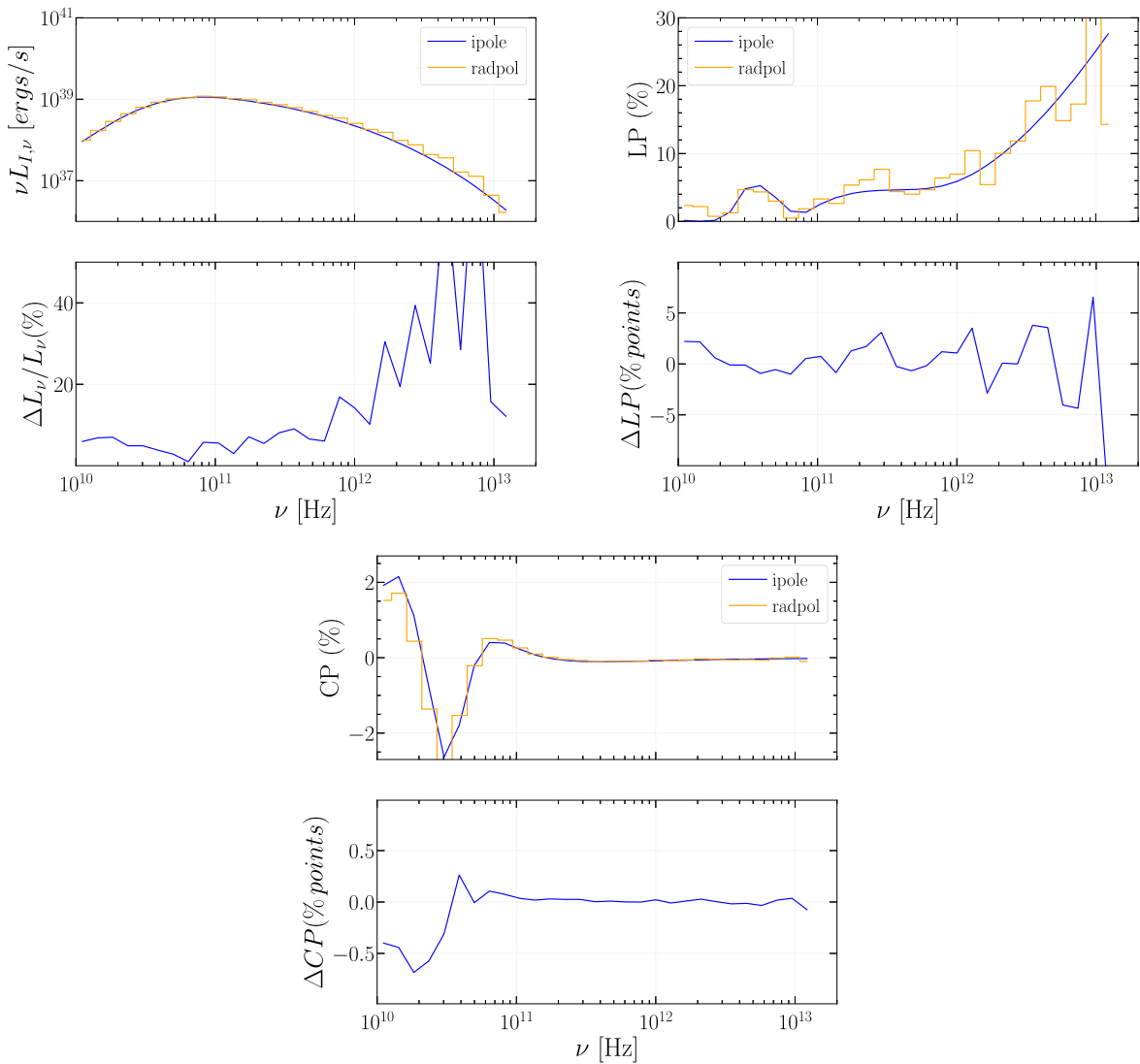
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## Appendix A

### Comparison of Polarized Spectra from Imaging and Monte Carlo Radiative Transfer Methods

In all imaging ray-tracing codes radiative transfer equations are solved along null geodesics that terminate at a “camera” at some large distance from the supermassive BHs and where a





**Figure 9.** Comparison of luminosity, fractional linear, and circular polarizations across synchrotron spectrum produced by `ipole` and `radpol` codes using SANE simulation with the same parameters. Both radiative transfer schemes show consistent amplitude of fractional linear and circular polarizations and agree on handedness of circular polarization.

polarization map at a chosen observing frequency is constructed. By integrating Stokes parameters over entire images made for different frequencies, one can also construct a polarized synchrotron spectral energy distribution of any model. However, instead of comparing model spectra from the discussed imaging codes, here we carry out an alternative comparison. Namely, we compare spectra produced by the `ipole` code to polarized spectra generated via Monte Carlo scheme `radpol`. In the Monte Carlo code the polarized radiative transfer integration scheme is conceptually distinct from all discussed imaging codes (for detailed description see Mościbrodzka 2020). Showing a convergence of two different approaches is an independent validation of emission produced by imaging codes. Hence, we compare spectra produced by `ipole` and `radpol` codes using the plasma model setup described in Section 3.3 (the test with  $\mathcal{M}_{\text{low}}$ ). In Figure 9, we show radio–millimeter spectra of Stokes  $I$  luminosity, fractional linear polarization, and circular polarizations. The relative difference between luminosities is less than 10%, except for high-frequency emission. Both codes show consistent amplitude of fractional linear and circular polarizations and agree on handedness of circular polarization.

## Appendix B

### Effect of Polarized Emissivity and Rotativity Fits

The emission, absorption, and rotation coefficients of a fluid are well determined for a particular distribution of electron energies. However, the integrations involved are numerically expensive; since the coefficients must be calculated at every step when integrating the radiative transfer equations, fitting functions have been developed to approximate the coefficients quickly. A few sets of such fitting functions exist applicable to our regime: one outlined in Dexter (2016), the other in Pandya et al. (2016) and Marszewski et al. (2021).

The differences between these functions at various points within a representative set of input parameters are given in their respective papers, but we wish to provide some intuition concerning the differences these functions make in practice, and consequently whether fitting accuracy might be a driving factor in code differences.

All of the images in this comparison were created using the coefficient fits from Dexter (2016). Substituting the coefficient fits from Pandya et al. (2016) produces results more dissimilar than the disagreement between codes on three metrics: the net

**Table 5**  
Comparison of Changes to the `ipole-IL` Result under Different Emission Coefficient Fitting Functions

<code>ipole-IL</code> Image Run...	NMSE $I$	NMSE $Q$	NMSE $U$	NMSE $V$	$\Delta m _{\text{net}}$ (%)	$\Delta\langle m \rangle$ (%)	$\Delta v_{\text{net}}$ (%)	$\Delta \beta_2 $	$\Delta\angle\beta_2$ (deg)
...with Pandya $j_s$	4.7e-05	0.0055	0.0059	0.0087	0.0246	1.78	0.127	0.0168	0.353
...with Dexter $\rho_V$	1.4e-09	2.6e-06	2.1e-06	6.8e-06	-0.00746	0.0186	-0.0031	0.000131	0.00225
...with approximate $K_n(x)$	3.9e-08	5.6e-05	2.5e-05	0.00014	-0.000404	0.0239	-0.00524	0.000123	-0.00336

**Note.** Each row lists a change made to the default emissivity values and the resulting differences between the new image and the one used in the comparison. Substituting emissivities from Pandya et al. (2016) changes the result by more than the overall level of code agreement, whereas substituting the  $\rho_V$  fit from Dexter (2016) changes almost nothing about the image. Approximating the Bessel functions  $K_n$  does not badly affect this image but can be a substantial source of error in images with higher Faraday rotation.

circular polarization at 0.13 points rather than 0.08 points, and average linear polarization fraction at 1.8 points rather than 0.7 points, and the  $|\beta_2|$  coefficient at 0.016 rather than 0.0026. This is due to significant differences between the fits in computing the emission coefficient for circularly polarized light,  $j_V$ .

Unlike most emission coefficients, Faraday rotation coefficient  $\rho_V$  does not go to zero with low temperature—therefore, the low-temperature behavior of fitting functions is important. In particular, the expression from Dexter (2016) for  $\rho_V$  should not be used at low temperature  $\Theta_e < 1$ , since it can produce a catastrophic cancellation not matching the desired limiting behavior of one of its quotients. Due to this instability, most codes either switch to the Shcherbakov fit at low temperature (e.g., `grtrans`) or use the Shcherbakov fit exclusively (e.g., `ipole-IL`). As illustrated in Table 5, these approaches produce nearly identical results, different by an NMSE less than  $10^{-5}$  in the worst case.

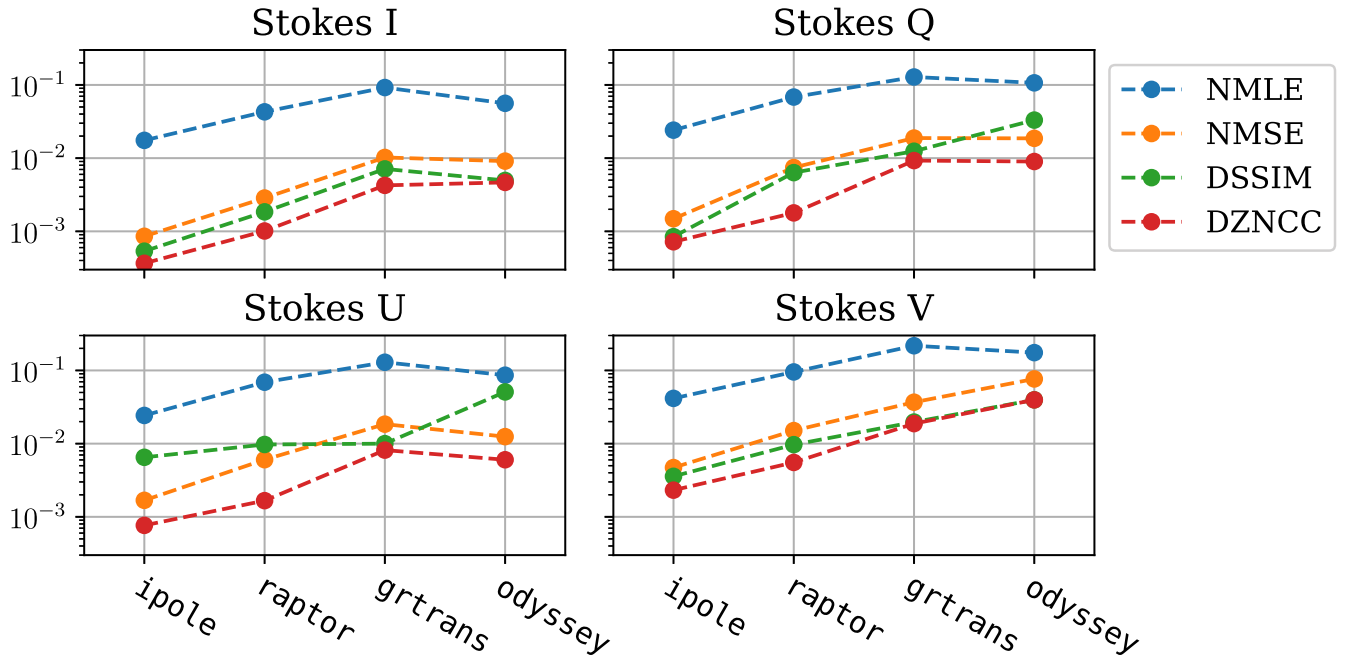
In addition, both the expressions from Shcherbakov (2008) and Dexter (2016) involve Bessel functions, which are tempting to approximate by assuming that emission is exclusive to the regime  $\Theta_e > 1$  in order to avoid unnecessary

computation. However, this approximation produces clearly incorrect limiting behavior for  $\rho_V$  at low temperature. Thus, the Faraday rotation is misapplied, producing too little rotation in the EVPA, or in some cases, rotation in the wrong direction. In the sample image this is a minor effect due to an overall small Faraday rotation, but it can severely affect SANE disks seen from larger observer angles.

### Appendix C Comparison between Image Difference Metrics

In addition to the NMSE, several other metrics could be used to gauge image dissimilarity between codes. Three additional metrics were evaluated in the context of this comparison: the normalized mean linear error (NMLE), structural dissimilarity (DSSIM), and inverse zero-normalized cross-correlation (DZNCC). These are defined as follows:

$$\text{NMLE}(A, B) = \frac{\sum_j |A_j - B_j|}{\sum_j |A_j|} \quad (\text{C1})$$



**Figure 10.** Each Stokes parameter of the `ipole-IL` image result for the high- $\mathcal{M}$  GRMHD snapshot test, evaluated against each other code using four different metrics (all normalized): the mean linear error (MLE), mean squared error (MSE), structural dissimilarity (DSSIM), and inverse zero-normalized cross-correlation (DZNCC).

$$\text{SSIM}(A, B) = \left( \frac{2\mu_A\mu_B}{\mu_A^2 + \mu_B^2} \right) \times \left( \frac{\frac{2}{N} \sum_j (A_j - \mu_A)(B_j - \mu_B)}{\sigma_A^2 + \sigma_B^2} \right) \quad (\text{C2})$$

$$\text{DSSIM}(A, B) = \frac{1}{|\text{SSIM}(A, B)|} - 1 \quad (\text{C3})$$

$$\text{ZNCC}(A, B) = \frac{1}{N} \sum_j \frac{1}{\sigma_A \sigma_B} (A_j - \mu_A)(B_j - \mu_B) \quad (\text{C4})$$

$$\text{DZNCC}(A, B) = \frac{1}{|\text{ZNCC}(A, B)|} - 1, \quad (\text{C5})$$

where  $\mu_X$  is the average pixel value of an image and  $\sigma_X$  is the standard deviation of the pixel values.

In this comparison, most images were very similar—for this limited case, the various similarity metrics were found to correlate strongly in ordering of similarity between all images, and usually even in relative magnitude, as shown in Figure 10. The absolute values of the constraint metrics mean little without the context of Section 5.2, so any particular metric could fill the role of a similarity gauge to compare to image variation from other sources.

#### Appendix D Comparison of Images over Full GRMHD Run

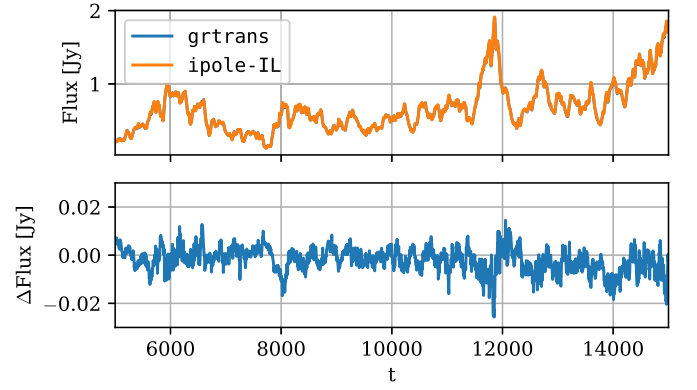
While the GRMHD snapshot file used for the test in Section 3.3 reflects the simulations and imaging parameters used in practice by the EHTC in studies of M87\*, there is always the chance that the snapshot itself is a particularly simple case, not reflective of average code differences in practice. Furthermore, in characterizing the impact of code differences on metric-based model comparisons, it would be useful to have an idea of what portion of code differences in metrics are due to systematic errors, versus stochastic products of limited accuracy parameters or sampling differences.

To measure the variation in results of this test over a typical variety of GRMHD states, two codes with substantially different algorithms, *ipole-IL* and *grtrans*, were compared across 2000 snapshots of a GRMHD simulation used in generating the EHT image libraries. This particular simulation represented a magnetically arrested disk (MAD) state about a BH of spin  $a_* = 0.9375$ , and the 2000 snapshots shown represent the entire quiescent portion of the simulation from  $5000 r_g/c$  to  $15,000 r_g/c$ . Details of the initial conditions, resolution, etc., are available in Wong et al. (2022).

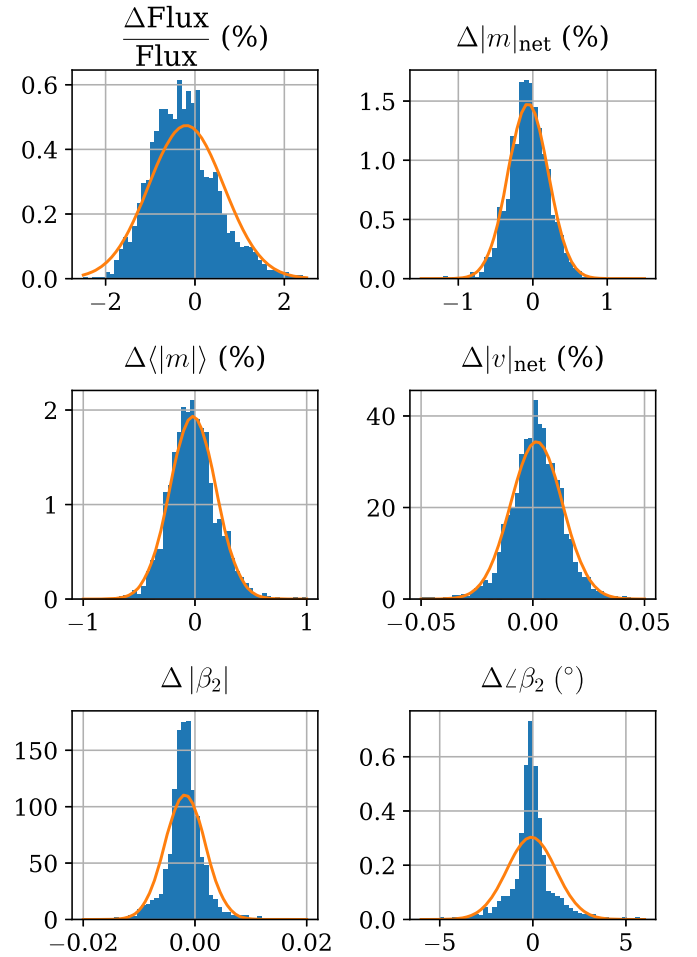
**Table 6**  
Mean Values and Standard Deviations of the Distributions in Figure 12

Variable	$\mu$	$\sigma$	$\mu/\sigma$
$\frac{\Delta \text{Flux}}{\text{Flux}} (\%)$	-0.197	0.841	0.234
$\Delta  m _{\text{net}} (\%)$	-0.0627	0.27	0.232
$\Delta \langle  m  \rangle (\%)$	-0.0182	0.206	0.0884
$\Delta  v _{\text{net}} (\%)$	0.00168	0.0116	0.145
$\Delta  \beta_2 $	-0.00178	0.00361	0.493
$\Delta \angle \beta_2$ (deg)	-0.0757	1.31	0.0576

**Note.** The last column lists the proportion  $\mu/\sigma$ , which can be taken as an estimate of the relevance of systematic versus stochastic errors in describing differences between these codes.



**Figure 11.** Comparison of total flux density computed by *ipole-IL* and *grtrans* over  $10,000 r_g/c$  of an MAD GRMHD simulation. The top panel shows the total (Stokes  $I$ ) flux density produced by each code, and the bottom panel shows the absolute difference in flux densities.



**Figure 12.** Histograms comparing image metrics between corresponding images over the entire window. Each histogram is computed with a total of 50 bins across the domain shown, with any values outside the range added to the final bins.

Figure 11 compares the total unpolarized flux computed by *ipole-IL* and *grtrans* over the entire window. Figure 12 provides histograms of the differences in all image-integrated values over the window, along with Gaussian functions following their means and standard deviations. Table 6 lists the mean (i.e., average difference) and standard deviation (i.e., span of

differences) between codes in each metric. The parameters  $\langle |m| \rangle$  and  $\angle \beta_2$  appear to be almost entirely stochastic. Flux,  $|m|_{\text{net}}$  and  $v_{\text{net}}$  are approximately a quarter systematic, and  $|\beta_2|$  is half systematic (though the error itself is minuscule).

### ORCID iDs

- Ben S. Prather <https://orcid.org/0000-0002-0393-7734>  
 Jason Dexter <https://orcid.org/0000-0003-3903-0373>  
 Monika Moscibrodzka <https://orcid.org/0000-0002-4661-6332>  
 Hung-Yi Pu <https://orcid.org/0000-0001-9270-8812>  
 Thomas Bronzwaer <https://orcid.org/0000-0003-1151-3971>  
 Jordy Davelaar <https://orcid.org/0000-0002-2685-2434>  
 Ziri Younsi <https://orcid.org/0000-0001-9283-1191>  
 Charles F. Gammie <https://orcid.org/0000-0001-7451-8935>  
 Roman Gold <https://orcid.org/0000-0003-2492-1966>  
 George N. Wong <https://orcid.org/0000-0001-6952-2147>  
 Kazunori Akiyama <https://orcid.org/0000-0002-9475-4254>  
 Antxon Alberdi <https://orcid.org/0000-0002-9371-1033>  
 Juan Carlos Algaba <https://orcid.org/0000-0001-6993-1696>  
 Richard Anantua <https://orcid.org/0000-0003-3457-7660>  
 Keiichi Asada <https://orcid.org/0000-0001-6988-8763>  
 Rebecca Azulay <https://orcid.org/0000-0002-2200-5393>  
 Uwe Bach <https://orcid.org/0000-0002-7722-8412>  
 Anne-Kathrin Baczko <https://orcid.org/0000-0003-3090-3975>  
 Mislav Baloković <https://orcid.org/0000-0003-0476-6647>  
 John Barrett <https://orcid.org/0000-0002-9290-0764>  
 Michi Bauböck <https://orcid.org/0000-0002-5518-2812>  
 Bradford A. Benson <https://orcid.org/0000-0002-5108-6823>  
 Lindy Blackburn <https://orcid.org/0000-0002-9030-642X>  
 Raymond Blundell <https://orcid.org/0000-0002-5929-5857>  
 Katherine L. Bouman <https://orcid.org/0000-0003-0077-4367>  
 Geoffrey C. Bower <https://orcid.org/0000-0003-4056-9982>  
 Hope Boyce <https://orcid.org/0000-0002-6530-5783>  
 Christiaan D. Brinkerink <https://orcid.org/0000-0002-2322-0749>  
 Roger Brissenden <https://orcid.org/0000-0002-2556-0894>  
 Silke Britzen <https://orcid.org/0000-0001-9240-6734>  
 Avery E. Broderick <https://orcid.org/0000-0002-3351-760X>  
 Dominique Brogiere <https://orcid.org/0000-0001-9151-6683>  
 Sandra Bustamante <https://orcid.org/0000-0001-6169-1894>  
 Do-Young Byun <https://orcid.org/0000-0003-1157-4109>  
 John E. Carlstrom <https://orcid.org/0000-0002-2044-7665>  
 Chiara Ceccobello <https://orcid.org/0000-0002-4767-9925>  
 Andrew Chael <https://orcid.org/0000-0003-2966-6220>  
 Chi-kwan Chan <https://orcid.org/0000-0001-6337-6126>  
 Dominic O. Chang <https://orcid.org/0000-0001-9939-5257>  
 Koushik Chatterjee <https://orcid.org/0000-0002-2825-3590>  
 Shami Chatterjee <https://orcid.org/0000-0002-2878-1502>  
 Ming-Tang Chen <https://orcid.org/0000-0001-6573-3318>  
 Yongjun Chen (陈永军) <https://orcid.org/0000-0001-5650-6770>  
 Xiaopeng Cheng <https://orcid.org/0000-0003-4407-9868>  
 Ilje Cho <https://orcid.org/0000-0001-6083-7521>  
 Pierre Christian <https://orcid.org/0000-0001-6820-9941>  
 Nicholas S. Conroy <https://orcid.org/0000-0003-2886-2377>  
 John E. Conway <https://orcid.org/0000-0003-2448-9181>  
 James M. Cordes <https://orcid.org/0000-0002-4049-1882>  
 Thomas M. Crawford <https://orcid.org/0000-0001-9000-5013>  
 Geoffrey B. Crew <https://orcid.org/0000-0002-2079-3189>  
 Alejandro Cruz-Orsorio <https://orcid.org/0000-0002-3945-6342>  
 Yuzhu Cui (崔玉竹) <https://orcid.org/0000-0001-6311-4345>  
 Mariafelicia De Laurentis <https://orcid.org/0000-0002-9945-682X>  
 Roger Deane <https://orcid.org/0000-0003-1027-5043>  
 Jessica Dempsey <https://orcid.org/0000-0003-1269-9667>  
 Gregory Desvignes <https://orcid.org/0000-0003-3922-4055>  
 Vedant Dhruv <https://orcid.org/0000-0001-6765-877X>  
 Sheperd S. Doeleman <https://orcid.org/0000-0002-9031-0904>  
 Sean Dougal <https://orcid.org/0000-0002-3769-1314>  
 Sergio A. Dzib <https://orcid.org/0000-0001-6010-6200>  
 Ralph P. Eatough <https://orcid.org/0000-0001-6196-4135>  
 Razieh Emami <https://orcid.org/0000-0002-2791-5011>  
 Heino Falcke <https://orcid.org/0000-0002-2526-6724>  
 Joseph Farah <https://orcid.org/0000-0003-4914-5625>  
 Vincent L. Fish <https://orcid.org/0000-0002-7128-9345>  
 Ed Fomalont <https://orcid.org/0000-0002-9036-2747>  
 H. Alyson Ford <https://orcid.org/0000-0002-9797-0972>  
 Raquel Fraga-Encinas <https://orcid.org/0000-0002-5222-1361>  
 Per Friberg <https://orcid.org/0000-0002-8010-8454>  
 Christian M. Fromm <https://orcid.org/0000-0002-1827-1656>  
 Antonio Fuentes <https://orcid.org/0000-0002-8773-4933>  
 Peter Galison <https://orcid.org/0000-0002-6429-3872>  
 Roberto García <https://orcid.org/0000-0002-6584-7443>  
 Olivier Gentaz <https://orcid.org/0000-0002-0115-4605>  
 Boris Georgiev <https://orcid.org/0000-0002-3586-6424>  
 Ciriaco Goddi <https://orcid.org/0000-0002-2542-7743>  
 Arturo I. Gómez-Ruiz <https://orcid.org/0000-0001-9395-1670>  
 José L. Gómez <https://orcid.org/0000-0003-4190-7613>  
 Minfeng Gu (顾敏峰) <https://orcid.org/0000-0002-4455-6946>  
 Mark Gurwell <https://orcid.org/0000-0003-0685-3621>  
 Kazuhiro Hada <https://orcid.org/0000-0001-6906-772X>  
 Daryl Haggard <https://orcid.org/0000-0001-6803-2138>  
 Michael H. Hecht <https://orcid.org/0000-0002-4114-4583>  
 Ronald Hesper <https://orcid.org/0000-0003-1918-6098>  
 Dirk Heumann <https://orcid.org/0000-0002-7671-0047>  
 Luis C. Ho (何子山) <https://orcid.org/0000-0001-6947-5846>  
 Paul Ho <https://orcid.org/0000-0002-3412-4306>  
 Mareki Honma <https://orcid.org/0000-0003-4058-9000>  
 Chih-Wei L. Huang <https://orcid.org/0000-0001-5641-3953>  
 Lei Huang (黄磊) <https://orcid.org/0000-0002-1923-227X>  
 Shiro Ikeda <https://orcid.org/0000-0002-2462-1448>  
 C. M. Violette Impellizzeri <https://orcid.org/0000-0002-3443-2472>  
 Makoto Inoue <https://orcid.org/0000-0001-5037-3989>  
 Sara Issaoun <https://orcid.org/0000-0002-5297-921X>  
 David J. James <https://orcid.org/0000-0001-5160-4486>  
 Buell T. Jannuzi <https://orcid.org/0000-0002-1578-6582>  
 Michael Janssen <https://orcid.org/0000-0001-8685-6544>

- Britton Jeter <https://orcid.org/0000-0003-2847-1712>  
Wu Jiang (江悟) <https://orcid.org/0000-0001-7369-3539>  
Alejandra Jiménez-Rosales <https://orcid.org/0000-0002-2662-3754>  
Michael D. Johnson <https://orcid.org/0000-0002-4120-3029>  
Svetlana Jorstad <https://orcid.org/0000-0001-6158-1708>  
Abhishek V. Joshi <https://orcid.org/0000-0002-2514-5965>  
Taehyun Jung <https://orcid.org/0000-0001-7003-8643>  
Mansour Karami <https://orcid.org/0000-0001-7387-9333>  
Ramesh Karuppusamy <https://orcid.org/0000-0002-5307-2919>  
Tomohisa Kawashima <https://orcid.org/0000-0001-8527-0496>  
Garrett K. Keating <https://orcid.org/0000-0002-3490-146X>  
Mark Kettenis <https://orcid.org/0000-0002-6156-5617>  
Dong-Jin Kim <https://orcid.org/0000-0002-7038-2118>  
Jae-Young Kim <https://orcid.org/0000-0001-8229-7183>  
Jongsoo Kim <https://orcid.org/0000-0002-1229-0426>  
Junhan Kim <https://orcid.org/0000-0002-4274-9373>  
Motoki Kino <https://orcid.org/0000-0002-2709-7338>  
Jun Yi Koay <https://orcid.org/0000-0002-7029-6658>  
Prashant Kocherlakota <https://orcid.org/0000-0001-7386-7439>  
Shoko Koyama <https://orcid.org/0000-0002-3723-3372>  
Carsten Kramer <https://orcid.org/0000-0002-4908-4925>  
Michael Kramer <https://orcid.org/0000-0002-4175-2271>  
Thomas P. Krichbaum <https://orcid.org/0000-0002-4892-9586>  
Cheng-Yu Kuo <https://orcid.org/0000-0001-6211-5581>  
Noemi La Bella <https://orcid.org/0000-0002-8116-9427>  
Tod R. Lauer <https://orcid.org/0000-0003-3234-7247>  
Daeyoung Lee <https://orcid.org/0000-0002-3350-5588>  
Sang-Sung Lee <https://orcid.org/0000-0002-6269-594X>  
Po Kin Leung <https://orcid.org/0000-0002-8802-8256>  
Aviad Levis <https://orcid.org/0000-0001-7307-632X>  
Zhiyuan Li (李志远) <https://orcid.org/0000-0003-0355-6437>  
Rocco Lico <https://orcid.org/0000-0001-7361-2460>  
Greg Lindahl <https://orcid.org/0000-0002-6100-4772>  
Michael Lindqvist <https://orcid.org/0000-0002-3669-0715>  
Mikhail Lisakov <https://orcid.org/0000-0001-6088-3819>  
Jun Liu (刘俊) <https://orcid.org/0000-0002-7615-7499>  
Kuo Liu <https://orcid.org/0000-0002-2953-7376>  
Elisabetta Liuzzo <https://orcid.org/0000-0003-0995-5201>  
Wen-Ping Lo <https://orcid.org/0000-0003-1869-2503>  
Andrei P. Lobanov <https://orcid.org/0000-0003-1622-1484>  
Laurent Loinard <https://orcid.org/0000-0002-5635-3345>  
Colin J. Lonsdale <https://orcid.org/0000-0003-4062-4654>  
Ru-Sen Lu (路如森) <https://orcid.org/0000-0002-7692-7967>  
Nicholas R. MacDonald <https://orcid.org/0000-0002-6684-8691>  
Jirong Mao (毛基荣) <https://orcid.org/0000-0002-7077-7195>  
Nicola Marchili <https://orcid.org/0000-0002-5523-7588>  
Sera Markoff <https://orcid.org/0000-0001-9564-0876>  
Daniel P. Marrone <https://orcid.org/0000-0002-2367-1080>  
Alan P. Marscher <https://orcid.org/0000-0001-7396-3332>  
Iván Martí-Vidal <https://orcid.org/0000-0003-3708-9611>  
Satoki Matsushita <https://orcid.org/0000-0002-2127-7880>  
Lynn D. Matthews <https://orcid.org/0000-0002-3728-8082>  
Lia Medeiros <https://orcid.org/0000-0003-2342-6728>  
Karl M. Menten <https://orcid.org/0000-0001-6459-0669>  
Daniel Michalik <https://orcid.org/0000-0002-7618-6556>  
Izumi Mizuno <https://orcid.org/0000-0002-7210-6264>  
Yosuke Mizuno <https://orcid.org/0000-0002-8131-6730>  
James M. Moran <https://orcid.org/0000-0002-3882-4414>  
Kotaro Moriyama <https://orcid.org/0000-0003-1364-3761>  
Cornelia Müller <https://orcid.org/0000-0002-2739-2994>  
Alejandro Mus <https://orcid.org/0000-0003-0329-6874>  
Gibwa Musoke <https://orcid.org/0000-0003-1984-189X>  
Ioannis Myserlis <https://orcid.org/0000-0003-3025-9497>  
Andrew Nadolski <https://orcid.org/0000-0001-9479-9957>  
Hiroshi Nagai <https://orcid.org/0000-0003-0292-3645>  
Neil M. Nagar <https://orcid.org/0000-0001-6920-662X>  
Masanori Nakamura <https://orcid.org/0000-0001-6081-2420>  
Ramesh Narayan <https://orcid.org/0000-0002-1919-2730>  
Gopal Narayanan <https://orcid.org/0000-0002-4723-6569>  
Iniyan Natarajan <https://orcid.org/0000-0001-8242-4373>  
Joey Neilsen <https://orcid.org/0000-0002-8247-786X>  
Roberto Neri <https://orcid.org/0000-0002-7176-4046>  
Chunchong Ni <https://orcid.org/0000-0003-1361-5699>  
Aristeidis Noutsos <https://orcid.org/0000-0002-4151-3860>  
Michael A. Nowak <https://orcid.org/0000-0001-6923-1315>  
Jungwhan Oh <https://orcid.org/0000-0002-4991-9638>  
Hiroki Okino <https://orcid.org/0000-0003-3779-2016>  
Héctor Olivares <https://orcid.org/0000-0001-6833-7580>  
Gisela N. Ortiz-León <https://orcid.org/0000-0002-2863-676X>  
Tomoaki Oyama <https://orcid.org/0000-0003-4046-2923>  
Feryal Özel <https://orcid.org/0000-0003-4413-1523>  
Daniel C. M. Palumbo <https://orcid.org/0000-0002-7179-3816>  
Georgios Filippas Paraschos <https://orcid.org/0000-0001-6757-3098>  
Jongho Park <https://orcid.org/0000-0001-6558-9053>  
Harriet Parsons <https://orcid.org/0000-0002-6327-3423>  
Nimesh Patel <https://orcid.org/0000-0002-6021-9421>  
Ue-Li Pen <https://orcid.org/0000-0003-2155-9578>  
Dominic W. Pesce <https://orcid.org/0000-0002-5278-9221>  
Richard Plambeck <https://orcid.org/0000-0001-6765-9609>  
Oliver Porth <https://orcid.org/0000-0002-4584-2557>  
Felix M. Pötzl <https://orcid.org/0000-0002-6579-8311>  
Jorge A. Preciado-López <https://orcid.org/0000-0002-4146-0113>  
Dimitrios Psaltis <https://orcid.org/0000-0003-1035-3240>  
Venkatesh Ramakrishnan <https://orcid.org/0000-0002-9248-086X>  
Ramprasad Rao <https://orcid.org/0000-0002-1407-7944>  
Mark G. Rawlings <https://orcid.org/0000-0002-6529-202X>  
Alexander W. Raymond <https://orcid.org/0000-0002-5779-4767>  
Luciano Rezzolla <https://orcid.org/0000-0002-1330-7103>  
Angelo Ricarte <https://orcid.org/0000-0001-5287-0452>  
Bart Ripperda <https://orcid.org/0000-0002-7301-3908>  
Freek Roelofs <https://orcid.org/0000-0001-5461-3687>  
Alan Rogers <https://orcid.org/0000-0003-1941-7458>  
Eduardo Ros <https://orcid.org/0000-0001-9503-4892>  
Cristina Romero-Cañizales <https://orcid.org/0000-0001-6301-9073>  
Arash Roshanineshat <https://orcid.org/0000-0002-8280-9238>  
Alan L. Roy <https://orcid.org/0000-0002-1931-0135>

Ignacio Ruiz  <https://orcid.org/0000-0002-0965-5463>  
 Chet Ruzsarczyk  <https://orcid.org/0000-0001-7278-9707>  
 Kazi L. J. Rygl  <https://orcid.org/0000-0003-4146-9043>  
 Salvador Sánchez  <https://orcid.org/0000-0002-8042-5951>  
 David Sánchez-Argüelles  <https://orcid.org/0000-0002-7344-9920>  
 Miguel Sánchez-Portal  <https://orcid.org/0000-0003-0981-9664>  
 Mahito Sasada  <https://orcid.org/0000-0001-5946-9960>  
 Kaushik Satapathy  <https://orcid.org/0000-0003-0433-3585>  
 Tuomas Savolainen  <https://orcid.org/0000-0001-6214-1085>  
 Jonathan Schonfeld  <https://orcid.org/0000-0002-8909-2401>  
 Karl-Friedrich Schuster  <https://orcid.org/0000-0003-2890-9454>  
 Lijing Shao  <https://orcid.org/0000-0002-1334-8853>  
 Zhiqiang Shen (沈志强)  <https://orcid.org/0000-0003-3540-8746>  
 Des Small  <https://orcid.org/0000-0003-3723-5404>  
 Bong Won Sohn  <https://orcid.org/0000-0002-4148-8378>  
 Jason SooHoo  <https://orcid.org/0000-0003-1938-0720>  
 Kamal Souccar  <https://orcid.org/0000-0001-7915-5272>  
 He Sun (孙赫)  <https://orcid.org/0000-0003-1526-6787>  
 Fumie Tazaki  <https://orcid.org/0000-0003-0236-0600>  
 Alexandra J. Tetarenko  <https://orcid.org/0000-0003-3906-4354>  
 Paul Tiede  <https://orcid.org/0000-0003-3826-5648>  
 Remo P. J. Tilanus  <https://orcid.org/0000-0002-6514-553X>  
 Michael Titus  <https://orcid.org/0000-0001-9001-3275>  
 Pablo Torne  <https://orcid.org/0000-0001-8700-6058>  
 Efthalia Traianou  <https://orcid.org/0000-0002-1209-6500>  
 Sascha Trippe  <https://orcid.org/0000-0003-0465-1559>  
 Matthew Turk  <https://orcid.org/0000-0002-5294-0198>  
 Ilse van Bommel  <https://orcid.org/0000-0001-5473-2950>  
 Huib Jan van Langevelde  <https://orcid.org/0000-0002-0230-5946>  
 Daniel R. van Rossum  <https://orcid.org/0000-0001-7772-6131>  
 Jesse Vos  <https://orcid.org/0000-0003-3349-7394>  
 Jan Wagner  <https://orcid.org/0000-0003-1105-6109>  
 Derek Ward-Thompson  <https://orcid.org/0000-0003-1140-2761>  
 John Wardle  <https://orcid.org/0000-0002-8960-2942>  
 Jonathan Weintraub  <https://orcid.org/0000-0002-4603-5204>  
 Norbert Wex  <https://orcid.org/0000-0003-4058-2837>  
 Robert Wharton  <https://orcid.org/0000-0002-7416-5209>  
 Maciek Wielgus  <https://orcid.org/0000-0002-8635-4242>  
 Kaj Wiik  <https://orcid.org/0000-0002-0862-3398>  
 Gunther Witzel  <https://orcid.org/0000-0003-2618-797X>  
 Michael F. Wondrak  <https://orcid.org/0000-0002-6894-1072>  
 Qingwen Wu (吴庆文)  <https://orcid.org/0000-0003-4773-4987>  
 Paul Yamaguchi  <https://orcid.org/0000-0002-6017-8199>  
 Aristomenis Yfantis  <https://orcid.org/0000-0002-3244-7072>  
 Doosoo Yoon  <https://orcid.org/0000-0001-8694-8166>  
 André Young  <https://orcid.org/0000-0003-0000-2682>  
 Ken Young  <https://orcid.org/0000-0002-3666-4920>  
 Wei Yu (于威)  <https://orcid.org/0000-0002-5168-6052>  
 Feng Yuan (袁峰)  <https://orcid.org/0000-0003-3564-6437>

Ye-Fei Yuan (袁业飞)  <https://orcid.org/0000-0002-7330-4756>  
 J. Anton Zensus  <https://orcid.org/0000-0001-7470-3321>  
 Shuo Zhang  <https://orcid.org/0000-0002-2967-790X>  
 Guang-Yao Zhao  <https://orcid.org/0000-0002-4417-1659>  
 Shan-Shan Zhao (赵杉杉)  <https://orcid.org/0000-0002-9774-3606>

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