Forecasting International REITs Volatility: The Role of Oil-Price Uncertainty#

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Abstract

We forecast realized variance (RV) of Real Estate Investment Trusts (REITs) for ten leading markets and regions, derived from 5-minutes-interval intraday data, based on the information content of two alternative metrics of daily oil-price uncertainty. Based on the period of the analysis covering January 2008 to July 2020, and using variants of the popular MIDAS-RV model, augmented to include oil market uncertainties, captured by its RV (also derived from 5-minute intraday data) and implied volatility (i.e., the oil VIX), we report evidence of significant statistical and economic gains in the forecasting performance. The result is robust to the size of the forecasting samples, including that of the COVID-19 period, jump risks, lag-length, nonlinearities, and asymmetric effects, and forecast horizon. Our results have important implications for investors and policymakers.

JEL Classifications: C22, C53, G15, Q02.

Keywords: REITs; International data; Realized volatility; Oil-Price Uncertainty, Forecasting

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1. Introduction

Real Estate Investment Trusts (REITs), associated with asset allocation, risk reduction, and diversification, have grown substantially during the last decade as an investment vehicle. According to recent figures, the total market capitalization stands at over US $1.9 trillion involving 40 countries, with the United States (US) as the leader among the REITs markets, given a market capitalization of US of over $1.15 trillion (European Public Real Estate Association (EPRA), 2020).[1] The success in attracting such a large scale of investment capital is mainly because REITs are accessible to all investors irrespective of portfolio size (Akinsomi et al., 2016). Naturally, accurate forecasting of REITs volatility is an important issue for investors, given that volatility, as a measure of risk, plays a critical role in portfolio diversification, derivatives pricing, hedging and financial risk management. Furthermore, REITs returns do not suffer from measurement error and high transaction costs compared to other real estate investments and provide a perfect high-frequency proxy for the overall real estate market, since REITs earn most of their income from investments in real estate being exchange-traded funds, and also because trading occurs as common stocks (Marfatia et al., 2017). Given these properties, and the fact that the Global Financial Crisis (GFC) had its roots in the collapse and the resulting uncertainty in the global real estate sector, forecastability of volatility of a relatively homogenous REITs sector, which is possible at a high-frequency unlike the heterogeneous housing market, is an important issue for policymakers too in allowing them to design appropriate policies to circumvent the potential negative impact of uncertainty in the REITs sector on the real economy.

Given the current emphasis[2] that intraday data leads to more precise estimates and forecasts for the volatility of the REITs returns (see, for example, Zhou (2017), Bonato et al. (2021a, b, c)), we contribute to this burgeoning line of research by predicting the

[1] See: https://prodapp.epra.com/media/EPRA_Total_Markets_Table_-_Q4-2020_1611762538108.pdf for more details.

[2] Earlier studies on modeling and forecasting of REITs volatility were primarily based on Generalized Autoregressive Conditional Heteroscedasticity (GARCH)-type models (see, for example, Bredin et al. (2007), and Zhou and Kang (2011).

realized variance (RV) of the US and other developed and developing REITs markets, where we estimate RV by using 5-minute-interval intraday data for the period from January 2008 to July 2020. Since Corsi (2009) relied on the heterogeneous market hypothesis to construct the Heterogeneous Autoregressive (HAR)-RV model, it has become the popular benchmark in the volatility forecasting field owing to the advantage, such as, easily to implement and can capture the long memory. Some studies extend the HAR-RV model with exogenous variables to explore whether they can improve the accuracy of REITs volatility forecasting, i.e., newspaper-based index of uncertainty associated with infectious diseases (Bonato et al., 2021a), realized skewness and kurtosis (Bonato et al., 2021b).

Although HAR-RV model indeed has huge advantage in forecasting volatility, some studies criticize the fixed lag structure[3] (Audrino and Knaus, 2016). To this end, the MIDAS model[4] with "smooth" distributed lag polynomials is powerful to represent the dynamic dependence (Bollerslev et al., 2018). Note that the HAR-RV model evolved from the MIDAS-RV model and is a special form of the latter, i.e., the MIDAS-version is the more general in the class of RV models (Bollerslev et al., 2018). Additionally, some existing studies on volatility forecasting have recorded the superior forecasting performance of MIDAS model used in this study, i.e., Ma et al. (2019, 2020, 2021), Wang et al. (2020) and Liang et al. (2021). More specifically, we extend the basic MIDAS-RV model to incorporate information on daily oil-price volatility, also captured by its RV (derived from 5-minute intraday data as well) or its implied volatility (IV), and examine the forecasting power of these metrics capturing oil market uncertainty (Wang et al., 2018; Liang et al., 2020; Wang et al., 2020) in extensive out-of-sample testing procedures. Given that the ultimate test of any predictive model, in terms of the econometric methodologies and predictors employed, is its out-of-sample

3 Specifically, the specification of HAR-RV model can be written as $\mathrm{RV}_{t+1} = \beta_0 + \beta_d \mathrm{RV}_t + \beta_w \mathrm{RVW}_t + \beta_m \mathrm{RVM}_t + \varepsilon_{t+1}$, where the $\mathrm{RVW}_t = \frac{1}{5}\sum_{i=1}^{5} \mathrm{RV}_i$, $\mathrm{RVM}_t = \frac{1}{22}\sum_{i=1}^{22} \mathrm{RV}_i$.

4 The MIDAS approach comprises of two modeling issues simultaneously. The first is the specification of "smooth" distributed lag polynomials for representing the dynamic dependencies. While the second deals with the use of data sampled at different frequencies, and the choice of sampling-frequency for the predictor variables. In this study, we mainly focus on the first aspect of the MIDAS approach. The reader is referred to Section 3.5 of Bollerslev et al. (2018) for further details on these issues.

performance, we focus on the predictive analysis from an out-of-sample perspective.

Our decision to introduce metrics of oil volatility into the MIDAS-RV model of REITs emanates from the following reasons. First, the close volatility and return linkage exist between oil and REITs markets (Nazlioglu et al., 2016; Nazlioglu et al., 2020). Specifically, Nazlioglu et al. (2016) examined the role of oil price and volatility on the first and second-moments of six REITs categories of the US: Residential, Hotel, Healthcare, Retail, Mortgage, and Warehouse/Industrial REITs. The results showed bi-directional volatility transmission between the oil market and all the REITs. Similarly, Nazlioglu et al. (2020) provided an international dimension by analyzing price and volatility transmissions between nineteen REITs and the oil markets. Oil prices are primarily found to predict REITs prices in mature REITs markets, but the feedback from REITs to oil prices is weak. In sum, these studies showed significant impact of oil price and volatility on the corresponding first- and second-moments of US and international REITs (and also indicated of possible feedbacks). In other words, the information flow can transmit from oil market to international REITs market.

Second, existing studies argue that oil volatility (including realized volatility and option-implied volatility) exerts a significantly negative impact on the macroeconomy and real economic activity (Hamilton, 1983; Elder and Serletis, 2010; Gao et al., 2022), and hence can serve as a high-frequency global metric containing leading information of slow-moving local macroeconomic variables. Specifically, Gao et al. (2022) find that the implied oil volatility contains powerful negative information in predicting economic growth. The increasing oil volatility can predict a reduction in oil consumption and an increase in oil inventories. The negative impact is often explained by irreversible investments theory, that is investor will delay their investments when oil volatility (uncertainty) rises resulting in an adverse effect on economic activity. Now since, rising oil-price uncertainty, captured through its volatility, results in growing uncertainty of macroeconomic conditions, consequently, the second-moment of oil price is expected to predict rising volatility in the assets, including the REITs market.[5]

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[5] In this regard, the effect of oil-price volatility on the second-moments of REITs can be explained based on the seminal work of Schwert's (1989) discounted cash flow model, where the price of an asset is the

Third, cross-market volatility effects exist between crude oil and equity market (Phan et al., 2016; Wang et al., 2018; Wang et al., 2020). For example, Wang et al. (2020) explore the forecasting performance of realized oil volatility information in the U.S. equity market using MIDAS regression, the results indicate that the oil RV can successfully predict the short-term stock volatility. Since the REITs has recently received increasing number of attentions from investors, researchers, financial institutions, and central banks, and its behave commonly like equities, one interesting question emerge. Can oil information still can drive the REITs volatility like drive equity volatility? The objective of this study is to address this meaningful issue, which matter to academics, investors, and traders in their quest to more accurate REITs' volatility forecasts.

The contribution of this study on REITs volatility forecasting by following aspects. First, this study is closely related to the studies that explore whether the exogenous variables can improve the accuracy of REITs volatility forecasting (Bonato et al., 2021a; Bonato et al., 2021c). To the best of our knowledge, our study is the first attempt to forecast the RV of international REITs returns based on oil RV or its implied volatility. The empirical results indicate that the oil volatility can extremely drive the international REITs volatility. Second, our paper is tied to the literature of information flows across REITs and other financial markets (Nazlioglu et al., 2016; Nazlioglu et al., 2020). We

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sum of the discounted expected cash flows. Given this, the volatility of the price of an asset depends on the volatility (or dispersion) of expectations about future cash flows and discount rates. Therefore, time variation in asset market volatility is linked to the time varying degree of uncertainty regarding future discount factors and expected cash flows. Since both interest rates and expected cash flows depend on the state (health) of the economy, then it is plausible that a change in the level of uncertainty about future macroeconomic conditions would cause a proportional change in the asset (REITs) returns volatility, as outlined in Schwert (1989). According to this, if some macroeconomic series could provide information regarding the dispersion of expectations (or uncertainty) about future cash flows or discount rates, then these series could be determinants of the time variation in REITs market volatility. As increasing oil-price volatility results in growing uncertainty about discount factors via increasing uncertainty about real interest rates and expected inflation, and future cash flows, the variance of oil price is likely to cause rising volatility in the REITs market. While ideally, it would be best-suited to look at metrics of interest rate uncertainty directly, but since our data sample involves REITs of not only individual countries, but also regions, finding measures of interest rate uncertainty for the latter would be difficult, especially based on intraday data. In light of this, we resort to the usage of oil RV, which acts as a common high-frequency global proxy for uncertainty, given its well-established impact on (at timeslow-frequency) macroeconomic variables (real activity, inflation and interest rates).

confirm the oil volatility can transmit to international REITs from the forecasting horizon. The results show that the oil realized volatility can lead to a reduction of MSPE between 3.447% and 9.799%, and the oil implied volatility can produce a reduction of MSPE between 0.348% and 9.065%, respectively. This study provides a new insight to explore the cross-market effect across REITs and oil markets. Third, we further discuss the performance of oil volatility for forecasting REITs volatility in terms of regime switching technique and asymmetric effect. Specifically, we observe that the regime switching technique can improve the accuracy of volatility forecasting for 8 of 10 international REITs. Interestingly, the superior forecasting performance of oil volatility extremely is reflected during high volatility level.

The remainder of the paper is organized as follows: Section 2 outlines the methodologies, while Section 3 presents the data. Section 4 is devoted to our various econometric results, with a wide-array of robustness checks involving model specifications, forecast horizons, and data samples, including an analysis associated with the outbreak of the COVID-19 pandemic. Section 5 concludes the paper.

2. Methodologies

2.1 Realized measure of volatility

The superior ex post variance, realized variance (RV), is commonly used as proxy for risk in financial markets such as stock market, crude oil futures market and among others due to it contains less noise and is easy to implement (Andersen and Bollerslev, 1998). For a specific day *t*, this ex post measure of variance is given by:

$$\mathrm{RV}_t = \sum_{j=1}^{M} r_{t,j}^2, \tag{1}$$

where $r_{t,j}$ represents the j^{th} intraday return of day *t*; $M = 1/\Delta$, and Δ is the sampling rate. Following the influential work of Liu et al. (2015), we adopt the 5-min sampling frequency to construct RV. According to the arguments of Andersen et al. (2007), the distribution of RV generated from Equation (1) is leptokurtic. To this end, we employ the natural logarithm of RV in the forecasting process, the distribution of which is approximately Gaussian[6].

2.2 Predictive regressions

We implement the mixed data sampling (MIDAS) regression to generate the one-day-ahead forecast. The superior performance of MIDAS framework has been recorded in growing number of studies associated with volatility forecasting (Bollerslev et al., 2018; Ma et al., 2019, 2020, 2021; Wang et al., 2020; Liang et al., 2021). The standard benchmark model to predict international REITs volatility, i.e., realized variance, at the horizon of a trading day is the following MIDAS-RV model:

Model 1: MIDAS-RV

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_{\mathrm{RV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1} + \varepsilon_{t+1}, \tag{2}$$

where RV_{t-k+1} represents the k-order lags of RV. We set the $k^{max} = 40$ and ω_k denotes the respective weights for different frequency components. Along the lines of Ghysels et al. (2006, 2007), the weight function is measured by following beta function:

$$b\left(k, \theta_1^{\mathrm{RV}}, \theta_2^{\mathrm{RV}}\right) = f\left(\frac{k}{k^{max}}, \theta_1, \theta_2\right) / \sum_{i=1}^{k^{max}} f\left(\frac{k}{k^{max}}, \theta_1, \theta_2\right), \tag{3}$$

where $f(x, y, z) = x^{y-1}(1-x)^{z-1}/\beta(y, z)$ and $\beta(y, z)$ is evaluated by $\beta(y, z) = \Gamma(y)\Gamma(z)/\Gamma(y+z)$. Following the work of Ghysels et al. (2009), Ma et al. (2021), Li et al. (2022) and among others, the maximum likelihood estimation is used to estimate the coefficients of MIDAS regressions. Mathematically, the likelihood function (L) can be written as,

$$L = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{\left(\mathrm{RV}_{t+1} - \beta_0 - \beta_{\mathrm{RV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1}\right)^2}{2\sigma^2}\right\}. \tag{4}$$

The goal of our study is to examine the role of crude oil volatility in forecasting international REITs volatility, hence, we extend the benchmark MIDAS-RV by incorporating RV of oil (ORV) or implied volatility of oil (OIV) as a predictor, to give us the following augmented-MIDAS-RVs:

Model 2: MIDAS-RV-ORV

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[6] To assess the out-of-sample performance, we transform the logarithm of forecasts into the original variance form using the bias-corrected approach of Proietti and Lütkepohl (2013).

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_{\mathrm{RV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1} + \beta_{\mathrm{ORV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{ORV}_{t-k+1} + \varepsilon_{t+1}. \quad (5)$$

Model 3: MIDAS-RV-OIV

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_{\mathrm{RV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1} + \beta_{\mathrm{OIV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{OIV}_{t-k+1} + \varepsilon_{t+1}. \quad (6)$$

2.3 Forecast evaluation

Along the lines of Rapach et al., (2010) and Wang et al., (2018), we employ the out-of-sample R^2 test to assess the forecasting quality, which basically evaluates the percent reduction of mean squared predictive error (MSPE) of the extended model ($\mathrm{MSPE}_{\mathrm{model}}$) relative to the MSPE of benchmark ($\mathrm{MSPE}_{\mathrm{bench}}$). The R_{OOS}^2 is defined as,

$$R_{oos}^2 = 1 - \frac{\mathrm{MSPE}_{\mathrm{model}}}{\mathrm{MSPE}_{\mathrm{bench}}}, \quad (7)$$

where $\mathrm{MSPE}_i = \frac{1}{T-M} \sum_{t=M+1}^{T} (\mathrm{RV}_t - \widehat{\mathrm{RV}}_{t,i})^2$ ($i = \mathrm{model}, \mathrm{bench}$), T and M are the lengths of the full-sample and the estimation window period. Furthermore, for assessing whether heterogeneous predictive performance exists across different models, we consider the MSPE-adjusted statistic of Clark and West (2007). Intuitively, a competing model is superior to the benchmark if the R_{OOS}^2 value is positive owing to the lower MSPE from the competing model.

Besides statistical evaluation, economic gain from the predictor is of vital important to investors. Therefore, we also look at economic value analysis, which allows us to compare the economic gains from each predictive regression. A mean-variance method is used to compare the difference of economic value obtained from all models that we consider, whereby the investor allocates her/his wealth to REITs or a risk-free asset. According to Bollerslev et al. (2018), expected utility obtained by averaging the corresponding realized expressions over the out-of-sample forecasts of RV can be written as follows[7]:

$$\bar{U}\left(\widehat{\mathrm{RV}}_{t+1}\right) = \frac{1}{q} \sum_{t=m+1}^{m+q-1} \frac{\mathrm{SR}^2}{\gamma} \left(\frac{\sqrt{\mathrm{RV}_{t+1}}}{\sqrt{\widehat{\mathrm{RV}}_{t+1}}} - \frac{1}{2} \frac{\mathrm{RV}_{t+1}}{\widehat{\mathrm{RV}}_{t+1}} \right), \quad (8)$$

[7] More technical details about this economic analysis can be found in Bollerslev et al. (2018).

where γ and SR are risk aversion coefficient and the Sharpe ratio. Along the lines of Bollerslev et al. (2018) and Liang et al. (2020), we set the annualized Sharpe ratio SR equal to 0.40, and the coefficient of relative risk aversion as $\gamma = 2$.

3. Data description

3.1 REITs data

We use 5-minute-interval intraday data on the REITs indexes to construct daily measure of RV, outlined in equation (1). Besides the FTSE Nareit All REITs (FNAR) Index for the US, which is the most prominent REITs market, we also investigate the role of oil uncertainty (the data for which we discuss below) on the REITs markets covering other developed and developing countries and regions (for which intraday data is available) namely, the FTSE Nareit Developed Asia (EGAS) Index, FTSE Nareit North America Asia (EGNA) Index, FTSE Nareit Australia (ELAU) Index, FTSE Nareit Hong Kong (ELHK) Index, FTSE Nareit Japan (ELJP) Index, FTSE Nareit UK (ELUK) Index, FTSE Nareit Developed Markets (ENGL) Index, FTSE Nareit Eurozone (EPEU) Index, FTSE Nareit Emerging Markets (FENEI) Index. The price data for all these indexes, in a continuous format, are obtained from Bloomberg terminal. Figure 1 depicts the RV series of 10 international REITs indexes over full sample period. Clearly, the distributions of 10 RV series are leptokurtic with fat-tails, which is consistent with the fact of financial time series. Besides, it is worth noting that Figure 1 reveals that during the global financial crisis in 2008 and the COVID-19 pandemic in 2020, almost all REITs markets suffered huge shocks and their volatility trends were very similar.

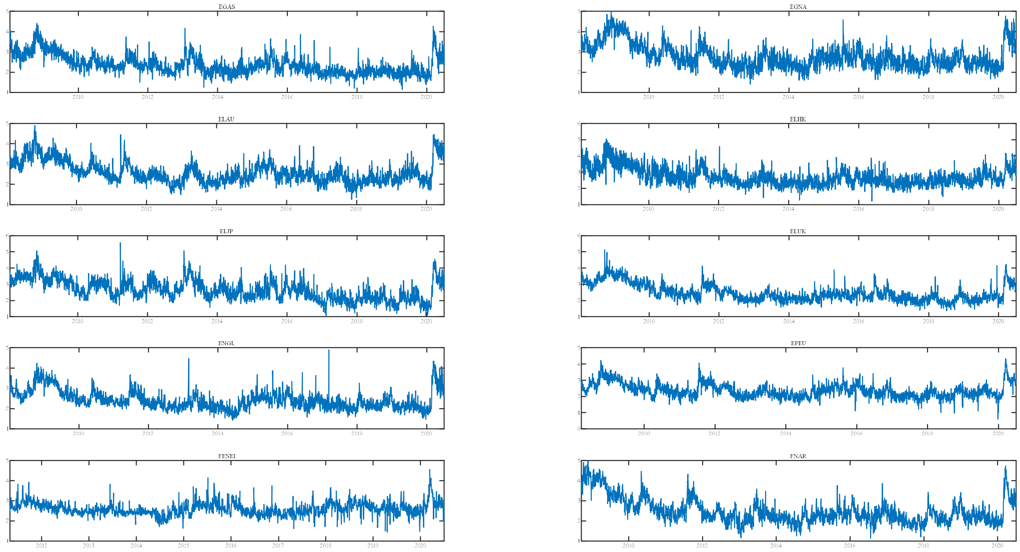

Figure 1. Realized volatility series

3.2 Oil data

Our oil-based dataset consists of realized variance of crude oil futures (ORV) and implied volatility of crude oil futures (OIV). For ORV, the 5-minute intraday data of the front-month West Texas Intermediate (WTI) oil futures is derived from the NYMEX-CME. Such data frequency as a rule of thumb can offer a balance between market microstructure noise and predictive improvement (Liu et al., 2015). And, we also use the measure of implied volatility of crude oil futures based on the Crude Oil Volatility index (OVX) of the Chicago Board of Options Exchange (CBOE), as a predictor capturing oil market volatility, in an attempt to ensure robustness of our findings. The OVX is an annualized index that measures the market's expectation of 30-day volatility of crude oil prices. The index is available from the FRED database of the Federal Reserve Bank of St. Louis at: https://fred.stlouisfed.org/series/OVXCLS.

Table 1 reports the descriptive statistics of the series of international REITs volatility, ORV and OIV. Obviously, all the series show significantly right-skewed and leptokurtic. Moreover, the results of Jarque-Bera statistic test demonstrate all the series are non-normally distributed, while they are stationary at the 1% significance level from Augmented Dickey-Fuller (ADF) test. The Ljung-Box test is used to evaluate the autocorrelation up to the 20^{th} (40^{th}) order. The results of Q (20) and Q (40) indicate that

Table 1. Descriptive statistics.

Variable	Full sample period	Mean	Std.dev	Skewness	Kurtosis	Jarque-Bera	ADF	Q (5)	Q (40)
EGAS	2008.01.17-2020.07.01	0.798	1.548	6.947	69.003	628822.605 ***	-23.974 ***	1080.074 ***	5564.075***
EGNA	2008.01.24-2020.07.01	2.554	5.771	5.465	41.949	239924.411 ***	-20.594 ***	10030.131***	54308.137***
ELAU	2008.02.06-2020.07.01	1.451	3.195	8.626	123.666	1930066.123 ***	-20.590 ***	10781.408***	60111.433***
ELHK	2008.01.28-2020.07.01	1.921	5.110	8.857	106.577	1476613.364 ***	-34.890 ***	9361.145***	54474.597***
ELJP	2008.01.23-2020.07.01	1.969	6.692	25.219	899.512	96318528.057 ***	-42.614 ***	8948.986***	49657.808***
ELUK	2008.01.23-2020.07.01	1.401	13.716	51.082	2730.881	945659780.218 ***	-53.907 ***	9597.713***	55358.547***
ENGL	2008.01.09-2020.07.01	0.920	2.172	14.847	394.457	20114070.816 ***	-34.305 ***	8785.733***	46057.798***
EPEU	2008.03.25-2020.07.01	0.718	1.226	8.090	99.214	1276664.313 ***	-17.526 ***	10821.042***	64460.884***
FENEI	2011.05.02-2020.07.01	0.923	1.261	13.300	302.662	8835141.252 ***	-28.458 ***	8015.423***	37767.539***
FNAR	2008.09.19-2020.07.01	1.914	5.448	6.440	57.042	419502.299 ***	-18.602 ***	3213.648***	11966.600***
ORV	2008.01.09-2020.07.01	6.515	61.939	48.427	2534.791	827112398.390 ***	-42.765 ***	379.913***	599.382***
OIV	2008.01.09-2020.07.01	38.362	19.939	4.301	33.491	153698.468 ***	-6.698 ***	13384.589***	65787.945***

Notes: This table reports the descriptive statistics of RV of 10 international REITs index and oil volatility (ORV and OIV). Columns show variable, abbreviation, observation, mean, standard deviation (Std.dev), skewness, kurtosis, Jarque-Bera test and Augmented Dickey-Fuller test (ADF). *** denote rejection of null hypothesis at the 1% level of significance.

all the series exist auto-correlation, which reconfirms the well-known property (long-memory) of volatility series.

4. Empirical results

4.1 Primary results

To generate our volatility forecasts at the horizon of one trading day, we consider the rolling window method. Although our ten international REITs indexes have different start and end dates, we set the first 50% observations as the estimation period and the last 50% observations as the out-of-sample forecasting sample.

Recall that, the primary objective of our study is to use oil-market uncertainties (ORV and OIV) to predict the realized volatility of international REITs indexes. Table 2 presents the out-of-sample R^2 test statistics and economic value analysis. The column named Out-of-sample R^2 test of Table 2 provides the $R_{\mathrm{OOS}}^2(\%)$, the MSPE-adjusted statistics and the corresponding p-values of including with ORV or OIV relative to the benchmark (MIDAS-RV model). We first focus on the forecasting performance of ORV. The values of $R_{\mathrm{OOS}}^2(\%)$ suggest that the forecasting model with ORV can lead to a reduction of MSPE between 3.447% and 9.799% for volatility forecasts of the 10 international REITs indexes that we consider. The p-values of MSPE-adjust statistic indicate that ORV can significantly improve the forecast accuracy of REITs volatility. Similar results are also obtained with the OIV. Specifically, the MIDAS-RV-OIV model can produce a reduction of MSPE between 0.348% and 9.065% over the forecasting period, with the p-values of the MSPE-adjusted test statistic being significant (except for the ENGL index) as well.

"Portfolio Exercise" column of Table 2 shows the results of the economic value analysis. Obviously, the percent realized utility of extending model with ORV and OIV are higher relative to the benchmark model for all international REITs indices. The results suggest the investors are willing to pay additional fee to access the models with information on ORV or OIV rather than simply using the benchmark (MIDAS-RV) model when dealing with one-day-ahead RV forecasting of the REITs markets. In other

Table 2. Forecasting performance.

REITs index	Out-of-sample R^2 test						Portfolio Exercise (%)		
	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)					
	R^2_{OOS}(%)	MSPE-Adj.	p-value	R^2_{OOS}(%)	MSPE-Adj.	p-value	Bench	ORV	OIV
EGAS	5.656	2.122	0.017	5.744	2.724	0.003	3.586	3.623	3.611
EGNA	4.539	2.592	0.005	3.387	2.819	0.002	3.260	3.304	3.309
ELAU	6.946	2.477	0.007	6.945	2.673	0.004	3.575	3.598	3.589
ELHK	8.745	5.442	0.000	5.887	5.674	0.000	3.697	3.867	3.745
ELJP	6.934	2.493	0.006	9.065	2.545	0.005	3.256	3.285	3.271
ELUK	9.799	2.373	0.009	6.305	2.724	0.003	3.669	3.677	3.677
ENGL	3.447	2.410	0.008	0.348	0.802	0.211	3.603	3.632	3.629
EPEU	6.910	2.417	0.008	5.125	2.525	0.006	3.666	3.682	3.689
FENEI	7.560	2.438	0.007	3.486	2.211	0.014	3.579	3.614	3.589
FNAR	6.161	2.265	0.012	5.962	2.546	0.005	3.626	3.639	3.640

Notes: The table represents the out-of-sample performance. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

words, ORV or OIV can help the investor achieve higher realized utility from an economic point of view.

Overall, the results based on statistical and economic evaluation, suggests that ORV or OIV can successfully produce statistical and economic gains for the investors including REITs in their portfolios.

4.2 Robustness

4.2.1 Alternative forecasting window

Rossi and Inoue (2012) suggest that the choice of window size plays an important role forecasting results. In light of this, we consider different window sizes, which involves including the last 70% and 60% of observations as out-of-sample period. Table 3 and Table 4 reports the evaluation results associated with the out-of-sample R^2 test, and also the associated economic value analyses. The results provide strong empirical evidence that the extending the MIDAS-RV model with ORV and OIV outperforms the benchmark model, which is consistent with the previous findings with a 50% split, and confirm that our results are robust to forecasting-sample periods.

In the context of the size of the out-of-sample periods, we also decided to closely analyze the forecasting ability of the models during the COVID-19 pandemic outbreak, which has resulted in an unprecedented shock to real economic activities, financial market and public lives (Baker et al., 2020). This section investigates the forecasting performance of ORV or OIV during the COVID-19 period, which following the work of Ji et al. (2020), we set to cover from 1 January 2020 to 1 July 2020, during which oil market witnessed heightened variability. Table 5 reports the forecasting performance of ORV and OIV for this period. Several interesting findings emerge. First, the values of R^2_{OOS} provide evidence that ORV and OIV continue to reduce the MSPE for the volatility of the 10 international REITs indexes, in line with our previous findings. Second, we find that oil implied volatility is superior in forecasting international REITs volatility than oil realized volatility for most cases, as the R^2_{OOS} values of the predictive regression model with OIV are greater than those with ORV. One possible reason of this observation is possibly due to the fact that that implied volatility is associated with the future 30-days market expectations.

Table 3. Forecasting performance with alternative forecasting window.

Equity index	Out-of-sample R^2						Portfolio Exercise		
	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)					
	R^2_{OOS}(%)	MSPE-Adj.	p-value	R^2_{OOS}(%)	MSPE-Adj.	p-value	Bench	ORV	OIV
EGAS	-0.106	0.750	0.227	5.713	2.432	0.008	3.587	3.606	3.605
EGNA	5.020	2.404	0.008	8.072	2.797	0.003	3.307	3.338	3.329
ELAU	3.745	1.885	0.030	7.986	2.215	0.013	3.623	3.642	3.638
ELHK	5.650	4.279	0.000	7.477	3.911	0.000	1.035	1.925	1.506
ELJP	1.471	2.146	0.016	4.501	2.935	0.002	3.271	3.282	3.285
ELUK	11.239	2.193	0.014	9.931	2.385	0.009	3.699	3.705	3.702
ENGL	3.344	2.570	0.005	3.209	2.612	0.004	3.537	3.542	3.536
EPEU	3.673	1.969	0.024	4.337	1.505	0.066	3.706	3.722	3.719
FENEI	2.658	1.732	0.042	4.876	2.407	0.008	3.579	3.582	3.595
FNAR	7.836	1.790	0.037	8.502	1.823	0.034	3.612	3.613	3.610

Notes: The table represents the out-of-sample performance with alternative forecasting window. The forecasting window covers at last 60% observations for 10 REITs index, respectively.

Table 4. Forecasting performance with alternative forecasting window.

REITs index	Out-of-sample R^2 test						Portfolio Exercise (%)		
	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)					
	R^2_{OOS}(%)	MSPE-Adj.	p-value	R^2_{OOS}(%)	MSPE-Adj.	p-value	Bench	ORV	OIV
EGAS	4.360	1.881	0.030	7.000	2.463	0.007	3.582	3.606	3.600
EGNA	5.987	2.694	0.004	6.455	3.052	0.001	3.272	3.304	3.295
ELAU	6.118	2.254	0.012	8.119	2.527	0.006	3.598	3.617	3.610
ELHK	9.199	4.522	0.000	8.978	4.316	0.000	3.620	3.076	3.831
ELJP	2.899	1.942	0.026	7.317	2.338	0.010	3.287	3.308	3.301
ELUK	11.239	2.193	0.014	9.931	2.385	0.009	3.699	3.705	3.702
ENGL	3.344	2.570	0.005	3.209	2.612	0.004	3.537	3.542	3.536
EPEU	7.130	2.260	0.012	8.435	2.190	0.014	3.695	3.709	3.710
FENEI	5.624	2.178	0.015	-0.657	0.546	0.293	3.551	3.565	3.553
FNAR	8.355	2.152	0.016	8.057	2.301	0.011	3.627	3.637	3.636

Notes: The table represents the out-of-sample performance with alternative forecasting window. The forecasting window covers at last 40% observations for 10 REITs index, respectively.

Table 5. Forecasting performance during COVID-19 period.

Equity index	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)		
	R^2_{OOS}(%)	MSPE-Adj.	p-value	R^2_{OOS}(%)	MSPE-Adj.	p-value
EGAS	4.315	1.503	0.066	5.481	1.816	0.035
EGNA	3.095	1.815	0.035	5.609	2.194	0.014
ELAU	4.369	1.783	0.037	5.611	2.103	0.018
ELHK	-0.866	3.070	0.001	13.677	3.336	0.000
ELJP	4.231	1.543	0.061	6.462	1.741	0.041
ELUK	7.372	1.412	0.079	5.518	1.189	0.117
ENGL	-0.417	0.690	0.245	4.063	1.986	0.024
EPEU	3.592	1.935	0.027	7.329	1.853	0.032
FENEI	7.487	2.252	0.012	5.390	2.262	0.012
FNAR	-5.026	-1.878	0.970	5.168	1.501	0.067

Notes: The table represents the out-of-sample performance during COVID-19 period.

4.2.2 Alternative $\boldsymbol{k^{max}}$

Recall that previous sections consider $\boldsymbol{k^{max}} = 40$. In this subsection, we reinvestigate the forecasting ability from oil volatility to REITs volatility by considering different $\boldsymbol{k^{max}}$, as another robustness test. Panels A and Panel B of Table 6 reports the statistical evaluation results by considering $\boldsymbol{k^{max}} = 20$ and $\boldsymbol{k^{max}} = 60$, respectively. Indeed, we find that the ORV or OIV can significantly reduce the MSPEs for forecasting volatility of the REITs considered. The results provide strong evidence that our findings are robust to different $\boldsymbol{k^{max}}$.

Table 6. Forecasting performance with alternative k^{max}.

Equity index	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)		
	R^2_{OOS}(%)	MSPE-Adj.	p-value	R^2_{OOS}(%)	MSPE-Adj.	p-value
Panel A: $k^{max}=20$						
EGAS	4.678	1.960	0.025	7.079	2.509	0.006
EGNA	6.505	2.767	0.003	6.767	3.097	0.001
ELAU	6.542	2.315	0.010	8.392	2.547	0.005
ELHK	9.568	4.556	0.000	9.244	4.359	0.000
ELJP	1.127	1.912	0.028	2.779	2.242	0.012
ELUK	11.918	2.285	0.011	10.394	2.443	0.007
ENGL	4.080	2.708	0.003	3.888	2.711	0.003
EPEU	7.747	2.288	0.011	8.494	2.138	0.016
FENEI	6.473	2.452	0.007	4.229	2.297	0.011
FNAR	8.702	2.161	0.015	8.638	2.371	0.009
Panel B: $k^{max}=60$						
EGAS	3.829	1.817	0.035	6.317	2.416	0.008
EGNA	5.824	2.646	0.004	6.215	3.009	0.001
ELAU	6.245	2.318	0.010	8.405	2.594	0.005
ELHK	9.669	4.546	0.000	9.085	4.354	0.000
ELJP	2.773	1.928	0.027	7.208	2.322	0.010
ELUK	10.850	2.157	0.015	9.539	2.320	0.010
ENGL	2.899	2.463	0.007	2.815	2.513	0.006
EPEU	6.201	2.228	0.013	7.421	2.118	0.017
FENEI	5.609	2.150	0.016	-3.723	-2.281	0.989
FNAR	8.806	2.125	0.017	7.802	2.228	0.013

Notes: The table represents the out-of-sample performance with alternative k^{max}. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

4.2.3 Alternative regressions model

As discussed in the part of introduction, the HAR-RV model of Corsi (2009) has become the popular benchmark in the area of volatility forecasting[8]. To this end, this subsection, we construct the HAR-type models to re-explore whether the oil volatility can improve the accuracy of international REITs volatility forecasting. The predictive regressions are defined as,

HAR-RV:

$$V_{t+1} = \beta_0 + \beta_{\mathrm{HAR}}^{(d)} V_t + \beta_{\mathrm{HAR}}^{(w)} \mathrm{VW}_t + \beta_{\mathrm{HAR}}^{(m)} \mathrm{VM}_t + \varepsilon_{t+1}, \tag{9}$$

[8] We are grateful to an anonymous referee for this constructive suggestion.

where V_t, VW_t and VM_t denote the logarithmic RV[9] for daily, weekly and monthly volatility components, respectively; moreover, $\mathrm{VW}_t = \log(\frac{1}{5}\sum_{i=1}^{5} RV_i)$ and $\mathrm{VM}_t = \log(\frac{1}{22}\sum_{i=1}^{22} RV_i)$.

HAR-ORV:

$$V_{t+1} = \beta_0 + \beta_{\mathrm{HAR}}^{(d)} V_t + \beta_{\mathrm{HAR}}^{(w)} \mathrm{VW}_t + \beta_{\mathrm{HAR}}^{(m)} \mathrm{VM}_t + \beta_{\mathrm{ORV}}^{(d)} \mathrm{ORV}_t + \beta_{\mathrm{ORV}}^{(w)} \mathrm{ORVW}_t + \beta_{\mathrm{ORV}}^{(m)} \mathrm{ORVM}_t + \varepsilon_{t+1}, \tag{10}$$

HAR-OIV:

$$V_{t+1} = \beta_0 + \beta_{\mathrm{HAR}}^{(d)} V_t + \beta_{\mathrm{HAR}}^{(w)} \mathrm{VW}_t + \beta_{\mathrm{HAR}}^{(m)} \mathrm{VM}_t + \beta_{\mathrm{OIV}}^{(d)} \mathrm{OIV}_t + \beta_{\mathrm{OIV}}^{(w)} \mathrm{OIVW}_t + \beta_{\mathrm{OIV}}^{(m)} \mathrm{OIVM}_t + \varepsilon_{t+1}, \tag{11}$$

Table 7 reports the out-of-sample results using HAR-type models. Obviously, we can observe that the predictive regression with ORV and OIV can consistently generate positive and significant values of $R_{\mathrm{OOS}}^2(\%)$ for 10 international REITs. The out-of-sample results from HAR-type models reconfirm our previous findings, that oil volatility information can successfully predict the international REITs volatility.

Table 7. Forecasting performance using HAR-type models.

REITs index	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)		
	$R_{\mathrm{OOS}}^2(\%)$	MSPE-Adj.	*p*-value	$R_{\mathrm{OOS}}^2(\%)$	MSPE-Adj.	*p*-value
EGAS	1.692	2.184	0.014	8.764	2.984	0.001
EGNA	1.578	4.861	0.000	1.307	5.314	0.000
ELAU	2.579	3.917	0.000	7.083	4.163	0.000
ELHK	1.514	2.037	0.021	7.971	3.323	0.000
ELJP	0.725	1.686	0.046	4.464	3.024	0.001
ELUK	2.798	4.122	0.000	7.256	4.115	0.000
ENGL	1.074	1.782	0.037	7.999	3.354	0.000
EPEU	1.561	2.926	0.002	4.787	3.212	0.001
FENEI	2.900	3.322	0.000	8.815	3.267	0.001
FNAR	1.594	2.114	0.017	7.538	3.324	0.000

Notes: The table represents the out-of-sample performance using HAR-type models. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

[9] The OLS regression directly uses the volatility series will lead to a misleading coefficient owing to the distribution of RV is leptokurtic. As such, we consider the natural logarithm of the RV ($V_t = \log(\mathrm{RV}_t)$) to obtain precise estimates, which is widely used in the existing literature on volatility forecasting.

4.2.4 Controlling VIX effect

Undoubtfully, the CBOE VIX index is a superior predictor in forecasting the stock volatility (Wang et al., 2020). Given the behavior of REITs increasingly like equity, this subsection we further focus on the forecasting performance of oil volatility after controlling VIX effect[10]. Specifically, we add the VIX index into the right-hand side of three predictive regressions to construct the MIDAS-RV-VIX, MIDAS-ORV-VIX and MIDAS-OIV-VIX model. Considering the MIDAS-RV-VIX model as an example, the regression can be defined as,

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_{\mathrm{RV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1} + \beta_{\mathrm{VIX}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{VIX}_{t-k+1} + \varepsilon_{t+1}, \quad (12)$$

Table 8 reports the out-of-sample evaluation resutls. Obviously, we can observe that the MIDAS-RV-ORV-VIX can generate the postive and signifanct $R^2_{\mathrm{OOS}}(\%)$ values for 3 of 10 international REITs volatility, suggesting that the oil RV information contains less incremental information than VIX index. A possible reason is that the VIX is often considered as the "fear index", which can reflect the expectations of stock market volatility over 30 days. Thus, the VIX index indeed contains more incremental information than oil RV in forecasting the REITs volatility. Being of our interest, the $R^2_{\mathrm{OOS}}(\%)$ values are positive and significant for 9 of 10 international REITs index using MIDAS-RV-OIV-VIX index, implying that the oil implied volatility still can successfully predict the international REITs volatility after controlling the effect of VIX.

[10] We are grateful to an anonymous referee for this valuable suggestion.

Table 8. Forecasting performance after controlling VIX.

REITs index	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)		
	R^2_{OOS}(%)	MSPE-Adj.	p-value	R^2_{OOS}(%)	MSPE-Adj.	p-value
EGAS	1.588	1.426	0.077	2.030	2.100	0.018
EGNA	2.003	1.434	0.076	1.019	1.398	0.081
ELAU	1.062	1.874	0.030	0.590	1.602	0.055
ELHK	0.231	1.075	0.141	0.374	1.982	0.024
ELJP	0.360	0.833	0.202	3.835	2.695	0.004
ELUK	1.696	0.970	0.166	1.239	0.981	0.163
ENGL	1.508	1.102	0.135	2.100	1.643	0.050
EPEU	0.046	0.823	0.205	2.919	1.342	0.090
FENEI	-0.994	-1.477	0.930	3.880	2.088	0.018
FNAR	-0.082	0.093	0.463	1.535	1.827	0.034

Notes: The table represents the out-of-sample performance after controlling VIX. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

4.2.5 Alternative evaluation method

In this subsection, we further consider another evaluation method, model confidence set (MCS) of Hansen et al. (2011), that is widely used in the literature of volatility forecasting. To determine the forecasting performance of predictive regressions, the stationary bootstrap approach[11] is used to evaluate the interpretation of the MCS test p-value. The confidence level of α is set to 0.25 to ascertain the superior model set. This indicates that the predictive regressions can pass the MCS test if its p-values of MCS test are over 0.25. The following two prevailing loss functions is introduced to assess forecasting quality,

$$\text{QLIKE} = \frac{1}{T-M}\sum_{t=M+1}^{T}(\ln(\widehat{\text{RV}}_t) + (\frac{\text{RV}_t}{\widehat{\text{RV}}_t})), \tag{13}$$

$$\text{MSE} = \frac{1}{T-M}\sum_{t=M+1}^{T}(\text{RV}_t - \widehat{\text{RV}}_t)^2. \tag{14}$$

Furthermore, besides the MIDAS-RV-type models, we further consider the HAR-RV model to ascertain superior performance of MIDAS-RV-type models[12]. The results of MCS test are shown in Table 9. Some interesting findings emerge. First, the HAR-RV model hardly to fall into the MCS test under the QLIKE and MSE loss functions

[11] Further technical details regarding the evaluation of the MCS test p value can be found in Hansen et al. (2011).

[12] We are grateful to an anonymous referee for this valuable suggestion.

(except EPEU), suggesting that the MIDAS-RV-type models outperform the peers. Second, being of our interest, the predictive regressions incorporate the oil realized volatility (ORV) or oil implied volatility (OIV) can consistently yield the largest p-values of 1 for 9 of 10 international REITs. The results reconfirm our previous findings, that the oil volatility information can extremely drive the international REITs volatility.

Table 9. Evaluation results of MCS test.

Regressions	QLIKE		MSE		QLIKE		MSE	
	Range	SeimQ	Range	SeimQ	Range	SeimQ	Range	SeimQ
REITs asset	EGAS				ELUK			
HAR-RV	0.010	0.004	0.112	0.073	0.001	0.002	0.044	0.045
MIDAS-RV	0.005	0.003	0.112	0.130	0.122	0.089	0.088	0.116
MIDAS-ORV	**<u>1.000</u>**	**<u>1.000</u>**	**0.949**	**0.949**	0.899	0.899	**<u>1.000</u>**	**<u>1.000</u>**
MIDAS-OIV	0.049	0.049	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	0.239	0.239
REITs asset	EGNA				ENGL			
HAR-RV	0.010	0.005	0.204	0.184	0.029	0.047	0.057	0.042
MIDAS-RV	0.001	0.002	0.204	0.184	**0.479**	**0.601**	0.142	0.143
MIDAS-ORV	**<u>1.000</u>**	**<u>1.000</u>**	0.204	0.184	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**
MIDAS-OIV	0.055	0.055	**<u>1.000</u>**	**<u>1.000</u>**	**0.479**	**0.601**	**0.871**	**0.871**
REITs asset	ELAU				EPEU			
HAR-RV	0.016	0.007	0.218	0.135	**<u>1.000</u>**	**<u>1.000</u>**	**0.275**	**0.269**
MIDAS-RV	0.036	0.020	0.218	0.183	0.000	0.003	**0.275**	0.206
MIDAS-ORV	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**0.285**	**0.343**	**<u>1.000</u>**	**<u>1.000</u>**
MIDAS-OIV	0.085	0.085	0.990	0.990	**0.641**	**0.641**	**0.493**	**0.493**
REITs asset	ELHK				FENEI			
HAR-RV	0.019	0.034	0.003	0.003	0.083	0.083	0.142	0.073
MIDAS-RV	0.019	0.034	0.088	0.074	0.017	0.039	0.142	0.087
MIDAS-ORV	0.019	0.034	0.270	0.270	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**
MIDAS-OIV	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	0.017	0.039	0.142	0.087
REITs asset	ELJP				FNAR			
HAR-RV	0.018	0.064	0.152	0.078	0.002	0.002	0.179	0.133
MIDAS-RV	0.010	0.017	0.152	0.114	0.002	0.002	0.179	0.095
MIDAS-ORV	**<u>1.000</u>**	**<u>1.000</u>**	**0.316**	**0.316**	**0.869**	**0.869**	**<u>1.000</u>**	**<u>1.000</u>**
MIDAS-OIV	0.018	0.064	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**<u>1.000</u>**	**0.819**	**0.819**

Notes: The table represents the out-of-sample performance using MCS test of Hansen et al. (2011). The p-values are calculated according to the range and semiquadratic (SeimQ) statistics. Bold indicates p-values > 0.25, and a p-value of 1 is indicated in bold and underlined. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

4.3 The nonlinear oil-REITs volatility relationship

4.3.1 Regime-Switching

Given the evidence that the nexus between oil and REITs volatilities are nonlinear (Nazlioglu et al., 2016, 2020), we re-conduct our analysis using the two-stage Markov switching model, as outlined by Ma et al. (2017), Wang et al. (2018) and Wang et al. (2020) as follows:

Model 7: MRS-MIDAS-ORV

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_{\mathrm{RV},S_t} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1} + \beta_{\mathrm{ORV},S_t} \sum_{k=1}^{k^{max}} \omega_k \mathrm{ORV}_{t-k+1} + \varepsilon_{t+1}, \quad (15)$$

Model 8: MRS-MIDAS-OIV

$$\mathrm{RV}_{t+1} = \beta_0 + \beta_{\mathrm{RV},S_t} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1} + \beta_{\mathrm{OIV},S_t} \sum_{k=1}^{k^{max}} \omega_k \mathrm{OIV}_{t-k+1} + \varepsilon_{t+1}, \quad (16)$$

Note that $S_t = 0$ and $S_t = 1$ indicates the low- and high-volatility regimes, respectively. We compare the forecasting performance of MRS-MIDAS-ORV (MRS-MIDAS-OIV) model with benchmark of MIDAS-ORV (MIDAS-OIV). Table 10 shows the forecasting results from the predictive regressions with and without regime-switching. Clearly, the MRS-MIDAS-ORV can further help to improve the accuracy of volatility forecasting for 8 out of 10 international REITs indexes including EGAS, EGNA, ELAU, ELJP, ELUK, EPEU, FENEI and FNAR. Similarly, the MRS-MIDAS-OIV model can outperform the benchmark for 7 out of 10 international REITs indexes including EGNA, ELAU, ELJP, ELUK, EPEU, FENEI and FNAR. The empirical results provide strong evidence that regime switching can further improve the accuracy of volatility forecasting for most cases of international REITs indexes.

To delve into this issue further, we divide the volatility forecasts over the out-of-sample period into high- and low-volatility level by median value of actual volatility for each REITs index. Table 11 presents the results of out-of-sample R^2 test during high, i.e., above-median and low, i.e., below-median, volatility levels. We find very strong evidence of forecasting ability from ORV and OIV for REITs volatility during the high-volatility regime, with weaker results under the low-volatility conditions.

Table 10. Forecasting performance with regime switching models.

REITs index	MIDAS-ORV vs. MRS-MIDAS-ORV			MIDAS-OIV vs. MRS-MIDAS-OIV		
	$R^2_{OOS}(\%)$	MSPE-Adj.	p-value	$R^2_{OOS}(\%)$	MSPE-Adj.	p-value
EGAS	5.110	2.542	0.006	-0.331	0.712	0.238
EGNA	0.434	1.430	0.076	1.143	2.759	0.003
ELAU	9.760	1.963	0.025	5.168	1.884	0.030
ELHK	-2.732	0.432	0.333	-2.265	0.280	0.390
ELJP	0.548	1.869	0.031	3.981	1.332	0.091
ELUK	16.520	2.034	0.021	6.700	1.526	0.064
ENGL	-0.952	-1.856	0.968	-0.791	-1.762	0.961
EPEU	18.425	2.015	0.022	19.065	1.865	0.031
FENEI	1.000	1.921	0.027	0.819	1.393	0.082
FNAR	5.943	1.580	0.057	9.774	1.822	0.034

Notes: The table represents the out-of-sample performance with regime switching. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

Table 11. Forecasting performance with high and low volatility level.

REITs index	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)		
	$R^2_{OOS}(\%)$	MSPE-Adj.	p-value	$R^2_{OOS}(\%)$	MSPE-Adj.	p-value
Panel A: High Volatility Level						
EGAS	4.425	1.887	0.030	7.070	2.469	0.007
EGNA	6.966	2.938	0.002	6.734	3.132	0.001
ELAU	6.185	2.262	0.012	8.228	2.517	0.006
ELHK	10.327	4.579	0.000	9.618	4.313	0.000
ELJP	0.998	1.820	0.034	2.798	2.249	0.012
ELUK	11.357	2.195	0.014	10.046	2.388	0.008
ENGL	3.959	2.906	0.002	3.400	2.711	0.003
EPEU	7.281	2.273	0.012	8.543	2.189	0.014
FENEI	5.939	2.225	0.013	-0.537	0.574	0.283
FNAR	8.466	2.143	0.016	8.164	2.300	0.011
Panel B: Low Volatility Level						
EGAS	-5.046	-0.752	0.774	-3.054	-0.745	0.772
EGNA	-71.327	-1.001	0.841	-15.314	-1.017	0.845
ELAU	-0.374	1.033	0.151	0.520	1.337	0.091
ELHK	-70.022	-0.625	0.734	-36.623	0.601	0.274
ELJP	0.896	2.007	0.022	1.170	4.029	0.000
ELUK	-1.557	0.537	0.296	-2.705	-0.094	0.537
ENGL	-48.739	-0.977	0.836	-12.898	-1.044	0.852
EPEU	-5.459	-0.820	0.794	-1.081	0.982	0.163
FENEI	-12.248	-1.527	0.937	-8.315	-1.950	0.974
FNAR	1.230	2.541	0.006	-2.008	-0.017	0.507

Notes: The table reports the evaluation results during high and low volatility level. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

These results suggest that investors in the REITs market are more sensitive to oil market uncertainty when volatility in the REITs sector is already high, i.e., agents are more worried about risk spillovers when the current volatility is in its higher rather than lower state, and hence aim to utilize the information content of oil uncertainty during this phase of the market to gauge whether the future risk is going to increase further or not to possibly assist in their investment decision and portfolio allocation. Similar concerns do not seem to arise at the lower-state of REITs volatility, even though increases in oil-price uncertainty is perceived as bad news, given that the underlying risk in international REITs is low, possibly due to initial low-levels of volatility in the oil market itself. This finding is also important from the perspective of policymakers who closely aim to monitor the volatility in the real estate sector following the GFC. Now policy authorities would know that future volatility in REITs is likely to increase further due to hikes in oil price uncertainty, particularly when the current uncertainty in the real estate market is already high, and in turn would require expansionary monetary policy to diffuse the risks in the market and in turn prevent a deep recession.

4.3.2 Asymmetric effect

Mork (1989) initially document the asymmetric effect of oil price on real economy. Specifically, the author argued that the oil price increase exerts significantly negative effect on the U.S. GDP, while the effect of oil price decrease is minor. While the role of oil uncertainty on the forecastability of the REITs market conditional on its state is an important issue, an equally pertinent question for both investors and policymakers is whether there is a role of asymmetry associated with positive or negative oil price movements in the resulting volatility process while forecasting REITs RV? For ORV, we construct "good" and "bad" volatility following the work of Patton and Sheppard (2015) as follows:

$$\mathrm{PSV} = \sum_{j=1}^{\frac{1}{\Delta}} r_{(t-1)+j*\Delta}^{2}\, I\big(r_{(t-1)+j*\Delta} > 0\big), \tag{17}$$

$$\mathrm{NSV} = \sum_{j=1}^{\frac{1}{\Delta}} r_{(t-1)+j*\Delta}^{2}\, I\big(r_{(t-1)+j*\Delta} < 0\big), \tag{18}$$

For OIV, we consider an indicator of OIV on positive oil returns day as PIV ($\mathrm{PIV} = \mathrm{OI}V_t * I(r_t \geq 0)$), and an indicator of OIV on negative returns day as NIV

($\mathrm{NIG} = \mathrm{OIV}_t * I(r_t < 0)$. Then we extend the benchmark model with PSV, NSV, PIV or NIV to examine the asymmetric effect of oil volatility in forecasting international REITs RV.

Table 12 reports the evaluation results with "good" and "bad" oil volatility. We first look at PSV and NSV. The value of R^2_{OOS} is roughly equivalent when we construct regression models with PSV or NSV. We find no evidence that decomposing the ORV into "good" and "bad" components can further improve the forecasting accuracy. However, the forecasting performance of NIV is a bit weaker, as the R^2_{OOS} of regression model with NIV is negative for 4 of the 10 REITs indexes. The R^2_{OOS} value of PIV and NIV suggests the regression model with PIV can outperform NIV. This provides some evidence in terms of implied volatility, that increases in oil market uncertainty resulting from increases in oil price and/or returns, has a stronger predictive content than when volatility results from oil price and/or returns declines.

Table 12. Forecasting performance with 'good' and 'bad' oil volatility.

Equity index	ORV				OIV			
	PSV		NSV		PIV		NIV	
	R^2_{OOS}(%)	*p*-value	R^2_{OOS}(%)	*p*-value	R^2_{OOS}(%)	*p*-value	R^2_{OOS}(%)	*p*-value
EGAS	3.873	0.030	4.167	0.029	2.097	0.086	0.682	0.229
EGNA	6.522	0.004	4.606	0.004	0.226	0.252	-1.275	0.679
ELAU	6.187	0.012	5.733	0.013	1.228	0.106	-0.384	0.547
ELHK	8.371	0.000	8.900	0.000	3.234	0.001	0.502	0.021
ELJP	3.865	0.021	4.718	0.015	3.025	0.022	-0.537	0.389
ELUK	10.279	0.013	10.055	0.014	3.205	0.015	1.719	0.066
ENGL	3.171	0.010	2.687	0.004	1.417	0.063	1.346	0.083
EPEU	7.326	0.021	5.847	0.003	0.724	0.127	-1.126	0.531
FENEI	6.141	0.023	6.716	0.027	5.962	0.009	1.261	0.031
FNAR	7.714	0.029	6.141	0.010	-0.585	0.591	0.027	0.417

Notes: The table represents the out-of-sample performance with "Good" and "Bad" volatility. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

Note that, even though we define oil volatility associated with oil returns hikes as good volatility, considering the issue from the perspective of the oil trader, oil price increases (due to supply, oil-specific-consumption and precautionary demand) are generally viewed as bad news for the overall economy, unless it is due to a growing global economy (Demirer et al., 2020). Given this, oil uncertainty associated with positive oil returns is likely to affect REITs volatility relatively more, via the leverage effect that has been shown to be strongly present in international REITs markets (Liow and Huang, 2018), than when increases in oil price volatility occurs due to oil price decreases, i.e., good news.

4.4 Forecasting performance at longer horizons

After ensuring that our results are robust to the size of the forecasting window, jump risks, and lag-length, as well as nonlinearities and asymmetric effects, we turn to the fact that investors not only focus on volatility forecasts at a-day-ahead, but also at longer horizons. To further investigate the forecasting performance of ORV or OIV, we replace the left-hand side of Model 1, 2 and 3 by $\mathrm{RV}_{t+1,h}$, and consider the forecasting horizons of 5, 10 and 22 trading days i.e., h = 5, 10 and 22. Considering MIDAS-RV as an example, it can be written as,

$$\mathrm{RV}_{t+1,h} = \beta_0 + \beta_{\mathrm{RV}} \sum_{k=1}^{k^{max}} \omega_k \mathrm{RV}_{t-k+1} + \varepsilon_{t+1,h}, \tag{19}$$

where $\mathrm{RV}_{t+1,h}$ represent the h-day-ahead RV at time t+1, $\mathrm{RV}_{t+1,h} = \frac{1}{h}\sum_{i=1}^{h} \mathrm{RV}_{t+i}$. For example, when h = 5, the left side of regression model is equal to $\mathrm{RV}_{t+1,5} = \frac{1}{5}(\mathrm{RV}_{t+1} + \mathrm{RV}_{t+2} + \mathrm{RV}_{t+3} + \mathrm{RV}_{t+4} + \mathrm{RV}_{t+5})$. Table 13 reports the forecasting results of the predictive regression models at longer horizons. We first look at out-of-sample R^2 test, to find that the results suggest both ORV and OIV can improve the accuracy of volatility forecasting for most international REITs index at the forecasting horizons of 5, 10 and 22 days. The results of the economic value analysis are also consistent with our previous findings for one-step-ahead, as they show that oil volatility can offer additional realized utility relative to the regressions without the information on oil uncertainty.

Table 13. Forecasting performance for longer horizons.

REITs index	Oil Realized Volatility (ORV)			Oil Implied Volatility (OIV)			Portfolio Exercise		
	R^2_{OOS}(%)	MSPE-Adj.	p-value	R^2_{OOS}(%)	MSPE-Adj.	p-value	Bench	ORV	OIV
Panel A: h = 5									
EGAS	8.113	2.777	0.003	11.581	2.857	0.002	3.697	3.719	3.734
EGNA	6.348	3.616	0.000	11.510	3.987	0.000	3.574	3.585	3.609
ELAU	1.557	2.822	0.002	9.091	3.037	0.001	3.733	3.761	3.763
ELHK	5.942	4.810	0.000	10.164	4.510	0.000	3.595	3.682	3.661
ELJP	1.067	2.074	0.019	2.394	2.700	0.003	3.494	3.528	3.532
ELUK	5.769	2.061	0.020	5.085	1.897	0.029	3.784	3.790	3.791
ENGL	2.598	3.006	0.001	2.990	3.002	0.001	3.324	3.299	3.336
EPEU	6.464	2.552	0.005	4.613	1.896	0.029	3.794	3.801	3.812
FENEI	-3.647	0.337	0.368	0.926	1.687	0.046	3.780	3.782	3.787
FNAR	4.666	2.526	0.006	7.318	2.510	0.006	3.650	3.657	3.667
Panel B: h = 10									
EGAS	4.691	2.539	0.006	6.864	2.731	0.003	3.650	3.662	3.715
EGNA	5.566	4.409	0.000	9.883	4.706	0.000	3.525	3.529	3.570
ELAU	4.177	4.128	0.000	10.494	3.919	0.000	3.665	3.707	3.715
ELHK	7.604	5.655	0.000	11.357	5.155	0.000	3.651	3.741	3.723
ELJP	1.370	2.079	0.019	2.055	2.777	0.003	3.434	3.489	3.512
ELUK	3.883	2.228	0.013	1.977	1.799	0.036	3.738	3.751	3.755
ENGL	-0.263	-0.777	0.781	2.668	3.275	0.001	3.354	3.318	3.376
EPEU	4.322	2.560	0.005	1.912	1.295	0.098	3.776	3.778	3.794
FENEI	-0.567	0.737	0.231	0.134	1.320	0.093	3.769	3.766	3.771
FNAR	2.425	2.475	0.007	3.663	2.329	0.010	3.535	3.547	3.558
Panel C: h = 22									
EGAS	2.187	3.139	0.001	0.958	3.025	0.001	3.343	3.359	3.505
EGNA	3.694	5.452	0.000	8.255	5.119	0.000	3.247	3.253	3.338
ELAU	10.728	5.747	0.000	14.023	4.933	0.000	3.304	3.388	3.431

ELHK	17.435	7.516	0.000	13.269	7.197	0.000	3.678	3.754	3.738
ELJP	2.122	2.571	0.005	1.077	2.744	0.003	2.807	2.908	3.038
ELUK	2.824	3.218	0.001	-0.526	2.280	0.011	3.563	3.582	3.598
ENGL	-0.860	-2.233	0.987	2.473	4.222	0.000	3.082	2.993	3.155
EPEU	0.252	5.596	0.000	-4.935	-0.416	0.661	3.590	3.584	3.616
FENEI	-2.038	0.906	0.182	-1.417	1.584	0.057	3.779	3.768	3.768
FNAR	2.949	3.517	0.000	2.690	3.133	0.001	2.898	2.910	2.987

Notes: The table represents the out-of-sample performance for longer horizon. The forecasting window covers at last 50% observations for 10 REITs index, respectively.

5. Summary and concluding remarks

Existing in-sample evidence indicate that causal effects from oil market uncertainty onto REITs market volatilities are exceptionally strong. Given that in-sample evidence does not necessarily translate into out-of-sample gains, in this paper we forecast realized variance (RV) of international REITs, derived from 5-minutes-interval intraday data. Based on the period of the analysis covering January, 2008 to July, 2020, and using variants of the popular MIDAS-RV model, augmented to include oil market uncertainty captured by its RV (ORV; derived from 5-minute intraday data) and implied volatility (OIV; obtained from oil VIX), we report evidence of significant statistical and economic gains in the forecasting performance emanating from these two metrics relative to the benchmark that excludes these predictors. The result is robust to the size of the forecasting samples, including that of the COVID-19 period, jump risks, lag-length, nonlinearities and asymmetric effects, and forecast horizon.

Given the tremendous growth of REITs as an asset class globally and, hence, the importance of accurate volatility forecasts as inputs for optimal asset-allocation decisions our findings suggest that incorporating ORV or OIV, in volatility forecasting models can help to improve the design of portfolios that include REITs across various investment horizons and countries, especially when the existing volatility in the REITs markets is high, and the oil uncertainty emanates from oil price increases. Further, with the future path of REITs volatility providing a high-frequency measure of uncertainty in the housing sector for which only low-frequency data is traditionally available, would allow policymakers to design timely responses to circumvent the negative influence on the real economy, given that the real estate sector is known to lead macroeconomic variables (Segnon et al., 2021). More specifically, policymakers need to be aware that oil market uncertainty spillover to the real estate sector particularly strongly at their respective higher ends, and can intensify the deepening of the recession that might have originated from oil uncertainty (Bernanke, 1983).

As part of future research, it would be is interesting to extend our study to sectoral REITs, as different REITs sectors are heterogeneously sensitive to the oil market.

Moreover, given the evidence of bi-directional causality in the volatility processes of oil and the REITs markets, an analysis of REITs of which economies and sectors can accurately forecast oil market volatility would also be an important area to delve into, especially given the financialization of the oil market post-2008 (Bonato, 2019).

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