A Bayesian ARMA-GARCH EWMA monitoring scheme for long run: A case study on monitoring the USD/ZAR exchange rate

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Abstract

Statistical process monitoring (SPM) offers an important toolkit used to monitor the stability of a process to improve the quality of outputs and / or services. More often, the design of control charts requires the estimation of the probability density function that involves selecting a common distribution that facilitates the estimation of the process parameters. The Bayesian approach is one of the most efficient techniques used in such instances. It incorporates informative and non-informative priors, i.e., uses information on past data and charting structures, to estimate parameters more efficiently than classical approaches. Bayesian approaches reduce the total expected cost over a finite horizon or the long-run expected average cost. This paper introduces a new Bayesian exponentially weighted moving average (EWMA) monitoring scheme for long runs based on an ARMA-GARCH model. The properties of the new monitoring scheme are investigated in terms of the run-length distribution. A case study on monitoring the USD to ZAR exchange rate is provided using the proposed Bayesian ARMA-GARCH EWMA monitoring scheme.

Keywords: ARMA; ARMA-GARCH; Bayesian approach; Control chart; EWMA; GARCH; Financial data; Prior distribution; Posterior distribution; Statistical process monitoring

1. Introduction

In production and manufacturing industries, it is important to produce goods and products of high quality on a regular basis. Statistical process monitoring (SPM) provides essential tools that help to control the stability of a process by spotting any abnormality that could ruin the outputs. One of the most popular tools used in SPM is the control chart (or monitoring scheme); see for instance, Montgomery (2020). Any process possesses a natural variability known as a "stable system of chance causes" which are an inherent part of the process itself. However, another type of variability can arise due to "special causes" resulting in the process operating in an out-of-control (OOC) state. These special causes of variation are also called "assignable causes" and they are often responsible for the deterioration of the quality of outputs or services; see for example, Montgomery (2020). The sooner a monitoring scheme detects assignable causes of variation, the more efficient it is and the better the outputs.

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In SPM, there are two types of monitoring schemes, namely parametric and nonparametric monitoring schemes. Parametric monitoring schemes are typically based on the assumption of normality or a specific probability distribution (such as the binomial, Poisson, exponential, etc.); see for example, Sheu and Lin (2003) and Gadde et al. (2019). However, nonparametric monitoring schemes do not require any particular assumption of the underlying process distribution. They are mostly used when the nature of the underlying process distribution is unknow or when there is not enough information on the shape of the distribution; see for example, Mabude et al. (2020) and Celano and Chakraborti (2021). Very often, the parameters of the underlying process distribution are unknown. In this case, they must be estimated before the actual process monitoring starts. Thus, in practice, monitoring schemes are usually implemented in two stages known as phase I and phase II. Phase I (also known as the retrospective phase), uses historical data to estimate the parameters and control limits when a process is assumed to be in-control. In phase II, the parameters and control limits found in phase I are then used for monitoring purposes. For more details on monitoring schemes with estimated process parameters (i.e., Case U for unknown); see for example, Maravelakis et al. (2002), Owlia et al. (2017) and Abbas et al. (2019a, b)). In cases where historical data and information on the underlying process distribution and the parameters or target values are available, monitoring schemes are designed under the parameter known case (i.e., Case K for known); see for example, Malela-Majika (2022a, b). Otherwise, parametric and/or nonparametric monitoring schemes under the parameter unknown case are considered. In this case, process parameters are estimated using statistical techniques (such as maximum likelihood estimation (MLE), Bayesian estimation, bootstrapping, etc.); see for example, Ibazizen and Fellag (2003), Owlia et al. (2017) and Imran et al. (2022). For cost effective monitoring schemes, the Bayesian approach is used which is based on subjective probability that could include uncertainty into the model (see Abbasi et al. (2018) and Aunali and Venkatesan (2019)). The use of Bayesian methods in SPM is mostly in the areas that aim to estimate the monitoring schemes' parameters more efficiently, considering both the cost of sampling and chart performance. It is well-known that the performance of a monitoring scheme is considerably affected by the parameter estimation and the dependency between and within samples (Zhou and Qui (2022) and Imran et al. (2022)). Therefore, the choice of appropriate methods that handles both autocorrelation and estimation issues is very important.

In practice, data are often characterised by a serial dependency (or autocorrelation) between successive observations. Such data are efficiently monitored using time series monitoring schemes such as the autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) schemes (see, Alwan (1992), Jiang et al. (2000), and Ibazizen and Fellag (2003)). The choice of the time series model to use also depends on the type of data. Al-Osh and Alzaid (1987) introduced a model for stationary sequences of integer valued random variables with a lag-one dependence referred to as the integer-value autoregressive of order one (denoted as INAR(1)) model. They reported that the correlation structure and the distributional properties of the INAR(1) model are similar to those of continuous valued AR(1) model. For more flexibility, Biswas and Song (2009) presented a unified framework of stationary ARMA processes for discrete-valued time series based on a stochastic operator. Weiß and Testik (2012) proposed a cumulative sum (CUSUM) control chart for integer-valued autoregressive conditional heteroscedasticity (INARCH) model to detect abrupt changes in count data time series. Vanli et al. (2019) proposed a CUSUM for Poisson integer-valued generalised autoregressive conditional heteroscedasticity (INGARCH) model. They showed that the CUSUM chart for INGARCH model based on a likelihood ratio can significantly provide improved performances in applications where serial correlation or seasonality is prevalent. For more details on time series for count data, readers are referred to McKenzie (1988), Woodall (1997), Ferland et al. (2006) and Xu et al. (2023). For continuous time series, Kim (2015) used an ARMA model to estimate asymmetric cost-sensitive loss functions for financial data. Tan et al. (2022) associated an autoregressive integrated moving average (ARIMA) model with a control chart to monitor stock price and trading volume at the same time. Using real-life stock exchange data, they demonstrated the effectiveness of the ARIMA monitoring scheme as compared to the performances of the volume-weighted moving average (VWMA) and relative strength index (RSI) schemes. Jones et al (2023) proposed the CUSUM and exponentially weighted moving average (EWMA) monitoring schemes using a Bayesian approach, where posterior predictive distributions are found using the squared error, precautionary and linex loss functions criteria. For more details on time series models for continuous data in SPM, readers are referred to Alwan and Roberts (1988), Stone (1995) and the review article by Knoth and Schmid (2004).

Data on exchange rates and other financial transactions often exhibit a serial dependency and a considerable unstable clustering where periods of high volatility are quickly followed by periods of low volatility and vice versa. Very often, such processes do not have a constant mean. In this instance, time series schemes such as the AR, MA, and ARMA monitoring schemes are not recommended. Instead, the literature recommends the use of the Generalised Auto-Regressive Conditional Heteroskedasticity (GARCH) monitoring scheme to model the data in order to effectively monitor the stability of the process.

This paper introduces a new long run Bayesian ARMA-GARCH EWMA monitoring scheme for monitoring, as an example, the United State Dollar (USD) to the South African Rand (ZAR) exchange rate. The remainder of this paper is organised as follows: Section 2 presents the USD/ZAR exchange rate data and explains how to get stationary time series data. Section 3 provides mathematical background of the existing times series monitoring schemes. In addition, the design and background of the existing Bayesian time series monitoring schemes is also given in Section 3. Section 4 develops a new Bayesian ARMA-GARCH EWMA (BAG EWMA) monitoring scheme. The performance of the proposed monitoring scheme is investigated in Section 5. The implementation and application of the new scheme is provided in Section 6. Section 7 presents the concluding remarks, directions for future research works and recommendations.

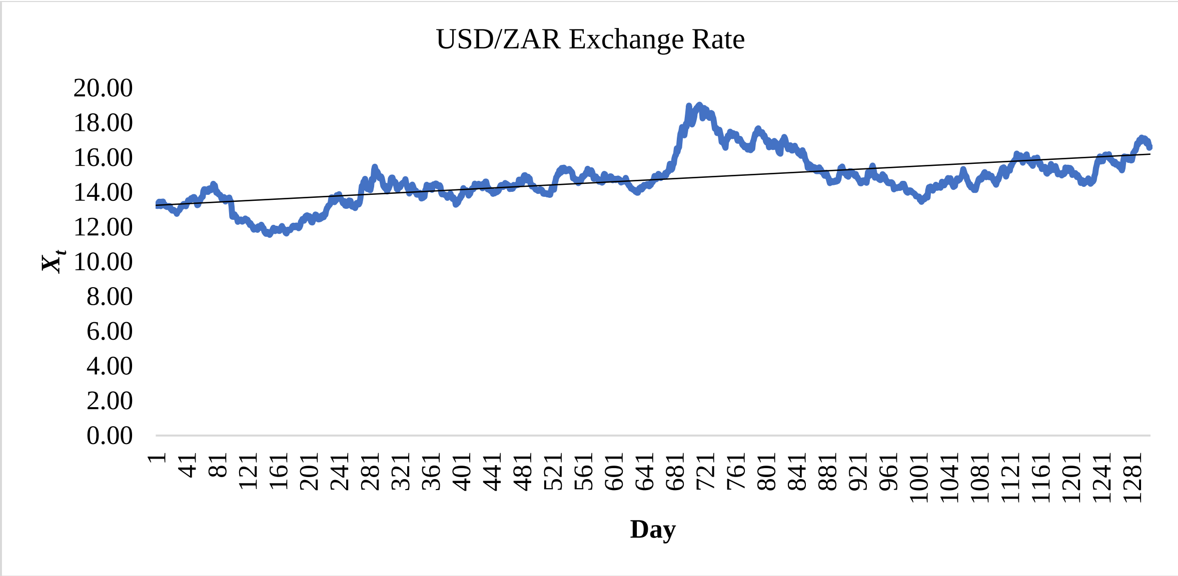
2. Monitoring the USD/ZAR exchange rate

The purpose of this paper is to design a monitoring scheme that is able and efficient in monitoring the USD/ZAR exchange rate. Let X_t^* ($t = 1,2.3,\ldots$) represents the USD/ZAR exchange rate at time t. Before the monitoring phase (i.e. phase II) can begin, it is necessary to visualise the data to have an idea of the long-term trend (see the solid line on Figure 1 (a)). The USD/ZAR exchange rate is displayed in Figure 1 (a) from the 31st July 2017 to the 29th July 2022; the data was sourced from the USA Economic Research Division[3]. This figure shows that the data are non-stationary and have an upwards trend. Time series analytical techniques assume that each observation is independent of one another. Stationarity of the data is one of the most important criteria to confirm this assumption. In this case study, the Augmented Dickey-Fuller (ADF) test is used to confirm the non-stationarity of the original data (i.e. the exchange rate data); see Kwiatkowski et al. (1992), Lopez (1997) and Paparoditis and Politis (2018) for more details and on the ADF test. Because it is found that the $p - value$ is larger than 0.05 it confirms that the original data are in fact non-stationary. In other words, we cannot reject the null hypothesis; and therefore, conclude that the time series has a unit root. Thus, before the monitoring process begins, it is important to transform the underlying process data into a stationary series. In this case, the data are transformed by computing the percentage change in the exchange rate, denoted as X_t, which is computed as follows:

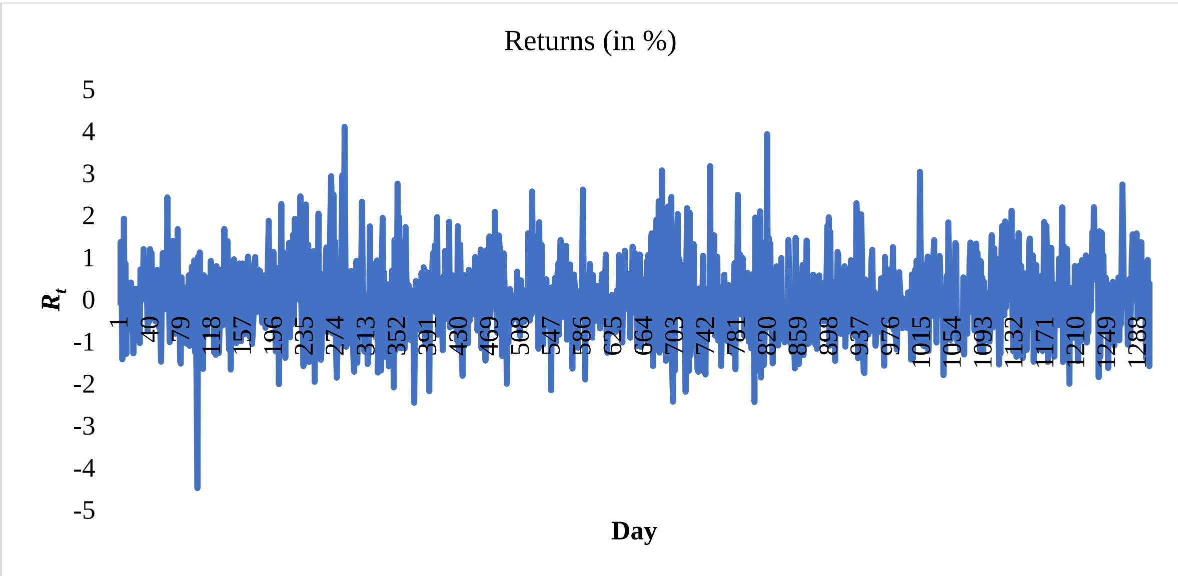
$$X_t = \left(\frac{X_t^* - X_{t-1}^*}{X_{t-1}^*} \right) \times 100. \tag{1}$$

Figure 1 (b) displays the percentage change in the USD/ZAR exchange rate from the 1st August 2017 to the 29th July 2022. It can be clearly noticed that the process is now stationary, i.e., the mean and the variance do not change over time. Using the ADF test, it is found that the $p - value$ is now less than 0.05 which means that the transformed data (the percentage changes) are stationary.

[3] Federal Reserve Economic Data (FRED); Link: https://fred.stlouisfed.org



(a) Trend of the exchange rate USD/ZAR



(b) Daily return of USD/ZAR exchange rate

Figure 1. USD/ZAR exchange rate data from July 31st 2017 to July 29th 2022

Since the transformed data are stationary the process can now be monitored using an efficient monitoring scheme for which the control limits and parameter estimates of the time series model are found using the phase I analysis. For the in phase I, we used the data from the 30th July 2014 to the 30th July 2017 (i.e. 1097 days).

3. Statistical background on financial time series models

3.1 ARMA model

Assume that $\{X_t, t = 1,2,3, \ldots\}$ is a sequence of autocorrelated normally distributed observations with mean 0 and variance σ_X^2. Then, the first order ARMA (i.e. ARMA (1,1)) model for stationary processes is defined by

$$Y_t = \theta_0(X_t - \vartheta X_{t-1}) + \phi Y_{t-1} \tag{2}$$

with $\theta_0 = 1 + \theta - \phi$, where ϕ and θ are the coefficients of the ARMA (1,1) process, $\vartheta = \theta/\theta_0$ and the stationarity and invertibility constraints of the process require $|\phi| < 1$ and $|\vartheta| < 1$, respectively.

Let us assume that the underlying process, Y_t, is characterised by the autocorrelation structure ρ_t with

$$\rho_\tau = \frac{\gamma_\tau}{\gamma_0}, \tag{3}$$

where

$$\gamma_\tau = cov(Y_t, Y_{t+\tau})$$

$$= cov\left(\theta_0 X_t + \alpha \sum_{k=1}^{t-1} \phi^{k-1} X_{t-k}, \theta_0 X_{t+\tau} + \alpha \sum_{k=1}^{\tau+t-1} \phi^{k-1} X_{t+\tau-k}\right)$$

$$= \left[\theta_0^2 \rho_\tau + \theta_0 \alpha \left(\sum_{k=1}^{t-1} \phi^{k-1} \rho_{t+k} + \sum_{k=1}^{t+\tau-1} \phi^{k-1} \rho_{t-k}\right) + \alpha^2 \sum_{i=1}^{t-1} \sum_{j=1}^{t+\tau-1} \phi^{i+j-2} \rho_{t-j+i}\right] \sigma_X^2,$$

with $\alpha = \phi\theta_0 - \theta$ and θ_0 is defined earlier.

From Equation (3), we have that $\rho_k = \rho_{-k}$ and therefore without loss of generality ρ_{t+k} and ρ_{t-k} can simply be denoted as ρ_k. Then, when $\tau = 0$, Jiang et al. (2000) showed that the variance of Y_t is:

$$Var(Y_t) = \sigma_{Y_t}^2 = \left(\theta_0^2 + 2\theta_0 \alpha \sum_{k=1}^{t-1} \phi^{k-1} \rho_k + \alpha^2 \sum_{i=1}^{t-1} \sum_{j=1}^{t-1} \phi^{i+j-2} \rho_{j-i}\right) \sigma_X^2. \tag{4}$$

When the scheme has been running for a long time, i.e. $t \to \infty$, the variance of Y_t simplifies to

$$Var(Y_\infty) = \sigma_{Y_\infty}^2 = \left(\theta_0^2 + \frac{\alpha^2}{1-\phi^2} + 2\left(\theta_0 \alpha + \frac{\phi \alpha^2}{1-\phi^2}\right) \sum_{k=1}^{\infty} \phi^{k-1} \rho_k\right) \sigma_X^2, \tag{5}$$

where $\sum_{k=1}^{\infty} \phi^k \rho_k$ converges towards some value ψ.

3.2 GARCH model

A stochastic process $\{Y_t\}$ is called a first order GARCH (i.e. GARCH (1,1)) process if it is defined by

$$Y_t = \mu_0 + \varepsilon_t h_t, \quad h_t > 0, t \geq 1$$

and (6)

$$h_t^2 = \omega + \beta (Y_{t-1} - \mu_0)^2 + \alpha h_{t-1}^2,$$

where h_t^2 is the conditional variance, α and β are the coefficients of the GARCH (1,1) process such that $\omega > 0, \alpha \geq 0, \beta \geq 0$, the random variables ε_t are independent and identically distributed (i.i.d.) normal variables with mean 0 and variance 1 (i.e. standard normal innovation). The stationarity constraint requires that $\alpha + \beta < 1$. Then, it follows that $E(Y_t) = \mu_0$, $cov(Y_t, Y_s) = 0 \; \forall \; t \neq s$ with $s \geq 1$. Hence,

$$\sigma^2 = Var(Y_t) = E(\varepsilon_t^2 h_t^2) = \frac{\omega}{1 - \alpha - \beta}. \tag{7}$$

3.3 First order ARMA-GARCH model

A linear regression model with a first order ARMA-GARCH (ARMA (1,1)-GARCH (1,1)) error can be defined as follows:

$$X_t = c + \alpha(X_{t-1} - c) - \beta \varepsilon_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sqrt{h_t} \eta_t$$

and (8)

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},$$

where c is the mean level, h_t is defined in Equation (6), η_t is the innovation and α and β are the coefficients of the GARCH(1,1) process. In this paper, the innovation terms independently and identically follow a standard normal distribution.

Let $\boldsymbol{\varphi} = (\phi, \theta, \alpha, \beta)$ is the vector of parameters where ϕ and θ are the coefficients of the ARMA (1,1) process defined in Equation (2) and α and β are defined in Equations (6) and (8). The parameters of the above process can be estimated using the maximum likelihood estimation (MLE) technique. Thus, the likelihood function can be written as

$$L(\boldsymbol{\varphi}) = L(\boldsymbol{\varphi}, \varepsilon_1, \varepsilon_2, \ldots \varepsilon_m) = \prod_{t=1}^{m} \frac{1}{\sqrt{2\pi h_t}} \, exp\left(\frac{\varepsilon_t^2}{2h_t}\right), \tag{9}$$

where

$$\varepsilon_t = \beta^{-1}(L)\alpha(L)(X_t - c)$$

is the residual of the ARMA part in which

$$\beta(L) = (1 + \beta L), \quad \alpha(L) = (1 - \alpha L)$$

and L is the lag operator.

Using the log likelihood function, Equation (9) becomes:

$$logL(\boldsymbol{\varphi}) = -\frac{m}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{m}\left(\log(h_t) + \frac{\varepsilon_t^2}{h_t}\right). \tag{10}$$

Maximising Equation (10) is equivalent to minimising the following expression:

$$\widehat{\boldsymbol{\varphi}} = argmin\, I(\boldsymbol{\varphi}), \tag{11}$$

where $I(\boldsymbol{\varphi}) = m^{-1}\sum_{t=1}^{m}\psi_t$ and $\psi_t = \varepsilon_t^2/h_t + \log(h_t)$.

4. Design of the proposed Bayesian ARMA-GARCH EWMA monitoring scheme

The Bayesian design of a monitoring scheme is based on the properties of the posterior density distribution defined by

$$p(\boldsymbol{\varphi}|X) = \frac{L(Y|\boldsymbol{X},\boldsymbol{\varphi})p(\boldsymbol{\varphi})}{\int L(Y|\boldsymbol{X},\boldsymbol{\varphi})p(\boldsymbol{\varphi})d\boldsymbol{\varphi}}\ , \tag{12}$$

where $\boldsymbol{\varphi}$ is the set of parameters of the first order Bayesian ARMA-GARCH (hereafter, BAG) model as defined above, $L(Y|\boldsymbol{X},\boldsymbol{\varphi})$ is the likelihood function, and $p(\boldsymbol{\varphi})$ is the prior density. In this paper, we use the following proper prior for simplicity:

$$p(\boldsymbol{\varphi}) = N\left(\mu_{\varepsilon_0}, \sigma_{\varepsilon_0}^2\right) \times N(\mu_v, \sigma_v^2) \times N\left(\mu_\phi, \sigma_\phi^2\right)I_\phi(C_1) \times \ldots \times N\left(\mu_\beta, \sigma_\beta^2\right)I_\beta(C_2), \tag{13}$$

where ε_0 is the pre-sample error, the prior parameters μ_ϕ and μ_β were chosen to be equal to zero (i.e. $\mu_\phi = \mu_\beta = 0$), $\sigma_\phi^2 = \sigma_\beta^2 = 1$, and $I_{\boldsymbol{\varphi}}(C_j)$ (j = 1, 2, …, 4) is the indicator function which is equal to one if the constraint C_j holds; otherwise zero.

In this paper, the constraints on the parameters in a BAG model are defined as follows (see also Jiang et al. (2000)):

(i) C_1: all the roots of $1 - \alpha(L) = 0$ are outside the unit circle,

(ii) C_2: all the roots of $1 - \beta(L) = 0$ are outside the unit circle,

(iii) C_3: $\alpha \geq 0$,

(iv) C_4: $\beta \geq 0$, and

(v) C_4: $\alpha + \beta < 1$,

where L is the lag operator.

The expected value and variance of a function of parameters are given by

$$E[f(\boldsymbol{\varphi})] = \int f(\boldsymbol{\varphi})\, p(\boldsymbol{\varphi}|\boldsymbol{X})\, d\boldsymbol{\varphi} \tag{14}$$

and

$$Var[f(\boldsymbol{\varphi})] = E[f^2(\boldsymbol{\varphi})] - \left[E\left(f(\boldsymbol{\varphi})\right)\right]^2, \tag{15}$$

where the integral symbol actually denotes a quadruple integral for each of the parameters ϕ, θ, α and β of the ARMA-GARCH model, and $f(\boldsymbol{\varphi})$ depends on the type of inference under consideration (i.e. probabilistic or non-probabilistic). In the ARMA-GARCH model it is analytically difficult to evaluate the quadruple integral defined in Equation (14). Therefore, numerical integration methods such as Monte Carlo integration have to be used to solve this integral.

Then, Equation (14) can be approximated by Monte Carlo simulation using

$$E[f(\boldsymbol{\varphi})] = \frac{1}{m}\sum_{i=1}^{m} f(\boldsymbol{\varphi}^{(i)}), \tag{16}$$

where $\boldsymbol{\varphi}^{(1)}$, $\boldsymbol{\varphi}^{(2)}$, …, $\boldsymbol{\varphi}^{(m)}$ are m samples of the parameter vector $\boldsymbol{\varphi}$ generated from the posterior distribution. This is done using the Metropolis-Hasting (MH) algorithm which is a Markov chain sampling method. For more details on the MH algorithm, readers are referred to Brooks (1998) and Luengo et al. (2020).

4.1 Metropolis-Hasting algorithm

We suggest to use the following MH algorithm to estimate the parameters of the ARMA-GARCH model:

Step 1 Select the initial $\boldsymbol{\varphi}$, denoted as $\boldsymbol{\varphi}^{(0)}$,

Step 2 For $i = 1, \ldots, m$,

(i) Draw candidate $\boldsymbol{\varphi}^{(i)}$, denoted as $\boldsymbol{\varphi}^*$, $\varphi^* \sim g\left(\boldsymbol{\varphi}^* | \boldsymbol{\varphi}^{(i-1)}\right)$ where $p(\boldsymbol{\varphi}^{(i)}) \propto g(\boldsymbol{\varphi}^{(i)})$.

(ii) Determine α:

$$\alpha = \frac{g(\boldsymbol{\varphi}^*)/g\left(\boldsymbol{\varphi}^* | \boldsymbol{\varphi}^{(i-1)}\right)}{g(\boldsymbol{\varphi}^{(i-1)})/g(\boldsymbol{\varphi}^{(i-1)} | \varphi^*)} = \frac{g(\boldsymbol{\varphi}^*)g\left(\boldsymbol{\varphi}^{(i-1)} | \boldsymbol{\varphi}^*\right)}{g(\boldsymbol{\varphi}^{(i-1)})g(\boldsymbol{\varphi}^* | \boldsymbol{\varphi}^{(i-1)})}$$

(iii) If $\alpha \geq 1$ accept $\boldsymbol{\varphi}^*$ and set $\boldsymbol{\varphi}^{(i)} = \boldsymbol{\varphi}^*$.

However, if $0 < \alpha < 1$, then accept $\boldsymbol{\varphi}^*$ and set $\boldsymbol{\varphi}^{(i)} = \boldsymbol{\varphi}^*$ with probability α.

Otherwise, reject $\boldsymbol{\varphi}^*$ and set $\boldsymbol{\varphi}^{(i)} = \boldsymbol{\varphi}^{(i-1)}$ with probability $1 - \alpha$.

Step 3 Repeat Steps 1 and 2 a certain number of times (e.g., 10000 iterations).

Step 4 Compute the average for elements of $\boldsymbol{\varphi}^{(i)}$ denoted as $\hat{\boldsymbol{\varphi}}$.

Step 5 Record the result.

To investigate the performance of the proposed BAG EWMA scheme, we first need to estimate the ARMA-GARCH model of the USD/ZAR exchange rate using the expression given in Equation (8). The Bayesian estimation of the ARMA-GARCH model is obtained by using the Markov chain Monte Carlo (MCMC) method explained above. The number of iterations of the Markov chain sampling is 10000 and the size of the Monte Carlo samples is 1000 to guarantee the convergence of the parameters to their true values. A kernel smoothing method with a Gaussian kernel is used to estimate the strict stationarity and ergodicity of the GARCH model. To estimate the posterior probabilities, we generate $X \sim N(0{,}1)$.

4.2 Phase I study

In this section, we explain how the parameters of the models were estimated using the in-control phase I sample, and also how the size of this sample was chosen. Note that the estimation of the parameters of the time series model were done in phase I using data from the 30th July 2014 to the 30 July 2017 (i.e. 1097 days). The phase I data were collected on a long-term period in order to see a trend in the time series data. We used the ***rugarch*** and ***brms*** packages in R and every time that there was an OOC signal, the sample was discarded and the search for new parameters was initiated. The parameters found when the process was declared in-control were recorded and used to compute the control limits. Note that the prior parameters μ_ϕ and μ_β were chosen to be equal to zero. For the simulation, the initial parameters were also set to zero. Table 1 presents the results found using the MH algorithm for two different cases by imposing some constraints on the parameters in the ARMA-GARCH model. The first case (i.e. Case 1) forces the approximation to be near the unit root and the second case (i.e. Case 2) imposes the variance to be strictly stationary and ergodic. Using Geweke' s criterion, we noticed that all the *t* statistics lie within [-1.96, 1.96], which indicates that the estimates efficiently converge to their true values.

Table 1. Parameter estimates when $m = 1000$ with 10000 replications

Cases	$\hat{\upsilon}$	$\hat{\omega}$	$\hat{\boldsymbol{\varphi}}$			
			$\hat{\phi}$	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\beta}$
Case 1	1.045	2.857	0.017	0.557	0.561	0.110
Case 2	0.612	0.413	0.022	0.583	1.450	0.308

4.3 Bayesian ARMA-GARCH EWMA monitoring scheme

Thus, once the parameter estimates $\hat{\boldsymbol{\varphi}}$, $\hat{\varepsilon}_t$ and $\hat{\sigma}_t^2$ are found, then the BAG statistic is given by

$$Z_t = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_t} \tag{17}$$

and the proposed BAG EWMA monitoring scheme is constructed as follows:

- The charting statistic of the BAG EWMA scheme, denoted as $BAGEWMA_t$, is defined by

$$BAGEWMA_t = \lambda Z_t + (1-\lambda)BAGEWMA_{t-1}, \tag{18}$$

where t $(t = 1,2,\ldots)$ and Z_t is defined as in Equation (17), λ (with $0 < \lambda \leq 1$) is the smoothing parameter and the starting value $BAGEWMA_0$ is set to be equal to zero.

- Then, for the time varying BAG EWMA scheme, the process is considered to be OOC if

$$|BAGEWMA_t| \geq c\sqrt{\frac{\lambda}{(2-\lambda)}(1-(1-\lambda)^{2t})}\,, \tag{19}$$

where $c > 0$, the control limit constant, is selected such that the attained in-control ARL is as close as possible to a pre-specified ARL_0 value.

- When the process has been running for a long time, an OOC situation is simply triggered if

$$|BAGEWMA_t| \geq c\sqrt{\frac{\lambda}{(2-\lambda)}}\,. \tag{20}$$

5. Results and discussion

The performance of the proposed BAG EWMA monitoring scheme is evaluated using the characteristics of the run-length distribution which is the number of rational subgroups to be plotted on the scheme before the first OOC signal. Here we use, the average run-length (*ARL*), the standard deviation of the run-length (*SDRL*) and five percentiles $P_{0.05}$, $P_{0.25}$, $P_{0.5}$, $P_{0.75}$, and $P_{0.95}$ of the run-length (*PRL*) which are the most popular run-length characteristics used in SPM.

Let *N* be the run-length of the proposed BAG EWMA scheme. Then, *N* can be defined as follows:

$$N = \inf\left\{t \geq 1 \middle| |BAGEWMA_t| \geq c\sqrt{\frac{\lambda}{(2-\lambda)}(1-(1-\lambda)^{2t})}\right\}. \quad (21)$$

5.1 In-control performance

In this subsection, the in-control properties of the proposed scheme under both Case 1 and Case 2 (see Table 1) are investigated in terms of the in-control *ARL*, *SDRL* and *PRL* profiles, and the results are summarised in Table 2 for a nominal pre-specified ARL (ARL_0) $= 370$. The findings in Table 2 reveal that as the magnitude of λ increases, the control limits get wider for both time varying and asymptotic cases. In addition, the control limits are wider when the conditional variance is strictly stationary and ergodic as compared to a near unit root situation. In terms of the *SDRL* profile, it can be seen that the proposed scheme is less likely to give more false alarms when it has been running for a long time (i.e. for an asymptotic case). However, in terms of the *PRL* profile, the proposed scheme is likely to detect a signal a bit sooner when it has been running for a long time.

Table 2. Case 1 and Case 2: *ARL*, *SDRL* and *PRL* profiles of the BAG EWMA scheme when $ARL_0 = 370$

λ	c	ARL	$SDRL$	PRL				
				$P_{0.05}$	$P_{0.25}$	$P_{0.5}$	$P_{0.75}$	$P_{0.95}$
Case 1 In-control asymptotic control limits								
0.05	1.2764	370.10	357.81	31	115	259	507	1087
0.10	1.3693	370.10	361.37	25	112	258	515	1085
0.20	1.4298	370.06	366.47	23	108	258	511	1093
0.35	1.4652	370.48	370.21	21	104	256	514	1122
0.50	1.4980	370.02	374.31	19	103	252	514	1126
0.75	1.5543	370.61	370.06	18	107	254	514	1119
0.90	1.5731	370.40	372.27	18	107	256	510	1121
Case 2 In-control asymptotic control limits								
0.05	1.8178	370.07	361.72	31	115	260	506	1089
0.10	1.9156	370.06	363.28	25	113	260	510	1096
0.20	2.0012	370.02	372.78	17	104	256	508	1125
0.35	2.1873	370.02	379.72	13	102	255	518	1108
0.50	2.3855	370.06	381.29	10	100	253	521	1124
0.75	2.6174	370.34	381.46	9	98	250	527	1131
0.90	2.6831	370.24	381.24	9	98	251	524	1138
Case 1 In-control time varying control limits								
0.05	1.2974	370.01	392.40	4	87	247	520	1183
0.10	1.3782	370.07	377.99	9	98	253	523	1115
0.20	1.4339	370.08	374.83	15	102	257	514	1111
0.35	1.4670	369.96	374.92	16	100	2523	515	1128
0.50	1.4989	370.08	377.53	16	101	251	515	1130
0.75	1.5545	370.50	370.86	18	107	254	514	1119
0.90	1.5731	370.04	371.81	18	107	255	510	1121
Case 2 In-control time varying control limits								
0.05	1.8736	370.03	436.57	1	50	230	529	1245
0.10	1.9515	370.01	425.16	1	60	235	531	1215
0.20	2.0248	370.40	417.33	1	69	239	520	1207
0.35	2.2016	370.02	405.50	1	83	245	522	1157
0.50	2.3948	370.06	391.56	2	91	249	525	1144
0.75	2.5673	370.64	382.87	7	97	251	527	1133
0.90	2.6833	370.02	381.72	8	98	250	524	1137

5.2 Out-of-control performance

In this subsection, the OOC performance of the proposed scheme is investigated in terms of the OOC ARL, *SDRL* and $P_{0.5}$ profiles, respectively, with simultaneous shifts in the mean level of the process ($\mu_1 = \mu_0 + \delta\sigma_0 = \delta$ since $\mu_0 = 0$ and $\sigma_0 = 1$) and error variance ($\sigma_1 = \Delta\sigma_0 = \Delta$ since σ_0 is taken to be equal to 1 and $\Delta > 1$). In addition to the run-length characteristics, the quality loss function (*QLF*) is used to study the overall performance of the proposed scheme. A *QLF* describes the relationship between the shift size and the quality impact. Thus, the *EQL* is mathematically defined by

$$EQL(\delta) = \frac{1}{\delta_{max}(\Delta_{max}-1)} \int_0^{\delta_{max}} \int_1^{\delta_{max}} w(\delta,\Delta).ARL(\delta,\Delta) \quad d\delta \, d\Delta, \tag{22}$$

where δ_{max} and Δ_{max} are the upper boundaries of the range of shifts in the mean and error variance, respectively, and $w(\delta,\Delta)$ (with $w(\delta,\Delta) = \delta^2 + \Delta^2 - 1$) represents the weight function associated with δ and Δ.

The expression of the *EQL* given in Equation (22) can also be written as follows

$$EQL(\delta,\Delta)=\frac{1}{\delta_{max}(\Delta_{max}-1)}\sum_{\delta=0}^{\delta_{max}}\sum_{\Delta=1}^{\Delta_{max}}(\delta^2+\Delta^2-1)\times ARL(\delta,\Delta). \quad (23)$$

Note that the minimum value of the EQL implies the best performance.

Tables 3 and 4 present the results of the OOC properties of the proposed scheme when $\lambda \in$ {0.05, 0.10, 0.20, 0.35, 0.50, 0.90} and $ARL_0 = 370$ for an asymptotic case. The results in Tables 3 and 4 can be summarized as follows:

- When the mean level is in-control and $\Delta \in \{1.5, 2, 3.5, 5, 5.5, 8\}$, the proposed scheme performs better for large values of λ in terms of the ARL profile. However, in terms of the $P_{0.5}$ profile, the smaller the smoothing parameter the better the performance of the proposed scheme.
- In terms of the ARL and $P_{0.5}$ profiles, when there is a simultaneous shift in the mean level and the error standard deviation, the smaller the value of λ the more efficient the proposed scheme is.
- The proposed scheme becomes more sensitive when there is a simultaneous shift in the process mean and error variance.
- The $SDRL$ profile reveals that the proposed scheme is likely to give more false alarms for large values of λ.
- In many situations, the performance of the proposed scheme deteriorates in Case 2 as compared to Case 1.

Table 3. Case 1: ARL, $SDRL$ and $P_{0.5}$ profiles for the asymptotic BAG EWMA scheme with shifts in the error variance term (Δ) and mean level (δ) for different λ values and $ARL_0 = 370$

$\boldsymbol{\delta}$	$\boldsymbol{\Delta}$	$\boldsymbol{\lambda}$																				
		0.05			**0.10**			**0.20**			**0.35**			**0.50**			**0.75**			**0.90**		
		ARL	$SDRL$	$P_{0.5}$	ARL	$SDRL$	$P_{0.5}$	ARL	$SDRL$	$P_{0.5}$	ARL	$SDRL$	$P_{0.5}$	ARL	$SDRL$	$P_{0.5}$	ARL	$SDRL$	$P_{0.5}$	ARL	$SDRL$	$P_{0.5}$
0	1.5	78.4	71.0	57.0	58.4	54.2	42.0	42.1	39.7	30.0	32.0	31.3	22.0	28.2	28.2	20.0	27.3	27.7	19.0	27.4	27.7	19.0
	2.0	38.3	34.1	28.0	26.7	23.6	20.0	17.2	16.1	12.0	12.0	11.7	8.0	10.1	10.1	7.0	9.3	9.8	6.0	9.3	9.9	6.0
	3.5	14.4	13.3	10.0	8.7	8.7	6.0	5.3	5.4	3.0	3.7	3.7	2.0	3.1	3.0	2.0	2.8	2.6	2.0	2.7	2.6	2.0
	5.0	8.3	8.5	5.0	5.0	5.3	3.0	3.1	3.1	2.0	2.3	2.0	1.0	2.0	1.7	1.0	1.9	1.5	1.0	1.8	1.4	1.0
	5.5	7.0	7.4	4.0	4.3	4.7	2.0	2.7	2.6	2.0	2.0	1.7	1.0	1.9	1.5	1.0	1.7	1.3	1.0	1.7	1.3	1.0
	8.0	4.2	5.0	2.0	2.8	3.1	2.0	1.9	1.8	1.0	1.6	1.1	1.0	1.5	0.9	1.0	1.4	0.8	1.0	1.4	0.8	1.0
$\boldsymbol{EQL}$		37.3	38.4	22.5	24.0	24.9	15.6	15.6	15.1	9.9	11.9	10.1	6.8	10.7	8.6	6.6	9.9	7.8	6.4	9.8	7.7	6.4
0.2	1.0	97.4	80.8	74.0	118.0	106.7	85.0	157.1	151.9	111.0	204.4	199.1	144.0	246.1	242.3	173.0	294.3	292.0	206.0	306.3	305.2	211.0
	1.5	56.0	47.5	42.0	45.3	40.3	33.0	35.9	33.7	25.0	28.6	27.4	20.0	26.1	25.9	18.0	26.1	26.5	18.0	26.6	26.8	18.0
	2.0	34.7	30.3	25.0	24.8	22.5	18.0	16.3	15.5	12.0	11.5	11.4	8.0	9.8	9.7	7.0	9.2	9.5	6.0	9.1	9.4	6.0
	3.5	13.9	13.0	10.0	8.8	8.5	6.0	5.3	5.2	3.0	3.7	3.6	2.0	3.1	3.0	2.0	2.8	2.59	2.0	2.8	2.5	2.0
	5.0	8.1	8.4	5.0	4.9	5.2	3.0	3.1	3.2	2.0	2.3	2.1	1.0	2.0	1.7	1.0	1.9	1.5	1.0	1.9	1.4	1.0
	5.5	7.1	7.4	4.0	4.3	4.7	2.0	2.8	2.8	2.0	2.1	1.8	1.0	1.9	1.5	1.0	1.7	1.3	1.0	1.7	1.3	1.0
	8.0	4.2	5.0	2.0	2.7	3.0	1.0	1.9	1.7	1.0	1.6	1.1	1.0	1.5	1.0	1.0	1.4	0.8	1.0	1.4	0.9	1.0
$\boldsymbol{EQL}$		35.9	37.0	21.8	23.3	24.0	12.9	15.7	15.1	9.9	12.1	10.4	7.0	10.9	9.1	6.8	10.3	8.2	6.7	10.3	8.3	6.7
0.4	1.0	36.2	23.4	30.0	40.3	31.5	31.0	52.6	47.1	38.0	79.6	76.8	57.0	114.8	114.0	79.0	177.8	179.9	123.0	202.0	206.6	137.0
	1.5	31.4	23.9	24.0	27.7	23.0	21.0	24.1	21.3	18.0	21.6	20.4	15.0	21.2	20.5	15.0	22.5	22.9	15.0	23.0	23.4	16.0
	2.0	26.4	21.3	20.0	20.2	17.4	15.0	14.2	13.0	10.0	10.5	10.2	7.0	9.1	8.9	6.0	8.6	8.9	6.0	8.9	9.0	6.0
	3.5	13.4	12.2	10.0	8.4	8.0	6.0	5.1	5.1	3.0	3.7	3.6	2.0	3.1	3.0	2.0	2.8	2.6	2.0	2.8	2.6	2.0
	5.0	8.2	8.3	5.0	5.0	5.3	3.0	3.1	3.2	2.0	2.3	2.1	1.0	2.0	1.7	1.0	1.9	1.5	1.0	1.9	1.4	1.0
	5.5	7.0	7.4	4.0	4.3	4.6	2.0	2.8	2.7	2.0	2.1	1.8	1.0	1.9	1.5	1.0	1.8	1.3	1.0	1.8	1.3	1.0
	8.0	4.3	5.1	2.0	2.7	3.0	2.0	1.9	1.7	1.0	1.6	1.1	1.0	1.5	0.9	1.0	1.4	0.8	1.0	1.4	0.8	1.0
$\boldsymbol{EQL}$		34.4	35.2	20.7	22.2	22.7	14.6	15.1	14.3	9.6	12.0	10.3	6.9	11.1	9.0	6.9	10.9	8.7	7.1	11.1	8.8	7.2

Table 4. Case 2: ARL, $SDRL$ and $P_{0.5}$ profiles for the asymptotic BAG EWMA scheme with shifts in the error variance term (Δ) and mean level (δ) for different λ values and $ARL_0 = 370$

$\boldsymbol{\delta}$	$\boldsymbol{\Delta}$	$\boldsymbol{\lambda}$																				
		0.05			**0.10**			**0.20**			**0.35**			**0.50**			**0.75**			**0.90**		
		$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$
0	1.5	152.2	146.0	107.0	116.4	117.8	80.0	84.2	91.6	55.0	76.9	87.9	49.0	76.8	88.9	48.0	77.1	90.2	48.0	77.7	91.5	48.0
	2.0	101.1	99.9	70.0	66.3	72.2	43.0	38.8	49.4	21.0	33.1	45.4	14.0	33.6	46.6	14.0	34.5	47.3	14.0	34.7	47.6	15.0
	3.5	50.4	63.8	27.0	27.4	41.7	6.0	13.8	26.0	2.0	10.3	20.7	2.0	10.3	21.2	1.0	10.6	21.5	1.0	10.9	21.6	1.0
	5.0	30.9	50.4	5.0	14.5	29.2	2.0	7.2	17.1	1.0	5.7	14.4	1.0	5.5	14.6	1.0	5.5	14.5	1.0	5.8	15.2	1.0
	5.5	27.5	48.9	3.0	13.1	27.5	2.0	6.5	16.4	1.0	4.6	12.4	1.0	4.4	12.4	1.0	4.8	13.4	1.0	4.8	13.4	1.0
	8.0	16.5	37.6	1.0	8.1	21.7	1.0	3.5	10.5	1.0	2.6	7.4	1.0	2.4	6.9	1.0	2.5	8.4	1.0	2.5	8.0	1.0
$\boldsymbol{EQL}$		130.2	221.7	32.8	67.6	132.3	16.6	34.3	75.2	9.7	26.7	59.1	8.6	25.9	58.5	8.2	26.8	63.1	8.2	27.2	62.9	8.3
0.2	1.0	102.7	86.5	77.0	124.9	115.2	90.0	187.6	187.5	126.5	280.3	286.6	193.0	315.0	322.8	215.0	331.0	343.0	225.0	336.0	343.4	229.0
	1.5	94.8	86.2	68.0	83.5	81.8	58.0	69.0	74.1	45.0	67.4	78.8	41.0	71.4	82.9	44.0	73.6	86.6	45.0	74.0	86.5	45.0
	2.0	81.0	79.1	57.0	57.8	62.0	38.0	36.5	46.6	20.0	33.0	44.8	14.0	33.2	46.5	14.0	34.9	49.4	14.0	35.6	50.2	15.0
	3.5	48.5	61.9	26.0	25.9	39.7	6.0	12.8	23.8	2.0	9.8	19.9	2.0	10.0	20.2	1.0	10.5	21.4	1.0	11.0	22.2	1.0
	5.0	31.4	51.0	6.0	14.5	29.7	2.0	7.2	17.6	1.0	5.4	13.7	1.0	5.1	13.2	1.0	5.5	14.7	1.0	5.4	14.0	1.0
	5.5	27.7	47.4	3.0	13.2	28.2	2.0	6.2	15.1	1.0	4.5	12.1	1.0	4.3	11.8	1.0	4.4	12.3	1.0	4.6	12.7	1.0
	8.0	16.2	37.4	1.0	7.7	20.6	1.0	3.5	10.4	1.0	2.5	7.0	1.0	2.5	7.5	1.0	2.5	7.6	1.0	2.5	7.8	1.0
$\boldsymbol{EQL}$		125.3	215.2	30.5	64.3	128.1	15.4	33.1	72.7	9.4	26.0	57.1	8.6	25.9	58.0	8.4	26.8	61.1	8.5	27.3	61.7	8.6
0.4	1.0	38.7	25.2	32.0	43.8	34.3	33.0	70.0	66.1	50.0	159.7	164.9	109.0	219.7	228.2	149.0	261.9	273.8	180.0	270.0	281.4	186.0
	1.5	46.8	37.1	36.0	44.5	39.8	33.0	43.7	46.1	29.0	55.8	66.0	34.0	62.7	76.1	39.0	67.9	82.0	41.0	69.0	83.2	42.0
	2.0	51.1	46.1	38.0	40.7	42.1	28.0	29.8	37.0	16.0	29.2	41.2	12.0	31.2	43.7	12.0	33.1	46.6	13.0	34.0	47.7	14.0
	3.5	41.5	52.1	23.0	24.3	37.2	6.0	12.4	23.2	2.0	9.5	19.7	2.0	10.1	21.1	2.0	10.6	22.3	1.0	10.5	21.8	1.0
	5.0	28.5	46.1	5.0	14.4	28.4	2.0	7.0	16.0	1.0	5.4	14.0	1.0	5.3	13.9	1.0	5.4	14.3	1.0	5.5	14.3	1.0
	5.5	26.3	45.1	3.0	12.4	26.4	2.0	6.3	15.6	1.0	4.7	12.8	1.0	4.3	11.7	1.0	4.7	13.6	1.0	4.6	13.1	1.0
	8.0	15.5	35.3	1.0	7.2	19.2	1.0	3.6	11.0	1.0	2.6	8.3	1.0	2.4	7.3	1.0	2.4	7.2	1.0	2.5	7.4	1.0
$\boldsymbol{EQL}$		112.4	195.2	25.4	58.7	117.7	13.5	31.8	71.3	8.5	26.4	61.2	8.7	26.6	59.4	9.2	28.0	62.9	9.1	28.4	62.9	9.3

Tables 5 and 6 present the results of the OOC properties of the proposed scheme when $\lambda \in$ {0.05, 0.1, 0.2, 0.35, 0.5, 0.9} for a nominal $ARL_0 = 370$ for time varying case. The results in Tables 5 and 6 can be summarized as follows:

- When the mean level is in-control and $\Delta \in$ {1.5, 2, 3.5,5,5.5,8}, the proposed scheme performs better for moderate values of λ in terms of the ARL profile. However, in terms of the $P_{0.5}$ profile, the smaller the smoothing parameter the better the performance of the proposed scheme.
- In terms of the ARL and $P_{0.5}$ profiles, when there is a simultaneous shift in the mean level and the error standard deviation, the smaller the value of λ the more efficient the proposed scheme is.
- The proposed scheme becomes more sensitive as the effect of the combined shift increases.
- The $SDRL$ profile reveals that the proposed scheme is likely to give more false alarms for large and very small values of λ.
- It is also noticed that in many situations, the proposed scheme performs better in Case 1 as compared to Case 2.

Table 5. Case 1: ARL, $SDRL$ and $P_{0.5}$ profiles for the time-varying BAG EWMA scheme with shifts in the error variance term (Δ) and mean level (δ) for different λ values and $ARL_0 = 370$

$\boldsymbol{\delta}$	$\boldsymbol{\Delta}$	$\boldsymbol{\lambda}$																				
		0.05			**0.10**			**0.20**			**0.35**			**0.50**			**0.75**			**0.90**		
		$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$
0	1.5	54.4	71.6	26.0	46.2	54.7	28.0	36.3	39.5	24.0	29.6	31.5	20.0	27.0	28.2	18.0	27.1	27.8	19.0	27.3	27.8	19.0
	2.0	18.9	29.3	6.0	16.3	22.0	7.0	12.6	15.2	7.0	10.1	11.4	6.0	9.3	10.1	6.0	9.2	9.8	6.0	9.3	9.9	6.0
	3.5	3.9	7.1	1.0	3.9	5.8	2.0	3.4	4.3	2.0	3.0	3.3	2.0	2.8	2.8	2.0	2.7	2.6	2.0	2.7	2.6	2.0
	5.0	2.0	2.9	1.0	2.1	2.7	1.0	2.0	2.1	1.0	2.0	1.8	1.0	1.9	1.6	1.0	1.8	1.5	1.0	1.9	1.5	1.0
	5.5	1.9	2.5	1.0	1.9	2.3	1.0	1.9	1.8	1.0	1.8	1.5	1.0	1.7	1.4	1.0	1.7	1.3	1.0	1.7	1.3	1.0
	8.0	1.4	1.3	1.0	1.5	1.3	1.0	1.4	1.1	1.0	1.4	0.9	1.0	1.4	0.9	1.0	1.4	0.8	1.0	1.4	0.8	1.0
$\boldsymbol{EQL}$		12.9	17.2	6.4	12.5	14.8	7.0	11.2	11.3	6.8	10.4	9.1	6.5	9.9	8.3	6.4	9.7	7.8	6.4	9.9	7.8	6.4
0.2	1.0	90.7	86.0	66.0	114.6	110.3	81.0	154.7	155.3	107.0	203.1	201.4	143.0	245.2	243.5	172.0	294.3	292.4	206.0	306.2	305.2	211.0
	1.5	38.2	46.8	21.0	35.3	40.0	22.0	30.8	33.6	20.0	26.4	27.4	18.0	24.9	25.9	17.0	25.8	26.6	18.0	26.5	26.8	18.0
	2.0	16.9	26.2	5.0	14.7	19.8	6.0	12.4	14.9	7.0	9.9	11.2	6.0	9.0	9.6	6.0	9.1	9.5	6.0	9.0	9.4	6.0
	3.5	3.8	6.9	1.0	3.7	5.5	2.0	3.4	4.3	2.0	2.9	3.2	2.0	2.8	2.9	2.0	2.8	2.6	2.0	2.8	2.5	2.0
	5.0	2.1	3.0	1.0	2.1	2.7	1.0	2.0	2.2	1.0	1.9	1.8	1.0	1.9	1.6	1.0	1.9	1.5	1.0	1.9	1.4	1.0
	5.5	1.9	2.4	1.0	1.9	2.1	1.0	1.9	1.9	1.0	1.8	1.5	1.0	1.7	1.4	1.0	1.7	1.3	1.0	1.7	1.3	1.0
	8.0	1.4	1.2	1.0	1.4	1.1	1.0	1.4	1.1	1.0	1.4	1.0	1.0	1.4	0.8	1.0	1.4	0.8	1.0	1.4	0.8	1.0
$\boldsymbol{EQL}$		12.2	15.7	6.2	11.8	13.4	6.7	11.2	11.5	6.8	10.4	9.4	6.6	10.2	8.4	6.6	10.3	8.2	6.7	10.3	8.1	6.7
0.4	1.0	30.5	25.3	24.0	37.1	32.5	28.0	51.0	47.7	36.0	78.4	77.7	55.0	114.1	114.7	78.0	177.6	180.1	123.0	201.9	206.6	137.0
	1.5	20.8	23.9	13.0	21.2	23.5	14.0	20.2	21.1	14.0	19.7	20.3	13.0	20.3	20.5	14.0	22.3	22.9	15.0	23.0	23.4	16.0
	2.0	12.7	18.4	5.0	12.2	16.0	6.0	10.6	12.6	6.0	9.1	10.2	6.0	8.4	8.9	5.0	8.4	8.8	5.0	8.8	9.0	6.0
	3.5	3.6	6.4	1.0	3.7	5.4	1.0	3.3	4.1	2.0	3.0	3.1	2.0	2.7	2.7	2.0	2.7	2.6	2.0	2.8	2.6	2.0
	5.0	2.1	3.2	1.0	2.1	2.6	1.0	2.0	2.1	1.0	1.9	1.8	1.0	1.9	1.6	1.0	1.8	1.4	1.0	1.8	1.4	1.0
	5.5	1.9	2.5	1.0	1.9	2.3	1.0	1.8	1.8	1.0	1.8	1.6	1.0	1.7	1.3	1.0	1.7	1.3	1.0	1.8	1.3	1.0
	8.0	1.4	1.3	1.0	1.4	1.2	1.0	1.4	1.0	1.0	1.4	0.9	1.0	1.4	0.9	1.0	1.4	0.8	1.0	1.4	0.8	1.0
$\boldsymbol{EQL}$		11.1	14.4	5.9	11.1	12.7	6.1	10.6	10.4	6.6	10.4	9.1	6.6	10.3	8.6	6.7	10.7	8.6	7.0	11.0	8.8	7.2

Table 6. Case 2: ARL, $SDRL$ and $P_{0.5}$ profiles for the time-varying BAG EWMA scheme with shifts in the error variance term (Δ) and mean level (δ) for different λ values when $ARL_0 = 370$

$\boldsymbol{\delta}$	$\boldsymbol{\Delta}$	$\boldsymbol{\lambda}$																				
		0.05			**0.10**			**0.20**			**0.35**			**0.50**			**0.75**			**0.90**		
		$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{P_{0.5}}$
0	1.5	95.0	152.9	18.0	80.6	122.1	24.5	64.7	92.9	25.0	66.6	88.2	33.5	72.0	89.1	42.0	76.4	90.5	47.0	77.6	91.5	48.0
	2.0	41.8	85.3	2.0	33.4	64.1	2.0	26.4	47.1	3.0	27.3	45.0	4.0	31.1	47.2	9.0	33.9	47.3	13.0	34.6	47.5	15.0
	3.5	11.4	37.8	1.0	8.9	26.2	1.0	6.7	18.2	1.0	7.4	17.7	1.0	8.2	19.1	1.0	10.4	21.6	1.0	10.8	21.9	1.0
	5.0	5.6	22.7	1.0	4.5	16.6	1.0	3.6	11.4	1.0	3.5	10.5	1.0	4.3	12.2	1.0	5.4	14.6	1.0	5.8	15.0	1.0
	5.5	4.3	19.7	1.0	3.9	15.7	1.0	3.0	9.8	1.0	3.1	9.2	1.0	3.8	11.3	1.0	4.6	12.8	1.0	4.8	13.4	1.0
	8.0	2.2	10.7	1.0	2.2	9.0	1.0	1.9	6.0	1.0	1.9	5.3	1.0	2.1	6.5	1.0	2.4	7.5	1.0	2.5	7.7	1.0
$\boldsymbol{EQL}$		27.5	95.3	5.6	23.6	73.7	5.9	18.9	50.0	6.0	19.4	46.4	6.5	22.2	53.6	7.4	26.1	60.5	8.0	27.1	62.2	8.3
0.2	1.0	90.2	98.0	60.0	116.8	128.0	77.0	184.3	208.1	116.0	276.9	301.0	182.0	315.7	334.4	212.0	332.1	344.3	226.0	336.0	343.4	229.0
	1.5	56.3	86.0	14.0	55.5	82.3	18.0	53.1	75.3	21.0	59.2	79.9	28.0	67.2	83.3	38.0	72.7	86.5	44.0	73.9	86.5	45.0
	2.0	32.8	67.3	2.0	30.1	56.6	2.0	23.7	42.8	2.0	26.1	42.3	4.0	30.5	46.3	8.0	34.0	49.2	12.0	35.5	50.2	15.0
	3.5	10.7	34.6	1.0	8.4	24.9	1.0	6.5	17.5	1.0	7.4	18.2	1.0	8.7	19.6	1.0	9.9	20.9	1.0	11.0	22.1	1.0
	5.0	5.2	21.3	1.0	4.4	15.6	1.0	3.6	11.8	1.0	3.9	11.6	1.0	4.3	12.2	1.0	5.4	14.8	1.0	5.3	13.9	1.0
	5.5	4.1	18.7	1.0	3.4	12.7	1.0	3.0	9.8	1.0	3.2	9.8	1.0	3.8	11.4	1.0	4.4	12.5	1.0	4.6	12.7	1.0
	8.0	2.3	10.8	1.0	2.1	8.2	1.0	1.8	5.9	1.0	1.8	5.2	1.0	2.1	6.6	1.0	2.2	6.6	1.0	2.6	8.1	1.0
$\boldsymbol{EQL}$		24.5	87.5	5.5	21.4	65.3	5.7	18.2	49.1	5.9	19.7	48.0	6.5	22.8	54.5	7.5	25.7	58.9	8.2	27.4	62.3	8.6
0.4	1.0	29.7	28.6	23.0	37.6	37.6	27.0	65.6	72.8	42.0	155.5	171.9	102.0	219.4	234.6	147.0	261.4	274.9	179.0	269.8	281.5	186.0
	1.5	26.5	38.2	8.0	28.3	40.0	11.0	33.0	45.9	13.0	48.9	66.1	23.0	59.1	76.8	33.0	67.2	82.2	39.0	68.9	83.1	42.0
	2.0	19.9	37.1	2.0	20.2	36.1	2.0	18.9	33.2	2.0	23.3	38.9	3.0	28.2	43.3	6.0	32.5	46.4	12.0	33.9	47.6	13.0
	3.5	8.9	27.4	1.0	7.5	21.8	1.0	6.4	17.1	1.0	7.0	17.0	1.0	8.5	19.7	1.0	10.2	21.5	1.0	10.5	21.8	1.0
	5.0	5.0	20.7	1.0	4.4	16.2	1.0	3.5	11.2	1.0	3.7	11.2	1.0	4.2	11.7	1.0	5.1	13.6	1.0	5.5	14.3	1.0
	5.5	4.2	18.0	1.0	3.6	13.0	1.0	3.0	10.1	1.0	3.2	9.8	1.0	3.6	10.4	1.0	4.4	12.5	1.0	4.6	13.1	1.0
	8.0	2.1	10.5	1.0	2.0	7.6	1.0	1.8	5.8	1.0	2.0	6.2	1.0	2.2	6.9	1.0	2.5	7.7	1.0	2.5	7.5	1.0
$\boldsymbol{EQL}$		20.8	77.9	5.3	19.1	60.0	5.5	17.0	46.8	5.7	19.9	49.6	6.7	23.2	54.7	7.8	27.4	61.9	8.9	28.3	63.1	9.2

5.3. Performance of the proposed scheme under different prior distributions

In this section, specific and overall performances of the proposed scheme in terms of the ARL and EQL profiles are evaluated using different prior distributions when $\lambda \in \{0.1, 0.5, 0.9\}$ for a nominal $ARL_0 =$ 370. For illustration purpose, the sensitivity of the proposed scheme using the proper prior introduced earlier (normal prior hereafter) is compared to its sensitivity under the Student' s t and gamma prior distributions with degrees of freedom $df = 5$ and parameters (a, b) =(1,1), respectively. These prior distributions are denoted as $t(df)$ and *Gamma*(a,b).

The results in Table 7 reveal that in terms of the ARL and EQL profiles, the proposed scheme performs better under the normal prior as compared to its performance under the *t*(5) and *Gamma*(1,1) prior distributions. The proposed scheme performs worst under skewed prior distributions (see large ARL and EQL values under the *Gamma*(1,1) prior distribution). Under heavy-tailed and skewed prior distributions the performance of the proposed scheme degrades as λ increases.

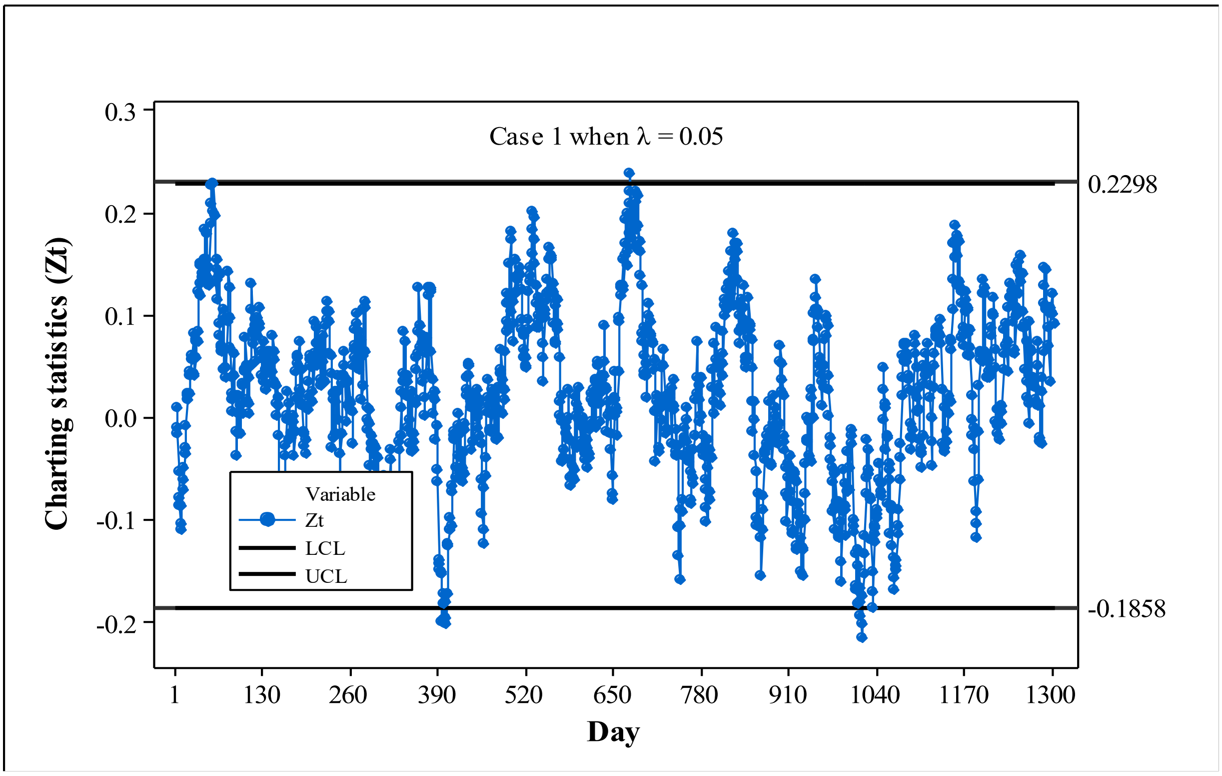
Table 7. Case 1: *ARL* profiles for the time-varying BAG EWMA scheme with shifts in the error variance term (Δ) and mean level (δ) for $\lambda \in \{0.10, 0.5, 0.9\}$ values when $ARL_0 = 370$

δ	Δ	The proposed proper prior			*t*(5)			*Gamma*(1,1)		
		0.1	0.5	0.9	0.1	0.5	0.9	0.10	0.5	0.9
0	1.5	46.2	27.0	27.3	46.5	40.2	80.1	55.1	57.8	79.0
	2.0	16.3	9.3	9.3	17.5	20.8	39.3	19.4	27.9	25.7
	3.5	3.9	2.8	2.7	3.5	4.5	12.1	5.6	6.7	10.2
	5.0	2.1	1.9	1.9	2.2	2.5	5.6	2.9	2.9	8.1
	5.5	1.9	1.7	1.7	1.4	1.9	4.5	2.2	2.3	7.0
	8.0	1.5	1.4	1.4	1.3	1.5	2.3	1.9	1.9	3.9
EQL		12.5	9.9	9.9	11.6	13.3	27.3	15.8	17.4	33.4
0.2	1.0	114.6	245.2	306.2	110.4	219.3	316.3	125.5	295.9	350.4
	1.5	35.3	28.9	26.5	39.3	29.9	32.1	40.6	32.3	38.9
	2.0	14.7	9.0	9.0	15.9	13.7	14.0	18.2	16.4	20.7
	3.5	3.7	3.2	2.8	3.8	4.3	5.2	4.3	6.0	9.1
	5.0	2.1	1.9	1.9	1.9	2.2	2.7	3.1	3.1	4.3
	5.5	1.9	1.7	1.7	1.5	1.9	1.9	2.3	2.4	3.5
	8.0	1.4	1.4	1.4	1.4	1.5	1.5	2.1	2.0	2.8
EQL		11.8	10.5	10.3	11.6	12.2	13.2	15.7	15.8	21.9
0.4	1.0	37.1	114.1	201.9	40.6	120.7	233.4	45.4	136.6	267.6
	1.5	21.2	24.3	23.0	24.1	26.9	29.6	26.4	33.3	36.4
	2.0	12.2	10.4	8.8	11.4	13.2	12.1	14.3	14.4	20.4
	3.5	3.7	2.7	2.8	3.4	3.1	3.7	4.8	6.6	8.2
	5.0	2.1	1.9	1.8	1.9	2.1	2.6	2.0	4.6	5.1
	5.5	1.9	1.7	1.8	1.7	1.7	2.0	1.6	2.7	3.0
	8.0	1.4	1.4	1.4	1.3	1.5	1.7	1.5	1.8	2.2
EQL		11.1	10.7	11.0	10.5	11.8	13.9	11.9	17.6	21.5

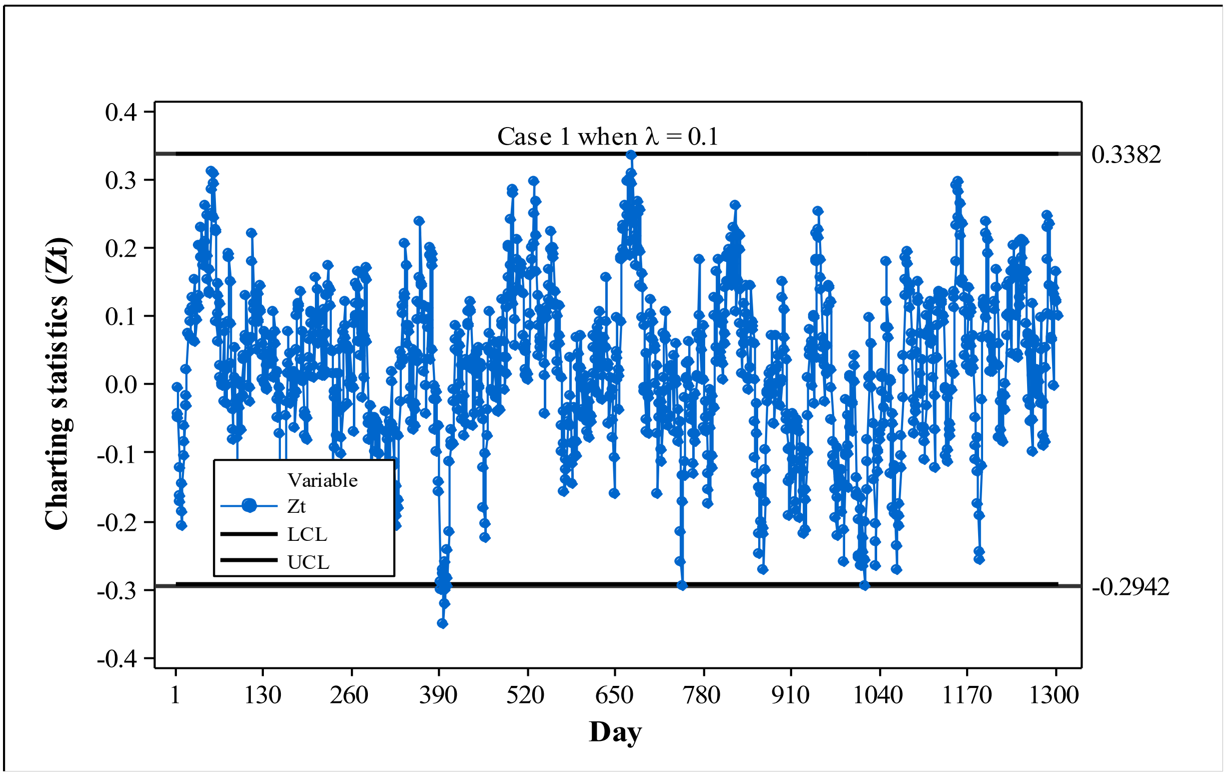
6. Case study: monitoring the USD/ZAR exchange rate using the proposed Bayesian ARMA-GARCH monitoring scheme

Volatility is an integral part of financial time series analysis and can be well-handled by models such as ARCH, GARCH and other variants of time series models; see, for example, Lange (2011) and Lawrance (2013). In financial industries, it is very important to detect a change (or shift) in the process as soon as they occur. This is made possible through the use of appropriate flexible and more efficient time series monitoring schemes. The combination of ARMA and GARCH models for the innovations results, denoted as ARMA-GARCH model, is more recommended by econometrician in such instances. This model is used because of its flexibility to model both the mean and variance. In this section, the proposed BAG EWMA scheme is used to monitor the USD/ZAR exchange rate. In this case study, it is necessary to detect any departure from the target process parameter as soon as possible. A change in the process parameter can be due to or lead to significant changes in the exchange rate and affects businesses in two ways: i) by changing the cost of supplies that are purchased from a different country, and ii) by changing the attractiveness of their products to overseas customers.

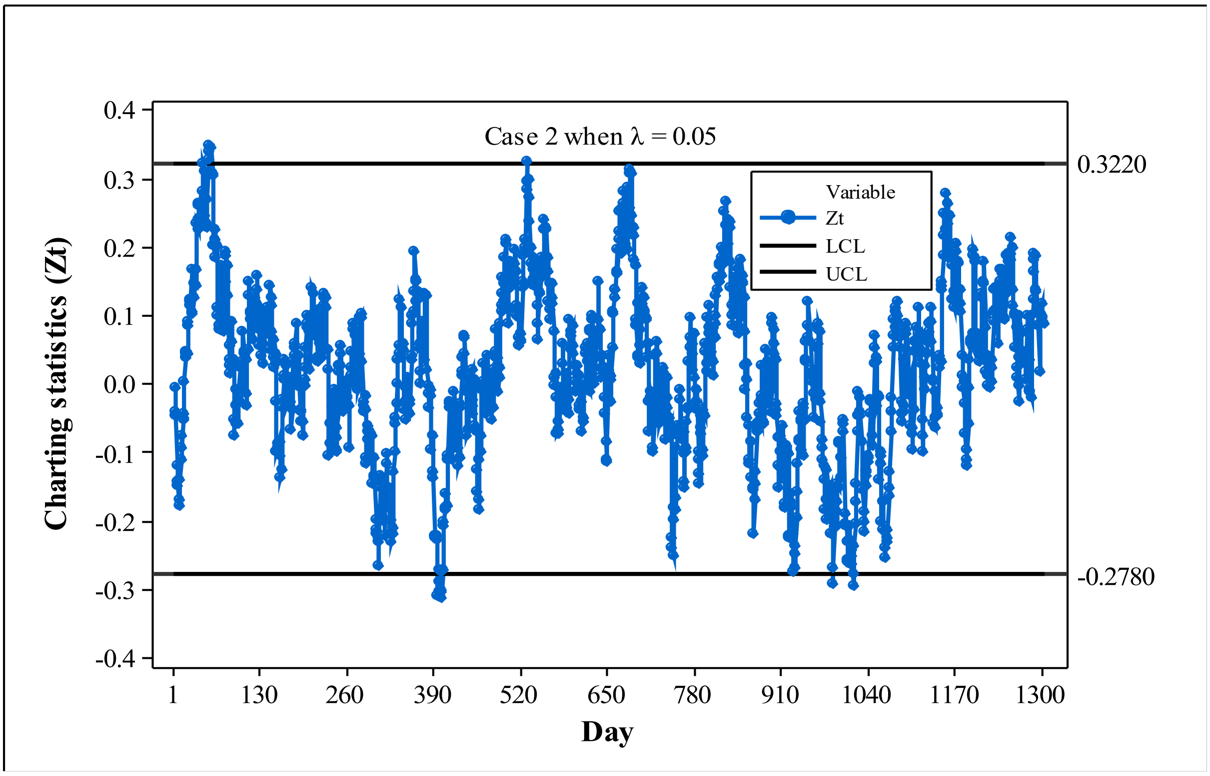
The charting statistics of the proposed BAG EWMA scheme are displayed in Figure 2. For Case 1, the control limit constants used to compute the asymptotic control limits of the proposed BAG EWMA scheme are 1.2764 and 1.3693 when $\lambda = 0.05$ and 0.1, respectively; while the ones for Case 2 are given by 1.8178 and 1.9156, respectively (see Table 2); the asymptotic control limits are displayed in the figures below. Figures 2 (a)-(d) show that for Case 1, when $\lambda = 0.05$ and 0.10, the proposed monitoring scheme gives a signal on the 55th and 391st samples in the prospective phase. These samples correspond to the 16th of Oct 2017 and 29th of January 2019, respectively. However, for Case 2, when $\lambda = 0.05$ and 0.10, the proposed scheme gives a signal on the 36th and 393rd samples which corresponds to the 19th of September 2017 and the 31st of Jan 2019. It can be seen that the proposed BAG EWMA scheme performs better for small smoothing parameters when the variance is strictly stationary and ergodic, i.e. Case 2. Apart from the general deterioration in the South African Rand, as shown in steady upward trend observed in Figure 1(a), it is interesting to note that both charts signal before the start of the Covid 19 pandemic. We can thus conclude that despite the severe negative impact the pandemic had, it did not lead to any statistically significant changes in the ZAR/USD exchange rate as monitored using the BAG EWMA scheme.



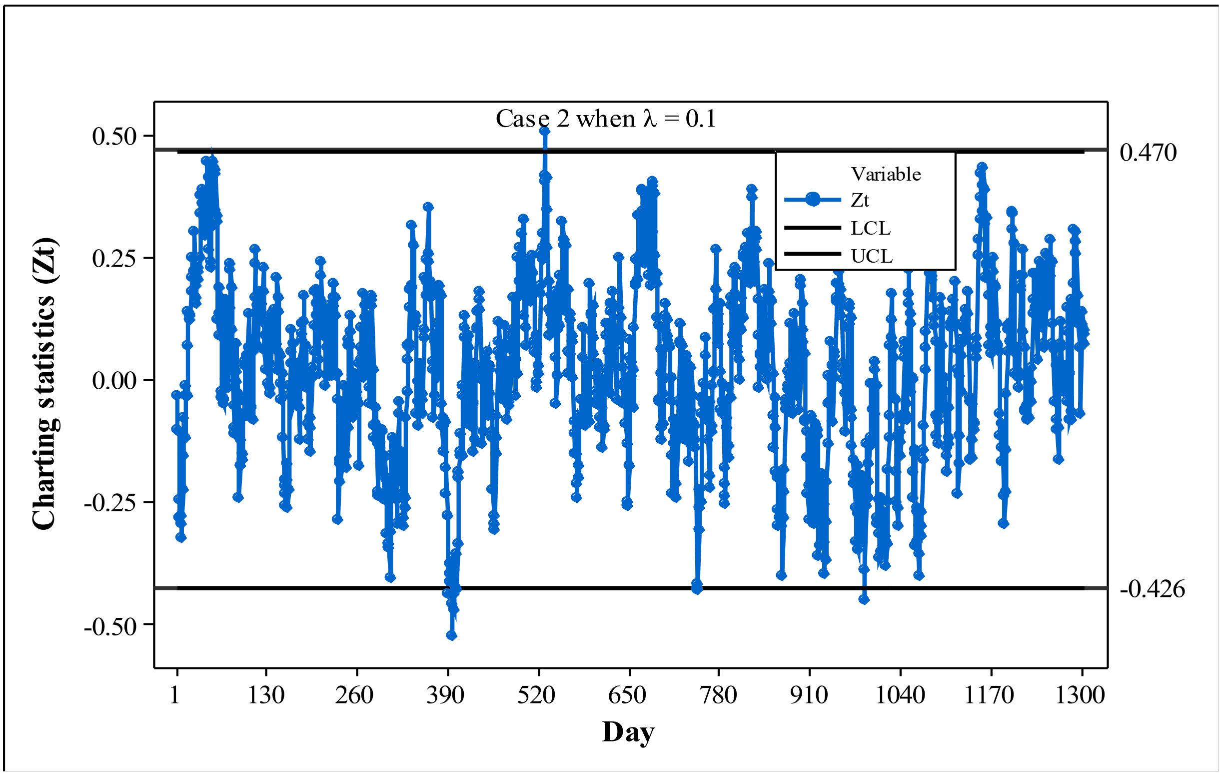
(a) Case 1 when $\lambda = 0.05$ (OOC 55th sample)



(b) Case 1 when $\lambda = 0.1$ (OOC 391st sample)



(c) Case 2 when $\lambda = 0.05$ (OOC 36th sample)



(d) Case 2 when $\lambda = 0.1$ (OOC 393rd sample)

Figure 2. BAG EWMA monitoring scheme for the returns of the USD/ZAR exchange rate data

7. Conclusion

This paper introduces a new ARMA-GARMA EWMA monitoring scheme for volatile long run processes using the Bayesian approach where the method is implemented utilising the MH algorithm to find the posterior and parameters of the model. It is found that, in many cases, the proposed monitoring scheme performs better when the approximation is done near the unit root. However, the case study revealed that the ability of the proposed scheme increases when the variance is strictly stationary and ergodic with small smoothing parameters. Thus, the proposed scheme is mostly recommended when the process is significantly volatile.

Note that the performance of the proposed monitoring scheme was investigated for two cases, i.e. when the estimation is done near the unit root and when the variance is strictly stationary and ergodic. For other cases and assumptions, the results might differ from the findings observed in this paper. Therefore, the performance of the proposed scheme needs to be investigated according to assumptions.

In future, researchers can investigate the performance and robustness of the proposed monitoring scheme using different priors and assumptions. Researchers can also investigate the performance of the proposed monitoring schemes for higher order ARMA models, i.e., when $p > 1$ and $q > 1$ and also look at the design of the ARMA-GARMA CUSUM monitoring scheme using the Bayesian approach.

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