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Effects of higher education subsidies on equity and efficiency across developmental stages[☆]

Yoseph Getachew

Department of Economics, University of Pretoria, 0028, Pretoria, South Africa

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ABSTRACT

This study investigates the impact of taxes and subsidies for higher education on equity–efficiency trade-offs under various phases of higher education development. A scholarship program emerges as the most effective subsidy for higher education across all developmental stages. During both early and later phases, a universal grant proves more effective than a scholarship grant in reducing inequality. At the later stages of higher education development, the enrollment rate increases for universal grants but decreases for other policies, implying that the recent shift away from universal grant schemes in the UK could lead the enrollment rate to decline.

1. Introduction

The debate over whether the government should subsidize higher education financing revolves around regressivity and externality effects. On the one hand, higher education subsidies and grants become a concern of transferring resources away from unskilled to skilled workers (Hansen and Weisbrod, 1969; Fernandez and Rogerson, 1995; Garcia-Penalosa and Wälde, 2000; De Fraja, 2002). On the other hand, they are justified based on the externality effects of human capital[1] and the pervasiveness of borrowing constraints that prevent individuals from investing optimally by borrowing against future human capital (Barham et al., 1995; Fender and Wang, 2003). Furthermore, a third case for education subsidies exists, alleviating the distortions in human capital owing to redistributive policies such as progressive taxation (Benabou, 2002; Bovenberg and Jacobs, 2005; Krueger and Ludwig, 2016).

A common feature of the literature is its failure to account for the different forms of the higher education system. The structure of a country's higher education system, particularly the developmental stage in which it exists, primarily determines the equity and efficiency impact of any higher education financing policy it adopts. When two countries are at different stages of higher education development, they will have different enrollment rates and class compositions, which in turn may create disparities in the effectiveness of higher education policies. Several countries in the developing world are at stages where higher education is a luxury consumption good enjoyed by a few elites.

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E-mail address: yoseph.getachew@up.ac.za.

[1] There is some support from the empirical literature concerning human capital externality, but not without dispute. Moretti (2004) estimated human capital externality (the effects of one more year of average education on income) up to 25% for the US. In contrast, Krueger and Lindahl (2001) and Acemoglu and Angrist (2000) argued that the difference between the social and private education returns is not significantly different from zero for the US. Benhabib and Spiegel (1994) found no relationship between human capital and growth but a positive relationship between human capital and total factor productivity.

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In contrast, for economies in the developed world, the "massification" of higher education is at an advanced stage where most of their population has access to it.[2]

This study investigates how alternative higher education subsidies and grants impact efficiency and equity when accounting for the *transition* and *different phases* of higher education.[3] In doing so, it develops a model that captures the endogenous transformation of higher education development. Subsequently, it offers a comprehensive analysis of the effects of alternative higher education subsidies on efficiency, equity, and enrollment rates. In the model, agents are heterogeneous in their initial human capital and learning ability. Individuals are differentiated as college-educated and non-college-educated based on their family background and as high and regular ability based on their learning ability. Children of parents who can afford to pay the minimum tuition fee up-front will join college. However, those whose parents cannot afford to pay the college tuition fee will join the unskilled labor force and earn a lower wage income. Further, individuals with a college education receive an additional skill premium. Skilled labor (human capital) and raw labor are the only factors of production at the aggregate level. Parents' ability to pay for children's education depends on their income and their children's ability.

The economy starts from Stage I – an early stage of development – and evolves endogenously afterward. The aggregate human capital is extremely low at this stage; hence, productivity and income are low. Only the upper class (wealthy families with high-ability children) can afford to pay the college tuition fee while the rest of the population does not. As the economy continues to grow and individuals' productivity and income increase due to positive human capital externality effects, more individuals could afford college tuition fees. In Stages II and III, the middle class (affluent families with regular-ability children) and lower-middle class (low-income families with high-ability children) can afford to pay the tuition fees. In Stage IV, aggregate productivity is large enough for everyone (including low-income families with regular-ability children) to afford the college tuition fee. The different phases of higher education can be associated with the stages of economic development that today's higher-, upper- and lower-middle, and lower-income countries exist, as reflected in the data (Fig. 1).

The government could adopt one of the three commonly practiced tuition subsidy programs in any developmental stage and finance it with flat-rate taxes. It can apply a universal tuition grant scheme that targets any individual who joins college, a scholarship grant scheme that targets high-ability individuals, or a means-tested grant scheme that targets high-ability individuals from poor backgrounds. I examine how these policies affect individuals' ability and decision to invest in higher education and their implications for equity–efficiency trade-offs at the different phases of higher education development. Furthermore, I compare each policy with a laissez-faire education system.[4]

Among the findings, a scholarship program is the most efficient higher education subsidy program at all phases of higher education development as it mobilizes resources to high-ability individuals in the economy. Moreover, a means-tested grant is the least efficient policy in the early Stages I and II, as few are eligible for this program during these stages. However, it is the most efficient one (along with the scholarship scheme) in Stages III and IV through mobilizing resources for the ablest individuals in the economy. Laissez-faire is the least efficient in Stage III when high-ability individuals of low-income families have access to higher education. At this stage, government intervention is required to ensure that resource-poor but high-ability individuals would not be left behind. A universal subsidy scheme performs as the second-best in most developmental phases.

In the early stages (Stage II),[5] the universal grant scheme is the most effective in reducing inequality, followed by laissez-faire. Since individuals in lower income groups do not invest in education at this stage, a scholarship grant or means-tested may aggravate inequality. The former is the most regressive as it mobilizes resources to the high-ability individuals in the top-income class, and means-tested leaves everyone worse off, as none of the groups who invest in education at this stage qualify for the program. However, taxation (not followed by subsidy) seems to hurt the middle-income group more. In Stage III, the means-tested grant is best in reducing inequality, followed by the scholarship program. Both programs (particularly means-tested) selectively target high-ability individuals of low-income families who begin investing in higher education during this stage. Laissez-faire is the worst in Stage III; however, in Stage IV, the universal subsidy grant scheme is the most effective in reducing inequality, followed by means-tested, as it targets individuals in the bottom income group who begin investing in higher education at this stage.

Regarding college enrollment rate, in Stage I, universal and scholarship grant schemes (vis-à-vis laissez-faire) have similar positive effects. Further, I find that the enrollment rate increases in universal subsidy but decreases in other policies in Stage IV. This result primarily confirms other studies that find the policy shift in 2012 has led to a decline in the enrollment rate in the UK (Geven, 2015).

This study is closely related to two strands of literature. Notably, it is related to the literature that compares the efficiency and equity effects of different financing systems such as Garcia-Penalosa and Wälde (2000), Caucutt and Kumar (2003), Akyol and Athreya (2005), Cigno and Luporini (2009), Del Rey and Racionero (2010), Abbott et al. (2019). For example, Garcia-Penalosa and Walde (2000) examine the equity and efficiency effects of general tax subsidies, pure and income-contingent loan schemes, and graduate tax. They argue that efficiency targets could be achieved with the universal tax-subsidies scheme but not equity and efficiency targets simultaneously, as the scheme is regressive. A more comprehensive and unifying work has been done by Abbott et al. (2019), who consider individuals' decisions through different life cycle stages—from high school to retirement; whether to attend high school and college; and whether to complete or drop out of high school and college. Moreover, they consider uncertain returns to investment in education, endogenous life span, and parental transfer of resources. They calibrate their model for the US economy and conclude that the current financial system in the US is welfare-improving. However, they do not address equity issues. An important difference between this study and the literature is that human capital externality is modeled here as the driving force of economic development, enabling the examination of the equity and efficiency effects of higher education subsidy policies considering the different phases of higher education development that countries go through.

This study is also related to the unified growth theory and the literature that focuses on altruistic parents who face a warm glow utility and human capital investment threshold (Galor and Zeira, 1993; Moav, 2002; Galor and Moav, 2004; Getachew, 2016; Zeng and Zhang, 2022), which defines individual investment and consumption decision. However, this literature abstracts from higher education policies but inequality and growth issues.

[2] Tables 11 and 12 in Appendix F demonstrate that the gross enrollment ratio in tertiary education of some low-income countries in 2013 is comparable to that in high-income countries registered in 1971.

[3] Almost half a century ago, Trow (1973), in a seminal work, predicted the transformation of the higher education system of today's advanced economies from an elite to a mass and a universal phase. Trow (1973) identifies the elite phase, where less than 15% of the high school cohorts move beyond the secondary level; the mass phase, where 16%–50% of high school graduates continue their educations; and the universal phase, where over 50% of graduates continue their higher education. See also Trow (2007).

[4] The analysis does not include student loans, which are extensively studied in the literature (Garcia-Penalosa and Walde, 2000; De Fraja, 2002; Del Rey and Racionero, 2010; Gary-Bobo and Trannoy, 2015; Heijdra et al., 2017; and Abbott et al. 2019).

[5] In Stage I, only one group of individuals (the upper class) invests in human capital.

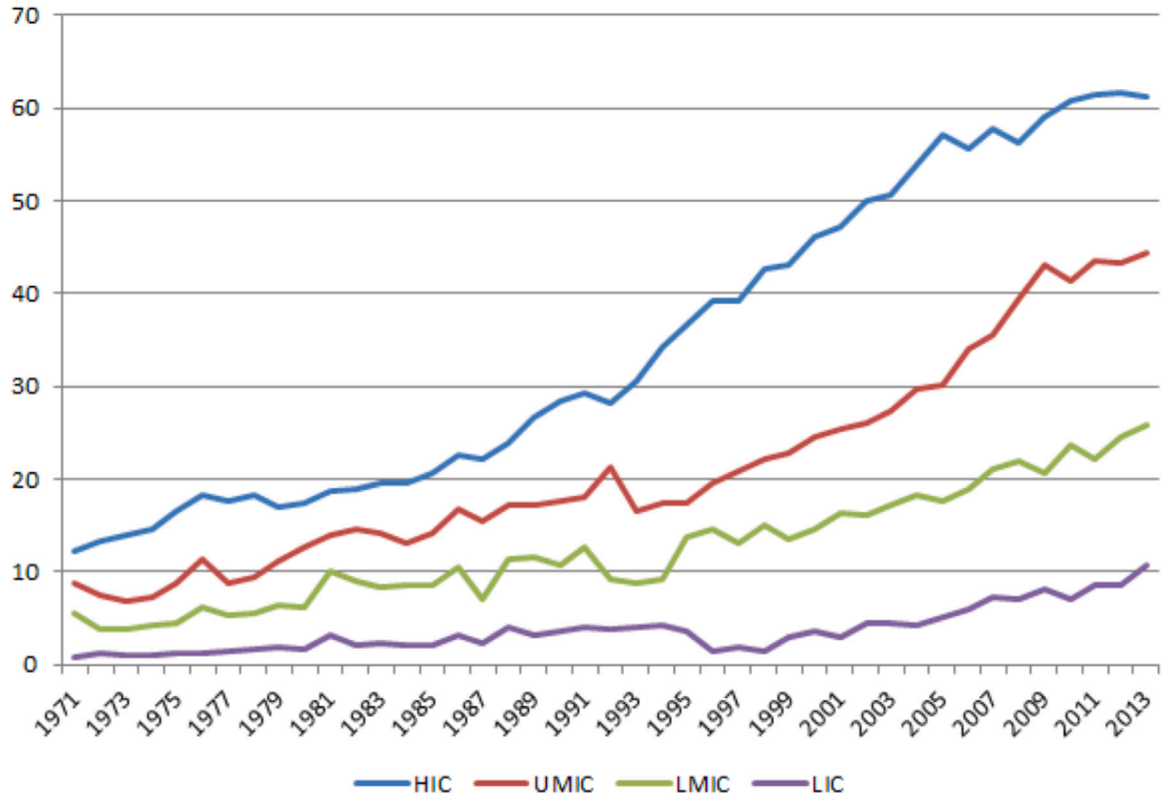

Fig. 1. Evolution and stages of higher education of countries at different stages of development.
Note: UIC, Upper income countries; HMIC, Upper middle-income countries; LMIC, Lower-middle income countries; LIC, Lower-income countries.

2. The model

Suppose heterogeneous agents in an overlapping generations model. The size of the population is one. In the beginning, at time $t = 0$, λ number of individuals are college-educated; therefore, the remaining $1 - \lambda$ are non-college-educated. Moreover, I assume the offspring of these individuals differ in their learning ability, denoted as high and regular ability, and their sizes are represented by p and $1 - p$, respectively. Following Galor and Moav (2004), individuals are assumed to be *ex-ante homogeneous within a group*, ensuring their offspring are also homogeneous, thereby dividing the population into four classes: (loosely identified as) the upper, middle, lower-middle, and lower classes. In every period, any individual is identified as a descendant of one of these groups. For example, a fraction of λp adults are identified as descendants of the upper class. Similarly, a fraction of $(1 - p)\lambda$, $(1-\lambda)p$, and $(1-\lambda)(1-p)$ adults are known as descendants of the middle, lower-middle, and lower class, respectively.[6]

Individuals live for two periods, namely, as a young person and as an adult. Each individual is born with a unit of time. Conditioned on parental investment (covering a fixed college tuition fee plus other variable costs such as books and laptops), they could accumulate human capital by joining a college. A college education is possible only if the minimum tuition fee is paid up-front. Therefore, only households that can afford the tuition fee (and find it optimal to do so) will send their children to college. Otherwise, the child joins the unskilled labor force when they become an adult.

2.1. Human capital and preferences

The human capital of an individual with family education background i and ability j who is born at date t is given as follows:

$$h^j_{it+1} = \epsilon^j e^j_{it} \tag{1}$$

where e^j_{it} represents additional parental investment (other than a fixed tuition cost) in education. Implicit in Eq. (1) is that human capital will be fully depreciated at the end of each period. Such specification is not as restrictive as may at first appear. It is appropriate given that human capital is embedded in individuals who have a finite life.[7] Parents who send their children to college must pay a fixed tuition fee up-front. If a parent chooses $e^j_{it} = 0$, they do not need to pay the tuition fee, but their child grows as an unskilled worker with no human capital $h^j_{it+1} = 0$. ϵ^j represents the learning ability of a child, where $j \equiv \{g, r\}$. g and r denote high and regular ability, respectively. $i \equiv \{c, n\}$ denotes the agents' educational background, where c and n represent college-educated and non-college-educated, respectively.

Suppose the following warm-glow utility function with logarithmic preferences.[8]

$$u^j_{it} \equiv \ln c^j_{it} + \beta \ln\left(h^j_{it+1} + 1\right) \tag{2}$$

[6] Thus, as we see later, growth comes in this economy either from similar individuals (who belong to the same descendant) who continue investing in human capital or/and from other groups (who are not previously investing in human capital) joining them in investing in human capital.

[7] Besides, it helps to obtain closed-form solutions without loss of generality. Incorporating parental human capital in the production function to capture intergenerational externality, for example, will not change the main results.

[8] The use of logarithmic utility function is ubiquitous in the literature (Glomm and Ravikumar, 1992; Galor and Zeira, 1993; Banerjee and Newman, 1993; Benabou, 2000; Moav, 2002; Galor and Maov, 2004). Its main advantage (*vis á vis* other dynastic altruistic models that assume parents derive utility from the utility of their children) is its greater analytical tractability while keeping the qualitative results of the model unaffected.

The utility function is thus set up to allow for the corner solution $h^j_{it+1} = 0$. Such utility functions are commonly applied in the growth and inequality literature that relies on specific threshold requirements to generate long-lasting inequality. For example, Moav (2002), Fishman and Simhon (2002), Galor and Moav (2004), and Asiedu et al. (2021) applied similar utility functions to generate a convex bequest (savings) and human capital accumulation functions.

The budget constraints are given as follows:

$$c^j_{it} + \widetilde{s}_t.\mathbf{1}_{\left\{e^j_{it}>0\right\}} + e^j_{it} \equiv I^j_{it} = \begin{cases} (1-\tau)\,w_t \text{ if } h^j_{it} = 0 \\ (1-\tau)\,w_t + (1-\tau)\,\phi_t h^j_{it} \text{ if } h^j_{it} > 0 \end{cases} \tag{3a}$$

where

$$\widetilde{s}_t \equiv \begin{cases} s_t - x_t \text{ if eligible for subsidy} \\ s_t \text{ otherwise} \end{cases} \tag{3b}$$

and

$$c^j_{it} \geq 0,\ e^j_{it} \geq 0 \tag{3c}$$

$$\epsilon^j > 0 \tag{3d}$$

where x_t represents a per capita tuition grant provided by the government to eligible individuals.[9] τ denotes the fixed tax rates imposed on income. c^j_{it} is the household's consumption, and I^j_{it} is the disposable income of the adult. An individual's income is determined based on the individual's educational background, that is, whether they received a college education as a child at date $t-1$ (or equivalently whether they are skilled at date t). w_t and ϕ_t are the wage rate per unit of labor and the skill premium, respectively. A skilled individual's disposable income constitutes labor income and skill premium minus the respective labor and capital taxes; for an unskilled person, it includes their after-tax labor income. The tuition cost that an eligible household has to pay up-front if they choose to send the child to college ($e^j_{it} > 0$) is $\widetilde{s}_t \equiv s_t - x_t$. I make two assumptions here. First, tuition cost is proportional to aggregate human capital (it increases with economic development), and second, it is different at different stages.

$$s_t = s_k + sh_t$$

where $s > 0$ is a parameter. $s_k > 0$ is the exogenous component of the tuition cost where $k \in \{I, II, III, IV\}$ reflects that it could differ at different stages of development. Therefore, with government intervention, the tuition fee for those eligible individuals who invest in education is reduced by x_t. However, individual incomes that are available for investment are reduced due to tax duties. Ineligible households who send their children to college, however, incur the full tuition cost ($\widetilde{s}_t \equiv s_t \neq 0$) and still pay their taxes accordingly. Meanwhile, families who do not invest in higher education ($e^j_{it} = 0$) will not pay the tuition fee ($\widetilde{s}_t \equiv s_t = 0$) and consume the full amount of their after-tax income.

2.2. The firm

There is a representative firm that operates in a perfectly competitive market. The firm uses both skilled and unskilled labor to produce the final product, where the latter is augmented by the aggregate capital stock in the economy (in the spirit of Romer (1986)). With Cobb–Douglas technology, the production function could take the form $y_t = A\left(h_t\right)^{\alpha}\left(l_t h_t\right)^{1-\alpha}$ where y_t is aggregate output, produced using aggregate human capital (h_t) and augmented-labor ($l_t h_t$), A is a constant total factor productivity (TFP), and α is a factor share.

Prices per unit of labor and human capital are thus given as follows, respectively:

$$w_t = (1-\alpha)\,Ah_t \tag{4a}$$

$$\phi = \alpha A \tag{4b}$$

where $l_t = 1$. Implicit in condition (4b) is perfect substitutability (or homogeneity) among skilled workers. Both high- and regular-ability individuals receive similar rates per unit of human capital holdings. The only difference between these individuals is thus the quantity but not the quality of human capital they possess.

2.3. Government budget

Given that there are λ college-educated and $1-\lambda$ non-college-educated individuals at time t, the total number of tax-payer individuals is unity, implying that in a balanced budget, the total government revenue, which is the sum of taxes collected from the labor income of skilled (λw_t) and unskilled individuals ($(1-\lambda)\,w_t$) and human capital incomes ($\lambda\phi h^j_{it} = \phi h_t$), equals to the total education expenditure (z_t).

$$z_t \equiv \tau w_t + \tau\phi h_t \tag{5a}$$

Using (4) this can be rewritten as follows:

$$z_t = \tau A h_t \tag{5b}$$

where τ is the grant ratio—the fraction of aggregate income used for public subsidy. Note that z_t is the aggregate tuition grant available at time t and could be different from the amount of tuition subsidy available per person (x_t).[10]

2.4. Optimal education investment

The solution for the ith household education investment is given as follows:

$$e^{j*}_{it} = b\left(I^j_{it} - \widetilde{s}_t\right) - b/\left(\beta\epsilon^j\right) \tag{6a}$$

where $b \equiv \beta/(1+\beta)$ and I^j_{it} is defined in Eq. (3a).

Three observations immediately follow, from Eq. (6a). First, individuals with a total income below the tuition fee ($\widetilde{s}_t$) *cannot afford* to send their children to college, considering that they face borrowing constraints. Second, even those who could afford the fixed college tuition fee may not necessarily invest in higher education, as they may not find it optimal. Third, agents with high income and high-ability children are more likely to send their children to college than their counterparts. Therefore, income, tuition fees, and ability are crucial factors in determining whether a child will have a college education.

Thus, effective college investment is given as follows:

$$e^j_{it} = \max\left(0, e^{j*}_{it}\right) \tag{6b}$$

The economy thus features two types of households. The first are households whose consumption decision entails consuming the full amount of their income and do not invest in education, because their income falls short of the tuition fee, it is not optimal to invest in education, or both. The second are those who send their children to college. The optimal human capital is as follows:

$$h^j_{it+1} = \max\left(0, h^{j*}_{it+1}\right) \tag{7a}$$

Condition (7a) includes the corner solution for individuals' human capital and follows (6b).

From (1), (3), (4) and (6a), the optimal human capital of a young individual born at time t and who receives a college education during the same period is given as follows:

$$h^{j*}_{it+1} = \begin{cases} \epsilon^j b\left(A'h_t - \widetilde{s}_t\right) + b - 1 \text{ if } h^j_{it} = 0 \\ \epsilon^j b\left(B'h_t - \widetilde{s}_t\right) + b - 1 \text{ if } h^j_{it} > 0 \end{cases} \tag{7b}$$

[9] I defer the definition and discussion of x_t to Section 4, where I study the equilibrium conditions under government interventions.

[10] See Section 4.

and[11]

$$A' \equiv (1-\tau)(1-\alpha)A$$
$$B' \equiv (1-\tau)(1-\alpha+\alpha/\lambda)A$$

The first and the second lines in Eq. (7b) denote the human capital of a young individual with unskilled and skilled parents, respectively. In addition to their family background, individuals differ in their learning ability (indicated by the superscript j). $A'h_t - \widetilde{s}_t$ and $B'h_t - \widetilde{s}_t$ are the *average after-tax* income of non-college-educated and college-educated parents, respectively, net of college tuition fees and subsidies.

3. Laissez-faire

First, I analyze the economy based on a laissez-faire condition. Subsequently, I introduce government interventions in the next section. With no government interventions, both taxes and expenditures are nil.

$$\tau = 0 \text{ and } z_t = x_t = 0 \Leftrightarrow \widetilde{s}_t = s_t = s_k + sh_t \tag{8}$$

3.1. Education investment threshold

Since household education investment is a function of their labor income, depending on aggregate productivity, the level of aggregate human capital, in essence, determines individuals' education investment. Considering (3), (4), (6a), and (8), the threshold levels of aggregate human capital in the economy below which individuals do not invest in education under a laissez-faire condition are as follows:

$$\overline{h}_c^j(l) = \left(\frac{1}{\beta\epsilon^j} + s_k\right)(B-s)^{-1} \tag{9a}$$

$$\overline{h}_n^j(l) = \left(\frac{1}{\beta\epsilon^j} + s_k\right)((1-\alpha)A-s)^{-1} \tag{9b}$$

where[12]

$$\chi \equiv 1-\alpha+\alpha/\lambda$$
$$B \equiv \chi A$$

B is the average income share of a college-educated parent at time t.

$\overline{h}_n^j(l)$ and $\overline{h}_c^j(l)$ represent the threshold levels of aggregate human capital beyond which non-college-educated and college-educated parents invest in their children's education, respectively.[13] The superscript j indicates that the thresholds are different for agents with varying ability. If there are no differences in the parents' educational level, the parents with high-ability children are likelier to invest in their children than those with lower ability. However, if there are differences, the parents' educational background (whether they are college-educated) and their children's abilities are important in determining who is more likely to attend college. In any group, parents invest in education more likely if they have high-ability children, they are more altruistic, and there is a lower tuition fee or a higher TFP. The higher the labor factor shares, the more likely unskilled individuals are to invest in education.

It can be easily shown from (9) that $\overline{h}_c^g(l) < \overline{h}_c^r(l)$ and $\overline{h}_n^g(l) < \overline{h}_n^r(l)$ hold given $\epsilon^g > \epsilon^r$; however, the comparison between $\overline{h}_n^g(l)$ and $\overline{h}_c^r(l)$ is relatively less clear. Nevertheless, in general, I consider the case $\overline{h}_c^j(l) < \overline{h}_n^j(l)$, implying that regardless of their ability, poor individuals are less likely to afford a college education by themselves.[14] Therefore, considering that the following relation holds:

$$\overline{h}_c^g(l) < \overline{h}_c^r(l) < \overline{h}_n^g(l) < \overline{h}_n^r(l) \tag{10}$$

Eq. (10) implies that parents with a college education and high-ability children are most likely to invest in children's education. Meanwhile, parents with no college education and regular-ability children are least likely to invest in education. Wealthy parents are more likely to invest in college education than poor parents, regardless of ability differences.

3.2. Aggregate capital dynamics

The aggregate human capital is the total human capital in the economy with regular and high-ability individuals in the population, with skilled and unskilled parents. The aggregate human capital (h_{t+1}) in the economy at time $t+1$ is given as follows:

$$h_{t+1} = \lambda\left(ph^g_{ct+1} + (1-p)h^r_{ct+1}\right) + (1-\lambda)\left(ph^g_{nt+1} + (1-p)h^r_{nt+1}\right) \tag{11}$$

where h^g_{ct+1} and h^r_{ct+1} represent the human capital of high and regular-ability individuals within the group of skilled parents, respectively. Furthermore, h^g_{nt+1} and h^r_{nt+1} represent the human capital of high and regular ability individuals within the group of unskilled parents, respectively.

The first term (in the big bracket) thus captures the aggregate human capital of individuals with college-educated parents, and the second is the aggregate human capital of individuals with non-college-educated parents. Each group has high-ability individuals with size p and regular-ability individuals with size $1-p$. Eq. (11) implicitly assumes that all individuals in the economy invest in education. If only part of the population invests in education, the aggregate human capital becomes smaller, accordingly.[15]

3.3. Stage of development and aggregate human capital dynamics

Using Eqs. (7b) to (11), the dynamic system that characterizes the economy's developmental stages under a laissez-faire condition can be derived (see Appendix A.1 for details):

$$h_{t+1} = \begin{cases} h_{t+1} = 0 \text{ if } h_t < \overline{h}_c^g(l) \\ b\lambda p\epsilon^g\left(Bh_t - s_t\right) + \lambda p(b-1) \text{ if } \overline{h}_c^g(l) < h_t < \overline{h}_c^r(l) \\ b\lambda\left(p\epsilon^g + \epsilon^r(1-p)\right)\left(Bh_t - s_t\right) + \lambda(b-1) \text{ if } \overline{h}_c^r(l) < h_t < \overline{h}_n^g(l) \\ b\left(p\epsilon^g\left(Ah_t - s\right) + \lambda(1-p)\epsilon^r\left(Bh_t - s_t\right)\right) + \omega(b-1) \text{ if } \overline{h}_n^g(l) < h_t < \overline{h}_n^r(l) \\ b\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(Ah_t - s_t\right) + b - 1 \text{ if } h_t > \overline{h}_n^r(l) \end{cases} \tag{12}$$

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[11] In deriving the second line of Eq. (7b), I use the fact that $h^j_{it} = h_t/\lambda$, for a college-educated parent. First, recall that at time t, there are λ and $1-\lambda$ college-educated and non-college-educated individuals. Therefore, aggregate income is the sum of the total income of these individuals.

$$y_t = (1-\lambda)w_t + \lambda\left(w_t + \phi h^j_{it}\right)$$
$$= w_t + \lambda\phi h^j_{it} = Ah_t$$

where w_t and $w_t + \phi h^j_{it}$ are before-tax income of a skilled and unskilled individual, respectively. Then, substituting (4) into the above gives $h^j_{it} = h_t/\lambda$.

[12] The investment threshold associated with the ith individual of ability j is derived by applying $e^{j*}_{it} = 0$ in (6a); considering $\widetilde{s}_t = s_t = s_k + sh_t$ and $\lambda h^j_{it} = h_t$; using (3), (4) and (8); and solving for h_t. For example, (9b) – the investment threshold of the agent with non-college-educated parent – is derived as follows:

$$e^{j*}_{it} = 0 = b\left(I^j_{it} - \widetilde{s}_t\right) - b/\left(\beta\epsilon^j\right)$$
$$\Leftrightarrow \overline{h}_n^j(l) \equiv h_t = \left(\frac{1}{\beta\epsilon^j} + s_k\right)((1-\alpha)A-s)^{-1}$$

That is, if the aggregate human capital is less than or equal to $\overline{h}_n^j(l)$, the agent will not invest in college education.

[13] Investment thresholds are commonly applied in unified growth and inequality literature with multiple equilibria (Banerjee and Newman, 1993; Galor and Zeira, 1993; Moav, 2002; Galor and Moav, 2004).

[14] The sufficient condition for $\overline{h}_c^r(l) < \overline{h}_n^g(l)$, given (9), is $\epsilon^r((1-\alpha)A - s + \alpha A/\lambda) > ((1-\alpha)A - s)\epsilon^g$.

[15] For example, if only parents with college education invest in education, then $h^g_{nt+1} = h^r_{nt+1} = 0$ and the total human capital in the economy becomes $h_{t+1} = \lambda\left(ph^g_{ct+1} + (1-p)h^r_{ct+1}\right)$.

where

$$\omega \equiv \lambda + (1-\lambda)\,p$$

The developmental stage is associated with the evolution of higher education enrollment. The economy starts from an early phase where only a limited elites have access to higher education, continues to evolve endogenously, and ends up in a highly advanced economy where all individuals invest in higher education.

The first line of Eq. (12) shows that aggregate human capital is zero when the initial aggregate capital is too small, below the threshold level $\overline{h}_c^g(l)$. Even the rich and high-ability individuals do not find it optimal to invest in education. The second line shows that some individuals, specifically college-educated parents with high-ability children, begin investing in college education when the initial aggregate human capital stock is greater than $\overline{h}_c^g(l)$. The total aggregate education investment is $b\lambda p\epsilon^g\left(Bh_t - s_t\right) + \lambda p(b-1)$: $\lambda p\epsilon^g$ implies only the rich, with high-ability children of size p, send their children to college. $Bh_t - s_t$ is the average income of college-educated parents, net of the college tuition fee.

When the economy's capital passes the threshold $\overline{h}_c^r(l)$, children with regular ability and college-educated parents join college (third line). When it passes the threshold $\overline{h}_n^g(l)$, high-ability children with non-college-educated parents begin joining college (fourth line). $Ah_t - s_t$ is the average income of all parents, net of the college tuition fee. The first term denotes education investment by all types of parents with high-ability children, while the second captures investment by wealthy parents with regular-ability children. Only when the aggregate human capital stock passes $\overline{h}_n^r(l)$, non-college-educated parents with regular children start to invest in education (fifth line). At this stage, education investment in the economy is simply a fraction of aggregate income net of the tuition fee.

Fig. 2 demonstrates the different developmental stages that the economy experiences based on Eq. (12). As shown in the horizontal line, $h_{t+1} = 0$ for any initial capital $h_t < \overline{h}_c^g(l)$. However, if $\overline{h}_c^g(l) < h_t < \overline{h}_c^r(l)$, the economy will be in Stage I, where $h_{t+1} > 0$ because some individuals, specifically parents with college education backgrounds and high-ability children, begin investing in education. However, if the initial capital stock is not sufficiently high, the dynamic could go back to the stable equilibrium, $h_{t+1} = 0$.

As a necessary condition for a growing economy, the following parameter restriction is imposed.

$$b\lambda p\epsilon^g (B - s) - 1 > 0 \tag{13}$$

It implies that the slope of the curve in the initial stage of the economy shall be greater than unity. A further restriction should be imposed for the economy to escape the low equilibrium. The associated threshold could be computed from Eq. (12) second line, as $h^T(l) \equiv h_{t+1} = h_t$.

$$h^T(l) = \frac{b\lambda p\left(\epsilon^g s_k - 1/\beta\right)}{\epsilon^g b\lambda p (B - s) - 1} \tag{14}$$

The economy continues to evolve endogenously as long as the initial aggregate human capital is greater than these threshold levels, that is, $h_0 > h^T(l)$ and (13) is satisfied. It eventually passes the thresholds required for other individuals to begin investing in human capital through productivity spillover that boosts individual labor income. Stage II of development begins when $\overline{h}_c^r(l) < h_t < \overline{h}_n^g(l)$. At this stage, all individuals with college-educated parents invest in college education. This is followed by Stages III and IV, when $\overline{h}_n^g(l) < h_t < \overline{h}_n^r(l)$ and $h_t > \overline{h}_n^r(l)$, respectively. The latter represents the long-run path of the economy where all individuals (rich and poor) invest in college education. Meanwhile, the former represents a middle stage where all rich and poor households with high-ability children invest in college education.

Table 1
Per capita tuition subsidy at different stages of development and grant scheme.

Stages	Grant available per person (x_t)		
	Universal	*Scholarship*	*Means-tested*
Stage I	$z_t/(\lambda p)$	$z_t/(\lambda p)$	0
Stage II	z_t/λ	$z_t/(\lambda p)$	0
Stage III	z_t/ω	z_t/p	$z_t/((1-\lambda)p)$
Stage IV	z_t	z_t/p	$z_t/((1-\lambda)p)$

4. Higher education policy

In this section, I introduce a government that offers different types of higher education tuition grants, which will be financed through taxes. I consider three higher education policies that are commonly applied, namely, (i) a universal, (ii) scholarship, and (iii) means-tested grant scheme. The policies differ in terms of eligibility criteria. In the first case, the grant is available for any individual who enrolls in higher education, and the government does not know individuals' abilities and backgrounds and thus provides grants for anyone who joins college. In the second case, a tuition grant is available based on merit, and the government knows individuals' abilities but does not know their family background. In the third case, the government provides tuition grants for high-ability individuals from low-income families, as it knows their abilities and family backgrounds.

4.1. Per capita tuition subsidy

Before characterizing the different phases of higher education development under government interventions, it might be necessary to explicitly define the per capita tuition subsidy (x_t). The per capita tuition subsidy is the total tuition subsidy (z_t) divided by the number of eligible individuals enrolled in college. Thus, the size of x_t depends on (i) the enrollment rate and (ii) eligibility. These, in turn, depend on the stage of the country's higher education development and the type of grant scheme.

The per capita tuition grant could thus be different at different phases of higher education and for different grant schemes due to variations in the number of eligible individuals enrolled in higher education. For example, consider an economy with a higher education level of Stage II (where only the rich invest in college education). Suppose the tuition grant is a universal grant scheme. In that case, the number of eligible individuals with college access is λ, and the amount of tuition subsidy available to an individual is $x_t = z_t/\lambda$. However, if it is a scholarship scheme, the number of eligible individuals with college access is λp; hence, the per capita tuition grant is $x_t = z_t/(\lambda p)$. If the program is a means-tested grant, then no one receives grants as no one is eligible: $x_t = 0$.

In determining the values of x_t, I adopt the same enrollment trend that I have in the laissez-faire case (10).[16] That is, grant or no grant, the upper class (λp) would most likely invest in college, followed by the middle class, $\lambda(1-p)$, and then the lower middle class, $p(1-\lambda)$. Meanwhile, the lower class, $(1-\lambda)(1-p)$, are the least likely ones to invest in higher education. Table 1 summarizes the per capita allocation of tuition grants that are available at different phases of higher education development and for different grant schemes.

[16] This makes a comparison with the laissez-faire case possible.

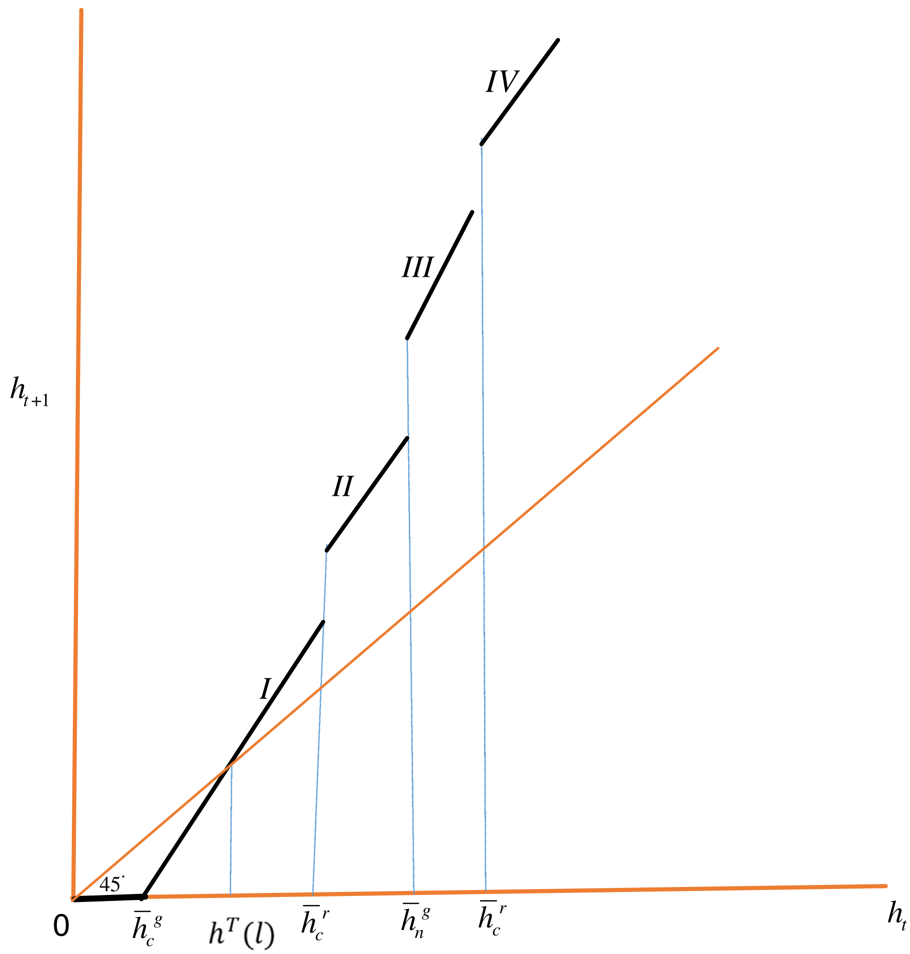

Fig. 2. Stages of higher education development: The economy kicks off only if the initial capital stock exceeds the threshold capital ($h^T(l)$).

ω is the number of individuals who enroll in college at Stage III.[17] One immediately sees that during the same stage, the per capita tuition grant varies as the number of eligible individuals changes over the type of tuition grant scheme. For example, in the universal grant scheme, z_t/ω tuition grant is available for an individual who joins college. In contrast, in the scholarship and means-tested schemes, the per capita tuition grants are higher, z_t/p and $z_t/((1-\lambda)p)$, respectively.

By substituting each column's x_t from Table 1 into (7b), and considering (3b), one obtains an individual's optimal human capital associated with the different grant plans. For example, substituting x_t from column 2 in (7b) gives an individual's optimal human capital investment under the universal grant scheme. In contrast, substituting it from columns 3 and 4 gives the human capital investment under the scholarship and means-tested programs, respectively.

5. Phases of higher education with government intervention

I characterize the different phases of higher education development under government intervention as similar to the one in laissez-faire. The difference from laissez-faire is that this time, the dynamics reflect the taxes individuals pay and the tuition grant they may receive under various grants such as a universal grant, scholarship, or means-tested grant.

5.1. Universal grant scheme

From (3b), (7b), (11) and Table 1 (*column 2*), the dynamics of aggregate human capital under the universal grant scheme are given as follows (see Appendix A.2):

$$h_{t+1}=\begin{cases} b\lambda p\epsilon^g\left(B'h_t-s_t\right)+b\epsilon^g z_t+\lambda p(b-1) \text{ if } \bar{h}_c^g(u)<h_t<\bar{h}_c^r(u) \\ b\lambda\left(p\epsilon^g+(1-p)\epsilon^r\right)\left(B'h_t-s_t\right)+b\left(p\epsilon^g+(1-p)\epsilon^r\right)z_t+\lambda(b-1) \\ \text{if } \bar{h}_c^r(u)<h_t<\bar{h}_n^g(u) \\ b\left(p\epsilon^g\left((1-\tau)Ah_t-s_t\right)+\lambda(1-p)\epsilon^r\left(B'h_t-s_t\right)\right)+\frac{\vartheta b z_t}{\omega}+\omega(b-1) \\ \text{if } \bar{h}_n^g(u)<h_t<\bar{h}_n^r(u) \\ b\left(p\epsilon^g+(1-p)\epsilon^r\right)\left(Ah_t-s_t\right)+b-1 \text{ if } h_t>\bar{h}_n^r(u) \end{cases} \tag{15}$$

where

$\vartheta \equiv p\epsilon^g+\lambda\epsilon^r(1-p)$

Eq. (15), which is comparable to Eq. (12), characterizes the dynamics of an economy that goes through four phases of higher education development under a universal tuition grant scheme. The first terms, from Stage I to III, represent fractions of after-tax average income invested in education; the second terms show the amount of tuition grant provided.[18]

[17] It is the sum of the lower-middle- $p(1-\lambda)$, middle- $\lambda(1-p)$, and upper-class (λp) individuals.

[18] In Stages I and II, λp and λ individuals invest in education while each receives $z_t/(p\lambda)$ and z_t/λ per capita tuition grants, respectively. In Stages III and IV, as increased individuals invest in education, per capita tuition grant

Similar to the laissez-faire case, if the initial capital at the economy level is smaller than the investment threshold ($\overline{h}_c^g(u)$), then no one in the economy will enroll in higher education (i.e., $h_{t+1} = 0$ if $h_t < \overline{h}_c^g(u)$). However, the economy will be in Stage I if the current aggregate capital is greater than the minimum investment threshold ($h_t > \overline{h}_c^g(u)$).[19] In Stage I, only parents with college education and high-ability children invest in education. As the economy continues growing, if the growth condition is satisfied, other families will join in education investment (through productivity spillover).[20] If not, the dynamics could go back to the stable equilibrium, $h_{t+1} = 0$.

The *threshold human capital* for takeoff ($h^T(u) \equiv h_{t+1} = h_t$) is identified using similar logic as in the preceding section.

$$h^T(u) = \frac{\lambda p b\left(\epsilon^g s_k - 1/\beta\right)}{\lambda p b \epsilon^g (B' - s) + b\epsilon^g \tau A - 1} \tag{16}$$

Since $h^T(u) < h^T(l)$, takeoff starts earlier under the universal tuition grant compared with the laissez-faire case.

During the transition periods of the economy (Stages I—III), growth is relatively higher than the ones in laissez-faire.[21] That is, the laissez-faire conditions are inferior in every stage of the higher education development process (except at the last stage, where all individuals enroll in college), as seen from comparing Eqs. (12) and (15). Under the universal grant, additional resources are mobilized for college education investment from individuals who are not joining college and consume the full amount of their income. Therefore, individuals who send their children to college and those who do not invest in college education bear the cost of tuition subsidies.

In Stage IV, the growth rates are similar for laissez-faire and universal grants. The intuition for that boils down to two factors. First, in this stage, the universal grant is similar to a lump sum transfer, as everyone receives a certain amount of transfer equally. Second, both taxes are, in nature, labor income taxes; since there are no labor leisure choices, the taxes are not distortionary. Therefore, the aggregate dynamics remain the same regardless of taxes and transfers.

5.2. Scholarship grant scheme

With the scholarship program, the dynamics of aggregate human capital are given by from (3b), (7b), (11), and Table 1 (*column 3*), (see Appendix A.3):

$$h_{t+1} = \begin{cases} b\lambda p\epsilon^g\left(B'h_t - s_t\right) + b\epsilon^g z_t + \lambda p(b-1) \ \text{if}\ \overline{h}_c^g(s) < h_t < \overline{h}_c^r(s) \\ b\lambda\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(B'h_t - s_t\right) + b\epsilon^g z_t + \lambda(b-1) \ \text{if}\ \overline{h}_c^r(s) < h_t < \overline{h}_n^g(s) \\ b\left(p\epsilon^g\left((1-\tau)Ah_t - s_t\right) + \lambda(1-p)\epsilon^r\left(B'h_t - s_t\right)\right) + b\epsilon^g z_t + \omega(b-1) \\ \quad \text{if}\ \overline{h}_n^g(s) < h_t < \overline{h}_n^r(s) \\ b\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(Ah_t - s_t\right) + b(1-p)\left(\epsilon^g - \epsilon^r\right)z_t + b - 1 \ \text{if}\ h_t > \overline{h}_n^r(s) \end{cases} \tag{17}$$

Eq. (17) represents the different phases of higher education development when the government provides a scholarship—a tuition grant that targets high-ability individuals. Stage I is attained if the initial aggregate capital is greater than the minimum investment threshold ($h_t > \overline{h}_c^g(s)$). Furthermore, the economy continues to evolve, similar to what is described above.[22] Note also that in Stage I, aggregate human capital is similar to the universal grant case due to the similarity in the amount of per capita grants available during this time ($z_t/(\lambda p)$). This also implies that the two economies face similar take-off conditions, defined in (16) ($h^T(u) = h^T(s)$) while (13) remains to be the sufficient condition for the slope of the curve be greater than unity.

The first terms in the big brackets, in Stages I–III, show the after-tax average income invested in education by λp, λ, and ω individuals, respectively. The second term, $b\epsilon^g z_t$, captures the total tuition grants provided to high-ability individuals at each stage. Unlike the previous cases, in the last stage of development (Stage IV), there is a redistribution from regular to high-ability individuals who invest in college.[23]

5.3. Means-tested grant scheme

Similarly, from (3b), (7b), (11), and Table 1 (*column 4*), the different stages of higher education development for the case where the government provides tuition subsidies based on both merit and need basis are given as follows (see Appendix A.4):

$$h_{t+1} = \begin{cases} b\lambda p\epsilon^g\left(B'h_t - s_t\right) + \lambda p(b-1) \ \text{if}\ \overline{h}_c^g(m) < h_t < \overline{h}_c^r(m) \\ b\lambda\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(B'h_t - s_t\right) + \lambda(b-1) \ \text{if}\ \overline{h}_c^r(m) < h_t < \overline{h}_n^g(m) \\ bp\epsilon^g\left((1-\tau)Ah_t - s_t\right) + \lambda(1-p)\epsilon^r\left(B'h_t - s_t\right) + b\epsilon^g z_t + \omega(b-1) \\ \quad \text{if}\ \overline{h}_n^g(m) < h_t < \overline{h}_n^r(m) \\ b\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(Ah_t - s_t\right) + (1-p)b\left(\epsilon^g - \epsilon^r\right)z_t + b - 1 \ \text{if}\ h_t > \overline{h}_n^r(m) \end{cases} \tag{18}$$

While the minimum threshold to be satisfied for the economy to be at Stage I is $h_t > \overline{h}_c^g(m)$, the same mechanism described above applies to the evolution of the economy.[24]

As in the previous two cases, the first terms in the big brackets show after-tax average income. The second terms (if any) show total tuition grants. In Stages I and II, the government provides no tuition grants, as none of the individuals investing in education are not qualified for the grant. These are the cases where the government collects taxes, and the revenues are "thrown to the ocean".[25] Indeed, this would have the immediate effect of lowering aggregate efficiency during these stages. Consequently, the economy may take off much later than any of the earlier cases. The respective threshold to take off can easily be computed as in the above cases.

$$h^T(m) = \frac{\lambda p b\left(\epsilon^g s_k - 1/\beta\right)}{\lambda p b \epsilon^g (B' - s) - 1} \tag{19}$$

Furthermore, the necessary condition for a growing economy is more restrictive.

$$b\lambda p\epsilon^g\left(B' - s\right) - 1 > 0 \tag{20}$$

Individuals eligible for the tuition grants begin to invest in education only in Stage III. At this stage and the next, the government's revenue will be available as tuition grants for these households.

6. Efficiency

Different grant schemes may have different implications for aggregate productivity and welfare due to differences in their eligibility

reduces to z_t/ω and z_t, respectively. ω is the number of eligible individuals for the grant in Stage III. Moreover, ϑ shows that the grant is distributed to p poor and high ability and $(1-p)\lambda$ rich and regular ability individuals.

[19] The threshold levels related to different stages of development are derived in Eq. (33), Appendix B.

[20] Note that (13) is a sufficient condition for the slope of the curve in the initial stage of the economy under the universal grant scheme to be greater than unity.

[21] In Stage IV, the growth rates are similar.

[22] The thresholds related to Stages I–IV are given in (34), Appendix B.

[23] If $\epsilon^g = \epsilon^r$, aggregate investment in education becomes similar to the previous cases.

[24] The threshold levels related to Stages I–IV when the grant scheme is means-tested are given in (35), Appendix B.

[25] It might be questionable, however, why the government behaves in such a counterintuitive manner. An alternative will be to consider the case where there is no government involvement in Stages I and II but only in the later stages. In this case, in the first two stages, aggregate capital dynamics are identical to the laissez-faire case.

Table 2
Productivity and welfare differences of moving from laissez-faire to the universal scheme.

Stages	Welfare gain	Productivity gain
Stage I	$(1+\beta)\epsilon^g(1-\lambda p\chi)z_t$ $+(1-\lambda+\lambda(1-p))\ln(1-\tau)$	$b\epsilon^g(1-\chi p\lambda)z_t$
Stage II	$(1+\beta)(p\epsilon^g+(1-p)\epsilon^r)z_t(1-\lambda\chi)$ $+(1-\lambda)\ln(1-\tau)$	$b(p\epsilon^g+(1-p)\epsilon^r)(1-\lambda\chi)z_t$
Stage III	$(1+\beta)p\epsilon^g(1-\omega)z_t/\omega+$ $(1+\beta)\lambda(1-p)\epsilon^r(1-\omega\chi)z_t/\omega$ $+(1-\omega)\ln(1-\tau)$	$bp\epsilon^g(1-\omega)z_t/\omega$ $+b\lambda(1-p)\epsilon^r(1-\omega\chi)z_t/\omega$
Stage IV	0	0

criteria and capacity to mobilize resources. In Stages I–III, there is a productivity gain from laissez-faire to the universal grant scheme. These can easily be computed by taking the differences between the right-hand sides of Eqs. (12) and (15) for each stage. The results are shown in the last column of Table 2.

The productivity gain primarily comes from resource mobilization and redistribution from those who do not invest in college education to those who do.[26] In Stage I, for example, the tax contribution by the $p\lambda$ elite is $p\lambda\chi z_t$ but the same individuals receive $z_t/(p\lambda)$ each or z_t in total. Similarly, in Stage II, the tax contribution by the wealthy λ individuals is $\lambda\chi z_t$, whereas the same individuals receive z_t/λ each or z_t in total. There are $1-p\lambda$ individuals in Stage I and $1-\lambda$ individuals in Stage II who pay taxes but do not invest in higher education; therefore, they do not receive any grant. Thus, resources are redistributed regressively to the upper and middle class in the form of tuition grants.

In Stage III, $(1-\lambda)(1-p)$ individuals pay taxes but do not have college access. The first and second terms capture *net* grants received by p high-ability poor individuals and $(1-p)\lambda$ regular-ability wealthy individuals, respectively. Meanwhile, in Stage IV, all individuals who pay taxes send their children to college. Generally, the gain in productivity would reduce when moving up stages, which disappears eventually as the number of college participants increases.

Column 2 of Table 2 shows a positive welfare gain in Stages I–III when moving from laissez-faire to the universal grant scheme (see Appendix E for details). It is similar to the last column except for the last terms. These terms are negative and reflect the relative advantage of individuals who do not invest in children's education (consume the full amount of their income) from not paying taxes in laissez-faire since these individuals would have paid taxes regardless of their situations in any of the grant programs. However, they are extremely small to create any difference, particularly given this is a growing economy.

Table 3 shows that the scholarship program is the most productive and efficient. Comparing Eq. (15) to Eq. (17), the latter is greater at every stage of development, except in the first stage where they are tied . There are $b(1-p)(\epsilon^g-\epsilon^r)z_t$ and $(1+\beta)(1-p)(\epsilon^g-\epsilon^r)z_t$ gain in productivity and efficiency by moving from a universal education grant to a scholarship program, in Stages II and IV.[27]

The efficiency gain comes from mobilizing resources to high-ability individuals. As the skill gap $(\epsilon^g-\epsilon^r)$ widens, it becomes increasingly efficient to shift to the scholarship program. Considering that $\frac{\lambda}{\omega}<1$, the gap in efficiency gain between the two programs is smaller in Stage III. The means-tested grant is the least efficient grant scheme in Stages I and II, as everyone pays taxes, but no one qualifies to receive

[26] It is straightforward that the first and the second equations in the last column of Table 2 are positive since $\chi\lambda<1$. The first term for the third equation is positive since $\omega<1$. The second term is also positive given $\omega\chi=(\lambda+p(1-\lambda))(\lambda+\alpha(1-\lambda))<1$. This is because $\omega\chi$ increases in α and p and attains its maximum value of unity when α and p approach unity.

[27] See Appendix E for the derivation and analysis of the welfare effects.

Table 3
Productivity and welfare differences of moving from universal to scholarship grant scheme.

Stage	Welfare gain	Productivity gain
I	0	0
II	$(1+\beta)(1-p)(\epsilon^g-\epsilon^r)z_t$	$b(1-p)(\epsilon^g-\epsilon^r)z_t$
III	$(1+\beta)\lambda\frac{1-p}{\omega}(\epsilon^g-\epsilon^r)z_t$	$b\lambda\frac{1-p}{\omega}(\epsilon^g-\epsilon^r)z_t$
IV	$(1+\beta)(1-p)(\epsilon^g-\epsilon^r)z_t$	$b(1-p)(\epsilon^g-\epsilon^r)z_t$

Table 4
Ranking of higher education grant schemes for impacts on aggregate efficiency.

	Laissez-faire	Universal grant	Scholarship	Means-tested
Stage I	2nd	1st	1st	3nd
Stage II	3rd	2nd	1st	4th
Stage III	3rd	2nd	1st	1st
Stage IV	2nd	2nd	1st	1st

grants during those stages. Notably, the aggregate human capital under the means-tested grant is similar to that of the scholarship program in Stages III and IV. Aggregate welfare is also the same during these advanced stages of higher education development.

6.1. Policy ranking

The scholarship program is the most efficient education subsidy program regardless of the higher education developmental stage of the economy. Table 4 ranks the public programs based on their efficiency at each developmental phase based on the foregoing discussion.

The following proposition summarizes Table 4 and the foregoing discussion.

Proposition 1.

(i) *The universal and scholarship grants are the most efficient in Stage I, followed by laissez-faire and means-tested grants.*
(ii) *In Stage II, the means-tested grant is the least efficient; the scholarship is the most efficient, followed by the universal grant scheme.*
(iii) *The scholarship and means-tested programs are the most efficient in Stages III and IV.*
(iv) *In Stage III, the universal grant is the second, and laissez-faire is the last, whereas they are tied in Stage IV.*

7. Equity

In this section, I investigate how higher education grant schemes impact each group's relative college education investment at the various stages of higher education development. An analytical comparison between the inequity impacts of each program at every stage may not be feasible. Therefore, I conduct the analysis quantitatively. Thus, I first construct the Gini coefficients for each phase of higher education development associated with the various tuition grant schemes based on an illustrative calibration and then compare them accordingly.

7.1. Higher education investment by income group

As noted earlier, only the upper and middle classes invest in higher education during the early stages (Stages I and II). However, in the later stages of higher education development (Stages III and IV), most of the population starts to invest in higher education. Tables 5 and 6 present human capital investment by type of individual associated with Stages I and II of higher education development. In Stage I, Table 5, all human capital investment is made by the top $100\lambda p$ percentile of the population. The rest of the population does not invest in education

Table 5
Individual human capital: Stage I.

Schemes	Human capital investment by individual type			
	λp	$\lambda(1-p)$	$(1-\lambda)p$	$(1-\lambda)(1-p)$
Laissez-faire	$\epsilon^g b\left(Bh_t - s_t\right) + b - 1$	0	0	0
Universal	$\epsilon^g b\left(B'h_t - s_t + z_t/(\lambda p)\right) + b - 1$	0	0	0
Scholarship	$\epsilon^g b\left(B'h_t - s_t + z_t/(\lambda p)\right) + b - 1$	0	0	0
Means-tested	$\epsilon^g b\left(B'h_t - s_t\right) + b - 1$	0	0	0

Table 6
Individual human capital: Stage II.

Schemes	Human capital investment by individual type			
	λp	$\lambda(1-p)$	$(1-\lambda)p$	$(1-\lambda)(1-p)$
Laissez-faire	$\epsilon^g b\left(Bh_t - s_t\right) + b - 1$	$\epsilon^r b\left(Bh_t - s_t\right) + b - 1$	0	0
Universal	$\epsilon^g b\left(B'h_t - s_t + z_t/\lambda\right) + b - 1$	$\epsilon^r b\left(B'h_t - s_t + z_t/\lambda\right) + b - 1$	0	0
Scholarship	$\epsilon^g b\left(B'h_t - s_t + z_t/(\lambda p)\right) + b - 1$	$\epsilon^r b\left(B'h_t - s_t\right) + b - 1$	0	0
Means-tested	$\epsilon^g b\left(B'h_t - s_t\right) + b - 1$	$\epsilon^r b\left(B'h_t - s_t\right) + b - 1$	0	0

or has no human capital. In Stage II, Table 6, the 100λ percentile of the population invests in education. Therefore, the comparison is for how the different policies impact the middle class (the $100\lambda(1-p)$ percentile), and the upper class (the $100\lambda p$ percentile), as the remaining $100(1-\lambda)$ percentile do not invest in education and remain unaffected by any policy.

Human capital investment by type of individuals associated with Stages III and IV of higher education development are presented in Appendix C. In Stage III, the lower-middle class (the $100(1-\lambda)p$ percentile), middle class, and upper class invest in higher education. In Stage IV, all individuals, including the lower class (the $100(1-\lambda)(1-p)$ percentile), invest in higher education.

7.2. Quantitative analysis

We can have further insight into the inequality impacts of the different policies and compare policies at different stages of development quantitatively by specifying parameter values. However, some caveats must be noted before proceeding with the quantitative analysis. First, estimates are only available for certain parameter values. Therefore, the interpretation of the results should be made more cautiously. Second, considering that this is a growing economy, the quantitative analysis cannot be conducted at specific stationary points but between two threshold points.[28] My approach to dealing with these problems is to conduct a sensitivity analysis considering various parameter values and initial aggregate human capital.

Starting with the calibration, I set $\lambda = 0.3$ and $p = 0.4$. This implies that in Stage I, only 12% of the population can access higher education. In Stages II and III, 30% and 58% of the population have access to higher education, respectively.[29] The capital share (α), and the preference parameter (β) are set at standard values of $\alpha = 0.36$, and $\beta = 0.3$ (de la Croix and Michel, 2002). Furthermore, I set $\tau = 0.05$, given public investment in education often ranges between 5%–6% of GDP. No estimates for the ability and threshold parameters, ϵ^j, s and s_k, are available. I chose their values, together with the TFP parameter A, targeting initial conditions that satisfy the necessary conditions for a growing economy (Eqs. (13) and (20)), the threshold conditions for the initial aggregate human capital (Eqs. (14), (16), and (19)), and the threshold conditions of individuals' investment hierarchy (Eq. (10)). I thus set $\epsilon^g = 6$ and $\epsilon^r = 4.8$, which implies a 25% productivity gap between the high-ability individuals and the rest of the population.[30] Finally, I let A and s take the values of 8 and 2 and s_I, s_{II}, s_{III} and s_{IV} take the values of 1, 2, 3 and 4, respectively.

Table 7
Threshold levels for aggregate human capital.

	Laissez-faire	Universal	Scholarship	Means-tested
Stage I	0.12	0.10	0.10	0.13
Stage II	0.21	0.20	0.22	0.22
Stage III	1.14	1.00	0.92	0.83
Stage IV	1.50	1.44	1.64	1.64

Note: Individual investment thresholds based on Eq. (9), (33) to (35).

These values lead to the threshold levels of aggregate human capital reported in Table 7. For example, the initial aggregate human capital should be greater than 0.12 for the elite λp individuals to invest in higher education if the government policy is laissez-faire. Similarly, $h_0 < 0.1$ implies no education investment in any program. The threshold for the means-tested scheme is higher at Stage I due to taxes (without subsidy). As expected, the thresholds at the universal and scholarship programs are small due to subsidies that reduce the tuition fees. When $h > 0.22$, the $(1-p)\lambda$ individuals with regular ability and from wealthy backgrounds start investing in education in Stage II. When the aggregate capital passes the 1.14 threshold, individuals with high ability and from poor backgrounds start investing in education. The final stage of higher education development takes place when the aggregate capital passes the 1.64 threshold. I set aggregate human capital in Stages I–IV slightly higher than these values at 0.15, 0.23, 1.20, and 2.0, respectively. Moreover, these initial conditions satisfy the threshold conditions for escaping the poverty trap, which is implied by the above-specified parameters, namely, $h^T(l) = 0.066$, $h^T(u) = h^T(s) = 0.048$, and $h^T(m) = 0.075$.[31]

[28] Particularly, there is no stationary aggregate human capital. Growth becomes stationary only at the last stage of development.

[29] This is in line with Trow's (1973) stylized facts of the transformation of higher education. See the discussion in Section 1 and Footnote 3.

[30] We see later that the difference is more important than the actual values of ϵ^j when it comes to the inequality impact of different policies.

[31] The conditions for a growing economy are also satisfied by the parameter values. Eq. (20) is easily satisfied since $b\lambda p\epsilon^g\left(B' - s\right) - 1 = 0.99 > 0$, indicating that (13) is satisfied.

Table 8
Gini coefficients.

	Laissez-faire	Universal	Scholarship	Means-tested
Stage I	0.8785	0.8793	0.8793	0.8778
Stage II	0.7515	0.7403	0.8586	0.7990
Stage III	0.7072	0.6808	0.6694	0.6361
Stage IV	0.5696	0.5428	0.5738	0.5461

Table 8 reports Gini coefficients based on the specified values. Inequality in human capital is higher at the early stages, that is, 88% of the Gini coefficient in Stage I. In this stage, no difference exists between the inequality impact of the universal and scholarship grant schemes (Table 5); both leave the top-income group better off compared with the other programs. Compared with laissez-faire, the means-tested program makes the middle and top-income groups worse off. However, the impact of the fiscal policy on inequality seems trivial at this stage since only one group of people invests in human capital.

In Stage II, the universal grant tops in reducing inequality, followed by laissez-faire. The scholarship program is the most regressive one, since it benefits only high-ability individuals or individuals in the top income class (the top $100\lambda p$ percentile). Moreover, means-tested grants leave everyone worse off, as none of the groups who invest in education are eligible for the program. Despite all paying taxes and none being eligible for the grant scheme, taxation seems to hurt individuals in the middle-income group more.

In Stage III, the means-tested program ranks first, followed by the scholarship program, whereas the universal grant performs better than laissez-faire. The scholarship program at this stage performs better than universal grants and laissez-faire, as it targets high-ability individuals from poor backgrounds who are joining higher education at this stage. The means-tested program, however, targets this group of individuals and provides them with relatively more resources.

In Stage IV, the universal grant scheme is the most effective in reducing inequality, as it targets individuals in the bottom income group who begin investing in higher education at this stage. Means-tested is second, followed by laissez-faire, as it targets individuals in the lower-middle class. The scholarship program ranks last based on its impact on inequality, as it also targets the top-income group.

The results are robust for alternative parameter specifications. I have experimented with the range of values of aggregate human capital within the threshold limits of Table 7 and for different parameter value specifications. While the magnitudes of the Gini coefficients change, the qualitative results (not reported) remain similar.

8. Enrollment

In this section, by comparing the education investment thresholds associated with the different grant schemes to laissez-faire, I examine how different higher education policies influence the college enrollment rate. As college access is categorized based on class in each phase of higher education development, I can compare the thresholds associated with each policy for the group of individuals with access to a college education for the first time at that stage. In Stage I, for example, the elite will have access to higher education for the first time; thus, I examine how a given policy (compared with laissez-faire) affects their likelihood of enrolling in higher education. Similarly, in Stage II, individuals with regular ability but from affluent families will have access to higher education for the first time. Hence, the question will be the following. How does each policy affect the investment thresholds of this group of individuals? In Stages III and IV, high- and regular-ability individuals from low-income families will have first-time access to college. Therefore, I examine how the investment thresholds of high- and regular-ability individuals are affected by each policy in Stages III and IV.

The investment thresholds related to the grant schemes at different stages of higher education development are derived in Appendix B. The following propositions compare these thresholds to the investment threshold associated with laissez-faire.

Proposition 2.

(i) Stage I: The universal and scholarship programs have a similar positive effect on enrollment rate; means-tested grant has a negative effect.

(ii) Stage II: The enrollment rate increases in the universal grant scheme but decreases in other policies.

(iii) Stage III: Means-tested grant is the best in increasing the college enrollment rate; scholarship and universal grants are the second and third best, respectively.

(iv) Stage IV: The enrollment rate increases in the universal grant scheme but decreases in other policies.

In Stage I, individuals likely to enroll in college do not qualify for the means-tested grant scheme despite paying taxes. Moreover, in the universal and scholarship programs, individuals with access to a college education are better off than laissez-faire, as the tuition grants they receive are more than the taxes they pay. In Stages II and IV, the investment thresholds are associated with regular-ability individuals not qualified for the scholarship and means-tested programs despite paying taxes. They are thus better off with the universal grant scheme, as the grants they receive are higher than the taxes they pay. In Stage II, the additional fund comes from those who do not enroll in college, and in Stage IV, it comes from individuals with a rich background (from the capital tax revenue). Furthermore, in Stage III, the investment thresholds are associated with high-ability individuals with a low-income family background. The means-tested program is the most effective one for boosting the enrollment rates of this group of individuals as the whole fund is available for them. The fund is distributed among a larger section of society in the scholarship or universal grant schemes.

9. Conclusion

This study has comprehensively analyzed the effects of alternative higher education financing policies on efficiency, equity, and enrollment rates. Furthermore, it has ranked different higher education grant schemes based on their impact on efficiency and equity vis-à-vis a laissez-faire approach. What makes the work unique is that all the analyses are conducted while considering the different phases of higher education development that countries may have faced. Many of today's industrialized economies, more or less, have gone through what is well known in the education literature as Trow's phases of higher education development – the transformation of the higher education system from the elite to the mass and the universal system – since the Second World War. The simple model employed herein accounts for the "massification" of higher education. Simultaneously, it results in closed-form solutions and offers a rich analysis of many aspects of higher education grants. In particular, this study captured the four phases and endogenous transition of the higher education system that starts from an early stage where only a few elites have access to higher education and evolves endogenously to eventually become a highly advanced economy, where all individuals invest in higher education.

The analysis was conducted under both government intervention and laissez-faire systems. In the former, the dynamics and equilibrium reflect the taxes individuals must pay and the tuition grants they could receive under alternatives such as universal, scholarship, and means-tested grants. Moreover, different grant schemes are found to have various implications for efficiency, equity, and enrollment due to differences in their eligibility criteria and capacity to mobilize resources from individuals who do not invest in college to those who do and from regular- to high-ability individuals.

Some primary results are that a scholarship program is the most efficient higher education subsidy program at all stages of higher education development. Means-tested grants are equally good during the advanced stages. Both programs increase aggregate efficiency by mobilizing resources to high-ability individuals within the economy. In the early stages, universal grants can be an alternative to scholarship; however, means-tested is the worst.

Regarding the impact on inequality, universal grants, followed by means-tested, are best at reducing inequality in the latest stage of development, as they reach the majority who have access to higher education. If the bottom income group has no access to higher education, means-tested, followed by scholarship, is the best, as it targets individuals with high ability but lower income. During the early phase, the universal grant is the best followed by laissez-faire. Scholarship is worst because during this stage, only high-ability individuals from the top-income group have access to higher education. At the later stages of higher education development, the enrollment rate increases in the universal grant scheme but decreases in other policies. The results suggest that the recent shift away from the universal grant scheme in the UK could go wrong on at least two fronts—it could lead to a decline in the college enrollment rate and aggravate some of the equity issues in higher education.

Declaration of competing interest

None.

Data availability

The matlab code is available on Mendeley.

Matlab code for Effects of Higher Education Subsidy (Original data) (Mendeley Data)

Appendix A. Aggregate dynamics

A.1. Laissez-faire

A.1.1. Stage I

To derive the second line of Eq. (12), substitute the second line from Eq. (7b) into the first term of Eq. (11) and use Eq. (8) to obtain the following.

$$\begin{aligned} h_{t+1} &= \lambda p h^g_{ct+1} \\ &= \lambda p \left(\epsilon^g b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right) \\ &= \lambda p \epsilon^g b \left(B h_t - s_t\right) + \lambda p (b-1) \end{aligned} \tag{21}$$

A.1.2. Stage II

To derive the third line of Eq. (12), substitute the second line from Eq. (7b) into the first two terms of Eq. (11) and use again Eq. (8) to obtain the following.

$$\begin{aligned} h_{t+1} &= \lambda p h^g_{ct+1} + \lambda(1-p) h^r_{ct+1} \\ &= \lambda p \left(\epsilon^g b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right) + \lambda(1-p)\left(\epsilon^r b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right) \\ &= b\lambda (p\epsilon^g + (1-p)(\epsilon^r))\left(B h_t - s_t\right) + \lambda(b-1) \end{aligned} \tag{22}$$

A.1.3. Stage III

I derive the fourth line of Eq. (12) from Eqs. (7b), (8), (11), and (22).

$$\begin{aligned} h_{t+1} &= b\lambda (p\epsilon^g + (1-p)(\epsilon^r))\left(B h_t - s_t\right) + \lambda(b-1) + (1-\lambda) p h^g_{nt+1} \\ &= b\lambda (p\epsilon^g + (1-p)(\epsilon^r))\left(B h_t - s_t\right) + \lambda(b-1) \\ &+ (1-\lambda) p \left(\epsilon^g b \left(A h_t - s_t\right) + b - 1\right) \\ &= p\epsilon^g b \left(A h_t - s_t\right) + b\lambda(1-p)\epsilon^r \left(B h_t - s_t\right) + \omega(b-1) \end{aligned} \tag{23}$$

A.1.4. Stage IV

I derive the fifth line of Eq. (12) using Eqs. (7b), (8), (11), and (23).

$$\begin{aligned} h_{t+1} &= p\epsilon^g b \left(A h_t - s_t\right) + b\lambda(1-p)\epsilon^r \left(B h_t - s_t\right) + \omega(b-1) + (1-\lambda)(1-p) h^r_{nt+1} \\ &= p\epsilon^g b \left(A h_t - s_t\right) + b\lambda(1-p)\epsilon^r \left(B h_t - s_t\right) \\ &+ \omega(b-1) + (1-\omega)\left(\epsilon^r b \left(A' h_t - \widetilde{s}_t\right) + b - 1\ \right) \\ &= p\epsilon^g b \left(A h_t - s_t\right) + b\lambda(1-p)\epsilon^r \left(B h_t - s_t\right) \\ &+ (1-\omega)\epsilon^r b \left((1-\alpha) A h_t - s_t\right)\ \ + b - 1 \\ &= b\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(A h_t - s_t\right) + b - 1 \end{aligned} \tag{24}$$

A.2. Universal grant

A.2.1. Stage I

In deriving the first line of Eq. (15), note that in Stage I, only λp number of high-ability individuals from college-educated parents have access to higher education. Therefore, from Eq. (11), the aggregate human capital in Stage I is given as follows:

$$h_{t+1} = \lambda p h^g_{ct+1}$$

Then, substitute the second line from Eq. (7b) into the above to obtain the following.

$$h_{t+1} = \lambda p \left(\epsilon^g b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right)$$

Under the universal grant scheme, anyone who enrolls in college is eligible for tuition grants; therefore, given Eq. (3b), $\widetilde{s}_t = s_t - x_t$ where $x_t = z_t/(\lambda p)$ (which is, from

Table 1, the value of x_t for Stage I and the universal grant scheme). Substituting that into the above gives the first line of Eq. (15):

$$\begin{aligned} h_{t+1} &= \lambda p \left(\epsilon^g b \left(B' h_t - s_t + z_t/(\lambda p)\right) + b - 1\right) \\ &= b\lambda p \epsilon^g \left(B' h_t - s_t\right) + b\epsilon^g z_t + \lambda p (b-1) \end{aligned} \tag{25}$$

A.2.2. Stage II

In Stage II, from Table 1, the value of x_t for Stage II and the universal grant scheme is $x_t = z_t/\lambda$. To derive the second line of Eq. (15), substitute the second line from Eq. (7b) into the first two terms of Eq. (11) to obtain the following.

$$\begin{aligned} h_{t+1} &= \lambda\left(p h^g_{ct+1} + (1-p) h^r_{ct+1}\right) \\ &= \lambda p \left(\epsilon^g b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right) + \lambda(1-p)\left(\epsilon^r b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right) \\ &= \lambda p \epsilon^g b \left(B' h_t - s_t + z_t/\lambda\right) + \lambda(1-p)\epsilon^r b \left(B' h_t - s_t + z_t/\lambda\right) + \lambda(b-1) \\ &= b\lambda\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(B' h_t - s_t\right) + b\left(p\epsilon^g + (1-p)\epsilon^r\right) z_t\ \ + \lambda(b-1) \end{aligned} \tag{26}$$

A similar procedure can be used to derive the rest of the equations.

A.3. Scholarship

Under the scholarship program, in State I and II, λp individuals are eligible for the tuition grants, while in Stage III and IV, p individuals are eligible. Therefore, the respective per capita tuition grants are, considering (3b), $x_t = z_t/(\lambda p)$ and $x_t = z_t/p$ (see Table 1, scholarship).

A.3.1. Stage i

This further implies that aggregate capital dynamics in Stage I, under the scholarship and universal grant schemes, are similar since, in both cases, $x_t = z_t/(\lambda p)$.

A.3.2. Stage II

Note that in Stage II, under the scholarship grant schemes, while λ individuals invest in higher education, only λp are eligible for grants. To derive the aggregate dynamics for this stage, substitute the second line from Eq. (7b) into the first two terms of Eq. (11), and use $x_t = z_t/(\lambda p)$.

$$h_{t+1} = \lambda\left(p h^g_{ct+1} + (1-p) h^r_{ct+1}\right)$$

$$= \lambda p \left(\epsilon^g b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right) + \lambda (1-p) \left(\epsilon^r b \left(B' h_t - \widetilde{s}_t\right) + b - 1\right)$$
$$= \lambda b p \epsilon^g \left(B' h_t - s_t + z_t/\lambda p\right) + \lambda (1-p) \epsilon^r b \left(B' h_t - s_t\right) + \lambda (b-1)$$
$$= \lambda b \left(\left(p\epsilon^g + (1-p)\epsilon^r\right)\left(B' h_t - s_t\right) + 1\right) + b\epsilon^g z_t + \lambda (b-1) \quad (27)$$

A similar procedure can be applied to derive the rest of the equations.

A.4. Means-tested grant

Under the means-tested grant scheme, in State I and II, no individual with college access is eligible for the grants, while in Stage III and IV, $p(1-\lambda)$ individuals are eligible. Therefore, the respective per capita tuition grants are, considering Eq. (3b), $x_t = 0$ and $x_t = z_t/((1-\lambda)p)$ (Table 1, *column 4*).

A.4.1. Stage i and II

Therefore, at the early stages, aggregate dynamics are similar to laissez-faire except that disposable income is income less taxes. That is, aggregate dynamics in Stage I is as follows:

$$h_{t+1} = \lambda p \epsilon^g b \left(B' h_t - s_t\right) + \lambda p (b-1) \quad (28)$$

and in Stage II is given as follows:

$$h_{t+1} = b\lambda \left(p\epsilon^g + (1-p)(\epsilon^r)\right)\left(B' h_t - s_t\right) + \lambda (b-1) \quad (29)$$

A.4.2. Stage III

I derive the aggregate dynamics in Stage III, from Eqs. (7b), (11), and (29) while considering $\widetilde{s}_t = s_t - x_t$ and $x_t = z_t/(p(1-\lambda))$.

$$h_{t+1} = b\lambda \left(p\epsilon^g + (1-p)(\epsilon^r)\right)\left(B' h_t - s_t\right) + \lambda (b-1) + (1-\lambda) p h^g_{nt+1}$$
$$= b\lambda \left(p\epsilon^g + (1-p)(\epsilon^r)\right)\left(B' h_t - s_t\right) + \lambda (b-1) + (1-\lambda) p \left(\epsilon^g b \left(A' h_t - \widetilde{s}_t\right) + b - 1\right)$$
$$= b \left(p\epsilon^g \left((1-\tau) A h_t - s_t\right) + \lambda (1-p) \epsilon^r \left(B' h_t - s_t\right)\right) + b\epsilon^g z_t + \omega (b-1) \quad (30)$$

Follow a similar procedure to derive the dynamics for Stage IV.

Appendix B. Investment thresholds under higher education policy

I derive the investment threshold levels, determining individuals' college education investment, associated with the different grant schemes in a similar fashion to the laissez-faire case (see (9)). That is, the investment threshold associated with the ith individual of ability j is derived by substituting $e^j_{it} = 0$ and $\widetilde{s}_t = s_t - x_t$ into (6a) and solving for h_t. The value of x_t is determined from (5b) and Table 1 , accordingly.

But note that the per capita tuition grant (x_t) is different, for different grant schemes and at different phases of higher education development, as shown in Table 1. Furthermore, it may differ among individuals due to differences in eligibility. Therefore, in contrast to laissez-faire, one may end up having different threshold levels for different phases of higher education development when applying the same policy.

B.1. Universal grant

The investment threshold of the j ability agents with non-college-educated ($\overline{h}^j_n(u)$) and college-educated ($\overline{h}^j_c(u)$) agents are derived by substituting $e^j_{it} = 0$, $\widetilde{s}_t = s_t - x_t$ and $s_t = s_k + s h_t$ into (6a), considering $\lambda h^j_{it} = h_t$, and using (3) and (4).

$$\overline{h}^j_n(u) : \left(A' h_t - s_k - s h_t + x_t\right) - 1/\left(\beta\epsilon^j\right) = 0 \quad (31)$$

$$\overline{h}^j_c(u) : \left((1-\tau) B' h_t - s_k - s h_t + x_t\right) - 1/\left(\beta\epsilon^j\right) = 0 \quad (32)$$

Then, solving for h_t in Eqs. (31) and (32), considering Eq. (10), Table 1, column 2, and Eq. (5b), gives the following.[32]

$$\overline{h}^g_c(u) \equiv \left(\frac{1}{\epsilon^g\beta} + s_I\right)\left(B' + A\tau/(\lambda p) - s\right)^{-1} \quad (33a)$$
$$\overline{h}^r_c(u) \equiv \left(\frac{1}{\beta\epsilon^r} + s_{II}\right)\left(B' + A\tau/\lambda - s\right)^{-1} \quad (33b)$$
$$\overline{h}^g_n(u) \equiv \left(\frac{1}{\beta\epsilon^j} + s_{III}\right)\left(A' + A\tau/\omega - s\right)^{-1} \quad (33c)$$
$$\overline{h}^r_n(u) \equiv \left(\frac{1}{\beta\epsilon^r} + s_{IV}\right)\left(A' + A\tau - s\right)^{-1} \quad (33d)$$

I follow similar steps to derive the investment thresholds associated with the scholarship and means-tested grant schemes.

B.2. Scholarship

By solving for h_t in Eqs. (31) and (32), considering Eq. (10), Table 1, column 3, and Eq. (5b), one derives individual investment thresholds associated with the scholarship grant.

$$\overline{h}^g_c(s) \equiv \left(\frac{1}{\epsilon^g\beta} + s_I\right)\left(B' + A\tau/(\lambda p) - s\right)^{-1} \quad (34a)$$
$$\overline{h}^r_c(s) \equiv \left(\frac{1}{\epsilon^r\beta} + s_{II}\right)\left(B' - s\right)^{-1} \quad (34b)$$
$$\overline{h}^g_n(s) \equiv \left(\frac{1}{\beta\epsilon^g} + s_{III}\right)\left(A' + A\tau/p - s\right)^{-1} \quad (34c)$$
$$\overline{h}^r_n(s) \equiv \left(\frac{1}{\beta\epsilon^r} + s_{IV}\right)\left(A' - s\right)^{-1} \quad (34d)$$

$\overline{h}^j_i(s)$ is the threshold associated with the ith person of j ability if the grant scheme is scholarship. The number of eligible individuals for the tuition grants is different from the college enrollment rate as the scheme naturally excludes some individuals. In Stage I, λp individuals enroll in college where all are eligible for tuition grants. Meanwhile, in Stage II, λ individuals enroll in college but only the λp high-ability individuals are eligible for the tuition grants. In Stages III and IV, ω and one unit of individuals enroll in college, respectively, but only the p high-ability individuals are eligible for the tuition grants.

B.3. Means-tested grant

By solving for h_t in Eqs. (31) and (32), considering Eq. (10), Table 1, column 4, and Eq. (5b), one derives the investment thresholds associated with the means-tested grant program.

$$\overline{h}^g_c(m) = \left(\frac{1}{\epsilon^g\beta} + s_I\right)\left(B' - s\right)^{-1} \quad (35a)$$
$$\overline{h}^r_c(m) = \left(\frac{1}{\epsilon^r\beta} + s_{II}\right)\left(B' - s\right)^{-1} \quad (35b)$$
$$\overline{h}^g_n(m) = \left(\frac{1}{\beta\epsilon^g} + s_{III}\right)\left(A' + A\tau/((1-\lambda)p) - s\right)^{-1} \quad (35c)$$
$$\overline{h}^r_n(m) = \left(\frac{1}{\beta\epsilon^r} + s_{IV}\right)\left(A' - s\right)^{-1} \quad (35d)$$

where $\overline{h}^j_i(m)$ is the threshold associated with the ith person of j ability if the grant scheme is means-tested. No one is eligible for this scheme in Stages I and II. However, in Stages III and IV, there are $(1-\lambda)p$ high-ability individuals who are eligible to the program.

[32] Recall that the enrollment rate at each stage is similar to the number of eligible individuals because, in the universal grant scheme, anyone who enrolls in college is automatically eligible for the tuition grants.

Table 9
Individual human capital: Stage III

Schemes	Human capital investment by individual type			
	λp	$\lambda(1-p)$	$(1-\lambda)p$	$(1-\lambda)\times(1-p)$
Laissez-faire	$\epsilon^g b(Bh_t - s_t) + b - 1$	$\epsilon^r b(Bh_t - s_t) + b - 1$	$\epsilon^g b(A(1-\alpha)h_t - s_t) + b - 1$	0
Universal	$\epsilon^g b(B'h_t - s_t + z_t/\omega) + b - 1$	$\epsilon^r b(B'h_t - s_t + z_t/\omega) + b - 1$	$\epsilon^g b(A'h_t - s_t + z_t/\omega) + b - 1$	0
Scholarship	$\epsilon^g b(B'h_t - s_t + z_t/p) + b - 1$	$\epsilon^g b(B'h_t - s_t) + b - 1$	$\epsilon^g b(A'h_t - s_t + z_t/p) + b - 1$	0
Means-tested	$\epsilon^g b(B'h_t - s_t) + b - 1$	$\epsilon^g b(B'h_t - s_t) + b - 1$	$\epsilon^g b(A'h_t - s_t + z_t/(1-\lambda)p) + b - 1$	0

Table 10
Individual human capital: Stage IV.

Schemes	Human capital investment by individual type			
	λp	$\lambda(1-p)$	$(1-\lambda)p$	$(1-\lambda)(1-p)$
Laissez-faire	$\epsilon^g b(Bh_t - s_t) + b - 1$	$\epsilon^r b(Bh_t - s_t) + b - 1$	$\epsilon^g b(A(1-\alpha)h_t - s_t) + b - 1$	$\epsilon^r b(A(1-\alpha)h_t - s_t) + b - 1$
Universal	$\epsilon^g b(B'h_t - s_t + z_t) + b - 1$	$\epsilon^r b(B'h_t - s_t + z_t) + b - 1$	$\epsilon^g b(A'h_t - s_t + z_t) + b - 1$	$\epsilon^r b(A'h_t - s_t + z_t) + b - 1$
Scholarship	$\epsilon^g b(B'h_t - s_t + z_t/p) + b - 1$	$\epsilon^g b(B'h_t - s_t) + b - 1$	$\epsilon^g b(A'h_t - s_t + z_t/p) + b - 1$	$\epsilon^r b(A'h_t - s_t) + b - 1$
Means-tested	$\epsilon^g b(B'h_t - s_t) + b - 1$	$\epsilon^g b(B'h_t - s_t) + b - 1$	$\epsilon^g b(A'h_t - s_t + z_t/(1-\lambda)p) + b - 1$	$\epsilon^r b(A'h_t - s_t) + b - 1$

Appendix C. Equity

Tables 9 and 10 show human capital investment individual type associated with Stages III and IV of the higher education development.

Appendix D. Proofs

This section provides the proofs for the Propositions.

D.1. Proposition 1

Proof. It is straightforward that it follows Table 4 and the preceding discussion. □

D.2. Proposition 2

Proof. Comparing the investment thresholds associated with the universal $\overline{h}_i^j(u)$, scholarship $\overline{h}_i^j(s)$, and means-tested $\overline{h}_i^j(m)$ grant schemes in Eqs. (33), (34), and (35), respectively, to the thresholds associated with laissez-faire $\overline{h}_i^j(l)$ in Eq. (9), one notes the following.

(1) In Stage I, $\overline{h}_c^g(m) > \overline{h}_c^g(l) > \overline{h}_c^g(u) = \overline{h}_c^g(s)$; the investment threshold associated with the means-tested program is the largest followed by the one for laissez-faire.
(2) In Stage II, $\overline{h}_c^g(s) = \overline{h}_c^g(m) > \overline{h}_c^r(l) > \overline{h}_c^r(u)$; the investment threshold for the universal grant scheme is the smallest followed by the one for laissez-faire.
(3) In Stage III, $\overline{h}_n^g(l) > \overline{h}_n^g(u) > \overline{h}_n^g(s) > \overline{h}_n^g(m)$; the investment threshold associated with the means-tested program is the smallest followed by the one for the scholarship program. The threshold associated with laissez-faire is the largest.
(4) In Stage IV, $\overline{h}_n^r(s) = \overline{h}_n^r(m) > \overline{h}_n^r(l) > \overline{h}_n^r(u)$; the threshold for the universal grant scheme is the smallest followed by the one for laissez-faire. □

Appendix E. Welfare

In this section, I derive individual and aggregate welfare functions. Depending on individual's educational background, whether they invest in their children's education, and their ability level, the welfare functions for them may vary.

E.1. Individual welfare

Individuals who do not invest in children's education consume the full amount of their income. Thus, their welfare is the log of their income.

$$u_{it}^j \equiv \ln c_{it}^j = \ln I_{it}^j$$

Substituting Eqs. (3) and (4) into the above, one gets the welfare of college- and non-college-educated individuals who do not invest in children's education.

$$u_{it}^j = \begin{cases} \ln\left(A'h_t\right) & \text{if } h_{it}^j = 0 \\ \ln\left(B'h_t\right) & \text{if } h_{it}^j > 0 \end{cases} \tag{36}$$

For individuals who invest in children's education, their welfare function is derived from Eqs. (1), (2) and (6a).

$$\begin{aligned} u_{it}^j &\equiv \ln c_{it}^j + \beta \ln\left(h_{it+1}^j + 1\right) \\ &= \ln\left(I_{it}^j - e_{it}^j - \widetilde{s}_t\right) + \beta \ln\left(\epsilon^j e_{it}^j + 1\right) \\ &= (1+\beta)\ln\left(\epsilon^j\left(I_{it}^j - \widetilde{s}_t\right) + 1\right) + \beta \ln b + \ln\frac{b}{\beta} - \ln\epsilon^j \end{aligned}$$

Substituting (3) and (4) into the above, one gets the welfare of college-educated and non-college-educated individuals who invest in their children's education.

$$u_{it}^j = \begin{cases} (1+\beta)\ln\left(\epsilon^j\left(A'h_t - \widetilde{s}_t\right) + 1\right) - \ln\epsilon^j + \overline{b} & \text{if } h_{it}^j = 0 \\ (1+\beta)\ln\left(\epsilon^j\left(B'h_t - \widetilde{s}_t\right) + 1\right) - \ln\epsilon^j + \overline{b} & \text{if } h_{it}^j > 0 \end{cases} \tag{37}$$

where

$$\overline{b} \equiv \beta \ln b + \ln\frac{b}{\beta}$$

Aggregate welfare is computed by aggregating the welfare of all individuals.

$$U_t = \lambda\left[pu^g_{ct} + (1-p)u^r_{ct}\right] + (1-\lambda)\left[pu^g_{nt} + (1-p)u^r_{nt}\right] \tag{38}$$

Using Eqs. (36), (37) and (38), I derive the aggregate welfare functions for different public programs and stages of development.

E.2. Aggregate welfare

I derive the utilitarian social welfare function, following a similar approach to the literature in standard heterogeneous agent models (Aiyagari and McGrattan, 1998; Gibson and Rioja, 2019). Notably, aggregation is conducted after equal weight is applied to each agent. Benabou (2002) argues that such aggregation overestimates the efficiency value of redistribution due to the concavity of the utility function. However, this is not an issue here, given the main focus is not on quantifying the welfare impact of redistribution but on conducting a qualitative comparison.

E.2.1. Stage I

In Stage I, only λp rich individuals with high-ability children invest in education. To derive the aggregate welfare in Stage I, first, substitute the second lines of Eqs. (36) and (37) in the first two terms of Eq. (38). Then, substitute the first line of Eq. (36) into the last two terms of Eq. (38) to get the following.

$$\begin{aligned} U^I_t &= \lambda p\left[(1+\beta)\ln\left(\epsilon^g\left(B'h_t - \widetilde{s}_t\right)+1\right) - \ln\epsilon^g + \bar{b}\right] \\ &+ \lambda(1-p)\ln\left(B'h_t\right) + (1-\lambda)\ln\left(A'h_t\right) \end{aligned}$$

This can be further simplified using the relation $\ln(y+1)\approx y$.

$$\begin{aligned} U^I_t &\approx \lambda p\left[(1+\beta)\epsilon^g\left(B'h_t - s_t + x_t\right) - \ln\epsilon^g + \bar{b}\right] \\ &+ \lambda(1-p)\ln\left(B'h_t\right) + (1-\lambda)\ln\left(A'h_t\right) \end{aligned} \tag{39}$$

I use the above to derive the aggregate welfare functions under the three grant schemes and laissez-faire in Stage I.

Laissez-faire. In the laissez-faire case, substituting Eq. (8) into the above, one obtains the following.

$$\begin{aligned} U^I_{t,l} &\approx \lambda p(1+\beta)\epsilon^g\left(Bh_t - s_t\right) + \lambda(1-p)\ln\left(Bh_t\right) \\ &+ (1-\lambda)\ln\left((1-\alpha)Ah_t\right) - \lambda p\ln\epsilon^g + \lambda p\bar{b} \end{aligned} \tag{40}$$

Universal and scholarship grants. Aggregate welfare is the same under the universal and scholarship grant schemes, given $x_t = z_t/(\lambda p)$. Substituting $x_t = z_t/(\lambda p)$ in Eq. (39) leads to the following.

$$\begin{aligned} U^I_{t,u} &\approx \lambda p(1+\beta)\epsilon^g\left(B'h_t - s_t\right) + (1+\beta)\epsilon^g z_t \\ &+ \lambda(1-p)\ln\left(B'h_t\right) + (1-\lambda)\ln\left(A'h_t\right) - \lambda p\ln\epsilon^g + \lambda p\bar{b} \end{aligned} \tag{41}$$

Thus, $U^I_{t,s} = U^I_{t,u}$.

Means-tested. Under the means-tested program, in Stage I, the agents who join college are not eligible for tuition subsidy thus $x_t = 0$; this implies the following.

$$\begin{aligned} U^I_{t,m} &\approx \lambda p(1+\beta)\epsilon^g\left(B'h_t - s_t\right) + \lambda(1-p)\ln\left(B'h_t\right) \\ &+ (1-\lambda)\ln\left(A'h_t\right) - \lambda p\ln\epsilon^g + \lambda p\bar{b} \end{aligned} \tag{42}$$

E.2.2. Stage II

In Stage II, the λ rich individuals invest in their children's education. To derive the aggregate welfare in this stage, first substitute the second line of Eq. (37) in the first two terms of Eq. (38) and then substitute the first line of Eq. (36) into the last two terms of Eq. (38) to obtain the following.

$$\begin{aligned} U^{II}_t &= \lambda p\left[(1+\beta)\ln\left(\epsilon^g\left(B'h_t - \widetilde{s}_t\right)+1\right) - \ln\epsilon^g + \bar{b}\right] \\ &+ \lambda(1-p)\left[(1+\beta)\ln\left(\epsilon^r\left(B'h_t - \widetilde{s}_t\right)+1\right) - \ln\epsilon^r + \bar{b}\right] + (1-\lambda)\ln\left(A'h_t\right) \end{aligned}$$

This can be further approximated.

$$\begin{aligned} U^{II}_t &\approx \lambda p(1+\beta)\epsilon^g\left(B'h_t - s_t + x_t\right) + \lambda(1-p)(1+\beta)\epsilon^r\left(B'h_t - s_t + x_t\right) \\ &+ (1-\lambda)\ln\left(A'h_t\right) + \lambda\bar{b} - \lambda\left(p\ln\epsilon^g + (1-p)\ln\epsilon^r\right) \end{aligned} \tag{43}$$

I use the above to derive the aggregate welfare functions under the three grant schemes, and laissez-faire, in Stage II.

Laissez-faire. In the Laissez-faire case, given Eq. (8), one obtains the following.

$$\begin{aligned} U^{II}_{t,l} &\approx \lambda\left[p\epsilon^g + (1-p)\epsilon^r\right](1+\beta)\left(Bh_t - s_t\right) \\ &+ (1-\lambda)\ln\left((1-\alpha)Ah_t\right) + \lambda\bar{b} - \lambda\left(p\ln\epsilon^g + (1-p)\ln\epsilon^r\right) \end{aligned} \tag{44}$$

Universal grant. Under the universal grant scheme, in Stage II, $x_t = z_t/\lambda$. Substituting that into Eq. (43) and considering that all individuals are eligible for the grant, one obtains the following.

$$\begin{aligned} U^{II}_{t,u} &\approx \lambda\left[p\epsilon^g + (1-p)\epsilon^r\right](1+\beta)\left(B'h_t - s_t\right) + \left[p\epsilon^g + (1-p)\epsilon^r\right](1+\beta)z_t \\ &+ (1-\lambda)\ln\left(A'h_t\right) + \lambda\bar{b} - \left(\lambda p\ln\epsilon^g + \lambda(1-p)\ln\epsilon^r\right) \end{aligned} \tag{45}$$

Scholarship. Under the scholarship program, only individuals with high ability are eligible for the grant. Thus, I apply $x_t = z_t/(p\lambda)$ in the first line of (43) and $x_t = 0$ in the second line.

$$\begin{aligned} U^{II}_{t,s} &\approx \left(p\epsilon^g + (1-p)\epsilon^r\right)\lambda(1+\beta)\left(B'h_t - s_t\right) + (1+\beta)\epsilon^g z_t \\ &+ (1-\lambda)\ln\left(A'h_t\right) + \lambda\bar{b} - \lambda\left((1-p)\ln\epsilon^r + p\ln\epsilon^g\right) \end{aligned} \tag{46}$$

Means-tested. Under the means-tested grant, in Stage II, both types of agents who join college are not eligible for the program thus $x_t = 0$, implying the following.

$$\begin{aligned} U^{II}_{t,m} &\approx \lambda\left[p\epsilon^g + (1-p)\epsilon^r\right]\left(B'h_t - s_t\right)(1+\beta) \\ &+ (1-\lambda)\ln\left(A'h_t\right) + \lambda\bar{b} - \lambda\left(p\ln\epsilon^g + (1-p)\ln\epsilon^r\right) \end{aligned} \tag{47}$$

E.2.3. Stage III

In Stage III, ω number of individuals invest in their children's education. To derive the aggregate welfare for this stage, first substitute Eq. (37) into the first three terms of Eq. (38) and then substitute the first line of Eq. (36) into the last term.

$$\begin{aligned} U^{III}_t &= \lambda p\left[(1+\beta)\ln\left(\epsilon^g\left(B'h_t - s_t + x_t\right)+1\right) - \ln\epsilon^g + \bar{b}\right] \\ &+ \lambda(1-p)\left[(1+\beta)\ln\left(\epsilon^r\left(B'h_t - s_t + x_t\right)+1\right) - \ln\epsilon^r + \bar{b}\right] \\ &(1-\lambda)p\left[(1+\beta)\ln\left(\epsilon^g\left(A'h_t - s_t + x_t\right)+1\right) - \ln\epsilon^g + \bar{b}\right] \\ &+ (1-\omega)\ln\left(A'h_t\right) \end{aligned}$$

This can be further simplified using similar procedures as above.

$$\begin{aligned} U^{III}_t &\approx \lambda p(1+\beta)\epsilon^g\left(B'h_t - s_t + x_t\right) \\ &+ \lambda(1-p)(1+\beta)\epsilon^r\left(B'h_t - s_t + x_t\right) \\ &+ (1-\lambda)p(1+\beta)\epsilon^g\left(A'h_t - s_t + x_t\right) \\ &+ (1-\omega)\ln\left(A'h_t\right) - \left(\lambda(1-p)\ln\epsilon^r + p\ln\epsilon^g\right) + \omega\bar{b} \end{aligned} \tag{48}$$

I use the above to derive the aggregate welfare functions under the three grant schemes, and laissez-faire, in Stage III.

Laissez-faire. In the laissez-faire case, substituting Eq. (8) into Eq. (48) gives the following.

$$\begin{aligned} U^{III}_{t,l} &\approx p\epsilon^g(1+\beta)\left(\lambda Bh_t + (1-\lambda)(1-\alpha)Ah_t - s_t\right) \\ &+ \lambda(1-p)(1+\beta)\epsilon^r\left(Bh_t - s_t\right) \\ &+ (1-\omega)\ln\left((1-\alpha)Ah_t\right) - \left(\lambda(1-p)\ln\epsilon^r + p\ln\epsilon^g\right) + \omega\bar{b} \end{aligned}$$

$$\begin{aligned} U^{III}_{t,l} &\approx \lambda(1-p)(1+\beta)\epsilon^r\left(Bh_t - s_t\right) \\ &+ p\epsilon^g(1+\beta)\left(Ah_t - s_t\right) \end{aligned}$$

$$+\ (1-\omega)\ln\left((1-\alpha)Ah_t\right)-\left(\lambda(1-p)\ln\epsilon^r+p\ln\epsilon^g\right)+\omega\bar{b} \tag{49}$$

since $\lambda Bh_t+(1-\lambda)(1-\alpha)Ah_t=Ah_t$.

Universal grant. Under the universal grant scheme, in Stage III, $x_t = z_t/\omega$. Substituting that into Eq. (48) and considering the fact that all individuals are eligible for the grant, one obtains the following.

$$\begin{aligned}U_{t,u}^{III} \approx\ & \lambda p(1+\beta)\epsilon^g\left(B'h_t-s_t\right)+\vartheta(1+\beta)z_t/\omega\\ &+\lambda(1-p)(1+\beta)\epsilon^r\left(B'h_t-s_t\right)\\ &+(1-\lambda)p(1+\beta)\epsilon^g\left(A'h_t-s_t\right)\\ &+(1-\omega)\ln\left(A'h_t\right)-\left(\lambda(1-p)\ln\epsilon^r+p\ln\epsilon^g\right)+\omega\bar{b}\end{aligned}$$

$$\begin{aligned}U_{t,u}^{III} \approx\ & \lambda(1-p)\epsilon^r(1+\beta)\left(B'h_t-s_t\right)+\vartheta(1+\beta)z_t/\omega\\ &+p(1+\beta)\epsilon^g\left((1-\tau)Ah_t-s_t\right)+(1-\omega)\ln\left(A'h_t\right)\\ &-\left(\lambda(1-p)\ln\epsilon^r+p\ln\epsilon^g\right)+\omega\bar{b}\end{aligned} \tag{50}$$

where $\vartheta\equiv p\epsilon^g+\lambda\epsilon^r(1-p)$ and given $\lambda B'h_t+(1-\lambda)A'h_t=(1-\tau)Ah_t$.

Scholarship. Under the scholarship program, only high-ability individuals are eligible for the grant. Thus, I apply $x_t = z_t/p$ for high-ability individuals (ϵ^g) in Eq. (48) and $x_t = 0$ for regular-ability individuals (ϵ^r).

$$\begin{aligned}U_{t,s}^{III} \approx\ & \lambda p(1+\beta)\epsilon^g\left(B'h_t-s_t\right)+\lambda p(1+\beta)\epsilon^g z_t/p+(1-\lambda)p(1+\beta)\epsilon^g z_t/p\\ &+\lambda(1-p)(1+\beta)\epsilon^r\left(B'h_t-s_t+x_t\right)+(1-\lambda)p(1+\beta)\epsilon^g\left(A'h_t-s_t\right)\\ &+(1-\omega)\ln\left(A'h_t\right)-\left(\lambda(1-p)\ln\epsilon^r+p\ln\epsilon^g\right)+\omega\bar{b}\end{aligned}$$

$$\begin{aligned}U_{t,s}^{III} \approx\ & \lambda(1-p)\epsilon^r(1+\beta)\left(B'h_t-s_t\right)+(1+\beta)\epsilon^g z_t\\ &+p(1+\beta)\epsilon^g\left((1-\tau)Ah_t-s_t\right)+(1-\omega)\ln\left(A'h_t\right)\\ &-\left(\lambda(1-p)\ln\epsilon^r+p\ln\epsilon^g\right)+\omega\bar{b}\end{aligned} \tag{51}$$

Means-tested. Under means-tested, in Stage III, only poor individuals with high ability are eligible for the program; thus, $x_t = z_t/((1-\lambda)p)$ is applied in the third line of Eq. (48) whereas $x_t = 0$ is applied in the rest.

$$\begin{aligned}U_{t,m}^{III} \approx\ & \lambda(1-p)\epsilon^r(1+\beta)\left(B'h_t-s_t\right)+(1+\beta)\epsilon^g z_t\\ &+p(1+\beta)\epsilon^g\left((1-\tau)Ah_t-s_t\right)+(1-\omega)\ln\left(A'h_t\right)\\ &-\left(\lambda(1-p)\ln\epsilon^r+p\ln\epsilon^g\right)+\omega\bar{b}\end{aligned} \tag{52}$$

E.2.4. Stage IV

To derive the aggregate welfare in Stage IV, substitute (36) and (37) into (38) to obtain the following.

$$\begin{aligned}U_t^{IV} =\ & \lambda p\left((1+\beta)\ln\left(\epsilon^g\left(B'h_t-\tilde{s}_t\right)+1\right)-\ln\epsilon^g+\bar{b}\right)\\ &+\lambda(1-p)\left((1+\beta)\ln\left(\epsilon^r\left(B'h_t-\tilde{s}_t\right)+1\right)-\ln\epsilon^r+\bar{b}\right)\\ &+(1-\lambda)p\left((1+\beta)\ln\left(\epsilon^g\left(A'h_t-\tilde{s}_t\right)+1\right)-\ln\epsilon^g+\bar{b}\right)\\ &+(1-\lambda)(1-p)\left((1+\beta)\ln\left(\epsilon^r\left(A'h_t-\tilde{s}_t\right)+1\right)-\ln\epsilon^r+\bar{b}\right)\end{aligned}$$

Further simplifying the above gives the following.

$$\begin{aligned}U_t^{IV} \approx\ & (1+\beta)\lambda\left[p\epsilon^g\left(B'h_t-s_t+x_t\right)+(1-p)\epsilon^r\left(B'h_t-s_t+x_t\right)\right]\\ &+(1-\lambda)(1+\beta)\left[p\epsilon^g\left(A'h_t-s_t+x_t\right)+(1-p)\epsilon^r\left(A'h_t-s_t+x_t\right)\right]\\ &-\left(p\ln\epsilon^g+(1-p)\ln\epsilon^r\right)+\bar{b}\end{aligned} \tag{53}$$

Applying the same procedures, I derive the aggregate welfare functions associated with all the schemes in Stage IV.

Laissez-faire and universal grant. Aggregate welfare functions for laissez-faire ($U_{t,u}^{IV}$) and universal grants ($U_{t,l}^{IV}$) are the same in Stage IV.

$$\begin{aligned}U_{t,l}^{IV} \approx\ & \left(p\epsilon^g+(1-p)\epsilon^r\right)(1+\beta)\left(Ah_t-s_t\right)\\ &-\left(p\ln\epsilon^g+(1-p)\ln\epsilon^r\right)+\bar{b}\end{aligned} \tag{54}$$

where $U_{t,u}^{IV}=U_{t,l}^{IV}$.

Table 11
Gross enrollment ratio in tertiary education—low-income countries.

Country	2013
Benin	15.3628
Burkina Faso	4.77591
Burundi	4.40817
Congo, Dem. Rep.	6.64076
Guinea	10.3789
Madagascar	4.24579
Mozambique	5.04323
Rwanda	7.52925
Tanzania	3.64732
Togo	10.0422
Zimbabwe	5.87175

Table 12
Gross enrollment ratio in tertiary education—high-income countries.

Country	1971	2013
Argentina	15.3701	79.9867
Australia	17.0328	86.5546
Austria	12.2113	80.3868
Belgium	16.8641	72.3096
Chile	11.1577	83.8164
Czech Republic	8.92373	65.3774
Denmark	18.8583	81.237
Finland	13.1341	91.0658
France	18.5413	62.1469
Hong Kong SAR, China	6.83597	67.2759
Hungary	10.0217	57.0167
Ireland	10.5903	73.1685
Italy	16.8803	63.4551
Japan	17.6406	62.4116
Korea, Rep.	7.24645	95.3454
Malta	6.51885	45.6805
New Zealand	16.9108	79.7143
Norway	15.7949	76.1179
Panama	10.3144	38.7393
Poland	13.3588	71.1587
Portugal	7.27266	66.2216
Spain	8.66966	87.0658
Sweden	21.7328	63.3929
Switzerland	10.0385	56.2682
United Kingdom	14.5679	56.8701
United States	47.3235	88.8086

Scholarship and means-tested. Meanwhile, aggregate welfare functions for the scholarship ($U_{t,s}^{IV}$) and means-tested grants ($U_{t,m}^{IV}$) are the same.

$$\begin{aligned}U_{t,s}^{IV} \approx\ & \left(p\epsilon^g+(1-p)\epsilon^r\right)(1+\beta)\left((1-\tau)Ah_t-s_t\right)+(1+\beta)\epsilon^g z_t\\ &-\left(p\ln\epsilon^g+(1-p)\ln\epsilon^r\right)+\bar{b}\end{aligned} \tag{55}$$

where $U_{t,m}^{IV}=U_{t,s}^{IV}$.

E.3. Welfare comparison

I derive the welfare impact of the different policies simply by looking at the difference in aggregate welfare between the programs. An alternative or a more standard approach is to measure the welfare effects using an equivalent variation—the percentage of wealth the average consumer must give up to be indifferent in the other state (Conesa et al., 2018). For example, a comparison between laissez-faire and universal grant can be made using the latter approach as $U_{t,l}(h) = U_{t,u}((1-cv)h)$, where cv is the percentage of wealth the average consumer should give up to be in the laissez-faire state. Qualitatively, the two approaches provide the same results, although they could differ quantitatively. In the first approach, $U_{t,u}(h) > U_{t,l}(h)$, implying that the consumer is better off with the universal grant, hereby $cv > 0$. Given

$U_t(h)$ increases in h, for $U^I_{t,l}(h) = U^I_{t,u}((1-cv)h)$ to hold, given the consumer is better off in universal grant, $1-cv<1$ or $cv>0$.[33]

The welfare gain when moving from laissez-faire to the universal grant in column 2 of Table 2 is derived by taking the differences between Eqs. (40) and (41) for Stage I ($U^I_{t,u} - U^I_{t,l}$), between Eqs. (44) and (45) for Stage II ($U^{II}_{t,u} - U^{II}_{t,l}$), and between Eqs. (49) and (50) for Stage III ($U^{III}_{t,u} - U^{III}_{t,l}$). Moreover, for Stage IV, aggregate welfare is similar for laissez-faire and the universal grant ($U^{IV}_{t,u} = U^{IV}_{t,l}$); thus, the difference is zero.

The welfare gain when moving from the universal grant to the scholarship grant, column 2 of Table 3, using the first approach, is derived as follows. For Stage I, aggregate welfare is similar for both the universal and scholarship grants as shown in Eq. (41) ($U^I_{t,s} = U^I_{t,u}$); thus, the difference is zero. For Stage II, take the difference between Eqs. (45) and (46) ($U^{II}_{t,s} - U^{II}_{t,u}$); for Stage III, take the difference between Eqs. (50) and (51) ($U^{III}_{t,s} - U^{III}_{t,u}$); and finally, for Stage IV, take the difference between Eqs. (54) and (55) ($U^{IV}_{t,s} - U^{IV}_{t,u}$).

Appendix F. Gross enrollment ratio in tertiary education

Tables 11 and 12 show the number of individuals enrolled in tertiary education as a percentage of the total population of the five-year age group following on from leaving secondary school.[34]

References

Abbott, B., Gallipoli, G., Meghir, C., Violante, G.L., 2019. Education policy and intergenerational transfers in equilibrium. J. Polit. Econ. 127 (6), 2569–2624.

Acemoglu, D., Angrist, J., 2000. How large are human-capital externalities? Evidence from compulsory schooling laws. NBER Macroecon. Annu. 15, 9–59.

Aiyagari, S.R., McGrattan, E.R., 1998. The optimum quantity of debt. J. Monetary Econ. 42 (3), 447–469.

Akyol, A., Athreya, K., 2005. Risky higher education and subsidies. J. Econom. Dynam. Control 29 (6), 979–1023.

Asiedu, E., Azomahou, T.T., Getachew, Y., Yitbarek, E., 2021. Share the love: Parental bias, women empowerment and intergenerational mobility. J. Econ. Behav. Organ. 191, 846–867.

Banerjee, A.V., Newman, A.F., 1993. Occupational choice and the process of development. J. Polit. Econ. 101 (2), 274–298.

Barham, V., Boadway, R., Marchand, M., Pestieau, P., 1995. Education and the poverty trap. Eur. Econ. Rev. 39 (7), 1257–1275.

Benabou, R., 2000. Unequal societies: Income distribution and the social contract. Amer. Econ. Rev. 90 (1), 96–129.

Benabou, R., 2002. Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? Econometrica 70 (2), 481–517.

Benhabib, J., Spiegel, M.M., 1994. The role of human capital in economic development evidence from aggregate cross-country data. J. Monet. Econ. 34 (2), 143–173.

Bovenberg, A.L., Jacobs, B., 2005. Redistribution and education subsidies are Siamese twins. J. Public Econ. 89, 11–12.

Caucutt, E.M., Kumar, K.B., 2003. Higher education subsidies and heterogeneity: A dynamic analysis. J. Econom. Dynam. Control 27 (8), 1459–1502.

Cigno, A., Luporini, A., 2009. Scholarships or student loans? Subsidizing higher education in the presence of moral hazard. J. Public Econ. Theory 11 (1), 55–87.

Conesa, J.C., Costa, D., Kamali, P., Kehoe, T.J., Nygard, V.M., Raveendranathan, G., Saxena, A., 2018. Macroeconomic effects of medicare. J. Econ. Ageing 11, 27–40.

De Fraja, G., 2002. The design of optimal education policies. Rev. Econ. Stud. 69 (2), 437–466.

de la Croix, D., Michel, P., 2002. A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations. Cambridge University Press, Cambridge.

Del Rey, E., Racionero, M., 2010. Financing schemes for higher education. Eur. J. Political Econ. 26 (1), 104–113.

Fender, J., Wang, P., 2003. Educational policy in a credit constrained economy with skill heterogeneity. Internat. Econom. Rev. 44 (3), 939–964.

Fernandez, R., Rogerson, R., 1995. On the political economy of education subsidies. Rev. Econ. Stud. 62 (2), 249–262.

Fishman, A., Simhon, A., 2002. The division of labor, inequality and growth. J. Econ. Growth 7, 117–136.

Galor, O., Moav, O., 2004. From physical to human capital accumulation: Inequality and the process of development. Rev. Econ. Stud. 71 (4), 1001–1026.

Galor, O., Zeira, J., 1993. Income distribution and macroeconomics. Rev. Econ. Stud. 60 (1), 35–52.

Garcia-Penalosa, C., Wälde, K., 2000. Efficiency and equity effects of subsidies to higher education. Oxf. Econ. Pap. 52 (4), 702–722.

Gary-Bobo, R.J., Trannoy, A., 2015. Optimal student loans and graduate tax under moral hazard and adverse selection. Rand J. Econ. 46 (3), 546–576.

Getachew, Y.Y., 2016. Credit constraints, growth and inequality dynamics. Econ. Model. 54, 364–376.

Geven, K., 2015. How did the latest increase in fees in England affect student enrolment and inequality? In: The European Higher Education Area. Springer, pp. 479–500.

Gibson, J., Rioja, F., 2019. The welfare effects of infrastructure investment in a heterogeneous agents economy. BE J. Macroecon. 20 (1), 20190095.

Glomm, G., Ravikumar, B., 1992. Public versus private investment in human capital: Endogenous growth and income inequality. J. Polit. Econ. 100 (4), 818–834.

Hansen, W.L., Weisbrod, B.A., 1969. The distribution of costs and direct benefits of public higher education: The case of California. J. Hum. Resour. 17, 6–191.

Heijdra, B.J., Kindermann, F., Reijnders, L.S., 2017. Life in shackles? The quantitative implications of reforming the educational financing system. Rev. Econ. Dyn. 25, 37–57.

Krueger, A.B., Lindahl, M., 2001. Education for growth: Why and for whom? J. Econ. Lit. 39 (4), 1101–1136.

Krueger, D., Ludwig, A., 2016. On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium. J. Monetary Econ. 77, 72–98.

Moav, O., 2002. Income distribution and macroeconomics: The persistence of inequality in a convex technology framework. Econom. Lett. 75 (2), 187–192.

Moretti, E., 2004. Human capital externalities in cities. In: Handbook of Regional and Urban Economics, vol. 4, Elsevier, pp. 2243–2291.

Romer, P.M., 1986. Increasing returns and long-run growth. J. Polit. Econ. 94 (5), 1002–1037.

Trow, M., 1973. Problems in the Transition from Elite to Mass Higher Education. ERIC.

Trow, M., 2007. Reflections on the transition from elite to mass to universal access: Forms and phases of higher education in modern societies since WWII. In: International Handbook of Higher Education. Springer, pp. 243–280.

Zeng, J., Zhang, J., 2022. Education policies and development with threshold human capital externalities. Econ. Model. 108, 105744.

[33] For example, in Stage I, $cv = 0.0894$, using parameter values in Section 7. The average consumer will be indifferent between laissez-faire and universal grants if aggregate human capital is lower by about 9% in the latter.

[34] Source: https://ourworldindata.org/tertiary-education#enrollment-in-tertiary-education