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The usefulness of the van Hiele model for geometric reasoning in Engineering Graphics and Design

by

Vernon John Candiotes

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Supervisor: Prof W J Rauscher

Co-supervisor: Prof S van Putten

October 2023

Declaration

I declare that the dissertation/thesis, which I hereby submit for the degree Philosophiae Doctor at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.”

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Dedication

I dedicate this research to my Creator, my Enabler;

My wife, Katherine Candiotes, and the children God gave us, Kimon, Alex, and Sarah.

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To have achieved this milestone in my life, I would like to express my sincere gratitude to:

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Last, but not the least, my family for believing in me.

Abstract

For many students, conceptual understanding in Engineering Graphics and Design (EGD) is problematic, where well-develop reasoning skills are essential but lacking. Through visuospatial and analytical reasoning, the EGD practitioner produces graphical solutions to engineering problems within the shared axiomatic system of geometry. Currently there is no learning model in terms of visuospatial and analytical reasoning within EGD's system of conventions that supports conceptual understanding. A model is needed to facilitate the development of conceptual understanding as the foundation upon which procedural knowledge is built. The value of the van Hiele model of geometric reasoning is well known but its useful application in EGD has not been demonstrated. This study's purpose was to investigate the usefulness of a particularised version of the van Hiele model in an EGD context.

The van Hiele model of geometric reasoning was used as a conceptual framework by aligning EGD content with the hierarchical cognition levels of the model. In constructing a conceptual framework that added three-dimensional reasoning, I integrated the New Typology proposed by Newcombe and Shipley. This inclusion allowed for the cognitive descriptors based on Euclidean geometry to bridge the two-dimensional limitations of the van Hiele model. This study followed a sequential-explanatory mixed-methods approach by employing a single case-study design, and a pragmatist epistemology guided me. Thirty-eight secondary EGD school teachers participated in the study. Data was collected through a mixed-methods approach by utilising a pretest-posttest instrument and a survey on solid geometry thought. Quantitative task and test scores were used to qualitatively explain the degree of van Hiele level acquisition through key cognitive descriptors particularised for EGD.

Although reasoning in EGD encompasses the constructs of visuospatial reasoning, analytical reasoning within a system of subject-specific conventions, I only reported on visuospatial reasoning as a medium to demonstrate the usefulness of the van Hiele model of geometric reasoning. The results obtained from the data analysis suggests that the van Hiele model relates conceptually well to EGD content. In demonstrating the usefulness of the particularised model, I was able to apply the models' hierarchical cognitive descriptors to explain the instances and types of cognitive deficiencies that proved to be persistent.

Within the scope of the EGD content this study was limited to, the particularised model confirmed that participants displayed shallow conceptual understanding and that their range of procedural knowledge was limited to familiar classroom content. Although this study focused on visuospatial reasoning, the particularised cognitive descriptors identified analytical reasoning and convention knowledge to be equally shallow. The study's main conclusion is that the van Hiele model of geometric reasoning is well suited to be particularised to EGD. Furthermore, it embodies the inclusion of visuospatial reasoning, analytical reasoning, and convention knowledge as major constructs to be included in a signature pedagogy for EGD. The particularisation process I used in this study can be extrapolated to all content areas of EGD in combination with the learning phases espoused by the original van Hiele model of geometric reasoning. The cognitive descriptors utilised in this study are not all-inclusive and further research is needed to develop the complete scope of cognition in EGD based on visuospatial reasoning, analytical reasoning and convention systems.

The van Hiele model of geometric reasoning shows a strong relationship with reasoning in EGD. Future educators and students of EGD could benefit from a signature pedagogy where a teaching and learning model forms the core of hierarchical progression through EGD’s reasoning levels.

Key Terms:

Spatial ability, visuospatial reasoning, teaching and learning models, conceptual understanding, Engineering Graphics and Design.

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This certificate confirms that the following manuscript has been checked for standard and appropriate academic English usage, and edited accordingly. Changes made after the issuing of this certificate are not covered by it.

Title: **The usefulness of the van Hiele model for geometric reasoning in Engineering Graphics and Design**

Student: Vernon John Candiotes

Student number: 10685210

L. Perkins

Date: 19/10/2023

Luke Perkins

M.Sc (Wildlife Management) (University of Pretoria)
B.Sc. (Hons) Wildlife Management (University of Pretoria)
B.Sc. (Hons) Industrial Psychology (University of the Witwatersrand)
Diploma (Economics and Public Finance) (UNISA)
H.E.D (UNISA)

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Member no. PER003
Cell 083 388 3861
Email lukeprkns@gmail.com

List of abbreviations

VSR	Visuospatial reasoning
AR	Analytical reasoning
CK	Convention knowledge
EGD	Engineering Graphics and Design
DBE	Department of basic education
MGSLG	Matthew Goniwe School of Leadership and Governance
VH	Van Hiele levels of reasoning
TVET	Technical Vocational Education and Training
STEM	Science, Technology, Engineering, and Mathematics
SPSS	Statistical Package for Social Sciences

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Orientation to the study Chapter 1

1.1 Overview of this chapter

This chapter provides a brief contextual background on Engineering Graphics and Design (EGD) concerning specific problems that are evident in South Africa and globally. The introduction orientates the reader with regard to the research problem, which is rooted in visuospatial reasoning (VSR). However, the constructs of analytical reasoning (AR) and convention knowledge (CK) play supplementary roles in effective spatial reasoning, and although their importance is acknowledged, the focus of this study is on VSR. The introduction leads the reader to the rationale for undertaking the research and the statement of this study's problem. The study's purpose, aims and objectives are stated next, followed by the research questions and the significant impact that findings could offer the EGD community. Subsequently, I explain how my philosophical stance underpinned the research design and methodology, the process of participant selection, and data collection and analysis. Finally, the choices and strategies that were followed to ensure the highest possible quality output, and ensure ethical behaviour throughout the study, are briefly discussed. The chapter concludes with an outline of the content for the remaining four chapters.

1.2 Introduction and background of the study

One of the core purposes of EGD is to offer the world of technology its own language, complete with its own operational symbols, lines and grammatical rules (Bertoline et al., 2011). This language enables people from different parts of the world to communicate non-verbally and convey technical ideas without ambiguity. The South Africa Curriculum and Assessment Policy Statement (CAPS) states that EGD adheres to internationally acknowledged principles in the teaching of academic and technical applications (DBE, 2011a). The CAPS for EGD includes specific foundational knowledge and various drawing techniques and skillsets to enable learners to interpret and produce accurate drawings within the contexts of Mechanical Technology, Civil Technology and Electrical Technology (DBE, 2011a).

Yet, for many students, performance in EGD has been problematic, where well-developed reasoning skills are essential but lacking (Khoza, 2014; Sotsaka, 2020). Annual reports from National Senior Certificate (NSC) examiners and moderators for EGD indicate that Grade 12 learners are persistently unable to read and interpret engineering drawings in their final examinations (Department of Basic Education, 2011–2018). In addition, the DBE report for the 2021 NSC notes that learners have difficulty in analysing and planning, and display poor knowledge of conventions as contained in the South African National Standards (SANS 10111 and SANS 10143, South African Bureau of Standards, 2013) codes for EGD practice. This report also states that many learners display poor visualisation abilities in navigating between 2D and 3D spaces. The 2021 DBE report also alludes to problems with teacher competence, as is evidenced by the typical errors produced by learners from certain institutions. Despite such persistent problems, national statistics for Grade 12 learners at secondary schools in SA have recorded an average pass rate for EGD of 93.3% between 2017 and 2021. The CAPS for EGD guides assessment of the subject to pitch 30% of the content as lower order, 40% as medium order, and 30% as higher order. Candidates are able to pass the subject by mastering lower order content, some medium order content and specific higher order content that they may feel comfortable with. This strategy may account for a high pass rate but resulting in shallow conceptual understanding. I address the apparent tension between high pass rates and poor conceptual understanding further in Chapter 5. However, the throughput rate for Drawing Office Practice (similar to EGD) at Technical Vocational Education and Training (TVET) Colleges in South Africa (SA) is only 4,4% for the period 2017 to 2019 (Khuluvhe & Mathibe, 2022). The subject “Drawing Office Practice” is offered on the National Certificate Vocational (NCV) on three levels that are parallel to Grades 10, 11, and 12 as offered by the DBE. The results for the period 2020 to 2022 are not yet available. Reasons for the difference in performance between EGD as offered by the DBE and Drawing Office Practice as offered by TVET colleges are not clear. Future explanatory studies are much needed to address this gap in our understanding.

A similar phenomenon is observed globally, where many EGD learners graduate from the educational system with poorly developed reasoning skills (Branoff & Dobelis, 2012; Ali et al., 2016; Lee & Widad, 2004; National Science Foundation, 2006; Potter & van der Merwe, 2010; Sorby, 2009; Metraglia et al., 2015). My experience is congruent with Hurrel's (2021) contention that schooling systems that focus more on procedural knowledge than conceptual knowledge are prone to encourage superficial learning, which impacts the transfer of learning negatively.

Research findings suggest that it is common for teachers of EGD to lack the teaching and learning skills required for learners to develop visuospatial reasoning (VSR) skills (Singh-Pillay & Sotsaka, 2016; Khoza, 2014; Perez & Serrano, 2012). Teachers are often unable to master certain spatial and geometry concepts themselves and may, therefore, by default, impart the same misconceptions to their students (Khoza, 2017; Cochiarella, 2015; Clark & Scales, 2004; Kell et al., 2013; Verdine et al., 2017). In addition, EGD teachers often experience difficulties with certain subject content and lack the understanding to plan teaching and learning experiences differently (Khoza, 2017; Marunic & Glazar, 2014). Several researchers mention a gap between understanding the complex cognitive involvement with VSR tasks and the absence of a unifying framework to classify the different cognitive processes (Buckley & Seery, 2016; Pittalis & Christou, 2010; Newcombe & Shipley, 2014; Seery et al., 2015).

The design of effective instructional systems is impossible unless the nature and character of VSR, and the interrelatedness of cognitive processes, are clearly understood (National Research Council, 2006). Understanding the interrelatedness of spatial factors has numerous implications for teaching and learning within Science, Technology, Engineering, and Mathematics (STEM) education (Delahunty et al., 2016).

My position with this study is that instructional strategies ought to be focused on identifying conceptual deficiencies in practitioners of EGD by utilising a hierarchical classification framework of cognitive skills. An effective teaching and learning framework must engage students in relevant cognitive activities and provide evidence of cognitive skills acquisition (Taylor & Hutton, 2013; Uttal et al., 2013; Pittalis & Christou, 2013). Not only should the development of VSR be central in such a framework, but equal space should be afforded to the EGD conventions of CK (Clark & Ernst, 2012) and AR (Hegarty, 2010).

Research into classical tests of VSR shows that during test-taking, the use of visualisation is an important process that is also accompanied by AR strategies (Marunic & Glazar, 2014). Ruckpaul et al. (2015) conducted research in AR with technical drawings and found that practitioners of EGD reasoned on a continuum of strategies to understand the drawings and their context. This continuum operates hierarchically from novice EGD practitioners to expert practitioners. McClaren (2008) found that students lacked an understanding of certain fundamental aspects of EGD, including projection methodology (projection of features across orthographic views), geometric constructions and CK. Metraglia et al. (2011) made a similar claim and argued that CK, geometrical constructions and the basic fundamentals of EGD as an engineering language are missing in educational practice.

Shulman (2005) proposed a signature pedagogy for STEM education that he structured on three categories of desirable outcomes:

1. a surface structure, including specific and discernible acts of teaching and learning
2. a deep structure, focused on how to best convey and teach the content to students
3. an implicit structure inclusive of desirable beliefs, attitudes, values, and dispositions that are important to STEM professions.

A well-established pedagogical approach to teaching and learning geometry exists in the field of mathematics that is congruent with Shulman's (2005) principles of subject-specific pedagogy and is known as the van Hiele (1957) model of geometric reasoning[1]. As a model of mathematical skill acquisition, the van Hiele model describes learners' cognitive reasoning on five hierarchical levels (level 1 to level 5). To optimally proceed through geometry curricula, learners should acquire the cognitive reasoning described in each level before moving on to the next level. The model provides a structured, hierarchical teaching framework where learner progress is scaffolded between cognitive levels, and progression occurs when the previous cognitive level has been acquired. Studies have shown that certain cognitive aspects per level may remain unacquired as opposed to others that have been acquired (Gutierrez et al., 1991). This phenomenon brings about a jagged non-linear progression. The van Hiele model has been applied with great success across the globe for teaching and learning geometry (de Villiers, 2010).

1.3 Rationale and significance of this study

The literature referred to in the previous section points to several obstacles in STEM education and, in particular, to EGD where research is needed to provide effective teaching and learning. Buckley (2018) conducted intensive research on the factors that encompass our understanding of spatial ability and states that a unifying framework is still lacking, although it is under development. Delahunty et al. (2016) state that the interrelatedness of spatial ability factors has numerous implications for STEM education. Compounding our as-yet incomplete understanding of VSR, research supports the argument that both STEM teachers and learners generally find VSR to be problematic (Mclaren, 2008; Metraglia et al., 2015; Ruckpaul et al., 2015; Clark & Ernst, 2012).

Absent from the literature are frameworks specific to the effective teaching and learning of EGD, where VSR features as the central construct. Also absent from the literature are studies where the van Hiele model has been particularised for EGD or any other STEM fields besides mathematics. Researchers agree that effective teaching and learning can be enhanced by systematic instructional strategies and by incorporating the principles of scaffolded learning (Shulman, 2005; Vygotsky, 1978; Wood et al., 1976). The van Hiele model has proven its usefulness in geometry education by virtue of its structured, hierarchical framework where cognitive skills are acquired through a scaffolded teaching and learning process (Alex & Mammen, 2014; Tutak & Birgin, 2008; Karakus & Peker, 2015).

A particularised version of the van Hiele model could offer EGD practitioners insight into their VSR competence and areas of reasoning deficiency. Success in EGD relies largely on VSR skills, yet it should be noted that VSR in turn relies on well-developed AR, and the correct

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[1] The van Hiele (1957) model of geometric reasoning will be hereafter referred to as the van Hiele model

application of CK (Metraglia et al., 2015; Tumkor & deVries, 2015; Villa et al., 2018). I had the choice of simply applying existing VSR tests, such as the Purdue battery of spatial tests (Bodner & Guay, 1997) or using performance scores to determine participants' varying competences. While performance scores are undoubtedly valuable, I purposed to determine participants' performances against specific cognitive reasoning metrics as espoused by EGD content.

With this study, I set out to demonstrate how the van Hiele model can be particularised for EGD by using the topic of solid geometry as an example. In Chapter 5, I review the usefulness of the particularised framework and propose a refined framework that could form the core within a tailored pedagogy for the effective teaching and learning of EGD. Even though the van Hiele model is utilised as the conceptual framework for this study, my refined framework satisfies the stipulations of Shulman (2005) with regard to a tailored pedagogy for STEM subjects such as EGD. Should the van Hiele model prove to have merit as a teaching and learning framework for the development of reasoning skills particular to EGD, some future benefits could be the following:

- Re-skilling teachers with regard to the main cognitive constructs of EGD reasoning.
- Conceptual deficiencies related to reasoning could be identified in greater detail than in a conventional test-scoring rubric in EGD. ↓
- Intervention strategies and standard instructional strategies could benefit from early and accurate detection of conceptual obstacles within EGD content.
- Teaching and learning activities can be scaffolded more effectively by using a tailored pedagogy consisting of learning phases designed for multi-level progression.

The framework has the potential to assess conceptual shortcomings in both teachers and learners with remedial action through the proposed learning phases. Consequently, teachers may be able to overcome their own conceptual obstacles and teach concepts with greater understanding. Learners could cycle and re-cycle through the learning phases to address the non-acquired conceptual skills until they achieve competence. One of the main aims of the van Hiele model is to improve students' geometric reasoning between levels by arranging the learning environment according to their levels of acquisition (Karakus & Peker, 2015). Subsequent content that builds on previous-level cognitive acquisitions can be integrated into the learner's prior body of knowledge in a more controlled manner without having to go through the hurdles of being taught new content while foundational concepts may still be lacking.

1.4 The problem statement

In a study conducted by Alex and Mammen (2014) and in support of the findings of Atebe and Schafer (2010), school learners were found to function on a very low level of geometry ability in South Africa. Alex and Mammen (2018) found that the teaching and learning experience regarding geometry in South Africa is not satisfactory and that studies are needed to assess teacher deficiencies for the purpose of designing intervention strategies. Research by Makgato and Khoza (2016) on preservice EGD teachers found that they are generally underprepared in their VSR ability. Potter et al. (2009) conducted longitudinal studies on EGD over a period of two decades and found that most first-year university students experienced difficulties with VSR.

A similar situation exists in other parts of the world where preservice teachers were found not to know the basic concepts well enough to understand complex concepts and to teach at the correct level (Couto & Vale, 2014; Cunningham & Roberts, 2010). Globally, spatial training is historically missing from most curricula (Taylor & Hutton, 2013; Uttal et al., 2013; Potter et al., 2009). As an unintended consequence, many students in STEM fields who display low levels of spatial ability are dropping out of tertiary education (de Rosa & Fontaine, 2018). As a compounding factor, the White Paper on Post-school Education and Training (DHET, 2013, p.12) hints at current lecturer shortcomings, especially in STEM subjects.

Although considerable progress has been made to increase enrolment in STEM fields, drop-out rates are undermining attempts to retain and grow student numbers in such programmes (Pinxten et al., 2015). To curtail the drop-out trend in STEM fields, it is vital to employ strategies for the early detection of at-risk students regarding spatial ability (Potter et al., 2009; Khoza, 2014; Pinxton et al., 2015). Battista (2007, 2011) argues for the need to understand students' thought processes in order to provide them with a meaningful education. It is essential to identify the cognitive processes that underlie geometry processes in learners in order to determine the nature of the difficulty they are experiencing (Duval, 2006; Lithner, 2000; Battista, 2007; Makina & Wessels, 2009). Scaffolding the development of students' spatial abilities has become a crucial concern in spatial ability research (Buckley, 2018; Luh & Chen, 2013; Alex & Mammen, 2014; Gal & Linchevski, 2010; Uttal et al., 2013). However, Taylor and Hutton (2013) argue that efforts towards developing spatial ability should focus on the teaching process rather than modified or additional content.

Much research has been conducted around aspects of VSR in EGD and related subjects, yet there is a paucity of research that addresses comprehensive variables in the teaching and learning of EGD. My response to this gap in EGD research was to conduct a study to assess the usefulness of the van Hiele model in an EGD context. I selected the van Hiele model as the conceptual framework for this study as it embodies EGD's main foundational construct of VSR and related spatial factors. With a view to achieving the significant benefits suggested in the previous section, the study's purpose was to demonstrate the usefulness of the van Hiele model by particularising its cognitive descriptors for EGD.

1.5 Purpose, aims and objectives

The purpose of this study was to demonstrate how the van Hiele model could be particularised for EGD by foregrounding the construct of VSR to address conceptual deficiencies in EGD. My approach was to particularise van Hiele's cognitive descriptors per hierarchical level to one content area of EGD, namely solid geometry. My protracted long-term aim is to develop a teaching and learning framework inclusive of the whole EGD content based on the foundation laid by this study and my three objectives.

My first objective was to develop a quantitative instrument (Instrument 1, Appendix A) to compare traditional rubric-item scores with the particularised cognitive descriptors derived from the van Hiele model. My second objective was to develop a qualitative instrument (Instrument 2, Appendix B) to investigate the transfer of learning after an intervention. My third objective was to critically describe the strengths and weaknesses of the particularised framework and make recommendations.

1.6 Research questions

In undertaking this study, I was guided by the following main research question:

How can a particularised version of the van Hiele model for EGD provide a deep understanding of conceptual deficiencies in cognitive reasoning?

I posed the following four secondary research questions as a means to answer the main research question.

1. How can the van Hiele model relate to EGD?
2. How can student performance in VSR be described in terms of the hierarchical levels of the van Hiele model?
3. How can student performance in VSR be described in terms of the van Hiele descriptors?
4. How can the van Hiele model allow for the identification of deficiencies in conceptual understanding?

1.7 Clarification of concepts

1.7.1 Orthographic projection

When the six individual faces (Nets) of a cube are drawn in 2D as perpendicular views to each of the six planes, they are referred to as multiviews and align with each other through parallel projection lines. All six views are positioned with reference to one principal view according to two systems known as first angle and third angle orthographic projections, as they were derived from two of the four quadrants of the Cartesian plane. Figure 1 of Appendix I serves as an example.

1.7.2 Axonometric projection

Axonometric drawings are 3D drawings that are classified as belonging to the orthographic category of drawings due to lines on each of three axes being drawn parallel to each other. There are three projection types, classified as isometric projections, dimetric projections, and trimetric projections. Unlike dimetric and trimetric projections, the three isometric axes are equally separated by 120°. Two other non-axonometric types of 3D projections are known as oblique projections, which adhere to the concept of parallel lines and perspective projections which do not adhere to the concept of parallel lines.

1.7.3 Isometric projection

Isometric projection is an axonometric drawing method in 3D that is classified as part of the category of orthographic projections. The height of objects in the isometric plane is represented by the y-axis on the Cartesian plane, and the length and width of objects are drawn at 30° to the left and the right of the y-axis, respectively, but with reference to the x-axis. All vertical and horizontal lines in the orthographic representation of multiviews are parallel to each other in the three isometric axes. All vertical and horizontal multiview dimensions are preserved in the isometric plane but appear equally foreshortened. Multiview lines that are not vertical or horizontal are called non-isometric lines and appear deformed in the isometric plane, and the dimensions of such lines are not preserved in the isometric plane.

1.7.4 Particularisation

To particularise means to describe something in detail without deviating from the core intention (as in transliteration). Cognitive descriptors from the mathematics domain that aligned with thinking/reasoning in EGD were directly adopted (particularised). Where wording leant vernacularly towards the subject of mathematics, such words were transliterated to fit in with EGD vernacular without compromising the original meaning of cognitive definitions.

1.7.5 Euclidean geometry

The study of shapes and figures in plane geometry (2D) and solid geometry (3D) that are guided by a set of propositions based on five postulates formulated by the ancient Greek mathematician Euclid is known as Euclidean geometry.

1.8 The research design and methodology

1.8.1 Research paradigm

A pragmatist paradigm was used to guide this study regarding the nature of reality and what can be known about the problem under investigation, as I had to frequently alter my methods of data collection and analysis to answer my research questions. In the pragmatist's search to find solutions, the researcher utilises various data collection strategies according to the utility of *"what works"* to address the research question (Creswell & Plano Clark, 2007, p. 24). I transitioned between quantitative and qualitative data collection techniques and applied various data analysis techniques, which included statistical procedures, document analysis, and thematic content analysis.

1.8.2 Research design

Research designs are strategic action plans that link research questions with strategies of data collection, documentation and analysis (Durrheim, 2006). I employed a sequential-explanatory mixed-methods design and collected both quantitative and qualitative data. Both quantitative and qualitative methods may follow a case study design, which endorses the mixed method approaches (Yin, 2013). Furthermore, case-study designs provide thick, detailed descriptions of social phenomena as they exist in real-world contexts (Yin, 2013). This design was the most suitable for administering two different instruments and allowed me the freedom to change between quantitative and qualitative strategies of data collection and analysis, within instruments, and across instruments.

1.8.3 Working assumptions

Based on the literature I consulted on teaching and learning theories and human cognitive development, I undertook this study bearing the following assumptions in mind:

- The quality of the participants' teacher training, with EGD as a speciality, was of a similar quality, implying that their understanding of cognitive processes in the execution of a solid geometry task was similar.

- Participants followed the instructions explicitly for completing the solid geometry task of Instrument 2.
- Participants were honest and thorough in rating their perceived difficulty of the solid geometry task.
- Participants from secondary schools in South Africa were the most appropriate sample selection from the greater population of EGD teachers in South Africa as teachers from TVET Colleges and Universities are not always formally trained as teachers (Blom, 2016a, 2016b).
- Participants had fully acquired the thinking associated with level 1 of the van Hiele model (VH1) which includes the recognition of and classification of different geometrical shapes.

1.8.4 Phases of the study, data collection, documentation and analysis

This study followed a linear progression over four phases. Data collection and documentation were achieved by performing a sequential data collection process (Creswell & Plano Clark, 2007), as illustrated in Figure 1.1. See Appendix C for a detailed alignment of the study's phases, the administration of instruments for answering the four sub-questions, and the data analyses employed for the different data strands.

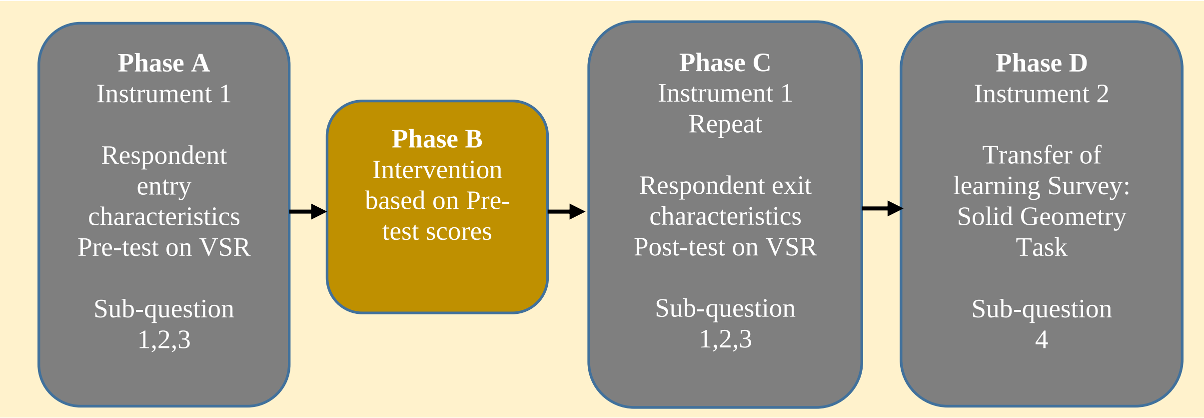


Figure 1.1: Strategy for administering the data collection instruments

A pretest was administered (Phase A), followed by an intervention (Phase B), and the same test was administered as a posttest (Phase C) at the conclusion of the intervention. A post-posttest was administered (Phase D) to measure the transfer of learning.

1.8.4.1 Pretest and Posttest (Instrument 1)

The same test was used for both the pretest and posttest and consisted of 12 test items representative of the CAPS for EGD and spanned the cognitive areas of VSR (Appendix A). Twelve questions were related to VSR, and the other twelve were related to CK. However, as mentioned before, AR and CK were excluded from demonstrating the usefulness of the van Hiele model for the sake of condensing the study's content.

Descriptive and correlational statistics were employed to answer the first two secondary questions. Each test item was meticulously analysed for its cognitive type and aligned with the particularised cognitive descriptors typifying the van Hiele model. In this manner, van Hiele levels could be assigned to each test item and added an additional qualitative strand to what was essentially a quantitative instrument. Quantitative test scores were qualitatively analysed against the category of cognitive descriptors that pertained to each level, resulting in a narrative description of reasoning levels that had not yet been fully acquired.

1.8.4.2 Cognitive survey on solid geometry (Instrument 2)

In a quest to understand the way in which people create, modify, and interpret the world in which they find themselves (Yin, 2013), I included both quantitative and qualitative aspects of data collection. Instrument 2 consisted of a solid geometry pentagonal pyramid, where six questions had to be answered and submitted in drawing format (Appendix B).

Participants were asked to record detailed procedural steps of their reasoning as they proceeded to draw the solutions (Appendix G). At the same time, participants were required to rate the perceived difficulty per cognitive action on a nine-point scale. This strategy yielded four strands of data: (1) quantitative data by assessing the drawings using an assessment rubric; (2) quantitative data from perceived difficulty on a Likert scale; (3) qualitative data by performing document analysis on the drawings, and (4) qualitative data from the record of cognitive actions.

I followed a process of thematic inductive content analysis to categorise the data into broad themes and to populate each theme with content that uniquely belonged to each category. From this process emerged a cognition memorandum consisting of 68 cognitive steps that were followed in solving the six questions (Appendix F). I made abbreviations so that “van Hiele level 1” becomes “VH1”, and the other four levels are likewise named VH2, VH3, VH4, and VH5. Each cognitive step was meticulously analysed against the particularised van Hiele model to assign a van Hiele level (VH2, VH3, and VH4) accordant with cognitive descriptors per level. Due to the nature of the content, both instruments excluded instances that aligned with VH1 and VH5 descriptors. This strategy was essentially the same process I used for instrument 1, and enabled me to describe degrees of cognitive acquisition with a narrative approach. Descriptive statistics were used to analyse the quantitative strands of data.

1.8.5 Participant selection

Two subject specialists from the Gauteng Department of Education (GDE) assisted me in selecting high school EGD teachers in Gauteng, South Africa, as participants. Thirty-eight teachers were selected on the preconditions that they were experienced in teaching EGD on Grades 10, 11 or 12 level; that they possessed a suitable teacher's qualification for EGD as an elective subject; and that they resided in close proximity to the University of Pretoria.

1.9 Quality criteria

1.9.1 Quantitative rigour

Reliability refers to the quality of a measuring instrument to the extent to which it will produce the same results over repeated trials (Maree & Pietersen, 2010). The contents of Instruments 1 and 2 were derived from textbooks that are aligned with the CAPS for EGD (DBE, 2011a) and representative of common tasks and exercises that feature in South Africa's secondary school curriculum (van Leeuwen & du Plooy, 2014). Three EGD colleagues evaluated the contents of the two instruments, and no modifications were made as all test items were considered as historical usage in EGD. Similar test items frequently appear in South Africa's National Examinations and are readily available on the internet (DBE/November 2014, Engineering Graphics and Design, paper 1). As recommended by Ivankova (2014), a pilot test was administered to a group of senior EGD students and no ambiguity was reported by the test-takers.

Validity indicates the degree to which instruments measure what they are intended to measure (Higgins & Straub, 2006; Maree & Pietersen, 2010; Shadish et al., 2002). I created Instruments 1 and 2 based on the content of textbooks that subscribe to the CAPS for EGD. I have administered these two instruments over four years to different groups of EGD students and have consistently found similar results and made similar inferences. Shadish et al. (2002) state that validity is not a property of designs or methods, but rather a property of inferences, as opting to use the same design under different circumstances may contribute to more or less valid inferences. The instruments measure what they are supposed to measure, based on: (1) the content being derived from CAPS textbooks; (2) the present author having administered the test consistently over four years; and (3) having it reviewed by professional EGD practitioners followed by appropriate modifications.

1.9.2 Qualitative rigour

According to Niewenhuis (2010b), the trustworthiness of a study refers to a researcher's ability to persuade the reader that the research is of high quality and meets the four criteria (bold and italicised) that are briefly discussed in this section. In addition, the use of multiple methods of data collection is reasonably accepted to enhance trustworthiness. The use of multiple or mixed methods supports ***transferability*** in the sense that the findings of this study may possibly be transferred to other similar contexts for further research (Kelly, 2006a). I provide detailed descriptions of the sampling criteria and purposive sampling strategy, and thick, detailed descriptions of the two data collection instruments and the particularisation process. Van der Riet and Durrheim (2008) refer to ***dependability*** as the degree to which readers believe that a study and its findings did emerge as reported. My chosen method of data collection and analysis supports dependability through generating rich, detailed descriptions of cognitive reasoning in solid geometry. I followed a process of coding-recoding, kept a complete audit trail, explained my methods in detail, and applied data triangulation where such instances were possible (Ary et al., 2010; Van der Riet & Durrheim, 2008).

Confirmability refers to the extent to which results can be supported and confirmed by others or confirm general findings in the field (Ary et al., 2010). My quantitative results confirm previously reported findings on the obstacles presented by under-developed VSR and CK. With the particularisation strategy employed in this study, qualitative findings support quantitative findings by adding rich, detailed cognitive descriptors. In addition, I included methods such as triangulation, control bias, code-recode, and audit trails to enhance the confirmability of findings. According to Shenton (2004), ***credibility*** refers to the conscious effort made by researchers to attain confidence in data interpretation. In an effort to attain credibility with my qualitative content analysis, I was transparent in the process to code-recode, particularise and draw inferences from the data (Zhang & Wildemuth, 2008). I applied methodological triangulation on specific objectives, especially where the particularisation of cognitive descriptors featured, and allowing multiple methods to converge and support the inferences I made (Cohen et al., 2018).

1.10 Ethical considerations

To meet the ethical requirements for social research, the following ethical aspects discussed by Kruger (2003) were adhered to:

Trust: All the participants in this study were informed beforehand regarding the nature, objectives and benefits of the study, and their permission was sought in this regard. The potential benefits to their organisation and to them as teachers and the contribution to this particular field of science were explained to them. Participants had an open channel of communication with me as the researcher and Enterprises University of Pretoria (Pty) Ltd telephonically and via email, and were encouraged to voice their opinions and concerns at any time (Human-Vogel, 2004).

Informed consent: Participants were informed before consenting to participate in the study regarding a report on the influence of the intervention that was required by the programme sponsor. The participants were also informed that the findings from the study would be used in a PhD dissertation and future articles or conferences on the subject whilst maintaining all the confidentiality restrictions that I, as the researcher, had committed to.

Confidentiality: Violation of privacy/anonymity/confidentiality: No real names of participants were or ever will be used. Each respondent was assigned a sequential pseudonym number and their identities were and will not be made available to anybody. The document that matches each respondent with their pseudonym number is only available to me as the researcher and is kept in a safe electronic storage device and is password-protected. Anonymity of information was and will continue to be preserved and only information for which permission had been obtained was used. In complying with the ethics of data sharing, data was carefully scrutinised regarding whether its inclusion might have violated participants' anonymity, and avoided at all such times (Chowdhury, 2014).

Voluntary participation: Participants were informed that they had the right to discontinue their participation at any point should they feel threatened in any way.

Safety in participation: Participants could in some instances be placed in a vulnerable position, by perceiving that the information required from them could be held against them by employers. No such incidents occurred, but sensitivity towards their fears was maintained throughout and the information pertaining to them will remain in a safe place.

Bias: As the only person involved with data collection, I endeavoured to remain detached from the participants by strictly adhering to the pseudonym system during assessment in pursuance of reaching unbiased conclusions (Durrheim & Painter, 2006). As the focus of this study was to illustrate the particularisation of the van Hiele model of geometric reasoning and not the effectiveness of the programme, any possible subconscious temptation to skew the results positively was curtailed to the best of my knowledge.

1.11 Outline and organisation of this study

Following Chapter 1, Chapter 2 contains the literature I reviewed on VSR, and geometry reasoning in the context of EGD. Aspects of AR and CK are also discussed due to its interwovenness with VSR. Literature pertaining to how the brain processes spatially-infused subject content typical of STEM education is critically compared. Theories on how different knowledge dimensions are developed in the context of EGD are explored through the literature. Theories on spatial development are explored and the conceptual framework arising from this exploration is defended for its suitability in guiding this study.

In Chapter 3, I briefly reintroduce the problem and purpose under investigation and justify the research design, processes and methodological choices I made based on my paradigmatic stances. I explain the selection of participants and the research questions arising from the problem. An overview of the conceptual framework is followed by my approach to instrument development, data collection, documentation and strategies of analyses. Finally, I elaborate on the quality measures I took to ensure rigour and the ethical considerations I upheld.

In Chapter 4, I present both the quantitative results obtained from pretests and posttests (Instrument 1, Appendix A) as well as the van Hiele levels per question item and the subsequent expansion of the test scores by means of cognitive descriptors derived from the van Hiele model. Subsequent quantitative and qualitative results obtained from Instrument 1 are used to explain participant performance through a particularised version of the van Hiele model. In addition, I present the findings of the solid geometry test (Instrument 2, Appendix B), accompanied by the cognition survey. This instrument performed the function of a test, but focused on the participants' cognitive reasoning while solving a solid geometry problem. In this chapter, I discuss how applying the particularised van Hiele model in EGD identified persistent conceptual deficiencies that add rich qualitative detail to quantitative numbers.

In Chapter 5, I draw conclusions based on the findings of Chapter 4. I address the research questions, highlight the contribution of the study and reflect on the limitations of the research. I present a proposed modified framework based on the van Hiele model as a functional hierarchical model that could serve as a tailored pedagogy for EGD. In addition, I discuss possible contributions made by this study. Finally, I make recommendations for validating the particularised model and equipping the model with level-specific assessment tools. I also describe what future research should entail to develop the learning phases described by the model for the optimum and efficient development of comprehensive reasoning in EGD.

1.12 Summary

In this chapter, I presented problems experienced in EGD from a VSR context. I also drew the reader’s attention to the fact that AR and CK are equally important for effective reasoning in EGD. However, the purpose of this study was to demonstrate the usefulness of a particularised version of the van Hiele model for EGD, and as such I focused only on one of the main constructs of EGD, namely VSR. Chapter 1 outlines the research design, data collection instruments and the data analysis strategy that were used to answer all the posed questions in support of addressing the study's problem and purpose.

Literature review and conceptual framework Chapter 2

2.1 Overview of this chapter

This chapter commences with an explanation of the purpose and nature of EGD, the cognitive processes practitioners engage with and the common obstacles prohibiting effective teaching and learning. As the theoretical underpinnings of reasoning in EGD are predicated on the constructs of visuospatial reasoning (VSR), analytical reasoning (AR), and convention knowledge (CK), those three constructs are reviewed from the literature. The focus then shifts to teaching and learning theories that are grounded on spatial concepts to utilise as elements of a conceptual framework. The van Hiele model of geometric thinking (1957) is comprehensively explored and critiqued from seminal and current literature. Finally, I explain the conceptual framework as a composite of the van Hiele model and the New Typology by Newcombe and Shipley (2014) and how it links to the research questions.

2.2 The purpose and nature of EGD

EGD is a core STEM subject as it provides a medium of graphical communication to convey ideas and information between and across STEM disciplines (Bertoline et al., 2011; DBE, 2011a; Chen et al., 2017). Furthermore, EGD stands at the forefront of the design process and manufacturing industries (Oviawe, 2020; Garland et al., 2017). Engineering drawings have been the backbone of communicating technical product specification over the past four centuries and were first formally standardised at the beginning of the twentieth century (Garland et al., 2017).

Tversky (2005) suggests that image properties can be compared with the words that make up a language and its semantic structure. Spatial relations between figure properties constitute a rudimentary syntax which expresses the relations among such properties. EGD is often referred to as a language consisting of its own operational symbols, lines and grammatical rules (Mclaren, 2008; Clark & Ernst, 2010). The most common STEM disciplines where practitioners produce or use engineering drawings include Architects, Engineers, Technologists, Designers, Craftsmen, Artisans, Operators, and Manufacturers (Oviawe, 2020).

Since 1947, the International Organisation of Standardization (ISO) has established engineering standards, symbols and rules which are represented in the South African National Standards (SANS) codes (International Organization for Standardization [ISO], 2020). According to the South Africa CAPS document for EGD, curricula are designed to teach the symbols, lines and grammar (CK) for both academic and pragmatic outcomes (DBE, 2011a, p.8). Educational institutions in South Africa adhere strictly to the SANS codes and, in particular, to the SANS 10111-3: 2011 and SANS 10143: 1980 as prescribed by the CAPS for EGD (South African Bureau of Standards, 2013).

2.3 Cognitive constructs in EGD

In this section, the three main constructs of cognition (VSR, AR, and CK) are discussed as they appear in an EGD context. In Section 2.4, the same three constructs are further explained from the perspective of cognitive psychology.

Courses in EGD are based on geometry, where geometrical constructions, projection methods and basic engineering standards are combined to produce engineering drawings (Chen et al., 2017). Students are introduced to Cartesian systems, where orthographic (parallel) and axonometric (non-parallel) systems form the foundation for skilful production and interpretation of engineering drawings. Curricula are designed to cultivate students' spatial (VSR) and analytical (AR) abilities through geometric problems of general spaces. By applying the basic principles of orthographic and axonometric projection systems (CK), students develop the ability to rationally express non-verbal ideas through manual drawing or CAD production (Chen et al., 2017; Tumkor & de Vries, 2015). The ability to visualise physical objects or construct mental images from text descriptions as objects in space places a high cognitive load on reasoning processes, which are mostly of a spatial and analytical nature (Tumkor & de Vries, 2015; Villa et al., 2018). EGD incorporates many of the principles of the mathematics branch of geometry, and as such, many of the research findings on reasoning and spatiality apply equally well to EGD as to the STEM fields of education (Pietropaolo & de Oliveira, 2020). VSR is a factor in explaining human intelligence in terms of activities that contain a high degree of visual matter but function in concert with other reasoning abilities that lie on the periphery of spatial thinking (Owens, 2015; Bobek & Tversky, 2016; Shah & Miyake, 2005; McGrew, 2006).

People approach problem-solving situations from multiple reasoning perspectives and employ various reasoning skills according to their conceptual knowledge capacity and how their associative skills relate to the problem at hand (Briscoe & Stout, 2001; Lithner, 2000; Olkun, 2003, 2022; Ramey, Stevens & Uttal, 2020).

2.3.1 Visuospatial reasoning (VSR)

Through the use of VSR, people create mental images and simulations to process spatial tasks (Schneider & McGrew, 2018) whilst observing spatial relationships in imagined and real spaces to manipulate and organise their mental imagery (Newcombe & Shipley, 2014). An important element of VSR is to maintain directionality and reasoning within dynamic (motion) and static (stationary) spatial frames (Carroll, 1993). With EGD content, practitioners constantly swap between 2D and 3D thought while maintaining mentally conceived reference points across orthographic and isometric configurations. They follow a process of component-mapping in relation to the object properties within the object and with other related objects (Chen et al., 2017). Other spatial abilities that are called upon are mental rotation of objects, identification of solid geometry nets, surface development, imagining obscured detail through sectional planes, and mentally transforming objects into new forms of the original (Tumkor & de Vries, 2015; Marunic & Glazar, 2014; Cheng & Mix, 2014). The development of spatial abilities and related reasoning processes are major predictors of future success in STEM disciplines (Sorby et al., 2018; Buckley, 2018).

Although spatial ability and reasoning skills are malleable, very carefully designed instructional practices are required to develop such skills (Uttal et al., 2013; Sorby et al., 2018; Meneghetti et al., 2016). It is well documented that strong spatial abilities are vital to producing and interpreting engineering drawings (Konadu-Yiadom, 2016; Makgato & Khoza, 2016). Van Hiele (1957) classifies the ability to distinguish between the properties of geometrical figures and spatially track them across transformations on a hierarchy of five reasoning levels. However, the encoding of spatial orientations and positions of object features may become weakened during the drafting process. Glazek (2012) and Ogawa et al. (2010) explain that such instances can be attributed to the extent and complexity of eye and hand movements that must be made whilst maintaining mental images of a given scenario.

Battista (2007) foregrounds VSR and AR as the core of engaging conceptually with geometric systems, which leads to the development of inventive skills in the investigation of shape and space. However, Gunhan (2014) found that most learners lack conceptual understanding and display deficient reasoning skills with geometry content. Sketching is a formidable enhancer of VSR development as it promotes the construction of abstract concepts and encourages the exploration of new concepts (Delahunty et al., 2013; McLaren, 2008; Sorby et al., 2018; Uttal et al., 2013; La Verne & Meyers, 2007). The field of cognitive psychology shows that human movement (tactile abilities) and the formation of mental faculties are directly linked (Veide, 2015). Hand movements, in conjunction with the development of visual estimation, lead to the formation of spatial representations, where external space is mentally manipulated to form the basis of internal cognition. Veide and Strozheva (2015) argue that in this regard, drawings should initially be performed manually as basic sketching.

In recognition of the value of tactile abilities, Chen et al. (2017) state that once the basic theories and knowledge of EGD are acquired and spatial abilities are well developed through manual drawing techniques, CAD exposure may be considered. Once students have mastered component disassembly and mapping of indices and can seamlessly swap between 2D and 3D drawings, practitioners may progress to CAD. Hao et al. (2019) advocate for extensive exposure to produce axonometric drawings with freehand techniques. This is a basic skill required in industry and essential in linking index points across drawing views during attribute-mapping strategies. Sun and Yin (2016) agree with Luh and Chen (2011) and call on teachers to return to the roots of EGD and demonstrate how to make freehand drawings instead of relying solely on computer-aided instruction (CAI).

2.3.2 Analytical reasoning (AR)

Analytical reasoning is separated into inductive and deductive reasoning for this study. Inductive reasoning relies on prior knowledge and observations to discover new principles, whereas deduction relies on established principles to discover new facts through ordered sequential reasoning steps (Kahneman, 2011). Kahneman (2011) explains how the brain prefers heuristic systems (inductive reasoning) as its normal mode of reasoning, but switches to deductive reasoning strategies when solutions to problems cannot be generated through prior knowledge or observations.

According to Hahne (2012), AR is an important aspect of producing and interpreting engineering drawings, and this reasoning skill develops over time and exposure to active involvement with engineering drawings. Where heuristic systems of reasoning become inadequate to generate solution paths, people focus on the analysis of established rules and principles to deduce incremental steps in solving problems (Helldin, 2016). Cognitive activities such as visualisation, mental rotations, mental sectioning, and mental transformations are the most important spatial abilities required for seemingly simple drawing tasks. Yet, according to Hegarty (2010), people displaying expert graphicacy tend to combine AR with VSR during spatial problem-solving. The ISO furnishes EGD with orthographic projection systems and conventions that are vital for the engineering language to function without ambiguity (Bertoline et al., 2011). Continued exposure to solid geometry tasks will eventually improve sensory processes, modify brain structures, improve visual attention and working memory and enhance analytical reasoning (Li et al., 2009; Draganski & May, 2008; Green & Bavelier, 2003, 2007; Pedreau & Cavanagh, 2015; Ruckpaul et al., 2015).

2.3.3 Convention Knowledge (CK)

The correct application of subject conventions (CK) is indispensable in the execution of drawing tasks (Metraglia et al., 2015). CK provides the platform of rules, regulations and symbols where geometrical axioms are used to make assertions about geometrical scenarios. It is on this platform that VSR and AR interweave to solve problems. Yet, the manufacturing industry reports that many novice engineers display a lack of mastery of EGD fundamentals and often fail to comply with rules, conventions and standards (Metraglia et al., 2015; Sun & Yin, 2016; Chen et al., 2017; Brown, 2009; McLaren, 2008). Chen et al. (2017) explain how EGD's systems and conventions provide the fabric onto which AR and VSR can be woven.

2.4 The theoretical underpinnings of VSR, AR, and CK

As posited by Buckley (2018), in defining human intelligence, the most appropriate strategy is to consult empirical evidence on the scope of factors related to the way humans reason. Thurstone (1938) is credited for being the major contributor to the development of factor-analytic methods, and although his methods have been refined over the years, the basic principles remain unchanged. Gustafsson (1984, p.181) states that “Thurstonian Multiple Factor analysis has evolved into the dominating factor analytic technique”.

Historical developments in exploratory and confirmatory factor analysis have resulted in new ways to advance the understanding of intelligence through novel batteries of psychometric tests (Schneider & McGrew, 2012). Exploratory factor analysis is employed to group together principal component variables and to seek out fragmented variables in veiled patterns for proper classification (Cohen et al., 2018). The stringent process of confirmatory factor analysis compares existing factors against hypothesised models of classifications and interrelationships. According to Cohen et al. (2018): "Such a model derives from pre-established theory which informs the generation of the model, and the confirmatory factor analysis tests a theory of the latent processes and relationships" (p 818). To provide structure for cognitive abilities pertaining to EGD from the field of cognitive psychology, I consulted the Cattell-Horn-Carroll (CHC) theory of intelligence. The CHC theory is currently regarded as the most comprehensive theory of intelligence (Schneider & McGrew, 2012).

In human intelligence research, the general factor for intelligence quotient (IQ) tests is referred to as the (g) factor (Horn & McArdle, 2007). Similarly, additional broad and narrow abilities are denoted by parenthesised single or double characters. For instance, where the general intelligence factor is denoted by the convention (g), VSR carries the notation (Gv) (Horn & McArdle, 2007). The convention symbols used in IQ research appear in parentheses in the discussion that follows the different factors of intelligence that apply to EGD.

A synthesis of the Cattell-Horn theory and Carroll theory was proposed by McGrew (1997) to form a single taxonomy of human intelligence known as the CHC theory (Buckley, 2018) and is in accordance with the hierarchical three-stratum principle of Carroll (1993). Figure 2.1 portrays Spearman's (1904) general intelligence factor (g) as the only third-stratum factor representing the broadest level of human intelligence (Schneider & McGrew, 2012). In

Schneider and McGrew's (2018) updated and reconceptualised version of the CHC theory, they have grouped the 16 second-order factors according to their conceptual and functional attributes. They used the four categories of Motor Abilities, Perceptual Processing, Controlled Attention, and Acquired knowledge (Figure 2.2) to graphically portray how the 16 second-order factors relate to each other (Schneider & McGrew, 2018). Note, henceforth, I refrain from referring to the levels as "strata" and instead will use the terms "first-order cognitive factors" and "second-order cognitive factors". This is in line with the terminology for factor analysis.

The theory contains 16 cognitive factors similar to Thurstone's (1938) conception of broad abilities on the second stratum (Buckley, 2018; Schneider & McGrew, 2012). There are currently 84 first-order cognitive factors which load on the 16 second-order cognitive factors and are generally referred to as narrow abilities. These narrow abilities are the focus of many psychometric assessments. However, as the main constructs of VSR, AR, and CK reside in the second stratum, only those factors relevant to this study are further explored.

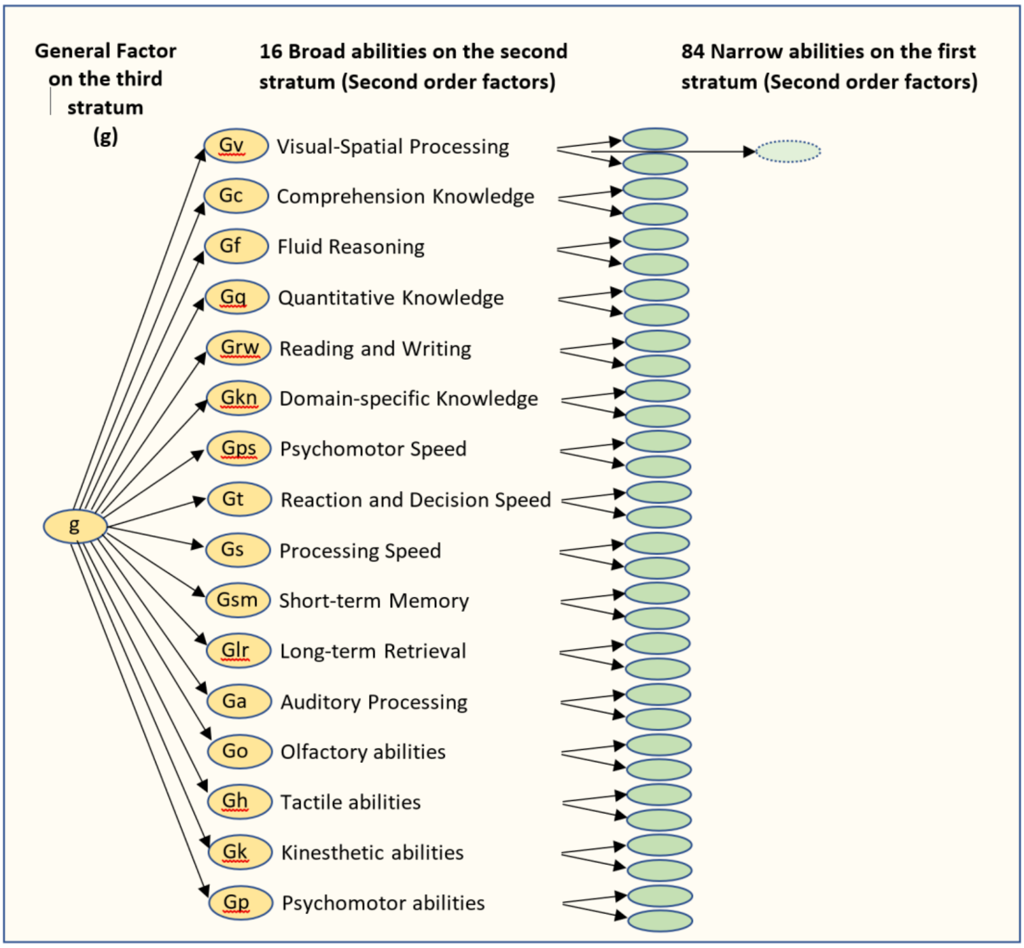


Figure 2.1: The Cattell–Horn–Carroll theory of intelligence

Figure 2.2 portrays the conceptual groupings' broad abilities according to Schneider and McGrew's (2018) latest re-conceptualisation of broad abilities. Note that the Long-term retrieval factor (Glr) has been separated under the two conventions of Retrieval Fluency (Gr) and Learning Efficiency (Gl). This change was previously proposed by Schneider and McGrew (2012, p. 117) because, as they argued, “there is a major division within Glr that was always implied in CHC theory, but we are making it more explicit here. Some Glr tests require efficient learning of new information, whereas others require fluent recall of information already in long-term memory”. For that reason, Figure 2.2 contains 17 second-order cognitive factors as opposed to the previous 16 cognitive factors. Only the broad factors (in orange) and their associated narrow factors pertaining to the focus of this study will be discussed further. The factors pertaining to this study appear in orange circles.

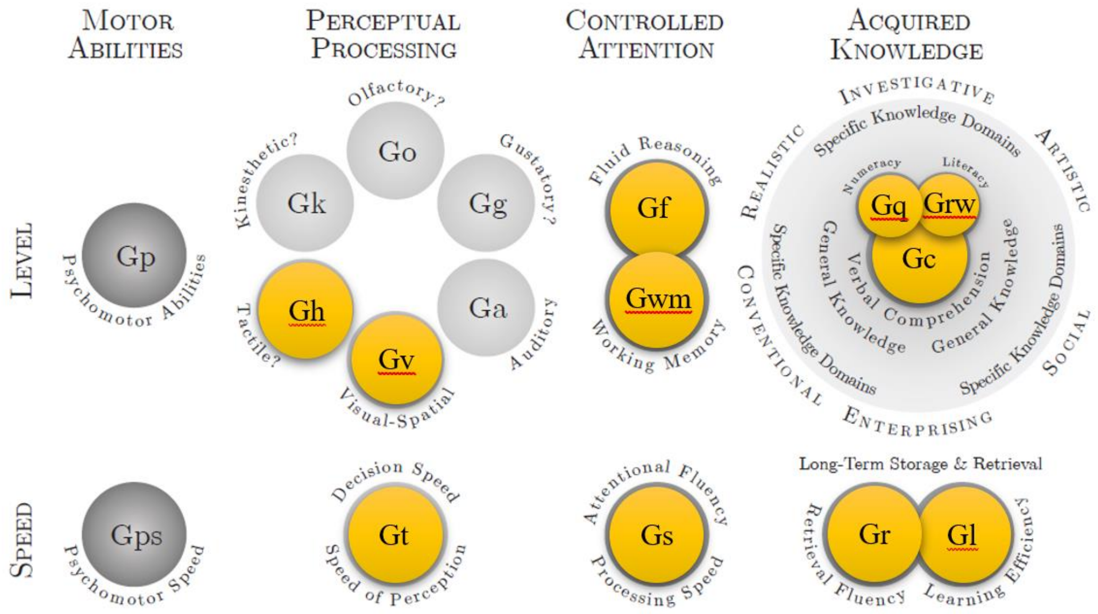


Figure 2.2: Conceptual groupings of CHC abilities (Schneider & McGrew, 2018, p.75)

In the following sections, I focus more on Visual-spatial processing (Gv), Fluid reasoning (Gf), and Crystallised intelligence (Gc) as the constructs of VSR, AR, and CK pertaining to this study reside under those second-order cognitive factors. Schneider and McGrew (2018) use the term "visual-spatial processing" (Gv), which is congruent with the wording "visuospatial reasoning" (VSR). To avoid confusion, the terms "visuospatial" and "visual-spatial" are interchangeable (Kreutzer et al., 2011). Visuospatial reasoning is discussed comprehensively in Section 2.4.1. Fluid Intelligence (Gf) and Crystalised Intelligence (Gc) feature in Sections 2.4.2 and 2.4.3. As a precursor to those two concepts, it is noteworthy to explain the relationship between (Gf) and (Gc). With Fluid Intelligence (Gf), people are able to perceive conceptual relationships in situations that are new to them and respond insightfully to phenomena that are complex in nature. This ability infers “the unstated rules that govern their behaviour, and exploiting this knowledge to deduce the best course of action to take”

(Schneider & McGrew, 2018, p. 90). Crystallized Intelligence (Gc) is the capacity of accumulated knowledge that was generated through fluid intelligence. Through (Gc), people generate solutions from prior experience that best match current needs. A high degree of (Gc) allows people within their cultural context to develop useful knowledge that is broad and deep through a comprehensive repertoire of skillsets (Schneider & McGrew, 2018).

Santarnecchi et al. (2017, p. 10) report on the different patterns of cortical activation discovered in a meta-analysis of brain scans for both "**process** (induction vs deduction) and **content** (verbal vs visuospatial) facets of (Gf)". From Carroll's (1993) analyses, recently confirmed by Weiss et al. (2013, p. 126), "visuospatial (Gv) ability tests often load on the (Gf) factor, and nonverbal (Gf) tests often load on the (Gv) factor". Schneider and McGrew (2018) observe that factors tend to group together when testing for fluid reasoning (Gf) when similar content is used, whether it is quantitative, spatial or verbal. Tests in deductive and inductive reasoning highlight disparities in the content matter, which implies "that the verbal, figural, and numeric content facets in (Gf) simply represent factor impurities from (Gc), (Gv), and quantitative reasoning (Gq), respectively" (Schneider and McGrew, 2018, p. 95).

Depending on task requirements, effective reasoning does not depend on one factor only, but rather on various factors operating in concert until a solution to a problem has been found (Schneider & McGrew, 2018). In the following three sections, VSR, AR, and CK will be discussed as they are positioned under *Perceptual Processing, Controlled Attention and Acquired Knowledge* in the CHC theory.

2.4.1 Perceptual processing: Visuospatial abilities (Gv)

Sir Francis Galton (1883) is credited for originally conceiving the construct of spatial ability and conducted mental imagery studies as early as 1880 (Mohler, 2008). The domain of spatial ability is contentious, as much research has been conducted on it, yet definitional attributes appear fragmented, and a unifying definition has not yet been conceded to (Seery et al., 2015).

Thorndike (1921), Kelley (1928), El Koussy (1935) and Thurstone (1938) laid a solid foundation for future researchers to discover that spatial ability must be seen as a separate ability from Spearman's (1927) general intelligence factor (g). Spatial ability was initially regarded as inferior, and the growing community of spatial researchers could not reach a consensus on naming and defining spatial factors and constantly added new factors (D'Oliveira, 2004; Lohman, 1979a; Hegarty & Waller, 2005). Spearman (1927) focused on intelligence as a single factor (g) with the support of British researchers (such as Burt, 1949 and Vernon, 1950). Researchers in the U.S. favoured multiple factors of intelligence, with

Thurstone (1950) leading the initiative, followed by Guilford (1967) and Cattell (1971). Through Thorndike's (1921) work, "mechanical intelligence" was defined as the ability to visualise relationships among objects and comprehend various functions of the physical world. Thurstone (1938) suggested a "space" factor as a mental ability to process spatial and/or visual images. He was among the first to propose that intelligence is not a singular factor and demonstrated his ideas through his Multiple Factors theory. Thurstone (1950) used abstract terms (S1, S2, and S3) to propose three factors central to spatiality, but these were subsequently replaced with terms most people can relate to. Mental rotation (S1) is the ability to maintain recognition of objects when a change in their orientation takes place. Spatial visualisation (S2) was defined as being able to recognise the components of objects being displaced from their original position. Spatial perception (S3) became defined as relating to spatial orientation by manipulating one's own body orientation. Definitions for spatial ability remain in flux and some modern interpretations are offered in the following paragraphs.

Arcavi (2003) states:

> Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (p. 217)

Clements and Battista (1992, p. 420) state that "spatial reasoning consists of the set of cognitive processes by which mental representation for spatial objects, relationships, and transformations are constructed and manipulated". Similarly, geometrical reasoning requires one to generate images; analyse these images to retrieve essential information; transform these images and maintain these images for further mental processing.

In defining spatial ability, Linn and Petersen (1985, p. 1482) suggested "the skill in representing, transforming, generating and recalling symbolic, non-linguistic information". In general terms, it is the capacity to visualise objects in space and to be perceptive of internal and external relationships and to transform and manipulate them. To be able to rotate an object in "the mind's eye" in an effort to form 2D and 3D images from multiple directions and orientations serves as an example of spatial abilities.

Spatial skills are mental functions to reason about spatial relationships in imagined and real spaces and manipulate and organise mental images (Newcombe & Shipley, 2014; Uttal et al., 2013).

Sorby (2020) defines spatial ability as:

> *Mental rotation*, the ability to visualise a well-structured image and mentally rotate it to a new orientation, *Spatial visualisation*, the ability to discern, analyse and manipulate complex spatial patterns and *Spatial orientation*, the ability to perceive objects in space and visualise how they, and/or you as the observer, are repositioned relative to each other from different perspectives. (p. 21)

Stemming from all the different spatial factors that exist and the resulting fragmentation in definitions, it is not easy to define VSR in a simple sentence. Owens (2015) argues that VSR is the encompassing term that describes human reasoning that is inclusive of the fragmented descriptions and definitions of spatiality. Tversky (2005) refers to VSR as a cognitive collage, a diverse but organised array of visual, spatial, and semantic information based on specific stimuli that capture visuospatial properties of the world.

Tversky (2005, p. 232) offers the following insight on VSR:

> Spatial thought, spatial language, and spatial graphics reflect the importance and prevalence of visuospatial reasoning in our lives, from knowing how to get home to knowing how to design a house, from explaining how to find the freeway to explaining how the judicial system works, from understanding basic science to inventing new conceptions of the origins of the universe.

According to Tversky's (2005, p. 216) cognitive collage approach to VSR, complex inferences can be made by combining single inferences on simple mental transformations. She proposes the separate and combinable factors belonging to VSR as the following: ***The factors when determining static properties of entities****:* "figure/ground, symmetry, shape, internal configuration, size, colour, texture, and more." ***The factors when determining relations between static entities and with respect to a frame of reference****:* "location, direction, distance, and more". ***The factors when determining relations between static entities and with respect to other entities*****: "**comparing size, colour, shape, texture, location, orientation, similarity, and other attributes". ***The factors when determining relations of dynamic/static entities with respect to other entities or to a reference frame***: "direction, speed, acceleration, manner, intersection/collision". ***The factors when performing transformations on entities****:* "change location (scanning), change perspective, orientation, size, shape; moving wholes, reconfiguring parts, zooming, and enacting". ***The factors when performing transformations on self****:* "change of perspective, change of location, change of size, shape, reconfiguring parts, enacting".

Drawing inspiration from the works of Tversky, Newcombe and Shipley (2014, p. 2) argue against the notion of a unitary approach to VSR and state: "Sadly, the truth is that a hundred years or so of work with existing tests and statistical techniques has not arrived at a cohesive view of the structure of spatial intellect". Through research in the STEM disciplines, Newcombe and Shipley (2014, p. 3) developed what they refer to as a "New Typology" for describing VSR centred on the differential of "spatial representations that are intrinsic" to objects (forms and components) and "those that are extrinsic" (relations between objects, intrinsic to objects and reference points). Information on these objects can either be static, or transformed dynamically, by mentally bending, moving or rotating objects. Newcombe and Shipley (2014, p. 5) describe four categories of VSR as portrayed in Table 2.1.

Table 2.1: Newcombe and Shipley's (2014) VSR descriptors for their "New Typology"

VSR	Description
Intrinsic-Static.	Forming a mental image is a process of "coding the spatial configuration (or shape) of objects; picking shapes out from overlapping objects or other perceptual information; identifying regions of space as constituting categories".
Intrinsic-Dynamic	Transformation of objects based on previously encoded spatial information, "including expansions or reductions in size, rotation, cross-sectioning, folding, bending, breaking and sliding; accumulating sequences of such changes and visualising change over time and an end product; relating 2- and 3-dimensional views to each other".
Extrinsic-Static.	"Coding the spatial location (or position) of objects relative to other objects or to a reference frame, including gravity; aligning location coding that differs in scale".
Extrinsic-Dynamic	For objects in motion, interrelations become transformed and the viewer must maintain stable representations of the scenario whilst considering different perspectives.

Schneider and McGrew (2018, p. 125) define visuospatial reasoning as an ability to create mental images and simulations to process spatial tasks through perceptions, manipulations, differentiating, and retrieving non-verbal images "in the mind's eye". Visual information enters the eyes, whereafter the brain spontaneously performs several low-key analyses (edge-detection, contrast-perception, colour-differentiation, and motion-detection). Results stemming from these low-key analyses are further analysed by various advanced processes "to infer more complex aspects of the visual image" such as object recognition, motion prediction, and mental modelling of spatial configurations. VSR is also referred to as spatial intelligence, spatial thinking, spatial cognition and spatial expertise (Hegarty, 2010). Carroll (1993) spoke of VSR as being able to orientate oneself in the surrounding space whilst maintaining directionality and reasoning within dynamic and static spatial frames.

Visualisation (Vz) is the core ability of (Gv) (Yilmaz, 2009). Schneider and McGrew (2018) state that (Gv) is a narrow ability that loads onto (Gv) and defines visualisation (Vz) as the capacity to be perceptive of complex patterns and to mentally generate/simulate transformed images of what is perceived *(mental rotation, twisting, surface development, inverting, re-sizing, obscured detail, sectional planes, swapping between 2D and 3D).* Yet, there are many other narrow abilities that do not form part of this study’s discussions. In his extensive studies on spatiality, Buckley (2018) proposes a framework that includes many narrow, first-order factors but which are too expansive for the scope of this study. See Appendix J for a diagrammatic structure of Buckley's (2018) spatial factor framework.

2.4.2 Controlled attention: (Analytical reasoning (AR))

According to Schneider and McGrew (2018, p. 93), "If we can translate Spearman's insights into more modern terms, fluid reasoning (Gf) is the process by which we extract new knowledge from the information we already have". To process bits of information, some knowledge about the bits and perceptions of the relations between the bits are required to be held in immediate awareness for sufficient periods. Fluid reasoning (Gf) is flexibly used in tandem or in alternating sequences, but apart from prior knowledge, to find solutions to tasks (Schneider & McGrew, 2018). Fluid reasoning (Gf) distinguishes between the **process** (induction vs deduction) and **content** (verbal vs visuospatial) facets of (Gf) (Santarnecchi et al., 2017). In functional CHC nomenclature, Schneider and McGrew (2018) denote reasoning as (Gr), inductive/deductive reasoning as (Gr-ID), and contextual reasoning as (GR-CR).

Inductive reasoning: Inductive reasoning (I) or rule inference is the ability to scrutinise phenomena to uncover the foundational rules and principles that constitute its construction. People who are well-versed in inductive reasoning perceive patterns and regularities in situations that might otherwise seem intractable. The first-angle and third-angle orthographic projection systems provide a platform for heuristic behaviour. The orthographic systems provide a "formula" that is replicable for any drawing. New, unfamiliar objects can be heuristically positioned correctly according to this "formula". Previous engagement with this "formula" strengthens this heuristic and solutions can easily be found by observing how the "formula" worked in previous scenarios. Both heuristic and analytic processes precipitate reasoning, where each process potentially judges the strength of an argument. Hayes and Heit (2018) state that inductive reasoning is influenced by rapid heuristic processes driven by information on associated contexts and similarities but without being able to guarantee the logical validity of an argument. It thus makes sense for EGD practitioners to rely on heuristics for planning and positioning drawings as the orthographic system is formulaic in how views are rotated in incremental steps of 90°. By using the Structure of the Observed Learning Outcome Taxonomy (SOLO), researchers have shown that inductive reasoning can be measured on hierarchical levels (Gagani & Misa, 2017). Heuristics serve the EGD practitioner well in performing easier tasks such as planning drawing positions, yet even though rotations are heuristically applied, higher order VSR is required and deductive reasoning becomes necessary for more complex manipulations.

Gagani and Misa (2017) believe that students who fail to achieve a simple task become frustrated with higher sub-problems in the same task that have to be performed. Students are not ready to grasp the relationship between the sub-problems during an inductive reasoning task when their prior knowledge pertaining to the task is sketchy. Some of the errors that were made by the participants during the execution of drawings pivoted on incomplete understanding of EGD’s axiomatic system. Their incomplete knowledge is rooted in how orthographic and isometric projections relate in terms of their derivation from the Cartesian Plane. For teachers to be effective in their daily facilitations, they have to first determine a baseline of prior knowledge to determine on what level the class as a collective can reason. Gagani and Misa (2017) argue that a hierarchical diagnostic system is useful in establishing such baselines onto which future instruction may be pivoted. Yet few teachers are equipped to embark on such diagnostic endeavours, which call for purposeful teacher training to equip teachers in their dualistic role of diagnosticians and remedial instructors (Gagani & Misa, 2017; Misrom et al., 2020; Tóth & Pogatsnik, 2023).

In contrast to inductive reasoning, deduction judgements are influenced by slower, effortful analytic processes comprising more deliberative and more accurate reasoning (Kahneman, 2011; Evans & Stanovich, 2013; Hayes et al., 2018). However, according to Mansiet al. (2022), inductive and deductive reasoning are based on multiple overlapping networks, supporting the notion of a single mechanism for these two types of logical reasoning.

Deductive reasoning: This is a rule-based ability, using known principles and premises in logical reasoning. Inductive reasoning uses prior knowledge to discover new principles, unlike deductive reasoning (general sequential reasoning (RG)), where established principles are used to discover new facts. Kahneman (2011) suggests that the brain uses heuristic systems (inductive reasoning) as its normal mode in argumentation and associatively learned responses. Yet, when necessary, deductive reasoning interrupts automatic heuristic processes and takes control to explore and evaluate alternative solutions. Deductive reasoning could sometimes cause a higher cognitive load, which explains why humans rely on default automatic processes (inductive) whenever possible (Kahneman, 2011).

No definitive consensus has been reached on the cognitive mechanisms used in inductive and deductive reasoning. Yet, the distinction can be attributed to autonomous processes versus high-level cognitive mechanisms respectively, where working memory and long-term memory form part of this complex cognitive network (Evans & Stanovich, 2013b; Pennycook, 2017). Heuer (2003, p.2) exemplifies analytic reasoning with this statement: "Thinking analytically is a skill like carpentry or driving a car. It can be taught, it can be learned, and it can improve with practice. But like many other skills, such as riding a bike, it is not learned by sitting in a classroom and being told how to do it". Reasoning about a problem is a cyclic, iterative process where numerous questions about the problem at hand are generated. Cook and Thomas (2005), as mentioned by Helldin (2016), stipulate that analytic reasoning is a three-step process: (1) Evaluate the problem, understand all its elements and explain what is known about it; (2) perform a SWOT analysis; and (3) propose several options, and assess their effectiveness and implications. To reason effectively in EGD, it is necessary to make use of inductive reasoning's heuristic processes for the sake of executing drawings speedily and according to the rules and formulas of conventional standards. From the foregoing discussion on the differences between inductive and deductive reasoning, the reasoner will have to, at some point, set heuristic systems of reasoning aside and introduce deductive reasoning processes (Kahneman, 2011).

Analytical reasoning is enabled by working memory (Gwm), which is defined as the sustainability of manipulating information while actively and with sharp attention attending to a task (Schneider & McGrew, 2018). Working memory forms the core of logical reasoning during the learning processes, being attentive in verbal and non-verbal language. It is the capacity to synthesise and reconfigure constructs and conventions. Baddeley (2012) explains how the central processor is responsible for focusing, dividing, and relaying attention intermittently between focus points while constantly interfacing with long-term memory (Gr & Gl). Because of the high cognitive load that imagery can induce on the memory system, EGD practitioners must utilise the conventions of EGD fully, which allows for a certain degree of heuristic thinking to ameliorate higher-order thinking.

The central processor is responsible for binding memory features together. For example, if a square is blue and a circle is yellow, the central processor associates yellow with a circle and blue with a square (Schneider & McGrew, 2018). People who can sustain focused attention on various objects for a long time can differentiate more extensive and complex relationships within objects and among objects. Visuospatial short-term memory refers to encoding visual stimuli and sustaining such information in primary memory, whereas visual memory (MV) refers to being able to hold complex images in "the mind's eye" for short periods. Attentional control (MS and WM) is the ability to manipulate focused attention on task stimuli that are relevant and ignore task stimuli that are irrelevant. Working memory capacity (Gwm) is the ability to manipulate content in primary memory.

2.4.3 Acquired knowledge: (Convention Knowledge (CK))

Schneider and McGrew (2018) make mention of the myriad of knowledge types that have been identified by cognitive psychologists. Vukić et al. (2020) state that learning outcomes stem from different knowledge levels, which can be comprehensively described through the revised version of Bloom's taxonomy (1956). Krathwohl (2002) proposed a revised structure of four broad knowledge areas based on Bloom's Taxonomy (1956) which include Declarative knowledge, Conceptual knowledge, Procedural knowledge, and Metacognitive knowledge.

2.4.3.1 Declarative knowledge

Vukić et al. (2020, p. 2) agree with Krathwohl (2002) by stating that declarative knowledge, which is also known as factual knowledge, "captures discrete, isolated content elements (terminology and knowledge of specific details and elements)". Schneider and McGrew (2018) classify procedural and factual knowledge as comprehension knowledge (Gc) and describe it as the ability to understand and interact with knowledge that is valuable for different cultures. For instance, the EGD practitioner cannot follow the heuristic procedures of producing orthographic multiviews, without knowing the facts about first- and third-angle orthographic projections, as the correct rotation and positioning of views are predicated on this CK (Bertoline et al., 2011; Giesecke, 2016). Domain-Specific Knowledge (Gkn) refers to the mastery of specialised declarative and procedural knowledge and is normally acquired through career activities, hobbies, and passionate interests (Schneider & McGrew, 2020). The axiomatic system of orthographic and isometric projections exemplifies domain-specific knowledge for EGD. Yet, a notable difference between comprehension knowledge (Gc) and domain-specific knowledge (Gkn) is how they relate to working memory. Similarly, Schipolowski et al. (2014, p. 157) state that "verbal ability (Grw) and factual knowledge are closely related but factorially distinct facets of the (Gc) construct". Schneider and McGrew (2018) attribute these relationships to the level of expertise and knowledge a person possesses and states that experts in a field “seem to be able to access large amounts of specialized knowledge very quickly in long-term memory” (p. 117). Experts in their field can hold domain-specific knowledge (stored in long-term memory) in immediate awareness as though it was stored in working memory to solve complex problems effectively and efficiently.

Specific to EGD is the narrow ability known as “mechanical knowledge” (MK), which Schneider and McGrew (2018) define as knowing the terminology pertaining to ordinary equipment and how they operate and what their functions are. This factor is imperative for success in EGD as drawings are produced on and about such items (Oviawe, 2020; Chen et al., 2017; Sotsaka, 2019). Quantitative knowledge (Gq) is invaluable to EGD and straddles declarative and procedural knowledge as they relate to the mathematical symbols used in EGD. Mathematical symbols such as π, +, –, ×, ÷, √, º, R, ø, %, ß, ά, θ and many others are used in both geometry and EGD. In addition, EGD requires knowledge to calculate scales, ratios, angles, areas, volumes, dimensions, etc. In contrast, the narrow ability known as “mathematical knowledge”

(KM) does not refer to the academic skills of applying mathematical operators or solving mathematics problems but is rather concerned with "what" rather than "how" knowledge *(e.g., "What is isometric scale"?" "What is a Net?").* I.e., it refers to the general knowledge about mathematics and geometry (Schneider & McGrew, 2018).

2.4.3.2 Conceptual Knowledge

Richland et al. (2012, p. 190) define conceptual knowledge as the ability to expertly navigate the structured concepts of a domain. They expand this definition by stating that "This level of understanding allows learners to think generatively within that content area, enabling them to select appropriate procedures for each step when solving new problems, make predictions about the structure of solutions, and construct new understandings and problem-solving strategies".

Hurrel (2021) states:

> In essence a concept is an idea that is well enough understood to allow other ideas to be connected with it and become part of a web of understanding. Such connections and webs often lead to the formation of conceptual knowledge. (p. 59)

Bruner (1966) foregrounded four characteristics of concepts that organise perceptions and underpin understanding: they 1) provide domain-specific structures; 2) provide a map for finer details to aid understanding and retention; 3) provide conduits for the transfer of learning; and 4) provide a framework towards lifelong learning. Krathwohl (2002) states that knowledge of interrelationships between the core elements within a larger schema that enable such elements to operate together is conceptual knowledge. Schemas include categories of knowledge, knowledge of classifications, principles, generalisations, theories, models, and conceptual frameworks (Vukić et al., 2020).

Hurrell (2021) explains how students who performed seemingly well in high school often require remedial assistance in university programmes due to a poorly developed conceptual understanding of quantitative subject matter. The data gathered in this study seem to corroborate Hurrell’s (2021) statement in the previous sentence. In South Africa, the pass rate for EGD on the Grade 12 level was reported as 93.3% between 2017 and 2021. This pass rate seems to imply that there are no problems regarding EGD in the secondary school system. Yet, the data collected by myself and my colleagues from first-year preservice teachers’ proficiency tests suggest that school-leavers with recent EGD experience display deficient knowledge and reasoning in many areas of the secondary school syllabi for EGD. It appears as though educators favour procedural knowledge at the expense of conceptual knowledge.

2.4.3.3 Procedural Knowledge

This refers to the "how to" of doing different things: directing inquiries; criteria for applying discipline-specific skills; discipline-specific techniques; and methods and algorithms for different procedures (Krathwohl, 2002). Although I describe the four knowledge categories as individual elements of the cognitive domain, they function together during episodes of knowledge acquisition and knowledge transfer (Makransky & Petersen, 2021). Makransky and Petersen (2021) add that the transfer of learning could manifest as a conceptual endeavour through a mental tour of seeking solution paths or in the execution of a physical procedure. It should be noted that the four knowledge dimensions of Krathwohl (2002) reside on a hierarchy, with factual knowledge on the lowest level. Through factor analysis, Anis et al. (2020) have demonstrated the progression of the hierarchy and concluded that all other knowledge dimensions are difficult to acquire when factual knowledge is inadequate. Through procedural knowledge, a sequence of steps and actions is employed to solve a problem (Rittle-Johnson, 2017; Rittle-Johnson et al., 2015). Hurrel (2021, p. 59) explains how the application of “rules without reason” in a mechanical fashion often leads to confusing solutions. Richland et al. (2012) ascribe such reasoning to a lack of conceptual understanding of key subject matter. Givvin et al. (2011) found that students who portrayed such ineffective reasoning mostly conducted faulty procedures and lacked conceptual understanding. Hurrel (2021) explains how dependency on procedural knowledge leads to superficial subject knowledge and how reliance on conceptual understanding leads to deep subject knowledge and positive transfer of learning.

2.4.3.4 Metacognitive Knowledge

With reference to the previous sections on procedural and conceptual knowledge, Guven and Kosa (2008) state that teachers often encourage learners to rote-learn procedures which impacts the development of spatial abilities and conceptual understanding negatively.

Capps and Crawford (2018) put it this way:

> We suggest that a teacher who thinks about and reflects on her teaching and takes a metacognitive stance by being aware of the complexities of the classroom and her role as a teacher is likely to achieve a higher ability to translate inquiry/scientific practices into the classroom than a teacher who does not. (p. 17)

Metacognitive behaviour does not only pertain to learners but is equally important for teachers and should be explicitly taught. Metacognitive regulation strategies such as planning, monitoring, and evaluation help learners to navigate their declarative knowledge (about &

what), their procedural knowledge (how), and their strategic knowledge (why & when) towards finding solution paths to problems (Wengrowicz et al., 2018).

2.5 The van Hiele model of geometric reasoning

Lowrie et al. (2019, p. 745) define geometric reasoning as "the transformation and manipulation of spatial properties, while providing the supportive scaffolds to access more complex geometric concepts". Clements and Battista (1992) state that geometrical reasoning requires one to generate images, analyse these images to retrieve essential information, transform these images, and maintain these images for further mental processing. From these definitions and the definitions for visuospatial reasoning (Gv) in Section 2.4.1, it seems that geometrical reasoning is congruent with visuospatial reasoning (Gv). The discussions that follow on the van Hiele model are based on the assumption that this congruency is a true reflection of spatial reasoning in EGD.
According to Lehrer et al. (1998), "the pioneering efforts of Piaget and van Hiele remain the most extensive sources of information about school-age children's initial conceptions about space and corresponding trajectories of change" (p. 137). Yet, there is tension between the two theories in terms of when development takes place (Papademetri-Kachrimani, 2012). The most inherent difference between van Hiele and Piaget is that van Hiele believed that progression is instruction-based, while Piaget believed that progression occurs strictly through the four chronological stages of development (George, 2017).

Two researchers from the Netherlands, Pierre van Hiele (1957) and Dina van Hiele-Geldof (1957) postulated a theory for the teaching and learning of geometry based on their research of how learners progressed through learning geometry concepts. Pierre and Dina van Hiele (husband and wife) believed that most of the problems experienced by students are situated in instructional practices rather than in the cognitive processes that describe hierarchical reasoning (Pegg, 2020). Pierre van Hiele stated that "the levels are situated not in the subject matter but in the thinking of man" (Van Hiele, 1986, p. 41). This theory is commonly known as the van Hiele theory (Pegg & Davey, 1998).

The van Hiele model is designed to nurture learners' skills acquisition by arranging the learning environment to accommodate their geometric thinking ability on five hierarchical levels (de Villiers, 2010; Alex & Mammen, 2018). Each level represents a cognitive developmental level by describing specific cognitive skills that define each level. According to the van Hiele model, the levels are mastered in linear order from Level 1 to Level 5. Yet, research findings support the notion that progression is not always linear and that different aspects of the five levels can be simultaneously present within a learners' frame of reasoning (Gutierrez et al., 1991). However, de Villiers (2010) points out that research supports the principle of designing instruction on a linear scale of cognitive acquisition, even though some higher-level cognitive skills may have already formed. Figure 2.3 represents the hierarchical structure of the van Hiele model's cognitive levels of thinking.

2.5.1 Hierarchical levels of reasoning

According to Pegg (2020), most researchers direct their efforts at the models' hierarchical levels of cognitive reasoning, as depicted in Figure 2.3. These hierarchical levels of thinking are learner-centred and provide cognitive descriptors for the way learners develop geometric thought. The five hierarchical levels through which learners move to learn geometry are known as *Recognition, Analysis, Informal deduction, Formal deduction, and Rigour* (Pegg, 2020).

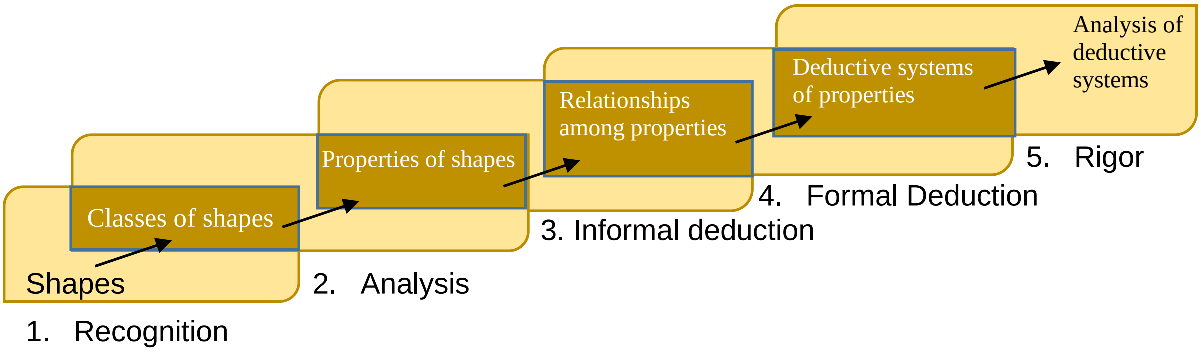


Figure 2.3: The Van Hiele theory of geometric reasoning (Van de Walle, 2004, p. 347)

Level 1: Recognition: Figures are visually recognised by their global characteristics. Figures such as squares, triangles and parallelograms are recognised according to their shape, but students are not yet able to identify figure properties explicitly. This level is characterised by learners' ability to observe shapes, take them apart, and rebuild or manipulate them somehow. The focus is on the **classification of shapes** by exploring their similarities and differences physically and mentally (van de Walle et al., 2019).

Level 2: Analysis: Learners start to move from considering classes of shapes by analysing the properties of figures and learning to describe them appropriately by their technical terminology, yet are unable to interrelate the properties of figures or figures with figures. Learners focus on one class of shapes and can reason that rectangles are rectangles because they have four sides, with opposites being parallel and with four 90° angles, their opposite sides equal lengths, and diagonals being congruent. Size and orientation are not as relevant and are not in focus. Learners begin to grasp those shapes that belong to a certain class, such as cubes, and share corresponding properties. For example, the six congruent faces of cubes are all square (van de Walle et al., 2019).

Level 3: Informal deduction: Learners move beyond considering figure properties and start to explore the relationships between objects and properties. Learners apply short deduction steps to arrange the properties of figures logically, and they can grasp such concepts as class inclusions and other interrelationships between figures. When learners no longer fixate on one particular shape and start to identify other object properties, relationships between various properties of an object and related objects start to make sense. According to van de Walle et al. (2019, p.505), "If all four angles are right angles, the shape must be a rectangle. If it is a square, all angles are right angles. If it is a square, it must be a rectangle." When learners start to engage in "if–then reasoning", they are able to use a minimum set of defining attributes to classify shapes. The ability to engage in informal logical reasoning is a signature attribute of level 3 reasoning. Because they comprehend various properties of shapes, the time has come to encourage conjecture and to ask "Why?" or "What if?"

Level 4: Formal deduction: Learners start to comprehend the significance of deductive reasoning as evidenced by the development of longer sequential statements, and start to function with axioms, proofs and theorems. Students can now analyse informal arguments and the structures of systems inclusive of their axioms. Geometric truth emerges as they begin to postulate and develop definitions, theorems and corollaries.

Level 5: Rigour: Students move beyond reasoning within one axiomatic system and start to compare and contrast various axiomatic systems of geometry. On the most advanced level of the van Hiele hierarchy, axiomatic systems are no longer just the deductions within a system, but the actual axiomatic systems become the focal point of interest. Mathematics majors at university operate on this level.

De Villers (2010) explains the importance of acquiring the technical language on each of the five hierarchical levels. This domain-specific knowledge is essential for describing the properties of various concepts to make progression from a previous level to the next level possible. Besides acquiring the specific technical language per level, in order to make the transition between levels possible, learners have to recognise new relationships between constructs and refine and renew existing constructs. Researchers found that an important factor for transitioning between van Hiele levels was the consistent and continuous sequencing and development of concepts from lower grades through to higher grades (de Villiers, 2010). There is a second aspect to the van Hiele model learning phases, which is equally important but not as widely acknowledged or scrutinised.

2.5.2 Phases of learning

Through Dina van Hiele's research, five instructional phases that are teacher-centred were developed to facilitate learner movement between hierarchical levels (Pegg, 2020). The most effective way to progress through the five levels is for teachers to arrange the learning environment through five phases of learning, known as: 1) Inquiry; 2) Guided orientation; 3) Explicitation; 4) Free orientation; and 5) Integration. Figure 2.5 illustrates how Pegg (2020) sees the cyclic and iterative progression between Informal deduction (VH3, Stage 1) and Formal deduction (VH4, Stage 2). When all the cognitive aspects of one level are satisfied, the cycle repeats itself for the higher level. In this manner, new knowledge is integrated with prior knowledge to "bind learning gaps" (Pegg, 2020).

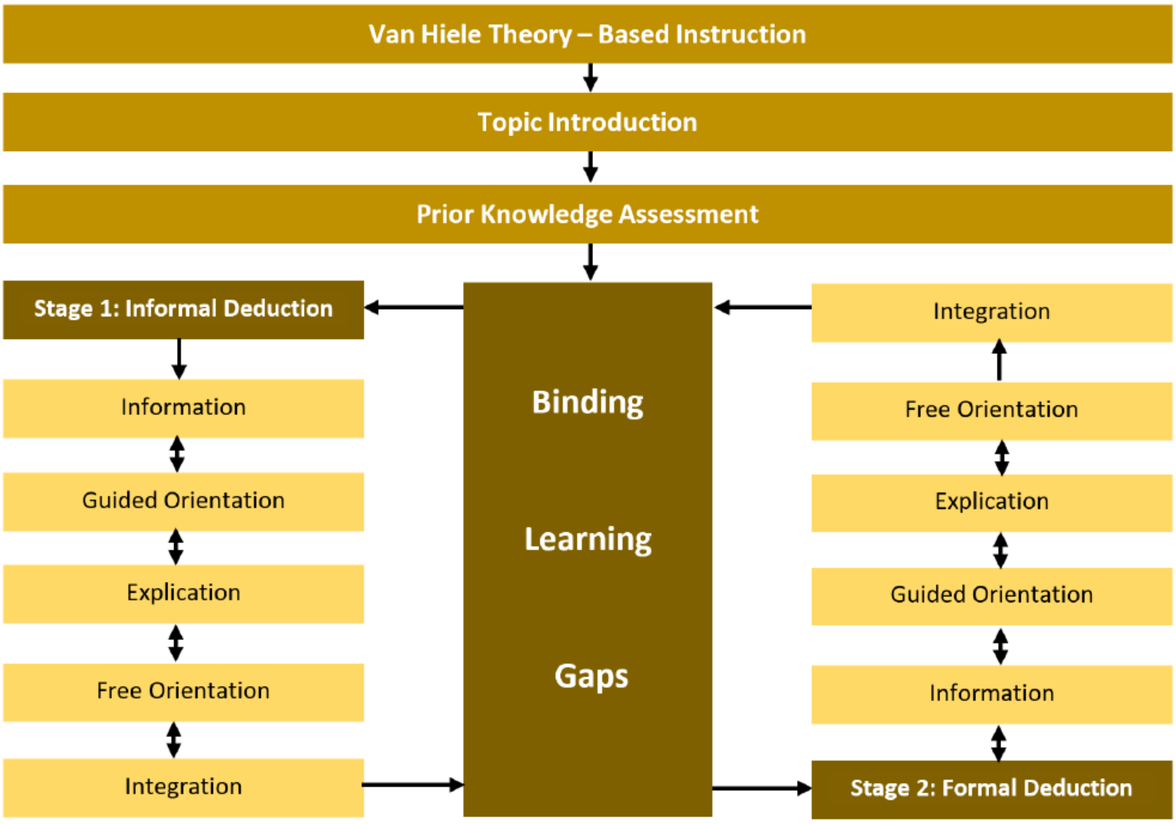


Figure 2.4: The Van Hiele phases of learning (Pegg, 2020)

Malloy (2002) highlights two important points for instructional practice regarding the van Hiele model. First, teachers have to grasp their students' acquisition of van Hiele levels, and second, they should facilitate their students' processes of acquisition through these levels to prepare them for deductive reasoning within an axiomatic system. School geometry revolves around Euclidean geometry, where learners are expected to reason on a formal deductive level (Alex & Mammen, 2016)

1. Inquiry/Information: The first part of the learning phase is a discussion between the teacher and student to discover what the topic is about. The teacher poses questions to trigger an interactive discussion with the learners. Through this discussion, prior knowledge about a topic is identified, and students can be oriented to the new topic. This first step is used to plan a scaffolded lesson for the next session.

2. Guided orientation: based on the discussions in phase 1, students' attention is directed to the new topic. Carefully structured experimental tasks allow students to fold, rotate, measure, draw and construct (Pegg, 2020). Spatial reasoning skills begin to develop when students are afforded the opportunity to see and manipulate shapes and/or objects (Howse & Howse, 2014).

3. Explicitation: The simple tasks and manipulatives provided by the teacher move student involvement along where they engage in talks and discussions on the subject matter. Students start to form opinions on geometry tasks, and once students can describe in their own words what they have learned about the topic and have formed networks of relations around the topics, only then can the teacher introduce specific mathematical terminology on the new topic (Pegg, 2020). Ding and Jones (2007) remind us how Hoffer (1983) noted that the third phase, commonly known as "Explication", was incorrectly translated by Wirszup (1976, p. 83). Dina van Hiele originally named the third phase "Explicitation", with Hoffer (1983, p. 207) stressing the intention of "Explicitation" in that "students make the observations explicitly rather than receive lectures (explanations) from the teacher".

4. Free Orientation: At this stage, students begin to engage independently in more advanced and complex activities and negotiate their own solution path. Teachers must encourage inventiveness and facilitate different solution paths. Empowered by newly acquired understandings of relationships, students become able to engage in more open-ended tasks and propose solution paths (Pegg, 2020).

5. Integration: Students now comprehend more fully the purpose of the content and form synopses of the new content by themselves and can progress to the next level. Students reflect on the new learning, make their own summaries to integrate prior knowledge with new knowledge, and form new schemas of objects and relations (Pegg, 2020; Schneider & McGrew, 2018). Some students may be required to cycle repeatedly through some of the five phases until new concepts are completely acquired (Mason, 2009).

These five teaching and learning phases provide a framework of instruction for systematising introductory discussions, with the scaffolded progression of selected activities and/or exercises. The five phases encourage language development and multipath solution development with students actively reviewing and overviewing newly learned topics. The five phases provide a logical, fluent journey to overcome common difficulties teachers and students face where diversified classrooms will benefit from an instructional strategy that accommodates differential cognitive development (Pegg, 2020).

According to Mason (2009), most students commencing with high school geometry courses function at VH1 or VH2, whereas teachers function at VH4 or VH5. Teachers are familiar with subject terminology, understand that different terms belong to different levels and should guard against using terms that are not yet familiar to students for the level they find themselves on. When teachers are insensitive to this principle and use terminology indiscriminately, students will not understand the new content. Often, in such cases, students tend to memorise the material, resulting in a false sense of mastery and a lack of deep understanding. Teachers must also understand that students may display inconsistent understandings of content strands based on having more experience with certain content and less in others. However, where students are advanced in one content strand, it is easier for them to function at that level in other areas, because schemas of relationships between figures and properties are already well-established for that content (Mason, 2009; Gutierrez et al., 1991). Discussions and verbalising concepts are important aspects of scaffolding learning in the explicitation and integration phases of learning (de Villiers, 2010). By talking about them, students clarify and construct their own knowledge.

2.5.3 The conditions for level progression

Usiskin (1982, p.4) highlighted four important characteristics of the theory, namely: 1) Fixed order: Progression is sequential and invariant. Learners cannot progress to the next level without having satisfied the conditions of the previous level; 2) Adjacency: where levels of thought on previous levels were intrinsic, they convert to being extrinsic on the next level; 3) Distinction: Technical language and symbols are unique on each level, and relationships that interconnect these symbols operate on a unique scheme; 4) Separation: Learners who function on different levels are not able to understand each other. Similarly, teachers function on higher VH levels and may impose separation by not being sensitive to learners' levels (Mason, 2009).

Pegg (2020) lists nine important features of the van Hiele model regarding the five levels of geometric thought:

1. Progression to a higher level is subject to attaining lower levels first. Yet, Fuys et al. (1988) found that learners can straddle levels and can appear to function at higher levels by rote-learning rules and definitions and by applying routine heuristics that are not properly

understood. The van Hiele model is concerned with the development of understanding of higher-order constructs based on understanding structures on lower levels. (George, 2017; Gutierrez et al., 1991). Schunk (2014) states that higher-order skills are easier to learn once lower-order skills have been mastered. Schunk (2014) states that the hierarchy of skills proposed by Gagne (1965) resembles the hierarchical levels of van Hiele as both models represent advancement from lower-order to higher-order skills. Yet, they differ distinctly in that van Hiele believed that each level's descriptors had to be fully acquired for progression to the next level (Usiskin, 1982). According to Vygotsky (1978), learning precedes development, as opposed to Piaget's belief that development comes before learning; however, certain cognitive structures have to be established for specific types of learning to take place (Slavin, 2018).

2. For learners to progress to the next level requires considerable time for direct instruction, exploration, and reflection for this growth to occur. Usiskin (1982) believes that cognitive development in geometry can be accelerated through the van Hiele levels by purposeful instruction. This belief is in agreement with Bruner's (1966) hypothesis that given the right teaching, learners can learn any task. Bruner et al. (1995) noted: "The van Hiele model of mathematical reasoning has become a proven descriptor of the progress of students' reasoning in geometry and is a valid framework for the design of teaching sequences in school geometry" (p. 592). Berger (1966) and Ostroff (2012) suggest that instructional sequences must be purposefully tailored to move learners from a teacher-centred approach to where learners can become fully autonomous. Slavin (2018) explains that for learners to mature cognitively to transfer newly acquired skills, learning must be scaffolded on a continuum of purposefully planned assistance to total independence. Wood et al. (1976, p. 90) coined the term "scaffolding" and described it as a process "that enables a child or novice to solve a task or achieve a goal that would be beyond his unassisted efforts." The concept of scaffolding originated from Vygotsky's sociocultural theory (Radford et al., 2014). Scaffolding provides optimal, systematic pedagogical support to the learners where prior knowledge becomes subsumed and supports learners to progress from the concrete to the abstract (Yazdani, 2008). Scaffolding is enabled by purposeful instruction to move students from simple concepts and principles to more advanced ones with the help of effective visual support and visual organisers (Alex & Mammen, 2016). Cooperative learning strategies aid the scaffolding process where learners assist one another in the learning process (Slavin, 2014; Webb, 2008). Peers find themselves sharing the same zone of proximal development (see point 4 on the next page), which positions them perfectly to learn from each other (Gredler, 2009).

Gagne (1965) proposed three categories of instruction with nine sub-phases, where each phase is subject to modification as a function of internal conditions and learning outcomes. The modification strategy is hierarchically organised according to intellectual skills, where the targeted skill resides as the highest skill in the hierarchy (Schunk, 2014). When a new hierarchy is designed, teachers must formulate specific prerequisites for the target skill and determine essential prior knowledge and skills before learning the new target skill. This is a top-down process and is repeated for all prerequisite skills and continues down the hierarchy until skills that learners can perform currently are arrived at (Schunk, 2014). In order to learn a higher-order skill, two or more prerequisite skills must be applied where none of the prerequisite skills depends on the other, as these hierarchies do not function in linear order.

3. Learners may function on different levels for different concepts, but once reasoning about one concept has progressed to a higher level, it takes less time to start reasoning about other related concepts to also progress to the higher level. Gagné (1984, p. 377) identified five different types of learning outcomes known as "intellectual skills, verbal information, cognitive strategies, motor skills, and attitudes" that aid in future learning. Yet, a myriad of external and internal variables may enhance or impede the way reasoning develops for learners (Gagné & Glaser, 1987). Intrinsic conditions are prerequisite skills across a range of cognitive processing requirements, while extrinsic conditions are environmental stimuli that support learners' cognitive processes. Both types of conditions must be specified as completely as possible when instructional strategies are designed. Internal conditions refer to learners' current capabilities and knowledge stored in long-term memory (LTM), which are activated through learning materials and instructional cues from teachers (Gagné & Glaser, 1987). External conditions are functions related to the teacher, and the internal conditions relate to learner attributes and are diversified (Gagné et al., 2005).

4. Learners have to confront a personal "crisis of thinking" when advancing through the levels. This condition is congruent with Piaget's concept of disequilibrium, where current knowledge proves to be insufficient for problem-solving. Equilibrium is restored through "fitting external reality to the existing cognitive structure" and "changing internal structures to provide consistency with external reality" (Schunk, 2014, p. 238). Teachers may not force learners to reason on a higher level when they have not yet attained that level. Such teaching strategies can inhibit learning and curb the learner's potential. Vygotsky (1978, p. 84) theorised that "learning takes place when children are within their zone of proximal development". When learners cannot yet execute tasks independently but depend on competent peers or adults, then learning can take place in the zone of proximal development (ZPD). Development takes place when children can independently and with self-regulation internalise domain-specific concepts during problem-solving.

5. "Level reduction" occurs when schemas on a higher level are restructured for a lower level by teachers to promote learner performance. Such approaches may be counterproductive because the stimulus is removed for learners to form higher-order schemas on their own

terms. Vygotsky (1978) agrees with the danger of level reduction and states that effective teaching and learning take place within the period of ZPD, and content that is too difficult or too easy negates effective learning.

6. Students that reason on different levels experience difficulty in understanding one another due to each level having its own language and linguistic symbols as they exist between learners and/or learners and teachers.

7. Each level is unique in how relationships are structured, resulting in some tension between levels. This aligns with Piaget's disequilibrium principle; what is correct for one level may not be acceptable for another. In comparing the ideas of van Hiele and Piaget, Clements and Battista (1992) suggest that both theories focus on building understanding by promoting learner ownership. They state that "both theorists believe that a critical instructional dilemma is teaching about objects that are not yet objects of reflection for students" (p. 437). In an effort to ameliorate the tension between levels, Van Merriënboer and Sweller (2005) suggest that instructional methods should focus on decreasing extraneous cognitive load to allow for existing resources to be devoted to learning. Extraneous cognitive load depends on how content is presented and learner activities are designed (Bruning et al., 2004). The process of unlocking new content according to a hierarchy (see point 2) demands expert content knowledge to formulate prerequisite skills in succession and to establish how instruction must be sequentially arranged over the scope of the hierarchy (Schunk, 2014). In line with Gagné's theory, simple-to-complex sequencing of content affords learners time to focus their resources on the intrinsic demands of learning and reduces the extrinsic load (van Merriënboer et al., 2003). Tasks that are significantly contextualised to align with the real world help to minimise extrinsic load because they limit the need to engage in extraneous processing to understand the context. With the increase in cognitive load, individual learning becomes less effective and efficient, which provides opportunities for collaborative learning (Kirschner et al., 2009).

8. The learning process is "discontinuous". When a level of thinking has been acquired, learners remain at that level periodically, as if being in "incubation", or a gap of undetermined duration. Learners cannot be expected to function at an advanced level, and neither can teaching be directed at advanced levels. The maturation process had not yet occurred; therefore, failure should be expected. Of particular interest is that cognitive development is not necessarily age-related as is evidenced by learners of different ages functioning on similar levels (Lunetta, 2014; Battista, 2007; Van de Walle, 2004). This is discussed further in Section 2.5.4.

9. The use of routine algorithms or heuristics and rote learning, in the absence of understanding, does not constitute cognitive acquisition for satisfying level requirements. As

mentioned earlier, Fuys et al. (1988) found that learners can appear to function at higher levels by rote-learning rules and definitions and by applying routine heuristics that are not properly understood.

2.5.4 Critique of the van Hiele model

Even though van Hiele called for his model to be applied in other subjects (Fuys, 1988), the cognitive descriptors are based on Euclidean geometry and, as such, cannot be particularised for 3D geometry with clarity (Gutierrez, 1992). Some researchers believe that the nature of the pedagogical sequence is not clear. As the model is a suggested process and not a fixed formula, it is unclear if teachers have to go through each learning phase (Pegg, 1997; Clements, 2003; Ding & Jones, 2007). Hershkowitz (1998) questioned the relationship between the learning context and the development of mathematical reasoning. However, this uncertainty could stem from insufficient research on the van Hiele model (Ding & Jones, 2007). Although most researchers agree that class inclusion occurs on VH3, some confusion persists, especially since the van Hiele literature themselves perpetuate confusion and clear contradictions appear in some of van Hiele's statements (Fuys et al., 1988).

Burger and Shaughnessy (1986) have questioned the discreteness of the levels. Research seems to indicate that learners move from one level to another through a series of small steps instead of one big jump (Battista, 2007). Pusey (2003) questioned the discontinuity between levels as evidenced by the number of learners who find themselves in between levels. They appear to have acquired partial degrees of acquisition on two or more levels. Wang and Kinzel (2014) have recently reported similar findings. The extent of this problem prompted Battista (2007) to argue that it was impossible to assign a specific level of acquisition to a learner at any given time. Pegg (1997) became aware of this problem much earlier and attempted to remedy it by merging the van Hiele model with the SOLO Taxonomy. In an attempt to address the discontinuity, Battista (2007) fragmented the levels into sub-levels and similarly, Gutierrez et al. (1991) assigned four degrees of acquisition to specific levels. In contrast, Papademetri-Kachrimani (2012) does not favour a hierarchical model for describing geometrical thinking and calls on researchers to propose alternate models that move away from levels. Atebe and Schafer (2011) developed a checklist of learning phase descriptors and reported positive findings in support of the general van Hiele model. From research performed by Atebe (2008) with Nigerian and South African school children, Atebe and Schafer (2011) point out that many factors can lead to differential performances. Culture and social elements are such factors to consider, but the van Hiele literature provides little insight on such variables. In addition, it is also not clear how factors such as culture and social elements can contribute to differential results in the classroom. We should not lose focus of the fact that mathematics and its peripheral areas of interest are products of human activity and as such are shaped by social and cultural contexts (Hulme, 2012).

Clements (2003) found that younger learners functioned below the lowest level and could therefore not be described adequately, with the implication that no level could be assigned to them. This inadequacy resulted in a reformat of how to number the levels. The lowest level was designated as level 1, increasing to level 5. The number (0) was reserved for learners who did not reach level 1 (Steyn, 2016). A very significant concern amongst researchers revolves around the assessment of levels. It remains difficult to assess the cognitive processes even though much headway in this regard has been made (Steyn, 2016). Fuys et al. (1988) call for the development of "easy-to-use" diagnostic instruments that can reliably be used as baseline tests to determine the entry level of learners before they commence with a programme. I have questioned the reliability of the multiple-choice questions I used in this study, and Fuys et al. (1988, p. 187) warn against such tests by stating that answers to such questions may not "reflect a certain level hypothesised for that item"

Researchers such as Usiskin (1982) and Idris (2005) state that level 5 cannot be tested as it provides theoretical value only and falls outside the ambit of school geometry. Level 5 did not feature in my own study, but it may still be valuable for university-level geometry and/or EGD. Through the research of Fuys et al. (1988), it became evident that metacognition plays a large role in the development of geometry thinking. Although elements of metacognition seem to exist in van Hiele's writings, it is not conceptually foregrounded and provided for in the model. Fuys et al. (1988) recommend that metacognitive language should be made more explicit in how it is incorporated into the level descriptors.

2.6 Conceptual Framework

Underlying causal factors precipitating poor academic performance can be classified as either internal or external factors that affect reasoning. Educators do not have any control over the historical internal and external characteristics of new cohorts of students, which leaves only current external factors open for possible intervention. Teachers remain the most influential variables in the teaching and learning experience (Karakus & Peker, 2015). My response to the need for enhanced reasoning in EGD through this research effort was to search for an existing teaching and learning model that could be adapted to the EGD context.

My literature review includes relevant aspects related to three main constructs: VSR, AR, and CK. From this premise, I identified the van Hiele model as suitable for EGD due to its focus on the spatial aspects of the mathematical branch of geometry. According to Maxwell (2013, p. 51), "a conceptual framework for your research is something that is constructed, not found.

It incorporates pieces that are borrowed from elsewhere, but the structure, the overall coherence, is something that you build, not something that exists ready-made".

Having compared the ideas of Piaget, Vygotsky, Gagne, Bruner, Bloom and van Hiele, I conclude that the van Hiele model, in the main, is the most suitable fit for the scope of this study. The van Hiele model contains cognitive descriptors on five hierarchical levels that encompass the constructs of VSR most comprehensively. Because the model is designed for reasoning in geometry, it fits well with EGD where most activities revolve around geometrical constructions. However, it does lack in its capacity for 3D reasoning. I address this matter in the next section.

2.6.1 A composite conceptual framework

Having chosen the van Hiele model as this study's guiding theory, I made adaptations according to the scope of my research. The van Hiele model is based on Euclidean geometry (2D geometry) where much of EGD content involves drawings of both 2D and 3D objects. Although the van Hiele model is based on Euclidean geometry, Gutierrez (1992) demonstrated how a reinterpretation of the model can lead to the understanding and structuring of learning three-dimensional Geometry. Fuys et al. (1988) call for 3D research on the van Hiele model and encourage exploration of its useful fit for other subjects. Due to the fact that the van Hiele model contains terminology specific to 2D geometry, I searched for a way to particularise the cognitive descriptors of van Hiele for EGD by using a compatible framework containing cognitive descriptors that are 3D in nature. Newcombe and Shipley (2014) proposed a "New Typology" (Table 2.1) for framing spatial factors within the four dimensions of "extrinsic/intrinsic" and "static/dynamic" reasoning. The conceptual framework for this study is portrayed in Figure 2.5.

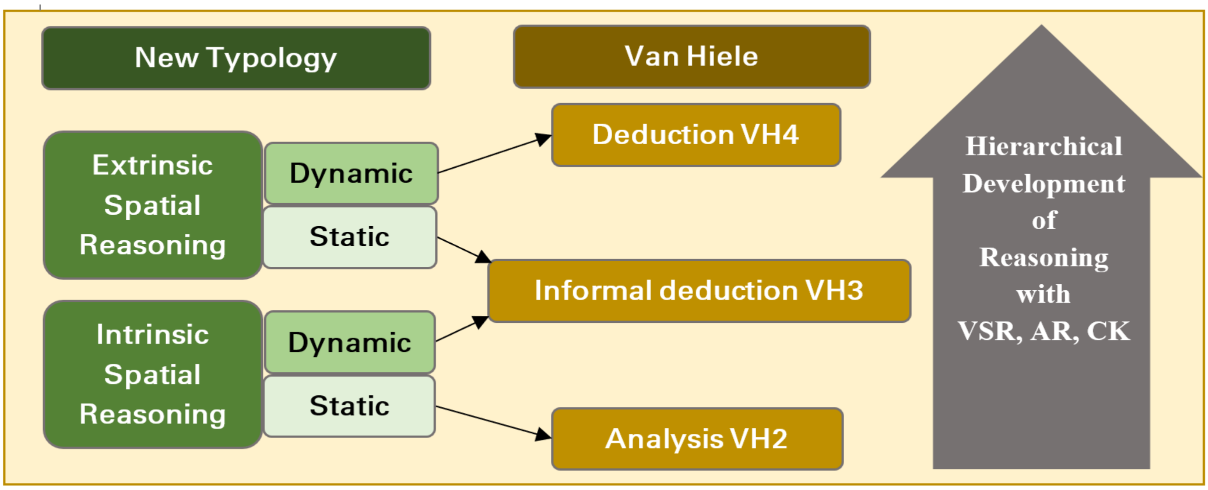


Figure 2.5: Conceptual framework: van Hiele (1957) and Newcombe and Shipley (2014)

Figure 2.5 contains elements of both the van Hiele model (1957) as well as the "New Typology" proposed by Newcombe and Shipley (2014). Recent studies employing factor analytic methods have indicated that the "New Typology" progresses hierarchically through cognitive development (Meinhardt et al., 2021; Jung et al., 2020). The cognitive levels that applied to this study included VH2, VH3, and VH4. As previously mentioned, VH1 and VH5 were excluded due to the cognitive nature of the two data collection instruments. From Table 2.1, the cognitive descriptors formulated by Newcombe and Shipley (2014) follow a parallel hierarchical progression with the van Hiele levels and are graphically portrayed in Figure 2.5. Table 2.2 shows the comparative cognitive descriptors for the van Hiele model and the New Typology of Newcombe and Shipley (2014).

2.6.2 Particularisation method

In Section 2.6.1, I demonstrated the composite structure of my conceptual framework as it was derived from the van Hiele model and the New Typology. I only looked at factors of VSR in the analysis of data in Chapter 4 in order to keep this manuscript compact, with sufficient evidence for the usefulness of the van Hiele model in EGD. Since Hoffer's (1981) initial attempt at characterising the van Hiele levels for 3D geometry, there remains a paucity of research in this regard. The reason for merging the New Typology with the van Hiele model was to add a layer of rigour to the model's potential to be used in an EGD context, as EGD contains both 2D geometry (Euclidean) and 3D geometry (orthographic, isometric and axonometric).

Table 2.2: Comparison of cognitive descriptors between van Hiele and the New Typology for VH2, VH3, and VH4

Van Hiele descriptors for geometry	Newcombe & Shipley: New Typology
VH2	**Intrinsic static**
1. Can differentiate between types of shapes 2. Classify types of shapes according to governing properties 3. Reasons inductively (informally) 4. Can recognise many properties of shapes but do not fully grasp the relationships between them	a) Coding the spatial properties of objects with reference to dimensions and configuration b) Classify objects according to their code c) Picking shapes out from overlapping objects or other perceptual information d) Identifying regions of space as constituting categories
VH3	**Intrinsic dynamic**
5. Recognise the importance of properties, the relationships between them. Can recognise a square as also being a rectangle by definition. 6. Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning. 7. Can order geometric properties and connect them deductively through logical arguments	e) Transformation of objects based on spatial information that were already encoded f) Visualise dimensional transformation, rotation, cross-sectioning, folding, bending, breaking and sliding g) Mentally track accumulating sequences of transformations and visualise change over time and an end product h) Transition between 2D and 3D views while maintaining relationships
	Extrinsic static
	i) Coding the spatial location of objects relative to other objects or to a reference frame j) Aligning location coding that differs in scale
VH4	**Extrinsic dynamic**
8. Grasps the significance of deduction. Can reason formally within the context of a mathematical system (axiomatic), complete with undefined terms, axioms, and underlying logical systems with definitions and theorems. 9. Properties can now be structured to derive further information from given data. Use logic more than intuition. 10. Manipulates intrinsic characteristics of relations	k) Transformation of the interrelations of objects when they move l) Transformation of the interrelations of objects when the viewer moves m) Maintain a stable representation of the world during navigation. n) Be able to take perspective of changes in objects during their motion and maintain their motion and own motion from a reference point

Research design and methodology Chapter 3

3.1 Overview of this chapter

This chapter commences with a description of this study's research paradigm in terms of ontology, epistemology, methodology and axiology. This is followed by a rationalisation of the research design, and how the participants were selected. I detail sources of data, instrument design, and how multiple strands of data were collected and analysed through four phases. Finally, details are provided on how the validity and reliability of the instruments were dealt with, and ethical considerations for this study are detailed. The chapter concludes with a brief summary of the remaining four chapters.

3.2 Research paradigm

A research paradigm constitutes a unique worldview or frame of reference which orientates research activities across a diverse spectrum in a logical and convincing manner (Willis et al., 2007). Research paradigms steer thoughts, intentions and actions according to individual world views and serve as a lens through which phenomena are studied (Mertens & Hesse-Biber, 2012; Mackenzie & Knipe, 2006). The interrelatedness of research variables peculiar to each research setting requires the adoption of fundamental assumptions germane to the nature of reality, its meaning and how it can be discovered (Willis et al., 2007). In my quest to study the nature of reality and what can be known about the problem in its social setting, I formulated a set of interrelated assumptions according to Ponterotto's (2005) suggestions, to best fit the pragmatic paradigm. The pragmatist approach allowed me to frequently alter my methods of data collection and analysis to answer my research questions.

3.2.1 Pragmatism

According to Creswell and Creswell (2018), world views are formed by factors such as discipline orientations, individual actions, situations, prior experiences, advisors and mentors, to name but a few. Guided by the pragmatist perspective of "what works" (Creswell & Creswell, 2018, p. 48; Kaushik & Walsh, 2019), I employed various methods by designing two research instruments to find solutions to the problem and research questions. Pragmatism removes the limitations of mono-methods and provides space for deep exploration and contextual understanding of the connections between knowledge and action (Biesta, 2010). Table 3.1 offers a comparison between the key elements of the pragmatic approach and the qualitative and quantitative mono-method approaches.

Table 3.1: The pragmatic research approach (Morgan, 2007, p. 71)

	Pragmatic approach	Qualitative approach	Quantitative approach
Connecting theory with data	Abductive	Inductive	Deductive
Nature of the research process	Intersubjective	Subjective	Objective
Data inferences	Transferability	Contextual	Generality

My endeavour to demonstrate how the van Hiele model could be applied in EGD was largely a conceptual effort, as I could not find similar studies to provide structure and referential value to my approach. My journey through the chapters of this study was one of constant transitioning between how I approached the data and theories, and the freedom of the pragmatic approach assisted me greatly. Researchers rely on a version of abductive reasoning that transitions between induction and deduction when the pragmatic approach is adopted (Morgan, 2007). Theories are initially generated from observations, whereafter they are assessed through appropriate actions. Morgan (2007) explains that abduction is commonly used in pragmatic reasoning by evaluating the ability of previous inductions to predict the feasibility of future trends of behaviour. Researchers such as Ivankova et al. (2006), who combine quantitative and qualitative methods sequentially, endorse the abductive process. They use the qualitative approach’s inductive results as input data for the quantitative approach’s deductive goals and vice versa. The differences between induction and deduction, as portrayed in Table 3.1, are key to distinguishing qualitative and quantitative research methodology, yet the process of navigating between data and theory is never unidirectional (Morgan, 2007). Pragmatic researchers do not embrace a purely data-driven or theory-driven approach in their research designs. Instead of questioning the validity and intrinsic value of various methodologies, the pragmatist predicates prerogatives on the significance of methods to transition between practice and theory (Kelemen & Rumens, 2012). Kelly and Cordeiro (2020) agree with this sentiment and advise that complex study contexts require investigations of multiple perspectives by developing variance into techniques and analytical proposals.

As suggested by Arnett et al. (2020) regarding constraints imposed by real-life situations, my study followed a mixed-methods approach based on my logical choices of "what works" (Creswell & Plano Clark, 2007, p. 24). The pragmatic approach embraces multiple worldviews (Creswell & Cresswell, 2018), which afforded me various options to understand how to best apply the particularisation process in answering my research questions. As intimated by Bazeley (2004), mixing methods can be challenging as the researcher has to understand the varying paradigmatic assumptions and procedures of analysis and acquire an appropriate working knowledge of multiple methods.

I strove to provide the reader with unambiguous details pertaining to the manner in which I mixed quantitative and qualitative data. This case study was based on a pragmatic paradigm which defined and oriented the study by way of adopting assumptions along four core philosophical dimensions. These are the dimensions of axiology, ontology, epistemology and methodology (Mertens & Wilson, 2012; Terre Blanche & Durrheim, 2006).

3.2.2 Ontological assumptions

What individuals believe about reality determines what they deduce as legitimate knowledge and how it can be obtained, which consequently defines their approach to scientific investigation and the sequence in which research techniques are applied (Guba & Lincoln, 1994). I.e., ontology informs one's epistemology, which leads to methodological choices. The participants in this study do not perceive reality in the same manner as me. In keeping with the suggestions of Maarouf (2019), I remained sensitised to the disparities in people's perceptions of reality, when I switched between objectivity and subjectivity as I applied different data collection and analysis methods. In searching for the truth and being aware of my own natural proclivities towards data analyses, I remained respectful of participants' perceptions during qualitative analysis and was objective about quantitative analysis. My ontological assumptions played an important role in delineating the research sample as participants had to conform to certain academic and experiential criteria.

Specific participant criteria were necessary for the data to be realistic and in accordance with the cognitive demands of solid geometry. Selecting formally educated participants for this study was an important prerequisite, as a certain "reality" had to be shared by all the participants. That non-negotiable "reality" of participants' being formally trained in EGD and possessing teaching experience in EGD did not remove their freedom to express their opinions on cognitive involvement in solid geometry. During the qualitative phases of this study, I remained cognisant that only one reality could contextually exist, and that participants would reveal multiple perceptions of this reality without my own construction of reality interfering with the process (Maarouf, 2019).

3.2.3 Epistemological assumptions

Epistemology is the theory of knowledge with reference to how and through which sources knowledge is gathered (Maarouf, 2019; Al-Ababaneh, 2020). My view of the world and of knowledge influences my interpretation of data, and I accept that knowledge may be both observable and unobservable. My understanding of how knowledge is embedded in theoretical perspectives afforded me the freedom to choose methods that were most appropriate to my ontological position and best served my research objectives. My epistemology and ontology are closely linked, and this allowed me to deduce thin, *a-priori* insights from quantitative analysis with the administration of Instrument 1 (Appendix A). During the abductive process of switching between objectivity and subjectivity and with the administration of Instrument 2 (Appendix B), I made thick, *posteriori* inferences that modified my initial understanding of the phenomenon. Hypotheses were formed before the collection of data and were tested and justified by using statistical methods. Qualitative methods were used later in the study to explain convergences and divergences in the data.

3.2.4 Axiological assumptions

I acknowledge that it is impossible to be either value-free or bias-free due to my prior knowledge and experience, and this may have affected my choice of research questions, objectives, data collection, analysis, and interpretation (Maarouf, 2019). Having been aware of my perceptions and predispositions toward the study's topic, I embraced the broad scope of mixed datasets and sought to add more insights to the multifaceted research problem. Being sensitive to the principles of justice, beneficence, and respect, I strove to minimise risk to anyone and produce favourable outcomes for the participants and communities of interest. I remained respectful and courteous to all people irrespective of social class and the possible power differential in my research setting. I remained sensitive for participants to be beneficiaries in the face of possible risks by following non-exploitative procedures.

3.3 Research design and methodological strategies

In this section, I present my research design within the context of my research questions and explain how I went about delineating participant characteristics and sampling procedures.

3.3.1 Research design: A sequential-explanatory mixed-methods design

Durrheim (2006) explains how research questions can be linked with strategies of data collection, documentation and analysis through strategic action plans according to a research design. I designed a mixed-methods, single-case quasi-experiment as endorsed by Yin (2013) and collected both quantitative and qualitative data through two instruments. I favoured the case-study design as it can provide thick, detailed descriptions of social phenomena in real-world contexts (Yin, 2013; du Plooy-Cilliers & Cronje, 2014). In addition, this design afforded me a variety of detailed data and documentation procedures (Botma et al., 2010) which I employed over an eight-month period. Creswell and Plano Clark (2007) suggest that mixed-methods research designs could effectively ameliorate the weaknesses of both quantitative and qualitative research.

The present study included secondary EGD school teachers who chose to participate in a re-skilling programme regarding conceptual deficiencies in solid geometry. My purpose with the intervention programme was to gauge the pre- and post-intervention performance of the participants in the case and to evaluate their transfer of learning at the conclusion of the programme. My data collection strategy was fashioned around the requirements of performing traditional rubric-type assessments of drawings as well as cognitive descriptors pertaining to the particularised van Hiele model.

3.3.2 Mixed-methods approach

Figure 3.1 depicts my mixed-methods approach over time and offers a diagrammatic view of enquiry based on the research questions and data collection instruments. See Appendix C, which provides an alignment between the research questions and the phased data collection and analysis.

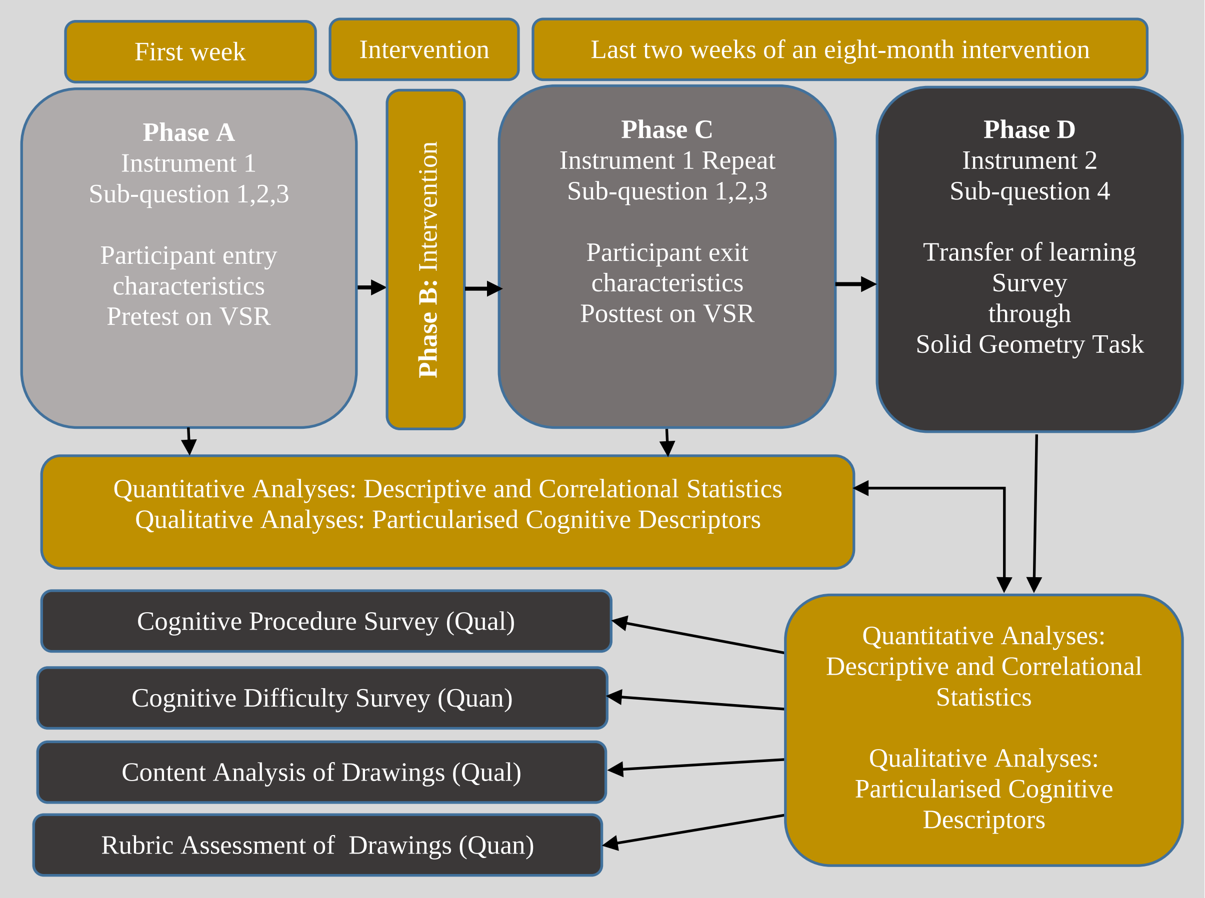


Figure 3.1: Strategy for administering the data collection instruments

With Instrument 1 (Appendix A), I collected quantitative data by conducting a pretest/intervention/posttest (Phases A, B, C) and used IBM's Statistical Package for the Social Sciences (SPSS), (version 28.0.0.1.0 (142)) to describe participants' VSR statistically.

By applying the particularisation process of the van Hiele model to the 12 VSR question items of the pretest-posttest, a parallel qualitative data strand was added to the original quantitative dataset. This strategy allowed me to answer the first three research questions by comparing traditional test scores with the particularised van Hiele model. By mixing methods in this manner, I was able to illustrate the relationship between EGD and the van Hiele model. At the conclusion of the intervention (eight months later) during Phase D, I administered a solid geometry test (Instrument 2) consisting of six drawing questions and a cognitive difficulty survey. The cognitive difficulty survey consisted of a perceived cognitive difficulty component and a cognitive procedure component. My intention was for participants to write down their reasoning steps in a procedural format, evaluate the difficulty of each reasoning/procedural step and rate the perceived difficulty on a nine-point Likert-type scale. Two strands of quantitative data were obtained by applying rubric scores to drawings and quantifying the cognitive difficulty survey. Through the cognitive difficulty survey and by applying the particularisation process to the quantitative rubric data, two additional qualitative strands of data were added. Content analysis was performed on the solid geometry drawing task, which yielded an additional qualitative data strand at the conclusion of Phase D. The data collected during Phase D enabled me to answer the fourth sub-question in terms of persistent reasoning deficiencies.

3.3.3 Overview of the research process

The Van Hiele model was designed to classify and separate the various cognitive processes at work when solving geometry problems. In its particularised format for EGD, it could offer practitioners insight into their VSR competence and accentuate areas of deficiency. Apart from well-developed AR, success in EGD relies predominantly on VSR and the correct application of CK (Metraglia et al., 2015; Tumkor & deVries, 2015; Villa et al., 2018). For this study, it would have been possible to simply apply existing VSR tests, such as the Purdue battery of spatial tests (Bodner & Guay, 1997) and use performance scores to determine participants' varying competence. However, I did not intend to simply determine performance scores but rather to determine participants' performance against specific cognitive reasoning metrics according to EGD content. All question items were based on the CAPS content for EGD to align with what the DBE prescribes. The question items chosen for data collection informed on the VSR, AR, and CK of participants and yielded insights into areas where high acquisition and low acquisition of cognitive skills were achieved. Yet, only VSR data was used in this study to demonstrate the usefulness of the van Hiele model. The Van Hiele model offered specific cognitive metrics that made it possible to translate performance scores into rich narrative descriptions of cognitive reasoning deficiencies.

3.4 Population and sample selection

3.4.1 Criteria for sample selection

Participants were selected according to three criterion stipulations: (1) participants had to be formally trained as teachers; (2) be trained in the teaching of EGD; and (3) they had to have current experience in teaching EGD. With the assistance of two subject advisors from the GDE, it was possible to proceed with a small sample (n=38) from the population of EGD high school teachers residing in Gauteng, South Africa. I followed the five principles suggested by Mertens and Wilson (2012) in selecting suitable participants for data collection.

1. EGD teachers within the experimentally accessible target population of high schools in Gauteng were invited to attend an EGD intervention programme. The focus of re-skilling was on VSR, AR and CK, in accordance with the CAPS (DBE, 2011a) for EGD.
2. A total of 38 EGD teachers registered for the intervention programme, which started in March 2018 and ended in October 2018. During the course of the programme, three teachers discontinued their participation, and another five did not fulfil the requirements of Instrument 2. Complete datasets were thus collected from thirty teachers for data analysis and interpretation.
3. In complying with the ethical requirements of anonymity, participants refrained from identifying themselves on any of the data collection forms and were assigned numbers as identifiers.
4. All participants received the same intervention treatment by the same trainer over the eight-month period, which comprised an equal mix of contact sessions and virtual/online sessions, and all participants received the same set of learning materials.
5. All participants received the same pretest-posttest, solid geometry task, cognitive survey, and cognitive difficulty survey.

The sample was diverse in terms of race, ethnicity, culture, socio-economic status, education, experience, age and gender. It seems that the research sample presented a cross-section of the average EGD school teacher in South Africa. Even though the sample was small, the generalisability of the data could prove useful in extrapolating the findings to a larger population of schoolteachers that adhere to the stipulated sample requirements.

3.4.2 The sample profile

The sample of 38 participants taking part in this study was purposively selected by two EGD subject advisors employed by the GDE towards re-skilling secondary school EGD teachers. The sample of teachers comprised 19 males and 11 females with an average of 15 years of teaching experience. Fourteen of the participants spoke Afrikaans as their mother tongue and the other 16 participants spoke one of the official African languages as their mother tongue. The average teaching years for males were 16 years and for females, 14 years.

3.5 Instruments, data collection and analysis

This study's main question was: *How can a particularised version of the van Hiele model of geometric reasoning for EGD provide a deep understanding of conceptual deficiencies in cognitive reasoning?* Four sub-questions were formulated to answer the main question using quantitative and qualitative research. I posed the following four sub-questions as a means to answer the main research question.

1. How can the van Hiele model relate to EGD?
2. How can student performance in VSR be described in terms of the hierarchical levels of the van Hiele model?
3. How can student performance in VSR be described in terms of the van Hiele descriptors?
4. How can the van Hiele model allow for the identification of deficiencies in conceptual understanding?

An alignment document based on the four sub-questions is available as Appendix C, detailing how data for the study's four phases (A, B, C and D) were sourced and the types of data analyses that were used.

As mentioned before, success in EGD is predominately reliant on VSR, AR and CK. However, only VSR data was used to demonstrate the usefulness of the van Hiele model in EGD. The choice to set aside CK and AR data was purely pragmatic in order to provide a focused demonstration of the usefulness of the van Hiele model. The particularised levels of the van Hiele model were used in a quasi-experiment by way of a pretest – intervention - posttest design (Instrument 1). Instrument 2 was administered with the transfer of learning in focus. The two instruments are described in Sections 3.5.1 and 3.5.2 to explain how the research questions were addressed.

3.5.1 Instrument 1 (Quasi-experimental Pretest-Posttest)

In this section, and with reference to Figure 3.1, Phases A, B, and C of the quasi-experimental part of this study are explained. The test answers were scored using standard EGD assessment strategies (rubrics) and then compared with the van Hiele level ratings and associative cognitive descriptors per level. The pretest-posttest was designed by choosing 12 exercise items from the Grade 10, 11 and 12 CAPS syllabi, and they were divided in the following manner: four questions on VSR requiring drawing solutions and eight questions on VSR requiring mental solving.

3.5.1.1 Data collection and documentation for Instrument 1

Quantitative data were collected by administering the test of Instrument 1 (Appendix A) to 38 participants. Each question carried its own quantitative weight as indicated by the marks allocated to the question and was assessed and scored accordingly and then transmuted to a percentage figure. Appendix D contains the captured data for the 12 VSR questions for Instrument 1 (Appendix A).

3.5.1.2 Data analysis for Instrument 1

The cognitive descriptors assigned to each VH level are comprehensively discussed according to Table 4.2 (Appendix E). Qualitative findings were based on the quantitative results and were expressed through the VH rating per question item. Based on my research design and methodological approach, I followed a sequence whereby I first collected the data, and then analysed it. The analysis of Instrument 1's quantitative data was instrumental in the design, collection and analysis of the quantitative and qualitative data of Instrument 2, and in keeping with Creswell & Plano Clark's (2007) guidance, the latter supplemented the former.

I analysed and assigned appropriate VH levels to the 12 VSR questions. While sub-question 1 was conceptually answered by comparing the van Hiele cognitive descriptors with EGD's cognitive descriptors, sub-questions 2 and 3 were qualitatively informed by comparing the quantitative test-item scores to the particularised cognitive descriptors as hierarchical instances of VH2, VH3, or VH4. VH1 and VH5 levels did not apply to this study. By mixing quantitative and qualitative data in this method of analysis, I was able to demonstrate the usefulness of particularising van Hiele's cognitive descriptors towards EGD. Note: While question items carry a VH rating that may seem fixed and in absolute terms, I acknowledge that the cognition of lower VH levels are subsumed and form part of one's reasoning. In other words, if a question item is say rated as VH3, the cognitive descriptors of VH3 are used to describe that level of reasoning even though cognitive descriptors of VH1 and VH2 are also part of the reasoning process.

Quantitative analysis

Each participant's test was scored according to a test memorandum (Appendix A). The scores of the 12 VSR questions were transmuted to percentage values for each individual participant. Averages per test item per participant were computed; the total average for VSR was respectively computed per participant; and the total average for the whole test was computed.

The same averages were also computed for the sample as a whole and applied to both the pretest and posttest. All these values were captured in one Microsoft Excel dataset, and a record of this data set is available as Appendix D. The data was subsequently exported to SPSS and appropriately captured according to variable views and data views. Statistical procedures were carried out with the assistance of Professor M Graham from the Department of Mathematics, Science and Technology Education of the University of Pretoria. Descriptive statistics were carried out to answer sub-question 1 of this study: *"How can the van Hiele model relate to EGD?"*

Bearing in mind the small sample (n=30), histograms and the outcome of the Shapiro-Wilk test (p > .05), non-parametric statistics were used throughout this study. Comparisons were also made to seek relationships between variables such as individual test-item scores and teaching experience using Spearman's rank-order correlations. Charts generated by Excel were used throughout as graphical representations of statistical findings. A Cronbach Alpha reliability test was carried out on the instruments, and a Wilcoxon signed rank test was used to describe the significance of scores before and after the intervention.

Qualitative analysis

Sub-question 2 guided the procedures of analysis for the qualitative component of Instrument 1, and was stated as follows: *"How can student performance in VSR be described in terms of the van Hiele descriptors?"*

From the wording of this question, it is clear that the quantitative values of the previous section played a role in the qualitative procedures that followed. Each of the 12 VSR test items was particularised and assigned a VH level (VH2, VH3 or VH4) according to the procedure explained in Section 2.7.1 of Chapter 2.

The process of answering sub-question 2 was carried out by applying the cognitive descriptors in Table 4.2 (Appendix E) per question item to explain how the implied cognitive reasoning levels were applied by participants. This process made it possible to determine the degree to which reasoning levels had been acquired. Note: I acknowledge that each progressive cognitive reasoning level subsumes the cognition of levels below them. However, discussions are based on the highest rating per question item. I.e. if a question item is rated as VH3, the cognitive descriptors for VH1 and VH2 are not included in the discussion as the objective is to isolate the cognitive descriptors per type and report systematically on each one. In addition, some question items contain specific cognitive descriptors related to the drawing process where others may lean more towards mental activities. By their definitional nature, the 10 cognitive descriptors of Table 4.2 lie on a continuum of inductive/deductive reasoning with

VSR factors increasing progressively in intensity across the VH spectrum. VH3 can be seen as a pivot point that separates inductive reasoning from deductive reasoning but not in absolute terms as people do not reason alike. True to its description of “Informal deduction”, VH3 marks the development stage where mechanical executions of pencil and paper drawings gradually become more mentally demanding especially in the sense of increased VSR. From Table 4.2, the expanded descriptors for VH3 are clearly more VSR intensive.

3.5.2 Instrument 2: Solid geometry task and cognitive survey

Instrument 2 (Phase D) was designed in response to the analysis and findings of the quantitative and qualitative data of Instrument 1 and is available as Appendix B. Instrument 2 data were used to answer the fourth sub-question: *“How can the van Hiele model allow for the identification of deficiencies in conceptual understanding?”*

Instrument 2 formed the mechanism to provide additional information on the transfer of learning that goes beyond the findings of Instrument 1. Quasi-experiments in the form of pretests and posttests are mostly useful in arguing the merits of a programme (Cohen et al., 2018). However, diagnostic tests such as Instrument 1, as performed in this study, may highlight problem areas within the boundaries of the test instruments’ scope but may not be able to provide evidence for the transfer of learning (Caffarella & Daffron, 2013; Cohen et al., 2018). Application of what was learnt can only be demonstrated where the task is divorced from the boundedness of the pretest-posttest yet still representative of all the original elements contained in the intervention (Caffarella & Daffron, 2013; Kirkpatrick, 2009).

A note to the reader: Please refer to Appendix B and in particular to the detailed instructions that participants were required to follow. Participants were required to answer the six questions by physically drawing the solutions on paper with precision drawing instruments over a two-week period. Concurrently, participants were required to describe their cognitive involvement with the task by listing the steps taken in the solution in fine detail and rating each cognitive step against its perceived cognitive difficulty on a nine-point scale. By allowing ample time to complete the task and reason it out, most participants produced fairly accurate and complete drawings as per the task instructions. Each participant’s cognitive steps were compared against a Cognition Memorandum which represented the complete and ideal cognitive journey through the six questions. The Cognition Memorandum is presented in Appendix F, and its formation is detailed in the next section.

3.5.2.1 Data collection and documentation for Instrument 2

Participants were provided with a form to record their cognitive reasoning steps (Appendix G). Participants were required to label each sub-section according to the way the six questions were posed, as exemplified in Appendix G. Underneath the column denoted as “Cognitive Step”, they had to list every step taken in minute detail while drawing the solution conventionally with pencil and paper. To the right of each step was space provided on a nine-point scale and divided into three groups of “Low order”, “Mid order”, and “High order”. Within each of those three categories of cognitive difficulty, three additional sections of refinement were added for making a choice about the perceived difficulty of a step. I provided these choices as most teachers are familiar with the cognitive orders of Bloom’s taxonomy (1956), and the CAPS for EGD requires school-based-assessment (SBA) to be structured in accordance with Bloom’s hierarchy of cognitive difficulty. For example, if a participant was unsure about whether a step was low-order or medium-order, they could rate it as 3 or 4, which makes for a refined choice. The same could be reasoned for choices between mid-order and high-order, where 6 or 7 would make choices easier.

I obtained the academic acumen of three EGD colleagues and asked each one to perform the same process of drawing the six solutions, listing the cognitive steps in minute detail and rating their perceived cognitive difficulty. Any valid cognitive steps that were not yet part of the Cognition Memorandum were subsequently coded and added to the existing themes. With reference to Appendix G, the column on the far left (VH level) was added at the conclusion of the Cognition Memorandum for the sake of particularising each cognitive step.

Participants were afforded two weeks to complete this task. It was imperative for them to draw the six solutions accurately and completely and, whilst drawing, to think about and list their cognitive steps in minute detail. In addition, participants were required to rate the perceived difficulty of each thinking/reasoning step on a nine-point scale. When the drawing tasks and accompanying surveys were returned to me after two weeks, and following the interpretivist philosophy (Niewenhuis, 2010b), I immediately started with the process of thematic inductive content analysis. This process started with the organisational principles suggested by Cohen et al. (2007) in noting patterns, themes, similarities and regularities. This process helped me to account for all datasets with a view to understanding participants’ definitions of reasoning steps according to their intended meanings.

Being guided by the rules of inferences as described by Terre Blanche et al. (2006), I followed the principles of inferring general rules from specific instances to identify themes and related sub-themes. I studied each participant’s data from different perspectives to isolate different meanings in the text that could lead to a deeper understanding and interpretation of the raw data (Niewenhuis, 2010b). Each participant's cognitive survey form was read carefully, and their steps were added to the Cognition Memorandum under a cognitive description where it belonged. As I read and re-read the documents, the steps contained in the Cognition Memorandum gradually grew as the participants varied quite significantly in their descriptions of their cognitive reasoning. Yet, their intended meanings were obvious, and I had no difficulty in categorising them under emerging themes. The process that I followed is described in greater detail in Table 3.2, as proposed by Braun and Clarke (2006).

Table 3.2: Steps in thematic analysis (Braun & Clarke, 2006, p.87)

	Stages	Process description
1	Become familiar with the data.	Transcribe data, read/re-read the data, and identify emerging constructs. Glance over the drawings.
2	Create initial codes	Code and arrange intriguing elements of the data methodically.
3	Identify themes	Arrange codes into possible themes.
4	Review themes	Test themes accordant with coded excerpts. Thematic map of analysis.
5	Define themes with appropriate titles.	Refine the specifics of each theme whilst preserving the central emerging story. Clearly define and entitle each theme.
6	Create Cognition Memorandum	Final analysis of rich, convincing excerpts. Align analyses with the research question and literature.

My procedure aligned with Niewenhuis's (2010a) suggestion in that I first coded the data by reading/re-reading the text data (cognitive steps) whilst being cognisant of the visual data (drawings). After the coding process, and having had to recode at times, I classified the data into meaningful analytical units and established themes through inductive reasoning. As I started to approach saturation with no new themes emerging, emergent themes were evaluated by checking the correctness of the data capturing process. Following Niewenhuis's (2010b) advice, I searched for gaps in the data to possibly revisit and collect additional data. Of the 38 original participants, 30 managed to complete the task and return them on time. Three participants discontinued their participation towards the middle of the intervention for personal reasons and another five did not fulfil the requirements of Instrument 2. The 30 datasets that were used were complete, obviating the need to revisit the participants for clarification or additional information. As thematic analyses result in an organised amalgamation of the data set, it can highlight convergences and divergences across datasets, and provide unexpected revelations. Such instances are discussed in Chapter 5. Thematic analysis could prove to have limited explanatory power and be reduced to descriptive utility should the conceptual framework not underpin the assertions of analysis that were made (Braun & Clarke, 2006). I foregrounded the cognitive descriptors in my conceptual framework (van Hiele model) to negate such possibilities and demonstrate the usefulness of van Hiele's cognitive descriptors.

By the time the Cognition Memorandum reached its final state of 68 cognitive steps, it had proved to be a very comprehensive and ideal document for describing the collective cognitive reasoning that was followed in drawing the six solutions of the solid geometry task. I relied on the experience of three EGD colleagues to help me categorise certain cognitive steps that were perhaps ambiguous or where a single statement included more than one cognitive type. In cases where more than one cognitive step was mentioned in the same statement, they were separated, coded and added to the corresponding cognitive type under an appropriate theme.

After completion of the thematic and content analysis process of the cognitive survey, only three themes were identified and proved to support the literature perfectly. The three identified themes were Analysis (AR), Visualisation (VSR), and Convention knowledge (CK). A total of 68 cognitive steps were recorded across the six questions and were suitably grouped together under each of the three themes. Instrument 2 provided four strands of data which are detailed in the alignment document (Appendix C).

3.5.2.2 Data analysis for Instrument 2

With Instrument 2, quantitative and qualitative data were collected in four data strands by means of a solid geometry task and accompanied by a cognitive survey. The four data strands are: *Rubric assessment of drawings; Cognitive difficulty survey; Content analysis of drawings; Cognitive steps survey*.

Quantitative analysis

Data strand 1: Rubric-item assessment of drawings

The solid geometry task was assessed according to the rubric in Appendix H. Rubric items 1 and 2 counted five marks each and applied to all six questions. Rubric items 3, 4 and 6 counted five marks each and applied to Question A only. Rubric items 5, 7, 8 and 9 counted five marks each and applied to all six questions, which accounted for 30 marks per item. Rubric item 10 counted five marks and only applied to Question F. Rubric item 11 counted five marks and applied to all six questions, which accounted for a total of 30 marks for this item. All scores were later transmuted as percentage figures for all items. Each rubric item was expressed and tabled as a percentage value per participant with a final percentage value for the whole drawing task. The quantitative dataset for Instrument 2 can be found in Appendix L. SPSS was used to produce descriptive statistics, frequencies, charts, and correlations.

The rubric item “Precision” was not assigned a VH level, as the precision with which the EGD practitioner draws is a product of most of the foregoing rubric items, drawing instrument quality and dexterity of hand. Precision is usually assessed by content analysis of the physical drawings with the help of a transparent memorandum of the drawing solution. By overlaying the transparency on top of the participants' drawings, imperfections in the drawing technique can easily be detected. Precision is not necessarily linked to conceptual competence or cognitive acquisition of skills, but rather refers to the participants' fine motor skills and dexterity of hand. Precision can, however be adversely affected by the lack of CK elements such as reference points, annotation system and projection system.

Data strand 2: Cognitive difficulty survey

The cognitive reasoning/procedure survey exemplified by Appendix G produced quantitative and qualitative data strands. Participants listed their cognitive steps and assigned a perceived cognitive difficulty rating to each step on a scale of 1 to 9. These nine levels were later reduced to three levels of High Order, Mid Order and Low Order. With reference to Appendix K, this qualitative section of Instrument 2 was quantified and where participants neglected to mention a step, the prefix "NR" (No Response) was tabled in the place of a cognitive rating. For correlational purposes, Mid Order was excluded as very few participants rated cognitive difficulty as Mid Order. Low Order was excluded in favour of including High Order as the focus was on cognitive steps that were perceived as high in cognitive difficulty, which may possibly refer to the poor acquisition of cognitive skills.

Qualitative analysis

Data strand 3: Content analysis of drawings

The complete drawing memorandum is available as part of Appendix B. Although the tasks were quantitatively assessed, they were also qualitatively assessed by visual inspection (content analysis) to provide information on nuanced aspects of the drawing and how it was executed. From my experience in teaching and assessing EGD, I propose that the following nuanced aspects can include:

- Light construction lines are indicative of pre-planning.
- Erased lines leave traces of error behind and inform on reasoning processes.
- Geometrical constructions can be evaluated against the pinpricks made by compass work and the dexterous manner in which instruments such as compasses were handled.
- Planning, spacing and positioning of views can only be assessed by visual inspection.
- Overall neatness and mentality with regard to the precision nature of EGD can be determined by content analysis.
- Common errors can easily be traced to their origin by analysing the matrix of light projecting lines to multiple views.

By analysing the content of participants' drawings, I was able to trace and link features in drawings to other strands of the collected data. For example: Where participants were negligent in mentioning the step "Analyse and Plan" in their cognitive survey, analysis of their drawing content affirmed their inadequate acquisition of this cognitive skill. This process was helpful in performing the triangulation of data.

Data strand 4: Analysis of Cognitive survey

Section 3.5.2.1 contains an explanation of the inductive thematic content analysis process that was followed for Instrument 2. Appendix G contains the cognitive survey form participants used to record their detailed cognitive steps in solving the six questions. It also includes a column on the left indicating the VH level per step and columns on the right for rating each step according to its perceived cognitive difficulty on a 9-point rating scale. The final Cognition Memorandum consisted of 68 steps (Appendix F). Each of the six questions contained duplications of some of the steps where certain cognitive reasoning steps were similar or identical. The Cognition Memorandum of 68 steps was essential in describing the cognitive behaviour espoused by participants. I was able to demonstrate the usefulness of the particularised framework in its ability to provide detailed descriptions of instances where cognitive reasoning was lacking or where reasoning per VH level had not been acquired. The Cognition Memorandum was used to determine the degree of thoroughness by which the participants listed the 68 cognitive steps. From the documented "NR" values, I was able to qualitatively describe the cognitive steps that were omitted, and simultaneously support my descriptions with quantitative frequency counts.

3.6 Quality Assurance: Quantitative

Koonin (2014) suggests that the principles of validity and reliability in research can be equated with the way humans develop trust in each other. Quantitative research employs the terms "validity" and "reliability" to describe the quality of findings. Establishing quality in mixed-methods research can be fraught with challenges because of the intentional blending of quantitative and qualitative methods as a means to derive credible meta-inferences (Ivankova, 2014). Meta-inferences are conclusionary statements of integrating the inferences that were obtained from mixing the quantitative and qualitative strands of a study (Teddlie & Tashakkori, 2010). Hence, during the integration of deductive and inductive inferences in this mixed-methods study, I adhered to the rigorous quality standards of validity and reliability to ensure the highest possible quality output for such inferences.

3.6.1 Reliability of quantitative data

Reliability refers to how dependable research instruments are in producing similar results over repeated administrations (Durrheim & Painter, 2006). The content of Instrument 1 was derived from educational material such as textbooks and previous exams and in accordance with the CAPS for EGD (DoBE, 2011a). The content was selected across Grades 10, 11 and 12 syllabi for EGD. The reliability of a test is determined as a coefficient which can be any value between 0 and 1 (Van Zyl, 2012). Tests are deemed reliable when the differential in test-takers' scores in repeated administrations of the test is small (Pietersen & Maree, 2010). The pretest/posttest was administered to 119 students of EGD over a four-year period. The test consisted of twenty items and the value for Cronbach's Alpha for the test was $\alpha = 7$.

To ensure that the instruments were valid and remained true to the VH hierarchical system of assessment, the cognitive descriptors defining the differences in the VH levels were particularised to the cognitive thinking in solid geometry. First, the literature on cognitive descriptors per level was consulted, and the five levels were tabled with a comprehensive list of descriptors per level (Table 4.2, Appendix E). Subsequently, solid geometry problems in EGD were analysed for the types of reasoning that were associated with them and these reasoning types were then matched (particularised) with the closest van Hiele descriptor over the five hierarchical levels. Three EGD colleagues were involved in verifying whether they agreed with the correctness of the particularisation process and outcome. Some disagreements were voiced with regard to border cases, i.e., whether a descriptor suited a higher or lower level better. At the end of the process, consensus on all the cognitive descriptors was reached and the particularisation process was accepted as a true reflection of the original van Hiele intention. Note: Only VH2, VH3, and VH4 are featured in this study. As previously mentioned, the cognitive descriptors for VH1 and VH5 did not align with the content of the two data collection instruments.

3.6.2 Validity of quantitative data

3.6.2.1 Construct validity:

Construct validity relates to how accurately constructs are defined and whether data collection instruments actually measure the construct they are designed to measure (Higgins & Straub, 2006). Validity is not a property of the of data collection instrument itself but implies that inferences drawn about test scores must be justified by the congruence between the concept being studied and the tests' intentions (Heale & Twycross, 2015; Kimberlin & Winterstein, 2008). My intentions with the two data collection instruments were to focus on VSR constructs to demonstrate the van Hiele models' usefulness in another subject. The conceptual framework used for this study (the van Hiele model) was initially designed for scaffolding geometry learning, and as such, the constructs of VSR, AR, and operational rules (CK) are sufficiently described by established cognitive descriptors. This study was firmly set in the everyday teaching and learning experience of EGD within the parameters of the CAPS content, where VSR, AR and CK were interwoven throughout. The two research instruments were embedded within this context and the test items were derived from participants' everyday work activities as prescribed by the CAPS. Kimberlin and Winterstein (2008) stress the importance of translating constructs into operational definitions that are directly linked to a conceptual framework. The strategy I employed to particularise the van Hiele model to EGD supports construct validity as the three constructs are firmly entrenched in operational definitions inherent to the theoretical framework of the van Hiele model.

With regard to inferences made, descriptive statistics cannot infer or predict and should strictly report on what was found in a variety of ways (Kimberlin & Winterstein, 2008). Descriptive statistics did answer certain aspects of my research questions, where I described participants' performance characteristics from a quantitative viewpoint (Cohen et al., 2018). Thoughtful rendering of descriptive data can add considerable value, but inferential statistics are often more valuable by virtue of its advanced mathematical power in making inferences and predictions on quantitative datasets. Inferential statistics were used to explore relationships between variables as well as to render verdicts on the significance of the instruments that were applied (Cohen et al., 2018). A professional statistician was consulted to assist in validating interpretations of the numbers, and consequently, I am optimistic that the findings are representative of the best possible truth.

Construct validity follows a protracted process of multiple studies and repeated measurements over time for revising the meaning and indicators of constructs. Higgins and Straub (2006) state that when the measured relationships of a theoretical framework's concepts are consistent, and a degree of validity and reliability is present, construct validity is enhanced. As mentioned before, all test items were derived from the CAPS content where such content is a historically well-established representation of cognitive involvement with EGD. In addition, the van Hiele model and its associated cognitive descriptors are considered to accurately represent a wide scope of cognitive involvement with geometry and have been validated by researchers all over the world (de Villiers, 2010). To the best of my knowledge, the van Hiele model has never been particularised for EGD before, and therefore, the methodology and methods applied during this study cannot be supported by previous attempts at similar studies.

Construct validity of a study's outcome measures may be compromised because the same individual who administered the intervention also administered the outcome measures; they may have unintentionally behaved in a way that made the intervention results look favourable (Shadish et al., 2002). The purpose of this study was on the particularisation of cognitive descriptors from the van Hiele model to the subject of EGD, and not on programme evaluation. Judgements about the effectiveness of the programme were not the focus, although the particularisation process did shed light on intervention shortcomings. The constructs of VSR that were central to this study's inquiry are well defined in the literature review (Chapter 2) and the particularisation process was closely moulded around those definitions to remain true to the principles of construct validity (Cohen et al., 2018).

As a final measure to enhance construct validity, I relied on the expertise of three EGD colleagues to scrutinise my two instruments in relation to the purpose of my study. No conflict between the items of both instruments, the conceptual framework and the intentions of the instruments were reported.

3.6.2.2 Internal validity

Internal validity is a quality assurance mechanism that focuses on the truth value of causal claims (Bartels, Hastie & Urminsky, 2018). Both internal and external validity are essential for research, yet they are often found to be inversely related (Bartels et al., 2018). Increases in internal validity tend to lead to a decrease in external validity, and vice versa. Attempts to maximise both internal and external validity in any study could lead to inevitable compromises. Pretests serve as baseline evaluations against which one can measure the effectiveness of an intervention programme (Fitzpatrick, 2012). Test items were directly derived from the CAPS content to reflect the constructs of VSR as accurately as possible. Thus, in support of content validity all test items measured the constructs they were designed to measure (Leedy & Ormrod, 2019).

According to Leedy and Ormrod (2019), the validity of test results will be enhanced should an uninterrupted intervention span over a short period and thereby limiting possible extraneous influences on the intervention programme's effect. As this intervention spanned a period of eight months between the pretest and the posttest, certain threats to the internal and external validity of the study could have occurred. Causation in favour of the intervention cannot be claimed as threats such as "history, maturation, statistical regression, testing, instrumentation, and experimental mortality" may have skewed the results (Cohen et al., 2018). All attempts were made at the proposal writing stage (*a-priori*) to control threats to internal validity and unexpected results were explained as best possible after the data collection (*posteriori*) (Larossa et al., 2016). Table 3.4 contains common factors that threaten internal validity and the steps taken to counter those threats are described.

Table 3.4: Factors threatening internal validity and steps taken to enhance the internal validity of the QUAN data set (Maree & Pietersen, 2010)

Threat	Description	Strategy
History	Unplanned influences between the pretest-posttest.	The pretest was administered before the intervention, and the posttest was administered eight months later which weakens causality. As the goal was not to prove causality, this threat is not harmful in demonstrating the application of van Hiele levels in EGD.
Selection	Participant selection compared to prerequisites.	Participants satisfied both criteria of formal teacher training in EGD and experience in teaching EGD.
Maturation	Influences on study results due to changes in participants over time. Cognitive, social, mental and physical changes	Participants were all at different high schools and often worked in online groups, which allowed for changes to occur that were outside the ambit of the intervention. This wasn't a problem as the focus wasn't so much on what was learnt, but rather on what wasn't learnt.
Mortality	Participants who could not complete the intervention.	Of the 38 participants, eight datasets could not be used for various reasons.

3.6.2.3 External validity

External validity is a quality assurance mechanism focusing on whether an experimental effect is generalisable and, if so, to what populations, situations, or treatment and measurement variables (Shadish et al., 2002). According to Higgins and Straub (2006), external validity is concerned with (1) generalising ***to*** particular people, situations and times and (2) ***across*** types of people, situations and times. The first construct is concerned with whether research goals for a certain population have been met; and the second one, assessing how far one can generalise. It is probable that another case study, with different participants in another province, would yield similar results as the sampling technique would feature the same bounded prerequisites, and the same CAPS content would apply. Future studies would not seek to generalise across types of people as the context would be completely different. Although the sample for this study was relatively small, the possibility exists for this study's findings to be valid for a similar cross-section elsewhere in South Africa, where teachers have undergone equal training.

This study was essentially a conceptual endeavour in the sense that a theory to particularise the cognitive descriptors of the van Hiele model was tested or illustrated as a possible teaching and learning framework for EGD. The study's immediate purpose was not to generalise to a larger population, but to illustrate the use of a novel reasoning framework focused on the constructs of VSR. It is my contention that any suitably qualified sample of teachers anywhere in the world would constitute an acceptable case for demonstrating the useful application of the van Hiele model in EGD. I base the previous statement on the assumption that the independent variables of EGD content, teacher training, and the van Hiele model ought to be similar in quality and nature across the world.

3.7 Quality Assurance: Qualitative

The overarching quality mechanism in qualitative research is known as *trustworthiness* which includes the four main criteria of *credibility, transferability, dependability, and confirmability (*Koonin, 2014). From the study's outset, I endeavoured to apply the four criteria with a view of establishing trust in the data's integrity and the honesty of my analysis and inferences (Niewenhuis, 2010b; Cohen et al., 2018). In addition, the use of multiple data collection methods is widely accepted to enhance trustworthiness.

3.7.1 Credibility

Credibility is a truth value concerned with the accuracy and consistency with which data was interpreted (Koonin, 2014; Maree, 2007). I aimed to enhance credibility by reading and rereading participants' opinions in the cognitive survey in order to understand their perceptions in context and gain insight into their intended meanings. Following the advice of Van der Riet and Durrheim (2008), I continuously searched for discrepant evidence during the research process to ensure rich and credible results. I made use of methodological triangulation on specific objectives, especially where the particularisation of cognitive descriptors featured, by checking the validity of inferences across multiple methods (Cohen et al., 2018). A case in point is where participants scored poorly in the pretest–posttest on geometrical constructions; they reflected this deficiency by neglecting to mention the importance of geometrical constructions in the cognitive survey, and this deficiency was confirmed during content analysis of their drawings. By using triangulation as a quality strategy, different strands of data helped me to confirm several inferences. Adding to the rigour of triangulation, I was sensitive to applying self-reflection before, during and after the data collection phases to control my bias and provided thick descriptions of the process across all data strands (Creswell & Creswell, 2018).

3.7.2 Transferability

Transferability takes place when findings are generalised to similar situations and similar findings are established (Koonin, 2014; Shenton, 2004; Kelly, 2006). My purpose with this study excluded the need for generalising findings to other populations. I provided detailed descriptions of the purposive sampling technique and specific sampling prerequisites. Likewise, I provided detailed and thick descriptions of the two data collection instruments, the particularisation process that resulted in multiple additional data strands and how I proceeded to analyse the data and made accordant inferences.

3.7.3 Dependability

Dependability is concerned with the consistency of how integration between methods of data collection, data analysis procedures, and theories is handled (Koonin, 2014). I provided detailed descriptions of how my own opinions and actions and those of participants were intertwined and developed within the study's context and setting. I followed a process of coding-recoding, kept a complete audit trail, explained my abductive methods in detail and applied data triangulation where such instances were possible (Ary et al., 2010; Van der Riet & Durrheim, 2006). My chosen methods of multiple strands of data collection required multiple methods of data analysis in support of the particularisation process and enabled me to provide rich, detailed descriptions of cognitive reasoning in solid geometry.

3.7.4 Confirmability

Koonin (2014) explains confirmability as the quality-process of data collection and its congruence with the flow of findings, interpretations and inferences made by the researcher. Shenton (2004) states that confirmability relates to a researcher's objectivity and neutrality to ensure that findings reflect the participants' opinions and experiences rather than the researchers' biased interpretations and predilections. I remained sensitive to control bias throughout the study to provide results that could be confirmed by others or confirm general findings in the field (Ary et al., 2010). My quantitative results largely confirm other studies that have previously reported on the obstacles presented by under-developed VSR, AR and CK. My sensitivity to self-reflection, control of bias, triangulation and establishing a complete audit trail was helpful in attaining this criterion of quality.

3.8 Ethical considerations

One of the main reasons for adhering to ethical rules and guidelines is to avoid harming any participant (Creswell, 2010). The Faculty of Education of the University of Pretoria prescribes strict principles of ethical conduct to ensure the best possible outcome for all stakeholders (https://www.up.ac.za/en/faculty-of-education/article/30611/research-ethics). This research was conducted in a manner compliant with the principles of anonymity and confidentiality, trust, voluntary participation and informed consent.

3.8.1 Permission to conduct research and voluntary participation

Participants have the right to freedom and self-determination, which places a responsibility on the researcher to educate them on the facts that would potentially influence their decisions (Cohen et al., 2010). I considered four elements in providing information about the study to the participants. First, the information had to be complete and accurate for the participants to make a *competent* choice about whether they wanted to participate or not. Second, the participants were afforded the freedom to choose to participate or not according to the principle of *voluntarism*. Third, *full information*, lacking nothing and not hiding anything deliberately, was made available to the participants. Lastly, and according to the principle of *comprehension*, the participants needed a complete understanding of the nature of the research project without any ambiguity (Cohen et al., 2010, p.52).

Due to the fact that the intervention programme was funded by the Matthew Goniwe School of Leadership and Governance (MGSLG) and administered by the Enterprises University of Pretoria (Pty) Ltd, written permission was obtained from these two institutions to use the data yielded for application in this study. The participants were invited to two separate information sessions where representatives from MGSLG and Enterprises University of Pretoria (Pty) Ltd addressed them with regard to the responsibilities assumed by all the stakeholders. The requirements from MGSLG regarding evidence of return on investment (ROI) were explained to the participants upfront before the programme commenced. Everything that was explained in these two sessions was captured in an official contract and signed by the participants individually and representatives from MGSLG and the Enterprises University of Pretoria (Pty) Ltd. All of the participants applied for permission from their line managers and principals to attend the eight-month long programme.

3.8.2 Confidentiality, anonymity and respect for privacy

Only information that participants had given permission for was used in recognition of confidentiality and anonymity (Cohen et al., 2010). Although I had access to the true identities of participants, I did not violate their rights to confidentiality and instead made use of pseudonym identities throughout the study. The ROI report required by MGSLG did not contain true identities either, as this was agreed upon by MGSLG and the participants. Each participant received a pseudonym number, and in all references to individuals, that pseudonym number was used instead of their true identities.

Apart from confidentiality and anonymity, and to safeguard participants' privacy (Cohen et al., 2010), In instances where participants decided to discontinue their participation, I removed their names and data from all Excel and SPSS electronic files. Throughout the study, data and actions in violation of participants' anonymity were carefully considered, and such data were avoided at all times (Chowdhury, 2014). I followed the ethical guidelines as required by the University of Pretoria (UP) in maintaining confidentiality and made provision for all data to be stored in a safe place.

3.8.3 Trust

In recognising the importance of remaining truthful to participants (Cohen et al., 2010), I informed the participants fully of the study's purpose and explained schedules and the research process before I commenced to collect data. At no point were the participants deceived and instead were clearly and unambiguously informed about what their participation would entail, negating the formation of stress or suspicion among them. During the study and afterwards, I maintained relationships of trust with the funders, the administrators and the participants. I remained sensitised to the possible effects of power relationships and how my perceptions and views as a researcher could impact my various approaches.

3.9 Conclusion

In this chapter, I explained why the pragmatic stance was adopted to answer the research questions through an abductive, mixed methods research design utilising a quasi-experiment and a cognitive opinion survey in a single case study. I explained the design of the two instruments and how the data was collected and analysed. The novel manner in which the van Hiele model was particularised to be used as an interpretive tool for understanding conceptual deficiencies was thoroughly explained as it formed the core instrument for answering the research questions.

In Chapter 4, the results of the data analyses for Instrument 1 and Instrument 2 are presented and the findings are discussed in light of particularised cognitive descriptors as derived from the van Hiele model. Instrument 1 was mainly responsible for demonstrating the relationship between EGD and the van Hiele model. Instrument 2 served the purpose of identifying areas of persistent conceptual deficiency by scrutinising the depth of learning transfer through the particularised van Hiele model.

Results and Discussion Chapter 4

4.1 Introduction

Chapter 3 served as an outline for this study's research process, design and methodological strategies employed in this study. The outline in Chapter 3 was framed considering the purpose of this study, namely, to explore the usefulness of employing a reasoning framework in Engineering Graphics and Design (EGD) based on the Van Hiele model for geometric thinking. In this chapter, the results for Instrument 1 and Instrument 2 are presented and discussed. In Section 4.2, I discuss the relationship between the van Hiele model and EGD. My particularisation process is explained and forms the basis for discussions in the rest of this chapter. In Sections 4.3 and 4.4, I present, discuss and compare the quantitative data resulting from quasi-experimental pre- and post-intervention tests on a standard solid geometry intervention I employed (Instrument 1). Refer to Chapter 1 on how the pretest (Instrument 1) was conceived and how it gave rise to the intervention content. In Section 4.5, I discuss the findings of Instrument 2, where I collected data through a solid geometry task that participants completed at home.

Section 4.2 answers the first sub-question: *How can the van Hiele model relate to EGD?*

Section 4.3 answers the second sub-question: *How can student performance in VSR be described in terms of the hierarchical levels of the van Hiele model?*

Section 4.4 answers the third sub-question: *How can student performance in VSR be described in terms of the van Hiele descriptors?*

Section 4.5 answers the fourth sub-question: *How can the van Hiele model allow for the identification of deficiencies in conceptual understanding?*

The aim of the study was to demonstrate how the Van Hiele model of geometric reasoning could be useful in describing levels of reasoning in EGD as espoused by my main research question: *How can a particularised version of the van Hiele model for EGD provide a deep understanding of conceptual deficiencies in cognitive reasoning?*

Test items were developed to assess and clarify EGD teachers' visuospatial reasoning skills (VSR) and convention knowledge (CK) through a traditional pretest-intervention-posttest design. The test items were analysed according to their cognitive reasoning types and aligned with the cognitive descriptors of the Van Hiele model of geometric thinking in pursuance of assigning a Van Hiele level to each test item. Descriptive statistics were used to describe the frequencies of test items in terms of participant performance.

By virtue of aligning van Hiele's hierarchical thinking levels with each test item, it was theoretically possible to demonstrate how participants reasoned in terms of the cognitive descriptors associated with each level. By comparing pretest scores with posttest scores, a determination could firstly be made whether the intervention had a significant effect on the improvement of scores. Secondly, because of the Van Hiele level assigned to each test item, it was possible to identify and describe instances of high and low acquisition per reasoning level. Note: The reader should note that although it may seem that question items are rated in absolute VH terms, the complete reasoning process includes lower cognitive levels that have been subsumed and which are necessary for the next, higher level to develop. However, if a question item carries a rating of say VH3, the cognitive descriptors for VH3 are isolated from VH1 and VH2 descriptors for the discussion on that question item to provide sharp focus on certain level descriptors. For the sake of fluent reading, I use the terms VH1, VH2, VH3, VH4 and VH5 to abbreviate the van Hiele levels of geometric thinking/reasoning. Table 4.1 shows how my abbreviations relate to the hierarchical levels of van Hiele’s levels of geometric thinking.

Table 4.1: Abbreviations used for van Hieles hierarchical acquisition of cognitive levels

Van Hiele thinking level	Level descriptor	Abbreviation
1	Recognition	VH1
2	Analysis	VH2
3	Informal deduction	VH3
4	Formal deduction	VH4
5	Rigour	VH5

VH1 and VH5 levels did not apply to the instruments I used. According to the cognitive descriptors of the van Hiele theory, neither one of the two instruments utilised in this study contained instances that included descriptors on those two levels. Even though VH1 does not appear as an item of discussion in this study, it remains the case that shapes and figures must still be recognised and differentiated from one another. Yet, one of the study’s assumptions was that all thinking on VH1 (Recognition) had already been acquired by all participants. Researchers such as Usiskin (1982) and Idris (2005) believe that thinking on VH5 only holds theoretical value and will be very difficult to test.

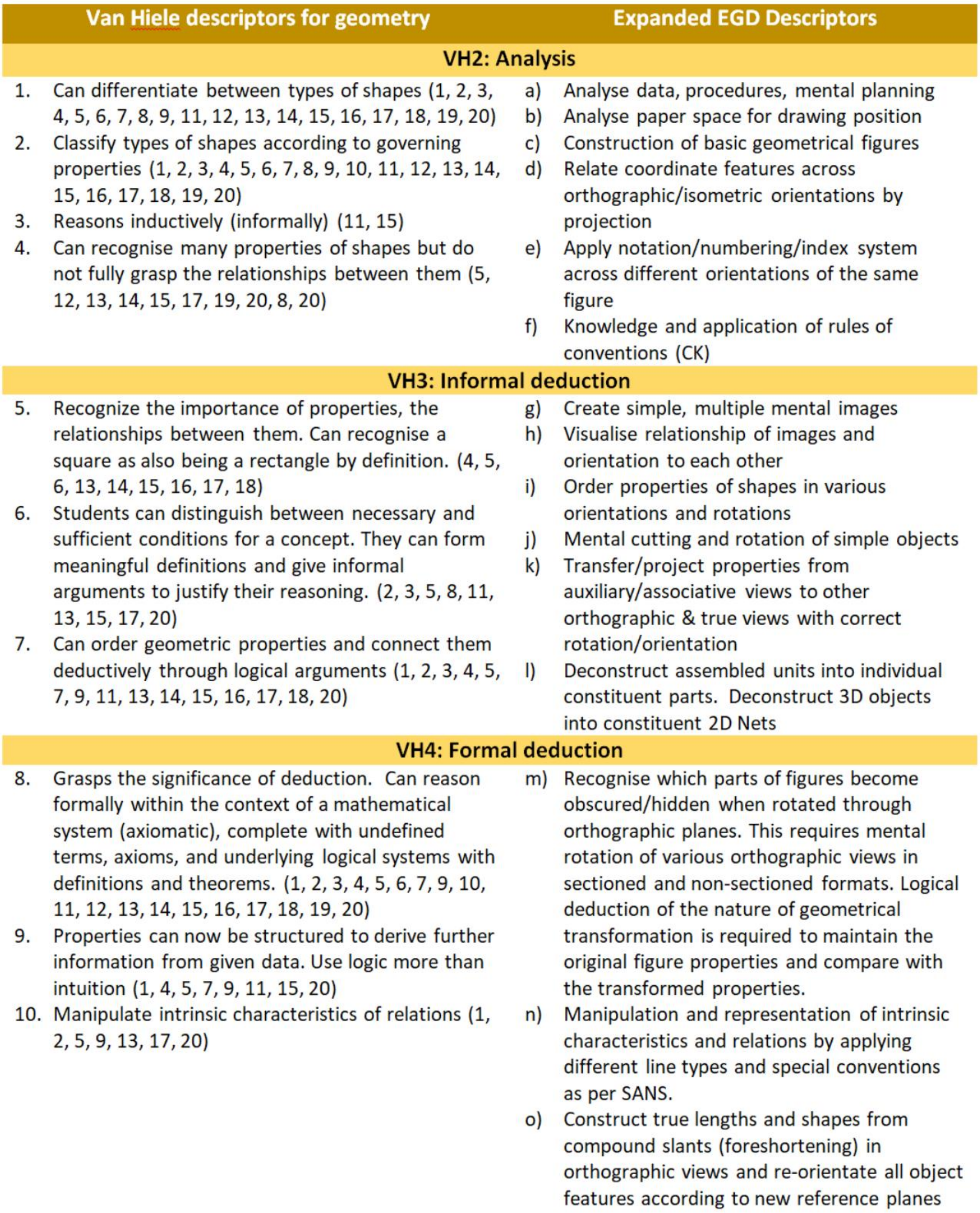
In the next section, I table the cognitive descriptors of van Hiele for VH2, VH3, and VH4 as they appear in the literature. In addition, I provide a simple orthographic drawing to demonstrate how thinking in EGD can be related to the van Hiele model.

4.2 The relationship between EGD and the van Hiele model

4.2.1 Cognitive descriptors of the van Hiele model

Table 4.2 is key to the particularisation process that follows on the next page.

Table 4.2: Particularisation of van Hiele cognitive descriptors for EGD



Van Hiele descriptors for geometry	Expanded EGD Descriptors
VH2: Analysis	
1. Can differentiate between types of shapes (1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) 2. Classify types of shapes according to governing properties (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) 3. Reasons inductively (informally) (11, 15) 4. Can recognise many properties of shapes but do not fully grasp the relationships between them (5, 12, 13, 14, 15, 17, 19, 20, 8, 20)	a) Analyse data, procedures, mental planning b) Analyse paper space for drawing position c) Construction of basic geometrical figures d) Relate coordinate features across orthographic/isometric orientations by projection e) Apply notation/numbering/index system across different orientations of the same figure f) Knowledge and application of rules of conventions (CK)
VH3: Informal deduction	
5. Recognize the importance of properties, the relationships between them. Can recognise a square as also being a rectangle by definition. (4, 5, 6, 13, 14, 15, 16, 17, 18) 6. Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning. (2, 3, 5, 8, 11, 13, 15, 17, 20) 7. Can order geometric properties and connect them deductively through logical arguments (1, 2, 3, 4, 5, 7, 9, 11, 13, 14, 15, 16, 17, 18, 20)	g) Create simple, multiple mental images h) Visualise relationship of images and orientation to each other i) Order properties of shapes in various orientations and rotations j) Mental cutting and rotation of simple objects k) Transfer/project properties from auxiliary/associative views to other orthographic & true views with correct rotation/orientation l) Deconstruct assembled units into individual constituent parts. Deconstruct 3D objects into constituent 2D Nets
VH4: Formal deduction	
8. Grasps the significance of deduction. Can reason formally within the context of a mathematical system (axiomatic), complete with undefined terms, axioms, and underlying logical systems with definitions and theorems. (1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) 9. Properties can now be structured to derive further information from given data. Use logic more than intuition (1, 4, 5, 7, 9, 11, 15, 20) 10. Manipulate intrinsic characteristics of relations (1, 2, 5, 9, 13, 17, 20)	m) Recognise which parts of figures become obscured/hidden when rotated through orthographic planes. This requires mental rotation of various orthographic views in sectioned and non-sectioned formats. Logical deduction of the nature of geometrical transformation is required to maintain the original figure properties and compare with the transformed properties. n) Manipulation and representation of intrinsic characteristics and relations by applying different line types and special conventions as per SANS. o) Construct true lengths and shapes from compound slants (foreshortening) in orthographic views and re-orientate all object features according to new reference planes

I collected a large volume of literature on the van Hiele theory, which included research on the five hierarchical levels of geometrical reasoning. From this content, I selected 20 sources that offered comprehensive information on cognitive descriptors, copied those descriptors verbatim into twenty groups, and noted each author’s definition/s for VH2, VH3, and VH4. From this collection of definitions, I followed a process of thematic inductive content analysis to link similar descriptors together and consolidated the many duplications. I collated a final list of ten descriptors under the headings of VH2, VH3, and VH4. Table 4.2 shows the spread of the ten van Hiele descriptors on the left, with expanded cognitive descriptors on the right. I solicited advice from a leading geometrician on the particularisation process of Table 4.2. This academic validated my application of the van Hiele cognitive descriptors in EGD. The descriptors in Table 4.2 (Appendix E) are henceforth used to provide qualitative explanations based on quantitative data.

I have numbered the van Hiele descriptors (1–10) and used alphabetical characters (a–o) in denoting the expanded EGD descriptors as identifying markers, to explain how they relate to the execution of a drawing. In the next section, I describe the step-by-step solution of the drawing in Figure 4.1. I use parenthesised numbers or alphabetical characters next to each thinking operation that links to Table 4.2. Next to each one of the ten van Hiele cognitive descriptors appear numbers in parentheses within the range of 1–20. These numbers correspond with the list of 20 authors in Appendix E, from which I derived the ten cognitive descriptors. To make sense of the particularisation process in Table 4.2, the reader is guided through a step-by-step procedure in how the isometric drawing shown in Figure 4.1 was executed. This process is explained in Section 4.2.2.

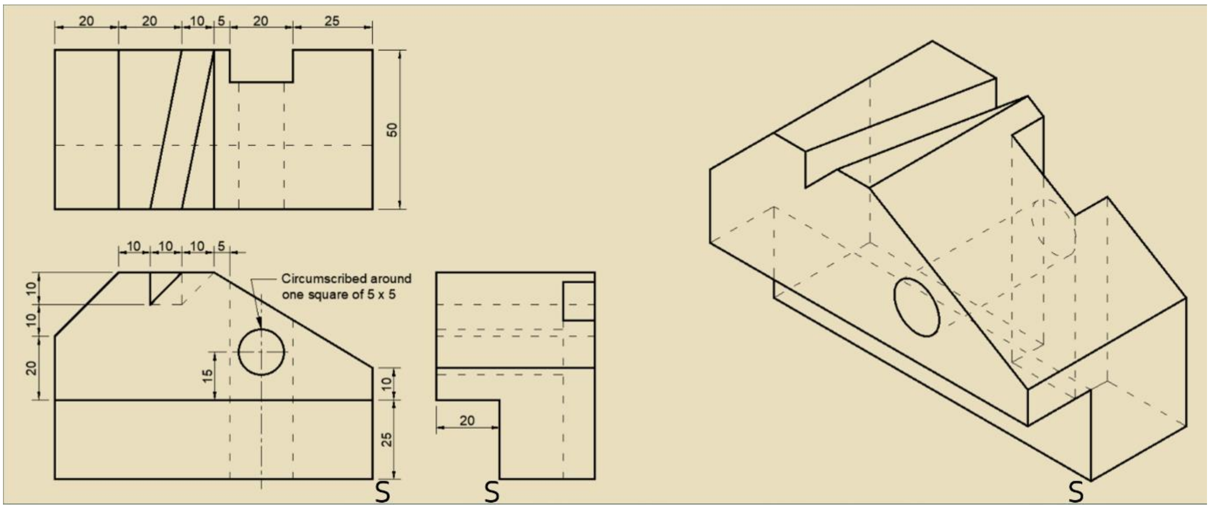


Figure 4.1: Third-angle orthographic multiviews and an isometric view of a machine casting

4.2.2 Relating the van Hiele levels to the solution process of solid geometry

Figure 4.1 shows three multiviews (front, right, and top) of a machine casting in third angle orthographic projection on the left and an isometric view of the object on the right. For this example, the objective was to draw the isometric view of the object (shown on the right) from the orthographic multiviews, orientated for “S” to be the lowest point. Following is a step-by-step explanation of how the reasoning listed in Table 4.2 links with the procedures of drawing the isometric projection. Note: Not everyone may follow the same exact steps as indicated in the “Drawing process” related to Figure 4.1. My objective with this procedure is simply to demonstrate how thinking in EGD relates to the van Hiele model of geometric thinking. In addition, where I previously mentioned that VH ratings per question item may appear as absolute and exclusive of other levels of cognition, the drawing process that follows contains reasoning elements on all three VH levels under discussion.

Drawing process

Step 1: Analyse the given graphical data (a) by identifying the orthographic Net-shapes (1) according to their governing properties (2) for mentally planning (a) the drawing extents on the available paper space (b).

Step 2: Number (e) all coordinate points on all three orthographic (f) views to determine shared coordinates (d,k). The numbering system provides an inductive/deductive (3,7) reasoning platform for relating figure properties with other figure properties (4).

Step 3: Analyse the scenario (a,b), and create a mental model (g,h,i) of how the 2D Nets (1,2) satisfy sufficient conditions (6) and how their properties (5,7) can be structured (9,l) to form (k) an associative 3D form. This step implies mental rotation (j) and the ability to manipulate intrinsic characteristics of relations (10)

Step 4: From the mental analysis (h,i) of Step 3, determine which three geometric Nets (1,2) and their properties are ordered (7) in adjacent proximity (d) to establish the referential (o) index point “S” from multiviews.

Step 5: Expand (7,8) the mental model already held in the mind’s eye to conceptually understand (6) which other Net-properties share the same coordinates in 3D space (9,10). Reorientate (h,i,o) all object features according to the new singular isometric reference point “S”. This process requires rapid scanning (h) of the given orthographic conditions (6) to maintain (i) and adjust (l,m) the mental model that is in flux.

Step 6: Construct (c) a cubic figure according to the given length, width, and height. Construct (c) portions of the front view and right view onto the cubic figure where they share point “S” and populate (l) those two planes as completely as possible by projecting (d) the given dimensions.

Step 7: Non-isometric lines (o) must be drawn in when all vertical and horizontal lines from the front view and right view have been positioned isometrically. The axiomatic system (8) of EGD specifies how lines (o) in the orthographic system appear vectorially different (10) in the isometric projection system.

Step 8: Through mental imagery, construct (l) the missing top view by aligning (i) figure properties (h) that meet the conditions (6) of true lengths/shapes (o) to derive sufficient information (9) for linking (7) isometric lines with non-isometric lines (o).

Step 9: Complete all visible and obscured (m) geometrical features by their object properties (n) and relevant position (h,i) in 3D space (g,j,7,8) by applying line types (f,n) according to convention.

Step 10: Construct (c) and position (d,I,k) the through-hole on the front view (7).

Step 11: Evaluate the solution by comparing it with the given data and task specifications. This consolidation process requires an interwoven effort of VSR, AR, and CK by rapid and cyclic utilisation of the cognitive descriptors across the range of (1–10) and (a–o) listed in Table 4.2.

4.2.3 Findings for the first sub-question:

The first sub-question: *How can the van Hiele model relate to EGD?*

From Table 4.2 (Appendix E) and for VH2, I used four cognitive descriptors as found in the literature on the van Hiele model. I relate the van Hiele cognitive descriptors with the Expanded EGD Descriptors through parenthesised alphabetical characters as they appear in Table 4.2. The following summaries of the four cognitive descriptors for VH2 are based on the discussions of Sections 4.2.1 and 4.2.2 and should be read alongside Table 4.2:

Cognitive descriptor 1: *Can differentiate between types of shapes*. With reference to the Expanded EGD Descriptors of Table 4.2 (Appendix E), practitioners of EGD need to be able to differentiate between types of shapes in the analysis (a) of a given drawing scenario to make sense of what is asked and what is required. This basic knowledge is also necessary to identify and analyse (a) different Nets in both orthographic and isometric layouts and plays a role in the construction (c) of various geometrical figures. Being able to distinguish and classify shapes is a basic requirement when engaging in mental planning (a) for the analysis of available paper space (b) and cardinal referential lines for positioning (b) the required views.

Cognitive descriptor 2: *Classify types of shapes according to governing properties*. Factual and conceptual knowledge of the properties that govern geometrical shapes is part and parcel of the ability to accurately construct (c) geometrical figures. This descriptor implies the ability to relate (d) orthographic coordinate features across other multiviews and isometric views of the same object. EGD utilises a notation system (e,f) where key coordinates provide referential points across all projections (d) of views and types of drawings of the same object.

Cognitive descriptor 3: *Reasons inductively (informally)*. Inductive reasoning is located on VH2 (Curran, 2015; Burger & Shaughnessy, 1986), but researchers such as Fuys et al. (1988) and Ding and Jones (2007) found that learners display a preference for either inductive or deductive reasoning even on VH2. Gutierrez (1991, p. 237) reported similar findings and proposed the idea of varying "degrees of level acquisition". Fuys et al. (1988) trained learners with a preponderance towards inductive reasoning on how to apply deductive reasoning. Learners were at first reluctant to deviate from their reasoning preference but eventually managed to use deductive reasoning strategies. Hayes and Heit (2018) explain how learners who are well-versed in inductive reasoning perceive patterns and regularities in situations that might otherwise seem intractable. Yet, Gagani and Misa (2017) state that inductive reasoning leads to poor results where learners' prior knowledge of a topic is incomplete. Kahneman (2011) suggests that humans rely on default automatic processes (inductive) whenever possible yet, when necessary, deductive reasoning interrupts automatic heuristic processes (inductive) and takes control to explore and evaluate alternative solutions. Where inductive reasoning relies on prior knowledge and observations to discover new principles, deductive reasoning relies on established principles to derive new facts from (Kahneman, 2011). From the findings of these researchers, it follows that learners ought to be trained on when and how to use these forms of analytical reasoning. According to Hegarty (2010), learners with advanced reasoning abilities tend to combine both VSR and AR techniques during problem solving. The conventions (f) of EGD are not only a means of standardising drawings, but also provide the mental fabric onto which VSR and AR can be woven to seek solution paths. A combination of inductive and deductive reasoning is essential for functional reasoning with the range of descriptors (a–f) of Table 4.2 (Appendix E).

Cognitive descriptor 4: *Can recognise many properties of shapes but do not fully grasp the relationships between them.* Learners who have not yet fully acquired this level of thinking may display difficulties in planning (a) the multiviews of an object and position the drawings incorrectly on the available paper space (b). They may not be able to maintain proportionality between views of the same object, as the mental tracking (d) of figure properties across multiviews implies a conceptual understanding of the properties of figures in their original and transformed guises.

Cognitive descriptor 5: *Recognize the importance of properties, the relationships between them. Can recognise a square as also being a rectangle by definition.* Conceptual understanding of the relationships (h) between figure properties enables the EGD practitioner to create mental images (g), and order figure properties (i) to visualise different orientations, transformations (j) across various projections (k). This kind of dynamic visualisation often entails the deconstruction/reconstruction (l) of figural Nets for relating 2D and 3D views of the same object with one another to maintain a true mental view (k) of the object in transformation.

Cognitive descriptor 6: *Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning.* When presented with a drawing task, learners must possess factual and conceptual knowledge of the scenario to evaluate whether the conditions are sufficient for creating (g) dynamic images (h) to order and project (k) figure properties (i) across multiple views and orientations (h). This process implies mental rotations (j), relating image properties with one another (h) and comparing 2D and 3D visualisations of the object (l). Baddeley (2012) explains how the brain focuses, divides, and relays attention intermittently between these focus points while constantly interfacing with long-term memory.

Cognitive descriptor 7: *Can order geometric properties and connect them deductively through logical arguments.* Learners are often presented with scenarios where information is sparse and reasoning can become intractable. To navigate one’s thoughts through cognitive descriptors (g–l) requires a process of logical deductive reasoning. Kahneman (2011) describes such logical deductive reasoning or general sequential reasoning as a rule-based ability by utilising known principles and premises. Tversky (2005) explains how complex VSR is predicated on a logical series of singular-sequential inferences regarding the static and dynamic properties of a given spatial scenario. Tversky (2005) states that image properties are comparable with the semantic structure of language, where spatial relations between figure properties constitute a rudimentary syntax.

Cognitive descriptor 8: *Grasps the significance of deduction. Can reason formally within the context of a mathematical system (axiomatic), complete with undefined terms, axioms, and underlying logical systems with definitions and theorems.* EGD utilises much of the axiomatic system of geometry. As such, applicable axioms can be used in EGD to argue a solution and “prove” the solution in EGD’s extended axiomatic system of orthographic and isometric projections. Through the utility of this axiomatic system, the EGD practitioner reasons deductively to derive information for the construction (o) of obscured or seemingly missing features (m) through complex VSR procedures (n).

Cognitive descriptor 9: *Properties can now be structured to derive further information from given data. Use logic more than intuition.* This cognitive operation flows from the previous one in the sense that “missing” information (m) can be derived deductively by comparing original figure properties with the transformed properties instigated by foreshortening, or sectional planes (o).

Cognitive descriptor 10: *Manipulate intrinsic characteristics of relations.* Through deductive reasoning, figure properties that changed due to transformations imposed on the original object must be tracked across orthographic planes (m). The intrinsic vector characteristics (n) of the original object are carefully manipulated according to the specifications of the transformation and represented on paper by assigning line types (n) to denote the new attributes of the object.

In Section 4.3, the procedural steps for the scenario in Figure 4.1 and their cognitive reasoning descriptors are summarised in a narrative format for VH2, VH3, and VH4. In addition, the performance of participants on these three VH levels is statistically discussed to provide insight on how they reasoned per VH level. In Section 4.4, participant performance is discussed in terms of the 10 cognitive descriptors.

4.3 Instrument 1: Pre- and post-intervention quantitative results

Following is a narrative summary of how EGD practitioners think in terms of the van Hiele model’s descriptors for VH2, VH3, and VH4. This section answers the second sub-question: *How can student performance in VSR be described in terms of the van Hiele levels?*

4.3.1 EGD thinking on VH2, VH3, and VH4

VH2: Analysis

On VH2, EGD practitioners base their reasoning primarily on visual observations and intuitive understanding and not on formal deductive arguments. They seek to identify the given geometric shapes and then strive to analyse their governing properties to determine the extents of the object and how it should be positioned on paper. Their visual analysis of the object and its figure properties are used to accurately construct the figure with the use of drawing instruments. EGD practitioners may display aspects of inductive and deductive reasoning such as recognising and relating figure-properties across multi views. However, their reasoning is typically more intuitive and informal rather than following formal logical steps.

With reference to Figure 4.1, the governing properties of the object are fragmented across the three orthographic multiviews, and therefore the shared coordinate features have to be mentally related to one another by means of a projection system. Novice EGD practitioners need to use an annotation system to number object features on one of the multiviews, and then assign the same annotation to corresponding features on the other multiviews. This method reduces the VSR load by employing a systematic analytical reasoning (AR) system.

A systematic annotation system needs to be established on the conventions of first and third angle orthographic projection methodology. On VH2, properties of multiviews can be recognised but not necessarily related to one another, which makes a systematic annotation system essential.

VH3: Informal deduction

The analysis process, as described under VH2, takes place by mentally "assembling" the multiviews into a single 3D object and rotating it in the mind's eye. From this process, it is logical that some of the cognitive descriptors of VH3 straddle the descriptors of VH2. The process of creating mental imagery is iterative and requires rapid and repeated visual inspections of the orthographic views until the "assembled" views can be mentally held and rotated in 3D space as a single object. It must be noted, however that the success of this process is reliant on one's short-term memory (STM) capacity (Schneider & McGrew, 2018).

On VH3, EGD practitioners do not only relate orthographic features with one another but logically order their properties as they relate to one another to sequentially construct the correct 3D view through VSR. The EGD practitioner has to identify at least two multiviews that contain sufficient information to start the sequence of mentally constructing the 3D view. Mental rotations are significant on this level as each rotational iteration brings about a measure of correction of the previous mental model that was held in 3D space. Multiple mental iterations are required. Orthographic multiviews represent the 3D object’s geometrical Nets (excluding foreshortened lines), and as such, the EGD practitioner follows a mental process of constructing a 3D image from 2D Nets. The developing 3D view must periodically be deconstructed as it remains in a state of flux until the 2D Nets mentally assume their correct positions for the 3D view that is held in the mind's eye to make logical sense.

On VH3, EGD practitioners may not officially engage in deductive reasoning but rather utilise AR techniques such as notation systems. According to van Hiele, they may demonstrate some aspects of deductive reasoning, but it is not the primary mode of thinking on this level. However, Fuys et al. (1988) found that learners of the same age could display varying degrees of deductive reasoning. According to Kahneman (2011), our brains use heuristic systems (inductive reasoning) as its normal reasoning mode, yet, when necessary, deductive reasoning supersedes automatic heuristic processes and controls the exploration and evaluation of alternative solutions. Kahneman (2011) explains how deductive reasoning could account for higher cognitive load, which causes humans to favour default automatic processes (inductive) whenever possible.

On VH3, learners focus on the recognition, analysis and manipulation of properties of geometric shapes. They develop a deeper understanding of how the shapes of multiviews can be classified and grouped based on associative attributes. The properties of shapes, such as symmetry, angles, and lengths, are used to make comparisons and create links between multiviews of the same object and its isometric views. On VH3, EGD practitioners are securing a foundation towards advanced geometric thinking, which leads to the refinement of their deductive reasoning ability. However, the Van Hiele levels of thinking about geometry do not necessarily follow a strict hierarchical order in a linear fashion (Gutierrez, 1991). Some learners may progress through the levels at different rates and display varying degrees of having acquired deductive reasoning abilities. Gutierrez (1991, p.237) puts it like this: “Although most students show a dominant level of thinking when answering open-ended questions, a large number of them clearly reflect in their answers the presence of other levels, and there are some students whose answers show two consecutive dominant levels of reasoning simultaneously”. Progression through the levels depends on many internal and external factors, such as instructional practice, experience, and personal cognitive development.

VH4: Formal deduction

On VH4, learners grasp the significance of deductive reasoning and can make logical connections between figure properties across various types of drawings. They engage in deductive arguments by scrutinising the conditions applicable to objects in space and their multi views. On VH4, learners fully understand the structure of geometric systems such as 1st and 3rd angle orthography, and they can relate the axioms, theorems, and geometric statements of pure geometry with comparable instances of EGD content. They are now able to use logical arguments within the formal language and symbolism of geometry.

Deductive reasoning is typified by the construction of logical arguments and making inferences based on appropriate rules and theorems in the context of EGD. The process of converting orthographic multiviews into isometric views requires a high degree of deductive reasoning to maintain original figure properties in the transformation process. Certain features may become obscured, and lines may become foreshortened/distorted or even disappear where sectional planes are applied. EGD practitioners can manipulate and represent intrinsic characteristics of relations by specific EGD conventions, such as alternating line types as symbols of figure properties. At this stage, orthographic lines that are non-isometric lines can be accurately constructed according to their vector properties (true lengths and shapes) when represented in 3D space. Even though proofs are not typical for EGD content, the mathematical theory underpinning proofs and theorems is understood and can be used in logical argumentation in linking object properties with one another in the same view, and across orthographic and isometric views for the same object. Axiomatic systems for EGD are fully grasped.

Table 4.3 shows the descriptive statistics of the pretest and posttest results for VSR for the entire sample. Note that only the 12 VSR test items were used to demonstrate the usefulness of the van Hiele model. The inclusion of CK and AR data would inflate the volume of this study without providing different insights into the usefulness of the van Hiele model in EGD.

4.3.2 Statistical results for participant performance on VH2, VH3, and VH4

Table 4.3: Descriptive statistics for VSR

Pretest (N = 30)				Posttest (N = 30)			
Min	Max	Mean	Std	Min	Max	Mean	Std
VSR							
11	91	49	18,50	33	100	76	19,89

The average score for the VSR pretest was 49%, which improved to 76% with the posttest. The same test was used before and after the intervention. A Wilcoxon signed rank test was performed, and it showed that there was a significant difference ($Z = 4.783$, $p < 0.001$) between pretest scores and posttest scores for VSR. Without considering other external factors of possible causation, the intervention seems to have made a positive difference. The purpose of this study was to demonstrate the usefulness of the particularised van Hiele model and factors of causation fall outside the focus of this study and will not be explored further. Table 4.4 provides a broad overview of the pretest and posttest scores where the results for VSR have been separated under the headings of VH2, VH3, and VH4 for the entire sample.

Table 4.4: Pretest and Posttest descriptive statistics for VSR divided into VH2, VH3, and VH4

VH2				VH3				VH4			
Min	Max	Ave	SD	Min	Max	Ave	SD	Min	Max	Ave	SD
Pretest											
40	100	79	17,5	3	84	45	22,24	0	100	39	31,78
Posttest											
50	100	95	10,94	27	100	77	20,63	0	100	63	36,51

For VSR, the group averages for the pretest were 79% for VH2, 45% for VH3, and 39% for VH4. Post-intervention averages were 95% for VH2, 77% for VH3 and 63% for VH4. Post-intervention performance on the VH4 level appear significantly less than for VH2 and VH3. Yet, improvement on VH4 (24%) and VH3 (32%) is more than for VH2 (16%). Question 13 was the only VH4 question. Participants had to find the true length of a line by construction; Questions 25 and 26 were both surface development questions.

The chart in Figure 4.2 provides a graphical representation of the pretest and posttest results for VSR per individual participant. The data is arranged with the VSR pretest scores (Orange line) to follow in ascending order and posttest data (Blue line) is accordant with this arrangement. Participant pseudonym numbers follow this arrangement on the horizontal axis.

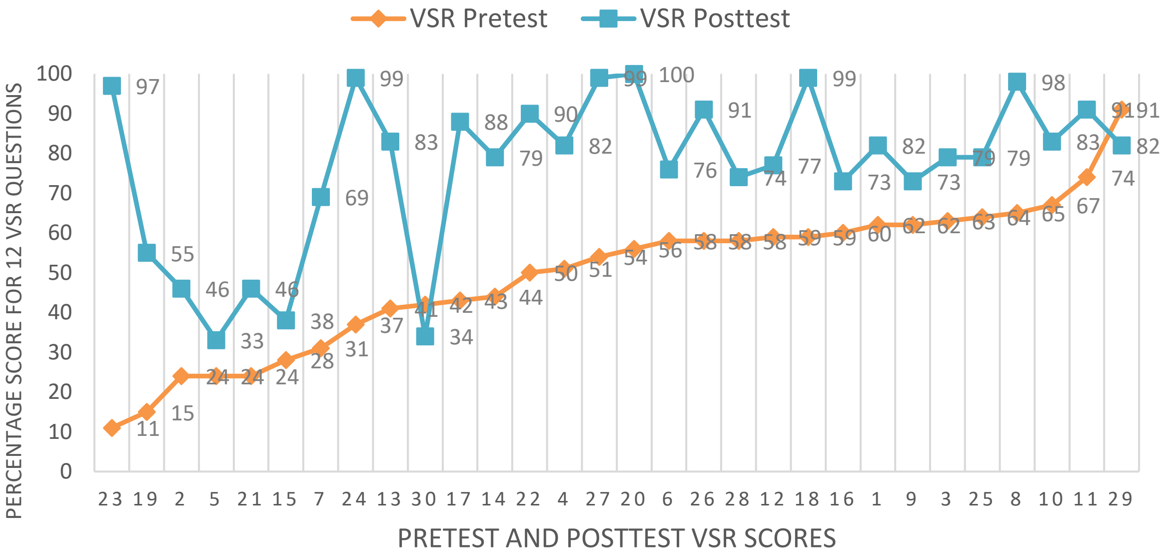


Figure 4.2: Pretest and Posttest for VSR arranged according to ascending VSR Pretest

A cursory visual inspection of Figure 4.2 reveals that significant improvements were recorded on the posttests for VSR. With a view to explaining VSR performance on the pretest and posttest, participants 2, 6, 23, 24, 29, and 30 will be briefly discussed as they represent the low, mid, and high performers in the group. Table 4.5 presents the average performance for these six participants and lists the averages for VH2, VH3, and VH4 from the data obtained through Instrument 1.

Table 4.5: VH performance (%) for participants 23, 2, 24, 30, 6, and 29

Participant	VSR average		VH2 average		VH3 average		VH4 average	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
23	11	97	40	100	7	95	0	100
2	24	46	70	94	6	52	33	0
24	37	99	73	100	43	98	0	100
30	42	34	79	82	21	35	67	0
6	58	76	100	100	57	73	33	67
29	91	82	100	100	84	84	100	67

Participant 23, with two years of EGD teaching experience, recorded the worst score of 11% for the VSR pretest and improved the most by achieving 97% for the VSR posttest. This participant improved significantly, especially on VH3 and VH4. This participant was quite upset about his/her performance and vowed to make the most of the intervention. None of the other participants displayed this kind of motivation throughout the intervention. This participant also asked the most questions and delved deeper into the content than anyone else. In addition, this participant had, up to the time of the intervention, only been involved with Grade 10 EGD and aspired to also teach EGD on Grade 11 and Grade 12.

Participant 2, with 16 years of EGD teaching experience, increased VSR performance from 24% for the pretest to 46% for the posttest. This participant improved slightly on VH2 and VH3 reasoning levels, but displayed a regression in reasoning on the VH4 level.

Participant 24, with 8 years of EGD teaching experience, improved their VSR score from 37% for the pretest to 99% for the posttest. This participant improved significantly on all three reasoning levels.

Participant 30, with 10 years of EGD teaching experience, decreased in VSR performance from 42% for the pretest to 34% for the posttest. This participant improved very slightly on VH2 and VH3 reasoning levels but, in the same vein as Participant 2, displayed regression on the VH4 reasoning level. This particular participant was a foreign national and experienced difficulty with English as a primary means of communication.

Participant 6, with 31 years EGD teaching experience, increased VSR performance from 58% for the pretest to 76% for the posttest. This participant received a 100% score for VH2, but displayed progressively lower levels of improvement on VH3 and VH4.

Participant 29, with six years EGD teaching experience, scored the highest for the VSR pretest at 91%, yet performance dropped to 82% in the posttest. This participant received a 100% score on VH2, showed no improvement on VH3, and performed more poorly on VH4. It is possible that this participant made "lucky" guesses with some of the multiple-choice items and guessed wrong during the posttest. This possibility brings the validity of multiple-choice questions into question.

From the graph in Figure 4.2, Participants 5, 21, and 15 not only scored poorly on the pretest, but also displayed very little improvement on the posttest, especially on VH3 and VH4 reasoning levels. Participant 15, with 33 years of teaching experience, performed more poorly on VH2 in the posttest than in the pretest. Participant 21, with 42 years’ teaching experience, improved marginally on VH2, fair on VH3, and showed no improvement on VH4. Figure 4.2 reflects this trend, where improvement gradually decreases as the VH levels increase.

4.3.3 Statistical results per question item

As mentioned before, my strategy in demonstrating the usefulness of the van Hiele model focuses only on VSR data. Table 4.6 contains the pretest and posttest descriptive statistics for VSR per question item, and the data for the 12 VSR questions are graphically represented in Figure 4.3. Only the 12 VSR question items out of the original 24 questions were used, which accounts for the non-chronological order of the numbers in Table 4.6. The 12 missing numbers belong to the 12 CK items which have been excluded. In Figure 4.3, the VH level per question item is shown next to each question number on the horizontal axis. For example, 1VH2 refers to Question item 1, which is rated as van Hiele level 2. VH2 levels are designated red in colour, VH3 are blue, and VH4 are green.

Table 4.6: Pretest and Posttest descriptive statistics for VSR

		Pretest (N = 30)				Posttest (N = 30)			
Question	VH	Min	Max	Mean	Std	Min	Max	Mean	Std
1	2	0	100	89	22,51	0	100	96	18,26
3	3	0	100	48	37,92	19	100	91	21,17
4	3	0	100	52	40,44	0	100	92	26,53
6	2	0	100	69	31,41	41	100	93	14,34
7	3	0	100	14	31,27	0	100	64	42,73
8	3	0	93	39	40,56	0	100	78	34,05
9	3	0	100	49	39,49	0	100	68	38,70
13	4	0	100	37	49,01	0	100	63	49,01
24	3	0	100	70	46,61	0	100	83	37,90
25	4	0	100	47	50,74	0	100	63	49,01
26	4	0	100	33	47,95	0	100	63	49,01
27	3	0	100	43	50,40	0	100	63	49,01

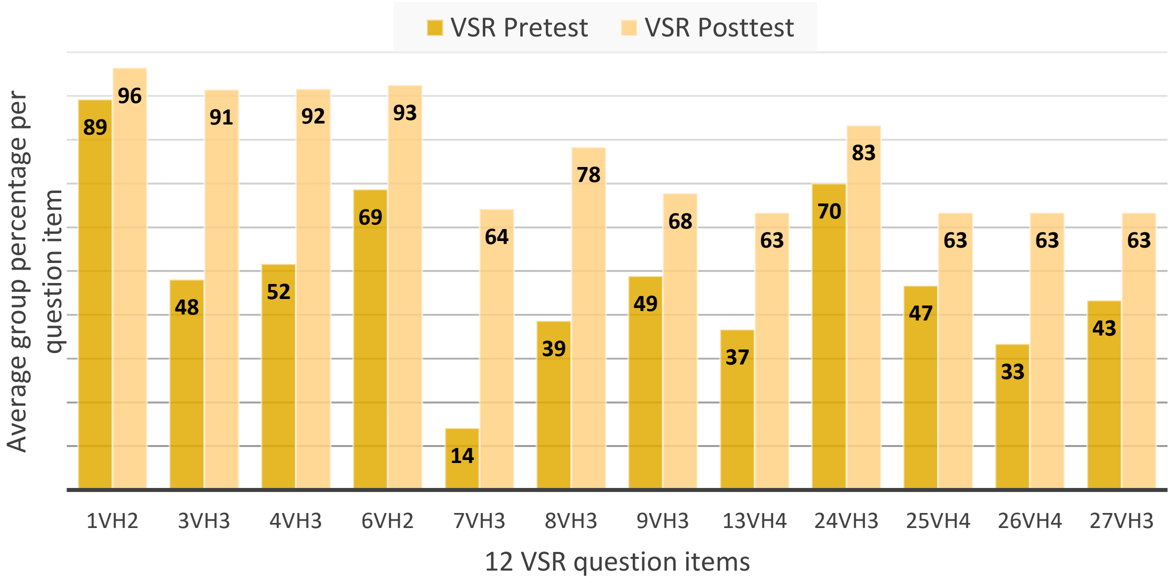


Figure 4.3: Group performance for the 12 VSR questions

Although the posttest scores had improved significantly for all question items, the six lowest-scoring pretest items recorded lower posttest scores than the remaining six question items

(Question items 7VH3, 8VH3, 13VH3, 25VH4, 26VH4, and 27VH4). The group pre-average for Question 1 was 89%, and 69% for Question 6. The smaller gains made with Question 1VH2 can be explained by this observation from Marsden and Torgerson (2012, p.583), "learners with higher baseline scores consistently made smaller gains than those with lower baseline scores, demonstrating that regression to the mean (RTM) is clearly observable in single group, pre-posttest designs". The opposite is implied for Question items with low pretest scores such as Question 7. Question 7 is further discussed in Section 4.4. Questions 1 and 6 were the only VSR questions on VH2, and although they were conceptually almost identical, participants clearly perceived Question 6 as more demanding. The same peculiar relationship seems to be shared between Questions 4 and 7; Questions 24 and 27; and Questions 25 and 26. These eight question items are further discussed in Section 4.4.

Question item 3VH3 saw a marked improvement from 48% to 91% and similarly, group-performance for Question item 8VH3 improved from 39% to 78%. For both these items, participants were required to draw freehand isometric drawings from multiview orthographic drawings. After the tests were assessed and participants were allowed to peruse their papers, the steps in solving the two drawings were comprehensively discussed and as the posttest contained the same drawings, participants responded positively on the posttest. Yet, Question item 8VH3 was perceived as more difficult than Question item 3VH3 for both pretest and posttest. This disparity is possibly due to the heightened orthographic illusion of what Newcombe and Shipley (2014) refer to as "to distinguish from overlapping patterns" in the intrinsic/static domain. Schneider and McGrew (2018) refer to such optical trickery as "perceptual illusions".

Quantitative scores are useful in differentiating performance between question items. Yet, the scores themselves cannot provide reasons for the differences in performance from a reasoning perspective. Possible reasons for the variance between these question items are qualitatively discussed in Section 4.4 to demonstrate the explanatory utility of the particularised van Hiele model. Figure 4.4 represents participant performance per VH level for the VSR pretest.

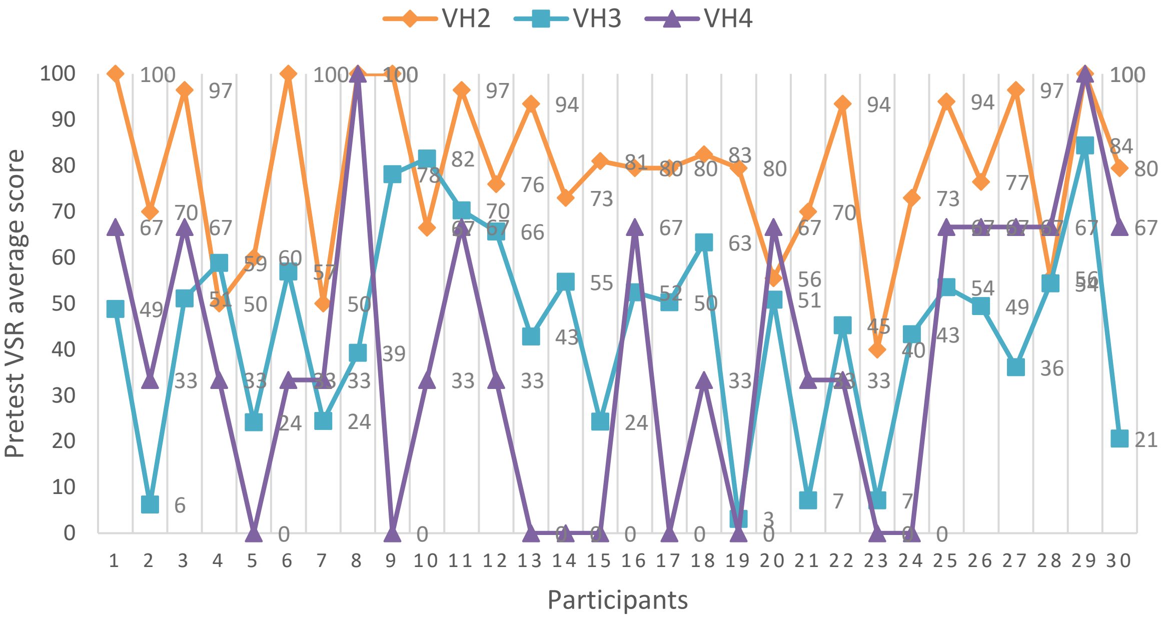


Figure 4.4: Individual participant performance per VH level for the VSR pretest

Figure 4.4 portrays the variance in participants' performance across the three VH levels and shows that nine of the 30 participants scored 0% for VH4 on the pretest for VSR. Only two of the 30 participants scored 100% for VH4. From this pre-intervention data, it seems that most participants had not acquired the kind of reasoning associated with VH3 and VH4.

Figure 4.5 represents participant performance per VH level for the VSR posttest. Although posttest performance had improved significantly, the data seems to indicate that deficiencies are persistent in VH3 and VH4.

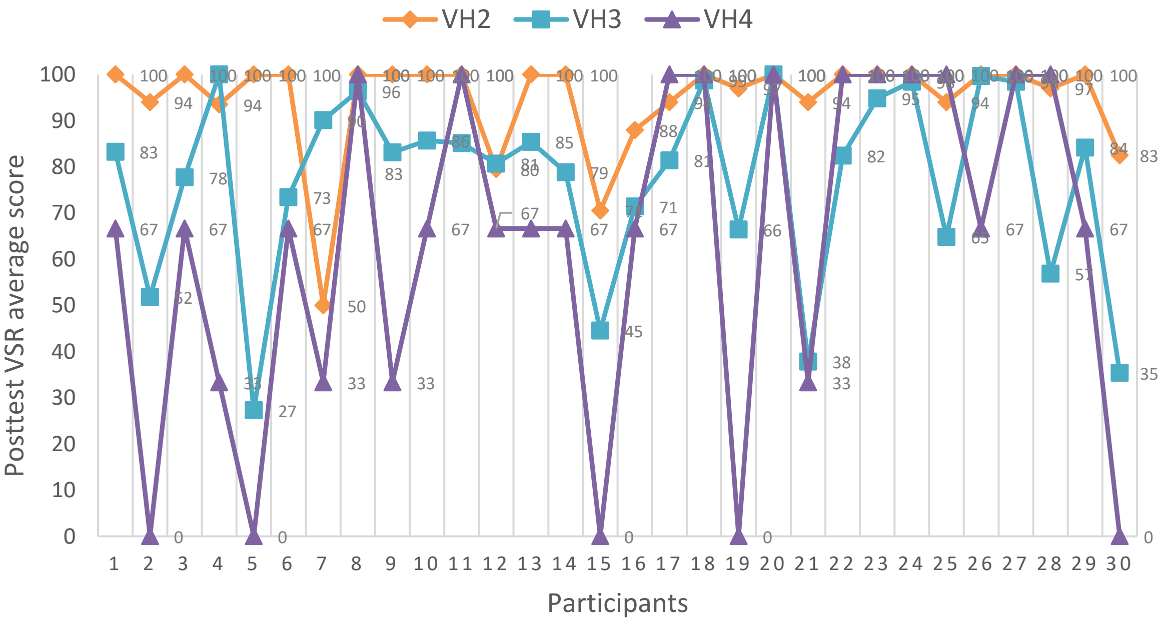


Figure 4.5: Individual participant performance per VH level for the VSR posttest

In congruence with Table 4.7, clear improvements were made on all three VH levels but more so for VH3 and VH4, as VH2 was already on quite a high level on the pretest. Of the nine participants who scored 0% on VH4, three participants showed no improvement on this level. Participant 2 decreased in VH4 performance from 33% to 0% and participant 30 decreased performance on VH4 from 67% to 0%. Question items 25 and 26 represented two of the three instances of VH4 type questions and required participants to mentally solve a rotation problem and choose between A, B, C, or D as the correct answer. A possible explanation for the reduction in performance of Participant 2 and Participant 30 may be that they guessed correctly by chance for the pretest, and incorrectly for the posttest. From this finding, the reliability of multiple-choice items becomes questionable.

In contrast, Question 13 was a VH4 type question where a true length had to be determined by construction and guesswork was not possible. Participant 30 scored 100% for this item in the pretest and 0% in the posttest. Participant 30 may have run out of time as this question was not attempted in the posttest.

4.3.4 Findings for the second sub-question

The second sub-question: *How can student performance in VSR be described in terms of the hierarchical levels of the van Hiele model?*

The group average for the VSR pretest improved from 49% to 76%. This improvement is significant, but the results indicate a gradual decrease in performance as the VH levels progressed hierarchically, with VH2 improving from 79% to 95%; VH3 improving from 45% to 77%; and VH4 improving from 39% to 63%. This sliding scale is observable in both the pretest and posttest as the VH levels progress hierarchically. This trend may appear differently in a follow-up pretest/posttest where a broader range of question items are included and equalised in terms of the number of items for VH2, VH3, and VH4 type questions. Question items where participants scored higher on the pretest than the posttest should be scrutinised for reliability. The possibility exists for multiple-choice type questions to allow too much room for "guessing" answers correctly.

It is interesting to note that performance became poorer as the VH levels became higher which is congruent with other studies that were conducted in geometry (Gunhan, 2014; Alex & Mammen, 2018). From the sliding scale in performance for VH2, VH3, and VH4 as noted in the previous paragraph, it seems that deficiencies exist on VH3 and are more pronounced on VH4. From the cognitive descriptors listed in Table 4.2 (Appendix E), I can infer the following main relationships between the average scores and descriptors of cognitive thinking for VH2, VH3, and VH4:

On VH2, some participants were lacking in their construction of geometrical figures and the use of auxiliary views for proper figure orientation. On VH3, deficiencies were observed in projection system methodology, as was evidenced by the lack of using a system to link figure properties across multi views. This problem was especially evident where simple 2D orthographic views had to be converted to 3D isometric views (Questions 3 and 8).

On VH4, participants displayed difficulties in identifying obscured features and foreshortening of lines in 3D space. Most participants (63%) performed poorly in constructing the true length of slanted lines in the orthographic plane during the pretest, and 37% still got it wrong in the posttest. The construction of true lengths and shapes requires a high degree of VSR and AR as constructions have to be performed on at least two multiviews where their geometric properties must be simultaneously related to each other. Section 4.4 relates the van Hiele cognitive descriptors more specifically to reasoning per question type for question items 1, 4, 6, 7, 24, 25, 26, and 27.

4.4 Instrument 1: Comparing test scores with Van Hiele levels

In this section, the third sub-question is answered namely: *How can student performance in VSR be described in terms of the van Hiele descriptors?*

With the view to discover relationships between the van Hiele cognitive descriptors and the decision pathways typical for reasoning with EGD content, I analysed each of the 12 VSR question items and tabulated a solution process for each instance. I followed the same process as I did with Figure 4.1, where Table 4.2 (Appendix E) provides the cognitive descriptors.

Table 4.7 presents the mean percentages, VH levels and summarised cognitive descriptors (from Table 4.2, Appendix E) for the eight of the 12 VSR pretest items that were chosen for qualitative discussion. These eight VSR test items are the same as those discussed in Section 4.3.1, and the reason for choosing these items refers to their peculiarities as explained in those sections.

Table 4.7: VSR test items aligned with van Hiele levels and cognitive descriptors

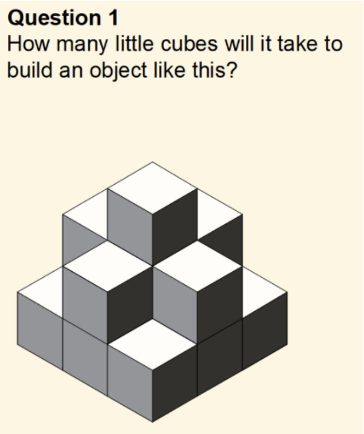
VH2		
Items	**Mean**	**Cognitive descriptors**
1 6	89 69	1. Analyse questions, their given data, and paper space for drawing position. 2. Differentiate between types of shapes. 3. Construct basic geometrical figures according to their governing properties. 4. Reason inductively. 5. Relate simple coordinate features across orthographic views by projection system. 6. Apply notation/numbering system across different views of the same figure. Describe conventions according to their function and apply them correctly
VH3		
Items	**Mean**	**Cognitive descriptors**
4 7 24 27	52 14 70 43	7. Follow logical arguments in simple deductive reasoning. 8. Create multiple mental images. Mentally rotate and section images. 9. Visualise relationship of images and how they are orientated to each other. 10. Order figure-properties in their various orientations and rotations and connect them deductively. 11. Transfer properties from an auxiliary view to any other orthographic view or true view with correct rotation/orientation. 12. Deconstruct 3D objects into their constituent 2D Nets
VH4		
Items	**Mean**	**Cognitive descriptors**
25 26	47 33	13. Manipulate intrinsic characteristics of relations. 14. Recognise which parts of figures become obscured/hidden when rotated through orthographic planes. 15. Logical deduction of the nature of geometrical transformation is required to maintain the original figure properties and compare the transformed properties. 16. Manipulate intrinsic characteristics of relations by applying different line types. 17. Construct true lengths from compound slants in orthographic views. 18. Surface development.

Table 4.7 shows that there were two test items on VH2, four test items on VH3, and two items on VH4. The cognitive descriptors in Table 4.7 were derived from the complete list of cognitive descriptors as contained in Table 4.2 (Appendix E). Table 4.7 forms the core of the qualitative discussion in Section 4.4.1 of the eight question items selected for qualitative analysis.

4.4.1 Analysis of Visuospatial test items using VH cognitive descriptors

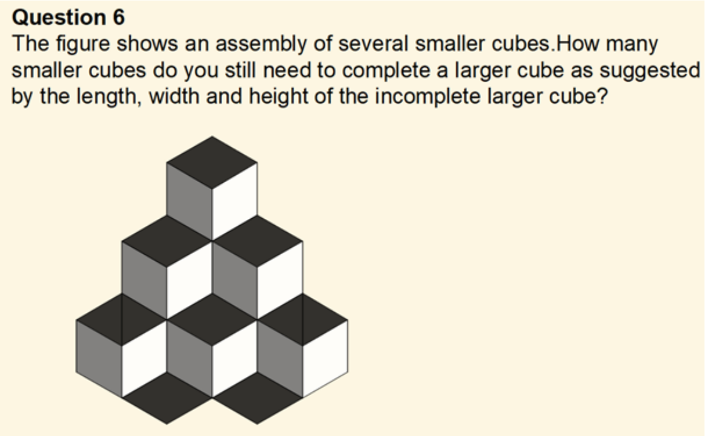
4.4.1.1 Test items 1 and 6 (VH2)

For test item 1, the group scored a pre-average of 89% and a post-average of 96%. For test item 6, the group scored a pre-average of 69% and post average of 93%. These two test items are shown in Figures 4.6 and 4.7.



Question 1
How many little cubes will it take to build an object like this?

Figure 4.6: VSR mental solving



Question 6
The figure shows an assembly of several smaller cubes.How many smaller cubes do you still need to complete a larger cube as suggested by the length, width and height of the incomplete larger cube?

Figure 4.7: VSR mental solving

From Table 4.7, under the VH2 heading, the minimum cognitive skills required to solve Questions 1 and 6 would include knowledge of the governing properties of a cube, its geometrical construction and inductive/deductive reasoning. The reasoning process may switch rapidly and iteratively between inductive and deductive reasoning. Inductive reasoning uses prior knowledge to discover new principles, unlike deductive reasoning, where established principles are used to discover new facts (Kahneman, 2011). Table 4.8 shows the reasoning processes for solving these two questions. Note: According to the van Hiele model, deductive reasoning is not yet possible for VH2 type questions. This notion has been questioned and dispelled by researchers as people develop differently and do not necessarily acquire the van Hiele levels of cognition in a linear manner. Gutierrez refers to this phenomenon as ‘degrees of acquisition’ as people may display fragmented acquisition per level without having acquired any level fully. The implication is that some people may have acquired deductive reasoning skills ahead of the proposed hierarchical model. This is especially true for adult participants as is the case with this study. There is a need to develop test items that distinguish between inductive and deductive reasoning in an EGD context.

Table 4.8: Solution process for Question 1 and Question 6

Question 1 (Pretest = 89% and Posttest = 96%)		
Steps	Cube Quantity	Reasoning
Bottom layer	9	Visual analysis and inductive reasoning: 3 x 3 = 9
Middle layer	5	Visual analysis and inductive reasoning: 4 outers + one hidden centre
Top layer	1	Visual analysis and inductive reasoning: I visible
Total	15	9 + 5 + 1 = 15
Question 6 (Pretest = 69% and Posttest = 93%)		
Steps	Cube Quantity	Reasoning
Larger cube	27	Visualise the invisible, larger cube to be 3x3x3 = 27.
Bottom layer	6	Visual analysis and inductive reasoning: 3 visible + 3 hidden
Middle layer	3	Visual analysis and inductive reasoning: 2 visible + 1 hidden = 3
Top layer	1	Visual analysis and inductive reasoning: I visible
Total	17	27 – 6 – 3 – 1 = 17

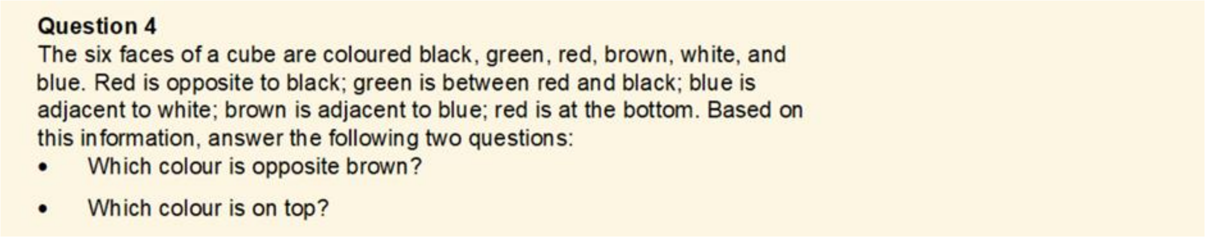
Qualitative analysis of the solution processes for Questions 1 and 6 through the van Hiele cognitive descriptors

In many instances, graphical problems do not require any EGD teaching experience or well-developed VSR, as they can be resolved by analytical reasoning (AR) (Hegarty, 2015) or heuristic processes (Hayes & Heit, 2018). From Table 4.8, it follows that Question 1 is solvable through visual analysis of Figure 4.6 by adding the cubes on the three **visible layers** through a simple inductive reasoning process. The cognitive difficulty with Question 6 is perceived as higher in the sense that the answer constitutes the quantity of **invisible cubes.** In this case, an inductive/deductive reasoning process may be followed, with the realisation that the invisible larger cube can be determined by calculating L x W x H as is suggested by the visible cubes in Figure 4.7. Through logical reasoning, the answer is provided by subtracting the sum of the visible cubes from the cubes making up the complete, larger invisible cube. Alternatively, a simple heuristic process may be followed by mentally adding cubes and noting the quantity required to complete the pattern. Having scored significantly lower on Question 6, and referring to the cognitive descriptors in Table 4.7 for VH2, the following diagnostic statement could be made: A deficiency exists in analysing the given data and/or determining the number of cubes per layer by reasoning inductively and/or deductively. For both questions, the posttest scores indicate that most participants (93%) had mastered this question.

However, as argued by Hegarty (2010), people use a variety of methods to reason, and often combine VSR and AR to process solutions to problems. It is possible that some participants lacked the level of thinking that allowed them to "see" that for Question 1, the bottom layer consisted of nine cubes. They may also have lacked the ability to visualise the fifth cube on the middle layer as obscured by the single cube on the top layer. Should this be the case, then the possibility exists that those participants had not yet acquired the level of thinking as suggested by VH3 cognitive descriptors.

4.4.1.2 Test items 4 and 7 (VH3)

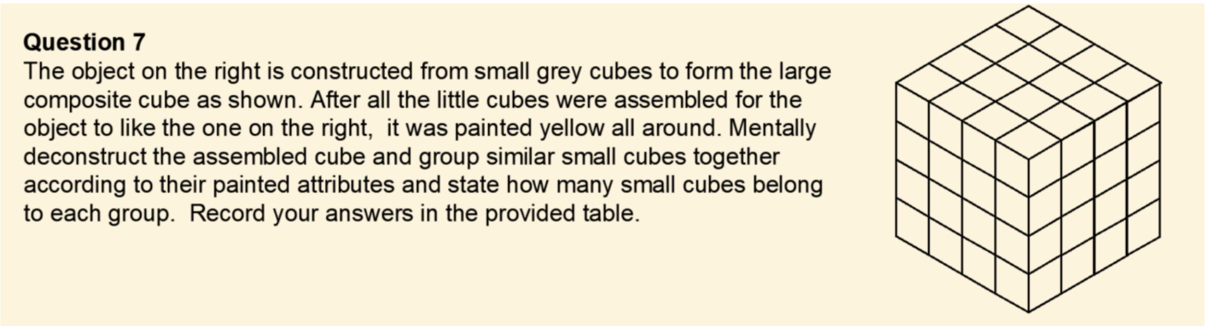
For test item 4 (Figure 4.8) the group scored a pre-average mark of 52% and a post-average mark of 92%. For test item 7 (Figure 4.9) the pre-average was 14% and the post-average was 64%.



Question 4
The six faces of a cube are coloured black, green, red, brown, white, and blue. Red is opposite to black; green is between red and black; blue is adjacent to white; brown is adjacent to blue; red is at the bottom. Based on this information, answer the following two questions:

- Which colour is opposite brown?
- Which colour is on top?

Figure 4.8: VSR mental solving



Question 7
The object on the right is constructed from small grey cubes to form the large composite cube as shown. After all the little cubes were assembled for the object to like the one on the right, it was painted yellow all around. Mentally deconstruct the assembled cube and group similar small cubes together according to their painted attributes and state how many small cubes belong to each group. Record your answers in the provided table.

Figure 4.9: VSR mental solving

Question 4 constituted a single cube, and Question 7 constituted a single cube assembled from 64 smaller cubes. On VH3, and with reference to Table 4.7, the cognitive involvement with these two questions includes: Create simple, multiple mental images; Visualise relationships of images and how they are orientated to each other; Order figure-properties in their various orientations and rotations; Order geometric properties and connect them deductively. In addition to VH3 descriptors, when solving graphical problems, it is essential to make use of short-term memory (STM) to "hold" figures in the mind's eye for the purpose of manipulating them, transforming them, and rotating them at will. Table 4.9 contains the solution path for solving Question 4.

Table 4.9: Solution process for Question 4

Question 4 (Pretest = 52% and Posttest = 92%)	
Steps	Reasoning
1	Create a mental image of the cube according to the given scenario
2	Visualise the cube and place red at the bottom. Deduce that black is on top according to the description in the question.
3	Place white and brown either side of blue as they are adjacent. Deduce that green must be opposite blue and therefore white must be opposite brown.
Answer	White is opposite to brown, and black is on top.

Qualitative analysis of the solution processes for Question 4 through the van Hiele cognitive descriptors

Most participants answered this question correctly in the posttest, which leads to the following diagnostic statement: Participants displayed improved VSR skills and their ability to order geometric properties improved to the point where they could accurately determine the relationships between figure properties. It should be noted that individuals with lower VSR and STM skills rely on increased AR and simple drawings to solve graphical solutions (Hegarty, 2010). A useful follow-up study could include participant observation during the solution process to determine reasoning preferences. Table 4.10 contains the solution process for Question 7.

Table 4.10: Solution process for Question 7

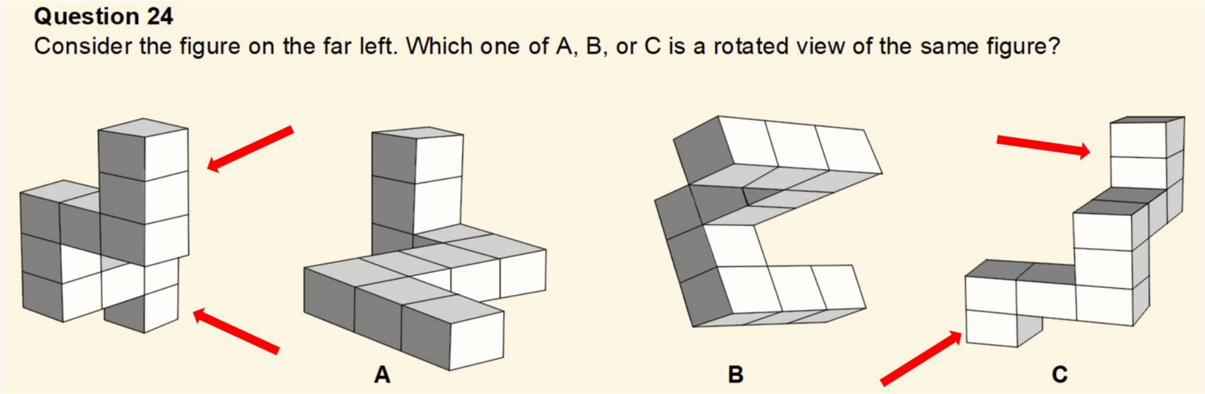
Question 7 (Pretest = 14% and Posttest = 64%)	
Steps	Reasoning
1	Create a mental image of the unpainted, assembled cube consisting of 64 smaller cubes
2	By visual analysis, deduce that there are 8 corner cubes with 3 painted faces
3	By visual analysis, deduce that there are 24 cubes with 2 painted faces
4	By visual analysis, deduce that there are 24 middle pieces with 1 painted face
5	By VSR and analysis of painted cubes, deduce that there are 8 unpainted, hidden cubes
Answer	8 (3 faces) + 24 (2 faces) + 24 (1 face) + 8 (unpainted, hidden) = 64

Qualitative analysis of the solution processes for Question 7 through the van Hiele cognitive descriptors

Participants were unable to solve this problem during the pretest, but improved significantly in the posttest. Reasoning deficiencies accounting for the poor pretest performance could include the inability to visualise a large cube consisting of smaller cubes and to deconstruct the assembled unit into individual constituent parts. Participants had difficulty in visualising the relationships between small cubes and their orientation to each other, and failed to effectively order the properties of shapes regarding their various orientations and locations within the larger cube.

4.4.1.3 Test items 24 and 27 (VH3)

For test item 24, the group scored a pre-average mark of 70% and a post-average mark of 83%. For test item 27, the pre-average was 43% and the post-average was 63%. These two test items are shown in Figures 4.10 and 4.11.



Question 24
Consider the figure on the far left. Which one of A, B, or C is a rotated view of the same figure?

Figure 4.10: VSR mental solving: Rotations

The cognitive descriptors applying to Question items 24 and 27 are the same as mentioned in Section 4.4.1.2 (Question items 4 and 7) as they are VH3-type questions. These two questions both test VSR ability in the form of rotations, but participants recorded significantly lower scores for Question 27 than for Question 24. Table 4.11 contains the solution process for Question 24.

Table 4.11: Solution process for Question 4

Question 24 (Pretest = 70% and Posttest = 83%)	
Steps	Reasoning
1	Verify that images A, B, and C contain 10 cubes each.
2	Consider the figure properties of the image on the left: 2 cubes on one end and 1 cube on the other end
3	Compare figure properties of the left image with options A, B, and C
Answer	The figure properties of option C conforms to the image on the left. See the red arrows

Qualitative analysis of the solution processes for Question 24 through the van Hiele cognitive descriptors

Using the cognitive descriptors as a diagnostic tool, participants proved to be deficient in their ability to visualise the relationships between the four images and how they were orientated to each other. In addition, the figure properties of each of the four images with regard to their geometric formation within their various orientations and rotations were incorrectly ordered due to poor visualisation and inductive/deductive reasoning.

Although Question 27, shown in Figure 4.11, is also rated as VH3, it carries a higher degree of difficulty than Question 24. In this case, Gutierrez et al. (1991) suggest that varying degrees of level acquisition exist which can account for difficulty differentials on the same level.

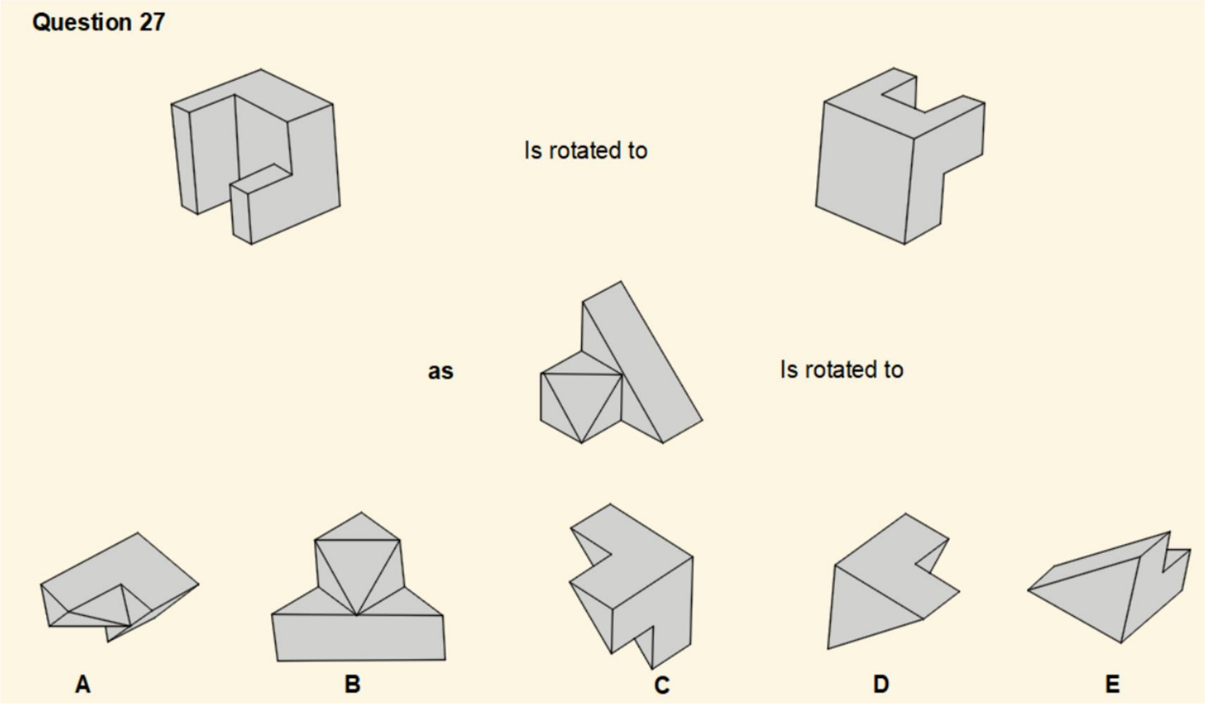


Figure 4.11: VSR mental solving: Rotations

Question 24 featured only one item, consisting of 10 identical cubes arranged to form different shapes, with possible variations of the same object rotated to different orientations. Question 27, on the other hand, featured two simple objects but each shaped intricately. The first object is rotated through a certain angle in a certain direction and the objective is to rotate the second object through the same angle and in the same direction. The new orientation must be selected from five possible answers, as shown in Figure 4.11. Table 4.12 lists the solution process for Question 27

Table 4.12: Solution process for Question 27

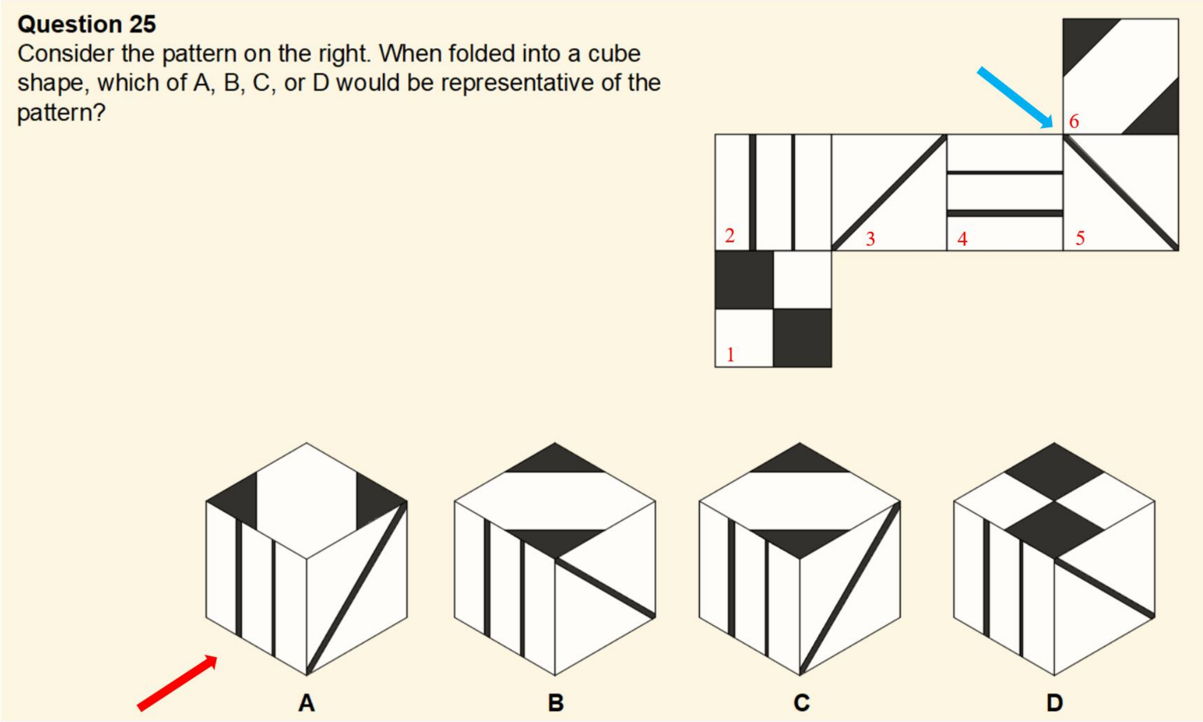
Question 27 (Pretest = 43% and Posttest = 63%)	
Steps	Reasoning
1	Study the sample scenario and deduce that the object is rotated 180° clockwise
2	Rotate the question item in the middle clockwise through 180°
3	By VSR and deduction, decide which side of the object faces 180° away before rotation
4	Answer: D resembles the mental image in step 3 rotated through 180° clockwise

Qualitative analysis of the solution processes for Question 27 through the van Hiele cognitive descriptors

From the VH3 cognitive descriptors in Table 4.7, and judging from the pretest results, participants were deficient in ordering the figure properties of both objects and maintaining relationships between figure properties in different orientations. Participants failed to order the figure properties before rotation and could therefore not correctly relate the obscured images with the visible images at a clockwise rotation of 180°. During the intervention, participants were taught the importance of first establishing the plane, magnitude and direction of rotation of the sample object before trying to visualise the correct answer. Participants were further taught to establish the plane through which the rotation must take place in order to identify the side of the object that faced 180° away before rotation. By only focusing on that single view out of the six possible sides of the object, a simple, mental rotation of 180° would reveal the answer as option D. From the foregoing explanation, it should be noted that AR and VSR are interwoven, and learners should be explicitly taught to combine strategies for improving both visualisation and deductive reasoning skills.

4.4.1.4 Test items 25 and 26 (VH4)

Figures 4.12 and 4.13 show the details for Questions 25 and 26 respectively. On the VH4 level, Question items 25 and 26 were both surface development questions where participants scored 47% for the pretest and 63% for the posttest for Question 25. For Question 26, participants scored an average of 33% for the pretest and 63% for the posttest. From Table 4.7, the cognitive descriptors applicable to these two questions include: The ability to structure figure properties to derive further information from given data; Manipulating intrinsic characteristics of relations; Recognising which parts of figures become obscured/hidden when rotated through orthographic planes; Logical deduction of the nature of geometrical transformations to maintain the original figure properties and compare with the transformed properties; Re-orientating all object features according to new reference planes.



Question 25
Consider the pattern on the right. When folded into a cube shape, which of A, B, C, or D would be representative of the pattern?

Figure 4.12: VSR mental solving: Surface development

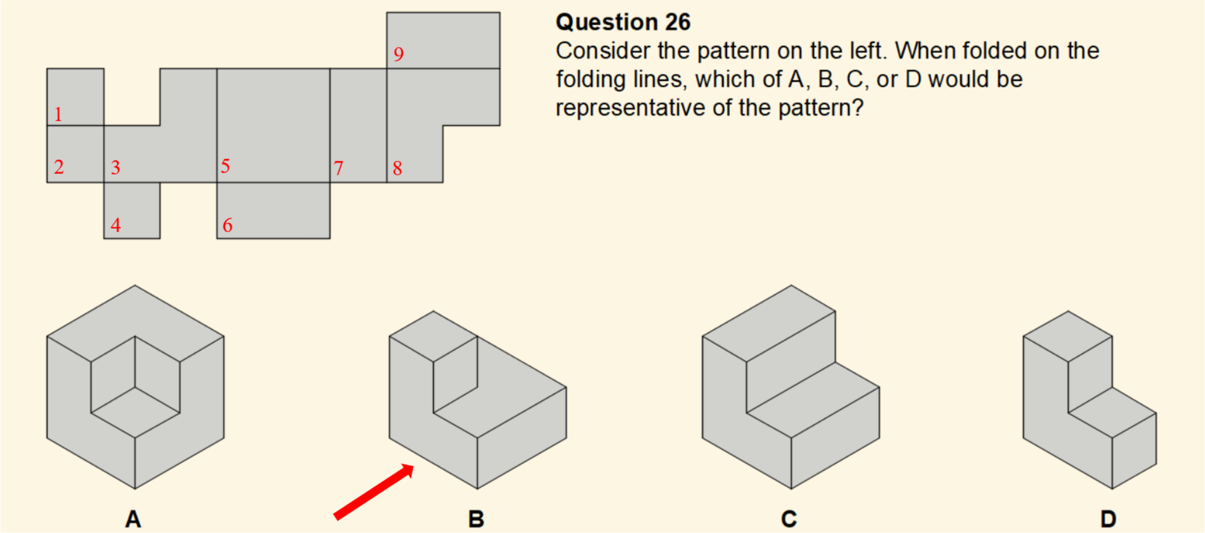
Table 4.13 contains the solution process for Question 25.

Table 4.13: Solution process for Question 25

Question 25 (Pretest = 47% and Posttest = 63%)	
Steps	Reasoning
1	Scrutinise the 4 possible answers and select a common reference plane: See Red Arrow. This plane shares the same ordered figure properties: Thick and thin vertical lines
2	Find this reference plane on the six Nets of the surface development: Net 2. Study Net 2 carefully and mentally orientate the property features of the other 5 Nets accordingly.
3	Fold Net 1 and 3 through 90°. Net 1 is at the bottom and Net 3 is on the right if the reference Net 2 is in the position of the red arrow. So far, option A satisfies the fold.
4	Fold Nets 4, 5, and 6 through 90° with respect to each other to complete the cube. Net 4 is obscured at the back right, Net 5 is obscured at the back left, and Net 6 is on top.
5	At this point, options A and C are likely answers. The position of Net 6 is determined by Net 5 which is in turn determined by Net 4 which is in turn determined by Net 3. Figure properties are now ordered according to each Net.
6	Use the corner where Nets 5 and 6 meet as a reference point and deduce that the thick line on Net 5 runs diagonally from the top to the bottom as a mirror image of Net 3.
7	From the mental process of Step 6, and considering the relations between Nets 5 and 6, it can now be deduced that option A is the correct answer.

Qualitative analysis of the solution processes for Question 25 through the van Hiele cognitive descriptors

The poor performance in the pretest for Question 25 can possibly be ascribed to a lack of logical thinking in participants not having identified common geometrical properties in the four possible answers. The identification of common properties on one 3D plane across the four 3D options creates an essential referential link to relate the six orthographic Nets of the 2D surface development with the 3D options. Participants failed to maintain Net-properties in the process of transforming the 2D surface development into a 3D cube and could, therefore, not make accurate comparisons to re-orientate Net properties in their transformed positions. The process of logical deduction becomes a guessing game in the absence of systematic cognitive reasoning. Although the posttest recorded an increase of 16%, it is clear that VH4 reasoning is not fully acquired. Table 4.14 on the next page contains the solution process for Question 26.



Question 26
Consider the pattern on the left. When folded on the folding lines, which of A, B, C, or D would be representative of the pattern?

Figure 4.13: VSR mental solving: Surface development

Table 4.14: Solution process for Question 26

Question 26 (Pretest = 33% and Posttest = 63%)	
Steps	Reasoning
1	Scrutinise cubes A, B, C, and D and select a common reference plane: Red Arrow. This plane shares the same ordered figure properties: L-shape.
2	At this point it is clear that the 2D surface development contains two L-shapes inverted to each other which obviates option A as a possibility. Option A has three visible L-shapes.
3	Net 5 is a square which obviates option D as a possible answer and option C can also be obviated by virtue of the absence of Net-properties 1, 2, and 4 as suggested by the 2D Nets. Also, unfolding C and D won't reveal inverted L-shapes.
4	Option B is the only possible answer due to an elimination process based on the Net properties of the 2D development and relating them to the four 3D options.

Qualitative analysis of the solution processes for Question 26 through the van Hiele cognitive descriptors

Participants scored significantly lower on Question 26 than on Question 25 in the pretest. The cognitive difficulty for Question 26 is considerably lower than for Question 25 due to the fact that options A, C, and D can easily be disqualified through a simple deductive reasoning process, as explained in Table 4.15. Yet, should one attempt to find a solution by only using VSR, it could well be more difficult than Question 25. It is therefore clear that participants were unable to identify a common reference plane among the four options, and were equally unable to relate 3D figure properties to the nine 2D Net-properties. Participants failed to transform the nine Nets into a 3D shape and logically relate the three visible and three obscured planes to the nine Nets in the surface development through mental rotation. An inability to relate Net properties to adjacent Net properties is also evident. However, if options A, B, C, and D all contained the nine Net properties, but in different arrangements and/or orientations, Question 26 could have been perceived as more difficult than Question 25. In such a case, a long, tedious process of eliminating options would have been required as the plane properties of Question 26 did not contain unique features (lines, triangles and chequered patterns) as with Question 25. In addition, what potentially increases the difficulty of Question 26 is the fact that the options all have different shapes resulting in three additional 2D Nets (Question 25 = 6 Nets). Considering the cognitive operators (mental rotation, orientations, obscure/visible, deductive reasoning, reference plane, transform, 3D/2D) one can infer that these cognitive operators are deficient on the VH4 level. Although the score difference (30%) between the pretest and the posttest can be considered significant, the posttest score of 63% suggests that participants are in need of additional intervention on VH4.

4.4.2 Findings for the third sub-question

In Section 4.4, the third sub-question was answered, namely: *How can student performance in VSR be described in terms of the van Hiele descriptors?*

Figure 4.14 represents the data listed in Table 4.5 on page 83 as stacked bar charts for participants 23 and 29. Each participant's pretest and posttest performance is represented by two stacked bars, the first being the posttest and the second being the pretest. I chose these two cases because of the significant differences between their pretest and posttest performances.

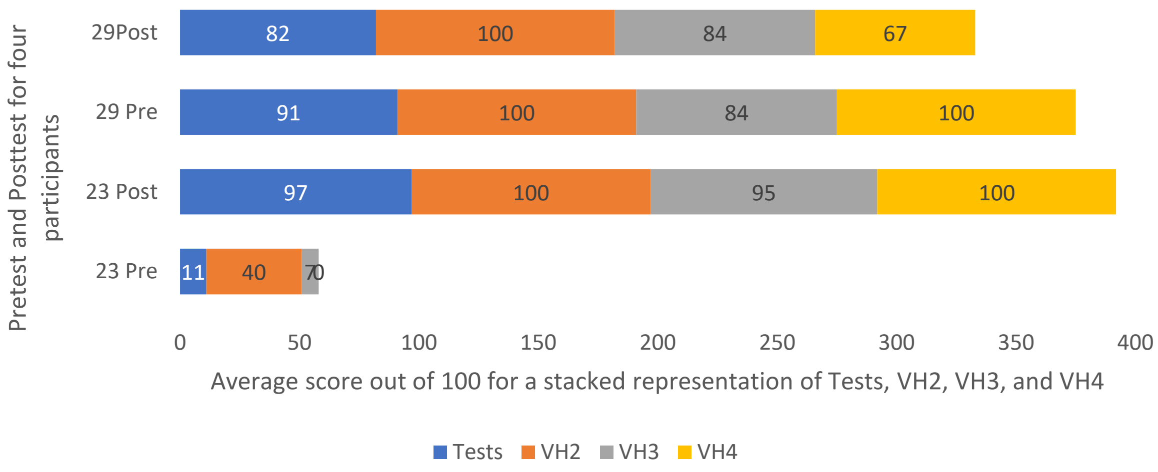


Figure 4.14: Stacked representation of Pretests, Posttests, VH2, VH3, and VH4 for participants 23 and 29

The reasoning levels as listed in Table 4.7 are used to describe the cognitive behaviour (from pretest and posttest data) of these two participants before and after the intervention. The data in Figure 4.14 represents the pretest and posttest scores for participants 23 and 29. These two cases represent significant improvement as opposed to very little improvement to demonstrate how the van Hiele descriptors could be used to describe their VSR performance with geometrical descriptors. Participant 23 performed the worst, with 11% for the VSR pretest, but then achieved one of the highest scores, 97%, for the posttest. Participant 23's reasoning on VH2 improved from 40% to 100%. From Table 4.7, this participant improved in the following areas:

- Analysis of given data, paper space and planning for multiview drawing positions.
- Geometrical constructions with reference to figure properties.
- Inductive reasoning.
- Relating simple figure properties across multiviews by using a conventional projection system and annotation system.

This participant thus demonstrated a high degree of reasoning according to VH2 cognitive descriptors.

Participant 23's reasoning on VH3 improved from 7% to 95%, which translates to:

- An improvement in deductive reasoning, forming and manipulating mental images whilst ordering and tracking figure properties with the same view and across multiviews.
- Performing rotations and mental transformations from cutting planes with reference to auxiliary views.
- Deconstructing 3D objects into their constituent geometrical 2D Nets.

Thus, Participant 23 attained a high degree of reasoning according to VH3 cognitive descriptors.

On the VH4 level, Participant 23 improved from 0% to 100% in the posttest. Improved VSR on the VH4 level includes the following:

- The ability to isolate visible and obscured features from each other and represent them by using correct line types.
- VSR improvement by holding multiple original and transformed images of objects in the mind's eye for prolonged periods.
- Relating all figure properties accurately during mental transformation of views.
- Noting lines that undergo foreshortening coupled with an understanding of how to construct true lengths and shapes from at least two multiviews.

Participant 23 attained a high degree of reasoning on VH4 according to the cognitive descriptors. In addition, improvements on VH4 go hand in hand with acquiring an axiomatic system of reasoning which underpins deductive reasoning.

Participant 29 achieved 91% for the pretest, decreasing to 82% with the posttest. I ascribe this anomaly to the possible influence of "choosing correctly by chance" where some questions allowed for multiple-choice answers. Participant 29 achieved 100% for both the pretest and posttest on the VH2 level. Participant 29 achieved 84% for both the pretest and posttest on VH3 level. Improvement of reasoning on the VH3 level is thus still possible for this participant. It seems that Participant 29 did not attain any additional reasoning skills on both VH2 and VH3. It is not possible to tell where on the VH3 level this participant requires improvement unless a detailed document analysis is performed on his/her answer sheets. This limitation of pinpointing areas of improvement on any VH level can be remedied by designing test items that are more inclusive and representative of the scope of reasoning per level.

With regard to the average test performance, Participant 29 regressed from 91% to 82%. This regression is mainly due to performance on VH4, where the pretest score of 100% dwindled to 67% in the posttest. The likelihood of multiple-choice questions being guessed incorrectly could perhaps explain this anomaly. If this is true, then pretest and posttest scores become questionable. However, from this data, Participant 29 could be said to not have attained the required level of reasoning as suggested by the cognitive descriptors for VH4.

For the van Hiele approach in EGD to be effective, practitioners should be evaluated on a wide range of cognitive reasoning types as suggested by those listed in Table 4.7. The range of questions per VH level was not comprehensive enough to represent reasoning per VH level with the focus on specific deficiencies. For the van Hiele model to function properly as a diagnostic tool, test instruments need to be designed that are focused on every aspect of cognitive reasoning per level. This study has demonstrated the utility of the van Hiele cognitive descriptors in an EGD context, but the format of the instrument is inadequate to be used as a comprehensive diagnostic instrument. Thus, it is possible to relate some of the van Hiele cognitive descriptors to how participants reasoned in this particular test. The limitations inherent to Instrument 1 make it impossible to reliably pinpoint reasoning deficiencies. I expand on this issue in Chapter 5. Yet, from the demonstrations I provided in answering sub-questions 2 and 3, I argue that a framework for a clear relationship between the van Hiele cognitive descriptors and reasoning in EGD could be established.

4.5 Instrument 2: Exploring deficiencies in conceptual understanding

The pretest and posttest intervention design (Phases A, B, and C) was helpful in providing insight into the way participants used their visuospatial reasoning (VSR) in solving EGD problems. Appendix B contains the complete task requirements for Instrument 2. Posttest results showed much improvement in academic performance when applying the same test which was used before the intervention. Yet, in acknowledgement of the limitations of Instrument 1 and to better understand deficiencies in conceptual understanding, an additional instrument was applied to answer sub-question 4: *How can the van Hiele model allow for the identification of deficiencies in conceptual understanding?*

4.5.1 Quantitative assessment of the solid geometry task

The solid geometry task (Appendix B) was assessed according to the rubric in Table 4.15. The average score for the solid geometry task for the group was 70%. The rubric items in Table 4.15 and Appendix B (the task memorandum) should be read together to align the descriptions of the rubric items with the drawing features. Note that Table 4.15 does not contain any VH3 rubric items. The rubric items were assessed according to the descriptors in Table 4.2 (Appendix E) and aligned with an appropriate VH level. It is difficult to access VSR thinking from document analyses, and for that reason there are no VSR rubric items.

Table 4.15 Assessment rubric for solid geometry task

	Item	VH Level	Description	marks
1	Analyse and Plan	2	General	5
2	Projection System	2	General	5
3	Auxiliary construction	2	Question 1 only	5
4	Auxiliary Annotation	2	Question 1 only	5
5	True Nets	4	Question 6 only	5
6	Section A-A	2	Question 1 only	5
7	Number all points	2	5 marks for each of the six questions	30
8	X-Y Reference	2	5 marks for each of the six questions	30
9	Label Cutting Points	2	5 marks for each of the six questions	30
10	Evaluate Lines & Types	4	5 marks for each of the six questions	30
11	Correctness		5 marks for each of the six questions	30
	Total			180

Figure 4.15 shows how participants performed when assessed against the first six rubric items. Reasoning deficiencies related to performance are discussed item by item by referring to the cognitive descriptors of Table 4.7. Each of the 11 rubric items is named and described by including the item number and the VH level. For example, Item 1 is written in the following manner: ***Analyse and Plan*** (Item 1/VH2). With this format, the type of cognitive action is tied to the item number and the van Hiele reasoning level. The same format is used for 10 rubric items. Item 11 was not assigned a VH level, as the correctness of a drawing does not assume one reasoning level, but rather represents a collective of all VH levels in varying degrees.

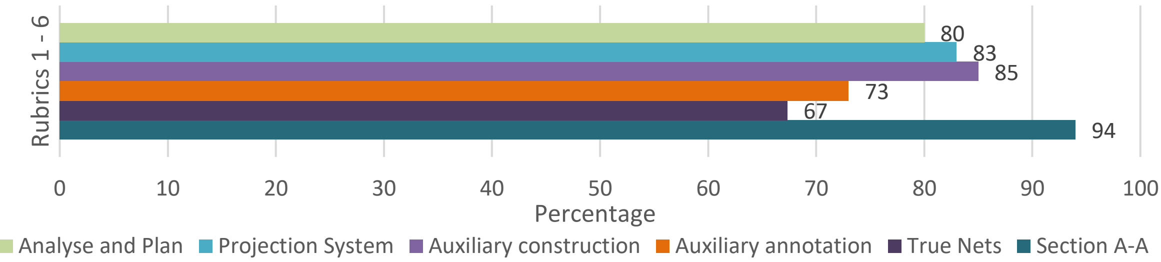


Figure 4.15: Graphical representation of the first six rubric scores for the whole group

Cognitive analysis for rubric Item 1: Analyse and Plan

For the rubric item ***Analyse and Plan*** (Item 1/VH2), participants scored an average of 80%. Most participants read and understood the questions, but 17 of the 30 participants exhibited poor planning in the layout of their drawings. Twelve participants scored full marks for this rubric item, but 18 participants presented inconsistent or haphazard use of this vital EGD convention. During the process of analysing the question specifications and planning the drawing layout, VSR plays an important role. It may be possible that the poor planning stems from poor visualisations of the proposed views in the mind's eye, but it is not distinguishable from document analyses. In Chapter 5, I make recommendations for future studies to utilise the Think Aloud Protocol (TAP) method for accessing VSR reasoning.

Cognitive analysis for rubric Item 2: Projection system

For the rubric item ***Projection system*** (Item 2/VH2), participants scored an average of 83%. The use of a projection system is meant to relate the governing properties of objects with one another through multiple orthographic views. The six questions produced six drawings of which all represented multiple views of one object. The data suggests that some participants are deficient in their understanding and application of linking object properties across multiple views.

Cognitive analysis for rubric Item 3: Auxiliary construction

For the rubric item ***Auxiliary construction*** (Item 3/VH2), participants scored an average of 85%. Ten participants presented fair to poor methodology in constructing the pentagon base and orientating it according to the given front view. These 10 participants can be said to be deficient in their knowledge of geometrical constructions. Three of the 10 participants orientated the pentagonal base incorrectly, which points to a deficiency in relating the object's front view with its base geometry. In addition, these three participants displayed deficient VSR ability by not being able to make a two-step compound rotation of the pyramid to find the correct placement of the auxiliary view as it related to the pentagon base.

Cognitive analysis for rubric Item 4: Auxiliary annotation

For the rubric item ***Auxiliary Annotation*** (Item 4/VH2), participants scored an average of 73%. According to EGD convention, the detailed annotations of the auxiliary view are to be extrapolated to all the other multiviews of the same object. Auxiliary views are critical and foundational for the production of correctly drawn views (Madsen & Madsen, 2012; Rathnam, 2018), yet eight of the 30 participants received 0% for this item. Most of the other participants (73%) scored between 80% and 100% for this item. Deficiencies exist in understanding and applying a notation/numbering system, which in turn may result in incorrectly relating coordinate features across multiple views of the same object.

Cognitive analysis for rubric Item 5: Section A-A

Most participants (94%) executed the convention for ***Section A-A*** (Item 5/VH2) correctly. Five participants made slight errors in interpreting the given front view, which points to poor analysis of the given information.

Cognitive analysis for rubric Item 6: True Nets

For the rubric item ***True Nets*** (Item 6/VH4), participants scored an average of 67%. The construction of true lengths and true shapes is historically perceived to be difficult drawing content to process (McManus, 2010; Papakostas, Troussas, Krouska, & Sgouropoulou, 2021; Martin-Gutierrez, 2010), evidenced by a relatively low score of 67%. ***True Nets*** are rated as VH4 and only applied to Question F; the total average score for Question F was 50%, which makes it the lowest scoring of all six questions. From the cognitive descriptors in Table 4.7, participants seem to be deficient on the VH4 level with regard to:

- Mental rotation of various orthographic views in sectioned and non-sectioned formats.
- Logical deduction of the nature of geometrical transformation to maintain the original figure properties and compare with the transformed properties.
- Visualisation the relationship of images and orientation to each other.
- Ordering properties of shapes in various orientations and rotations.
- Mental cutting and rotation of simple objects.
- The transfer of properties through a projection system from auxiliary views to other orthographic & true views with correct rotation/orientation.

Figure 4.16 represents the average scores for rubric item 7, **Numbering all points**, for the whole group for Questions A to F.

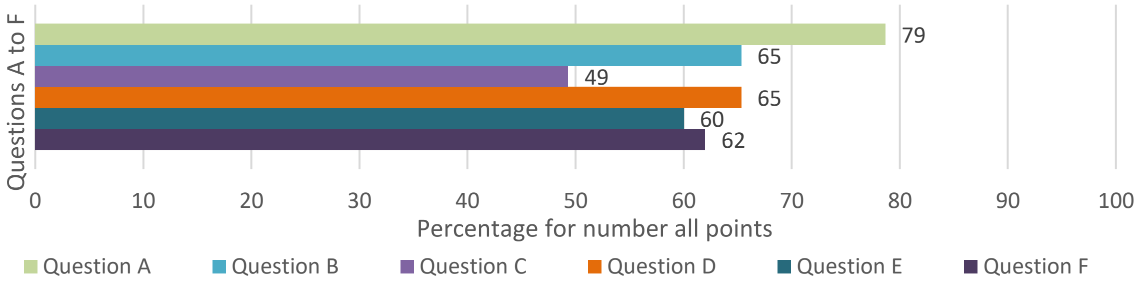


Figure 4.16: Graphical representation of scores for rubric item 7 for the whole group

Cognitive analysis for rubric Item 7: Numbering all points

The rubric item for ***Numbering all Points*** (Item 7/VH2) shows a group average of 79% for Question A, but average performance drops to between 62% and 65% for the remaining questions, with Question C at a low of 49%. The construction of the answer for Question C was virtually a mirror image of Question B, yet participants numbered it less correctly. The numbering of key coordinate points on any EGD drawing is an essential convention requirement for VSR purposes and should be executed at all times (van Leeuwen & du Plooy, 2014). Object properties within the same question/view and across other questions/views can easily be related to one another when a proper numbering or labelling convention is followed. It is especially critical for novice learners of EGD to be taught the convention of annotation, numbering, and labelling in minute detail, as it constitutes an AR method to lessen cognitive demand for both VSR and STM (Metraglia et al., 2015). Drawings can become webs of confusing lines when cross-referencing conventions are absent. Only 11 of the 38 participants received a 100% score for ***Numbering all Points*** (Item 7/VH2). One may conclude that most participants have not yet acquired the necessary conceptual understanding of this convention and how to apply it correctly.

Figure 4.17 represents the average **X-Y References** scores for rubric item 8 for the whole group for Questions A to F.

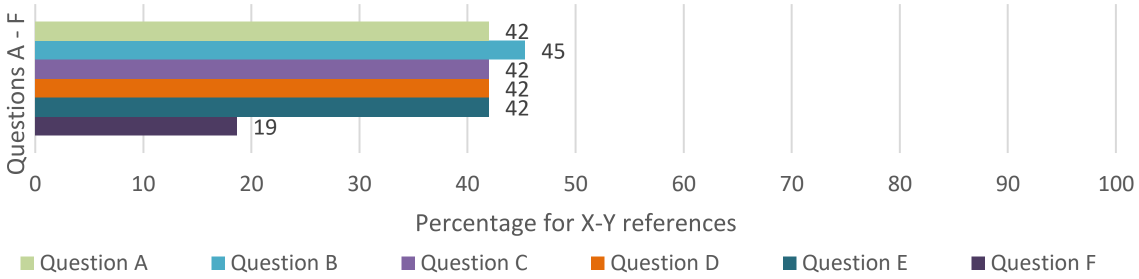


Figure 4.17: Graphical representation of scores for rubric item 8 for the whole group

Cognitive analysis for rubric Item 8: X-Y References

A vital convention in EGD is to establish indexable reference points/datum lines in the form of ***X-Y References*** (Item 8/VH2). These lines and points are essential index markers from which to project XYZ-coordinates across multiple views for relating object properties with one another (Madsen and Madsen, 2016). Without index markers for transferring measurements accurately, it becomes a guessing game for positioning projected points accurately. This is especially true for instances where lines become foreshortened, and where it becomes obligatory to fix their coordinate endpoints through at least two adjacent orthographic vectors.

The average score for Question F was 19% for rubric item 8, and for the remaining five questions, performance varied between 42% and 45%. From the cognitive descriptors in Table 4.7, the results suggest that most participants are deficient in establishing reference markers for relating coordinate features and governing properties within an object and across multiple views of the same object. It seems that a conceptual understanding of differentiating true sizes from foreshortened sizes is lacking in most participants. In addition, it appears as though participants could not relate foreshortened lines with their respective adjacent vector lines per endpoint.

Figure 4.18 represents the average scores for **Label section points** for rubric item 9 for the whole group for Questions A to F.

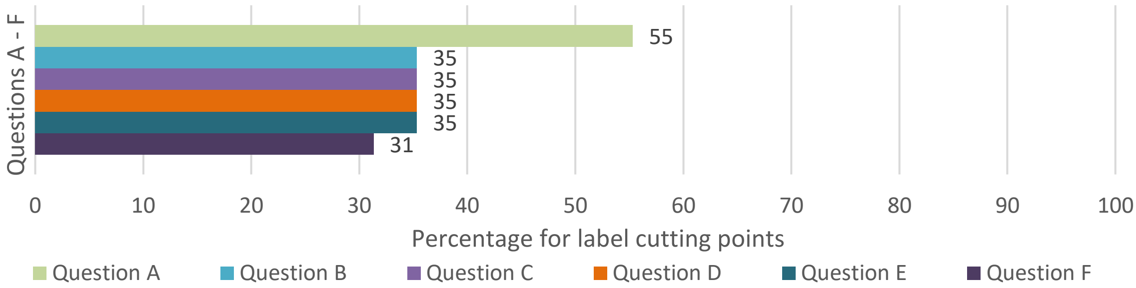


Figure 4.18: Graphical representation of scores for rubric item 9 for the whole group

Cognitive analysis for rubric Item 9: Label section points

The average group score for ***Label Section Points*** (Item 9/VH2) for Question A was 55%, with the remainder of the questions (B to F) for the same item scoring between 31% and 35%. Question F (surface development) is historically known to fall in a category of drawing solutions that are perceived as difficult. The data seems to point to the use of EGD conventions as being generally problematic, especially when it comes to annotation, numbering, labelling, and establishing reference points. Poor or absent labelling of section points may exacerbate VSR and AR in terms of the practitioner's inability to relate the governing properties of transformed geometry (due to the section plane) with the original object features across multiple views. Participants seem to be deficient in the essential labelling conventions meant to assist in identifying and aligning object properties.

Figure 4.19 represents the average scores for **Evaluating line types** for rubric item 10 for the whole group for Questions A to F.

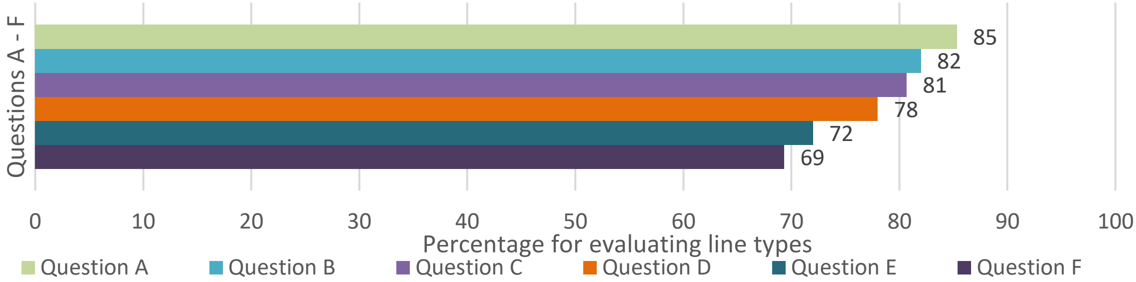


Figure 4.19: Graphical representation of scores for rubric item 10 for the whole group

Cognitive analysis for rubric Item 10: Evaluating line types

The evaluation of lines and their types relies on well-established knowledge of drawing conventions (Khoza, 2017). From Khoza's (2017) research with preservice EGD teachers, convention application is contingent on well-developed VSR, as the position of obscured lines in 3D space has to be correctly identified before applying the correct line types. ***Evaluating line types*** (Item 8/VH4) and applying them correctly is rated as VH4 due to the required combination of different reasoning levels. Yet for this item, the group performed the best of all the rubric items by achieving a group average of 78%. Question A received the highest score of 85%, where performance degenerated gradually to 69% for Question F. A trend seems to be evident of attention to detail tapering down gradually as participants progressed through the six questions.

From Table 4.7 (cognitive descriptors), the evaluation of line types requires the EGD practitioner to reason on VH2, VH3, and VH4 levels. This process requires relating governing properties with each other and across multiple views by VSR, which includes rotations and mental cutting. Logical deduction of the nature of geometrical transformation is required to maintain the original figure properties and compare them with the transformed properties. By reasoning on multiple VH levels, intrinsic characteristics of geometrical features can be represented by the correct line types as they become visible or obscured in 2D and 3D space.

At this point, it should be noted that VH4 items seem to be less problematic than VH2 items. One would expect the higher VH levels to be more problematic as they are purported to belong to a higher order of cognitive acquisition. However, some studies have shown that hierarchical–linear progression through the VH levels is not a constant, as the degree of skill acquisition varies dynamically for different people (Gutierrez, 1991). Having said that, the scope of question items utilised in this study may not be comprehensive enough to have yielded sufficient data on casting inferences on which level may or may not present more challenges.

Figure 4.20 represents the average scores for rubric item 11 for **Correctness** for the whole group for Questions A to F. Notice that rubric item 11 did not carry a VH rating as ***Correctness*** refers to the physically correct attributes of the drawing answer and is, in essence, a product of the procedures represented by the foregoing 10 rubric items. The item ***Correctness*** thus implies reasoning on all levels to have been acquired in order to achieve a 100% score for this item.

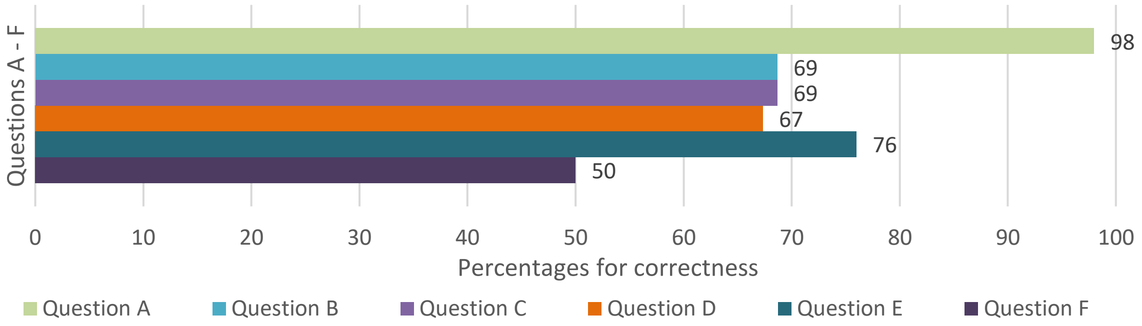


Figure 4.20: Graphical representation of scores for rubric item 11 for the whole group

Cognitive analysis for rubric Item 11: Correctness

From Figure 4.20, it is clear that Question A, at 98%, was executed correctly by most of the group. Question A required a correctly drawn front view from the given specifications and any errors in this drawing view stemmed from inaccurately drawn auxiliary views. However, performance significantly dropped over the remaining five questions. Some participants drew the orthographic views in the incorrect system or even mixed first- and third- angle placements of views. In most cases, incorrectness stemmed from errors in not adhering to the proper projection system, not numbering the governing features correctly before and after the section, transferring sizes without the use of X-Y reference lines, and not completing the line types correctly.

Even though my efforts in this study focused on demonstrating the van Hiele framework through the lens of VSR, most of the incorrect drawings could be related to poor usage of EGD conventions (CK) as they underpin the functional axiomatic systems. In the main, the conventions that are deficient are those that provide reference markers for relating object properties with each other, within an object and across multiple views. In short, they are *Auxiliary annotation, Number all points, X-Y references, and Label section points*. As a final summary of deficiencies in reasoning levels, and as derived from the cognitive descriptors listed in Table 4.7, the following reasoning types on the different VH levels are deficient in varying degrees among most participants:

- Relating governing features of objects within itself and across multiple views of the same object due to the absence of various index/reference markers.

- VSR deficiencies in rotations, ordering of object properties before and after figure transformation, identification of obscured features and mental cutting.
- Misconceptions with regard to true lengths, foreshortened lengths and transfer of appropriate measurements across various views.
- Orientation of all object features according to new reference planes and geometrical transformations.

4.5.2 Findings for the fourth sub-question

In Section 4.5, the fourth sub-question was answered, namely: *How can the van Hiele model allow for the identification of deficiencies in conceptual understanding?*

4.5.2.1 The effect of a structured axiomatic system in task execution

Figure 4.21 is arranged in ascending order of average task performance for the 30 participants. The orange line represents the ascending order of average task performance.

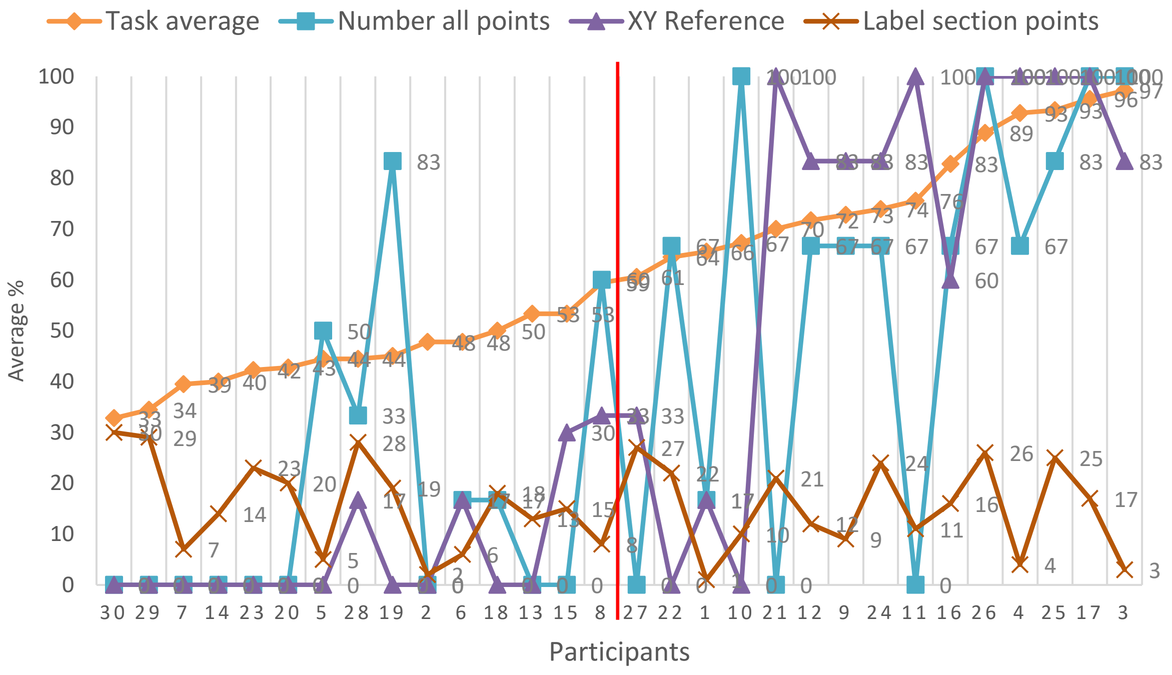


Figure 4.21: Participant task score compared with three lowest achieving rubric items

On visual inspection of the graph, it can be seen that nine of the 15 lowest-performing participants scored 0% for the rubric item **Number All Points**. Note the red vertical dividing line in the middle of the graph. Only three of the 15 higher-performing participants achieved 0% for this rubric item. For the rubric item **X-Y References**, 11 of the 15 lower performing participants scored 0% for this item and only two of the 15 higher performing participants scored 0% for this item. For the rubric item **Label Section Points**, all 30 participants performed poorly, with the highest score being 30%. For Number All points, only four of the 30 participants scored 100% and for X-Y References, only six participants scored 100%. For both these rubric items, the 100% scores were recorded within the range of 15 high-performing participants. From this data, 80% of the participants had not yet acquired the level of reasoning associated with Number All points; 87% had not yet acquired the reasoning associated with establishing X-Y Reference Points; and 100% of the participants are still deficient on the reasoning level Label Section Points.

The three lowest-scoring rubric items (Number All points, X-Y Reference Points, and Label Section Points) were all rated as VH2 reasoning level, and they constitute essential procedural steps in finding solution paths. It is, however, important to mention that a conceptual understanding of the function of such procedures is essential for VSR to be used effectively. In essence, it means that conceptual understanding forms the foundations on which the three legs of reasoning in EGD rest, namel: VSR, AR, and CK.

From the cognitive descriptors of Table 4.7, and bearing in mind that the three lowest scoring rubric items are central to the first- and third- angle orthographic systems of projection, persistent reasoning deficiencies on VH2 level could be stated as:

- Construction of basic geometrical figures according to their governing properties becomes challenging as index points are missing from which to reference a myriad of mental actions.
- To reason inductively, the EGD practitioner relies on previous situations where simple figures were transformed mentally and in freehand drawings. By virtue of this previous experience and observations, mental predictions are made to maintain figure properties in their original and transformed states. EGD prescribes a convention system whereby simple coordinate features across orthographic views can be related to one another by a projection system.
- Such procedures constitute the axiomatic system of EGD, where 3D objects are mentally manipulated within a 3D matrix where lines are parallel, and views are generated through stepped angles of ninety degrees.
- The level of reasoning as suggested by VH2 cognitive descriptors, places a high load on one's short-term memory (STM). As such, the conventions represented by the three low-scoring rubric items are historically known to provide analytical structure to both inductive and deductive reasoning.
- The annotation system encapsulation of the three low-scoring rubric items can only be applied effectively when a conceptual understanding of the whole axiomatic system is in place. From document analysis, it is clear that participants implement the annotation

system correctly in Question A, but they fail in relating object properties across all six questions. The errors that gradually creep in from question to question can be ascribed to the collapse of the system that was initially decided upon in the construction of the auxiliary view.

4.5.2.2 The effect of procedural understanding in task execution

Participants completed the solid geometry task of Instrument 2 at home over a period of two weeks. During the execution of the six questions, participants were tasked to keep a step-by-step record of their solutions paths as a detailed analysis of their thinking/reasoning. From the 30 documents that were returned in this regard, I employed a thematic inductive analysis of the record of their thinking. This process resulted in a cognition memorandum that described solution paths to the six questions in minute detail. The cognition memorandum is available as Appendix F. The cognition memorandum is thus a document that reflects the collective procedural understanding of the 30 participants. The cognition memorandum consisted of 68 procedural steps across the six questions, of which there were 48 on VH2, 13 on VH3, and 7 on VH4. Ideally, each participant should have listed these 68 steps as a complete account of thinking/reasoning about the procedures to follow for solving the six questions.

Each participant's procedural record was compared with the cognition memorandum and where they had omitted certain steps, those steps were assigned with an NR code which stands for "No Response". Figure 4.22 represents the percentage of NR responses per participant for VH2, VH3, and VH4 compared to the task average. The data on average task score is arranged in ascending order from lowest scoring participant to highest scoring participant. On visual inspection of Figure 4.22, it can be seen that only Participants 20 and 22 returned complete records of VH4 cognitive steps, while Participant 22 returned close to perfect records for VH2 and VH3 as well. Participant 12 recorded the highest NR codes and yet he/she achieved a score of 71% for the overall task average. I conducted a Spearman correlation coefficient, but no significant relationships exist between each participant's task score and NR record. Even though NR records do not seem to correlate with task performance, they may correlate with yet-to-be-collected data on teaching performance.

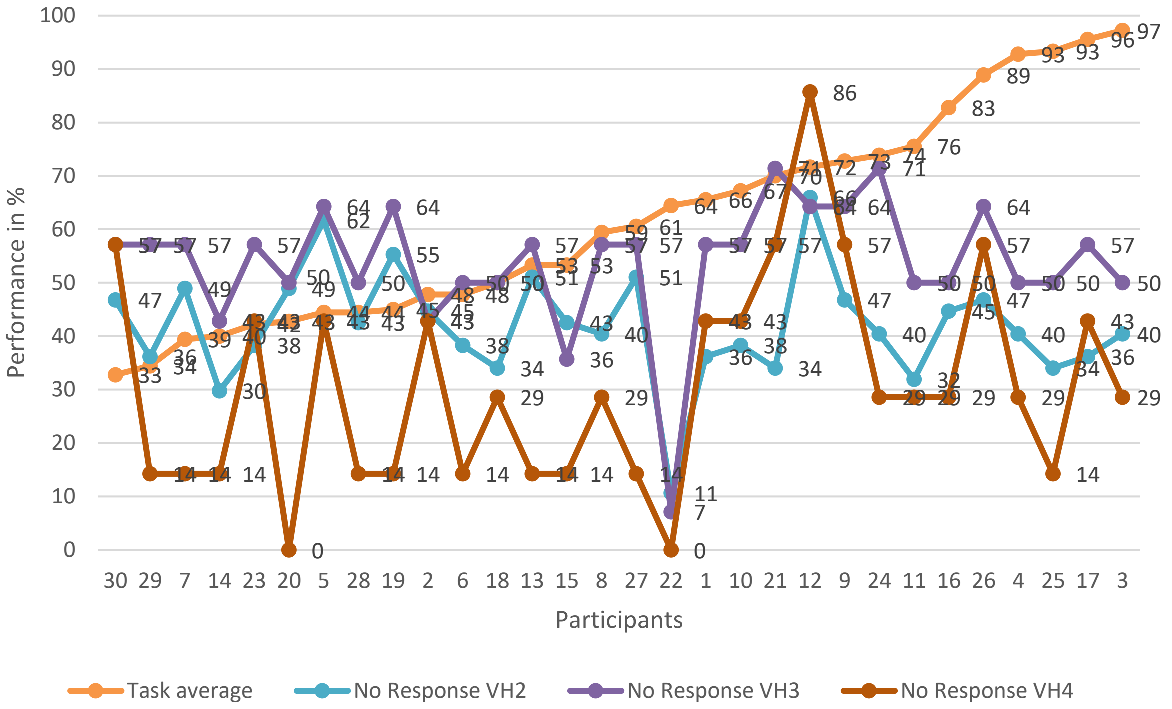


Figure 4.22: Participant task score compared with *No Response* record per VH level

Figure 4.23 compares the average task score for the 15 low-scoring (Orange line) and 15 high-scoring (Blue line) participants with the three levels of NR as well as the three lowest-scoring rubric items. From this graph, it can be seen that the group average for VH2 and VH3 NR do not deviate much between the 15 low-scoring participants and 15 high-scoring participants. On Vh4, the high-scoring participants omitted 37% of the cognitive steps, and the lower-scoring 15 participants were slightly more thorough in their cognitive record at 24% omissions. However, the data records significant differences between the 15 low-scoring participants and 15 high-scoring participants for the three rubric items where the lowest scores were recorded. From empirical data, it is thus clear that performance related to rubric items 7, 8, and 9 (**Number All Points, X-Y References**, **Label Section Points)** account significantly for the difference in performance between the 15 low-scoring participants and 15 high-scoring participants. The NR variable does not seem to point to any cognitive levels of reasoning that are not yet fully acquired. Yet, I believe the need exists to investigate whether high or low NR records affect participants' efficacy in teaching EGD content, and what the nature and extent of possible differentials in that regard may be. Such an investigation should focus on the relationship between the procedural and conceptual understanding of participants. It may be that participants lack a conceptual understanding of procedures, which may account for the high record of NR associated with some of the participants.

The available data from Instrument 1 and Instrument 2 are not appropriate for making a determination on how participants reason. Sub-question three can thus not be fully answered due to the nature of the data. A future Talk Aloud Protocol (TAP) study ought to provide sufficient data on how participants reason while establishing solution paths to similar solid geometry tasks and should be designed to also gauge participants ability to explain such solution paths in terms of the cognition memorandum for solid geometry.

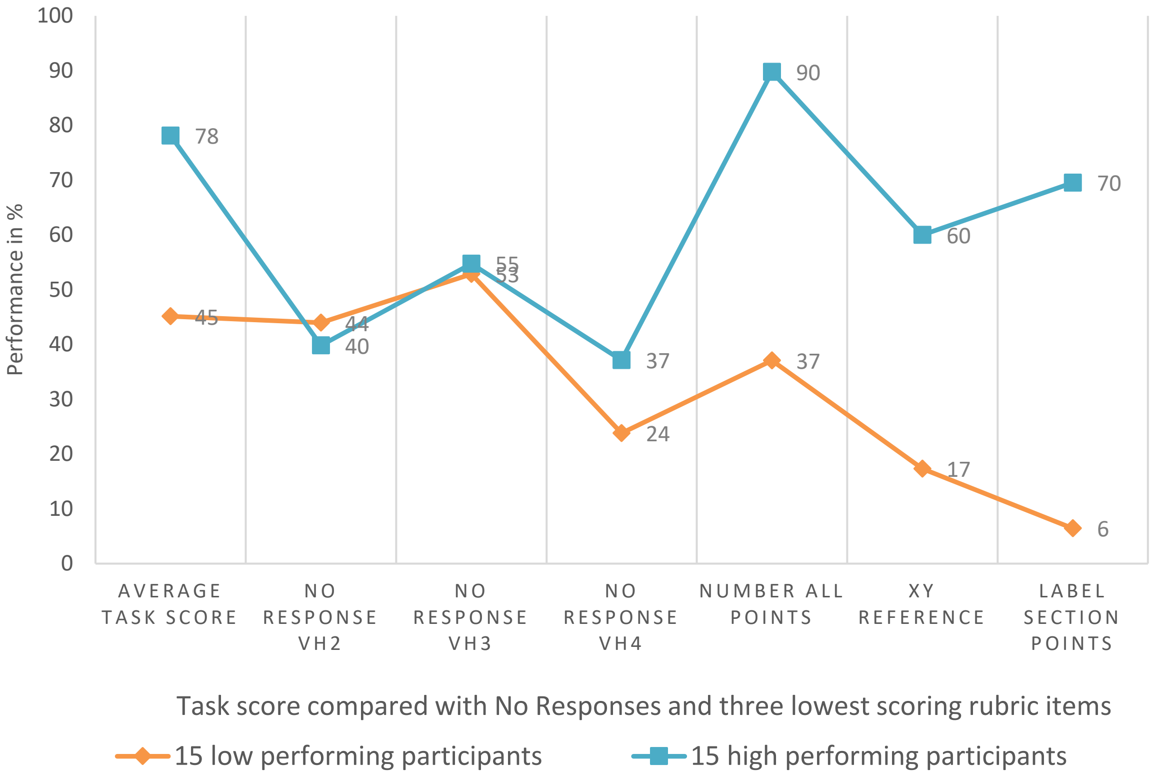


Figure 4.23: Participant task score compared with NR records and three lowest achieving rubric items for the 15 low and 15 high-scoring participants.

4.6 In summary

The nature of the cognitive scope of the two instruments employed in this study negated the lowest and highest levels of the van Hiele model, namely VH1 and VH5. I have provided a wide range of quantitative data spanning the two instruments that offered a platform onto which I demonstrated the application of the van Hiele levels of reasoning. Where quantitative data are useful in determining cognitive areas of concern, well-defined cognitive descriptors per reasoning type/level are essential in pinpointing deficiencies in conceptual understanding. Armed with clear indicators of conceptual deficiencies due to reasoning levels that may be partially acquired, this study provides the first footings for the development of a comprehensive teaching and learning framework for EGD.

With this study, I first demonstrated how the van Hiele model can be related to EGD content. Second, I showed how performance in EGD can be qualitatively described in terms of a hierarchy of cognitive reasoning descriptors. Last, I demonstrated how the van Hiele model can pinpoint persistent areas of conceptual deficiency. I acknowledge the weaknesses and limitations of the instruments used for data collection to fully describe all cognitive areas of concern. I am, however, satisfied that a clear and useful relationship exists between the van Hiele model of geometric reasoning and the subject of EGD. Failings and limitations in soliciting exact descriptions of reasoning on a hierarchical structure do not point to the weakness of the van Hiele model but rather point to the inadequacies of the two research instruments.

This study did not aim to provide a full diagnostic report on the reasoning levels of participants. The purpose of this study was to demonstrate the usefulness of the van Hiele model in EGD. For the van Hiele model to be truly effective in EGD, diagnostic instruments should be designed to measure level-specific reasoning on a hierarchy that spans the five van Hiele levels. Yet, the inclusion of VH5 is debatable on the grounds that it may only hold theoretical value with little pragmatic utility. In addition, cognitive descriptors per level need to be more specific to reduce the straddling effect of multiple modes of reasoning. Finally, the acquisition of thinking associated with cognitive descriptors has to be aligned with the five learning phases to remedy conceptual deficiencies as effectively and efficiently as possible.

Conclusions and recommendations Chapter 5

5.1 Introduction and overview of the preceding chapters

In Chapter 4, I presented the results obtained from data I collected with two instruments and related the results to the literature of Chapter 2.

Chapter 1 introduced the background to my study by defining the purpose of EGD and certain problems that affect the teaching and learning experience. I made specific reference to the three essential constructs that reasoning in EGD encompasses, namely VSR, AR, and CK. I provided a research design, instruments, methods and methodology to answer the study's research questions within the van Hiele model of geometrical reasoning. I framed my paradigmatic approach and working assumptions within the van Hiele model, which served as my conceptual framework. As noted before in previous chapters, I only used VSR data to demonstrate the usefulness of the van Hiele model in an EGD context. Finally, I described the ethical principles I adhered to and how I implemented quality criteria towards a rigorous and trustworthy study.

In Chapter 2, I scrutinised the nature of EGD with a focus on the cognitive demand practitioners of EGD face. I provided literature on current obstacles in teaching and learning EGD, explored differences experienced at school level and university level, and showed how newly qualified EGD practitioners are viewed in the job markets. Appropriate teaching and learning theories centred on VSR, AR, and CK acquisition were explored and applicable elements were foregrounded that underpin the theoretical principles of the van Hiele model. I concluded Chapter 2 by merging the static and dynamic elements of the "New Typology" proposed by Newcombe and Shipley (2014) as an extension to the van Hiele model as my conceptual framework.

In Chapter 3, I explained my pragmatic stance of using a mixed-methods approach with a single case study based on a solid geometry intervention programme. I explained how I selected participants and how the involvement of MGSLG as the programme funder affected the criteria stipulated for an appropriate sample. I explained the collection, documentation and analysis of data by utilising two instruments and the particularisation strategy I used to demonstrate the usefulness of the van Hiele model. I explained the validity and reliability, and quality criteria of the quantitative and qualitative data strands that the two instruments yielded, and concluded the chapter with ethical considerations that guided my interaction with the participants and the data.

In Chapter 4, I presented the VSR data collected through Instrument 1 and Instrument 2 and subsequent results and findings.

For Instrument 1, the quantitative pretest/posttest scores were qualitatively interpreted by means of the cognitive descriptors derived from the van Hiele model (Table 4.2, Appendix E). The quantitative results were converted to qualitative descriptors through the particularisation process to demonstrate the usefulness of the van Hiele model as adapted (particularised) to EGD. Instrument 1 was designed to test VSR and CK specifically, but with the application of the cognitive descriptors stemming from the particularisation process, instances of AR were identified but not elaborated upon. I only used VSR data to answer the four sub-questions. The purpose of this study was to demonstrate how the van Hiele model could be usefully particularised to EGD by foregrounding the construct of VSR to address conceptual deficiencies in EGD. Instrument 1 data enabled me to answer the second and third sub-questions.

With Instrument 2, I presented a variety of data strands that were specifically designed to provide deeper insight into the transfer of learning from the intervention. In addition, Instrument 2 provided a qualitative strand of reasoning steps from which the Cognition Memorandum (Appendix F) was derived, but also provided a quantitative strand of difficulty perception. The cognitive descriptors that were derived from the van Hiele model were used to assign hierarchical levels of reasoning, which were denoted as VH2, VH3, and VH4. I used these cognitive descriptors to describe participants' reasoning levels and their perception of each reasoning step's cognitive difficulty to demonstrate the usefulness of the van Hiele model. This strategy made it possible to explicate test and task scores diagnostically, and it provided clear information on cognitive skills that were acquired and not acquired.

Instrument 2 provided answers to sub-question four and showed that despite a seemingly successful intervention, certain levels of reasoning had not been fully acquired. By applying the particularisation process to the van Hiele model, I was able to adequately define those persistent reasoning deficiencies within the limitations of my Instruments. By application of the hierarchical VH levels, the study provided useful information on the constructs of VSR, AR, and CK and accurately pinpointed the types of cognitive skills at play in EGD. This process highlighted the inadequacies of the intervention and inherently contains the elements for designing a focused approach to the future teaching and learning of solid geometry. In the next sections, I make proposals for how an adapted van Hiele model could be incorporated into a tailor-made pedagogy for EGD. These proposals are squarely based on the findings of Chapter 4 and the scope of existing theories that I explored in Chapter 2.

In the previous chapters, I mainly focused on VSR constructs. In this chapter, I pursue the interwoven relationship between VSR, AR, and CK as inseparable strands of finding solution paths in EGD. In Chapter 1, I mentioned that my third objective with this study was to critically describe the strengths and weaknesses of the particularised framework and propose a modified version for future use.

5.2 Addressing the secondary research questions

As mentioned before, the purpose of this study was to demonstrate how the van Hiele model could be particularised to EGD by foregrounding the construct of VSR to address conceptual deficiencies in EGD*.* To that extent, and having chosen the van Hiele model of geometric reasoning as a conceptual framework, I formulated the following primary research question to guide this study:

This study's main question was: *How can a particularised version of the van Hiele model of geometric reasoning for EGD provide a deep understanding of conceptual deficiencies in cognitive reasoning?*

Four sub-questions were formulated to answer the main question using quantitative and qualitative research. I discuss these four sub-questions in the following sections.

5.2.1 Secondary research question 1: How can the van Hiele model relate to EGD?

In order to answer this question, I had to consider the fundamental underpinnings of both the van Hiele model and reasoning in EGD to explore their relationships. In Section 4.2 of Chapter 4, I explained the process that led me to the list of 10 cognitive descriptors of Table 4.2 (Appendix E) that span the three levels of VH2, VH3, and VH4. Level 5 (VH5), known as rigour, is generally ignored by most researchers as it does not fall within the ambit of secondary-school geometry or even first-year university level. I discounted VH1 and VH5 as the EGD content featuring in the two data collection instruments for this study excluded reasoning on these two levels. What was missing at this point was a similar list of cognitive descriptors for reasoning with EGD content. From the solid geometry scenario posed in Figure 4.1 of Chapter 4, I created an isometric drawing from the given orthographic multiviews and recorded my own reasoning in a step-by-step format as accurately and comprehensively as I could. I executed an 11-step process to draw the isometric view. Afterwards, I compared each step with the 10 van Hiele cognitive descriptors, which I divided between the three VH levels; VH2 containing four descriptors, VH3 three descriptors, and VH4 three descriptors. From the 11 steps I followed to draw the isometric view, I derived 15 expanded EGD cognitive descriptors for the solid geometry exercise. From a cursory inspection of the 10 van Hiele descriptors and 15 expanded EGD descriptors of Table 4.2 (Appendix E), the reader may infer that the relationship between the two lists is quite weak, when comparing the wording. However, by careful inspection of the underlying intent inherent to each of the combined 25 descriptors, a strong relationship becomes clear. Section 4.2 of Chapter 4 provides a detailed description of the relationship between the 10 VH descriptors and the 15 EGD descriptors. For the sake of clarification, I provide an analysis of Step 3 of drawing the isometric view. The example for analysis is taken verbatim from Section 4.2 in Chapter 4:

Step 3: Analyse the scenario (a,b), and create a mental model (g,h,i) of how the 2D Nets (1,2) satisfy sufficient conditions (6) and how their properties (5,7) can be structured (9,l) to form (k) an associative 3D form. This step implies mental rotation (j) and the ability to manipulate intrinsic characteristics of relations (10).

The numbers and alphabetical characters appearing in parentheses correspond with the cognitive descriptors in Table 4.2 (Appendix E), where the numbers are used for the 10 van Hiele descriptors and alphabetical characters are used for the 15 expanded EGD descriptors. To demonstrate the relationship between the seven van Hiele descriptors and eight EGD descriptors denoted by the parentheses (see Step 3 above), Step 3 can be expressed in the following narrative form:

After analysing (a,b), the scenario’s conditions (6), the creation of a mental model (g) requires one to visualise (h) the orientation and relationship between orthographically projected (k) multiviews by ordering (5,7,i) the properties (2) of Net-shapes (1) deductively from their 2D representation to form a 3D shape (l). The process of deconstructing/reconstructing 2D Nets (l) implies the manipulation of intrinsic characteristics of relations (10).

It should be noted that the 10 van Hiele descriptors and 14 expanded EGD descriptors of Table 4.2 (Appendix E) are not purported to be comprehensive representations of reasoning descriptors. Cognitive descriptors for reasoning in EGD may include other cognition that is not mentioned in this study, and such missing descriptors may expand the range of levels to at least include VH1.

I conclude that the cognitive descriptors of the van Hiele model are directly relatable to the way EGD practitioners reason. I reached this conclusion by analysing the cognitive reasoning associated with the 11-Step solution path of Figure 4.1. A direct comparison of the 10 van Hiele cognitive descriptors with the cognitive descriptors belonging to each of the 11 steps confirms a strong relationship. Yet, additional studies are necessary to explore and compare the full content of EGD for nuanced aspects that may add additional insights.

5.2.2 Secondary research question 2: How can student performance in VSR be described in terms of the van Hiele levels?

In answering sub-question 1, I referred to Figure 4.1 of Chapter 4 and focused on Step 3 of the 11-step solution to demonstrate the strong relationship between the van Hiele model and EGD. In answering the second research question, the reader should turn to the 11-step solution process of Section 4.2.2 as well as the cognitive descriptors of Table 4.2. In order to describe student performance in terms of the van Hiele levels, I transposed the parenthesised numbers and alphabetical characters from the 11 steps as they appear in Table 4.2 (Appendix E) to Table 5.1. Table 5.1 shows how the combined van Hiele and EGD cognitive descriptors accounting for each of the 11 steps were spread over the range of VH2, VH3, and VH4. Through the 11-step process, 23 cognitive descriptors featured on the VH2 reasoning level, of which 15 were van Hiele descriptors and 8 were EGD descriptors. For VH3, there were 36 descriptors, of which 24 were van Hiele descriptors and 12 were EGD descriptors. For VH4, there were 17 descriptors, of which 9 were van Hiele descriptors and 8 were EGD descriptors. The 36 cognitive descriptors recorded for VH3 suggest that VH3 reasoning has high incidences of VSR. This observation should be further explored in follow-up research. Step 11 was a final evaluation step that involved measuring the drawing outcome against the given data and specifications. The evaluation process implies a consolidation of the input with the output of the drawing scenario, which required rapid mental switching between all cognitive descriptors in order to validate the solution path that was followed.

Table 5.1: Spread of cognitive descriptors over VH2, VH3, and VH4 (From Table 4.2 Chapter 4)

Drawing steps of Section 4.2.2 of Chapter 4	Reasoning types per VH level for van Hiele and EGD descriptors		
	VH2	**VH3**	**VH4**
1	a, 1, 2, a, b		
2	e, f, d, 3, 4	k, 7	
3	a, b, 1, 2,	g, h, i, 5, 7, l, k, j	10
4	1, 2, d	h, i, 7	o
5		7, 6, h, i, h, 6, l, l	8, 9, 10, o, m
6	c, c, d	l	
7			o, 8, o, 10
8		l, i, h, 6, 7	o, 9
9	f	6, h, i, g, j, 7	m, n, 8, n
10	c, d	l, k, 7	
11	This evaluation step implies a mix of all the cognitive descriptors		
Quantity	23	36	17

It is interesting to note that the 11-step process for Figure 4.1 required twice as many cognitive descriptors for VH3 than for VH4 and 56% more descriptors than for VH2. It is likely that this particular pattern will remain similar for other questions of this nature, but will likely differ in the degree of difficulty per question item. This could be further explored in follow-up studies.

From the pretest/posttest VSR data in Table 4.4 of Chapter 4, the average for VH2 improved from 79% to 95% (16% difference); the average for VH3 improved from 45% to 77% (32% difference); and the average for VH4 improved from 39% to 63% (24% difference). For VSR, participant performance in terms of the van Hiele levels seems to follow a gradual order of descent where scores are highest on VH2 and the least on VH4 with VH3 averaging somewhere in between. It appears that the decrease in performance through the hierarchy of reasoning levels is in alignment with other research where the van Hiele model was applied in geometry research (Alex & Mammen, 2014; Patkin & Barkai, 2014). Judging from the group average of 95% for the posttest on VH2, it appears that the 23 cognitive descriptors on that level did not pose conceivable obstacles for the participants. Although participant performance improved by 32% for VH3, and 24% for VH4, the posttest averages of 77% for VH3 and 63% for VH4 suggest that performance per VH level should be scrutinised on the basis of the individual descriptors per level.

By assigning van Hiele levels to each of the 12 VSR questions of Instrument 1, it was possible to show how participant performance was spread over VH2, VH3, and VH4. This strategy provided insight into cognitive groupings where participants fared better, on which level cognition may not have been fully acquired; and where reasoning deficiencies seem to be nested. The effect that individual cognitive descriptors per VH level have on participant performance is discussed in Section 5.2.3.

5.2.3 Secondary research question 3: How can student performance in VSR be described in terms of the van Hiele descriptors?

In Chapter 4, I selected eight of the 12 VSR question items to demonstrate the usefulness of the particularised van Hiele framework, by qualitatively analysing the quantitative scores in terms of the cognitive descriptors for VH2, VH3, and VH4. In answering sub-question 3, I describe the difference in how participants performed on Questions 1 and 6 in the light of the van Hiele cognitive descriptors and the expanded EGD descriptors of Table 4.2 (Appendix E). The parenthesised numbers and alphabetical characters of Table 4.2 denote the cognitive descriptors that apply in solving these questions.

Questions 1 and 6 (Section 4.4.1 of Chapter 4) were nearly identical; yet for the pretest, participants scored 89% on Question 1 and 69% on Question 6. For Question 1, participants had to analyse (a) the given cubic formation by the governing properties (2) of the individual constituent cubes. From prior knowledge of cube properties (2) and their construction (c) participants could inductively (3) determine the number of cubes per layer by observing that the incomplete larger cube followed a 3x3x3 arrangement. This simple inductive procedure led to the correct answer of 15 cubes.

Question 6 was very similar, yet performance dropped by 20%. For Question 6, the same reasoning as for Question 1 had to be followed to determine that the given scenario contained 10 small cubes. An additional step was however required to determine that an additional 17 small cubes were required to complete a larger cube. Although the solution was possible through pure inductive reasoning, it appears that Question 6 contains a higher level of visualisation (g,h) and deconstruction/construction of assembled units into individual constituent parts (l). From this analysis, it appears that my initial assignment of VH2 for Question 6 may have been wrong, and that it belongs on VH3 due to deductive reasoning in connecting the visible 10 cubes to the invisible 17 cubes to complete a 3x3x3 formation. Through the step-by-step application of cognitive descriptors, I have answered sub-question 3 by showing how the drop in performance between the two questions is likely due to Question 6 requiring a higher degree of VSR and the inclusion of both inductive and deductive reasoning strategies. In addition, the cognitive descriptors led me to the conclusion that Question 6 may actually be rated as a VH3 question, or that sub-levels should be applied per VH level. Several researchers have questioned the discreteness of VH levels and suggested the inclusion of sub-levels (Gutierrez et al., 1991; Burger & Shaughnessy, 1986; Battista, 2007; Pusey, 2003; Wang & Kinzel, 2014).

By focusing on the inherent meaning of each of the 10 van Hiele cognitive descriptors, it is possible to describe the difference in participant performance, especially in questions that are near identical. The answer to this question bolsters the usefulness of the van Hiele model in EGD, as it provides a cognitive explanation across a range of cognitive descriptors to explain student performance. Van Hiele level 3 (VH3) accounts for reasoning where deductive reasoning becomes more intense and formal than in VH2. The nuanced differences between VH2 and VH3 account for the performance differences between Questions 1 and 6. From a diagnostic point of view, it is very advantageous to classify performance on hierarchical levels, where such hierarchies are able to clearly distinguish between inductive/deductive reasoning and higher-order VSR. Deductive reasoning requires more effort than inductive reasoning (Kahneman, 2011); the process of transitioning between the two reasoning processes is systematic, and should be taught specifically (Heuer, 2003; Helldin, 2016).

5.2.4 Secondary research question 4: How does the van Hiele model allow for the identification of deficiencies in conceptual understanding?

At the completion of the intervention, and after the posttest had been administered, I administered Instrument 2, which consisted of a solid geometry task that required four orthographic multiviews, a true shape and a surface development. I selected Participant 30’s solid geometry task to demonstrate how the van Hiele model allows for the identification of deficiencies in conceptual understanding. There are many errors in this particular participant’s drawing, but I discuss only a selected number of errors to answer sub-question 4. A copy of Participant 30’s drawing is available as Appendix M. From document analysis of Participant 30’s drawings, the following analysis can be made according to the van Hiele cognitive descriptors:

This participant constructed (c) the pentagonal auxiliary view incorrectly. EGD practitioners follow various geometrical construction methods to produce geometrical figures according to figure properties (2) and the relationships (4) between the figure properties. This participant displays partial conceptual understanding of how to construct a pentagon. Analysis (a) of what was given and what was asked was not carried out properly, as Question E contains a true section only, but the rest of the object as it appears in 3D is missing. The coordinate points numbered x, y, and z do not align (7) according to the orthographic projection system (d). The coordinate points v_2 and w_2 also do not align. Points v_1 and w_1 share the same coordinate properties (7) but the drawing shows them to be apart. Incorrect linework (n) is applied to differentiate obscured lines from visible lines. There is a complete absence of using conventions (f) such as numbering coordinate points (e) for the original geometry and transformed geometry due to sectional lines. X-Y Reference lines between views are also missing.

The errors mentioned above can be described in terms of the van Hiele descriptors as follows: This participant shows deficiencies in geometrical constructions (c) and does not seem to grasp the relationship between figure properties (4). This participant lacks conceptual understanding of the purpose of a numbering/notation/index system (e) and how to use it to connect geometrical properties (7) across multiviews through a projection system (d). Likewise, the lack of utilising notation conventions (e) resulted in incorrect transfer (d) of figure properties from the auxiliary view (k) to the other views. This participant is deficient in manipulating intrinsic characteristics of relations (10) by not being able to use the correct line types to differentiate visible features from hidden features. From the above descriptions, one can deduce that this participant's errors were not made through haste, but are rooted in a lack of conceptual understanding of geometrical concepts and convention concepts for relating object properties with one another. VSR and AR can be applied with great effectiveness and efficiency when fundamental constructs such as a notational reference network is established and applied correctly.

Traditionally, an assessment rubric will be used to score a drawing and the student should note from the marks per rubric item on which aspects he/she should improve on. The van Hiele model allows for a detailed assessment to identify errors according to a hierarchical range of cognitive descriptors.

5.3 Contributions of the study

The findings of this study are quite diverse and may benefit the EGD community and the wider STEM community where reasoning with geometry content is in focus. Theories underpinning VSR, AR, and CK may receive positive insights from this study, especially in the way the data highlighted the strong cohesion between constructs. This study may benefit research methodology as well as teaching and learning methodologies in a positive manner.

5.3.1 Theoretical contribution

At the time of this writing, I did not become aware of instances where the van Hiele model of geometric reasoning was previously applied or particularised for EGD. Since the mathematical concepts foundational to geometry are equally shared by EGD content, it seemed appropriate to use the van Hiele model as a conceptual framework. I searched thoroughly in my effort to populate the three van Hiele levels that applied to this study with appropriate and accurate cognitive descriptors (Table 4.2, Appendix E). By studying the underlying cognitive behaviour described by the van Hiele descriptors, I particularised the ones that aligned with cognitive behaviour in EGD and used EGD terminology to describe them. This exercise provided cognitive descriptors for VH2, VH3, and VH4 levels that proved useful in explaining quantitative scores in a qualitative manner. I should caution that the cognitive descriptors in Table 4.2 (Appendix E) are not purported to be comprehensive or complete, and that much more research is needed to validate the existing ones and add new ones, with a fresh focus on defining specific cognitive actions. This process should span the earliest modules of EGD offerings (Grade 8) through to university offerings. A comprehensive and detailed analysis of cognitive involvement ought to be explored to establish a hierarchy of cognitive descriptors unique to EGD, which also represents the most appropriate sequence of progressing through the learning phases.

From the existing literature, it is clear that most geometry and EGD research is focused on VSR. While I focused on VSR in answering my research questions, it is clear that AR and CK constructs are inextricably intertwined and that reasoning in EGD depends on the strategic and complimentary use of both VSR and AR within a CK framework. This study has reinforced my contention that a deep understanding of CK and its correct application provides the fabric onto which diverse reasoning skills are woven. Having said that, I recognise that the three constructs can stand separately on their own, and I encourage the development of test items that can uniquely measure each construct in detail. Correlation studies between the three constructs will certainly add much value to our understanding of individual reasoning processes. Buckley (2018) developed a spatial factor framework (Appendix J) in this regard, and in recognition of his ideas, Schneider and McGrew (2018) call on researchers to develop instruments to validate those proposed spatial factors.

I propose that an important benefit of the particularisation process is the ability of the proposed model to supplement traditional test scores with a parallel field of cognitive reasoning types that span the constructs of VSR, AR, and CK. Test scores may be useful to identify areas of acquisition and non-acquisition of skills, but not with the diverse precision offered by hierarchical levels of cognitive skill descriptors such as were derived from the van Hiele model. I am confident that I have demonstrated the usefulness of adopting the principles of my research for further research in developing a more complete model for EGD.

5.3.2 Research methodology

The particularisation process at the core of this study brought about opportunities to utilise quantitative data (test scores) and superimpose a second qualitative data layer over the primary dataset. This strategy supplemented my primary data strand by virtue of hierarchical categories of cognitive variables that were at play in participant performance. The particularisation process proved that VH2, VH3, and VH4 levels applied to the content of both instruments. This strategy proved to be a good fit for the mixed-methods approach I followed, and once the particularisation process was complete for the three VH levels, it provided parallel strands of qualitative data to other quantitative datasets. This strategy obviated the need for additional data collection instruments. I have described my methodology and methods thoroughly, and the particularised cognitive descriptors ought to be reliably applied in similar future studies or at least provide a platform onto which new studies could be designed.

Although certain findings were corroborated between the two instruments, triangulation proved that there were divergences and convergences in the data. With Instrument 2, the apparent success of the intervention became questionable, which places a burden on evaluating the validity of the pretest/posttest outcome. Instrument 2 (Phase D) was designed to measure the transfer of learning from the intervention (Phases A, B, and C). I am of the opinion that participants became familiar with the content of the pretest over the eight-month period and subconsciously or consciously studied questions that were similar to the questions in the pretest, which may account for the significantly improved scores in the posttest. If this is the case, then the use of pretest/posttest methodology should be carefully reconsidered for future studies. The data from Instrument 2 suggests that the transfer of learning was quite poor in contrast with the significant finding of the Wilcoxon signed rank test. This anomaly may contribute to the design of future studies where programme evaluation is the focus, as results may inadvertently be misleading when employing pretests/posttests.

5.3.3 Teaching and learning methodology

My purpose with this study was to demonstrate how the van Hiele model can be particularised for EGD by using the topic of solid geometry as an example. I propose that the format of the van Hiele model as used in this study can be further developed and refined to be used as an effective teaching and learning framework within a tailored pedagogy for EGD and accordant with the principles proposed by Shulman (2005). Such a framework can benefit the EGD community twofold, as a diagnostic tool and an instructional tool.

5.3.3.1 Diagnostic framework

By using the cognitive descriptors derived from the van Hiele model, I have demonstrated that the degree of cognitive acquisition, as suggested by quantitative scores, can be qualitatively analysed to serve as a diagnostic tool. The hierarchical levels of different types of reasoning provide specific categories of cognitive skills that have to be acquired by EGD practitioners in their journey of learning EGD. When a singular drawing task is analysed against the cognitive descriptors as represented by Table 4.2, Appendix E, the different cognitive activities for that task can be assigned with appropriate VH levels and grouped under VSR, AR, and CK. Cognitive descriptors for each VH level represent different cognitive skills, yet it should be made clear that certain skills could straddle two or more levels concurrently by virtue of their uniqueness and the task requirements.

Previous studies have shown that cognitive skills are not necessarily acquired on a linear path, and skills on higher levels may be acquired while skills on lower levels may still be lacking (Patkin & Barkai, 2014). Once the task is fully described by valid cognitive descriptors spanning different VH levels, the quantitative score for that item serves to inform which cognitive skills were acquired and which ones had not yet been acquired. The framework has the potential to assess conceptual deficiencies in both teachers and learners and can be used to inform remedial action plans. Teachers should understand their own and their learners' cognitive processes in terms of appropriate learning theories and with special reference to the cognitive demands imposed by VSR, AR, and CK across hierarchical levels of acquisition. Once a complete diagnosis of reasoning deficiencies has been made, special intervention programmes and day-to-day classroom instruction can be designed around the five learning phases. The five learning phases are discussed in the next section.

5.3.3.2 Instructional framework

The van Hiele model characterises students' learning of geometry on hierarchical levels of reasoning (Fuys et al., 1988). The model also proposes five learning phases to improve students' acquisition of reasoning levels by arranging the learning environment according to their prior learning and ability to acquire new levels of reasoning (Karakus & Peker, 2015). Content that builds onto previous-level cognitive acquisitions can be strategically integrated into the learner's prior body of knowledge by taking cognisance of foundational concepts that should already be acquired. Most studies based on the van Hiele model have only considered the learner-centred part of the model that focuses on cognitive diagnostics (Slavin, 2018).

Guided by Slavin’s (2018) thoughts on van Hiele’s five learning phases, it is my contention that the teacher-centred part of the van Hiele model is just as important, and efforts to focus purely on diagnostics will not necessarily lead to practices of improved teaching and learning. The van Hiele's developed the hierarchical levels to separate different cognitive skills and to demonstrate how instructional strategies should be designed for learners to progress through the levels. The process allows learners to cycle and re-cycle through the learning phases to address the non-acquired conceptual skills until they achieve competence. From my experience with this study, the five instructional/learning phases promoted by the van Hiele model should incorporate the following cognition areas through the hierarchy of cognitive levels of reasoning:

1. Recognise and differentiate between 2D shapes and 3D objects according to their order of governing properties.
2. The principles of both informal and formal inductive and deductive reasoning to relate figure properties within same objects and across different 2D and 3D views of the same object.
3. Identify and distinguish between the necessary and sufficient conditions for a concept to form meaningful definitions towards formal arguments to justify a reasoning path.
4. Critical, logical reasoning through theorems, axioms, and definitions in the context of EGD's axiomatic systems.
5. Logical reasoning in structuring figure properties and manipulating intrinsic characteristics of relations to derive further information from given data (transition pieces with branches of interpenetration require such reasoning).
6. Technical language acquisition by which the properties of concepts can be described.
7. New relationships between concepts must be foregrounded whilst maintaining, refining and renewing existing concepts. For students to progress through the levels of reasoning, conceptual understanding of such re-arrangements must occur.
8. Van Hiele (1973, p. 94), as translated from Dutch to English by de Villiers (2010, p. 3) stressed the importance of progression with the following quote:

> The network of relations on Level 3 can only be meaningfully established, when the network of relations at Level 2 are adequately established. When the second network of relations are present in such an adequate form, that its structure becomes apparent and one can talk about it with others, then the building blocks for Level 3 are ready.

I propose a particularised version for EGD where all hierarchical levels have to reflect the cognitive progression of VSR, AR, and CK. I also foresee the possible inclusion of VH1 and VH5 levels, which were not applicable to this study due to the cognitive scope of the content under discussion. The van Hiele level proposes five teacher-driven learning stages that are carefully designed to introduce subject terminology and constructs in adherence with cognitive skills that build logically onto each other. It is incumbent on the teacher to understand the natural progression of cognitive skills, associative terminology and conventions that represent each hierarchical level. I have demonstrated that many of the cognitive descriptors that apply to geometry apply equally to EGD, but those cognitive descriptors do not constitute the full spectrum of EGD reasoning.

Further research is needed to establish hierarchical levels that represent cognitive skills from Grade 8 level to university level. From an analysis of the current South African syllabi, Level 1 will span the scope of Grade 8 and 9 EGD content with elements of Level 2 present. Levels 2, 3, and 4 will represent Grades 10, 11, and 12 content with certain elements overlapping for those three levels. Level 5 will not apply to high school and represent university level content. Once the hierarchical levels are properly defined, EGD content that represents instances of VSR, AR, and CK per level has to be identified and minutely described in terms of the cognitive levels they represent. Only then can batteries of tests be developed that accurately represent instances of VSR, AR, and CK per level. Such tests could be useful as diagnostic instruments for measuring degrees of cognitive acquisition across hierarchical levels with specific categories for VSR, AR, and CK. Having said that, it is important to remain cognisant of the interwoven nature of the three main constructs of EGD.

In the next section, I propose a graphical representation of the relationships between VSR, AR, and CK and how their individual development may influence the cognitive profile of the EGD practitioner.

5.4 Limitations

As this study unfolded, I became aware of certain unintended limitations that I had not foreseen during the design of the data collection instruments. I designed and administered the pretest before I compiled the list of cognitive descriptors spanning VH1 to VH5. After analysis of the question items, and having identified VH2, VH3, and VH4 as the only applicable levels, I realised that the 12 question items did not represent the three VH levels equally. For VSR, there were two VH2 items, seven VH3 items and three VH4 items. The unequal distribution of question items per level could have weakened the statistical results and skewed the overall performance of participants. It is also not prudent to statistically compare two VH3 items with ten VH2 items due to the unequal nature of the data. I should have first compiled the list cognitive descriptors across the five VH levels. Alternative, yet similar, question items could have been designed to ensure a balanced spread across the three VH levels.

The particularisation method was limited in the sense that the question items chosen from Grade 11 and Grade 12 CAPS content effectively cancelled opportunities to measure VH1 and VH5. In defence of the choices I made regarding the test items and intervention content, I was bound by an agreement with MGSLG to design and present a programme for reskilling high school EGD teachers on the most appropriate content. Before the intervention programme commenced, I met with the 38 participants on two separate occasions to discuss their needs and interests, and from those discussions the format of the programme came into being. The data is rich and diverse but may not technically be the best for positioning performance on VH levels against each other. Having said that, I am confident that I have suitably demonstrated the usefulness of the particularised framework, and in having done so, the purpose of the study was fulfilled.

The intervention ended when the workload for teachers peaked due to examination responsibilities. Because of that intense period, I was unable to conduct interviews. The absence of formal interview data is unfortunate, and I have only been able to offer anecdotal evidence from casual discussions with participants. However, such data would have mainly served to improve the intervention programme and falls outside the purpose of this study.

Due to participant workload at the conclusion of the programme, eight participants did not manage to complete the final solid geometry assignment. The programme commenced with a small sample of 38 participants and the reliability of the data could have been enhanced by a larger sample. I had to accept the small sample as only 38 teachers were willing to commit to the rather lengthy programme, which was offered as a hybrid online and face-to-face programme.

The fact that this case study consisted of a single group placed limitations on the type of statistical processes that were available to me. I made use of descriptive statistics to answer parts of the research questions. In addition, the cognitive descriptors that were compiled (Appendix E) are not purported to be inclusive of all types of cognition in the reasoning processes presented by participants on the three VH levels. In hindsight, my initial conception of cognitive behaviour was quite rudimentary. I searched the literature for as many cognitive descriptors as I could find that were put forward by various researchers of geometry, and particularised the ones that proved to be a good fit for EGD. As the study unfolded, I realised that many more descriptors could be valid for EGD.

Although VSR was rated as VH3 for this study, I found that the constructs of VSR cannot be limited to one level only. VSR abilities span a wide range of graphicacy factors and as such should be represented on more than one level. For instance, VH1 is traditionally the level where figures are recognised by their appearance; VH2 is characterised by learners being more aware of the properties of figures; and VH3 is where the properties can be ordered and linked across different figures through deductive reasoning. VSR should thus feature on all levels, but in the context of EGD, which is a purely graphical subject, there should be precise definitions to distinguish between levels of VSR behaviour.

For example, through the orthographic way of thinking in EGD, 2D and 3D views are mostly rotated through 90° in a certain plane. But as transformations become more complex, multiple rotations of 90° through multiple planes and directions are required to manipulate objects.

Thus, VSR lies on a continuum of VH1 to VH5, which is in congruence with the ideas of Hegarty et al. (2013; 2015). As most CK applications are procedural in nature and based on declarative knowledge, the continuum could perhaps be described as ranging from VH2 to VH3. My findings are in agreement with the ideas of Ramful et al. (2015) in the sense that AR compliments VSR and can be observed to also lie on a continuum of VH1 to VH5. Much further exploration is required to classify the different types of spatial thinking and determine their position on a reasoning hierarchy. Schneider and McGrew (2018) and Buckley (2018) have made significant inroads in this regard, and their theoretical perspectives are worth pursuing.

5.5 Recommendations

5.5.1 Recommendations for teaching and learning

In the absence of a fully-developed tailored pedagogy for EGD, I recommend the following to teachers of EGD who want to implement some of the principles I have put forward:

First and foremost, teachers should understand their own cognitive processes and those of their learners to properly understand the cognitive underpinnings of success and failure with diverse tasks. Teachers should understand the cognitive scope of diverse tasks in EGD and perform their own detailed cognitive analyses on diverse tasks in different content areas in pursuance of understanding the cognitive demand of EGD. Teachers should take cognisance of the three intertwined legs that supports EGD: VSR, AR, and CK. I have not come across literature where these three constructs are properly unpacked as a holistic approach to the teaching and learning of EGD. Future interventions should focus on the dynamic interwovenness of the three constructs that supports EGD reasoning (VSR, AR, and CK). Teachers should explicitly teach these constructs towards building the capacity for holistic reasoning. Taking the lead from the teaching and learning of languages, EGD teachers should adopt a similar approach, with VSR, AR, and CK representing the vocabulary, rules, symbols, and grammatical structures of successful EGD communication. Inductive and deductive reasoning processes, and the different spatial factors constituting VSR, should be purposefully taught in a manner which develops these highly malleable skills. This study specifically points to a hiatus in CK. Teachers must purposefully learn the ISO conventions as contained in the appropriate SANS codes of drawing conduct and deliberately teach these conventions. Successful EGD practitioners artfully interweave the constructs of VSR, AR, and CK and the finesse of this process is displayed in the fabric of their practice.

5.5.2 Recommendations for further research

Research is needed that focuses on classroom practice with an emphasis on how learners are guided through the many reasoning steps that represent VSR, AR, and CK. Research questions which could drive future research may ask whether teachers facilitate the learning of concepts in a logical, sequential manner accordant with the principles of scaffolding and whether teachers provide development opportunities for VSR, AR, and CK over and above the CAPS content. It is my contention that future research should reflect parity between these three constructs as a collective, and recognise them as essential coexisting constructs for reasoning in the context of EGD. Although my focus in this study was the constructs of VSR, it became clear that that the interactions of VSR with AR and CK are generally neglected in research, and their combined effect on EGD performance should be the focus of future studies.

In South Africa, instructional practices by teachers of EGD pivot on workbooks that contain exercises and elements of the procedural way in which EGD tasks are approached. Formal textbooks have largely been abandoned in favour of workbooks. This phenomenon has been anecdotally confirmed by casual discussions with teachers and students over many years. The reader should note that no empirical evidence can be provided for this phenomenon due to the absence of research in this regard. Few high school learners in South Africa are required to obtain textbooks, as EGD content is facilitated through these workbooks, with instruction being based on their limited content. Entire workbook syllabi are backed up by complete memoranda and step-by-step PowerPoint presentations for every exercise, which significantly reduces the need for lesson preparation. This system provides teachers with a tremendous body of resources and saves precious time. However, I share the opinion of some EGD teachers in stating that such excellent and complete resources have caused many teachers to become "blunted" to the deeper skills of VSR, AR, and CK. I have personally witnessed preservice teachers utilising these well-structured workbooks and teacher aids in their delivery of seemingly excellent micro lessons on content that they have not yet fully mastered. Yet, when tested on similar content, these same preservice teachers tend to perform poorly.

In Chapter 2, I cited literature on how all knowledge dimensions are difficult to acquire when factual knowledge is inadequate (Anis et al., 2020; Rittle-Johnson, 2017; Rittle-Johnson et al., 2015), and how "rules without reason" in a mechanical fashion lead to unsatisfactory outcomes (Hurrel, 2021, p. 59). Hurrel (2021) ascribes such reasoning to a lack of knowledge of key subject matter and found that students who portrayed such ineffective reasoning mostly conducted faulty procedures and lacked conceptual understanding. A dependency on procedural knowledge curtails conceptual understanding and leads to superficial subject knowledge and negative transfer of learning (Hurrel, 2021; Richland et al., 2012; Givvin et al., 2011). Research is needed to explore the “workbook without textbook” practice, and useful data collected to prove or disprove this approach as harmful towards the acquisition of conceptual understanding.

5.5.3 Recommendations for policy and practice

Learning of EGD should commence and progress along a logical path of acquiring cognitive skills. From the data and informal discussions with participants and students of EGD, it is evident that teacher training institutions focus mostly on satisfying the content of curricula. Yet, the constructs of VSR, AR, and CK are not purposefully taught, and preservice teachers do not possess sufficient knowledge of cognitive processes when they commence with service. Through my own informal research, I have found that first-year students with an EGD background lack essential conceptual knowledge and understanding. The same can be said for the 30 participants in this study. Although teachers may be highly skilled and effective in their own EGD practice, they need to understand the diverse and unequal cognitive reasoning processes of their learners. A pedagogy that offers a framework of hierarchical reasoning processes coupled with systematic and strategic learning phases ought to be at the core of teacher training programmes. I agree with researchers such as Mclaren (2008), Ernst and Clark (2010), and Metraglia et al. (2011) on the fact that EGD should be seen as a language and taught in similar terms.

5.6 Conclusion

By virtue of the nature of EGD content, drawing tasks are traditionally quantitatively assessed at the hand of rubrics and are sometimes accompanied by short notes from the assessor. Rubric scores may well point to areas of poor performance but do not provide comprehensive information on the acquisition or non-acquisition of essential levels of cognitive reasoning.

Application of the van Hiele model across the world has been instrumental in ameliorating reasoning deficiencies in geometry by way of the models' dualistic utility. First, in a diagnostic sense, it measures reasoning on a hierarchy of cognitive competence and second, it provides learning phases for learners to cycle through content until the required cognitive skills for a particular level are acquired. A similar model for EGD may provide a much-needed utility to diagnose cognitive obstacles and accelerate cognitive skill acquisition.

I have demonstrated that a particularisation of the van Hiele model of geometric reasoning is possible for VSR. I have demonstrated that student performance can be classified per VH level and that reasoning deficiencies can be identified through a range of cognitive descriptors. In the context of EGD, essential cognitive skills are bounded by the constructs of VSR, AR, and CK. Follow-up studies may prove whether the van Hiele model could be applied across all three constructs in the same way it was applied for VSR.

Many researchers report that VSR skills are not explicitly taught (Metraglia et al., 2016; Chen et al., 2017; Mclaren, 2008; Ernst & Clark, 2010). It is equally important for AR skills to be taught (Hegarty, 2010; Marunic & Glazar, 2014; Ruckpaul et al., 2015). Knowledge of conventions (CK) is not taught with the explicitness that it deserves, and I have concluded that CK is intuitively acquired as a "by the way" skill. Knowledge of conventions is in fact the foundation of EGD's axiomatic system. A framework based on the van Hiele model could be used by teachers to assess their learners’ cognition on hierarchical levels in order to arrange their learning experience for a balanced acquisition of visuospatial reasoning, analytical reasoning and convention knowledge. Complete acquisition of reasoning skills may limit the trend to apply “rules without reason” and reinforce conceptual understanding.

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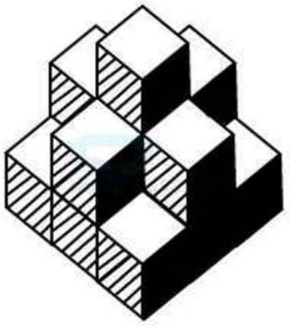
Appendices

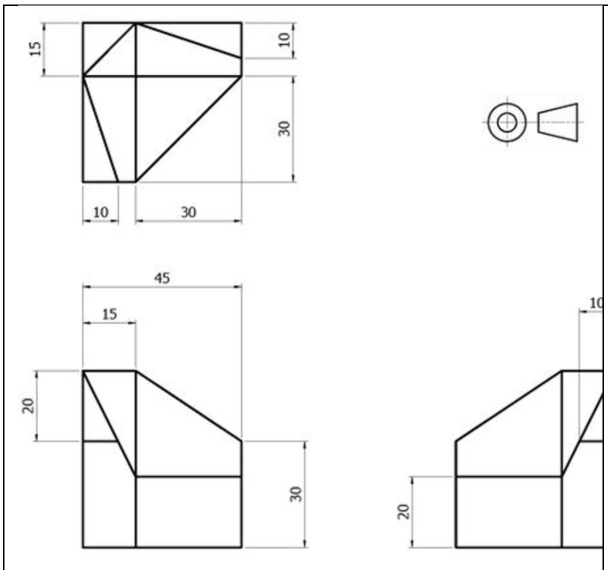
Appendix A
Instrument 1
Question paper and Memorandum for 12 Pretest and Posttest VSR Questions

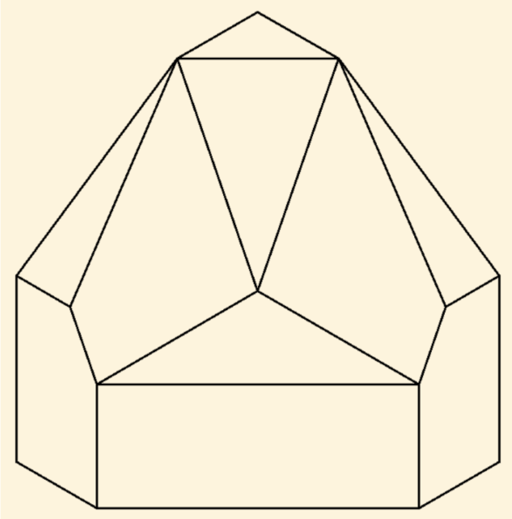
UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

Student number: ____________________

Question 1 How many little cubes will it take to build the figure as shown?  Answer 15	
Question 3 Three views of a casting are shown. Draw freehand, an isometric view of the casting provided on the grid on the right. Position the isometric view to show maximum visible detail. Isometric grid scale is 5mm per unit.	





Question 4

The six faces of a cube are coloured black, green, red, brown, white and blue. Red is opposite to black; green is between red and black; blue is adjacent to white; brown is adjacent to blue; red is at the bottom. Based on this information, answer the following two questions:

- Which colour is opposite brown?

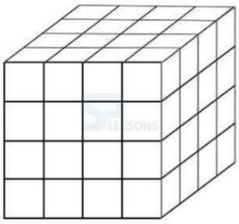
Answer
White

- Which colour is on top?

Answer
Black

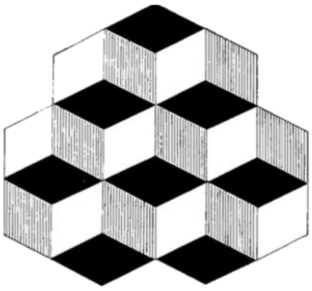
Question 7

The object below is constructed from small non-painted cubes to form a larger cube and the assembled cube is painted all around. Deconstruct the assembled cube and group similar small cubes together according to their painted attributes and state how many small cubes belong to each group. Use the table below to record your answer.



8	Corners cubes with three painted sides
8	Non-painted cubes hidden in the centre
24	Edge cubes with two painted faces (12 x 2)
24	Middle cubes with one painted face (4 x 6)

Question 6



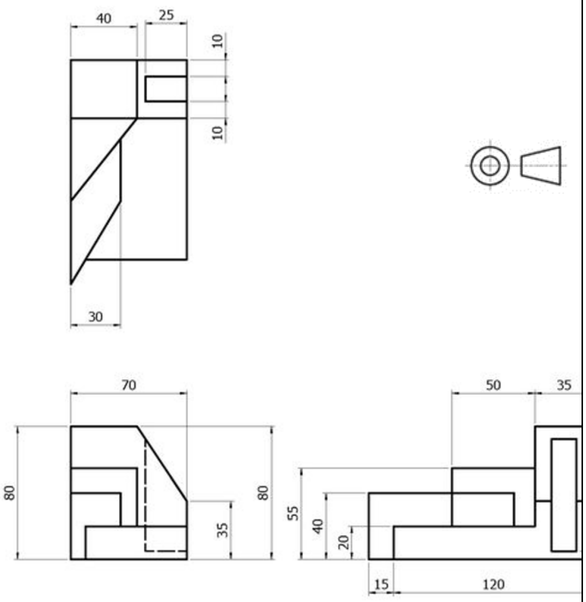
Answer

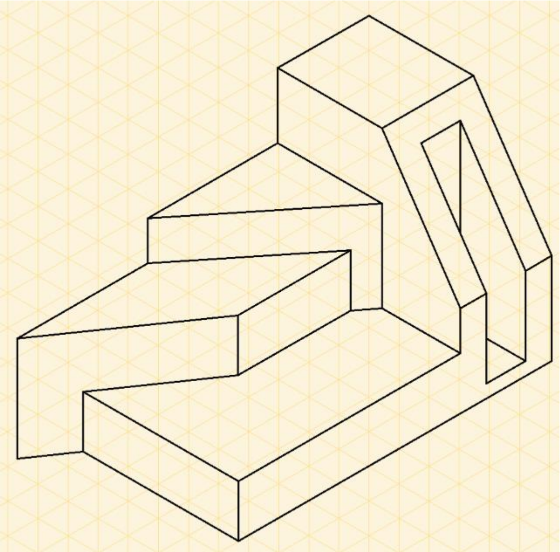
17

The figure shows an assembly of several smaller cubes. How many smaller cubes do you still need to complete a larger cube as suggested by the length, width and height of the incomplete larger cube?

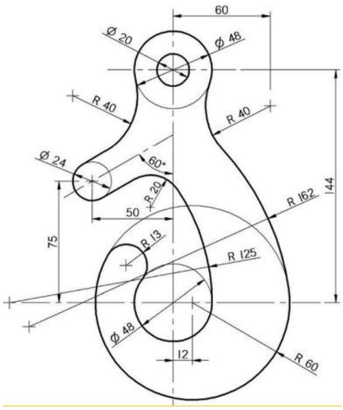
Question 8

Three views of a casting are shown. Draw freehand, an isometric view of the casting provided on the grid on the right. Position the isometric view to show maximum visible detail. Isometric grid scale is 10mm per unit.

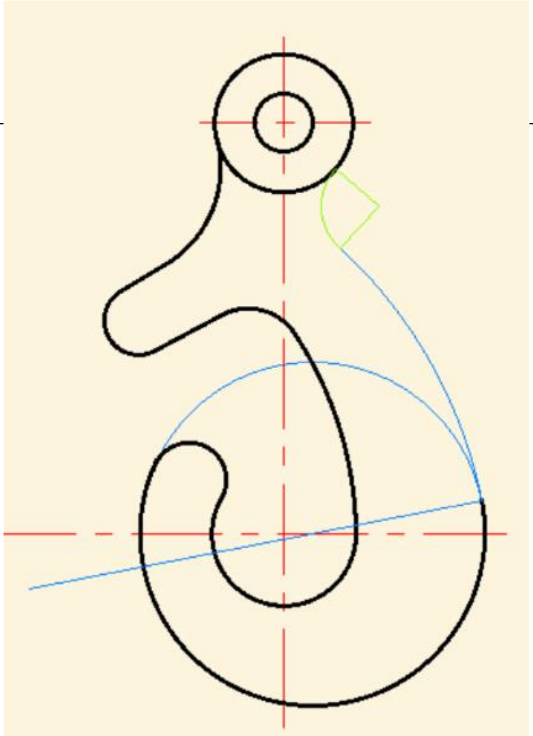




Question 9

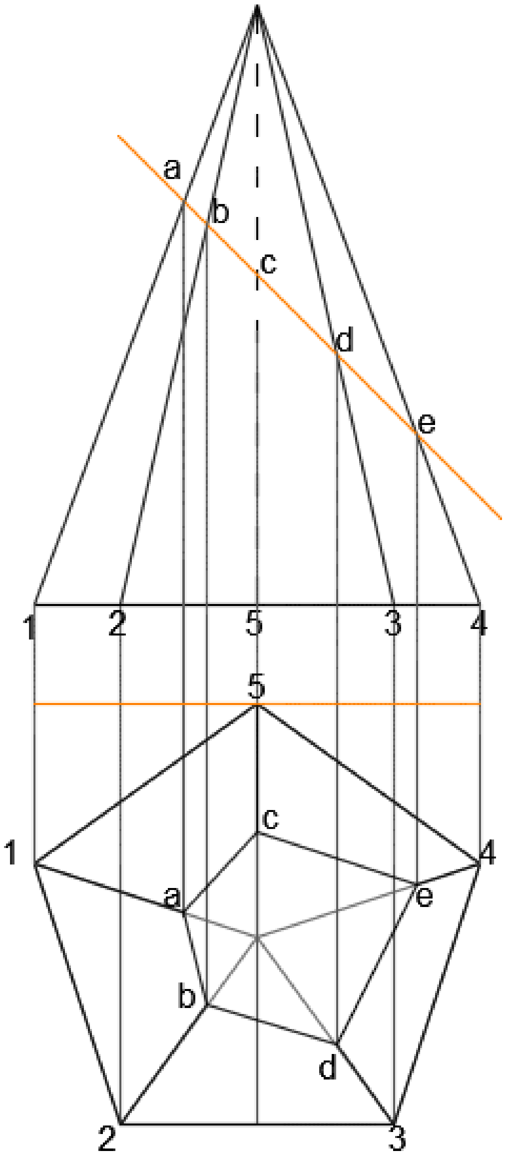


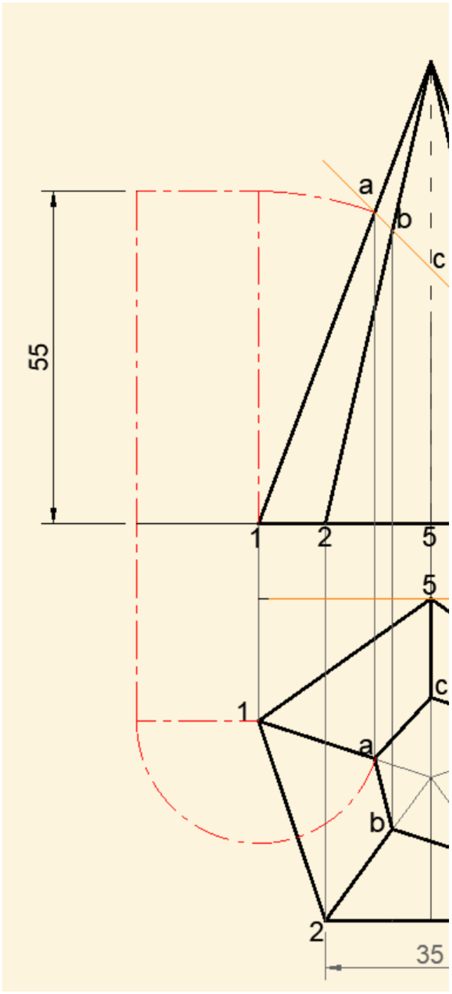
On the left is a drawing of a hook. A larger copy of the hook is shown on the right. By construction, draw in the missing lines with a compass as per the method of tangency. Use a scale factor of 1:3 as per the photocopy reduction. Show all construction lines.



Question 13

Find the true length by construction and measurement of the line-segment 1a.

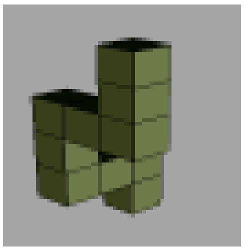


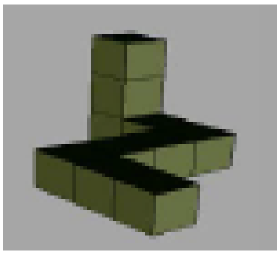


Answer: 55mm
± 1 is acceptable

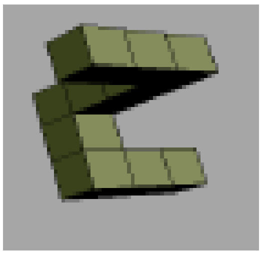
Question 24

Consider the figure on the far left. Which one of a, b or c is a rotated view of the same figure?

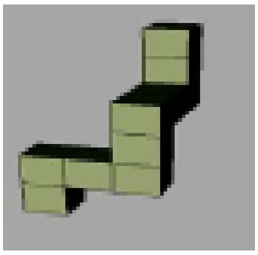




A



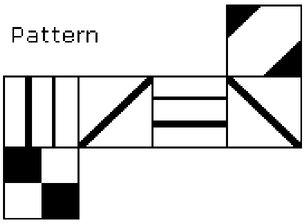
B

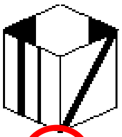


[✓ C]

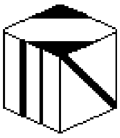
Question 25

Consider the pattern below. When folded into a cube shape, which of a, b, c or d would be representative of the pattern?

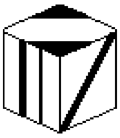




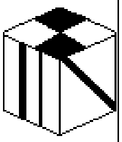
[✓ A]



B



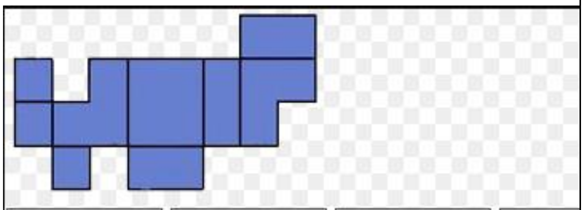
C

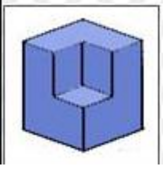


D

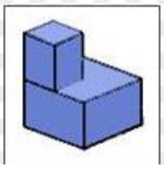
Question 26

Consider the pattern below. When folded on the folding lines, which of a, b, c or d would be representative of the pattern?

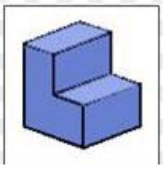




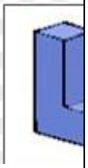
A



[✓ B]



C



D

Question 27

Answer:
D

IS ROTATED TO

AS

IS ROTATED TO

A

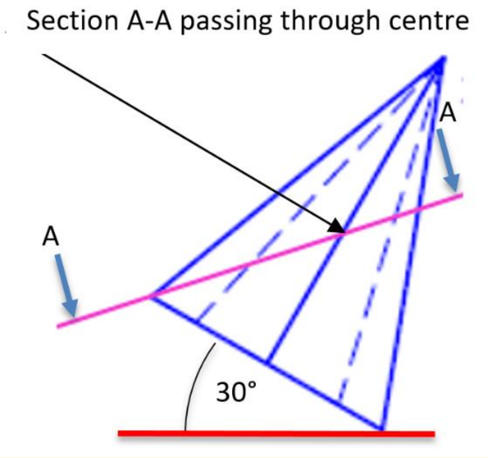
B

C

[✓ D]

E

Appendix B
Instrument 2: Solid Geometry Task



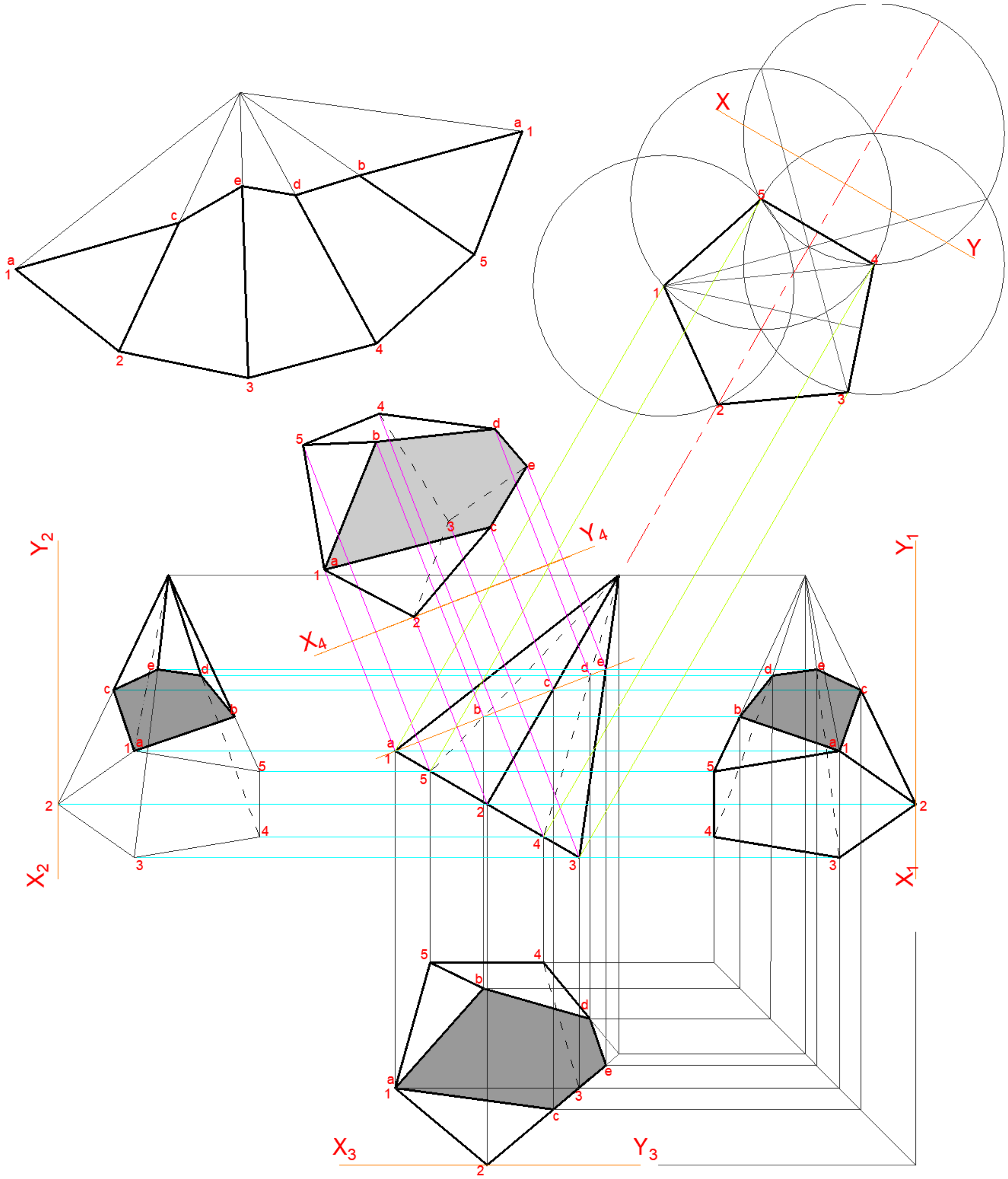
A pentagonal pyramid (Pivoting on one of the five base points) with base sides of 40mm and height of 80mm is tilted at 30° with the horizontal. Section A-A carry outs through the centre point of the figure and through one of the five base points. The following views are required in first angle orthographic projection:

A. Draw a front view (FV)
B. Draw a sectional left view (LV) (Of the lower part)
C. Draw a sectional right view (RV) (Of the upper part)
D. Draw a sectional top view (TV) (Of the lower part)
E. Draw a view of the true shape of the section (TS) (Show the remainder of the lower part also and include hidden detail)
F. Develop the surface of the figure

Instructions to participants:

The purpose of this assignment is for you to analyse your cognitive behaviour/reasoning as an analytical and visuospatial exercise in how reasoning takes place in Engineering Graphics and Design. In the execution of the drawings, analyse each cognitive activity and allocate a lower, mid or higher order cognitive involvement to that activity on a nine-point scale. Think very carefully while drawing as to what you are thinking about for each step. Some steps require a fair amount of visualisation, mental planning, mental rotation etc. Mention those instinctive steps as well. Execute in table form as per the example below and mention every conceivable step in the solution. Pretend you are teaching the solution to someone and include all the steps for maximum clarity.

Memorandum for Instrument 2



Appendix C

Alignment document

Research questions	Phase	Type	Instrument	Data analysis	Software	Purpose
Sub-question 1 How can the van Hiele model relate to EGD? Section 4.2	A	Quan		Thematic content analysis of 20 Authors' descriptions of VH2, VH3, and VH4.	None	1. In answering this question, I employed a conceptual strategy. 2. I gathered data from 20 authors to generate a list of 10 cognitive descriptors spanning VH2, VH3, and VH4. 3. Afterwards, I recorded the cognitive descriptors particular to EGD in solving the isometric scenario of Figure 4.1. 4. By comparing the two lists of cognitive descriptors, I demonstrated the relationship between the van Hiele model and EGD.
Sub-question 2 How can student performance in VSR be described in terms of the hierarchical levels of the van Hiele model? Section 4.3	A C	Quan Qual	Instrument 1: Pretest Instrument 1: Posttest	Rubric scoring Van Hiele's cognitive descriptors particularised for EGD	SPSS: Descriptive statistics, Frequencies	1. To determine on which levels the particularised cognitive descriptors for EGD featured. 2. To determine to what extent cognition featured per level.
Sub-question 3 How can student performance in VSR be described in terms of the van Hiele descriptors? Section 4.4	A C	Quan Qual	Instrument 2: Solid geometry task	Rubric scoring Van Hiele's cognitive descriptors particularised for EGD.		1. To determine the kind of reasoning used in following procedural steps when solving a solid geometry question. 2. To analyse each step to identify the cognitive descriptors that belonged to each step.
Sub-question 4 To what extent does the van Hiele model allow the identification of deficiencies in conceptual understanding? Section 4.5	D	Quan and Qual	Instrument 2: Solid geometry task	Rubric scoring,	SPSS: Descriptive statistics, Frequencies	1. Score 6 drawings according to 10 key rubric items. 2. To understand reasoning steps perceived as difficult/easy, and to seek relationships between appropriate variables. 3. To scrutinise aspects of drawing competence that evade the scope of assessment rubrics. Such as drawing quality, analysis, visualisation skills. 4. To produce a cognition memorandum of reasoning steps in solid geometry classified within themes/categories

Appendix D

Dataset for Instrument 1: VSR Pretest and Posttest

Participant	Yearsteaching	Gender	Pre1	Pre3	Pre4	Pre6	Pre7	Pre8	Pre9	Pre13	Pre24	Pre25	Pre26	Pre27	PreaverageVSR	Post1	Post3	Post4	Post6	Post7	Post8	Post9	Post13	Post24	Post25	Post26	Post27	PostAverageVSR
1	2	1	100	52	0	100	50	62	78	100	100	0	100	0	62	100	100	50	100	50	83	100	100	100	0	100	100	82
2	16	2	93	0	0	47	0	0	44	0	0	0	100	0	24	100	100	100	88	13	83	67	0	0	0	0	0	46
3	20	2	93	14	100	100	0	0	44	100	100	100	0	100	63	100	100	100	100	100	0	44	100	100	100	0	100	79
4	10	1	100	62	50	0	0	0	100	100	100	0	0	100	51	93	100	100	94	100	100	100	100	100	0	0	100	82
5	10	1	20	19	50	100	0	0	0	0	100	0	0	0	24	100	19	100	100	25	14	33	0	0	0	0	0	33
6	31	1	100	91	50	100	0	91	67	100	100	0	0	0	58	100	100	100	100	25	100	89	100	100	100	0	0	76
7	20	2	0	71	0	100	0	0	0	0	100	100	0	0	31	0	100	100	100	100	98	33	100	100	0	0	100	69
8	26	2	100	0	100	100	75	0	100	100	0	100	100	0	65	100	100	100	100	75	100	100	100	100	100	100	100	98
9	14	2	100	57	100	100	100	79	11	0	100	0	0	100	62	100	100	100	100	100	93	89	0	100	100	0	0	73
10	15	1	80	100	100	53	0	93	78	100	100	0	0	100	67	100	100	100	100	0	100	100	100	100	0	100	100	83
11	32	1	93	100	50	100	75	67	0	0	100	100	100	100	74	100	76	100	100	100	98	22	100	100	100	100	100	91
12	22	2	93	100	0	59	0	93	67	0	100	100	0	100	59	100	100	100	59	0	98	67	0	100	100	100	100	77
13	16	1	93	0	100	94	100	0	0	0	100	0	0	0	41	100	100	100	100	100	98	0	0	100	100	100	100	83
14	6	2	93	81	50	53	0	52	100	0	100	0	0	0	44	100	100	100	100	100	52	100	0	100	100	100	0	79
15	33	1	80	81	0	82	0	0	89	0	0	0	0	0	28	100	100	0	41	0	12	100	0	100	0	0	0	38
16	4	1	100	76	0	59	0	91	0	0	100	100	100	100	60	100	100	100	76	0	100	0	0	100	100	100	100	73
17	5	1	100	81	0	59	0	93	78	0	100	0	0	0	43	100	100	100	88	75	95	0	100	100	100	100	100	88
18	10	1	100	43	100	65	0	0	100	0	100	0	100	100	59	100	100	100	100	100	91	100	100	100	100	100	100	99
19	14	1	100	0	0	59	0	0	22	0	0	0	0	0	15	100	95	100	94	75	95	100	0	0	0	0	0	55
20	10	1	93	0	50	18	0	17	89	0	100	100	100	100	56	100	100	100	100	100	100	100	100	100	100	100	100	100
21	42	1	93	0	50	47	0	0	0	0	0	100	0	0	24	100	48	0	88	0	17	100	100	100	0	0	0	46
22	5	2	93	67	50	94	0	0	0	0	100	0	100	100	50	100	86	100	100	100	91	0	100	100	100	100	100	90
23	1	1	80	0	50	0	0	0	0	0	0	0	0	0	11	100	100	100	100	100	86	78	100	100	100	100	100	97
24	8	2	93	71	50	53	0	26	56	0	0	0	0	100	37	100	100	100	100	100	100	89	100	100	100	100	100	99
25	41	1	100	5	100	88	0	81	89	100	100	100	0	0	64	100	100	100	88	13	52	89	100	100	100	100	0	79
26	9	1	100	57	100	53	25	64	0	100	0	100	0	100	58	100	100	100	100	100	98	100	100	100	0	100	100	91
27	9	2	93	38	0	100	0	71	44	100	100	100	0	0	54	100	100	100	100	100	100	89	100	100	100	100	100	99
28	5	1	93	76	50	18	0	88	67	0	100	100	100	0	58	100	100	100	94	100	98	0	100	0	100	100	0	74
29	6	2	100	100	100	100	0	91	100	100	100	100	100	100	91	100	91	100	100	0	98	100	0	100	100	100	100	82
30	10	1	100	0	100	59	0	0	44	100	0	100	0	0	42	100	29	100	65	75	0	44	0	0	0	0	0	34

Appendix E

Appendix E is the same as Table 4.2: Particularisation of van Hiele cognitive descriptors for EGD

Van Hiele descriptors for geometry	Expanded EGD Descriptors
VH2: Analysis	
1. Can differentiate between types of shapes (1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) 2. Classify types of shapes according to governing properties (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) 3. Reasons inductively (informally) (11, 15) 4. Can recognise many properties of shapes but do not fully grasp the relationships between them (5, 12, 13, 14, 15, 17, 19, 20, 8, 20)	a) Analyse data, procedures, mental planning b) Analyse paper space for drawing position c) Construction of basic geometrical figures d) Relate coordinate features across orthographic/isometric orientations by projection e) Apply notation/numbering/index system across different orientations of the same figure f) Knowledge and application of rules of conventions (CK)
VH3: Informal deduction	
5. Recognize the importance of properties, the relationships between them. Can recognise a square as also being a rectangle by definition. (4, 5, 6, 13, 14, 15, 16, 17, 18) 6. Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning. (2, 3, 5, 8, 11, 13, 15, 17, 20) 7. Can order geometric properties and connect them deductively through logical arguments (1, 2, 3, 4, 5, 7, 9, 11, 13, 14, 15, 16, 17, 18, 20)	g) Create simple, multiple mental images h) Visualise relationship of images and orientation to each other i) Order properties of shapes in various orientations and rotations j) Mental cutting and rotation of simple objects k) Transfer/project properties from auxiliary/associative views to other orthographic & true views with correct rotation/orientation l) Deconstruct assembled units into individual constituent parts. Deconstruct 3D objects into constituent 2D Nets
VH4: Formal deduction	
8. Grasps the significance of deduction. Can reason formally within the context of a mathematical system (axiomatic), complete with undefined terms, axioms, and underlying logical systems with definitions and theorems. (1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) 9. Properties can now be structured to derive further information from given data. Use logic more than intuition (1, 4, 5, 7, 9, 11, 15, 20) 10. Manipulate intrinsic characteristics of relations (1, 2, 5, 9, 13, 17, 20)	m) Recognise which parts of figures become obscured/hidden when rotated through orthographic planes. This requires mental rotation of various orthographic views in sectioned and non-sectioned formats. Logical deduction of the nature of geometrical transformation is required to maintain the original figure properties and compare with the transformed properties. n) Manipulation and representation of intrinsic characteristics and relations by applying different line types and special conventions as per SANS. o) Construct true lengths and shapes from compound slants (foreshortening) in orthographic views and re-orientate all object features according to new reference planes

The 10 van Hiele cognitive descriptors in the table above were compiled from a refined list of 20 authors. The parenthesised numbers next to each of the 10 cognitive descriptors indicate which authors made mention of each descriptor. The 20 authors are listed in the following table.

No.	References for van Hiele descriptors
1	Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2019). *Elementary and middle school mathematics*. Pearson.
2	Patkin, D., & Barkai, R. (2014). Geometric thinking levels of pre-and in-service mathematics teachers at various stages of their education. *Educational Research Journal*, *29*(1/2), 1-26.
3	Karakuş, F., & Peker, M. (2015). The effects of dynamic geometry software and physical manipulatives on pre-service primary teachers’ van Hiele levels and spatial abilities. Turkish Journal of Computer and Mathematics Education (TURCOMAT), 6(3), 338-365.
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Appendix F
Cognition Memorandum

The Table in Appendix F is divided into the three categories of VH2. VH3, and VH4. The six questions of Instrument 2 are denoted as A, B, C, D, E, and F. Please note that each alphabetical is followed by a number. These numbers denote the steps that were taken for each question. For example, Question A has 14 steps, yet A2 and A10 are not listed under "Cognitive Reasoning Steps VH2". Those two steps are VH3 level steps and appear under the heading ""Cognitive Reasoning Steps VH3".

	Question	**Cognitive Reasoning Steps VH2**
	A1	**Analyse** the question, its subsets and the given data
	A3	**Analyse** paper space for fitting the 5 views and the AV
	A4	**Convention:** Draw XY reference line for AV on 30° with horizontal
	A5	**Convention:** Draw perpendicular centre line to 30° line
	A6	**Analyse and Identify** pyramid pivot point to orientate pentagon
	A7	**Convention:** Draw 40mm line in correct place and angle as construction
	A8	**Identify** appropriate construction method and **execute** in LCL
	A9	**Convention/Visuo:** Number all 5 points and apex
	A11	**Convention/Visuo:** Project from AV 5 points + apex down to FV in LCL
	A12	**Convention:** Mark off base and apex according to height
	A13	**Convention/Visuo:** Connect 5 points to apex
	A14	**Convention:** Place line for Section A-A
	B1	**Analyse and Plan** procedure
	B2	**Convention:** X1Y1 Reference
	B3	**Convention/Visuo:** Project apex and base points to LV from FV
	B4	**Convention/Visuo:** From XY, transfer all points to X1Y1 reference
	B5	**Convention/Visuo:** Number the 6 coordinates according to AV
	B7	Convention/Visuo: Join apex to 5 basepoints in LCL
	B10	**Convention/Visuo:** Join a, b, c, d, e
	C2	**Convention:** X2Y2 Reference
	C3	**Convention/Visuo:** Project apex and base points to RV from FV
	C5	**Convention/Visuo:** Number the 6 coordinates according to AV
	C7	**Convention/Visuo:** Join apex to 5 basepoints in LCL
	C9	**Convention/Visuo:** Label cut points on FV and RV as a, b, c, d, e
	C10	**Convention/Visuo:** Join a, b, c, d, e
	D1	**Analysis and Mental planning** of procedure
	D2	**Convention:** X3Y3 Reference
	D3	**Convention/Visuo:** Project base points and apex vertically down

	D4	**Convention/Visuo:** Apply 45° line for 1st angle from LV or RV. Project 5 base points to apex
	D5	**Convention/Visuo:** Apply 45° line for 1st angle from LV or RV. Project 5 cut points and apex to TV
	D7	**Convention/Visuo:** Number all points according to AV and A-A
	D8	**Convention/Visuo:** Join apex to 5 base points in LCL
	D10	**Convention/Visuo:** Join lines to form the cutting plane
	E1	**Analysis and Mental planning** of position and procedure
	E2	**Convention/Visuo:** X4Y4 Reference parallel with section line
	E3	**Convention/Visuo:** Project 5 base point and apex to X4Y4 perpendicularly
	E4	**Convention/Visuo:** Project 5 cut points on A-A to X4Y4 and beyond
	E6	**Convention/Visuo:** Number all points according to AV and A-A
	E7	**Convention/Visuo:** Join 5 base points to the apex
	E9	**Convention/Visuo:** Connect a, b, c, d, e to form the true shape
	E10	**Convention/Evaluate/Visuo:** Show hidden and visible detail with line types
	F1	**Analysis and Mental planning** of procedure **(Visuo)**
	F2	**Convention:** X5Y5 Reference
	F6	**Convention:** Draw triangular net with true lengths on X5Y5
	F7	**Convention:** Draw four more identical nets rotating around the apex
	F8	**Convention/Visuo:** Number the apex and 5 base points of the nets and according to the point where the join will be the shortest
	F9	**Convention/Visuo:** Mark off the 5 true length section points from the apex and number them
	F10	**Convention:** Connect the section points

	Question	**Cognitive Reasoning Steps VH3**
	A2	Create a **mental image** of the five views and their positioning
	A10	Form **mental image** of rotated auxiliary view as the pyramid base
	B6	Form **mental image** of LV with and without section
	B8	**Convention/Visuo (mental cutting):** Project cutting points from the FV to the LV in LCL
	B9	**Convention/Visuo:** Label cut points on FV and LV as a, b, c, d, e
	C4	**Convention/Visuo:** From XY, transfer all points to X2Y2 reference
	C6	Form **mental image** of RV with and without section and as a mirror of the left
	C8	**Convention/Visuo:** Project cutting points from the FV to the RV in LCL
	D6	**Convention/Visuo:** Use compass to transfer sizes from the AV from XY line and X3Y3 line
	D9	Form **mental image** of the cutting plane
	E5	**Convention/Visuo:** From XY on AV, transfer all points to X4Y4 reference perpendicularly
	E8	**Visualise and rotate** pyramid mentally to see the shape
	F3	**Identify** 5 triangular sides of pyramid **(Visuo)**

	Question	**Cognitive Reasoning Steps VH4**
	B11	**Convention/Evaluate/Visuo:** Obscured/hidden detail with correct line types
	C11	**Convention/Evaluate/Visuo:** Obscured/hidden detail with correct line types
	D11	**Convention/Evaluate/Visuo:** Obscured/hidden detail with correct line types
	E4	**Convention/Evaluate/Visuo:** Obscured/hidden detail with correct line types
	F4	**Convention/Evaluate/Visuo:** Construct the true length of the triangular sides (nets) of the pyramid on the FV
	F5	**Convention/Evaluate/Visuo:** Construct the true lengths of the 5 cut points from the apex on the FV
	F11	**Convention/Evaluate/Visuo:** Obscured/hidden detail

Appendix G

Cognitive survey form to capture cognitive reasoning/procedural steps for questions A to F of the solid geometry task for Instrument 2

VH Level	**A. Copying the given front view**										
	Cognitive Step		Low Order			Mid Order			High Order		
			1	2	3	4	5	6	7	8	9
2	1	Analyse the questions and the given data			X						
3	2	Mental image of the 6 views and positioning							X		
2	3	Convention: Draw XY reference for AV		X							

The example above served to inform participants on how to capture their reasoning/procedural steps in minute detail. To the right, participants were required to evaluate each step according to its perceived difficulty on a nine-point scale.

The column on the left was added later on for th sake of rating each step according to the van Hiele descriptors of geometric reasoning.

The forms collected from 30 participants were subjected to inductive thematic content analysis and resulted in the Cognition Memorandum which is available as Appendix F.

Appendix H

Assessment Rubric for Instrument 2: Solid Geometry Task

	Item	VH Level	Description	marks
1	Analyse and Plan	2	General	5
2	Projection System	2	General	5
3	Auxiliary construction	2	Question 1 only	5
4	Auxiliary Annotation	2	Question 1 only	5
5	True Nets	4	Question 6 only	5
6	Section A-A	2	Question 1 only	5
7	Number all points	2	5 marks for each of the six questions	30
8	X-Y Reference	2	5 marks for each of the six questions	30
9	Label Cutting Points	2	5 marks for each of the six questions	30
10	Evaluate Lines & Types	4	5 marks for each of the six questions	30
11	Correctness		5 marks for each of the six questions	30
	Total			180

Appendix I

Third angle orthographic multiviews related to an isometric view of a cube

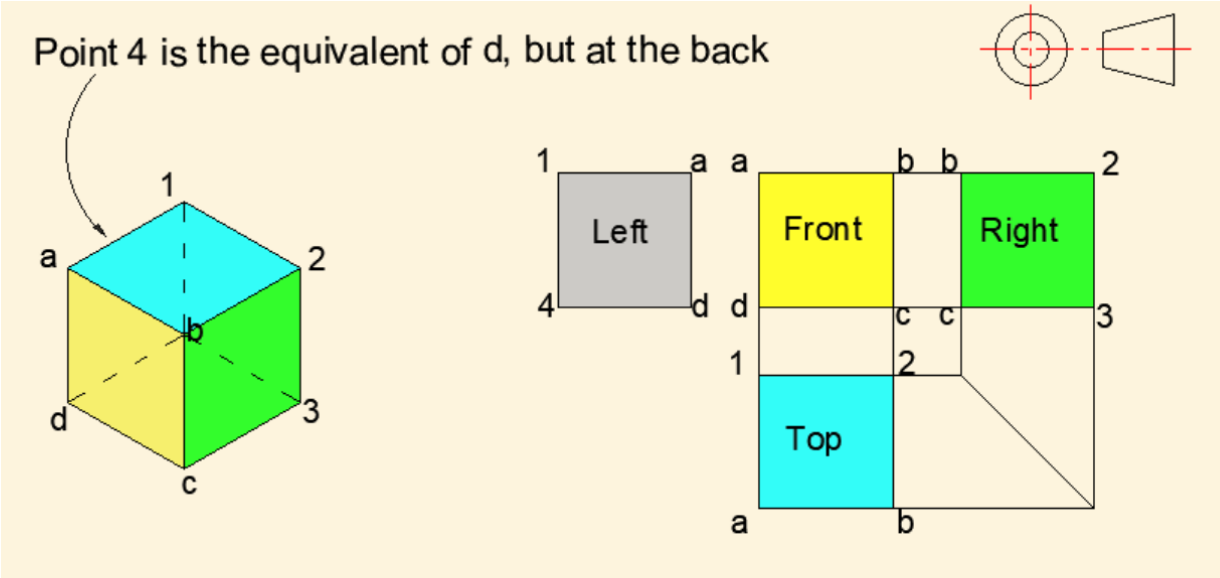

Figure 1: Third angle orthographic multiviews of a cube with a corresponding isometric view of the cube.

Appendix J
Buckley's (2018, p.26) Spatial factor framework

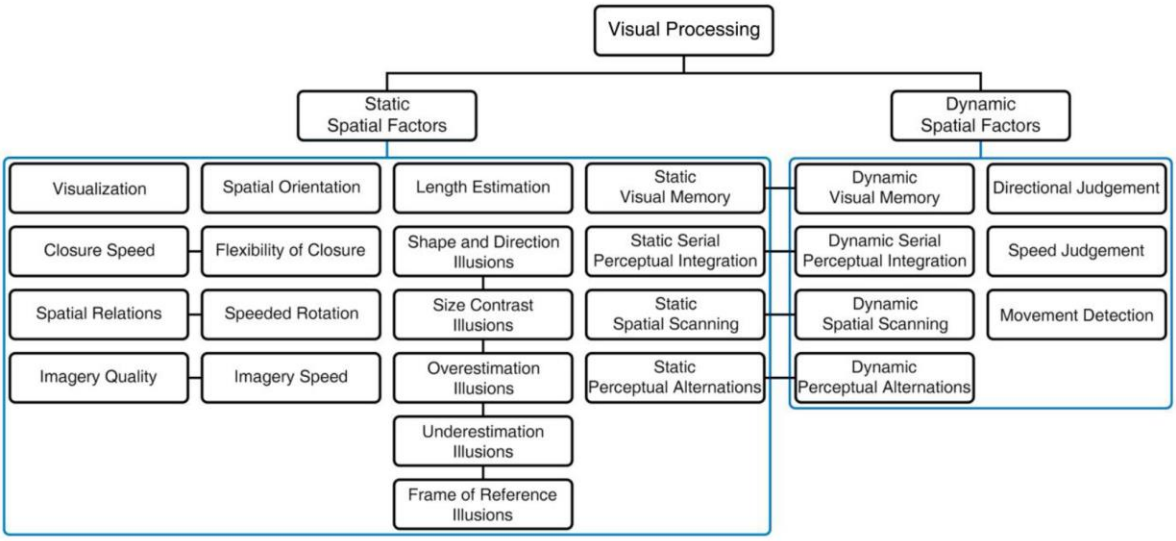

Figure 10. Spatial factor framework presented in Paper III.

Reference

Buckley, J. (2018). *Investigating the role of spatial ability as a factor of human intelligence in technology education: towards a casual theory of the relationship between spatial ability and STEM education.* KTH Royal Institute of Technology.

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Appendix K

Quantitative Dataset for Instrument 2: No Response (NR) data for Difficulty Perception on Solid Geometry Task

Participant	PreVSR	PostVSR	NRVH2	%	HighVH2	%	NRVH3	%	HighVH3	%	NRVH4	%	HighVH4	%
1	62	82	17	36	6	13	8	57	1	7	3	43	4	57
2	24	46	21	45	7	15	6	43	4	29	3	43	0	0
3	63	79	19	40	10	21	7	50	0	0	2	29	2	29
4	51	82	19	40	14	30	7	50	5	36	2	29	2	29
5	24	33	29	62	8	17	9	64	5	36	3	43	2	29
6	58	76	18	38	4	9	7	50	5	36	1	14	4	57
7	31	69	23	49	4	9	8	57	3	21	1	14	5	71
8	65	98	19	40	12	26	8	57	1	7	2	29	5	71
9	62	73	22	47	4	9	9	64	1	7	4	57	3	43
10	67	83	18	38	5	11	8	57	0	0	3	43	4	57
11	74	91	15	32	11	23	7	50	3	21	2	29	3	43
12	59	77	31	66	6	13	9	64	5	36	6	86	1	14
13	41	83	24	51	5	11	8	57	2	14	1	14	5	71
14	44	79	14	30	21	45	6	43	8	57	1	14	6	86
15	28	38	20	43	5	11	5	36	4	29	1	14	4	57
16	60	73	21	45	2	4	7	50	0	0	2	29	4	57
17	43	88	17	36	19	40	8	57	4	29	3	43	3	43
18	59	99	16	34	20	43	7	50	6	43	2	29	5	71
19	15	55	26	55	2	4	9	64	3	21	1	14	5	71
20	56	100	23	49	7	15	7	50	4	29	0	0	6	86
21	24	46	16	34	9	19	10	71	4	29	4	57	2	29
22	50	90	5	11	11	23	1	7	7	50	0	0	6	86
23	11	97	18	38	11	23	8	57	1	7	3	43	4	57
24	37	99	19	40	4	9	10	71	2	14	2	29	4	57
25	64	79	16	34	16	34	7	50	5	36	1	14	2	29
26	58	91	22	47	5	11	9	64	2	14	4	57	3	43
27	54	99	24	51	4	9	8	57	3	21	1	14	5	71
28	58	74	20	43	17	36	7	50	4	29	1	14	3	43
29	91	82	17	36	11	23	8	57	1	7	1	14	5	71
30	42	34	22	47	10	21	8	57	1	7	4	57	3	43
	49	77		42		19		54		22		30		52

Appendix I
Quantitative Dataset for Instrument 2: Rubric Scores on Solid Geometry Task

Respondent	Analyse and plan		Projection		Aux construction		Aux numbering		Number all points							Section A-A		X-Y Reference							Label cutting points							Evaluate lines & types							True Nets		Correctness							Solid Geometry score
		%		%		%		%	Q1	Q2	Q3	Q4	Q5	Q6	%	Q1	%	Q1	Q2	Q3	Q4	Q5	Q6	%	Q1	Q2	Q3	Q4	Q5	Q6	%	Q1	Q2	Q3	Q4	Q5	Q6	%	Q6	%	Q1	Q2	Q3	Q4	Q5	Q6	%	
1	3	60	5	100	5	100	5	100	5	5	5	5	5	5	100	5	100	0	5	0	0	0	0	17	5	0	0	0	0	0	17	5	5	5	5	1	5	87	5	100	5	5	5	5	0	4	80	66
2	5	100	4	80	3	60	5	100	5	5	0	5	5	5	83	4	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4	4	4	80	5	100	5	0	0	3	3	0	37	48
3	5	100	5	100	5	100	5	100	5	5	5	5	5	5	100	5	100	5	5	5	5	5	5	100	5	5	5	5	5	0	83	5	5	5	5	5	5	100	5	100	5	5	5	5	5	5	100	97
4	4	80	5	100	3	60	5	100	5	5	5	5	5	5	100	5	100	0	5	5	5	5	0	67	5	5	5	5	5	5	100	5	5	5	5	5	5	100	5	100	5	5	5	5	5	5	100	93
5	5	100	5	100	5	100	0	0	0	0	0	0	0	0	0	5	100	3	3	3	3	3	0	50	0	0	0	0	0	0	0	4	4	4	4	0	4	67	0	0	5	5	5	5	5	0	83	44
6	3	60	5	100	5	100	5	100	5	5	5	5	5	5	100	5	100	0	0	0	0	5	0	17	5	0	0	0	0	0	17	3	3	3	3	3	3	60	0	0	5	0	0	0	0	0	17	48
7	5	100	5	100	5	100	0	0	0	0	0	0	0	0	0	5	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	4	3	4	4	73	5	100	5	5	5	5	0	4	80	39
8	3	60	3	60	5	100	5	100	5	0	0	0	0	5	33	5	100	3	3	3	3	3	3	60	5	0	0	0	0	5	33	3	4	4	4	3	4	73	5	100	5	3	3	3	5	2	70	59
9	3	60	3	60	4	80	5	100	5	5	5	5	5	0	83	5	100	4	4	4	4	4	0	67	5	5	5	5	5	0	83	4	5	4	4	5	0	73	0	0	5	3	3	3	5	0	63	73
10	4	80	5	100	2	40	5	100	5	0	0	5	5	5	67	3	60	5	5	5	5	5	5	100	0	0	0	0	0	0	0	4	4	3	4	4	4	77	5	100	5	3	3	3	5	5	80	67
11	3	60	5	100	5	100	5	100	5	5	5	5	5	5	100	5	100	0	0	0	0	0	0	0	5	5	5	5	5	5	100	5	3	3	3	5	5	80	5	100	5	3	3	3	5	5	80	76
12	3	60	3	60	5	100	5	100	4	4	4	4	4	0	67	5	100	4	4	4	4	4	0	67	5	5	5	5	5	0	83	5	5	5	5	5	0	83	0	0	5	3	3	3	4	0	60	72
13	3	60	4	80	5	100	5	100	5	5	5	5	5	5	100	5	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3	60	5	100	5	3	3	0	5	5	70	53
14	5	100	4	80	0	0	0	0	4	4	0	0	0	4	40	4	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3	60	5	100	5	3	3	3	5	5	80	40
15	3	60	4	80	5	100	5	100	5	5	0	5	5	5	83	5	100	0	0	0	0	0	0	0	5	0	0	0	0	4	30	4	4	4	4	4	4	80	0	0	5	3	3	3	2	0	53	53
16	5	100	5	100	5	100	5	100	5	5	5	5	5	5	100	5	100	0	5	5	5	5	0	67	3	3	3	3	3	3	60	5	5	5	5	5	5	100	3	60	4	3	3	3	5	5	77	83
17	5	100	5	100	5	100	5	100	5	5	5	5	5	5	100	5	100	5	5	5	5	5	5	100	5	5	5	5	5	5	100	5	5	5	5	5	5	100	5	100	5	3	3	3	5	3	73	96
18	3	60	4	80	4	80	0	0	5	5	5	5	0	5	83	5	100	5	0	0	0	0	0	17	0	0	0	0	0	0	0	4	4	4	4	4	4	80	4	80	5	4	4	2	0	1	53	50
19	3	60	3	60	5	100	0	0	0	0	0	0	0	0	0	5	100	5	5	5	5	5	0	83	0	0	0	0	0	0	0	5	5	5	3	3	0	70	0	0	5	3	3	3	5	0	63	45
20	4	80	4	80	3	60	5	100	0	0	0	0	0	0	0	5	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	3	3	3	5	3	73	5	100	5	5	5	5	5	4	97	43
21	3	60	5	100	5	100	0	0	5	5	0	5	4	4	77	5	100	0	0	0	0	0	0	0	5	5	5	5	5	5	100	4	4	4	4	4	4	80	3	60	5	5	5	5	5	3	93	70
22	5	100	4	80	5	100	5	100	5	5	5	5	5	5	100	0	0	4	4	4	4	4	0	67	0	0	0	0	0	0	0	4	4	4	4	4	4	80	5	100	3	3	3	3	3	3	60	64
23	3	60	3	60	5	100	5	100	5	0	0	0	0	0	17	5	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	3	4	4	3	4	77	5	100	5	3	3	3	4	4	73	42
24	3	60	3	60	5	100	4	80	5	5	5	4	4	0	77	5	100	5	5	5	5	0	0	67	5	5	5	5	5	0	83	4	4	4	4	4	0	67	0	0	5	5	5	5	5	0	83	74
25	5	100	5	100	5	100	5	100	5	5	5	5	5	5	100	5	100	5	5	5	5	0	5	83	5	5	5	5	5	5	100	5	5	5	5	5	5	100	5	100	5	3	3	3	5	4	77	93
26	5	100	4	80	1	20	5	100	5	5	5	5	5	5	100	5	100	5	5	5	5	5	5	100	5	5	5	5	5	5	100	5	5	5	5	5	5	100	1	20	5	3	3	3	5	0	63	89
27	5	100	3	60	5	100	5	100	5	5	0	5	3	5	77	5	100	0	0	0	0	0	0	0	5	0	0	0	0	5	33	5	5	5	5	0	3	77	5	100	5	5	5	5	0	5	83	61
28	4	80	4	80	5	100	5	100	5	0	0	0	0	0	17	5	100	5	0	0	0	5	0	33	5	0	0	0	0	0	17	3	3	3	3	3	3	60	0	0	5	3	3	3	5	0	63	44
29	5	100	4	80	4	80	0	0	0	0	0	0	0	0	0	5	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	3	2	2	3	60	5	100	5	3	3	3	4	3	70	34
30	5	100	4	80	4	80	0	0	0	0	0	0	0	0	0	5	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	3	2	2	3	60	5	100	5	3	3	3	4	0	60	33
Average		80		83		85		73	79	65	49	65	60	62	63		94	42	45	42	42	42	19	39	55	35	35	35	35	31	38	85	82	81	78	72	69	78		67	98	69	69	67	76	50	71	62

Appendix M

Solid geometry drawing for Participant 30