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Primary school mathematics teachers' planning and teaching of word problems

By

Lehlohonolo V Dlamini

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Co-supervisor: Prof UI Ogbonnaya

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Declaration

I, Lehlohonolo Vytalisce Dlamini (18396756), declare that the dissertation titled "Primary school mathematics teachers' planning and teaching of word problems" which I hereby submit for the degree in Magister Educationis at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

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Dedication

I dedicate this study to my sons Junior, Liam, and Leo in appreciation of their unfailing inspiration, support, and motivation. I appreciate your patience while I conducted my research and was frequently unavailable. I hope you will be pleased with what your father has accomplished.

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Abstract

Word problems in mathematics often pose a problem for both teachers and learners because they are language-rich and based on real-life contexts. The purpose of this study was to explore how primary school mathematics teachers plan and teach word problems. I used a qualitative interpretivist case study involving two Grade 7 teachers from different schools to gain insight into this problem, and I was guided by two theoretical lenses, namely Realistic Mathematics Education and Pedagogical Content Knowledge. Data collection was done through lesson observations and document analysis. The findings revealed that in situations where lesson planning was done, the important features of a lesson plan, such as lesson objective, prior knowledge, and learner engagement (teacher's and learners' activities) were omitted; however, learners were actively involved in the lesson presentation. In addition, although learners were able to translate mathematical word problems into mathematical symbols, they were not conscious of translating their solution into the context in which the question was posed and thereby answering the question based on a mathematical word problem in real-life context. I conclude that lack of proper and thorough lesson planning can compromise the effective teaching (and learning) of word problems because teachers tend to teach word problems in an unstructured and haphazard manner. Word problems, by their nature, are context-embedded and language-rich, and therefore, require thorough planning to enable learners navigate between real-life context, everyday language, and mathematical language.

Keywords: Mathematical word problems, teacher planning, schematisation, realistic mathematics education, pedagogical content knowledge

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To Whom It May Concern:

This letter is to confirm that *Primary school mathematics teachers' planning and teaching of word problems* by Lehlohonolo V. Dlamini was edited by a professional language practitioner. It requires further work by the author in response to my suggested edits. I cannot be held responsible for what the author does from this point onward.

Regards,

Karien Hurter

List of Abbreviations

CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
MSI	Math Scene Investigator
PCK	Pedagogical Content Knowledge
RME	Realistic Mathematics Education
SQR	Survey Question Read
SRQ	Secondary Research Question
STAR	Search Translate Answer Review

Description of Key Terms

Learner: In the South African context, a learner is defined as any person receiving or obliged to receive education at school level. Other countries use the term student or pupil to refer to a learner, and therefore, I used the term *learner* as a synonym for *student* and *pupil*.

Educator: Any person who teaches learners at any of the public schools or private schools in South Africa and appointed legally in the post under the Employment of Educators Act, no. 76 of 1998. The term *educator* can also be used to refer to a teacher, and therefore, I used the term *teacher* as a synonym for *educator* in this study.

Mathematization: The process of interpreting or expressing something mathematically, or the situation that is being thought about or explained mathematically.

Real-life context: is a teaching strategy that emphasises practical application of mathematical concepts (in this case mathematical word problems) within the environment that learners are familiar with.

Word problems: consist of mathematical tasks that are portrayed as narratives or actual situations.

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CHAPTER 1: GENERAL ORIENTATION OF THE STUDY

1.1 Introduction and background of study

When developing their mathematical skills at primary school level, learners are introduced to word problems, which are among the most challenging and complex types of problems (Daroczy et al., 2015), but the ability to solve problems is among the most crucial elements of mathematics education. According to National Council of Teachers of Mathematics (2000), learners must take a variety of actions in order to create extractions and be offered mathematical support.

According to Adler and Pillay (2016), the teaching of mathematics in South Africa aims to foster learners' creativity as well as their capacity for problem-solving and communication. In addition, mathematics teaching ought to help learners develop their spatial and numerical awareness as well as their appreciation of patterns and geometrical structures. Further, to understand science subjects, instructors expect learners to be able to employ mathematics and mathematical reasoning in everyday situations (Saragih & Napitupulu, 2015). According to Phonapichat et al. (2014), the main goal of teaching mathematics is to equip students with the skills necessary to address problems in everyday life. Being able to solve mathematical problems is not only a goal in mathematics education but also has great significance in daily life and the workplace (Amador, 2022).

Word problems, particularly those with a realistic context, are essential for motivating learners, helping them understand where and how to apply newly learned mathematical concepts and skill development in real life and at work, and giving newly learned mathematical concepts and skill development meaningful context (Depaepe et al., 2015). The use of word problems in mathematics lessons helps learners explore mathematical relationships and organisation; however, dealing with problems from the real world might cause a number of difficulties for learners (Goulet-Lyle et al., 2020). Learners tend to leap into computations using the available figures and well-known processes and operations or rely on keywords rather than consider the problems' structure as a technique for solving the word problem (Lee & Hwang, 2022). When converting utterances into symbolic language, people frequently only remember information that is congruent with their existing schemas (Cruz & Lapinid, 2014).

In the current study, I explored the instructional practices used by teachers in Grade 7 mathematics classes during the teaching and learning of mathematical word problems. The majority of the national early grade mathematics curriculum place high demands on students to be adept at solving a wide range of word problems, and instructors are expected to document their progress (Roberts, 2016). As a result, advice on the necessary kinds of word problems is given to help teachers assign word-problem tasks and keep track of student progress. The Mathematics Curriculum and Assessment Policy Statement (CAPS) for Foundation Phase (Grades 1–3) outlines the current intended curriculum for mathematics in the early years of primary schooling in South Africa.

The Annual National Assessment diagnostic reports for Foundation Phase (Department of Basic Education [DBE], 2012, 2013, 2014) consistently identified learner difficulty with word problems as a weakness. Additionally, there is proof that learners have problems with word problems in the Intermediate Phase (Grades 4–6; Cobbe et al., 2021). Further, Senior Phase (Grades 7–9) learners in South Africa tend to struggle with word problems, whether they are given in English or their native language (Sepeng & Madzorera, 2014). According to Sepeng and Madzorera (2014), learners struggles with word problems are related to both language and mathematics, and they start showing up in elementary school and continue throughout secondary school.

1.2 Problem statement

In the context of mathematics education, word problems serve a number of different roles, such as introducing diversity to the repetition of fundamental processes and preparing students to apply their knowledge outside of the classroom (Cobbe et al., 2021). Learners often have difficulty solving mathematical word problems since they can be linguistically complex and necessitate learners to understand and evaluate the issue presented in the narrative (words). According to Zagala (2016), the key to successful problem-solving is the capacity to represent ideas mathematically.

The DBE (2011a, 2012, 2013, 2014) identified word problems as a recurrent difficulty for learners. These DBE reports claimed that learners demonstrate incompetence in word problem solving by failing to translate problem text into mathematical expression and write correct mathematical sentences (DBE, 2011a, 2013), fail to apply knowledge in each context (DBE, 2012), and generally fail to respond to non-routine word problems (DBE, 2014). The reports were created every year from 2011 to 2014 after learners took the Annual National Assessment examination with the intention of making a significant contribution to better learning in classrooms (DBE, 2010). The Annual National Assessment study identified areas of mathematics knowledge and proficiency where learners displayed poor levels of aptitude (DBE, 2013). In fact, learners' difficulties with word problems was also revealed in the Trends in Mathematics and Science Study diagnostic reports where weak language proficiency among learners was cited as one of the contributing factors(Graven et al., 2022).

While the problem with mathematical word problem focuses on learners, an area that is often not explored is how teachers teach word problems. Thus, in order to improve learners' success in word problem solving activities and their mathematical performance, teachers' planning and teaching of word problems must be taken into consideration. Teaching word-problem solving to students is a challenge that many teachers face daily (Andam, 2018), and their performance may be hampered by a lack of competency in preparing to teach mathematical word-problem exercises (Barlow & Cates, 2006). The lack of subject-matter knowledge among teachers result in learners learning incorrect beliefs, conceptions, and interpretations about the material while it is being presented (Amini et al., 2019)

Some learners struggle to solve word problems because they are unable to translate them into a mathematical form (Cruz & Lapinid, 2014). Since teachers and learners are accustomed to using algorithms while teaching and learning mathematics, it can be difficult for them to translate a word problem into a mathematical equation, solve it, and answer the question posed in real-life context. The premise for efforts to analyse word-problem teaching and learning is the disparity between word-problem solutions and mathematical concerns. My interest was piqued to carry out this study after reading several papers and noticing learners' difficulties in solving word problems.

1.3 Purpose and rationale of the study

The goal of this research was to explore how primary school teachers plan and teach mathematical word problems. I explored this subject from the following three perspectives: Teachers' planning to teach mathematical word problems; facilitating the learning of mathematical word problems; and schematising word problems presented in real-life context. I was able to gain insight into the instructional practices primary school mathematics teachers use to teach mathematical word problems.

It is essential for both teachers and learners to be able to use correct techniques or approaches to answer mathematical word problems. These skills will be useful to learners in more than mathematical word problems in mathematics and it may help learners build practical abilities like critical thinking, comprehensive thinking, and analysing skills if teachers use effective pedagogy to teach mathematical word problems.

1.4 Research questions

The following is the primary research question used to guide this study:

- Which instructional practices do primary school mathematics teachers use to teach mathematics word problems?

The following are the secondary research questions (SRQs) that guided this study:

- **SRQ1**: How do primary school teachers plan to teach mathematical word problems?
- **SRQ2**: How do primary school teachers facilitate the learning of mathematical word problems?
- **SRQ3**: How do learners mathematically schematise word problems presented in real-life context?

Through SRQ1, I explored and gained insight into the intricacies of planning lessons based on mathematical word problems. In doing so, I paid attention to teachers' pedagogical content knowledge (PCK), which is an essential part of the actual facilitation of learning, and should therefore feature prominently in lesson plans.

Through SRQ2, I explored and gained insight into the teachers' presentation of the lesson based on mathematical word problems. Entailed in this research question was the practical execution of PCK to help learners solve mathematical word problems.

Through SRQ3, my focus was on how learners solve mathematical word problems based on how they were taught. In other words, although the study mainly focused on teachers, my view is that learners' solutions of mathematical word problems mirror how teaching unfolded.

To summarise the intricacies of my research questions, PCK was the main focus of exploration while realistic mathematics education (RME) framed the context of the mathematical problems used, which was real-life mathematical word problems.

1.5 Literature review

In order to obtain insight into previously conducted studies related to the topic of my research, I reviewed the literature comprehensively in Chapter 2. However, in this section, I give a summary or a sneak preview of what is covered in Chapter 2.

1.5.1 Understanding the concept of word problem in mathematics

Several definitions of mathematical word problems have been provided in the literature. According to Verschaffel et al. (2020), a mathematical word problem is a verbal representation of a problem situation in which one or more questions are addressed and solutions can be found using mathematical operations on the numerical information provided in the problem statement. According to Polya (2004), problem-solving is a useful skill. Ferretti (2020) claimed that in order to solve word problems, learners must comprehend what they are reading because learners' comprehension of word problems affects their capacity to provide answers to word problems.

1.5.2 Teaching and learning word problems

In South Africa, teaching and learning mathematics are guided by the CAPS curriculum, and according to the CAPS (2012), developing problem-posing and problem-solving abilities requires acquiring the specialised information and abilities to use mathematics to solve real-world, physical, social, and mathematical challenges.

According to Driver and Powell (2017), if teachers give their learners rich and pertinent learning experiences, learners will build a solid conceptual knowledge of mathematical word problems. For instance, learners' interest is aroused and they are motivated to apply more significant and sophisticated mathematical reasoning when given the chance to create their own word problems.

1.5.3 Errors made by teachers and learners in word-problem solving

Yusuf et al. (2021) asserted that learners' errors are not the product of their incompetence but rather of a lack of problem-solving techniques. According to Yusuf et al., learners are unable to employ the correct answer strategies because they cannot understand the questions. However, according to Powell et al. (2021), the failure of learners to solve word problems is due to their lack of understanding of the problem. Another aspect of learners' difficulties in word-problem solving was illustrated by Ahmad et al. (2020), who found that Malacca's Form 4 learners made errors in their solution processes because they found it difficult to correctly convert word representations into mathematical representation, and therefore, learners' failure to develop "concrete images" or envision concepts frequently leads to them being unable to make mathematical arguments.

1.5.4 Strategies for solving word problems

According to Kalinec-Craig (2017), educators should encourage their learners to establish plans, which start by repeatedly reading the word problem on their own; next, they draw attention to the crucial information in the demand and the query; then they choose which number sentence to employ; they solve the issue independently; and finally, confirm that the result meets the requirements. Batrisyia et al. (2020) proposed using a model-drawing or box diagram strategy to solve the mathematical word problems. They asserted that this approach helps learners comprehend mathematical word problems by simplifying their complexity and semantic structure.

1.5.5 Theoretical framework

Based on the literature review, I opted to approach the study through two theoretical lenses, namely RME (Freudenthal et al., 1976) and PCK (Shulman, 1986), to explore the instructional practices uses by primary school teachers to teach mathematical word problems. Using two theoretical frameworks rather than one was advantageous because they supported one another. Briefly stated, PCK is relevant when mathematics teachers present a new concept to their class through a word problem and the learners are required to read the word problem with understanding while applying the new concept they learned. The RME framework is relevant here because learners who read and understand word problems must choose an acceptable approach to solve the problem before comparing their conclusions to the original query.

1.6 Research methodology

The research onion, as advocated by Saunders et al. (2015), served as the foundation for the methodology chapter of the current study. In Chapter 3, the research methodology is covered in detail.

1.6.1 Research philosophy

The research philosophy, which establishes the groundwork for ontology (or the nature of reality), epistemology (or sources of knowledge), and axiology (or values, beliefs, and research ethics), was the first layer that needed to be addressed in Saunders et al.’s (2015) research onion. The interpretivist research theory served as the basis for the current investigation.

1.6.2 Approach to theory development

The second layer of the research onion that needed to be peeled back in current research study was the research methodology (Saunders et al., 2015). There are times when using the term *approach* in research might be perplexing because the terms *approach*, *methodology*, and *paradigm* can all be used interchangeably (Tuffour, 2017). According to Teherani et al. (2015), the general premise of an approach is a line of thought or course of action. I chose to use an inductive research strategy, which opened the door to the qualitative research techniques covered in the following layer of the research onion.

1.6.3 Methodological choice

The third layer of the research onion is the methodological decision (Saunders et al., 2015). I chose qualitative research methods over mixed methods and quantitative research approaches. The goal of qualitative methodology is to provide in-depth and descriptive data to understand the various aspects of the subject under investigation. Therefore, qualitative research is concerned with aspects of reality that cannot be defined and is focused on understanding and describing the dynamics of social relationships (Queirós et al., 2017).

1.6.4 Strategies

The research strategy I used to conduct the study is the fourth layer of the research onion (Saunders et al., 2015). The case study approach was used for the current investigation. Heale and Twycross (2018) defined a case study as in-depth research of one person, one group of individuals, or one unit with the goal of generalising it to other units.

1.6.5 Time horizons

The study's time horizons are discussed in the sixth layer of the research onion. Saunders et al. (2015) proposed two time horizons, namely a cross-sectional time horizon and a longitudinal time horizon. Longitudinal time horizons are for long-term studies, and cross-sectional time horizons are for short-term studies. The time horizon adopted in this study was cross-sectional.

1.6.6 Technique and procedures

The methods and processes I used to carry out the current study make up the sixth and last layer of the research onion (Saunders et al., 2015). The levels of methodologies and procedures cover sample selection, data collecting, and data analysis (Sahay, 2016). Due to their accessibility and interest in the study, two distinct primary school mathematics teachers who are teaching Grade 7 learners were chosen as participants for the current study.

Classroom observation and document analysis were used to acquire the data for the current study. According to Davie and Wyatt (2021), document analysis is a methodical process used to study and/or assess printed or electronic materials. Observation is a data collecting strategy that involves noting a subject's, object's, or event's behaviour without necessarily asking them a question or interacting with them (Anis et al., 2021). The lesson plan for the teachers was the subject of the document analysis, and the observations focused on how the teachers' helped learners learn mathematical word problems and how learners schematised the learning of mathematical word problems presented in a real-world setting. Additionally, I designed two data collection tools to help me (see Appendix A for the observation data collection tool and Appendix B for the document analysis data collection tool).

I employed thematic data analysis for the current study. According to Braun and Clarke (2019), thematic analysis is a process to methodically identify, gather, and offer insight into patterns of meaning (themes) throughout a data set. Finding specific and unusual interpretations and experiences that are only present in one data item is not the aim of thematic analysis. As a result, I was able to interpret commonalities in the way a topic is talked or written about by employing this method as a researcher (Braun & Clarke, 2021).

1.7 Quality criteria

In contrast to the validity and reliability used in quantitative studies, trustworthiness is the key idea in qualitative research (Laumann, 2020). Credibility, transferability, dependability, and conformability are the four fundamental ideas I identified to demonstrate and improve quality.

1.8 Ethical considerations

To stop atrocities, research should always abide by ethical principles (Mertens, 2019). The teachers and students in the two Grade 7 mathematics classed from two different schools within the same district Mpumalanga were treated with the utmost respect in order to ensure ethical accountability. The district, school principal, subject teacher, parents, and learners who participated in the study were all asked for their consent to proceed. Finally, pseudonyms were used to hide the identities of the district, schools, teachers, and learners.

1.9 The outline of the chapters

The whole study is structured into five chapters that are briefly described here:

- *Chapter 1: General orientation of the study:* This chapter provides a summary of the research and includes background information, an introduction, a problem statement, justification for the study's existence, research questions, a literature review, a theoretical framework, an explanation of key terms used in the study, methodology, quality standards, and ethical considerations.
- *Chapter 2: Literature review and theoretical framework:* This chapter provides an overview of some of the literature associated with the topic so readers can acquire insight into the research on mathematical word problems and how they are taught. Additionally, the theoretical framework for this investigation is described.
- *Chapter 3: Research methodology:* The approach and the methods used to acquire the data for the study are explained and described in this chapter. The research methodology was guided by the research onion advocated by Saunders et al. (2015).
- *Chapter 4: Presentation of findings:* The data used to compile the findings were gathered through document analysis and classroom observations with primary mathematics teachers. The data were examined using thematic data analysis, and the findings are discussed and evaluated in light of the literature and theoretical underpinnings.
- *Chapter 5: Discussion of the findings, recommendations, and conclusions:* The findings are discussed in this chapter to address the research questions. The discussion also takes into consideration the usefulness of the theoretical framework. In addition, the study's limitations, suggestions for implementation, and ideas for further research are included in this chapter.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

The major objective of this chapter is to review studies on mathematical word problems and how they are taught in mathematics classroom settings and to get insight into what mathematical word problems are. Problem-solving, which is a major focus in teaching and learning mathematics, is one of the topics that should be emphasised in mathematics classroom (Ministry of Education, 2002). Understanding what mathematical word problems are is the first topic I examine in the literature review. I then move on to the teaching and learning of word problems, look more closely at the errors that teachers and learners make while teaching and learning word problems, and finally review the strategies used to solve word problems.

2.2 Understanding the concept of word problem in mathematics

Word problems have been defined in a variety of ways by researchers. For instance, Allen et al. (2019) defined word problems a mathematical problems that are presented within the context of a story or a real-world scenario. They emphasised that in order to translate the given information into mathematical symbols, learners need to have certain abilities. According to Ahmad et al. (2020), a word problem is a story problem in which learners must change the concrete into the abstract and the abstract into the tangible. They asserted that problem-solving involves a rigorous intellectual approach combined with real imagery. Hoogland et al. (2018, p. 60) claimed that mathematical word problems are simply mathematical tasks that have been “dressed” up in a real-world context that learners must “undress” and answer. The resolution of word problems is one of the major subjects covered in mathematics curricula (Vicente et al., 2022).

Singer et al. (2019) defined word problems as inquiries that require the application of mathematical knowledge to answer, stating that the information from a specific text must be extracted. On the other hand, Dhami (2021) defined word problems as “problems articulated in English and solved by expressing them structurally and solving those using basic operations” (p. 1). In addition, Pongsakdi et al. (2020)

defined word problems as “a mathematical verbal problem in written form that combines the percentages symbol (%), and other mathematical symbols such as +, - , ÷, ×” (p. 4). I inferred from the definitions given here that word problems are mathematical questions expressed in phrases and that answering them necessitates a thorough understanding of the situation to identify the mathematical concepts needed.

Van Dooren et al. (2019) described word problems as “verbal descriptions of problem situations in which one or more questions are addressed and answers can be discovered by applying mathematical operations to numerical data accessible in the problem statement” (p. 87). In other words, scientists and mathematicians turn linguistic difficulties into mathematical problems, which they subsequently answer with the aid of mathematical techniques; the mathematical solution is then transformed back into its original form, which completes the solution. Fuchs et al. (2020) claimed that number-based problems are simpler to solve than typical word problems. I concur with Fuchs et al.’s claim that learners find it easier to solve numerical problems than verbal ones because learners find it simpler to analyse and interpret numerical problems than those that are expressed in words.

However, Kazemi (2017) emphasised that learners must grasp what they are reading in order to solve word problems when assessing how learners’ ability to answer word problems is influenced by their knowledge of word problems. It is challenging for learners to solve word problems if they do not understand the problems. It is therefore essential that teachers and learners study and understand problems before attempting to solve them. If learners are only given one word problem to study at a time, they will succeed. In addition to giving learners the ability to assess their own strengths and weaknesses and increase their sense of independence, this makes it simple for teachers to keep track of learners’ development.

De Koning et al. (2017) categorised mathematical word problems into the following three categories: combine, change, and compare word problems. A superset or subset must be computed while attempting to solve combine word problems, according to their conclusion, given the information of two other sets. It necessitates understanding the relationship between individual components and the idea that the whole is equal to the sum of its parts. They further explained that the addition or subtraction of items from a starting set is required in change word problems, as well as the computation of the cardinality of a start set, transfer set, or outcome set using knowledge of two sets. Lastly, in compare word problems, one set serves as the comparison set and the other as the referent set, and the cardinality of one set must be established by comparing the data provided about the relative sizes of the other set sizes.

Word problems must be read and understood before a strategy is developed and used to provide a solution, and the answer must then be evaluated to determine whether it produced the desired outcome (Verdine et al., 2017). Given that word problems challenge learners to use their expertise, they help them acquire new mathematical concepts. Mathematical word problems were divided into the following four categories by Ogan-Bekiroglu and Dulger (2018):

- Exercise problems: After practice, learners can recall an algorithm for a solution right away and use it.
- General well-structured problems: Learners can easily understand the problem and aim of a generic, well-structured problem. In this situation, students must comprehend the issue, remember an algorithm, use it to address the issue, and then consider the resolution.
- Tougher well-structured problems: A more challenging, well-structured task with a distinct problem and goal that learners are unfamiliar with. Additionally, students must comprehend the issue, come up with or remember a solution, use it to solve the issue, and then consider their choices.
- Ill-structured problems: An ill-structured problem is a mathematical word problem where the problem and aim are unclear. Without the necessary knowledge, learners must discover the problem’s goal, create an algorithm, and use it to solve the problem.

Translating words into symbols is one of the essential and critical steps in solving word problems, making it one of the trickiest areas for learners and a significant obstacle to learning mathematics. The ability to convert word problems into symbols depends on conceptual understanding, and therefore, it is simpler for learners to do so if they understand fundamental mathematical concepts.

2.3 Teaching and learning word of problems

As a preface to gaining insights into the literature on teaching and learning word problems, I first explored the South African curriculum prescripts on mathematical word problems. In South Africa, the CAPS (DBE, 2011b) for mathematics informs the content (knowledge and skills) to be taught and learned. The CAPS states that the purpose of mathematics instruction and learning is to foster the development of the specific knowledge and skills necessary for applying mathematics in different contexts, such as physical and social contexts. In fact, learners ought to be exposed to mathematical problems that are both context-rich and context-free (DBE, 2011b). The latter focuses on word problems embedded in real-life contexts.

Sun et al. (2017) stressed that mathematical problem-solving techniques help develop learners' ways of thinking, their tenacity and curiosity, and their confidence in unfamiliar situations, and they will be able to use these abilities outside of the mathematics classroom to solve problems. According to Sun et al. (2017) and Edwards and Vale (2016), problem-solving is a crucial mathematical activity since it requires learners to combine their knowledge and concepts to deal with specific mathematical situations. It is essential to teach learners problem-solving skills since doing so enables them to develop the mental habits necessary for problem-solving.

Teachers strongly emphasise the importance of going beyond merely following directions and using algorithms while instructing and studying mathematics (Little, 2020). The tight adherence to the material in CAPS, including the algorithms teachers use, is one of the mistakes teachers make because in order to successfully teach mathematics, teachers must draw on their own experience and develop lesson plans that enable learners to apply their newly acquired knowledge to practical circumstances. Word problems are crucial to develop mathematical concepts (Khoshaim, 2020).

Traditionally, lesson led by teachers feature prominently in the classroom, and well-crafted lesson plans are rarely implemented (Iqbal et al., 2021), and because there are no instructional plans, learners play a passive role in the classroom (Milkova, 2012). Teachers are required to create lesson plans that serve the needs of learners because doing so tends to help learners understand the lesson taught. When teachers plan lessons, it shows confidence in their knowledge of the subject matter (Bin-Hady

& Abdulsafi, 2019). According to Bin-Hady and Abdulsafi (2019), a good lesson plan should include the following elements: objectives for student learning, teaching/learning activities, and strategies to check learners' understanding. According to Fujii (2019), lesson plans are teachers' roadmap for what their class will cover and how it will be done efficiently. The learning objectives must be determined before designing a lesson. After that, the teacher can create instructive learning exercises and find methods for getting feedback on learners' academic progress.

According to McCormick (2016), exercises that involve problem-solving can be used to gauge how well learners understand mathematical concepts. In fact, using word problems to teach mathematics has been called the "heart of math" (Sa'dijah et al., 2021). It has been demonstrated that learners who can master word-problem solving can also master mathematical concepts and solve word problems successfully (Sa'dijah et al., 2021). The role of the teacher in the classroom is to guide learners and determine the most important lessons for them to learn. The first stage is to increase learners' self-confidence because teaching word problems may be difficult. Sahin et al. (2020) asserted that learners develop a solid conceptual understanding of word problems when teachers provide them with rich, relevant learning experiences. For instance, encouraging learners to come up with their own word problems grab their interest and encourage them to engage in more practical and insightful mathematical thinking. According to Craig (2018), teachers should help learners develop plans for how to approach word problems that call for careful reading and repeated reading. Firstly, learners must read the word problem aloud many times on their own, and then, in the demand and question, they highlight the key details. Next they decide which number sentence to use, work the problem out on their own, and then verify that the solution complies with the criteria.

According to Silaban and Surya (2018), mathematics knowledge can be divided into three interrelated categories, namely concepts, procedures, and problem-solving. Dealing with concepts helps learners understand "what something?" (Avcu, 2018), such as realising a square has four sides. Learners prove conceptual understanding when they can "use concepts and their representations to debate or classify mathematical facts" (Dossey, 2017). In other words, conceptual understanding is used to contrast and compare items as well as to draw links between concepts and ideas. When learners "select and apply procedures correctly", they display procedural knowledge (Dossey, 2017). The third aspect of mathematical understanding is problem-solving aptitude. Learners must be able to perceive situations, abstract their basic elements, model the relationships in play, control those interactions, and communicate the results in order to solve problems (Amiripour et al., 2017).

Baraké et al.'s (2015) findings reinforce the notion that conceptual knowledge and choosing the best way to solve a problem are essential elements of teaching and learning word problems. It is crucial to give learners word problems that accurately reflect the scenarios they face frequently. This helps learners understand the difficulties more clearly and enables them to represent the situations using symbols.

Prior to translating a verbal problem into a mathematical problem, learners must first read and comprehend the verbal problem (Hadi & Wijaya, 2020). A great way to start is by inquiring about the learners' level of difficulty with a specific word problem. The bulk of problems arise when learners struggle and become bewildered about which number sentence to write because they are unable to comprehend the context in which the word problem is presented. Therefore, teachers' job is to devise novel ways to teach the information (Moleko & Mosimege, 2021). It is critical to understand learners' viewpoints on the challenges they are facing when solving word problems. This helps teachers create lessons and make sure that learners are fully responsible for their education, which is not the responsibility of the teacher.

De Koning and van der Schoot (2019) contended that if teachers give their learners rich and fulfilling learning opportunities, learners build deep conceptual knowledge of word problems. When learners are invited to create their own word problems, for instance, their interest is peaked and they are encouraged to engage in more crucial and significant mathematical thinking. A personal experience, like a trip or a football game should be the basis for word problems learners are asked to write. Learners learn more and develop better problem-solving skills and attitudes toward mathematics if they can design their own mathematical word problems (de Koning & van der Schoot, 2019).

While the Department of Basic Education (DBE) in South Africa advises that students be taught in their mother tongue until Grade 3, schools are free to select the language of teaching. Given that there are eleven official languages in South Africa, curriculum content ought to be taught in these languages in schools. Because of the perceived social capital that learning English would provide for their children in the future, more and more South African parents are choosing to send their children to schools where it is taught.

While mandatory kindergarten is somewhat less widespread in public schools, it is typically attended by students at private schools prior to Grade 1 (Suna et al., 2020). The four years that make up the "foundation phase" (also known as the elementary school phase) include kindergarten, the first year of education (Suna et al., 2020). Certain parents in rural areas opt to send their children to school only for Grade 1 due to transportation and educational expenses. Even though kindergarten attendance became mandatory in 2014, a lot of parents continue to follow the previous policy, which stated that kindergarten is optional, and that Grade 1 is the first year of school (South African Schools Act, 1996).

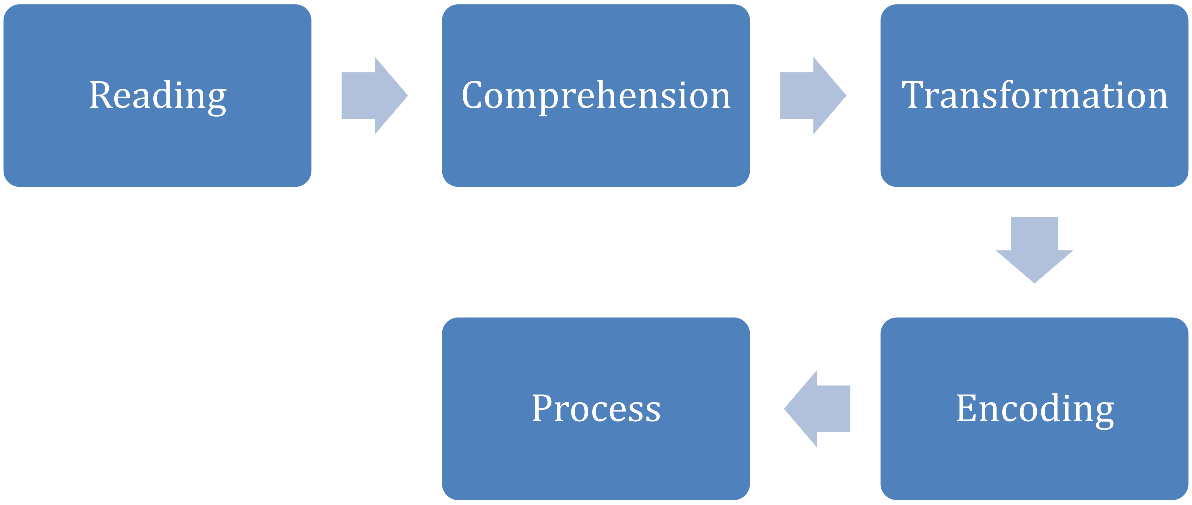
2.4 Errors made by teachers and learners in word problem solving

Yusuf et al. (2021) asserted that learners' errors are not the product of their incompetence but rather of a lack of problem-solving techniques. According to Yusuf et al., learners are unable to employ the correct answer strategies because they cannot understand the questions. Zakaria and Yusoff (2009) noted that learners' incapacity to relate to the problem description is caused by exposure to solving procedural problems with few steps. Furthermore, they noted that errors occur when learners implement regulations without fully understanding the guiding ideas.

Hatisaru (2022) asserted that learners can learn complicated mathematical concepts if they put in the necessary effort to hone their skills, including reading more frequently and analysing simple arithmetic problems with a sense of curiosity about why certain things occur. Perhaps mathematics teachers do not provide their learners with enough activities to practice decoding and interpreting, which are two skills necessary for problem-solving. Most of the time, teachers ask their learners to solve word problems without them being proficient in these skills. However, Powell et al. (2021) claimed learners' failure to solve word problems is due to their lack of understanding of the problem. Another aspect of learners' difficulties in word-problem solving was illustrated by Ahmad et al. (2020), who found that Malacca's Form 4 learners make errors in their solution processes because they find it difficult to correctly convert word representations into mathematical representation. Learners' failure to develop "concrete images" or envision concepts frequently leads to their incapacity to make mathematical arguments. Sepeng and Sigola (2013) asserted that when addressing word problems, learners choose to ignore practical knowledge because the learners see no connection between the mathematics taught in the classroom and the mathematics they apply in their daily lives.

Newman (1977) examined the mistakes learners make when attempting to solve word problems, and the results were supported by Hadi et al. (2018). The Newman error analysis model was created to categorise mistakes and postulates the stages to follow as shown in Figure 2.1.

Figure 2.1
Newman error analysis model



The steps can be explained as follows:

- Reading skills: Can the learners truly decode the question? Do they comprehend the question's words or symbols?
- Comprehension: Can the learners comprehend the question both in terms of a basic understanding of the mathematical topic and in terms of specific mathematical expressions and symbols after having decoded the words or symbols?
- Transformation: Can the learners select a suitable method or algorithm to tackle the issue?

- Process Skills: Can the learners accurately do the operation(s) they choose in the transformation step?
- Encoding: Can the learners connect their response to the original query in order to record it in the correct format? (Adapted from Dickson et al., 1984)

Mukunthan (2013) and Reid O'Connor and Norton (2020) used Newman's error analysis model to analyse their data, and they discovered that the reading, comprehension, and transformation stages are where 66.67% of mistakes are made when answering word problems. This means that the errors occur before the learners do any mathematics. According to the findings of these two studies, more than 60% of learners make mistakes when conducting any calculations.

According to Akhtar et al. (2020), one of the mistakes learners make when attempting to solve word problems is making translational errors, which have been noted in a number of equation writing activities. Adu et al. (2015) also found that learners consistently misunderstand the concepts used because mathematical language is disregarded. Word problems are said to have historically been the bane of many algebra learners. Transforming the narrative into the proper algebraic equations is the main challenge that learners face when solving algebraic word problems (Tonui et al., 2018).

According to Adu et al. (2015), learners have trouble grasping the particular phrases used in mathematics as well as translating word problems into mathematical form. This failure can be the result of teachers not placing enough focus on their learners' knowledge of mathematical language. It may also be due to teachers' lack of or incapacity to ensure each learner is proficient in reading with understanding before introducing new concepts. For instance, Haryanti et al. (2019) found that transformation errors and process skill errors are the most common types of error learners made when answering mathematical word problems or other mathematical problems. Additionally, according to Malibiran et al. (2019), learners with high achievement make careless errors when solving non-routine word problems, learners with average achievement make procedural mistakes, and learners with low achievement have the most trouble visualising and representing the problem.

Sepeng and Sigola (2013) conducted a study of mistakes made by Grade 9 learners when solving mathematical word problems, and their results showed that the learners' mistakes resulted from a lack of knowledge of the mathematical terminology used in the problem. Uyen et al. (2021) found that a variety of factors, including negligence, subjectivity, incorrect application of the calculation rules, incorrect identification of problem types, and inaccurate calculation, contribute to learners' errors when solving mathematical problems. Based on his findings, mistakes that are often made by learners when solving mathematical problems are errors in interpreting problem sentences and making mathematical models.

Similar findings from another study indicated that most research participants erred in selecting and determining problem-solving solutions, and one of the mistakes involved mathematical statements that contained modelling problems (Maulyda et al., 2020). This mistake leads to improper mathematics communication in writing. One of the most challenging steps for learners to complete is modelling. Learners' blunders in expressing their mathematical ideas are impacted by errors in correctly comprehending problem statements (Son & Fatimah, 2019).

2.5 Strategies for solving word problem

The *Principles and Standards for School Mathematics* advised learners to improve their problem-solving skills, especially for word problems in mathematics (National Council for Teachers of Mathematics, 2000). According to Spencer et al. (2022), in order for learners to master solving mathematical word problems, they need the support of thinking strategies that regulate the interpretation and manipulation of information through linguistic capabilities and visual capabilities in working memory. This is because mathematical word problems contain worded pieces and the way they are structured make them difficult to solve.

Examining and understanding word problems are necessary to make decisions of strategies learners may use in solving mathematical word problems(Clinton et al., 2018). The development of learners' mathematical problem-solving skills requires guidance and exposure to strategic thinking and representation strategies (Jitendra et al., 2015). In addition to language and manipulation skills, it is crucial to develop thought processes and representational abilities to help connect all of the important parts of a problem. According to Abdullah et al. (2014), visual thinking can aid in the successful resolution of mathematical word problems by offering concepts and promoting a deeper understanding of connections. In the first several years of school, learners learn basic math skills. The concepts of addition, subtraction, multiplication, and division are explained and practiced in great detail. During these years learners are only offered mathematics problems expressed in mathematical symbols and teachers frequently teach learners how to solve these questions. These early experiences give learners the notion that mathematics can be mastered by memorising processes and procedures.

The keywords technique was recommended to help learners answer mathematical word problems question(Riccomini et al., 2016).This technique teaches learners to quickly skim the sentences to identify any important terms that alert them to the appropriate operations. This method demonstrates that learners do not approach the questions with as much thought as they could have. The keywords tactic has also caused problems, for instance, learners may rush in finding key details in the problem without understanding the entirely problem situation before the answer the mathematical word problems(Riccomini et al., 2016).

Said and Tengah (2021) applied a del-drawing or box diagram approach to help learners solve mathematical word problems. They claimed this approach helps learners grasp mathematical word problems better because it makes their complexity and semantic structure more clear.

In his 1945 book *How to Solve It*, Polya offered steps learners can use to help them answer mathematical word problems. According to Polya, there are four principles that serve as the foundation for problem-solving. The first is understanding the problem, which requires learners to read the problem with understanding until they get the meaning of the problem. The second is creating a plan, during which learners start examining the various approaches to problem-solving to determine which approach is best for that problem. In the third step, the learners implement their plan. Finally, in the last step, they evaluate whether they have answered the problem in a way that satisfies the requirements of the question by reviewing the question and the solution.

In an effort to make their tactics more understandable, other researchers developed their respective approaches to solve mathematical word problems (Faucette & Pittman, 2015; Hains-Wesson, 2013; Krulik & Rudnick, 1987; Maccini & Gagnon, 2006; Snyder, 1988). Similar to Polya (1945), Krulik and Rudnick (1987) proposed four problem-solving techniques; Snyder (1988) introduced STAR strategies, which stands for *search*, *translate*, *answer*, and *review*. Maccini and Gagnon (2006) discussed RIDGES; and Hains-Wesson (2013) introduced typical first steps in problem-solving. Faucette and Pittman (2015) demonstrated two techniques: The first is K.N.W.S and has four phases, and the second is SQRQSQ and has six steps. These tactics are intended to help learners read and learn mathematics with a primary focus on word problems, and also to help teachers assess learners' learning and any potential misconceptions they have regarding a given word problem (Faucette & Pittman, 2015, cited by Raoano, 2016).

According to Krulik and Rudnick (1987), their approaches were meant to "explore, select a technique, address the problem, and review and extend" (p. 87). The RIDGE acronym used by Snyder (1988) stands for *read the problem statement*, *draw a picture*, *goal statement*, *equation development*, and *solve the equation* (Snyder, 1988). The acronym STAR used by Cole and Wasburn-Moses (2010) stands for *search the term problem*, *translate the problem*, *answer the problem*, and *review the solution*.

Learners must understand the mathematical tasks required of them to build on the strategies of Polya (1945) by reading between the lines (Barwell, 2011). They must list what is known and what is unknown after carefully reading the problem to find what needs to be solved or found. Every fact, no matter how irrelevant, should be stated (Xin, 2019). They can also state the problem in their own words to show that they understand it. According to Gavin and Renzulli (2021), learners must understand the problem at hand and be driven to discover a solution in order to complete this phase of problem-solving successfully. Bergqvist and Bergqvist (2020) noted that the stages involving comprehension of the problem process are frequently absent in general practice. The Florida Department of Education (2010) identified this step as the most neglected phase of the problem-solving process, even though it is really a more advanced version of the first step in which learners recognise the problem and depict it in a way that is understandable (Krulik & Rudnick, 1987). This step calls for an early understanding of the givens, resources, and purpose (Moursund & Albrecht, 2011).

The Florida Department of Education (2010) listed the following strategies that teachers can use to support learners through this first step of problem-solving:

- Survey, Question, Read (SQR): The *S* is for skimming the problem rapidly to determine its nature, the *Q* is for deciding what is being asked, or in other words, "what is the problem?", and the *R* is for reading for details and interrelationships.
- Frayer vocabulary model: The Frayer model is a concept map that enables learners to make relational connections with vocabulary words.
- Mnemonic devices: These are strategies that learners and teachers can create to help learners remember content. They are memory aids in which specific words are used to remember a concept or a list.
- Graphic organisers: These are diagrammatic illustrations designed to help learners represent patterns, interpret data, and analyse information relevant to problem-solving.
- Paraphrase: This strategy is designed to help learners restate mathematics problems in their own words, strengthening their comprehension of the problem.
- Visualise: Learners visualise and then draw the problem, allowing them to obtain a clearer understanding of what the problem is asking, and in a way learners practice creating pictorial representations of mathematical problems.

The Florida Department of Education (2010) also suggested that teachers base their decisions on how well their learners understand the material. Teachers should also help learners step by step in choosing appropriate strategy in solving mathematical word problems (Abdullah et al., 2014). After understanding the problem, the next step is to develop an opinion or a hypothesis about how to solve the problem in light of what was learned in step one (Gurat, 2018). The equation can then be created from the problem's translation using various designs, such as turning the problem into a picture, and from there word problem is created (Florida Department of Education, 2010; Ran et al., 2021; Snyder, 1988).

According to Polya (1945), when learners are aware of the calculations, computations, or structures that must be made in order to access the unknown, the plan is readily available. Before choosing a plan, learners should establish a link between the data and the variable, but this will only work if the concept is founded on prior experience and formal education (Florida Department of Education, 2010; Marcera, 2020).

Polya (1945) listed a few partial strategies that can assist in planning to solve the problem at this stage, including guess and check, drawing a picture or diagram, looking for a pattern, creating a table, using a variable, creating an organised list, eliminating possibilities, using logical reasoning, and working backwards. Now that the information has been provided, it can be plugged into the equation to solve the unknown and provide the solution. Since there is a risk that learners would forget the plan if they heard it from the teacher, it is best if they create the idea themselves based on prior knowledge (Kaur, 2019).

Kaur (2019) stated that it is advisable to read the word problem again and verify that the provided solution makes sense. By doing this, learners re-evaluate the outcomes and the steps that led to the answer or solution. This can also enhance their problem-solving skills and consolidate their knowledge.

Learners must be able to recognise a variety of word problems as well as the appropriate response to each (Powell et al., 2022). The goal of employing multiple approaches to solve word problems is to teach learners to become more autonomous learners rather than to rely on a single method to solve problems. For example, a cognitive strategy lesson can be used to educate young children how to solve word problems to increase their learning and performance (Powell et al., 2022). Learners use cognitive strategies to help them learn more effectively, which include organising new language, summarising meaning, estimating context meaning, and using imagery for memorisation (Marcelino et al., 2019).

Teaching cognitive and metacognitive methods to increase learning and performance is cognitive strategy instruction (Powell et al., 2022). Numerous studies such as (Cook et al., 2020 and Swanson et al., 2021) addressed the teaching of cognitive strategies in problem-solving. These studies covered teaching learners how to use a cognitive strategy (such as a visual strategy), how to finish all steps within a problem, and how to improve self-regulation. It is crucial for learners to use a cognitive approach to remember information and advance comprehension of word-problem types. Cognitive strategy education directs learners' attention to the linguistic and semantic details of a problem's structure (such as join and separate problems) and offers a solution, thereby improving learners' comprehension of the problem's meaning and problem-solving skills (Swanson et al., 2021).

Additionally, learners can improve their problem-solving approach by planning, monitoring, and changing using metacognitive methods, also known as 'thinking about thinking', such self-regulation (Dinc et al., 2020). It has been demonstrated that when cognitive methods (read, plan, solve) are combined with metacognitive strategies, learners' comprehension and problem-solving skills improve (Montague et al., 2011). One cognitive technique that can be used to teach primary-level learners with learning disabilities and math challenges how to answer elementary word problems is the Maths Scene Investigator (MSI). The Early Numeracy Intervention program, which includes the MSI, was validated through trials that had largely favourable results (Bryant et al., 2014, as cited in Elassabi & Kaçar, 2020). The MSI is a cognitive tool for learners in elementary school that can be used to solve a variety of word problems found in textbooks. Both verbal and visual (e.g., manipulatives) solutions have been shown to be effective in helping learners with significant math difficulty solve word problems (Swanson et al., 2021). The MSI strategy addresses the following six components of word-problem solving: (a) State the question being asked and the important units in the question, (b) identify important numbers, (c) explain what the question is trying to answer, (d) select the operation needed to solve, (e) create the picture or computational strategy used to solve, and (f) discern distractible or unimportant information (Chang et al., 2019). Table 2.1 sets out the MSI strategy.

Table 2.1
MSI strategy

STEP	ACTIONS WITHIN EACH STEP
Step 1: Inspect and find clues; read the problem	Underline the question and the unit Circle important information Cross out distractible information
Step 2: Plan and solve; write the equation	Draw a picture to solve
Step 3: Retrace	Write the inverse equation Recount picture drawn Check to see if question was answered

The MSI strategy is built on learners acting as the detective to solve a mystery. With respect to the six components of word-problem solving, there are three main processes (see Table 2.1). Each phase includes specific actions that help learners determine the kind of problem and the approach to solving it. The first stage is to look over the problem and look for indications using conversational techniques. During the inspection process, learners read the word problem, emphasise the question, include the unit, circle crucial terms and figures, and cross out superfluous details. Based on the question's unit, the learners decide which words to circle because they are very significant. The unit is what the word problem is about; for instance, if the question is

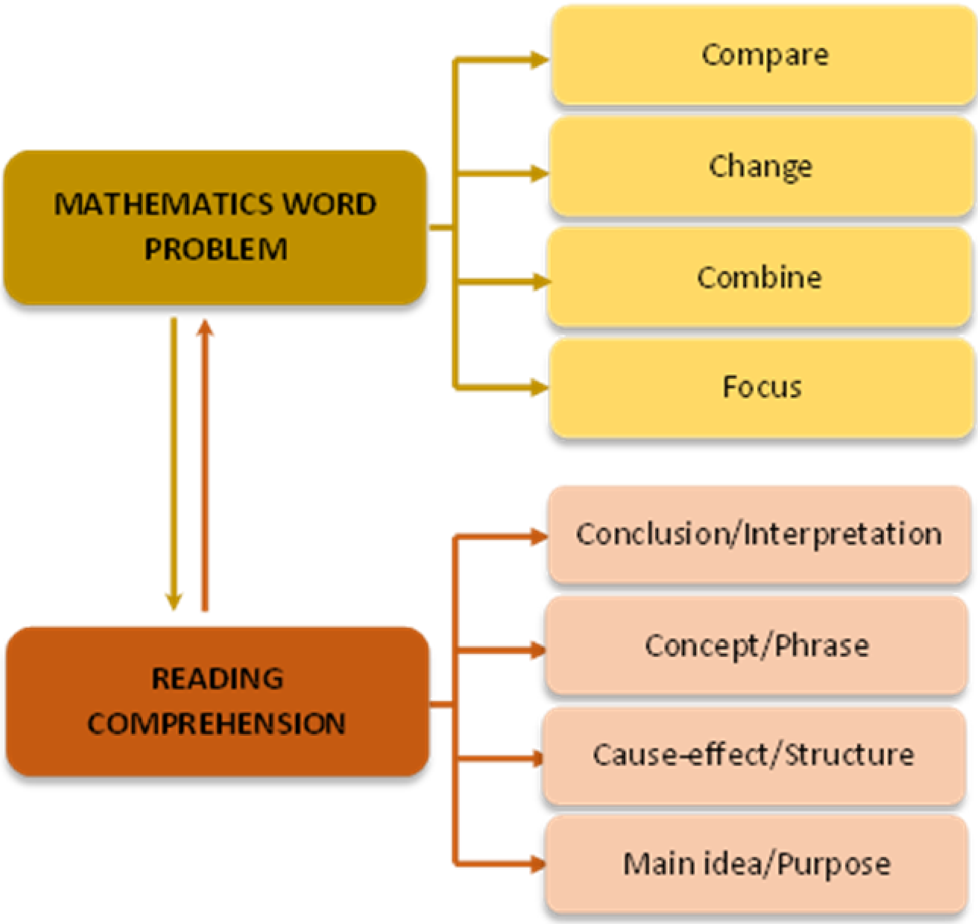
'How many oranges are in the tree?', the unit is oranges. The second phase is plan and solve, which is a visual technique. Writing an equation and drawing a picture are the two steps in planning and solving. After highlighting the crucial details, learners develop an equation. Learners choose how to solve a problem based on the link between the parts and the whole by formulating an equation after reading. In order to develop algebra ready skills, learners are trained to use a question mark that functions as a variable to denote the absence of a part or a whole in the equation. Although the word *variable* is not used, MSI includes the idea of solving for an unknown quantity. Retracing or checking the work, is the last phase. In order to verify that the question was addressed, learners now create the inverse fact equation and recount the illustration.

De Koning et al. (2017) categorised mathematical word problems into the following four categories: change, equalise, combine, and compare word problems. According to Mostert (2019), the term *change* is better than *join/separate* since it emphasises the shared action. Both *collection* and *part-part-whole*, which emphasise a structure shared by all additive related issue types, are preferred to *combine*, which indicates an action despite being a static problem type. The following are examples of the different types of problems:

- **Change:** Leo had 5 cakes. Then Lee gave him 6 more cakes. How many cakes does Leo have now?
- **Equalise:** Leo has 6 cakes. Lee has 9 cakes. How many cakes do Leo need to have as many as Lee?
- **Combine:** Leo has 10 cakes. Lee has 3 cakes. How many cakes do they have altogether?
- **Compare:** Leo has 9 cakes. Lee has 4 cakes. How many cakes do Leo have more than Lee?

Vilenius-Tuohimaa et al. (2008) took a combination of these techniques into consideration, and their findings were supported by Reynders (2014) with some modifications. Vilenius-Tuohimaa et al. claimed that reading generally works on two levels and promotes understanding. The reader first determines the sentence's meaning before applying their existing general and domain-specific knowledge to the content at hand. Vilenius-Tuohimaa et al. classified four categories of reading comprehension questions, namely conclusion/interpretation, concept/phrase, cause-and-effect/structure, and core idea/purpose. To address one of their research questions, namely whether text comprehension abilities and performance on mathematical word problems are related, they established a schematic model (Figure 2.2) for mathematical word-problem components like compare, change, combine, and focus.

Figure 2.2
A schematic model for mathematics word problems and reading (adapted from Vilenius-Tuohimaa et al., 2008)



2.6 Theoretical framework

I investigated how primary school teachers plan and teach mathematical word problems using two theoretical lenses, namely RME (Freudenthal, 1968) and PCK (Shulman, 1986). Using two theoretical frameworks rather than one was advantageous because they supported one another.

RME is a principle that teachers must use while teaching word problems and learners must use when solving word problems, even though mathematics education is realistic. In RME, mathematics teachers present a new concept to the class through a word problem, and the learners are required to read the word problem with understanding while applying the new concept they have learned that is when PCK engaged. The RME framework engaged at this point when learners read and understand word problems, they must use the correct strategy to solve the mathematical problem before comparing their solutions to the original query.

2.6.1 Pedagogical content knowledge

According to Shulman (1987), the “missing paradigm” in teacher education research is the paucity of studies on how teachers turn their subject-matter expertise into courses that their learners can understand. He discussed the concept of PCK in his 1985 presidential address to the American Educational Research Association and said the following:

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one’s subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations in a word, the ways of representing and formulating the subject that make it comprehensible to others … Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (Shulman, 1986, p. 9)

According to Shulman (2015), PCK entails the following components: (a) knowledge of themes generally taught in one’s subject area; (b) knowledge of techniques to communicate those concepts; and (c) knowledge of students’ understanding of the issues (Shulman, 1986). Gudmundsdottir and Shulman (1987) added the following to this definition of PCK: (a) Knowledge of the major topics, concepts, and areas of the subject matter that may be and are taught to learners; (b) expertise in the use of metaphors, similes, illustrations, and analogies to convey concepts to learners, which is influenced by content knowledge; (c) knowledge of learners impacts knowledge of the subjects that learners find interesting, challenging, or easy to learn; (d) knowledge of learners impacts on knowledge of the various ways topics can be taught, and the advantages and disadvantages of each approach that is influenced by general pedagogical knowledge; and (e) knowledge of students' preconceptions or misconceptions about the topics they learn.

Following his introduction of the theory of PCK in 1986, Shulman (1986, 1987) advocated that teaching expertise should be articulated and assessed in terms of PCK. Shulman (1986) offered a paradigm for analysing instructors' knowledge and distinguished between various forms of information that may serve a variety of functions in training. The current study demonstrated the following for PCK (Shulman, 1986): It is crucial to comprehend how particular concepts, ideas, and strategies in a particular topic area are learned and how susceptible it is to forgetting or being misconstrued. Such information comprises the classifications that can be used to group similar problem types or conceptualisations (for example, what are the 10 most common forms of algebra word problems?), the psychology of learning them, and which verbal structures are the least understood? (Shulman, 1986). According to Shulman (1986), pedagogical topic knowledge embodies the aspects of the subject matter that are most crucial to its teachability.

It is possible for teachers to "transform their subject-matter expertise into forms that are pedagogically potent and yet adaptive to the differences in ability and background given by learners" using PCK, a specialised field of subject-matter expertise (Shulman, 1987, p. 15). It covers information on pedagogical representations, learners' prior knowledge, learning challenges, misconceptions, as well as instructional techniques that build on prior knowledge and solve learning challenges and misconceptions. To put it another way, it entails understanding the most potent analogies, illustrations, examples, explanations, and demonstrations as well as the most successful ways to depict the most taught concepts in one's subject area.

Another part of pedagogical subject knowledge is understanding the notions and preconceptions that learners of various ages and backgrounds bring to the learning of those most frequently taught topics and courses. These are the elements that affect how easy or difficult a subject is to learn. Teachers need to be aware of the strategies that are most likely to be successful organising learners' knowledge if these beliefs are misconceptions, which they commonly are, as it is unlikely that they will approach them with a clean slate (Shulman, 1986).

Shulman's (1986) work is arguably the most significant of the attempts to map out teachers' professional knowledge. In it, he defined three key domains: Subject Matter Knowledge (SMK), Pedagogical Content Knowledge (PCK), and Curricular Knowledge (CK). The addition of PCK and the subject matter to be taught as the defining characteristics of teachers' knowledge was Shulman's most important contribution. Many approaches to conceptualizing teachers' knowledge have been put forth since Shulman's ground-breaking work was published; these approaches emphasize different components and aspects for examples,.(Baumert & Kunter, 2013; Davis & Simmt, 2006; Ma, 1999; Rowland, 2009; Schoenfeld & Kilpatrick, 2008)

The Mathematical Knowledge for Teaching (MKT) paradigm developed by Ball et al. (2008) is particularly noteworthy. MKT takes PCK and SMK into account. Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), and Knowledge of Content and Curriculum (KCC) constitute PCK; while Specialized Content Knowledge (SCK), Common Content Knowledge (CCK), and Horizon Content Knowledge (HCK) constitute SMK. CCK is defined as the knowledge of the relevant educational level possessed by an adult who is well-educated. In contrast to the mathematical knowledge needed by other professionals who utilize mathematics, SCK acknowledges the specific nature of a teacher's mathematical understanding. The idea that a teacher ought to show some understanding of the connections between school mathematics is reflected in HCK.

KCS encompasses the teacher's ability to predict what the pupils will find simple, tough, fascinating, or motivating. KCT considers the information that helps teachers negotiate the particulars of the lesson, like how to highlight or explain a certain mathematical concept. KCC consists of the knowledge that is used to decide what kind of content and in which direction students should be learning. The identification of a kind of knowledge unique to instructors (SCK), based on the notion that teaching necessitates specialized knowledge that other professions do not, is one of the important contributions made.

2.6.2 Realistic mathematics education

According to Van den Heuvel-Panhuizen and Drijvers (2020), RME is a domain-specific education theory for mathematics that was created in the Netherlands. The importance of rich, realistic scenarios in the learning process is unique to RME. In word problems, the RME technique focuses on helping learners go beyond what they are taught in the classroom and link it to real-life problems (Suaebah et al., 2020).

The idea of mathematization comes from the RME paradigm (Van den Heuvel-Panhuizen & Drijvers, 2020). It describes the process of using mathematics to organise and investigate any form of reality. For example, it describes converting a real-world problem into a symbolic mathematical problem and vice versa as well as rearranging and (re)constructing problems within the context of mathematics. Depending on how important and possible they are to the learners, *reality* may refer to real life, a fantasy world, or mathematical problems, for instance, because the learners had already encountered and comprehended its key components (Freudenthal, 1991; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2001; Van den Heuvel-Panhuizen & Drijvers, 2020). There are two types of mathematization, namely horizontal mathematization and vertical mathematization (Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).

Horizontal mathematization is the process of converting practical problems into symbolic mathematical problems through inductive reasoning, investigation, and observation (Da, 2022). Horizontal mathematization is characterised by activities such as locating the specific mathematics in a general framework, schematising, phrasing and picturing a problem in several ways, and uncovering relationships (De Lange, 1987). The process of horizontal mathematization can be used to solve word problems, particularly those involving both symbolic and natural language.

Vertical mathematization, which comprises problem-solving, generalising the solution, and further formalising the result, is the process of restructuring and (re)constructing inside the world of symbols (Treffers, 1987). Vertical mathematization is characterised by tasks such as modifying and improving mathematical models, using various models, combining and integrating models, and generalising (De Lange, 1987). According to Pratiwi and Waziana (2018), vertical mathematization encompasses both mechanical, or automated, processes and holistic features of restructuring and (re)constructing within the universe of symbols. In vertical mathematization, "symbols are mechanically shaped, reshaped, and altered in order to understand and reflect" (Freudenthal, 1991, p.42).

The word *realistic* in RME comes from the Dutch phrase *zich REALISEren*, which literally means "to imagine" (Van den Heuvel-Panhuizen & Drijvers, 2020). The emphasis on making something real in the minds of learners gives RME it is name. As a result, RME challenges can come from the real world, the fantasy world of fairy tales, or the formal world of mathematics, as long as the difficulties are experientially real in the eyes of the learners (Van den Heuvel-Panhuizen & Drijvers, 2020). These scenarios serve as a jumping-off point for the development of mathematical concepts, methods, and procedures, as well as a framework within which learners can apply their mathematical knowledge, which has become more formal and generic over time and less context specific (Suaebah et al., 2020). Although *real-world* conditions are important in RME, the term *realistic* has a broader connotation here (Van den Heuvel-Panhuizen & Drijvers, 2020).

RME is unquestionably a product of its time and is inextricably linked to the global reform movement in mathematics education that has occurred in recent decades. As a result, RME's current approach to math instruction is comparable to that of other countries. Despite this, RME is intricately tied to a few essential concepts in mathematics education. Most of these essential teaching ideas were first articulated by Treffers (1978) and were changed over time, most notably by Treffers himself. The following six principles can be identified:

- Learners at RME are viewed as active learners according to the **activity principle**. It emphasises that doing mathematics is the best method to learn it, which is illustrated by Freudenthal (1979) and Van den Heuvel-Panhuizen (2015) viewing mathematics as a human activity and by Freudenthal's and Treffers' (1978) idea of mathematization.
- The **reality principle** is recognised by RME in two ways. To begin with, it emphasises the importance of the mathematics education goal, which involves learners' ability to apply mathematics to *real-life* problems. Secondly, it proposes that learners' mathematics education begin with relevant problem scenarios, allowing them to attach meaning to the mathematical constructions they create when solving problems. RME begins with problems in rich contexts that require

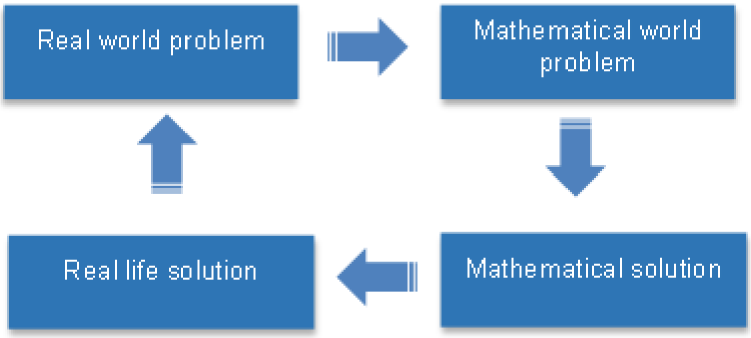
mathematical organisation, or in other words, that can be mathematized, and sets learners on the path of informal context-related solution strategies as a first step in the learning process, rather than beginning with abstractions or definitions that will be applied later.

- The **level principle** highlights that learning mathematics requires learners to go through multiple stages of comprehension, from informal context-related solutions, establishing various levels of shortcuts and schematisations to understanding how ideas and approaches are related. Models are essential for bridging the gap between informal and formal mathematics. Streefland (1996) explained that models must transcend from being a *model of* a given scenario to being a *model for* a wide variety of similar but unrelated scenarios to execute this bridging function (Van den Heuvel-Panhuizen, 2003). This level principle is embodied in Treffers' (1987) didactical method of *progressive schematisation* in which visible whole number methods are employed to teach operating with numbers.
- The **intertwinement principle** states that mathematical topic categories such as number, geometry, measurement, and data management are not treated as separate curricular chapters but rather as tightly linked together. Learners are presented with a variety of problems in which they can apply their mathematics skills and knowledge. This principle holds true within domains as well. Within the realm of number sense, for example, mental arithmetic, estimating, and algorithms are all taught in tandem.
- According to RME's **interaction principle**, learning mathematics is both an individual and a social activity. Therefore, RME encourages whole class discussions and group work, allowing students to share their ideas and skills with others. This is a good technique for students to get ideas for improving their tactics.
- The **guidance principle** refers to Freudenthal's (1979) concept of *directed re-invention* of mathematics. It suggests that in RME, teachers should play an active part in their students' learning, and that educational programs should incorporate scenarios that can change learners' perceptions. To achieve this, teaching and learning should be built on long-term, consistent teaching-learning trajectories.

The two methods of mathematization complement one another in all stages of mathematical endeavour (De Lange, 1987). The overall concept of the horizontal and vertical mathematization activity is shown in Figure 2.3. According to De Lange (1987), the process of mathematization learners engage in during the learning process is individual and may follow many paths based on how they perceive a genuine scenario, their talents, and their capacity for problem-solving. De Lange (2006) asserted that the mathematization process that leaners engage in has a cyclical nature. The learners who assume the role of the problem solver begins the process by comprehending the problem and finding the pertinent mathematical principles inside it. The problem solver then trims away the extraneous aspects that exist by putting the problem into a mathematical model based on the discovered mathematical ideas.

Figure 2.3

Cyclical nature of mathematization (adapted from Suaebah et al., 2020, p. 2)



The first two steps, which concern horizontal mathematization, convert a practical situation into a symbolic mathematical problem. The third stage, which defines vertical mathematization, occurs in the symbolic mathematical realm. Step four, which once more involves horizontal mathematization, involves interpreting the mathematical solution in terms of the practical solution. Vertical mathematization is involved if the construction of the realistic solution in terms of the original realistic problem comprises validating all conditions in the problem, generalising the solution technique, and recognising a potential application of this procedure in other comparable problems.

The function of horizontal and vertical mathematization is explained in the model in Figure 2.3. Horizontal mathematizing begins with a realistic context (word problems) expressed in English, and then learners engage with the challenge by translating it into a mathematical model. As a result, after grasping the problem, learners proceed to solve it using mathematical objects, structures, or other solutions, which is vertical mathematization. Learners must return to a realistic setting after completing word problems using mathematics symbols and explain their conclusions in a relevant manner.

2.6.3 Relevance of the two theoretical lenses to my study

In this study, I used PCK and RME to demonstrate how mathematics word problems should be planned and taught in a classroom setting. For example, PCK informs educators about the knowledge and skills they must possess in order to teach mathematics word problems, specifically content knowledge. One of the research questions asked how primary school teachers facilitate the learning of mathematical word problems, which led to PCK. A mathematics educator must have knowledge on how to teach word problems and how to present the content to learners. RME emerged in this study when I examined how math teachers teach word problems and how learners solve them. The relevance of RME is determined by the type of mathematical word problem presented in class, whether it is applicable to real-world situations, and whether the question makes sense to the learners. It is also based on an analysis of how learners convert mathematical word problems into mathematical symbols and how they approach word problem solving. The usefulness of RME and PCK in this study can be summed up as follows: RME provided the guidance on how realist mathematics education correlate with mathematical word problems, especially word problems that link what is happening in the classroom and what is happening in the reality; and PCK reviewed what content knowledge mathematics teachers have with regard to planning for and teaching mathematical word problems.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

I used Saunders et al.’s (2015) research onion to guide the structure of the research methodology chapter. According to Melnikovas (2018), the research onion provides a detailed description of the numerous phases required to systematically produce a research report. The sequential layers of the research onion proposed by Saunders et al. (2015) include the following: Research philosophy; research approach; methodological choice; research strategy; time horizon; and techniques and procedures. In the following sections, I address each layer of the research onion as it applies to my study. I start with an explanation of the layer and then articulate the context in which I used each layer in my study to justify my choice.

3.2 Research philosophy

According to Saunders et al. (2015), a research philosophy establishes the ontology as nature of reality, the epistemology as sources of knowledge, and the axiology as values, beliefs, and ethics of the research. The origin, nature, and growth of knowledge are some of the elements of a research philosophy. A research philosophy can be defined as a set of beliefs regarding the methods that should be used to gather, analyse, and apply evidence about a topic (Melnikovas, 2018). The value of philosophy for human existence is how it interacts with, dissects, or examines concepts so that it may be used to evaluate, appraise, and interpret phenomena and does not rest in its definition but in its utility for human existence (Melnikovas, 2018).

Phenomenology, an approach that promotes the necessity to consider the subjective interpretations of people and their perceptions of their lives, had a significant influence on interpretivism (Bell & Waters, 2018). Reality, according to interpretivists, is socially constructed rather than objectively determined (Berger & Luckmann, 2016). Consequently, starting with people in their social contexts or habitats will give researchers a better opportunity to understand how they see their own behaviours (Bell & Waters, 2018)

In order to understand why the Grade 7 mathematics teachers use certain instructional practices in teaching and learning mathematical word problems, I used interpretivism as my research paradigm or research philosophy. Interpretivists advocate for individuals' capacity to create meaning (Alharahsheh & Pius, 2020). Furthermore, interpretivism contends that methods for understanding knowledge in the human and social sciences cannot be the same as those used in the natural sciences since humans interpret and act on their environment (Pham, 2018). As a result, interpretivists adopt a relativist ontology in which a single experience can have several interpretations as opposed to a single reality revealed through measurement. Considering this philosophical underpinning, interpretivism bases its ontological premises on the notion that there are numerous perspectives on an event because multiple individuals interpret the same event in distinct ways. In order to understand how people create meaning, a researcher must step inside this environment and analyse it from within through the actual experience of the individuals.

Through the ontological standpoint of interpretivism, I collected data from two different schools. I allowed the two teachers to share their opinions and experiences about how mathematical word problems are taught in Grade 7 by observing them and analysing their respective written materials. Interpretivists have come to the conclusion that there are numerous realities based on human interpretation and experience because of the potential for differences in interpretations and experiences (Mertens, 2016). According Saunders et al. (2015), reality can only be fully grasped by the subjective intervention in it as well as its interpretation.

In the interpretivist epistemological perspective, I presented myself as a researcher in a classroom setting and observed mathematics teachers teach mathematical word problems. For me to understand how the participants interpret and comprehend a mathematical word problems, I communicated with them and respected their opinions and experiences regarding the way that Grade 7 learners are taught mathematical word problems. In the interpretivist methodological perspective, I used the qualitative approach as a method to help me respond to my research questions (section 3.3.). In accordance with the axiology of interpretivism, I made sure that as a researcher I valued and respected the perspectives, convictions, and opinions that the participants shared with regard to the teaching and learning of mathematical word problems. I give more details regarding axiological issues in section 3.11.

Interpretivism allowed me to better understand how knowledge is socially constructed and its complexity in its specific context rather than seeking to generalise the basis of knowing for the entire population (Fetters & Molina-Azorin, 2019). The benefit of using the interpretivism paradigm in my research was that it offered a variety of perspectives on the phenomenon since interpretivist researchers can describe things, people, or events while also truly understanding them in their social context.

3.3 Research approach

Before I delve into the intricacies of the research approach I chose for the study, I must demystify *research approach* as used in the research onion advocated by Saunders et al. (2015). Generally, the term research approach is used in literature to refer to either a qualitative, quantitative or mixed methods approach (Melnikovas, 2018). However, Saunders et al. used *research approach* to refer to either a deductive, inductive, or abductive approach. For Saunders et al., the terms qualitative, quantitative, or mixed method approach are categorised as a methodological choice (section 3.4). In the current section, therefore, I align myself with Saunders et al. and use the term research approach to refer to a deductive or inductive approach.

According to Saunders et al. (2015), there are three basic methods for developing theories, namely deductive, inductive, and abductive approaches. Deductive approach refers to the process of reasoning that moves from a broad rule to a specific law-like inference and is typically used for hypothesis testing. By starting with a specific observation and forming a general rule from it, inductive reasoning is a method used to develop theories.

Qualitative research aligns with inductive thinking because it assumes a progression from the particular to the universal (Bezuidenhout et al., 2017). It encourages subjectivity and meaningful deductions regarding the underlying causes or justifications for the observed phenomena (Feldbacher-Escamilla & Gebharter, 2019). An inductive approach seeks to develop patterns, consistencies, and meaning. It enables the researcher to find a unifying principle (Morgan et al., 2017). It is closely related to the everyday reasoning that people use in their relationships. According to Mousa and Molnár (2020), induction involves a number of cognitive processes, including decision-making, classification, probability assessment, analogy reasoning, and scientific inference. This enables the researcher to create patterns as well as spot data anomalies (Bunge & Leib, 2020).

According to Khemlani and Johnson-Laird (2016), using a deductive approach often entails coming to "legitimate conclusions", and it asserts that there is only one valid truth. An inductive approach, which accepts numerous realities, contrasts sharply with this view (Blundell & Harwell, 2016). In order to arrive at a sound conclusion, a deductive approach requires the establishment of a true premise. Hypotheses and their testing, confirmation, or rejection are favoured by researchers using a deductive approach. Deduction is the process of drawing conclusions from known facts or presumptions or inferences by applying general principles to specific situations (Abazi, 2018).

An abductive approach is the process of inferring a theory from data that is already known (Bennett et al., 2018). Abduction creates a midway ground between inductive and deductive thinking (Fazey et al., 2018). The purpose of abductions is to deduce something about the hidden causes or justifications behind the occurrences that are being seen (Fazey et al., 2018). So, in processes of discovery and knowledge development, abduction serves as a discovery tool (Karlgren et al., 2020). Because of this, it may be thought of as the type of thinking that is used most frequently in a mixed methods approach (Awuzie & McDermott, 2017). It is connected to pragmatism (Mitchell & Education, 2018).

The current study was done using an inductive research approach; however, there was room for the deductive approach because it was supported by a theoretical framework that improved my understanding of what to anticipate in the teaching and learning of word problems as a researcher. Therefore, my study was mainly driven by an inductive approach, with a deductive approach playing a complementary role; hence, I used an inductive-deductive approach. An inductive approach is consistent with interpretivism, a subjective philosophy that emphasises making judgements about the underlying causes or explanations of perceptible occurrences (Azungah, 2018). The fundamental argument for using an inductive approach in this study was that it freed me as researcher from being constrained by organised methods to derive conclusions from the common, dominant, or noteworthy themes present in the raw data. Using an inductive approach in my research also helped me identify guiding concepts and make attempts to identify patterns, consistencies, and meaning (Astroth & Chung, 2018).

Deductive reasoning was reinforced by my theoretical framework, which informed me about how to teach mathematical word problems and what to anticipate prior to data collection. Deduction is therefore the process of drawing conclusions based on generalisations or assumptions, or inference from generals to specifics (Rahmah, 2017). One drawback of using an inductive approach is that it is time consuming as it takes a long time to formulate the themes for analysing data, and as a researcher, I had to depend on the participants to find fresh information (Azungah, 2018). In order to keep my study on track, I had to manage those constraints by adhering to a specific timeline in order to avoid wasting time while gathering data. The fact that my theoretical framework and paradigm were founded on participant experience in the teaching and learning of mathematical word problems also made my interpretation of it arbitrary.

3.4 Methodological choice

Saunders et al. (2015) describe methodological choices with reference to the employment of quantitative and qualitative research methods, as well as the simple or complicated combination of both, or the use of one approach. Quantitative research techniques use statistics and mathematical procedures, whereas qualitative techniques call for gathering a large amount of descriptive data.

The goal of qualitative research is to comprehend a topic at a deeper level rather than to accurately depict it numerically. When conducting qualitative research, the researcher himself or herself serves as both his subject and his object of study (Queirós et al., 2017). In this scenario, the researcher is led by the study’s objectives and research questions. The objective of the qualitative methodology is to produce in-depth and descriptive information in order to understand the various features of the problem being investigated. Because of this, qualitative research focuses on comprehending and explaining the dynamics of social relationships and is concerned with parts of reality that cannot be quantified (Queirós et al., 2017). According to Maxwell (2021), qualitative research focuses on the universe of meanings, motives, aspirations, beliefs, values, and attitudes, which relates to a deeper realm of relationships, processes, and phenomena that cannot be reduced to the operationalisation of variables. Alternatively, the data might be quantified in quantitative research. The findings are interpreted as though they provided a broad and sufficiently detailed view of the entire population because the samples are often big and considered emblematic of the population (Courtney, 2021). In the process of analysing and generalising the data gained, fields such as statistics and mathematics have a fundamental significance (Queirós et al., 2017). According to Queirós et al. (2017), when it is possible to gather quantifiable measurements of variables and interpretations from samples of a population, quantitative research, which is focused on objectivity, is particularly appropriate. In quantitative research, the procedures and equipment for gathering data are institutionalised. The data collection process is thorough and objective. Statistical procedures are also used to analyse numerical data.

In line with the interpretivist paradigm which allows for multiple interpretations of the same event, I opted for qualitative research method. By gathering and analysing non-numerical data, qualitative research seeks to better understand the thoughts, beliefs, or experiences of Grade 7 teachers about the teaching and learning of mathematical word problems (Kamran et al., 2021). I was able to gain in-depth understanding of individuals' experiences, feelings, and perspectives using qualitative research.

3.5 Research strategy

Some scholars, such as Bloomfield and Fisher (2019), referred to this layer as the research design, but Saunders et al. (2015), with whom I aligned myself, referred to it as the research strategy. According to Saunders et al. (2015), a research strategy helps the researcher select the primary data collection methods or sets of procedures needed to address the research question and achieve the study's goals. Research design is the overall approach taken to do research and establishes a clear and logical plan to address predetermined research question(s) through the collection, interpretation, analysis, and discussion of data (Sileyew, 2019).

In this research, I used a case study to explore primary school mathematics teachers' planning and teaching of word problems. According to Heale and Twycross (2018), a case study is a thorough investigation into one individual, group of people, or unit. A case study can also be defined as a thorough systematic assessment of a single person, group, community, or other unit in which the researcher looks at extensive data relating to a number of variables (Thomas, 2021).

Since the phenomenon that was explored, namely teachers' planning and teaching of word problems, lacked a distinct and unified set of outcomes, I chose an exploratory case study design (Daniel, 2019). This allowed me to emphasise a concern in a bounded system in order to better comprehend the phenomenon that was the subject of my investigation (Daniel, 2019). I used an exploratory case study with the aim of answering *how* and *what* questions by exploring primary school mathematics teachers' word problems instructional practices (Sjoberg et al., 2020). There are two different kinds of case studies, namely single case studies and multiple case studies (Gustafsson, 2017). A single case study focuses on one particular group, individual, or even set of cases, and a multiple case study focuses on two or more cases (Cheung & Hennebry-Leung, 2023). According to Rashid et al. (2019), single case study is recommended if a researcher wishes to investigate a specific occurrence that results from a particular entity since it would allow for a thorough comprehension of that phenomenon.

The current study supported single case study for which two mathematics teachers at two primary schools were selected, observed while presenting classes, and asked to avail their lesson plans for documents analysis. By incorporating a case study into my research, I managed to gain practical, contextual, and in-depth understanding of teachers' instructional approaches to teaching word problems. Additionally, it enabled me to focus more on the case's essential traits, interpretations, and consequences while keeping my research organised and manageable.

3.6 Time horizon

Saunders et al. (2015) asserted that temporal horizons in research studies often refer to periods to be researched or chronological horizons of variable breadth. Three fundamental time frames were identified by Dean (2019), namely short-term, defined as up to 10 years; medium-term, defined as 25 years or more, and long-term, defined as more than 25 years. The research-related timescale is known as the time horizon in the research onion (Su et al., 2019). In this time frame, the researcher is interested in examining the population. Depending on the goals of the study and the sort of investigation, the researcher chooses the time frame (Bigoni et al., 2015). A population study at a particular time or a population study across time may be of interest to the researcher (Sankoff, 2019).

There are two types of studies that are time-bound, namely longitudinal research and cross-sectional research, depending on the time horizon (Bigoni et al., 2015). Cross-sectional research is characterised by the need to examine samples or collect data at a certain point in time (a short-term time frame), and in longitudinal studies, samples are tracked and data collected over a period of time (medium- to long-term time frame). Essentially, cross-sectional research does not repeat the process of collecting data from samples (Su et al., 2019).

For this study, I opted for a cross-sectional time frame. Cross-sectional studies are generally less expensive and quicker to execute than longitudinal studies (Da et al., 2021).

3.7 Sampling

The two main constraints on a study's ability to involve the full population are time and cost (Zhao et al., 2019). As a result, sampling is a necessary component of most studies, especially when a sizeable population is being studied (Stratton, 2021). Sampling techniques fall into two categories, namely probability techniques and non-probability techniques (Gabriel et al., 2019). According to Lehdonvirta et al. (2021), non-probability sampling is a sampling technique that uses factors other than randomness to determine the sample size, such as availability, proximity to the study subject, or subject-matter expertise. When the population parameters cannot be individually identified or are unknown, non-probability sampling is used (Berndt, 2020). In contrast to non-probability approaches, probability methods are founded on the concepts of randomness and probability theory. As a result, probability samples meet the criteria for using probability theory to accurately generalise to the population.

Since this study did not use a random selection of population elements, a non-probability sampling method was used. Non-probability sampling methods come in four primary categories, namely convenience, quota, snowball, and purposive sampling (Tutz, 2023). The best sampling strategy for my investigation was purposive sampling. This type of sampling, also known as judgement sampling, involves the researcher using their expertise to choose a sample that is most pertinent to the study's objectives (Campbell et al., 2020). Purposive sampling's major objective is to concentrate on key demographic features that are interesting and will help the researcher respond to their research questions (Denieffe, 2020). Purposive sampling is generally applicable to conducting qualitative research and case study research designs (Ames et al., 2019). Purposive sampling enables researchers to select instances that are information rich; in other words, the participants are chosen because they have important knowledge about the topic being investigated (Gentles et al., 2015). As a researcher, a well-designed sample aided my ability to learn a great deal about mathematical word-problem teaching and learning.

The sample consisted of two mathematics teachers and their Grade 7 learners from two separate public primary schools located in one education district. Both schools are in a rural area and have two Grade 7 classes; however, I only collected my data in one class per school. The class used for this study was chosen by the teacher depending on a convenient time for me to collect data because the same content is taught in both classes. The reason for selecting Grade 7 was that it is the exit grade in primary school and the entry grade in the Senior Phase (Grades 7 to 9), and therefore, the perfect grade to examine learners' word-problem knowledge before they enter high school in South Africa's schooling system. In addition, as indicated in the problem statement (section 1.2), Trends in Mathematics and Science Study diagnostic reports revealed that learners struggle with word problems.

3.8 Data collection

I describe the procedures and methods used to obtain data from the participants in this section. In line with interpretivist philosophy, I used a variety of data collection strategies for triangulation and crystallisation in order to support my findings and develop a thorough grasp of the phenomenon. Data for the current study were collected using document analysis and non-participant observation to achieve triangulation. These collection methods are discussed in the following sections.

3.8.1 Observation

Observation is the data collection technique of noting the behavioural patterns of subjects, things, and events without necessarily asking them questions or interacting with them (Anis et al., 2021). Maree and Omlin (2022) cited observation as the most crucial method of data collection and as essential to human interactions, and added that first-hand data must be created for events as they happen. Observation was employed as a data collection strategy in this study to capture the main raw data.

During lesson observations, I gathered data using a field notes and observation schedule (Appendix A). I observed one lesson presentation on word problems for each of the two participants. Since I collected data from two participants from different schools, each participant gave me information about the time they would be teaching mathematical word problems.

I concentrated on the problems presented in the context of the use of operations and language richness and what types of questions are posed in the mathematics classroom as I observed the participants engage in teaching and learning activities related to mathematical word problems. Additionally, I observed how the participants used horizontal and vertical mathematization, which is the process by which they translate mathematical word problems into mathematical symbols, solve mathematical word problems using mathematical operations, and finally, translate the mathematical solution into the context of the real world. The use of observation as a qualitative data-gathering technique allowed me to obtain a deeper understanding of the phenomenon (primary school mathematics teachers' planning and teaching of word problems).

I took on the character of an observer who did not participate in the class presentations and did not speak with the teachers, but I noted their actions (Sileyew, 2019). The case study research design, qualitative research methodology, and interpretivist paradigm allowed me to observe events in their natural environments. To this effect, the choice of observation as one of the ways for collecting data was fitting (Martinko & Gardner, 2019). According to Ncala (2020), while employing observation as a method of data collection, a researcher must be judicious about the information they gather. Only pertinent phenomena at the research sites were included in the observed data by using the research questions as a guide. I captured only the teacher-related activities that entailed completing mathematical word problems in field notes and audio recordings. The foundations of non-participant observation were followed, including maintaining a low profile and avoiding any contact with the course presentation that would have influenced the activities in order to prevent biases (Johnson et al., 2020).

3.8.2 Document analysis

Document analysis is a data-gathering method that focuses on all written communications that could shed light on the phenomenon being studied (Owen, 2014). Document analysis is a methodical process used to review or assess documents, both printed and electronic (computer-based and Internet-transmitted). Data must be examined and interpreted in order to elicit meaning, gain understanding, and produce empirical knowledge when using document analysis, just like when using other analytical techniques in qualitative research (Busetto et al., 2020). I collected second-hand data from the participants via document analysis with the aim to understand how mathematical word problems are taught, for example, how educators prepare to deliver the lesson to the learners and how learners apply the material they were taught to solve word problems (Appendix B).

By reading through learners' written work, I was able to gain insight into how they approach word problems. Additionally, each teacher gave their lesson plans to me to analyse in order to gain insight into how they had prepared to teach their lessons on mathematical word problems. Documents (textual data) focus on all types of written correspondence that can shed light on the subject being examined as a technique of gathering information (Benoit et al., 2018). Using documents as a data source can help reconstruct events and important situations.

3.9 Data analysis

Analysing qualitative data requires conscious effort (Mihas, 2019), and therefore, a system to categorise participant comments was created to assess the data (Jackson & Bazeley, 2019). The research questions, design, and technique must all be compatible with the data analysis approach used. Thematic data analysis was used for this study.

Thematic data analysis was used to examine the data gathered for this study through document analysis and instructional observation. According to Braun and Clarke (2019), thematic analysis is used to methodically locate, compile, and provide insight into patterns of meaning (themes) in a data set. They further defined thematic analysis as a technique for finding, analysing, classifying, and reporting themes present in a data set. Thematic analysis allowed me to see and understand communal or shared meanings and experiences and to focus on meaning across the data set (Braun & Clarke, 2021). It is not the goal of thematic analysis to pinpoint particular and peculiar meanings and experiences that are found exclusively in a single data item. Therefore, using this approach helped me find commonalities in the way a subject was discussed or written about and interpret them (Braun & Clarke, 2021).

The theoretical frameworks and literature discussed in Chapter 2 and the inductive approach influenced and predetermined the themes. The literature review helped me relate what other scholars had said to my study, and the theoretical framework guided me on the methodology to use to collect data. Following the principles of thematic analysis, the information gathered for this study was arranged in accordance with pertinent topics that addressed the stated research questions (Nowell et al., 2017). One of my justifications for using thematic analysis is that the present study adhered to a qualitative research approach, and experts generally concurred that thematic analysis is congruent with qualitative research (Nowell et al., 2017). I used thematic analysis as the primary data analysis technique because when done successfully, it ensures the trustworthiness of research findings (Braun & Clarke, 2021)

According to Nowell et al. (2017), there are six crucial steps that should be logically followed when doing thematic analysis. They also pointed out that there is no clearly defined customary approach to use to analyse qualitative data in the literature. Another benefit of using thematic analysis for data analysis is its adaptability. The steps I took using thematic data analysis, as described by Nowell et al. (2017) and Braun and Clarke (2019), are summarised in the following sections. I took advantage of known themes that were pertinent to the issues I was researching for my secondary sources, which were lesson plans. The information was separated into the following three categories because I had three secondary research questions: Understanding of mathematical word problems; teaching and learning mathematical word problem; and strategies for solving mathematical word problems.

3.9.1 Stage 1: Familiarise yourself with your data

Researchers use this phase to review or listen to audio recordings obtained during observations and document analysis to become fully immersed in the gathered data (Braun & Clarke, 2019). I asked the teachers to supply their lesson plans and learners' textbooks for the purpose of document analysis. The teachers were observed teaching mathematical word problems in their classrooms for one lesson. I took my time reading the data I had obtained and eliminated irrelevant data that had no bearing on my secondary research questions.

3.9.2 Stage 2: Generate initial codes

Braun and Clarke (2021) asserted that interaction, in-depth reading, and comfort with the content are the first steps in the Stage 2. I created a preliminary list of ideas that addressed the intriguing aspects of the data. During this stage, I created codes that explained how teachers teach mathematical word problems, how they assisted students in learning mathematical word problems, and how learners mathematically schematised word problems that were presented in a real-world setting. For instance, planning of the lesson, using mathematical operations, and changing mathematical word problems to mathematical symbols and learners doing calculations.

3.9.3 Stage 3: Search for themes

After the data have been coded, the third stage of establishing the theme began. Stage 3 involves grouping the numerous codes created in stage 2 into potential themes, which refocuses the study on themes rather than codes, which is at a higher level of abstraction (Braun & Clarke, 2021). I categorised the codes into many themes that directly addressed the research problem regarding the current study. For instance, Teachers' planning for teaching mathematical word problems, Facilitating the learning of mathematical word problems, and Schematisation of the word problems presented in real-life context.

3.9.4 Stage 4: Review themes

In Stage 4, Nowell et al. (2017) assert that the researcher decides which themes to maintain and which to eliminate from the list of prospective themes since they are neither themes nor pertinent to the study. The themes in my study were my primary focus, and I extensively scrutinised the data within the themes. I maintained the topics that were pertinent and eliminated all of the concepts that did not significantly contribute to addressing my research questions.

3.9.5 Stage 5: Define and name themes

According to Braun and Clarke (2021), this phase logically follows from examining the concepts and developing a thematic map. I provided a description and relevant details for each theme (see Chapter 4). I broke down the information in the themes into subthemes and categories to generate in-depth descriptions of the themes. In total, three themes emerged from the study, and they include: Teachers' planning for teaching mathematical word problems, Facilitating the learning of mathematical word problems, and Schematisation of the word problems presented in real-life context.

3.9.6 Stage 6: Produce the report

I arranged in tables the three themes, together with their respective subthemes, which completed the report's analysis and summary (Braun & Clarke, 2019). Chapter 5 constitutes the report in the form of the research findings. Chapter 5 was informed by the themes presented in paragraph 3.9.5.

3.10 Quality criteria

According to Laumann (2020), qualitative research employs the concept of trustworthiness rather than the validity and reliability used in quantitative investigations. The key determinants of trustworthiness are credibility, transferability, dependability, and conformability (Patias & Hohendorff, 2019). Curiously, Laumann (2020) asserted that a qualitative study is only trustworthy if the reader thinks it is credible. I upheld the four criteria for trustworthiness throughout the whole research process, from data collection to thematic analysis.

3.10.1 Credibility

Credibility is concerned with how closely the findings match reality (Laumann, 2020). By using data collection techniques that had been used in research similar studies, and using document analysis and non-participant observation to ensure triangulation, the present study's credibility was ensured. This helped produce data that are entirely representative of the real-life context in which mathematical word problems are implemented successfully in math classrooms (Laumann, 2020). According to Patias and Hohendorff (2019), triangulation may involve the use of numerous methods, including observation, focus groups, and individual interviews, which represent the major data-gathering procedures for much qualitative research.

The observations and documents analysis gathering procedures allowed me to completely immerse myself in the phenomenon's context as I observed mathematics teachers teach and read their documentation. I provided the participants with the transcripts of the data I had gathered for member verification, which allowed them to certify that the data had not been altered and were unaffected by researcher bias (Nowell et al., 2017).

3.10.2 Transferability

According to Nowell et al. (2017), external validity is the degree to which a study's findings can be extrapolated to apply to different situations, that is, to a larger population. It is impossible to generalise the findings from qualitative research because studies focus on a constrained context and fewer participants. However, readers who identify with the study's surroundings may think the results apply to them. Nowell et al. (2017) contended that this can be viewed as transferability and asserted that it is the reader who can discern whether the findings are applicable outside of the researcher's own context.

A thorough explanation of the contexts from which the data were collected was intended to guarantee transferability of the present study. It will allow readers to comprehend the contexts, compare the research's contextual variables to their own contextual factors, and establish the necessary connections. The practice of using mathematical word problems in mathematics is the phenomenon under research. The study's setting provides a detailed description of the phenomenon to allow readers to compare it to their own context's teaching methodologies (Patias & Hohendorff, 2019). The research complied with the following important elements that help ensure the findings are transferable (Yadav, 2022):

- The number of participating organisations and their locations were as follows: It is explicitly stated in the current study that I reviewed documents and observed two elementary mathematics teachers.
- Any limitations on the people who provided the data: The present study only included primary school teachers who teach mathematics to learners in Grade 7. Two mathematics educators and their learners were chosen as participants in this study, which involved a large number of participants in the fieldwork.
- The data-gathering techniques used: In the first section of this chapter, I detailed and explained the various data collection techniques I used for my qualitative research. I gathered data for the study via non-participant observation and document analysis.

- The quantity and duration of data collecting sessions: For each of the two participant teachers, one lesson observation was conducted, followed by a document analysis.
- The length of the data collection period in this study: The data were collected over two months, which also contributed to the validity of the study's conclusions.

3.10.3 Dependability

Dependability is used to describe the consistency of the findings through time. It involves participants' appraisal of the findings as well other scholar's interpretation and recommendations for future research, which ensure all conclusions are supported by the data gathered from the study's participants (Grant & Lincoln, 2021). Reliability is used by quantitative researchers to describe the consistency of their studies' findings when they are repeated, and dependability is used by qualitative researchers. A qualitative study should demonstrate a high degree of confirmability in order to obtain dependability (Johnson et al., 2020).

The entire research process and its many stages are described in the current study. In addition to properly stating and describing how interpretivism was applicable to the study, a detailed explanation of the research methodology choices, such as the paradigm, is given. A description of the study's case study research design and qualitative research methods is provided. In addition, I describe the methods for data collection, which were document analysis and observation. I also clearly explain thematic data analysis. An audit trail, as Nowell et al. (2017) referred to it, is a comprehensive explanation of the methodology that will enable other researchers to conduct comparable studies with the potential to produce comparable results.

3.10.4 Confirmability

According to Moon et al. (2016), the confirmability of a study's findings in a qualitative setting is a sign that the participants and circumstances of the inquiry, and not the researcher's biases, motivations, interests, or perspectives, were the only factors influencing the study's conclusions. Parallel to this, Johnson et al. (2020) contended that confirmability is associated with demonstrating that the data and findings are not merely the product of the inquirer's fantasies but are directly attributable to the data.

I used qualitative research data collecting techniques in this study to attain confirmability and explained them well in this chapter. Purposive sampling was applied as the sampling technique, and I collected data through observing lesson presentations by mathematics teachers and conducting document analysis.

3.11 Ethical consideration

Research should always adhere to ethical standards to prevent atrocities (Mertens, 2019). The study's activities involved mathematics teaching and learning and were supported by qualitative data. All participants were informed of the study objectives and given the option to decline participation if they desired. By taking the participants' ethics into account, I answered their ethical concerns. This covered voluntariness, confidentiality, anonymity, and the right of participants to withdraw from the study at any time during the investigation.

Ethics are implemented to guarantee study participants' safety and prevent disparities in the findings. To protect the participants' identities, I used the pseudonyms. I sought written consent from the participants and secured approval from the headmaster and the DBE to undertake the study. The University of Pretoria ethics committee granted approval for data collection, and later issued an ethical clearance certificate. Each participant received an informed consent form and signed the form before the data collection started to show that they had read and understood it.

CHAPTER 4: PRESENTATION OF FINDINGS

4.1 Introduction

In this chapter, I present the findings of my study. I present the research findings according to the themes that define the instructional strategies that primary school mathematics teachers use to teach mathematical word problems. These themes were generated from the study's three sub-questions. To support the findings presented under the themes and subthemes, I provide various snippets of teachers' oral and written questions and associated learner responses. To avoid the possibility of misrepresenting the teachers' and learners' utterances, I chose to use the transcripts verbatim. In addition, where appropriate and necessary, I used vignettes of learners' solutions to substantiate the findings.

The primary goal of the study was to address the following research question: Which instructional practices do primary school mathematics teachers use to teach mathematics word problems? The study addressed the following three secondary research questions; I briefly explain the purpose of each question:

a) How do primary school teachers plan to teach mathematical word problems?
   - The purpose of this question was to gain insight into how primary school teachers plan their lessons to teach mathematical word problems, in other words, how they apply their PCK during lesson planning. The data were collected from the lesson plans during document analysis.

b) How do primary school teachers facilitate the learning of mathematical word problems?
   - The purpose of this question was to gain insight, through observation, into primary school teachers' facilitation of the learning of mathematical word problems based on their lesson planning. In other words, how they put their PCK into action during teaching

c) How do learners mathematically schematise word problems presented in real-life context?
   - The purpose of this question was to explore how learners solve mathematical word problems as they grapple with transitioning from real-life context to the mathematical realm (horizontal mathematization) and then apply mathematical processes to seek solutions (vertical

mathematization). Data to answer this question were collected through lesson observation and document analysis of learners' exercise books.

The following themes aligned to the secondary research questions:

- Theme 1: Teachers' planning for teaching mathematical word problems
- Theme 2: Facilitating the learning of mathematical word problems
- Theme 3: Schematisation of the word problems presented in real-life context

To give a detailed explanation of the findings, the themes were separated into subthemes. Details regarding the themes and subthemes are presented in the following section. Two primary mathematics teachers from two schools participated in the study. These participants were given pseudonyms to protect their identity and are referred to as Teacher A from School A and Teacher B from School B. Both schools are public schools and located in a rural area. A detailed description of the settings of the two schools where the participating teachers taught their lessons on mathematical word problems are included in the presentation of the study's findings.

4.2 Synopsis of themes

The themes generated from the data were based on and guided by the secondary research questions to give context and to make it easier for readers to fully understand the study's findings. The subthemes that were created during data collection are meant to give a thorough explanation of the themes. Each theme is informed by a specific secondary research question, and therefore, they were identified a priori. For instance, Theme 1, teachers' planning for teaching mathematical word problems, flows from the SRQ1.

Through SRQ1, I was interested in the aspects of mathematical word problems that teachers weave into the lesson plans that would later guide the presentation of the lesson. Theme 1 therefore has subthemes according to which the findings of the main theme were organised and presented. The following subthemes frame the presentation of the findings related to Theme 1:

- Objective of the lesson plan
- Building on prior knowledge
- Involvement/engagement of learners in a lesson

- Consolidation

Similarly, I derived Theme 2 from SRQ2, and this theme has the following three subthemes:

- Use of operation/s
- Learner's involvement/engagement
- Teaching strategies

SRQ3 was used to create Theme 3 and the following subthemes:

- Horizontal mathematization
- Vertical mathematization
- Translating maths solutions to real-life context

4.3 Theme 1: Teachers' planning for teaching mathematical word problems

According to Bin-Hady and Abdulsafi (2019), a lesson plan is a schedule that guides teachers on what to do and when during teaching. It can also be described as the strategy or framework used to design every class for related subjects from the first meeting during teaching and learning until final examinations. Additionally, lesson plans help teachers develop the logical, systematic learning approach necessary to ensure learners can understand the content they are being taught.

According to Bin-Hady and Abdulsafi (2019), the ideal lesson plan should include the following four components: teaching objectives, a warm-up activity, techniques and procedures that can be used to accomplish the teaching objectives, and assessment to determine whether the techniques and procedures successfully accomplished the teaching objectives. In line with Bin-Hady and Abdulsafi's view on the components of the lesson plan, the relevant data for the Theme 1 on teachers' planning for teaching mathematical word problems included teaching objectives, prior knowledge, learner involvement/engagement, and consolidation. This component that has been mentioned are the important PCK, because they reveal how teach should plan and teach mathematical word problems. The first sub-theme is objectives of the lesson and focuses on what teachers intend their learners to achieve by the end of the lesson on word problems. The second sub-theme is building on prior knowledge and introduction and focuses on the teachers' plans for building on learners' prior knowledge of mathematical word problems. Any data not related to any prior knowledge plan for mathematical word problems were excluded. The third sub-theme is involvement/engagement of learners in a lesson and focuses on how teachers plan to involve learners in a lesson based on mathematical word problems. The last sub-theme is consolidation and focuses on information that includes the consolidations of the lesson that teachers plan to emphasise based on mathematical word problems.

The following subsections discuss the subthemes for Theme 1.

4.3.1 The objectives of the lesson

As a teacher, it is essential to establish the goals for a lesson presentation before beginning with the lesson presentation. The success of a lesson is measured through the achievement of its objectives. Sadly, neither participant provided me with a comprehensive lesson plan that guided their teaching. Teacher A insisted that she did not have a lesson plan, and Teacher B provided me with a lesson plan that was limited to class activities. As a result, neither had set objectives to achieve during the lesson.

4.3.2 Building on prior knowledge and introduction of the lesson

During the teaching and learning of mathematical word problems, one of the two participants had prepared a lesson. I evaluated the lesson plan to ascertain what the teacher intended to teach and how the lesson would be delivered. Unfortunately, Teacher A did not prepare any lesson plan and Teacher B’s lesson plan did not contain all components of the lesson because it contained only class activities and examples. The lesson plan created by the Teacher B is presented in figure 4.1. The lesson plan Teacher B provided only included the examples that would be used in class and the activities that would be used to evaluate the learners. In summary, neither teacher demonstrated how the lesson on mathematical word problems would be introduced.

It is crucial to assess what learners already know and to use their prior knowledge as a foundation for the new topic or concept being taught. Because neither participant provided me with a complete lesson plan that stated how they planned to incorporate their learners prior knowledge, I was unable to ascertain how they planned to build on what their learners already knew. As a result, neither teacher made any preparations for their lessons nor a plan for how they would build on learners’ prior knowledge.

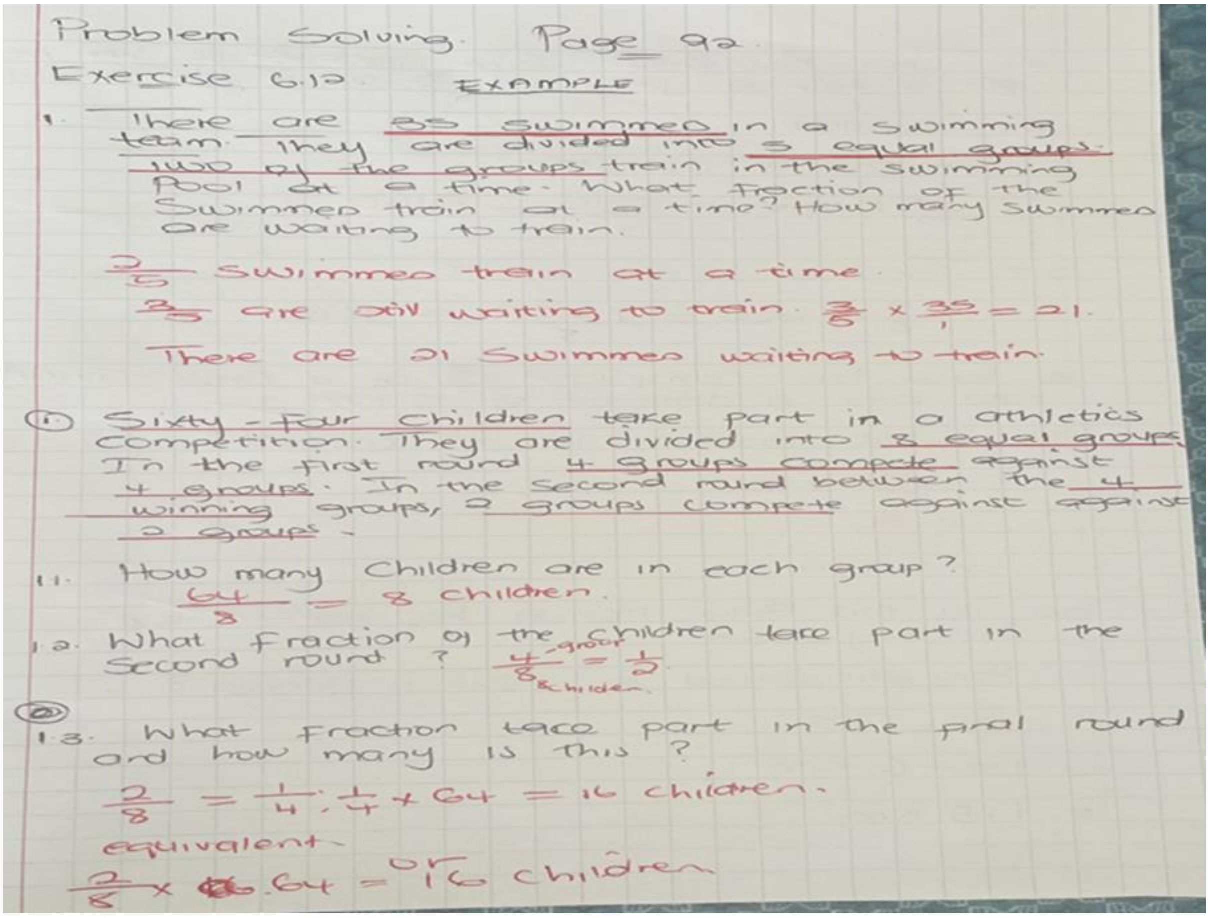
4.3.3 Involvement/engagement of learners in a lesson

When developing a lesson plan, teachers ought to consider their learners' needs, such as how to involve/engage them in the lesson. The only items shown in the lesson preparation by Teacher B were examples and activities that would be carried out in class; however, she did not explain how learners would be kept actively involved during the teaching and learning of mathematical word problems. Figure 4.1 depicts the exercises and scenarios that Teacher B planned to use to keep her learners actively involved in the teaching and learning of mathematical word problems. She employed a keyword approach while responding to inquiries as well as writing statements and questions in black ink and responses in red ink.

In conclusion, Teacher A ideas to involve learners in the class were not supported by any evidence. Teacher B did not make it clear what exactly learners would be doing when they were being taught and learning because writing down examples and activities does not provide a clear picture of how a teacher intends to keep learners engaged during the lesson.

Figure 4.1

Examples of Teacher B's lesson plan.



Problem Solving. Page 92.
Exercise 6.12 EXAMPLE
1. There are 35 swimmers in a swimming team. They are divided into 5 equal groups. Two of the groups train in the swimming pool at a time. What fraction of the swimmers train at a time? How many swimmers are waiting to train.
$\frac{2}{5}$ swimmers train at a time.
$\frac{3}{5}$ are still waiting to train. $\frac{3}{5} \times \frac{35}{1} = 21$.
There are 21 swimmers waiting to train.
1) Sixty-four children take part in a athletics competition. They are divided into 8 equal groups. In the first round 4 groups compete against 4 groups. In the second round between the 4 winning groups, 2 groups compete against 2 groups.
1.1. How many children are in each group? $\frac{64}{8} = 8$ children.
1.2. What fraction of the children take part in the second round? $\frac{4}{8} = \frac{1}{2}$ (group, 8 children)
1.3. What fraction take part in the final round and how many is this?
$\frac{2}{8} = \frac{1}{4}; \frac{1}{4} \times 64 = 16$ children.
equivalent
$\frac{2}{8} \times 64 =$ or 16 children

4.3.4 Consolidation

Neither Teachers A nor B had any consolidation or focal points in their lesson plans. As a result, they did not provide a clear picture of what they specifically wanted their students to understand about mathematical word problems or what was significant about them. Finally, neither recommendations nor accents were included in the lesson plan created by Teacher B.

4.3.5 Summary of theme one

The secondary research question asked how primary school teachers plan to teach mathematical word problems and provided the framework for the first theme, which focused on how teachers planned to teach mathematical word problems. Unfortunately, neither of the teachers provided a clear plan for how they would introduce the lesson or integrate the learners' prior knowledge in the instruction on solving mathematical word problems. Even though there was no clear indication of how Teacher B planned to keep actively engaged in the lesson, she provided a lesson plan that only had examples and activities for the class setting. She mentioned no consolidations or accents in her lesion plan that she intended to focus on during the teaching and learning of mathematical word problems.

4.4 Theme 2: Facilitating the learning of mathematical word problems

The purpose of this theme was to gain insight, through observation, into primary school teachers' facilitation of the learning of mathematical word problems based on their lesson planning. I intended to gain in-depth knowledge on how teachers keep learners actively involved in the lesson when teaching mathematical word problems; how they bridge the gap between what learners know and do not know in word problems; and what teaching strategies they use to help themselves and the learners achieve the objectives of the lesson.

Theme 2 has four subthemes that focuses on answering SRQ2 (How do primary schools teachers facilitate the learning of mathematical word problems?) The first sub-theme is the use of operation/s and focuses on data about the use of operation/s during the teaching and learning of mathematical word problems; any information that does not relate to the use of operation/s was excluded during data collection. The second sub-theme is teaching strategies and focuses on the teaching strategies that were used during teaching and learning of mathematical word problems; any data that does not involve the use of teaching strategies were excluded from this sub-theme. The third sub-theme is learners' involvement and focuses on data related to learners' involvement during the teaching and learning of mathematical word problems; any data not related to learners' involvement during teaching and learning of mathematical word problems were excluded during data collection.

4.4.1 The use of operation/s during the teaching and learning of mathematical word problems

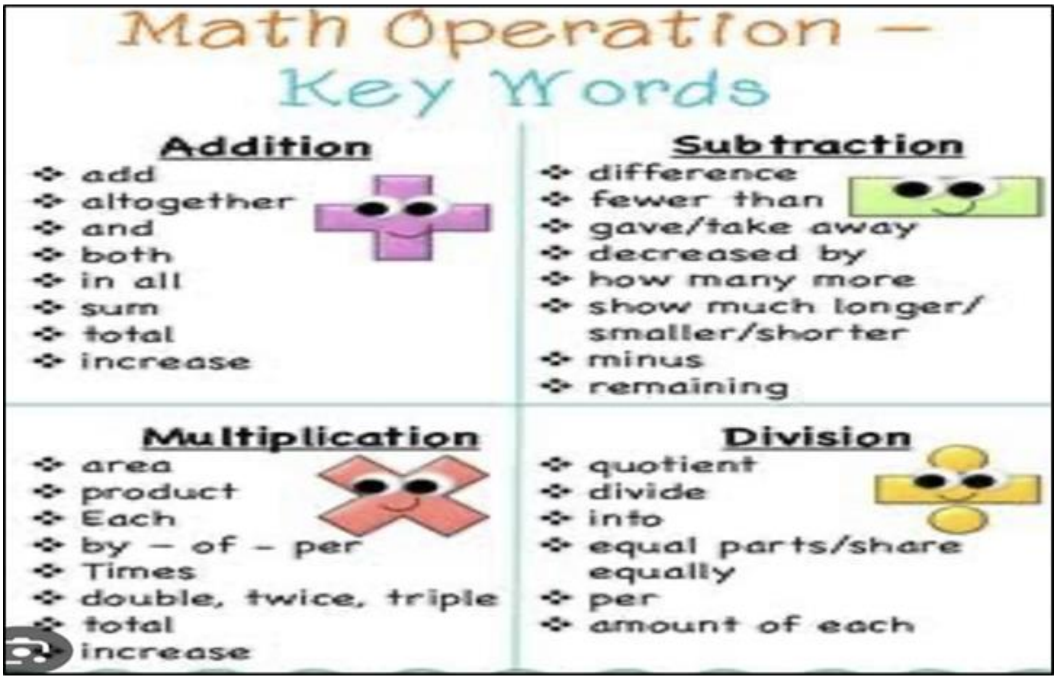
In the process of teaching and learning how to solve mathematical word problems, it is crucial that learners appropriately apply the four basic mathematical operations, namely addition, subtraction, multiplication, and division (Amini et al., 2019). During my observations, the teachers gave learners the chance to discuss among themselves and to show on the chalkboard which mathematical operations were appropriate for a particular mathematical word problem that was posed in the classroom. My observations are discussed in the following subsections.

4.4.1.1 Observation of Teacher A's lesson

During the teaching and learning of mathematical word problems, Teacher A gave the learners papers that described the mathematical operations and what they meant (Figure 4.2). The symbols that the teacher gave to the learners included × for multiplication, ÷ for division, + for addition, and - for subtraction. The teacher informed the learners that they need to understand these operations so they can use them appropriately when solving word problems in class.

Figure 4.2

Teacher's illustration of the four basic operations.



Teacher A explained how these operations must be used in order to solve any mathematical word problems as follows:

> You must look at a notion like multiply, product, of, and times. Divide, quotient, and share are concepts you need to look for when doing division. Add, more than, increased, entirely, total, and plus are concepts you need to look for when doing addition. Subtract, minus, difference, less than, and decrease are concepts you need to look for when doing subtraction.

After explaining to the learners how to use the mathematical operations, the teacher provided them with a mathematical word problem to see if they understood. The teacher wrote the following statement and question on the board: “Simphiwe watered half of his vegetable garden on Monday and $\frac{3}{8}$ of the garden on Tuesday. What fraction did he watered altogether?” In order to answer the given mathematical word problem, the teacher asked the class which mathematical operation should be applied. The whole class collectively answered, “Addition”. She then asked them to name any phrase that helped them realise what they needed to add, and one student responded, “Altogether”. The learners in the class all responded “yes” when the teacher asked them to confirm that the plus sign on the papers she had given them earlier, which said “altogether”, meant addition. In order to solve the above mathematical word problem, the teacher instructed the learners to add both fractions after concluding that the word *altogether* denotes addition. Figure 4.3 shows a learner who was given the opportunity to solve the mathematical word problem on the chalkboard.

Figure 4.3

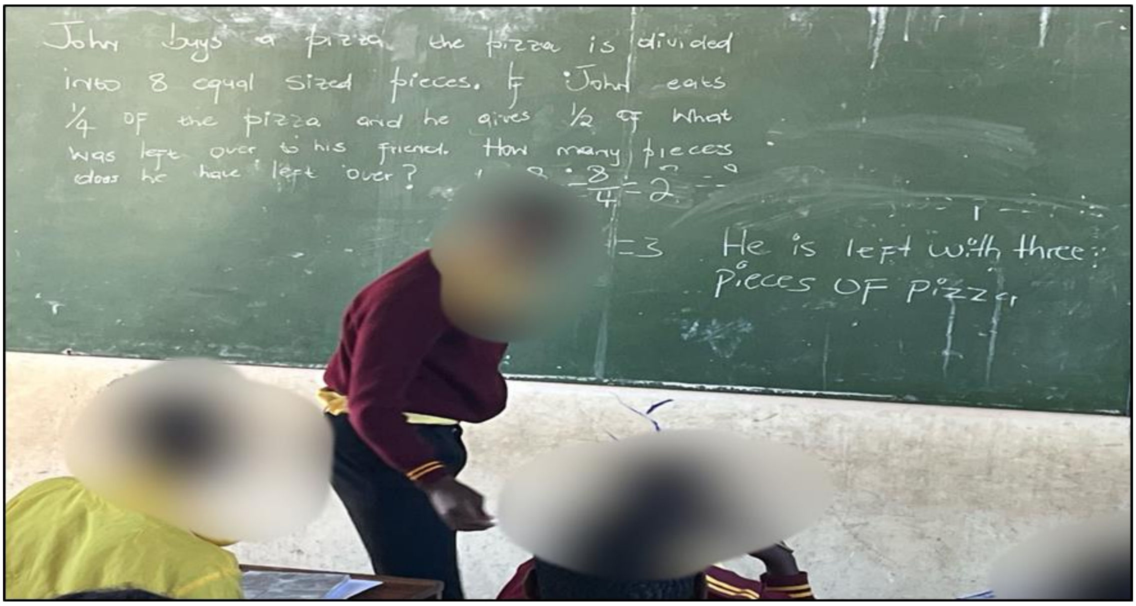
Learner solving mathematical word problem on the chalkboard.

The teacher emphasised to the learners that they should evaluate the mathematical word problem they are given and analyse it in order to have an idea of the mathematical operation(s) to be used. The teacher also gave the following example that was still based on the use of operations to solve mathematical word problems: "What is $\frac{2}{3}$ of 40?" The teacher questioned the learners on how to approach this mathematical problem. The majority of learners said they should multiply. She continued by inquiring where the learners had learned that multiplication was necessary. The learners justified their response by saying that the word *of* denotes multiplication. The teacher told the learners that the word *of* becomes the descriptor of the multiplication symbol (×).

The teacher gave the learners further activities that were also based on mathematical word problems in which the learners had to decide which mathematical operation to apply to solve the problem. One of the activities was presented thus: "John buys a pizza, and it is divided into 8 equal pieces. If John ate $\frac{1}{4}$ of the pizza and he gives $\frac{1}{2}$ of what is left to his friends. How many pieces does he have left over?" The learners' response was that was John left with three pizzas. The teacher asked the class to identify the mathematical operation(s) they applied to find the solution, and the learners began to guess which mathematical operation would be appropriate to use to solve the given mathematical word problem. The teacher instructed the class to go back and review the question. After repeated attempts, the learners' results revealed that they had employed addition, subtraction, and division to solve the given mathematical word problem. The teacher affirmed their responses and emphasised that, as shown in the activity given, it is possible to apply more than one mathematical operation in a single mathematical word problem. Figure 4.4 show the learners solving the mathematical word problem on the chalkboard.

Figure 4.4

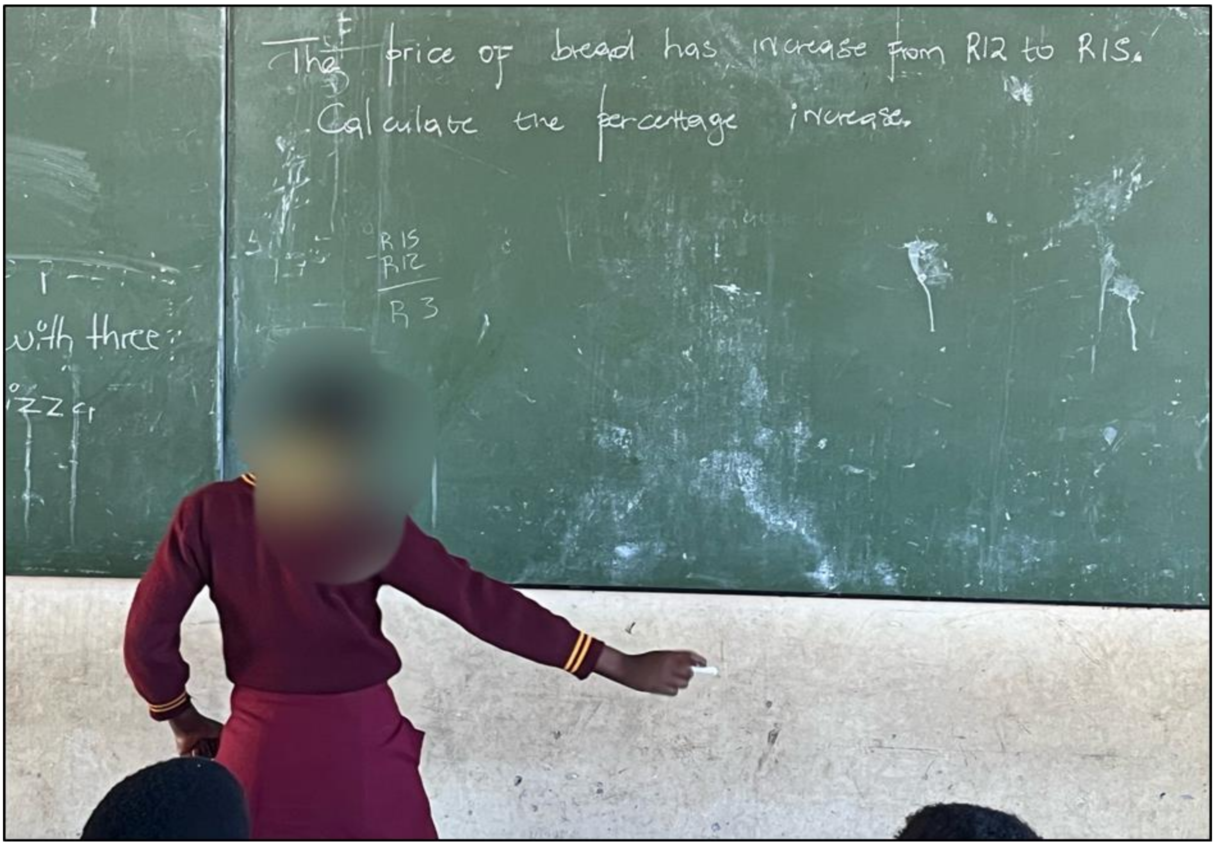
Learner solving mathematical word problem on the chalkboard.



Next, the teacher presented the following problem: “The price of bread has increased from R12 to R15. How much money they have added from the previous price of the bread? Calculate the percentage increased” (Figure 4.5). The teacher asked the learners with how many Rands bread had increased from R12 to R15. In response, the learners said R3. The learners confidently responded “increased” when the teacher asked them which element in the statement meant addition. In addition to acknowledging the learners’ comprehension, the teacher remarked, “Some of you can read without understanding, and some of them pass through when they read, which causes them to make mistakes in solving mathematical word problems”.

Figure 4.5

Learner solving a mathematical word problem on the chalkboard.



The teacher gave the learners the following last example: "I have 100 sweets for my children, I brought in the packet of 100 sweets and I am having only 4 children. How much sweets will each child have if they share equally?" She prompted learners to think about what mathematical operation they should use to solve the problem before they attempt to answer it. Some learners gave the answer "addition" and others said "division". "I am not conducting music, please raise your hands", the teacher said as she signalled for the learners to raise their hands. In order to solve the provided mathematical word problem, she again asked the learners which mathematical operation should be applied. One learner was chosen to respond to the question, and she chose to apply division in her solution. The teacher emphasised that reading, analysing, checking for mathematical concepts, and answering the problem come in that order when solving mathematical word problems. The teacher assessed learners to determine whether they had understood after providing multiple examples. She wrote the following statement on the chalkboard: "Adam and Nana decided to work in shift at the local shop for a total of 6 hours, Nana works $\frac{1}{3}$ of the time and Adam works $\frac{1}{4}$ of the time". Next, she asked the learners the following questions:

- "How many hours Nana have to work in the local shop?"
- "Calculate the time that Adams will have to spend at the local shop."
- "The manager pays them R120 in total for the 6 hours. How much money did Nana earns for her fraction of work?"

The learners were asked how they planned to figure out Nana's working hours. One learner raised their hands, but another replied, "I'm going to use division". "We must use addition", argued a different learner, and another learner said, "We need to multiply". The teacher acknowledged the learners' responses; however, instead of asking them why and/or how they chose a particular operation, she instructed them to multiply $\frac{1}{3}$ to determine how many hours Nana spent working at the local shop, and then multiply $\frac{1}{4}$with 6 hours to determine how long Adams spent working there. She then instructed them to multiply $\frac{1}{3}$ by R120 to determine how much money Nana will receive.

4.4.1.2 Observation of Teacher B's lesson

Teacher B informed the class that the topic of the day was based on mathematical word problems. She said, "There will be times in life when you will need to solve problems with a mathematical context; they could come from subtraction, addition, multiplication, or fraction". She instructed the class to look at the example given on page 92 of their books. She reiterated the need for learners to underline key words as mathematics concepts in order to answer mathematical word problems and asked them to read the following example aloud: "There are 35 swimmers in the swimming team, teams are divided into 5 equal groups. Only 2 of the groups train in the swimming pool at the time. How many swimmers are waiting to train?" The teacher asked the learners how they would approach this problem. Learners randomly shouted, "Divide". She acknowledged their response before doing the class work together to solve a problem in which they divided 35 into 5 groups, and then multiplied 2 groups by 5 as follows: $35 \div 5 = 7$ groups of 5 swimmers each, and therefore, 2 groups of 5 each gives 10 swimmers who can train in the swimming pool at the time. Since the example was already written in the learner's textbook, she instructed the class to look at Exercise 6.1.2. The learners then read the following question in their book:

> 64 children take part in an athletics competition. They are divided into 8 equal groups. In the first round, 4 groups compete against 4 groups. In the second round between the 4 winning groups, 2 groups compete against 2 groups. How many children are in each group?

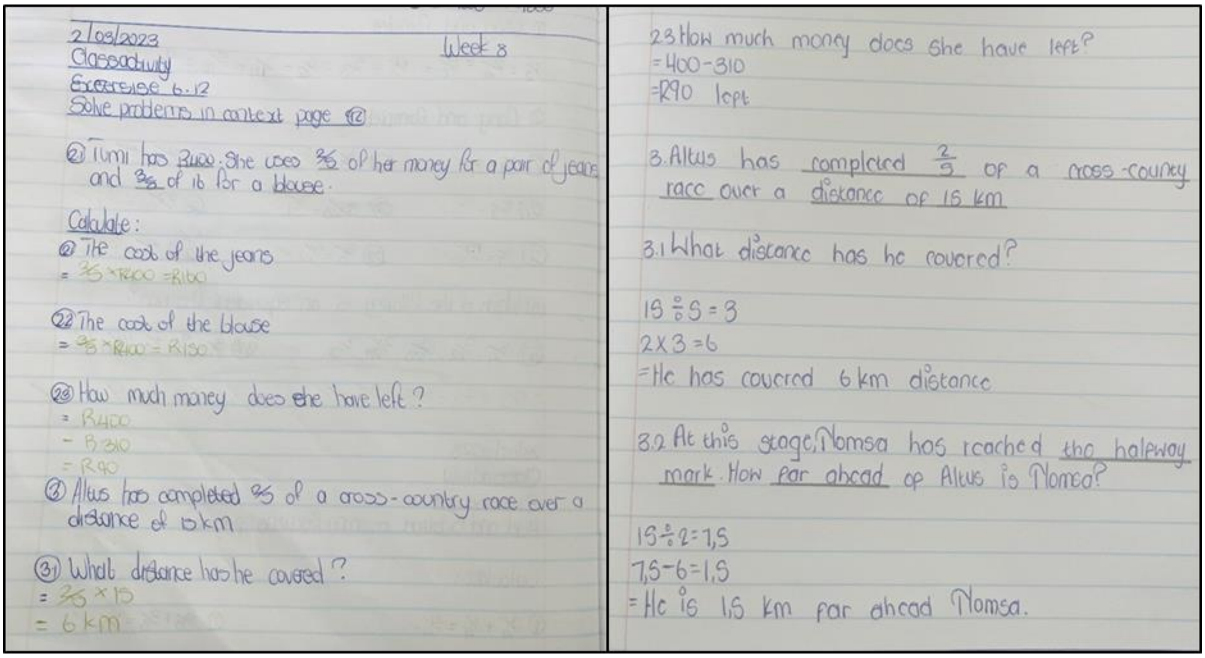
The teacher asked the class what the important key words were that made up the statement's essence after they had read the statement and the corresponding question in their books. In response, the learners stated that it is divide and that the total number of children is 64. Additionally, after reading the problem description, the learners underlined any important key words that could help them solve the mathematical word problem. The teacher asked the learners, "How many children are in each group?" One learner responded, "8 children". The teacher said, "Can you elaborate?" The learner explained that she divided 68 by 8 groups to get the number of children in each group. The teacher acknowledged the learner's response by pointing out that in the statement, it was mentioned that there are 64 children, and that they are divided into 8 equal groups. She then asked the class to clap their hands for the learner who gave the correct response, and the entire class does so. After two examples were done as a class, the teacher began to assess learners to check their understanding of solving mathematical word problems using the correct mathematical operation(s).

Both teachers used distinct approaches to teach their learners to solve mathematical word problems. Teacher A emphasised that reading, analysing, checking for mathematical concepts, and answering the problem come in that order when solving mathematical word problems. As an illustration, after reading a mathematical word problem, learners must analyse it to determine what is required of them to answer it and how they can apply any mathematical ideas that are included in the problem. Once the learners have read aloud with comprehension, finished their analysis of the word-problem statements, and located the mathematical ideas in the problem statements, they can continue to solve the mathematical word problem.

Teacher B employed a keyword teaching style. In order to try to comprehend what is required of them to solve the problem, she allowed her learners to read the statements of the mathematical word problems. They had to highlight the keywords they discovered in the problem statement as they read it. After highlighting the keywords, learners went back and read the word-problem statement to make sure they highlighted the correct keywords to help solve the mathematical word problems. Figure 4.6 is an example of the work of a learner at School B that shows how they solved the mathematical word problems using the keyword strategy.

Figure 4.6

Excerpt of learner's work using the keyword strategy.



Teacher B's approach was to solve mathematical word problems using the keyword technique, which entails reading the problem, underlining the relevant information, analysing it, determining whether the underlined material correlates with the problem statement, and then starting to solve the problem. When learners begin to solve a mathematical word problem, they go through a process called horizontal mathematization in which they convert the word problem from the context of real life to mathematical terminology. Then they resolve the mathematical word problem that is described mathematically by completing vertical mathematization. Both teachers used the methodologies that worked for them and that apparently made it easy for their learners to understand solving mathematical word problems.

4.4.2 Learners involvement during teaching and learning of mathematical word problems

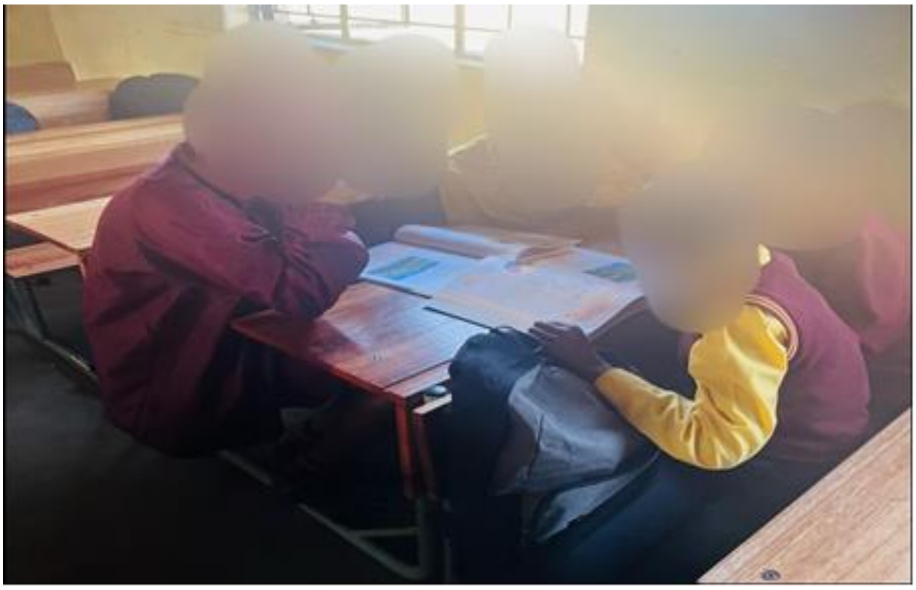
I observed the teachers while they worked with learners to solve mathematics word problems throughout my observation period. The learners' participation is crucial because it determines whether the learners understand the day's lesson and allows the teachers to pinpoint learners' areas of weakness and strength. The learners' engagement in the teaching and learning of mathematical word problems also affects whether the lesson's objectives are met, and if so, how successfully.

4.4.2.1 Observation of learners at School A

By interacting with her learners through questioning, group discussions, and sharing of ideas on how to solve mathematical word problems, the teacher was able to keep her learners engaged during the teaching and learning of mathematical word problems. For instance, the teacher ensured every learner had the chance to read and solve the word problem, and learners were given an opportunity to ask questions if there was a misunderstanding about a problem. When the learners were taught how to solve mathematical word problems at School A, they were permitted to talk about mathematical word problems among themselves and express their thoughts to the class. If the teacher asked the class a question, she would read the problem out loud and ask the learners if they understood the question and what solutions they had in mind for it. Figures 4.7 and 4.8 show learners engaged in group discussion during the teaching and learning of mathematical word problems.

Figure 4.7

Learners engaged in group discussion.



As I previously stated, each learner was given the chance to share their opinions on how to solve mathematical word problems, and they were also given the opportunity to solve mathematical word problems on the chalkboard. Even though some of the learners' answers were incorrect, others were able to correct them under the teacher's supervision, and the teacher gave a reasoned justification for why they disagreed with the learners who wrote the incorrect answers without offending any learner. Learners had the opportunity to ask questions if they did not understand the problem presented in class. Those who did understand the problem, shared their understanding with others and also offered additional ways or strategies that they can use to solve the mathematical word problem.

Figure 4.8

Teacher A involving her learners in group discussion.

4.4.2.2 Observation of learners at School B

Learners actively participated in the lesson as mathematical word problems were taught and learned. The teacher offered her learners a chance to read the problem statement in their books. Following this, the learners analysed the question by highlighting the key points and shared with the class how they intended to address the mathematical word problem.

There were disagreements and agreements during the sharing of ideas on how to solve the given mathematical word problem but an explanation was provided to those who misunderstood the question. Learners were also allowed to share ideas and help one another solve the word problems. They were also engaged in group discussions based on solving word problems and shared opinions with the class. The learners respected each other's opinions, listened to one another without offending anyone, and corrected one another, demonstrating effective learner-teacher teamwork. Figures 4.9 and 4.10 show learners engaged in group discussion and sharing ideas on the chalkboard during the teaching and learning of word problems.

Figure 4.9

Learners sharing ideas on the chalkboard.

Figure 4.10

Learners involved in a group discussion.

4.4.3 Summary of Theme 2

Both teachers used different PCK to teach mathematical word problems, but they both emphasised that learners must look for the terms that define the mathematical operations while using the procedures. If learners understand terms like *plus sign*, *sum* and *increase*, they can apply the appropriate operation to solve word problems. Both teachers applied different PCK approaches to tackle mathematical word problems, but Teacher A encouraged her learners to read, consider, and solve the problems. Teacher B taught them to solve word problems using the keyword technique in which they read the problem and highlight important details. As a result of their participation in the lesson and mutual respect for each other's opinions, the learners from the two schools found it easy to present their ideas to the class. The teachers often led group discussions with their class to foster positive relationships and teamwork among learners.

4.5 Theme 3: Schematising mathematical word problems presented in real-life context

Given that mathematical word problems are context-based and language-rich, it is necessary for students to understand, analyse, and solve them (Nag Chowdhuri, 2022). Mathematical word problems help teachers and learners apply what is being taught in the classroom to real-life problems.

In this theme, I share my findings on how learners mathematically schematise word problems that are given in practical settings. The theme was divided into three subthemes that focus on answering SRQ3, which asks how learners mathematically schematise the word problem presented in real-life context. The first sub-theme is horizontal mathematization and focused on how learners translate mathematical word problems presented in real-life context to mathematical symbols; any information that does not relate to the translation of mathematical word problems to mathematical symbols was excluded. The second sub-theme is based on vertical mathematization and focused on how learners solved mathematical word problems presented in mathematical symbols; any data that do not involve learners solving a mathematical word problem were excluded. The last sub-theme is translating mathematics solution to real-life context of the problem and looks at how learners translate maths solutions to real-life context; any data that do not involve learners translating maths solutions to real-life context were excluded. This theme is explained using data gathered through observation and document analysis of learners' workbooks.

4.5.1 Translating mathematical word problems to mathematical symbols (horizontal mathematization)

Learners from School A and School B were able to read and convert mathematical word problems given in language to mathematical symbols before solving the problems. I observed that both teachers permitted their learners to solve mathematical word problems on the chalkboard: One learner stood up and went to chalkboard to solve word problem and the other learners helped the learner who was solving the mathematical word problem on the chalkboard. Most of the time during teaching and learning, teachers helped the learners convert word problems into mathematical symbols. For instance, Teacher A would pose a problem statement and a question based on a mathematical word problem before showing her learners how to convert the words into symbols. Figures 4.11 and 4.12 show how learners at School A translate mathematical word problems expressed in real-life context into mathematical symbols.

Figure 4.11

School A learner's work showing horizontal mathematization.

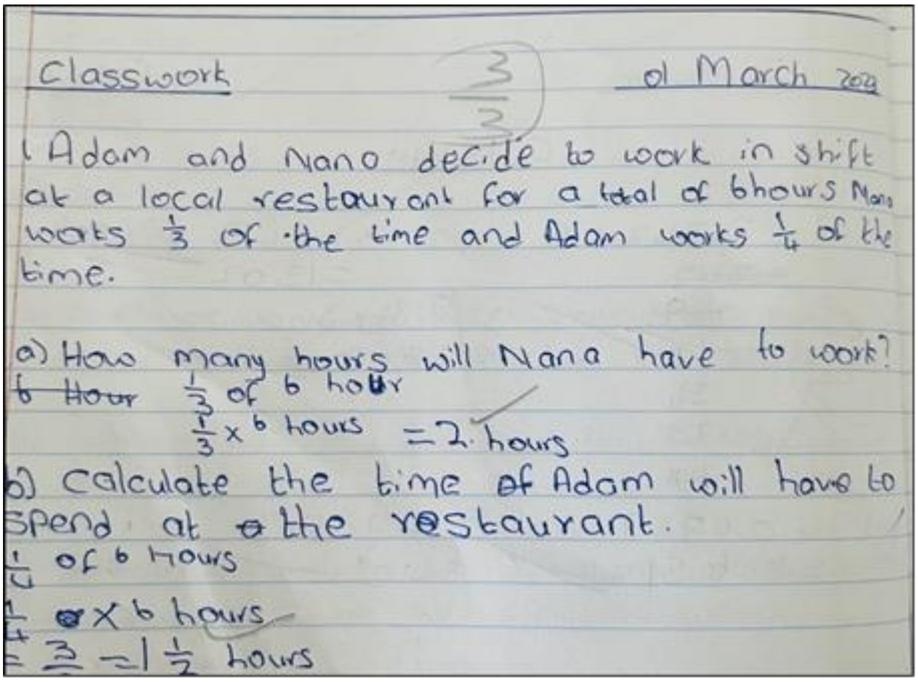
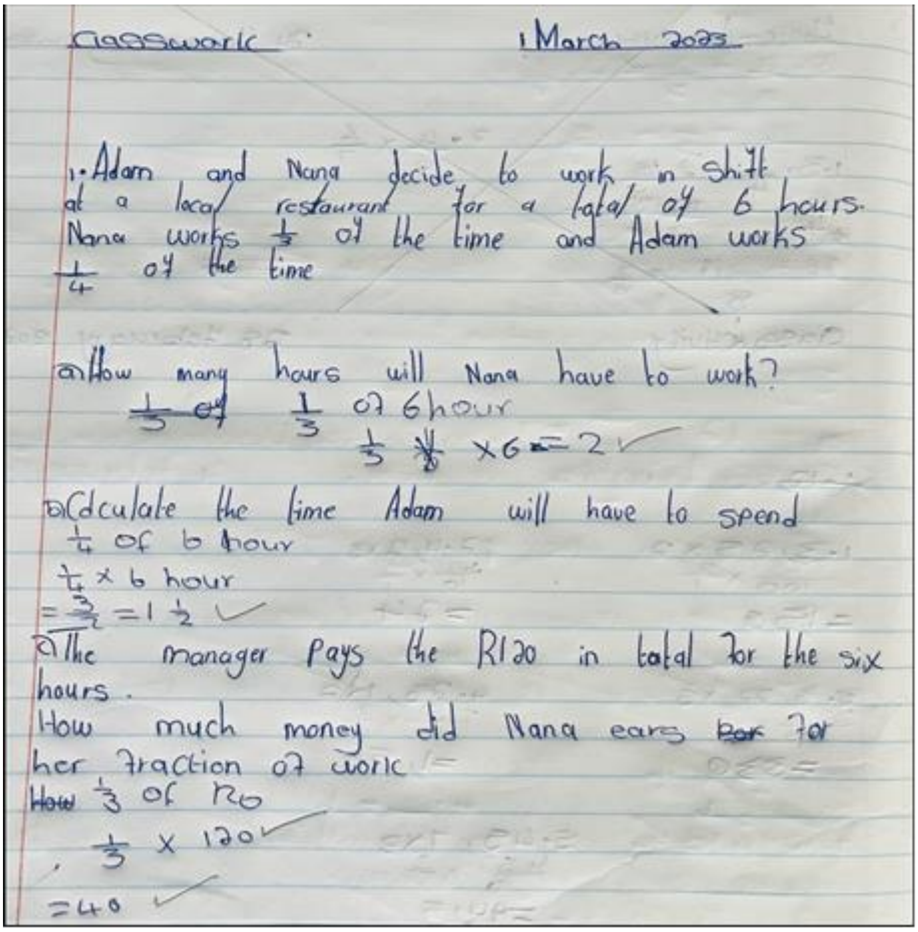


Figure 4.12

School A learner's work showing horizontal mathematization.


Classwork 1 March 2023

1. Adam and Nana decide to work in shift at a local restaurant for a total of 6 hours. Nana works $\frac{1}{3}$ of the time and Adam works $\frac{1}{4}$ of the time

a) How many hours will Nana have to work?
~~$\frac{1}{3}$ of~~ $\frac{1}{3}$ of 6 hour
$\frac{1}{3} \times 6 = 2$

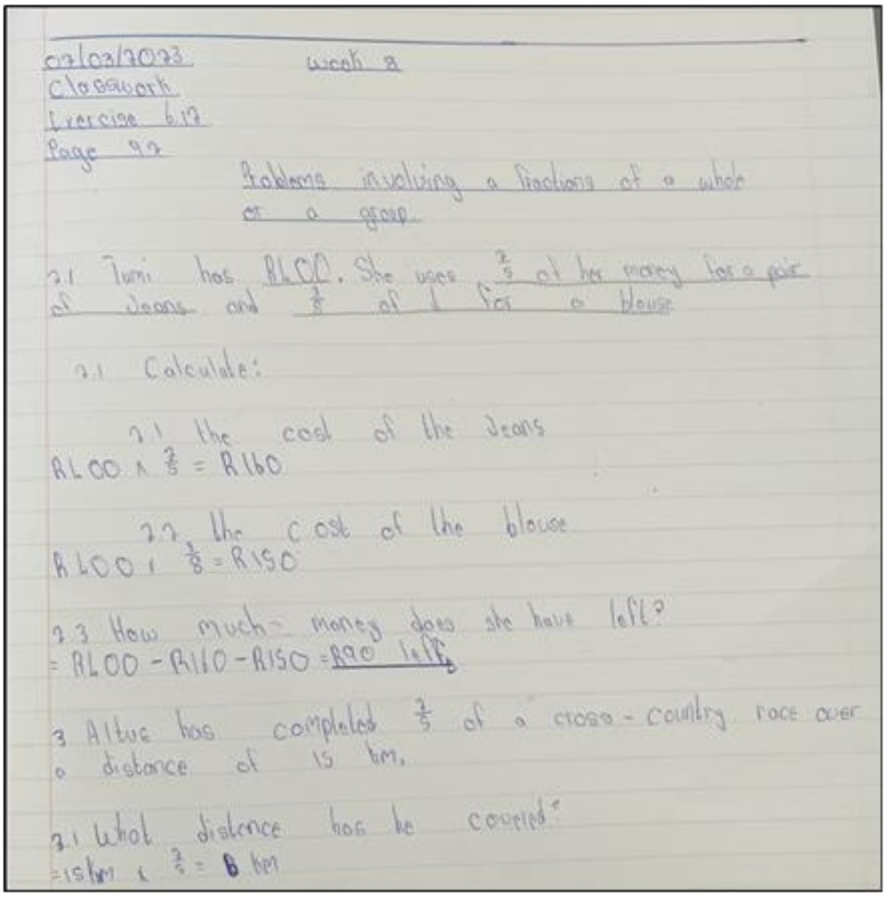
b) Calculate the time Adam will have to spend
$\frac{1}{4}$ of 6 hour
$\frac{1}{4} \times 6$ hour
$= \frac{3}{2} = 1\frac{1}{2}$

c) The manager pays the R120 in total for the six hours.
How much money did Nana earn for her fraction of work
$\frac{1}{3}$ of R0
$\frac{1}{3} \times 120$
$= 40$

Teacher B instructed learners to read the problem statement and look at their books before instructing them to apply the keyword method to convert the mathematical word problem into mathematical symbols so they could start solving it. The learners were able to transform verbal expressions into mathematical symbols under the teacher's guidance. Figure 4.13 show a learner from School B translates mathematical word problems into mathematical symbols using the keyword technique.

Figure 4.13

School B learner's work showing horizontal mathematization.



4.5.2 *Solving mathematical word problems expressed in mathematical symbols (vertical mathematization)*

Following the conversion of verbally expressed mathematical word problems into mathematical symbols, learners started to solve the mathematical problems. After converting to mathematical symbols, learners from both Schools A and B were able to solve the word problems. Figures 4.14 and 4.15 shows examples from School A and School B of learners' efforts to solve word problems expressed in mathematical symbols.

Figure 4.14

School A learner's work showing vertical mathematization.

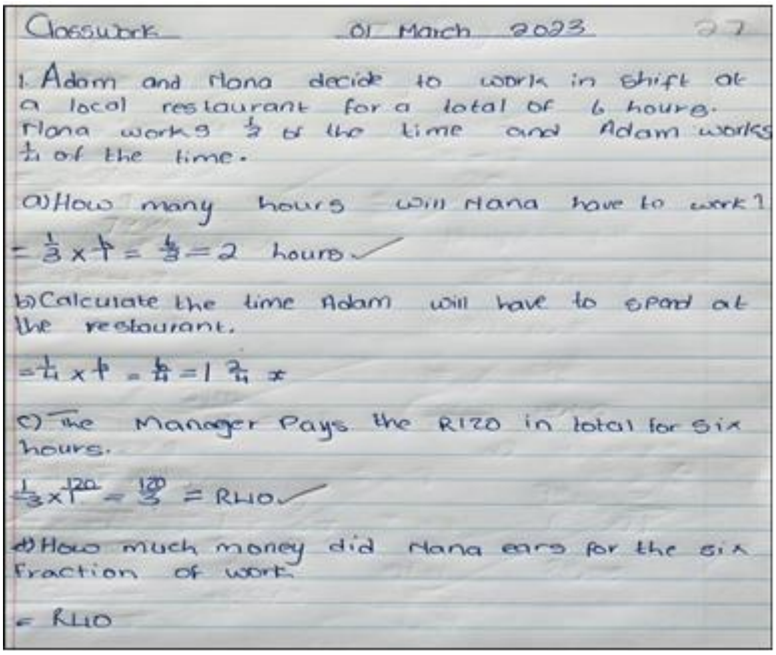


Figure 4.15

School B learners' work showing vertical mathematization.

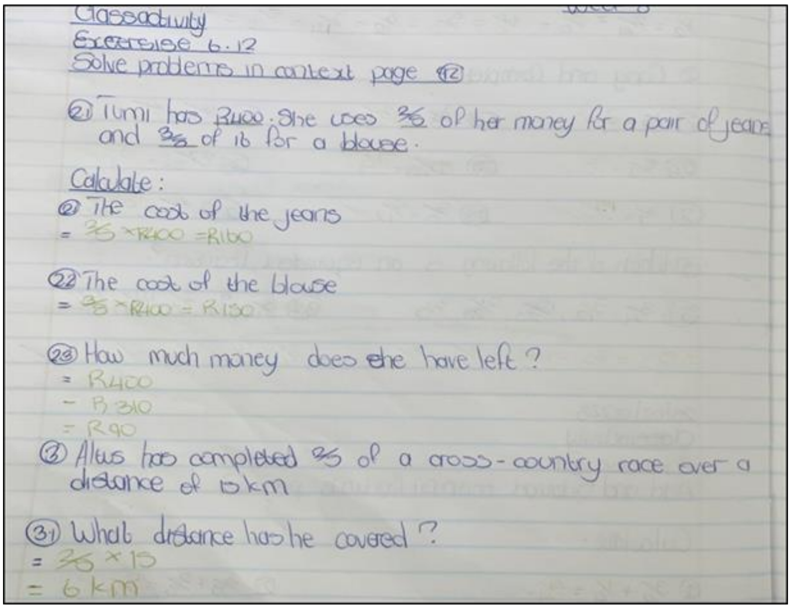


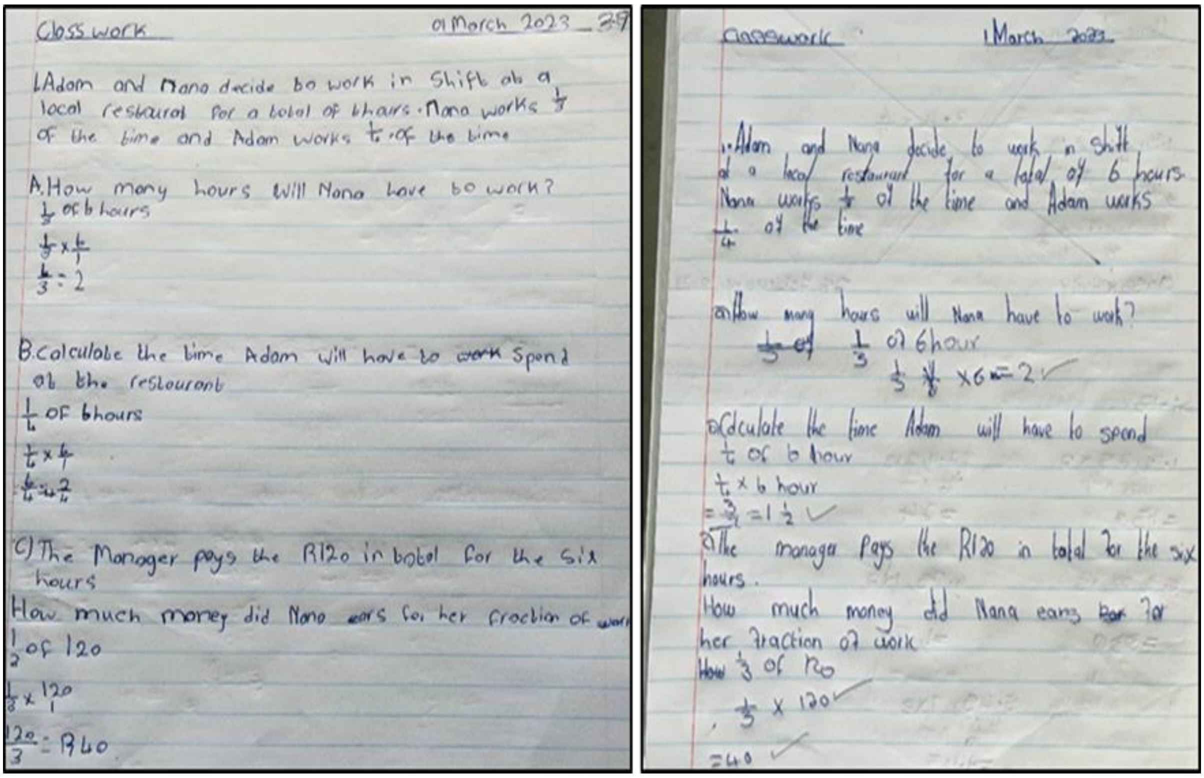
Figure 4.15 shows how the learner used two different colours ink: The questions and statements are written in blue, and the solutions are written in green, indicating that the student is now solving a problem.

4.5.3 Translating maths solution to real-life context of the problem

Most learners did not relate their solutions to real-life context after solving the mathematical word problem that was presented in a real-world setting. This means that the learners merely converted mathematical word problems into mathematical symbols, solved them, and left their solutions expressed in mathematical symbols, neglecting to read the problem and the question again to ensure that they had answered it as it had been asked. Nearly none of the learners in School A translated their mathematical solutions into real-world context. Most of the learners in School B were able to convert their mathematical answers into language within a mathematical context. However, some learners from School B did not translate their mathematical answers into real-life context and they left them in mathematical symbols. Figures 4.16 and 4.17 show how the learners from School A and School B performed their calculations.

Figure 4.16

Learner's work from School A.



Both images in Figure 4.16 demonstrate how the learners converted mathematical word problems from real-life context into mathematical symbols and then calculated the answer using mathematical operations. However, while solving a mathematical word problem that was articulated in a real-life context, the learners did not convert their answers from mathematical symbols to mathematical words that addressed the question. For instance, the first query in Figure 4.16 reads "How many hours will Nana have to work?" The learners responded by doing the calculation and writing "2", which is correct, but the question asked how many hours Nana will have to work. Therefore, the learner should have responded: Nana will have to work 2 hours of her shift in the restaurant.

Figure 4.17

Learner's work from School B.

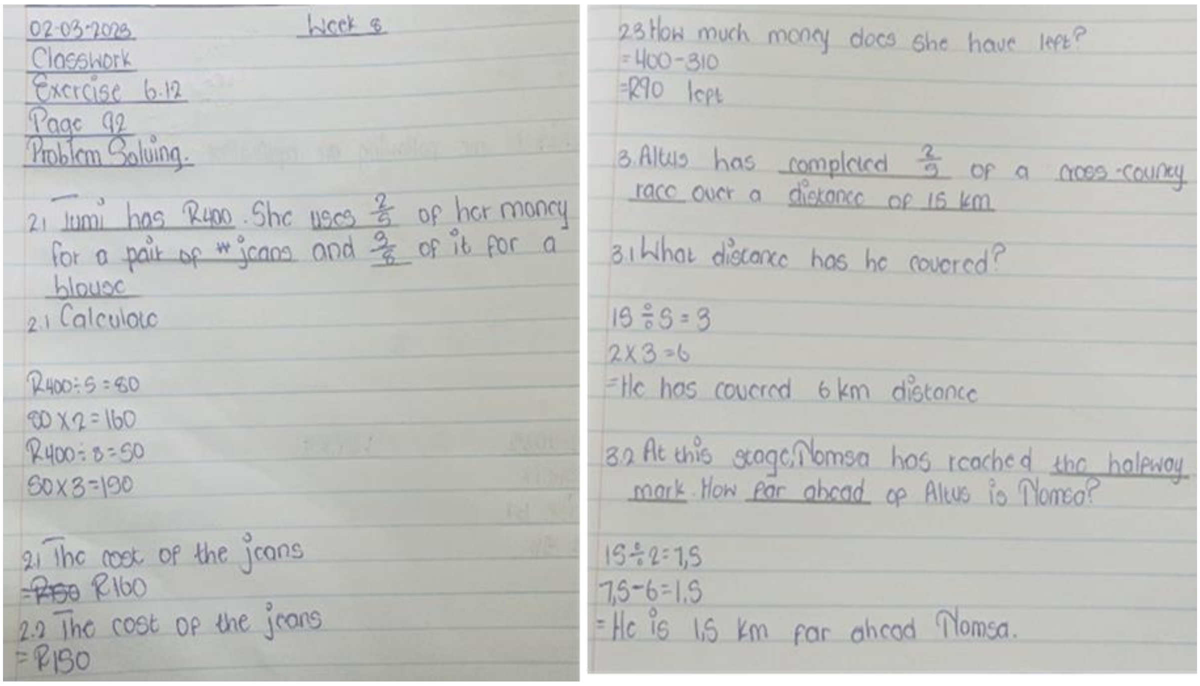


Figure 4.17 shows how a learner performed horizontal and vertical mathematization, but for questions 2.1–2.3, the learner did not put their answers into mathematical words to answer the question. In questions 3.1–3.2, the learner did succeed in translating the solution into mathematical words.

4.5.4 Summary of Theme 3

Nearly all learners were able to read and understand the mathematical word problem because they were able to translate the problem into mathematical symbols to solve it. Learners were assessed to determine how well they understood the mathematical word problems during the teaching and learning process. The ability to read with comprehension and analyse problems was demonstrated by the learners' successful conversion of verbal expressions into mathematical symbols. Most learners were unable to translate their mathematical solutions into real-world context, and they tended to leave their replies in mathematical symbols rather than translate them into words, which would have demonstrated their understanding of the subject.

CHAPTER 5: DISCUSSION OF FINDINGS, RECOMMENDATIONS, AND CONCLUSIONS

5.1 Introduction

This chapter presents a discussion of the findings, limitations, and recommendations for the study. After addressing the research questions and providing an explanation of the study's methodology, this chapter opens with a synopsis of the previous four chapters. Following this summary, I go over the findings' significance and ramifications, as well as the study's advantages and disadvantages, and I offer some suggestions for future research. The study's findings are outlined in the chapter's last section.

Since many academics are attempting to understand how to help learners and teachers comprehend and teach mathematical word problems so they can attain higher performance in mathematics, mathematical word problems have attracted a lot of attention in South Africa. Therefore, it was essential to document this study's methodological contribution so other researchers can expand the techniques needed for studies of this kind. The literature review presented in Chapter 2 is relevant to how the results were interpreted and discussed. Research methodology, which deals with how data were gathered and analysed, is connected to the discussion of procedures and methods in Chapter 3. The study's limitations, findings, and a statement intended to capture the major thesis of the current study are given, followed by suggestions for future research and limitations of the study.

5.2 Discussion of the findings

The following subsections discuss the findings under each theme presented in Chapter 4. Each theme relates to a research question, as explained in section 4.2. Therefore, each of the following subsections responds to the relevant research question.

5.2.1 Teachers' planning for teaching mathematical word problems

Traditionally, most classes are led by teachers and well-crafted lesson plans are rarely implemented (Iqbal et al., 2021), which leads to learners playing a passive role in lessons (Milkova, 2012). Teachers are required to create lesson plans that focus on the needs of learners because it tends to help learners understand the lesson taught. When teachers plan lessons, it shows their confidence in their subject knowledge (Bin-Hady & Abdulsafi, 2019). According to Bin-Hady and Abdulsafi (2019), a good lesson plan should include objectives for student learning, teaching/learning activities, and strategies to check learners' understanding. According to Fujii (2019), lesson plans are teachers' roadmap for what the class will cover and how it will be done efficiently. The learning objectives for the class must be determined before the teacher can design the lesson. After that, they can create instructive learning exercises and come up with methods to get feedback on students' academic progress.

The findings of my study revealed that the primary school mathematics teachers who participated in the study used a variety PCK tools to prepare their lessons. For instance, Teacher B prepared her lesson using only examples and class activities that relate to the content she would be teaching. On the other hand, the lesson plan of Teacher A lacked explicit details such as objectives for student learning, teaching/learning activities, and strategies to check learners' understanding, and she appeared to rely on her experience of teaching mathematical word problems.

Teacher B used a technique called lesson preparation instead of a lesson plan when she prepared to teach her learners. However, the only activities and class examples included in her lesson plan were those relevant to the content being taught. The difference between a lesson plan and lesson preparations is that in a lesson plan, teachers meticulously outline the material they will teach, how it will be delivered, and how the learning process will be assessed. A lesson plan essentially consists of a clearly laid out schedule that must be followed during each lesson, and consequently, a lesson plan aids teachers to deliver more effective lessons. Lesson preparation, on the other hand, refers to a document in which the teacher describes the structure of the lesson and how it will affect the learners. Lesson preparations are typically handwritten documents in which teachers record what will be taught and the activities the learners will take part in. Teacher B did not identify other elements in her lesson preparations, such as objectives, learners' prior knowledge, and strategies to check learners' understanding. If the lesson objectives are not stated in the lesson plan, the teacher is likely to teach learners with no vision and navigate the teaching and learning process haphazardly. Teacher A had no lesson plan or any resources she used to organise her lesson presentations. Therefore, I inferred that she drew on her experience of teaching word problems.

From my perspective, the teachers contribute towards learners' poor performance in mathematical word problems since they presented the lessons without a lesson plan. Without a well conceptualised and detailed lesson plan, teachers may find it difficult to effectively teach learners and help those who lag in their learning. However, I am not implying that, strictly speaking, lesson plans help learners perform better, but I am asserting that if a teacher does not adequately plan what they will teach, the lesson presentation may spiral out of control.

This theme allowed me to answer the secondary research question that asked how primary school teachers plan to teach mathematical word problems. The lesson plans constituted the artefacts from which data were collected through document analysis. To this effect, the primary school teachers used lesson preparation and not lesson planning to teach mathematical word problems. Additionally, the lesson preparation included exercises and examples of the material to be covered to engage students and assess their comprehension of the content. In addition, given the scanty information in their lesson plans, it seemed that the mathematics teachers relied on their prior experience of teaching mathematical word problems. They seemed not to find value in writing down their ideas in the form of a lesson plan and rather relied on their memorised experiences, and therefore, used lesson preparation.

5.2.2 Facilitating the learning of mathematical word problems

The results of the current study showed that the two teachers facilitated the lesson on mathematical word problems in different ways. For instance, Teacher A emphasised that when solving mathematical word problems, the steps of reading, analysing, checking for mathematical concepts, and answering the problem should be taken in that order; while Teacher B encouraged her learners to read the problem statement several times to comprehend the statement and underline keywords that can help them solve the word problem. Teacher A's approach is consistent with that of Sahin et al. (2020), who claimed that when teachers give their students rich, relevant learning experiences, they will gain a strong conceptual comprehension of word problems. Allowing learners to read the problem statement aloud helps them become more familiar with the mathematical ideas that will help them analyse the problem.

When teaching and learning mathematical word problems, conceptual understanding and procedural knowledge are crucial (Baraké et al., 2015). For instance, it is easier for learners to use the proper mathematical operations if they have a basic understanding of the other concepts necessary for fundamental operations like multiplication. In order to encourage learners to be very active and develop conceptual knowledge, Teacher A encouraged her learners to exchange ideas on the chalkboard and participate in group discussions. Learners were given the chance to ask questions, and if they did not understand the material being taught, Teacher A allowed other learners to answer the question. If there was no response, she addressed the issue so that no learner were left behind. By doing this, the teacher conveyed her desire for every one of her learners to comprehend and learn the topic being taught in class.

Teacher B used a keyword technique when facilitating her lesson. She gave her learners the opportunity to read the statements of the mathematical word problems in order to attempt to understand what was required of them to solve the problem. As they read the problem statement, they underlined relevant words they identified. After underlining the crucial terms, the learners re-read the word-problem statement to confirm that the words they had underlined were still relevant and could help them solve the problem. The results support de Koning and van der Schoot's (2019) claim that learners develop profound conceptual understandings of word problems if teachers provide them with rich and satisfying learning opportunities, which in this context, is the identification of keywords.

The learners had the opportunity to read a problem for themselves, and Teacher B made sure to underline the crucial elements that served as the cornerstone for solving word problems. The learners were also reminded by Teacher B that reading comprehension and underlining the key ideas that they think will help them solve the word problems can help them develop conceptual understanding of mathematics, which is necessary for solving word problems. The results support Baraké et al.'s (2015) statement that conceptual understanding and selecting the appropriate approach to a problem are crucial components of teaching and learning word problems.

In my opinion, learners must understand the basic ideas, such as that for addition operations words such as *add*, *altogether*, *sum*, and *increase* can be used. Any approach to solve mathematical word problems can be useful provided learners are familiar with the underlying ideas. The conceptual depth of mathematics is the foundation of all mathematical word problems. In order to help learners improve their conceptual understanding of mathematics, they should be exposed to mathematical concepts and provided with the freedom to explore synonyms. One of the things I noticed during data collection is that neither of the teachers let the learners come up with their own mathematical word problems. Although learners were very engaged in responding to the teachers' questions, they should also develop their own mathematical word problems to show their mastery of mathematical concepts. De Koning and van der Schoot (2019) pointed out that if learners create their own mathematical word problems, they learn more, improve their problem-solving abilities, and change their attitudes about mathematics. This is also consistent with the claim made by Sahin et al. (2020) that learners acquire a strong conceptual knowledge of word problems when teachers give them rich, applicable learning experiences. For instance, allowing learners to create their own word problems will pique their attention and motivate them to use more useful and analytical mathematical reasoning. According to Craig (2018), teachers should help learners create plans for how to handle word problems that need careful reading and repeated reading.

This theme allowed me to answer my second secondary research question that asked how primary school teachers facilitate the learning of mathematical word problems. This question was intended to give me insight through observation into how primary school teachers facilitate the learning of mathematical word problems based on their lesson planning. In order to be able to solve mathematical word problems, both primary teachers demonstrated how important it is for learners to have conceptual knowledge of mathematics. Both teachers actively involved their learners in acquiring a conceptual understanding of mathematics. During the teaching and learning of mathematical word problems, the teachers allowed learners to discuss the mathematical word problems that were posed. Both teachers taught their learners how to approach solving mathematical word problems although they used different approaches.

5.2.3 Schematising mathematical word problems presented in real-life context

Schematising word problems focuses on how learners approach and complete mathematical word problems, such as the steps they take. I examined how the learners solved mathematical word problems given in a real-life context. The RME framework I employed for this study serves as a guide for how teachers and learners can approach word problems in mathematics. Adopting the RME technique for word problems is intended to help learners connect what they learn in the classroom to challenges they encounter in real life (Suaebah et al., 2020). RME offers a step-by-step guide for solving mathematical word problems through horizontal and vertical mathematization, which aligns with schematisation.

The concepts of horizontal and vertical mathematization originate from the RME (Van den Heuvel-Panhuizen & Drijvers, 2020). For instance, it describes the transformation or schematisation of a symbolic mathematical problem into a real-world problem and vice versa. According to the results of the current study, learners can do horizontal mathematization to translate mathematical word problems that are language- and context-rich into mathematical symbols.

The focus of applying these ideas of horizontal and vertical mathematization of mathematical word problems is how learners break down the solution of mathematical word problems that are expressed in real-life context, demonstrating the relationship between schematisation and correlation. Therefore, horizontal and vertical mathematization are a form of schematisation mathematical word problems. For example, when learners perform horizontal mathematization, they convert a mathematical word problem from words into mathematical symbols, which is reorganising, regrouping, and rearranging; and when learners perform vertical mathematization, they solve a mathematical word problem that is now in the world of mathematics by restructuring and reconstructing it, which is how they schematise mathematical word problems that are presented in real-life context.

As an illustration, when they schematised a word problem, they rearranged, regrouped, and reorganised the problem. In contrast, understanding the word problem (i.e. question posed) and becoming familiar with the mathematical concepts are two important variables that helped the learners translate mathematical word problems presented in real-life context into mathematical language. The teachers encouraged the learners to study the problem situation and highlight the important ideas at the heart of solving mathematical word problems before engaging in horizontal mathematization. Learners transformed mathematical word problems that were written in language into symbols using different coloured ink. For instance, a learner wrote the problem statement in black ink, underlined the main ideas with another ink, and used a third ink to solve the mathematical word problem using mathematical symbols. The findings of this study showed that learners were successful in doing both horizontal and vertical mathematization when being taught and learning mathematical word problems. According to Pratiwi and Waziana (2018), vertical mathematization entails both mechanical, as in automated processes, and holistic features of restructuring and (re)constructing using mathematical symbols.

Vertical mathematization, to put it simply, is the process through which learners start to solve word problems and includes mathematical symbols. After finishing a word problem involving numbers, learners had to perform another round of horizontal mathematization, sometimes known as 'verification' to translate their answer, which was in mathematical language, into real-world context. The findings showed that learners were unaware that they had to change their answers from symbolic to verbal form. For instance, the following mathematical word problem was presented: "John buys a pizza, and it is divided into 8 equal pieces. If John ate $\frac{1}{4}$ of the pizza and he gives $\frac{1}{2}$ of what is left to his friends. How many pieces does he have left over?" The learner answered 2, which is correct, but the question asked 'number of pieces of a pizza', and therefore, during the verification stage, the learners should have gone back and reviewed the question and problem statement to refresh their memory of what was being asked. The learner should have answered 'John will be left with 2 pieces of pizza' to address the question.

From my vantage point, if teachers focused more on problem-solving in realistic mathematics instruction, it might help learners answer mathematical word problems successfully. While the learners were able to solve the word problems, they often did not provide a complete response to the question because they left the answer in the mathematical realm. Teacher B left her solution as symbols instead of translating them to mathematical words, which shows why her learners have this problem. For learners to connect the questions to their own experiences, the teachers can let them design their own mathematical word problems that are relevant to their lives. Teachers should present mathematical word problems that are connected to current events because in some circumstances, learners better understand scenarios they encounter in real life.

This theme helped me answer the third secondary research question that asked how learners mathematically schematise the word problem presented in real-life context. The aim of this question was to investigate how learners approach solving mathematical word problems as they struggle with the transition from the real world to the mathematical domain (horizontal mathematization) and the subsequent use of mathematical techniques to find solutions (vertical mathematization). Learners were given an opportunity to read the problems several times until they understood what the question required, and underlined the key terms that helped them choose the best operation to use as they solve the problem. After that, learners carried out horizontal mathematization by changing the word problem's letters to symbols. The learners then started seeking a solution after transforming a mathematical word problem into mathematical symbols using their preferred method of solving. The only area where most failed was going back to the question and placing their replies in a mathematical context; most learners left their answers as mathematical symbols. Simply put, learners successfully schematised the mathematical word problem presented in real-life context by doing vertical mathematization and horizontal mathematization, but they neglected to double-check their answers and transformed their solutions to words.

5.3 Responding to the primary research question

This study was based on the following research question: Which instructional practices do primary school mathematics teachers use to teach mathematics word problems? In this study, I explored how primary school mathematics teachers prepare to teach mathematical word problems, how they facilitate the learning of word problems, and how learners mathematically schematise word problems given in practical situations. The answers to the main research questions were found in all these areas of study. Data were gathered to accomplish this through observation and document analysis. RME and PCK, the two lenses that framed the study, offered theoretical direction on how mathematical word problems should be solved. The literature review gave a clear picture of the research done in the area of my study. Data-gathering, analysis, and discussion, as well as the answer to the main study question, were all influenced by how all the factors integrated.

The results showed that primary school mathematics teachers do not use lesson plans, which must include the lesson's goal, relevant background information, teaching and learning activities, and methods for assessing learners' understanding. However, they used lesson preparations, which included exercises and illustrations of the content they want to present in the classroom context. Additionally, they teach learners without a lesson plan by drawing on their experience teaching mathematics. In order to help learners learn about mathematical word problems, the teachers adopted a variety of instructional strategies. For example, they emphasised the need for learners to read the provided problem several times, analyse it, and identify any key ideas or words that will aid them in solving the problem.

During the teaching and learning of mathematical word problems, the teachers encouraged learners to share their thoughts or illustrate how they can answer the posed mathematical word problems on the chalkboard. This encouraged learner's participation in group discussions and helped them develop self-confidence. When teaching and learning mathematical word problems, there is a power imbalance that can be seen in the teacher-centred approach used at the beginning of the lesson presentation during which the teacher speaks to the learners while the learners passively listen. In order to put learners in charge of their learning, teachers used a learner-centred approach when evaluating their learners. For example, learners shared opinions on the chalkboard, asked questions about the content they had just learned, and the teacher acted as a facilitator, directing learners on how to solve mathematical word problems. Additionally, learners were challenged to solve mathematical word problems that were presented in a real-world context. Learners were able to translate mathematical word problems into mathematical symbols and start to solve the problem expressed in symbols, but learners frequently left their solutions in mathematical symbols without realising that the question was originally asked in words, which required them to convert their answers back to words.

The answer to the main research question that asked which instructional practices primary school mathematics teachers use to teach mathematics word problems is that the teachers stressed reading, analysing, locating essential concepts or emphasising key facts when answering mathematical word problems. They drew on their experiences and lesson preparations to teach mathematical word problems. They encouraged group discussion among the learners and gave them the opportunity to voice their thoughts on the chalkboard.

5.4 Nexus between the theoretical framework and the study

The purpose of this section is to show how the theoretical framework helped me carry out my research. To examine the instructional practices used by primary school teachers to teach mathematical word problems, I chose two theoretical perspectives, namely RME and PCK. These theoretical views helped me conceptualise the research questions; uncover pertinent literature to understand the study's focus and what other studies have highlighted about mathematical word problems and how teachers teach mathematical word problems; analyse and discuss the findings; and respond to the research questions. RME and PCK were essential in answering the main research question that asked which instructional practices primary school mathematics teachers use to teach mathematical word problems. PCK was mainly concerned with how primary school teachers plan to teach mathematical word problems and teachers facilitate the learning of mathematical word problems. RME was concerned with how learners mathematically schematise word problems presented in real-life context.

Concerning the review of the literature, the theoretical framework helped me explore and select pertinent literature relating to the teaching and learning of mathematical word problems and the skills learners should possess to solve mathematical word problems. The theoretical framework also helped me by providing direction on how to organise the subheadings in the literature review chapter.

The interpretivist paradigm was selected as the research philosophy through which the study was interpreted due to the alignment of RME; for instance, the interpretivism paradigm asserts that participants have multiple realities in which they participate and have their own experiences. RME encourages the use of mathematics word problems that are connected to real-world events, thereby allowing teachers to guide learners to emulate mathematicians by re-inventing mathematics. The interpretivist paradigm helped me design data collection tools that allowed me to systematically gather, evaluate, and interpret data by guiding my methodological decisions. This framework was appropriate for the study's objectives since I looked at which instructional practices primary school mathematics teachers use to teach mathematical word problems. PCK and RME also helped me during data collection; for instance, PCK helped me review how primary school teachers plan to teach mathematical word problems and how they understood the concept of mathematical word problems. RME, on other hand, helped me observe how learners solved mathematical word problems and gain insight into how they schematised the mathematical word problems they had to solve. During the presentation of the findings and the discussion, the theoretical framework helped me focus on my main research question.

5.5 Limitations of the study

Since the case study was chosen as the research strategy, the first drawback of this study is that it offers minimal basis for generalisation (Crowe et al., 2011), making it impossible to generalise the study's results to the wider population. The second constraint was caused by my personal prejudice as a mathematics teacher who also teaches in primary schools when I was observing and conducting document analysis. I have my own ideas about and experiences of how mathematical word problems should be planned and taught. However, I tried to reduce my bias by employing thematic data analysis and having my co-workers examine my data. The absence of interviews from the study is the third constraint. I would have learned more about the instructional strategies that primary school teachers use to teach arithmetic word problems had I interviewed the participants in depth.

Due to the lack of lesson plans, rather than lesson preparation, from the teachers, I also found it difficult to gather a lot of information about how they prepared their lessons. I would have had a better understanding of their instructional practices if I had spent more time observing and if the teachers had planned the lessons.

5.6 Recommendations

The study found that teachers relied on their prior experiences teaching learners about mathematical word problems when delivering lessons without a lesson plan. Although I could not establish the motive for not developing lesson plans, I recommend that teachers should receive focused training on how to develop lesson plans and how to use them effectively to facilitate the learning of mathematical word problems. Relying on experience alone is not enough since different learners learn differently, and therefore teachers' pedagogical knowledge ought to be customised for each cohort of learners. Additionally, learners were not given the chance to design their own mathematical word problems. Therefore, I recommend that curriculum developers consider the practical aspects that affect teaching and learning in the educational setting, such as reviewing the amount of work that has to be completed for the grade and the time allotted.

The study uncovered that teachers may not put much effort into word problems involving mathematics because according to the mathematics CAPS policy, problem-solving is only weighted 10% of bloom taxonomy. Therefore, I recommend that curriculum designers should assess the impact the percentage weighting has on performance, teaching, and learning.

Since I identified some limitations related to the current study, I recommend that more studies be considered to gain more insight into the teaching and learning of word problems. For instance, researchers can use interviews to gain more information about the instructional practices used by teachers to teach mathematical word problems. I also recommend that a similar study be conducted by a researcher who is not a mathematics educator to reduce the bias in the study.

5.7 Reflection

When I decided to pursue my master’s degree, I promised myself to research an area that interests me. I felt motivated to work hard on my studies because I am interested in mathematical word problems. I have learned a lot about myself and developed academically, emotionally, and professionally through my work as a mathematics teacher. I am committed to develop as a reflective, successful teacher for my learners and have a desire to advance personally. Therefore, I decided to pursue postgraduate courses in the hopes that it would allow me to learn more about how to effectively teach mathematical word problems.

Public schools in South Africa are receiving more consideration from the DBE, policy makers, and media on a much larger scale. I began to see the value of mathematics and how it affected our community as a mathematics teacher. It is my sincere hope that the results of this study will help the DBE and mathematics teachers improve their knowledge about mathematical word problems and expertise of various teaching approaches and teaching strategies as these factors influence the choice, adaptation, and implementation of assessments in the mathematics classroom. I maintain that the data were collected in the schools I listed in Chapter 3 under sampling, not at the school I was directed to. Additionally, as I am mathematics teacher with experience in teaching mathematical word problems, I did not allow my personal experience and view interfere with data collection, analysis, or interpretation process. I adhered to strict ethical practices as required by the University of Pretoria to avoid any potential bias.

5.8 Conclusion

The current study examined how primary school mathematics teachers plan and teach word problems. Given that it was a qualitative study using a case study design and the interpretivist paradigm, the chosen sample was examined in their natural environment. RME, which provided the theoretical basis for solving mathematical word problems, and PCK, which is mostly concerned with the instructional practices of mathematical word problems, served as the study's theoretical lenses.

The following is an overview of the study's main findings. Firstly, teachers frequently teach mathematical word problems without first creating a lesson plan, and instead, they rely on their teaching experiences, thereby choosing to use lesson preparations that merely provide classroom activities and illustrations of the material to be taught. Secondly, the teachers used a variety of teaching techniques when presenting and having learners learn mathematical word problems. For example, one teacher emphasised the importance of having learners read the problem repeatedly until they understood what was required of them in order to solve the problems; and find the mathematical concepts contained in the problem that can help them choose an appropriate approach to solve the problem. The other teacher adhered to the keyword method, which requires learners to read the problem and underline any crucial information that can help them solve the problem. Lastly, learners were successful in reading the mathematical word problems provided in real-life context and translating the problems into mathematical symbols. However, the learners did not apply their mathematically generated solutions to real-world scenarios.

Despite the fact the that that the implications of the instructional strategies employed by teachers to teach mathematical word problems based on the case study cannot be generalised, I can draw a few conclusions from my observations of the teaching experiences of mathematical word problems. Firstly, mathematical word problems are indeed challenging for teachers to teach effectively. Teachers do not plan or have lesson plans that show what they intend to teach learners; instead, they rely on lesson preparation that consists of classroom activities and examples of the lesson's content rather than essential elements of a lesson plan like objectives, background knowledge, teaching/learning activities, and methods for assessing learners' understanding. A critical shift is necessary within the community of mathematics educators, particularly among mathematics teachers, in how lesson plans are developed. By relying on experience, teachers limit their learners' ability to grasp new mathematical concepts. Although it is common knowledge that teachers detest paperwork, this should not be a reason to neglect lesson planning.

Secondly, by completely involving learners in the teaching and learning of mathematical word problems, learners are more likely to understand mathematical conceptual knowledge. Additionally, in order to empower learners to make responsible decisions when solving mathematical word problems, teachers must use a learner-centred approach. In order for learners to develop their confidence and have a better knowledge of and ability to use mathematical concepts, teachers must also provide opportunities for learners to generate their own word problems.

Lastly, the schematisation of mathematical word problems that are given in a real-world context must be in line with the teaching strategy employed by the teacher. According to the findings of this research, schematising mathematical word problems helped learners have a more original approach to addressing these mathematical word problems as well as a better understanding of mathematical concepts.

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Appendix A: Observation sheet

Content to be observed	Comments
Problem posed in the context Utilization of operation/s Language richness Types of questions (realistic or unrealistic)	
Horizontal mathematization Translation from word to mathematical symbol	
Vertical mathematization Solving word problem using mathematical operation and translating maths solution to realm context	

Appendix B: Documental analysis

Content to be analysed	Comments
Analyse lesson plan Introduction of the lesson Formulating of questions Implementation of strategies Consolidation	
Analyse workbooks How do learners solve word problems (Implementation of horizontal and vertical mathematization process)	

Appendix C: District Director letter

education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

Ikhamanga Building, Government Boulevard, Riverside Park, Mpumalanga Province
Private Bag X11341, Mbombela, 1200.
Tel: 013 766 5552/5115, Toll Free Line: 0800 203 116

Litiko le Temfundvo, Umnyango we Fundo | Departement van Onderwys | Ndzawulo ya Dyondzo

MR L Dlamini

Plazaview
Acornhoek
1360
U18396756@tuks.co.za

Dear Sir

APPROVAL OF REQUEST TO CONDUCT RESEARCH

Kindly be informed of the approval of your request to conduct research in Primary School Mathematics Teachers' Word problems instrumental practices in [redacted] Research to be based on how word problems are taught and Learned in Mathematics lessons for Learners Grade 7.

Please be informed that upon completion of your research the Mpumalanga Department of Education will require access to your research findings and recommendations. You are advised to communicate with your chosen schools and ensure that no inconvenience is experienced at any given time. Teaching and learning must not be negatively affected as a result of this Research.

Your professionalism in this regard will be highly appreciated. Good luck on your research; your interest on matters of the District is applauded.

[redacted]

22/02/2023
DATE

MPUMALANGA

Appendix D: Principal consent letters

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquiries: Mr Lehlohonolo Dlamini
Plazaview
Acornhoek, 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF CONSENT TO CONDUCT THE RESEARCH STUDY

I, [redacted]………., principal of [redacted], voluntarily and willingly permit Mr L.V. Dlamini to conduct a research study titled: ***Primary school mathematics teachers' word problems instructional practices.*** I understand that the participation of both grade 7 learners and mathematics teachers in the afore-mentioned study to which I am consenting, will involve:

a) teachers and learners being observed during the lesson presentation.
b) teachers availing their lesson plan for analysis.
c) learners providing their mathematics exercise books for analysis

I declare that I understand the purpose of the study and that you (the researcher) subscribe to the ethical research principles, including the informed consent, safety, privacy (confidentiality and anonymity) and trust.

In addition, I grant the University of Pretoria permission to use data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I grant you permission to conduct your study in our school.

[redacted]	[redacted]	01/03/2023
(**Name** and **surname**)	Signature	Date

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquiries: Mr Lehlohonolo Dlamini
Plazaview
Acornhoek, 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF CONSENT TO CONDUCT THE RESEARCH STUDY

I,…[redacted]……., principal of …[redacted]…, voluntarily and willingly permit Mr L.V. Dlamini to conduct a research study titled: ***Primary school mathematics teachers' word problems instructional practices.*** I understand that the participation of both grade 7 learners and mathematics teachers in the afore-mentioned study to which I am consenting, will involve:

a) teachers and learners being observed during the lesson presentation.
b) teachers availing their lesson plan for analysis.
c) learners providing their mathematics exercise books for analysis

I declare that I understand the purpose of the study and that you (the researcher) subscribe to the ethical research principles, including the informed consent, safety, privacy (confidentiality and anonymity) and trust.

In addition, I grant the University of Pretoria permission to use data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I grant you permission to conduct your study in our school.

[redacted]

28/02/2023

(Name and surname) Signature Date

Appendix E: Teachers consent letters

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquiries: Mr L.V. Dlamini
40122 Plazaview
Acornhoek 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF CONSENT TO CONDUCT THE RESEARCH STUDY

[redacted]..., teacher at [redacted], voluntarily and willingly permit Mr L.V. Dlamini to conduct a research study titled: ***Primary school mathematics teachers' word problems instructional practices..*** I understand that the participation of both learners and I in the afore-mentioned study to which I am consenting, will involve:

a) Learners and I being observed during the lesson presentation.

b) Availing the collaboratively prepared lesson plan for analysis.

I declare that I understand the purpose of the study and that you (the researcher) will subscribe to the ethical research principles, including the informed consent, safety, privacy (confidentiality and anonymity) and trust.

In addition, I grant the University of Pretoria permission to use data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I grant you permission to conduct your study in our school.

[redacted]	[redacted]	01/03/2023
(Name and surname)	Signature	Date

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquiries: Mr Lehlohonolo Dlamini
40122 Plazaview
Acornhoek, 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF CONSENT TO CONDUCT THE RESEARCH STUDY

I [redacted]...... teacher a[redacted] voluntarily and willingly permit Mr L.V. Dlamini to conduct a research study titled: ***Primary school mathematics teachers' word problems instructional practices.*** I understand that the participation of both learners and I in the afore-mentioned study to which I am consenting, will involve:

a) Learners and I being observed during the lesson presentation.

b) Availing the collaboratively prepared lesson plan for analysis.

I declare that I understand the purpose of the study and that you (the researcher) will subscribe to the ethical research principles, including the informed consent, safety, privacy (confidentiality and anonymity) and trust.

In addition, I grant the University of Pretoria permission to use data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I grant you permission to conduct your study in our school.

[redacted]	[redacted]	02/03/2023
(Name and **surname)**	Signature	Date

Appendix F: Parents' consent letters

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquiries: Mr Lehlohonolo Dlamini
Plazaview
Acornhoek, 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF CONSENT FOR MY CHILD TO PARTICIPATE IN THE RESEARCH STUDY

I, [redacted] ,parent of [redacted] voluntarily and willingly permit my child to participate in Mr L.V. Dlamini research study titled: ***Primary school mathematics teachers' word problems instructional practices.*** I understand that the participation of my child in the afore-mentioned study to which I am granting permission, will involve being observed during the lesson taught by their teacher, and analysing their mathematics activities in their exercise books. I declare that I understand the purpose of the study and that you subscribe to the ethical research principles, including *informed consent, safely, privacy* and *trust*.

In addition, I grant the University of Pretoria permission to use the data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I give permission for my child's participation in the study.

[redacted]		01-03-23
(Name and surname)	Signature	Date

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquiries: Mr Lehlohonolo Dlamini
Plazaview
Acornhoek, 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF CONSENT FOR MY CHILD TO PARTICIPATE IN THE RESEARCH STUDY

[redacted]..,parent of [redacted]…..voluntarily and willingly permit my child to participate in Mr L.V. Dlamini research study titled: ***Primary school mathematics teachers' word problems instructional practices.*** I understand that the participation of my child in the afore-mentioned study to which I am granting permission, will involve being observed during the lesson taught by their teacher, and analysing their mathematics activities in their exercise books. I declare that I understand the purpose of the study and that you subscribe to the ethical research principles, including *informed consent, safely, privacy* and *trust.*

In addition, I grant the University of Pretoria permission to use the data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I give permission for my child's participation in the study.

[redacted]	[redacted]	01-03-2023
(Name and **surname)**	Signature	Date

Appendix G: Learners consent letters

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquires: Mr Lehlohonolo Dlamini
Plazaview
Acornhoek, 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF ASSENT TO PARTICIPATE IN THE RESEARCH STUDY

I [redacted], Grade 7 learner at [redacted], voluntarily and willingly agree to participate in the study titled: ***Primary school mathematics teachers' word problems instructional practices.*** I understand that I will be observed during teaching and the activities in my mathematics exercise book will be analysed.
I declare that I understand the purpose of the study and that you subscribe to the ethical research principles, including the informed consent, safety, privacy (confidentiality and anonymity) and trust.

In addition, I grant the University of Pretoria permission to use the data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I voluntarily agree to participate in the study.

[redacted]		01-03-2023
(Name and surname)	Signature	Date

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Enquires: Mr Lehlohonolo Dlamini
Plazaview
Acornhoek, 1360
Email: u18396756@tuks.co.za

Dear Mr L.V. Dlamini

LETTER OF ASSENT TO PARTICIPATE IN THE RESEARCH STUDY

I [redacted]........... Grade 7 learner at.[redacted]................., voluntarily and willingly agree to participate in the study titled: ***Primary school mathematics teachers' word problems instructional practices.*** I understand that I will be observed during teaching and the activities in my mathematics exercise book will be analysed.
I declare that I understand the purpose of the study and that you subscribe to the ethical research principles, including the informed consent, safety, privacy (confidentiality and anonymity) and trust.

In addition, I grant the University of Pretoria permission to use the data provided for this study, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

Given the above information, I voluntarily agree to participate in the study.

[redacted]	[redacted]	01/03/2023
(**Name** and **surname**)	Signature	Date