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An Economic Model Predictive Control for Knowledge Transmission Processes in Multilayer Complex Networks

Jun Mei, Sixin Wang, Xiaohua Xia, and Weifeng Wang

***Abstract*—In this article, we study the optimal feedback control problems of knowledge dissemination processes in multilayer complex networks. First, a node-based model is established in multilayer complex networks and two collaborative control strategies are exerted to increase the scope and speed of knowledge dissemination, forming a closed-loop control system. Then, we develop a two-layer optimal control framework. At the upper level, the optimal solution of the control system is solved and sent to the lower layer. At the lower level, a model predictive controller (MPC) receives input information from the upper level and is formulated to decide on the network and then transmits it to its heterogeneous networks which can reduce control resources and computation complexity. Finally, numerical simulations are conducted to confirm the theoretical results.**

***Index Terms*—Knowledge transmission, model-predictive control (MPC), multilayer complex networks, Pontryagin's maximum principle.**

I. Introduction

Knowledge dissemination is crucial for knowledge management [1], technological innovation [2], and the response to natural disasters [3]. Currently, the primary emphasis of knowledge dissemination is on the factors that underlie knowledge diffusion and the effect of the network structure. Given the similarities and differences between knowledge transmission and infectious disease transmission [4], [5], [6], improved infectious disease models have been widely used in knowledge transmission based on their intrinsic mechanisms, including internalization mechanisms [7], leadership mechanisms [8], self-learning mechanisms [9], review mechanisms [10], friendship-based altruistic mechanism [2], etc. The research also revealed that scale-free networks were the most effective for knowledge dissemination [11]. In addition, a multilayer complex network can capture the coupling characteristics of several targets and different propagation channels, in contrast to single-layer networks. Moreover, multilayer networks can also assess capacity development in an organization [12]. A multiplexed network can be seen as a set of interconnected layered networks, where each layer might have unique properties that distinguish it apart from the others and support various dynamic activities. Therefore, researchers are trying to extend the propagation model from a single network to multilayer networks to study the spread of epidemics [13], information [13], and knowledge [14]. Multilayer network theory provided a valuable perspective for studying how knowledge is disseminated through multiple channels. Inspired by the above results, a multilayer complex network is constructed according to the way knowledge is acquired.

Due to the positive impact of knowledge dissemination in economic, cultural, and social aspects, it is of certain practical significance to study its dynamic processes. Furthermore, increasing the number of information disseminators and broadening the area of knowledge dissemination are efficient approaches to boost people's knowledge stores in the face of unanticipated calamities. The control system ideology has the ability to accomplish these tasks [15], [16], [17], [18]. However, the introduction of an intervention control strategy to the knowledge transmission process will increase resource costs inevitably. According to the optimal control theory, control inputs that match initial and terminal conditions can be specified to minimize execution costs or maximize operational benefits [19], [20], which is intimately allied to enhancing the effectiveness of knowledge distribution. But optimal control theory for nonlinear systems is rarely exploited in the process of knowledge transmission. Currently, optimal controllers for various applications in complex or social networks are only obtainable using the maximum principle (such as Pontryagin's maximum principle) [4], [21]. Even though global optimization is achieved, the maximum principle generally does not yield feedback controllers that implement online real-time control based on real-time data to adapt to changing circumstances. That is to say, even if the knowledge dissemination system satisfies the global optimal conditions, the population performing the control task

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Jun Mei, Sixin Wang, and Weifeng Wang are with the School of Mathematics and Statistics, South-Central MinZu University, Wuhan 430074, China (e-mail: meij0000@163.com; wangsx_789@163.com; wwf87487643@163.com).

Xiaohua Xia is with the Centre of New Energy Systems, Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa.

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cannot achieve real-time adjustment of the control task in the short term. As a result, the effectiveness of the control measures will unarguably decrease and the cost of control will increase.

In this article, we extend the framework developed in [22] to address the problem of optimal feedback control for nonlinear knowledge transmission systems, that is, the problem of finding state-feedback control laws that minimize a given performance measure and gradually exponential convergence of the closed-loop control system. Specifically, a novel knowledge transmission model is modeled by taking into account knowledge obtained, absorbed, and forgotten. To continuously and rapidly enhance knowledge dissemination, two intervention control strategies are designed based on their physical connotation. Furthermore, to obtain performance improvement with the proposed control strategy compared to that with conventional economic optimal control over a finite-time optimal horizon, a model predictive control (MPC) is introduced into the promotion of the knowledge transmission process for the first time. MPC can be used to optimize a system in a finite-time domain to maintain local optimization and to roll online in acclimatizing to unpredictable surroundings [23]. However, an economic MPC with a long prediction horizon may be inappropriate for adoption in real-time optimization due to the computational time needed to seek the optimization problem of the MPC [24]. In addition, based on the needs of knowledge dissemination processes, we expect that the interest problem is usually to find a specific control action that enables rapid knowledge diffusion while consuming a small number of economic resources. To overcome these challenges, the economic optimization problem is divided into a two-layer control scheme. In the upper layer, a maximum principle is employed to optimize the objective function over a long horizon. In the lower layer, an MPC with a shorter prediction horizon and a smaller sampling period calculates optimal control inputs that can be used to feedback on the process. In this manner, the lower layer MPC is designed to enhance the robustness of the closed-loop system while improving knowledge transmission ability.

The main contributions of this article are summarized as follows.

1) A novel knowledge transmission model in the multilayer complex networks is proposed taking into consideration how knowledge is obtained, absorbed, and forgotten.
2) We first introduce two intervention control strategies to the knowledge dissemination model to formulate a closed-loop control system. Each controller is endowed with its physical connotation to facilitate knowledge transmission and to qualify control actions.
3) The application of MPC to the continuous-time knowledge transmission process in complex networks is innovative to the literature. A two-layer control structure consisting of long-horizon optimization and shorter horizon prediction is formed to accomplish feedback control while guaranteeing both theoretical existence feasibility and stability. MPC provides real-time optimization and interference resistance of knowledge dissemination by designing an explicit feedback controller.

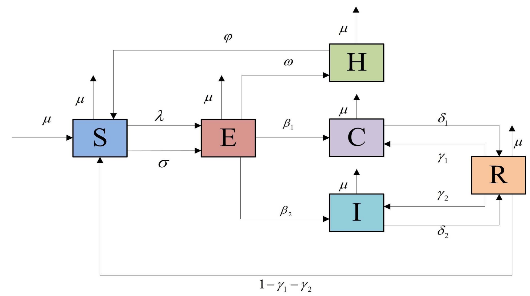

Fig. 1. Diagram of knowledge transmission.

4) Numerical simulations verify that the proposed method with two control strategies has a significant effect on improving knowledge dissemination, which shows that the control strategy we proposed is satisfactory.

The remainder of this article is organized as follows. In Section II, a knowledge dissemination model in the multilayer complex networks is built, and the stability of two equilibrium points based on the basic regeneration number is proved. In Section III, it is proposed two-layer control structures for the model, and the existence of the optimal control solution is proved. In Section IV, we conduct an extensive numerical simulation of the model and its optimal control problem. The conclusion is shown in Section V.

II. Preliminaries

A. Dynamic Model of Knowledge Transmission

To build a mathematical knowledge transmission model that considers the way of knowledge acquisition and human psychology is the purpose of this section. A two-layer network consisted of the knowledge base layer and the individual interaction layer is established. The knowledge base layer is a homogeneous network, and the individual interaction layer is a heterogeneous network. People's interpersonal relationships have little change in a short period, so it is assumed that the edges of the two networks are the same and undirected. Nodes in a network represent individuals, and edges equate to communication between two people [25]. Assuming that the population is sufficiently mixed, all nodes are statistically equal, so that the likelihood of a connection between two individuals is equal. A knowledge dissemination system is shown in Fig. 1, in which the population is divided into six states according to behavior performance. The knowledge recipient (S) is the one who desires to acquire it. Knowledge contacts (E) are people who are exposed to knowledge. Knowledge refusers (H) are those who do not have further knowledge due to a lack of interest and other practical factors. Knowledge conservatives (C) are those who choose to further digest and understand but do not disseminate knowledge after exposure to knowledge. Knowledge communicators (I) are people who not only further digest but also spread knowledge after exposure to knowledge, and knowledge forgetters (R) are those who forget knowledge. Obviously, $S_k(t)+E_k(t)+H_k(t)+C_k(t)+I_k(t)+R_k(t)=1$. It is assumed that people are free to enter and exit the system and both the entry rate and exit rate are $\mu(0 \le \mu \le 1)$. The rules for the dissemination of knowledge are expressed as follows.

1) *Knowledge Acquisition:* There are two main ways for knowledge recipients to acquire knowledge: one is to acquire knowledge through contact and communication with others, and the contact rate is $\lambda(0 \le \lambda \le 1)$. The other is to browse the knowledge base to obtain knowledge, and the browsing rate is $\sigma(0 \le \sigma \le 1)$. In this article, the knowledge base refers to books, publications, network resources, and other shared static knowledge sources. The knowledge base is actually established, supplemented, and revised by the knowledge disseminator, so the knowledge base can be seen as a linear combination of knowledge communicators, that is, $B(t) = \alpha \Sigma_k p(k) I_k(t)$, where $\alpha(0 \le \alpha \le 1)$ is the contribution rate of the knowledge disseminator to the knowledge base and $p(k)$ is the degree distribution of the network.
2) *Knowledge Rejection and Digestion:* Knowledge contacts will refuse to learn more about the knowledge and become knowledge refusers after taking into account practical factors, such as no interest or no assistance with future development, the rejection rate is $\omega(0 \le \omega \le 1)$. At the same time, knowledge contacts may also choose to digest and understand that knowledge. However, considering psychological factors, some knowledge contacts will become knowledge conservatives who are reluctant to share knowledge, and this digestion rate is $\beta_1(0 \le \beta_1 \le 1)$, and the rest of the knowledge contacts will become knowledge disseminators who are willing to spread knowledge, and this digestibility rate is $\beta_2(0 \le \beta_2 \le 1)$. $\omega + \beta_1 + \beta_2 = 1$.
3) *Knowledge Forgetting and Reviewing:* Over time, both knowledge conservatives and knowledge disseminators have the potential ability to dilute the memory of knowledge. The forgetting rate for knowledge conservatives is $\delta_1(0 \le \delta_1 \le 1)$, and the forgetting rate for knowledge disseminators is $\delta_2(0 \le \delta_2 \le 1)$. In fact, the process of disseminating knowledge is the process of deepening memory, so $\delta_1 > \delta_2$. At the same time, knowledge forgetters can regain knowledge by reviewing knowledge. Similarly, the knowledge forgetters may become knowledge conservatives with a review rate $\gamma_1(0 \le \gamma_1 \le 1)$, and the knowledge forgetters can also become knowledge disseminators after acquiring knowledge, and the review rate is $\gamma_2(0 \le \gamma_1 \le 1)$. In addition, $\gamma_1 + \gamma_2 \le 1$.
4) *Knowledge Degradation:* If the knowledge refuser and the knowledge forgetter do not come into contact with knowledge for a long time, they will degenerate into knowledge recipients, and the degradation rate is $\varphi(0 \le \varphi \le 1)$ and $1 - \gamma_1 - \gamma_2$, respectively.

In the individual interaction layer, the probability that any selected edge points to the knowledge communicator with degree k is $q(t) = kp(k)I_k(t)$ [26]. Thus, the probability that any given edge pointing to a knowledge communicator node at time t is

$$\Theta(t) = \frac{\sum_k kp(k)I_k(t)}{\langle k \rangle} \tag{1}$$

where $\langle k \rangle$ is the average degree of the network, and $\langle k \rangle = \sum kp(k)$. According to the above description, the dynamic knowledge recipients-knowledge contacts-knowledge refusers-knowledge conservatives-knowledge disseminators-knowledge forgetters (SEHCIR) model can be represented as follows by the mean-field theory:

$$\begin{cases} \frac{dS_k(t)}{dt} = \mu + \varphi H_k(t) - \mu S_k(t) - \sigma S_k(t) B(t) \\ \qquad - \lambda k S_k(t) \Theta(t) + (1 - \gamma_1 - \gamma_2) R_k(t) \\ \frac{dE_k(t)}{dt} = \lambda k S_k(t) \Theta(t) + \sigma S_k(t) B(t) - (\omega + \beta_1 \\ \qquad + \beta_2 + \mu) E_k(t) \\ \frac{dH_k(t)}{dt} = \omega E_k(t) - (\varphi + \mu) H_k(t) \\ \frac{dC_k(t)}{dt} = \beta_1 E_k(t) + \gamma_1 R_k(t) - (\delta_1 + \mu) C_k(t) \\ \frac{dI_k(t)}{dt} = \beta_2 E_k(t) + \gamma_2 R_k(t) - (\delta_2 + \mu) I_k(t) \\ \frac{dR_k(t)}{dt} = \delta_1 C_k(t) + \delta_2 I_k(t) - (1 + \mu) R_k(t). \end{cases} \tag{2}$$

The feasible region of the model (2) is $\Omega = \{(S_k(t), E_k(t), H_k(t), C_k(t), I_k(t), R_k(t)) \in \mathbb{R}_+^6, 0 \le S_k(t), E_k(t), H_k(t), C_k(t), I_k(t), R_k(t) \le 1\}$.

Remark 1: Traditional knowledge dissemination models focused on basic mechanisms, such as acquisition [9], [2]; forgetting [27]; and reviewing [10] while skipping some practical and psychological factors. In general, knowledge may be rejected if someone is uninterested in it or believes it will not aid future progress. Furthermore, those with the ability to exchange intelligence may be inclined to keep silent. In comparison to previous models, the SEHCIR model includes knowledge rejection and conservatism, which is more resemblant to the real propagation process.

B. Stability Analysis of Knowledge Transmission Model

In order to facilitate the dissemination of knowledge, it is necessary to study the conditions of knowledge dissemination and the final state of the model. In this regard, infectious disease research has accumulated a lot of theoretical knowledge, and the basic regeneration number (R_0) is one of the concepts. The basic regeneration number refers to the average number of people infected by an infected person during the infection cycle when all the people around them are susceptible, which is an important indicator of disease outbreak or extinction [28]. The basic regeneration number is deduced by the next-generation matrix method [29]. Let the left-hand side of (2) equal to 0, and a zero solution ($Q^0(\overbrace{1, 0, 0, 0, 0, 0, \ldots, 1, 0, 0, 0, 0, 0}^{6n})$), which is called the knowledge-free equilibrium (KFE), can be obtained by direct observation, where n is the maximum value of the degree k of the node. For simplicity, S_k, E_k, H_k, C_k, I_k, and R_k are used to replace $S_k(t), E_k(t), H_k(t), C_k(t), I_k(t)$, and $R_k(t)$, respectively. Only compartments E_k, C_k, and I_k, $k = 1, \ldots, n$, can be involved in the calculation of R_0, so their differential equations can be rewritten as

$$\begin{pmatrix} \frac{dE_k}{dt} & \frac{dC_k}{dt} & \frac{dI_k}{dt} \end{pmatrix}' = F_1(t) - V_1(t).$$

F and V are the derivatives of F_1 and V_1 with respect to E_k, C_k, and I_k, $k = 1, 2, \ldots, n$, respectively. The basic regeneration number R_0 satisfies $R_0 = \rho(FV^{-1})$, where $\rho(\cdot)$ is the spectral

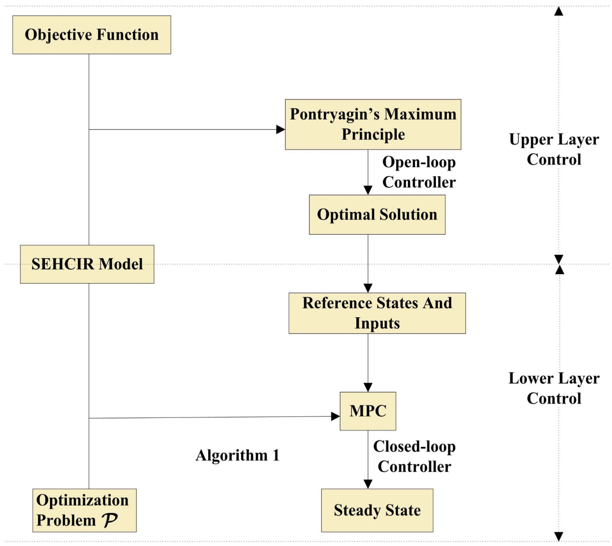

Fig. 2. Diagram of a two-layer control scheme.

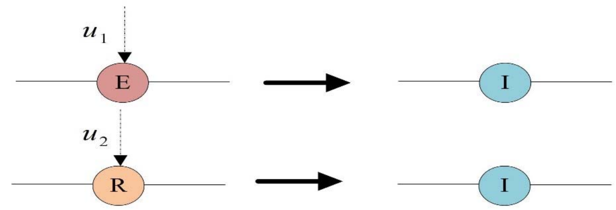

Fig. 3. Diagram of state transitions of E and R with two controllers in the individual interaction layer.

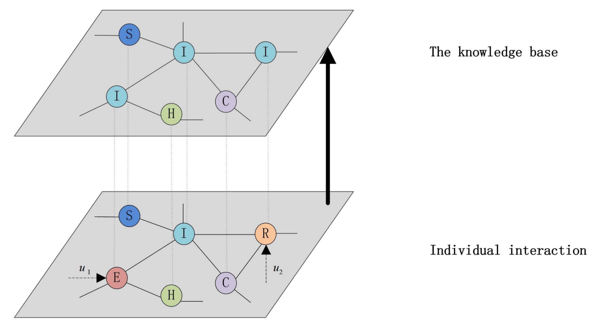

Fig. 4. Diagram of state transitions of E and R with two controllers in a two-layer network.

radius of the matrix FV^{-1}. Thus

$$R_0 = \frac{\sqrt{\left(\frac{\lambda\langle k^2\rangle}{\langle k\rangle} - \sigma\alpha\right)^2 + 2\sigma\alpha\langle k\rangle(2\lambda + 1 - \sigma\alpha)}}{2} + \frac{\frac{\lambda\langle k^2\rangle}{\langle k\rangle} + \sigma\alpha}{2}.$$

The obtained basic regeneration number R_0 can be applied to analyze the stability problem of the knowledge-free equilibrium and knowledge-endemic equilibrium. There are many relevant works on the following stability results [30], [31], we can draw the following conclusions directly.

Theorem 1: When $R_0 < 1$, the KFE Q^0 of system (2) is globally asymptotically stable, and Q^0 is unstable when $R_0 > 1$.

From the above analysis, it can be noted that the Q^0 is unstable when $R_0 > 1$. What is certain, however, is that there exists at least one equilibrium point $Q^*(\overbrace{S_1^*, E_1^*, H_1^*, C_1^*, I_1^*, R_1^*, \ldots, S_n^*, E_n^*, H_n^*, C_n^*, I_n^*, R_n^*}^{6n})$ so that knowledge can be disseminated among people when $R_0 > 1$. Q^* is called the knowledge-endemic equilibrium (KEE).

Theorem 2: When $R_0 > 1$, the KEE Q^* of system (2) is globally asymptotically stable for a finite time.

Remark 2: When $R_0 > 1$ and $t \to +\infty$, the density of all six groups tends to be a positive value. Knowledge lasts forever.

III. Proposed Solution

In this section, in order to facilitate the knowledge transmission processes in complex networks, we first develop two collaborative control strategies to enable people in states R and E to become knowledge disseminators, respectively, and simultaneously. Then, a two-layer control scheme is designed as shown in Fig. 2. In the upper layer, Pontryagin's maximum principle [32] is employed to optimize the objective function over a long horizon. In the lower layer, an MPC with a shorter prediction horizon and a smaller sampling period calculates optimal control inputs that can be used to feedback on the process.

A. Optimal Control Problem of Knowledge Dissemination

Knowledge dissemination plays an important role in enhancing personal value, promoting economic development, and maintaining social stability. Furthermore, optimal control can play a better performance of the system under certain prerequisites. The premise of this section is $R_0 > 1$. In order to increase the number of knowledge disseminators, it makes sense that optimal control strategies are imposed. As shown in Fig. 1, increasing the digestibility of choosing to be a communicator (β_2) and the review rate of becoming a communicator (γ_2) can increase the number of knowledge disseminators. In practice, the review is the process of reducing forgetting. That is, our model assumes the existence of a set of control actions $\mathcal{A}$. It is assumed that each action a entirely determines the set of system states through a known map $\Phi(a) = \{\beta_2(a), \gamma_2(a)\}$ and the cost of employing action a is denoted by $J(a)$, which is a known function as follows (8). In this study, we will analyze which actions can be applied to the network on some predetermined set of times $T_{\Delta t} \triangleq \{t \in \mathbb{R}_{\geq 0} | t = \Delta tk, k \in \mathbb{Z}_{\geq 0}\}$. Once observing the state of the system Ω, an explicit action employed to the network model is calculated by the control strategy u. Therefore, our controller function is a sampled-data controller for the knowledge dissemination process. We study the case in which two collaborative control strategies are introduced for the sake of expanding the scale of knowledge dissemination simultaneously. Two control strategies are elaborated as follows and the proposed control framework is shown in Figs. 3 and 4.

1) Digestibility: Continue to gain insight into the knowledge and give material rewards to improve the digestibility of knowledge contacts who choose to be knowledge communicators. When keep learning and receiving material rewards, the probability of knowledge contacts choosing to become knowledge communicators is $\iota_1(0 \leq \iota_1 \leq 1)$ under a predefined period of time. Furthermore, continuing learning intensity is denoted by u_1. The minimum sleep time for people is 6 h [33]. We assume that the time spent engaging in activities necessary for survival is 1 h and that people are fully engaged in learning. Therefore, the maximum learning intensity $u_{1\max}$ is 17 h per day, and u_1 can be expressed as

$$u_1 = \frac{\overline{u_1}}{u_{1\max}} \tag{3}$$

where $\overline{u_1}$ is the current learning intensity.

2) Reinforcement: For improving the review rate of the knowledge forgetters of people who choose to become knowledge disseminators, increasing the frequency of review and advocating win–win ideas are applied at the same time. Similarly, the probability of a knowledge forgetter becoming a knowledge communicator after completing the above behavior is $\iota_2(0 \leq \iota_2 \leq 1)$ under a predefined period of time. In addition, the review frequency is denoted by u_2. The shortest memory cycle is 5 min [34]. Therefore, the maximum revision frequency $u_{2\max}$ is 288 per day, and u_2 can be expressed as

$$u_2 = \frac{\overline{u_2}}{u_{2\max}} \tag{4}$$

where $\overline{u_2}$ is the current review frequency.

According to the above description, only E and R undergo state transitions under the control strategies at the personal interaction layer, as shown in Fig. 3. After two controllers are exerted in the individual interaction layer, the knowledge base layer receives data from the underlying and directly follows the result, as shown in Fig. 4.

Furthermore, a controlled model can be obtained as

$$\begin{cases} \frac{dS_k(t)}{dt} = \mu + \varphi H_k(t) - \sigma S_k(t)B(t) \\ \qquad + (1-\gamma_1-\gamma_2-\iota_2 u_2)R_k(t) \\ \qquad - \lambda k S_k(t)\Theta(t) - \mu S_k(t) \\ \frac{dE_k(t)}{dt} = \lambda k S_k(t)\Theta(t) + \sigma S_k(t)B(t) - (\omega+\beta_1 \\ \qquad +\beta_2 + \iota_1 u_1 + \mu)E_k(t) \\ \frac{dH_k(t)}{dt} = \omega E_k(t) - (\varphi+\mu)H_k(t) \\ \frac{dC_k(t)}{dt} = \beta_1 E_k(t) + \gamma_1 R_k(t) - (\delta_1+\mu)C_k(t) \\ \frac{dI_k(t)}{dt} = (\beta_2+\iota_1 u_1)E_k(t) + (\gamma_2+\iota_2 u_2)R_k(t) \\ \qquad - (\delta_2+\mu)I_k(t) \\ \frac{dR_k(t)}{dt} = \delta_1 C_k(t) + \delta_2 I_k(t) - (1+\mu)R_k(t) \end{cases} \tag{5}$$

where the control domain is given by

$$\Psi = \{(u_1, u_2) | 0 \leq u_i \leq 1, u_i \text{ is lebesgue measurable } i = 1, 2\} \tag{6}$$

and the feasible region of states in the model (5) is

$$\Omega = \Big\{(S_k(t), E_k(t), H_k(t), C_k(t), I_k(t), R_k(t)) \in \mathbb{R}_+^6 \; 0 \leq S_k(t), E_k(t), H_k(t), C_k(t), I_k(t), R_k(t) \leq 1\Big\}. \tag{7}$$

Remark 3: $u_1 = 1$ and $u_2 = 1$ mean that the learning intensity and review frequency reach the maximum. Moreover, $u_1 = 0$ and $u_2 = 0$ indicate that no control is applied to the system.

Controllers are proposed to increase the number of knowledge communicators, but the imposition of controllers in practice will increase the cost of the system. Therefore, the cost function should be set to the number of knowledge propagators minus the cost of the controller. The definition of the objective function is as follows:

$$J(u_1, u_2) = \int_0^{t_f} \Big(AI_k - \frac{c_1}{2}u_1^2 - \frac{c_2}{2}u_2^2\Big)dt \tag{8}$$

where t_f represents the final time, A is a positive weighting constant for knowledge spreaders, $c_1 > 0$, and $c_2 > 0$ are weighing constants for improving digestibility and increasing review rate, respectively. It is to design a quadratic form to measure the control costs. The terms $(c_1/2)u_1^2$ and $(c_2/2)u_2^2$ are accounted for the costs of two controllers.

Remark 4: The purpose of the market-determined weight factors A, c_1, and c_2 is to evaluate the relative influence of the knowledge disseminator and resource consumption by the two controllers on the objective function. If higher gains of knowledge dissemination are foreseen, A can be raised while c_1 and c_2 can be decreased; otherwise, the reverse.

The main goal is to maximize the number of knowledge communicators by application of optimal control theory. Thus, it is of great significance to seek optimal controls u_1^* and u_2^* so that the objective function (8) satisfies the following equation:

$$J\big(u_1^*, u_2^*\big) = \max_{u_1, u_2 \in \Psi} J(u_1, u_2). \tag{9}$$

We first need to prove the existence of optimal controls u_1^* and u_2^* that satisfy (9) and the control domain in (6).

Theorem 3: With the optimal control problem (9) defined on the control set Ψ and model (5), there exist optimal controls u_1^* and u_2^* such that $\max_{u_1, u_2 \in \Psi} J(u_1, u_2) = J(u_1^*, u_2^*)$.

Proof: The proof of Theorem 3 is in Appendix A. ■

Note that the optimization problem (8) is an economic optimal control problem. In this article, we develop an economic MPC to optimize the economic performance of the controlled SEHCIR system. In order to guarantee the system properties on stability, feasibility, and optimal solution to the optimization problem, the economic MPC controller is decomposed into a maximum optimal controller and an MPC controller.

B. Optimal Design

Since there exists an optimal control sequence $u^*(t) = (u_1^*, u_2^*)$ over a total horizon t_f to improve the performance of knowledge transmission processes, the maximizing problem (9) with regard to system (5) can be turned into another problem of maximizing a Hamiltonian by applying Pontryagin's maximum principle [32]. Then, the Hamiltonian, denoted by $\mathcal{H}$, is given by

$$\mathcal{H} = AI_k - \frac{c_1}{2}u_1^2 - \frac{c_2}{2}u_2^2 + \lambda_1\frac{dS_k}{dt} + \lambda_2\frac{dE_k}{dt} + \lambda_3\frac{dH_k}{dt} + \lambda_4\frac{dC_k}{dt} + \lambda_5\frac{dI_k}{dt} + \lambda_6\frac{dR_k}{dt} \tag{10}$$

where λ_1, λ_2, λ_3, λ_4, λ_5, and λ_6 represent co-state variables for the state variables S_k, E_k, H_k, C_k, I_k, and R_k, respectively.

The following result show the characteristics of optimal controls u_1^* and u_2^* that satisfy (9) and the control domain in (6).

Theorem 4: Given the optimal controls $u_1^* \in \Psi$ and $u_2^* \in \Psi$ satisfying (9) subject to the system (5), then there exist co-state variables λ_1, λ_2, λ_3, λ_4, λ_5, and λ_6 which satisfy the following co-state system:

$$\begin{cases} \frac{d\lambda_1}{dt} = (\lambda_1 - \lambda_2)(\lambda k\Theta + \sigma B) + \lambda_1\mu \\ \frac{d\lambda_2}{dt} = (\lambda_2 - \lambda_3)\omega + (\lambda_2 - \lambda_4)\beta_1 \\ \qquad +(\lambda_2 - \lambda_5)(\beta_2 + \iota_1 u_1) + \lambda_2\mu \\ \frac{d\lambda_3}{dt} = (\lambda_3 - \lambda_1)\varphi + \lambda_3\mu \\ \frac{d\lambda_4}{dt} = (\lambda_4 - \lambda_6)\delta_1 + \lambda_4\mu \\ \frac{d\lambda_5}{dt} = -A + (\lambda_1 - \lambda_2)\left(\frac{\lambda\langle k^2\rangle}{\langle k\rangle} + \sigma\alpha\right)S_k \\ \qquad +(\lambda_5 - \lambda_6)\delta_2 + \lambda_5\mu \\ \frac{d\lambda_6}{dt} = (\lambda_1 - \lambda_4)\gamma_1 + (\lambda_1 - \lambda_5)(\gamma_2 + \iota_2 u_2) \\ \qquad +(\lambda_6 - \lambda_1) + \lambda_6\mu \end{cases} \tag{11}$$

with transversal conditions $\lambda_i(t_f) = 0, i = 1, 2, \ldots, 6$. Thus, the optimal controls u_1^* and u_2^* are characterized by

$$\begin{cases} u_1^* = \max\left\{0, \min\left\{\frac{(\lambda_5-\lambda_2)\iota_1 E_k}{c_1}, 1\right\}\right\} \\ u_2^* = \max\left\{0, \min\left\{\frac{(\lambda_5-\lambda_1)\iota_2 R_k}{c_2}, 1\right\}\right\}. \end{cases} \tag{12}$$

Proof: The proof of Theorem 4 is in Appendix B. ■

Since the improvement of knowledge transmission processes is achieved by a system of differential equations, any optimal intervention strategy cannot accurately capture the real users' information. In addition, two intervention control strategies might be different in achieving control targets on difficulty level and period. For the purpose of maintaining the proposed intervention strategies for the knowledge transmission processes, feedback control is one of the effective strategies.

C. Problem Statement of Model Predictive Control

For the sake of increasing the digestibility rate and expanding the review rate, and transmitting reliable information to the heterogeneous network, we propose an efficient nonlinear feedback control mechanism for the knowledge transmission in multilayer complex networks, as shown in Fig. 4. Note that MPC is capable of predicting and making the optimal actions over a predefined period, this section develops an MPC to facilitate the controlled network model. Specifically, completing a round of feedback control requires five steps, as shown in Fig. 5. In the first step, the controller receives the information of the knowledge dissemination, that is, the value of $S_k(t)$, $E_k(t)$, $H_k(t)$, $C_k(t)$, $I_k(t)$, and $R_k(t)$. In the second step, the controller sends control schemes to E and R after assessing the number of six populations. In the third step, E and R perform the received control tasks, respectively, that is, E increases the intensity of continued learning, and R increases the frequency of review. In the fourth step, after E and R complete the task, the SEHCIR system needs to output the number of six populations. In the fifth step, the results are sampled and feedback

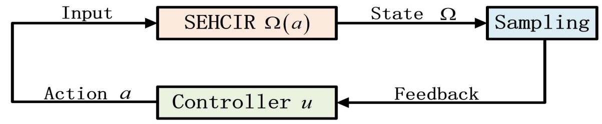

Fig. 5. Schematic of feedback control in this article. The state Ω of the process is observed by a sampled-data controller u, which then performs an action a to the SEHCIR process, which will generate a set of populations to disseminate the process forward.

to the controller. The controller repeatedly receives feedback and solves the optimal problem during the prediction horizon [35]. It is worth noting that the number of six populations is measurable.

In this article, the controller we study is a nonlinear MPC controller which develops a feasible and optimal solution to the optimization problem $\mathcal{P}$ as follows:

$$\min_{a\in\mathcal{A}}\{\mathcal{J}(a)|C(\Omega(t), a) \le 0\} \tag{13}$$

at all time t in the predetermined set of time $T_{\triangle t} \triangleq \{t \in R_{\ge 0}|t = \triangle tk, k \in Z_{\ge 0}\}$, $C(\Omega(t), a)$ is the stability constraint function and $\mathcal{J}(a)$ is a quadratic objective function. In the closed-loop feedback of the controlled SEHCIR process, the purpose of (5) is to automatically generate actions from an implicit nonlinear controller u_{mpc}. Although it may be difficult to seek a feasible control action for (5) in general care, we assume that the control law can pursue a state auxiliary controller u_{aux} that supplies such action in any state. This hypothesis is universal in the nonlinear MPC design because the nonconvexity caused by the nonlinear property of dynamics is such that it is difficult to achieve global optimity. Furthermore, in the field of the knowledge transmission process in complex networks, as mentioned above, we expect to have such intervention strategies to be easy implement-if we are increasing the frequency of review and advocating win–win ideas, we may forget less knowledge, and if we are going to improve digestibility, we may keep learning and receiving material rewards to everyone. In all of these cases, if we do this, the knowledge-endemic states will reach equilibrium quickly. That is, in the context of knowledge transmission processes, we hope that the interest problem is usually not to find an abstract control strategy that enables rapid knowledge diffusion, but to determine how to effectively implement control actions to improve rapid knowledge dissemination while using a small number of economic resources.

To construct a robust MPC for the SEHCIR process for knowledge-endemic states quickly, while constraining the rate of economic resources adapted by the intervention control law. This article addresses the following two key points to knowledge transmission by applying MPC.

1) Approximating the evolution process of (2) to design a specific and effective fixed control action to reach the tracking target.
2) Studying the convergence of the closed-loop control algorithm to provide a strict guarantee for the rapid diffusion of knowledge from the networks.

D. Model Predictive Control Design

The main challenge to employing MPC is the guarantee of closed-loop stability, the computation performed by the standard MPC is done based on local linearization on the system model. In addition, a specific and fixed control action is considered. In this context, the Lyapunov-based MPC [36] based on the SEHCIR control system (5) is employed to stabilize the controlled SEHCIR model (2) in the stability region and meanwhile force the system to realize the optimal solutions of input and state calculated by the maximum optimization problem. We begin our development by designing an auxiliary feedback controller for the SEHCIR dynamics. To consider the developed controller in a stabilizing manner, we first need to give the following assumptions.

Assumption 1 [36]: There exists a local Lipschitz feedback controller $u = h(x)$ satisfying $h(0) = 0$, constants $\rho > 0$, d_i $(i = 1, 2, 3, 4)$ and a continuous differential function $V : R^n \to R_+$ for (19) such that the following inequalities hold for all $x \in \Omega_\rho$: 1) $d_1|x|^2 \leq V(x) \leq d_2|x|^2$; 2) $([\alpha V(x)]/\alpha x) f_0(t, x, h(x)) \leq -d_3|x|^2$; and 3) $|[\alpha V(x)]/\alpha x| \leq d_4|x|$. Then, the controller can force the origin of the closed-loop system locally exponentially stable.

In this article, an auxiliary feedback controller u_{aux} is designed to generate a stability region Ω_ρ and provide a specific control action a for the evolution of the controlled SEHCIR process in the optimization problem $\mathcal{P}$, under satisfying inequality conditions of Assumption 1. Under the controller u_{aux}, the following results can be achieved.

Theorem 5: If a symmetric positive-definite matrix L, a positive-definite matrix P and a scalar $c > 0$, any appropriate dimension matrices K and B, degrees k and R_0 are existent to satisfy $k > 2$, $R_0 > 1$, and

$$\begin{pmatrix} PBK + K^T B^T P + L^T P + PL & \sqrt{c} I \\ * & -P^{-1} \end{pmatrix} < 0$$

where I is an appropriate identity matrix, then the auxiliary controller $u_{\text{aux}} = (\iota_1 E_k u_1, \iota_2 R_k u_2)^T$ is designed to ensure that the system (5) is exponential stable.

Proof: The proof of Theorem 5 is in Appendix C. ■

Exponential stability ensures the rate at which the control system (5) converges under the controller (26), which witnesses the speed of knowledge dissemination.

Under the design of two auxiliary controllers u_{aux}^1 and u_{aux}^2, the stability of the evolution of the closed-loop system (5) is guaranteed, and they can make the state Ω in the stability region Ω_ρ for all times if any initial condition sets $x_0 \in \Omega_\rho$. Moreover, the specific control law u and the probability of knowledge contacts and forgetters becoming knowledge communicators is directly determined by the auxiliary control u_{aux}. Consequently, a stabilizing Lyapunov-based MPC is proposed to optimize knowledge transmission processes.

The formulation of MPC optimization problem $\mathcal{P}$ is then expressed as follows:

$$\mathcal{J} = \min_{u \in S(\Delta_t)} \int_{t_k}^{t_k + T^p} \ell(x(t_k), u(s; t_k)) dt \tag{14a}$$

$$\text{s.t. } \dot{\tilde{x}}(s; t_k) = f_1(\tilde{x}(s; t_k)) + f_2(\tilde{x}(s; t_k)) u(s; t_k) \tag{14b}$$

$$\tilde{x}(t_k; t_k) = x(t_k) \tag{14c}$$

Algorithm 1 Lyapunov-Based MPC Implementation Strategy

1: **Initialization** For system (19), at the lower layer sampling period Δ_t, the lower layer receives the optimal state and input trajectories, $x^{*u}(t_k)$, $u^{*u}(t_k)$ calculated by the upper layer, and initials the prediction horizon $T^p = \widehat{\Delta_t}/\Delta_t$, set the weighting matrices Q, R, other design parameters and $k = 0$;
2: Sample the state $x(t_k)$ of the system;
3: Computes the stability constraint (14e) by using a stabilizing feedback controller (26);
4: The lower layer computes its Lyapunov-based MPC optimization problem $\mathcal{P}$ (14);
5: Apply the optimal input trajectory $u^{*l}(s; t_k)$ $s \in [t_k, t_k + \Delta_t]$ to the control actuators in a sample-and-hold fashion from t_k to t_{k+1};
6: Set $k = k + 1$, and turn to step 2.

$$u(s; t_k) \in \mathcal{U} \quad \forall\, t \in \left[t_k, t_k + T^p\right) \tag{14d}$$

$$\dot{\mathcal{V}}(x(t_k), u(t_k)) \leq \dot{\mathcal{V}}(x(t_k), u_{\text{aux}}(t_k)) \text{ if } x(t_k) \in \Omega_\rho \backslash \Omega_{\rho_e}, \tag{14e}$$

$$\mathcal{V}(\tilde{x}(t)) \leq \rho_e, \ \forall\, t \in \left[t_k, t_k + T^p\right) \quad \text{if } x(t_k) \in \Omega_{\rho_e} \tag{14f}$$

where $s \in [t_k, t_k + T^p]$, T^p is the prediction horizon and less than t_f. $\tilde{x}$ is the predicted state trajectory, $S(\Delta_t)$ represents the set of piecewise-constant functions with sampling period Δ_t, that is, $t_{k+1} = t_k + \Delta_t$ and T^p is the prediction horizon. The control input u and auxiliary control law u_{aux} are described in (19) and (26), respectively. In the optimization problem (14a), the cost function is described as $\ell(x(t_k), u(s; t_k)) = ||x(t) - x^{*u}||_Q + ||u(t) - u^{*u}||_R$, where x^{*u} and u^{*u} denote optimal state and input generated by the upper layer control, and Q and R are the weighting matrix. The constraint of (14b) in the system model of (19) is used to predict state trajectories of the closed-loop system. The condition (14c) denotes the initial condition $\tilde{x}(t_k; t_k)$ of (14b) which is the state measurement at time $t = t_k$. The condition (14d) is the input constraint over the prediction horizon, which guarantees the computed control input within the bounds of the available control action. If $x(t_k) \in \Omega_\rho \setminus \Omega_{\rho_e}$, the constraint (14e) implies that the closed-loop state $x(t_k)$ can be forced to move toward the origin; however, if $x(t_k)$ enters Ω_{ρ_e}, the state predicted will be maintained at Ω_{ρ_e} for the total prediction horizon. $u^{*l}(s; t_k)$ and $x^{*l}(s; t_k)$ are optimal control solution and optimal state of the optimization problem $\mathcal{P}$ of the closed-loop system, respectively. In addition, based on the derivative of the associated Lyapunov function (28), and (19), (26), and the stability constraint (14e) at time instant t_k is expressed as follows:

$$\begin{aligned} &2x^T P(f_1(t, x) + f_2(t, x) u(t_k)) \\ &\quad \leq x^T P(f_1 + BKx) + (f_1 + BKx)^T P x \end{aligned} \tag{15}$$

where $u(t_k)$ denotes the optimization control vector.

The Lyapunov-based MPC control is implemented by the sample-and-hold fashion, for knowledge transmission processes in multilayer complex networks and summarized in Algorithm 1 as follows.

To facilitate the Lyapunov-based MPC scheme, it is important to ensure that the optimization problem $\mathcal{P}$ admits a

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solution at each time instant. Therefore, the feasibility and stability of the knowledge transmission system under the predictive control algorithm are required and presented as follows.

Theorem 6: For the knowledge transmission control system (19), and computes optimization problem $\mathcal{P}$ at each sampling instant t_k, Algorithm 1 is recursively feasible if there exists a feasible solution at an initial point t_0, then the optimization problem admits a solution for $\forall t_k \geq t_0$. The optimal control of the knowledge transmission closed-loop system can be obtained with the controller calculated by Algorithm 1.

Proof: The proof of Theorem 6 is in Appendix D. ■

IV. Numerical Simulation

In this section, the preceding theoretical results and also the comparison between the model (2) and the controlled model (5) are drawn by numerical results. In the first case, the multilayer complex networks are established. Compared to other complex networks, the performance of knowledge propagation is maximized in scale-free networks [11]. Thus, the personal interaction layer is set to a scale-free network where the degree distribution satisfies the power-law distribution, and the knowledge base layer is designed as a random network with a degree distribution that obeys the Poisson distribution. In both the scale-free network and the random network, there are five initial seeds, which are randomly connected, and then the two networks connect five new nodes with five edges according to their respective laws of degree distribution until 2000 nodes are reached. Second, the fourth-order Runge–Kutta ($RK4$) scheme is proposed to solve the differential (2).

A. Knowledge Transmission Without Control

In the beginning, the spread of knowledge was caused by a small number of original knowledge propagators, who were called the seeds of knowledge transmission. Thus, $S_k(0) = 0.95$, $E_k(0) = 0.01$, $H_k(0) = 0.01$, $C_k(0) = 0.01$, $I_k(0) = 0.01$, and $R_k(0) = 0.01$. The degree of the nodes is taken as 15. Considering the scenario that $\mu = 0.02$, $\varphi = 0.8$, $\gamma_1 = 0.4$, $\gamma_2 = 0.1$, $\lambda = 0.01$, $\sigma = 0.1$, $\omega = 0.4$, $\beta_1 = 0.4$, $\beta_2 = 0.2$, $\alpha = 0.6$, $\delta_1 = 0.6$, and $\delta_2 = 0.4$, then $R_0 = 0.7392 < 1$. As shown in Fig. 6, the system finally evolved to the point where only the knowledge recipients remained, and the other five groups went perished. Thus, when $R_0 < 1$, knowledge will be unable to propagate and will become extinct. Theorem 1 is confirmed and it takes about 40 days for six groups to reach their steady-state. In addition, the case of $\mu = 0.02$, $\varphi = 0.8$, $\gamma_1 = 0.1$, $\gamma_2 = 0.4$, $\lambda = 0.3$, $\sigma = 0.2$, $\omega = 0.3$, $\beta_1 = 0.3$, $\beta_2 = 0.4$, $\alpha = 0.6$, $\delta_1 = 0.6$, and $\delta_2 = 0.2$ is taken into account, then $R_0 = 4.1571 > 1$. As shown in Fig. 7, the six groups of participants in this system tend to have a positive value, and the knowledge diffusion process may keep functioning regularly and permanently. Then, knowledge can be spread permanently. Theorem 2 is validated and the time that it takes for six groups to reach a steady state is about 51 days. In the final stable state, the densities of the six groups are ranked as $I_k > S_k > R_k > E_k > C_k > H_k$.

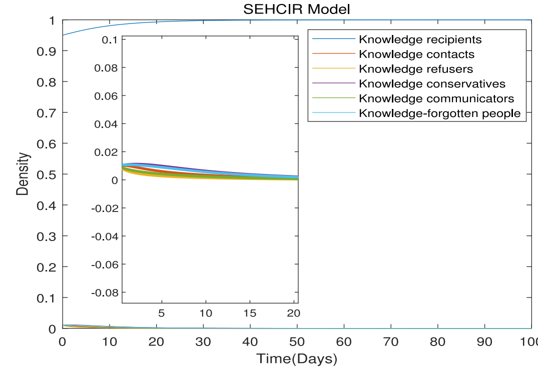

Fig. 6. Diagram of knowledge dissemination when the basic regeneration number is less than 1.

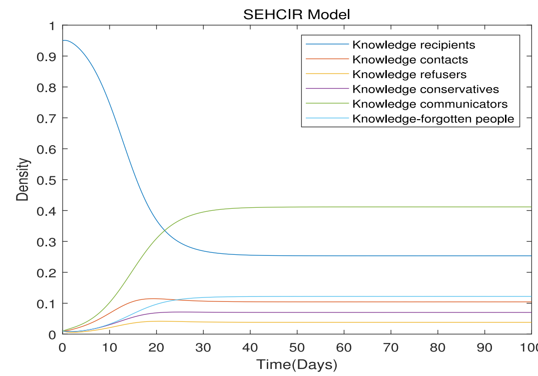

Fig. 7. Diagram of knowledge dissemination when the basic regeneration number is greater than 1.

TABLE I
Change Rate of Six States Under Different Control Strategies

	S	E	H	C	I	R
IV-B1	37.35%	0.07%	35.07%	51.14%	34.07%	2.30%
IV-B2	56.26%	22.51%	30.10%	47.29%	46.38%	12.21%
IV-B3	64.03%	29.41%	77.23%	80.90%	64.30%	11.97%

B. Knowledge Transmission With Control

Regarding system (5) and optimal control problem (9), the Runge–Kutta forward–backward method [37] is used to find the optimal solution. Under different control strategies, the density of the six populations will also be compared. Set $A = 0.02$, $c_1 = 2$, $c_2 = 10$, $\iota_1 = 0.5$, $\iota_2 = 0.5$, $k = 4$, $\mu = 0.02$, $\varphi = 0.8$, $\gamma_1 = 0.1$, $\gamma_2 = 0.4$, $\lambda = 0.3$, $\sigma = 0.2$, $\omega = 0.1$, $\beta_1 = 0.1$, $\beta_2 = 0.3$, $\alpha = 0.6$, $\delta_1 = 0.6$, and $\delta_2 = 0.2$, then $R_0 = 6.8076 > 1$. Various strategies based on the combination of the control for knowledge transmission are considered. Specifically, the rates of change of each state under different control measures are shown in the Table I. The details are outlined in the following.

1) Increasing the Digestibility of Being Spreader: In this strategy, we use an active control u_1, which is improving digestibility by increasing the intensity of learning and material rewards. The control profile is depicted in Fig. 8. It is noticed that the controller u_1 should be maintained at upper limit 1 during the first three days before declining to lower limit 0. The changes in the six populations are shown in

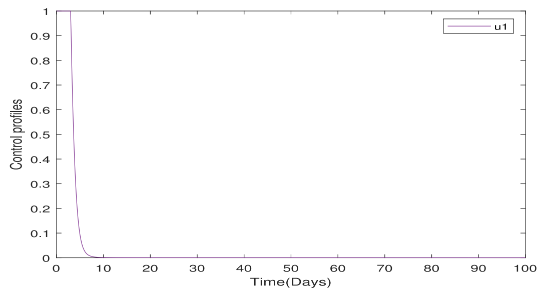

Fig. 8. Transmission control profile u_1.

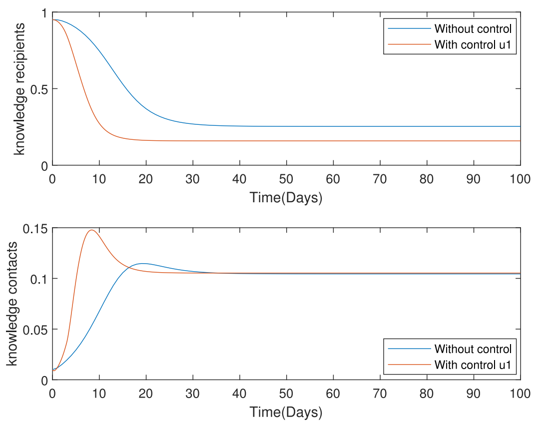

Fig. 9. Diagram of the evolution of S and E with u_1.

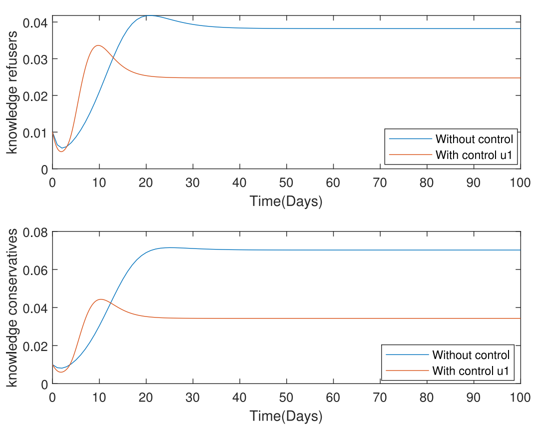

Fig. 10. Diagram of the evolution of H and C with u_1.

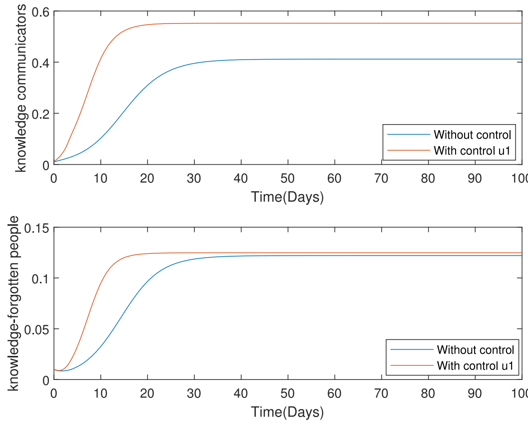

Fig. 11. Diagram of the evolution of I and R with u_1.

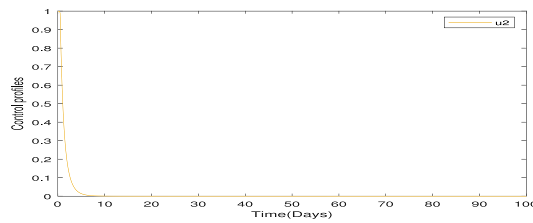

Fig. 12. Transmission control profile u_2.

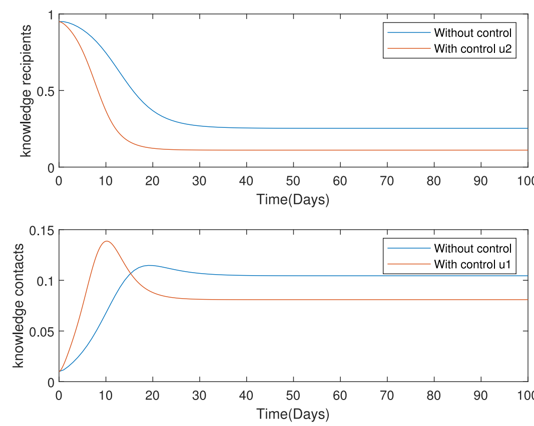

Fig. 13. Diagram of the evolution of S and E with u_2.

Figs. 9–11. Under the action of the controller, the knowledge recipient declines slowly until it stabilizes at a positive value. The knowledge contact increases rapidly to a peak in the first 8 days and then slowly declines until it stabilizes to a positive. This is because a large number of knowledge recipients evolve into knowledge contacts at the commencement of dissemination, and as the number of information recipients diminishes, the number of knowledge contacts reduces under the management of the controller until they are stable. Both knowledge deniers and knowledge conservatives decrease to a minimum in the first two days and then slowly decrease until they stabilize at a positive value. Knowledge disseminators and knowledge forgetters slowly increase until they stabilize at a positive point. Fewer knowledge contacts turn into knowledge deniers and conservatives in the beginning, while their knowledge is forgotten and deteriorated. However, as the number of information connections grows, so does the number of knowledge doubters and conservatives.

2) Expanding Review Rate of Being Spreader: The expansion of knowledge disseminators can also be achieved by promoting the revision rate u_2, which is fulfilled mainly by increasing the frequency of revision and promoting a combination of win–win ideas. As shown in Fig. 12, the controller u_2 remains at 1 for the first 0.4 days and then slowly descends until it reaches 0. The changes in the six populations are revealed in Figs. 13–15. Under this strategy, it takes 35 days for the system to remain stable and in equilibrium. In summary, The application of controller u_2 over the process

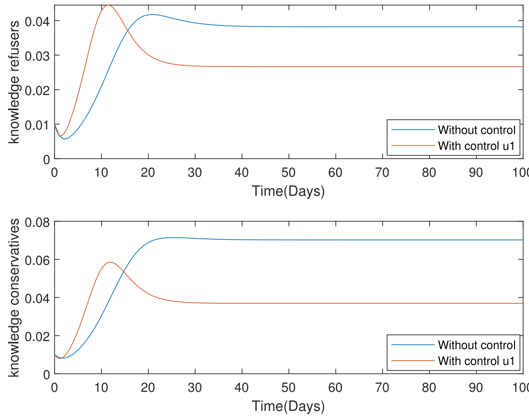

Fig. 14. Diagram of the evolution of H and C with u_2.

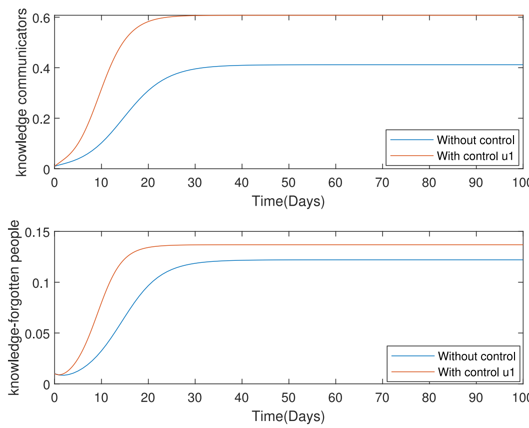

Fig. 15. Diagram of the evolution of I and R with u_2.

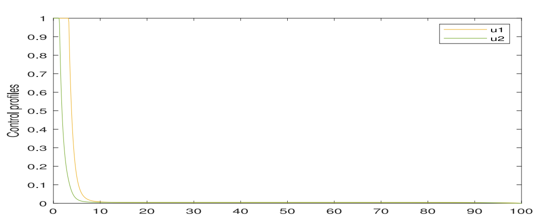

Fig. 16. Transmission control profiles u_1 and u_2.

effectively improves the performance of knowledge dissemination. Furthermore, the appearance of the evolution curve of the six populations under this control legislation is similar to Section IV-B1.

3) Increasing the Digestibility and Expanding Review Rate of Being Spreader: In this strategy, improving digestibility u_1 and promoting review rates u_2 are taken into account, which can be taken together by taking material rewards, promoting learning intensity, and increasing the frequency of revision. The control profile is depicted in Fig. 16. Controller u_1 maintains upper bound 1 for the first three days and then slowly descents until it reaches 0, while controller u_2 maintains upper bound 1 in the first one day and then slowly decreases to 0. Figs. 17–19 show the changes in the six populations. Under this strategy, it takes 26 days for the system to remain stable and in equilibrium. Specifically, the rates of change of each state under different control measures are shown in the Table I. Compared with the previous two strategies, under the joint action of controllers u_1 and u_2, the performance of knowledge dissemination plays the best. Furthermore, the appearance of the evolution curve of the six populations under this control legislation is similar to Section IV-B1. However, the performance of the knowledge dissemination model of simultaneous action of the two controllers is significantly better than that of the single controller, since more knowledge

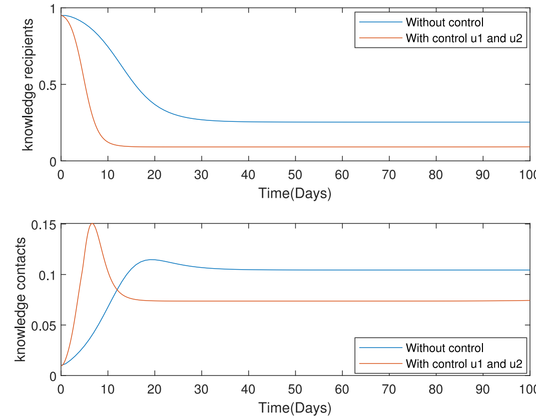

Fig. 17. Diagram of the evolution of S and E with u_1 and u_2.

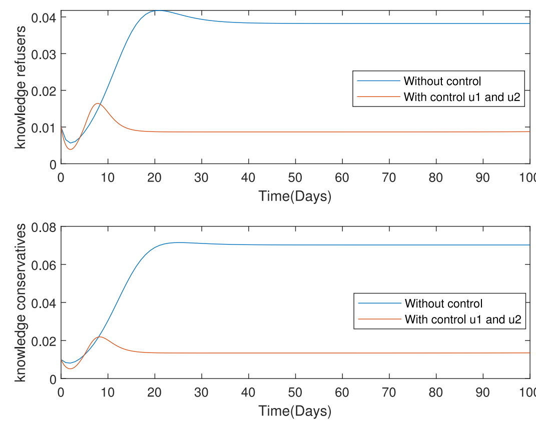

Fig. 18. Diagram of the evolution of H and C with u_1 and u_2.

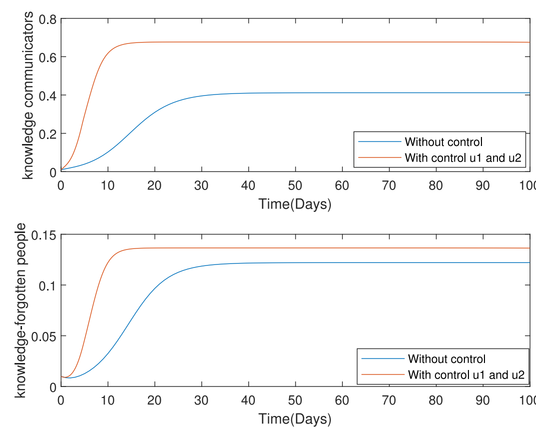

Fig. 19. Diagram of the evolution of I and R with u_1 and u_2.

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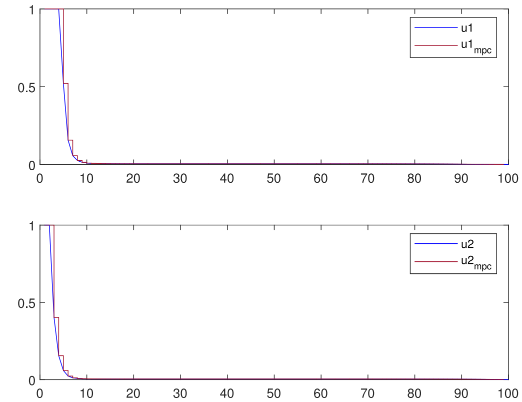

Fig. 20. Comparison of optimal control and MPC.

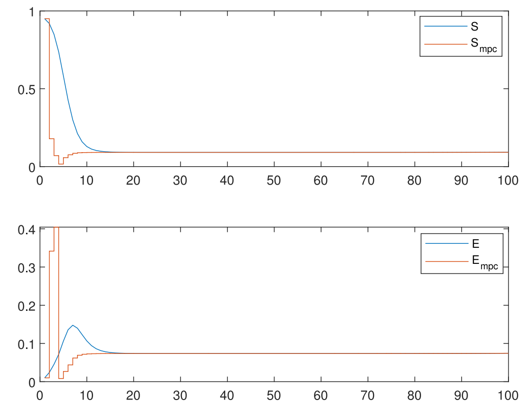

Fig. 21. Comparison of S and E with optimal control and MPC under system disturbances.

contacts and knowledge forgetters evolve into knowledge disseminators, reducing the distribution of the other five groups of people.

C. Knowledge Transmission With MPC

In the above analysis, optimal control based on Pontryagin's maximum principle is proposed. However, due to its poor anti-interference, the accuracy rate needs to be strengthened. Therefore, an MPC with prediction, feedback, anti-interference, and real-time optimization is studied in this article. The optimal solution in Section IV-B3 is used as a reference trajectory. The MPC parameter settings are as follows: the prediction horizon $T^p = 10$ days, the sampling period $\Delta = 1$ day, final time $t_f = 100$ days, and the weighting matrix $Q = \text{diag}(1, 1, 1)$ and $R = \text{diag}(1, 1, 1)$. The comparison of two controllers under the maximization principle and MPC is shown in Fig. 20. For about the first ten days, the trajectories of the two controllers differed from the reference trajectories, but the trajectories were the same after that. All the parameters in the system (5) will change with time, they are not static, which constitutes a parameter disturbance within the system. In addition, Figs. 21–23 show the comparison of the state in the maximization principle with the MPC under system disturbances. There was variation between the trajectories of the six states and the reference trajectories around the first 13 days. However, the trajectory of the states floats on the reference trajectory after 13 days, which means that the MPC has good immunity to interference and a high accuracy rate. The errors of both controllers and six states are rather large in the first ten days since the sampling period is shorter than the prediction period, but even in this scenario, the pattern of the model prediction curve is compatible with the direction of the reference curve.

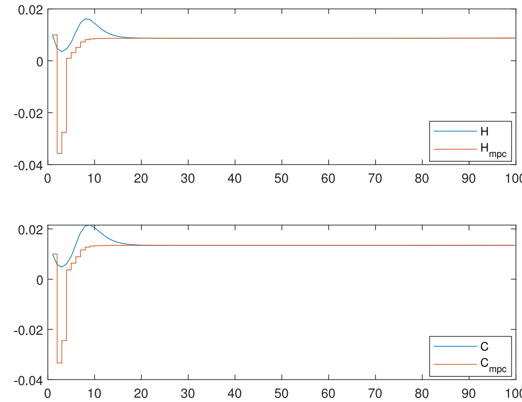

Fig. 22. Comparison of H and C with optimal control and MPC under system disturbances.

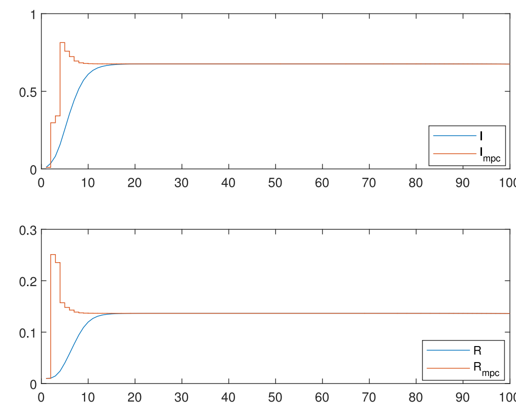

Fig. 23. Comparison of I and R with optimal control and MPC under system disturbances.

V. CONCLUSION

The density of the six populations in the system tends to be positive when $R_0 > 1$ and knowledge can persist in the system permanently. When $R_0 < 1$, only the recipients of the knowledge survive over time, with the other five states converging to zero and knowledge finally fading. Different control mechanisms were shown to boost the volume and speed of information diffusion to a certain extent when compared to the performance of the uncontrolled model. It is crucial to highlight that applying two controls simultaneously enhances the performance of the information dissemination model. MPC with short prediction and sampling periods is able to seamlessly trace the optimal solution, which is calculated using the maximization principle. Moreover, under the disturbance of the system parameters, the tracking effect is robust and the six states are asymptotically stable. Therefore, the two-layer control structure not only reduces control resources and computational complexity but also improves the robustness of the system.

However, this article simply simulated the dynamic process of knowledge dissemination in artificial complex networks, which fails naturalism. In addition, assessing the entire state, which requires a significant amount of control resources, is a massive undertaking. Therefore, output feedback control of the knowledge dissemination in real-world networks is the subject of our future research.

Appendix A

The [38, Corollary 4.1] is applied to prove the existence of the optimal control problem (8), (9) can be written as

$$J\left(u_1^*, u_2^*\right) = \min_{u_1, u_2 \in \Psi} -J(u_1, u_2) = \min_{u_1, u_2 \in \Psi} \int_0^{t_f} \left(\frac{c_1}{2}u_1^2 + \frac{c_2}{2}u_2^2 - AI_k\right)dt \quad (16)$$

and the Lagrangian is

$$\mathcal{L} = \frac{c_1}{2}u_1^2 + \frac{c_2}{2}u_2^2 - AI_k. \quad (17)$$

In short, (16) holds when the following conditions are satisfied.

1) The sets of controls and state variables are nonempty.
2) The admissible control Ψ is closed and convex.
3) The right-hand side of system (5) is bounded by a linear function in the state and control variables.
4) $\mathcal{L}$ is convex on Ψ.
5) $\mathcal{L}$ is bounded below by $e_1\|u\|_2^{\varrho} - e_2$, where $e_1, e_2 > 0$ and $\varrho > 1$.

Next, the satisfaction of five conditions is proved in turn.

1) Apparently, the six equations in the system (5) are uniformly Lipschitz continuous, so Ψ and the set of optimal solutions are nonempty, so the first condition is satisfied [39].
2) From the definition of Ψ, it can be directly concluded that Ψ is closed and convex.
3) Let $u = (u_1, u_2)^T$, $x = (S_k, E_k, H_k, C_k, I_k, R_k)^T$, and $f_0(t, x, u)$ be the right-hand side of system (5) given by

$$f_0(t, x, u) = \left[D_1\ D_2\ D_3\ D_4\ D_5\ D_6\right]^T \quad (18)$$

where $D_1 = \mu + \varphi H_k + (1 - \gamma_1 - \gamma_2 - \iota_2 u_2)R_k - \lambda k S_k \Theta - \sigma S_k B - \mu S_k$, $D_2 = \lambda k S_k \Theta + \sigma S_k B - (\omega + \beta_1 + \beta_2 + \iota_1 u_1 + \mu)E_k$, $D_3 = \omega E_k - (\varphi + \mu)H_k$, $D_4 = \beta_1 E_k + \gamma_1 R_k - (\delta_1 + \mu)C_k$, $D_5 = (\beta_2 + \iota_1 u_1)E_k + (\gamma_2 + \iota_2 u_2)R_k - (\delta_2 + \mu)I_k$, and $D_6 = \delta_1 C_k + \delta_2 I_k - (1 + \mu)R_k$. Clearly, $f_0(t, x, u)$ can be written as

$$f_0(t, x, u) = f_1(t, x) + f_2(t, x)u \quad (19)$$

where

$$f_1(t, x) = \left[G_1 \quad G_2 \quad G_3 \quad G_4 \quad G_5 \quad G_6\right]^T \quad (20)$$

where $G_1 = \mu + \varphi H_k + (1 - \gamma_1 - \gamma_2)R_k - \lambda k S_k \Theta - \sigma S_k B - \mu S_k$, $G_2 = \lambda k S_k \Theta + \sigma S_k B - (\omega + \beta_1 + \beta_2 + \mu)E_k$, $G_3 = \omega E_k - (\varphi + \mu)H_k$, $G_4 = \beta_1 E_k + \gamma_1 R_k - (\delta_1 + \mu)C_k$, $G_5 = \beta_2 E_k + \gamma_2 R_k - (\delta_2 + \mu)I_k$, $G_6 = \delta_1 C_k + \delta_2 I_k - (1 + \mu)R_k$, and

$$f_2(t, x) = \begin{pmatrix} 0 & -\iota_2 R_k \\ -\iota_1 E_k & 0 \\ 0 & 0 \\ 0 & 0 \\ \iota_1 E_k & \iota_2 R_k \\ 0 & 0 \end{pmatrix}. \quad (21)$$

Hence

$$\|f_0(t, x, u)\| \le \|f_1(t, x)\| + \|f_2(t, x)\|\|u\| \quad (22)$$

where $\|f_1(t, x)\|$ and $\|f_2(t, x)\|$ are non-negative constants and are the Frobenius norm of matrices $f_1(t, x)$ and $f_2(t, x)$, respectively. Thus, the third condition is satisfied.
4) Since $(\partial^2\mathcal{L}/\partial u_1^2) = c_1 \ge 0$ and $(\partial^2\mathcal{L}/\partial u_2^2) = c_2 \ge 0$, $\mathcal{L}$ is convex on Ψ.
5) Since $I_k \ge 0$, $\mathcal{L} \ge -A + (c_1/2)u_1^2 + (c_2/2)u_2^2 \ge (1/2)\min\{c_1, c_2\}\|u\|_2^2 - A$. Let $e_1 = (1/2)\min\{c_1, c_2\}$, $e_2 = A$, and $\varrho = 2$, $\mathcal{L} \ge c_1\|u\|_2^{\varrho} - c_2$.

Based on the above discussion, the existence of an optimal solution is guaranteed. The proof is completed.

Appendix B

Pontryagin's maximum principle is introduced to the Hamiltonian (10) for finding the optimal solution. The necessary conditions for the optimal solution to optimal control problems are given by [38]. To obtain the maximum value of $J(u_1, u_2)$, it is only need to maximize $\mathcal{H}(t, x, u, \lambda)$. If (x, u) is an optimal solution of the optimal control problem (9), in which $x = (S_k, E_k, H_k, C_k, I_k, R_k)^T$ and $u = (u_1, u_2)^T$. Hence, there exists a nonlinear vector function $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)$ which satisfies the equalities as follows:

$$\begin{cases} \frac{dS_k}{dt} = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial\lambda_1} \\ \frac{dE_k}{dt} = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial\lambda_2} \\ \frac{dH_k}{dt} = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial\lambda_3} \\ \frac{dC_k}{dt} = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial\lambda_4} \\ \frac{dI_k}{dt} = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial\lambda_5} \\ \frac{dR_k}{dt} = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial\lambda_6} \end{cases}, \quad \begin{cases} \frac{d\lambda_1}{dt} = -\frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial S_k} \\ \frac{d\lambda_2}{dt} = -\frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial E_k} \\ \frac{d\lambda_3}{dt} = -\frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial H_k} \\ \frac{d\lambda_4}{dt} = -\frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial C_k} \\ \frac{d\lambda_5}{dt} = -\frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial I_k} \\ \frac{d\lambda_6}{dt} = -\frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial R_k} \end{cases} \quad (23)$$

with transversal conditions $\lambda_i(t_f) = 0, i = 1, 2, 3, 4, 5, 6$. In addition, the optimality condition $([\partial\mathcal{H}(t, x, u, \lambda)]/[\partial u]) = 0$ is used to seek optimal controls u_1^* and u_2^*

$$\begin{cases} 0 = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial u_1} \\ 0 = \frac{\partial\mathcal{H}(t,x,u,\lambda)}{\partial u_2}. \end{cases} \quad (24)$$

Then, we obtain

$$u_1^* = \begin{cases} 0, & u_1 \le 0 \\ \frac{(\lambda_5 - \lambda_2)\iota_1 E_k}{c_1}, & 0 < u_1 < 1 \\ 1, & u_1 \ge 1 \end{cases}$$

$$u_2^* = \begin{cases} 0, & u_2 \le 0 \\ \frac{(\lambda_5 - \lambda_1)\iota_2 R_k}{c_2}, & 0 < u_2 < 1 \\ 1, & u_2 \ge 1. \end{cases}$$

In brief, the optimal control problem has a unique optimal solution $(S_k^*(t), E_k^*(t), H_k^*(t), C_k^*(t), I_k^*(t), R_k^*(t))$ associated with (u_1^*, u_2^*) on $[0, t_f]$ to maximize the objective function (8).

Appendix C

From (20)–(22), system (19) can be rewritten as the following linear state feedback control system:

$$\dot{x} = f_0(t, x, u) = f_1(t, x) + Bu_{\text{aux}} \tag{25}$$

where $u_{\text{aux}} = (u_{\text{aux}}^1, u_{\text{aux}}^2)^T = (\iota_1 E_k u_1, \iota_2 R_k u_2)^T$ is an auxiliary controller, $f_1(t, x) \leq Lx$, and

$$B = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{pmatrix}'.$$

For system (25), we design the auxiliary controller as

$$u_{\text{aux}} = Kx \tag{26}$$

where K is the control gain matrix, then the system is rewritten as follows:

$$\dot{x} = f_1(t, x) + BKx. \tag{27}$$

Define a Lyapunov function $\mathcal{V} = x^T P x > 0$, where P is a positive-definite matrix. It is noted that this Lyapunov function is continuously differentiable and radically unbounded. Then, there exists a class of $\mathcal{K}_\infty$ function $\kappa_i(\cdot), i = 1, 2$ such that the function $\mathcal{V}$ satisfies: $\kappa_1(\|z\|) \leq \mathcal{V}(t) \leq \kappa_2(\|z\|)$.

The derivative of the Lyapunov function along the system (25) under the controller (26), we have

$$\begin{aligned} \dot{\mathcal{V}} &= \dot{x}^T P x + x^T P \dot{x} \\ &= f_1^T P x + x^T K^T B^T P x + x^T P f_1 + x^T PBKx \\ &\quad + cx^T P x - cx^T P x \\ &\leq x^T L^T P x + x^T K^T B^T P x + x^T P L x \\ &\quad + x^T PBKx + cx^T P x - cx^T P x \\ &= x^T \Omega x - cx^T P x \end{aligned} \tag{28}$$

where $\Omega = L^T P + PL + K^T B^T P + PBK + cP$. By the third condition, $\Omega < 0$. Thus

$$\dot{\mathcal{V}} \leq -c\mathcal{V}(t). \tag{29}$$

Consequently, system (5) is exponential stable.

Appendix D

To obtain the recursive feasibility of the optimization problem $\mathcal{P}$, it is necessary to find control input sequences that satisfy (14d) and (14e). Actually, the inequality constraint (14d) is established if we assume the controller as the stabilizing feedback controller, that is, $u(t_{k+1}; t_{k+1}) = u_{\text{aux}}(t_{k+1})$. Note that the stabilizing feedback controller $u_{\text{aux}}(t_{k+1})$ is obtained by the controller at time instant t_{k+1}. The optimal input trajectory is obtained as $u^{*l}(s; t_k), s \in [t_k, t_k + T^p]$. At the time instant t_{k+1}, the feasible control law can be given by

$$u(s; t_{k+1}) = \begin{cases} u_{\text{aux}}(t_{k+1}), & s \in [t_{k+1} \\ & \quad t_{k+1} + \Delta] \\ u^{*l}(s; t_k), & s \in [t_{k+1} + \Delta \\ & \quad t_k + T^p] \\ u^{*l}(t_k + T^p; t_k), & s \in [t_k + T^p \\ & \quad t_{k+1} + T^p]. \end{cases} \tag{30}$$

Since the control input sequence is within the constraint (14d), Hence, the recursive feasibility is satisfied.

Note that Theorem 5 holds, we have $\dot{V}|_{u_{\text{aux}}(t_k)} \leq 0$. Using optimal control law u^{*l} to be formulated in the optimization problem $\mathcal{P}$ at each sampling instant, then $\dot{V}|_{u^{*l}(t_k)} \leq \dot{V}|_{u_{\text{aux}}(t_k)} \leq 0$.

The optimal control of knowledge transmission processes in complex networks via predictive feedback controller solved by Algorithm 1 is satisfied. Therefore, the stability of the closed-loop system under the receding horizon can be guaranteed. This completes the proof.

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