Supplementary Information

A photo-identification-based assessment model of southern right whales *Eubalaena australis* surveyed in South African waters, with a focus on recent low counts of mothers with calves

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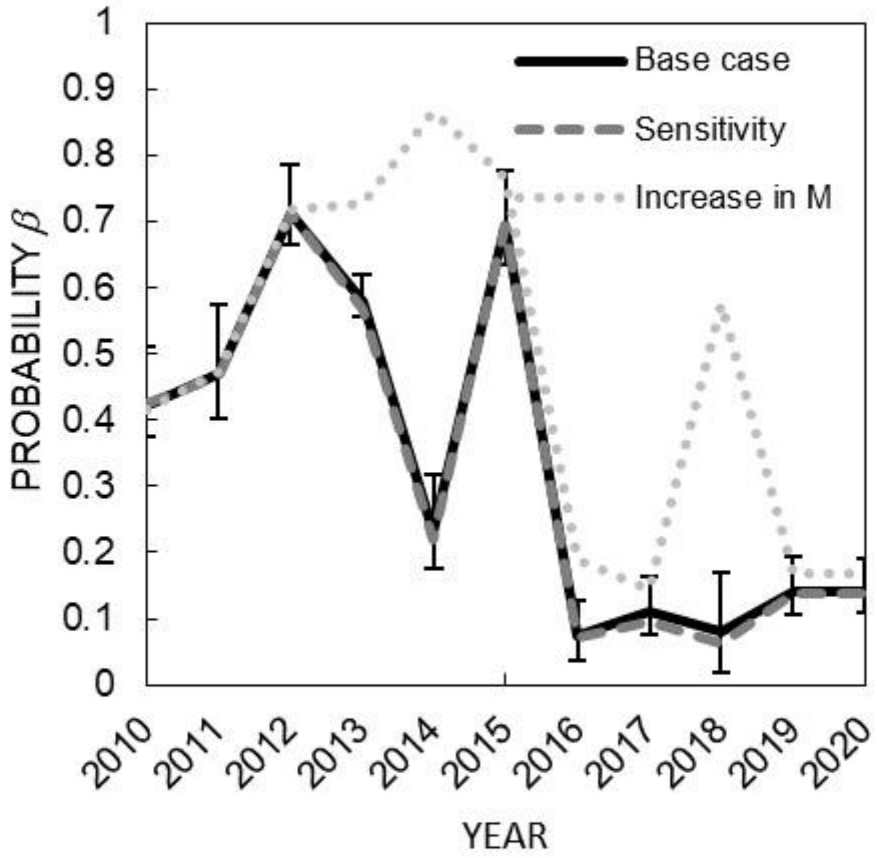
This Supplementary Information is split into two sections, with the first providing some additional results to those given in the main text (one additional table and two additional figures), and the second providing the details and equations for the calculations of the probabilities for the sighting histories, which are used to develop the likelihood maximised in the model-fitting process.

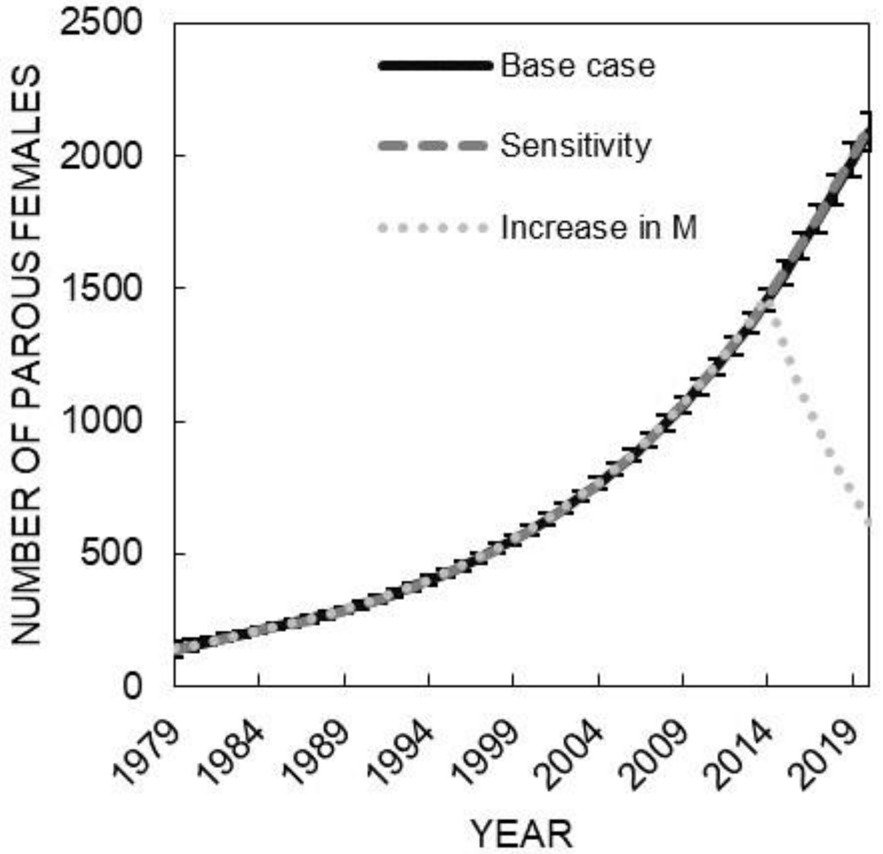
Section 1: Additional Results

Supplementary Table S1: Estimates of the δ parameters for 2014 to 2019 (the values plotted in Figure 9 of the main paper) for right whales off South Africa under the new model for the Base case and the Sensitivity (see the main text and the following section for explanation of symbols). The quantities in brackets are Hessian-based estimates of standard errors

Parameter	Model	
	Base case	Sensitivity
$\boldsymbol{\delta_{2014}}$	0.501 (0.093)	0.489 (0.084)
$\boldsymbol{\delta_{2015}}$	0.867 (0.018)	0.865 (0.015)
$\boldsymbol{\delta_{2016}}$	0.764 (0.034)	0.756 (0.026)
$\boldsymbol{\delta_{2017}}$	0.155 (0.070)	0.530 (0.028)
$\boldsymbol{\delta_{2018}}$	0. 721 (0.057)	0.834 (0.014)
$\boldsymbol{\delta_{2019}}$	0.829 (0.023)	0.868 (0.011)



Supplementary Figure S1: Time-varying estimates of the probabilities (β) that a resting whale will rest in the following year under the new model for the Base case and the Sensitivity, as well as when a recent increase in natural mortality (M) is considered. The error bars represent the range of one Hessian-based standard error added to and subtracted from the estimate concerned



Supplementary Figure S2: Estimated total number of females having reached the age at first parturition under the new model for the Base case and the Sensitivity, as well as when a recent increase in natural mortality (M) is considered. The error bars represent the range of one Hessian-based standard error added to and subtracted from the estimate concerned

Section 2: Probability calculations for sighting histories

At any particular point in time, an adult female can be in one of three states: resting (1), receptive (2), or calving (3). When a female with a calf was sighted (i.e. calving state 3), this was recorded as '1' in the sighting history for that female. For years in which the female was not sighted, however, several possible events could have occurred, including death if the female had not been sighted again to date.

Therefore, this section of the Supplementary Information outlines the methodology used to obtain all possible scenarios for a given sighting history, to calculate the associated probability.

Supplementary Figure S3 shows the possible directions in which a female can move from one state to another. It should be noted that the following assumptions have been made:

1. A resting female has to first become receptive before calving (i.e. there is no flow from state 1 to state 3).
2. A female that is resting in one year may remain in the resting state the following year.
3. A receptive female that has an early abortion can become receptive again the following year.

In the equations that follow,

α_y is the probability that a mature female whale that calves in year y ovulates the next year;

β_y is the probability that a resting mature female whale in year y rests for a further year;

γ_y is the probability that a pregnant whale in year y rests rather than calves the next year (i.e. following a late abortion, or if the calf dies soon after birth before the cow-calf pair can be sighted);

δ_y is the probability that a pregnant whale in year y is pregnant the next year (i.e. following an early abortion);

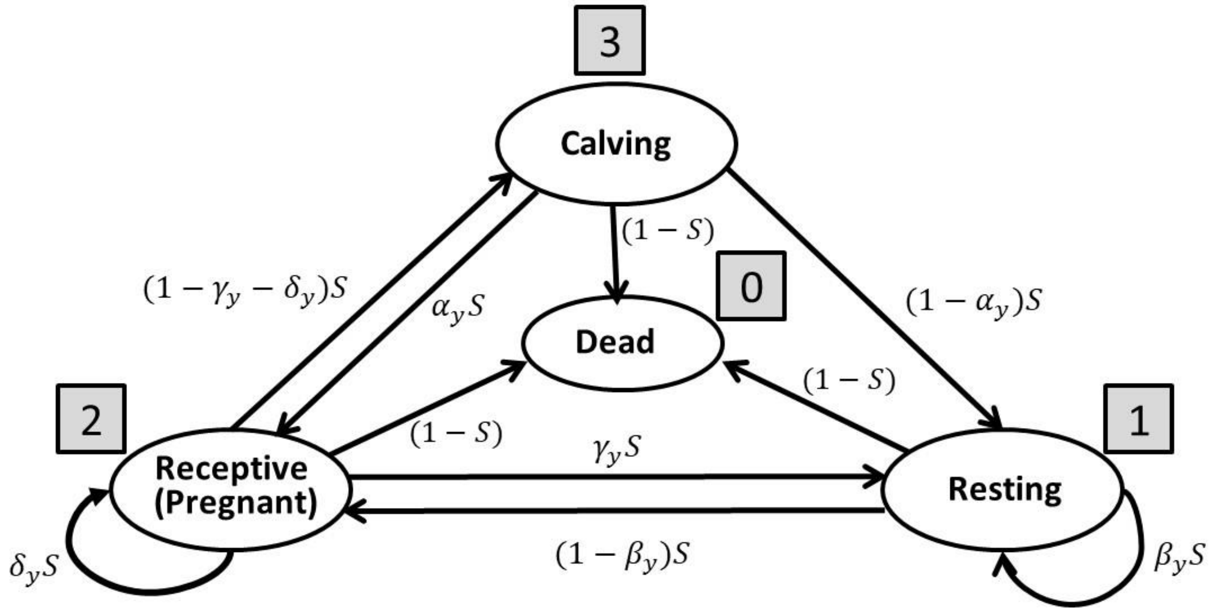
S is the post-first-year annual female survival proportion; alternatively expressed as an annual natural mortality rate M, where $S = e^{-M}$;

S_j is the first-year female survival proportion;

P_y^h is the probability of observing a sighting history h in year y;

$\hat{P}_y^A$ is the probability that a female whale with a calf is seen in year y; and

ϕ_a is the probability that an immature female whale of age a becomes receptive the following year.



Supplementary Figure S3: (Repeat of Figure 4 of the main text.) Flow diagram showing the possible ways in which a mature female southern right whale can move from one reproductive stage to another

Sighting histories where the female is first seen as an adult with calf ("mature algorithms")

Given a particular sighting history, the computation starts with the first sighting, and then proceeds through the rest of the sighting history using various algorithms based on when the next sightings occur. The sighting history is essentially broken into segments, and each unique segment is associated with a unique algorithm that was constructed based on all the possible scenarios that could have produced that segment. These scenarios consist of all the possible sequences of states (receptive, calving, resting) that could produce a particular segment, bearing in mind that a '0' in a sighting history could mean that a female was (a) without calf, (b) with calf but not sighted, or (c) dead (if there were no further sightings in the sighting history). Probabilities were calculated taking all scenarios into account.

There are three basic kinds of algorithms:

1. "Calving Algorithm (CA)", which is used at the start of the sighting history, as well as after a female has been sighted with a calf. There are four different CAs, which differ depending on when and if the female is sighted again with a calf in the following four years.
2. "Mature Normal Algorithm (MNA)", which is used when the female was not sighted in the previous year, or the following three years.
3. "Mature Upcoming Calf Algorithm (MUCA)", which is used to compute probabilities when the female is sighted with calf in three years' time.

Each algorithm keeps track of the possible states (receptive, calving or resting) that a whale could be in given the segment of the sighting history in question, as well as the associated probabilities. Thus, the computation will move from one algorithm to the next depending on the placement of '1's and '0's in the sighting history until the last year is reached. Supplementary Figure S4 illustrates the manner in which one algorithm flows to the next. Details of each algorithm, the corresponding sighting history segment, the possible states for that segment, along with the associated probabilities, are given in Supplementary Tables S2–S7.

Sighting histories where the female is first seen as a calf ("immature algorithms")

For females that are first seen as calves, and later as adults with own calves, a fourth state "immature" needs to be taken into account, as it is not known exactly when the female matures. The following assumptions have been made:

1. The youngest possible age for a female to produce a calf is 6 years. Therefore, the youngest possible age for a female to become receptive is age 5 years.
2. From age 15, all females are considered mature.

The algorithms for a female sighted first as a calf follow similar logic to those for a female first seen as an adult. The following basic algorithms have been implemented.

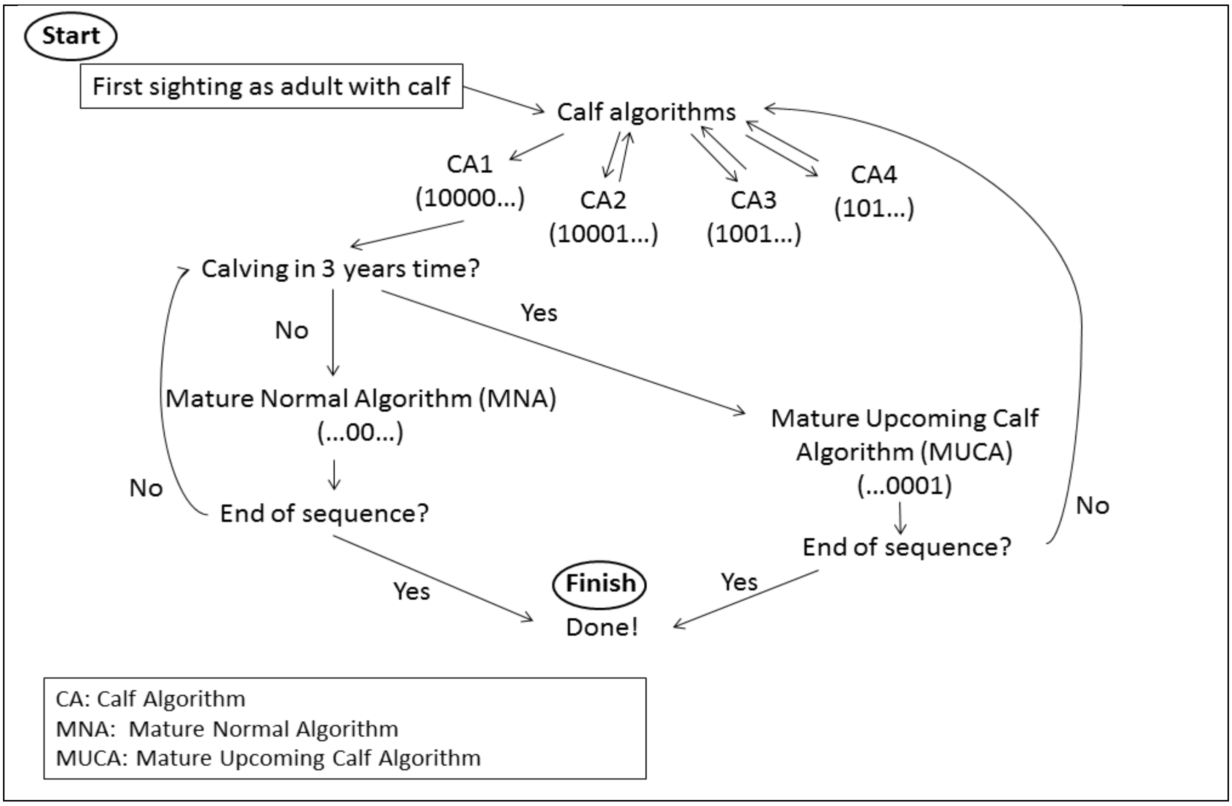
1. "First Calf Algorithm (FCA)", which was used only at the start of the sighting history when the female was first seen as a calf. There are four different FACs depending on when and if the female was first resighted with a calf.
2. "Immature Normal Algorithm (INA)", which was used when a female had not yet been sighted with her own calf, and was not sighted in the next three years.
3. "First Upcoming Calf Algorithm (FUCA)", which was used when the female was spotted for the first time with a calf after three years' time.
4. Two additional algorithms for when a whale had reached age 13, to ensure that the whale is mature by age 15.

Once a female had been sighted with its own calf, the algorithms proceed with the “mature algorithms”. The flow diagram is given in Supplementary Figure S5, and the probabilities and possible scenarios associated with each algorithm are given in Supplementary Tables S8–S15.

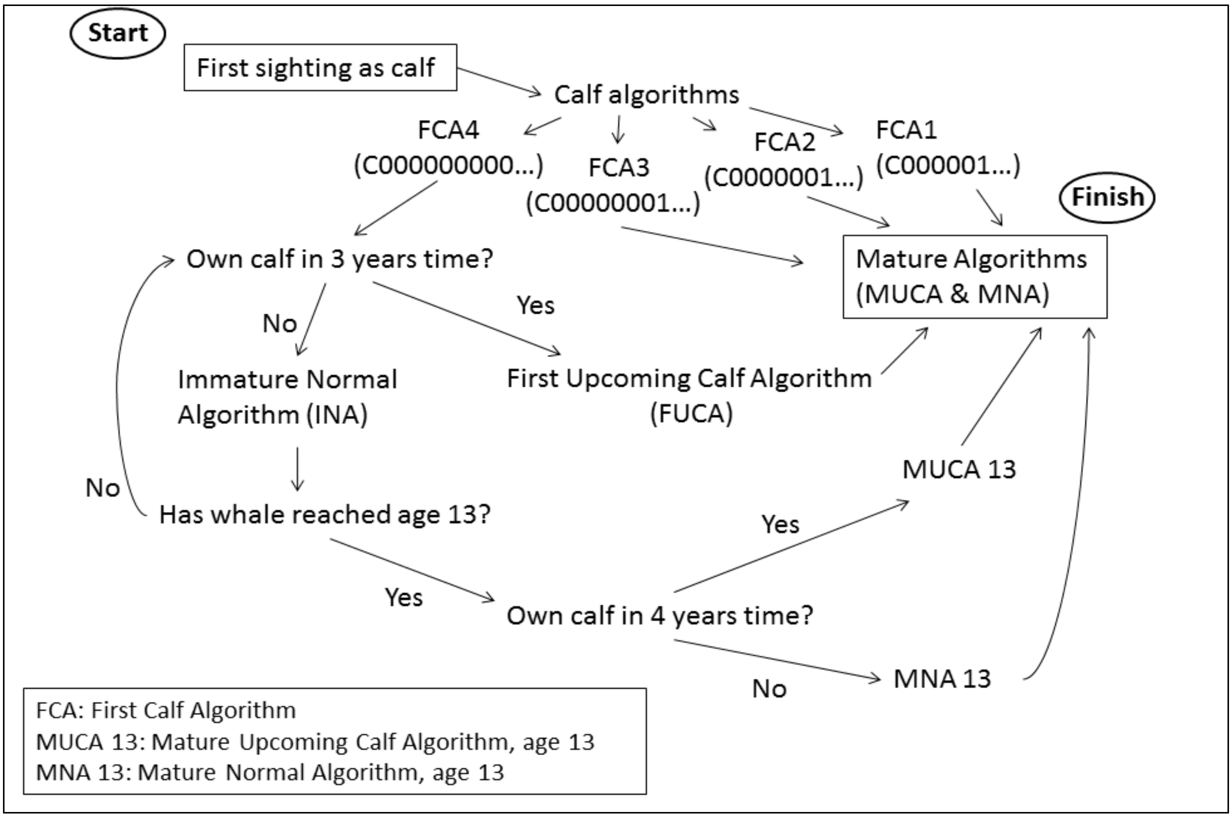
Note that if a calf was not seen again as an adult, then the probability of death needed to be taken into account:

$$P_{y+1}^{0} = (1 - S^{*}\rho)$$

where S^{*} is S_j for the first year and S afterwards.



Supplementary Figure S4: Flow chart for female whales first seen as an adult with calf. Details for algorithms are given in Supplementary Tables S2–S7



Supplementary Figure S5: Flow chart for female whales first seen as a calf, and later seen as an adult with a calf. Details for the algorithms are given in Supplementary Tables S8–S15

Supplementary Table S2: Calf Algorithm CA1. For any sequence starting off with 10000..., where '1' is a sighting of the adult female with a calf. Brackets around a '3' indicate a possible calving that occurred but was not sighted

CA1	1	0	0	Probability	
	y	y+1	y+2	y+1	y+2
Calving	3		(3)	$P^3_{y+1} = 0$	$P^3_{y+2} = P^2_{y+1}(1-\gamma-\delta)S(1-\hat{P}^A_{y+2})$
Receptive		2	2	$P^2_{y+1} = P^3_y(\alpha S)$	$P^2_{y+2} = P^1_{y+1}(1-\beta)S + P^2_{y+1}(\delta S)$
Resting		1	1	$P^1_{y+1} = P^3_y(1-\alpha)S$	$P^1_{y+2} = P^2_{y+1}(\gamma S) + P^1_{y+1}(\beta S)$
Dead		0	0	$P^0_{y+1} = P^3_y(1-S)$	$P^0_{y+2} = \sum_{i=1}^{2} P^i_{y+1}(1-S)$ if there are no further sightings. $P^0_{y+2} = 0$ otherwise.

Supplementary Table S3: **Calf Algorithm CA2.** For any sequence starting off with 10001..., where '1' is a sighting of the adult female with a calf. Brackets around a '3' indicate a possible calving that occurred but was not sighted. Note that because the whale was sighted in year $y+4$, the probability of it being dead is zero

CA2	1	0	0	0	1	Probability			
	y	y+1	y+2	y+3	y+4	y+1	y+2	y+3	y+4
Calving	3		(3)		3	$P^3_{y+1} = 0$	$P^3_{y+2} = P^2_{y+1}(1-\gamma-\delta)S(1-\hat{P}^A_{y+2})$	$P^3_{y+3} = 0$	$P^3_{y+4} = P^2_{y+3}(1-\gamma-\delta)S(\hat{P}^A_{y+4})$
Receptive		2	2	2		$P^2_{y+1} = P^3_y(\alpha S)$	$P^2_{y+2} = P^2_{y+1}(\delta S)$	$P^2_{y+3} = P^3_{y+2}(\alpha S) + P^2_{y+2}(\delta S) + P^1_{y+2}(1-\beta)S$	$P^2_{y+4} = 0$
Resting		1	1			$P^1_{y+1} = P^3_y(1-\alpha)S$	$P^1_{y+2} = P^2_{y+1}(\gamma S) + P^1_{y+1}(\beta S)$	$P^1_{y+3} = 0$	$P^1_{y+4} = 0$
Dead						$P^0_{y+1} = 0$	$P^0_{y+2} = 0$	$P^0_{y+3} = 0$	$P^0_{y+4} = 0$

Supplementary Table S4: Calf Algorithm CA3. For any sequence starting off with 1001..., where '1' is a sighting of the adult female with a calf. Note that because the whale was sighted in year $y+3$, the probability of it being dead is zero

CA3	1	0	0	1	Probability		
	y	y+1	y+2	y+3	y+1	y+2	y+3
Calving	3			3	$P^3_{y+1} = 0$	$P^3_{y+2} = 0$	$P^3_{y+3} = P^2_{y+2}(1-\gamma-\delta)S(\hat{P}^A_{y+3})$
Receptive		2	2		$P^2_{y+1} = P^3_y(\alpha S)$	$P^2_{y+2} = P^1_{y+1}(1-\beta)S + P^2_{y+1}(\delta S)$	$P^2_{y+3} = 0$
Resting		1			$P^1_{y+1} = P^3_y(1-\alpha)S$	$P^1_{y+2} = 0$	$P^1_{y+3} = 0$
Dead					$P^0_{y+1} = 0$	$P^0_{y+2} = 0$	$P^0_{y+3} = 0$

Supplementary Table S5: Calving Algorithm CA4. For any sequence starting off with 101..., where '1' is a sighting of the adult female with a calf. Note that because the whale was sighted in year $y+2$, the probability of it being dead is zero

CA4	1	0	1	Probability	
	y	y+1	y+2	y+1	y+2
Calving	3		3	$P^3_{y+1} = 0$	$P^3_{y+2} = P^2_{y+1}(1-\gamma-\delta)S(\hat{P}^A_{y+2})$
Receptive		2		$P^2_{y+1} = P^3_y(\alpha S)$	$P^2_{y+2} = 0$
Resting				$P^1_{y+1} = 0$	$P^1_{y+2} = 0$
Dead				$P^0_{y+1} = 0$	$P^0_{y+2} = 0$

Supplementary Table S6: Mature Normal Algorithm (MNA). No calf in the previous year or the following three years. Brackets around a '3' indicate a possible calving that was not sighted

MNA	0	0	Probability
	y	y+1	y+1
Calving	(3)	(3)	$P^3_{y+1} = P^2_y(1-\gamma-\delta)S(1-\hat{P}^A_{y+1})$
Receptive	2	2	$P^2_{y+1} = P^3_y(\alpha S) + P^2_y(\delta S) + P^1_y(1-\beta)S$
Resting	1	1	$P^1_{y+1} = P^3_y(1-\alpha)S + P^2_y(\gamma S) + P^1_y(\beta S)$
Dead	0	0	$P^0_{y+1} = \sum_{i=1}^{3} P^i_y(1-S)$ if there are no further sightings. $P^0_{y+1} = 0$ otherwise.

Supplementary Table S7: Mature Upcoming Calf Algorithm (MUCA). Calving in three years' time, but none in the previous three years. Brackets around a '3' indicate a possible calving that was not sighted. Note that because the whale was sighted in year $y+3$, the probability of it being dead is zero

MUCA	0	0	0	1	Probability		
	y	y+1	y+2	y+3	y+1	y+2	y+3
Calving	(3)	(3)		3	$P^3_{y+1} = P^2_y(1-\gamma-\delta)S(1-\hat{P}^A_{y+1})$	$P^3_{y+2} = 0$	$P^3_{y+3} = P^2_{y+2}(1-\gamma-\delta)S(\hat{P}^A_{y+3})$
Receptive	2	2	2		$P^2_{y+1} = P^3_y\alpha S + P^2_y(\delta S) + P^1_y(1-\beta)S$	$P^2_{y+2} = P^3_{y+1}(\alpha S) + P^2_{y+1}(\delta S) + P^1_{y+1}(1-\beta)S$	$P^2_{y+3} = 0$
Resting	1	1			$P^1_{y+1} = P^3_y(1-\alpha)S + P^2_y(\gamma S) + P^1_y(\beta S)$	$P^1_{y+2} = 0$	$P^1_{y+3} = 0$
Dead					$P^0_{y+1} = 0$	$P^0_{y+2} = 0$	$P^0_{y+3} = 0$

Supplementary Table S8: First Calf Algorithm FCA1. The sequence C000001, where 'C' is the first sighting as a calf, and '1' is the first resighting as an adult with a calf. Note that because the whale was sighted in year $y+6$, the probability of it being dead is zero

FCA1	C	0	0	0	0	0	1	Probability		
	y	y+1	y+2	y+3	y+4	y+5	y+6	y+1	y+5	y+6
Immature	Im	Im	Im	Im	Im			$P^{\text{Im}}_{y+1} = S_j\rho$	$P^{\text{Im}}_{y+5} = 0$	$P^{\text{Im}}_{y+6} = 0$
Calving							3	$P^{3}_{y+1} = 0$	$P^{3}_{y+5} = 0$	$P^{3}_{y+6} = P^{2}_{y+5}(1-\gamma-\delta)S(\hat{P}^{A}_{y+6})$
Receptive						2		$P^{2}_{y+1} = 0$	$P^{2}_{y+5} = P^{\text{Im}}_{y+4}(\varphi_4 S)$	$P^{2}_{y+6} = 0$
Resting								$P^{1}_{y+1} = 0$	$P^{1}_{y+5} = 0$	$P^{1}_{y+6} = 0$

Supplementary Table S9: First Calf Algorithm FCA2. The sequence C0000001, where 'C' is the first sighting as a calf, and '1' is the first resighting as an adult with a calf. Note that because the whale was sighted in year $y+7$, the probability of it being dead is zero

FCA2	C	0	0	0	0	0	0	1	Probability		
	y	y+1	y+2	y+3	y+4	y+5	y+6	y+7	y+5	y+6	y+7
Immature	Im	Im	Im	Im	Im	Im			$P^{\text{Im}}_{y+5} = P^{\text{Im}}_{y+4}(1-\varphi_4)S$	$P^{\text{Im}}_{y+6} = 0$	$P^{\text{Im}}_{y+7} = 0$
Calving								3	$P^{3}_{y+5} = 0$	$P^{3}_{y+6} = 0$	$P^{3}_{y+7} = P^{2}_{y+6}(1-\gamma-\delta)S(\hat{P}^{A}_{y+7})$
Receptive							2		$P^{2}_{y+5} = 0$	$P^{2}_{y+6} = P^{\text{Im}}_{y+5}(\varphi_5 S)$	$P^{2}_{y+7} = 0$
Resting									$P^{1}_{y+5} = 0$	$P^{1}_{y+6} = 0$	$P^{1}_{y+7} = 0$

Supplementary Table S10: First Calf Algorithm FCA3. The sequence C00000001, where 'C' is the first sighting as a calf, and '1' is the first resighting as an adult with a calf. Brackets around a '3' indicate a possible calving that was not sighted

FCA3	C	...	0	0	0	1
	Y	...	y+5	y+6	y+7	y+8
Immature	Im	...	Im	Im		
Calving				(3)		3
Receptive			2	2	2	
Resting				1		

Probability			
y+5	y+6	y+7	y+8
$P^{\text{Im}}_{y+5} = P^{\text{Im}}_{y+4}(1-\varphi_4)S$	$P^{\text{Im}}_{y+6} = P^{\text{Im}}_{y+5}(1-\varphi_5)S$	$P^{\text{Im}}_{y+7} = 0$	$P^{\text{Im}}_{y+8} = 0$
$P^{3}_{y+5} = 0$	$P^{3}_{y+6} = P^{2}_{y+5}(1-\gamma-\delta)S(1-\hat{P}^{A}_{y+6})$	$P^{3}_{y+7} = 0$	$P^{3}_{y+8} = P^{2}_{y+7}(1-\gamma-\delta)S(\hat{P}^{A}_{y+8})$
$P^{2}_{y+5} = P^{\text{Im}}_{y+4}(\varphi_4 S)$	$P^{2}_{y+6} = P^{\text{Im}}_{y+5}(\varphi_5 S) + \mathrm{P}^{2}_{y+5}(\delta S)$	$P^{2}_{y+7} = P^{\text{Im}}_{y+6}(\varphi_6 S) + P^{3}_{y+6}(\alpha S) + \mathrm{P}^{2}_{y+6}(\delta S) + P^{1}_{y+6}(1-\beta)S$	$P^{2}_{y+8} = 0$
$P^{1}_{y+5} = 0$	$P^{1}_{y+6} = P^{2}_{y+6}(\gamma S)$	$P^{1}_{y+7} = 0$	$P^{1}_{y+8} = 0$

Supplementary Table S11: First Calf Algorithm FCA4. The sequence C000000001, where 'C' is the first sighting as a calf, and '1' is the first resighting as an adult with a calf. Brackets around a '3' indicate a possible calving that was not sighted

FCA4	C	...	0	0	0	0	0	1	Probability		
	y	...	y+4	y+5	y+6	y+7	y+8	y+9	y+5	y+6	y+7 to y+9
Immature	Im	...	Im	Im	Im	Im			$P^{\text{Im}}_{y+5} = P^{\text{Im}}_{y+4}(1-\varphi_4)S$	$P^{\text{Im}}_{y+6} = P^{\text{Im}}_{y+5}(1-\varphi_5)S$	
Calving					(3)	(3)		3	$P^{3}_{y+5} = 0$	$P^{3}_{y+6} = P^{2}_{y+5}(1-\gamma-\delta)S(1-\hat{P}^{A}_{y+6})$	Proceed with Upcoming Calf Algorithm (FUCA)
Receptive				2	2	2	2		$P^{2}_{y+5} = P^{\text{Im}}_{y+4}(\varphi_4 S)$	$P^{2}_{y+6} = P^{\text{Im}}_{y+5}(\varphi_5 S) + \mathrm{P}^{2}_{y+5}(\delta S)$	
Resting					1	1			$P^{1}_{y+5} = 0$	$P^{1}_{y+6} = P^{2}_{y+6}(\gamma S)$	

Supplementary Table S12: Immature Normal Algorithm (INA). Not sighted yet with her own calf, and no calf in the following three years. Brackets around a '3' indicate a possible calving that was not sighted

INA	0	0	Probability
	y	y+1	y+1
Immature	Im	Im	$P_{y+1}^{\mathrm{Im}} = P_{y}^{\mathrm{Im}}(1-\varphi_a)S$
Calving	(3)	(3)	$P_{y+1}^{3} = P_{y}^{2}(1-\gamma-\delta)S(1-\hat{P}_{y+1}^{A})$
Receptive	2	2	$P_{y+1}^{2} = P_{y}^{\mathrm{Im}}\varphi_a S + P_{y}^{3}(\alpha S) + \mathrm{P}_{y}^{2}(\delta S) + P_{y}^{1}(1-\beta)S$
Resting	1	1	$P_{y+1}^{1} = P_{y}^{3}(1-\alpha)S + P_{y}^{2}(\gamma S) + P_{y}^{1}(\beta S)$

Supplementary Table S13: First Upcoming Calf Algorithm (FUCA). Calving in three years' time, but none in the previous three years. Brackets around a '3' indicate a possible calving that was not sighted. Note that because the whale was sighted in year $y+3$, the probability of it being dead is zero

FUCA	0	0	0	1	Probability		
	y	y+1	y+2	y+3	y+1	y+2	y+3
Imm.	Im	Im			$P_{y+1}^{\mathrm{Im}} = P_{y}^{\mathrm{Im}}(1-\varphi_a)S$	$P_{y+2}^{\mathrm{Im}} = 0$	$P_{y+3}^{\mathrm{Im}} = 0$
Calving	(3)	(3)		3	$P_{y+1}^{3} = P_{y+2}^{2}(1-\gamma-\delta)S(1-\hat{P}_{y+1}^{A})$	$P_{y+2}^{3} = 0$	$P_{y+3}^{3} = P_{y+2}^{2}(1-\gamma-\delta)S(\hat{P}_{y+3}^{A})$
Rec.	2	2	2		$P_{y+1}^{2} = P_{y}^{\mathrm{Im}}\varphi_a S + P_{y}^{3}(\alpha S) + \mathrm{P}_{y}^{2}(\delta S)+P_{y}^{1}(1-\beta)S$	$P_{y+2}^{2} = P_{y+1}^{\mathrm{Im}}\varphi_a S + P_{y+1}^{3}(\alpha S) + \mathrm{P}_{y}^{2}(\delta S) + P_{y+1}^{1}(1-\beta)S$	$P_{y+3}^{2} = 0$
Resting	1	1			$P_{y+1}^{1} = P_{y}^{3}(1-\alpha)S + P_{y}^{2}(\gamma S) + P_{y}^{1}(\beta S)$	$P_{y+2}^{1} = 0$	$P_{y+3}^{1} = 0$

Supplementary Table S14: Mature Upcoming Calf Algorithm 13 (MUCA13). Whale aged 13, with sighting with her own calf in four years' time. Brackets around a '3' indicate a calving that was not sighted

MUCA13	C	0	0	0	1
	y (13)	y+1 (14)	y+2 (15)	y+3 (16)	y+4 (17)
Immature	Im				
Calving	(3)	(3)	(3)		3
Receptive	2	2	2	2	
Resting	1	1	1		

Probability			
y+1	y+2	y+3	y+4
$P_{y+1}^{\mathrm{Im}} = 0$	$P_{y+2}^{\mathrm{Im}} = 0$	$P_{y+3}^{\mathrm{Im}} = 0$	$P_{y+4}^{\mathrm{Im}} = 0$
$P_{y+1}^{3} = P_{y}^{2}(1-\gamma-\delta)S(1-\hat{P}_{y+1}^{A})$	$P_{y+2}^{3} = P_{y+1}^{2}(1-\gamma-\delta)S(1-\hat{P}_{y+2}^{A})$	$P_{y+3}^{3} = 0$	$P_{y+4}^{3} = P_{y+3}^{2}(1-\gamma-\delta)S(\hat{P}_{y+4}^{A})$
$P_{y+1}^{2} = P_{y}^{\mathrm{Im}}(\varphi_{13}S) + P_{y}^{3}(\alpha S) + P_{y}^{2}(\delta S) + P_{y}^{1}(1-\beta)S$	$P_{y+2}^{2} = P_{y+1}^{3}(\alpha S) + P_{y+1}^{2}(\delta S) + P_{y+1}^{1}(1-\beta)S$	$P_{y+3}^{2} = P_{y+2}^{3}(\alpha S) + P_{y+2}^{2}(\delta S) + P_{y+2}^{1}(1-\beta)S$	$P_{y+4}^{2} = 0$
$P_{y+1}^{1} = P_{y}^{3}(1-\alpha)S + P_{y}^{2}(\gamma S) + P_{y}^{1}(\beta S)$	$P_{y+2}^{1} = P_{y+1}^{3}(1-\alpha)S + P_{y+1}^{2}(\gamma S) + P_{y+1}^{1}(\beta S)$	$P_{y+3}^{1} = 0$	$P_{y+4}^{1} = 0$

Supplementary Table S15: Mature Normal Algorithm 13 (MNA13). Whale aged 13, with no sighting with calf in the following four years. Brackets around a '3' indicate a calving that was not sighted

MNA13	0	0	0	0	0	Probability		
	y (13)	y+1 (14)	y+2 (15)	y+3 (16)	y+4 (17)	y+1	y+2	y+3 to y+4
Immature	Im	Im				$P_{y+1}^{\mathrm{Im}} = P_{y}^{\mathrm{Im}}(1-\varphi_{13})S$	$P_{y+2}^{\mathrm{Im}} = 0$	Proceed with Mature Normal Algorithm (MNA)
Calving	(3)	(3)	(3)	(3)	(3)	$P_{y+1}^{3} = P_{y}^{2}(1-\gamma-\delta)S(1-\hat{P}_{y+1}^{A})$	$P_{y+2}^{3} = P_{y+1}^{2}(1-\gamma-\delta)S(1-\hat{P}_{y+2}^{A})$	
Receptive	2	2	2	2	2	$P_{y+1}^{2} = P_{y}^{\mathrm{Im}}(\varphi_{13}S) + P_{y}^{3}(\alpha S) + P_{y}^{2}(\delta S) + P_{y}^{1}(1-\beta)S$	$P_{y+2}^{2} = P_{y+1}^{\mathrm{Im}}(\varphi_{14}S) + P_{y+1}^{3}(\alpha S) + P_{y+1}^{2}(\delta S) + P_{y+1}^{1}(1-\beta)S$	
Resting	1	1	1	1	1	$P_{y+1}^{1} = P_{y}^{3}(1-\alpha)S + P_{y}^{2}(\gamma S) + P_{y}^{1}(\beta S)$	$P_{y+2}^{1} = P_{y+1}^{3}(1-\alpha)S + P_{y+1}^{2}(\gamma S) + P_{y+1}^{1}(\beta S)$	