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TEACHING MEASUREMENT: THE ROLE OF MATHEMATICS TEACHERS' ENACTED PCK ON LEARNER OUTCOMES

Hanlie Botha*, Corene Coetzee, Liezell Zweers
University of Pretoria, South Africa

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ABSTRACT

Teaching is a challenging profession where teachers must create valuable learning opportunities to enhance learners' conceptual understanding. Apart from mathematical content knowledge, teachers use their pedagogical content knowledge (PCK) that develops as they reflect on previously taught lessons and learner responses in assessments. International and national assessment studies showed that South African learners perform poorly in, among other topics, Measurement. Thus, we determined the gain in learner outcomes as revealed in a pre-and post-test on Measurement and studied one of the PCK domains, namely teachers' enacted PCK as informed by the baseline assessment learner outcomes. The aim was to determine how teachers' enacted PCK relate to learner outcomes. Underpinned by a social constructivist paradigm, the study used a mixed-method research approach. Data were gathered from a pre-and post-test written by 124 Grade 9 learners taught by two experienced mathematics teachers in a city school in South Africa. Findings revealed that although some improvements are evident after the topic has been taught, the test was still experienced as difficult by almost all the learners. However, from the observations, there is little evidence that the experienced teachers extensively used the baseline assessment outcomes to inform their teaching.

Corresponding Author:

Hanlie Botha,
Department of Science, Mathematics and Technology Education,
University of Pretoria
Groenkloof Campus, Corner of George Storrar Drive and Leyds Street, Groenkloof, Pretoria, South Africa.
Email: hanlie.botha@up.ac.za

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1. INTRODUCTION

One of the core aspects of maximising learners' learning opportunities, is quality instruction using tuition time effectively. The driving forces behind teachers' quality instruction are appropriate goals, developed knowledge, and positive beliefs regarding mathematics and the teaching thereof (Artzt et al., 2015). Regarding a teacher's knowledge, Hill et al. (2008), distinguish between mathematical content knowledge and pedagogical content knowledge (PCK). Other components that enhance teachers' instruction are their ability to meaningfully use assessment, but also to create a positive classroom environment, create opportunities for meaningful discourse in class, and use of purposive activities (Artzt et al., 2015). Good instruction starts with teacher clarity on teaching intentions, and continue by using appropriate teaching and learning strategies, well-designed tasks, meaningful classroom discourse, multiple representations, various resources and manipulatives (Fisher et al., 2017). These aspects are encompassed in Hill et al.'s (2008) components of PCK namely a teacher's knowledge of the learners, knowledge of teaching mathematics, and knowledge of the curriculum. Apart from mathematical content knowledge as a knowledge base, it is sound teacher's PCK, and in particular, topic-specific PCK that makes for good instruction. The importance of exploring topic-specific PCK is underlined by Kirschner et al. (2015, p. 234) when saying that "… one major, if not the main theoretical premise behind studying PCK, is that teachers with higher levels of PCK are better able to help students learn". Geddis (1993) purported that PCK is embedded in a teacher's knowledge of learner understanding and misconceptions and strategies towards sound conceptual understanding by using different representations. These are accepted as knowledge components of PCK at topic level (Mavhunga, 2019). A valuable source to inform a teacher's knowledge about learner understanding is the outcome of baseline assessment. Sadler and Sonnert (2016) found in a study conducted to 620 senior phase science teachers in 589 schools, that teachers having sound topic-specific PCK regarding their learners' misconceptions, are more likely to increase their learners' knowledge, than those teachers lacking that knowledge.

PCK is a knowledge base that distinguishes the teacher from the mere subject specialist (Shulman, 1986). Shulman (1986, p. 9) described it as knowledge about "the ways of representing and formulating the subject that make it comprehensible to others". In the years following Shulman's conceptualization of PCK, researchers have embarked on studies using PCK as a lens to investigate teaching and learning. Veal and MaKinster (1999) developed a hierarchical taxonomy of PCK in which they suggested that PCK can be investigated at different levels, namely general, domain specific, and topic-specific PCK. For investigating PCK at topic level, Mavhunga and Rollnick (2013) proposed five components from which the transformation of teaching content develops. The components that are most relevant to the current study are: teachers' knowledge of (i) learners' understanding of the topic at hand, and (ii) representations, such as diagrams and models that can be used in a conceptual teaching strategy. In 2019 a revised consensus model of PCK was developed that places what happens in the classroom at the centre of the model (Carlson et al., 2019) where the enacted PCK (ePCK) of the teacher is situated. The ePCK of a teacher is a subset of the personal PCK (pPCK) of the teacher which is informed by the collective PCK (cPCK); a PCK belonging to the profession. Pedagogical reasoning, that "takes place during all aspects of the teaching are unique to each teacher and every teaching moment" (Carlson et al., 2019, p. 83), shapes a teacher's enacted PCK and resonates with Dewey's (1933) idea of reflective thought.

Dewey (1933) introduced the concept of reflective thought, describing reflection as "an active and deliberative cognitive process which involves the sequence of interconnected ideas that take into account the underlying beliefs and knowledge" (Pedro, 2006, p. 130). Reflection before teaching (reflection for action), when a teacher employs PCK for the personal and the enacted realms of PCK, can be done when a teacher is planning a lesson, to anticipate the possible outcomes of the lesson or how learners will act in the lesson. The learner outcomes of a baseline test could inform the teacher's instruction and ePCK in terms of the learners' pre-concepts and prior knowledge, appropriate modes of representation, teaching strategies, and integration with other mathematical topics and disciplines.

Findings from the Trends in International Mathematics and Science Study (TIMSS) (Reddy et al., 2020), and the Southern and Eastern Africa Consortium for Monitoring

Educational Quality IV (SACMEQ, 2017), two international assessment studies conducted in 2019 and 2013 respectively, revealed general poor performance from Grade 9 mathematics learners. This was in line with the outcomes of the Grade 9 Annual National Assessment (ANA) (Department of Basic Education, 2014a, 2014b), a terminated national assessment study. According to Ijeh and Nkopodi (2013, p. 473), the findings from the TIMSS and SACMEQ assessments concluded that factors influencing the quality of mathematics and science education "are likely to be deeply rooted in the learner, national curriculum, subject matter and pedagogical flexibility of teachers". It has been posited that the problem lies in teachers having "limited content knowledge and ineffective teaching approaches and unprofessional attitudes" (Kriek & Grayson, 2009, p. 185) and this is still echoed in the TIMSS report (Reddy et al., 2020).

1.1. Purpose of study

To shed light on the role of teachers' enacted PCK on learner understanding, the curriculum topic of Measurement has been chosen for this study, being identified in the TIMSS Item Diagnostic Report: South Africa Grade 9 Mathematics (Mosimege et al., 2016), as well as the Annual National Assessment of 2014 Diagnostic Report Intermediate and Senior Phases Mathematics (Department of Basic Education, 2014a), as one of the topics South African learners find most challenging. This poor performance in Measurement is not restricted to South Africa, but is a global tendency. Sisman and Aksu (2016, p. 1294) referred to several international studies indicating that learners "have poor and superficial understanding of length, area, and volume measurement". In their study conducted on Grade 6 learners, they found learners specifically lack comprehension of the concepts; they confuse the perimeter with the area formula, and the volume with the surface area formula. Even first-year students in tertiary institutions revealed the same kind of errors and misconceptions. A study conducted in the United Kingdom, investigating 326 first-year bioscience students' mathematical errors and misconceptions, revealed that "a high proportion (52–95%) of students encountered difficulties with individual questions involving the calculation of volume or surface area, the conversion of units of measurement, working with ratios, proportions and powers of 10, and determining magnification or magnitude" (Tariq, 2008, p. 889). Tariq (2008, p. 889) further mentioned that "many of the errors and misconceptions students exhibited were similar to those reported previously as made commonly by 13–14-year-old children".

In the South African school curriculum, two-dimensional (2D) shapes and three-dimensional (3D) objects form a prominent part from the beginning of the Intermediate phase (Grades 4-6) to the end of the Senior phase (Grades 7-9). The Department of Basic Education's (2011a) reason for its prominence is that "it relates directly to the learner's scientific, technological and economic worlds, enabling the learner to make sensible estimates and be alert to the reasonableness of measurements and results" (p. 6). From the report about the findings of Grade 9 mathematics learners' performances in international assessments, Mosimege et al. (2016) pointed out that due to an overload of content in the Intermediate phase curriculum, there is not enough teaching and consolidation time. Due to limited tuition time, teachers do not involve learners in activities building 3D objects from their nets and having the opportunity to "touch and see the object to remember" (Mosimege et al., 2016, p. 72). Consequently, learners' poor prior knowledge and conceptual understanding are carried over to the Senior phase, and seemingly not addressed in the first two grades of the Senior phase. The Annual National Assessment 2014 Diagnostic Report Intermediate and Senior Phases Mathematics (Department of Basic Education, 2014a) indicated two areas of weaknesses. The first area was knowledge of properties of 2D shapes and 3D objects, where learners "had difficulty in identifying regular and irregular 2D shapes" (Department of Basic Education, 2014a, p. 21), "could not name a common 3D object and displayed limited knowledge in finding the number of faces and naming the shapes" (Department of Basic Education, 2014a, p. 34). The second area was conversion of units in measurement where learners generally failed in "converting meters and centimeters to [only] centimeters" (Department of Basic Education, 2014a, p. 36), and to "convert the given mass from kilograms to grams" (Department of Basic Education, 2014a, p. 49). Finding the perimeter and area of shapes, as well as using terminology and definitions in geometry, are also areas of weakness in Grade 9. Learners were, for example, required to "demonstrate knowledge of and skills in finding the perimeter and area of 2-D shapes" (Department of Basic Education, 2014a, p. 53).

This article is part of a larger study, of which one of the aims was to shed some light on the role of teachers' enacted PCK on learner outcomes. In this paper, the first objective was to determine the gain in learner outcomes as revealed in a baseline (pre) and a post assessment. The second objective associated with this aim was to study teachers' enacted PCK, as informed by baseline assessment learner outcomes, when teaching Measurement. The components of teachers' enacted PCK under investigation were teachers' knowledge and skills related to learner understanding of the topic, and teachers' use of conceptual teaching strategies. The research questions we posed were: 1) How did learners' performance change after instruction? and 2) How can the teachers' enacted PCK in relation to learner outcomes be described?

1.2. Conceptual frameworks

The Refined Consensus Model (Carlson et al., 2019) identifies three realms of PCK: collective PCK (cPCK), personal PCK (pPCK) and enacted PCK (ePCK). Collective PCK constitutes the canonical PCK and is informed by research and discussion amongst peers. At the centre of the model is the enacted PCK of the teachers which a teacher reveals in planning and executing lessons. Both the pPCK and ePCK of a teacher develop as a teacher reflects and reasons about lessons previously taught, learners' responses in assessments and evidence of learning (Carlson et al., 2019). In this study the focus was on the enacted PCK of the teachers as revealed through classroom observations after they had access to learners' responses in a baseline assessment on measurement. The components of PCK that were particularly under investigation were teachers' knowledge and skills related to student understanding of the topic and knowledge and skills related to conceptual teaching strategies (Chan et al., 2019). Knowledge and skills related to student understanding involves a teacher's understanding of the pre-conceptions, naïve ideas and possible misconceptions learners come to class with and the teacher's competence in addressing those. Knowledge and skills related to conceptual teaching strategies refers to any appropriate and valuable strategy a teacher uses to create meaningful learning opportunities and a teacher's ability to select and use appropriate representations (examples, models, diagrams etc.).

2. METHOD

The study was conducted in a social constructivism research paradigm where we constructed our knowledge of the phenomenon by being actively involved in two Grade 9 mathematics teachers' practices from the same school. The purpose was to explore teachers' enacted PCK, as informed by baseline assessment learner outcomes when teaching Measurement. The assumption was that, knowledge of the outcomes of the baseline test, will inform teachers' knowledge of learner understanding of the topic which will, in turn, inform their instructional strategies. A mixed method was used where the quantitative part of the study used a Rasch analysis to determine the change in learner performances after instruction. The other part consisted of a qualitative exploratory case study where an in-depth investigation was done on the teachers' enacted PCK with focus on teachers' knowledge of learner understanding and their use of conceptual teaching strategies. We used purposive and convenient sampling in selecting the school. Through purposive sampling schools were selected that conform to the following inclusion criteria: (1) diverse in terms of social, economic and racial backgrounds and consequently also divers in terms of first language, (2) had an average performance in the 2014 ANA results, and (3) follow the DBE's curriculum. From this list, the school nearest to our working place was conveniently chosen. The two teachers were purposively chosen based on their experience, Alice having four years' and Mary having nine years' Grade 9 mathematics teaching experience. Two classes per teacher with a total of 124 participating learners were purposefully chosen based on their performances – one poor performing and one average/high performing class. This serves as a good representation of the group of Grade 9 mathematics learners. The data were collected using a baseline test, classroom observations, and a formative test.

2.1. Tests

The two tests were similar, except for given measurements in three of the questions. The tests were marked by us and handed back to the teachers prior to their instruction. The test items that were set by us and approved by the teachers, were taken from standardized tests for this grade based on the official Department of Basic Education's Curriculum and Assessment Policy Statement (CAPS). The purpose of the tests was to test learners' knowledge and understanding, prior and after instruction, of:

a) certain pre-concepts, by explaining in their own words the meaning of surface area, volume and capacity;
b) nets of a rectangular container and being able to say in words how the net is used to calculate the surface area, to use that explanation to write down a formula for surface area, and to finally calculate the surface area of the container with given measurements.
c) conversions between SI units: 1 m^3 to cm^3; 1 m^3 to *liter*; and a certain number of *ml* to cm^3, as well as finding the volume of the container, volume not occupied after a certain amount of water is poured in, and lastly the height of the water in the container.

2.2. Observations

There were five classroom observations for Alice and three classroom observations for Mary enabling them to complete the teaching of surface area, volume and capacity of prisms. Unlike Mary, Alice did not teach in her mother tongue. These lessons were video recorded, and afterwards transcribed verbatim. A deductive data analysis approach was used, where the observation data were analysed according to the components of enacted PCK as indicated in the conceptual framework.

2.3. Rasch analysis

Learner performance in the pre- and post-test were analysed using Rasch analysis employing the Rasch Unidimensional Measurement Models (RUMM2030) software. In this study the attribute being measured is learners' performance (person ability) before and after instruction of the topic. The person ability of a participant refers to his/her competence as measured by the instrument. During Rasch analysis person ability and item difficulty are analysed simultaneously and placed on the same numerical scale (Wright & Mok, 2004).

The ability measure of a person is defined by the position on the numerical scale. This information is displayed on a person-item map (see Figure 1) generated by the RUMM software, with person locations on the left-hand side and item locations on the right-hand side. A person with average ability has a 50% chance of getting the item at the 0.00 item location right. Such a person will have a higher chance of getting the items below 0.00 correct. In general, it shows that the items at the bottom of the diagram (low item location) are experienced as easy by the average student and the ones at the top of the diagram as difficult.

In a research field such as mathematics and sciences, it is important that the instrument used for measurement does not change. However, in a study such as this, one expects that participants' perception of the instrument changes because of the intervention, in the sense that, although the test stays the same, the students may find the items easier. A technique in Rasch analysis called racking the data enables the researcher to determine the change in item-difficulty from the pre-test to the post-test as perceived by the participants (Wright, 2003). With this analysis, inferences can be made about what knowledge was acquired and what was not acquired. Data is racked when the pre- and post-test are analysed simultaneously as one test with labels to distinguish between pre- and post-test items (for Example 1a and 1b). The pre- and post-test were designed such that items with the same number assess knowledge about the same concept. Consequently, with racked data the item location (and thus the perceived difficulty) of an item before and after the intervention can be tracked and compared.

3. RESULT AND DISCUSSION

In this study data were collected by a baseline test (pre-test) and a post-test consisting of 13 items. After the teachers had insight into the learners' responses to the baseline test, they taught the topic (considered as an intervention) after which the test was repeated (post-test). The responses were analysed using Rasch analysis and discussed qualitatively. Based on the assumption that learner outcomes should improve after instruction, we were interested in analysing teachers' enacted PCK when addressing content related to items that showed little or no improvement. Items were categorised in four categories, namely: (a) Conversions of units (Items 9, 10, 11); (b) Understanding concepts (Items 1, 2, 3, 4, 5, 6); (c) Calculations (Items 7, 8); and (d) Higher order application (Items 12, 13).

Before inferences can be made from the Rasch analysis we needed to establish that the research instrument and the sample fit the Rasch model to ensure the validity of the inferences. With the first run of the Rasch analysis, items 5 and 13 were flagged as extreme items as almost no participants obtained any marks for these items in both the pre- and post-test. Item 5 expected learners to explain in their own words how to use a net to calculate the surface area of a container, while Item 13 expected learners to determine the height of a given amount of water in a container of which the total capacity (in *ml*) was known. Consequently, these items were removed from the test post hoc. Although these items were not included in further Rasch analysis, the fact that they were labelled as extreme, indicated that learners experienced serious difficulties with these items and they will be discussed qualitatively.

The RUMM software also provides individual item and person fit residuals. A residual is related to the difference between the expected value and the observed value for a particular person or item. In the RUMM software, these values are set to be highlighted when they fall outside the -2.5 to 2.5 interval. A value outside this interval indicates substantial deviation from the model. The individual item fit residuals for all items in this analysis (except Items 5 and 13 that were labelled as extreme) lie between -2.17 and 1.50 and the individual person fit residual between -1.31 and 1.41 with two extreme persons removed; indicating that items and persons behaved as expected by the model.

Response dependence between items can be a threat to validity of the analysis because these items may measure the same concept and one of them can be considered as redundant. The response correlation matrix provided by the RUMM software indicated correlation between 0.5 and 0.6 for Items 6 and 7. This is to be expected since Item 6 required learners to give a formula for calculating surface area and Item 7 required learners to use the same formula by substituting appropriate values. It was decided not to delete these items for the analysis because of the differences detected in the responses between the pre- and post-test for these items.

The person-item-map obtained with racked data after the deletion of the extreme items is shown in Figure 1. This map enabled us to compare the item difficulties as experienced by students before and after the intervention. It can be seen that all but five participants' person location is below 0.00, which means that the learners in the sample performed poorly in both the pre- and the post-test. The obvious conclusion is that the test was far too difficult for this particular group of learners and that the expectations were unrealistic. However, keeping in mind that test items were taken from standardized tests for this grade based on CAPS, the investigation of the problem of poor performance leads us to investigate what happened in the classroom and with teachers' enacted PCK.

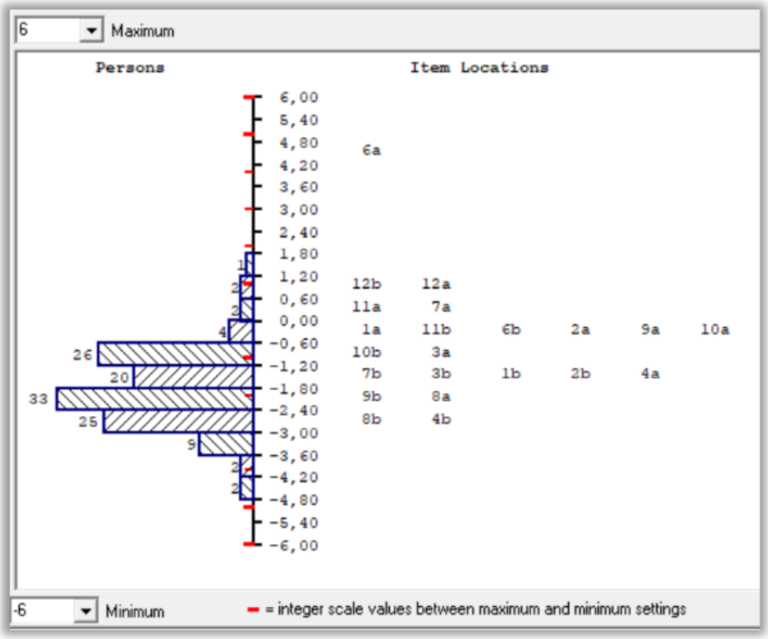

Figure 1. Person-Item map generated by RUMM2030

In the RUMM software we were able to distinguish between learners from the two teachers by using person factors. We investigated the difference in performance of the two teachers' groups per item by comparing the Item Characteristic Curves (ICC) (see Figure 3) of the pre -and post-test items. The Rasch program group the participants (persons) in three class intervals according to the person locations (person ability). The class interval structure for the current sample (N=124) is shown in Figure 2. For example, CInt1 is the group of participants with lowest person location and can therefore be considered as the participants with lowest performance in the test.

Teacher-by-CInt		CInt1	CInt2	CInt3
Mary	[55]	10	20	25
Alice	[69]	26	19	24
TOTAL	124	36	39	49

Figure 2. Class interval structure of the sample (N= 124)

3.1. Conversions (Item 9, 10, and 11)

In these items, learners were required to convert from 1 m^3 to cm^3 (Item 9); 1 m^3 to litre (Item 10); and 500 *ml* to cm^3 (Item 11). For Item 11 learners could obtain either 1 or 0 marks. In the ICC curve (see Figure 3) the person location is given on the horizontal axis while the average performance in the test for the item is on the vertical axis. The light grey curve represents the performance as expected by the model. Mary's learners are shown with circles and Alice's learners with crosses. The position of three markers (circles and crosses) on the lines show the performance of the three class intervals of each teacher. It can be seen that the learners in the first- and second-class interval in Alice's group received zero out of one for this item in the pre-test (Item 11a). The average for the learners in the highest-class interval was 0.2 out of 1. Mary's group performed slightly better but an ANOVA ($p = 0.842$) shows no significant difference between the learners of the two teachers.

The ICC curve for Item 11 in the post-test (Item 11b) shows that the learners in the highest-class interval of Alice's group performed better (with an average just above 0.6 out of 1) than in the pre-test and also significantly better than Mary's highest group ($p = 0.000$). This was however, only for the highest group; the learners in the first- and second-class intervals still did not obtain marks for the item.

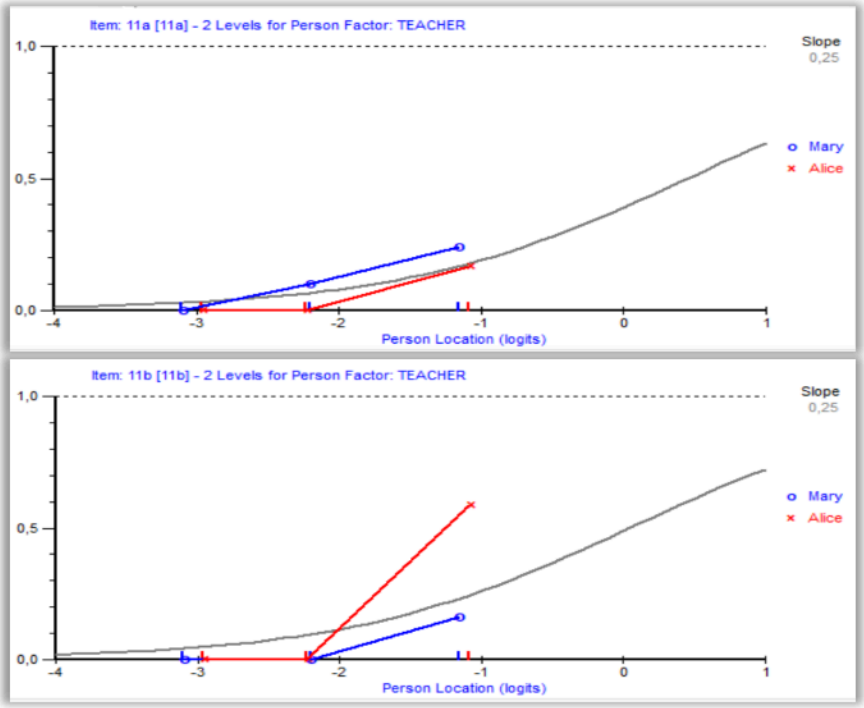

Figure 3. ICC curves for Item 11 in the pre- and post-assessments

This tendency that only the highest-class interval of Alice's group shows improvement, is also visible in Items 9 and 10. A qualitative discussion of what transpired in the classes about conversions, follow.

Alice addressed conversions in Lessons 1 and 5. She started using a rhyme: "Kids hate doing maths during cloudy Mondays" and wrote the symbols used for the units of measurement of length on the board: K H D m d c m, indicating kilometres, hectometres, decametres, meters, decimetres and metres. She continued: "If you go from millimetres to metres, you go left and each jump counts one zero. If you work with square metres (and she added the squares above), each jump counts two zeros (indicated it on board). Now this was also in the test, if you have a cm by a cm by a cm, it is 1 ml. (and she wrote c^3 d^3 m^3 below c^2 d^2 m^2 and wrote *ml* below c^3). Now 10 *cm* by 10 *cm* by 10 *cm*, that is a litre (wrote l on the board). And then a metre by a metre by a metre is a kl. So, if you have to go from cubed cm to ml, it is exactly the same. So, if you have to go from millilitres to cubed meters, in other words, from small to large (demonstrate with arms crossed to arms open), you have to divide with three, six zeros (wrote ÷1 000 000 on the board)." In Lesson 5 she revised the conversions between capacity and volume and told them to memorise that and to remember that each jump counts three zeros. She did three more examples in the same manner as in Lesson 1: convert from cubed *cm* to *kl*; from *l* to cubed *cm*; and 3 *l* to cubed centimetre. Mary attended to this question only in Lesson 3. A similar homework problem that she discussed, required the volume of a rectangular prism and afterwards the capacity of the same container. She demonstrated the solution on the board, ending with an answer of 832000 cm^3. To find the capacity, she said: "We need to do a conversion now and a 1000 cm^3 is 1 litre (Wrote 1000 cm^3=1*l* on the board). This you need to know please. How will I convert from cm^3 to *l*?" Learners: "Divide with a 1000." Teacher: "So, the answer is 832000 divided by a 1000, and it is 832 *l*."

Although conversions is prescribed in the curriculum for Grades 7 to 9, the focus in the classrooms was on procedural knowledge and memorisation of rhymes and rules, resulting in learners lacking conceptual understanding thereof. Although Alice used a table, the information on the table was confusing as the symbol m was used for both metres and millimetres. Analysis shows that only the learners in Alice's most abled group showed improved outcomes and that learners in the least and average abled groups of both teachers showed no improvement at all. If we compare the teaching of the two teachers, the difference may be due to the fact that Alice spent more time on the concept and have more elaborate explanations and examples of conversions. Although Alice spent more time on explaining conversions, we do not have enough evidence to conclude that this is the reason why her highest class interval performed so much better, because the lower class intervals still performed very poorly.

The teachers had access to the responses of learners in the baseline test and, as such, to their prior and alternative conceptions. According to Duit and Treagust (2003), the learners should become dissatisfied with their prior conceptions during instruction and be willing to replace it with an intelligible and plausible idea. Only then accommodation of the new concept will follow. The explanations of both teachers did not support conceptual development for most of the learners. Apart from our finding that learners find conversions challenging, our experience at a tertiary institution also reveal that undergraduate students find conversions difficult. This concurs with the finding of Tariq (2008) who found that a high percentage of first year Bioscience students had difficulties with the conversion of units of measurement.

3.2. Understanding the concepts (Item 1, 2, 3, 4, 5, and 6)

These items assess learners' understanding of the concepts surface area, volume and capacity where they had to explain the concepts in their own words, draw the net of an open rectangular container and write down the formulae. All items were experienced difficult by the majority of learners. There was a slight improvement after the intervention in Item 4 where learners had to draw a net of a rectangular container with an open top. When looking at the item map (see Figure 1), it seems that Item 6 has the biggest improvement, but it should be mentioned the improvement was from zero to six learners answering it correctly in the pre- and post-test respectively. Even though responses to the items seem to have improved, all these items were still experienced as very difficult by most learners. The two questions with the lowest improvement were Items 3 and 5, with Item 5 being flagged as an extreme item. In an attempt to understand this poor improvement, we qualitatively analysed the two teachers' enacted PCK on these two items.

In Item 3, learners had to explain capacity in their own words. Alice spent about half a minute in both Lessons 4 and 5 to revise the definition of capacity. In Lesson 4 she said: "Volume is the space it takes up. Capacity talks about fluids. It's the same thing, it is also what is inside, we calculate it the same way, it's just the unit that is going to change. So, if I fill it up with chocolates, its volume, and if I fill it up with melted chocolate, its capacity." Only a few learners actually listened as there were a number of poor behaving learners disrupting all her lessons. The result was that the lack of discipline caused the teacher to be very irritated. Later in Lesson 5 she said: "If we look at capacity, as we said in the beginning, volume is the space it wells up. Capacity is the amount that wells inside. Capacity is fluids. Its measured in millilitres, litres, kilolitres." One may consider the fact that Alice is teaching in her second language. In this explanation she attempted to use the word 'well' as a verb and it could be understood as follows: Volume is the space [in which] it [the liquid] wells up. Capacity is the amount [of liquid] that wells up inside. It was in Lesson 3 that Mary marked a homework problem based on volume and capacity that she mentioned the difference between capacity and volume by saying: Volume is the amount of space the box occupies, then, how much fluids or whatever fits in the box, is the capacity.

Even though Alice gave a more extensive explanation than Mary of the difference between capacity and volume, there is a slight, but not significant difference between the two teachers' groups, with Mary's class slightly better. While Alice is teaching in her second language, Mary is teaching in her mother tongue and Alice's explanation may have been confusing to the learners.

In Item 5, learners were required to explain how the net of an open rectangular container, asked in Item 4, will be used to calculate the surface area. Alice covered the concepts related to these items in the first three lessons. Regarding the surface area, she said in lesson 1: "It is the area of all the faces added together. So, it is all the sides (showing the different faces), we add it together. In Lesson 2 she repeated how to find the surface area of a cube with length l, saying the area of one face is $l \times l$, and there are 6, therefore $6 \times l \times l$." Alice then had the formula of a rectangular prism on the board: $l \times w \times 2 + l \times h \times 2 + w \times h \times 2$, and by using a box, she explained the formula. A learner then asked: "If there are different rectangular prisms, will the formula change?" Alice answered: "When it does not have a top, then it is going to change." In Lesson 3 she had the formula written on the board: Surface area = $2(lw+lh+wh)$, saying: "You must know, if this one isn't there (showing a side of the box), you have to take one of them out", and she drew a cross over $w \times h$ inside the brackets and added $w \times h$ after the bracket. This explanation was however built on using the formula and not a net to calculate the surface area. Mary covered concepts related to this item only in the first lesson. The quotes are direct translations from Afrikaans to English. Learners had to write Rectangular prism in their books. Teacher: "It consists of different faces (and she indicated to the steel cabinet in the corner of the class)." Teacher: "How many?" Learners: "Six." Teacher: "Are all the same size?" Learners: "No." The teacher then used a net and showed that the opposite faces are the same and said: "There are three sets of rectangles that look alike. And remember, the rectangle is 2D. How do I calculate the area of a rectangle? It is $l \times b$. But I cannot say times six because they are not all alike." She then wrote three separate formulae on the board under the headings: top and bottom; front and back; and two sides, and then combined it in a final formula for the surface area.

Responses to the test item that required learners to draw the net improved significantly from the pre- to post-test, but despite of this the learners did not improve on using that net to explain in words how the net will be used to calculate the surface area. A possible reason can be that learners lack appropriate language to express their reasoning. There are two concepts involved in this question: how to use the net to calculate the surface area; and to do this for an open container. To conclude, conceptual understanding of a concept, as required in the last two items, is the foundation for learners to develop further knowledge and skills.

The document Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding (Department of Basic Education, 2018), emphasizes the necessity that learners should be involved through activities, communication and the use of concrete material to develop conceptual understanding. Furthermore, Gooding and Metz (2011, p. 36) claimed that teachers should "facilitate their [learners'] learning and encourage student discourse, individual reflection, metacognition, and acceptance of alternatives". However, there was no such evidence of the teachers attempting to establish learners' understanding of the topic. There was no meaningful oral discourse or any activity where learners were actively involved in doing and discussing in order to enhance their understanding, instead, only the teachers were talking and demonstrating. Regarding representations, the teachers used either just words, or words and models respectively when addressing these two questions.

3.3. Calculations (Item 7 and 8)

The ICCs for Items 7a and 7b (see Figure 4) show a similar improvement from pre- to post-test for both teachers. Item 8 followed the same tendency, but was perceived as one of the easiest items. It is evident that the greatest improvement was shown by participants in the highest class-interval but that the average achievement was still far from full marks. Both items involved the substitution of given values into a formula. Item 7 expected learners to determine the surface area of a rectangular container with an open top. The learners used the formula of the surface area of a rectangular container but did not know how to deal with the open top as can be seen by the fact that the most able learners obtained only one out of two marks (see Figure 4).

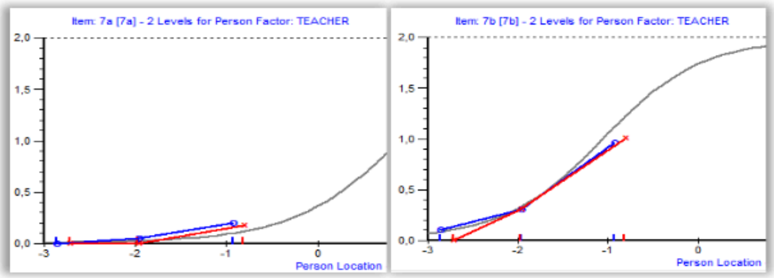

Figure 4. ICCs for items 7a and 7b

Item 8 required learners to calculate the volume of the same container where the open top was not an issue to be considered when applying the formula. In the post-test learners performed the best in this item with only 25 learners out of 214 still not able to achieve any marks for the item. These items resort under the first (Item 8) and second (Item 7) cognitive levels described by CAPS (Department of Basic Education, 2011b) where only knowledge recall and routine procedures are expected.

3.4. Applications (Item 12 and 13)

In Item 12, learners had to determine the volume of a container not fully occupied with water. Neither Mary nor Alice addressed this issue in their teaching. Both teachers only discussed and marked homework where calculation of the entire volume was required and the three dimensions given. Evidence shows that learners in the least and average abled groups of both teachers showed no improvement at all, obtaining no marks in the question in both the pre-and post-tests. Although the group of most abled learners in Alice's class showed a slight improvement, it was actually only six learners who improved in this question. This was one of the most difficult items for the learners as it requires learners to analyse the question and consider the relevance of previous answers. They should have realised they had to subtract the answer from Item 11 (volume of water converted from capacity) from the answer to Item 8 (volume of container), thus resulting in a higher order question. To find the volume of the container NOT occupied by water, learners needed to realise the question is about volume and not capacity and that the unit of measurement is cm^3.

Not one of the teachers referred to Item 13 in the baseline test. This problem integrates with Algebra where the subject of the equation needs to be changed. Learners further need to know that height is one-dimensional, and the unit is cm. This item was flagged as an extreme item, because there was almost no correct answer in the pre- or post-test. It is evident that learner performance in higher order application problems did not improve during the teaching of the topic. Learners were not challenged with such level of application in the classroom. These items are considered to be at the highest cognitive level namely problem solving according to CAPS (Department of Basic Education, 2011b). Mosimege et al. (2016) report that South African learners find questions difficult that require detailed reading, interpretation and problem-solving. They conclude that problem-solving is not sufficiently addressed in many mathematics classrooms in the country.

4. CONCLUSION

The mean person location (an indication of learner ability) improved from -2.37 (sd =1.27) for the pre-test to -1.42 (sd = 1.30) for the post-test. The baseline performance was extremely low despite the fact that the content for the Grade 9 learners had been taught in increasingly difficulty levels since Grade 4. Although some improvement is evident after the topic has been taught, the test was still experienced as difficult by almost all the learners. We were interested in exploring the teachers' enacted PCK and learn from the creative and conceptual teaching approaches teachers would use to address these errors and misconceptions.

As such, the study explored how the baseline assessment informed teachers' instruction after they have had access to the outcomes of the test. Our working assumptions were that experienced teachers possess rich PCK regarding their learners' common errors and misconceptions, but also learners' understanding of difficult concepts. We assumed that they will reflect on the responses received in the baseline assessment and that they will draw from their pPCK to address the difficulties. The baseline assessment outcomes were intended to provide the teachers with an opportunity to identify specific aspects or concepts where their learners lacked prior knowledge and demonstrated poor understanding. This practice of interpreting and using baseline assessment is expected from South African mathematics teachers as underlined in the CAPS document (Department of Basic Education, 2011b, p. 223), "Knowing learners' level of proficiency in a particular mathematics topic enables the teacher to plan her/his Mathematics lesson appropriately and to pitch it at the appropriate level. Baseline assessment, as the name suggests, should therefore be administered prior to teaching a particular mathematics topic".

However, from the observations there is little evidence that the experienced teachers extensively used the baseline assessment outcomes to inform their teaching. For example, the two most difficult items, Items 12 and 13 were never addressed in any of their lessons. Although Mary's learners showed a greater overall gain in learner outcomes after instruction, it is minimal. Evidence demonstrated during their instruction, reveal that their pPCK does not inform their ePCK or that there are filters present (Carlson et al., 2019) that prevented them from drawing from their pPCK when teaching this topic; an aspect we did not explore in this study.

It should be mentioned that enacted PCK is subject to various complexities and challenges embedded in the context of mathematics teaching in South Africa. If it is true that the overload in the Intermediate phase mathematics curriculum causes insufficient time for teachers to develop learners' conceptual understanding (Mosimege et al., 2016), it suggests that different tuition approaches are required where learners can be creative, and be involved in discovery in order to learn with impact and understanding. Similar to the general findings of the TIMSS 2019 results (Reddy et al., 2020), the emphasis in these two classes was also on developing procedural instead of conceptual knowledge. Teachers should realise that a learner-centred approach where learners are actively involved in activities, allowing them the opportunity to discuss and discover, results in more sustainable knowledge development. Although the teachers used physical objects and nets of 3D objects to demonstrate, the learners should be engaged in meaningful tasks where they had to physically work with the nets of prisms. This kind of learner engagement will contribute to their curiosity and interest, allowing for meaningful discourse as they can ask questions, listen to others' thinking, justify their reasoning, answering questions, all contributing to developing conceptual understanding (Botha, 2012).

According to the CAPS for Intermediate phase Grades 4-6 (Department of Basic Education, 2011a), developing an understanding of the concepts of surface area and volume of 3-D objects begins in Grade 4. The relationship between surface area and volume of rectangular prisms (asked in the baseline test), is addressed from Grade 6 (Department of Basic Education, 2011a) and continues to be described in the CAPS Senior Phase Grades 7 to 9 (Department of Basic Education, 2011b). It should be borne in mind that Grade 9 is the last year this topic is explicitly taught for learners who continue with Mathematics to grade 12. With this in mind, it is evident that teachers' ePCK, the use of appropriate and effective teaching approaches and their use of the opportunities afforded by baseline assessment outcomes need to receive serious attention.

In the current study, certain aspects emerged that have a negative impact on the instruction and consequently learner understanding, namely, the language proficiency of a teacher and the lack of discipline in class due to bad learner behaviour. These factors may act as filters for transfer of pPCK into ePCK (Carlson et al., 2019). In Alice's class some of the explanations were unclear since she was explaining in her second language. Language of instruction, especially when teachers are not teaching in their mother tongue, plays an influencing role on learner performances (Blömeke et al., 2011). The role of language is therefore pivotal and it is the teachers' responsibility to "motivate learners in the use of comprehension skills in class when complex language and terminology are used" (Mosimege et al., 2016, p. 98). Language proficiency does not only refer to teachers, but to learners too. Learners need to develop their mathematical language skills and neither of the participating teachers gave learners the opportunity to express their understanding of a concept in their own words and it can therefore not be expected of learners to perform well in such items.

The important aspects that need to receive attention in pre-service and in-service teacher training are: (a) the use of the correct mathematical language in the class and the awareness teachers should have about the challenges learners have in learning in their second language; (b) the practical interpretation and use of baseline and formative assessments and how this should inform their instructional strategies; and (c) learner centered strategies, discourse in the mathematics classroom, contextualized learning and problem-solving skills.

In Alice's classroom, the difficult and challenging group of learners continually disrupting all her lessons, caused her to be irritated and angry, and she struggled to keep her calm. According to Carlson et al. (2019), learner attributes and the classroom environment influence the teaching and learning taking place, and TIMSS results (Reddy et al., 2020) also reported on the negative role of poor discipline in classrooms. According to Hodgen (2011), the resulting affective emotions experienced by a teacher, also have an influence on the teacher's enacted PCK. The outcomes of the current study and emergent findings therefore suggest further study in the amplifiers and filters that impact the transfer of a teachers' personal PCK to the PCK enacted in the classroom. The study points to aspects such as language proficiency of the teacher, discipline in the classroom and overload of the curriculum. To conclude, although Depaepe et al. (2013) found that several studies reported on the positive relation between teachers' PCK and learners' learning outcomes, we realise that the gain in learner outcomes depends on more than only the teaching intervention and that this leaves scope for further research.

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