# Security Event-trigger Based Distributed Energy Management Of Cyber-Physical Isolated Power System With Considering Non-smooth Effects

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Abstract—Due to cyber-physical fusion and non-smooth characteristics of energy management, this paper proposes a security event-trigger based distributed approach to address these issues with developed smoothing technique. To tackle with non-convex and non-differentiable issue, a randomized gradient-free based successive convex approximation is developed to smooth economic objective function. Due to resilience ability against security issue, a security event-triggered mechanism based distributed energy management is proposed to optimize social welfare, which coordinately controls both power generators and load demand. The security event-triggered mechanism is designed to reduce power system security risks, and relieve communication burden caused by smoothing calculation, the convergence of proposed distributed algorithm is also properly proved. According to those obtained results on both IEEE 9-bus and IEEE 39-bus systems, it reveals that the proposed approach can achieve good convergence performance and have less security risks than other alternatives, which also proves that the proposed approach can be a viable and promising way for tackling with energy management issue of cyber-physical isolated power system.

*Index Terms*—isolated power system, distributed energy management, randomized gradient-free, event-triggered mechanism, communication burden.

#### I. INTRODUCTION

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ITH the integration of increasing renewable energy resources and information technologies, energy management of isolated power system can be a complex cyberphysical issue [1], which mainly aims to ensure power system security and maximize economic profit by keeping power balance between power generation and load demand while satisfying basic cyber-physical control limits [2]. In past decades, many optimization approaches have been proposed to tackle with energy management problems [3]–[7]. In order to optimize the operation of micro-grid, an energy management approach is proposed in literature [3] with considering the feature of solar power output and wind power output. In literature [5], a robust optimal control approach is designed for optimization dispatch of hybrid wind/photovoltaic/hydro/thermal power systems, which makes an ideal compromise between reliability and economy of system dispatch. In [7], energy management system based on determinism is proposed according to different functions and divided into two parts: the central energy management of the smart grid and the local energy management of the user. The central and local management systems exchange data and orders via communication networks. However, those above mentioned methods are centralized measurements, the centralized methods place strict requirements on the communication bandwidth, computational ability and a high level of connectivity and is prone to be easily threatened with a single point of failure. Instead of those centralized method, distributed approaches can be more appropriate for power system control due to its better scalability, reliability and resilience.

Afterwards, many scholars have proposed various of distributed optimization algorithms for cyber-physical energy management problem [8]–[13]. In literature [8], a new coordinated controller is proposed for active power optimization of multiple generators to calculate optimal active power generation reference for each generator in distributed network. Literature [10] develops three coordination laws for optimal energy generation and energy distribution on both physical flow layer and cyber communication layer. Literature [12] proposes a two-layer network and distributed control method to tackle with power dispatch problem with a top-layer communication network over a bottom-layer smart grid. Literature [13] studies the resilient coordinated output regulation problem of a class of uncertain nonlinear multi-agent systems under denial-ofservice (DoS) attacks. It is the first attempt to investigate the cooperative output regulation problem for nonlinear MASs under DoS attacks, and a novel distributed control scheme consisting of a resilient distributed observer and a distributed adaptive controller is proposed. While power-generation side and demand-side power dispatching are carried out separately in those above researches, it is more realistic to consider both of them with distributed energy integration. In [14], a distributed energy management of micro-grid is proposed to control both power generation and load demand to achieve social welfare maximization with consideration of transmission loss. In [15], a distributed energy management approach is proposed to maximize social welfare by controlling power output and system load with considering direct-current power flow. Literature [16] designs a distributed coordination optimization to tackle with social welfare maximization problem with taking into account time constraints of wind power and demands. However, those above distributed optimization methods require convex and differentiable conditions of cost function, while some actual cost functions can be corrugated curves with many non-convex and non-differentiable points. Though some existing literatures have tackled with some non-convex or non-differentiable problems [17][18], the issue with non-convex and non-differentiable problem is seldom simultaneously solved in existing literatures. In this paper, a randomized gradient-free based successive convex approximation technique is proposed to smooth the non-convex and non-differentiable function, which can convert the nonsmooth problem into a convex problem for facilitating convex optimization.

As it is known that event-driven mechanisms can effectively save network communication resources. For this reason, a Zeno-free event-triggered mechanism is designed for each agent to generate the asynchronous trigger time instants [19]. In this paper, a security event-triggered mechanism is designed in distributed optimization approach to enhance the resilience against security issue. In comparison to those existing literatures [20]-[23], they mainly focus on the event-trigger mechanisms to reduce communication burden during coordinated optimization to improve optimal control performance, but few designed mechanisms take power system reliability or security into consideration. Here, the proposed security eventtrigger mechanism is designed with considering not merely control performance but also isolated power system security, which can reduce communication burden as well as ensure the security of isolated power system. The main contributions of this paper can be summarized as follows:

- Instead of merely addressing non-convex or nondifferentiable characteristics of economic cost function in existing literatures, a randomized gradient-free technique is developed to address this problem on the basis of successive convex approximation method, which can convert a non-convex and non-differentiable issue into a convex optimization problem without missing extreme values.
- Those existing event-triggered mechanisms mainly focus on the communication burden issue, but they hardly involve the power system security into the event-triggered

mechanisms. Hence, an improved event-triggered mechanism based distributed optimization approach is proposed to achieve consistency with well designed security event-triggered mechanisms, which can enhance optimization performance as well as ensure power system security.

3) The convergence of the proposed security event-trigger based distributed energy management algorithm with successive randomized gradient-free(SE-DEMA-SRGF) is strictly proved, and its optimal control efficiency is also verified on both standard IEEE 9-bus and IEEE 39bus systems.

The rest of this paper is organized as follows: The problem formulation is described in Section II, the proposed smoothing technique is presented in section III. The proposed eventtriggered method is presented in section IV. The simulation results and conclusion are shown in section V and section VI respectively.



Fig. 1. The structure of cyber-physical system.

### II. PROBLEM FORMULATION OF CYBER-PHYSICAL ENERGY MANAGEMENT IN ISOLATED POWER SYSTEM

### A. Communication Network Model

An isolated power system consists of distributed generators, and demands, as shown in Fig. 1. It is supposed that an isolated power system with  $\mathcal{N}_G$  distributed generators and  $\mathcal{N}_D$ demand loads indexed by  $1, 2, \ldots, \mathcal{N}_G$  and  $\mathcal{N}_G + 1, \ldots, \mathcal{N}$ , respectively, where  $\mathcal{N} = \mathcal{N}_G + \mathcal{N}_D$ . A connected undirected graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  denotes the communication topology of the network, where  $\mathcal{V}$  represents the set of  $\mathcal{N}$  nodes and each undirected edge  $\{i, j\} \in \mathcal{E}$  represents a communication link between nodes i and j. The generator set and demand set are denoted as  $\mathcal{V}_G$  and  $\mathcal{V}_D$ , respectively, and  $\mathcal{V} = \mathcal{V}_G \cup \mathcal{V}_D$ . It can be considered that node j is a neighbor of node i if  $\{i, j\} \in \mathcal{E}$ . Besides, the weight of undirected edge between node i and node j is noted as  $a_{ij}$ , where  $0 \le a_{ij} \le 1$  (there is no link between node *i* and node *j* especially when  $a_{ij} = 0$ . The matrix of elements  $a_{ij}$  is named as adjacent matrix A. For clarity, it is assumed that the communication network is fixed and connected.

### B. Social Welfare Maximization

In the isolated power system, all power generators and load demands must be well controlled to maximize total social welfare, which can be described as the summation of power generation welfare and load demand welfare. Due to power balance requirement, the social welfare maximization model can be expressed as follows:

$$\max J = \sum_{i \in \mathcal{V}_G} W_{G,i}(P_i, \rho) + \sum_{j \in \mathcal{V}_D} W_{D,j}(P_j, \rho)$$
(1a)

s.t. 
$$\sum_{i \in \mathcal{V}_G} P_i = \sum_{j \in \mathcal{V}_D} P_j \tag{1b}$$

$$P_i^{\min} \le P_i \le P_i^{\max}, i \in \mathcal{V}_G \tag{1c}$$

$$P_j^{\min} \le P_j \le P_j^{\max}, j \in \mathcal{V}_D$$
 (1d)

where J represents social welfare function,  $W_{G,i}(\cdot)$  and  $W_{D,j}(\cdot)$  denote the power generation welfare and demand welfare respectively,  $P_i$  and  $P_j$  represent power output of  $i \in \mathcal{V}_G$  th power generator and load demand of  $j \in \mathcal{V}_D$ th load demand,  $\rho$  denotes electricity price.  $P_i^{\min}$  and  $P_i^{\max}$  represent the lower bound and upper bound of *i*th power generator,  $P_j^{\min}$  and  $P_j^{\max}$  denote the minimum and maximum load demand of *j* th load demand.

1) Generation Welfare: The generation welfare function of each power generator can be expressed as follows:

$$W_{G,i}(P_i,\rho) = \rho * P_i - C_i(P_i), \ i \in \mathcal{V}_G$$
(2)

where  $C_i(\cdot)$  represents power generation cost of *i*th power generator. In general, the cost function of distributed power generation can be formulated with non-differentiable and non-convex effects as follows:

$$C_{i}(P_{i}) = \left[a_{i}P_{i}^{2} + b_{i}P_{i} + c_{i} + \left|d_{i}\sin\left\{e_{i}\left(P_{i}^{\min} - P_{i}\right)\right\}\right|\right]$$
(3)

where  $a_i$ ,  $b_i$ ,  $c_i$ ,  $d_i$  and  $e_i$  are the cost parameters.

*Remark 1:* As it is presented in (3) that cost function is both non-convex and non-differentiable, some techniques are required to polish it especially on those non-differentiable points, which can be denoted as:

$$\chi_i \triangleq \left\{ P_i \mid P_i = P_i^{\min} + \frac{k\pi}{e_i}, k \in \mathbb{Z} \right\}$$
(4)

2) *Demand Welfare:* The demand welfare function of each load can be expressed as follows:

$$W_{D,j}(P_j,\rho) = -\rho * P_j + U_j(P_j), \ j \in \mathcal{V}_D$$
(5)

where  $U_j(\cdot)$  represents the utility function of *j*th load demand. The utility function on the demand side denoted by  $U_j(P_j)$  describes the satisfaction level of the load power  $P_j$ ,  $j \in \mathcal{V}_D$ . Usually, the utility function  $U_j(P_j)$  satisfies the following properties [24].

- 1) The first-order derivative of the utility  $U_j(P_j)$  is non-negative.
- 2) The second-order derivative of the utility  $U_j(P_j)$  is non-positive, which means the utility function of load demand will increase as power consumption increases, and the increasing speed will slow down.

## 3) $U_j(0) = 0$ , indicating that the satisfaction on the demand side is zero with no load demand.

Here, the utility function  $U_j(P_j)$  can be specifically described as follows [25]:

$$U_{j}(P_{j}) = \left\{ \begin{array}{cc} -\kappa_{j}P_{j}^{2} + \varpi_{j}P_{j}, & P_{j} \leq \frac{\varpi_{j}}{2\kappa_{j}} \\ \frac{\varpi_{j}^{2}}{4\kappa_{j}}, & P_{j} > \frac{\varpi_{j}}{2\kappa_{j}} \end{array} \right\}$$
(6)

where  $\kappa_j > 0$  and  $\varpi_j$  are the parameters for the *j*th load. Combined with Lagrangian multiplier, the social welfare maximization model can be converted into minimization problem with satisfying power balance and basic limits as follows:

$$\begin{cases}
\min L(P_i, P_j, \lambda) = \sum_{i \in \mathcal{V}_G} C_i(P_i) - \sum_{j \in \mathcal{V}_D} U_j(P_j) \\
+\lambda (\sum_{j \in \mathcal{V}_D} P_j - \sum_{i \in \mathcal{V}_G} P_i) \\
P_i^{\min} \leq P_i \leq P_i^{\max}, i \in \mathcal{V}_G \\
P_j^{\min} \leq P_j \leq P_j^{\max}, j \in \mathcal{V}_D
\end{cases}$$
(7)

where  $\lambda \geq 0$  represents the Lagrangian multiplier. Due to nonconvex and non-differentiable characteristics of cost function  $C_i(\cdot)$ , the equivalent model can not be optimized directly by any convex optimization approaches before polishing those non-differentiable points and addressing non-convex characteristics.

### III. RANDOMIZED GRADIENT-FREE BASED SUCCESSIVE CONVEX APPROXIMATION METHOD

In this section, a randomized gradient-free based successive convex approximation method is proposed to tackled with an optimization problem which can be generally considered as follows:

$$\begin{array}{ll} \min_{\boldsymbol{x}} & C(\boldsymbol{x}) \\ \text{s.t.} & \boldsymbol{x} \in \mathcal{X} \end{array} \tag{8}$$

where  $\mathcal{X} \subseteq \mathbb{R}^n$  represents the general constraint set which is convex and closed, C(x) is Lipschitz continuous with Lipschitz constant  $L_C > 0$  and C(x) is a non-convex and nondifferentiable objective function, which can not be optimized directly by any convex optimization approaches. To address this problem, a Gaussian-smoothed technique is involved to transform original function C(x) into a convex and differentiable function. The optimization problem after implementing Gaussian-smoothed technique can be described as:

$$\begin{array}{ll}
\min & C_{\mu}(x) \\
\text{s.t.} & x \in \mathcal{X} \\
\end{array} \tag{9}$$

where  $C_{\mu}(x) = \frac{1}{\omega} \int_{\mathbb{R}^n} C(x + \mu\nu) e^{-\frac{1}{2} \|\nu\|^2} d\nu$  is the Gaussian approximation function of C(x),  $\omega = \int_{\mathbb{R}^n} e^{-\frac{1}{2} \|\nu\|^2} d\nu = (2\pi)^{\frac{n}{2}}$  is a smoothing parameter of function  $C_{\mu}(x)$ ,  $\mu \ge 0$  is a small control parameter,  $\nu$  is a normally Gaussian random variable. Then, the gradient of  $C_{\mu}(x)$  in (9) can be obtained as follows:

$$\nabla C_{\mu}(x) = \int_{\mathbb{R}^n} \frac{g_{\mu}(x)}{(2\pi)^{\frac{n}{2}}} \nu \mathrm{e}^{-\frac{1}{2} \|\nu\|^2} d\nu \tag{10}$$

where  $g_{\mu}(x)$  represents the two-sided gradient-free oracle, which can be described as:

$$g_{\mu}(x) = \frac{C(x + \mu\nu) - C(x - \mu\nu)}{2\mu}\nu$$
 (11)

To analyze the equivalent gradient of C(x) at nondifferentiable points in (8), the following lemma is provided to present some crucial properties of the function  $C_{\mu}(x)$  and the random gradient-free oracle  $g_{\mu}(x)$ .

Lemma 1 ([26]): For problem (8) and (9), it has the following characteristics:

1) If C(x) is non-differentiable, then  $C_{\mu}(x)$  is differentiable, and it satifies

$$C(x) \le C_{\mu}(x) \le C(x) + \sqrt{n\mu L_C}$$

2) The gradient  $\nabla C_{\mu}(x)$  satisfies

$$\nabla C_{\mu}\left(x\right) = \mathbb{E}\left[g_{\mu}\left(x\right)\right]$$

3) The random gradient-free oracle  $g_{\mu}(x)$  satisfies

$$\mathbb{E}\left[\left\|g_{\mu}(x)\right\|\right] \le nL_{C}$$
$$\mathbb{E}\left[\left\|g_{\mu}(x)\right\|^{2}\right] \le (n+4)^{2}L_{C}^{2}$$

*Remark 2:* From Lemma 1, it can be noticed that  $C_{\mu}(x)$  can substitute C(x) when  $\mu$  is a sufficiently small parameter. Thus, the solution of optimization problem (9) can approximates that of problem (8), and the gradient of  $C_{\mu}(x)$  at non-differentiable points in (8) can be calculated as follows:

$$\nabla C_{\mu}(x) = \mathbb{E}\left[\frac{C\left(x+\mu\nu\right) - C(x-\mu\nu)}{2\mu}\nu\right]$$
(12)

After smoothing the non-differentiable points of C(x), a successive convex approximation technique is used to address the non-convex objective function  $C_{\mu}(x)$  in the reformulated optimization problem (9). In the formula (9), a convex surrogate function  $ilde{C}_{\mu}(x;x^k)$  can be used to replace the non-convex function  $C_{\mu}(x)$  at each iteration k. Thus, the problem (9) at iteration k can be reformulated as follows:

min 
$$\tilde{C}_{\mu}(x; x^k)$$
  
s.t.  $x \in \mathcal{X}$  (13)

where  $\tilde{C}_{\mu}(x;x^k)$  represents the convex surrogate function of  $C_{\mu}(x)$  at current iteration  $x^k$ , which is a convex function that closely approximates the original function in a neighborhood of  $x^k$  this paper use a series of convex surrogate functions to iteratively approximate the original objective function and obtain the optimal solution. In addition, the surrogate function also satisfies some conditions, which can be presented in the following Assumption.

Assumption 1 ([27]): The key assumptions on the choice of the surrogate function are described as follows:

- C
   <sup>˜</sup><sub>μ</sub>(x; x<sup>k</sup>) is strongly convex in x, ∀x<sup>k</sup> ∈ X.
   C
   <sup>˜</sup><sub>μ</sub>(x; x<sup>k</sup>) is continuously differentiable in terms of x,  $\forall x^k \in \mathcal{X}.$
- 3) Function value consistency:  $\tilde{C}_{\mu}(x^k; x^k) = C_{\mu}(x^k), \quad \forall x^k \in \mathcal{X}.$

- 4) Gradient consistency:  $\nabla \tilde{C}_{\mu}(x^k; x^k) = \nabla C_{\mu}(x^k)$ , for any x<sup>k</sup> ∈ X at which C<sub>μ</sub>(x) is differentiable.
  5) Upper-bound: C̃<sub>μ</sub>(x; x<sup>k</sup>) ≥ C<sub>μ</sub>(x), ∀(x, x<sup>k</sup>) ∈ X.

Here, a strongly convex surrogate function is taken to approximate the non-convex function, it can be described as:

$$\tilde{C}_{\mu}(x;x^{k}) = C_{\mu}(x^{k}) + \nabla C_{\mu}(x^{k}) \left(x - x^{k}\right) + \beta (x - x^{k})^{2}$$
(14)

where  $\beta > 0$  is a sufficiently large constant to satisfy the item 5) of Assumption 1. The main idea of proposed smoothing technique is to approximate non-differentiable and non-convex function with convex surrogate functions at those non-differentiable and non-convex points, which have been shown in Fig. 2, and the pseudo code of proposed randomized gradient-free based successive convex approximation technique is presented in Algorithm 1.

Algorithm 1: Randomized gradient-free based successive convex approximation technique

Input:  $C(x), \mathcal{X}, \epsilon$ Output:  $x^*$ 

- 1 Initialization: Select  $x^0 \in \mathcal{X}$  and choose a step size  $\delta \in (0, 1]$  at iteration k = 0 randomly.
- 2 while  $\bar{x}(x^k) x^k < \epsilon$  do
- **1.**Select the surrogate function  $\tilde{C}_{\mu}(x; x^k)$  based on 3 Assumption 1.

4 **2.**Compute 
$$\bar{x}(x^k) = \underset{x \in \mathcal{X}}{\operatorname{arg\,min}} \hat{C}_{\mu}(x; x^k).$$

**3.**Update  $x^{k+1}$  according to:

$$x^{k+1} = x^k + \delta \left( \bar{x}(x^k) - x^k \right)$$

6 | 4. Set 
$$k = k + 1$$
  
7 end

5



Fig. 2. The successive convex approximation of  $C_{\mu}(x)$ .

### IV. SECURITY EVENT-TRIGGER BASED DISTRIBUTED ENERGY MANAGEMENT APPROACH

Due to power system security and communication burden issue, a security event-trigger based distributed energy management approach is proposed to address these problems. On the basis of consensus based energy management algorithm (CEMA), a security event-triggered mechanism is designed with considering both power supply deviation and communication burden caused by above smoothing technique, which can ensure the isolated power system security as well as reduce communication complexity.

# A. Distributed optimization with smoothing approximating technique

Combined with above polishing techniques on non-convex and non-differentiable characteristics, local cost function  $C_i(\cdot)$ can be converted into a convex and differentiable equivalent which can be described as  $C_{i,\mu^i}(\cdot)$ . Then the derivation of local objective function can be calculated as follows:

$$\begin{cases} \frac{\partial L(\lambda)}{\partial P_i} = \nabla C_{i,\mu^i}(P_i) - \lambda \\ \frac{\partial L(\lambda)}{\partial P_j} = -\nabla U_j(P_j) + \lambda \end{cases}$$
(15)

Define  $\lambda_i$  as the increment cost of each power generator which can be denoted as:

$$\lambda_i = \nabla C_{i,\mu^i}(P_i), i \in \mathcal{V}_G \tag{16}$$

Define  $\lambda_j$  as the increment utility of demand load which can be denoted as:

$$\lambda_{j} = \nabla U_{j} (P_{j})$$

$$= \begin{cases} -2\kappa_{j}P_{j} + \varpi_{j}, P_{j} \leq \frac{\varpi_{j}}{2\kappa_{j}} \text{ and } j \in \mathcal{V}_{D} \\ 0, P_{j} > \frac{\varpi_{j}}{2\kappa_{j}} \text{ and } j \in \mathcal{V}_{D} \end{cases}$$
(17)

For converting centralized problem (7) into a distributed version, it can be decomposed into  $\mathcal{N}$  sub-optimization problems combined with (16) and (17) considering power balance requirement between power generation and load demand as follows:

$$\min \quad f_i(P_i, \lambda_i) = C_{i,\mu^i}(P_i) - \lambda_i P_i$$
s.t. 
$$P_i^{\min} \le P_i \le P_i^{\max}, i \in \mathcal{V}_G$$
(18)

min 
$$f_j(P_j, \lambda_j) = \lambda_j P_j - U_j(P_j)$$
  
s.t.  $P_j^{\min} \le P_j \le P_j^{\max}, j \in \mathcal{V}_D$  (19)

If each variable  $P_l$ ,  $l \in \mathcal{V}$  minimizes above two optimization problems and the power balance constraint (1b) is also satisfied, then all  $\lambda$  can reach consensus which means the optimization problem (1) has been solved. Here, a distributed optimization with gradient descent can be described as:

$$\lambda_i^{k+1} = \lambda_i^k + \sum_{j \in \mathcal{V}} a_{ij} (\lambda_j^k - \lambda_i^k) - \varphi^k \nabla f_i(\lambda_i^k)$$
(20)

where  $a_{ij}$  denotes the (i, j)th nonnegative element of doubly stochastic weighting matrix A, it means the weight information between node i and its neighbor j,  $\varphi^k$  represents the step size, and  $\nabla f_i(\lambda_i^k)$  is the derivation of objective function  $f_i(\lambda_i^k)$ . Generally, traditional distributed energy management approach can achieve the optimal scheme with above iteration algorithm, the periodical communication based method has low optimization efficiency and also lacks of considering power system security.

# B. Security event-triggered based distributed optimization method

In order to reduce both security risk and communication burden, a security event-triggered mechanism is designed here. Combined with event-triggered communication and power supply security, the iterative updating formula of increment cost of each power generator or the increment utility of demand load can be presented as follows:

$$\lambda_{i}^{k+1} = \lambda_{i}^{k} + \sum_{j \in \mathcal{V}} a_{ij} (\tilde{\lambda}_{ji}^{k} - \lambda_{i}^{k}) - \varphi^{k} \frac{\partial f_{i}(P_{i}, \lambda_{i})}{\partial \lambda_{i}^{k}} + \gamma \zeta_{i}^{k}, i \in \mathcal{V}$$

$$(21)$$

where  $\tilde{\lambda}_{ji}^k$  represents the state that node j sends to its neighbor node i at the last triggering time, which can be expressed as:

$$\tilde{\lambda}_{ji}^{k} = \begin{cases} \lambda_{j}^{k}, & k \in \mathbb{k} = \mathbb{k}_{Opt} \cup \mathbb{k}_{Sec} \\ \tilde{\lambda}_{ji}^{k-1}, & \text{otherwise} \end{cases}$$
(22)

where k denotes the set of all event-triggering times. Due to the electronic protection device, the supply security must be very strictly satisfied within the feasible domain at each iteration. Hence, the event-triggering time set consists of optimization performance event-trigger time set  $k_{Opt}$  and security deviation event-trigger time set  $k_{Sec}$  as:

$$\begin{cases} \mathbb{k}_{Opt} = \{k || \lambda_j^k - \hat{\lambda}_{ji}^k| \ge E_i^k, \forall j \in \mathcal{V} \} \\ \mathbb{k}_{Sec} = \{k || P_j^k + \sum_{s \in \mathcal{V}_{j,G}} P_s^k - \sum_{s \in \mathcal{V}_{j,D}} P_s^k| \ge \epsilon_j^{sec}, \forall j \in \mathcal{V}_G \} \\ or = \{k || \sum_{s \in \mathcal{V}_{j,G}} P_s^k - P_j^k - \sum_{s \in \mathcal{V}_{j,D}} P_s^k| \ge \epsilon_j^{sec}, \forall j \in \mathcal{V}_D \} \end{cases}$$

$$(23)$$

where  $E_i^k$  and  $\epsilon_i^{sec}$  denote optimization performance and system security event-trigger thresholds,  $\mathcal{V}_{j,G}$  and  $\mathcal{V}_{j,D}$  represent the generator and demand neighbor set of *j*th node.  $\gamma$  and  $\zeta_i^k$  in (21) denotes the proportional feedback gain which satisfies  $0 < \gamma < 1$  and the local generation and load deviation of node *i* at time *k*, respectively. Generally, arbitrary agent *j* exchanges information with other agent without triggering events as:

$$\begin{cases} |\lambda_{j}^{k} - \tilde{\lambda}_{ji}^{k}| \leq E_{i}^{k} \quad \forall i \in \mathcal{V}, k \in \mathbb{N} \\ |P_{j}^{k} + \sum_{s \in \mathcal{V}_{j,G}} P_{s}^{k} - \sum_{s \in \mathcal{V}_{j,D}} P_{s}^{k}| \leq \epsilon_{j}^{sec}, j \in \mathcal{V}_{G} \\ |\sum_{s \in \mathcal{V}_{j,G}} P_{s}^{k} - P_{j}^{k} - \sum_{s \in \mathcal{V}_{j,D}} P_{s}^{k}| \leq \epsilon_{j}^{sec}, \forall j \in \mathcal{V}_{D} \end{cases}$$
(24)

Meanwhile, to ensure the convergence of (21), the following assumptions must be satisfied:

Assumption 2 ([28]): If  $\{i, j\} \in \mathcal{E}$ ,  $a_{ij} > \tau$ ,  $a_{ii} = 1 - \sum_{j \in \mathcal{V} \setminus \{i\}} a_{ij} > \tau$ , where  $\tau \in (0, 1)$ . If  $\{i, j\} \notin \mathcal{E}$ ,  $a_{ij} = 0$ . Since A is a doubly stochastic weighting matrix, it satisfies  $a_{ij} = a_{ji}$  and  $\sum_{i=1}^{\mathcal{N}} a_{ij} = 1$ ,  $\sum_{j=1}^{\mathcal{N}} a_{ij} = 1$ ,  $\forall i, j \in \mathcal{V}$ .

Assumption 3:  $\varphi^k > 0$  is a step size that decays exponentially and satisfies

$$\sum_{k=0}^{\infty} \varphi^k = \infty, \quad \sum_{k=0}^{\infty} (\varphi^k)^2 < \infty.$$

Assumption 4:  $E_i^k$ ,  $i \in \mathcal{V}$  satisfies the following properties.

$$E_i^k \le E^k$$
,  $\lim_{k \to \infty} E^k = 0$ ,  $\sum_{k=0}^{\infty} (E^k)^2 < \infty$ .

*Remark 3:* Assumption 2, 3, and 4 respectively indicate how the weight matrix A, iteration step and event-trigger threshold are selected, which can be convenient to ultimately converge to the optimal value. Since the network topology is a connected graph, assumption 2 is reasonable. The common form of iteration step size and event-trigger threshold is  $\frac{c}{(k+a)^b}$  where a, c are positive constants and  $0.5 < b \leq 1$ , in which assumptions 3 and 4 are also reasonable.

Moreover, the generation power  $P_i^k$ ,  $i \in \mathcal{V}_G$  can be updated according to the following iteration.

$$P_i^{k+1} = \underset{P_i \in \mathcal{P}_i}{\operatorname{arg\,min}} [C_{i,\mu^i}(P_i) - \lambda_i^{k+1} P_i], \quad i \in \mathcal{V}_G$$
(25)

Since  $C_{i,\mu^i}(P_i)$  is non-convex, (25) can be tackled with Algorithm 1 in Section III. Then, the load demand  $P_j^k$ ,  $j \in \mathcal{V}_D$ can be updated according to the following iteration.

$$P_j^{k+1} = \underset{\substack{P_j^{\min} \le P_j \le P_j^{\max}}}{\operatorname{arg\,min}} [\lambda_j^{k+1} P_j - U_j(P_j)], \quad j \in \mathcal{V}_D \quad (26)$$

The optimal solution of (26) can be calculated as:

$$P_{j}^{k+1} = \begin{cases} \frac{\varpi_{j} - \lambda_{j}^{k+1}}{2\kappa_{j}}, P_{j}^{\min} \leq \frac{\varpi_{j} - \lambda_{j}^{k+1}}{2\kappa_{j}} \leq P_{j}^{\max} \\ P_{j}^{\min}, P_{j}^{\min} > \frac{\varpi_{j} - \lambda_{j}^{k+1}}{2\kappa_{j}} \\ P_{j}^{\max}, P_{j}^{\max} < \frac{\varpi_{j} - \lambda_{j}^{k+1}}{2\kappa_{j}} \end{cases}$$
(27)

In order to balance power generation and load demand, the total deviation requires to converge. Then, the control parameter  $\zeta_i^{k+1}$   $(i \in \mathcal{V}_G)$  and  $\zeta_i^{k+1}$   $(j \in \mathcal{V}_D)$  of local generation and load deviation of node *i* and node *j* at step *k* can be deduced as follows respectively.

$$\begin{cases} \zeta_{i}^{k+1} = \sum_{s \in \mathcal{V}} a_{is} \zeta_{s}^{k} + (P_{i}^{k} - P_{i}^{k+1}), i \in \mathcal{V}_{G} \\ \zeta_{j}^{k+1} = \sum_{s \in \mathcal{V}} a_{js} \zeta_{s}^{k} + (P_{j}^{k+1} - P_{j}^{k}), j \in \mathcal{V}_{D} \end{cases}$$
(28)

where  $a_{is}$  and  $a_{js}$  also denote nonnegative elements of doubly stochastic weighting matrix A. The main procedures of SE-DEMA-SRGF have been presented as pseudo code in Algorithm 2.

#### C. The convergence and optimality analysis

For further analysis on convergence and optimality of proposed SE-DEMA-SRGF, Theorem 1 is presented as follows to ensure the convergence and optimality of algorithm 2.

*Theorem 1:* Suppose above mentioned assumptions and conditions hold, then it has following properties as:

For all i ∈ V, lim<sub>k→∞</sub> ||λ<sub>i</sub><sup>k</sup> − λ̄<sup>k</sup>|| = 0, where λ̄<sup>k</sup> is the average value of all nodes i, i ∈ V.

2) 
$$\lim_{k \to \infty} \left[ \sum_{j \in \mathcal{V}_D} P_j^k - \sum_{i \in \mathcal{V}_G} P_i^k \right] = 0$$

3) For all  $i \in \mathcal{V}$ ,  $\lim_{k\to\infty} P_i^k = P_i^*$ , where  $P_i^*$  is the optimal power of each node  $i \in \mathcal{V}$ .

*Proof*: The following crucial Lemmas involving the proof of Theorem 1 are presented as follows:

### Algorithm 2: The pseudo of the SE-DEMA-SRGF

**Input:** A,  $C_i(P_i)$ ,  $i \in \mathcal{V}_G$ ,  $U_j(P_j), j \in \mathcal{V}_D$ ,  $\epsilon_a$  and  $\epsilon_b$ **Output:**  $\lambda_i(k)$ ,  $P_i(k)$ ,  $i \in \mathcal{V}$ 

1 **Initialization**: Set initial value  $\lambda_i^0$  and  $\lambda_j^0$  as:

$$\begin{cases} \lambda_i^0 = \mathbb{E}[\frac{C_i(P_i^{\min} + \mu_i \nu) - C_i(P_i^{\min} - \mu_i \nu)}{2\mu_i}\nu], i \in \mathcal{V}_G\\ \lambda_j^0 = \nabla U_j(P_j^{\max}), j \in \mathcal{V}_D \end{cases}$$
(29)

 $\begin{array}{l|l} \text{Set } P_i^0 = 0 \text{ and } \zeta_i^0 = 0, \ i \in \mathcal{V}.\\ \text{2 while } \left|\lambda_i^k - \lambda_i^{k-1}\right| < \epsilon_a \ or \ \left|\zeta_i^k\right| < \epsilon_b \ \text{do}\\ \text{3} & \begin{array}{|l|l|l|l} \textbf{1. Update } \lambda_i^{k+1} \ \text{according to } (21).\\ \text{4} & \begin{array}{|l|l|l|l|l|l|l} \textbf{2. Update } P_i^{k+1} \ \text{according to } (25) \ \text{solved by}\\ \text{Algorithm 1 and and } P_j^{k+1} \ \text{according to } (27).\\ \text{5} & \begin{array}{|l|l|l|l|l|l} \textbf{3. Update } \zeta_i^{k+1} \ \text{and } \zeta_j^{k+1} \ \text{according to } (28).\\ \text{6 end} \end{array}$ 

*Lemma 2 ( [29]):* Combined with Assumption 2, for all  $i, j \in \mathcal{V}$  and  $k, t \in \mathbb{N}$  with  $k \geq t$ , it can obtain  $\left| \left[ A^{k-t+1} \right]_{ij} - \frac{1}{\mathcal{N}} \right| \leq \frac{1}{\eta} \eta^{k-t}$ , where  $\eta = 1 - \frac{\tau}{4\mathcal{N}^2}$ .

*Lemma 3* ([30]): Suppose  $\{\vartheta^k\}$  is a positive scalar sequence.

1) If 
$$\lim_{k\to\infty} \vartheta^k = 0, \ 0 < \eta < 1$$
, then  
$$\lim_{k\to\infty} \sum_{t=0}^k \eta^{k-t} \vartheta^t = 0$$

2) If  $\sum_{k=0}^{\infty} \vartheta^k < \infty$ ,  $0 < \eta < 1$ , then

$$\sum_{k=0}^{\infty} \sum_{t=0}^{k} \eta^{k-t} \vartheta^t < \infty$$

*Proof 1:* It mainly focuses on the convergence analysis of  $\lambda_i, i \in \mathcal{V}$  in formula (21) referring to [28]. Firstly, the event-trigger deviation term  $e_{ii}^k$  can be described as:

$$e_{ji}^k = \tilde{\lambda}_{ji}^k - \lambda_j^k \tag{30}$$

Then, formula (21) can be rewritten as follows:

$$\lambda_i^{k+1} = \sum_{j \in \mathcal{V}} a_{ij} \lambda_j^k + \gamma \zeta_i^k + \sum_{j \in \mathcal{V}} a_{ij} e_{ji}^k - \varphi^k \frac{\partial f_i(P_i, \lambda_i)}{\partial \lambda_i^k}, i \in \mathcal{V}.$$
(31)

By referring to the handling skills of [31] and [32], both Assumption 3 and Assumption 4 hold, it is assumed that the third and fourth terms on the right of equation (31) can be ignored after iteration  $\bar{k}$ . Then, the update rule of  $\lambda_i, i \in \mathcal{V}$  is similar to [14], then it can obtain that  $\lim_{k\to\infty} \zeta_i^k = 0$  by referring to the proof of [14]. Let  $\hat{e}_{ij}^k = \sum_{j\in\mathcal{V}} a_{ij}(e_{ji}^k - e_{ij}^k) + \gamma \zeta_i^k$ , therefore, the formula (21) can be rewritten as follows:

$$\lambda_i^{k+1} = \sum_{j \in \mathcal{V}} a_{ij} \lambda_j^k + \hat{e}_{ij}^k - \varphi^k \frac{\partial f_i(P_i, \lambda_i)}{\partial \lambda_i^k}, i \in \mathcal{V}.$$
(32)

Combined with the average vector  $\bar{\lambda}^k = \frac{1}{N} \sum_{i=1}^{N} \lambda_i^k$ , then,

 $\bar{\lambda}^{k+1}$  can be described as:

$$\bar{\lambda}^{k+1} = \bar{\lambda}^k + \frac{1}{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} \hat{e}_{ji}^k - \frac{\varphi^k}{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} \frac{\partial f_j(P_j, \lambda_j)}{\partial \lambda_j^k}$$
(33)

Define  $R_i^k = \hat{e}_{ij}^k - \varphi^k \frac{\partial f_i(P_i)}{\partial \lambda_i^k}$  as a consensus interference. According to formula (24), (30) and Assumption 4, it can obtain as:

$$\left\|e_{ij}^k\right\| \le E^k \quad \forall i \in \mathcal{V}, k \in \mathbb{N}$$
(34)

Since  $\lim_{k\to\infty} \zeta_i^k = 0$ ,  $\zeta_i^k$  is bounded and it denotes  $\zeta_i^{k,\max}$  as the upper bound of  $\zeta_i^k$ ,  $i \in \mathcal{V}$ , and  $\lim_{k\to\infty} \zeta_i^{k,\max} = 0$ . Then, it satisfies  $\|\hat{e}_{ij}^k\| \leq 2E^k + \gamma \zeta_i^{k,\max}$ . Thus, it can obtain:

$$\left\|R_{i}^{k}\right\| \leq 2E^{k} + \gamma \zeta_{i}^{k,\max} + P_{i}^{\max}\varphi^{k}, i \in \mathcal{V}$$
(35)

Let  $R_i^{k,\max} = 2E^k + \gamma \zeta_i^{k,\max} + P_i^{\max} \varphi^k$ . Combined with basic inequality, it can obtain:

$$(R_i^{k,\max})^2 \le 8(E^k)^2 + 4\gamma^2(\zeta_i^{k,\max})^2 + 4(P_i^{\max})^2(\varphi^k)^2$$
(36)

According to Assumption 3 and Assumption 4,  $R_i^{k,\max}$  satisfies the following:

$$\lim_{k \to \infty} R_i^{k,\max} = 0, \quad \sum_{k=0}^{\infty} (R_i^{k,\max})^2 < \infty$$
(37)

By introducing  $R_i^k$ , formula (32) can be rewritten as follows:

$$\lambda_i^{k+1} = \sum_{j \in \mathcal{V}} a_{ij} \lambda_j^k + R_i^k, i \in \mathcal{V}.$$
(38)

Then, formula (33) can be rewritten as follows:

$$\bar{\lambda}^{k+1} = \bar{\lambda}^k + \frac{1}{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} R_j^k, i \in \mathcal{V}.$$
(39)

It utilizes doubly stochastic matrix A to rewrite formula (38) and (39) into the other two forms as follows:

$$\lambda_i^{k+1} = \sum_{j=1}^{\mathcal{N}} [A^{k+1}]_{ij} \lambda_j^0 + \sum_{t=0}^{k-1} \sum_{j=1}^{\mathcal{N}} [A^{k-t}]_{ij} R_j^t + R_i^k \quad (40)$$

$$\bar{\lambda}^{k+1} = \bar{\lambda}(0) + \frac{1}{N} \sum_{t=0}^{k-1} \sum_{j=1}^{N} R_j^t + \frac{1}{N} \sum_{j=1}^{N} R_j^k$$
(41)

Furthermore, it can make a difference between above two equations and take the Euclidean norm as follows:

$$\begin{aligned} \left\| \lambda_{i}^{k+1} - \bar{\lambda}^{k+1} \right\| \\ &\leq \sum_{j=1}^{\mathcal{N}} \left| \left[ A^{k+1} \right]_{ij} - \frac{1}{\mathcal{N}} \right| \left\| \lambda_{j}^{0} \right\| + \frac{1}{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} \left\| R_{j}^{k} \right\| \\ &+ \sum_{t=0}^{k-1} \sum_{j=1}^{\mathcal{N}} \left| \left[ A^{k-t} \right]_{ij} - \frac{1}{\mathcal{N}} \right| \left\| R_{j}^{t} \right\| + \left\| R_{i}^{k} \right\| \end{aligned}$$
(42)

According to Lemma 2 and formula (35), it can obtain:

$$\begin{aligned} \|\lambda_{i}^{k+1} - \bar{\lambda}^{k+1}\| &\leq \eta^{k-1} \sum_{j=1}^{N} \|\lambda_{j}^{0}\| + 2R_{i}^{k,\max} \\ &+ \frac{\mathcal{N}}{\eta} \sum_{t=0}^{k-1} \eta^{k-t-1} R_{i}^{t,\max} \end{aligned}$$
(43)

Since  $\eta \in (0, 1)$ , it can obtain  $\lim_{k\to\infty} \eta^k = 0$ . According to formula (37), take the limit on both sides of formula (43) at the same time, it can obtain  $\limsup_{k\to\infty} \|\lambda_i^{k+1} - \bar{\lambda}^{k+1}\| \leq \frac{N}{\eta} \limsup_{k\to\infty} \sum_{t=0}^{k-1} \eta^{k-t-1} R_i^{t,\max}$ . Therefore, according to Lemma 3, it can be concluded that  $\lim_{k\to\infty} \|\lambda_i^k - \bar{\lambda}^k\| = 0$ . The remaining part of proof about Theorem 1 can be referred to [14] and [27].

#### V. CASE STUDY

For verifying performance of proposed method, it is implemented on both IEEE 9-bus and IEEE 39-bus systems. IEEE 9-bus system consists of 3 generators and 6 loads, the parameters of power generators and demand loads are shown in Table I and Table II, and the communication network of generators and loads is depicted by the red lines in Fig. 3. IEEE 39-bus system consists of 10 generators and 18 loads, the communication network of generators and loads is depicted by the red lines in Fig. 4, those related data details about generators and loads can be referred in [14]. It is worth noting that both in the IEEE 9-bus and IEEE 39-bus systems, generation units are considered as thermal units with nonconvex generation cost functions.



Fig. 3. IEEE 9-bus test system.

# A. The performance of SE-DEMA-SRGF on IEEE 9-bus system

The convergence curve of  $\lambda_i^k$  for each bus is shown in Fig. 5, and it can achieve the optimal price  $\lambda^* = 6.84$  (it can be considered as electricity price  $\rho$  after calculation) within 50 iterations. In Fig. 6, the deviation parameter  $\zeta_i$  of each bus is presented, it can be seen that local mismatch power

Generator	$a_i(\text{h})$	$b_i$ (\$/MWh)	$c_i \left( \text{\$/MW}^2 h \right)$	$d_i(\text{h})$	$e_i(\mathrm{rad}/\mathrm{MW})$	$P_i^{\min}(MW)$	$P_i^{\max}(MW)$
1	0.0064	5.56	30	100	0.062	60	339.69
2	0.0056	4.32	25	120	0.06	25	479.1
3	0.0072	6.6	25	140	0.058	28	290.4



Fig. 4. IEEE 39-bus test system.

TABLE II PARAMETERS OF LOADS

Load	$\varpi_j$ (\$/MWh)	$\kappa_j(\text{h})$	$P_j^{\min}(MW)$	$P_j^{\max}(MW)$
4	18.43	0.0545	40	306.31
5	13.17	0.0877	35	593.8
6	15.46	0.0547	29	137.19
7	10.03	0.1041	45	595.4
8	8.45	0.087	56	162.17
9	15.38	0.0984	12	165.1

between generators and loads can converge well to zero within 80 iterations, which also means that power balance constraint can be properly satisfied. The convergence performance of generators power and load demand are shown in Fig. 7, where load demand is considered as negative power output to distinguish from generators power for simplicity. From Fig. 7, it is clearly seen that the generated power and the load demand can eventually converge to the optimal solution. It compares the optimal solution of social welfare maximization between SE-DEMA-SRGF and CEMA in literature [14]. The comparison results are shown in Fig. 8, it can be noticed that the obtained total social welfare is 2668.83 by SE-DEMA-SRGF, which is very close to the optimal value 2669, while CEMA method converges to 2920.05, which has large deviation from optimal value.

Inspired by [15], in order to show the advantages of proposed event-triggered scheme over other alternatives, comparisons of communication number are made between the eventtriggered and periodic communication which is presented in



Fig. 5. The convergence of  $\lambda$  for each node.



Fig. 6. The convergence of deviation control parameter  $\zeta$ .



Fig. 7. The convergence of generators power and load demand.

Fig. 9. As it is shown in Fig. 9, the communication number can be tremendously reduced when adopting the proposed event-



Fig. 8. The comparison with other method on social welfare.

triggered mechanism. According to Fig. 10, the proposed security event-triggered mechanism can ensure smaller disturbance of total power mismatch in comparison to mere communication event-triggered based distributed energy management approach with successive randomized gradient-free (DEMA-SRGF). In addition, to verify the safety of the event-triggered mechanism proposed in this paper, the convergence curve of the power balance deviation for node 9 is presented in Figure. 11. As shown in the results, the traditional event-triggered mechanism only considers the difference between the current state and the state at the last triggering moment, which may lead to safety hazards. However, SE-DEMA-SRGF can avoid this issue. For further analysis on triggering time,  $\lambda_1$  is taken as the typical case to show triggering times which are shown in Fig. 12, where it can be clearly seen that security eventtriggered mechanism mainly occurs in first 50 iterations, and it seldom occurs after convergence.



Fig. 9. Comparisons of communication number during coordination among different nodes.

# B. The performance of SE-DEMA-SRGF under IEEE 39-bus system

To illustrate the scalability of SE-DEMA-SRGF, the proposed method is also implemented on IEEE 39-bus system. The convergence of  $\lambda_i$ ,  $P_i$ ,  $\zeta_i$  and triggering time is shown in Fig. 13, where it shows that SE-DEMA-SRGF still has



Fig. 10. The power mismatch during coordinated optimization



Fig. 11. The comparison with traditional methods on power balance deviation.



Fig. 12. The triggering time of  $\lambda_1$ .

an excellent performance on IEEE 39-bus system. It also compares the optimal solution of social welfare maximization between SE-DEMA-SRGF and CEMA. The comparison results are shown in Fig. 14, it can be noticed that the obtained total social welfare is 7931.89 by SE-DEMA-SRGF, which is very close to the optimal value 7930, while CEMA method converges to 8211.46, which has large deviation from optimal value. Meanwhile, the proposed event-triggered schemes can still reduce the communication numbers tremendously which can be seen in Fig. 15, and the total power mismatch has



Fig. 13. The convergence of  $\lambda_i$ ,  $P_i$ ,  $\zeta_i$  and triggering time.



Fig. 14. The comparison with other method on social welfare.



Fig. 15. Comparisons of communication number during coordination among different nodes.

smaller disturbance in comparison to other event-triggered mechanisms in Fig. 16, which can better ensure the power supply security of isolated power system. As in the previous set of experiments, the convergence curve of the power balance deviation for node 28 is presented in Figure 17. It can be seen that SE-DEMA-SRGF can effectively avoid the problem of a certain node's power balance deviation exceeding a safety threshold. According to obtained results on two test systems,



Fig. 16. The total power mismatch during coordinated optimization



Fig. 17. The comparison with traditional methods on power balance deviation.

it can reveal that the proposed method can tackle with energy management problem of cyber-physical isolated power system well with both reducing communication burden and ensuring system security.

### VI. CONCLUSION

Due to cyber-physical fusion and non-differentiable nonconvex characteristics of energy management of isolated power system, this paper proposes a security event-triggered mechanism based distributed optimization approach with successive randomized gradient-free technique to address this problem. According to both theoretical analysis and simulation results, some merits can be concluded as: (1) Randomized gradient-free based successive convex approximation method can deduce a convex and smooth equivalent for replacing economic objective function, which can provide an accurate objective function for seeking global optima. (2) The security event-triggered mechanism based distributed optimization method can ensure power system security well and also reduce communication burden caused by smoothing calculation. However, the current work lacks of reasonable strategy against cyber-physical coordinated attack, the future work is to design a resilient distributed energy management strategy under cyber-physical coordinated attack.

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