PSSA Conference

Stellenbosch (R.S.A.)   January 2024

Aristotle's "Sea Battle" Scenario :

A MATTER OF STRICT VERSUS LAZY EVALUATION

Conference Categories:

- Philosophy of Logic
- Philosophy of Science

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TALK SLIDES

CONTEXT [TOULMIN 1980]

» A static ontology and taxonomy was the means by which ARISTOTLE was able to harness his newly invented syllogistic process to his scientific world view, and so to make formal logic the prime instrument of scientific explanation.
It was no wonder that his philosophical successors fell into the habit of construing causal connections as though they were logical connections, so confusing the physically "Necessary" and the logically "Entailed". Within an evolutionary world picture, by contrast, NO descriptions can be trusted to hold good indefinitely, ... and our deductive conclusions can be trusted only to the extent that ... events have ... the character of "convergent" ... sequences. «

ARISTOTLE's "SEA BATTLE" PROBLEM
[de Interpretatione IX]

S = " TOMORROW there will be a sea battle OR
TOMORROW there will be NO sea battle." $(B \lor \neg B)$

must be **TRUE** in Tertium-Non-Datur logic (TND)
even though today we have no knowledge about
TOMORROW: **That is peculiar!** He worried about "Determinism".

- ARISTOTLE's "WAY OUT": Abandon the TND principle
as soon as a formal sentence S is materially loaded
with future contingency semantics. ✳

  — Still much discussed in the 20ᵗʰ Century
  [several literature references]: with formalisation
  attempts by various MODAL and/or HIGHER ORDER logics.

✳ N.B. (Aristotle illogically CONFLATED Logical "Truth" with Physical "Necessity")

TRIVALENT KLEENE LOGIC

- Propositional (not Modal, not Higher-Order).
- Semantics of "unknown" is captured by a 3rd truth value U.

$S =$ "tomorrow there will be a sea battle"

Interpretation: $\mathcal{I}(S) := U$.

NO Tertium-non-datur :
In Kleene-Interpretation
$(S \lor \neg S)$ is **no** longer
a formally true Tautology

A	B	A ∨ B	(Kleene)
F	U	U	
U	F	U	
U	U	U	
U	T	T	
T	U	T	

with $\neg U := U$

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Now the PHILOSOPHICAL QUESTIONS
(for the Philosophers' conference) :

- Are we anyhow "obliged" or "forced" to agree with Aristotle and to "follow" him in his decision to abandon the TND principle for all sentences with future contingency semantics ?

- Or do we have an alternative "option" to "rescue" the TND principle also for future contingencies ?

— IF "yes", THEN: how ?

— FORMALLY: Is it anyhow possible to "reconcile" the classical TND Tautology $(B \lor \neg B) \equiv T$ with the U value of the tri-valent KLEENE Logic ?

ANSWER = "YES" IF we agree to ALLOW
the techique of LAZY EVALUATION
(from Informatics) to be used as
a Method of Formal Reasoning !

That is a genuinely PHILOSOPHICAL
decision (so-to-say "conventional")
which is NOT neccessarily "enforced"
by the underlying Logical Calculus
per-se (as such / itself)

GIVEN : SOME COMPOSITE FORMAL TERM

$$t = (t' \; \boxed{OP} \; t'')$$

● STRICT Evaluation :

Both t' as well as t'' <u>must already</u> be evaluated <u>BEFORE</u> we are allowed to apply the OPERATION for the evaluation of t.

● LAZY Evaluation :

Under particular (well-defined) circumstances we may evaluate t by applying the OPERATION <u>without</u> having previously evaluated t' and t''.

"Sea Battle" $S = (B \lor \neg B)$
with KLEENE's u
$\begin{cases} \to (u \lor \neg u) \to u. \; (\text{"strict" Ev.}) \\ \to \text{TAUT.} \; (\text{"lazy" Evaluation}) \end{cases}$

EXAMPLES FROM THE HISTORY OF SCIENCE

1. Discovery of the "Irrational Number" $i = \sqrt[2]{-1}$ in the 16th Century [Literature Reference: in the Paper]

— Issue : Algebra was still interpreted geometrically in those days

— In geometrical terms (areas and volumes) the negative numbers did not "make sense"

— Attempt at solving the cubic equation $x^3 = 15x + 4$ lead to the term $(2 + \sqrt[2]{-1}) + (2 - \sqrt[3]{-1})$

— SOLUTION: Without worrying about the mysterious "meaning" of the Root of a negative number, treat $\sqrt[2]{-1}$ as "y": $\Rightarrow$ 4.

"lazy" 😊

EXAMPLES FROM THE History OF Science

2. Functional Programming in the 20th Century

[Literature Reference: in the paper]

INDERMARK's PROGRAM :

$$F(x) := G(H(x), H(x+3))$$

with

$$G(a,b) := \text{IF } a \leqslant 3 \text{ THEN } a \text{ ELSE } b$$

and

$$H(c) := \text{IF } c = 3 \text{ THEN } 0 \text{ ELSE } H(c+1).$$

What is $F(1)$?

STRICT Eval.:

$\Downarrow$

undef.

"∞"

LAZY Eval.:

$\Downarrow$

$H((1+1)+1)$
$=$
$H(3) = 0.$

THE MEANING IS NOT "in" THE FUNCTION ALONE !

CONCLUSION:

We are <u>not</u> mandatory obliged to "follow" Aristotle in his decision to abandon the TND Principle for propositional sentences with *future contingency* semantics: We <u>have</u> alternative *logical modelling* options that are <u>also</u> consistent and <u>also</u> "make good sense" from a philosophical point of view.
(Aristotle "created" the Problem by CONFLATING "Truth" and "Necessity")

FORTHCOMING "PAPER":

- Formal elaboration of the details : TND with Modified HILBERT calculus for tri-valent KLEENE semantics with Meta-Rule for LAZY Eval.;

- Further explanations and Literature References.

END